This is the Hald system of section 7 in Chorin05

Centre manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$$\dot{u}_1 = u_2$$

$$\dot{u}_2 = -\varepsilon u_1 u_3^2 - u_1$$

$$\dot{u}_3 = u_4$$

$$\dot{u}_4 = -\varepsilon u_1^2 u_3 - u_3$$

Centre subspace basis vectors

$$\vec{e}_1 = \{\{1, i, 0, 0\}, e^{ti}\}$$

$$\vec{e}_2 = \{\{1, -i, 0, 0\}, e^{-ti}\}$$

$$\vec{e}_3 = \{\{0, 0, 1, i\}, e^{ti}\}$$

$$\vec{e}_4 = \{\{0, 0, 1, -i\}, e^{-ti}\}$$

$$\vec{z}_1 = \{\{1/2, 1/2i, 0, 0\}, e^{ti}\}$$

$$\vec{z}_2 = \{\{1/2, -1/2i, 0, 0\}, e^{-ti}\}$$

$$\vec{z}_3 = \{\{0, 0, 1/2, 1/2i\}, e^{ti}\}$$

$\vec{z}_4 = \{\{0, 0, 1/2, -1/2i\}, e^{-ti}\}$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$u_1 = \varepsilon(1/8\, e^{-3ti} s_4^2 s_2 - 1/4\, e^{-ti} s_4^2 s_1 - 1/2\, e^{-ti} s_4 s_3 s_2 + 1/8\, e^{3ti} s_3^2 s_1 - 1/2\, e^{ti} s_4 s_3 s_1 - 1/4\, e^{ti} s_3^2 s_2) + e^{-ti} s_2 + e^{ti} s_1$

$u_2 = \varepsilon(- 3/8\, e^{-3ti} s_4^2 s_2 i - 1/4\, e^{-ti} s_4^2 s_1 i - 1/2\, e^{-ti} s_4 s_3 s_2 i + 3/8\, e^{3ti} s_3^2 s_1 i + 1/2\, e^{ti} s_4 s_3 s_1 i + 1/4\, e^{ti} s_3^2 s_2 i) - e^{-ti} s_2 i + e^{ti} s_1 i$

$u_3 = \varepsilon(1/8\, e^{-3ti} s_4 s_2^2 - 1/2\, e^{-ti} s_4 s_2 s_1 - 1/4\, e^{-ti} s_3 s_2^2 + 1/8\, e^{3ti} s_3 s_1^2 - 1/4\, e^{ti} s_4 s_1^2 - 1/2\, e^{ti} s_3 s_2 s_1) + e^{-ti} s_4 + e^{ti} s_3$

$u_4 = \varepsilon(- 3/8\, e^{-3ti} s_4 s_2^2 i - 1/2\, e^{-ti} s_4 s_2 s_1 i - 1/4\, e^{-ti} s_3 s_2^2 i + 3/8\, e^{3ti} s_3 s_1^2 i + 1/4\, e^{ti} s_4 s_1^2 i + 1/2\, e^{ti} s_3 s_2 s_1 i) - e^{-ti} s_4 i + e^{ti} s_3 i$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = \varepsilon^2(- 9/16 s_4^2 s_3^2 s_1 i - 1/4 s_4^2 s_1^3 i - 1/2 s_4 s_3^3 s_2 i - 9/8 s_4 s_3 s_2 s_1^2 i - 3/4 s_3^2 s_2^2 s_1 i) + \varepsilon(s_4 s_3 s_1 i + 1/2 s_3^2 s_2 i)$

$\dot{s}_2 = \varepsilon^2(1/2 s_4^3 s_3 s_1 i + 9/16 s_4^2 s_3^2 s_2 i + 3/4 s_4^2 s_2 s_1^2 i + 9/8 s_4 s_3 s_2^2 s_1 i + 1/4 s_3^2 s_2^3 i) + \varepsilon(- 1/2 s_4^2 s_1 i - s_4 s_3 s_2 i)$

$\dot{s}_3 = \varepsilon^2(- 3/4 s_4^2 s_3 s_1^2 i - 9/8 s_4 s_3^2 s_2 s_1 i - 1/2 s_4 s_2 s_1^3 i - 1/4 s_3^3 s_2^2 i - 9/16 s_3 s_2^2 s_1^2 i) + \varepsilon(1/2 s_4 s_1^2 i + s_3 s_2 s_1 i)$

$\dot{s}_4 = \varepsilon^2(1/4 s_4^3 s_1^2 i + 9/8 s_4^2 s_3 s_2 s_1 i + 3/4 s_4 s_3^2 s_2^2 i + 9/16 s_4 s_2^2 s_1^2 i + 1/2 s_3 s_2^3 s_1 i) + \varepsilon(- s_4 s_2 s_1 i - 1/2 s_3 s_2^2 i)$