A normal form of your dynamical system

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Generally, the lowest order, most important, terms are near the end of each expression.

off echo;

Specified dynamical system

$$\dot{x}_1 = -x_1 y_1 \varepsilon$$

$$\dot{y}_1 = w_1 \sigma + x_1^2 \varepsilon - 2y_1^2 \varepsilon - y_1$$
off echo;

Time dependent normal form coordinates

$$y_1 = O(\varepsilon^2, \sigma^2) + 4e^{-1t} \star w_1 Y_1 \sigma \varepsilon + e^{-1t} \star w_1 \sigma + X_1^2 \varepsilon + 2Y_1^2 \varepsilon + Y_1$$
$$x_1 = O(\varepsilon^2, \sigma^2) + e^{-1t} \star w_1 X_1 \sigma \varepsilon + X_1 Y_1 \varepsilon + X_1$$

Result normal form DEs

$$\dot{Y}_1 = O(\varepsilon^3, \sigma^3) + 8e^{-1t} \star w_1 w_1 Y_1 \sigma^2 \varepsilon^2 - 4w_1 Y_1 \sigma \varepsilon - 2X_1^2 Y_1 \varepsilon^2 - Y_1$$
$$\dot{X}_1 = O(\varepsilon^3, \sigma^3) + 2e^{-1t} \star w_1 w_1 X_1 \sigma^2 \varepsilon^2 - w_1 X_1 \sigma \varepsilon - X_1^3 \varepsilon^2$$