Centre manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

Example 8 with, e.g., c2=k2=k1=1
and using x=u1, y=u3 :

```
ff_:=tp mat((u2
,-(small*c1+mu*u1^2)*u2-u1-a*u1*u3-b*u1^3
,u4
,-u4-u3+c*u1^2)
))
freqm_:=mat((1,-1));
ee_:=tp mat((1,i,0,0),(1,-i,0,0));
zz_:=tp mat((1,i,0,0),(1,-i,0,0));
toosmall:=3;
```

The specified dynamical system

$\dot u_1 = u_2$

$\dot u_2 = -\varepsilon^2 c1 u_2 + \varepsilon(-a u_1 u_3 - b u_1^3 - \mu u_1^2 u_2) - u_1$

$\dot u_3 = u_4$

$\dot u_4 = \varepsilon c u_1^2 - u_3 - u_4$

Centre subspace basis vectors

$\vec e_1 = \{\{1, i, 0, 0\}, e^{ti}\}$

$\vec e_2 = \{\{1, -i, 0, 0\}, e^{-ti}\}$

$\vec z_1 = \{\{1/2, 1/2i, 0, 0\}, e^{ti}\}$

$\vec z_2 = \{\{1/2, -1/2i, 0, 0\}, e^{-ti}\}$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$$u_1 = \varepsilon(1/8\, e^{-3ti} s_2^3 b - 1/8\, e^{-3ti} s_2^3 i\mu - 3/4\, e^{-ti} s_2^2 s_1 b + 1/4\, e^{-ti} s_2^2 s_1 i\mu + 1/8\, e^{3ti} s_1^3 b + 1/8\, e^{3ti} s_1^3 i\mu - 3/4\, e^{ti} s_2 s_1^2 b - 1/4\, e^{ti} s_2 s_1^2 i\mu) + e^{-ti} s_2 + e^{ti} s_1$$

$$u_2 = \varepsilon(- 3/8\, e^{-3ti} s_2^3 bi - 3/8\, e^{-3ti} s_2^3 \mu - 3/4\, e^{-ti} s_2^2 s_1 bi - 1/4\, e^{-ti} s_2^2 s_1 \mu + 3/8\, e^{3ti} s_1^3 bi - 3/8\, e^{3ti} s_1^3 \mu + 3/4\, e^{ti} s_2 s_1^2 bi - 1/4\, e^{ti} s_2 s_1^2 \mu) - e^{-ti} s_2 i + e^{ti} s_1 i$$

$$u_3 = \varepsilon(2/13\, e^{-2ti} s_2^2 ci - 3/13\, e^{-2ti} s_2^2 c - 2/13\, e^{2ti} s_1^2 ci - 3/13\, e^{2ti} s_1^2 c + 2 s_2 s_1 c)$$

$$u_4 = \varepsilon(6/13\, e^{-2ti} s_2^2 ci + 4/13\, e^{-2ti} s_2^2 c - 6/13\, e^{2ti} s_1^2 ci + 4/13\, e^{2ti} s_1^2 c)$$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = \varepsilon^2(- 51/16 s_2^2 s_1^3 b^2 i + 1/2 s_2^2 s_1^3 b\mu - 11/16 s_2^2 s_1^3 i\mu^2 + 23/26 s_2 s_1^2 aci + 1/13 s_2 s_1^2 ac - 1/2 s_1 c1) + \varepsilon(3/2 s_2 s_1^2 bi - 1/2 s_2 s_1^2 \mu)$$

$$\dot{s}_2 = \varepsilon^2(51/16 s_2^3 s_1^2 b^2 i + 1/2 s_2^3 s_1^2 b\mu + 11/16 s_2^3 s_1^2 i\mu^2 - 23/26 s_2^2 s_1 aci + 1/13 s_2^2 s_1 ac - 1/2 s_2 c1) + \varepsilon(- 3/2 s_2^2 s_1 bi - 1/2 s_2^2 s_1 \mu)$$