Invariant manifold of your dynamical system

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9:45am, July 8, 2025

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$$\dot{u}_1 = I\varepsilon u_1 u_2 + l_{13}\varepsilon^2 u_3 + l_{12}\varepsilon^2 u_2 + l_{11}\varepsilon^2 u_1$$

$$\dot{u}_2 = -I\varepsilon u_1^2 - l_{12}\varepsilon^2 u_1 - \epsilon^{-1} u_2$$

$$\dot{u}_3 = -l_{13}\varepsilon^2 u_1 - 2\epsilon^{-1} u_3$$

Invariant subspace basis vectors

$$\vec{e}_1 = \{\{1, 0, 0\}, \exp(0)\}$$

$$\vec{z}_1 = \{\{1, 0, 0\}, \exp(0)\}$$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$$u_1 = O(\varepsilon^4) + s_1$$

$$u_2 = -2\epsilon^3 I^3 s_1^4 \varepsilon^3 + 2\epsilon^2 I l_{11} s_1^2 \varepsilon^3 - \epsilon I s_1^2 \varepsilon - \epsilon l_{12} s_1 \varepsilon^2 + O(\varepsilon^4)$$

$$u_3 = -1/2\epsilon l_{13} s_1 \varepsilon^2 + O(\varepsilon^4)$$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = -2\epsilon^3 I^4 s_1^5 \varepsilon^4 + 2\epsilon^2 I^2 l_{11} s_1^3 \varepsilon^4 - \epsilon I^2 s_1^3 \varepsilon^2 - 2\epsilon I l_{12} s_1^2 \varepsilon^3 + \epsilon(-1/2 l_{13}^2 s_1 \varepsilon^4 - l_{12}^2 s_1 \varepsilon^4) + l_{11} s_1 \varepsilon^2 + O(\varepsilon^5)$$

Normals to isochrons at the slow manifold Use these vectors: to project initial conditions onto the slow manifold; to project non-autonomous forcing onto the slow evolution; to predict the consequences of modifying the original system; in uncertainty quantification to quantify effects on the model of uncertainties in the original system. The normal vector $\vec{z}_j := (z_{j1}, \ldots, z_{jn})$

$$z_{11} = 16\epsilon^4 I^4 s_1^4 \varepsilon^4 - 4\epsilon^3 I^2 l_{11} s_1^2 \varepsilon^4 + 2\epsilon^2 I^2 s_1^2 \varepsilon^2 + 3\epsilon^2 I l_{12} s_1 \varepsilon^3 + \epsilon^2(1/4 l_{13}^2 \varepsilon^4 + l_{12}^2 \varepsilon^4) + O(\varepsilon^5) + 1$$

$$z_{12} = 4\epsilon^3 I^3 s_1^3 \varepsilon^3 + 10\epsilon^3 I^2 l_{12} s_1^2 \varepsilon^4 - \epsilon^2 l_{12} l_{11} \varepsilon^4 + \epsilon I s_1 \varepsilon + \epsilon l_{12} \varepsilon^2 + O(\varepsilon^5)$$

$$z_{13} = 7/4\epsilon^3 I^2 l_{13} s_1^2 \varepsilon^4 - 1/4\epsilon^2 l_{13} l_{11} \varepsilon^4 + 1/2\epsilon l_{13} \varepsilon^2 + O(\varepsilon^5)$$