J. B. van den Berg and J. D. M. James. Parameterization of slow-stable manifolds and their invariant vector bundles: Theory and numerical im- plementation. Discrete and Continuous Dynamical Systems, 36(9):4637– 4664, May 2016.

Centre manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$$\dot u_1 = k\varepsilon^2(-u_1^2 + u_1 - 2u_2^2 - 2u_3^2) + \varepsilon(-u_1^2 - 2u_2^2 - 2u_3^2) + u_1$$

$$\dot u_2 = k\varepsilon^2(-2u_1u_2 - 2u_2u_3 + u_2) + \varepsilon(-2u_1u_2 - 2u_2u_3)$$

$$\dot u_3 = k\varepsilon^2(-2u_1u_3 - u_2^2 + u_3) + \varepsilon(-2u_1u_3 - u_2^2) - 3u_3$$

Centre subspace basis vectors

$$\vec e_1 = \{\{0,1,0\}, e^{0i}\}$$

$$\vec z_1 = \{\{0,1,0\}, e^{0i}\}$$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$$u_1 = 2\varepsilon^2 s_1^2 + 2\varepsilon s_1^2$$

$$u_2 = s_1$$

$$u_3 = -1/3\varepsilon^2 s_1^2 - 1/3\varepsilon s_1^2$$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = -20/3\varepsilon^3 s_1^3 + \varepsilon^2(-10/3s_1^3 + s_1)$$

Normals to isochrons at the slow manifold Use these vectors: to project initial conditions onto the slow manifold; to project non-autonomous forcing onto the slow evolution; to predict the consequences of modifying the original system; in uncertainty quantification to quantify effects on the model of uncertainties in the original system. The normal vector $\vec{z}_j := (z_{j1}, \ldots, z_{jn})$

$$z_{11} = \varepsilon^3(-196/9s_1^3 - 2s_1) + 2\varepsilon^2 s_1 + 2\varepsilon s_1$$

$$z_{12} = -152/9\varepsilon^3 s_1^2 - 76/9\varepsilon^2 s_1^2 + 1$$

$$z_{13} = \varepsilon^3(160/27s_1^3 - 2/9s_1) - 2/3\varepsilon^2 s_1 - 2/3\varepsilon s_1$$

```
ff_:=tp mat(((1+small*k)*(u1-u1^2-2*u2^2-2*u3^2)
,-u2+(1+small*k)*(u2-2*u1*u2-2*u2*u3)
,-4*u3+(1+small*k)*(u3-2*u1*u3-u2^2)
));
freqm_:=mat((0));
ee_:=tp mat((0,1,0));
zz_:=tp mat((0,1,0));
toosmall:=4;
factor small,k;
```