Invariant manifold of your dynamical system

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8:13am, October 16, 2023

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = u_2$

$\dot{u}_2 = \varepsilon^2(-pu_1^3 + 3/2u_1u_2^2 + 8u_1u_2u_3) + \varepsilon(qu_1^2 - 3/2u_2^2 - 4u_2u_3) - u_1w^2$

$\dot{u}_3 = 0$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1, iw, 0\}, \exp(itw)\}$

$\vec{e}_2 = \{\{1, -iw, 0\}, \exp(-itw)\}$

$\vec{e}_3 = \{\{0, 0, 1\}, \exp(0)\}$

$\vec{z}_1 = \{\{1/(w^2+1), (iw)/(w^2+1), 0\}, \exp(itw)\}$

$\vec{z}_2 = \{\{1/(w^2+1), (-iw)/(w^2+1), 0\}, \exp(-itw)\}$

$\vec{z}_3 = \{\{0, 0, 1\}, \exp(0)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = (2s_3s_2i\varepsilon\exp(-itw)w)/(w^2+1)+(-2s_3s_1i\varepsilon\exp(itw)w)/(w^2+1)+s_2^2\varepsilon(-1/3\exp(-2itw)qw^{-2}-1/2\exp(-2itw))+s_2s_1\varepsilon(2qw^{-2}-3)+s_2\exp(-itw)+s_1^2\varepsilon(-1/3\exp(2itw)qw^{-2}-1/2\exp(2itw))+s_1\exp(itw)+O(\varepsilon^2)$

$u_2 = (-2s_3s_2\varepsilon\exp(-itw))/(w^2+1)+(-2s_3s_1\varepsilon\exp(itw))/(w^2+1)+s_2^2i\varepsilon(2/3\exp(-2itw)qw^{-1}+\exp(-2itw)w)-s_2i\exp(-itw)w+s_1^2i\varepsilon(-2/3\exp(2itw)qw^{-1}-\exp(2itw)w)+s_1i\exp(itw)w+O(\varepsilon^2)$

$u_3 = s_3+O(\varepsilon^2)$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = -2s_3^2s_1i\varepsilon^2w^{-1}-2s_3s_1\varepsilon+s_2s_1^2i\varepsilon^2(3/2pw^{-1}-5/3q^2w^{-3}+5/2qw^{-1}-9/4w)+O(\varepsilon^3)$

$\dot{s}_2 = 2s_3^2s_2i\varepsilon^2w^{-1}-2s_3s_2\varepsilon+s_2^2s_1i\varepsilon^2(-3/2pw^{-1}+5/3q^2w^{-3}-5/2qw^{-1}+9/4w)+O(\varepsilon^3)$

$\dot{s}_3 = O(\varepsilon^3)$