Invariant manifold of your dynamical system

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1:27pm, April 8, 2025

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot u_1 = \varepsilon(\frac{d u_1}{d x} - 1/2u_1^2) - 1/2u_1 + 1/2u_2$

$\dot u_2 = \varepsilon(- \frac{d u_2}{d x} + 1/2u_2^2) + 1/2u_1 - 1/2u_2$

Invariant subspace basis vectors

$\vec e_1 = \{\{1/2, 1/2\}, \exp(0)\}$

$\vec z_1 = \{\{1, 1\}, \exp(0)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = \varepsilon(1/2\frac{d s_1}{d x} - 1/8s_1^2) + O(\varepsilon^2) + 1/2s_1$

$u_2 = \varepsilon(- 1/2\frac{d s_1}{d x} + 1/8s_1^2) + O(\varepsilon^2) + 1/2s_1$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = \varepsilon^2\left(\frac{d^2 s_1}{dx^2} - \frac{d\, s_1}{d\, x} s_1 + 1/8 s_1^3\right) + O(\varepsilon^3)$$

Normals to isochrons at the slow manifold Use these vectors: to project initial conditions onto the slow manifold; to project non-autonomous forcing onto the slow evolution; to predict the consequences of modifying the original system; in uncertainty quantification to quantify effects on the model of uncertainties in the original system. The normal vector $\vec{z}_j := (z_{j1}, \ldots, z_{jn})$

$$z_{11} = \varepsilon^2(-\partial_{\dot{x}}^2 + \partial_{\dot{x}} s_1 - 1/4 s_1^2) + \varepsilon(\partial_{\dot{x}} - 1/2 s_1) + O(\varepsilon^3) + 1$$

$$z_{12} = \varepsilon^2(-\partial_{\dot{x}}^2 + \partial_{\dot{x}} s_1 - 1/4 s_1^2) + \varepsilon(-\partial_{\dot{x}} + 1/2 s_1) + O(\varepsilon^3) + 1$$