Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = \pi u_2$

$\dot{u}_2 = -\pi u_1 + \pi^{-1} \varepsilon u_1 u_5$

$\dot{u}_3 = \pi u_4$

$\dot{u}_4 = -\pi u_3 + \pi^{-1} \varepsilon u_3 u_5$

$\dot{u}_5 = 2\pi u_6$

$\dot{u}_6 = -2\pi u_5 + \pi^{-1} \varepsilon (1/2 u_1^2 + 1/2 u_3^2)$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1, i, 0, 0, 0, 0\}, \mathrm{e}^{i\pi t}\}$

$\vec{e}_2 = \{\{1, -i, 0, 0, 0, 0\}, \mathrm{e}^{-i\pi t}\}$

$\vec{e}_3 = \{\{0, 0, 1, i, 0, 0\}, \mathrm{e}^{i\pi t}\}$

$\vec{e}_4 = \{\{0, 0, 1, -i, 0, 0\}, \mathrm{e}^{-i\pi t}\}$

$\vec{e}_5 = \{\{0, 0, 0, 0, 1, i\}, \mathrm{e}^{2i\pi t}\}$

$\vec{e}_6 = \{\{0,0,0,0,1,-i\}, \mathrm{e}^{-2i\pi t}\}$

$\vec{z}_1 = \{\{1/2,1/2i,0,0,0,0\}, \mathrm{e}^{i\pi t}\}$

$\vec{z}_2 = \{\{1/2,-1/2i,0,0,0,0\}, \mathrm{e}^{-i\pi t}\}$

$\vec{z}_3 = \{\{0,0,1/2,1/2i,0,0\}, \mathrm{e}^{i\pi t}\}$

$\vec{z}_4 = \{\{0,0,1/2,-1/2i,0,0\}, \mathrm{e}^{-i\pi t}\}$

$\vec{z}_5 = \{\{0,0,0,0,1/2,1/2i\}, \mathrm{e}^{2i\pi t}\}$

$\vec{z}_6 = \{\{0,0,0,0,1/2,-1/2i\}, \mathrm{e}^{-2i\pi t}\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = \mathrm{e}^{-i\pi t}s_2 + \mathrm{e}^{i\pi t}s_1 + \pi^{-2}\varepsilon(1/4\,\mathrm{e}^{-i\pi t}s_6s_1 - 1/8\,\mathrm{e}^{-3i\pi t}s_6s_2 + 1/4\,\mathrm{e}^{i\pi t}s_5s_2 - 1/8\,\mathrm{e}^{3i\pi t}s_5s_1)$

$u_2 = i(-\mathrm{e}^{-i\pi t}s_2 + \mathrm{e}^{i\pi t}s_1) + \pi^{-2}i\varepsilon(1/4\,\mathrm{e}^{-i\pi t}s_6s_1 + 3/8\,\mathrm{e}^{-3i\pi t}s_6s_2 - 1/4\,\mathrm{e}^{i\pi t}s_5s_2 - 3/8\,\mathrm{e}^{3i\pi t}s_5s_1)$

$u_3 = \mathrm{e}^{-i\pi t}s_4 + \mathrm{e}^{i\pi t}s_3 + \pi^{-2}\varepsilon(1/4\,\mathrm{e}^{-i\pi t}s_6s_3 - 1/8\,\mathrm{e}^{-3i\pi t}s_6s_4 + 1/4\,\mathrm{e}^{i\pi t}s_5s_4 - 1/8\,\mathrm{e}^{3i\pi t}s_5s_3)$

$u_4 = i(-\mathrm{e}^{-i\pi t}s_4 + \mathrm{e}^{i\pi t}s_3) + \pi^{-2}i\varepsilon(1/4\,\mathrm{e}^{-i\pi t}s_6s_3 + 3/8\,\mathrm{e}^{-3i\pi t}s_6s_4 - 1/4\,\mathrm{e}^{i\pi t}s_5s_4 - 3/8\,\mathrm{e}^{3i\pi t}s_5s_3)$

$u_5 = \mathrm{e}^{-2i\pi t}s_6 + \mathrm{e}^{2i\pi t}s_5 + \pi^{-2}\varepsilon(1/16\,\mathrm{e}^{-2i\pi t}s_4^2 + 1/16\,\mathrm{e}^{-2i\pi t}s_2^2 + 1/16\,\mathrm{e}^{2i\pi t}s_3^2 + 1/16\,\mathrm{e}^{2i\pi t}s_1^2 + 1/2s_4s_3 + 1/2s_2s_1)$

$u_6 = i(-\mathrm{e}^{-2i\pi t}s_6 + \mathrm{e}^{2i\pi t}s_5) + \pi^{-2}i\varepsilon(1/16\,\mathrm{e}^{-2i\pi t}s_4^2 + 1/16\,\mathrm{e}^{-2i\pi t}s_2^2 - 1/16\,\mathrm{e}^{2i\pi t}s_3^2 - 1/16\,\mathrm{e}^{2i\pi t}s_1^2)$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = -1/2\pi^{-1}i\varepsilon s_5 s_2 + \pi^{-3}i\varepsilon^2(-1/16 s_6 s_5 s_1 - 1/4 s_4 s_3 s_1 - 1/32 s_3^2 s_2 - 9/32 s_2 s_1^2)$$

$$\dot{s}_2 = 1/2\pi^{-1}i\varepsilon s_6 s_1 + \pi^{-3}i\varepsilon^2(1/16 s_6 s_5 s_2 + 1/32 s_4^2 s_1 + 1/4 s_4 s_3 s_2 + 9/32 s_2^2 s_1)$$

$$\dot{s}_3 = -1/2\pi^{-1}i\varepsilon s_5 s_4 + \pi^{-3}i\varepsilon^2(-1/16 s_6 s_5 s_3 - 9/32 s_4 s_3^2 - 1/32 s_4 s_1^2 - 1/4 s_3 s_2 s_1)$$

$$\dot{s}_4 = 1/2\pi^{-1}i\varepsilon s_6 s_3 + \pi^{-3}i\varepsilon^2(1/16 s_6 s_5 s_4 + 9/32 s_4^2 s_3 + 1/4 s_4 s_2 s_1 + 1/32 s_3 s_2^2)$$

$$\dot{s}_5 = \pi^{-1}i\varepsilon(-1/4 s_3^2 - 1/4 s_1^2) + \pi^{-3}i\varepsilon^2(-1/16 s_5 s_4 s_3 - 1/16 s_5 s_2 s_1)$$

$$\dot{s}_6 = \pi^{-1}i\varepsilon(1/4 s_4^2 + 1/4 s_2^2) + \pi^{-3}i\varepsilon^2(1/16 s_6 s_4 s_3 + 1/16 s_6 s_2 s_1)$$