A normal form of your dynamical system

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Generally, the lowest order, most important, terms are near the end of each expression.

off echo;

Specified dynamical system

$$\dot{x}_1 = \varepsilon x_1 y_1$$

$$\dot{y}_1 = \sigma w_1 - y_1$$

off echo;

Time dependent normal form coordinates

$$y_{1} = \sigma e^{-1t} \star w_{1} + O(\varepsilon^{4}, \sigma^{2}) + Y_{1}$$

$$x_{1} = -1/2\sigma\varepsilon^{3}e^{-1t} \star w_{1} X_{1}Y_{1}^{2} + \sigma\varepsilon^{2}e^{-1t} \star w_{1} X_{1}Y_{1} - \sigma\varepsilon e^{-1t} \star w_{1} X_{1} - 1/6\varepsilon^{3}X_{1}Y_{1}^{3} + 1/2\varepsilon^{2}X_{1}Y_{1}^{2} - \varepsilon X_{1}Y_{1} + O(\varepsilon^{4}, \sigma^{2}) + X_{1}$$

Result normal form DEs

$$\dot{Y}_1 = O(\varepsilon^5, \sigma^3) - Y_1$$

$$\dot{X}_1 = \sigma \varepsilon w_1 X_1 + O(\varepsilon^5, \sigma^3)$$