Centre manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = \varepsilon u_2(1/3 u_1^3 u_3^3 - u_1^3 u_3)$

$\dot{u}_2 = 0$

$\dot{u}_3 = \varepsilon u_2(- 1/2 u_1 u_3^2 + u_1) - u_3$

Centre subspace basis vectors

$\vec{e}_1 = \{\{1, 0, 0\}, e^{0i}\}$

$\vec{e}_2 = \{\{0, 1, 0\}, e^{0i}\}$

$\vec{z}_1 = \{\{1, 0, 0\}, e^{0i}\}$

$\vec{z}_2 = \{\{0, 1, 0\}, e^{0i}\}$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$u_1 = s_1$

$$u_2 = s_2$$

$$u_3 = s_2^3\varepsilon^3(s_1^4 - 1/2s_1^3) + s_2\varepsilon s_1$$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = s_2^4\varepsilon^4(- s_1^7 + 5/6s_1^6) - s_2^2\varepsilon^2 s_1^4$$

$$\dot{s}_2 = 0$$

Normals to isochrons at the slow manifold Use these vectors: to project initial conditions onto the slow manifold; to project non-autonomous forcing onto the slow evolution; to predict the consequences of modifying the original system; in uncertainty quantification to quantify effects on the model of uncertainties in the original system. The normal vector $\vec{z}_j := (z_{j1}, \ldots, z_{jn})$

$$z_{11} = s_2^4\varepsilon^4(6s_1^6 - 7/2s_1^5) + s_2^2\varepsilon^2 s_1^3 + 1$$

$$z_{12} = s_2^3\varepsilon^4(5s_1^7 - 7/2s_1^6) + s_2\varepsilon^2 s_1^4$$

$$z_{13} = s_2^3\varepsilon^3(- 2s_1^6 + 2s_1^5) - s_2\varepsilon s_1^3$$

$$z_{21} = 0$$

$$z_{22} = 1$$

$$z_{23} = 0$$