Centre manifold of your dynamical system

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10:09 A.M., December 20, 2019

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = u_3$

$\dot{u}_2 = u_4$

$\dot{u}_3 = -\varepsilon^2 u_1^3 + \varepsilon u_5 - 2u_1 + u_2 - u_3 + u_4$

$\dot{u}_4 = -\varepsilon^2 u_2^3 + 2\varepsilon u_6 + u_1 - 2u_2 + u_3 - u_4$

$\dot{u}_5 = -u_7$

$\dot{u}_6 = -u_8$

$\dot{u}_7 = 16u_5$

$\dot{u}_8 = 64u_6$

Centre subspace basis vectors

$\vec{e}_1 = \left\{\{1, 1, -i, -i, 0, 0, 0, 0\}, e^{-ti}\right\}$

$\vec{e}_2 = \left\{\{1, 1, i, i, 0, 0, 0, 0\}, e^{ti}\right\}$

$\vec{e}_3 = \left\{\{0, 0, 0, 0, -1/4i, 0, 1, 0\}, e^{-4ti}\right\}$

$\vec{e}_4 = \{\{0,0,0,0,1/4i,0,1,0\}\,,\, e^{4ti}\}$

$\vec{e}_5 = \{\{0,0,0,0,0,-1/8i,0,1\}\,,\, e^{-8ti}\}$

$\vec{e}_6 = \{\{0,0,0,0,0,1/8i,0,1\}\,,\, e^{8ti}\}$

$\vec{z}_1 = \{\{1/4,1/4,-1/4i,-1/4i,0,0,0,0\}\,,\, e^{-ti}\}$

$\vec{z}_2 = \{\{1/4,1/4,1/4i,1/4i,0,0,0,0\}\,,\, e^{ti}\}$

$\vec{z}_3 = \{\{0,0,0,0,-2i,0,1/2,0\}\,,\, e^{-4ti}\}$

$\vec{z}_4 = \{\{0,0,0,0,2i,0,1/2,0\}\,,\, e^{4ti}\}$

$\vec{z}_5 = \{\{0,0,0,0,0,-4i,0,1/2\}\,,\, e^{-8ti}\}$

$\vec{z}_6 = \{\{0,0,0,0,0,4i,0,1/2\}\,,\, e^{8ti}\}$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$u_1 = \varepsilon^3(512738/81102607047\, e^{-10ti}s_5s_1^2i - 2079/78020786\, e^{-10ti}s_5s_1^2 +$
$515986/20925267867\, e^{-8ti}s_5s_2s_1i - 1464/15816529\, e^{-8ti}s_5s_2s_1 +$
$12338/400464015\, e^{-6ti}s_5s_2^2i - 105/1089698\, e^{-6ti}s_5s_2^2 +$
$25673/22344700\, e^{-6ti}s_3s_1^2i + 35/63842\, e^{-6ti}s_3s_1^2 +$
$38957/8143350\, e^{-4ti}s_3s_2s_1i + 156/54289\, e^{-4ti}s_3s_2s_1 +$
$1553/237660\, e^{-2ti}s_3s_2^2i + 45/7922\, e^{-2ti}s_3s_2^2 -$
$512738/81102607047\, e^{10ti}s_6s_2^2i - 2079/78020786\, e^{10ti}s_6s_2^2 -$
$515986/20925267867\, e^{8ti}s_6s_2s_1i - 1464/15816529\, e^{8ti}s_6s_2s_1 -$
$12338/400464015\, e^{6ti}s_6s_1^2i - 105/1089698\, e^{6ti}s_6s_1^2 -$
$25673/22344700\, e^{6ti}s_4s_2^2i + 35/63842\, e^{6ti}s_4s_2^2 -$
$38957/8143350\, e^{4ti}s_4s_2s_1i + 156/54289\, e^{4ti}s_4s_2s_1 -$
$1553/237660\, e^{2ti}s_4s_1^2i + 45/7922\, e^{2ti}s_4s_1^2) + \varepsilon^2(1/8\, e^{-3ti}s_1^3 -$
$3/4\, e^{-ti}s_2s_1^2 + 1/8\, e^{3ti}s_2^3 - 3/4\, e^{ti}s_2^2s_1) + \varepsilon(67/1002204\, e^{-8ti}s_5i -$
$2/3977\, e^{-8ti}s_5 + 107/6990\, e^{-4ti}s_3i + 1/233\, e^{-4ti}s_3 - 67/1002204\, e^{8ti}s_6i -$
$2/3977\, e^{8ti}s_6 - 107/6990\, e^{4ti}s_4i + 1/233\, e^{4ti}s_4) + e^{-ti}s_1 + e^{ti}s_2$

$u_2 = \varepsilon^3(36959441/324410428188\, e^{-10ti}s_5s_1^2i +$
$2079/78020786\, e^{-10ti}s_5s_1^2 + 14784557/41850535734\, e^{-8ti}s_5s_2s_1i +$
$1464/15816529\, e^{-8ti}s_5s_2s_1 + 495497/1601856060\, e^{-6ti}s_5s_2^2i +$
$105/1089698\, e^{-6ti}s_5s_2^2 + 1562/5586175\, e^{-6ti}s_3s_1^2i - 35/63842\, e^{-6ti}s_3s_1^2 +$

$7666/4071675\,e^{-4ti}s_3s_2s_1i - 156/54289\,e^{-4ti}s_3s_2s_1 +$
$602/59415\,e^{-2ti}s_3s_2^2i - 45/7922\,e^{-2ti}s_3s_2^2 -$
$36959441/324410428188\,e^{10ti}s_6s_2^2i + 2079/78020786\,e^{10ti}s_6s_2^2 -$
$14784557/41850535734\,e^{8ti}s_6s_2s_1i + 1464/15816529\,e^{8ti}s_6s_2s_1 -$
$495497/1601856060\,e^{6ti}s_6s_1^2i + 105/1089698\,e^{6ti}s_6s_1^2 -$
$1562/5586175\,e^{6ti}s_4s_2^2i - 35/63842\,e^{6ti}s_4s_2^2 - 7666/4071675\,e^{4ti}s_4s_2s_1i -$
$156/54289\,e^{4ti}s_4s_2s_1 - 602/59415\,e^{2ti}s_4s_1^2i - 45/7922\,e^{2ti}s_4s_1^2) +$
$\varepsilon^2(1/8\,e^{-3ti}s_1^3 - 3/4\,e^{-ti}s_2s_1^2 + 1/8\,e^{3ti}s_2^3 - 3/4\,e^{ti}s_2^2s_1) +$
$\varepsilon(1955/501102\,e^{-8ti}s_5i + 2/3977\,e^{-8ti}s_5 + 19/13980\,e^{-4ti}s_3i -$
$1/233\,e^{-4ti}s_3 - 1955/501102\,e^{8ti}s_6i + 2/3977\,e^{8ti}s_6 - 19/13980\,e^{4ti}s_4i -$
$1/233\,e^{4ti}s_4) + e^{-ti}s_1 + e^{ti}s_2$

$u_3 = \varepsilon^3(10395/39010393\,e^{-10ti}s_5s_1^2i + 5127380/81102607047\,e^{-10ti}s_5s_1^2 +$
$11712/15816529\,e^{-8ti}s_5s_2s_1i + 4127888/20925267867\,e^{-8ti}s_5s_2s_1 +$
$315/544849\,e^{-6ti}s_5s_2^2i + 24676/133488005\,e^{-6ti}s_5s_2^2 -$
$105/31921\,e^{-6ti}s_3s_1^2i + 77019/11172350\,e^{-6ti}s_3s_1^2 -$
$624/54289\,e^{-4ti}s_3s_2s_1i + 77914/4071675\,e^{-4ti}s_3s_2s_1 -$
$45/3961\,e^{-2ti}s_3s_2^2i + 1553/118830\,e^{-2ti}s_3s_2^2 -$
$10395/39010393\,e^{10ti}s_6s_2^2i + 5127380/81102607047\,e^{10ti}s_6s_2^2 -$
$11712/15816529\,e^{8ti}s_6s_2s_1i + 4127888/20925267867\,e^{8ti}s_6s_2s_1 -$
$315/544849\,e^{6ti}s_6s_1^2i + 24676/133488005\,e^{6ti}s_6s_1^2 + 105/31921\,e^{6ti}s_4s_2^2i +$
$77019/11172350\,e^{6ti}s_4s_2^2 + 624/54289\,e^{4ti}s_4s_2s_1i +$
$77914/4071675\,e^{4ti}s_4s_2s_1 + 45/3961\,e^{2ti}s_4s_1^2i + 1553/118830\,e^{2ti}s_4s_1^2) +$
$\varepsilon^2(-3/8\,e^{-3ti}s_1^3i - 3/4\,e^{-ti}s_2s_1^2i + 3/8\,e^{3ti}s_2^3i + 3/4\,e^{ti}s_2^2s_1i) +$
$\varepsilon(16/3977\,e^{-8ti}s_5i + 134/250551\,e^{-8ti}s_5 - 4/233\,e^{-4ti}s_3i +$
$214/3495\,e^{-4ti}s_3 - 16/3977\,e^{8ti}s_6i + 134/250551\,e^{8ti}s_6 + 4/233\,e^{4ti}s_4i +$
$214/3495\,e^{4ti}s_4) - e^{-ti}s_1i + e^{ti}s_2i$

$u_4 =$
$\varepsilon^3(-10395/39010393\,e^{-10ti}s_5s_1^2i + 184797205/162205214094\,e^{-10ti}s_5s_1^2 -$
$11712/15816529\,e^{-8ti}s_5s_2s_1i + 59138228/20925267867\,e^{-8ti}s_5s_2s_1 -$
$315/544849\,e^{-6ti}s_5s_2^2i + 495497/266976010\,e^{-6ti}s_5s_2^2 +$
$105/31921\,e^{-6ti}s_3s_1^2i + 9372/5586175\,e^{-6ti}s_3s_1^2 + 624/54289\,e^{-4ti}s_3s_2s_1i +$
$30664/4071675\,e^{-4ti}s_3s_2s_1 + 45/3961\,e^{-2ti}s_3s_2^2i + 1204/59415\,e^{-2ti}s_3s_2^2 +$
$10395/39010393\,e^{10ti}s_6s_2^2i + 184797205/162205214094\,e^{10ti}s_6s_2^2 +$
$11712/15816529\,e^{8ti}s_6s_2s_1i + 59138228/20925267867\,e^{8ti}s_6s_2s_1 +$
$315/544849\,e^{6ti}s_6s_1^2i + 495497/266976010\,e^{6ti}s_6s_1^2 - 105/31921\,e^{6ti}s_4s_2^2i +$
$9372/5586175\,e^{6ti}s_4s_2^2 - 624/54289\,e^{4ti}s_4s_2s_1i +$
$30664/4071675\,e^{4ti}s_4s_2s_1 - 45/3961\,e^{2ti}s_4s_1^2i + 1204/59415\,e^{2ti}s_4s_1^2) +$

$\varepsilon^2(-3/8\,e^{-3ti}s_1^3 i - 3/4\,e^{-ti}s_2 s_1^2 i + 3/8\,e^{3ti}s_2^3 i + 3/4\,e^{ti}s_2^2 s_1 i) + \varepsilon(-16/3977\,e^{-8ti}s_5 i + 7820/250551\,e^{-8ti}s_5 + 4/233\,e^{-4ti}s_3 i + 19/3495\,e^{-4ti}s_3 + 16/3977\,e^{8ti}s_6 i + 7820/250551\,e^{8ti}s_6 - 4/233\,e^{4ti}s_4 i + 19/3495\,e^{4ti}s_4) - e^{-ti}s_1 i + e^{ti}s_2 i$

$u_5 = -1/4\,e^{-4ti}s_3 i + 1/4\,e^{4ti}s_4 i$

$u_6 = -1/8\,e^{-8ti}s_5 i + 1/8\,e^{8ti}s_6 i$

$u_7 = e^{-4ti}s_3 + e^{4ti}s_4$

$u_8 = e^{-8ti}s_5 + e^{8ti}s_6$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = \varepsilon^4(-3973/168370272 s_6 s_5 s_1 i - 229/559200 s_4 s_3 s_1 i + 51/16 s_2^2 s_1^3 i) - 3/2\varepsilon^2 s_2 s_1^2 i$

$\dot{s}_2 = \varepsilon^4(3973/168370272 s_6 s_5 s_2 i + 229/559200 s_4 s_3 s_2 i - 51/16 s_2^3 s_1^2 i) + 3/2\varepsilon^2 s_2^2 s_1 i$

$\dot{s}_3 = 0$

$\dot{s}_4 = 0$

$\dot{s}_5 = 0$

$\dot{s}_6 = 0$