Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = -\varepsilon^2 u_1^2 u_2 - 1/2 u_1 - u_2$

$\dot{u}_2 = \varepsilon^2 \epsilon u_2 - \varepsilon u_2^2 - u_1 - 2u_2$

Invariant subspace basis vectors

$\vec{e}_1 = \left\{ \{1, 2\}, \mathrm{e}^{(-5t}/2) \right\}$

$\vec{z}_1 = \left\{ \{1/5, 2/5\}, \mathrm{e}^{(-5t}/2) \right\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = \varepsilon^3(53152/140625\,\mathrm{e}^{-10t} s_1^4 + 88/625\,\mathrm{e}^{-5t} s_1^2 \epsilon) +$
$\varepsilon^2(838/1875\,\mathrm{e}^{(-15t}/2) s_1^3 + 8/25\,\mathrm{e}^{(-5t}/2) s_1 \epsilon) + 8/25 \varepsilon\,\mathrm{e}^{-5t} s_1^2 + \mathrm{e}^{(-5t}/2) s_1$

$u_2 = \varepsilon^3(122444/140625\,\mathrm{e}^{-10t}s_1^4 + 76/625\,\mathrm{e}^{-5t}s_1^2\epsilon) +$
$\varepsilon^2(2116/1875\,\mathrm{e}^{(-15t}/2)s_1^3 - 4/25\,\mathrm{e}^{(-5t}/2)s_1\epsilon) + 36/25\varepsilon\,\mathrm{e}^{-5t}s_1^2 + 2\,\mathrm{e}^{(-5t}/2)s_1$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = -8/125\varepsilon^4 s_1\epsilon^2 + 4/5\varepsilon^2 s_1\epsilon$