Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$$\dot{u}_1 = \gamma\varepsilon^2\left(-\tfrac{1}{2}u_4 - \tfrac{1}{2}u_2\right) + \tfrac{1}{2}u_4 + \tfrac{3}{2}u_2$$

$$\dot{u}_2 = \gamma\varepsilon^2\left(\tfrac{1}{2}u_3 + \tfrac{1}{2}u_1\right) + \tfrac{1}{2}u_3 - \tfrac{1}{2}u_1$$

$$\dot{u}_3 = \gamma\varepsilon^2\left(\tfrac{1}{2}u_4 + \tfrac{1}{2}u_2\right) + \tfrac{1}{2}u_4 - \tfrac{1}{2}u_2$$

$$\dot{u}_4 = \gamma\varepsilon^2\left(-\tfrac{1}{2}u_3 - \tfrac{1}{2}u_1\right) + \mu\varepsilon^3 u_1 - \varepsilon^3 u_1^3 - \tfrac{3}{2}u_3 - \tfrac{1}{2}u_1$$

Centre subspace basis vectors

$$\vec{e}_1 = \left\{\left\{\tfrac{1}{2}, \tfrac{1}{2}i, -\tfrac{1}{2}, -\tfrac{1}{2}i\right\}, e^{it}\right\}$$

$$\vec{e}_2 = \left\{\left\{-\tfrac{3}{2}i, \tfrac{1}{2}, -\tfrac{1}{2}i, \tfrac{3}{2}\right\}, e^{it}\right\}$$

$$\vec{e}_3 = \left\{\left\{\tfrac{1}{2}, -\tfrac{1}{2}i, -\tfrac{1}{2}, \tfrac{1}{2}i\right\}, e^{-it}\right\}$$

$$\vec{e}_4 = \left\{\left\{\tfrac{3}{2}i, \tfrac{1}{2}, \tfrac{1}{2}i, \tfrac{3}{2}\right\}, e^{-it}\right\}$$

$$\vec{z}_1 = \left\{\left\{\tfrac{1}{4}, \tfrac{3}{4}i, -\tfrac{3}{4}, -\tfrac{1}{4}i\right\}, e^{it}\right\}$$

$\vec{z}_2 = \left\{\left\{-\frac{1}{4}i, \frac{1}{4}, -\frac{1}{4}i, \frac{1}{4}\right\}, e^{it}\right\}$

$\vec{z}_3 = \left\{\left\{\frac{1}{4}, -\frac{3}{4}i, -\frac{3}{4}, \frac{1}{4}i\right\}, e^{-it}\right\}$

$\vec{z}_4 = \left\{\left\{\frac{1}{4}i, \frac{1}{4}, \frac{1}{4}i, \frac{1}{4}\right\}, e^{-it}\right\}$

The Centre manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = \frac{3}{2}\, e^{-it} s_4 i + \frac{1}{2}\, e^{-it} s_3 - \frac{3}{2}\, e^{it} s_2 i + \frac{1}{2}\, e^{it} s_1$

$u_2 = \frac{1}{2}\, e^{-it} s_4 - \frac{1}{2}\, e^{-it} s_3 i + \frac{1}{2}\, e^{it} s_2 + \frac{1}{2}\, e^{it} s_1 i$

$u_3 = \frac{1}{2}\, e^{-it} s_4 i - \frac{1}{2}\, e^{-it} s_3 - \frac{1}{2}\, e^{it} s_2 i - \frac{1}{2}\, e^{it} s_1$

$u_4 = \frac{3}{2}\, e^{-it} s_4 + \frac{1}{2}\, e^{-it} s_3 i + \frac{3}{2}\, e^{it} s_2 - \frac{1}{2}\, e^{it} s_1 i$

Centre manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = -2\gamma\varepsilon^2 s_2 + \mu\varepsilon^3(\frac{3}{8}s_2 + \frac{1}{8}s_1 i) + \varepsilon^3(- \frac{81}{32}s_4 s_2^2 - \frac{27}{16}s_4 s_2 s_1 i + \frac{9}{32}s_4 s_1^2 + \frac{27}{32}s_3 s_2^2 i - \frac{9}{16}s_3 s_2 s_1 - \frac{3}{32}s_3 s_1^2 i)$

$\dot{s}_2 = \mu\varepsilon^3(- \frac{3}{8}s_2 i + \frac{1}{8}s_1) + \varepsilon^3(\frac{81}{32}s_4 s_2^2 i - \frac{27}{16}s_4 s_2 s_1 - \frac{9}{32}s_4 s_1^2 i + \frac{27}{32}s_3 s_2^2 + \frac{9}{16}s_3 s_2 s_1 i - \frac{3}{32}s_3 s_1^2)$

$\dot{s}_3 = -2\gamma\varepsilon^2 s_4 + \mu\varepsilon^3(\frac{3}{8}s_4 - \frac{1}{8}s_3 i) + \varepsilon^3(- \frac{81}{32}s_4^2 s_2 - \frac{27}{32}s_4^2 s_1 i + \frac{27}{16}s_4 s_3 s_2 i - \frac{9}{16}s_4 s_3 s_1 + \frac{9}{32}s_3^2 s_2 + \frac{3}{32}s_3^2 s_1 i)$

$\dot{s}_4 = \mu\varepsilon^3(\frac{3}{8}s_4 i + \frac{1}{8}s_3) + \varepsilon^3(- \frac{81}{32}s_4^2 s_2 i + \frac{27}{32}s_4^2 s_1 - \frac{27}{16}s_4 s_3 s_2 - \frac{9}{16}s_4 s_3 s_1 i + \frac{9}{32}s_3^2 s_2 i - \frac{3}{32}s_3^2 s_1)$