Normal form of your dynamical system

A. J. Roberts, University of Adelaide
http://orcid.org/0000-0001-8930-1552

6:44am, April 28, 2021

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

off echo;

Specified dynamical system

$\dot{x}_1 = \varepsilon(- x_1 y_1 + y_2)$

$\dot{x}_2 = \varepsilon(- x_2 y_1 - x_1 y_2 + y_3)$

$\dot{x}_3 = \sigma\varepsilon w_1 + \varepsilon(- x_3 y_1 - 2x_2 y_2 - x_1 y_3)$

$\dot{y}_1 = \varepsilon(x_2 - 1/2x_1^2 - 1/2y_1^2) - y_1$

$\dot{y}_2 = \varepsilon(x_3 - x_2 x_1 - y_2 y_1) - y_2$

$\dot{y}_3 = \sigma\varepsilon w_2 + \varepsilon(- x_3 x_1 - x_2^2 - y_3 y_1 - y_2^2) - y_3$

off echo;

Time dependent coordinate transform

$y_1 = \varepsilon(X_2 - 1/2X_1^2 + 1/2Y_1^2) + Y_1$

$y_2 = \varepsilon(X_3 - X_2 X_1 + Y_2 Y_1) + Y_2$

$$y_3 = \sigma\varepsilon \mathrm{e}^{-1t} \star w_2 \ + \varepsilon(- X_3X_1 - X_2^2 + Y_3Y_1 + Y_2^2) + Y_3$$

$$x_1 = \varepsilon(X_1Y_1 - Y_2) + X_1$$

$$x_2 = \varepsilon(X_2Y_1 + X_1Y_2 - Y_3) + X_2$$

$$x_3 = \varepsilon(X_3Y_1 + 2X_2Y_2 + X_1Y_3) + X_3$$

Result normal form DEs

$$\dot{Y}_1 = \varepsilon^2(- 1/2X_1^2Y_1 + 2X_1Y_2 - Y_3) - Y_1$$

$$\dot{Y}_2 = \varepsilon^2(- X_2X_1Y_1 + 2X_2Y_2 - 1/2X_1^2Y_2 + 2X_1Y_3) - Y_2$$

$$\dot{Y}_3 = -\sigma\varepsilon^2 w_2 Y_1 + \varepsilon^2(- X_3X_1Y_1 - X_3Y_2 - X_2^2Y_1 - 2X_2X_1Y_2 + X_2Y_3 - 1/2X_1^2Y_3) - Y_3$$

$$\dot{X}_1 = \varepsilon^2(X_3 - 2X_2X_1 + 1/2X_1^3)$$

$$\dot{X}_2 = \sigma\varepsilon^2 w_2 + \varepsilon^2(- 2X_3X_1 - 2X_2^2 + 3/2X_2X_1^2)$$

$$\dot{X}_3 = -\sigma\varepsilon^2 w_2 X_1 + \sigma\varepsilon w_1 + \varepsilon^2(- 3X_3X_2 + 3/2X_3X_1^2 + 3X_2^2X_1)$$