Centre manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$$\dot u_1 = \varepsilon\big(- \beta^{-1} u_3^2 + u_2 u_3\big) + \beta u_3 - u_1 - u_3$$

$$\dot u_2 = \varepsilon^2 u_3$$

$$\dot u_3 = \varepsilon^2 u_1$$

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ff_:=tp mat((u2*u3-u3^2/beta-(1-beta)*u3-u1
,small*u3
,small*u1 ));
freqm_:=mat((0,0));
ee_:=tp mat((0,1,0),(-1+beta,0,1));
zz_:=tp mat((0,1,0),(0,0,1));
toosmall:=5;
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Centre subspace basis vectors

$$\vec e_1 = \{\{0, 1, 0\}\,,\, e^{0i}\}$$

$$\vec e_2 = \{\{\beta - 1, 0, 1\}\,,\, e^{0i}\}$$

$$\vec z_1 = \{\{0, 1, 0\}\,,\, e^{0i}\}$$

$$\vec z_2 = \{\{0, 0, 1\}\,,\, e^{0i}\}$$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$$u_1 = \varepsilon^3\big(- 3s_2^2\beta^{-1} + 2s_2^2 - 2s_2s_1\beta + 2s_2s_1\big) + \varepsilon^2\big(- s_2\beta^2 + 2s_2\beta - s_2\big) + \varepsilon\big(- s_2^2\beta^{-1} + s_2s_1\big) + s_2\beta - s_2$$

Got much of (32)

$$u_2 = s_1$$

$$u_3 = s_2$$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$$\dot s_1 = \varepsilon^2 s_2$$

Half of (33)

$$\dot s_2 = \varepsilon^4\big(- s_2\beta^2 + 2s_2\beta - s_2\big) + \varepsilon^3\big(- s_2^2\beta^{-1} + s_2s_1\big) + \varepsilon^2\big(s_2\beta - s_2\big)$$

Normals to isochrons at the slow manifold Use these vectors: to project initial conditions onto the slow manifold; to project non-autonomous forcing onto the slow evolution; to predict the consequences of modifying the original system; in uncertainty quantification to quantify effects on the model of uncertainties in the original system. The normal vector $\vec z_j := (z_{j1},\ldots,z_{jn}\ )$

$$z_{11} = -\varepsilon^4$$

$$z_{12} = 1$$

$$z_{13} = \varepsilon^4(\beta - 1)$$

$$z_{21} = \varepsilon^4\big(- 2\beta + 2\big) + \varepsilon^2$$

$$z_{22} = -\varepsilon^3 s_2$$

$$z_{23} = \varepsilon^4(3\beta^2 - 6\beta + 3) + \varepsilon^3(2s_2\beta^{-1} - s_1) + \varepsilon^2\big(- \beta + 1\big) + 1$$