Centre manifold of your dynamical system

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4:18 P.M., January 15, 2016

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

u1=p1, u2=p2, u3=q1, u4=q2, u5=cos(t), u5=sin(t)

The specified dynamical system

$$\dot{u}_1 = \sigma\varepsilon(- u_1u_5 + u_2u_6) + \varepsilon^2(- 1/5u_1u_3 + 2/5u_1) - 2u_1 + 2u_2$$

$$\dot{u}_2 = \sigma\varepsilon(u_1u_5 - u_2u_6) + \varepsilon^2(- 1/10u_2u_4 + 3/10u_2) + 2u_1 - 2u_2$$

$$\dot{u}_3 = \sigma\varepsilon(- u_3u_6 + u_4u_5) + \varepsilon^2(1/2u_1u_3 - 1/10u_3) - 2u_3 + 2u_4$$

$$\dot{u}_4 = \sigma\varepsilon(u_3u_6 - u_4u_5) + \varepsilon^2(3/10u_4^2 - 1/5u_4) + 2u_3 - 2u_4$$

$$\dot{u}_5 = -u_6$$

$$\dot{u}_6 = u_5$$

Centre subspace basis vectors

$$\vec{e}_1 = \{\{1, 1, 0, 0, 0, 0\}, e^{0i}\}$$

$$\vec{e}_2 = \{\{0, 0, 1, 1, 0, 0\}, e^{0i}\}$$

$$\vec{e}_3 = \{\{0, 0, 0, 0, 1/2, -1/2i\}, e^{ti}\}$$

$$\vec{e}_4 = \{\{0, 0, 0, 0, 1/2, 1/2i\}, e^{-ti}\}$$

$$\vec{z}_1 = \{\{1/2, 1/2, 0, 0, 0, 0\}, e^{0i}\}$$

$$\vec{z}_2 = \{\{0, 0, 1/2, 1/2, 0, 0\}, e^{0i}\}$$

$\vec{z}_3 = \{\{0, 0, 0, 0, 1, -i\}, e^{ti}\}$

$\vec{z}_4 = \{\{0, 0, 0, 0, 1, i\}, e^{-ti}\}$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$u_1 = \sigma^3\varepsilon^3(- 11/3400\, e^{-3ti} s_4^3 s_1 i + 7/3400\, e^{-3ti} s_4^3 s_1 - 1/340\, e^{-ti} s_4^2 s_3 s_1 i - 1/170\, e^{-ti} s_4^2 s_3 s_1 + 11/3400\, e^{3ti} s_3^3 s_1 i + 7/3400\, e^{3ti} s_3^3 s_1 + 1/340\, e^{ti} s_4 s_3^2 s_1 i - 1/170\, e^{ti} s_4 s_3^2 s_1) + \sigma^2\varepsilon^2(3/170\, e^{-2ti} s_4^2 s_1 i + 7/340\, e^{-2ti} s_4^2 s_1 - 3/170\, e^{2ti} s_3^2 s_1 i + 7/340\, e^{2ti} s_3^2 s_1 + 1/68 s_4 s_3 s_1) + \sigma\varepsilon^3(169/46240\, e^{-ti} s_4 s_2 s_1 i - 45/9248\, e^{-ti} s_4 s_2 s_1 - 5/544\, e^{-ti} s_4 s_1 i - 3/544\, e^{-ti} s_4 s_1 - 169/46240\, e^{ti} s_3 s_2 s_1 i - 45/9248\, e^{ti} s_3 s_2 s_1 + 5/544\, e^{ti} s_3 s_1 i - 3/544\, e^{ti} s_3 s_1) + \sigma\varepsilon(3/34\, e^{-ti} s_4 s_1 i - 5/34\, e^{-ti} s_4 s_1 - 3/34\, e^{ti} s_3 s_1 i - 5/34\, e^{ti} s_3 s_1) + \varepsilon^2(- 1/80 s_2 s_1 + 1/80 s_1) + s_1$

$u_2 = \sigma^3\varepsilon^3(11/3400\, e^{-3ti} s_4^3 s_1 i - 7/3400\, e^{-3ti} s_4^3 s_1 + 1/340\, e^{-ti} s_4^2 s_3 s_1 i + 1/170\, e^{-ti} s_4^2 s_3 s_1 - 11/3400\, e^{3ti} s_3^3 s_1 i - 7/3400\, e^{3ti} s_3^3 s_1 - 1/340\, e^{ti} s_4 s_3^2 s_1 i + 1/170\, e^{ti} s_4 s_3^2 s_1) + \sigma^2\varepsilon^2(- 3/170\, e^{-2ti} s_4^2 s_1 i - 7/340\, e^{-2ti} s_4^2 s_1 + 3/170\, e^{2ti} s_3^2 s_1 i - 7/340\, e^{2ti} s_3^2 s_1 - 1/68 s_4 s_3 s_1) + \sigma\varepsilon^3(- 169/46240\, e^{-ti} s_4 s_2 s_1 i + 45/9248\, e^{-ti} s_4 s_2 s_1 - 3/544\, e^{-ti} s_4 s_1 i - 9/2720\, e^{-ti} s_4 s_1 + 169/46240\, e^{ti} s_3 s_2 s_1 i + 45/9248\, e^{ti} s_3 s_2 s_1 + 3/544\, e^{ti} s_3 s_1 i - 9/2720\, e^{ti} s_3 s_1) + \sigma\varepsilon(- 3/34\, e^{-ti} s_4 s_1 i + 5/34\, e^{-ti} s_4 s_1 + 3/34\, e^{ti} s_3 s_1 i + 5/34\, e^{ti} s_3 s_1) + \varepsilon^2(1/80 s_2 s_1 - 1/80 s_1) + s_1$

$u_3 = \sigma^3\varepsilon^3(11/3400\, e^{-3ti} s_4^3 s_2 i - 7/3400\, e^{-3ti} s_4^3 s_2 + 1/340\, e^{-ti} s_4^2 s_3 s_2 i + 1/170\, e^{-ti} s_4^2 s_3 s_2 - 11/3400\, e^{3ti} s_3^3 s_2 i - 7/3400\, e^{3ti} s_3^3 s_2 - 1/340\, e^{ti} s_4 s_3^2 s_2 i + 1/170\, e^{ti} s_4 s_3^2 s_2) + \sigma^2\varepsilon^2(- 3/170\, e^{-2ti} s_4^2 s_2 i - 7/340\, e^{-2ti} s_4^2 s_2 + 3/170\, e^{2ti} s_3^2 s_2 i - 7/340\, e^{2ti} s_3^2 s_2 - 1/68 s_4 s_3 s_2) + \sigma\varepsilon^3(- 1869/46240\, e^{-ti} s_4 s_2^2 i - 159/9248\, e^{-ti} s_4 s_2^2 - 57/9248\, e^{-ti} s_4 s_2 s_1 i - 143/9248\, e^{-ti} s_4 s_2 s_1 + 3/544\, e^{-ti} s_4 s_2 i + 9/2720\, e^{-ti} s_4 s_2 + 1869/46240\, e^{ti} s_3 s_2^2 i - 159/9248\, e^{ti} s_3 s_2^2 + 57/9248\, e^{ti} s_3 s_2 s_1 i - 143/9248\, e^{ti} s_3 s_2 s_1 - 3/544\, e^{ti} s_3 s_2 i + 9/2720\, e^{ti} s_3 s_2) + \sigma\varepsilon(- 3/34\, e^{-ti} s_4 s_2 i + 5/34\, e^{-ti} s_4 s_2 + 3/34\, e^{ti} s_3 s_2 i + 5/34\, e^{ti} s_3 s_2) + \varepsilon^2(- 3/80 s_2^2 + 1/16 s_2 s_1 + 1/80 s_2) + s_2$

$u_4 = \sigma^3\varepsilon^3(- 11/3400\, e^{-3ti} s_4^3 s_2 i + 7/3400\, e^{-3ti} s_4^3 s_2 - 1/340\, e^{-ti} s_4^2 s_3 s_2 i - 1/170\, e^{-ti} s_4^2 s_3 s_2 + 11/3400\, e^{3ti} s_3^3 s_2 i + 7/3400\, e^{3ti} s_3^3 s_2 + 1/340\, e^{ti} s_4 s_3^2 s_2 i - 1/170\, e^{ti} s_4 s_3^2 s_2) + \sigma^2\varepsilon^2(3/170\, e^{-2ti} s_4^2 s_2 i +$

$7/340\,e^{-2ti}s_4^2s_2 - 3/170\,e^{2ti}s_3^2s_2i + 7/340\,e^{2ti}s_3^2s_2 + 1/68s_4s_3s_2) + \sigma\varepsilon^3(- 2211/46240\,e^{-ti}s_4s_2^2i - 1653/46240\,e^{-ti}s_4s_2^2 + 57/9248\,e^{-ti}s_4s_2s_1i + 143/9248\,e^{-ti}s_4s_2s_1 + 5/544\,e^{-ti}s_4s_2i + 3/544\,e^{-ti}s_4s_2 + 2211/46240\,e^{ti}s_3s_2^2i - 1653/46240\,e^{ti}s_3s_2^2 - 57/9248\,e^{ti}s_3s_2s_1i + 143/9248\,e^{ti}s_3s_2s_1 - 5/544\,e^{ti}s_3s_2i + 3/544\,e^{ti}s_3s_2) + \sigma\varepsilon(3/34\,e^{-ti}s_4s_2i - 5/34\,e^{-ti}s_4s_2 - 3/34\,e^{ti}s_3s_2i - 5/34\,e^{ti}s_3s_2) + \varepsilon^2(3/80s_2^2 - 1/16s_2s_1 - 1/80s_2) + s_2$

$u_5 = 1/2\,e^{-ti}s_4 + 1/2\,e^{ti}s_3$

$u_6 = 1/2\,e^{-ti}s_4i - 1/2\,e^{ti}s_3i$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = \sigma^2\varepsilon^4(3/340s_4s_3s_2s_1 + 1/1360s_4s_3s_1) + \varepsilon^4(1/400s_2^2s_1 - 1/320s_2s_1^2 - 3/1600s_2s_1 + 1/1600s_1) + \varepsilon^2(- 3/20s_2s_1 + 7/20s_1)$

$\dot{s}_2 = \sigma^2\varepsilon^4(9/680s_4s_3s_2^2 - 1/68s_4s_3s_2s_1 - 1/1360s_4s_3s_2) + \varepsilon^4(9/800s_2^3 - 1/32s_2^2s_1 - 9/1600s_2^2 + 1/64s_2s_1^2 + 3/320s_2s_1 + 1/1600s_2) + \varepsilon^2(3/20s_2^2 + 1/4s_2s_1 - 3/20s_2)$

$\dot{s}_3 = 0$

$\dot{s}_4 = 0$