Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$$\dot{u}_1 = \varepsilon u_2$$

$$\dot{u}_2 = -\varepsilon u_1 u_3^2 w_1^2 - \varepsilon u_3^2 w_0 w_1$$

$$\dot{u}_3 = \varepsilon u_1 u_4 w_1 + u_4 w_0$$

$$\dot{u}_4 = -\varepsilon u_1 u_3 w_1 - u_3 w_0$$

Invariant subspace basis vectors

$$\vec{e}_1 = \{\{1, 0, 0, 0\}, \exp(0)\}$$

$$\vec{e}_2 = \{\{0, 1, 0, 0\}, \exp(0)\}$$

$$\vec{e}_3 = \{\{0, 0, 1/2, 1/2i\}, \exp(itw_0)\}$$

$$\vec{e}_4 = \{\{0, 0, 1/2, -1/2i\}, \exp(-itw_0)\}$$

$$\vec{z}_1 = \{\{1, 0, 0, 0\}, \exp(0)\}$$

$$\vec{z}_2 = \{\{0, 1, 0, 0\}, \exp(0)\}$$

$$\vec{z}_3 = \{\{0, 0, 1, i\}, \exp(itw_0)\}$$

$\vec{z}_4 = \{\{0, 0, 1, -i\}, \exp(-itw_0)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = s_1 + O(\varepsilon^2)$

$u_2 = -1/8 s_4^2 s_1 i \exp(-2itw_0)\varepsilon w_0^{-1} w_1^2 - 1/8 s_4^2 i \exp(-2itw_0)\varepsilon w_1 + 1/8 s_3^2 s_1 i \exp(2itw_0)\varepsilon w_0^{-1} w_1^2 + 1/8 s_3^2 i \exp(2itw_0)\varepsilon w_1 + s_2 + O(\varepsilon^2)$

$u_3 = 1/2 s_4 \exp(-itw_0) + 1/2 s_3 \exp(itw_0) + O(\varepsilon^2)$

$u_4 = -1/2 s_4 i \exp(-itw_0) + 1/2 s_3 i \exp(itw_0) + O(\varepsilon^2)$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = s_2 \varepsilon + O(\varepsilon^3)$

$\dot{s}_2 = -1/2 s_4 s_3 s_1 \varepsilon w_1^2 - 1/2 s_4 s_3 \varepsilon w_0 w_1 + O(\varepsilon^3)$

$\dot{s}_3 = s_3 s_1 i \varepsilon w_1 + O(\varepsilon^3)$

$\dot{s}_4 = -s_4 s_1 i \varepsilon w_1 + O(\varepsilon^3)$