Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

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The specified dynamical system

$\dot{u}_1 = \varepsilon^2(- u_1^3 - 3/2u_1^2u_2 - u_1u_2^2 + u_1 - 3/2u_2^3 + 3/2u_2) - u_2$

$\dot{u}_2 = \varepsilon^2(3/2u_1^3 - u_1^2u_2 + 3/2u_1u_2^2 - 3/2u_1 - u_2^3 + u_2) + u_1$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1/2, -1/2i\}, \exp(it)\}$

$\vec{e}_2 = \{\{1/2, 1/2i\}, \exp(- it)\}$

$\vec{z}_1 = \{\{1, -i\}, \exp(it)\}$

$\vec{z}_2 = \{\{1, i\}, \exp(- it)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = 1/2 \exp(-it)s_2 + 1/2 \exp(it)s_1 + O(\varepsilon^4)$

$u_2 = i(1/2 \exp(-it)s_2 - 1/2 \exp(it)s_1) + O(\varepsilon^4)$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = i\varepsilon^2(3/2s_2s_1^2 - 3/2s_1) + \varepsilon^2(-s_2s_1^2 + s_1) + O(\varepsilon^5)$

$\dot{s}_2 = i\varepsilon^2(-3/2s_2^2s_1 + 3/2s_2) + \varepsilon^2(-s_2^2s_1 + s_2) + O(\varepsilon^5)$