

Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

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The specified dynamical system

$$\dot{u}_1 = u_2$$

$$\begin{aligned}\dot{u}_2 = & \varepsilon^2 (-3/2 \exp(-it)u_1^2u_7 + 3/4 \exp(-it)u_3^2u_7 + 3/4 \exp(-it)u_5^2u_7 + 1/4 \exp(-2it)u_1u_7^2 - 3/2 \exp(it)u_1^2u_7 + \\ & 3/4 \exp(it)u_3^2u_7 + 3/4 \exp(it)u_5^2u_7 + 1/4 \exp(2it)u_1u_7^2 + 4u_1^3 - \\ & 6u_1u_3^2 - 6u_1u_5^2 + 1/2u_1u_7^2) + \varepsilon (-1/2 \exp(-it)u_1u_7 - \\ & 1/2 \exp(it)u_1u_7 + 3u_1^2 - 3/2u_3^2 - 3/2u_5^2) + u_1 + 2u_4\end{aligned}$$

$$\dot{u}_3 = u_4$$

$$\begin{aligned}\dot{u}_4 = & \varepsilon^2 (3/2 \exp(-it)u_1u_3u_7 + 1/4 \exp(-2it)u_3u_7^2 + 3/2 \exp(it)u_1u_3u_7 + \\ & 1/4 \exp(2it)u_3u_7^2 + b_2u_3 - 6u_1^2u_3 + 3/2u_3^3 + 3/2u_3u_5^2 + 1/2u_3u_7^2) + \\ & \varepsilon (-1/2 \exp(-it)u_3u_7 - 1/2 \exp(it)u_3u_7 - 3u_1u_3) - 2u_2 - 4/5u_3\end{aligned}$$

$$\dot{u}_5 = u_6$$

$$\begin{aligned}\dot{u}_6 = & \varepsilon^2 (3/2 \exp(-it)u_1u_5u_7 + 1/4 \exp(-2it)u_5u_7^2 + 3/2 \exp(it)u_1u_5u_7 + \\ & 1/4 \exp(2it)u_5u_7^2 + b_2u_5 - 6u_1^2u_5 + 3/2u_3^2u_5 + 3/2u_5^3 + 1/2u_5u_7^2) + \\ & \varepsilon (-1/2 \exp(-it)u_5u_7 - 1/2 \exp(it)u_5u_7 - 2u_1u_5) - 4u_5\end{aligned}$$

$$\dot{u}_7 = 0$$

Invariant subspace basis vectors

$$\vec{e}_1 = \{\{1, 2i, 5/4i, -5/2, 0, 0, 0\}, \exp(2it)\}$$

$$\vec{e}_2 = \{\{1, -2i, -5/4i, -5/2, 0, 0, 0\}, \exp(-2it)\}$$

$$\vec{e}_3 = \{\{0, 0, 0, 0, 1, 2i, 0\}, \exp(2it)\}$$

$$\vec{e}_4 = \{\{0, 0, 0, 0, 1, -2i, 0\}, \exp(-2it)\}$$

$$\vec{e}_5 = \{\{0, 0, 0, 0, 0, 0, 1\}, \exp(0)\}$$

$$\vec{z}_1 = \{\{16/205, 32/205i, 4/41i, -8/41, 0, 0, 0\}, \exp(2it)\}$$

$$\vec{z}_2 = \{\{16/205, -32/205i, -4/41i, -8/41, 0, 0, 0\}, \exp(-2it)\}$$

$$\vec{z}_3 = \{\{0, 0, 0, 0, 1/5, 2/5i, 0\}, \exp(2it)\}$$

$$\vec{z}_4 = \{\{0, 0, 0, 0, 1/5, -2/5i, 0\}, \exp(-2it)\}$$

$$\vec{z}_5 = \{\{0, 0, 0, 0, 0, 0, 1\}, \exp(0)\}$$

The invariant manifold These give the location of the invariant manifold in terms of parameters s_j .

$$\begin{aligned} u_1 = & e^2 \varepsilon^2 (-1073/49680 \exp(-4it)s_2 + 144679/1320200 \exp(-2it)s_2 - \\ & 1073/49680 \exp(4it)s_1 + 144679/1320200 \exp(2it)s_1 - 7/16s_2 - \\ & 7/16s_1) + e\varepsilon^2 (1/90 \exp(-it)s_4s_3 - 809/3680 \exp(-it)s_2s_1 - \\ & 589/34020 \exp(-5it)s_4^2 + 2097463/87635520 \exp(-5it)s_2^2 - \\ & 7763/37260 \exp(-3it)s_4^2 + 361381/596160 \exp(-3it)s_2^2 + \\ & 1/90 \exp(it)s_4s_3 - 809/3680 \exp(it)s_2s_1 - 589/34020 \exp(5it)s_3^2 + \\ & 2097463/87635520 \exp(5it)s_1^2 - 7763/37260 \exp(3it)s_3^2 + \\ & 361381/596160 \exp(3it)s_1^2) + e\varepsilon (-3/8 \exp(-it)s_2 + 157/920 \exp(- \\ & 3it)s_2 - 3/8 \exp(it)s_1 + 157/920 \exp(3it)s_1) + \varepsilon^2 (24419/156384 \exp(- \\ & 6it)s_3^2s_2 - 674395/2502144 \exp(-6it)s_2^3 - 33647/185976 \exp(- \\ & 2it)s_4^2s_1 - 19241/17220 \exp(-2it)s_4s_3s_2 + 11291051/14878080 \exp(- \\ & 2it)s_2^2s_1 + 1195/6888 \exp(-2it)s_2b_2 + 24419/156384 \exp(6it)s_3^2s_1 - \\ & 674395/2502144 \exp(6it)s_1^3 - 19241/17220 \exp(2it)s_4s_3s_1 - \\ & 33647/185976 \exp(2it)s_3^2s_2 + 11291051/14878080 \exp(2it)s_2s_1^2 + \\ & 1195/6888 \exp(2it)s_1b_2) + \varepsilon (19/162 \exp(-4it)s_4^2 - \end{aligned}$$

$$683/2592 \exp(-4it)s_2^2 + 19/162 \exp(4it)s_3^2 - 683/2592 \exp(4it)s_1^2 + 3s_4s_3 - 21/16s_2s_1) + \exp(-2it)s_2 + \exp(2it)s_1 + O(\varepsilon^3)$$

$$\begin{aligned} u_2 = & e^2 i \varepsilon^2 (1073/12420 \exp(-4it)s_2 + 15644/165025 \exp(-2it)s_2 - 1073/12420 \exp(4it)s_1 - 15644/165025 \exp(2it)s_1) + ei\varepsilon^2 (-1/90 \exp(-it)s_4s_3 + 809/3680 \exp(-it)s_2s_1 + 589/6804 \exp(-5it)s_4^2 - 2097463/17527104 \exp(-5it)s_2^2 + 7763/12420 \exp(-3it)s_4^2 - 361381/198720 \exp(-3it)s_2^2 + 1/90 \exp(it)s_4s_3 - 809/3680 \exp(it)s_2s_1 - 589/6804 \exp(5it)s_3^2 + 2097463/17527104 \exp(5it)s_1^2 - 7763/12420 \exp(3it)s_3^2 + 361381/198720 \exp(3it)s_1^2) + ei\varepsilon (3/8 \exp(-it)s_2 - 471/920 \exp(-3it)s_2 - 3/8 \exp(it)s_1 + 471/920 \exp(3it)s_1) + i\varepsilon^2 (-24419/26064 \exp(-6it)s_4^2s_2 + 674395/417024 \exp(-6it)s_2^3 - 18392/23247 \exp(-2it)s_4^2s_1 + 7673/4305 \exp(-2it)s_4s_3s_2 - 6433853/3719520 \exp(-2it)s_2^2s_1 - 85/1722 \exp(-2it)s_2b_2 + 24419/26064 \exp(6it)s_3^2s_1 - 674395/417024 \exp(6it)s_1^3 - 7673/4305 \exp(2it)s_4s_3s_1 + 18392/23247 \exp(2it)s_3^2s_2 + 6433853/3719520 \exp(2it)s_2s_1^2 + 85/1722 \exp(2it)s_1b_2) + ie(-38/81 \exp(-4it)s_4^2 + 683/648 \exp(-4it)s_2^2 + 38/81 \exp(4it)s_3^2 - 683/648 \exp(4it)s_1^2) + i(-2 \exp(-2it)s_2 + 2 \exp(2it)s_1) + O(\varepsilon^3)) \end{aligned}$$

$$\begin{aligned} u_3 = & e^2 i \varepsilon^2 (503/19872 \exp(-4it)s_2 - 44831/264040 \exp(-2it)s_2 - 503/19872 \exp(4it)s_1 + 44831/264040 \exp(2it)s_1 - 25/32s_2 + 25/32s_1) + ei\varepsilon^2 (-1/9 \exp(-it)s_4s_3 + 7/184 \exp(-it)s_2s_1 + 10/1701 \exp(-5it)s_4^2 + 3830/273861 \exp(-5it)s_2^2 + 277/1863 \exp(-3it)s_4^2 + 2663/14904 \exp(-3it)s_2^2 + 1/9 \exp(it)s_4s_3 - 7/184 \exp(it)s_2s_1 - 10/1701 \exp(5it)s_3^2 - 3830/273861 \exp(5it)s_1^2 - 277/1863 \exp(3it)s_3^2 - 2663/14904 \exp(3it)s_1^2) + ei\varepsilon (5/8 \exp(-it)s_2 - 37/184 \exp(-3it)s_2 - 5/8 \exp(it)s_1 + 37/184 \exp(3it)s_1) + ie(-3695/208512 \exp(-6it)s_4^2s_2 - 619025/3336192 \exp(-6it)s_2^3 + 148915/185976 \exp(-2it)s_4^2s_1 - 1801/3444 \exp(-2it)s_4s_3s_2 + 2323711/2975616 \exp(-2it)s_2^2s_1 - 725/6888 \exp(-2it)s_2b_2 + 3695/208512 \exp(6it)s_3^2s_1 + 619025/3336192 \exp(6it)s_1^3 + 1801/3444 \exp(2it)s_4s_3s_1 - 148915/185976 \exp(2it)s_3^2s_2 - 2323711/2975616 \exp(2it)s_2s_1^2 + 725/6888 \exp(2it)s_1b_2) + ie(-5/81 \exp(-4it)s_4^2 - 35/324 \exp(-4it)s_2^2 + 5/81 \exp(4it)s_3^2 + 35/324 \exp(4it)s_1^2) + i(-5/4 \exp(-2it)s_2 + 5/4 \exp(2it)s_1) + O(\varepsilon^3)) \end{aligned}$$

$$\begin{aligned} u_4 = & e^2 \varepsilon^2 (503/4968 \exp(-4it)s_2 + 27931/528080 \exp(-2it)s_2 + 503/4968 \exp(4it)s_1 + 27931/528080 \exp(2it)s_1) + e\varepsilon^2 (- \end{aligned}$$

$$\begin{aligned}
& 1/9 \exp(-it)s_4s_3 + 7/184 \exp(-it)s_2s_1 + 50/1701 \exp(-5it)s_4^2 + \\
& 19150/273861 \exp(-5it)s_2^2 + 277/621 \exp(-3it)s_4^2 + \\
& 2663/4968 \exp(-3it)s_2^2 - 1/9 \exp(it)s_4s_3 + 7/184 \exp(it)s_2s_1 + \\
& 50/1701 \exp(5it)s_3^2 + 19150/273861 \exp(5it)s_1^2 + \\
& 277/621 \exp(3it)s_3^2 + 2663/4968 \exp(3it)s_1^2) + e\varepsilon(5/8 \exp(-it)s_2 - \\
& 111/184 \exp(-3it)s_2 + 5/8 \exp(it)s_1 - 111/184 \exp(3it)s_1) + \varepsilon^2(- \\
& 3695/34752 \exp(-6it)s_4^2s_2 - 619025/556032 \exp(-6it)s_2^3 + \\
& 59585/371952 \exp(-2it)s_4^2s_1 - 11099/6888 \exp(-2it)s_4s_3s_2 + \\
& 7718189/5951232 \exp(-2it)s_2^2s_1 + 2225/13776 \exp(-2it)s_2b_2 - \\
& 3695/34752 \exp(6it)s_3^2s_1 - 619025/556032 \exp(6it)s_1^3 - \\
& 11099/6888 \exp(2it)s_4s_3s_1 + 59585/371952 \exp(2it)s_3^2s_2 + \\
& 7718189/5951232 \exp(2it)s_2s_1^2 + 2225/13776 \exp(2it)s_1b_2) + \varepsilon(- \\
& 20/81 \exp(-4it)s_4^2 - 35/81 \exp(-4it)s_2^2 - 20/81 \exp(4it)s_3^2 - \\
& 35/81 \exp(4it)s_1^2) - 5/2 \exp(-2it)s_2 - 5/2 \exp(2it)s_1 + O(\varepsilon^3)
\end{aligned}$$

$$\begin{aligned}
u_5 = & e^2\varepsilon^2(-1/60 \exp(-4it)s_4 + 4/75 \exp(-2it)s_4 - 1/60 \exp(4it)s_3 + \\
& 4/75 \exp(2it)s_3 + 1/12s_4 + 1/12s_3) + e\varepsilon^2(23/30 \exp(-it)s_4s_1 + \\
& 601/1035 \exp(-it)s_3s_2 - 302/7245 \exp(-5it)s_4s_2 - 1/2 \exp(- \\
& 3it)s_4s_2 + 601/1035 \exp(it)s_4s_1 + 23/30 \exp(it)s_3s_2 - \\
& 302/7245 \exp(5it)s_3s_1 - 1/2 \exp(3it)s_3s_1) + e\varepsilon(-1/6 \exp(- \\
& it)s_4 + 1/10 \exp(-3it)s_4 - 1/6 \exp(it)s_3 + 1/10 \exp(3it)s_3) + \varepsilon^2(- \\
& 205/5184 \exp(-6it)s_4^3 + 21125/82944 \exp(-6it)s_4s_2^2 - \\
& 281/1620 \exp(-2it)s_4^2s_3 - 193/480 \exp(-2it)s_4s_2s_1 + 1/10 \exp(- \\
& 2it)s_4b_2 - 17669/25920 \exp(-2it)s_3s_2^2 - 205/5184 \exp(6it)s_3^3 + \\
& 21125/82944 \exp(6it)s_3s_1^2 - 281/1620 \exp(2it)s_4s_3^2 - \\
& 17669/25920 \exp(2it)s_4s_1^2 - 193/480 \exp(2it)s_3s_2s_1 + \\
& 1/10 \exp(2it)s_3b_2) + \varepsilon(1/6 \exp(-4it)s_4s_2 + 1/6 \exp(4it)s_3s_1 - \\
& 1/2s_4s_1 - 1/2s_3s_2) + \exp(-2it)s_4 + \exp(2it)s_3 + O(\varepsilon^3)
\end{aligned}$$

$$\begin{aligned}
u_6 = & e^2i\varepsilon^2(1/15 \exp(-4it)s_4 + 2/75 \exp(-2it)s_4 - 1/15 \exp(4it)s_3 - \\
& 2/75 \exp(2it)s_3) + ei\varepsilon^2(-23/30 \exp(-it)s_4s_1 - 601/1035 \exp(- \\
& it)s_3s_2 + 302/1449 \exp(-5it)s_4s_2 + 3/2 \exp(-3it)s_4s_2 + \\
& 601/1035 \exp(it)s_4s_1 + 23/30 \exp(it)s_3s_2 - 302/1449 \exp(5it)s_3s_1 - \\
& 3/2 \exp(3it)s_3s_1) + ei\varepsilon(1/6 \exp(-it)s_4 - 3/10 \exp(-3it)s_4 - \\
& 1/6 \exp(it)s_3 + 3/10 \exp(3it)s_3) + i\varepsilon^2(205/864 \exp(-6it)s_4^3 - \\
& 21125/13824 \exp(-6it)s_4s_2^2 - 281/3240 \exp(-2it)s_4^2s_3 - \\
& 193/960 \exp(-2it)s_4s_2s_1 + 1/20 \exp(-2it)s_4b_2 - 17669/51840 \exp(- \\
& 2it)s_3s_2^2 - 205/864 \exp(6it)s_3^3 + 21125/13824 \exp(6it)s_3s_1^2 + \\
& 281/3240 \exp(2it)s_4s_3^2 + 17669/51840 \exp(2it)s_4s_1^2 +
\end{aligned}$$

$$193/960 \exp(2it)s_3s_2s_1 - 1/20 \exp(2it)s_3b_2) + i\varepsilon(-2/3 \exp(-4it)s_4s_2 + 2/3 \exp(4it)s_3s_1) + i(-2 \exp(-2it)s_4 + 2 \exp(2it)s_3) + O(\varepsilon^3)$$

$$u_7 = e + O(\varepsilon^3)$$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = e^2 i \varepsilon^3 (-17/210 s_4 s_3 + 4901/86940 s_3^2 + 112013/695520 s_2 s_1 + 112013/1391040 s_1^2) - 1011/3220 e^2 i \varepsilon^2 s_1 + i \varepsilon^2 (19/42 s_4 s_3 s_1 + 2615/2268 s_3^2 s_2 + 7691/36288 s_2 s_1^2 - 25/84 s_1 b_2) + O(\varepsilon^4)$$

$$\dot{s}_2 = e^2 i \varepsilon^3 (-4901/86940 s_4^2 + 17/210 s_4 s_3 - 112013/1391040 s_2^2 - 112013/695520 s_2 s_1) + 1011/3220 e^2 i \varepsilon^2 s_2 + i \varepsilon^2 (-2615/2268 s_4^2 s_1 - 19/42 s_4 s_3 s_2 - 7691/36288 s_2^2 s_1 + 25/84 s_2 b_2) + O(\varepsilon^4)$$

$$\dot{s}_3 = e^2 i \varepsilon^3 (4729/39744 s_4 s_1 - 1/64 s_3 s_2 - 1/64 s_3 s_1) - 2/15 e^2 i \varepsilon^2 s_3 + i \varepsilon^2 (281/648 s_4 s_3^2 + 17669/10368 s_4 s_1^2 + 193/192 s_3 s_2 s_1 - 1/4 s_3 b_2) + O(\varepsilon^4)$$

$$\dot{s}_4 = e^2 i \varepsilon^3 (1/64 s_4 s_2 + 1/64 s_4 s_1 - 4729/39744 s_3 s_2) + 2/15 e^2 i \varepsilon^2 s_4 + i \varepsilon^2 (-281/648 s_4^2 s_3 - 193/192 s_4 s_2 s_1 + 1/4 s_4 b_2 - 17669/10368 s_3 s_2^2) + O(\varepsilon^4)$$

$$\dot{s}_5 = O(\varepsilon^4)$$