Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$$\dot u_1 = u_3$$

$$\dot u_2 = u_4$$

$$\dot u_3 = p(\varepsilon^2 u_1 - \varepsilon^2 u_2) - u_1$$

$$\dot u_4 = p(- \varepsilon^2 u_1 + \varepsilon^2 u_2) + 2\varepsilon u_1 - u_2$$

Invariant subspace basis vectors

$$\vec e_1 = \{\{0, 1, 0, i\}, \exp(it)\}$$

$$\vec e_2 = \{\{0, 1, 0, -i\}, \exp(- it)\}$$

$$\vec e_3 = \{\{1, 0, i, 0\}, \exp(it)\}$$

$$\vec e_4 = \{\{1, 0, -i, 0\}, \exp(- it)\}$$

$$\vec z_1 = \{\{0, 1/2, 0, 1/2i\}, \exp(it)\}$$

$$\vec z_2 = \{\{0, 1/2, 0, -1/2i\}, \exp(- it)\}$$

$$\vec z_3 = \{\{1/2, 0, 1/2i, 0\}, \exp(it)\}$$

$\vec{z}_4 = \{\{1/2, 0, -1/2i, 0\}, \exp(-it)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$$u_1 = s_4 p(-1/4\exp(-it)\varepsilon^3 + 1/4\exp(-it)\varepsilon^2) + s_4\exp(-it) + s_3 p(-1/4\exp(it)\varepsilon^3 + 1/4\exp(it)\varepsilon^2) + s_3\exp(it) - 1/4 s_2 p\exp(-it)\varepsilon^2 - 1/4 s_1 p\exp(it)\varepsilon^2 + O(\varepsilon^4)$$

$$u_2 = s_4 p(1/2\exp(-it)\varepsilon^3 - 1/4\exp(-it)\varepsilon^2) + 1/2 s_4\exp(-it)\varepsilon + s_3 p(1/2\exp(it)\varepsilon^3 - 1/4\exp(it)\varepsilon^2) + 1/2 s_3\exp(it)\varepsilon + s_2 p(-1/4\exp(-it)\varepsilon^3 + 1/4\exp(-it)\varepsilon^2) + s_2\exp(-it) + s_1 p(-1/4\exp(it)\varepsilon^3 + 1/4\exp(it)\varepsilon^2) + s_1\exp(it) + O(\varepsilon^4)$$

$$u_3 = s_4 ip(-1/4\exp(-it)\varepsilon^3 + 1/4\exp(-it)\varepsilon^2) - s_4 i\exp(-it) + s_3 ip(1/4\exp(it)\varepsilon^3 - 1/4\exp(it)\varepsilon^2) + s_3 i\exp(it) - 1/4 s_2 ip\exp(-it)\varepsilon^2 + 1/4 s_1 ip\exp(it)\varepsilon^2 + O(\varepsilon^4)$$

$$u_4 = s_4 ip(1/2\exp(-it)\varepsilon^3 - 1/4\exp(-it)\varepsilon^2) + 1/2 s_4 i\exp(-it)\varepsilon + s_3 ip(-1/2\exp(it)\varepsilon^3 + 1/4\exp(it)\varepsilon^2) - 1/2 s_3 i\exp(it)\varepsilon + s_2 ip(-1/4\exp(-it)\varepsilon^3 + 1/4\exp(-it)\varepsilon^2) - s_2 i\exp(-it) + s_1 ip(1/4\exp(it)\varepsilon^3 - 1/4\exp(it)\varepsilon^2) + s_1 i\exp(it) + O(\varepsilon^4)$$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = 1/4 s_3 ip^2\varepsilon^4 + s_3 ip(1/4\varepsilon^4 - 1/2\varepsilon^3 + 1/2\varepsilon^2) - s_3 i\varepsilon - 1/4 s_1 ip^2\varepsilon^4 + s_1 ip(1/4\varepsilon^3 - 1/2\varepsilon^2) + O(\varepsilon^5)$$

$$\dot{s}_2 = -1/4 s_4 ip^2\varepsilon^4 + s_4 ip(-1/4\varepsilon^4 + 1/2\varepsilon^3 - 1/2\varepsilon^2) + s_4 i\varepsilon + 1/4 s_2 ip^2\varepsilon^4 + s_2 ip(-1/4\varepsilon^3 + 1/2\varepsilon^2) + O(\varepsilon^5)$$

$$\dot{s}_3 = -1/4 s_3 ip^2\varepsilon^4 + s_3 ip(1/4\varepsilon^3 - 1/2\varepsilon^2) + 1/4 s_1 ip^2\varepsilon^4 + 1/2 s_1 ip\varepsilon^2 + O(\varepsilon^5)$$

$$\dot{s}_4 = 1/4 s_4 ip^2\varepsilon^4 + s_4 ip(-1/4\varepsilon^3 + 1/2\varepsilon^2) - 1/4 s_2 ip^2\varepsilon^4 - 1/2 s_2 ip\varepsilon^2 + O(\varepsilon^5)$$