

# Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

## **The specified dynamical system**

$$\dot{u}_1 = \varepsilon^2 u_1 u_3 u_5 - \varepsilon u_2 u_5$$

$$\dot{u}_2 = \varepsilon^2 u_2 u_3 u_5 + \varepsilon u_1 u_5 - \varepsilon u_4 u_5$$

$$\dot{u}_3 = -u_1 - u_3$$

$$\dot{u}_4 = \varepsilon u_1 u_2 - u_4$$

$$\dot{u}_5 = 0$$

## **Invariant subspace basis vectors**

$$\vec{e}_1 = \{ \{1, 0, -1, 0, 0\}, \exp(0) \}$$

$$\vec{e}_2 = \{ \{0, 1, 0, 0, 0\}, \exp(0) \}$$

$$\vec{e}_3 = \{ \{0, 0, 1, 0, 0\}, \exp(-t) \}$$

$$\vec{e}_4 = \{ \{0, 0, 0, 1, 0\}, \exp(-t) \}$$

$$\vec{e}_5 = \{ \{0, 0, 0, 0, 1\}, \exp(0) \}$$

$$\vec{z}_1 = \{ \{1, 0, 0, 0, 0\}, \exp(0) \}$$

$$\vec{z}_2 = \{\{0, 1, 0, 0, 0\}, \exp(0)\}$$

$$\vec{z}_3 = \{\{1, 0, 1, 0, 0\}, \exp(-t)\}$$

$$\vec{z}_4 = \{\{0, 0, 0, 1, 0\}, \exp(-t)\}$$

$$\vec{z}_5 = \{\{0, 0, 0, 0, 1\}, \exp(0)\}$$

$$ds1dt = s_5^4(-7s_2^2s_1 - 35s_2s_1^3 + s_2s_1 - 14s_1^5 + 8s_1^3) + s_5^3(-5s_2s_1^2 - 3s_1^4 + s_1^2) + s_5^2(-s_2s_1 - s_1^3) + s_5(-s_2 - s_1^2)$$

$$ds2dt = s_5^4(-24s_2^3 - 194s_2^2s_1^2 + 5s_2^2 - 140s_2s_1^4 + 74s_2s_1^2 + 46s_1^4 - 4s_1^2) + s_5^3(-18s_2^2s_1 - 20s_2s_1^3 + 5s_2s_1 + 6s_1^3) + s_5^2(-2s_2^2 - 4s_2s_1^2 + s_1^2) + s_5(-2s_2s_1 + s_1)$$

$$ds3dt = s_5^4(-3s_4s_2 - 18s_4s_1^2 + s_4 - 3s_3s_2^2 + 18s_3s_2s_1^2 + 13s_3s_1^4 + s_3s_1^2) + s_5^3(-4s_4s_1 + 4s_3s_2s_1 + 5s_3s_1^3) + s_5^2(-s_4 + s_3s_2 + 2s_3s_1^2) + s_5s_3s_1$$

$$ds4dt = s_5^4(-2s_4s_2^2 + 44s_4s_2s_1^2 - s_4s_2 + 12s_4s_1^4 - 8s_4s_1^2 + 5s_3s_2^3 - s_3s_2^2s_1^2 - 10s_3s_2s_1^4 + 6s_3s_2s_1^2 - 14s_3s_1^4) + s_5^3(8s_4s_2s_1 + 5s_4s_1^3 - s_4s_1 + s_3s_2^2s_1 - 11s_3s_2s_1^3 - 2s_3s_1^3) + s_5^2(s_4s_2 + 2s_4s_1^2 - 5s_3s_2s_1^2) + s_5(s_4s_1 - 2s_3s_2s_1)$$

$$xc = 1/24 \exp(-4t)s_5^4s_3^4s_1 + \exp(-3t)s_5^4(11/36s_4s_3^2 - 13/18s_3^3s_1^2) - 1/6 \exp(-3t)s_5^3s_3^3s_1 + \exp(-2t)s_5^4(-6s_4s_3s_1 + 3/2s_3^2s_2s_1 + 19/4s_2^3s_1^3) + \exp(-2t)s_5^3(-3/4s_4s_3 + 7/4s_2^3s_1^2) + 1/2 \exp(-2t)s_5^2s_3^3s_1 + \exp(-t)s_5^4(-2s_4s_2 + 26s_4s_1^2 - s_4 + 5s_3s_2^2 - 16s_3s_2s_1^2 - 7s_3s_1^4 - 4s_3s_1^2) + \exp(-t)s_5^3(6s_4s_1 - 2s_3s_2s_1 - 4s_3s_1^3) + \exp(-t)s_5^2(s_4 - 2s_3s_1^2) - \exp(-t)s_5s_3s_1 + s_1$$

$$pc = \exp(-4t)s_5^4(-1/24s_4s_3^3 + 1/24s_3^4s_2) + \exp(-3t)s_5^4(19/36s_4s_3^2s_1 - 17/36s_3^3s_2s_1) + \exp(-3t)s_5^3(1/6s_4s_3^2 - 1/6s_3^3s_2) + \exp(-2t)s_5^4(1/4s_4^2 - 3/2s_4s_3s_2 - 15/4s_4s_3s_1^2 + 7/8s_4s_3 - 3/4s_2^3s_2^2 + 13/4s_2^3s_2s_1^2 + 1/8s_2^3s_1^2) + \exp(-2t)s_5^3(-3/2s_4s_3s_1 + 3/2s_3^2s_2s_1) + \exp(-2t)s_5^2(-1/2s_4s_3 + 1/2s_3^2s_2) + \exp(-t)s_5^4(18s_4s_2s_1 + 12s_4s_1^3 - 8s_4s_1 + 15s_3s_2^2s_1 - 3s_3s_2s_1^3 + 10s_3s_2s_1 - 14s_3s_1^3) + \exp(-t)s_5^3(2s_4s_2 + 5s_4s_1^2 - s_4 + 3s_3s_2^2 - 7s_3s_2s_1^2 - 2s_3s_1^2) + \exp(-t)s_5^2(2s_4s_1 - 3s_3s_2s_1) + \exp(-t)s_5(s_4 - s_3s_2) + s_2$$

$$yc = 1/72 \exp(-4t)s_5^4s_3^4s_1 + \exp(-3t)s_5^4(11/72s_4s_3^2 + 1/24s_3^3s_2 - 4/9s_3^3s_1^2) - 1/12 \exp(-3t)s_5^3s_3^3s_1 + \exp(-2t)s_5^4(-17/2s_4s_3s_1 - s_3^2s_2s_1 + 25/4s_2^3s_1^3 - 1/2s_2^3s_1) + \exp(-2t)s_5^3(-3/4s_4s_3 - 1/2s_2^3s_2 + 9/4s_2^3s_1^2) + 1/2 \exp(-2t)s_5^2s_2^3s_1 + \exp(-t)s_5^4(2s_4s_2 - 26s_4s_1^2 + s_4 - 5s_3s_2^2 + 16s_3s_2s_1^2 + 7s_3s_1^4 + 4s_3s_1^2) + \exp(-t)s_5^3(-6s_4s_1 + 2s_3s_2s_1 + 4s_3s_1^3) + \exp(-t)s_5^2(-s_4 + 2s_3s_1^2) + \exp(-t)s_5s_3s_1 + \exp(-t)s_3 +$$

$$s_5^4(-138s_2^2s_1 - 295s_2s_1^3 + 39s_2s_1 - 85s_1^5 + 73s_1^3 - s_1) + s_5^3(-7s_2^2 - 35s_2s_1^2 + s_2 - 14s_1^4 + 8s_1^2) + s_5^2(-5s_2s_1 - 3s_1^3 + s_1) + s_5(-s_2 - s_1^2) - s_1$$

$$\begin{aligned} qc = & \exp(-4t)s_5^4(5/24s_4s_3^3s_1 - 2/9s_3^4s_2s_1) + \exp(-3t)s_5^4(5/8s_4^2s_3 - \\ & 35/72s_4s_3^2s_2 - 125/36s_4s_3^2s_1^2 - 1/3s_3^3s_2^2 + 73/18s_3^3s_2s_1^2 + 1/3s_3^3s_1^2) + \\ & \exp(-3t)s_5^3(-7/12s_4s_3^2s_1 + 2/3s_3^3s_2s_1) + \exp(-2t)s_5^4(-47/4s_4^2s_1 + \\ & 16s_4s_3s_2s_1 + 85/4s_4s_3s_1^3 - 13/8s_4s_3s_1 + 65/4s_3^2s_2^2s_1 - 26s_3^2s_2s_1^3 + \\ & 8s_3^2s_2s_1 - 107/8s_3^2s_1^3) + \exp(-2t)s_5^3(-s_4^2 + 1/4s_4s_3s_2 + 7s_4s_3s_1^2 + \\ & 2s_3^2s_2^2 - 37/4s_3^2s_2s_1^2 - 2s_3^2s_1^2) + \exp(-2t)s_5^2(3/2s_4s_3s_1 - 2s_3^2s_2s_1) + \\ & \exp(-t)s_4 + s_5^4(552s_2^3s_1 + 2013s_2^2s_1^3 - 227s_2^2s_1 + 1107s_2s_1^5 - 955s_2s_1^3 + \\ & 16s_2s_1 - 415s_1^5 + 84s_1^3) + s_5^3(17s_2^3 + 159s_2^2s_1^2 - 4s_2^2 + 126s_2s_1^4 - 66s_2s_1^2 - \\ & 46s_1^4 + 4s_1^2) + s_5^2(13s_2^2s_1 + 17s_2s_1^3 - 4s_2s_1 - 6s_1^3) + s_5(s_2^2 + 3s_2s_1^2 - s_1^2) + s_2s_1 \end{aligned}$$