Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

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The specified dynamical system

$\dot{u}_1 = -e_0\varepsilon^2 u_1 - e_0\varepsilon \exp(0) s_0 + \varepsilon u_1 u_2 + s_0 u_2 + u_2$

$\dot{u}_2 = e_0\varepsilon^2 u_1 + e_0\varepsilon \exp(0) s_0 - \varepsilon u_1 u_2 - s_0 u_2 - 2u_2$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1, 0\}, \exp(0)\}$

$\vec{z}_1 = \{\{1, 0\}, \exp(0)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = O(\varepsilon^2) + s_1$

$u_2 = (e_0 \varepsilon s_0)/(s_0 + 2) + O(\varepsilon^2)$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = (- 2e_0 \varepsilon^2 s_1)/(s_0^2 + 4s_0 + 4) + (- e_0 \varepsilon s_0)/(s_0 + 2) + O(\varepsilon^3)$

Normals to isochrons at the slow manifold Use these vectors: to project initial conditions onto the slow manifold; to project non-autonomous forcing onto the slow evolution; to predict the consequences of modifying the original system; in uncertainty quantification to quantify effects on the model of uncertainties in the original system. The normal vector $\vec{z}_j := (z_{j1}, \ldots, z_{jn})$

$z_{11} = e_0 \varepsilon^2 (- 2s_0 - 2)/(s_0^3 + 6s_0^2 + 12s_0 + 8) + O(\varepsilon^3) + 1$

$z_{12} = e_0 \varepsilon^2 (- 2s_0^2 - 3s_0)/(s_0^4 + 8s_0^3 + 24s_0^2 + 32s_0 + 16) + (- \varepsilon^2 s_1^2)/(s_0^3 + 6s_0^2 + 12s_0 + 8) + (\varepsilon s_1)/(s_0^2 + 4s_0 + 4) + (O(\varepsilon^3) s_0 + 2O(\varepsilon^3) + s_0 + 1)/(s_0 + 2)$