Normal form of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

Specified dynamical system

$\dot{x}_1 = \sqrt{\tau} y_1 \varepsilon a - \varepsilon^2 \tau x_1$

$\dot{y}_1 = w_1 \sigma - y_1 + \sqrt{\tau} \varepsilon x_1$

off echo;

Time dependent coordinate transform

$y_1 = -\mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star w_1 \ \sigma \varepsilon^2 \tau a - \mathrm{e}^{-1t} \star w_1 \, \sigma \varepsilon^2 \tau a + \mathrm{e}^{-1t} \star w_1 \, \sigma + Y_1 + \sqrt{\tau} \varepsilon X_1$

$x_1 = -\sqrt{\tau} \mathrm{e}^{-1t} \star w_1 \, \sigma \varepsilon a - \sqrt{\tau} Y_1 \varepsilon a + X_1$

Result normal form DEs

$\dot{Y}_1 = -Y_1 \varepsilon^2 \tau a - Y_1$

$\dot{X}_1 = w_1 \sigma \varepsilon^3 \tau (- 2\sqrt{\tau} a^2 + \sqrt{\tau} a) + \sqrt{\tau} w_1 \sigma \varepsilon a + \varepsilon^2 \tau (X_1 a - X_1)$