A slow manifold of your dynamical system

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Generally, the lowest order, most important, terms are near the end of each expression.

Specified dynamical system

$$\dot u_1 = -1/2\sqrt{s_0}w_1\sigma\varepsilon u_1 s_0^{-1} - \sqrt{s_0}w_1\sigma - e_0\varepsilon^2 u_1 - e_0\varepsilon s_0 + \varepsilon u_2 u_1 + u_2 s_0 + u_2$$

$$\dot u_2 = 1/2\sqrt{s_0}w_1\sigma\varepsilon u_1 s_0^{-1} + \sqrt{s_0}w_1\sigma + e_0\varepsilon^2 u_1 + e_0\varepsilon s_0 - \varepsilon u_2 u_1 - u_2 s_0 - 2u_2$$

off echo;

Time dependent slow manifold parametrisation

$$u_1 = \mathrm{e}^{-s_0-2t}\star\mathrm{e}^{-s_0-2t}\star w_1{}^2\,\sigma^2\varepsilon(-s_0^3-2s_0^2-s_0)/(s_0^2+4s_0+4) + \mathrm{e}^{-s_0-2t}\star\mathrm{e}^{-s_0-2t}\star w_1\;\sigma\varepsilon(\sqrt{s_0}s_1s_0+\sqrt{s_0}s_1)/(s_0+2) + \mathrm{e}^{-s_0-2t}\star\mathrm{e}^{-s_0-2t}\star\mathrm{e}^{-s_0-2t}\star w_1\;w_1\,\sigma^2\varepsilon(s_0^2+s_0)/(s_0^2+4s_0+4) + \mathrm{e}^{-s_0-2t}\star\mathrm{e}^{-s_0-2t}\star w_1\,w_1\,\sigma^2\varepsilon(1/2s_0^2+1/2s_0)/(s_0^2+4s_0+4) + \mathrm{e}^{-s_0-2t}\star w_1{}^2\sigma^2\varepsilon(1/2s_0^2+1/2s_0)/(s_0^3+6s_0^2+12s_0+8) + \mathrm{e}^{-s_0-2t}\star w_1\,\sigma\varepsilon(-1/2\sqrt{s_0}s_1s_0-\sqrt{s_0}s_1s_0^{-1}-5/2\sqrt{s_0}s_1)/(s_0^2+4s_0+4) + \mathrm{e}^{-s_0-2t}\star w_1\,\sigma(-\sqrt{s_0}s_0-\sqrt{s_0})/(s_0+2) + e_0\varepsilon(-s_0^2-s_0)/(s_0^2+4s_0+4) + O(\varepsilon,\sigma^2) + s_1$$

$$u_2 = \mathrm{e}^{-s_0-2t}\star\mathrm{e}^{-s_0-2t}\star w_1{}^2\,\sigma^2\varepsilon(s_0^2+s_0)/(s_0+2) - \sqrt{s_0}\mathrm{e}^{-s_0-2t}\star\mathrm{e}^{-s_0-2t}\star w_1\;\sigma\varepsilon s_1 + (-\mathrm{e}^{-s_0-2t}\star\mathrm{e}^{-s_0-2t}\star\mathrm{e}^{-s_0-2t}\star w_1\;w_1\,\sigma^2\varepsilon s_0)/(s_0+2) + (-1/2\mathrm{e}^{-s_0-2t}\star\mathrm{e}^{-s_0-2t}\star w_1\,w_1\,\sigma^2\varepsilon s_0)/(s_0+2) + 1/2\sqrt{s_0}\mathrm{e}^{-s_0-2t}\star w_1\,\sigma\varepsilon s_1 s_0^{-1} + \sqrt{s_0}\mathrm{e}^{-s_0-2t}\star w_1\,\sigma + (e_0\varepsilon s_0)/(s_0+2) + O(\varepsilon,\sigma^2)$$

Result slow manifold DEs

$\dot{s}_1 = \mathrm{e}^{-s_0-2t}\star w_1\, w_1\sigma^2\varepsilon(- 1/2s_0 + 1/2)/(s_0^2 + 4s_0 + 4) + w_1\sigma\varepsilon(- \sqrt{s_0}s_1 s_0^{-1} + 1/2\sqrt{s_0}s_1)/(s_0^2 + 4s_0 + 4) + (- \sqrt{s_0}w_1\sigma)/(s_0 + 2) + (- e_0\varepsilon s_0)/(s_0 + 2) + O(\varepsilon^2, \sigma^3)$