Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = u_3$

$\dot{u}_2 = u_4$

$\dot{u}_3 = \varepsilon^2(-1/2u_1^3 - 3/10u_3 + 3/10u_4) - 2u_1 + u_2$

$\dot{u}_4 = \varepsilon^2(3/10u_3 - 3/5u_4) + u_1 - 2u_2$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1, 1, i, i\}, e^{ti}\}$

$\vec{e}_2 = \{\{1, 1, -i, -i\}, e^{-ti}\}$

$\vec{e}_3 = \left\{\left\{1, -1, \sqrt{3}i, -\sqrt{3}i\right\}, e^{\sqrt{3}ti}\right\}$

$\vec{e}_4 = \left\{\left\{1, -1, -\sqrt{3}i, \sqrt{3}i\right\}, e^{-\sqrt{3}ti}\right\}$

$\vec{z}_1 = \{\{1/4, 1/4, 1/4i, 1/4i\}, e^{ti}\}$

$\vec{z}_2 = \{\{1/4, 1/4, -1/4i, -1/4i\}, e^{-ti}\}$

$$\vec{z}_3 = \left\{\left\{1/4, -1/4, 1/12\sqrt{3}i, -1/12\sqrt{3}i\right\}, e^{\sqrt{3}ti}\right\}$$

$$\vec{z}_4 = \left\{\left\{1/4, -1/4, -1/12\sqrt{3}i, 1/12\sqrt{3}i\right\}, e^{-\sqrt{3}ti}\right\}$$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$$u_1 = e^{-\sqrt{3}ti}s_4 + e^{-ti}s_2 + e^{\sqrt{3}ti}s_3 + e^{ti}s_1$$

$$u_2 = -e^{-\sqrt{3}ti}s_4 + e^{-ti}s_2 - e^{\sqrt{3}ti}s_3 + e^{ti}s_1$$

$$u_3 = -\sqrt{3}\,e^{-\sqrt{3}ti}s_4 i - e^{-ti}s_2 i + \sqrt{3}\,e^{\sqrt{3}ti}s_3 i + e^{ti}s_1 i$$

$$u_4 = \sqrt{3}\,e^{-\sqrt{3}ti}s_4 i - e^{-ti}s_2 i - \sqrt{3}\,e^{\sqrt{3}ti}s_3 i + e^{ti}s_1 i$$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = 3/4 s_4 s_3 s_1 \varepsilon^2 i + 3/8 s_2 s_1^2 \varepsilon^2 i - 3/40 s_1 \varepsilon^2$$

$$\dot{s}_2 = -3/4 s_4 s_3 s_2 \varepsilon^2 i - 3/8 s_2^2 s_1 \varepsilon^2 i - 3/40 s_2 \varepsilon^2$$

$$\dot{s}_3 = 1/8\sqrt{3} s_4 s_3^2 \varepsilon^2 i + 1/4\sqrt{3} s_3 s_2 s_1 \varepsilon^2 i - 3/8 s_3 \varepsilon^2$$

$$\dot{s}_4 = -1/8\sqrt{3} s_4^2 s_3 \varepsilon^2 i - 1/4\sqrt{3} s_4 s_2 s_1 \varepsilon^2 i - 3/8 s_4 \varepsilon^2$$