Normal form of your dynamical system

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Specified dynamical system

$$\dot{x}_1 = s\sigma(-1/2w_2 - 1/2w_1) + \sigma w_1 + \varepsilon(-x_1y_2 - 2x_1y_1)$$

$$\dot{x}_2 = 0$$

$$\dot{y}_1 = s\varepsilon(-x_2y_2 - x_2y_1 - x_2) + \sigma(-w_3 + 1/2w_2 + 1/2w_1) + \varepsilon(-x_2x_1 - y_2^2 - 3y_2y_1 - 2y_1^2) - 2y_1$$

$$\dot{y}_2 = s\varepsilon(x_2y_2 + x_2y_1 + x_2) + \sigma w_3 + \varepsilon x_2x_1 - y_2$$

Time dependent coordinate transform

$$y_1 = -1/2X_2X_1\varepsilon + X_2s\sigma\varepsilon(e^{-2t}\star e^{-2t}\star w_3 - 1/2e^{-2t}\star e^{-2t}\star w_2 - 1/2e^{-2t}\star e^{-2t}\star w_1 - e^{-1t}\star w_3 + e^{-2t}\star w_3 - 1/4e^{-2t}\star w_2 - 1/4e^{-2t}\star w_1) + X_2s\varepsilon(-Y_2 - 1/2) + 1/2X_2\sigma\varepsilon e^{-2t}\star w_1 + \sigma\varepsilon(-2e^{-1t}\star e^{-1t}\star w_3\, Y_2 + 3e^{-1t}\star w_3\, Y_2 + 3e^{-1t}\star w_3\, Y_1 - 3e^{-2t}\star w_3\, Y_2 - 2e^{-2t}\star w_3\, Y_1 - 3/2e^{-1t}\star w_2\, Y_2 + 3/2e^{-2t}\star w_2\, Y_2 + e^{-2t}\star w_2\, Y_1 - 3/2e^{-1t}\star w_1\, Y_2 + 3/2e^{-2t}\star w_1\, Y_2 + e^{-2t}\star w_1\, Y_1) + \sigma(-e^{-2t}\star w_3 + 1/2e^{-2t}\star w_2 + 1/2e^{-2t}\star w_1) + \varepsilon(3Y_2Y_1 + Y_1^2) + Y_1$$

$$y_2 = X_2X_1\varepsilon + X_2s\sigma\varepsilon(e^{-1t}\star e^{-1t}\star w_3 - e^{-1t}\star w_3 + e^{-2t}\star w_3 + e^{-1t}\star w_2 - 1/2e^{-2t}\star w_2 + e^{-1t}\star w_1 - 1/2e^{-2t}\star w_1) + X_2s\varepsilon(-Y_1 + 1) - X_2\sigma\varepsilon e^{-1t}\star w_1 + \sigma e^{-1t}\star w_3 + Y_2$$

$x_1 = X_1\sigma\varepsilon(e^{-1t}\star w_3 - e^{-2t}\star w_3 + 1/2e^{-2t}\star w_2 + 1/2e^{-2t}\star w_1) + X_1\varepsilon(Y_2 + Y_1) + X_1 + s\sigma\varepsilon(-1/2e^{2t}\star w_2\, Y_1 - 1/2e^{t}\star w_2\, Y_2 - 1/2e^{2t}\star w_1\, Y_1 - 1/2e^{1t}\star w_1\, Y_2) + \sigma\varepsilon(e^{2t}\star w_1\, Y_1 + e^{t}\star w_1\, Y_2)$

$x_2 = X_2$

Result normal form DEs

$\dot{Y}_1 = X_2^2 s^2\varepsilon^2 Y_1 - 2X_2X_1\varepsilon^2 Y_1 + X_2 s\sigma\varepsilon^2(-1/4w_2Y_1 - 1/4w_1Y_1) + X_2 s\varepsilon^2(3Y_2^2 - Y_1) - X_2 s\varepsilon Y_1 + 3/2X_2\sigma\varepsilon^2 w_1Y_1 + \sigma^2\varepsilon^2(e^{-1t}\star w_3\, w_2Y_1 + e^{-1t}\star w_3\, w_1Y_1 - e^{-2t}\star w_3\, w_2Y_1 - e^{-2t}\star w_3\, w_1Y_1 + 1/2e^{-2t}\star w_2\, w_2Y_1 + 1/2e^{-2t}\star w_2\, w_1Y_1 + 1/2e^{-2t}\star w_1\, w_2Y_1 + 1/2e^{-2t}\star w_1\, w_1Y_1) + \sigma\varepsilon^2(7/2w_3Y_2^2 + 11/4w_2Y_2^2 + 11/4w_1Y_2^2) + \sigma\varepsilon(-w_3Y_1 - w_2Y_1 - w_1Y_1) - \varepsilon Y_2^2 - 2Y_1$

$\dot{Y}_2 = -X_2^2 s^2\varepsilon^2 Y_2 + X_2X_1\varepsilon^2 Y_2 + X_2 s\sigma\varepsilon^2(-1/2w_3Y_2 - 5/4w_2Y_2 - 5/4w_1Y_2) + X_2 s\varepsilon Y_2 + X_2\sigma\varepsilon^2 w_1Y_2 - Y_2$

$\dot{X}_1 = X_2X_1 s\sigma\varepsilon^2(-1/4w_2 - 1/4w_1) + 1/2X_2X_1\sigma\varepsilon^2 w_1 + X_2 s^2\sigma^2\varepsilon^2(1/2e^{-1t}\star e^{-1t}\star w_3\; w_2 + 1/2e^{-1t}\star e^{-1t}\star w_3\; w_1 + 1/2e^{-2t}\star e^{-2t}\star w_3\; w_2 + 1/2e^{-2t}\star e^{-2t}\star w_3\; w_1 - 1/4e^{-2t}\star e^{-2t}\star w_2\; w_2 - 1/4e^{-2t}\star e^{-2t}\star w_2\; w_1 - 1/4e^{-2t}\star e^{-2t}\star w_1\; w_2 - 1/4e^{-2t}\star e^{-2t}\star w_1\; w_1 - e^{-1t}\star w_3\, w_2 - e^{-1t}\star w_3\, w_1 + e^{-2t}\star w_3\, w_2 + e^{-2t}\star w_3\, w_1 + 1/2e^{-1t}\star w_2\, w_2 + 1/2e^{-1t}\star w_2\, w_1 - 3/8e^{-2t}\star w_2\, w_2 - 3/8e^{-2t}\star w_2\, w_1 + 1/2e^{-1t}\star w_1\, w_2 + 1/2e^{-1t}\star w_1\, w_1 - 3/8e^{-2t}\star w_1\, w_2 - 3/8e^{-2t}\star w_1\, w_1) + X_2 s\sigma^2\varepsilon^2(-e^{-1t}\star e^{-1t}\star w_3\; w_1 - e^{-2t}\star e^{-2t}\star w_3\; w_1 + 1/2e^{-2t}\star e^{-2t}\star w_2\; w_1 + 1/2e^{-2t}\star e^{-2t}\star w_1\; w_1 + 2e^{-1t}\star w_3\, w_1 - 2e^{-2t}\star w_3\, w_1 - e^{-1t}\star w_2\, w_1 + 3/4e^{-2t}\star w_2\, w_1 - 1/2e^{-1t}\star w_1\, w_2 - 3/2e^{-1t}\star w_1\, w_1 + 1/4e^{-2t}\star w_1\, w_2 + e^{-2t}\star w_1\, w_1) + X_2\sigma^2\varepsilon^2(e^{-1t}\star w_1\, w_1 - 1/2e^{-2t}\star w_1\, w_1) + X_1\sigma^2\varepsilon^2(1/2e^{-1t}\star w_3\, w_2 + 1/2e^{-1t}\star w_3\, w_1 - 1/2e^{-2t}\star w_3\, w_2 - 1/2e^{-2t}\star w_3\, w_1 + 1/4e^{-2t}\star w_2\, w_2 + 1/4e^{-2t}\star w_2\, w_1 + 1/4e^{-2t}\star w_1\, w_2 + 1/4e^{-2t}\star w_1\, w_1) + X_1\sigma\varepsilon(-1/2w_2 - 1/2w_1) + s\sigma^2\varepsilon(1/2e^{-1t}\star w_3\, w_2 + 1/2e^{-1t}\star w_3\, w_1 - 1/2e^{-2t}\star w_3\, w_2 - 1/2e^{-2t}\star w_3\, w_1 + 1/4e^{-2t}\star w_2\, w_2 + 1/4e^{-2t}\star w_2\, w_1 + 1/4e^{-2t}\star w_1\, w_2 + 1/4e^{-2t}\star w_1\, w_1) + s\sigma(-1/2w_2 - 1/2w_1) + \sigma^2\varepsilon(-e^{-1t}\star w_3\, w_1 + e^{-2t}\star w_3\, w_1 - 1/2e^{-2t}\star w_2\, w_1 - 1/2e^{-2t}\star w_1\, w_1) + \sigma w_1$

$\dot{X}_2 = 0$