Centre manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = -\varepsilon u_1^2 - u_1 + u_2$

$\dot{u}_2 = \varepsilon u_2^2 + u_1 - u_2$

Centre subspace basis vectors

$\vec{e}_1 = \{\{1, 1\}, e^{0i}\}$

$\vec{z}_1 = \{\{1/2, 1/2\}, e^{0i}\}$

The centre manifold   These give the location of the centre manifold in terms of parameters s_j.

$u_1 = 3/8\varepsilon^3 s_1^4 - 1/2\varepsilon s_1^2 + s_1$

$u_2 = -3/8\varepsilon^3 s_1^4 + 1/2\varepsilon s_1^2 + s_1$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = -3/4\varepsilon^4 s_1^5 + \varepsilon^2 s_1^3$$

Normals to isochrons at the slow manifold Use these vectors: to project initial conditions onto the slow manifold; to project non-autonomous forcing onto the slow evolution; to predict the consequences of modifying the original system; in uncertainty quantification to quantify effects on the model of uncertainties in the original system. The normal vector $\vec{z}_j := (z_{j1}, \ldots, z_{jn})$

$$z_{11} = 3/2\varepsilon^4 s_1^4 + 3/4\varepsilon^3 s_1^3 - 1/2\varepsilon^2 s_1^2 - 1/2\varepsilon s_1 + 1/2$$

$$z_{12} = 3/2\varepsilon^4 s_1^4 - 3/4\varepsilon^3 s_1^3 - 1/2\varepsilon^2 s_1^2 + 1/2\varepsilon s_1 + 1/2$$