A slow manifold of your dynamical system

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Generally, the lowest order, most important, terms are near the end of each expression.

Specified dynamical system

$\dot{u}_1 = \sigma(u_2 w_s - u_1 w_s) + u_2 - u_1$

$\dot{u}_2 = \rho\varepsilon^2 u_1 + \sigma u_1 w_\rho - \varepsilon u_3 u_1 - u_2 + u_1$

$\dot{u}_3 = -\sigma u_3 w_\beta + \varepsilon u_2 u_1 - u_3$

off echo;

Time dependent slow manifold parametrisation

$u_1 = -1/2\sigma \mathrm{e}^{-2t} \star w_\rho \, s_1 + O(\varepsilon^2, \sigma^2) + s_1$

$u_2 = 1/2\sigma \mathrm{e}^{-2t} \star w_\rho \, s_1 + O(\varepsilon^2, \sigma^2) + s_1$

$u_3 = \sigma\varepsilon(- \mathrm{e}^{-1t} \star w_\beta \, s_1^2 - \mathrm{e}^{-1t} \star w_\rho \, s_1^2) + \varepsilon s_1^2 + O(\varepsilon^2, \sigma^2)$

Result slow manifold DEs

$\dot{s}_1 = \rho\sigma^2\varepsilon^2(1/4\mathrm{e}^{-2t} \star \mathrm{e}^{-2t} \star w_\rho \; s_1 w_\rho - 1/2\mathrm{e}^{-2t} \star \mathrm{e}^{-2t} \star w_\rho \; s_1 w_s + 1/4\mathrm{e}^{-2t} \star w_\rho \, s_1 w_\rho - 1/8\mathrm{e}^{-2t} \star w_\rho \, s_1 w_s + 1/8\mathrm{e}^{-2t} \star w_s \, s_1 w_\rho - 1/4\mathrm{e}^{-2t} \star w_s \, s_1 w_s) + \rho\sigma\varepsilon^2(- 1/4 s_1 w_\rho + 1/4 s_1 w_s) + 1/2\rho\varepsilon^2 s_1 + \sigma^2\varepsilon^2(- 1/4\mathrm{e}^{-1t} \star \mathrm{e}^{-2t} \star w_\beta \; s_1^3 w_\rho + 1/2\mathrm{e}^{-1t} \star \mathrm{e}^{-2t} \star w_\beta \; s_1^3 w_s -$

$1/4\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\ s_1^3 w_\rho + 1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\ s_1^3 w_s - 1/4\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\ s_1^3 w_\rho +$
$1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\ s_1^3 w_s - 1/2\mathrm{e}^{-1t}\star w_\beta\, s_1^3 w_\beta - 1/12\mathrm{e}^{-1t}\star w_\beta\, s_1^3 w_\rho -$
$1/2\mathrm{e}^{-1t}\star w_\rho\, s_1^3 w_\beta - 1/12\mathrm{e}^{-1t}\star w_\rho\, s_1^3 w_\rho - 1/12\mathrm{e}^{-2t}\star w_\rho\, s_1^3 w_\beta -$
$25/48\mathrm{e}^{-2t}\star w_\rho\, s_1^3 w_\rho + 7/8\mathrm{e}^{-2t}\star w_\rho\, s_1^3 w_s - 1/8\mathrm{e}^{-2t}\star w_s\, s_1^3 w_\rho +$
$1/4\mathrm{e}^{-2t}\star w_s\, s_1^3 w_s) + \sigma^2(-1/4\mathrm{e}^{-2t}\star w_\rho\, s_1 w_\rho + 1/2\mathrm{e}^{-2t}\star w_\rho\, s_1 w_s) +$
$\sigma\varepsilon^2(1/2 s_1^3 w_\beta + 3/4 s_1^3 w_\rho - 1/4 s_1^3 w_s) + 1/2\sigma s_1 w_\rho - 1/2\varepsilon^2 s_1^3 + O(\varepsilon^3, \sigma^3)$

CAS code

```
% Example 5.1 of Potzsche & Rasmussen (2006)
in_tex "slowNonauto.tex"$
factor small,sigma,rho;
sigmae:=1+w(s);
rhoe:=1+small*rho+w(rho);
betae:=1+w(beta);
slownonauto(
    mat(( sigmae*(u2-u1)
    , rhoe*u1-u2-u1*u3
    , -betae*u3+u1*u2 )),
    mat((1,1,0)),
    mat((-2,-1)),
    mat((1,-1,0),(0,0,1)),
    3 )$
end;
```