A slow manifold of your dynamical system

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6:48am, June 2, 2023

Generally, the lowest order, most important, terms are near the end of each expression.

Specified dynamical system

$\dot{u}_1 = \sigma(u_2 w_s - u_1 w_s) + u_2 - u_1$

$\dot{u}_2 = \sigma u_1 w_\rho - \varepsilon u_3 u_1 - u_2 + u_1$

$\dot{u}_3 = -\sigma u_3 w_\beta + \varepsilon u_2 u_1 - u_3$

off echo;

Time dependent slow manifold parametrisation

$u_1 = \sigma^2\varepsilon^2(- 1/8\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star{w_\rho}^2\, s_1^3 - 1/8\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star{w_\rho}^2\ s_1^3 +$
$1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\beta\, w_\beta\ s_1^3 + 1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\beta\, w_\rho\ s_1^3 +$
$1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\beta\ s_1^3 + 1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\rho\ s_1^3 +$
$1/4\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\rho\ s_1^3 + 1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\rho\ s_1^3 -$
$1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\, w_s\ s_1^3 + 1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\, w_s\ s_1^3 +$
$\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\beta\ w_\rho\, s_1^3 + 1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\beta\ w_s\, s_1^3 +$
$\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\ w_\rho\, s_1^3 + 1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\ w_s\, s_1^3 +$
$\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\ w_\rho\, s_1^3 + 1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\ w_s\, s_1^3 +$
$1/4\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\beta\,\mathrm{e}^{-2t}\star w_\rho\ s_1^3 + 1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_\beta\, w_\beta\, s_1^3 +$
$1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_\beta\, w_\rho\, s_1^3 + 1/4\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\beta\, w_\rho\, s_1^3 +$
$1/4\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\,\mathrm{e}^{-2t}\star w_\rho\ s_1^3 + 1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_\rho\, w_\beta\, s_1^3 +$
$1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_\rho\, w_\rho\, s_1^3 + 1/4\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\rho\, s_1^3 + 1/4\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\beta\, s_1^3 +$

$$1/4\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star w_\rho\, w_\rho\, s_1^3+7/4\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\rho\, s_1^3-1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star w_\rho\, w_s\, s_1^3+$$
$$1/8\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\, w_s\, s_1^3+1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_s\, w_\rho\, s_1^3+1/4\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_s\, w_s\, s_1^3+$$
$$1/12\mathrm{e}^{-1t}\star w_\beta\,\mathrm{e}^{-2t}\star w_\rho\, s_1^3+1/12\mathrm{e}^{-1t}\star w_\rho\,\mathrm{e}^{-2t}\star w_\rho\, s_1^3-1/32\mathrm{e}^{-2t}\star {w_\rho}^2 s_1^3)+$$
$$\sigma^2(1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\rho\, s_1+1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\, w_s\, s_1)+\sigma\varepsilon^2(-$$
$$1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\beta\ s_1^3-1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\ s_1^3-1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\ s_1^3-$$
$$1/2\mathrm{e}^{-1t}\star w_\beta\, s_1^3-1/2\mathrm{e}^{-1t}\star w_\rho\, s_1^3-5/8\mathrm{e}^{-2t}\star w_\rho\, s_1^3-1/4\mathrm{e}^{-2t}\star w_s\, s_1^3)-$$
$$1/2\sigma\mathrm{e}^{-2t}\star w_\rho\, s_1+1/4\varepsilon^2 s_1^3+O(\varepsilon^2,\sigma^2)+s_1$$

$$u_2=\sigma^2\varepsilon^2(-1/8\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star {w_\rho}^2\, s_1^3+1/8\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star {w_\rho}^2\ s_1^3-$$
$$1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\beta\, w_\beta\ s_1^3-1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\beta\, w_\rho\ s_1^3-$$
$$1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\beta\ s_1^3-1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\rho\ s_1^3-$$
$$1/4\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\rho\ s_1^3-1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\rho\ s_1^3+$$
$$1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\, w_s\ s_1^3-1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\, w_s\ s_1^3-$$
$$\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\beta\ w_\rho\, s_1^3-1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\beta\ w_s\, s_1^3-$$
$$\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\ w_\rho\, s_1^3-1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\ w_s\, s_1^3-$$
$$\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\ w_\rho\, s_1^3-1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\ w_s\, s_1^3-$$
$$1/4\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\beta\,\mathrm{e}^{-2t}\star w_\rho\ s_1^3+1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_\beta\, w_\beta\, s_1^3+$$
$$1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_\beta\, w_\rho\, s_1^3-1/4\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\beta\, w_\rho\, s_1^3-$$
$$1/4\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\,\mathrm{e}^{-2t}\star w_\rho\ s_1^3+1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_\rho\, w_\beta\, s_1^3+$$
$$1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_\rho\, w_\rho\, s_1^3-1/4\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\rho\, s_1^3-1/4\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\beta\, s_1^3+$$
$$1/4\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star w_\rho\, w_\rho\, s_1^3-3/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\rho\, s_1^3-1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star w_\rho\, w_s\, s_1^3+$$
$$1/8\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\, w_s\, s_1^3-1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_s\, w_\rho\, s_1^3-1/4\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_s\, w_s\, s_1^3+$$
$$1/12\mathrm{e}^{-1t}\star w_\beta\,\mathrm{e}^{-2t}\star w_\rho\, s_1^3+1/12\mathrm{e}^{-1t}\star w_\rho\,\mathrm{e}^{-2t}\star w_\rho\, s_1^3-1/32\mathrm{e}^{-2t}\star {w_\rho}^2 s_1^3)+$$
$$\sigma^2(-1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\, w_\rho\, s_1-1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\, w_s\, s_1)+$$
$$\sigma\varepsilon^2(1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\beta\ s_1^3+1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\ s_1^3+1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\ s_1^3-$$
$$1/2\mathrm{e}^{-1t}\star w_\beta\, s_1^3-1/2\mathrm{e}^{-1t}\star w_\rho\, s_1^3+3/8\mathrm{e}^{-2t}\star w_\rho\, s_1^3+1/4\mathrm{e}^{-2t}\star w_s\, s_1^3)+$$
$$1/2\sigma\mathrm{e}^{-2t}\star w_\rho\, s_1-1/4\varepsilon^2 s_1^3+O(\varepsilon^2,\sigma^2)+s_1$$

$$u_3=$$
$$\sigma^2\varepsilon(-1/4\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star {w_\rho}^2\, s_1^2+\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_\beta\, w_\beta\, s_1^2+\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_\beta\, w_\rho\, s_1^2+$$
$$\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_\rho\, w_\beta\, s_1^2+\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_\rho\, w_\rho\, s_1^2+1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star w_\rho\, w_\rho\, s_1^2-$$
$$\mathrm{e}^{-2t}\star\mathrm{e}^{-1t}\star w_\rho\, w_s\, s_1^2)+\sigma\varepsilon(-\mathrm{e}^{-1t}\star w_\beta\, s_1^2-\mathrm{e}^{-1t}\star w_\rho\, s_1^2)+\varepsilon s_1^2+O(\varepsilon^2,\sigma^2)$$

Result slow manifold DEs

$$\dot{s}_1=\sigma^2\varepsilon^2(-1/4\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\beta\ s_1^3 w_\rho+1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\beta\ s_1^3 w_s-$$
$$1/4\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\ s_1^3 w_\rho+1/2\mathrm{e}^{-1t}\star\mathrm{e}^{-2t}\star w_\rho\ s_1^3 w_s-1/4\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\ s_1^3 w_\rho+$$
$$1/2\mathrm{e}^{-2t}\star\mathrm{e}^{-2t}\star w_\rho\ s_1^3 w_s-1/2\mathrm{e}^{-1t}\star w_\beta\, s_1^3 w_\beta-1/12\mathrm{e}^{-1t}\star w_\beta\, s_1^3 w_\rho-$$

$$1/2\mathrm{e}^{-1t}\star w_\rho\, s_1^3 w_\beta - 1/12\mathrm{e}^{-1t}\star w_\rho\, s_1^3 w_\rho - 1/12\mathrm{e}^{-2t}\star w_\rho\, s_1^3 w_\beta - 25/48\mathrm{e}^{-2t}\star w_\rho\, s_1^3 w_\rho + 7/8\mathrm{e}^{-2t}\star w_\rho\, s_1^3 w_s - 1/8\mathrm{e}^{-2t}\star w_s\, s_1^3 w_\rho + 1/4\mathrm{e}^{-2t}\star w_s\, s_1^3 w_s) + \sigma^2(-1/4\mathrm{e}^{-2t}\star w_\rho\, s_1 w_\rho + 1/2\mathrm{e}^{-2t}\star w_\rho\, s_1 w_s) + \sigma\varepsilon^2(1/2 s_1^3 w_\beta + 3/4 s_1^3 w_\rho - 1/4 s_1^3 w_s) + 1/2\sigma s_1 w_\rho - 1/2\varepsilon^2 s_1^3 + O(\varepsilon^3, \sigma^3)$$