Invariant manifold of your dynamical system

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12:18pm, December 7, 2025

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = \frac{\partial^2 u_1}{\partial x^2} D\varepsilon^2 - \frac{\partial u_1}{\partial x} A\varepsilon - \varepsilon D_{t,\pi/2}(u_1)u_1 - D_{t,\pi/2}(u_1)$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1\}, \exp(it)\}$

$\vec{e}_2 = \{\{1\}, \exp(-it)\}$

$\vec{z}_1 = \{\{1\}, \exp(it)\}$

$\vec{z}_2 = \{\{1\}, \exp(-it)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = \exp(-it)s_2 + \exp(-2it)s_2^2\varepsilon(1/5i + 2/5) + \exp(it)s_1 + \exp(2it)s_1^2\varepsilon(-1/5i + 2/5) + O(\varepsilon^2)$

$$u_1 = \exp(-it)s_2 + \exp(-2it)s_2^2\varepsilon(0.2i + 0.4) + \exp(it)s_1 + \exp(2it)s_1^2\varepsilon(-0.2i + 0.4) + O(\varepsilon^2)$$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = \frac{\partial^2 s_1}{\partial x^2}A^2\varepsilon^2(-6i\pi^4 + 8i\pi^2 - \pi^5 + 12\pi^3)/(\pi^6 + 12\pi^4 + 48\pi^2 + 64) + \frac{\partial^2 s_1}{\partial x^2}D\varepsilon^2(-2i\pi + 4)/(\pi^2 + 4) + \frac{\partial s_1}{\partial x}A\varepsilon(2i\pi - 4)/(\pi^2 + 4) + s_2 s_1^2\varepsilon^2(-2/5i\pi^5 - 12/5i\pi^4 - 16/5i\pi^3 - 96/5i\pi^2 - 32/5i\pi - 192/5i - 6/5\pi^5 + 4/5\pi^4 - 48/5\pi^3 + 32/5\pi^2 - 96/5\pi + 64/5)/(\pi^6 + 12\pi^4 + 48\pi^2 + 64) + O(\varepsilon^3)$$

$$\dot{s}_2 = \frac{\partial^2 s_2}{\partial x^2}A^2\varepsilon^2(6i\pi^4 - 8i\pi^2 - \pi^5 + 12\pi^3)/(\pi^6 + 12\pi^4 + 48\pi^2 + 64) + \frac{\partial^2 s_2}{\partial x^2}D\varepsilon^2(2i\pi+4)/(\pi^2+4) + \frac{\partial s_2}{\partial x}A\varepsilon(-2i\pi-4)/(\pi^2+4) + s_2^2 s_1\varepsilon^2(2/5i\pi^5 + 12/5i\pi^4 + 16/5i\pi^3 + 96/5i\pi^2 + 32/5i\pi + 192/5i - 6/5\pi^5 + 4/5\pi^4 - 48/5\pi^3 + 32/5\pi^2 - 96/5\pi + 64/5)/(\pi^6 + 12\pi^4 + 48\pi^2 + 64) + O(\varepsilon^3)$$

$$\dot{s}_1 = \frac{\partial^2 s_1}{\partial x^2}A^2\varepsilon^2(-0.1895i + 0.02476) + \frac{\partial^2 s_1}{\partial x^2}D\varepsilon^2(-0.453i + 0.2884) + \frac{\partial s_1}{\partial x}A\varepsilon(0.453i - 0.2884) + s_2 s_1^2\varepsilon^2(-0.2636i - 0.2141) + O(\varepsilon^3)$$

$$\dot{s}_2 = \frac{\partial^2 s_2}{\partial x^2}A^2\varepsilon^2(0.1895i + 0.02476) + \frac{\partial^2 s_2}{\partial x^2}D\varepsilon^2(0.453i + 0.2884) + \frac{\partial s_2}{\partial x}A\varepsilon(-0.453i - 0.2884) + s_2^2 s_1\varepsilon^2(0.2636i - 0.2141) + O(\varepsilon^3)$$