Centre manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot u_1 = u_2$

$\dot u_2 = \varepsilon(\alpha u_1 u_3 u_7 + \gamma u_2 u_7) - u_1$

$\dot u_3 = u_4$

$\dot u_4 = \varepsilon(- \beta u_4 u_7 + 2 d^3 u_1^2 u_3 u_7 + d^2 u_1^2 u_7 + u_5 u_7) - 4 u_3$

$\dot u_5 = u_6$

$\dot u_6 = -4 u_5$

$\dot u_7 = 0$

Centre subspace basis vectors

$\vec e_1 = \{\{1, i, 0, 0, 0, 0, 0\}, e^{ti}\}$

$\vec e_2 = \{\{1, -i, 0, 0, 0, 0, 0\}, e^{-ti}\}$

$\vec e_3 = \{\{0, 0, 1, 2i, 0, 0, 0\}, e^{2ti}\}$

$\vec e_4 = \{\{0, 0, 1, -2i, 0, 0, 0\}, e^{-2ti}\}$

$\vec e_5 = \{\{0, 0, 0, 0, 1, 2i, 0\}, e^{2ti}\}$

$\vec{e}_6 = \{\{0, 0, 0, 0, 1, -2i, 0\}, e^{-2ti}\}$

$\vec{e}_7 = \{\{0, 0, 0, 0, 0, 0, 1\}, e^{0i}\}$

$\vec{z}_1 = \{\{1/2, 1/2i, 0, 0, 0, 0, 0\}, e^{ti}\}$

$\vec{z}_2 = \{\{1/2, -1/2i, 0, 0, 0, 0, 0\}, e^{-ti}\}$

$\vec{z}_3 = \{\{0, 0, 1/5, 2/5i, 0, 0, 0\}, e^{2ti}\}$

$\vec{z}_4 = \{\{0, 0, 1/5, -2/5i, 0, 0, 0\}, e^{-2ti}\}$

$\vec{z}_5 = \{\{0, 0, 0, 0, 1/5, 2/5i, 0\}, e^{2ti}\}$

$\vec{z}_6 = \{\{0, 0, 0, 0, 1/5, -2/5i, 0\}, e^{-2ti}\}$

$\vec{z}_7 = \{\{0, 0, 0, 0, 0, 0, 1\}, e^{0i}\}$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$u_1 = s_7\varepsilon(-1/8\, e^{-3ti}s_4s_2\alpha + 1/4\, e^{-ti}s_4s_1\alpha - 1/4\, e^{-ti}s_2\gamma i - 1/8\, e^{3ti}s_3s_1\alpha + 1/4\, e^{ti}s_3s_2\alpha + 1/4\, e^{ti}s_1\gamma i) + e^{-ti}s_2 + e^{ti}s_1$

$u_2 = s_7\varepsilon(3/8\, e^{-3ti}s_4s_2\alpha i + 1/4\, e^{-ti}s_4s_1\alpha i + 1/4\, e^{-ti}s_2\gamma - 3/8\, e^{3ti}s_3s_1\alpha i - 1/4\, e^{ti}s_3s_2\alpha i + 1/4\, e^{ti}s_1\gamma) - e^{-ti}s_2 i + e^{ti}s_1 i$

$u_3 = s_7\varepsilon(-1/6\, e^{-4ti}s_4s_2^2d^3 + 1/10\, e^{-2ti}s_6 + 2/5\, e^{-2ti}s_4s_2s_1d^3 + 1/5\, e^{-2ti}s_4\beta i + 1/10\, e^{-2ti}s_2^2d^2 - 1/6\, e^{4ti}s_3s_1^2d^3 + 1/10\, e^{2ti}s_5 + 2/5\, e^{2ti}s_3s_2s_1d^3 - 1/5\, e^{2ti}s_3\beta i + 1/10\, e^{2ti}s_1^2d^2 + 1/2s_4s_1^2d^3 + 1/2s_3s_2^2d^3 + 1/2s_2s_1d^2) + e^{-2ti}s_4 + e^{2ti}s_3$

$u_4 = s_7\varepsilon(2/3\, e^{-4ti}s_4s_2^2d^3 i + 1/20\, e^{-2ti}s_6 i + 1/5\, e^{-2ti}s_4s_2s_1d^3 i - 1/10\, e^{-2ti}s_4\beta + 1/20\, e^{-2ti}s_2^2d^2 i - 2/3\, e^{4ti}s_3s_1^2d^3 i - 1/20\, e^{2ti}s_5 i - 1/5\, e^{2ti}s_3s_2s_1d^3 i - 1/10\, e^{2ti}s_3\beta - 1/20\, e^{2ti}s_1^2d^2 i) - 2\, e^{-2ti}s_4 i + 2\, e^{2ti}s_3 i$

$u_5 = e^{-2ti}s_6 + e^{2ti}s_5$

$u_6 = -2\, e^{-2ti}s_6 i + 2\, e^{2ti}s_5 i$

$u_7 = s_7$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = s_7^2\varepsilon^2(- 1/20 s_5 s_2 \alpha i - 1/16 s_4 s_3 s_1 \alpha^2 i - 1/4 s_4 s_1^3 \alpha d^3 i - 9/20 s_3 s_2^2 s_1 \alpha d^3 i - 1/10 s_3 s_2 \alpha\beta - 1/4 s_3 s_2 \alpha\gamma - 3/10 s_2 s_1^2 \alpha d^2 i - 1/8 s_1 \gamma^2 i) + s_7\varepsilon(- 1/2 s_3 s_2 \alpha i + 1/2 s_1 \gamma)$

$\dot{s}_2 = s_7^2\varepsilon^2(1/20 s_6 s_1 \alpha i + 1/16 s_4 s_3 s_2 \alpha^2 i + 9/20 s_4 s_2 s_1^2 \alpha d^3 i - 1/10 s_4 s_1 \alpha\beta - 1/4 s_4 s_1 \alpha\gamma + 1/4 s_3 s_2^3 \alpha d^3 i + 3/10 s_2^2 s_1 \alpha d^2 i + 1/8 s_2 \gamma^2 i) + s_7\varepsilon(1/2 s_4 s_1 \alpha i + 1/2 s_2 \gamma)$

$\dot{s}_3 = s_7^2\varepsilon^2(- 1/16 s_5 s_2 s_1 d^3 i + 1/32 s_5 \beta - 1/5 s_4 s_3 s_1^2 \alpha d^3 i - 1/4 s_4 s_1^4 d^6 i - 7/40 s_3^2 s_2^2 \alpha d^3 i - 5/12 s_3 s_2^2 s_1^2 d^6 i - 1/40 s_3 s_2 s_1 \alpha d^2 i - 3/20 s_3 s_2 s_1 d^3 \gamma - 1/16 s_3 \beta^2 i - 5/16 s_2 s_1^3 d^5 i + 1/32 s_1^2 \beta d^2 + 7/80 s_1^2 d^2 \gamma) + s_7\varepsilon(- 1/4 s_5 i - s_3 s_2 s_1 d^3 i - 1/2 s_3 \beta - 1/4 s_1^2 d^2 i)$

$\dot{s}_4 = s_7^2\varepsilon^2(1/16 s_6 s_2 s_1 d^3 i + 1/32 s_6 \beta + 7/40 s_4^2 s_1^2 \alpha d^3 i + 1/5 s_4 s_3 s_2^2 \alpha d^3 i + 5/12 s_4 s_2^2 s_1^2 d^6 i + 1/40 s_4 s_2 s_1 \alpha d^2 i - 3/20 s_4 s_2 s_1 d^3 \gamma + 1/16 s_4 \beta^2 i + 1/4 s_3 s_2^4 d^6 i + 5/16 s_2^3 s_1 d^5 i + 1/32 s_2^2 \beta d^2 + 7/80 s_2^2 d^2 \gamma) + s_7\varepsilon(1/4 s_6 i + s_4 s_2 s_1 d^3 i - 1/2 s_4 \beta + 1/4 s_2^2 d^2 i)$

$\dot{s}_5 = 0$

$\dot{s}_6 = 0$

$\dot{s}_7 = 0$