A normal form of your dynamical system

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Generally, the lowest order, most important, terms are near the end of each expression.

off echo;

Specified dynamical system

$\dot{x}_1 = \varepsilon y_1^2$

$\dot{y}_1 = \sigma w_1 - y_1$

off echo;

Time dependent normal form coordinates

$y_1 = \sigma e^{-1t} \star w_1 + O(\varepsilon^2, \sigma^2) + Y_1$

$x_1 = \sigma \varepsilon \left(-e^t \star w_1 \, Y_1 - e^{-1t} \star w_1 \, Y_1 \right) - 1/2 \varepsilon Y_1^2 + O(\varepsilon^2, \sigma^2) + X_1$

Result normal form DEs

$\dot{Y}_1 = O(\varepsilon^3, \sigma^3) - Y_1$

$\dot{X}_1 = \sigma^2 \varepsilon e^{-1t} \star w_1 \, w_1 + O(\varepsilon^3, \sigma^3)$