Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = \varepsilon^2(L_3u_3 + L_2u_2 + L_1u_1 + Iu_1u_2)$

$\dot{u}_2 = \varepsilon^2(- L_2u_1 - Iu_1^2) - \gamma_2u_2$

$\dot{u}_3 = -\varepsilon^2 L_3u_1 - \gamma_3u_3$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1, 0, 0\}, \exp(0)\}$

$\vec{z}_1 = \{\{1, 0, 0\}, \exp(0)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = O(\varepsilon^5) + s_1$

$u_2 = \varepsilon^4(L_2L_1\gamma_2^{-2}s_1 + 2L_1\gamma_2^{-2}s_1^2I) + \varepsilon^2(- L_2\gamma_2^{-1}s_1 - \gamma_2^{-1}s_1^2I) + O(\varepsilon^5)$

$$u_3 = \varepsilon^4 L_3 L_1 \gamma_3^{-2} s_1 - \varepsilon^2 L_3 \gamma_3^{-1} s_1 + O(\varepsilon^5)$$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = \varepsilon^4 (- L_3^2 \gamma_3^{-1} s_1 - L_2^2 \gamma_2^{-1} s_1 - 2 L_2 \gamma_2^{-1} s_1^2 I - \gamma_2^{-1} s_1^3 I^2) + \varepsilon^2 L_1 s_1 + O(\varepsilon^6)$$

Normals to isochrons at the slow manifold Use these vectors: to project initial conditions onto the slow manifold; to project non-autonomous forcing onto the slow evolution; to predict the consequences of modifying the original system; in uncertainty quantification to quantify effects on the model of uncertainties in the original system. The normal vector $\vec{z}_j := (z_{j1}, \ldots, z_{jn})$

$$z_{11} = \varepsilon^4 (L_3^2 \gamma_3^{-2} + L_2^2 \gamma_2^{-2} + 3 L_2 \gamma_2^{-2} s_1 I + 2 \gamma_2^{-2} s_1^2 I^2) + O(\varepsilon^6) + 1$$

$$z_{12} = -\varepsilon^4 L_2 L_1 \gamma_2^{-2} + \varepsilon^2 (L_2 \gamma_2^{-1} + \gamma_2^{-1} s_1 I) + O(\varepsilon^6)$$

$$z_{13} = -\varepsilon^4 L_3 L_1 \gamma_3^{-2} + \varepsilon^2 L_3 \gamma_3^{-1} + O(\varepsilon^6)$$