Centre manifold of your dynamical system

A. J. Roberts, University of Adelaide
http://www.maths.adelaide.edu.au/anthony.roberts

5:30 A.M., March 7, 2016

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = \varepsilon u_2(1/3 u_1^3 u_3^3 - u_1^3 u_3)$

$\dot{u}_2 = 0$

$\dot{u}_3 = \varepsilon u_2(- 1/2 u_1 u_3^2 + u_1) - u_3$

Centre subspace basis vectors

$\vec{e}_1 = \{\{1, 0, 0\}, e^{0i}\}$

$\vec{e}_2 = \{\{0, 1, 0\}, e^{0i}\}$

$\vec{e}_3 = \{\{0, 0, 1\}, e^{iti}\}$

$\vec{z}_1 = \{\{1, 0, 0\}, e^{0i}\}$

$\vec{z}_2 = \{\{0, 1, 0\}, e^{0i}\}$

$\vec{z}_3 = \{\{0, 0, 1\}, e^{iti}\}$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$u_1 = s_2^3\varepsilon^3(e^{iti}s_3s_1^6 - 2\,e^{iti}s_3s_1^5 - 5/1458\,e^{9iti}s_3^9s_1^7 + 5/54\,e^{7iti}s_3^7s_1^7 + 11/252\,e^{7iti}s_3^7s_1^6 - 5/6\,e^{5iti}s_3^5s_1^7 - 16/45\,e^{5iti}s_3^5s_1^6 - 1/10\,e^{5iti}s_3^5s_1^5 + 5/2\,e^{3iti}s_3^3s_1^7 - 41/54\,e^{3iti}s_3^3s_1^6 - 5/36\,e^{3iti}s_3^3s_1^5) + s_2^2\varepsilon^2(1/54\,e^{6iti}s_3^6s_1^5 - 1/3\,e^{4iti}s_3^4s_1^5 - 1/8\,e^{4iti}s_3^4s_1^4 + 3/2\,e^{2iti}s_3^2s_1^5 - 1/4\,e^{2iti}s_3^2s_1^4) + s_2\varepsilon(e^{iti}s_3s_1^3 - 1/9\,e^{3iti}s_3^3s_1^3) + s_1$

$u_2 = s_2$

$u_3 = s_2^3\varepsilon^3(1/756\,e^{8iti}s_3^8s_1^5 - 1/27\,e^{6iti}s_3^6s_1^5 - 19/720\,e^{6iti}s_3^6s_1^4 + 13/36\,e^{4iti}s_3^4s_1^5 + 25/108\,e^{4iti}s_3^4s_1^4 + 1/8\,e^{4iti}s_3^4s_1^3 - 3/2\,e^{2iti}s_3^2s_1^5 + 7/4\,e^{2iti}s_3^2s_1^4 - 1/2\,e^{2iti}s_3^2s_1^3 + s_1^4 - 1/2s_1^3) + s_2^2\varepsilon^2(- 1/72\,e^{5iti}s_3^5s_1^3 + 11/36\,e^{3iti}s_3^3s_1^3 + 1/4\,e^{3iti}s_3^3s_1^2) + s_2\varepsilon(1/2\,e^{2iti}s_3^2s_1 + s_1) + e^{iti}s_3$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$\dot s_1 = s_2^4\varepsilon^4(- s_1^7 + 5/6s_1^6) - s_2^2\varepsilon^2s_1^4$

$\dot s_2 = 0$

$\dot s_3 = s_2^4\varepsilon^4(s_3s_1^6 - 7/2s_3s_1^5 + 1/2s_3s_1^4) + s_2^2\varepsilon^2(s_3s_1^3 - s_3s_1^2)$