Normal form of your dynamical system

A. J. Roberts, University of Adelaide
http://orcid.org/0000-0001-8930-1552

7:18am, April 21, 2021

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

Specified dynamical system

$\dot{x}_1 = \varepsilon b_1 y_2 y_1$

$\dot{y}_1 = \sigma b_{21} w_2 + \varepsilon b_2 x_1 y_2 - y_1$

$\dot{y}_2 = \sigma b_{31} w_3 + \varepsilon b_3 x_1 y_1 - y_2$

off echo;

Time dependent coordinate transform

$y_1 = 1/4 Y_2^3 \varepsilon^3 b_2^2 b_1 X_1 + 1/4 Y_2^2 Y_1 \varepsilon^2 b_2 b_1 + Y_2^2 \sigma \varepsilon^3 (b_{31} b_2^2 b_1 \mathrm{e}^t \star \mathrm{e}^t \star \mathrm{e}^t \star w_3 \ X_1 + b_{31} b_2^2 b_1 \mathrm{e}^t \star \mathrm{e}^t \star w_3 \ X_1 + 1/4 b_{31} b_2^2 b_1 \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star w_3 \ X_1 + 3/4 b_{31} b_2^2 b_1 \mathrm{e}^t \star w_3 X_1 + 3/4 b_{31} b_2^2 b_1 \mathrm{e}^{-1t} \star w_3 X_1) + Y_2^2 \sigma \varepsilon^2 (1/2 b_{21} b_2 b_1 \mathrm{e}^t \star \mathrm{e}^{1t} \star w_2 \ + 1/4 b_{21} b_2 b_1 \mathrm{e}^t \star w_2 \ + 1/4 b_{21} b_2 b_1 \mathrm{e}^{-1t} \star w_2 \ ) + 1/4 Y_2 Y_1^2 \varepsilon^3 b_3 b_2 b_1 X_1 + Y_2 Y_1 \sigma \varepsilon^3 (b_{21} b_3 b_2 b_1 \mathrm{e}^t \star \mathrm{e}^t \star \mathrm{e}^t \star w_2 \ X_1 + 1/2 b_{21} b_3 b_2 b_1 \mathrm{e}^t \star \mathrm{e}^t \star w_2 \ X_1 + 1/2 b_{21} b_3 b_2 b_1 \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star w_2 \ X_1 + 1/2 b_{21} b_3 b_2 b_1 \mathrm{e}^t \star w_2 \ X_1 + 1/2 b_{21} b_3 b_2 b_1 \mathrm{e}^{-1t} \star w_2 \ X_1) + Y_2 Y_1 \sigma \varepsilon^2 (1/2 b_{31} b_2 b_1 \mathrm{e}^t \star \mathrm{e}^t \star w_3 \ + 1/2 b_{31} b_2 b_1 \mathrm{e}^t \star w_3 \ + 1/2 b_{31} b_2 b_1 \mathrm{e}^{-1t} \star w_3 \ ) + Y_1^2 \sigma \varepsilon^3 (1/4 b_{31} b_3 b_2 b_1 \mathrm{e}^t \star \mathrm{e}^{1t} \star w_3 \ X_1 + 1/4 b_{31} b_3 b_2 b_1 \mathrm{e}^t \star w_3 X_1 +$

$1/4b_{31}b_3b_2b_1\mathrm{e}^{-1t}\star w_3\ X_1) + Y_1 + \sigma\varepsilon^3 b_{31}b_3b_2^2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3\ \ X_1^3 + \sigma\varepsilon^2 b_{21}b_3b_2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2\ \ X_1^2 + \sigma\varepsilon b_{31}b_2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3\ X_1 + \sigma b_{21}\mathrm{e}^{-1t}\star w_2$

$y_2 = 1/4Y_2^2 Y_1\varepsilon^3 b_3b_2b_1 X_1 + Y_2^2\sigma\varepsilon^3(1/4b_{21}b_3b_2b_1\mathrm{e}^t\star\mathrm{e}^t\star w_2\ X_1 + 1/4b_{21}b_3b_2b_1\mathrm{e}^t\star w_2\ X_1 + 1/4b_{21}b_3b_2b_1\mathrm{e}^{-1t}\star w_2\ X_1) + 1/4Y_2 Y_1^2\varepsilon^2 b_3b_1 + Y_2 Y_1\sigma\varepsilon^3(b_{31}b_3b_2b_1\mathrm{e}^{1t}\star\mathrm{e}^t\star\mathrm{e}^t\star w_3\ X_1 + 1/2b_{31}b_3b_2b_1\mathrm{e}^t\star\mathrm{e}^t\star w_3\ X_1 + 1/2b_{31}b_3b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3\ X_1 + 1/2b_{31}b_3b_2b_1\mathrm{e}^t\star w_3\ X_1 + 1/2b_{31}b_3b_2b_1\mathrm{e}^{-1t}\star w_3\ X_1) + Y_2 Y_1\sigma\varepsilon^2(1/2b_{21}b_3b_1\mathrm{e}^t\star\mathrm{e}^t\star w_2\ + 1/2b_{21}b_3b_1\mathrm{e}^t\star w_2\ + 1/2b_{21}b_3b_1\mathrm{e}^{-1t}\star w_2\ ) + Y_2 + 1/4Y_1^3\varepsilon^3 b_3^2 b_1 X_1 + Y_1^2\sigma\varepsilon^3(b_{21}b_3^2 b_1\mathrm{e}^t\star\mathrm{e}^t\star\mathrm{e}^t\star w_2\ X_1 + b_{21}b_3^2 b_1\mathrm{e}^t\star\mathrm{e}^t\star w_2\ X_1 + 1/4b_{21}b_3^2 b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2\ X_1 + 3/4b_{21}b_3^2 b_1\mathrm{e}^t\star w_2\ X_1 + 3/4b_{21}b_3^2 b_1\mathrm{e}^{-1t}\star w_2\ X_1) + Y_1^2\sigma\varepsilon^2(1/2b_{31}b_3 b_1\mathrm{e}^t\star\mathrm{e}^t\star w_3\ + 1/4b_{31}b_3 b_1\mathrm{e}^t\star w_3\ + 1/4b_{31}b_3 b_1\mathrm{e}^{-1t}\star w_3\ ) + \sigma\varepsilon^3 b_{21}b_3^2 b_2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2\ \ X_1^3 + \sigma\varepsilon^2 b_{31}b_3b_2\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3\ X_1^2 + \sigma\varepsilon b_{21}b_3\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2\ X_1 + \sigma b_{31}\mathrm{e}^{-1t}\star w_3$

$x_1 = -1/16Y_2^3 Y_1\varepsilon^3 b_2 b_1^2 + Y_2^3\sigma\varepsilon^3(-1/4b_{21}b_2 b_1^2\mathrm{e}^{1t}\star\mathrm{e}^t\star w_2\ - 1/16b_{21}b_2 b_1^2\mathrm{e}^{3t}\star w_2 - 1/16b_{21}b_2 b_1^2\mathrm{e}^{-1t}\star w_2) - 1/4Y_2^2 Y_1\sigma\varepsilon^3(- 1/4b_{31}b_2 b_1^2\mathrm{e}^t\star\mathrm{e}^t\star w_3 - 1/16b_{31}b_2 b_1^2\mathrm{e}^{3t}\star w_3 - 1/8b_{31}b_2 b_1^2\mathrm{e}^t\star w_3 - 3/16b_{31}b_2 b_1^2\mathrm{e}^{-1t}\star w_3) - 1/4Y_2^2\varepsilon^2 b_2 b_1 X_1 - 1/16Y_2 Y_1^3\varepsilon^3 b_3 b_1^2 + Y_2 Y_1^2\sigma\varepsilon^3(- 1/4b_{21}b_3 b_1^2\mathrm{e}^t\star\mathrm{e}^t\star w_2 - 1/16b_{21}b_3 b_1^2\mathrm{e}^{3t}\star w_2 - 1/8b_{21}b_3 b_1^2\mathrm{e}^t\star w_2 - 3/16b_{21}b_3 b_1^2\mathrm{e}^{-1t}\star w_2) - 1/2Y_2 Y_1\varepsilon^3 b_3 b_2 b_1 X_1^2 - 1/2Y_2 Y_1\varepsilon b_1 + Y_2\sigma\varepsilon^3(- 1/2b_{21}b_3 b_2 b_1\mathrm{e}^{1t}\star\mathrm{e}^t\star\mathrm{e}^t\star w_2\ \ X_1^2 - 1/2b_{21}b_3 b_2 b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2\ \ X_1^2 - 1/2b_{21}b_3 b_2 b_1\mathrm{e}^t\star\mathrm{e}^t\star w_2\ X_1^2 - 1/2b_{21}b_3 b_2 b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2\ X_1^2 - 1/2b_{21}b_3 b_2 b_1\mathrm{e}^t\star w_2\ X_1^2 - 1/2b_{21}b_3 b_2 b_1\mathrm{e}^{-1t}\star w_2\ X_1^2) + Y_2\sigma\varepsilon^2(- 1/2b_{31}b_2 b_1\mathrm{e}^t\star\mathrm{e}^t\star w_3\ X_1 - 1/2b_{31}b_2 b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3\ X_1 - 1/2b_{31}b_2 b_1\mathrm{e}^t\star w_3\ X_1 - 1/2b_{31}b_2 b_1\mathrm{e}^{-1t}\star w_3\ X_1) + Y_2\sigma\varepsilon(- 1/2b_{21}b_1\mathrm{e}^t\star w_2 - 1/2b_{21}b_1\mathrm{e}^{-1t}\star w_2) + Y_1^3\sigma\varepsilon^3(- 1/4b_{31}b_3 b_1^2\mathrm{e}^t\star\mathrm{e}^{1t}\star w_3\ - 1/16b_{31}b_3 b_1^2\mathrm{e}^{3t}\star w_3 - 1/16b_{31}b_3 b_1^2\mathrm{e}^{-1t}\star w_3) - 1/4Y_1^2\varepsilon^2 b_3 b_1 X_1 + Y_1\sigma\varepsilon^3(- 1/2b_{31}b_3 b_2 b_1\mathrm{e}^t\star\mathrm{e}^t\star\mathrm{e}^t\star w_3\ \ X_1^2 - 1/2b_{31}b_3 b_2 b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3\ \ X_1^2 - 1/2b_{31}b_3 b_2 b_1\mathrm{e}^t\star\mathrm{e}^t\star w_3\ X_1^2 - 1/2b_{31}b_3 b_2 b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3\ X_1^2 - 1/2b_{31}b_3 b_2 b_1\mathrm{e}^t\star w_3\ X_1^2 - 1/2b_{31}b_3 b_2 b_1\mathrm{e}^{-1t}\star w_3\ X_1^2) + Y_1\sigma\varepsilon^2(- 1/2b_{21}b_3 b_1\mathrm{e}^t\star\mathrm{e}^t\star w_2\ X_1 - 1/2b_{21}b_3 b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2\ X_1 - 1/2b_{21}b_3 b_1\mathrm{e}^t\star w_2\ X_1 - 1/2b_{21}b_3 b_1\mathrm{e}^{-1t}\star w_2\ X_1) + Y_1\sigma\varepsilon(- 1/2b_{31}b_1\mathrm{e}^t\star w_3 - 1/2b_{31}b_1\mathrm{e}^{-1t}\star w_3) + X_1$

Result normal form DEs

$\dot{Y}_1 = Y_2\sigma^2\varepsilon^4(-3/2b_{31}b_{21}b_3b_2^2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\mathrm{e}^{-1t}\star w_3 \ w_2X_1^2 -$
$1/2b_{31}b_{21}b_3b_2^2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_2X_1^2 -$
$3/2b_{31}b_{21}b_3b_2^2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_3X_1^2 -$
$b_{31}b_{21}b_3b_2^2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_2X_1^2 - 3/2b_{31}b_{21}b_3b_2^2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_3X_1^2 -$
$3/2b_{31}b_{21}b_3b_2^2b_1\mathrm{e}^{-1t}\star w_3 \ w_2X_1^2 - 3/2b_{31}b_{21}b_3b_2^2b_1\mathrm{e}^{-1t}\star w_2 \ w_3X_1^2) +$
$Y_2\sigma^2\varepsilon^3(-b_{31}^2b_2^2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_3X_1 - 3/2b_{31}^2b_2^2b_1\mathrm{e}^{-1t}\star w_3 \ w_3X_1 -$
$b_{21}^2b_3b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_2X_1 - 1/2b_{21}^2b_3b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_2X_1 -$
$1/2b_{21}^2b_3b_2b_1\mathrm{e}^{-1t}\star w_2 \ w_2X_1) + Y_2\sigma^2\varepsilon^2(-1/2b_{31}b_{21}b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_2 -$
$1/2b_{31}b_{21}b_2b_1\mathrm{e}^{-1t}\star w_3 \ w_2 - 1/2b_{31}b_{21}b_2b_1\mathrm{e}^{-1t}\star w_2 \ w_3) + Y_2\varepsilon b_2X_1 +$
$Y_1\sigma^2\varepsilon^4(-3/2b_{31}^2b_3b_2^2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_3X_1^2 -$
$1/2b_{31}^2b_3b_2^2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_3X_1^2 - b_{31}^2b_3b_2^2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_3X_1^2 -$
$3/2b_{31}^2b_3b_2^2b_1\mathrm{e}^{-1t}\star w_3 \ w_3X_1^2 - b_{21}^2b_3^2b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_2X_1^2 -$
$1/2b_{21}^2b_3^2b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_2X_1^2) + Y_1\sigma^2\varepsilon^3(-$
$b_{31}b_{21}b_3b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_3X_1 -$
$1/2b_{31}b_{21}b_3b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_2X_1 - 1/2b_{31}b_{21}b_3b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_3X_1 -$
$1/2b_{31}b_{21}b_3b_2b_1\mathrm{e}^{-1t}\star w_3 \ w_2X_1 - 1/2b_{31}b_{21}b_3b_2b_1\mathrm{e}^{-1t}\star w_2 \ w_3X_1) +$
$Y_1\sigma^2\varepsilon^2(-1/2b_{31}^2b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_3 - 1/2b_{31}^2b_2b_1\mathrm{e}^{-1t}\star w_3 \ w_3) - Y_1$

$\dot{Y}_2 = Y_2\sigma^2\varepsilon^4(-b_{31}^2b_3b_2^2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_3X_1^2 -$
$1/2b_{31}^2b_3b_2^2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_3X_1^2 -$
$3/2b_{21}^2b_3^2b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_2X_1^2 -$
$1/2b_{21}^2b_3^2b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_2X_1^2 - b_{21}^2b_3^2b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_2X_1^2 -$
$3/2b_{21}^2b_3^2b_2b_1\mathrm{e}^{-1t}\star w_2 \ w_2X_1^2) + Y_2\sigma^2\varepsilon^3(-$
$b_{31}b_{21}b_3b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_2X_1 -$
$1/2b_{31}b_{21}b_3b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_2X_1 - 1/2b_{31}b_{21}b_3b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_3X_1 -$
$1/2b_{31}b_{21}b_3b_2b_1\mathrm{e}^{-1t}\star w_3 \ w_2X_1 - 1/2b_{31}b_{21}b_3b_2b_1\mathrm{e}^{-1t}\star w_2 \ w_3X_1) +$
$Y_2\sigma^2\varepsilon^2(-1/2b_{21}^2b_3b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_2 - 1/2b_{21}^2b_3b_1\mathrm{e}^{-1t}\star w_2 \ w_2) - Y_2 +$
$Y_1\sigma^2\varepsilon^4(-3/2b_{31}b_{21}b_3^2b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_3X_1^2 -$
$3/2b_{31}b_{21}b_3^2b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_2X_1^2 -$
$1/2b_{31}b_{21}b_3^2b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_3X_1^2 -$
$3/2b_{31}b_{21}b_3^2b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_2X_1^2 - b_{31}b_{21}b_3^2b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_3X_1^2 -$
$3/2b_{31}b_{21}b_3^2b_2b_1\mathrm{e}^{-1t}\star w_3 \ w_2X_1^2 - 3/2b_{31}b_{21}b_3^2b_2b_1\mathrm{e}^{-1t}\star w_2 \ w_3X_1^2) + Y_1\sigma^2\varepsilon^3(-$
$b_{31}^2b_3b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_3X_1 - 1/2b_{31}^2b_3b_2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_3 \ w_3X_1 -$
$1/2b_{31}^2b_3b_2b_1\mathrm{e}^{-1t}\star w_3 \ w_3X_1 - b_{21}^2b_3^2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_2X_1 -$
$3/2b_{21}^2b_3^2b_1\mathrm{e}^{-1t}\star w_2 \ w_2X_1) + Y_1\sigma^2\varepsilon^2(-1/2b_{31}b_{21}b_3b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star w_2 \ w_3 -$
$1/2b_{31}b_{21}b_3b_1\mathrm{e}^{-1t}\star w_3 \ w_2 - 1/2b_{31}b_{21}b_3b_1\mathrm{e}^{-1t}\star w_2 \ w_3) + Y_1\varepsilon b_3X_1$

$\dot{X}_1 = \sigma^2\varepsilon^4(1/2b_{31}^2b_3b_2^2b_1\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\star\mathrm{e}^{-1t}\mathrm{e}^{-1t}\star w_3 \ w_3X_1^3 +$

$$1/2b_{31}^2 b_3 b_2^2 b_1 \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star w_3 \ w_3 X_1^3 + 1/2b_{31}^2 b_3 b_2^2 b_1 \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star w_3 \ w_3 X_1^3 +$$

$$1/2b_{21}^2 b_3 b_2^2 b_1 \mathrm{e}^{-1t} \star w_3 \ w_3 X_1^3 + 1/2b_{21}^2 b_3^2 b_2 b_1 \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star w_2 \ \ w_2 X_1^3 +$$

$$1/2b_{21}^2 b_3^2 b_2 b_1 \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star w_2 \ w_2 X_1^3 +$$

$$1/2b_3^2 b_3^2 b_2 b_1 \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star w_2 \ w_2 X_1^3 + 1/2b_{21}^2 b_3^2 b_2 b_1 \mathrm{e}^{-1t} \star w_2 \ w_2 X_1^3) +$$

$$\sigma^2 \varepsilon^3 (1/2b_{31} b_{21} b_3 b_2 b_1 \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star w_3 \ w_2 X_1^2 +$$

$$1/2b_{31} b_{21} b_3 b_2 b_1 \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star w_2 \ w_3 X_1^2 +$$

$$1/2b_{31} b_{21} b_3 b_2 b_1 \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star w_3 \ w_2 X_1^2 +$$

$$1/2b_{31} b_{21} b_3 b_2 b_1 \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star w_2 \ w_3 X_1^2 + 1/2b_{31} b_{21} b_3 b_2 b_1 \mathrm{e}^{-1t} \star w_3 w_2 X_1^2 +$$

$$1/2b_{31} b_{21} b_3 b_2 b_1 \mathrm{e}^{-1t} \star w_2 w_3 X_1^2) + \sigma^2 \varepsilon^2 (1/2b_{31}^2 b_2 b_1 \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star w_3 \ w_3 X_1 +$$

$$1/2b_{21}^2 b_2 b_1 \mathrm{e}^{-1t} \star w_3 \ w_3 X_1 + 1/2b_{21}^2 b_3 b_1 \mathrm{e}^{-1t} \star \mathrm{e}^{-1t} \star w_2 \ w_2 X_1 +$$

$$1/2b_{21}^2 b_3 b_1 \mathrm{e}^{-1t} \star w_2 \ w_2 X_1) + \sigma^2 \varepsilon (1/2b_{31} b_{21} b_1 \mathrm{e}^{-1t} \star w_3 \ w_2 +$$

$$1/2b_{31} b_{21} b_1 \mathrm{e}^{-1t} \star w_2 \ w_3)$$