Normal form of your dynamical system

A. J. Roberts, University of Adelaide
http://www.maths.adelaide.edu.au/anthony.roberts

7:29am, February 3, 2021

Specified dynamical system

$\dot{x}_1 = \epsilon\varepsilon(x_1 y_1 \rho - x_1\rho^2 - 1/2 y_1\rho + y_1 + 1/2\rho^2 - 1/2\rho)$

$\dot{y}_1 = \varepsilon(-x_1 y_1\rho + x_1\rho^2) - y_1$

Time dependent coordinate transform

$y_1 = \epsilon\varepsilon^2(-X_1 Y_1^2\rho^2 + X_1\rho^4 + 1/2 Y_1^2\rho^2 - Y_1^2\rho - 1/2\rho^4 + 1/2\rho^3) - \varepsilon^2 X_1^2\rho^3 + \varepsilon X_1\rho^2 + Y_1$

$x_1 = \epsilon^2\varepsilon^2(1/2 X_1 Y_1^2\rho^2 - 1/4 Y_1^2\rho^2 + 1/2 Y_1^2\rho - 1/2 Y_1\rho^2) + \epsilon\varepsilon^2(X_1^2 Y_1\rho^2 - 1/2 X_1 Y_1\rho^2 + X_1 Y_1\rho) + \epsilon\varepsilon(-X_1 Y_1\rho + 1/2 Y_1\rho - Y_1) + X_1$

Result normal form DEs

$\dot{Y}_1 = -1/2\epsilon^2\varepsilon^3 Y_1\rho^4 + \epsilon\varepsilon^3(2X_1^2 Y_1\rho^4 - X_1 Y_1\rho^4 + 2X_1 Y_1\rho^3) + \epsilon\varepsilon^2(-X_1 Y_1\rho^3 + 1/2 Y_1\rho^3 - Y_1\rho^2) - \varepsilon X_1 Y_1\rho - Y_1$

$\dot{X}_1 = \epsilon^2\varepsilon^3(X_1^2\rho^5 - X_1\rho^5 + 3/2 X_1\rho^4 + 1/4\rho^5 - 3/4\rho^4 + 1/2\rho^3) + \epsilon\varepsilon^3(-X_1^3\rho^4 + 1/2 X_1^2\rho^4 - X_1^2\rho^3) + \epsilon\varepsilon^2(X_1^2\rho^3 - 1/2 X_1\rho^3 + X_1\rho^2) + \epsilon\varepsilon(-X_1\rho^2 + 1/2\rho^2 - 1/2\rho)$