Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

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The specified dynamical system

$\dot u_1 = -e_0\varepsilon^2 u_1 - e_0\varepsilon \exp(0) s_0 + \varepsilon u_1 u_2 + s_0 u_2 + u_2$

$\dot u_2 = e_0\varepsilon^2 u_1 + e_0\varepsilon \exp(0) s_0 - \varepsilon u_1 u_2 - s_0 u_2 - 2u_2$

Invariant subspace basis vectors

$\vec e_1 = \{\{1, 0\}, \exp(0)\}$

$\vec z_1 = \{\{1, 0\}, \exp(0)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = O(\varepsilon^4) + s_1$

$u_2 = (2e_0^2\varepsilon^3 s_0)/(s_0^4 + 8s_0^3 + 24s_0^2 + 32s_0 + 16) + (-2e_0\varepsilon^3 s_1^2)/(s_0^3 + 6s_0^2 + 12s_0 + 8) + (2e_0\varepsilon^2 s_1)/(s_0^2 + 4s_0 + 4) + (e_0\varepsilon s_0)/(s_0 + 2) + O(\varepsilon^4)$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = e_0^2\varepsilon^4(-4s_1 s_0^2 + 2s_1 s_0 + 4s_1)/(s_0^5 + 10s_0^4 + 40s_0^3 + 80s_0^2 + 80s_0 + 32) + e_0^2\varepsilon^3(2s_0^2 + 2s_0)/(s_0^4 + 8s_0^3 + 24s_0^2 + 32s_0 + 16) + (-2e_0\varepsilon^4 s_1^3)/(s_0^4 + 8s_0^3 + 24s_0^2 + 32s_0 + 16) + (2e_0\varepsilon^3 s_1^2)/(s_0^3 + 6s_0^2 + 12s_0 + 8) + (-2e_0\varepsilon^2 s_1)/(s_0^2 + 4s_0 + 4) + (-e_0\varepsilon s_0)/(s_0 + 2) + O(\varepsilon^5)$

Normals to isochrons at the slow manifold Use these vectors: to project initial conditions onto the slow manifold; to project non-autonomous forcing onto the slow evolution; to predict the consequences of modifying the original system; in uncertainty quantification to quantify effects on the model of uncertainties in the original system. The normal vector $\vec{z}_j := (z_{j1}, \ldots, z_{jn})$

$z_{11} = e_0^2\varepsilon^4(10s_0^2+8s_0-4)/(s_0^6+12s_0^5+60s_0^4+160s_0^3+240s_0^2+192s_0+64)+(-6e_0\varepsilon^4 s_1^2 s_0)/(s_0^5+10s_0^4+40s_0^3+80s_0^2+80s_0+32)+e_0\varepsilon^3(4s_1 s_0+2s_1)/(s_0^4+8s_0^3+24s_0^2+32s_0+16)+e_0\varepsilon^2(-2s_0-2)/(s_0^3+6s_0^2+12s_0+8)+O(\varepsilon^5)+1$

$z_{12} = e_0^2\varepsilon^4(10s_0^3 + 17s_0^2 - 8)/(s_0^7 + 14s_0^6 + 84s_0^5 + 280s_0^4 + 560s_0^3 + 672s_0^2 + 448s_0+128)+e_0\varepsilon^4(-6s_1^2 s_0^2+6s_1^2 s_0+16s_1^2)/(s_0^6+12s_0^5+60s_0^4+160s_0^3+240s_0^2+192s_0+64)+e_0\varepsilon^3(4s_1 s_0^2+s_1 s_0-6s_1)/(s_0^5+10s_0^4+40s_0^3+80s_0^2+80s_0+32)+e_0\varepsilon^2(-2s_0^2-3s_0)/(s_0^4+8s_0^3+24s_0^2+32s_0+16)+(-\varepsilon^4 s_1^4)/(s_0^5+10s_0^4+40s_0^3+80s_0^2+80s_0+32)+(\varepsilon^3 s_1^3)/(s_0^4+8s_0^3+24s_0^2+32s_0+16)+(-\varepsilon^2 s_1^2)/(s_0^3+6s_0^2+12s_0+8)+(\varepsilon s_1)/(s_0^2+4s_0+4)+(O(\varepsilon^5)s_0+2O(\varepsilon^5)+s_0+1)/(s_0+2)$