Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = u_2$

$\dot{u}_2 = -3/2\varepsilon^2 u_1^5 + \varepsilon(-1/2u_1^3 - u_2u_5 + u_4u_5) - 2u_1 + u_3$

$\dot{u}_3 = u_4$

$\dot{u}_4 = \varepsilon(u_2u_5 - 2u_4u_5) + u_1 - 2u_3$

$\dot{u}_5 = 0$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1, i, 1, i, 0\}, \mathrm{e}^{it}\}$

$\vec{e}_2 = \{\{1, -i, 1, -i, 0\}, \mathrm{e}^{-it}\}$

$\vec{e}_3 = \{\{0, 0, 0, 0, 1\}, \mathrm{e}^{0}\}$

$\vec{z}_1 = \{\{1/4, 1/4i, 1/4, 1/4i, 0\}, \mathrm{e}^{it}\}$

$\vec{z}_2 = \{\{1/4, -1/4i, 1/4, -1/4i, 0\}, \mathrm{e}^{-it}\}$

$\vec{z}_3 = \{\{0, 0, 0, 0, 1\}, \mathrm{e}^{0}\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = -1/8\,\mathrm{e}^{-it}s_3s_2\varepsilon i - 9/16\,\mathrm{e}^{-it}s_2^2s_1\varepsilon + \mathrm{e}^{-it}s_2 + 7/96\,\mathrm{e}^{-3it}s_2^3\varepsilon + 1/8\,\mathrm{e}^{it}s_3s_1\varepsilon i - 9/16\,\mathrm{e}^{it}s_2s_1^2\varepsilon + \mathrm{e}^{it}s_1 + 7/96\,\mathrm{e}^{3it}s_1^3\varepsilon$

$u_2 = -3/8\,\mathrm{e}^{-it}s_3s_2\varepsilon + 3/16\,\mathrm{e}^{-it}s_2^2s_1\varepsilon i - \mathrm{e}^{-it}s_2i - 7/32\,\mathrm{e}^{-3it}s_2^3\varepsilon i - 3/8\,\mathrm{e}^{it}s_3s_1\varepsilon - 3/16\,\mathrm{e}^{it}s_2s_1^2\varepsilon i + \mathrm{e}^{it}s_1i + 7/32\,\mathrm{e}^{3it}s_1^3\varepsilon i$

$u_3 = 3/8\,\mathrm{e}^{-it}s_3s_2\varepsilon i + 3/16\,\mathrm{e}^{-it}s_2^2s_1\varepsilon + \mathrm{e}^{-it}s_2 - 1/96\,\mathrm{e}^{-3it}s_2^3\varepsilon - 3/8\,\mathrm{e}^{it}s_3s_1\varepsilon i + 3/16\,\mathrm{e}^{it}s_2s_1^2\varepsilon + \mathrm{e}^{it}s_1 - 1/96\,\mathrm{e}^{3it}s_1^3\varepsilon$

$u_4 = 1/8\,\mathrm{e}^{-it}s_3s_2\varepsilon - 9/16\,\mathrm{e}^{-it}s_2^2s_1\varepsilon i - \mathrm{e}^{-it}s_2i + 1/32\,\mathrm{e}^{-3it}s_2^3\varepsilon i + 1/8\,\mathrm{e}^{it}s_3s_1\varepsilon + 9/16\,\mathrm{e}^{it}s_2s_1^2\varepsilon i + \mathrm{e}^{it}s_1i - 1/32\,\mathrm{e}^{3it}s_1^3\varepsilon i$

$u_5 = s_3$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = 1/32s_3^2s_1\varepsilon^2i - 3/16s_3s_2s_1^2\varepsilon^2 - 1/4s_3s_1\varepsilon + 805/256s_2^2s_1^3\varepsilon^2i + 3/8s_2s_1^2\varepsilon i$

$\dot{s}_2 = -1/32s_3^2s_2\varepsilon^2i - 3/16s_3s_2^2s_1\varepsilon^2 - 1/4s_3s_2\varepsilon - 805/256s_2^3s_1^2\varepsilon^2i - 3/8s_2^2s_1\varepsilon i$

$\dot{s}_3 = 0$