Centre manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot u_1 = \varepsilon u_2 u_3 - u_2$

$\dot u_2 = -\varepsilon u_1 u_3 + u_1$

$\dot u_3 = \varepsilon u_1^2 u_2^2 - u_3$

Centre subspace basis vectors

$\vec e_1 = \{\{1, -i, 0\}, e^{ti}\}$

$\vec e_2 = \{\{1, i, 0\}, e^{-ti}\}$

$\vec e_3 = \{\{0, 0, 1\}, e^{iti}\}$

$\vec z_1 = \{\{1/2, -1/2i, 0\}, e^{ti}\}$

$\vec z_2 = \{\{1/2, 1/2i, 0\}, e^{-ti}\}$

$\vec z_3 = \{\{0, 0, 1\}, e^{iti}\}$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$$u_1 = \varepsilon(- e^{it-ti} s_3 s_2 i + e^{it+ti} s_3 s_1 i) + e^{-ti} s_2 + e^{ti} s_1$$

$$u_2 = \varepsilon(e^{it-ti} s_3 s_2 + e^{it+ti} s_3 s_1) + e^{-ti} s_2 i - e^{ti} s_1 i$$

$$u_3 = \varepsilon(-4/17\, e^{-4ti} s_2^4 i - 1/17\, e^{-4ti} s_2^4 + 4/17\, e^{4ti} s_1^4 i - 1/17\, e^{4ti} s_1^4 + 2 s_2^2 s_1^2) + e^{iti} s_3$$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = -2\varepsilon^2 s_2^2 s_1^3 i$$

$$\dot{s}_2 = 2\varepsilon^2 s_2^3 s_1^2 i$$

$$\dot{s}_3 = 0$$