Centre manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = \varepsilon u_1 u_3 - u_2$

$\dot{u}_2 = \varepsilon u_2 u_3 + u_1$

$\dot{u}_3 = \varepsilon(- u_1^2 - u_2^2 + u_3^2) - u_3$

Centre subspace basis vectors

$\vec{e}_1 = \{\{1, -i, 0\}, e^{ti}\}$

$\vec{e}_2 = \{\{1, i, 0\}, e^{-ti}\}$

$\vec{e}_3 = \{\{0, 0, 1\}, e^{iti}\}$

$\vec{z}_1 = \{\{1/2, -1/2i, 0\}, e^{ti}\}$

$\vec{z}_2 = \{\{1/2, 1/2i, 0\}, e^{-ti}\}$

$\vec{z}_3 = \{\{0, 0, 1\}, e^{iti}\}$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$u_1 =$
$\varepsilon^2(e^{2it-ti} s_3^2 s_2 + e^{2it+ti} s_3^2 s_1) + \varepsilon(- e^{it-ti} s_3 s_2 - e^{it+ti} s_3 s_1) + e^{-ti} s_2 + e^{ti} s_1$

$u_2 = \varepsilon^2(e^{2it-ti} s_3^2 s_2 i - e^{2it+ti} s_3^2 s_1 i) + \varepsilon(- e^{it-ti} s_3 s_2 i + e^{it+ti} s_3 s_1 i) +$
$e^{-ti} s_2 i - e^{ti} s_1 i$

$u_3 = \varepsilon^2 e^{3iti} s_3^3 + \varepsilon(- e^{2iti} s_3^2 - 4 s_2 s_1) + e^{iti} s_3$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = -4\varepsilon^2 s_2 s_1^2$

$\dot{s}_2 = -4\varepsilon^2 s_2^2 s_1$

$\dot{s}_3 = 0$