Centre manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$$\dot{u}_1 = \gamma\varepsilon^2(-1/2u_2 - 1/2u_4) + 3/2u_2 + 1/2u_4$$

$$\dot{u}_2 = \gamma\varepsilon^2(1/2u_1 + 1/2u_3) - 1/2u_1 + 1/2u_3$$

$$\dot{u}_3 = \gamma\varepsilon^2(1/2u_2 + 1/2u_4) - 1/2u_2 + 1/2u_4$$

$$\dot{u}_4 = \gamma\varepsilon^2(-1/2u_1 - 1/2u_3) + \mu\varepsilon^3 u_1 - \varepsilon^3 u_1^3 - 1/2u_1 - 3/2u_3$$

Centre subspace basis vectors

$$\vec{e}_1 = \{\{1, i, -1, -i\}, e^{ti}\}$$

$$\vec{e}_2 = \{\{-3i, 1, -i, 3\}, e^{ti}\}$$

$$\vec{e}_3 = \{\{1, -i, -1, i\}, e^{-ti}\}$$

$$\vec{e}_4 = \{\{3i, 1, i, 3\}, e^{-ti}\}$$

$$\vec{z}_1 = \{\{1/8, 3/8i, -3/8, -1/8i\}, e^{ti}\}$$

$$\vec{z}_2 = \{\{-1/8i, 1/8, -1/8i, 1/8\}, e^{ti}\}$$

$$\vec{z}_3 = \{\{1/8, -3/8i, -3/8, 1/8i\}, e^{-ti}\}$$

$$\vec{z}_4 = \{\{1/8i, 1/8, 1/8i, 1/8\}, e^{-ti}\}$$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$$u_1 = 3\,e^{-ti} s_4 i + \,e^{-ti} s_3 - 3\,e^{ti} s_2 i + \,e^{ti} s_1$$

$$u_2 = \,e^{-ti} s_4 - \,e^{-ti} s_3 i + \,e^{ti} s_2 + \,e^{ti} s_1 i$$

$$u_3 = \,e^{-ti} s_4 i - \,e^{-ti} s_3 - \,e^{ti} s_2 i - \,e^{ti} s_1$$

$$u_4 = 3\,e^{-ti} s_4 + \,e^{-ti} s_3 i + 3\,e^{ti} s_2 - \,e^{ti} s_1 i$$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = -2\gamma\varepsilon^2 s_2 + \mu\varepsilon^3(3/8 s_2 + 1/8 s_1 i) + \varepsilon^3(-81/8 s_4 s_2^2 - 27/4 s_4 s_2 s_1 i + 9/8 s_4 s_1^2 + 27/8 s_3 s_2^2 i - 9/4 s_3 s_2 s_1 - 3/8 s_3 s_1^2 i)$$

$$\dot{s}_2 = \mu\varepsilon^3(-3/8 s_2 i + 1/8 s_1) + \varepsilon^3(81/8 s_4 s_2^2 i - 27/4 s_4 s_2 s_1 - 9/8 s_4 s_1^2 i + 27/8 s_3 s_2^2 + 9/4 s_3 s_2 s_1 i - 3/8 s_3 s_1^2)$$

$$\dot{s}_3 = -2\gamma\varepsilon^2 s_4 + \mu\varepsilon^3(3/8 s_4 - 1/8 s_3 i) + \varepsilon^3(-81/8 s_4^2 s_2 - 27/8 s_4^2 s_1 i + 27/4 s_4 s_3 s_2 i - 9/4 s_4 s_3 s_1 + 9/8 s_3^2 s_2 + 3/8 s_3^2 s_1 i)$$

$$\dot{s}_4 = \mu\varepsilon^3(3/8 s_4 i + 1/8 s_3) + \varepsilon^3(-81/8 s_4^2 s_2 i + 27/8 s_4^2 s_1 - 27/4 s_4 s_3 s_2 - 9/4 s_4 s_3 s_1 i + 9/8 s_3^2 s_2 i - 3/8 s_3^2 s_1)$$

INPUT TO THE WEB PAGE:

```
ff_:=tp mat(((3*u2+u4-gamma*small*(u2+u4))/2
,(-u1+u3+gamma*small*(u1+u3))/2
,(-u2+u4+gamma*small*(u2+u4))/2
,(-u1-3*u3-gamma*small*(u1+u3))/2+small^2*(mu*u1-u1^3)
));
freqm_:=mat((1,1,-1,-1));
ee_:=tp mat((1,i,-1,-i),(-3*i,1,-i,3)
,(1,-i,-1,i),(3*i,1,i,3));
zz_:=tp mat((1/8,3*i/8,-3/8,-i/8),(-i/8,1/8,-i/8,1/8)
,(1/8,-3*i/8,-3/8,i/8),(i/8,1/8,i/8,1/8));
toosmall:=4;
factor small,mu,gamma;
```