Normal form of your dynamical system

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Specified dynamical system

$$\dot{x}_1 = \varepsilon(- x_2 x_1 - x_2 y_1)$$

$$\dot{x}_2 = -\mu\varepsilon x_2 + \varepsilon(x_1^2 - y_1^2)$$

$$\dot{y}_1 = \sigma w_1 + \varepsilon x_2 x_1 - y_1$$

Time dependent coordinate transform

$$y_1 = \sigma e^{-1t} \star w_1 \ + \varepsilon X_2 X_1 + Y_1$$

$$x_1 = \sigma\varepsilon e^{-1t} \star w_1 \, X_2 + \varepsilon X_2 Y_1 + X_1$$

$$x_2 = \sigma\varepsilon(e^{t} \star w_1 \, Y_1 + e^{-1t} \star w_1 \, Y_1) + 1/2\varepsilon Y_1^2 + X_2$$

Result normal form DEs

$$\dot{Y}_1 = 2\sigma\varepsilon^2 w_1 X_1 Y_1 + \varepsilon^2 X_2^2 Y_1 - Y_1$$

$$\dot{X}_1 = \mu\sigma\varepsilon^2 w_1 X_2 - 1/2\sigma^2\varepsilon^2 e^{-1t} \star w_1 \, w_1 X_1 + \sigma\varepsilon^2(- w_1 X_2^2 - w_1 X_1^2) - \sigma\varepsilon w_1 X_2 - \varepsilon^2 X_2^2 X_1 - \varepsilon X_2 X_1$$

$$\dot{X}_2 = -1/2\mu\sigma^2\varepsilon^2 e^{-1t} \star w_1 \, w_1 - \mu\varepsilon X_2 - \sigma^2\varepsilon e^{-1t} \star w_1 \, w_1 + \varepsilon X_1^2$$