Invariant manifold of your dynamical system

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5:55am, November 5, 2025

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$$\dot{u}_1 = u_2$$

$$\dot{u}_2 = i\varepsilon(-1/2\exp(-it)b + 1/2\exp(it)b) + \varepsilon(1/2\exp(-it)a + 1/2\exp(it)a - C_1u_2 - 1/5u_1^3 + 3/5u_1^2u_3 - 3/5u_1u_3^2 + 1/5u_3^3) - u_1$$

$$\dot{u}_3 = u_4$$

$$\dot{u}_4 = \varepsilon(1/5u_1^3 - 3/5u_1^2u_3 + 3/5u_1u_3^2 - 2/5u_3^3 + 3/5u_3^2u_5 - 3/5u_3u_5^2 + 1/5u_5^3) - u_3 - 1/5u_4$$

$$\dot{u}_5 = u_6$$

$$\dot{u}_6 = \varepsilon(1/5u_3^3 - 3/5u_3^2u_5 + 3/5u_3u_5^2 - 1/5u_5^3) - u_5 - 3/10u_6$$

Invariant subspace basis vectors

$$\vec{e}_1 = \{\{1, i, 0, 0, 0, 0\}, \exp(it)\}$$

$$\vec{e}_2 = \{\{1, -i, 0, 0, 0, 0\}, \exp(-it)\}$$

$$\vec{z}_1 = \{\{1/2, 1/2i, 0, 0, 0, 0\}, \exp(it)\}$$

$$\vec{z}_2 = \{\{1/2, -1/2i, 0, 0, 0, 0\}, \exp(-it)\}$$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$$u_1 = \exp(-it)s_2 + \exp(it)s_1 + O(\varepsilon)$$

$$u_2 = i(-\exp(-it)s_2 + \exp(it)s_1) + O(\varepsilon)$$

$$u_3 = O(\varepsilon)$$

$$u_4 = O(\varepsilon)$$

$$u_5 = O(\varepsilon)$$

$$u_6 = O(\varepsilon)$$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = i\varepsilon(3/10 s_2 s_1^2 - 1/4a) + \varepsilon(-1/2 s_1 C_1 + 1/4b) + O(\varepsilon^2)$$

$$\dot{s}_2 = i\varepsilon(-3/10 s_2^2 s_1 + 1/4a) + \varepsilon(-1/2 s_2 C_1 + 1/4b) + O(\varepsilon^2)$$