# Centre manifold of your dynamical system

A. J. Roberts, University of Adelaide
http://www.maths.adelaide.edu.au/anthony.roberts

6:58 A.M., November 23, 2017

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

**The specified dynamical system**

$$\dot{u}_1 = u_2$$

$$\dot{u}_2 = \eta\varepsilon(D_{t,2\pi}(u_1)^2 - 2D_{t,2\pi}(u_1)u_1 + u_1^2) - 2\varepsilon^2\zeta u_2 - 3/8D_{t,2\pi}(u_1) - 5/8u_1$$

```
ff_:=tp mat((u2
,-5/8*u1-3/8*u1(2*pi)
-small*2*zeta*u2 +eta*(u1(2*pi)-u1)^2 ));
freqm_:=mat((1/2,1,-1/2,-1));
ee_:=tp mat((1,i/2),(1,i),(1,-i/2),(1,-i));
zz_:=tp mat((1,2*i),(1,i),(1,-i*2),(1,-i));
```

**Centre subspace basis vectors**

$$\vec{e}_1 = \left\{\{1, 1/2i\}\,,\, e^{t/2i}\right\}$$

$$\vec{e}_2 = \{\{1, i\}\,,\, e^{ti}\}$$

$$\vec{e}_3 = \left\{\{1, -1/2i\}\,,\, e^{(-ti}/2)\right\}$$

$$\vec{e}_4 = \{\{1, -i\}\,,\, e^{-ti}\}$$

$$\vec{z}_1 = \left\{\{1/2, i\}\,,\, e^{t/2i}\right\}$$

$$\vec{z}_2 = \{\{1/2, 1/2i\}\,,\, e^{ti}\}$$

$$\vec{z}_3 = \left\{\{1/2, -i\}\,,\, e^{(-ti}/2)\right\}$$

$$\vec{z}_4 = \{\{1/2, -1/2i\}\,,\, e^{-ti}\}$$

**The centre manifold** These give the location of the centre manifold in terms of parameters $s_j$.

$u_1 = s_4\,e^{-ti} + s_3^2\eta\varepsilon(24\,e^{-ti}i\pi + 64\,e^{-ti})/(9\pi^2+64) + 8s_3s_1\eta\varepsilon +$
$s_3\,e^{(-ti}/2) + s_2\,e^{ti} + s_1^2\eta\varepsilon(-24\,e^{ti}i\pi + 64\,e^{ti})/(9\pi^2+64) + s_1\,e^{t/2i}$

$u_2 = -s_4\,e^{-ti}i + s_3^2\eta\varepsilon(64\,e^{-ti}i - 24\,e^{-ti}\pi)/(9\pi^2+64) - 1/2s_3\,e^{(-ti}/2)i +$
$s_2\,e^{ti}i + s_1^2\eta\varepsilon(-64\,e^{ti}i - 24\,e^{ti}\pi)/(9\pi^2+64) + 1/2s_1\,e^{t/2i}i$

**Centre manifold ODEs** The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = s_3s_1^2\eta^2\varepsilon^2(-6144i\pi^2 - 4608\pi^3 - 16384\pi)/(81\pi^4+720\pi^2+1024) +$
$s_1\varepsilon^2\zeta(-12i\pi - 16)/(9\pi^2+16)$

$\dot{s}_2 = s_2\varepsilon^2\zeta(24i\pi - 64)/(9\pi^2+64) + s_1^2\eta\varepsilon(-128i - 48\pi)/(9\pi^2+64)$

$\dot{s}_3 = s_3^2s_1\eta^2\varepsilon^2(6144i\pi^2 - 4608\pi^3 - 16384\pi)/(81\pi^4+720\pi^2+1024) +$
$s_3\varepsilon^2\zeta(12i\pi - 16)/(9\pi^2+16)$

$\dot{s}_4 = s_4\varepsilon^2\zeta(-24i\pi - 64)/(9\pi^2+64) + s_3^2\eta\varepsilon(128i - 48\pi)/(9\pi^2+64)$