Invariant manifold of your dynamical system

A. J. Roberts, University of Adelaide
http://orcid.org/0000-0001-8930-1552

8:20am, April 9, 2021

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = u_2$

$\dot{u}_2 = -1/24\varepsilon^2 u_1^4 + 1/2\varepsilon u_1^2 + \sigma u_3$

$\dot{u}_3 = u_4$

$\dot{u}_4 = \varepsilon^2(-1/6\sigma^{-1}u_1^3 - 1/6u_1^3 u_3) + \varepsilon u_1 u_3 + \sigma^{-1}u_1$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{\sigma, i\sigma, -1, -i\}, \mathrm{e}^{it}\}$

$\vec{e}_2 = \{\{\sigma, -i\sigma, -1, i\}, \mathrm{e}^{-it}\}$

$\vec{z}_1 = \{\{1/4\sigma^{-1}, 1/4i\sigma^{-1}, -1/4, -1/4i\}, \mathrm{e}^{it}\}$

$\vec{z}_2 = \{\{1/4\sigma^{-1}, -1/4i\sigma^{-1}, -1/4, 1/4i\}, \mathrm{e}^{-it}\}$

off echo;

The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = \mathrm{e}^{-it} s_2\sigma - 1/5\,\mathrm{e}^{-2it} s_2^2\varepsilon\sigma^2 + \mathrm{e}^{it} s_1\sigma - 1/5\,\mathrm{e}^{2it} s_1^2\varepsilon\sigma^2 + 2s_2s_1\varepsilon\sigma^2$

$u_2 = -\mathrm{e}^{-it} s_2 i\sigma + 2/5\,\mathrm{e}^{-2it} s_2^2\varepsilon i\sigma^2 + \mathrm{e}^{it} s_1 i\sigma - 2/5\,\mathrm{e}^{2it} s_1^2\varepsilon i\sigma^2$

$u_3 = -\mathrm{e}^{-it} s_2 + 3/10\,\mathrm{e}^{-2it} s_2^2\varepsilon\sigma - \mathrm{e}^{it} s_1 + 3/10\,\mathrm{e}^{2it} s_1^2\varepsilon\sigma - s_2s_1\varepsilon\sigma$

$u_4 = \mathrm{e}^{-it} s_2 i - 3/5\,\mathrm{e}^{-2it} s_2^2\varepsilon i\sigma - \mathrm{e}^{it} s_1 i + 3/5\,\mathrm{e}^{2it} s_1^2\varepsilon i\sigma$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = -6/5 s_2 s_1^2\varepsilon^2 i\sigma^2$

$\dot{s}_2 = 6/5 s_2^2 s_1\varepsilon^2 i\sigma^2$