Invariant manifold of your dynamical system

A. J. Roberts, University of Adelaide
http://orcid.org/0000-0001-8930-1552

6:32am, June 22, 2022

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

off echo;

The specified dynamical system

$$\dot u_1 = \varepsilon(-u_1^3u_4 - u_1u_2^2u_4 + u_1u_4) + u_2$$

$$\dot u_2 = \varepsilon(-u_1^2u_2u_4 - u_2^3u_4 + u_2u_4) - u_1$$

$$\dot u_3 = \varepsilon(3u_1^3u_4 + u_1^3 + 3u_1^2u_2 - 3u_1^2u_3u_4 - 3u_2^2u_3u_4) - u_3$$

$$\dot u_4 = 0$$

Invariant subspace basis vectors

$$\vec e_1 = \{\{1/2, 1/2i, 0, 0\}, \exp(it)\}$$

$$\vec e_2 = \{\{1/2, -1/2i, 0, 0\}, \exp(-it)\}$$

$$\vec e_3 = \{\{0, 0, 1, 0\}, \exp(-t)\}$$

$$\vec e_4 = \{\{0, 0, 0, 1\}, \exp(0)\}$$

$$\vec z_1 = \{\{1, i, 0, 0\}, \exp(it)\}$$

$$\vec z_2 = \{\{1, -i, 0, 0\}, \exp(-it)\}$$

$\vec{z}_3 = \{\{0,0,1,0\}, \exp(-t)\}$

$\vec{z}_4 = \{\{0,0,0,1\}, \exp(0)\}$

off echo;

The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = 1/2 \exp(-it)s_2 + 1/2 \exp(it)s_1 + O(\varepsilon^4)$

$u_2 = -1/2 \exp(-it)s_2 i + 1/2 \exp(it)s_1 i + O(\varepsilon^4)$

$u_3 = s_4^3\varepsilon^3(81/32 \exp(-it)s_2^2 s_1 i - 81/32 \exp(-it)s_2^2 s_1 - 243/4000 \exp(-3it)s_2^3 i - 351/4000 \exp(-3it)s_2^3 - 81/32 \exp(it)s_2 s_1^2 i - 81/32 \exp(it)s_2 s_1^2 + 243/4000 \exp(3it)s_1^3 i - 351/4000 \exp(3it)s_1^3) + s_4^2\varepsilon^3(27/16 \exp(-it)s_2^2 s_1 i + 27/400 \exp(-3it)s_2^3 i - 9/100 \exp(-3it)s_2^3 - 27/16 \exp(it)s_2 s_1^2 i - 27/400 \exp(3it)s_1^3 i - 9/100 \exp(3it)s_1^3) + s_4^2\varepsilon^2(-27/16 \exp(-it)s_2^2 s_1 i - 27/400 \exp(-3it)s_2^3 i + 9/100 \exp(-3it)s_2^3 + 27/16 \exp(it)s_2 s_1^2 i + 27/400 \exp(3it)s_1^3 i + 9/100 \exp(3it)s_1^3) + s_4\varepsilon^2(-9/16 \exp(-it)s_2^2 s_1 i - 9/16 \exp(-it)s_2^2 s_1 - 9/80 \exp(-3it)s_2^3 i - 3/80 \exp(-3it)s_2^3 + 9/16 \exp(it)s_2 s_1^2 i - 9/16 \exp(it)s_2 s_1^2 + 9/80 \exp(3it)s_1^3 i - 3/80 \exp(3it)s_1^3) + s_4\varepsilon(9/16 \exp(-it)s_2^2 s_1 i + 9/16 \exp(-it)s_2^2 s_1 + 9/80 \exp(-3it)s_2^3 i + 3/80 \exp(-3it)s_2^3 - 9/16 \exp(it)s_2 s_1^2 i + 9/16 \exp(it)s_2 s_1^2 - 9/80 \exp(3it)s_1^3 i + 3/80 \exp(3it)s_1^3) + \varepsilon(3/8 \exp(-it)s_2^2 s_1 + 1/8 \exp(-3it)s_2^3 + 3/8 \exp(it)s_2 s_1^2 + 1/8 \exp(3it)s_1^3) + \exp(-t)s_3 + O(\varepsilon^4)$

$u_4 = s_4 + O(\varepsilon^4)$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = s_4\varepsilon(-s_2 s_1^2 + s_1) + O(\varepsilon^5)$

$\dot{s}_2 = s_4\varepsilon(-s_2^2 s_1 + s_2) + O(\varepsilon^5)$

$\dot{s}_3 = -3 s_4 \varepsilon s_3 s_2 s_1 + O(\varepsilon^5)$

$\dot{s}_4 = O(\varepsilon^5)$