Invariant manifold of your dynamical system

A. J. Roberts, University of Adelaide
http://orcid.org/0000-0001-8930-1552

2:49pm, November 25, 2021

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

off echo;

The specified dynamical system

$\dot{u}_1 = \varepsilon u_2 u_3$

$\dot{u}_2 = \varepsilon(- c^{-1} r1 u_1 u_2 + c^{-1} r2 u_2^2)$

$\dot{u}_3 = \varepsilon(c^{-1} r1 u_1 u_3 - c^{-1} r2 u_2 u_3 + u_1^2) - c u_3 - u_1$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1, 0, -c^{-1}\}, \exp(0)\}$

$\vec{e}_2 = \{\{0, 1, 0\}, \exp(0)\}$

$\vec{z}_1 = \{\{1, 0, 0\}, \exp(0)\}$

$\vec{z}_2 = \{\{0, 1, 0\}, \exp(0)\}$

off echo;

The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = O(\varepsilon^2) + s_1$

$u_2 = O(\varepsilon^2) + s_2$

$u_3 = \varepsilon(s_2 s_1 c^{-3} r2 - s_2 s_1 c^{-3} + s_1^2 c^{-1} - s_1^2 c^{-3} r1) + O(\varepsilon^2) - s_1 c^{-1}$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = \varepsilon^2(s_2^2 s_1 c^{-3} r2 - s_2^2 s_1 c^{-3} + s_2 s_1^2 c^{-1} - s_2 s_1^2 c^{-3} r1) - \varepsilon s_2 s_1 c^{-1} + O(\varepsilon^3)$

$\dot{s}_2 = \varepsilon(s_2^2 c^{-1} r2 - s_2 s_1 c^{-1} r1) + O(\varepsilon^3)$

Normals to isochrons at the slow manifold Use these vectors: to project initial conditions onto the slow manifold; to project non-autonomous forcing onto the slow evolution; to predict the consequences of modifying the original system; in uncertainty quantification to quantify effects on the model of uncertainties in the original system. The normal vector $\vec{z}_j := (z_{j1}, \ldots, z_{jn})$

$z_{11} = \varepsilon^2(-s_2^2 c^{-4} r2 + 3 s_2^2 c^{-4} - 2 s_2 s_1 c^{-2} + 2 s_2 s_1 c^{-4} r1) + \varepsilon s_2 c^{-2} + O(\varepsilon^3) + 1$

$z_{12} = \varepsilon^2(-s_2 s_1 c^{-4} r2 + s_2 s_1 c^{-4}) + O(\varepsilon^3)$

$z_{13} = 2\varepsilon^2 s_2^2 c^{-3} + \varepsilon s_2 c^{-1} + O(\varepsilon^3)$

$z_{21} = \varepsilon^2 s_2^2 c^{-4} r1 + O(\varepsilon^3)$

$z_{22} = O(\varepsilon^3) + 1$

$z_{23} = \varepsilon^2 s_2^2 c^{-3} r1 + O(\varepsilon^3)$