Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = \varepsilon u_2$

$\dot{u}_2 = -\varepsilon u_1 u_3^2 w_1^2 - \varepsilon u_3^2 w_0 w_1$

$\dot{u}_3 = \varepsilon u_1 u_4 w_1 + u_4 w_0$

$\dot{u}_4 = -\varepsilon u_1 u_3 w_1 - u_3 w_0$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1, 0, 0, 0\}, \exp(0)\}$

$\vec{e}_2 = \{\{0, 1, 0, 0\}, \exp(0)\}$

$\vec{e}_3 = \{\{0, 0, 1/2, 1/2i\}, \exp(itw_0)\}$

$\vec{e}_4 = \{\{0, 0, 1/2, -1/2i\}, \exp(-itw_0)\}$

$\vec{z}_1 = \{\{1, 0, 0, 0\}, \exp(0)\}$

$\vec{z}_2 = \{\{0, 1, 0, 0\}, \exp(0)\}$

$\vec{z}_3 = \{\{0, 0, 1, i\}, \exp(itw_0)\}$

$\vec{z}_4 = \{\{0, 0, 1, -i\}, \exp(-itw_0)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$$u_1 = -1/16 s_4^2 s_2 i \exp(-2itw_0)\varepsilon^3 w_0^{-3} w_1^2 - 1/8 s_4^2 s_1^2 \exp(-2itw_0)\varepsilon^3 w_0^{-3} w_1^3 + s_4^2 s_1(-1/8\exp(-2itw_0)\varepsilon^3 w_0^{-2} w_1^2 + 1/16\exp(-2itw_0)\varepsilon^2 w_0^{-2} w_1^2) + 1/16 s_4^2 \exp(-2itw_0)\varepsilon^2 w_0^{-1} w_1 + 1/16 s_3^2 s_2 i \exp(2itw_0)\varepsilon^3 w_0^{-3} w_1^2 - 1/8 s_3^2 s_1^2 \exp(2itw_0)\varepsilon^3 w_0^{-3} w_1^3 + s_3^2 s_1(-1/8\exp(2itw_0)\varepsilon^3 w_0^{-2} w_1^2 + 1/16\exp(2itw_0)\varepsilon^2 w_0^{-2} w_1^2) + 1/16 s_3^2 \exp(2itw_0)\varepsilon^2 w_0^{-1} w_1 + s_1 + O(\varepsilon^4)$$

$$u_2 = -1/256 s_4^4 s_1 i \exp(-4itw_0)\varepsilon^3 w_0^{-3} w_1^4 - 1/256 s_4^4 i \exp(-4itw_0)\varepsilon^3 w_0^{-2} w_1^3 - 1/32 s_4^3 s_3 s_1 i \exp(-2itw_0)\varepsilon^3 w_0^{-3} w_1^4 - 1/32 s_4^3 s_3 i \exp(-2itw_0)\varepsilon^3 w_0^{-2} w_1^3 + 3/16 s_4^2 s_2 s_1 \exp(-2itw_0)\varepsilon^3 w_0^{-3} w_1^3 + s_4^2 s_2(1/16\exp(-2itw_0)\varepsilon^3 w_0^{-2} w_1^2 - 1/16\exp(-2itw_0)\varepsilon^2 w_0^{-2} w_1^2) - 1/8 s_4^2 s_1^3 i \exp(-2itw_0)\varepsilon^3 w_0^{-3} w_1^4 + s_4^2 s_1^2 i(-1/8\exp(-2itw_0)\varepsilon^3 w_0^{-2} w_1^3 + 1/8\exp(-2itw_0)\varepsilon^2 w_0^{-2} w_1^3) + s_4^2 s_1 i(1/8\exp(-2itw_0)\varepsilon^2 w_0^{-1} w_1^2 - 1/8\exp(-2itw_0)\varepsilon w_0^{-1} w_1^2) - 1/8 s_4^2 i \exp(-2itw_0)\varepsilon w_1 + 1/32 s_4 s_3^3 s_1 i \exp(2itw_0)\varepsilon^3 w_0^{-3} w_1^4 + 1/32 s_4 s_3^3 i \exp(2itw_0)\varepsilon^3 w_0^{-2} w_1^3 + 1/256 s_3^4 s_1 i \exp(4itw_0)\varepsilon^3 w_0^{-3} w_1^4 + 1/256 s_3^4 i \exp(4itw_0)\varepsilon^3 w_0^{-2} w_1^3 + 3/16 s_3^2 s_2 s_1 \exp(2itw_0)\varepsilon^3 w_0^{-3} w_1^3 + s_3^2 s_2(1/16\exp(2itw_0)\varepsilon^3 w_0^{-2} w_1^2 - 1/16\exp(2itw_0)\varepsilon^2 w_0^{-2} w_1^2) + 1/8 s_3^2 s_1^3 i \exp(2itw_0)\varepsilon^3 w_0^{-3} w_1^4 + s_3^2 s_1^2 i(1/8\exp(2itw_0)\varepsilon^3 w_0^{-2} w_1^3 - 1/8\exp(2itw_0)\varepsilon^2 w_0^{-2} w_1^3) + s_3^2 s_1 i(-1/8\exp(2itw_0)\varepsilon^2 w_0^{-1} w_1^2 + 1/8\exp(2itw_0)\varepsilon w_0^{-1} w_1^2) + 1/8 s_3^2 i \exp(2itw_0)\varepsilon w_1 + s_2 + O(\varepsilon^4)$$

$$u_3 = 1/64 s_4^3 s_1 \exp(-3itw_0)\varepsilon^3 w_0^{-3} w_1^3 + 1/64 s_4^3 \exp(-3itw_0)\varepsilon^3 w_0^{-2} w_1^2 - 1/64 s_4^2 s_3 s_1 \exp(-itw_0)\varepsilon^3 w_0^{-3} w_1^3 - 1/64 s_4^2 s_3 \exp(-itw_0)\varepsilon^3 w_0^{-2} w_1^2 - 1/64 s_4 s_3^2 s_1 \exp(itw_0)\varepsilon^3 w_0^{-3} w_1^3 - 1/64 s_4 s_3^2 \exp(itw_0)\varepsilon^3 w_0^{-2} w_1^2 + 1/2 s_4 \exp(-itw_0) + 1/64 s_3^3 s_1 \exp(3itw_0)\varepsilon^3 w_0^{-3} w_1^3 + 1/64 s_3^3 \exp(3itw_0)\varepsilon^3 w_0^{-2} w_1^2 + 1/2 s_3 \exp(itw_0) + O(\varepsilon^4)$$

$$u_4 = -1/64 s_4^3 s_1 i \exp(-3itw_0)\varepsilon^3 w_0^{-3} w_1^3 - 1/64 s_4^3 i \exp(-3itw_0)\varepsilon^3 w_0^{-2} w_1^2 - 1/64 s_4^2 s_3 s_1 i \exp(-itw_0)\varepsilon^3 w_0^{-3} w_1^3 - 1/64 s_4^2 s_3 i \exp(-itw_0)\varepsilon^3 w_0^{-2} w_1^2 + 1/64 s_4 s_3^2 s_1 i \exp(itw_0)\varepsilon^3 w_0^{-3} w_1^3 + 1/64 s_4 s_3^2 i \exp(itw_0)\varepsilon^3 w_0^{-2} w_1^2 - 1/2 s_4 i \exp(-itw_0) + 1/64 s_3^3 s_1 i \exp(3itw_0)\varepsilon^3 w_0^{-3} w_1^3 + 1/64 s_3^3 i \exp(3itw_0)\varepsilon^3 w_0^{-2} w_1^2 + 1/2 s_3 i \exp(itw_0) + O(\varepsilon^4)$$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = s_2\varepsilon + O(\varepsilon^5)$$

$$\dot{s}_2 = 3/32 s_4^2 s_3^2 s_1^2 \varepsilon^4 w_0^{-3} w_1^5 + s_4^2 s_3^2 s_1 (1/8\varepsilon^4 w_0^{-2} w_1^4 - 1/32\varepsilon^3 w_0^{-2} w_1^4) + s_4^2 s_3^2 (1/32\varepsilon^4 w_0^{-1} w_1^3 - 1/32\varepsilon^3 w_0^{-1} w_1^3) - 1/2 s_4 s_3 s_1 \varepsilon w_1^2 - 1/2 s_4 s_3 \varepsilon w_0 w_1 + O(\varepsilon^5)$$

$$\dot{s}_3 = s_3 s_1 i\varepsilon w_1 + O(\varepsilon^5)$$

$$\dot{s}_4 = -s_4 s_1 i\varepsilon w_1 + O(\varepsilon^5)$$