Invariant manifold of your dynamical system

A. J. Roberts, University of Adelaide
http://orcid.org/0000-0001-8930-1552

5:54am, November 6, 2025

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = u_3$

$\dot{u}_2 = u_4$

$\dot{u}_3 = F_1\varepsilon^2(1/2\exp(-it)i - 1/2\exp(it)i) + \varepsilon^2(-1/2u_1^3 - 3/10u_3 + 3/10u_4) - 2u_1 + u_2$

$\dot{u}_4 = F_2\varepsilon^2(3/20\exp((-it)/2)u_3 - 3/10\exp((-it)/2)u_4 + 3/20\exp((it)/2)u_3 - 3/10\exp((it)/2)u_4) + u_1 - 2u_2$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1, 1, i, i\}, \exp(it)\}$

$\vec{e}_2 = \{\{1, 1, -i, -i\}, \exp(-it)\}$

$\vec{z}_1 = \{\{1/4, 1/4, 1/4i, 1/4i\}, \exp(it)\}$

$\vec{z}_2 = \{\{1/4, 1/4, -1/4i, -1/4i\}, \exp(-it)\}$

off echo;

The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$$u_1 = F_1\varepsilon^2(3/16\exp(-it)i - 3/16\exp(it)i) + F_2\varepsilon^2(4/55\exp((-it)/2)s_2 i - 4/25\exp((-3it)/2)s_2 i - 4/55\exp((it)/2)s_1 i + 4/25\exp((3it)/2)s_1 i) + \varepsilon^2(-9/16\exp(-it)s_2^2 s_1 + 7/96\exp(-3it)s_2^3 - 9/16\exp(it)s_2 s_1^2 + 7/96\exp(3it)s_1^3) + \exp(-it)s_2 + \exp(it)s_1 + O(\varepsilon^4)$$

$$u_2 = F_1\varepsilon^2(-1/16\exp(-it)i + 1/16\exp(it)i) + F_2\varepsilon^2(7/55\exp((-it)/2)s_2 i + 1/25\exp((-3it)/2)s_2 i - 7/55\exp((it)/2)s_1 i - 1/25\exp((3it)/2)s_1 i) + \varepsilon^2(3/16\exp(-it)s_2^2 s_1 - 1/96\exp(-3it)s_2^3 + 3/16\exp(it)s_2 s_1^2 - 1/96\exp(3it)s_1^3) + \exp(-it)s_2 + \exp(it)s_1 + O(\varepsilon^4)$$

$$u_3 = F_1\varepsilon^2(1/16\exp(-it) + 1/16\exp(it)) + F_2\varepsilon^2(2/55\exp((-it)/2)s_2 - 6/25\exp((-3it)/2)s_2 + 2/55\exp((it)/2)s_1 - 6/25\exp((3it)/2)s_1) + \varepsilon^2(3/16\exp(-it)s_2^2 s_1 i - 7/32\exp(-3it)s_2^3 i - 3/16\exp(it)s_2 s_1^2 i + 7/32\exp(3it)s_1^3 i) - \exp(-it)s_2 i + \exp(it)s_1 i + O(\varepsilon^4)$$

$$u_4 = F_1\varepsilon^2(-3/16\exp(-it) - 3/16\exp(it)) + F_2\varepsilon^2(7/110\exp((-it)/2)s_2 + 3/50\exp((-3it)/2)s_2 + 7/110\exp((it)/2)s_1 + 3/50\exp((3it)/2)s_1) + \varepsilon^2(-9/16\exp(-it)s_2^2 s_1 i + 1/32\exp(-3it)s_2^3 i + 9/16\exp(it)s_2 s_1^2 i - 1/32\exp(3it)s_1^3 i) - \exp(-it)s_2 i + \exp(it)s_1 i + O(\varepsilon^4)$$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = F_1\varepsilon^4(9/64 s_2 s_1 - 9/128 s_1^2 + 3/160 i) - 1/8 F_1\varepsilon^2 + 93/5500 F_2^2\varepsilon^4 s_1 i + \varepsilon^4(-155/256 s_2^2 s_1^3 i + 9/160 s_2 s_1^2) + 3/8\varepsilon^2 s_2 s_1^2 i + O(\varepsilon^5)$$

$$\dot{s}_2 = F_1\varepsilon^4(-9/128 s_2^2 + 9/64 s_2 s_1 - 3/160 i) - 1/8 F_1\varepsilon^2 - 93/5500 F_2^2\varepsilon^4 s_2 i + \varepsilon^4(155/256 s_2^3 s_1^2 i + 9/160 s_2^2 s_1) - 3/8\varepsilon^2 s_2^2 s_1 i + O(\varepsilon^5)$$