Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot u_1 = u_3$

$\dot u_2 = u_4$

$\dot u_3 = \varepsilon^2\big(\frac{3}{10}u_4 - \frac{3}{10}u_3 - \frac{1}{2}u_1^3\big) + u_2 - 2u_1$

$\dot u_4 = \varepsilon^2\big(- \frac{3}{5}u_4 + \frac{3}{10}u_3\big) - 2u_2 + u_1$

Centre subspace basis vectors

$\vec e_1 = \{\{1, 1, i, i\}, e^{it}\}$

$\vec e_2 = \{\{1, 1, -i, -i\}, e^{-it}\}$

$\vec e_3 = \Big\{\Big\{1, -1, \sqrt{3}i, -\sqrt{3}i\Big\}, e^{\sqrt{3}it}\Big\}$

$\vec e_4 = \Big\{\Big\{1, -1, -\sqrt{3}i, \sqrt{3}i\Big\}, e^{-\sqrt{3}it}\Big\}$

$\vec z_1 = \Big\{\Big\{\frac{1}{4}, \frac{1}{4}, \frac{1}{4}i, \frac{1}{4}i\Big\}, e^{it}\Big\}$

$\vec z_2 = \Big\{\Big\{\frac{1}{4}, \frac{1}{4}, -\frac{1}{4}i, -\frac{1}{4}i\Big\}, e^{-it}\Big\}$

$\vec{z}_3 = \left\{\left\{\frac{1}{4}, -\frac{1}{4}, \frac{1}{12}\sqrt{3}i, -\frac{1}{12}\sqrt{3}i\right\}, e^{\sqrt{3}it}\right\}$

$\vec{z}_4 = \left\{\left\{\frac{1}{4}, -\frac{1}{4}, -\frac{1}{12}\sqrt{3}i, \frac{1}{12}\sqrt{3}i\right\}, e^{-\sqrt{3}it}\right\}$

The Centre manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = \varepsilon^2(\frac{11}{80}\sqrt{3}\,e^{-\sqrt{3}it}s_4 i + \frac{5}{16}\,e^{-\sqrt{3}it}s_4^2 s_3 + \frac{5}{8}\,e^{-\sqrt{3}it}s_4 s_2 s_1 - \frac{9}{8}\,e^{-it}s_4 s_3 s_2 - \frac{9}{16}\,e^{-it}s_2^2 s_1 - \frac{3}{80}\,e^{-it}s_2 i + \frac{25}{1248}\,e^{-3\sqrt{3}it}s_4^3 + \frac{7}{96}\,e^{-3it}s_2^3 + \frac{11}{32}\sqrt{3}\,e^{-\sqrt{3}it-2it}s_4 s_2^2 - \frac{15}{32}\,e^{-\sqrt{3}it-2it}s_4 s_2^2 - \frac{11}{32}\sqrt{3}\,e^{-\sqrt{3}it+2it}s_4 s_1^2 - \frac{15}{32}\,e^{-\sqrt{3}it+2it}s_4 s_1^2 - \frac{37}{416}\sqrt{3}\,e^{-2\sqrt{3}it-it}s_4^2 s_2 + \frac{99}{416}\,e^{-2\sqrt{3}it-it}s_4^2 s_2 + \frac{37}{416}\sqrt{3}\,e^{-2\sqrt{3}it+it}s_4^2 s_1 + \frac{99}{416}\,e^{-2\sqrt{3}it+it}s_4^2 s_1 - \frac{11}{32}\sqrt{3}\,e^{\sqrt{3}it-2it}s_3 s_2^2 - \frac{15}{32}\,e^{\sqrt{3}it-2it}s_3 s_2^2 + \frac{11}{32}\sqrt{3}\,e^{\sqrt{3}it+2it}s_3 s_1^2 - \frac{15}{32}\,e^{\sqrt{3}it+2it}s_3 s_1^2 + \frac{37}{416}\sqrt{3}\,e^{2\sqrt{3}it-it}s_3^2 s_2 + \frac{99}{416}\,e^{2\sqrt{3}it-it}s_3^2 s_2 - \frac{37}{416}\sqrt{3}\,e^{2\sqrt{3}it+it}s_3^2 s_1 + \frac{99}{416}\,e^{2\sqrt{3}it+it}s_3^2 s_1 - \frac{11}{80}\sqrt{3}\,e^{\sqrt{3}it}s_3 i + \frac{5}{16}\,e^{\sqrt{3}it}s_4 s_3^2 + \frac{5}{8}\,e^{\sqrt{3}it}s_3 s_2 s_1 - \frac{9}{8}\,e^{it}s_4 s_3 s_1 - \frac{9}{16}\,e^{it}s_2 s_1^2 + \frac{3}{80}\,e^{it}s_1 i + \frac{25}{1248}\,e^{3\sqrt{3}it}s_3^3 + \frac{7}{96}\,e^{3it}s_1^3) + e^{-\sqrt{3}it}s_4 + e^{-it}s_2 + e^{\sqrt{3}it}s_3 + e^{it}s_1$

$u_2 = \varepsilon^2(\frac{1}{80}\sqrt{3}\,e^{-\sqrt{3}it}s_4 i + \frac{7}{16}\,e^{-\sqrt{3}it}s_4^2 s_3 + \frac{7}{8}\,e^{-\sqrt{3}it}s_4 s_2 s_1 + \frac{3}{8}\,e^{-it}s_4 s_3 s_2 + \frac{3}{16}\,e^{-it}s_2^2 s_1 + \frac{9}{80}\,e^{-it}s_2 i - \frac{1}{1248}\,e^{-3\sqrt{3}it}s_4^3 - \frac{1}{96}\,e^{-3it}s_2^3 + \frac{5}{32}\sqrt{3}\,e^{-\sqrt{3}it-2it}s_4 s_2^2 - \frac{9}{32}\,e^{-\sqrt{3}it-2it}s_4 s_2^2 - \frac{5}{32}\sqrt{3}\,e^{-\sqrt{3}it+2it}s_4 s_1^2 - \frac{9}{32}\,e^{-\sqrt{3}it+2it}s_4 s_1^2 + \frac{11}{416}\sqrt{3}\,e^{-2\sqrt{3}it-it}s_4^2 s_2 - \frac{21}{416}\,e^{-2\sqrt{3}it-it}s_4^2 s_2 - \frac{11}{416}\sqrt{3}\,e^{-2\sqrt{3}it+it}s_4^2 s_1 - \frac{21}{416}\,e^{-2\sqrt{3}it+it}s_4^2 s_1 - \frac{5}{32}\sqrt{3}\,e^{\sqrt{3}it-2it}s_3 s_2^2 - \frac{9}{32}\,e^{\sqrt{3}it-2it}s_3 s_2^2 + \frac{5}{32}\sqrt{3}\,e^{\sqrt{3}it+2it}s_3 s_1^2 - \frac{9}{32}\,e^{\sqrt{3}it+2it}s_3 s_1^2 - \frac{11}{416}\sqrt{3}\,e^{2\sqrt{3}it-it}s_3^2 s_2 - \frac{21}{416}\,e^{2\sqrt{3}it-it}s_3^2 s_2 + \frac{11}{416}\sqrt{3}\,e^{2\sqrt{3}it+it}s_3^2 s_1 - \frac{21}{416}\,e^{2\sqrt{3}it+it}s_3^2 s_1 - \frac{1}{80}\sqrt{3}\,e^{\sqrt{3}it}s_3 i + \frac{7}{16}\,e^{\sqrt{3}it}s_4 s_3^2 + \frac{7}{8}\,e^{\sqrt{3}it}s_3 s_2 s_1 + \frac{3}{8}\,e^{it}s_4 s_3 s_1 + \frac{3}{16}\,e^{it}s_2 s_1^2 - \frac{9}{80}\,e^{it}s_1 i - \frac{1}{1248}\,e^{3\sqrt{3}it}s_3^3 - \frac{1}{96}\,e^{3it}s_1^3) - e^{-\sqrt{3}it}s_4 + e^{-it}s_2 - e^{\sqrt{3}it}s_3 + e^{it}s_1$

$u_3 = \varepsilon^2(- \frac{7}{16}\sqrt{3}\,e^{-\sqrt{3}it}s_4^2 s_3 i - \frac{7}{8}\sqrt{3}\,e^{-\sqrt{3}it}s_4 s_2 s_1 i + \frac{3}{80}\,e^{-\sqrt{3}it}s_4 + \frac{3}{8}\,e^{-it}s_4 s_3 s_2 i + \frac{3}{16}\,e^{-it}s_2^2 s_1 i - \frac{9}{80}\,e^{-it}s_2 - \frac{25}{416}\sqrt{3}\,e^{-3\sqrt{3}it}s_4^3 i - \frac{7}{32}\,e^{-3it}s_2^3 i - \frac{7}{32}\sqrt{3}\,e^{-\sqrt{3}it-2it}s_4 s_2^2 i - \frac{3}{32}\,e^{-\sqrt{3}it-2it}s_4 s_2^2 i - \frac{7}{32}\sqrt{3}\,e^{-\sqrt{3}it+2it}s_4 s_1^2 i + \frac{3}{32}\,e^{-\sqrt{3}it+2it}s_4 s_1^2 i - \frac{161}{416}\sqrt{3}\,e^{-2\sqrt{3}it-it}s_4^2 s_2 i + \frac{123}{416}\,e^{-2\sqrt{3}it-it}s_4^2 s_2 i - \frac{161}{416}\sqrt{3}\,e^{-2\sqrt{3}it+it}s_4^2 s_1 i - \frac{123}{416}\,e^{-2\sqrt{3}it+it}s_4^2 s_1 i + \frac{7}{32}\sqrt{3}\,e^{\sqrt{3}it-2it}s_3 s_2^2 i - \frac{3}{32}\,e^{\sqrt{3}it-2it}s_3 s_2^2 i + \frac{7}{32}\sqrt{3}\,e^{\sqrt{3}it+2it}s_3 s_1^2 i + \frac{3}{32}\,e^{\sqrt{3}it+2it}s_3 s_1^2 i +$

$\frac{161}{416}\sqrt{3}\, e^{2\sqrt{3}it-it} s_3^2 s_2 i + \frac{123}{416}\, e^{2\sqrt{3}it-it} s_3^2 s_2 i + \frac{161}{416}\sqrt{3}\, e^{2\sqrt{3}it+it} s_3^2 s_1 i -$
$\frac{123}{416}\, e^{2\sqrt{3}it+it} s_3^2 s_1 i + \frac{7}{16}\sqrt{3}\, e^{\sqrt{3}it} s_4 s_3^2 i + \frac{7}{8}\sqrt{3}\, e^{\sqrt{3}it} s_3 s_2 s_1 i + \frac{3}{80}\, e^{\sqrt{3}it} s_3 -$
$\frac{3}{8}\, e^{it} s_4 s_3 s_1 i - \frac{3}{16}\, e^{it} s_2 s_1^2 i - \frac{9}{80}\, e^{it} s_1 + \frac{25}{416}\sqrt{3}\, e^{3\sqrt{3}it} s_3^3 i + \frac{7}{32}\, e^{3it} s_1^3 i) -$
$\sqrt{3}\, e^{-\sqrt{3}it} s_4 i - e^{-it} s_2 i + \sqrt{3}\, e^{\sqrt{3}it} s_3 i + e^{it} s_1 i$

$u_4 = \varepsilon^2(- \frac{5}{16}\sqrt{3}\, e^{-\sqrt{3}it} s_4^2 s_3 i - \frac{5}{8}\sqrt{3}\, e^{-\sqrt{3}it} s_4 s_2 s_1 i + \frac{33}{80}\, e^{-\sqrt{3}it} s_4 -$
$\frac{9}{8}\, e^{-it} s_4 s_3 s_2 i - \frac{9}{16}\, e^{-it} s_2^2 s_1 i + \frac{3}{80}\, e^{-it} s_2 + \frac{1}{416}\sqrt{3}\, e^{-3\sqrt{3}it} s_4^3 i + \frac{1}{32}\, e^{-3it} s_2^3 i -$
$\frac{1}{32}\sqrt{3}\, e^{-\sqrt{3}it-2it} s_4 s_2^2 i + \frac{3}{32}\, e^{-\sqrt{3}it-2it} s_4 s_2^2 i - \frac{1}{32}\sqrt{3}\, e^{-\sqrt{3}it+2it} s_4 s_1^2 i -$
$\frac{3}{32}\, e^{-\sqrt{3}it+2it} s_4 s_1^2 i + \frac{31}{416}\sqrt{3}\, e^{-2\sqrt{3}it-it} s_4^2 s_2 i - \frac{45}{416}\, e^{-2\sqrt{3}it-it} s_4^2 s_2 i +$
$\frac{31}{416}\sqrt{3}\, e^{-2\sqrt{3}it+it} s_4^2 s_1 i + \frac{45}{416}\, e^{-2\sqrt{3}it+it} s_4^2 s_1 i + \frac{1}{32}\sqrt{3}\, e^{\sqrt{3}it-2it} s_3 s_2^2 i +$
$\frac{3}{32}\, e^{\sqrt{3}it-2it} s_3 s_2^2 i + \frac{1}{32}\sqrt{3}\, e^{\sqrt{3}it+2it} s_3 s_1^2 i - \frac{3}{32}\, e^{\sqrt{3}it+2it} s_3 s_1^2 i -$
$\frac{31}{416}\sqrt{3}\, e^{2\sqrt{3}it-it} s_3^2 s_2 i - \frac{45}{416}\, e^{2\sqrt{3}it-it} s_3^2 s_2 i - \frac{31}{416}\sqrt{3}\, e^{2\sqrt{3}it+it} s_3^2 s_1 i +$
$\frac{45}{416}\, e^{2\sqrt{3}it+it} s_3^2 s_1 i + \frac{5}{16}\sqrt{3}\, e^{\sqrt{3}it} s_4 s_3^2 i + \frac{5}{8}\sqrt{3}\, e^{\sqrt{3}it} s_3 s_2 s_1 i + \frac{33}{80}\, e^{\sqrt{3}it} s_3 +$
$\frac{9}{8}\, e^{it} s_4 s_3 s_1 i + \frac{9}{16}\, e^{it} s_2 s_1^2 i + \frac{3}{80}\, e^{it} s_1 - \frac{1}{416}\sqrt{3}\, e^{3\sqrt{3}it} s_3^3 i - \frac{1}{32}\, e^{3it} s_1^3 i) +$
$\sqrt{3}\, e^{-\sqrt{3}it} s_4 i - e^{-it} s_2 i - \sqrt{3}\, e^{\sqrt{3}it} s_3 i + e^{it} s_1 i$

Centre manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = \varepsilon^2(\frac{3}{4} s_4 s_3 s_1 i + \frac{3}{8} s_2 s_1^2 i - \frac{3}{40} s_1)$

$\dot{s}_2 = \varepsilon^2(- \frac{3}{4} s_4 s_3 s_2 i - \frac{3}{8} s_2^2 s_1 i - \frac{3}{40} s_2)$

$\dot{s}_3 = \varepsilon^2(\frac{1}{8}\sqrt{3} s_4 s_3^2 i + \frac{1}{4}\sqrt{3} s_3 s_2 s_1 i - \frac{3}{8} s_3)$

$\dot{s}_4 = \varepsilon^2(- \frac{1}{8}\sqrt{3} s_4^2 s_3 i - \frac{1}{4}\sqrt{3} s_4 s_2 s_1 i - \frac{3}{8} s_4)$