Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = u_2$

$\dot{u}_2 = f_1\varepsilon(-1/2\exp(-it) - 1/2\exp(it)) + \varepsilon(-c_1u_2 - 1/5u_1^3 + 3/5u_1^2u_3 - 3/5u_1u_3^2 + 1/5u_3^3) - u_1$

$\dot{u}_3 = u_4$

$\dot{u}_4 = \varepsilon(1/5u_1^3 - 3/5u_1^2u_3 + 3/5u_1u_3^2 - 2/5u_3^3 + 3/5u_3^2u_5 - 3/5u_3u_5^2 - 1/5u_4 + 1/5u_5^3) - u_3$

$\dot{u}_5 = u_6$

$\dot{u}_6 = \varepsilon(1/5u_3^3 - 3/5u_3^2u_5 + 3/5u_3u_5^2 - 1/5u_5^3 - 3/10u_6) - u_5$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1, i, 0, 0, 0, 0\}, \exp(it)\}$

$\vec{e}_2 = \{\{1, -i, 0, 0, 0, 0\}, \exp(-it)\}$

$\vec{e}_3 = \{\{0, 0, 1, i, 0, 0\}, \exp(it)\}$

$\vec{e}_4 = \{\{0, 0, 1, -i, 0, 0\}, \exp(-it)\}$

$\vec{e}_5 = \{\{0, 0, 0, 0, 1, i\}, \exp(it)\}$

$\vec{e}_6 = \{\{0, 0, 0, 0, 1, -i\}, \exp(-it)\}$

$\vec{z}_1 = \{\{1/2, 1/2i, 0, 0, 0, 0\}, \exp(it)\}$

$\vec{z}_2 = \{\{1/2, -1/2i, 0, 0, 0, 0\}, \exp(-it)\}$

$\vec{z}_3 = \{\{0, 0, 1/2, 1/2i, 0, 0\}, \exp(it)\}$

$\vec{z}_4 = \{\{0, 0, 1/2, -1/2i, 0, 0\}, \exp(-it)\}$

$\vec{z}_5 = \{\{0, 0, 0, 0, 1/2, 1/2i\}, \exp(it)\}$

$\vec{z}_6 = \{\{0, 0, 0, 0, 1/2, -1/2i\}, \exp(-it)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$u_1 = \exp(-it)s_2 + \exp(it)s_1 + O(\varepsilon)$

$u_2 = i(-\exp(-it)s_2 + \exp(it)s_1) + O(\varepsilon)$

$u_3 = \exp(-it)s_4 + \exp(it)s_3 + O(\varepsilon)$

$u_4 = i(-\exp(-it)s_4 + \exp(it)s_3) + O(\varepsilon)$

$u_5 = \exp(-it)s_6 + \exp(it)s_5 + O(\varepsilon)$

$u_6 = i(-\exp(-it)s_6 + \exp(it)s_5) + O(\varepsilon)$

$u_1 = \exp(-it)s_2 + \exp(it)s_1 + O(\varepsilon)$

$u_2 = i(-\exp(-it)s_2 + \exp(it)s_1) + O(\varepsilon)$

$u_3 = \exp(-it)s_4 + \exp(it)s_3 + O(\varepsilon)$

$u_4 = i(-\exp(-it)s_4 + \exp(it)s_3) + O(\varepsilon)$

$u_5 = \exp(-it)s_6 + \exp(it)s_5 + O(\varepsilon)$

$u_6 = i(-\exp(-it)s_6 + \exp(it)s_5) + O(\varepsilon)$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = 1/4f_1i\varepsilon + i\varepsilon(- 3/10s_4s_3^2 + 3/5s_4s_3s_1 - 3/10s_4s_1^2 + 3/10s_3^2s_2 - 3/5s_3s_2s_1 + 3/10s_2s_1^2) - 1/2\varepsilon s_1c_1 + O(\varepsilon^2)$

$\dot{s}_2 = -1/4f_1i\varepsilon + i\varepsilon(3/10s_4^2s_3 - 3/10s_4^2s_1 - 3/5s_4s_3s_2 + 3/5s_4s_2s_1 + 3/10s_3s_2^2 - 3/10s_2^2s_1) - 1/2\varepsilon s_2c_1 + O(\varepsilon^2)$

$\dot{s}_3 = i\varepsilon(- 3/10s_6s_5^2 + 3/5s_6s_5s_3 - 3/10s_6s_3^2 + 3/10s_5^2s_4 - 3/5s_5s_4s_3 + 3/5s_4s_3^2 - 3/5s_4s_3s_1 + 3/10s_4s_1^2 - 3/10s_3^2s_2 + 3/5s_3s_2s_1 - 3/10s_2s_1^2) - 1/10\varepsilon s_3 + O(\varepsilon^2)$

$\dot{s}_4 = i\varepsilon(3/10s_6^2s_5 - 3/10s_6^2s_3 - 3/5s_6s_5s_4 + 3/5s_6s_4s_3 + 3/10s_5s_4^2 - 3/5s_4^2s_3 + 3/10s_4^2s_1 + 3/5s_4s_3s_2 - 3/5s_4s_2s_1 - 3/10s_3s_2^2 + 3/10s_2^2s_1) - 1/10\varepsilon s_4 + O(\varepsilon^2)$

$\dot{s}_5 = i\varepsilon(3/10s_6s_5^2 - 3/5s_6s_5s_3 + 3/10s_6s_3^2 - 3/10s_5^2s_4 + 3/5s_5s_4s_3 - 3/10s_4s_3^2) - 3/20\varepsilon s_5 + O(\varepsilon^2)$

$\dot{s}_6 = i\varepsilon(- 3/10s_6^2s_5 + 3/10s_6^2s_3 + 3/5s_6s_5s_4 - 3/5s_6s_4s_3 - 3/10s_5s_4^2 + 3/10s_4^2s_3) - 3/20\varepsilon s_6 + O(\varepsilon^2)$

$\dot{s}_1 = 0.25f_1i\varepsilon + i\varepsilon(- 0.3s_4s_3^2 + 0.6s_4s_3s_1 - 0.3s_4s_1^2 + 0.3s_3^2s_2 - 0.6s_3s_2s_1 + 0.3s_2s_1^2) - 0.5\varepsilon s_1c_1 + O(\varepsilon^2)$

$\dot{s}_2 = -0.25f_1i\varepsilon + i\varepsilon(0.3s_4^2s_3 - 0.3s_4^2s_1 - 0.6s_4s_3s_2 + 0.6s_4s_2s_1 + 0.3s_3s_2^2 - 0.3s_2^2s_1) - 0.5\varepsilon s_2c_1 + O(\varepsilon^2)$

$\dot{s}_3 = i\varepsilon(- 0.3s_6s_5^2 + 0.6s_6s_5s_3 - 0.3s_6s_3^2 + 0.3s_5^2s_4 - 0.6s_5s_4s_3 + 0.6s_4s_3^2 - 0.6s_4s_3s_1 + 0.3s_4s_1^2 - 0.3s_3^2s_2 + 0.6s_3s_2s_1 - 0.3s_2s_1^2) - 0.1\varepsilon s_3 + O(\varepsilon^2)$

$\dot{s}_4 = i\varepsilon(0.3s_6^2s_5 - 0.3s_6^2s_3 - 0.6s_6s_5s_4 + 0.6s_6s_4s_3 + 0.3s_5s_4^2 - 0.6s_4^2s_3 + 0.3s_4^2s_1 + 0.6s_4s_3s_2 - 0.6s_4s_2s_1 - 0.3s_3s_2^2 + 0.3s_2^2s_1) - 0.1\varepsilon s_4 + O(\varepsilon^2)$

$\dot{s}_5 = i\varepsilon(0.3s_6s_5^2 - 0.6s_6s_5s_3 + 0.3s_6s_3^2 - 0.3s_5^2s_4 + 0.6s_5s_4s_3 - 0.3s_4s_3^2) - 0.15\varepsilon s_5 + O(\varepsilon^2)$

$\dot{s}_6 = i\varepsilon(- 0.3s_6^2s_5 + 0.3s_6^2s_3 + 0.6s_6s_5s_4 - 0.6s_6s_4s_3 - 0.3s_5s_4^2 + 0.3s_4^2s_3) - 0.15\varepsilon s_6 + O(\varepsilon^2)$