Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = \varepsilon(- u_1 + u_2 u_3 - u_3) + 3u_1 + u_2 + 3u_3$

$\dot{u}_2 = \varepsilon(- u_1 u_3 + u_1 + u_3) - u_2$

$\dot{u}_3 = \varepsilon(- u_1 u_2 + u_1 + u_3) - 4u_1 - u_2 - 4u_3$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1, -1, -1\}, \mathrm{e}^{-t}\}$

$\vec{e}_2 = \{\{2/5, 7/5, -1\}, \mathrm{e}^{-t}\}$

$\vec{z}_1 = \{\{5/3, 0, 2/3\}, \mathrm{e}^{-t}\}$

$\vec{z}_2 = \{\{-5/3, 0, -5/3\}, \mathrm{e}^{-t}\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$$u_1 = \varepsilon(-51/25\,e^{-2t}s_2^2 - 6/5\,e^{-2t}s_2s_1 + 3\,e^{-2t}s_1^2) + 2/5\,e^{-t}s_2 + e^{-t}s_1$$

$$u_2 = \varepsilon(-2/5\,e^{-2t}s_2^2 - 7/5\,e^{-2t}s_2s_1 - e^{-2t}s_1^2) + 7/5\,e^{-t}s_2 - e^{-t}s_1$$

$$u_3 = \varepsilon(4\,e^{-2t}s_2^2 + 13/5\,e^{-2t}s_2s_1 - 5\,e^{-2t}s_1^2) - e^{-t}s_2 - e^{-t}s_1$$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = 3/5\varepsilon s_2$$

$$\dot{s}_2 = 0$$