Centre manifold of your dynamical system

A. J. Roberts, University of Adelaide
http://www.maths.adelaide.edu.au/anthony.roberts

5:34 A.M., November 14, 2017

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$$\dot u_1 = \varepsilon(k1u_1u_2 + k2u_1u_3 - u_1u_4) - b1u_1 - b2u_1 + u_4$$

$$\dot u_2 = c1\varepsilon u_3u_5 - c2u_2 + \varepsilon(-k1u_1u_2 + k3u_2u_3 - u_1u_2 - u_2^2 - u_2u_3 - u_2u_4) + b1u_1$$

$$\dot u_3 = -c1\varepsilon u_3u_5 + c2u_2 + \varepsilon(-k2u_1u_3 - k3u_2u_3) + b2u_1$$

$$\dot u_4 = \varepsilon u_1u_4 - u_4$$

$$\dot u_5 = 0$$

```
ff_:=tp mat((-(b1+b2)*u1+u4*(1-u1)+k1*u1*u2+k2*u1*u3
,b1*u1-u2*(u1+u2+u3+u4)-c2*u2+u5*c1*u3-k1*u1*u2+k3*u2*u3
,b2*u1+c2*u2-u5*c1*u3-k2*u1*u3-k3*u2*u3
,-u4*(1-u1),
0));
freqm_:=mat((0,0));
ee_:=tp mat((0,0,1,0,0),(0,0,0,0,1));
zz_:=tp mat((1,1,1,1,0),(0,0,0,0,1));
```

Centre subspace basis vectors

$$\vec e_1 = \{\{0,0,1,0,0\}, e^{0i}\}$$

$$\vec e_2 = \{\{0,0,0,0,1\}, e^{0i}\}$$

$$\vec z_1 = \{\{1,1,1,1,0\}, e^{0i}\}$$

$$\vec z_2 = \{\{0,0,0,0,1\}, e^{0i}\}$$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$u_1 = 0$

$u_2 = c2^{-1} s_2 s_1 c1\varepsilon$

$u_3 = -c2^{-1} s_2 s_1 c1\varepsilon + s_1$

$u_4 = 0$

$u_5 = s_2$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = -c2^{-1} s_2 s_1^2 c1\varepsilon^2$

$\dot{s}_2 = 0$

Normals to isochrons at the slow manifold Use these vectors: to project initial conditions onto the slow manifold; to project non-autonomous forcing onto the slow evolution; to predict the consequences of modifying the original system; in uncertainty quantification to quantify effects on the model of uncertainties in the original system. The normal vector $\vec{z}_j := (z_{j1}, \ldots, z_{jn})$

$z_{11} = (c2^{-1} s_2 s_1 c1\varepsilon^2)/(b1 + b2) + c2^{-2} s_2 s_1 c1\varepsilon^2 (2b1 + b2)/(b1 + b2) + (-c2^{-1} s_1^2 \varepsilon^2 b1k2)/(b1^2 + 2b1b2 + b2^2) + c2^{-2} s_1^2 \varepsilon^2 (-b1k3 + b1)/(b1 + b2) + (-c2^{-1} s_1 \varepsilon b1)/(b1 + b2) + 1$

$z_{12} = 2c2^{-2} s_2 s_1 c1\varepsilon^2 + c2^{-2} s_1^2 \varepsilon^2 (-k3 + 1) - c2^{-1} s_1 \varepsilon + 1$

$z_{13} = c2^{-2} s_2 s_1 c1\varepsilon^2 + 1$

$z_{14} = c2^{-1} s_2 s_1 c1\varepsilon^2 (b1^2 + 2b1b2 + b1 + b2^2 + b2)/(b1^2 + 2b1b2 + b2^2) + c2^{-2} s_2 s_1 c1\varepsilon^2 (2b1^2 + 3b1b2 + b2^2)/(b1^2 + 2b1b2 + b2^2) + (-c2^{-1} s_1^2 \varepsilon^2 b1k2)/(b1^2 + 2b1b2 + b2^2) + c2^{-2} s_1^2 \varepsilon^2 (-b1k3 + b1)/(b1 + b2) + (-c2^{-1} s_1 \varepsilon b1)/(b1 + b2) + 1$

$$z_{15} = c2^{-2} s_1^2 c1 \varepsilon^2$$

$$z_{21} = 0$$

$$z_{22} = 0$$

$$z_{23} = 0$$

$$z_{24} = 0$$

$$z_{25} = 1$$