Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = -5/3\varepsilon \frac{d\, u_2}{d\, x}$

$\dot{u}_2 = \varepsilon\left(- \frac{d\, u_1}{d\, x} - \frac{d\, u_3}{d\, x}\right)$

$\dot{u}_3 = -4/3\varepsilon \frac{d\, u_2}{d\, x} - \epsilon^{-1} u_3$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1, 0, 0\}, \exp(0)\}$

$\vec{e}_2 = \{\{0, 1, 0\}, \exp(0)\}$

$\vec{z}_1 = \{\{1, 0, 0\}, \exp(0)\}$

$\vec{z}_2 = \{\{0, 1, 0\}, \exp(0)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters $s_j := u_j$—the pressure and velocity, and their gradients.

$$u_1 = O(\varepsilon^4) + s_1$$

$$u_2 = O(\varepsilon^4) + s_2$$

$$u_3 = -4/9\varepsilon^3 \frac{d^3 s_2}{dx^3}\epsilon^3 - 4/3\varepsilon^2 \frac{d^2 s_1}{dx^2}\epsilon^2 - 4/3\varepsilon \frac{d\, s_2}{d\, x}\epsilon + O(\varepsilon^4)$$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = -5/3\varepsilon \frac{d\, s_2}{d\, x} + O(\varepsilon^5)$$

$$\dot{s}_2 = 4/9\varepsilon^4 \frac{d^4 s_2}{dx^4}\epsilon^3 + 4/3\varepsilon^3 \frac{d^3 s_1}{dx^3}\epsilon^2 + 4/3\varepsilon^2 \frac{d^2 s_2}{dx^2}\epsilon - \varepsilon \frac{d\, s_1}{d\, x} + O(\varepsilon^5)$$

Normals to isochrons at the slow manifold Use these vectors: to project initial conditions onto the slow manifold; to project non-autonomous forcing onto the slow evolution; to predict the consequences of modifying the original system; in uncertainty quantification to quantify effects on the model of uncertainties in the original system. The normal vector $\vec{z}_j := (z_{j1}, \ldots, z_{jn})$

$$z_{11} = -20/9\varepsilon^4 \partial_x^4 \epsilon^4 + O(\varepsilon^5) + 1$$

$$z_{12} = -20/9\varepsilon^3 \partial_x^3 \epsilon^3 + O(\varepsilon^5)$$

$$z_{13} = 35/9\varepsilon^4 \partial_x^4 \epsilon^4 - 5/3\varepsilon^2 \partial_x^2 \epsilon^2 + O(\varepsilon^5)$$

$$z_{21} = -4/3\varepsilon^3 \partial_x^3 \epsilon^3 + O(\varepsilon^5)$$

$$z_{22} = 8/9\varepsilon^4 \partial_x^4 \epsilon^4 - 4/3\varepsilon^2 \partial_x^2 \epsilon^2 + O(\varepsilon^5) + 1$$

$$z_{23} = \varepsilon^3 \partial_x^3 \epsilon^3 - \varepsilon \partial_x \epsilon + O(\varepsilon^5)$$