From section 8.7.2 in "Acoustics of Musical Instruments" by Chaigne and Kergomard.

Centre manifold of your dynamical system

A. J. Roberts, University of Adelaide
http://www.maths.adelaide.edu.au/anthony.roberts

12:13 A.M., May 26, 2016

Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$$\dot u_1 = u_2$$

$$\dot u_2 = \varepsilon^2(-3/2u_1^3 - 3/2u_1u_3^2) + \varepsilon(-3/2u_1^2 - 2u_1u_3 - 1/2u_3^2) - u_1$$

$$\dot u_3 = u_4$$

$$\dot u_4 = \varepsilon^2(-u_1^2u_3 - u_3^3) + \varepsilon(-u_1^2 - u_1u_3 - 3u_3^2) - 2u_3$$

Centre subspace basis vectors

$$\vec e_1 = \{\{1, i, 0, 0\}, e^{ti}\}$$

$$\vec e_2 = \{\{1, -i, 0, 0\}, e^{-ti}\}$$

$$\vec e_3 = \left\{\left\{0, 0, 1, \sqrt{2}i\right\}, e^{\sqrt{2}ti}\right\}$$

$$\vec e_4 = \left\{\left\{0, 0, 1, -\sqrt{2}i\right\}, e^{-\sqrt{2}ti}\right\}$$

$$\vec z_1 = \{\{1/2, 1/2i, 0, 0\}, e^{ti}\}$$

$$\vec z_2 = \{\{1/2, -1/2i, 0, 0\}, e^{-ti}\}$$

$$\vec z_3 = \left\{\left\{0, 0, 1/3, 1/3\sqrt{2}i\right\}, e^{\sqrt{2}ti}\right\}$$

$\vec{z}_4 = \left\{ \left\{ 0, 0, 1/3, -1/3\sqrt{2}i \right\}, e^{-\sqrt{2}ti} \right\}$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$u_1 = \varepsilon(1/14\, e^{-2\sqrt{2}ti} s_4^2 + 1/2\, e^{-2ti} s_2^2 + \sqrt{2}\, e^{-\sqrt{2}t-ti} s_4 s_2 - e^{-\sqrt{2}t-ti} s_4 s_2 - \sqrt{2}\, e^{-\sqrt{2}t+ti} s_4 s_1 - e^{-\sqrt{2}t+ti} s_4 s_1 - \sqrt{2}\, e^{\sqrt{2}t-ti} s_3 s_2 - e^{\sqrt{2}t-ti} s_3 s_2 + \sqrt{2}\, e^{\sqrt{2}t+ti} s_3 s_1 - e^{\sqrt{2}t+ti} s_3 s_1 + 1/14\, e^{2\sqrt{2}ti} s_3^2 + 1/2\, e^{2ti} s_1^2 - s_4 s_3 - 3 s_2 s_1) + e^{-ti} s_2 + e^{ti} s_1$

$u_2 = \varepsilon(- 1/7\sqrt{2}\, e^{-2\sqrt{2}ti} s_4^2 i - e^{-2ti} s_2^2 i - e^{-\sqrt{2}t-ti} s_4 s_2 i + e^{-\sqrt{2}t+ti} s_4 s_1 i - e^{\sqrt{2}t-ti} s_3 s_2 i + e^{\sqrt{2}t+ti} s_3 s_1 i + 1/7\sqrt{2}\, e^{2\sqrt{2}ti} s_3^2 i + e^{2ti} s_1^2 i) - e^{-ti} s_2 i + e^{ti} s_1 i$

$u_3 = \varepsilon(1/2\, e^{-2\sqrt{2}ti} s_4^2 + 1/2\, e^{-2ti} s_2^2 + 2/7\sqrt{2}\, e^{-\sqrt{2}t-ti} s_4 s_2 - 1/7\, e^{-\sqrt{2}t-ti} s_4 s_2 - 2/7\sqrt{2}\, e^{-\sqrt{2}t+ti} s_4 s_1 - 1/7\, e^{-\sqrt{2}t+ti} s_4 s_1 - 2/7\sqrt{2}\, e^{\sqrt{2}t-ti} s_3 s_2 - 1/7\, e^{\sqrt{2}t-ti} s_3 s_2 + 2/7\sqrt{2}\, e^{\sqrt{2}t+ti} s_3 s_1 - 1/7\, e^{\sqrt{2}t+ti} s_3 s_1 + 1/2\, e^{2\sqrt{2}ti} s_3^2 + 1/2\, e^{2ti} s_1^2 - 3 s_4 s_3 - s_2 s_1) + e^{-\sqrt{2}ti} s_4 + e^{\sqrt{2}ti} s_3$

$u_4 = \varepsilon(- \sqrt{2}\, e^{-2\sqrt{2}ti} s_4^2 i - e^{-2ti} s_2^2 i - 1/7\sqrt{2}\, e^{-\sqrt{2}t-ti} s_4 s_2 i - 3/7\, e^{-\sqrt{2}t-ti} s_4 s_2 i - 1/7\sqrt{2}\, e^{-\sqrt{2}t+ti} s_4 s_1 i + 3/7\, e^{-\sqrt{2}t+ti} s_4 s_1 i + 1/7\sqrt{2}\, e^{\sqrt{2}t-ti} s_3 s_2 i - 3/7\, e^{\sqrt{2}t-ti} s_3 s_2 i + 1/7\sqrt{2}\, e^{\sqrt{2}t+ti} s_3 s_1 i + 3/7\, e^{\sqrt{2}t+ti} s_3 s_1 i + \sqrt{2}\, e^{2\sqrt{2}ti} s_3^2 i + e^{2ti} s_1^2 i) - \sqrt{2}\, e^{-\sqrt{2}ti} s_4 i + \sqrt{2}\, e^{\sqrt{2}ti} s_3 i$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = \varepsilon^2(- 36/7 s_4 s_3 s_1 i - 2 s_2 s_1^2 i)$

$\dot{s}_2 = \varepsilon^2(36/7 s_4 s_3 s_2 i + 2 s_2^2 s_1 i)$

$\dot{s}_3 = \varepsilon^2(- 181/56\sqrt{2} s_4 s_3^2 i - 79/28\sqrt{2} s_3 s_2 s_1 i)$

$\dot{s}_4 = \varepsilon^2(181/56\sqrt{2} s_4^2 s_3 i + 79/28\sqrt{2} s_4 s_2 s_1 i)$