Centre manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = \varepsilon u_2 u_3$

$\dot{u}_2 = \varepsilon u_1 u_3 - u_1 u_4$

$\dot{u}_3 = 0$

$\dot{u}_4 = u_5$

$\dot{u}_5 = -u_4$

Centre subspace basis vectors

$\vec{e}_1 = \{\{1, 0, 0, 0, 0\}, e^{0i}\}$

$\vec{e}_2 = \{\{0, 1, 0, 0, 0\}, e^{0i}\}$

$\vec{e}_3 = \{\{0, 0, 1, 0, 0\}, e^{0i}\}$

$\vec{e}_4 = \{\{0, 0, 0, 1, i\}, e^{ti}\}$

$\vec{e}_5 = \{\{0, 0, 0, 1, -i\}, e^{-ti}\}$

$\vec{z}_1 = \{\{1, 0, 0, 0, 0\}, e^{0i}\}$

$\vec{z}_2 = \{\{0, 1, 0, 0, 0\}, e^{0i}\}$

$\vec{z}_3 = \{\{0, 0, 1, 0, 0\}, e^{0i}\}$

$\vec{z}_4 = \{\{0, 0, 0, 1/2, 1/2i\}, e^{ti}\}$

$\vec{z}_5 = \{\{0, 0, 0, 1/2, -1/2i\}, e^{-ti}\}$

The centre manifold These give the location of the centre manifold in terms of parameters s_j.

$$u_1 = s_3^3\varepsilon^3(1/36\,e^{-3ti}s_5^3 s_1 - 3/4\,e^{-2ti}s_5^2 s_2 i + 25/4\,e^{-ti}s_5^2 s_4 s_1 - 4\,e^{-ti}s_5 s_1 + 1/36\,e^{3ti}s_4^3 s_1 + 3/4\,e^{2ti}s_4^2 s_2 i + 25/4\,e^{ti}s_5 s_4^2 s_1 - 4\,e^{ti}s_4 s_1) + s_3^2\varepsilon^2(1/4\,e^{-2ti}s_5^2 s_1 - 2\,e^{-ti}s_5 s_2 i + 1/4\,e^{2ti}s_4^2 s_1 + 2\,e^{ti}s_4 s_2 i) + s_3\varepsilon(\,e^{-ti}s_5 s_1 + \,e^{ti}s_4 s_1) + s_1$$

$$u_2 = s_3^3\varepsilon^3(\,-1/144\,e^{-4ti}s_5^4 s_1 i - 5/18\,e^{-3ti}s_5^3 s_2 - 79/18\,e^{-2ti}s_5^3 s_4 s_1 i + 11/4\,e^{-2ti}s_5^2 s_1 i - 3\,e^{-ti}s_5^2 s_4 s_2 + 4\,e^{-ti}s_5 s_2 + 1/144\,e^{4ti}s_4^4 s_1 i - 5/18\,e^{3ti}s_4^3 s_2 + 79/18\,e^{2ti}s_5 s_4^3 s_1 i - 11/4\,e^{2ti}s_4^2 s_1 i - 3\,e^{ti}s_5 s_4^2 s_2 + 4\,e^{ti}s_4 s_2) + s_3^2\varepsilon^2(\,-1/12\,e^{-3ti}s_5^3 s_1 i - 5/4\,e^{-2ti}s_5^2 s_2 - 9/4\,e^{-ti}s_5^2 s_4 s_1 i + 2\,e^{-ti}s_5 s_1 i + 1/12\,e^{3ti}s_4^3 s_1 i - 5/4\,e^{2ti}s_4^2 s_2 + 9/4\,e^{ti}s_5 s_4^2 s_1 i - 2\,e^{ti}s_4 s_1 i) + s_3\varepsilon(\,-1/2\,e^{-2ti}s_5^2 s_1 i - \,e^{-ti}s_5 s_2 + 1/2\,e^{2ti}s_4^2 s_1 i - \,e^{ti}s_4 s_2) - \,e^{-ti}s_5 s_1 i + \,e^{ti}s_4 s_1 i + s_2$$

$u_3 = s_3$

$u_4 = \,e^{-ti}s_5 + \,e^{ti}s_4$

$u_5 = -\,e^{-ti}s_5 i + \,e^{ti}s_4 i$

Centre manifold ODEs The system evolves on the centre manifold such that the parameters evolve according to these ODEs.

$\dot{s}_1 = s_3\varepsilon s_2$

$\dot{s}_2 = s_3^3\varepsilon^3(\,-25/2 s_5^2 s_4^2 s_1 + 8 s_5 s_4 s_1) + s_3\varepsilon(\,-2 s_5 s_4 s_1 + s_1)$

$\dot{s}_3 = 0$

$\dot{s}_4 = 0$

$\dot{s}_5 = 0$