Normal form of your dynamical system

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Specified dynamical system

$\dot{x}_1 =$
$\varepsilon^2(-3/20x_2x_1 - 1/20x_2y_1 - 1/20x_1y_2 + 7/20x_1 - 3/20y_2y_1 + 1/20y_1)$

$\dot{x}_2 = \varepsilon^2(3/20x_2^2 + 1/4x_2x_1 - 3/10x_2y_2 + 1/4x_2y_1 - 3/20x_2 +$
$1/4x_1y_2 + 3/20y_2^2 + 1/4y_2y_1 + 1/20y_2)$

$\dot{y}_1 = \sigma(w_2x_1 - w_2y_1 - w_1x_1 - w_1y_1) + \varepsilon^2(-1/20x_2x_1 - 3/20x_2y_1 -$
$3/20x_1y_2 + 1/20x_1 - 1/20y_2y_1 + 7/20y_1) - 4y_1$

$\dot{y}_2 =$
$\sigma(-w_2x_2 - w_2y_2 + w_1x_2 - w_1y_2) + \varepsilon^2(-3/20x_2^2 + 1/4x_2x_1 + 3/10x_2y_2 +$
$1/4x_2y_1 + 1/20x_2 + 1/4x_1y_2 - 3/20y_2^2 + 1/4y_2y_1 - 3/20y_2) - 4y_2$

Time dependent coordinate transform

$y_1 = \sigma\varepsilon^2(3/20e^{-4t}\star e^{-4t}\star w_2\ X_2X_1 - 3/20e^{-4t}\star e^{-4t}\star w_1\ X_2X_1 -$
$1/32e^{4t}\star w_2\ Y_2Y_1 + 1/80e^{-4t}\star w_2\ X_2X_1 - 1/80e^{-4t}\star w_2\ X_2Y_1 +$
$1/80e^{-4t}\star w_2\ X_1Y_2 - 1/80e^{-4t}\star w_2\ X_1 + 1/160e^{4t}\star w_1\ Y_2Y_1 +$
$1/80e^{-4t}\star w_1\ X_2X_1 + 1/80e^{-4t}\star w_1\ X_2Y_1 - 1/80e^{-4t}\star w_1\ X_1Y_2 -$

$1/80e^{-4t}\star w_1\, X_1) + \sigma(e^{-4t}\star w_2\, X_1 - e^{-4t}\star w_1\, X_1) + \varepsilon^2(\,-1/80X_2X_1 + 1/80X_1 + 1/80Y_2Y_1) + Y_1$

$y_2 = \sigma\varepsilon^2(\,-3/20e^{-4t}\star e^{-4t}\star w_2\; X_2^2 + 1/4e^{-4t}\star e^{-4t}\star w_2\; X_2X_1 + 3/20e^{-4t}\star e^{-4t}\star w_1\; X_2^2 - 1/4e^{-4t}\star e^{-4t}\star w_1\; X_2X_1 - 9/160e^{4t}\star w_2\, Y_2^2 + 1/32e^{4t}\star w_2\, Y_2Y_1 + 3/80e^{-4t}\star w_2\, X_2^2 - 1/16e^{-4t}\star w_2\, X_2X_1 - 3/40e^{-4t}\star w_2\, X_2Y_2 + 1/16e^{-4t}\star w_2\, X_2Y_1 - 1/80e^{-4t}\star w_2\, X_2 - 1/16e^{-4t}\star w_2\, X_1Y_2 - 3/160e^{4t}\star w_1\, Y_2^2 + 3/32e^{4t}\star w_1\, Y_2Y_1 + 3/80e^{-4t}\star w_1\, X_2^2 - 1/16e^{-4t}\star w_1\, X_2X_1 + 3/40e^{-4t}\star w_1\, X_2Y_2 - 1/16e^{-4t}\star w_1\, X_2Y_1 - 1/80e^{-4t}\star w_1\, X_2 + 1/16e^{-4t}\star w_1\, X_1Y_2) + \sigma(\,-e^{-4t}\star w_2\, X_2 + e^{-4t}\star w_1\, X_2) + \varepsilon^2(\,-3/80X_2^2 + 1/16X_2X_1 + 1/80X_2 + 3/80Y_2^2 - 1/16Y_2Y_1) + Y_2$

$x_1 = \sigma\varepsilon^2(\,-3/80e^{8t}\star w_2\, Y_2Y_1 - 1/32e^{4t}\star w_2\, X_2Y_1 + 1/160e^{4t}\star w_2\, X_1Y_2 + 1/80e^{4t}\star w_2\, Y_1 - 3/160e^{-4t}\star w_2\, X_2Y_1 + 3/160e^{-4t}\star w_2\, X_1Y_2 - 1/80e^{-4t}\star w_2\, X_1 - 3/80e^{8t}\star w_1\, Y_2Y_1 + 1/160e^{4t}\star w_1\, X_2Y_1 - 1/32e^{4t}\star w_1\, X_1Y_2 + 1/80e^{4t}\star w_1\, Y_1 + 3/160e^{-4t}\star w_1\, X_2Y_1 - 3/160e^{-4t}\star w_1\, X_1Y_2 + 1/80e^{-4t}\star w_1\, X_1) + \varepsilon^2(1/80X_2Y_1 + 1/80X_1Y_2 + 3/160Y_2Y_1 - 1/80Y_1) + X_1$

$x_2 = \sigma\varepsilon^2(3/80e^{8t}\star w_2\, Y_2^2 + 1/16e^{8t}\star w_2\, Y_2Y_1 - 3/80e^{4t}\star w_2\, X_2Y_2 + 3/32e^{4t}\star w_2\, X_2Y_1 + 1/32e^{4t}\star w_2\, X_1Y_2 + 1/80e^{4t}\star w_2\, Y_2 - 3/40e^{-4t}\star w_2\, X_2^2 + 3/80e^{-4t}\star w_2\, X_2Y_2 + 1/32e^{-4t}\star w_2\, X_2Y_1 + 1/80e^{-4t}\star w_2\, X_2 - 1/32e^{-4t}\star w_2\, X_1Y_2 + 3/80e^{8t}\star w_1\, Y_2^2 + 1/16e^{8t}\star w_1\, Y_2Y_1 - 9/80e^{4t}\star w_1\, X_2Y_2 + 1/32e^{4t}\star w_1\, X_2Y_1 + 3/32e^{4t}\star w_1\, X_1Y_2 + 1/80e^{4t}\star w_1\, Y_2 + 3/40e^{-4t}\star w_1\, X_2^2 - 3/80e^{-4t}\star w_1\, X_2Y_2 - 1/32e^{-4t}\star w_1\, X_2Y_1 - 1/80e^{-4t}\star w_1\, X_2 + 1/32e^{-4t}\star w_1\, X_1Y_2) + \varepsilon^2(3/40X_2Y_2 - 1/16X_2Y_1 - 1/16X_1Y_2 - 3/160Y_2^2 - 1/32Y_2Y_1 - 1/80Y_2) + X_2$

Result normal form DEs

$\dot{Y}_1 =$
$\sigma^2\varepsilon^2(-1/32e^{4t}\star w_2\, w_2X_2Y_1 + 1/160e^{4t}\star w_2\, w_2X_1Y_2 + 1/80e^{4t}\star w_2\, w_2Y_1 + 1/32e^{4t}\star w_2\, w_1X_2Y_1 - 1/160e^{4t}\star w_2\, w_1X_1Y_2 - 1/80e^{4t}\star w_2\, w_1Y_1 - 1/32e^{-4t}\star w_2\, w_2X_2Y_1 + 1/32e^{-4t}\star w_2\, w_2X_1Y_2 + 1/160e^{-4t}\star w_2\, w_1X_2Y_1 -$

$1/160e^{-4t}\star w_2\, w_1X_1Y_2 + 1/160e^{4t}\star w_1\, w_2X_2Y_1 - 1/32e^{4t}\star w_1\, w_2X_1Y_2 + 1/80e^{4t}\star w_1\, w_2Y_1 - 1/160e^{4t}\star w_1\, w_1X_2Y_1 + 1/32e^{4t}\star w_1\, w_1X_1Y_2 - 1/80e^{4t}\star w_1\, w_1Y_1 + 1/32e^{-4t}\star w_1\, w_2X_2Y_1 - 1/32e^{-4t}\star w_1\, w_2X_1Y_2 - 1/160e^{-4t}\star w_1\, w_1X_2Y_1 + 1/160e^{-4t}\star w_1\, w_1X_1Y_2) + \sigma\varepsilon^2(1/40w_2X_2Y_1 - 1/80w_2Y_1 - 1/40w_1X_2Y_1 + 1/80w_1Y_1) + \sigma(-w_2Y_1 - w_1Y_1) + \varepsilon^2(-3/20X_2Y_1 - 3/20X_1Y_2 + 7/20Y_1) - 4Y_1$

$\dot{Y}_2 = \sigma^2\varepsilon^2(3/80e^{4t}\star w_2\, w_2X_2Y_2 - 3/32e^{4t}\star w_2\, w_2X_2Y_1 - 1/32e^{4t}\star w_2\, w_2X_1Y_2 - 1/80e^{4t}\star w_2\, w_2Y_2 - 3/80e^{4t}\star w_2\, w_1X_2Y_2 + 3/32e^{4t}\star w_2\, w_1X_2Y_1 + 1/32e^{4t}\star w_2\, w_1X_1Y_2 + 1/80e^{4t}\star w_2\, w_1Y_2 - 9/80e^{-4t}\star w_2\, w_2X_2Y_2 + 1/32e^{-4t}\star w_2\, w_2X_2Y_1 - 1/32e^{-4t}\star w_2\, w_2X_1Y_2 - 3/80e^{-4t}\star w_2\, w_1X_2Y_2 + 3/32e^{-4t}\star w_2\, w_1X_2Y_1 - 3/32e^{-4t}\star w_2\, w_1X_1Y_2 + 9/80e^{4t}\star w_1\, w_2X_2Y_2 - 1/32e^{4t}\star w_1\, w_2X_2Y_1 - 3/32e^{4t}\star w_1\, w_2X_1Y_2 - 1/80e^{4t}\star w_1\, w_2Y_2 - 9/80e^{4t}\star w_1\, w_1X_2Y_2 + 1/32e^{4t}\star w_1\, w_1X_2Y_1 + 3/32e^{4t}\star w_1\, w_1X_1Y_2 + 1/80e^{4t}\star w_1\, w_1Y_2 + 9/80e^{-4t}\star w_1\, w_2X_2Y_2 - 1/32e^{-4t}\star w_1\, w_2X_2Y_1 + 1/32e^{-4t}\star w_1\, w_2X_1Y_2 + 3/80e^{-4t}\star w_1\, w_1X_2Y_2 - 3/32e^{-4t}\star w_1\, w_1X_2Y_1 + 3/32e^{-4t}\star w_1\, w_1X_1Y_2) + \sigma\varepsilon^2(1/8w_2X_1Y_2 + 1/80w_2Y_2 - 1/8w_1X_1Y_2 - 1/80w_1Y_2) + \sigma(-w_2Y_2 - w_1Y_2) + \varepsilon^2(3/10X_2Y_2 + 1/4X_2Y_1 + 1/4X_1Y_2 - 3/20Y_2) - 4Y_2$

$\dot{X}_1 = \sigma^2\varepsilon^2(3/80e^{-4t}\star w_2\, w_2X_2X_1 - 1/80e^{-4t}\star w_2\, w_2X_1 - 3/80e^{-4t}\star w_2\, w_1X_2X_1 - 1/80e^{-4t}\star w_2\, w_1X_1 - 3/80e^{-4t}\star w_1\, w_2X_2X_1 + 1/80e^{-4t}\star w_1\, w_2X_1 + 3/80e^{-4t}\star w_1\, w_1X_2X_1 + 1/80e^{-4t}\star w_1\, w_1X_1) + \sigma\varepsilon^2(1/80w_2X_1 - 1/80w_1X_1) + \varepsilon^2(-3/20X_2X_1 + 7/20X_1)$

$\dot{X}_2 = \sigma^2\varepsilon^2(-3/80e^{-4t}\star w_2\, w_2X_2^2 - 1/16e^{-4t}\star w_2\, w_2X_2X_1 + 1/80e^{-4t}\star w_2\, w_2X_2 - 9/80e^{-4t}\star w_2\, w_1X_2^2 + 1/16e^{-4t}\star w_2\, w_1X_2X_1 + 1/80e^{-4t}\star w_2\, w_1X_2 + 3/80e^{-4t}\star w_1\, w_2X_2^2 + 1/16e^{-4t}\star w_1\, w_2X_2X_1 - 1/80e^{-4t}\star w_1\, w_2X_2 + 9/80e^{-4t}\star w_1\, w_1X_2^2 - 1/16e^{-4t}\star w_1\, w_1X_2X_1 - 1/80e^{-4t}\star w_1\, w_1X_2) + \sigma\varepsilon^2(3/40w_2X_2^2 - 1/80w_2X_2 - 3/40w_1X_2^2 + 1/80w_1X_2) + \varepsilon^2(3/20X_2^2 + 1/4X_2X_1 - 3/20X_2)$