Invariant manifold of your dynamical system

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Throughout and generally: the lowest order, most important, terms are near the end of each expression.

The specified dynamical system

$\dot{u}_1 = u_3$

$\dot{u}_2 = u_4$

$\dot{u}_3 = F_1\varepsilon^2 \exp(0) + \varepsilon^2(-1/2u_1^3 - 3/10u_3 + 3/10u_4) - 2u_1 + u_2$

$\dot{u}_4 = F_2\varepsilon^2(3/10u_3 - 3/5u_4) + u_1 - 2u_2$

Invariant subspace basis vectors

$\vec{e}_1 = \{\{1, 1, i, i\}, \exp(it)\}$

$\vec{e}_2 = \{\{1, 1, -i, -i\}, \exp(-it)\}$

$\vec{z}_1 = \{\{1/4, 1/4, 1/4i, 1/4i\}, \exp(it)\}$

$\vec{z}_2 = \{\{1/4, 1/4, -1/4i, -1/4i\}, \exp(-it)\}$

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The invariant manifold These give the location of the invariant manifold in terms of parameters s_j.

$$u_1 = -5/9\frac{\partial^2 F_1}{\partial t^2}\varepsilon^2 + \frac{\partial^2 F_2}{\partial t^2}\varepsilon^2(33/320\exp(-it)s_2 i - 33/320\exp(it)s_1 i) + \frac{\partial F_2}{\partial t}\varepsilon^2(3/32\exp(-it)s_2 + 3/32\exp(it)s_1) + 2/3F_1\varepsilon^2 + F_2\varepsilon^2(-3/80\exp(-it)s_2 i + 3/80\exp(it)s_1 i) + \varepsilon^2(-9/16\exp(-it)s_2^2 s_1 + 7/96\exp(-3it)s_2^3 - 9/16\exp(it)s_2 s_1^2 + 7/96\exp(3it)s_1^3) + \exp(-it)s_2 + \exp(it)s_1 + O(\varepsilon^4)$$

$$u_2 = -4/9\frac{\partial^2 F_1}{\partial t^2}\varepsilon^2 + \frac{\partial^2 F_2}{\partial t^2}\varepsilon^2(-39/320\exp(-it)s_2 i + 39/320\exp(it)s_1 i) + \frac{\partial F_2}{\partial t}\varepsilon^2(-9/160\exp(-it)s_2 - 9/160\exp(it)s_1) + 1/3F_1\varepsilon^2 + F_2\varepsilon^2(9/80\exp(-it)s_2 i - 9/80\exp(it)s_1 i) + \varepsilon^2(3/16\exp(-it)s_2^2 s_1 - 1/96\exp(-3it)s_2^3 + 3/16\exp(it)s_2 s_1^2 - 1/96\exp(3it)s_1^3) + \exp(-it)s_2 + \exp(it)s_1 + O(\varepsilon^4)$$

$$u_3 = 2/3\frac{\partial F_1}{\partial t}\varepsilon^2 + \frac{\partial^2 F_2}{\partial t^2}\varepsilon^2(63/320\exp(-it)s_2 + 63/320\exp(it)s_1) + \frac{\partial F_2}{\partial t}\varepsilon^2(-21/160\exp(-it)s_2 i + 21/160\exp(it)s_1 i) + F_2\varepsilon^2(-9/80\exp(-it)s_2 - 9/80\exp(it)s_1) + \varepsilon^2(3/16\exp(-it)s_2^2 s_1 i - 7/32\exp(-3it)s_2^3 i - 3/16\exp(it)s_2 s_1^2 i + 7/32\exp(3it)s_1^3 i) - \exp(-it)s_2 i + \exp(it)s_1 i + O(\varepsilon^4)$$

$$u_4 = 1/3\frac{\partial F_1}{\partial t}\varepsilon^2 + \frac{\partial^2 F_2}{\partial t^2}\varepsilon^2(-57/320\exp(-it)s_2 - 57/320\exp(it)s_1) + \frac{\partial F_2}{\partial t}\varepsilon^2(27/160\exp(-it)s_2 i - 27/160\exp(it)s_1 i) + F_2\varepsilon^2(3/80\exp(-it)s_2 + 3/80\exp(it)s_1) + \varepsilon^2(-9/16\exp(-it)s_2^2 s_1 i + 1/32\exp(-3it)s_2^3 i + 9/16\exp(it)s_2 s_1^2 i - 1/32\exp(3it)s_1^3 i) - \exp(-it)s_2 i + \exp(it)s_1 i + O(\varepsilon^4)$$

Invariant manifold ODEs The system evolves on the invariant manifold such that the parameters evolve according to these ODEs.

$$\dot{s}_1 = -531/12800\frac{\partial^2 F_2}{\partial t^2}F_2\varepsilon^4 s_1 i + \frac{\partial^2 F_2}{\partial t^2}\varepsilon^4(99/2560 s_2 s_1^2 + 9/320 s_1 i) + 9/256\frac{\partial F_2}{\partial t}F_2\varepsilon^4 s_1 + \frac{\partial F_2}{\partial t}\varepsilon^4(27/256 s_2 s_1^2 i - 9/400 s_1) + 9/640F_2^2\varepsilon^4 s_1 i + F_2\varepsilon^4(-9/80 s_2 s_1^2 - 9/800 s_1 i) - 3/40F_2\varepsilon^2 s_1 + \varepsilon^4(-155/256 s_2^2 s_1^3 i + 9/160 s_2 s_1^2) + 3/8\varepsilon^2 s_2 s_1^2 i + O(\varepsilon^5)$$

$$\dot{s}_2 = 531/12800\frac{\partial^2 F_2}{\partial t^2}F_2\varepsilon^4 s_2 i + \frac{\partial^2 F_2}{\partial t^2}\varepsilon^4(99/2560 s_2^2 s_1 - 9/320 s_2 i) + 9/256\frac{\partial F_2}{\partial t}F_2\varepsilon^4 s_2 + \frac{\partial F_2}{\partial t}\varepsilon^4(-27/256 s_2^2 s_1 i - 9/400 s_2) -$$

$9/640F_2^2\varepsilon^4 s_2 i + F_2\varepsilon^4(-9/80s_2^2 s_1 + 9/800 s_2 i) - 3/40F_2\varepsilon^2 s_2 + \varepsilon^4(155/256 s_2^3 s_1^2 i + 9/160 s_2^2 s_1) - 3/8\varepsilon^2 s_2^2 s_1 i + O(\varepsilon^5)$