Remarks on:

G. A. Jarrad and A. J. Roberts, "Smooth subgrid fields underpin rigorous closure in spatial discretisation of reaction-advection-diffusion PDEs"

In this work, the authors employ dynamical systems theory to improve the efficiency of spatial discretizations of non-autonomous reaction-advection-diffusion PDEs. They restrict their considerations to one spatial dimension. After discretizing in space, the resulting semi-discretized problem given by a set of ODEs is treated as a continuous time dynamical system.

The analysis is based upon center manifold theory. The theory is then illustrated on the nonlinear advection-diffusion Burgers' equation. Unlike the conventional numerical methods, the approach taken here is tailored to the PDE as it allows the equation to determine itself the subgrid fields. The authors also discuss how the theory can be generalized to include the second order in time PDEs. Dynamical stability of the proposed discretization approach is investigated as well and a comparison of the numerical behavior of the holistic models and other established models is given.

Overall, I find the paper well-organized and the proposed holistic approach promising. There are a couple of points which could use clarification. Please find my comments below.

Remarks

1. Are the initial conditions for $U_j(t)$ to be taken by sampling the field at grid points, i.e. $u(X_j, 0)$?

2. It is not quite clear what the assumptions on ν and α in (4) are. Are they allowed to vary in space?

3. Theorem 5 and its proof contain many references to previous works, which makes them difficult to follow. It is mentioned at the beginning of the paper that the smoothness conditions on F and G will be specified here, but they are not given in the formulation. You should state the preconditions taken from Haragus & Ioss in the formulation of the theorem to make it a bit more self-contained.

4. Which time stepping scheme was used in numerical simulations in Section 4 to get Figure 7? Was it the same for all models?

Minor remarks

1. On page 21, in the partial sum (26), I think it should be $\vec{g}^p$ instead of $\vec{g}^n$.

2. In (5)-(6), does it still hold that $j \in [0, N]$ as assumed at the beginning of Section 1 or should this change to $j \in [1, N-1]$?