

# Calculus II

## Exercise II, Kepler's Second Law, Part 2

In Page 880 of the textbook, we know that :  
the velocity vector is  $\mathbf{v} = \mathbf{r}'$  and the acceleration vector is  $\mathbf{a} = \mathbf{r}''$ . We use the following laws of Newton :

$$\text{Second Law of Motion : } \mathbf{F} = m\mathbf{a}$$

$$\text{Law of Gravitation : } \mathbf{F} = -\frac{GMm}{r^3}\mathbf{r}$$

where  $\mathbf{F}$  is the gravitational force on the planet,  $m$  and  $M$  are the masses of the planet and the sun,  $G$  is the gravitational constant, and  $r = |\mathbf{r}|$ .

By equating the expressions for  $\mathbf{F}$  in Newton's two laws, we find that

$$\mathbf{a} = -\frac{GM}{r^3}\mathbf{r}$$

and so  $\mathbf{a}$  is parallel to  $\mathbf{r}$ . It follows that  $\mathbf{r} \times \mathbf{a} = \mathbf{0}$ .