

Calculus II

Exercise II, Kepler's Second Law, Part 2

In Page 880 of the textbook, we know that :

The velocity vector is $\mathbf{v} = \mathbf{r}'$ and the acceleration vector is $\mathbf{a} = \mathbf{r}''$. We use the following laws of Newton :

Second Law of Motion : $\mathbf{F} = m\mathbf{a}$

Law of Gravitation : $\mathbf{F} = -\frac{GMm}{r^3}\mathbf{r}$

where \mathbf{F} is the gravitational force on the planet, m and M are the masses of the planet and the sun, G is the gravitational constant, $r = |\mathbf{r}|$.

By equating the expressions for \mathbf{F} in Newton's laws, we find that

$$\mathbf{a} = -\frac{GM}{r^3}\mathbf{r}$$

and so \mathbf{a} is parallel to \mathbf{r} . It follows that $\mathbf{r} \times \mathbf{a} = 0$.