Calculus II

Assignment 5

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Name : _____

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1. Find $\frac{dy}{dx}$.
   $y \cos x = x^2 + y^2$
2. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
   $yz + x \ln y = z^2$
3. Find the directional derivative of f at the given point in the direction indicated by the angle θ.
   $f(x, y) = x^3y^4 + x^4y^3,\ (1, 1),\ \theta = \frac{\pi}{6}$
4. (a) Find the gradient of f.
   (b) Evaluate the gradient at the point P.
   (c) Find the rate of change of f at P in the direction of the vector $\mathbf{u}$.
   $f(x, y) = \sin(2x + 3y),\ P(-6, 4),\ \mathbf{u} = \frac{1}{2}\left(\sqrt{3}\mathbf{i} - \mathbf{j}\right)$
5. Find the directional derivative of the function at the given point in the direction of the vector $\mathbf{v}$.
   $g(x, y) = \tan^{-1}(xy),\ (1, 2),\ \mathbf{v} = 5\mathbf{i} + 10\mathbf{j}$
   Hint : $\mathbf{v}$ is not a unit vector.
6. Suppose that over a certain region of space the electrical potential V is given by $V(x, y, z) = 5x^2 - 3xy + xyz$.
   (a) Find the rate of change of the potential at $P(3, 4, 5)$ in the direction of the vector $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$.
   (b) In which direction does change most rapidly at P?
   (c) What is the maximum rate of change at P?

Note :

Section 15.5, Equation 6 : $\frac{dy}{dx} = -\frac{F_x}{F_y}$.

Section 15.5, Equation 7 : $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$, $\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$.

Reading materials : Textbook Section 15.5 and 15.6.