38. Consider the equation

$$x\,y'' - (2+x)\,y' + 2\,y = 0, \quad x > 0.$$

(a) Show that $x = 0$ is a regular singular point. Find the indicial equation and the indicial roots, and conclude that we are in Case 3 of the Frobenius method.
(b) Use the method of Frobenius to find a Frobenius series solution corresponding to the larger root r_1.
(c) Identify the solution in (b) as $3!(e^x - 1 - x - \frac{x^2}{2})$.
(d) Even though the roots differ by an integer, there exists a second Frobenius series solution. Using the method of Frobenius, show that e^x or $1 + x + x^2/2$ can be taken as a second solution. [Hint: Argue that the coefficient b_3 is arbitrary.]

39. Consider the equation

$$xy'' - (n+x)\,y' + n\,y = 0, \quad x > 0,$$

where n is a nonnegative integer.
(a) Show that $x = 0$ is a regular singular point. Find the indicial equation and the indicial roots, and conclude that we are in Case 3 of the Frobenius method.
(b) Use the method of Frobenius to find a Frobenius series solution corresponding to the larger root r_1.
(c) Identify the solution in (b) as

$$(n+1)!(e^x - 1 - x - \frac{x^2}{2} - \cdots - \frac{x^n}{n!}).$$

(d) Using the method of Frobenius, show that e^x or $1 + x + x^2/2 + \cdots + \frac{x^n}{n!}$ can be taken as a second solution. [Hint: Argue that the coefficient b_{n+1} is arbitrary.]

APPENDIX B

TABLES OF TRANSFORMS

- **Table of Fourier Transforms**
- **Table of Fourier Cosine Transforms**
- **Table of Fourier Sine Transforms**
- **Table of Laplace Transforms**

Table of Fourier Transforms

	$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \widehat{f}(\omega) e^{ix\omega}\, d\omega$	$\widehat{f}(\omega) = \mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$
1.	$\begin{cases} 1 & \text{if } \lvert x\rvert < a \\ 0 & \text{if } \lvert x\rvert > a \end{cases}$	$\sqrt{\frac{2}{\pi}} \frac{\sin a\omega}{\omega}$
2.	$\begin{cases} 1 & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}$	$\frac{i\left(e^{-ib\omega} - e^{ia\omega}\right)}{\sqrt{2\pi}\omega}$
3.	$\begin{cases} 1 - \frac{\lvert x\rvert}{a} & \text{if } \lvert x\rvert < a \\ 0 & \text{if } \lvert x\rvert > a \end{cases} \quad a > 0$	$2\sqrt{\frac{2}{\pi}} \frac{\sin^2(\frac{a\omega}{2})}{a\omega^2}$
4.	$\begin{cases} x & \text{if } \lvert x\rvert < a \\ 0 & \text{if } \lvert x\rvert > a \end{cases} \quad a > 0$	$i\sqrt{\frac{2}{\pi}} \frac{a\omega\cos(a\omega) - \sin(a\omega)}{\omega^2}$
5.	$\begin{cases} \sin x & \text{if } \lvert x\rvert < \pi \\ 0 & \text{if } \lvert x\rvert > \pi \end{cases}$	$i\sqrt{\frac{2}{\pi}} \frac{\sin(\pi\omega)}{\omega^2 - 1}$
6.	$\begin{cases} \sin(ax) & \text{if } \lvert x\rvert < b \\ 0 & \text{if } \lvert x\rvert > b \end{cases} \quad a, b > 0$	$i\sqrt{\frac{2}{\pi}} \frac{\omega\cos(b\omega)\sin(ab) - a\cos(ab)\sin(b\omega)}{\omega^2 - a^2}$
7.	$\frac{1}{a^2 + x^2},\ a > 0$	$\sqrt{\frac{\pi}{2}} \frac{e^{-a\lvert\omega\rvert}}{a}$
8.	$\frac{x}{a^2 + x^2},\ a > 0$	$-i\sqrt{\frac{\pi}{2}} \operatorname{sgn}\omega\, e^{-a\lvert\omega\rvert}$
9.	$\sqrt{\frac{2}{\pi}} \frac{a}{1 + a^2x^2},\ a > 0$	$e^{-\frac{\lvert\omega\rvert}{a}}$
10.	$\frac{\sin ax}{x},\ a > 0$	$\begin{cases} \sqrt{\frac{\pi}{2}} & \text{if } \lvert\omega\rvert < a \\ \frac{1}{2}\sqrt{\frac{\pi}{2}} & \text{if } \lvert\omega\rvert = a \\ 0 & \text{if } \lvert\omega\rvert > a \end{cases}$
11.	$\frac{4}{\sqrt{2\pi}} \frac{\sin^2(\frac{1}{2}ax)}{ax^2},\ a > 0$	$\begin{cases} 1 - \frac{\lvert\omega\rvert}{a} & \text{if } \lvert\omega\rvert < a \\ 0 & \text{if } \lvert\omega\rvert > a \end{cases}$
12.	$\frac{4}{\sqrt{2\pi}} \frac{\sin^2(ax) - \sin^2(\frac{1}{2}ax)}{ax^2},\ a > 0$	$\begin{cases} 1 & \text{if } \lvert x\rvert < a \\ (-x + 2a)/a & \text{if } a < x < 2a \\ (x + 2a)/a & \text{if } a < x < 2a \\ 0 & \text{if } \lvert x\rvert > 2a \end{cases}$
13.	$e^{-a\lvert x\rvert},\ a > 0$	$\sqrt{\frac{2}{\pi}} \frac{a}{a^2 + \omega^2}$
14.	$\begin{cases} e^{-ax} & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases},\ a > 0$	$\frac{1}{\sqrt{2\pi}} \frac{1}{a + i\omega}$
15.	$\begin{cases} 0 & \text{if } x > 0 \\ e^{ax} & \text{if } x < 0 \end{cases},\ a > 0$	$\frac{1}{\sqrt{2\pi}} \frac{1}{a - i\omega}$
16.	$\lvert x\rvert^n e^{-a\lvert x\rvert},\ a > 0,\ n > 0$	$\frac{\Gamma(n+1)}{\sqrt{2\pi}} \left(\frac{1}{(a - i\omega)^{1+n}} + \frac{1}{(a + i\omega)^{1+n}} \right)$

Table of Fourier Transforms (continued)

	$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \widehat{f}(\omega) e^{ix\omega}\, d\omega$	$\widehat{f}(\omega) = \mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$
17.	$e^{-\frac{a}{2}x^2},\ a > 0$	$\frac{1}{\sqrt{a}} e^{-\frac{\omega^2}{2a}}$
18.	$e^{-ax^2},\ a > 0$	$\frac{1}{\sqrt{2a}} e^{-\frac{\omega^2}{4a}}$
19.	$xe^{-\frac{a}{2}x^2},\ a > 0$	$\frac{-i\omega}{a^{3/2}} e^{-\frac{\omega^2}{2a}}$
20.	$x^2 e^{-\frac{a}{2}x^2},\ a > 0$	$\frac{a - \omega^2}{a^{5/2}} e^{-\frac{\omega^2}{2a}}$
21.	$x^3 e^{-\frac{a}{2}x^2},\ a > 0$	$\frac{-i\omega(3a - \omega^2)}{a^{7/2}} e^{-\frac{\omega^2}{2a}}$
22.	$e^{-\frac{x^2}{2}} H_n(x)$, H_n, nth Hermite polynomial	$(-1)^n i^n e^{\frac{-\omega^2}{2}} H_n(\omega)$
23.	$J_0(x)$, Bessel function of order 0	$\begin{cases} \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{1-\omega^2}} & \text{if } \lvert\omega\rvert < 1 \\ 0 & \text{if } \lvert\omega\rvert > 1 \end{cases}$
24.	$J_n(x)$, Bessel function of order $n \geq 0$	$\begin{cases} \sqrt{\frac{2}{\pi}} \frac{(-i)^n}{\sqrt{1-\omega^2}} T_n(\omega) & \text{if } \lvert\omega\rvert < 1 \\ 0 & \text{if } \lvert\omega\rvert > 1 \end{cases}$ T_n, Chebyshev polynomial of degree n.

Special Transforms

25.	$\mathcal{F}(\delta_0(x))(\omega) = \frac{1}{\sqrt{2\pi}}$	**27.**	$\mathcal{F}(\sqrt{\frac{2}{\pi}} \frac{1}{x})(\omega) = -i \operatorname{sgn}\omega$
26	$\mathcal{F}(\delta_0(x - a))(\omega) = \frac{1}{\sqrt{2\pi}} e^{-ia\omega}$	**28.**	$\mathcal{F}(e^{iax})(\omega) = \sqrt{2\pi}\delta_0(\omega - a)$

Operational Properties

29.	$\mathcal{F}(af + bg)(\omega) = a\mathcal{F}(f) + b\mathcal{F}(g)$	**36.**	$\mathcal{F}(f\, g)(\omega) = \mathcal{F}(f) * \mathcal{F}(g)(\omega)$
30.	$\mathcal{F}(f')(\omega) = i\omega\mathcal{F}(f)(\omega)$	**37.**	$\mathcal{F}(f(x - a))(\omega) = e^{-ia\omega}\mathcal{F}(f)(\omega)$
31.	$\mathcal{F}(f'')(\omega) = -\omega^2\mathcal{F}(f)(\omega)$	**38.**	$\mathcal{F}(e^{iax}f(x))(\omega) = \mathcal{F}(f)(\omega - a)$
32.	$\mathcal{F}(f^{(n)})(\omega) = (i\omega)^n\mathcal{F}(f)(\omega)$	**39.**	$\mathcal{F}(\cos(ax)f(x))(\omega) = \frac{\mathcal{F}(f)(\omega-a)+\mathcal{F}(f)(\omega+a)}{2}$
33.	$\mathcal{F}(xf(x))(\omega) = i\frac{d}{d\omega}\mathcal{F}(f)(\omega)$	**40.**	$\mathcal{F}(\sin(ax)f(x))(\omega) = \frac{\mathcal{F}(f)(\omega-a)-\mathcal{F}(f)(\omega+a)}{2i}$
34.	$\mathcal{F}(x^n f(x))(\omega) = i^n \frac{d^n}{d\omega^n}\mathcal{F}(f)(\omega)$	**41.**	$\mathcal{F}(f(ax))(\omega) = \frac{1}{\lvert a\rvert}\mathcal{F}(f)(\frac{\omega}{a}),\ a \neq 0$
35.	$\mathcal{F}(f * g)(\omega) = \mathcal{F}(f)(\omega)\mathcal{F}(g)(\omega)$	**42.**	$f(x) = \mathcal{F}(\widehat{f})(-x),\ \mathcal{F}(\mathcal{F}(f)) = f(-x)$

Table of Fourier Cosine Transforms

	$f(x)=\sqrt{\frac{2}{\pi}}\int_0^\infty \mathcal{F}_c(f)(\omega)\cos\omega x\,d\omega,$ $0<x<\infty$	$\mathcal{F}_c(f)(\omega)=\widehat{f}_c(\omega)=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos\omega x\,dx,$ $0\le\omega<\infty$
1.	$\begin{cases}1 & \text{if } 0<x<a\\ 0 & \text{otherwise}\end{cases}$	$\sqrt{\frac{2}{\pi}}\frac{\sin a\omega}{\omega}$
2.	$e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{a}{a^2+\omega^2}$
3.	$x\,e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{a^2-\omega^2}{(a^2+\omega^2)^2}$
4.	$e^{-a\,x^2/2},\quad a>0$	$\frac{1}{\sqrt{a}}e^{-\omega^2/2a}$
5.	$\cos ax\,e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{a\omega^2+2a^3}{4a^4+\omega^4}$
6.	$\sin ax\,e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{2a^3-a\omega^2}{4a^4+\omega^4}$
7.	$\frac{a}{a^2+x^2},\quad a>0$	$\sqrt{\frac{\pi}{2}}e^{-a\,\omega}$
8.	$x^p,\quad 0<p<1$	$\sqrt{\frac{2}{\pi}}\frac{\Gamma(p)\cos(p\omega/2)}{\omega^p}$
9.	$\begin{cases}\cos x & \text{if } 0<x<a\\ 0 & \text{otherwise}\end{cases}$	$\frac{1}{\sqrt{2\pi}}\left[\frac{\sin a(1-\omega)}{1-\omega}+\frac{\sin a(1+\omega)}{1+\omega}\right]$
	Operational Properties	
10.	$\alpha f(x)+\beta g(x)$	$\alpha\mathcal{F}_c(f)(\omega)+\beta\mathcal{F}_c(g)(\omega)$
11.	$f(ax),\quad a>0$	$\frac{1}{a}\widehat{f}_c(\frac{\omega}{a})$
12.	$f'(x)$	$\omega\widehat{f}_s(\omega)-\sqrt{\frac{2}{\pi}}f(0)$
13.	$f''(x)$	$-\omega^2\widehat{f}_c(\omega)-\sqrt{\frac{2}{\pi}}f'(0)$
14.	$xf(x)$	$\left[\widehat{f}_s\right]'(\omega)$
15.	$\mathcal{F}_c(\mathcal{F}_c f)$	f

Table of Fourier Sine Transforms

	$f(x)=\sqrt{\frac{2}{\pi}}\int_0^\infty \mathcal{F}_s(f)(\omega)\sin\omega x\,d\omega,$ $0<x<\infty$	$\mathcal{F}_s(f)(\omega)=\widehat{f}_s(\omega)=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin\omega x\,dx,$ $0<\omega<\infty$
1.	$\begin{cases}1 & \text{if } 0<x<a\\ 0 & \text{otherwise}\end{cases}$	$\sqrt{\frac{2}{\pi}}\frac{1-\cos a\omega}{\omega}$
2.	$e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{\omega}{a^2+\omega^2}$
3.	$x\,e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{2a\omega}{(a^2+\omega^2)^2}$
4.	$\frac{e^{-ax}}{x},\quad a>0$	$\sqrt{\frac{2}{\pi}}\tan^{-1}\frac{\omega}{a}$
5.	$\frac{1}{2}x\,e^{-a\,x^2},\quad a>0$	$\frac{\omega}{a^{3/2}}e^{-\omega^2/2a}$
6.	$\cos ax\,e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{\omega^3}{4a^4+\omega^4}$
7.	$\sin ax\,e^{-ax},\quad a>0$	$\sqrt{\frac{2}{\pi}}\frac{2a^2\omega}{4a^4+\omega^4}$
8.	$\frac{x}{a^2+x^2},\quad a>0$	$\sqrt{\frac{\pi}{2}}e^{-a\,\omega}$
9.	$x^{p-1},\quad 0<p<1$	$\sqrt{\frac{2}{\pi}}\frac{\Gamma(p)\cos(\pi p/2)}{\omega^p}$
10.	$\begin{cases}\sin x & \text{if } 0<x<a\\ 0 & \text{otherwise}\end{cases}$	$\frac{1}{\sqrt{2\pi}}\left[\frac{\sin a(1-\omega)}{1-\omega}-\frac{sin a(1+\omega)}{1+\omega}\right]$
	Operational Properties	
11.	$\alpha\,f(x)+\beta\,g(x)$	$\alpha\,\mathcal{F}_s(f)(\omega)+\beta\,\mathcal{F}_s(g)(\omega)$
12.	$f(ax),\quad a>0$	$\frac{1}{a}\widehat{f}_s(\frac{\omega}{a})$
13.	$f'(x)$	$-\omega\widehat{f}_c(\omega)$
14.	$f''(x)$	$-\omega^2\widehat{f}_s(\omega)+\sqrt{\frac{2}{\pi}}\,\omega f(0)$
15.	$x\,f(x)$	$-\left[\widehat{f}_c\right]'(\omega)$
16.	$\mathcal{F}_s(\mathcal{F}_s f)$	f

Table of Laplace Transforms

	$f(t), \quad t \geq 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^\infty f(t)e^{-st}\,dt,$
1.	1	$\frac{1}{s}, \quad s > 0$
2.	$t^n, \quad n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}, \quad s > 0$
3.	$t^a \quad (a > -1)$	$\frac{\Gamma(a+1)}{s^{a+1}}, \quad s > 0$
4.	e^{at}	$\frac{1}{s-a}, \quad s > a$
5.	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
6.	$\frac{e^{at}-e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}, \quad s > \max(a, b)$
7.	$\frac{ae^{at}-be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}, \quad s > \max(a, b)$
8.	$\sin kt$	$\frac{k}{s^2+k^2}, \quad s > 0$
9.	$\cos kt$	$\frac{s}{s^2+k^2}, \quad s > 0$
10.	$e^{at}\sin kt$	$\frac{k}{(s-a)^2+k^2}, \quad s > a$
11.	$e^{at}\cos kt$	$\frac{s-a}{(s-a)^2+k^2}, \quad s > a$
12.	$t\sin kt$	$\frac{2ks}{(s^2+k^2)^2}, \quad s > 0$
13.	$t\cos kt$	$\frac{s^2-k^2}{(s^2+k^2)^2}, \quad s > 0$
14.	$\frac{1}{2a^3}(\sin at - at\cos at)$	$\frac{1}{(s^2+a^2)^2}, \quad s > 0$
15.	$\sinh kt$	$\frac{k}{s^2-k^2}, \quad s > \lvert k\rvert$
16.	$\cosh kt$	$\frac{s}{s^2-k^2}, \quad s > \lvert k\rvert$
17.	$e^{at}\sinh kt$	$\frac{k}{(s-a)^2-k^2} \quad s > a + \lvert k\rvert$
18.	$e^{at}\cosh kt$	$\frac{s-a}{(s-a)^2-k^2} \quad s > a + \lvert k\rvert$
19.	$t\sinh kt$	$\frac{2ks}{(s^2-k^2)^2}, \quad s > \lvert k\rvert$
20.	$t\cosh kt$	$\frac{s^2+k^2}{(s^2-k^2)^2}, \quad s > \lvert k\rvert$
21.	$\frac{1}{2k^3}(kt\cosh kt - \sinh kt)$	$\frac{1}{(s^2-k^2)^2}, \quad s > \lvert k\rvert$

Table of Laplace Transforms (continued)

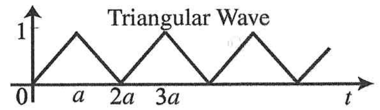
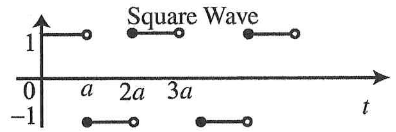
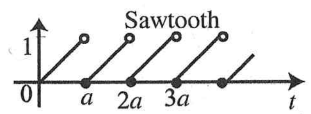
	$f(t), \quad t \geq 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^\infty f(t)e^{-st}\,dt,$
22.	$\delta_0(t - t_0), \quad t_0 \geq 0$	$e^{-t_0 s}, \quad s > 0$
23.	$\mathcal{U}_0(t-a) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t \geq a \end{cases} \quad (a > 0)$	$\frac{e^{-as}}{s}, \quad s > 0$
24.	$f(t+T) = f(t) \quad (T > 0)$	$\dfrac{1}{1-e^{-Ts}}\int_0^T e^{-st}f(t)\,dt$
25.	$f(t+T) = -f(t) \quad (T > 0)$	$\dfrac{1}{1+e^{-Ts}}\int_0^T e^{-st}f(t)\,dt$
26.		$\frac{1}{as^2}\left[\frac{1-e^{-as}}{1+e^{-as}}\right] = \frac{1}{as^2}\tanh\left(\frac{as}{2}\right), \quad s > 0$
27.		$\frac{1}{s}\left[\frac{1-e^{-as}}{1+e^{-as}}\right] = \frac{1}{s}\tanh\left(\frac{as}{2}\right), \quad s > 0$
28.		$\frac{1}{as^2} - \frac{e^{-as}}{s(1-e^{-as})}, \quad s > 0$
29.	$\frac{\sin at}{t}$	$\tan^{-1}(\frac{a}{s}), \quad s > 0$
30.	$J_0(at)$	$\frac{1}{\sqrt{s^2+a^2}}, \quad s > 0$
31.	$J_0(a\sqrt{t})$	$\frac{e^{-a^2/4s}}{s}, \quad s > 0$
32.	$t^p J_p(at) \quad (p > -\frac{1}{2})$	$\frac{2^p a^p \Gamma(p+\frac{1}{2})}{\sqrt{\pi}(s^2+a^2)^{p+\frac{1}{2}}}, \quad s > 0$
33.	$\frac{\sqrt{\pi}}{\Gamma(k)}\left(\frac{t}{2a}\right)^{k-\frac{1}{2}} J_{k-\frac{1}{2}}(at) \quad (k > 0)$	$\frac{1}{(s^2+a^2)^k}, \quad s > 0$
34.	$\frac{\sqrt{\pi}}{\Gamma(k)}a\left(\frac{t}{2a}\right)^{k-\frac{1}{2}} J_{k-\frac{3}{2}}(at) \quad (k > \frac{1}{2})$	$\frac{s}{(s^2+a^2)^k}, \quad s > 0$
35.	$2\sum_{m=1}^{n}\binom{2n-m-1}{n-1}\dfrac{t^{m-1}\cos(at - \frac{m\pi}{2})}{(2a)^{2n-m}(m-1)!}$ (n an integer ≥ 1)	$\frac{1}{(s^2+a^2)^n}, \quad s > 0$
36.	$\dfrac{1}{(n-1)!}\sum_{m=1}^{n-1}\dfrac{(2n-m-3)!}{(m-1)!(n-m-1)!}\dfrac{t^m\cos(at - \frac{m\pi}{2})}{(2a)^{2n-m-2}}$ (n an integer ≥ 2)	$\frac{s}{(s^2+a^2)^n}, \quad s > 0$

Table of Laplace Transforms (continued)

	$f(t), \quad t \geq 0$	$F(s) = \mathcal{L}(f)(s) = \int_0^\infty f(t)e^{-st}\,dt,$
37.	$\operatorname{erf}(at) \quad (a > 0)$	$\frac{1}{s}\, e^{s^2/4a^2} \operatorname{erfc}\left(\frac{s}{2a}\right), \quad s > 0$
38.	$\operatorname{erf}(a\sqrt{t})$	$\frac{a}{s\sqrt{s+a^2}}, \quad s > 0$
39.	$e^{-a^2t^2} \quad (a > 0)$	$\frac{\sqrt{\pi}}{2a} e^{s^2/4a^2} \operatorname{erfc}(\frac{s}{2a}), \quad s > 0$
40.	$\frac{1}{\sqrt{\pi t}} e^{-a^2/4t} \quad (a \geq 0)$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}, \quad s > 0$
41.	$\frac{a}{2\sqrt{\pi} t^{3/2}} e^{-a^2/4t} \quad (a > 0)$	$e^{-a\sqrt{s}}, \quad s > 0$
42.	$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right) \quad (a \geq 0)$	$\frac{1}{s}\, e^{-a\sqrt{s}}, \quad s > 0$
	Operational Properties	
43.	$\alpha f(t) + \beta g(t)$	$\alpha F(s) + \beta G(s)$
44.	$f'(t)$	$sF(s) - f(0)$
45.	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
46.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$
47.	$-tf(t)$	$F'(s)$
48.	$t^nf(t)$	$(-1)^nF^{(n)}(s)$
49.	$\int_0^t f(\tau)\,d\tau$	$\frac{1}{s}\,F(s), \quad s > 0$
50.	$\int_0^t \int_0^\tau f(\rho)\,d\rho d\tau$	$\frac{1}{s^2}\,F(s), \quad s > 0$
51.	$\frac{f(t)}{t}$	$\int_s^\infty F(u)\,du$
52.	$\frac{f(t)}{t^2}$	$\int_s^\infty \int_\sigma^\infty F(u)\,du d\sigma$
53.	$\mathcal{U}_0(t-a)f(t-a) \quad (a > 0)$	$e^{-as}F(s)$
54.	$e^{at}f(t)$	$F(s-a)$
55.	$f(ct) \quad (c > 0)$	$\frac{1}{c}F(\frac{s}{c})$
56.	$f * g(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$

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