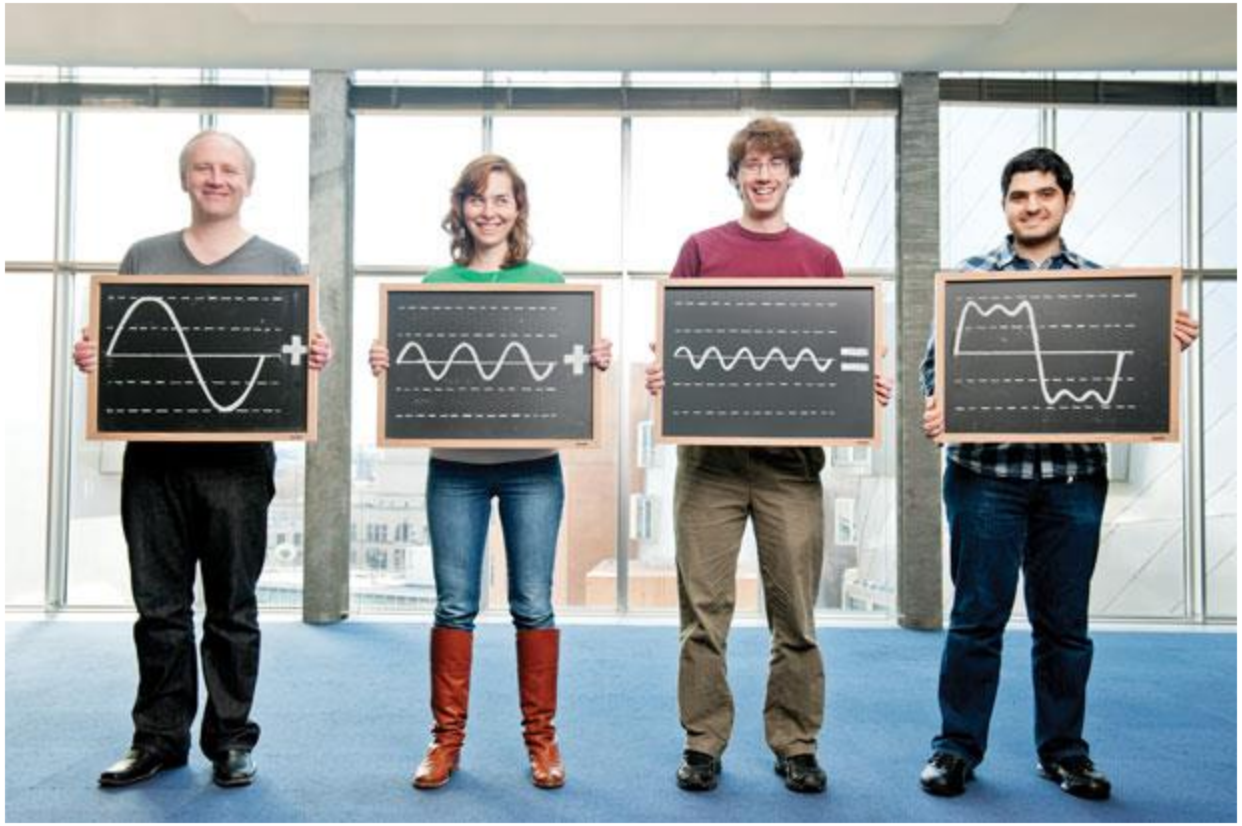
Fourier Analysis

Instructor: Xiang Li

Teaching Assistant: Lingjun Zhao



Class Information

Lecture (14): Monday (月曜日), Thursday (木曜日)

Grades: 10% Attendance (>2/3)

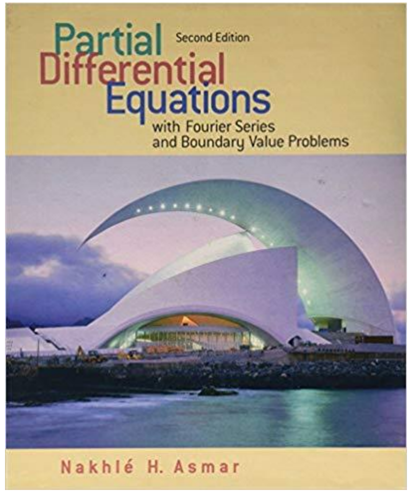
20% Assignment

70% Examination

Office hours: Afternoon, Monday and Thursday (Office: #247C)

Textbook: Partial Differential Equations with Fourier Series and Boundary Value Problems 2nd/3rd ,

Nakhlé H. Asmar, University of Missouri, USA



Prerequisites

M-3 **Calculus I** or M-4 **Calculus II**,

M-1 **Linear algebra** or M-2 **Linear algebra II**

Important related courses:

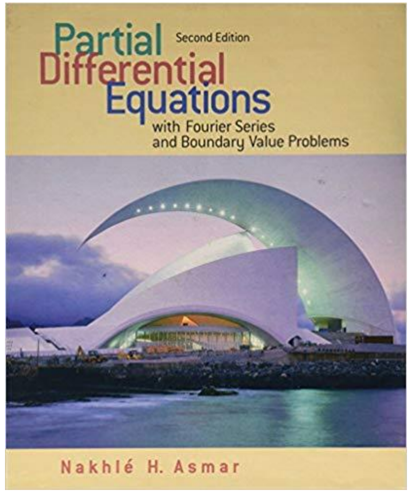
M-6 **Complex analysis**,

A-3 **Image processing**,

A-8 **Digital signal processing**

What we will cover

Full syllabus on course website



Chapter 2
- 1: Part1. Fourier series expansion (Orthogonal system of the function space)
- 2: Part1. Fourier series expansion (Fourier series of trigonometric functions)
- 3: Part1. Fourier series expansion (Exercise)
- 4: Part2. Properties of Fourier series (Convergence condition of Fourier series)
- 5: Part2. Properties of Fourier series (Parseval's theorem, Weierstrass' theorem)
- 6: Part2. Properties of Fourier series (Exercise)

Chapter 7
- 7: Part3. Fourier integral (Introduction from Fourier series, Fourier transform)
- 8: Part3. Fourier integral (Parseval's theorem, convolution)
- 9: Part3. Fourier integral (Exercise)

Chapter 8
- 10: Part4. Laplace transform (Introduction from Fourier transform)
- 11: Part4. Laplace transform (Ordinary differential equations of constant coefficients)
- 12: Part4. Laplace transform (Exercise)

Chapter 10
- 13: Part5. Discrete Fourier transform (Introduction from Fourier series)
- 14: Part5. Discrete Fourier transform (FFT(Fast Fourier Transform))

Fourier Series Expansion

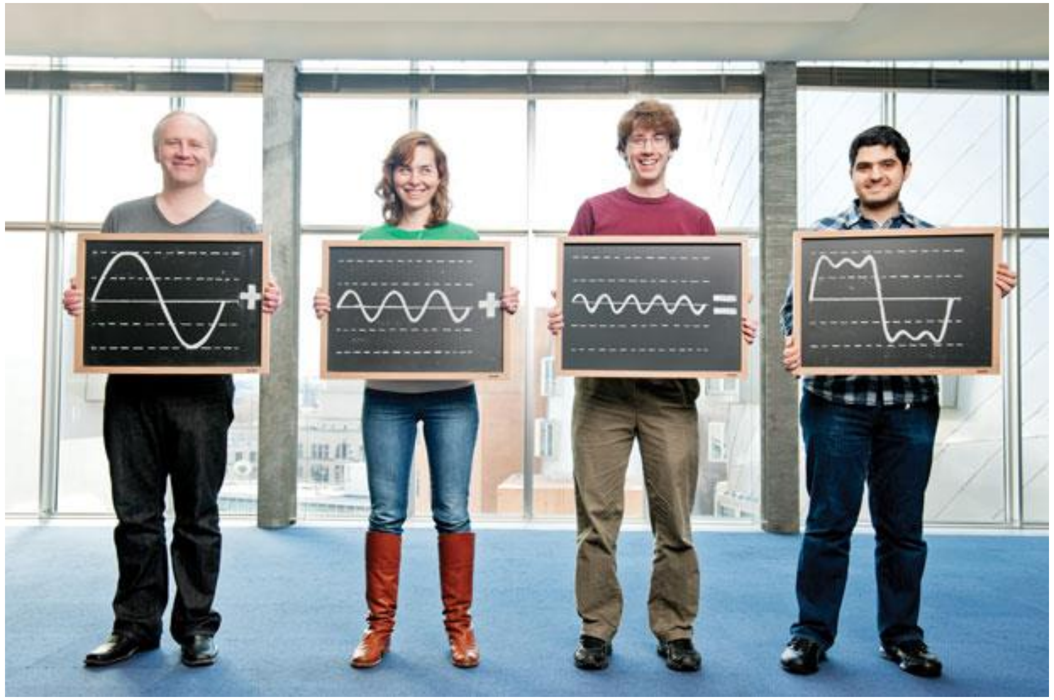
Properties of Fourier Series

Fourier Integral

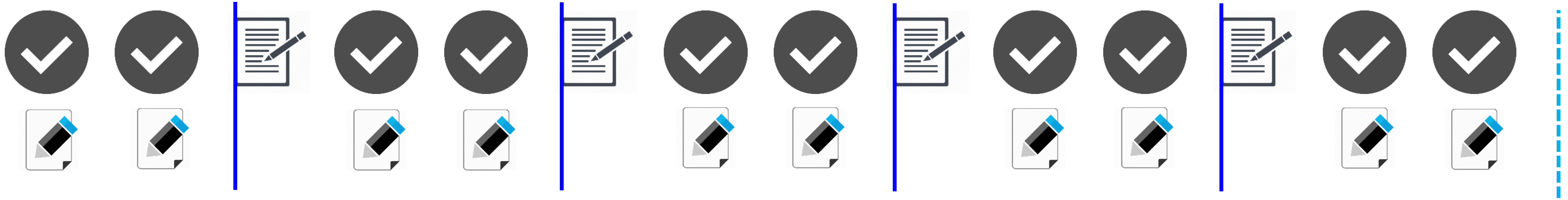
Laplace Transform

Discrete Fourier Transform

Fourier Analysis



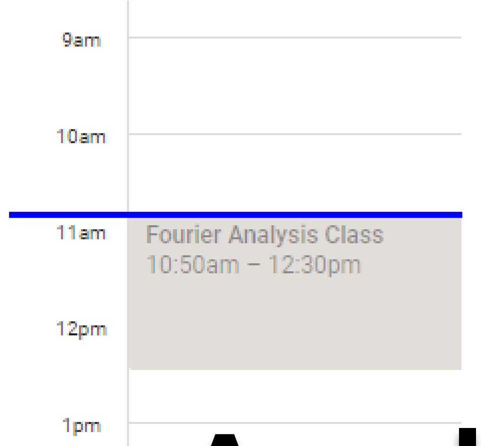
Deadline Oct 11 Deadline Oct 22 Deadline Nov 1 Deadline Nov 12 Deadline Day



Assignment (10)

NOTICE: The Deadline is **BEFORE** the EXERCISE CLASS beginning.
Day 3, 6, 9, 12 and an additional day of this course.

Deadline Hour



2018年10月

日	月	火	水	木	金	土
30	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31	1	2	3
4	5	6	7	8	9	10

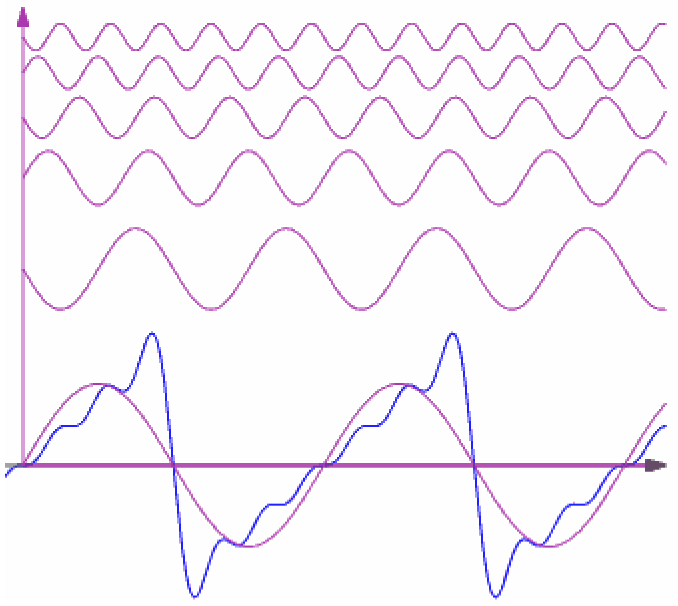
2018年11月

日	月	火	水	木	金	土
28	29	30	31	1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	1
2	3	4	5	6	7	8

Fourier Analysis

Lecture 1

Orthogonal System and Fourier Series



What you will learn in Lecture 1

I. Introduction of Fourier Analysis

II. Periodic Functions

III. Piecewise Continuous and Piecewise Smooth Functions

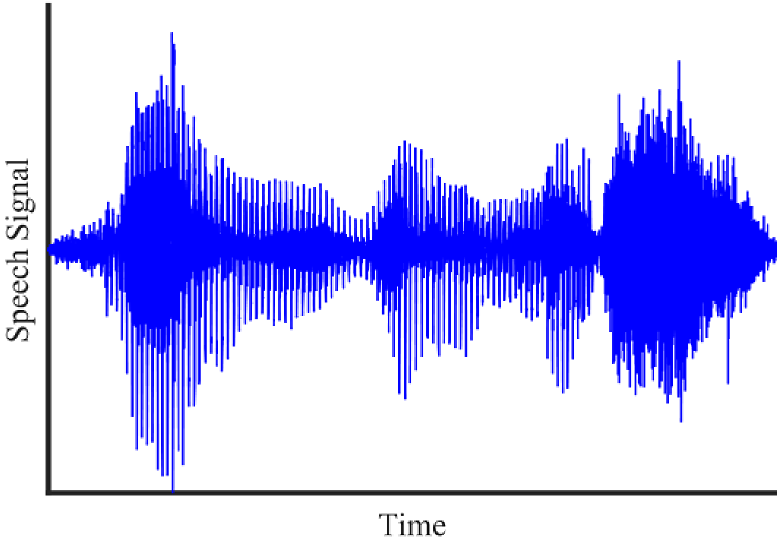
IV. The Trigonometric System and Orthogonality

V. Fourier Series

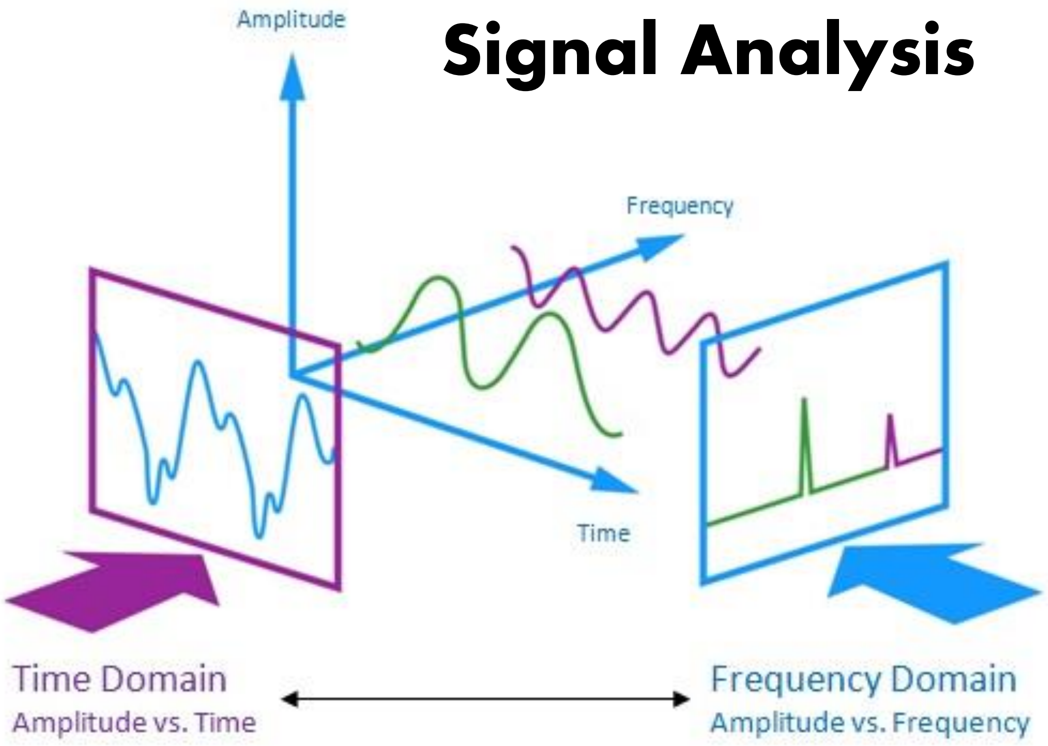
1.1 Introduction of Fourier Analysis

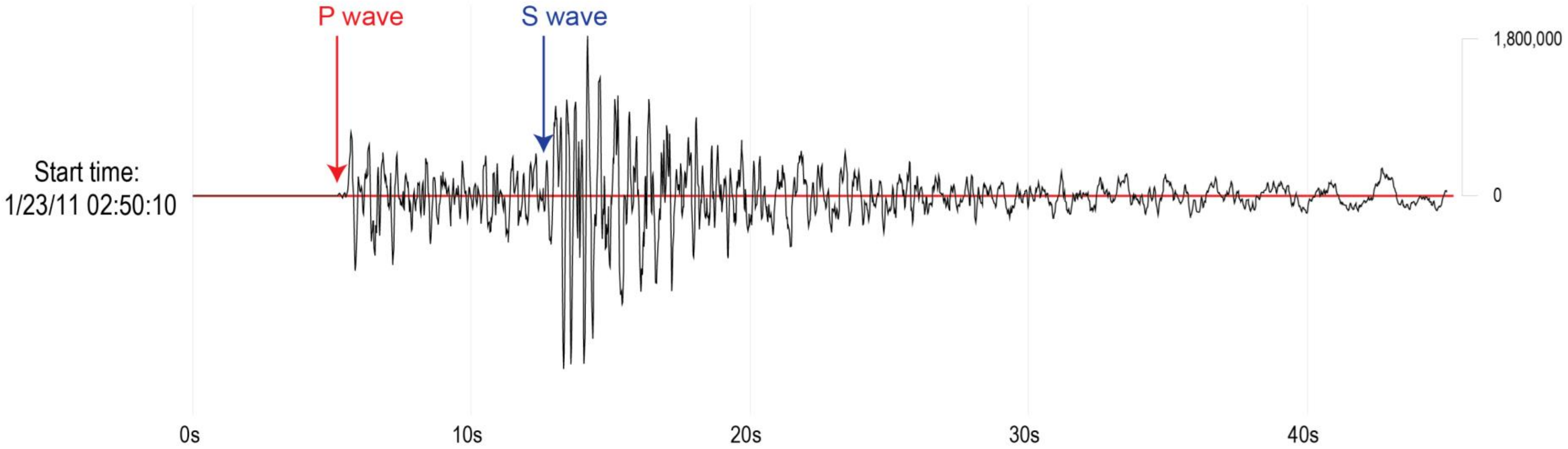
1.1 Introduction of Fourier Analysis

Signals in real-world



Signal Analysis





What is Signal?

"A signal is a source of information, generally a physical quantity, which varies with respect to time, space, temperature or like any other independent variable."

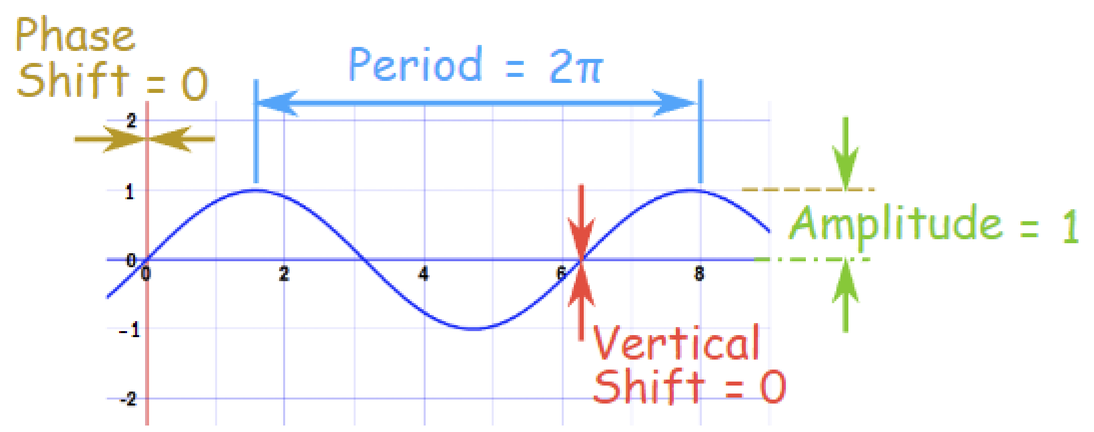
1.1 Introduction of Fourier Analysis

signal $y = A\sin(Bx + C) + D$

- amplitude is A
- period is $2\pi/B$
- phase shift is $-C/B$
- vertical shift is D

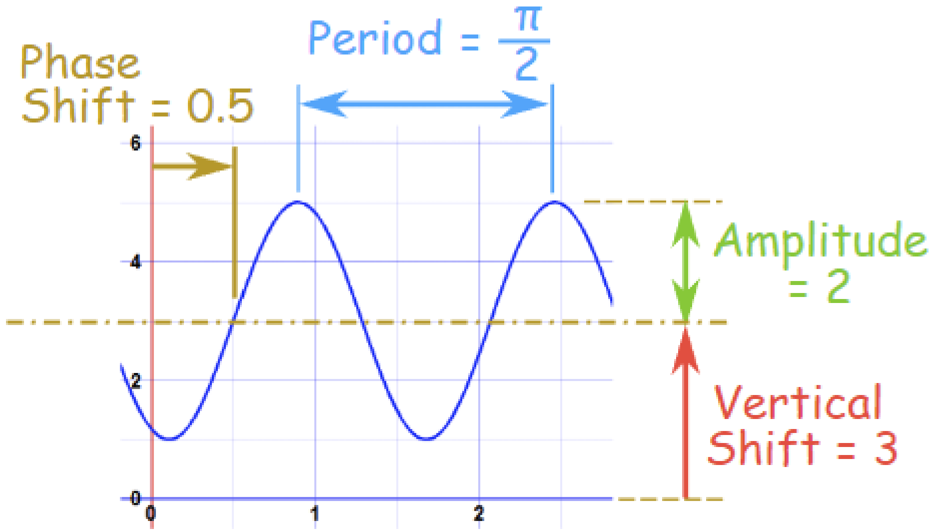
Example I: $\sin(x)$

- amplitude $A = 1$
- period $2\pi/B = 2\pi$
- phase shift $-C/B = -(0)/1 = 0$
- vertical shift $D = 0$



Example II: $2\sin(4x - 2) + 3$

- amplitude $A = 2$
- period $2\pi/B = 2\pi/4 = \pi/2$
- phase shift $-C/B = -(-2)/4 = 1/2 = 0.5$
- vertical shift $D = 3$



https://www.mathsisfun.com/algebra/amplitude-period-frequency-phase-shift.html

1.1 Introduction of Fourier Analysis

How to Represent Signals?

- Taylor series represents any function using polynomials.

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

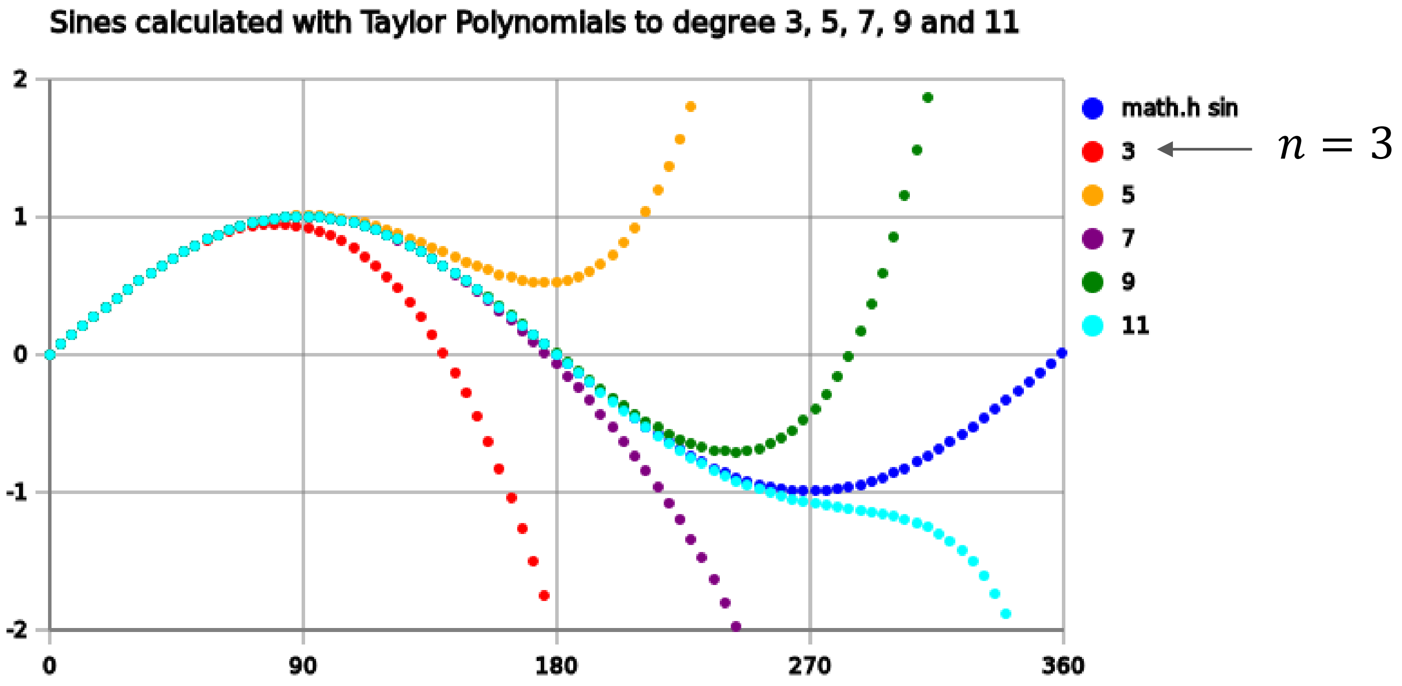
$$= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

The series is called the **Taylor series of the function** f at a (or **about** a or **centered at** a)

James Stewart, Calculus, 6th Edition, 2007

Polynomials are not the best - unstable and not very physically meaningful.

How to Represent Signals?



Taylor series of the function f at a

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

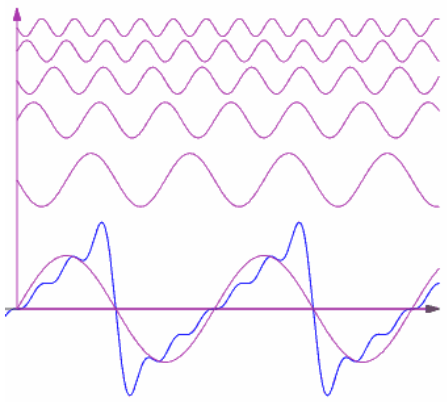
$$= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

1.1 Introduction of Fourier Analysis

How to Represent Signals?

Joseph Fourier had an amazing idea (1807):

"**Any** periodic function can be rewritten as a weighted sum of Sines and Cosines of different frequencies. "

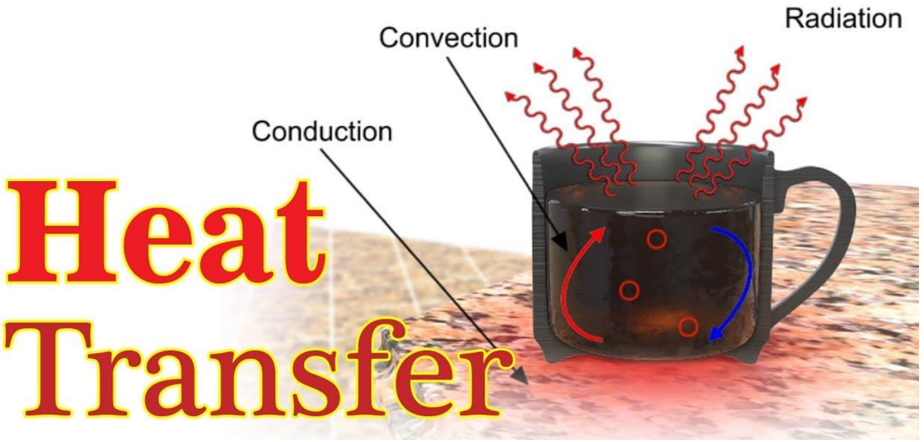


Joseph Fourier (1768-1830)

1.1 Introduction of Fourier Analysis

How to Represent Signals?

Joseph Fourier (1768-1830)

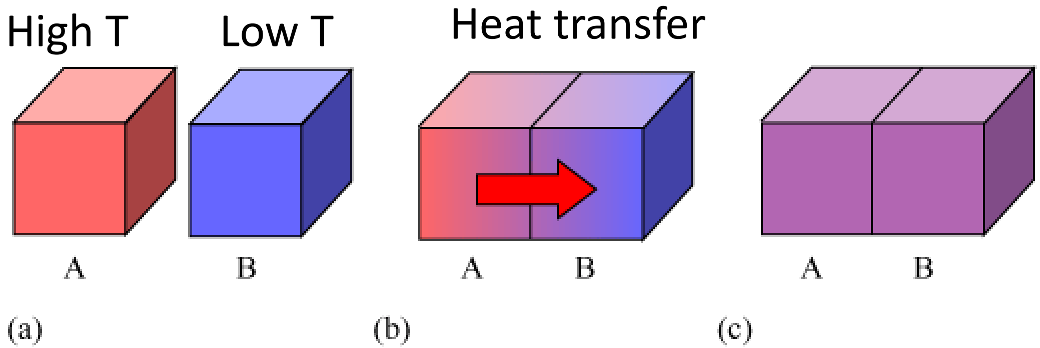


Partial Differential Equation: the Heat Equation

Solve $u(x,t)$: $$k\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < L, \quad t > 0$$

subject to : $$u(0,t) = 0, \quad u(L,t) = 0, \quad t > 0$$ Boundary condition

$$u(x,0) = f(x), \quad 0 < x < L$$ Initial condition



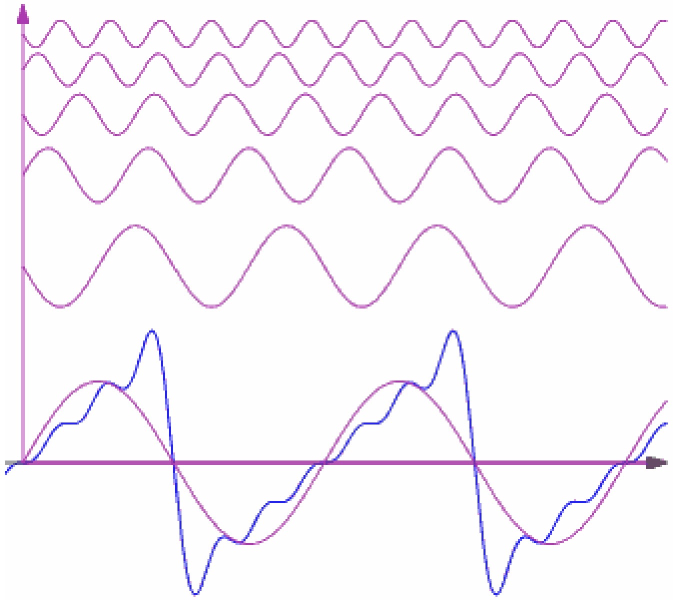
$$f(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi}{L}x\right)$$ for $0 < x < L$

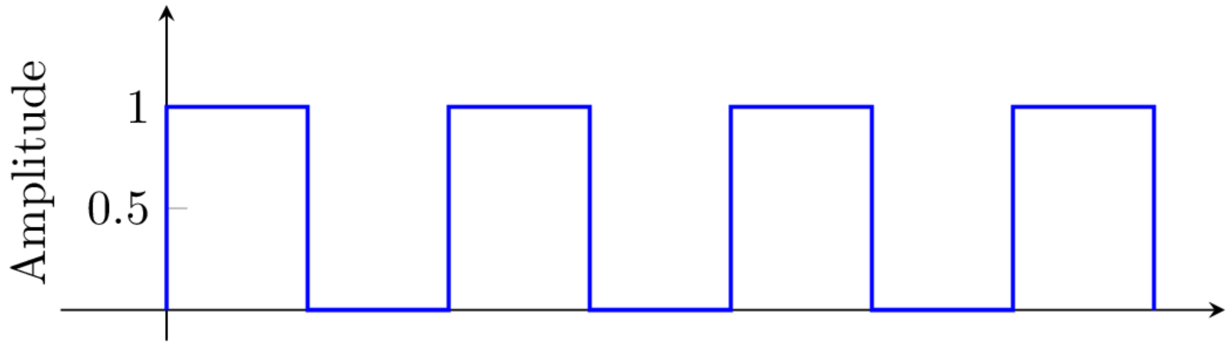
Read more:

1. I-HSIANG WANG, http://homepage.ntu.edu.tw/~ihwang/Teaching/Fall13/Slides/DE_Lecture_13_handout_v3.pdf
2. Chapter 1, Textbook

How to Represent Signals?

Fourier concludes that
an arbitrary wave can be represented **as a sum of an infinite number of weighted sinusoids**, i.e., sine and cosine waves.



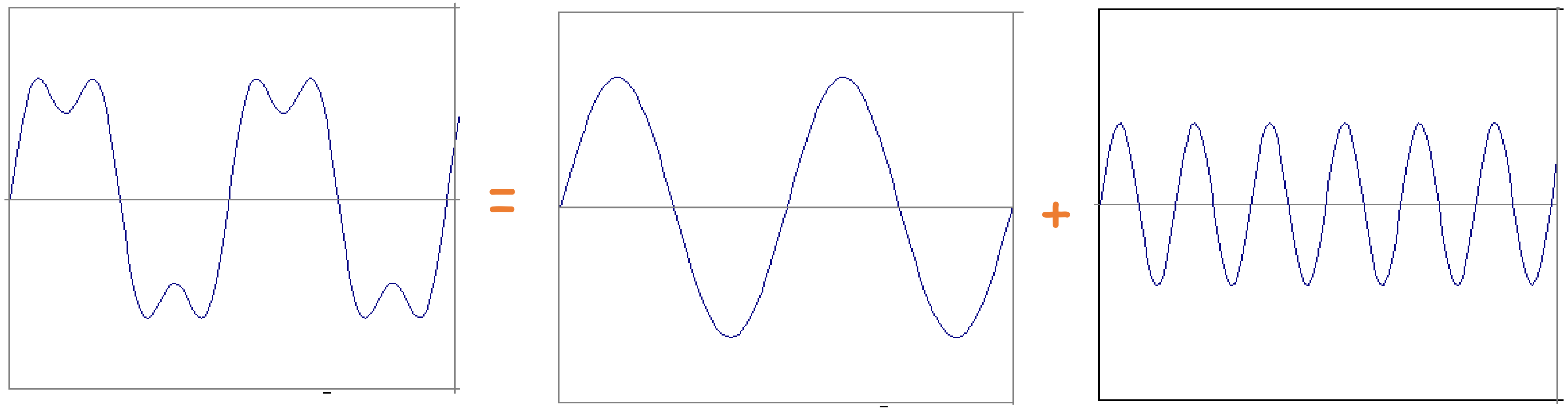


1.1 Introduction of Fourier Analysis

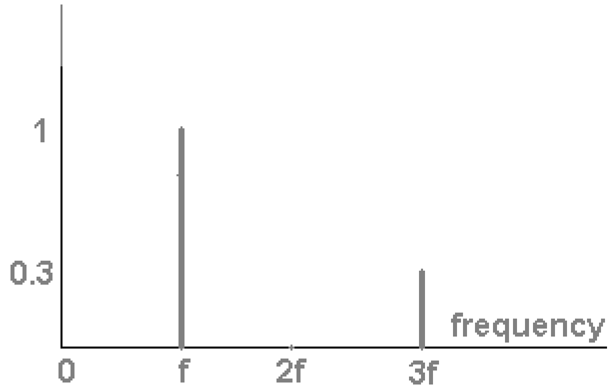
Time and Frequency

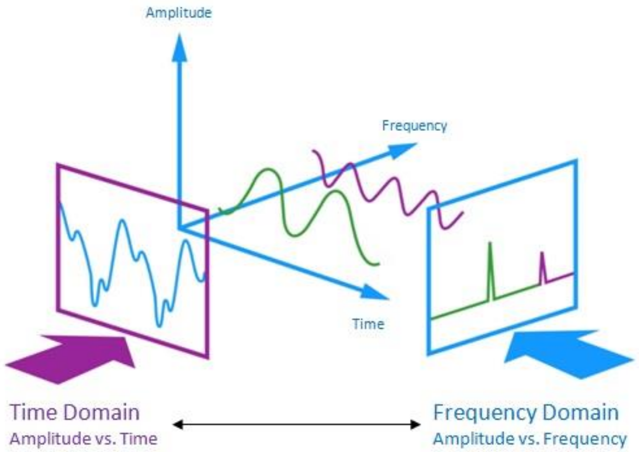
Example : $g(t) = \sin(2\pi f t) + \frac{1}{3}\sin(2\pi(3f)t)$

signal



Frequency Spectra

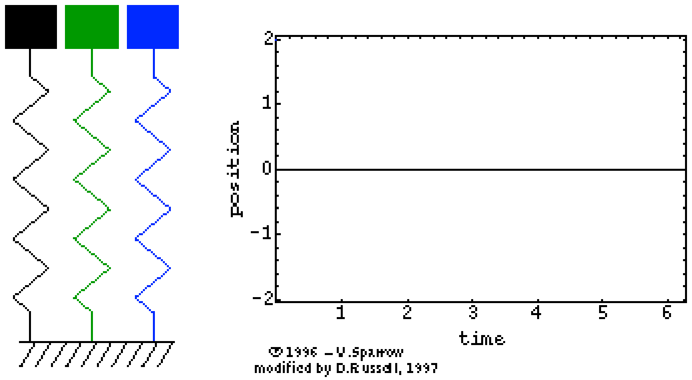




1.1 Introduction of Fourier Analysis

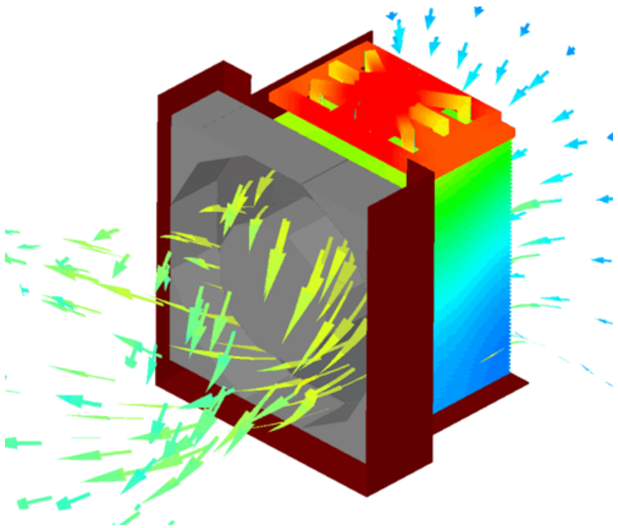
Fourier series are indeed the most suitable expansions for solving certain classical problems in **applied mathematics**.

They are fundamental to the **important physical phenomena**, such as

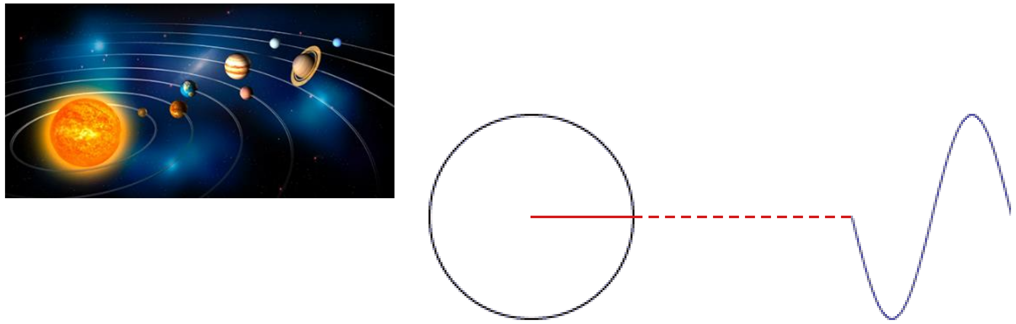


mechanical vibrations

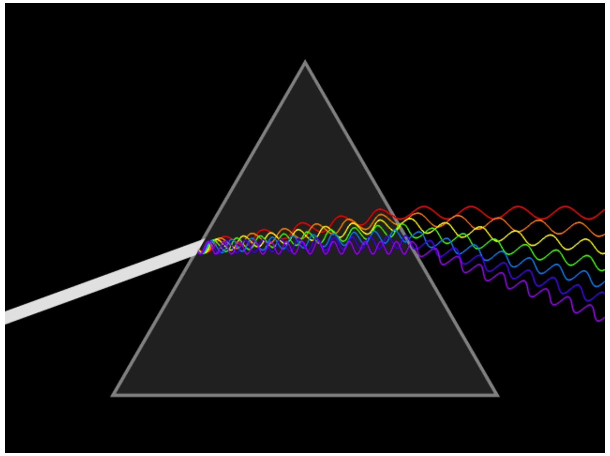
acoustic vibrations



heat transfer



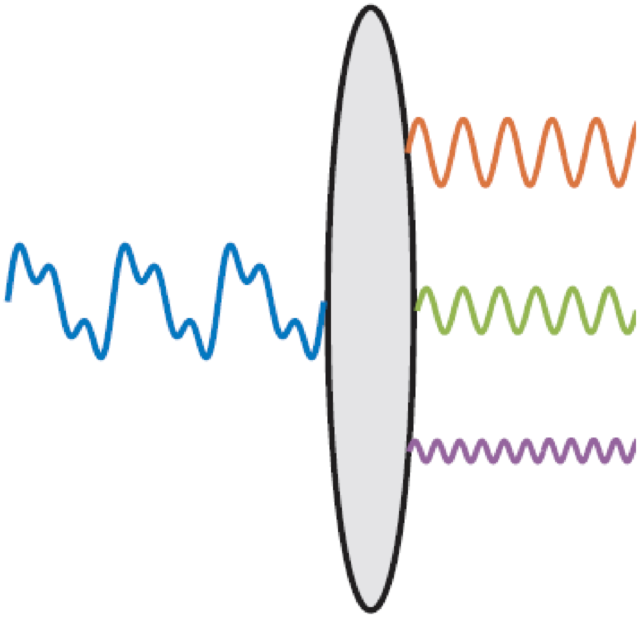
planetary motion



optics

1.1 Introduction of Fourier Analysis

Fourier **Analysis** | Fourier **Synthesis**



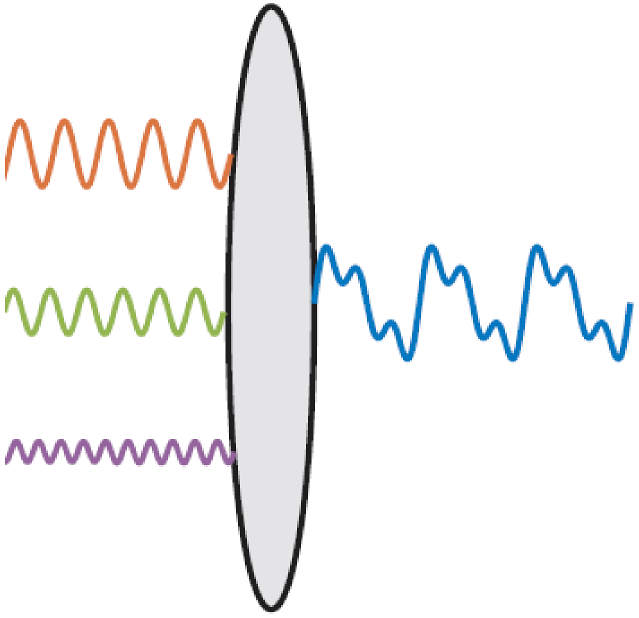


Figure: Fourier analysis is used to understand composite waves.

(a) Analysis: breaking a given signal into sine and cosine components

(b) Synthesis: adding certain sine and cosine to create a desired signal.

Charan Langton, Victor Levin, The Intuitive Guide to Fourier Analysis and Spectral Estimation, 2016

1.2 Periodic Functions

1.2 Periodic Functions

Periodic Functions

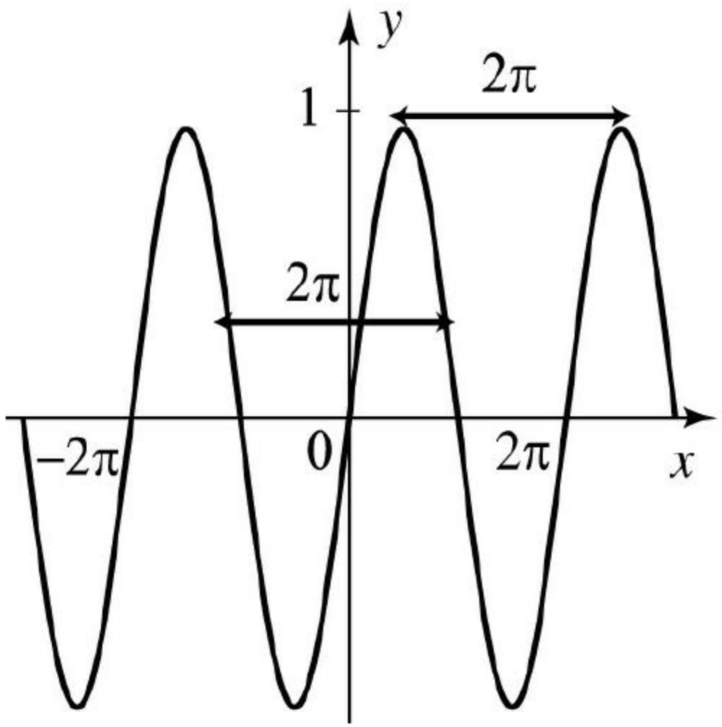


Figure 1 Graph of $\sin x$

$\sin x$ **repeat** every 2π units,
its graph is obtained **by repeating** the portion over any interval of length 2π.

This *periodicity* is expressed by the identity

$$\sin x = \sin(x + 2\pi) \quad \text{for all } x$$

1.2 Periodic Functions

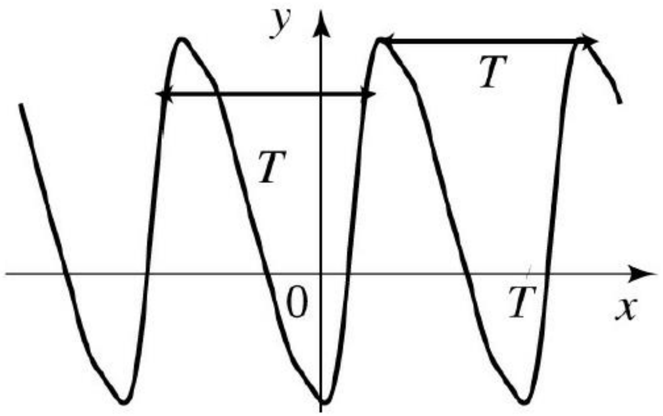


Figure 2 A T-periodic function

In general, a function f satisfying the identity

(1) $\quad f(x) = f(x+T) \quad$ for all x

where $T > 0$, is called **periodic**, or more specifically, T-**periodic**

The number $\boldsymbol{T}$ is called a **period** of f.

If f is nonconstant, we define the **fundamental period**, or simply, the period of f to be **the smallest positive number $\boldsymbol{T}$** for which (1) holds.

1.2 Periodic Functions

$$f(x) = f(x+T) = f(x+2T) = \cdots = f(x+nT)$$

Hence **if** T **is a period**, **then** nT **is also a period for any integer** $n > 0$.
In the case of the sine function, this amounts to saying that $2\pi, 4\pi,$ $6\pi, \ldots$ are all periods of $\sin x$, but only 2π is the fundamental period

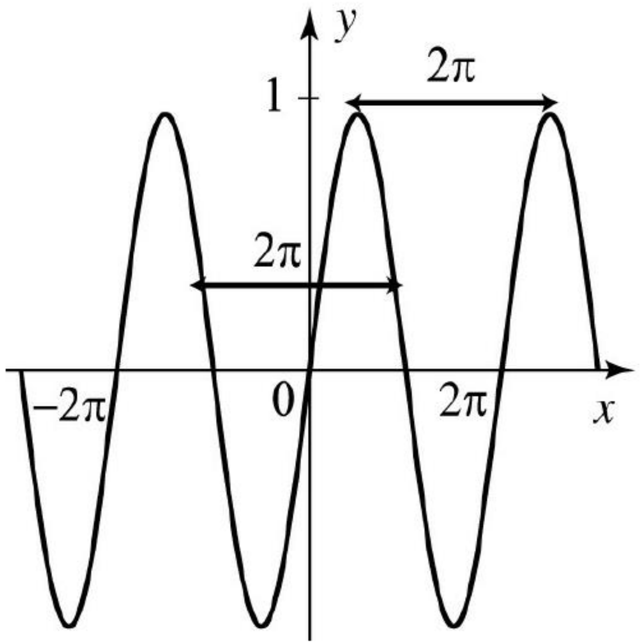


Figure 1 Graph of $\sin x$

EXAMPLE 1 Describing a periodic function

Describe the 2-periodic function f in Figure 3 in two different ways:

(a) by considering its values on the interval $0 \leq x < 2$;
(b) by considering its values on the interval $-1 \leq x < 1$.

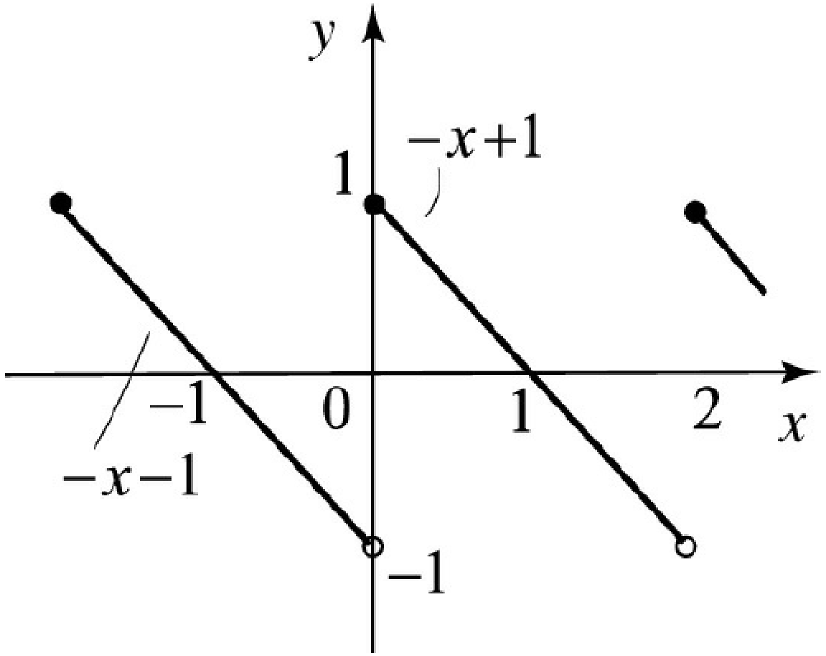


Figure 3 A 2-periodic function

Solution

(a) On the interval $0 \le x < 2$ the graph is a portion of the straight line $y = -x + 1$. Thus

$$f(x) = -x + 1 \quad \text{if } 0 \le x < 2$$

Now the relation $f(x + 2) = f(x)$ describes f for all other values of x.

(b) On the interval $-1 \le x < 1$, the graph consists of two straight lines (Figure 3). We have

$$f(x) = \begin{cases} -x - 1 & if \quad -1 \le x < 0 \\ -x + 1 & if \quad 0 \le x < 1 \end{cases}$$

As in part (a), the relation $f(x + 2) = f(x)$ describes f for all values of x outside the interval $[-1, 1)$.

Although the formulas in Example 1(a) and (b) are different, they describe the same periodic function.

1.2 Periodic Functions

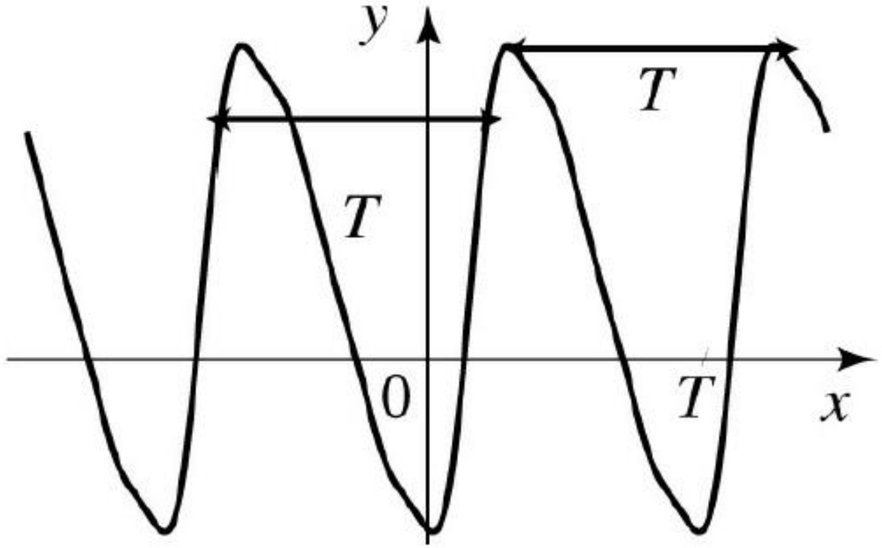


Figure 2 A T-periodic function

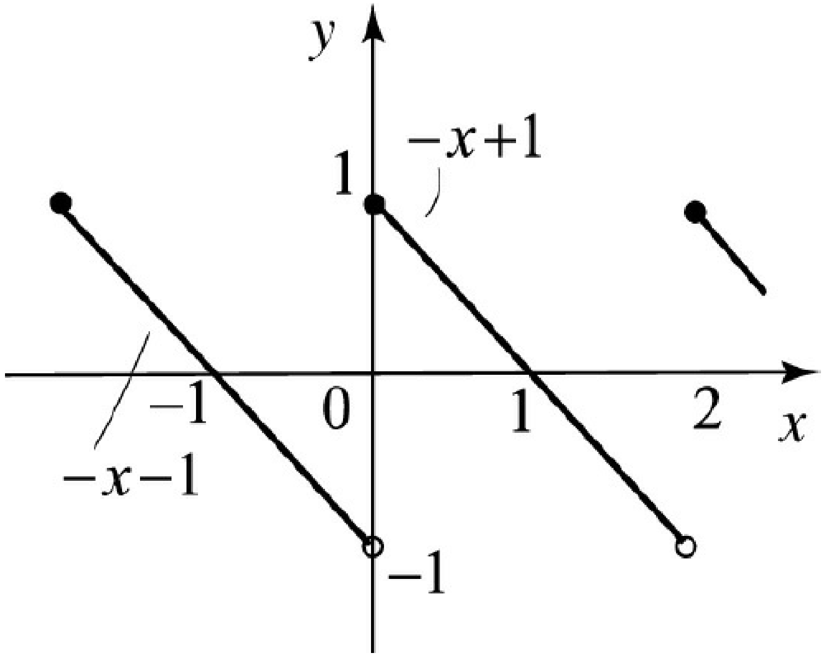


Figure 3 A 2-periodic function

continuous

Discontinuous at some certain points

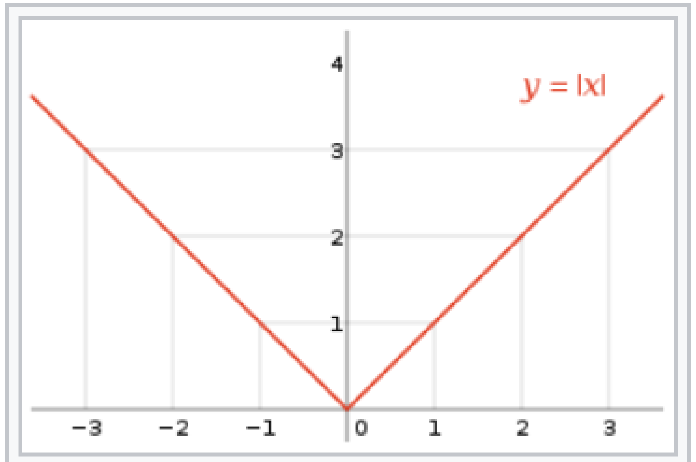
Piecewise Continuous and Piecewise Smooth Functions

1.2 Periodic Functions

A **piecewise-defined function** is defined by **multiple sub-functions**, **each sub-function** applying to **a certain interval** of the main function's domain, a sub-domain

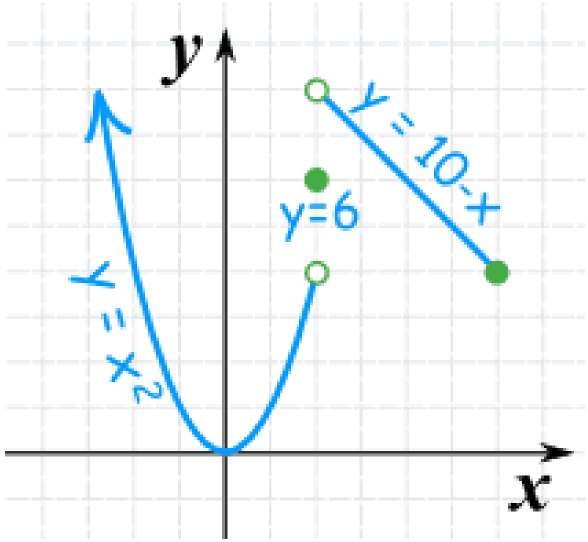
$$|x| = \begin{cases} -x & if\ x < 0 \\ x & if\ x \geq 0 \end{cases}$$

$$f(x) = \begin{cases} x^2 & if\ x < 2 \\ 6 & if\ x = 2 \\ 10 - x & if\ x > 2\ and\ x \leq 6 \end{cases}$$



Graph of the absolute value function, $y = |x|$.

From wikipedia



https://www.mathsisfun.com/sets/functions-piecewise.html

Discontinuity

Consider the function $f(x)$ in Figure 3. This function is not continuous at $x = 0, \pm 2, \pm 4, \ldots$.
Take a point of discontinuity, say $x = 0$. The limit of the function from the left is -1, while the limit from the right is 1.
Symbolically, this is denoted by

$$f(0-) = \lim_{x \to 0^-} f(x) = -1 \quad \text{and} \quad f(0+) = \lim_{x \to 0^+} f(x) = 1$$

In general, we write

$$f(c-) = \lim_{x \to c^-} f(x)$$

$$f(c+) = \lim_{x \to c^+} f(x)$$

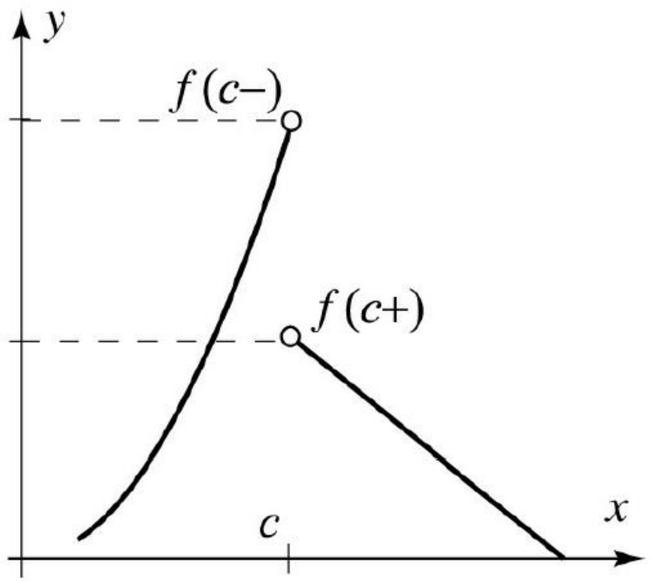


Figure 4 Left and right limit

Piecewise Continuous Functions

A function f is said to be **piecewise continuous** on the interval $[a, b]$ if $f(a+)$ and $f(b-)$ exist, and f is defined and continuous on (a, b) except at a finite number of points in (a, b) where the left and right limits exist.

Piecewise Continuous Functions

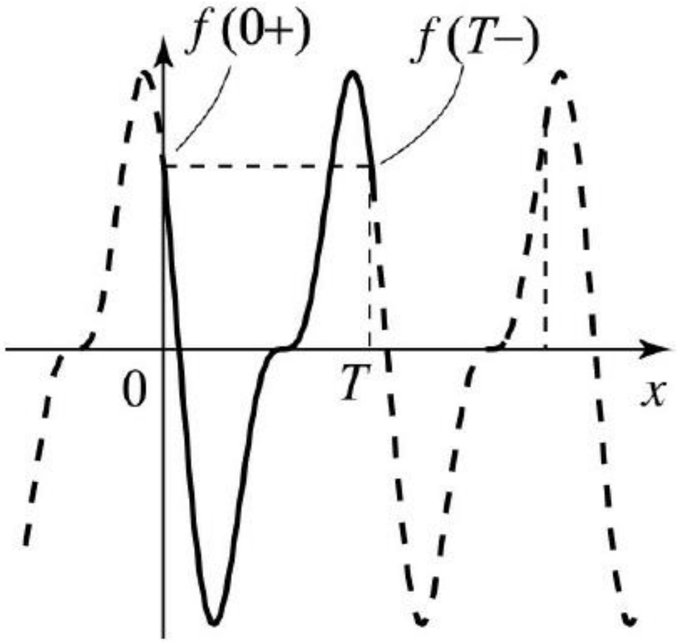


Figure 5 A continuous T-periodic function

At endpoints of the periodic function,

if f is T-periodic and continuous, then necessarily $f(0+) = f(T-)$

Piecewise Smooth Functions

A function f, defined on the interval $[a, b]$, is said to be **piecewise smooth** if f and f' are **piecewise continuous** on $[a, b]$.

Thus f is **piecewise smooth** if f is piecewise continuous on $[a, b]$, f' exists and is continuous in (a, b) except possibly at finitely many points c where the one-sided limits $\lim_{x \to c^-} f'(x)$ and $\lim_{x \to c^+} f'(x)$ exist.

Furthermore, $\lim_{x \to a^+} f'(x)$ and $\lim_{x \to b^-} f'(x)$ exist.

THEOREM 1: Integral over one period

Suppose that f is piecewise continuous and T-periodic. Then, for any real number a, we have

$$\int_0^T f(x)dx = \int_a^{a+T} f(x)dx$$

THEOREM 1: Integral over one period

Proof

(1) We have

$$\begin{aligned}\int_{nT}^{(n+1)T} f(x)\,dx &= \int_0^T f(s+nT)\,ds \quad (\text{let } x = s+nT,\ dx = ds)\\ &= \int_0^T f(s)\,ds \quad (\text{because } f \text{ is } T\text{-periodic})\\ &= \int_0^T f(x)\,dx.\end{aligned}$$

(2)

$$\begin{aligned}\int_{(n+1)T}^{a+T} f(x)\,dx &= \int_{nT}^{a} f(s+T)\,ds \quad (\text{let } x = s+T,\ dx = ds)\\ &= \int_{nT}^{a} f(s)\,ds \quad (\text{because } f \text{ is } T\text{-periodic})\\ &= \int_{nT}^{a} f(x)\,dx.\end{aligned}$$

(3)

$$\begin{aligned}\int_a^{a+T} f(x)\,dx &= \int_a^{(n+1)T} f(x)\,dx + \int_{(n+1)T}^{a+T} f(x)\,dx\\ &= \int_a^{(n+1)T} f(x)\,dx + \int_{nT}^{a} f(x)\,dx \quad (\text{by (b)})\\ &= \int_{nT}^{(n+1)T} f(x)\,dx = \int_0^T f(x)\,dx \quad (\text{by (a)}).\end{aligned}$$

■

EXAMPLE 2 Integrating periodic functions

Let f be the 2-periodic function in Example 1. Use **Theorem 1** to compute

(a) $\int_{-1}^{1} f^2(x)\,dx$

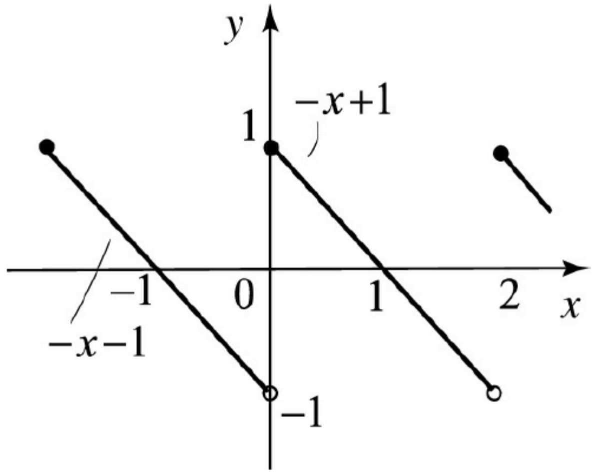


Figure 3 A 2-periodic function

Solution

(a) Observe that $f2(x)$ is also 2-periodic. Thus, by Theorem 1, to compute the integral in (a) we may choose any interval of length 2. Since on the interval $(0, 2)$ the function $f(x)$ is given by a single formula, we choose to work on this interval, and, using the formula from Example 1(a), we find

$$\int_{-1}^{1} f^2(x)\,dx = \int_{0}^{2} f^2(x)\,dx = \int_{0}^{2} (-x+1)^2\,dx = -\frac{1}{3}(-x+1)^3\Big|_0^2 = \frac{2}{3}$$

1.2 Periodic Functions

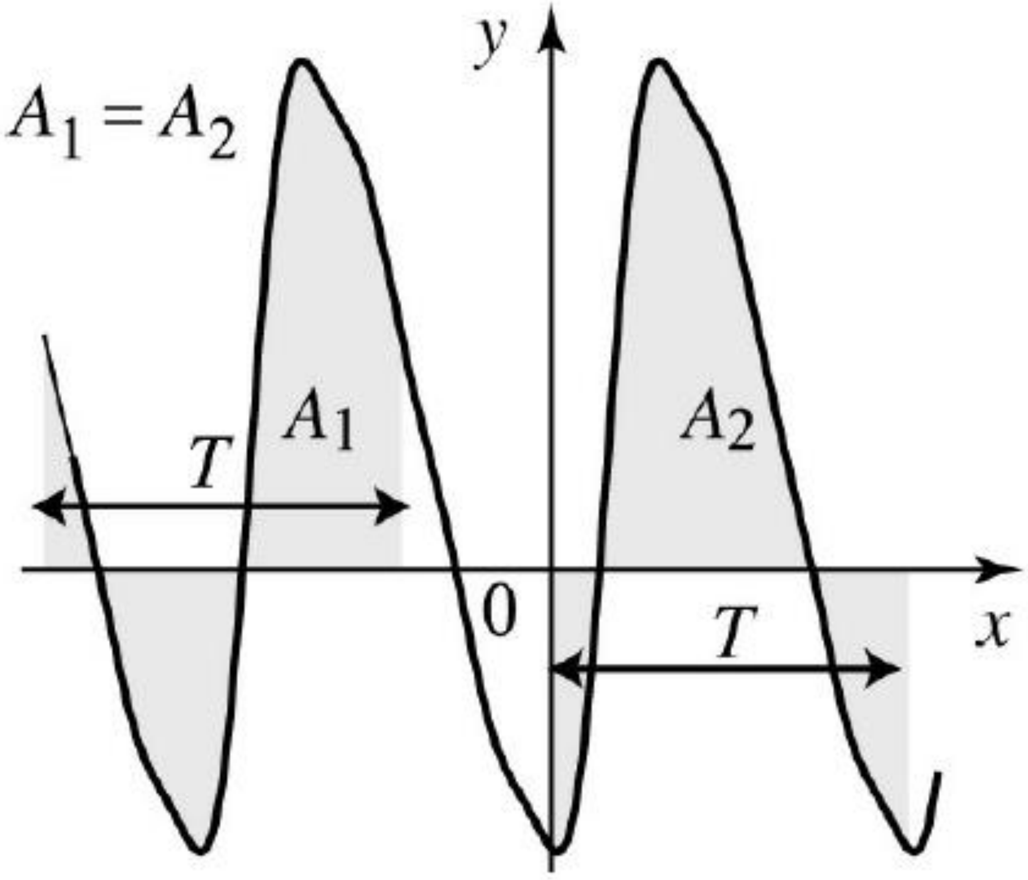


Figure 6 Areas over one period

1.3 The Trigonometric System and Orthogonality

1.3 The Trigonometric System and Orthogonality

The most important periodic functions are those in the (2π-periodic) **trigonometric system**

$$1, \cos x, \cos 2x, \cos 3x, \cdots, \cos mx, \cdots,$$
$$\sin x, \sin 2x, \sin 3x, \cdots \sin nx, \cdots$$

useful property: **orthogonality**

We say that **two functions f and g are orthogonal** over the interval $[a, b]$ if

$$\int_a^b f(x)g(x)dx = 0$$

1.3 The Trigonometric System and Orthogonality

Dot Product   Scalar product   Inner Product

If $\boldsymbol{a} = \langle a_1, a_2, a_3 \rangle$ and b $= \langle b_1, b_2, b_3 \rangle$, then the dot product of $\boldsymbol{a}$ and $\boldsymbol{b}$ is the number $\boldsymbol{a} \cdot \boldsymbol{b}$ given by

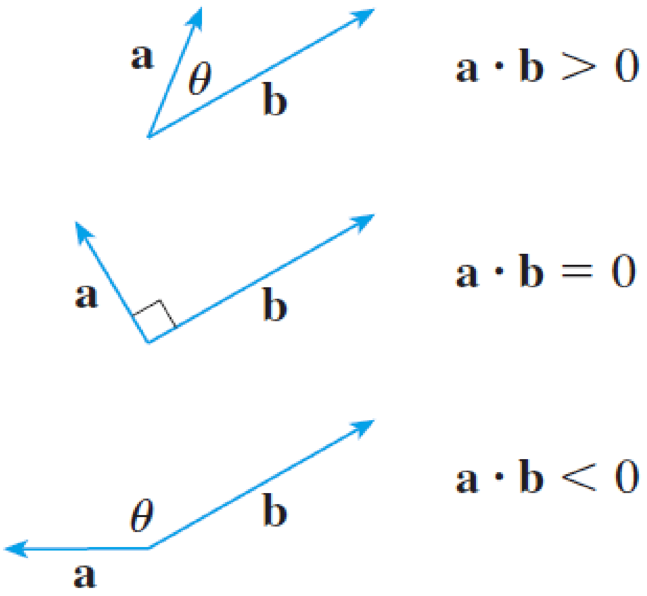
$$\boldsymbol{a} \cdot \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

If θ is the angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, then

$$\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$$

Two vectors and are orthogonal if and only if

$$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{0}$$



Note: Vector Length $\|\boldsymbol{a}\|$ can be expressed via inner product:

$$\|\boldsymbol{a}\|^2 = a_1^2 + a_2^2 + \cdots + a_n^2 = \langle \boldsymbol{a}, \boldsymbol{a} \rangle, \quad \text{so} \quad \|\boldsymbol{a}\|^2 = \sqrt{\langle \boldsymbol{a}, \boldsymbol{a} \rangle}$$

James Stewart, Calculus, 6th Edition, 2007

A. Kissinger, Matrix Calculations: Inner Products & Orthogonality, Radboud University Nijmegen

Definition (Inner Product of Functions)

The inner product of $f_1(x)$ and $f_2(x)$ on an interval $[a, b]$ is defined as

$$\langle f_1, f_2 \rangle := \int_a^b f_1(x) f_2(x)\, dx$$

Once inner product is defined, we can accordingly define **norm**.

Definition (Norm of a Function)

The norm of a function $f(x)$ on an interval $[a, b]$ is

$$||f(x)|| := \sqrt{\langle f, f \rangle} = \sqrt{\int_a^b (f(x))^2 \, dx}$$

I-Hsiang Wang, Chapter 11: Fourier Series, Differential Equations, National Taiwan University, 2013

1.3 The Trigonometric System and Orthogonality

Definition (Orthogonal Functions)

$f_1(x)$ and $f_2(x)$ are **orthogonal** on an interval $[a, b]$ if $\langle f_1, f_2 \rangle = 0$.

Definition (Orthogonal Set)

$\{\phi_0(x), \phi_1(x), \cdots\}$ are **orthogonal** on an interval $[a, b]$ if

$$\langle \phi_m, \phi_n \rangle = \int_a^b \phi_m(x)\phi_n(x)\, dx = 0, \quad m \neq n.$$

I-Hsiang Wang, Chapter 11: Fourier Series, Differential Equations, National Taiwan University, 2013

Example (Orthogonal or Not Depends on the Inverval)

The functions $f_1(x) = x$ and $f_2(x) = x^2$ are orthogonal on the interval $[a, b]$, $a < b$, only if $a = -b$.

Proof: When $a < b$,

$$\langle x, x^2 \rangle = \int_a^b x^3 \, dx = \left[\frac{1}{4} x^4 \right]_a^b = \frac{1}{4} \left(a^4 - b^4 \right) = 0 \iff a + b = 0$$

I-Hsiang Wang, Chapter 11: Fourier Series, Differential Equations, National Taiwan University, 2013

1.3 The Trigonometric System and Orthogonality

Orthogonality properties of the trigonometric system are expressed by

$$\int_{-\pi}^{\pi} \cos mx \cos nx \, dx = 0 \quad m \neq n$$

$$\int_{-\pi}^{\pi} \cos mx \sin nx \, dx = 0 \quad \text{for all } m \text{ and } n$$

$$\int_{-\pi}^{\pi} \sin mx \sin nx \, dx = 0 \quad m \neq n$$

For the case $n = m$

$$\int_{-\pi}^{\pi} \cos^2 mx \, dx = \int_{-\pi}^{\pi} \sin^2 mx \, dx = \pi \quad \text{for all } m \neq 0$$

1.3 The Trigonometric System and Orthogonality

$$\int_{-\pi}^{\pi} \cos mx \cos nx \, dx = 0 \quad m \neq n$$

Proof

Use a trigonometric identity and write

$$\cos mx \cos nx \, dx = \frac{1}{2}(\cos(m+n)\,x + \cos(m-n)\,x)$$

Since $m \pm n \neq 0$, we get

$$\int_{-\pi}^{\pi} \cos mx \cos nx \, dx = \int_{-\pi}^{\pi} \frac{1}{2}(\cos(m+n)\,x + \cos(m-n)\,x)\, dx$$

$$= \frac{1}{2}\left[\frac{1}{m+n}\sin(m+n)\,x + \frac{1}{m-n}\sin(m-n)\,x\right]_{-\pi}^{\pi} = 0 \qquad \blacksquare$$

1.4 Fourier Analysis

1.4 Fourier Analysis

Fourier series are special expansions of 2π-periodic functions of the form

$$(1) \qquad f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Q: If a function has a Fourier series, how do we compute the coefficients $a_0, a_1, a_2, \ldots, b_1, b_2, \ldots$?

Euler Formulas for the Fourier Coefficients

We proceed as Fourier himself did.

We integrate both sides of (1) over the interval $[-\pi, \pi]$

assuming term-by-term integration is justified, and get

$$\int_{-\pi}^{\pi} f(x)dx = \int_{-\pi}^{\pi} a_0 dx + \sum_{n=1}^{\infty} \int_{-\pi}^{\pi} (a_n \cos nx + b_n \sin nx) dx$$

But because

$$\int_{-\pi}^{\pi} \cos nx \, dx = \int_{-\pi}^{\pi} \sin nx \, dx = 0 \quad \text{for } n = 1, 2, \cdots$$

it follows that

$$\int_{-\pi}^{\pi} f(x)dx = \int_{-\pi}^{\pi} a_0 dx + 0 = 2\pi a_0 \quad \Rightarrow \quad a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

Note: a_0 is the average of f on the interval $[-\pi, \pi]$.

1.4 Fourier Analysis

Starting with (1), we **multiply both sides by** $\cos mx\ (m \geq 1)$, integrate term-by-term, use the **orthogonality**

$$\int_{-\pi}^{\pi} f(x)\cos mx\,dx = \overbrace{\int_{-\pi}^{\pi} a_0 \cos mx\,dx}^{=0} + \overbrace{\sum_{n=1}^{\infty}\int_{-\pi}^{\pi} a_n \cos nx \cos mx\,dx}^{=0\ \text{for}\ m \neq n}$$

$$+ \overbrace{\sum_{n=1}^{\infty}\int_{-\pi}^{\pi} b_n \sin nx \cos mx\,dx}^{=0}$$

$$= a_m \overbrace{\int_{-\pi}^{\pi} \cos^2 mx\,dx}^{=\pi\ \text{for}\ m = n} = \pi a_m$$

Hence $\quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \quad (n = 1, 2, \dots)$

1.4 Fourier Analysis

By a similar procedure, starting with (1), we **multiply both sides by** $\sin mx\ (m \geq 1)$, integrate term-by-term, use the **orthogonality**

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx \quad (n = 1, 2, \dots)$$

Euler Formulas for the Fourier Coefficients

Suppose that f has the Fourier series representation

$$(1) \qquad f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

Then the coefficients a_0, a_n, and b_n are called the Fourier coefficients of f and are given by the following Euler formulas:

$$(2) \qquad a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$(3) \qquad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx \quad (n = 1, 2, \dots)$$

$$(4) \qquad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\, dx \quad (n = 1, 2, \dots)$$

Euler Formulas for the Fourier Coefficients

Alternative Euler Formulas

(5) $$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)\,dx$$

(6) $$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx \quad (n = 1, 2, \dots)$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx \quad (n = 1, 2, \dots)$$

Review for Lecture 1

- Periodic Functions
- Piecewise Continuous Functions
- Piecewise Smooth Functions
- The Trigonometric System and Orthogonality
- Fourier Series

Exercises

Please Check https://github.com/uoaworks/FourierAnalysisAY2018

Reading: Section 2.1, Textbook

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