

# Fourier Analysis

## Assignment 2

20181009

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1. A  $2\pi$ -periodic function in Figure 1 is specified on the interval  $[-\pi, \pi]$ .
  - (a) Plot function on the interval  $[-3\pi, 3\pi]$ .
  - (b) Plot its Fourier series (without computing it) on the interval  $[-3\pi, 3\pi]$ .  
Hint : Theorem 1 and Figure 5 of Section 2.2 in the textbook.
2. The equation of a  $2\pi$ -periodic function is given on an interval of length  $2\pi$ . You are also given the Fourier series of the function, Derive the given Fourier series.  
 $f(x) = |x|$  if  $-\pi \leq x < \pi$ .  
Fourier series :  $\frac{\pi}{2} - \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} \cos(2k+1)x$   
Hint :  $|x|$  is an even function on the interval  $-\pi \leq x < \pi$ .
3. \*A  $2p$ -periodic function is given on an interval of length  $2p$ .
  - (a) State whether the function is even, odd, or neither.
  - (b) Derive the given Fourier series, and determine its values at the points of discontinuity.

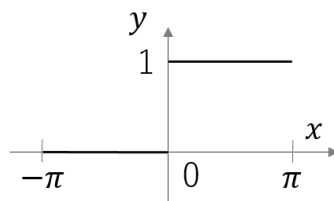


FIGURE 1 – For Exercise 1.

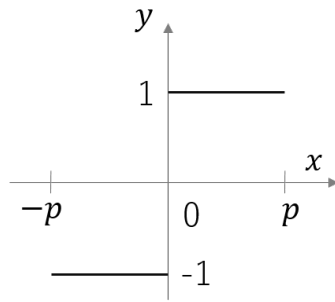


FIGURE 2 – For Exercise 3.

$$f(x) = \begin{cases} 1, & \text{if } 0 < x < p \\ -1, & \text{if } -p < x < 0 \end{cases}$$

$$\text{Fourier Series : } \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{1}{(2k+1)} \sin \frac{(2k+1)\pi}{p} x$$

4. \*Find the half-range expansions of the given function.

$$f(x) = \pi - x \text{ if } 0 \leq x \leq \pi$$

Notice : \* is an optional question.

Reading Materials : Section 2.2 (Example 1, 2, 4, 5), Section 2.3 (Example 1, 4, 5) and Section 2.4 (Example 1, 2) of the textbook.