Week 1: Context Free Grammars

We start with some revision of sets. If you are unfamiliar with any of the notation, please ask one of the TAs.

* 1. Write $\mathbb{N}$ for the set of natural numbers $0, 1, 2\ldots$, and write Σ for the alphabet $\{0, 1\}$. List the elements of the following sets in any order.

  (a) $\{1, 2, 3\} \cup \{2, 4, 6\}$

  (b) $\{1, 2, 3\} \cap \{2, 4, 6\}$

  (c) $\{1, 2, 3\} \times \{2, 4, 6\}$

  (d) $\{1, 2, 3\} \setminus \{2, 4, 6\}$

  (e) $\{2m \mid m \in \mathbb{N}, \, 0 \le m \le 5\}$

  (f) $\{uu \mid u \in \Sigma^*, |u| = 2\}$

  (g) $\{u0v \mid u \in \Sigma^*, \, v \in \Sigma^*, \, |uv| = 2\}$

  (h) $\{uvw \mid u \in \Sigma, \, v \in \Sigma, \, w \in \Sigma, \, w \text{ is the xor of } u \text{ and } v\}$

Solution

  (a) $1, 2, 3, 4, 6$

  (b) 2

  (c) $(1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 2), (3, 4), (3, 6)$

  (d) $1, 3$

  (e) $0, 2, 4, 6, 8, 10$

  (f) $0000, 0101, 1010, 1111$

  (g) $001, 100, 101, 000, 010, 011, 110$

  (h) $000, 011, 101, 110$

Recall the grammar from the notes for Boolean expressions.

$$
\begin{array}{rcl}
B & ::= & F \mid F \parallel B \\
F & ::= & L \mid L \ \&\& \ F \\
L & ::= & \text{true} \mid \text{false} \mid (B)
\end{array}
$$

* 2. Give derivations for the following strings which are in the language of Boolean expressions.

    (a) true && false

    (b) true || false && true

    (c) true && (false || true)

Solution

There are different possible derivations depending on which non-terminals are chosen to be replaced. I will generally replace leftmost non-terminals at each step.

(a)

$$
\begin{aligned}
B & \to F \\
& \to L \ \&\& \ F \\
& \to \text{true} \ \&\& \ F \\
& \to \text{true} \ \&\& \ L \\
& \to \text{true} \ \&\& \ \text{false}
\end{aligned}
$$

(b)

$$
\begin{aligned}
B & \to F \parallel B \\
& \to L \parallel B \\
& \to \text{true} \parallel B \\
& \to \text{true} \parallel F \\
& \to \text{true} \parallel L \ \&\& \ F \\
& \to \text{true} \parallel \text{false} \ \&\& \ F \\
& \to \text{true} \parallel \text{false} \ \&\& \ L \\
& \to \text{true} \parallel \text{false} \ \&\& \ \text{true}
\end{aligned}
$$

2

(c)

$$B \rightarrow F$$
$$\rightarrow L \text{ \&\& } F$$
$$\rightarrow \text{true \&\& } F$$
$$\rightarrow \text{true \&\& } L$$
$$\rightarrow \text{true \&\& } (B)$$
$$\rightarrow \text{true \&\& } (F \| B)$$
$$\rightarrow \text{true \&\& } (L \| B)$$
$$\rightarrow \text{true \&\& } (\text{false} \| B)$$
$$\rightarrow \text{true \&\& } (\text{false} \| F)$$
$$\rightarrow \text{true \&\& } (\text{false} \| L)$$
$$\rightarrow \text{true \&\& } (\text{false} \| \text{true})$$

* 3. Give three distinct derivations for the string true || false.

Solution

(a)

$$B \rightarrow F \| B$$
$$\rightarrow L \| B$$
$$\rightarrow \text{true} \| B$$
$$\rightarrow \text{true} \| F$$
$$\rightarrow \text{true} \| L$$
$$\rightarrow \text{true} \| \text{false}$$

(b)

$$B \rightarrow F \| B$$
$$\rightarrow F \| F$$
$$\rightarrow L \| F$$
$$\rightarrow \text{true} \| F$$
$$\rightarrow \text{true} \| L$$
$$\rightarrow \text{true} \| \text{false}$$

(c)

$$B \rightarrow F \| B$$
$$\rightarrow F \| F$$
$$\rightarrow F \| L$$
$$\rightarrow F \| \text{false}$$
$$\rightarrow L \| \text{false}$$
$$\rightarrow \text{true} \| \text{false}$$