

Week 2: Context Free Grammars

We start with some revision of sets. If you are unfamiliar with any of the notation, please ask one of the TAs.

- * 1. Write \mathbb{N} for the set of natural numbers $0, 1, 2, \dots$, and write Σ for the alphabet $\{0, 1\}$. List the elements of the following sets in any order.

- (a) $\{1, 2, 3\} \cup \{2, 4, 6\}$
- (b) $\{1, 2, 3\} \cap \{2, 4, 6\}$
- (c) $\{1, 2, 3\} \times \{2, 4, 6\}$
- (d) $\{1, 2, 3\} \setminus \{2, 4, 6\}$
- (e) $\{2m \mid m \in \mathbb{N}, 0 \leq m \leq 5\}$
- (f) $\{uu \mid u \in \Sigma^*, |u| = 2\}$
- (g) $\{u0v \mid u \in \Sigma^*, v \in \Sigma^*, |uv| = 2\}$
- (h) $\{uvw \mid u \in \Sigma, v \in \Sigma, w \in \Sigma, w \text{ is the xor of } u \text{ and } v\}$

Solution

- (a) 1, 2, 3, 4, 6
- (b) 2
- (c) (1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 2), (3, 4), (3, 6)
- (d) 1, 3
- (e) 0, 2, 4, 6, 8, 10
- (f) 0000, 0101, 1010, 1111
- (g) 001, 100, 101, 000, 010, 011, 110
- (h) 000, 011, 101, 110

Recall the grammar from the notes for Boolean expressions.

* 2. Give derivations for the following strings which are in the language of Boolean expressions.

(a) true && false

(b) true || false && true

(c) true && (false || true)

Solution

There are different possible derivations depending on which non-terminals are chosen to be replaced. I will generally replace leftmost non-terminals at each step.

(a)

$$\begin{aligned} B &\rightarrow F \\ &\rightarrow L \ \&\& \ F \\ &\rightarrow \text{true} \ \&\& \ F \\ &\rightarrow \text{true} \ \&\& \ L \\ &\rightarrow \text{true} \ \&\& \ \text{false} \end{aligned}$$

(b)

$$\begin{aligned} B &\rightarrow F \ || \ B \\ &\rightarrow L \ || \ B \\ &\rightarrow \text{true} \ || \ B \\ &\rightarrow \text{true} \ || \ F \\ &\rightarrow \text{true} \ || \ L \ \&\& \ F \\ &\rightarrow \text{true} \ || \ \text{false} \ \&\& \ F \\ &\rightarrow \text{true} \ || \ \text{false} \ \&\& \ L \\ &\rightarrow \text{true} \ || \ \text{false} \ \&\& \ \text{true} \end{aligned}$$

(c)

$$\begin{aligned} B &\rightarrow F \\ &\rightarrow L \ \&\& \ F \\ &\rightarrow \text{true} \ \&\& \ F \\ &\rightarrow \text{true} \ \&\& \ L \\ &\rightarrow \text{true} \ \&\& \ (B) \\ &\rightarrow \text{true} \ \&\& \ (F \ || \ B) \\ &\rightarrow \text{true} \ \&\& \ (L \ || \ B) \\ &\rightarrow \text{true} \ \&\& \ (\text{false} \ || \ B) \\ &\rightarrow \text{true} \ \&\& \ (\text{false} \ || \ F) \\ &\rightarrow \text{true} \ \&\& \ (\text{false} \ || \ L) \\ &\rightarrow \text{true} \ \&\& \ (\text{false} \ || \ \text{true}) \end{aligned}$$

* 3. Give three distinct derivations for the string $\text{true} \parallel \text{false}$.

Solution

(a)

$$\begin{aligned} B &\rightarrow F \parallel B \\ &\rightarrow L \parallel B \\ &\rightarrow \text{true} \parallel B \\ &\rightarrow \text{true} \parallel F \\ &\rightarrow \text{true} \parallel L \\ &\rightarrow \text{true} \parallel \text{false} \end{aligned}$$

(b)

$$\begin{aligned} B &\rightarrow F \parallel B \\ &\rightarrow F \parallel F \\ &\rightarrow L \parallel F \\ &\rightarrow \text{true} \parallel F \\ &\rightarrow \text{true} \parallel L \\ &\rightarrow \text{true} \parallel \text{false} \end{aligned}$$

(c)

$$\begin{aligned} B &\rightarrow F \parallel B \\ &\rightarrow F \parallel F \\ &\rightarrow F \parallel L \\ &\rightarrow F \parallel \text{false} \\ &\rightarrow L \parallel \text{false} \\ &\rightarrow \text{true} \parallel \text{false} \end{aligned}$$