

Week 4: Regular Languages

In the problems this week you will need to make use of the formal definition of a finite state automaton, given at <https://uob-coms20007.github.io/reference/regular/automata.html#finite-state-automaton>.

* 1. Draw the diagram of the following automata:

(a) $(\{e, o\}, \{0, 1\}, \{(e, 0, o), (e, 1, o), (o, 0, e), (o, 1, e)\}, e, \{e\})$

(b) $(Q, \{0, 1\}, \Delta, q_0, Q)$ where $Q = \{q_0, q_1, q_2, q_3\}$ and Δ is:

$$\{(q_0, 0, q_0), (q_0, 1, q_1), (q_1, 0, q_2), (q_1, 1, q_3), (q_2, 0, q_1), (q_3, 0, q_3)\}$$

(c) $(Q, \Sigma, \Delta, q_0, Q)$ where:

- $Q = \{1, 2, 3, 4, 5\}$
- $\Sigma = \{a, b\}$
- $\Delta = \{(i, a, i+1) \mid 1 \leq i \leq 5\} \cup \{(j, b, j) \mid j \text{ is even}\}$
- $q_0 = 1$
- $F = \{1, 3, 5\}$

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procedure cl( $X$ ):
   $X' := \emptyset$ 
  while  $X \neq X'$  do
     $X' := X$ 
    for each  $q \in X$ 
      if  $(q, \epsilon, q') \in \Delta$ 
         $X' := X' \cup \{q'\}$ 
  return  $X$ 

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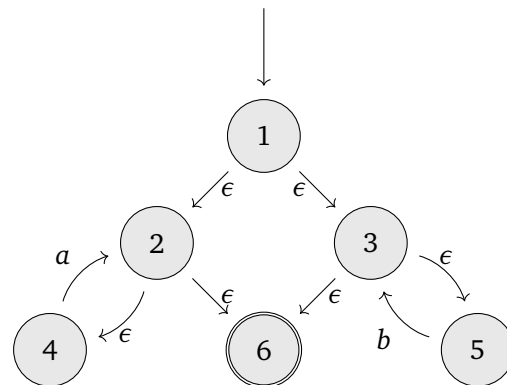


Figure 1: ϵ -closure of $X \subseteq Q$ wrt transitions Δ , and the automaton from Week 3 Q4(b)

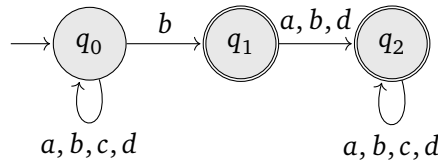
- * 2. Suppose M is a finite automaton with states Q . The ϵ -closure of a set of states $X \subseteq Q$ in M , written $\text{cl}(X)$, is the set of all states that can be reached from any state in X using only ϵ -transitions. It can be computed using the algorithm in Figure 1.

- (a) Construct a table with two columns. Each row of the table should contain a state of the automaton from Figure 1 in the first column and the ϵ -closure of that state in the second column.
- (b) Let the automaton in Figure 1 be $(Q, \{a, b\}, \Delta, 1, \{6\})$. Draw the diagram for the automaton $(Q', \{a, b\}, \Delta', 1, Q')$ where $Q' = \{\text{cl}(1), \text{cl}(2), \text{cl}(3)\}$ and:

$$\Delta' = \{(X, \ell, \text{cl}(j)) \mid \ell \in \{a, b\} \text{ and there is some } i \in X \text{ such that } (i, \ell, j) \in \Delta\}$$

- ** 3. Let $\text{rev}(w)$ be the reverse of the word w , e.g. $\text{rev}(abccd) = dccba$ and $\text{rev}(\epsilon) = \epsilon$.

Let P be the following automaton:



- (a) Construct another automaton that recognises $\{\text{rev}(w) \mid w \in L(P)\}$. Try not to think about what this language actually looks like, instead try to think how you could “reverse” the diagram, because, in the next part, you will not have a specific language.
- (b) Suppose $M = (Q, \Sigma, \Delta, q_0, F)$ is a finite automaton. By filling out (i)–(iii), complete the following definition of a finite automaton N in such a way that $L(N) = \{\text{rev}(w) \mid w \in L(P)\}$.

Let s be a new state not in Q . Then finite automaton N is $(Q', \Sigma, \Delta', q'_0, F')$ where:

- $Q' = Q \cup \{s\}$
- $\Delta' =$ (i)
- $q'_0 =$ (ii)
- $F' =$ (ii)

- (c) Argue that if A is a regular language, then so is $\{\text{rev}(w) \mid w \in A\}$.

- ** 4. Let $\text{tail}(w)$ be the tail of the word w , i.e:

$$\begin{aligned} \text{tail}(\epsilon) &= \epsilon \\ \text{tail}(a \cdot w) &= w \end{aligned}$$

By following a similar approach to parts (b) and (c) of the previous question, argue that if S is regular, then so is $\{\text{tail}(w) \mid w \in S\}$.

- ** 5. Show that language $S = \{w \in \{a, b\}^* \mid w = \text{rev}(w)\}$ is not regular.

*** 6. Prove that the language of squares (written in unary), $\{1^{n^2} \mid n \in \mathbb{N}\}$, is not regular.

| Hint $n^2 + m$ is not a square number whenever $0 < m \leq n$.