PROGRAMMING LANGUAGES AND COMPUTATION

Week 4: Regular Languages

In the problems this week you will need to make use of the formal definition of a finite state automaton, given at https://uob-coms20007.github.io/reference/regular/automata.html#finite-state-automaton.

* 1. Draw the diagram of the following automata:

  (a) $(\{e, o\}, \{0, 1\}, \{(e, 0, o), (e, 1, o), (o, 0, e), (o, 1, e)\}, e, \{e\})$

  (b) $(Q, \{0, 1\}, \Delta, q_0, Q)$ where $Q = \{q_0, q_1, q_2, q_3\}$ and Δ is:

  $$\{(q_0, 0, q_0), (q_0, 1, q_1), (q_1, 0, q_2), (q_1, 1, q_3), (q_2, 0, q_1), (q_3, 0, q_3)\}$$

  (c) $(Q, \Sigma, \Delta, q_0, Q)$ where:
   - $Q = \{1, 2, 3, 4, 5\}$
   - $\Sigma = \{a, b\}$
   - $\Delta = \{(i, a, i + 1) \mid 1 \leq i \leq 5\} \cup \{(j, b, j) \mid j \text{ is even}\}$
   - $q_0 = 1$
   - $F = \{1, 3, 5\}$

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procedure cl(X):
    X' := ∅
    while X ≠ X' do
        X' := X
        for each q ∈ X
            if (q, ε, q') ∈ Δ
                X' := X' ∪ {q'}
    return X
```
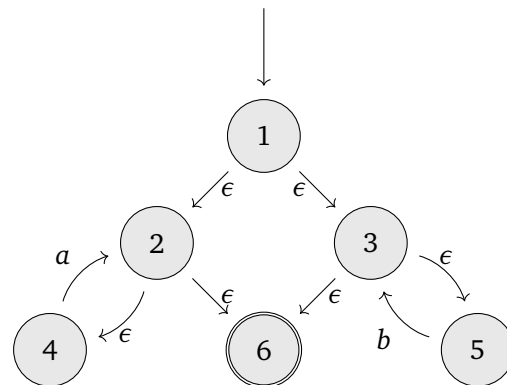

Figure 1: ϵ-closure of $X \subseteq Q$ wrt transitions Δ, and the automaton from Week 3 Q4(b)

* 2. Suppose M is a finite automaton with states Q. The *ϵ-closure* of a set of states $X \subseteq Q$ in M, written $cl(X)$, is the set of all states that can be reached from any state in X using only ϵ-transitions. It can be computed using the algorithm in Figure 1.

   (a) Construct a table with two columns. Each row of the table should contain a state of the automaton from Figure 1 in the first column and the ϵ-closure of that state in the second column.

   (b) Let the automaton in Figure 1 be $(Q, \{a, b\}, \Delta, 1, \{6\})$. Draw the diagram for the automaton $(Q', \{a, b\}, \Delta', 1, Q')$ where $Q' = \{cl(1), cl(2), cl(3)\}$ and:

   $$\Delta' = \{(X, \ell, cl(j)) \mid \ell \in \{a, b\} \text{ and there is some } i \in X \text{ such that } (i, \ell, j) \in \Delta\}$$

** 3. Let $rev(w)$ be the reverse of the word w, e.g. $rev(abccd) = dccba$ and $rev(\epsilon) = \epsilon$.

   Let P be the following automaton:
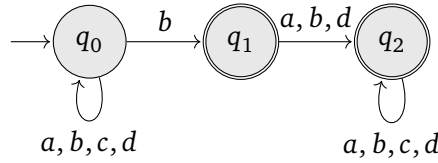
   (a) Construct another automaton that recognises $\{rev(w) \mid w \in L(P)\}$. Try not to think about what this language actually looks like, instead try to think how you could "reverse" the diagram, because, in the next part, you will not have a specific language.

   (b) Suppose $M = (Q, \Sigma, \Delta, q_0, F)$ is a finite automaton. By filling out (i)–(iii), complete the following definition of a finite automaton N in such a way that $L(N) = \{rev(w) \mid w \in L(P)\}$.

   Let s be a new state not in Q. Then finite automaton N is $(Q', \Sigma, \Delta', q_0', F')$ where:
   - $Q' = Q \cup \{s\}$
   - $\Delta' = $ (i)
   - $q_0' = $ (ii)
   - $F' = $ (ii)

   (c) Argue that if A is a regular language, then so is $\{rev(w) \mid w \in A\}$.

** 4. Let $tail(w)$ be the tail of the word w, i.e:

   $$tail(\epsilon) = \epsilon$$
   $$tail(a \cdot w) = w$$

   By following a similar approach to parts (b) and (c) of the previous question, argue that if S is regular, then so is $\{tail(w) \mid w \in S\}$.

** 5. Show that language $S = \{w \in \{a, b\}^* \mid w = rev(w)\}$ is not regular.

*** 6. Prove that the language of squares (written in unary), $\{1^{n^2} \mid n \in \mathbb{N}\}$, is not regular.

Hint $n^2 + m$ is not a square number whenever $0 < m \le n$.