

# Problem Sheet 11: The Halting Problem & Reductions

\*\* 1. Show that if  $f : U \lesssim V$  and  $g : V \lesssim W$  then  $g \circ f : U \lesssim W$ .

\*\* 2. Is the set

$$\text{LUCKY}_{127} = \{ \gamma(S) \mid \text{running } S \text{ on input 1 runs for at least 127 computational steps} \}$$

decidable? (Hint: if it is, describe a program that decides it. Think simply, write informally, and do not let the expressive poverty of While confine you.)

\*\*\* 3. Suppose we have a way of encoding every DFA  $M$  as a natural number  $\delta(M) \in \mathbb{N}$ . Is the predicate

$$\text{EMPTY} = \{ \delta(M) \mid L(M) = \emptyset \}$$

decidable? Why, or why not? (Hint: the advice from the previous question applies here too.)

\*\* 4. (Trick question.) Is it decidable whether God exists?

\*\* 5. Prove that if  $f : A \xrightarrow{\cong} B$  is a bijection, then so is its inverse  $f^{-1} : B \rightarrow A$ .

\*\*\* 6. Prove that the set

$$\mathbb{N} \rightarrow \mathbb{N} = \{ f \mid f : \mathbb{N} \rightarrow \mathbb{N} \}$$

is uncountable.

\*\*\*\* 7. Prove that the set

$$\text{ONE} = \{ \gamma(S) \mid \llbracket S \rrbracket_x(0) \downarrow \}$$

is undecidable by reduction from HALT.