Programming Languages and Computation

Week 11: The Halting Problem & Reductions

** 1. (Trick question.) Is it decidable whether God exists?

*** 2. Suppose we have a way of encoding every DFA M as a natural number $\delta(M) \in \mathbb{N}$. Is the predicate
$$\text{EMPTY} = \{\delta(M) \mid L(M) = \emptyset\}$$
decidable? [Hint: if it is, describe a program that decides it. Think simply, write informally, and do not let the syntactic poverty of While confine you.]

** 3. Show that if $f : U \lesssim V$ and $g : V \lesssim W$ then $g \circ f : U \lesssim W$.

** 4. Is the set
$$\text{LUCKY}_{127} = \{\ulcorner S \urcorner \mid \text{ running } S \text{ on input } 1 \text{ runs for at least } 127 \text{ computational steps }\}$$
decidable? [Hint: if it is, describe a program that decides it. Think simply, write informally, and do not let the syntactic poverty of While confine you.]

**** 5. Prove that the set
$$\text{ONE} = \{\ulcorner S \urcorner \mid \llbracket S \rrbracket_x (0) \downarrow\}$$
is undecidable by reduction from HALT.