Week 4: Regular Languages

In the problems this week you will need to make use of the formal definition of a finite state automaton, given at https://uob-coms20007.github.io/reference/regular/automata.html#finite-state-automaton.

* 1. Draw the diagram of the following automata:

(a) $(\{e, o\}, \{0, 1\}, \{(e, 0, o), (e, 1, o), (o, 0, e), (o, 1, e)\}, e, \{e\})$

(b) $(Q, \{0, 1\}, \Delta, q_0, Q)$ where $Q = \{q_0, q_1, q_2, q_3\}$ and Δ is:

$$\{(q_0, 0, q_0),\ (q_0, 1, q_1),\ (q_1, 0, q_2),\ (q_1, 1, q_3),\ (q_2, 0, q_1),\ (q_3, 0, q_3)\}$$

(c) $(Q, \Sigma, \Delta, q_0, Q)$ where:

- $Q = \{1, 2, 3, 4, 5\}$
- $\Sigma = \{a, b\}$
- $\Delta = \{(i, a, i+1) \mid 1 \leq i \leq 5\} \cup \{(j, b, j) \mid j \text{ is even}\}$
- $q_0 = 1$
- $F = \{1, 3, 5\}$

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procedure cl(X):
  X' := ∅
  while X ≠ X' do
    X' := X
    for each q ∈ X
      if (q, ϵ, q') ∈ Δ
        X' := X' ∪ {q'}
  return X
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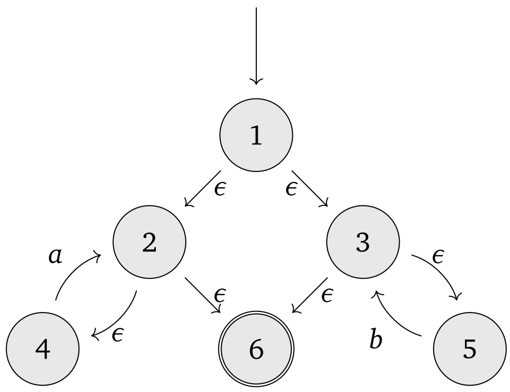


Figure 1: ϵ-closure of $X \subseteq Q$ wrt transitions Δ, and the automaton from Week 3 Q4(b)

* 2. Suppose M is a finite automaton with states Q. The *ϵ-closure* of a set of states $X \subseteq Q$ in M, written $\mathsf{cl}(X)$, is the set of all states that can be reached from any state in X using only ϵ-transitions. It can be computed using the algorithm in Figure 1.

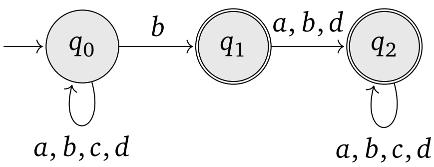
(a) Construct a table with two columns. Each row of the table should contain a state of the automaton from Figure 1 in the first column and the ϵ-closure of that state in the second column.

(b) Let the automaton in Figure 1 be $(Q, \{a, b\}, \Delta, 1, \{6\})$. Draw the diagram for the automaton $(Q', \{a, b\}, \Delta', \mathsf{cl}(1), Q')$ where $Q' = \{\mathsf{cl}(1), \mathsf{cl}(2), \mathsf{cl}(3)\}$ and:

$$\Delta' = \{(X, \ell, \mathsf{cl}(j)) \mid \ell \in \{a, b\} \text{ and there is some } i \in X \text{ such that } (i, \ell, j) \in \Delta\}$$

** 3. Let $\mathsf{rev}(w)$ be the reverse of the word w, e.g. $\mathsf{rev}(abccd) = dccba$ and $\mathsf{rev}(\epsilon) = \epsilon$.

Let P be the following automaton:



(a) Construct another automaton that recognises $\{\mathsf{rev}(w) \mid w \in L(P)\}$. Try not to think about what this language actually looks like, instead try to think how you could "reverse" the diagram, because, in the next part, you will not have a specific language.

(b) Suppose $M = (Q, \Sigma, \Delta, q_0, F)$ is a finite automaton. By filling out (i)–(iii), complete the following definition of a finite automaton N in such a way that $L(N) = \{\mathsf{rev}(w) \mid w \in L(P)\}$.

Let s be a new state not in Q. Then finite automaton N is $(Q', \Sigma, \Delta', q_0', F')$ where:

- $Q' = Q \cup \{s\}$
- Δ' = (i)
- q_0' = (ii)
- F' = (ii)

(c) Argue that if A is a regular language, then so is $\{\mathsf{rev}(w) \mid w \in A\}$.

** 4. Let $\mathsf{tail}(w)$ be the tail of the word w, i.e:

$$\mathsf{tail}(\epsilon) = \epsilon$$
$$\mathsf{tail}(a \cdot w) = w$$

By following a similar approach to parts (b) and (c) of the previous question, argue that if S is regular, then so is $\{\mathsf{tail}(w) \mid w \in S\}$.

** 5. Show that language $S = \{w \in \{a, b\}^* \mid w = \mathsf{rev}(w)\}$ is not regular.

*** 6. Prove that the language of squares (written in unary), $\{1^{n^2} \mid n \in \mathbb{N}\}$, is not regular.