Programming Languages and Computation

Week 5: Regular Languages

** 1. Suppose $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA. Construct a DFA $\overline{M}$ with $L(\overline{M}) = \{w \in \Sigma^* \mid w \notin L(M)\}$. You will not be able to answer this question with a diagram.

** 2.

(a) Given a word $w \in \Sigma^*$, and a letter $a \in \Sigma$, let $\mathsf{erase}_a(w)$ be the word w but with any occurrences of a erased. For example, for alphabet $\Sigma = \{0, 1, 2\}$:

$$\begin{aligned} \mathsf{erase}_1(01211210012) &= 022002 \\ \mathsf{erase}_1(101001222100) &= 00022200 \\ \mathsf{erase}_2(101001222100) &= 101001100 \\ \mathsf{erase}_2(101001100) &= 101001100 \end{aligned}$$

Define a function $\mathsf{erase}_a(R)$ that takes a regular expression R over Σ as input and returns a new regular expression S such that:

$$L(S) = \{\mathsf{erase}_a(w) \mid w \in L(R)\}$$

You need not justify your answer.

(b) Conclude that, if X is a regular language over Σ and $a \in \Sigma$, then $\{\mathsf{erase}_a(w) \mid w \in X\}$ is also a regular language over Σ.

** 3. Let $\mathsf{rev}(w)$ be the reverse of the word w, e.g. $\mathsf{rev}(abccd) = dccba$ and $\mathsf{rev}(\epsilon) = \epsilon$.

Show that language $S = \{w \in \{a, b\}^* \mid w = \mathsf{rev}(w)\}$ is not regular.

** 4. Show that the language $\{a^n b^m \mid n = 2 * m\}$ over $\{a, b\}$ is not regular.

*** 5. Prove that the language of squares (written in unary), $\{1^{n^2} \mid n \in \mathbb{N}\}$, is not regular.

*** 6. Show that the following language over $\{0, 1\}$ is regular and justify your answer:

$$\{0^k u 0^k \mid k \geq 1 \wedge u \in \{0, 1\}^*\}$$