

Miscellaneous Problems on Syntax

- * 1. A language is said to be *context-free* just if it is expressible by a context free grammar. For each of the following statements about languages over the alphabet $\{0, 1\}^*$, determine if it is true or false.

- (a) Every word of a context-free language is of finite length.
- (b) The language \emptyset is context free.
- (c) The language of all even length strings is context free.
- (d) Every finite subset of $\{0, 1\}^*$ is a context free.

- * 2. For each of the following statements, determine if it is true or false.

- (a) In the context-free grammar, there is exactly one rule for each nonterminal.
- (b) The language Σ^* is context free.
- (c) No context-free language can include the empty word.
- (d) In a context-free grammar, there are always more terminal symbols than non-terminal symbols.

- * 3. Consider the following grammar:

$$\begin{aligned} S &\longrightarrow 0 T 0 \mid 1 T 1 \\ T &\longrightarrow 0 T \mid 1 T \mid 0 \mid 1 \end{aligned}$$

For each of the following words, give a derivation to show that it is in the language of this grammar.

- (a) 0110
- (b) 00110
- (c) 11001

- * 4. For each of the following CFG over $\{a, b, c\}$, give (I) one word that is in the language and (II) one word that is not.

Both words should be over the alphabet $\{a, b, c\}$. Label the two words with (I) and (II) so that it is clear which is claimed to be in and which is claimed to be not in.

(a)

$$\begin{aligned} S &\longrightarrow XXX \\ X &\longrightarrow a \mid b \end{aligned}$$

(b)

$$\begin{aligned} S &\longrightarrow TS \mid \epsilon \\ T &\longrightarrow AbAbc \\ A &\longrightarrow aA \mid \epsilon \end{aligned}$$

(c)

$$\begin{aligned} S &\longrightarrow AC \mid BC \\ A &\longrightarrow a \mid aA \\ B &\longrightarrow b \mid bB \\ C &\longrightarrow c \mid cC \end{aligned}$$

(d)

$$S \longrightarrow aSa \mid bS \mid c$$

* 5. Consider the grammar for Lisp, given below.

$$\begin{aligned} S &\longrightarrow A \mid (E) \\ E &\longrightarrow SE \mid \epsilon \\ A &\longrightarrow \text{id} \mid \text{num} \end{aligned}$$

This grammar is over the 4 terminal symbols:

() id num

(a) Compute the nullable, first and follow maps for this grammar.

(b) Construct the parse table for this grammar.

(c) Is this grammar LL(1)?

** 6. For each of the following languages over $\{0, 1\}$, construct a CFG to express it.

(a) $\{uv^n \mid u \in \{0\}^*, v = 11, n \in \mathbb{N}\}$

(b) $\{w \mid w \text{ starts with } 1\}$

(c) $\{0u1v0 \mid u \text{ is } v \text{ reversed}\}$

** 7. For each of the following languages over $\{a, b\}$, construct a CFG to express it.

- (a) $\{w \mid \text{in } w \text{ every 'a' is followed immediately by a 'b'}\}$
- (b) $\{w \mid \text{the number of occurrences of 1 in } w \text{ is a multiple of 3}\}$
- (c) $\{w \mid w \text{ does not contain substring "ab"}\}$

** 8. Design CFG to express the following sets of strings over the alphabet of ASCII characters. Note: (a) you will find it convenient use some abbreviation (like \dots) to help present the expressions compactly and (b) this would not be given as an exam question without specifying the shape of the strings in each part more precisely.

- (a) Valid Bristol University usernames (two lowercase letters followed by five digits)
- (b) Valid 24 hour clock times in format HH:MM
- (c) Valid IPv4 addresses written in decimal

** 9. Construct a context-free grammar to recognise Haskell floating point literals, e.g. 2.99, 23.09e+34, 0.12e-200, 1.4e1.

A general description is as follows. A *decimal literal* is a non-empty sequence of digits (0–9). A *floating point literal* is either:

- a decimal literal followed by a decimal point followed by a decimal literal, optionally followed by an exponent
- or, a decimal literal followed by an exponent.

An exponent is the character e ; optionally followed by the character $+$ or the character $-$; followed in all cases by a decimal literal.

** 10. For each of the following, give an equivalent grammar which is LL(1).

(a)

$$\begin{aligned} S &\longrightarrow S \wedge S \mid G \Rightarrow \text{prop} \\ G &\longrightarrow G \wedge G \mid \text{prop} \end{aligned}$$

(b)

$$S \longrightarrow \text{int} \mid \text{string} \mid S \rightarrow S \mid S \times S \mid (S)$$

*** 11. Construct a CFG to express the following language over $\{0, 1\}$, and justify your answer:

$$\{0^k u 0^k \mid k \geq 1 \wedge u \in \{0, 1\}^*\}$$

- *** 12. Construct a CFG expressing the language of bit strings (strings over $\{0, 1\}$) that represent numbers written in binary that are divisible by three. For example, 10010 should be accepted because it represents the decimal number 18 written in binary and this number is divisible by 3. However, 101 should not be accepted, because this is the binary representation of the number 5, which is not divisible by 3.