PROGRAMMING LANGUAGES AND COMPUTATION

Problem Sheet 2: Deterministic Automata

* 1. For each of the automata in Figure 1, give (i) the trace of the automaton running on input 12212 and (ii) the language recognised by the automaton.

* 2. Give finite automata to recognise each of the following:

   (a) The language of all strings over $\{0, 1\}$ such that each string has even length.

   (b) The language of all strings over $\{0, 1\}$ such that each string has a number of 1s that is a multiple of three.

   (c) The language of all strings over $\{0, 1\}$ such that each string has even length and a number of 1s that is a multiple of three.

** 3. Construct a DFA to recognise Haskell floating point literals, e.g. `2.99`, `23.09e+34`, `0.12E-200`, `1.4e1`.

   A general description is as follows. A *decimal literal* is a non-empty sequence of digits (0–9). A *floating point literal* is either:

   • a decimal literal followed by a decimal point followed by a decimal literal, optionally followed by an exponent

   • or, a decimal literal followed by an exponent.

   An exponent is either the character e or the character E; optionally followed by the character + or the character −; followed in all cases by a decimal literal.

   Since the diagram for this automaton will be relatively complex, you may wish to adopt the following convention, which allows you to omit certain transitions:

   If a state in the diagram does not have an outgoing edge for every letter of the alphabet, then it is implied that the missing edges all lead to a distinguished "junk" state, which is not an accepting state.

** 4. Suppose $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA. Construct a DFA $\overline{M}$ with $L(\overline{M}) = \{w \in \Sigma^* \mid w \notin L(M)\}$. You will not be able to answer this question with a diagram.

Figure 1: Automata used in Question 1.
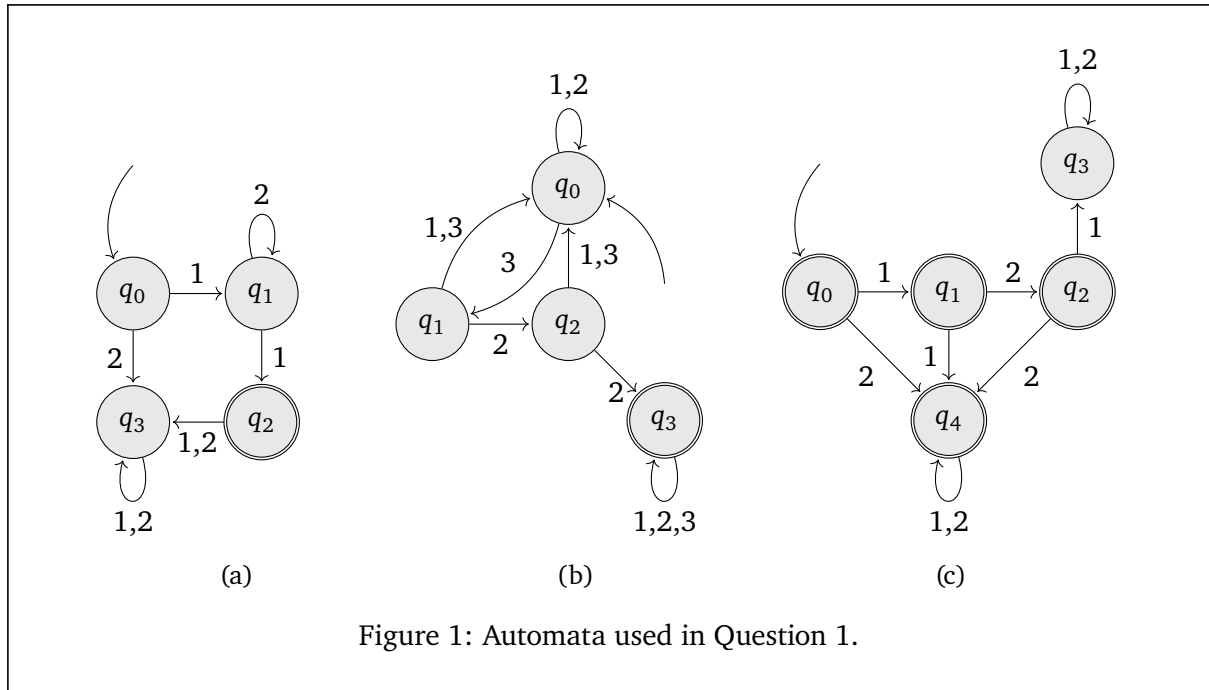
** 5. Construct a DFA to recognise the language of strings over {0, 1} such that any two occurrences of 1 are separated by an even number of letters.

Hint how many 1s can occur in a word of this language?

*** 6. Construct a DFA to recognise the language of strings over the alphabet $\{a, b\}$ containing at least three occurrences of three consecutive b, overlapping permitted (e.g. the word $bbbbb$ should be accepted).

**** 7. (Optional) Let Σ be the following set of binary vectors:

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

We use each word w over alphabet Σ to encode a pair of natural numbers, written $[\![w]\!]$, which is defined by the following recursive function:

$$
\begin{aligned}
[\![\epsilon]\!] &= (0,0) \\
[\![\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \cdot w]\!] &= (2*m + b_1, 2*n + b_2) \\
&\quad where\ (m, n) = [\![w]\!]
\end{aligned}
$$

For example:

$$\left[\!\left[\binom{0}{1}\binom{0}{0}\binom{1}{1}\binom{0}{1}\right]\!\right] = (4, 13)$$

$$\left[\!\left[\binom{0}{1}\binom{1}{1}\binom{0}{0}\binom{1}{0}\binom{1}{0}\right]\!\right] = (26, 3)$$

$$\left[\!\left[\binom{1}{1}\binom{0}{1}\binom{0}{1}\binom{0}{0}\binom{0}{1}\right]\!\right] = (1, 23)$$

Construct a DFA to recognise the language of "multiplication by 3":

$$\{w \in \Sigma^* \mid \exists m \in \mathbb{N}.\ [\![w]\!] = (m,\, 3 * m)\}$$