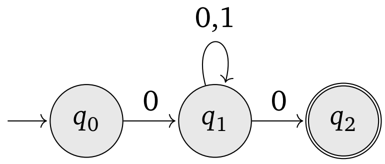
Programming Languages and Computation

Problem Sheet 4: Regular Languages

* 1. Construct the powerset automaton for the automaton over $\{0, 1\}$ that is drawn below:



** 2. Construct finite automata for each of the following sets:

(a) The set of strings over $\{a, b, c\}$ containing the substring ab.

(b) The set $\{w \mid \text{some occurrence of } b \text{ in } w \text{ is } \textit{not} \text{ followed immediately by } c\}$ which is a subset of all strings over the alphabet $\{a, b, c, d\}$.

(c) The set of finite sequences of ternary (base-3) digits, i.e. $\{0, 1, 2\}$, that represent numbers *not* divisible by four. We assume that sequences are given to the automaton with most-significant digit first, e.g. the word 201 represents the number written 19 in decimal notation.

** 3. Let $\mathsf{rev}(w)$ be the reverse of the word w, e.g. $\mathsf{rev}(abccd) = dccba$ and $\mathsf{rev}(\epsilon) = \epsilon$.

Show that if L is a regular language, then so is $\{\mathsf{rev}(w) \mid w \in L\}$.

** 4. Let $\mathsf{tail}(w)$ be the tail of the word w, i.e:

$$\begin{aligned}\mathsf{tail}(\epsilon) &= \epsilon \\ \mathsf{tail}(a \cdot w) &= w\end{aligned}$$

Show that if S is regular, then so is $\{\mathsf{tail}(w) \mid w \in S\}$.

*** 5. Prove that the language of squares (written in unary), $\{1^{n^2} \mid n \in \mathbb{N}\}$, is not regular.

$\boxed{\text{Hint}}$ $n^2 + m$ is not a square number whenever $0 < m \leq n$.

*** 6. Using the closure properties of regular languages, prove that the following language is *not* regular:

$$\{a^i b^j c^k \mid i, j, k \in \mathbb{N} \text{ and, if } i = 1 \text{ then } j \neq k\}$$

**** 7. (Optional) Assume that $M = (Q, \Sigma, \delta, q_0, F)$ is an automaton recognising the language L. Construct an automaton to recognise the language $\{v \mid \exists w.\ vw \in L \land |v| = |w|\}$.