Programming Languages and Computation

Problem Sheet 11: The Halting Problem & Reductions

** 1. Show that if $f : U \lesssim V$ and $g : U \lesssim W$ then $g \circ f : U \lesssim W$.

** 2. Is the set

$$\mathsf{LUCKY}_{127} = \{\ \gamma(S) \mid \text{ running } S \text{ on input 1 runs for at least 127 computational steps }\}$$

decidable? (Hint: if it is, describe a program that decides it. Think simply, write informally, and do not let the expressive poverty of While confine you.)

*** 3. Suppose we have a way of encoding every DFA M as a natural number $\delta(M) \in \mathbb{N}$.
Is the predicate

$$\mathsf{EMPTY} = \{\delta(M) \mid L(M) = \emptyset\}$$

decidable? Why, or why not? (Hint: the advice from the previous question applies here too.)

** 4. (Trick question.) Is it decidable whether God exists?

** 5. Prove that if $f : A \xrightarrow{\cong} B$ is a bijection, then so is its inverse $f^{-1} : B \to A$.

*** 6. Prove that the set

$$\mathbb{N} \to \mathbb{N} = \{f \mid f : \mathbb{N} \to \mathbb{N}\}$$

is uncountable.

**** 7. Prove that the set

$$\mathsf{ONE} = \{\ \gamma(S) \mid [\![\mathtt{S}]\!]_{\mathtt{x}}(0)\downarrow\}$$

is undecidable.