

Problem Sheet 11: The Halting Problem & Reductions

** 1. Show that if $f : U \lesssim V$ and $g : V \lesssim W$ then $g \circ f : U \lesssim W$.

** 2. Is the set

$$\text{LUCKY}_{127} = \{ \gamma(S) \mid \text{running } S \text{ on input 1 runs for at least 127 computational steps} \}$$

decidable? (Hint: if it is, describe a program that decides it. Think simply, write informally, and do not let the expressive poverty of While confine you.)

*** 3. Suppose we have a way of encoding every DFA M as a natural number $\delta(M) \in \mathbb{N}$. Is the predicate

$$\text{EMPTY} = \{ \delta(M) \mid L(M) = \emptyset \}$$

decidable? Why, or why not? (Hint: the advice from the previous question applies here too.)

** 4. (Trick question.) Is it decidable whether God exists?

** 5. Prove that if $f : A \xrightarrow{\cong} B$ is a bijection, then so is its inverse $f^{-1} : B \rightarrow A$.

*** 6. Prove that the set

$$\mathbb{N} \rightarrow \mathbb{N} = \{ f \mid f : \mathbb{N} \rightarrow \mathbb{N} \}$$

is uncountable.

**** 7. Prove that the set

$$\text{ONE} = \{ \gamma(S) \mid \llbracket S \rrbracket_x(0) \downarrow \}$$

is undecidable.