

Week 11: Undecidability and Reductions

* 1. [Trick question.] Is it decidable whether God exists?

* 2.

1. Is the set \emptyset decidable?

2. Is the set \mathbb{N} decidable?

** 3. Show that if $f : U \lesssim V$ and $g : V \lesssim W$ then $g \circ f : U \lesssim W$.

*** 4. Prove that the set

$$\text{ZERO} = \{ \ulcorner S \urcorner \mid \llbracket S \rrbracket_x(0) \downarrow \}$$

is undecidable by reduction from HALT.

*** 5. Prove that the predicate ZERO is undecidable, but using Rice's theorem instead.

**** 6. Show that the predicate

$$U = \{ \langle \ulcorner S \urcorner, \ulcorner T \urcorner \rangle \mid \llbracket S \rrbracket_x(0) \simeq \llbracket T \rrbracket_x(0) \}$$

is undecidable, by reduction from HALT.

*** 7. Is it possible to prove that the predicate U from the previous exercise is undecidable using Rice's theorem? If so, prove it. If not, explain why not.

**** 8. [Trick question.] Is the predicate

$$V = \{ \ulcorner S \urcorner \mid \forall n \in \mathbb{N}. \llbracket S \rrbracket_x(n) \downarrow \}$$

from the last lecture semi-decidable? Why or why not?