Programming Languages and Computation

Problem Sheet 2: Deterministic Automata

* 1. For each of the automata in Figure 1, give (i) the trace of the automaton running on input 12212 and (ii) the language recognised by the automaton.

* 2. Give finite automata to recognise each of the following:

(a) The language of all strings over $\{0,1\}$ such that each string has even length.

(b) The language of all strings over $\{0,1\}$ such that each string has a number of 1s that is a multiple of three.

(c) The language of all strings over $\{0,1\}$ such that each string has even length and a number of 1s that is a multiple of three.

** 3. Construct a DFA to recognise Haskell floating point literals, e.g. `2.99`, `23.09e+34`, `0.12E-200`, `1.4e1`.

A general description is as follows. A *decimal literal* is a non-empty sequence of digits (0–9). A *floating point literal* is either:

- a decimal literal followed by a decimal point followed by a decimal literal, optionally followed by an exponent
- or, a decimal literal followed by an exponent.

An exponent is either the character e or the character E; optionally followed by the character $+$ or the character $-$; followed in all cases by a decimal literal.

Since the diagram for this automaton will be relatively complex, you may wish to adopt the following convention, which allows you to omit certain transitions:

> If a state in the diagram does not have an outgoing edge for every letter of the alphabet, then it is implied that the missing edges all lead to a distinguished "junk" state, which is not an accepting state.

** 4. Suppose $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA. Construct a DFA $\overline{M}$ with $L(\overline{M}) = \{w \in \Sigma^* \mid w \notin L(M)\}$. You will not be able to answer this question with a diagram.

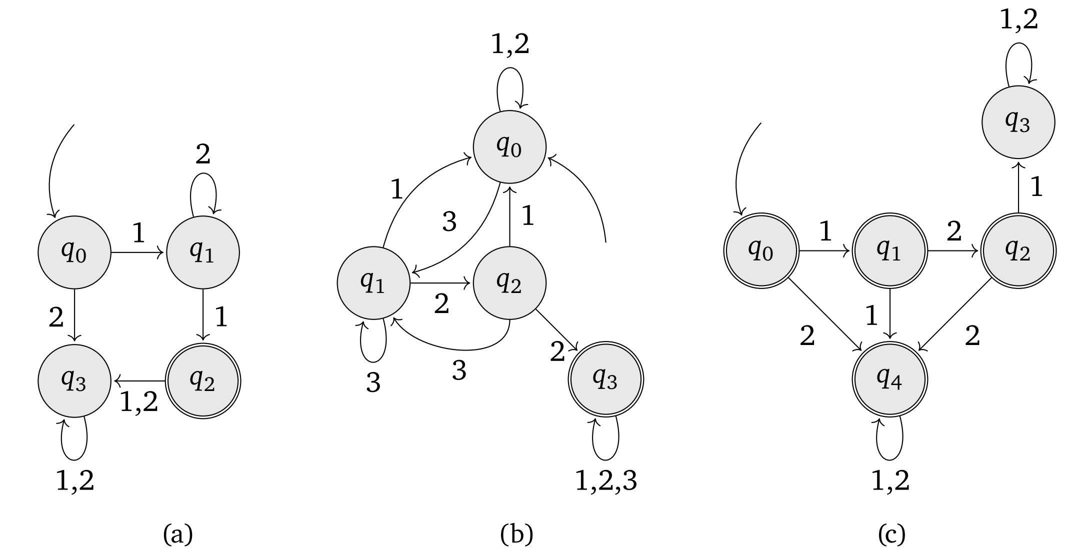

Figure 1: Automata used in Question 1.

** 5. Construct a DFA to recognise the language of strings over $\{0, 1\}$ such that any two occurrences of 1 are separated by an even number of letters.

Hint how many 1s can occur in a word of this language?

*** 6. Construct a DFA to recognise the language of strings over the alphabet $\{a, b\}$ containing at least three occurrences of three consecutive b, overlapping permitted (e.g. the word $bbbbb$ should be accepted).

**** 7. (Optional) Let Σ be the following set of binary vectors:

$$\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}$$

We use each word w over alphabet Σ to encode a pair of natural numbers, written $[\![w]\!]$, which is defined by the following recursive function:

$$\begin{array}{rcl} [\![\epsilon]\!] & = & (0,0) \\ [\![\begin{pmatrix}b_1\\b_2\end{pmatrix} \cdot w]\!] & = & (2*m+b_1,\, 2*n+b_2) \\ & & \qquad \textit{where } (m, n) = [\![w]\!] \end{array}$$

For example:

$$\left[\!\left[\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\right]\!\right] = (4,\ 13)$$

$$\left[\!\left[\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right]\!\right] = (26,\ 3)$$

$$\left[\!\left[\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\right]\!\right] = (1,\ 23)$$

Construct a DFA to recognise the language of “multiplication by 3”:

$$\{w \in \Sigma^* \mid \exists m \in \mathbb{N}.\ [\![w]\!] = (m,\ 3 * m)\}$$