# REDUCTS AND NORMALISATION

- If $M \twoheadrightarrow_\beta N$ then we say that $N$ is a ***reduct*** of $M$. If also $M \neq_\beta N$, we say that it is a ***proper reduct***.
- A term $M$ such that $M \twoheadrightarrow_\beta N$ for some normal form $N$ is said to ***have a normal form*** or be ***normalisable***.
- A term $M$ for which there is no infinite reduction sequence $M \to_\beta M_1 \to_\beta M_2 \to_\beta \cdots$ is said to be ***strongly normalisable***.

# STANDARD COMBINATORS

$$
\begin{aligned}
\mathbf{I} &:= \lambda x.x & \mathbf{I}\,M &\twoheadrightarrow_\beta M\\
\mathbf{K} &:= \lambda xy.x & \mathbf{K}\,M\,N &\twoheadrightarrow_\beta M\\
\mathbf{S} &:= \lambda xyz.xz(yz) & \mathbf{S}\,M\,N\,P &\twoheadrightarrow_\beta MP(NP)\\
\omega &:= \lambda x.xx & \omega\,M &\twoheadrightarrow_\beta M\,M\\
\Omega &:= \omega\omega & \Omega &\twoheadrightarrow_\beta \Omega\\
\Theta &:= (\lambda xy.y(xxy))(\lambda xy.y(xxy)) & \Theta\,M &\twoheadrightarrow_\beta M(\Theta M)
\end{aligned}
$$

**Theorem (Confluence of $\beta$)**

*If $M \twoheadrightarrow_\beta P$ and $M \twoheadrightarrow_\beta Q$ then there exists a term $N$ (not necessarily a normal form) such that $P \twoheadrightarrow_\beta N$ and $Q \twoheadrightarrow_\beta N$.*

# $\beta$-CONVERSION

Let $M$ and $N$ be terms. If $M$ and $N$ have a common reduct $P$, i.e. there is a term $P$ such that $M \twoheadrightarrow_\beta P$ and $N \twoheadrightarrow_\beta P$, then we say that $M$ and $N$ are $\beta$-convertible and write $M =_\beta N$.