

$$x : \forall \bar{a}. A \in \Gamma \quad \frac{}{\Gamma \vdash x : A[\bar{B}/\bar{a}]} \text{ (TVar)}$$

$$\frac{\Gamma \vdash M : B \rightarrow A \quad \Gamma \vdash N : B}{\Gamma \vdash MN : A} \text{ (TApp)}$$

$$x \notin \text{dom } \Gamma \quad \frac{\Gamma \cup \{x : B\} \vdash M : A}{\Gamma \vdash \lambda x. M : B \rightarrow A} \text{ (TAbs)}$$

BASIC PROPERTIES

Lemma

Suppose M is a term, Γ an environment and A a type. Then:

(Subterm Closure) *If $\Gamma \vdash M : A$ is derivable and N is a subterm of M then there is some $\Gamma' \supseteq \Gamma$ and some A' such that $\Gamma' \vdash N : A'$.*

(Relevance-1) *If $\Gamma \vdash M : A$, then $\text{FV}(M) \subseteq \text{dom}(\Gamma)$*

(Relevance-2) *If $\Gamma \vdash M : A$, then*
$$\{x : \forall \bar{b}. B \mid x : \forall \bar{b}. B \in \Gamma \wedge x \in \text{FV}(M)\} \vdash M : A$$

(Weakening) *If $\Gamma \vdash M : A$ and $\Gamma \subseteq \Gamma'$ then $\Gamma' \vdash M : A$.*

Theorem (Subject Reduction)

If $\Gamma \vdash M : A$ and $M \rightarrow_{\beta} N$ then $\Gamma \vdash N : A$

Lemma

If $\Gamma, x : B \vdash M : A$ and $\Gamma \vdash N : B$ then $\Gamma \vdash M[N/x] : A$.

