

MONOTYPES

We assume a countable set of **type variables** \mathbb{A} , ranged over by a , b , c and other lowercase letters from the start of the alphabet. The **monotypes**, written \mathbb{T} are a set of strings defined inductively by the following rules:

$$\begin{array}{ll} \text{(TyVar)} \quad \frac{}{a \in \mathbb{T}} \quad a \in \mathbb{A} & \text{(Arrow)} \quad \frac{A \in \mathbb{T} \quad B \in \mathbb{T}}{(A \rightarrow B) \in \mathbb{T}} \end{array}$$

The ***type schemes*** are pairs consisting of a finite set of type variables a_1, \dots, a_m and a monotype A , that we write suggestively as:

$$\forall a_1 \dots a_m. A$$

FREE TYPE VARIABLES

We define the set of a free type variables for a type (scheme) $\forall \bar{a}. A$, written $\text{FTV}(\forall \bar{a}. A)$, recursively on the syntax:

$$\begin{aligned}\text{FTV}(a) &= \{a\} \\ \text{FTV}(A \rightarrow B) &= \text{FTV}(A) \cup \text{FTV}(B) \\ \text{FTV}(\forall a_1 \dots a_m. A) &= \text{FTV}(A) \setminus \{a_1, \dots, a_m\}\end{aligned}$$

A ***type substitution*** is a total map $\sigma : \mathbb{A} \rightarrow \mathbb{T}$ from type variables to monotypes, with the property that $\sigma(a) \neq a$ only for finitely many $a \in \mathbb{A}$.

We will use σ , τ and θ to stand for type substitutions generically.

TYPE SUBSTITUTION COMPOSITION

We write $\sigma_1\sigma_2$ for the substitution obtained by **composing** σ_2 after σ_1 , defined as the following total function on type variables:

$$(\sigma_1\sigma_2)(a) := (\sigma_1(a))\sigma_2$$

