Types and $\lambda$-calculus

# Problem Sheet 6

* 1. Which of the following pairs of type constraints have solutions? Give the most general unifier for those that do.

(a) $a \stackrel{?}{=} b \to c$ and $b \to b \stackrel{?}{=} c$

(b) $a \to a \stackrel{?}{=} b \to c$ and $a \stackrel{?}{=} c \to c$

(c) $a \to b \stackrel{?}{=} b \to c \to d$ and $c \stackrel{?}{=} d$

** 2.

(a) Compute the set of type constraints that characterises valid types of closed term $\lambda xy.x$ using the constraint generation algorithm.

(b) Solve the constraints obtained in the previous part using the unification algorithm and exhibit the most general unifier.

(c) Give the principal type of this term.

** 3. Give an alternative proof that $\lambda x.xx$ is untypable by computing the set of type constraints using Hindley Milner type inference and showing, using unification, that these constraints are unsolvable.

Read the proof of the same result in Lemma 8.1 and reflect on the similarities.

** 4. Suppose closed term $M$ has a normal form $\lambda x.xx$. Prove that all reducts of $M$ are untypable, i.e. $M \twoheadrightarrow_\beta N$ implies $N$ untypable.

*** 5. The following property is a variation of subject reduction:

$$\text{if } M =_\beta N \text{ and } \Gamma \vdash M : A \text{ then } \Gamma \vdash N : A$$

Is this property true for our type system? Either prove it or give a counterexample.