FIXED POINT

A λ-term N is said to be a ***fixed point*** of another λ-term M just if $MN =_\beta N$.

Theorem (First Recursion Theorem)

Every λ-term posesses a fixed point.

CURRY'S PARADOXICAL COMBINATOR

$$\mathbf{Y} := \lambda f.(\lambda x.f(xx))(\lambda x.f(xx))$$

satisfies

$$\mathbf{Y}\,M =_\beta M\,(\mathbf{Y}\,M)$$

RECIPE

If you want an M such that (assuming $y \neq x_i$)...

$$M\, x_1 \cdots x_n =_\beta N[M/y]$$

...just choose $M \coloneqq \mathbf{Y}\,(\lambda y.\, \lambda x_1 \ldots x_n.\, N)$