

TYPES AND λ -CALCULUS

Problem Sheet 1

* 1. Which of the following are terms? For those that are terms, write out a proof tree justification.

- (a) $(\lambda x. ((xx)x))$
- (b) $(\lambda(\lambda x. x))$
- (c) $((xy)z)$

* 2. Write these terms using the minimum number of parentheses and λ , according to our conventions.

- (a) $(\lambda y. ((yy)(zz)))$
- (b) $(\lambda y. (((yy)y)y))$
- (c) $((xy)(\lambda y. (\lambda z. (z(xy))))))$

* 3. Write the term $(\lambda x y z. xy(xz))(\lambda xy. x)$ with all the parentheses and λ that we will usually omit tediously put back in.

** 4. Note that, by the conventions of logic, $A \Rightarrow B \Rightarrow C$ is a shorthand for $A \Rightarrow (B \Rightarrow C)$ and conjunction binds tighter than implication, so $A \wedge B \Rightarrow C$ means $(A \wedge B) \Rightarrow C$.

Give proofs of the following.

- (a) $\neg A \Rightarrow A \Rightarrow B$
- (b) $(A \wedge B \Rightarrow C) \Rightarrow A \Rightarrow B \Rightarrow C$
- (c) $\neg(A \wedge \neg A)$
- (d) $(A \Rightarrow B) \Rightarrow (B \Rightarrow C) \Rightarrow A \Rightarrow C$
- (e) $\neg A \wedge \neg B \Rightarrow \neg(A \vee B)$