TYPE ASSIGNMENT

A ***type assignment*** is a pair of a term M and a type A, written $M : A$. The term part of a type assignment is called the ***subject*** and the type part the ***predicate***.

TYPE ENVIRONMENT

A ***type environment***, generically written Γ, is a finite set of type assignments of the form $x : \forall\overline{a}.A$ which is, moreover, ***consistent***, in the sense that if $x : \forall\overline{a}.A \in \Gamma$ and $x : \forall\overline{b}.B \in \Gamma$, then $\forall\overline{a}.A = \forall\overline{b}.B$.

The ***subjects*** of Γ is the set, written $\operatorname{dom} \Gamma$, of those term variables x for which there is some $\forall\overline{a}.A$ such that $x : \forall\overline{a}.A \in \Gamma$.

TYPE SYSTEM

$$x : \forall \overline{a}.\, A \in \Gamma \quad \frac{}{\Gamma \vdash x : A[\overline{B}/\overline{a}]} \; \text{(TVar)}$$

$$\frac{\Gamma \vdash M : B \to A \quad \Gamma \vdash N : B}{\Gamma \vdash MN : A} \; \text{(TApp)}$$

$$x \notin \mathrm{dom}\ \Gamma \quad \frac{\Gamma \cup \{x : B\} \vdash M : A}{\Gamma \vdash \lambda x.\, M : B \to A} \; \text{(TAbs)}$$

In type theory, a proof tree justifying a type judgement is usually called a ***type derivation***.