CURRY-HOWARD

Theorem (Curry-Howard)

If we consider properly labelled proofs in the implicational fragment of propositional logic (i.e. the only connective is implication), then the following is true:

- From a constructive proof of A from starting assumptions Γ, one can extract a term M such that Γ ⊢ M : A.
- From each term M such that Γ ⊢ M : A, we can extract a constructive proof that A follows from assumptions Γ.

(This requires we accept that \rightarrow is an equally good way to write implication.)

PROOFS AS PROGRAMS

 $\begin{array}{ccc} \text{Types} & \longleftrightarrow & \text{Formulas} \\ \text{Programs} & \longleftrightarrow & \text{Proofs} \\ \text{Inhabitation} & \longleftrightarrow & \text{Provability} \end{array}$