β-REDEX

A term of the form $(\lambda x. M)N$ is called a β-***redex*** and we say that $M[N/x]$ is the ***contraction*** of the redex.

ONE-STEP β-REDUCTION

The ***one-step β-reduction*** relation:

$$\frac{}{(\lambda x.\, M)N \to_\beta M[N/x]}\ \text{(Redex)}$$

$$\frac{M \to_\beta M'}{MN \to_\beta M'N}\ \text{(AppL)} \qquad \frac{N \to_\beta N'}{MN \to_\beta MN'}\ \text{(AppR)}$$

$$\frac{M \to_\beta N}{\lambda x.\, M \to_\beta \lambda x.\, N}\ \text{(Abs)}$$

If $M \to_\beta N$ then we say that N is a ***reduct*** of M. A term M is said to be in β-***normal form*** just if there is no term N for which $M \to_\beta N$.

β-REDUCTION

Whenever there is a possibly empty sequence of consecutive one-step reductions:

$$M_0 \to_\beta M_1 \to_\beta \cdots \to_\beta M_{k-1} \to_\beta M_k$$

for $k \geq 0$, we say that M_0 β-reduces to M_k and write $M_0 \twoheadrightarrow_\beta M_k$. Note, we include the case that $k = 0$ and hence $M_0 = M_k$ (i.e. we include 0-step reductions).