SOLUTIONS

A ***unifier*** or ***solution*** to a finite set of type constraints $\{A_1 \stackrel{?}{=} B_1, \ldots, A_n \stackrel{?}{=} B_n\}$ is a type substitution σ such that:

$$\forall i \in \{1 \ldots n\}.\ A_i\sigma = B_i\sigma$$

We will write $\mathscr{C}\sigma$ for the application of σ to all the types in $\mathscr{C}$.

EXAMPLE SOLUTION σ

$$\sigma := \left[\begin{array}{rcl} a \to a & / & b \\ (a \to a) \to a \to a & / & c \\ a \to a & / & d \\ a \to a & / & e \\ a & / & f \end{array}\right]$$

EXAMPLE SOLUTION τ

$$\tau := \left[\begin{array}{rcl} b \to b & / & a \\ (b \to b) \to b \to b & / & b \\ ((b \to b) \to b \to b) \to (b \to b) \to b \to b & / & c \\ (b \to b) \to b \to b & / & d \\ (b \to b) \to b \to b & / & e \\ b \to b & / & f \end{array}\right]$$

SOLVED FORM

A set of type constraints of the shape $\mathscr{C} = \{a_1 \stackrel{?}{=} A_1, \ldots, a_m \stackrel{?}{=} A_m\}$ is in ***solved form*** just if each of the a_i are pairwise distinct variables none of which occurs in any of the A_i.

In such a case, we define $[\![\mathscr{C}]\!] := [A_1/a_1, \ldots, A_m/a_m]$.

ROBINSON'S ALGORITHM

$$
\begin{array}{rcll}
\{A \stackrel{?}{=} A\} \uplus \mathscr{C} & \Longrightarrow & \mathscr{C} & \\
\{A_1 \to A_2 \stackrel{?}{=} B_1 \to B_2\} \uplus \mathscr{C} & \Longrightarrow & \{A_1 \stackrel{?}{=} A_1, A_2 \stackrel{?}{=} B_2\} \uplus \mathscr{C} & \\
\{A \stackrel{?}{=} a\} \uplus \mathscr{C} & \Longrightarrow & \{a \stackrel{?}{=} A\} \uplus \mathscr{C} & \text{if } A \notin \mathbb{A} \\
\{a \stackrel{?}{=} A\} \uplus \mathscr{C} & \Longrightarrow & \{a \stackrel{?}{=} A\} \uplus \mathscr{C}[A/a] & \text{if } a \notin \mathrm{FTV}(A)
\end{array}
$$