Types and λ-calculus

Problem Sheet 7

** 1. Suppose closed term M has a normal form $\lambda x.xx$. Prove that all reducts of M are untypable, i.e. $M \rhd^* N$ implies N untypable.

** 2.

(a) Prove the following result by induction on M:

> Let B be a type and N a term. For all terms M, types A and environments Γ: if $\Gamma, x{:}B \vdash M : A$ and $\Gamma \vdash N : B$ then $\Gamma \vdash M[N/x] : A$.

(b) Prove the missing case in the proof of Lemma 12.2 from the notes: if $\Gamma \vdash (\lambda x.M)N : A$ then $\Gamma \vdash M[N/x] : A$.

** 3. Find pure terms that inhabit the following types (no need for justification):

(a) $(a \to a \to b) \to a \to b$

(b) $(a \to b) \to (a \to c) \to (b \to c \to d) \to a \to d$

(c) $(((a \to a \to b) \to a \to b) \to c) \to c$

** 4.

(a) Find a *pure* term that inhabits the type $(((a \to b) \to b) \to b) \to a \to b$.

(b) Give the corresponding proof of the corresponding formula.

*** 5. The reason that we don't study full PCF in connection with the Curry-Howard correspondence is the presence of fix.

(a) Use fix to show that every type is inhabited by some *PCF term* (not necessarily pure).

(b) What is the consequence for the Curry-Howard correspondence extended to full PCF?

*** 6. The following property is called Subject Invariance:

if $M \approx N$ and $\Gamma \vdash M : A$ then $\Gamma \vdash N : A$

Is this property true for our type system? Either prove it or give a counterexample.