EXAMPLE SOLUTION σ

$$\sigma := \begin{bmatrix} a \to a & / & b \\ (a \to a) \to a \to a & / & c \\ a \to a & / & d \\ a \to a & / & e \\ a & / & f \end{bmatrix}$$

EXAMPLE SOLUTION τ

$$\tau := \left[\begin{array}{rcl} b \to b & / & a \\ (b \to b) \to b \to b & / & b \\ ((b \to b) \to b \to b) \to (b \to b) \to b \to b & / & c \\ (b \to b) \to b \to b & / & d \\ (b \to b) \to b \to b & / & e \\ b \to b & / & f \end{array}\right]$$

MOST GENERAL UNIFIER

Let $\mathscr{C}$ be a set of type constraints. Then we say that σ is a ***most general unifier*** (mgu) of $\mathscr{C}$ just if:

(i) σ is a unifier of $\mathscr{C}$

(ii) and every unifier σ' of $\mathscr{C}$ is of shape $\sigma\sigma''$ for some σ''

On input closed term M:

1. Generate constraints $\mathscr{C}$ and type variable a using $\mathrm{CGen}(\emptyset, M)$.
2. Solve $\mathscr{C}$ using Robinson's algorithm to obtain mgu σ or deduce unsolvability.
3. If $\mathscr{C}$ has no solution then M is untypable. Otherwise return $\sigma(a)$.

Theorem (Principal Type Scheme Theorem)
If closed term M is typable, then Hindley-Milner type inference returns a type A that is ***principal*** *in the sense that:*

- $\vdash M : A$ *is derivable*
- *and, if some other* $\vdash M : B$ *is derivable, then there is a choice of monotypes* $C_1, \ldots, C_k$ *such that* $B = A[C_1/a_1, \ldots, C_k/a_k]$.