

$$\begin{aligned}
R(a) &= \{M \mid M \text{ is SN}\} \\
R(A \rightarrow B) &= \{M \mid \forall N \in R(A). MN \in R(B)\} \\
R(\forall \bar{a}. A) &= \{M \mid \text{for all } \bar{B} \in \mathcal{P}(\mathbb{T}), M \in R(A[\bar{B}/\bar{a}])\}
\end{aligned}$$

Lemma

for all A : $VHSN \subseteq R(A) \subseteq SN$

Lemma

If, for all $N \in R(B)$: $M[N/x] \in R(A)$, then for all N : $(\lambda x.M)N \in R(A)$.

Theorem

Suppose $\Gamma \vdash M : B$ and, for each variable $x \in \text{dom}(\Gamma)$, there is a term N_x such that $N_x \in R(\Gamma(x))$.

Then $M[N_x/x \mid x \in \text{dom}(\Gamma)] \in R(B)$.

Corollary (Strong Normalisation)

If $\Gamma \vdash M : A$ then M is SN.

