TYPES AND λ-CALCULUS

Problem Sheet 1

* 1. Which of the following are terms? For those that are terms, write out a proof tree justification.

(a) $(\lambda x.((xx)x))$
(b) $(\lambda(\lambda x.x))$
(c) $((xy)z)$

* 2. Write these terms using the minimum number of parentheses and λ, according to our conventions.

(a) $(\lambda y.((yy)(zz)))$
(b) $(\lambda y.(((yy)y)y))$
(c) $((xy)(\lambda y.(\lambda z.(z(xy)))))$

* 3. Write the term $(\lambda xyz.xy(xz))(\lambda xy.x)$ with all the parentheses and λ that we will usually omit tediously put back in.

** 4. Note that, by the conventions of logic, $A \Rightarrow B \Rightarrow C$ is a shorthand for $A \Rightarrow (B \Rightarrow C)$ and conjunction binds tighter than implication, so $A \wedge B \Rightarrow C$ means $(A \wedge B) \Rightarrow C$.

Give proofs of the following.

(a) $\neg A \Rightarrow A \Rightarrow B$
(b) $(A \wedge B \Rightarrow C) \Rightarrow A \Rightarrow B \Rightarrow C$
(c) $\neg(A \wedge \neg A)$
(d) $(A \Rightarrow B) \Rightarrow (B \Rightarrow C) \Rightarrow A \Rightarrow C$
(e) $\neg A \wedge \neg B \Rightarrow \neg(A \vee B)$