TYPES AND λ-CALCULUS

Problem Sheet 7

** 1.

(a) Find an inhabitant of the type $(((a \to b) \to b) \to b) \to a \to b$.

(b) Give the corresponding proof of the corresponding formula.

** 2. Sketch an algorithm to decide the following problem and justify that it works:

Given:	two typable terms M and N
Decide:	if $M =_\beta N$

** 3. A term M is a *fixed point combinator* just if, for all terms P, $M\ P =_\beta P\ (M\ P)$. In other words, M computes a fixed point of its argument.

Prove that no fixed point combinator is typable, i.e. if closed term M is a fixed point combinator, then M is not typable.

Hint: Try to arrive at a contradiction by obtaining a β-equality in which the two sides are distinct normal forms.

** 4. Suppose we add a fixed point combinator fix to our lambda calculus as a new primitive. In other words, we extend the syntax of terms by the rule:

$$\frac{}{\mathsf{fix} \in \Lambda}$$

and we extend the type system by the following rule:

$$\frac{}{\Gamma \vdash \mathsf{fix} : (A \to A) \to A}\,(\mathsf{TFix})$$

This type makes sense since fix takes a function as input and returns a fixed point of the function. (We should also extend the definition of β-reduction, but it is not important to this question.)

(a) Show that every type in this extended system is inhabited.

(b) What is the consequence for the Curry-Howard correspondence?