CHURCH LISTS

The ***Church encoding*** of a list xs is the term $\ulcorner xs \urcorner$ defined recursively by:

$$\begin{aligned} \ulcorner [] \urcorner &= \lambda cn.\, n \\ \ulcorner n : ns \urcorner &= \lambda cn.\, c\ \ulcorner n \urcorner\ (\ulcorner ns \urcorner\ c\ n) \end{aligned}$$

NIL AND CONS

$$\text{Nil} := \lambda cn.\, n \qquad \text{Cons} := \lambda xy.\, \lambda cn.\, c\, x\, (y\, c\, n)$$

$$\text{Sum} := \lambda xs.\, xs\ \mathbf{Add}\ \ulcorner 0 \urcorner$$