STRONG NORMALISATION

Theorem
If $\Gamma \vdash M : A$ then M is strongly normalising.

LOGICAL RELATION

Define $\mathrm{R} : \mathbb{T} \to \mathscr{P}(\Lambda)$ is a function from types to sets of terms defined recursively as follows.

$$
\begin{aligned}
\mathrm{R}(a) &= \{M \mid M \text{ is SN}\} \\
\mathrm{R}(A \to B) &= \{M \mid \forall N \in \mathrm{R}(A).\, MN \in \mathrm{R}(B)\} \\
\mathrm{R}(\forall \overline{a}.\, A) &= \{M \mid \text{for all } \overline{B} \in \mathscr{P}(\mathbb{T}),\ M \in \mathrm{R}(A[\overline{B}/\overline{a}])\}
\end{aligned}
$$

USEFUL LEMMAS

Lemma
for all A: $VHSN \subseteq R(A) \subseteq SN$

Lemma
If, for all $N \in \mathrm{R}(B)$*:* $M[N/x] \in \mathrm{R}(A)$*, then for all N:* $(\lambda x.M)N \in \mathrm{R}(A)$*.*