

TYPES AND  $\lambda$ -CALCULUS

## Problem Sheet 4

- \* 1. Put in all the implicit parentheses required by the official syntax of types for the following examples:
- (a)  $a \rightarrow b \rightarrow a$
  - (b)  $\forall bc. (b \rightarrow c) \rightarrow c$
  - (c)  $\forall ab. (c \rightarrow c \rightarrow d) \rightarrow a \rightarrow b$
  - (d)  $\forall a. (a \rightarrow b) \rightarrow a \rightarrow b$
- \* 2. Derive the induction principle for the inductively defined set of monotypes  $\mathbb{T}$ .
- \*\* 3. Prove, by induction on  $A$ , that  $A(\sigma_1 \sigma_2) = (A\sigma_1)\sigma_2$ .
- \*\* 4. Prove, by induction on  $M$ , that:
- $$\text{if } x \in \text{FV}(M) \text{ then } \text{FV}(M[N/x]) = (\text{FV}(M) \setminus \{x\}) \cup \text{FV}(N).$$
- Hint: You will want to use Lemma 6.1 of the notes.  
Hint: In the application case, consider splitting on whether  $x$  is free in the operator only, the operand only, or both.
- \*\* 5. Prove, by induction on  $M \rightarrow_\beta N$ , that:  $M \rightarrow_\beta N$  implies  $\text{FV}(N) \subseteq \text{FV}(M)$ .