INDUCTION ON Λ

Let Φ be some property of λ-terms. If the following conditions are all met:

(LI1) For all variables x, Φ holds of x.

(LI2) For all terms P and Q, if Φ holds of P and Φ holds of Q then Φ holds of PQ.

(LI3) For all terms P and variables x, if Φ holds of P then Φ holds of $\lambda x.P$.

Then it follows that Φ holds of all λ-terms.

REMINDER: FVS AND SUBSTITUTION

$$
\begin{aligned}
\mathrm{FV}(x) &= \{x\} \\
\mathrm{FV}(PQ) &= \mathrm{FV}(P) \cup \mathrm{FV}(Q) \\
\mathrm{FV}(\lambda x.N) &= \mathrm{FV}(N) \setminus \{x\}
\end{aligned}
$$

$$
\begin{array}{rcll}
y[N/x] & = & y & \text{if } x \neq y \\
y[N/x] & = & N & \text{if } x = y \\
(PQ)[N/x] & = & P[N/x]Q[N/x] & \\
(\lambda y.P)[N/x] & = & \lambda y.P & \text{if } y = x \\
(\lambda y.P)[N/x] & = & \lambda y.P[N/x] & \text{if } y \neq x \text{ and } y \notin \mathrm{FV}(N)
\end{array}
$$