TYPE SYSTEM

$$x : \forall \overline{a}.\, A \in \Gamma \quad \frac{}{\Gamma \vdash x : A[\overline{B}/\overline{a}]} \ \text{(TVar)}$$

$$\frac{\Gamma \vdash M : B \to A \quad \Gamma \vdash N : B}{\Gamma \vdash MN : A} \ \text{(TApp)}$$

$$x \notin \mathrm{dom}\ \Gamma \quad \frac{\Gamma \cup \{x : B\} \vdash M : A}{\Gamma \vdash \lambda x.\, M : B \to A} \ \text{(TAbs)}$$

TYPABILITY

We say that a closed term M is ***typable*** just if some type S such that $\vdash M : S$ is derivable in the type system.

Theorem (Inversion)
Suppose $\Gamma \vdash M : A$ (is derivable), then:

- *If M is a variable x, then there is a type scheme $\forall \overline{a}.B$ (with $\overline{a}$ possibly empty) and $A = B[\overline{C}/\overline{a}]$ for some monotypes $\overline{C}$.*
- *If M is an application PQ, then there is a type B such that $\Gamma \vdash P : B \to A$ and $\Gamma \vdash Q : B$.*
- *If M is an abstraction $\lambda x.P$, then there are types B and C such that $A = B \to C$, and $\Gamma, x : B \vdash P : C$.*

TYPING PROBLEMS

- **Typability.** Given a closed term M, is M typable?
- **Type checking.** Given a closed term M and a type A, is $\vdash M : A$ derivable?
- **Type inference.** Given a closed term M, compute all types A such that $\vdash M : A$.