MONOTYPES

We assume a countable set of ***type variables*** $\mathbb{A}$, ranged over by a, b, c and other lowercase letters from the start of the alphabet. The ***monotypes***, written $\mathbb{T}$ are a set of strings defined inductively by the following rules:

$$\text{(TyVar)}\ \frac{}{a \in \mathbb{T}}\ a \in \mathbb{A} \qquad \text{(Arrow)}\ \frac{A \in \mathbb{T} \quad B \in \mathbb{T}}{(A \to B) \in \mathbb{T}}$$

TYPE SCHEMES

The ***type schemes*** are pairs consisting of a finite set of type variables $a_1, \ldots, a_m$ and a monotype A, that we write suggestively as:

$$\forall a_1 \ldots a_m.\, A$$

FREE TYPE VARIABLES

We define the set of a free type variables for a type (scheme) $\forall \overline{a}.\ A$, written $\mathrm{FTV}(\forall \overline{a}.\ A)$, recursively on the syntax:

$$
\begin{array}{rcl}
\mathrm{FTV}(a) & = & \{a\} \\
\mathrm{FTV}(A \to B) & = & \mathrm{FTV}(A) \cup \mathrm{FTV}(B) \\
\mathrm{FTV}(\forall a_1 \ldots a_m . A) & = & \mathrm{FTV}(A) \setminus \{a_1, \ldots, a_m\}
\end{array}
$$

TYPE SUBSTITUTION

A ***type substitution*** is a total map $\sigma : \mathbb{A} \to \mathbb{T}$ from type variables to monotypes, with the property that $\sigma(a) \neq a$ only for finitely many $a \in \mathbb{A}$.

We will use σ, τ and θ to stand for type substitutions generically.

TYPE SUBSTITUTION COMPOSITION

We write $\sigma_1\sigma_2$ for the substitution obtained by ***composing*** σ_2 after σ_1, defined as the following total function on type variables:

$$(\sigma_1\sigma_2)(a) := (\sigma_1(a))\sigma_2$$