TERMS

Assume an infinite collection of ***variables*** x, y, $z \ldots \in \mathbb{V}$.

The set of ***terms***, written Λ, is the subset of strings over the alphabet $\mathbb{V} + \{\lambda, ., (,)\}$ that is defined inductively by the rules:

$$\frac{x \in \mathbb{V}}{x \in \Lambda}\ (\mathrm{Var})$$

$$\frac{M \in \Lambda \quad N \in \Lambda}{(MN) \in \Lambda}\ (\mathrm{App}) \qquad \frac{x \in \mathbb{V} \quad M \in \Lambda}{(\lambda x.\, M) \in \Lambda}\ (\mathrm{Abs})$$

Any substring of a term, except for the variable between the λ and the dot, that is itself a term we call a ***subterm***.

MEMBERSHIP VIA PROOF TREES

Membership in Λ is governed by the following *principle*:

> *A string M is a member of Λ iff there is a proof tree built from these rules with conclusion $M \in \Lambda$.*

A *proof tree* for Λ is finite tree labelled by statements $M \in \Lambda$ so that, a node is labelled $M \in \Lambda$ and its children are labelled $M_1 \in \Lambda \ldots M_k \in \Lambda$, only if:

$$\frac{M_1 \in \Lambda \cdots M_k \in \Lambda}{M \in \Lambda}$$

is an instance of one of the given rules: (Var), (App) or (Abs).

CONVENTIONS

- *We will omit the outermost parentheses.* For example, whenever we write MN to mean a term, the term that we mean is (MN).
- *In any subterm, we will assume that application associates to the left.* For example, whenever we write MNP, the term that we mean is $((MN)P)$.
- *In any subterm, we will assume that the body of an abstraction extends as far to the right as possible.* For example, when we write $\lambda x.\, MN$, the term that we mean is $(\lambda x.\,(MN))$.
- *In any subterm, iterated abstractions can be grouped.* For example, when we write $\lambda xy.\, M$, the term that we mean is $(\lambda x.\,(\lambda y.\, M))$