INDUCTION ON $\mathbb{N}$

Let Φ be a property of natural numbers. If all of the following conditions are met:

(NI1) Φ holds of 0

(NI2) For all $n \in \mathbb{N}$, if Φ holds of n then Φ holds of $n + 1$.

Then it follows that, for all natural numbers $n \in \mathbb{N}$, Φ holds of n.

INDUCTION METAPRINCIPLE

Suppose we have an inductive definition of a set S using rules $R_1, \ldots, R_k$. The induction principle for proving $\forall s \in S. \Phi(s)$, has k clauses, one for each of the rules.

If rule R_i has m premises and a side condition ψ:

$$\psi \quad \frac{s_1 \in S \quad \cdots \quad s_m \in S}{s \in S} \ (\mathrm{R}_i)$$

then the corresponding clause in the induction principle requires:

$$\text{if } \Phi(s_1) \text{ and } \cdots \text{ and } \Phi(s_m) \text{ and } \psi \text{ then } \Phi(s)$$

REMINDER: $\to_\beta$

$$\frac{}{(\lambda x. M)N \to_\beta M[N/x]}\ \text{(Redex)}$$

$$\frac{M \to_\beta M'}{MN \to_\beta M'N}\ \text{(AppL)} \qquad \frac{N \to_\beta N'}{MN \to_\beta MN'}\ \text{(AppR)}$$

$$\frac{M \to_\beta N}{\lambda x. M \to_\beta \lambda x. N}\ \text{(Abs)}$$