INHABITATION

The ***inhabitation problem***, is the problem of, given a type A, determining if there a closed term M such that $\vdash M : A$.

INHABITATION EXAMPLES

- $a \to a$
- $(a \to a \to b) \to a \to b$
- $((a \to a) \to b) \to b$
- $a \to b$ (with $a \neq b$)
- $(a \to b) \to b$

USEFUL THEOREMS

Theorem (Inversion)
Suppose $\Gamma \vdash M : A$ (is derivable), then:

- *If M is a variable x, then there is a type scheme $\forall \overline{a}. B$ (with $\overline{a}$ possibly empty) and $A = B[\overline{C}/\overline{a}]$ for some monotypes $\overline{C}$.*
- *If M is an application PQ, then there is a type B such that $\Gamma \vdash P : B \to A$ and $\Gamma \vdash Q : B$.*
- *If M is an abstraction $\lambda x. P$, then there are types B and C such that $A = B \to C$, and $\Gamma, x : B \vdash P : C$.*

Theorem (Subject Reduction)
If $\Gamma \vdash M : A$ and $M \to_\beta N$ then $\Gamma \vdash N : A$

Theorem (The Subformula Property)
Suppose $\vdash M : A$ with M in β-normal form. Then all types mentioned in the derivation are substrings of the types mentioned in the conclusion.

Theorem (Strong Normalisation)
If $\Gamma \vdash M : A$ then M is SN.

VALIDITY EXAMPLES

- $a \Rightarrow a$
- $(a \Rightarrow a \Rightarrow b) \Rightarrow a \Rightarrow b$
- $((a \Rightarrow a) \Rightarrow b) \Rightarrow b$
- $a \Rightarrow b$ (with $a \neq b$)
- $(a \Rightarrow b) \Rightarrow b$

COMPARISON

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x : a \to a \to b} \quad \dfrac{}{\Gamma \vdash y : a}}{\Gamma \vdash xy : a \to b} \quad \dfrac{}{\Gamma \vdash y : a}}{\Gamma \vdash xyy : b}}{x : a \to a \to b \vdash \lambda y.xyy : a \to b}}{\vdash \lambda xy.xyy : (a \to a \to b) \to a \to b}$$

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x : (a \to a) \to b} \qquad \dfrac{\dfrac{}{\Gamma, y : a \vdash y : a}}{\Gamma \vdash \lambda y.y : a \to a}}{\Gamma \vdash x(\lambda y.y) : b}}{\vdash \lambda x.x(\lambda y.y) : ((a \to a) \to b) \to b}$$