

FREE VARIABLES

The set of *free variables* of a term M is defined by recursion on the structure of M :

$$\begin{aligned} \text{FV}(x) &= \{x\} \\ \text{FV}(PQ) &= \text{FV}(P) \cup \text{FV}(Q) \\ \text{FV}(\lambda x. N) &= \text{FV}(N) \setminus \{x\} \end{aligned}$$

A term M without free variables is said to be *closed* or a *combinator*. The set of all closed terms is written Λ^0 .

SUBSTITUTION

We define *capture-avoiding substitution* of N for free variable x in term M , written $M[N/x]$, recursively on the structure of M :

$$\begin{aligned}y[N/x] &= y && \text{if } x \neq y \\y[N/x] &= N && \text{if } x = y \\(PQ)[N/x] &= P[N/x]Q[N/x] \\(\lambda y. P)[N/x] &= \lambda y. P && \text{if } y = x \\(\lambda y. P)[N/x] &= \lambda y. P[N/x] && \text{if } y \neq x \text{ and } y \notin \text{FV}(N)\end{aligned}$$

