

# Solutions: Theoretical Computer Science

## 1. Knights and Knaves

Answers must be correctly justified. A correct answer with incorrect justification does not show understanding.

### 1.1 The first encounter

No inhabitant would claim to be a knave, so Rémi could not have said that. Miguel must be lying, and hence a knave. Nothing can be determined about Rémi.

### 1.2 The second encounter

It may be tempting to respond “the inhabitant is a knight, and his second statement was ‘I am a knight.’”. However, this is wrong: both knights *and* knaves could say “I am a knight” as their second statement in this context. So, if that was the second statement, you would have no way of knowing if the inhabitant was a knight or a knave.

The inhabitant must be a knave, and his second statement must have been completely unrelated (eg. “I don’t have a ring in my pocket”). Since the second statement was unrelated, that makes the first statement a lie. A knight could have never lied in the first statement like this, so from these statements, you can deduce that the inhabitant must be a knave.

### 1.3 A conundrum

This statement is equivalent to the statement  $S =$  “both knights and knaves can make this statement.”

Assume  $S$  is true. Then both knights and knaves could make it. But then we would have knaves making a true statement, which is a contradiction.

Assume  $S$  is false. If we assume that knaves can say it, the interpretation appears consistent [this isn’t very rigorous].

$\therefore$  knaves can say the statement (and  $S$  is false).

## 2. SKI Combinator Calculus

### 2.1 False

$F = KI$ .

### 2.2 Idiot

$I = SKx$ , where  $x$  is any arbitrary expression. For example,  $SKK$ ,  $SKS$ , or  $SK(K(SKK))$  are all solutions.

### 2.3 Compose

$B = S(KS)K$