**Worked Example of 3x3**

In this document I will show the analysis of the 3x3 design with dummy variables. For each parameter I will show the cells in the mean classification table used in the calculation, the relation to the dummy variable coding, and the working to solve for the beta coefficients from the cell means.

The workings follow the 2x2 case, in that we are looking at the differences between cell means for values coded 1 against the reference group, and for the interactions, we are identifying 4 cells, and considering the difference in differences.

*Full equation:*

**INTERCEPT: b0**

Mean of reference group

*Dummy Variable Coding*

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Row** | **Column** | **Row/Column Dummies** | | | | **Interactions Dummies** | | | |
| **Levels** | **Levels** | **D1**  **(b1)** | **D2**  **(b2)** | **D3**  **(b3)** | **D4**  **(b4)** | **D1D3 (b5)** | **D1D4 (b6)** | **D2D3 (b7)** | **D2D4 (b8)** |
| A | A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | B | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| A | C | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| B | A | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | B | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| B | C | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| C | A | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | B | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| C | C | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

*Classification of means*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** |  |  | **A** | **B** | **C** |
| **A** | 12.4 | 12.7 | 12.9 |  | **A** | AA (1,1) | AB (1,2) | AC (1,2) |
| **B** | 11.5 | 11.1 | 13.4 |  | **B** | BA (2,1) | BB (2,2) | BC (2,3) |
| **C** | 10.7 | 16.1 | 35.4 |  | **C** | CA (3,1) | CB (3,2) | CC (3,3) |

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| **Solved** |
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**CONDITIONAL MAIN EFFECTS: b1**

Difference between row level A and row level B.

*Dummy Variable Coding*

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Row** | **Column** | **Row/Column Dummies** | | | | **Interactions Dummies** | | | |
| **Levels** | **Levels** | **D1**  **(b1)** | **D2**  **(b2)** | **D3**  **(b3)** | **D4**  **(b4)** | **D1D3 (b5)** | **D1D4 (b6)** | **D2D3 (b7)** | **D2D4 (b8)** |
| A | A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | B | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| A | C | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| B | A | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | B | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| B | C | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| C | A | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | B | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| C | C | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

*Classification of means*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** |  |  | **A** | **B** | **C** |
| **A** | 12.4 | 12.7 | 12.9 |  | **A** | AA (1,1) | AB (1,2) | AC (1,2) |
| **B** | 11.5 | 11.1 | 13.4 |  | **B** | BA (2,1) | BB (2,2) | BC (2,3) |
| **C** | 10.7 | 16.1 | 35.4 |  | **C** | CA (3,1) | CB (3,2) | CC (3,3) |

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| **Solved** |
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**CONDITIONAL MAIN EFFECTS: b2**

Difference between row level A and row level C

*Dummy Variable Coding*

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Row** | **Column** | **Row/Column Dummies** | | | | **Interactions Dummies** | | | |
| **Levels** | **Levels** | **D1**  **(b1)** | **D2**  **(b2)** | **D3**  **(b3)** | **D4**  **(b4)** | **D1D3 (b5)** | **D1D4 (b6)** | **D2D3 (b7)** | **D2D4 (b8)** |
| A | A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | B | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| A | C | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| B | A | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | B | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| B | C | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| C | A | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | B | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| C | C | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

*Classification of means*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** |  |  | **A** | **B** | **C** |
| **A** | 12.4 | 12.7 | 12.9 |  | **A** | AA (1,1) | AB (1,2) | AC (1,2) |
| **B** | 11.5 | 11.1 | 13.4 |  | **B** | BA (2,1) | BB (2,2) | BC (2,3) |
| **C** | 10.7 | 16.1 | 35.4 |  | **C** | CA (3,1) | CB (3,2) | CC (3,3) |

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| **Solved** |
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**CONDITIONAL MAIN EFFECTS: b3**

Difference between column level A and column level B

*Dummy Variable Coding*

|  |  |  |  |  |  |  |  |  |  |
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| **Row** | **Column** | **Row/Column Dummies** | | | | **Interactions Dummies** | | | |
| **Levels** | **Levels** | **D1**  **(b1)** | **D2**  **(b2)** | **D3**  **(b3)** | **D4**  **(b4)** | **D1D3 (b5)** | **D1D4 (b6)** | **D2D3 (b7)** | **D2D4 (b8)** |
| A | A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | B | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| A | C | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| B | A | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | B | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| B | C | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| C | A | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | B | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| C | C | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

*Classification of means*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** |  |  | **A** | **B** | **C** |
| **A** | 12.4 | 12.7 | 12.9 |  | **A** | AA (1,1) | AB (1,2) | AC (1,2) |
| **B** | 11.5 | 11.1 | 13.4 |  | **B** | BA (2,1) | BB (2,2) | BC (2,3) |
| **C** | 10.7 | 16.1 | 35.4 |  | **C** | CA (3,1) | CB (3,2) | CC (3,3) |

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| **Solved** |
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**CONDITIONAL MAIN EFFECTS: b4**

Difference between column level A and column level C

*Dummy Variable Coding*

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Row** | **Column** | **Row/Column Dummies** | | | | **Interactions Dummies** | | | |
| **Levels** | **Levels** | **D1**  **(b1)** | **D2**  **(b2)** | **D3**  **(b3)** | **D4**  **(b4)** | **D1D3 (b5)** | **D1D4 (b6)** | **D2D3 (b7)** | **D2D4 (b8)** |
| A | A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | B | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| A | C | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| B | A | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | B | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| B | C | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| C | A | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | B | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| C | C | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

*Classification of means*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** |  |  | **A** | **B** | **C** |
| **A** | 12.4 | 12.7 | 12.9 |  | **A** | AA (1,1) | AB (1,2) | AC (1,2) |
| **B** | 11.5 | 11.1 | 13.4 |  | **B** | BA (2,1) | BB (2,2) | BC (2,3) |
| **C** | 10.7 | 16.1 | 35.4 |  | **C** | CA (3,1) | CB (3,2) | CC (3,3) |

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| **Solved** |
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**INTERACTIONS: b5**

Difference between row level A and row level B, is different across column level A and column level B

*Dummy Variable Coding*

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Row** | **Column** | **Row/Column Dummies** | | | | **Interactions Dummies** | | | |
| **Levels** | **Levels** | **D1**  **(b1)** | **D2**  **(b2)** | **D3**  **(b3)** | **D4**  **(b4)** | **D1D3 (b5)** | **D1D4 (b6)** | **D2D3 (b7)** | **D2D4 (b8)** |
| A | A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | B | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| A | C | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| B | A | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | B | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| B | C | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| C | A | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | B | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| C | C | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

*Classification of means*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** |  |  | **A** | **B** | **C** |
| **A** | 12.4 | 12.7 | 12.9 |  | **A** | AA (1,1) | AB (1,2) | AC (1,2) |
| **B** | 11.5 | 11.1 | 13.4 |  | **B** | BA (2,1) | BB (2,2) | BC (2,3) |
| **C** | 10.7 | 16.1 | 35.4 |  | **C** | CA (3,1) | CB (3,2) | CC (3,3) |

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| **Solved** |
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**INTERACTIONS: b6**

Difference between row level A and row level B, is different across column level A and column level C

*Dummy Variable Coding*

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Row** | **Column** | **Row/Column Dummies** | | | | **Interactions Dummies** | | | |
| **Levels** | **Levels** | **D1**  **(b1)** | **D2**  **(b2)** | **D3**  **(b3)** | **D4**  **(b4)** | **D1D3 (b5)** | **D1D4 (b6)** | **D2D3 (b7)** | **D2D4 (b8)** |
| A | A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | B | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| A | C | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| B | A | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | B | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| B | C | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| C | A | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | B | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| C | C | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

*Classification of means*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** |  |  | **A** | **B** | **C** |
| **A** | 12.4 | 12.7 | 12.9 |  | **A** | AA (1,1) | AB (1,2) | AC (1,2) |
| **B** | 11.5 | 11.1 | 13.4 |  | **B** | BA (2,1) | BB (2,2) | BC (2,3) |
| **C** | 10.7 | 16.1 | 35.4 |  | **C** | CA (3,1) | CB (3,2) | CC (3,3) |

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| **Solved** |
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**INTERACTIONS: b7**

Difference between row level A and row level C, is different across column level A and column level B

*Dummy Variable Coding*

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Row** | **Column** | **Row/Column Dummies** | | | | **Interactions Dummies** | | | |
| **Levels** | **Levels** | **D1**  **(b1)** | **D2**  **(b2)** | **D3**  **(b3)** | **D4**  **(b4)** | **D1D3 (b5)** | **D1D4 (b6)** | **D2D3 (b7)** | **D2D4 (b8)** |
| A | A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | B | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| A | C | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| B | A | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | B | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| B | C | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| C | A | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | B | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| C | C | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

*Classification of means*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** |  |  | **A** | **B** | **C** |
| **A** | 12.4 | 12.7 | 12.9 |  | **A** | AA (1,1) | AB (1,2) | AC (1,2) |
| **B** | 11.5 | 11.1 | 13.4 |  | **B** | BA (2,1) | BB (2,2) | BC (2,3) |
| **C** | 10.7 | 16.1 | 35.4 |  | **C** | CA (3,1) | CB (3,2) | CC (3,3) |

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| **Solved** |
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**INTERACTIONS: b8**

Difference between row level A and row level C, is different across column level A and column level C

*Dummy Variable Coding*

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Row** | **Column** | **Row/Column Dummies** | | | | **Interactions Dummies** | | | |
| **Levels** | **Levels** | **D1**  **(b1)** | **D2**  **(b2)** | **D3**  **(b3)** | **D4**  **(b4)** | **D1D3 (b5)** | **D1D4 (b6)** | **D2D3 (b7)** | **D2D4 (b8)** |
| A | A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | B | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| A | C | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| B | A | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | B | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| B | C | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| C | A | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| C | B | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| C | C | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |

*Classification of means*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **A** | **B** | **C** |  |  | **A** | **B** | **C** |
| **A** | 12.4 | 12.7 | 12.9 |  | **A** | AA (1,1) | AB (1,2) | AC (1,2) |
| **B** | 11.5 | 11.1 | 13.4 |  | **B** | BA (2,1) | BB (2,2) | BC (2,3) |
| **C** | 10.7 | 16.1 | 35.4 |  | **C** | CA (3,1) | CB (3,2) | CC (3,3) |

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| **Solved** |
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