Phase shift of contact for modulated scattering length

Tilman Enss & Keisuke Fujii, 5 Oct 2023

Following [1], a time dependent scattering length $a(t)$ leads to the following time dependence of the contact density $\mathcal{C}(t)$ in $d = 3$ dimensions:

$$\mathcal{C}(t) = \mathcal{C}_{\text{eq}}(a(t)) + 36\pi m\zeta\dot{a}(t) + \mathcal{O}(\delta^2 a, \delta\ddot{a}) \tag{1}$$

in terms of the equilibrium contact density $\mathcal{C}_{\text{eq}}(a)$ and the bulk viscosity $\zeta(n, T, a)$. Numerical values for the degenerate, strongly correlated Fermi gas are given in Fig. 3 of Ref. [2]. We consider a scattering length that is modulated periodically with dimensionless amplitude A (on the $1/(k_F a)$ axis) around a background scattering length a_0 as

$$a^{-1}(t) = a_0^{-1} + Ak_F \sin\omega t, \tag{2}$$

such that $\dot{a}(t) = -a^2(t)\frac{d}{dt}a^{-1}(t)$. In response, also the contact density will oscillate with the same frequency. The dissipative term ζ leads to a phase lag with respect to the drive, which we will estimate in the following.

For small drive amplitude A the equilibrium contact density will change to linear order as

$$\mathcal{C}_{\text{eq}}(a^{-1}(t)) = \mathcal{C}_{\text{eq}}(a_0^{-1}) + Ak_F \sin(\omega t)\frac{\partial\mathcal{C}(a^{-1})}{\partial a^{-1}}\Big|_{a_0^{-1}} \tag{3}$$

$$= \mathcal{C}_{\text{eq}}(a_0^{-1}) + Ak_F \sin(\omega t)\, 36\pi m a^2 S(a_0^{-1}) \tag{4}$$

in terms of the "scale susceptibility" S from Eq. (7) in [2]. We thus find for the contact density

$$\mathcal{C}(t) = \mathcal{C}_{\text{eq}}(a_0^{-1}) + 36\pi Amk_F a^2 S(a_0^{-1})\sin(\omega t) - 36\pi Amk_F\omega a^2\zeta(a_0^{-1})\cos(\omega t) \tag{5}$$

$$= \mathcal{C}_{\text{eq}}(a_0^{-1}) + \frac{6}{\pi}Ak_F^4\Big[\frac{(k_F a)^2 S}{nE_F}\,\sin(\omega t) - \frac{\hbar\omega}{E_F}\,\frac{(k_F a)^2\zeta}{\hbar n}\,\cos(\omega t)\Big]. \tag{6}$$

We define the dimensionless frequency-dependent viscosity $z = \frac{(k_F a)^2\zeta(\omega)}{\hbar n}$ plotted in Fig. 3a of [2] and the dimensionless scale susceptibility (sum rule) $s = \frac{(k_F a)^2 S}{nE_F}$ plotted in Fig. 3c (solid), as well as the dimensionless contact density $c = \mathcal{C}/k_F^4$ plotted in Fig. 3c (dotted), and find

$$c(t) = c_{\text{eq}}(a_0^{-1}) + \frac{6}{\pi}A\big[s\,\sin(\omega t) - \frac{\hbar\omega}{E_F}\,z\,\cos(\omega t)\big] \tag{7}$$

$$= c_{\text{eq}}(a_0^{-1}) + \frac{6}{\pi}A\sqrt{s^2 + (\hbar\omega/E_F)^2 z^2}\,\sin(\omega t - \varphi). \tag{8}$$

The oscillations have amplitude $\frac{6As}{\pi\cos(\varphi)}$ directly proportional to the drive A (in contrast to the dissipative heating, which scales as A^2) and a dissipative phase shift

$$\varphi = \arctan\Big(\frac{\hbar\omega}{E_F}\,\frac{z}{s}\Big) = \arctan\Big(\frac{\omega\zeta(\omega)}{S}\Big) = \arctan\Big(\frac{\omega\zeta(\omega)}{\frac{2}{\pi}\int_0^\infty d\omega\,\zeta(\omega)}\Big). \tag{9}$$

With the numerical values $s(T/T_F = 0.16) = 0.23$, $s(T/T_F = 0.25) = 0.18$ and $s(T/T_F = 0.58) = 0.14$, we find phase shifts up to $\varphi \approx 0.35$ at intermediate frequencies, as shown in Fig 1. The phase shift initially increases linearly with frequency and peaks at a frequency of roughly $\omega \sim T$, which arises from the quantum critical scaling of the width of the Drude peak in the bulk viscosity.

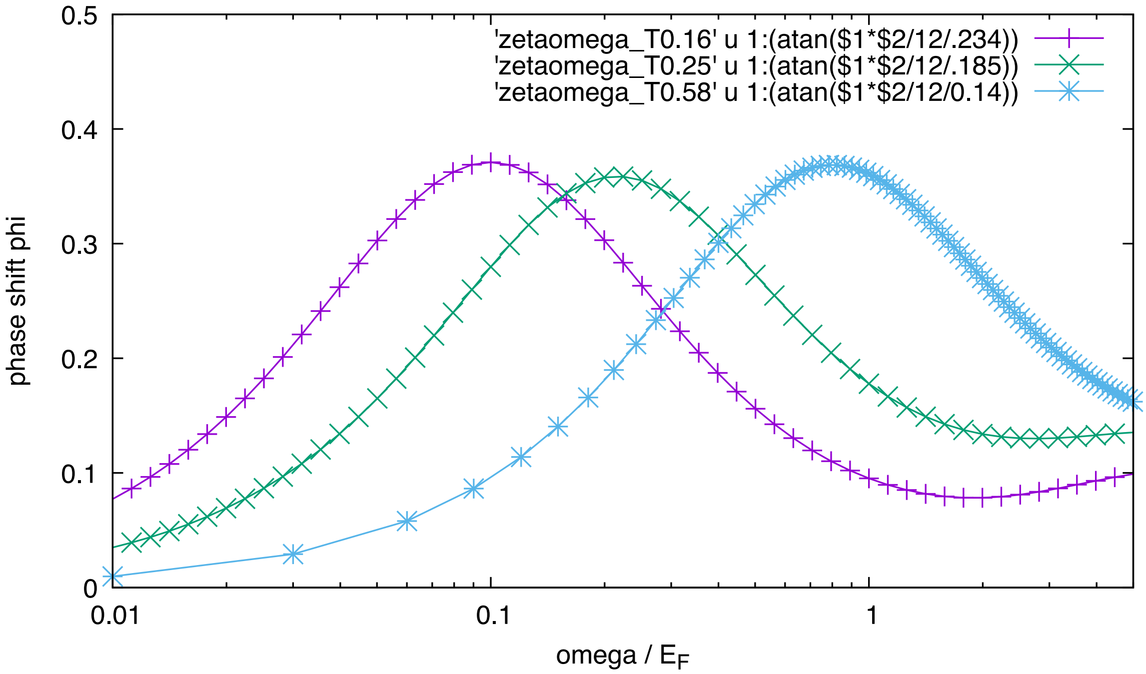

Figure 1: Phase shift φ at unitarity as a function of drive frequency $\hbar\omega/E_F$ for 3 different temperatures, $T/T_F = 0.16$, 0.25, 0.58.

References

[1] K. Fujii and Y. Nishida, *Hydrodynamics with spacetime-dependent scattering length*, Phys. Rev. A **98**, 063634 (2018).

[2] T. Enss, *Bulk Viscosity and Contact Correlations in Attractive Fermi Gases*, Phys. Rev. Lett. **123**, 205301 (2019).