Contact Correlations Notes

Various chip lab members
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I. AC DIMER ASSOCIATION

At unitarity for an interacting ab spin mixture, the first-moment in frequency of the transfer rate to a weakly interacting probe state c is given by [cite?]

$$\frac{\bar{\omega}}{E_F/\hbar} = -\frac{1}{\pi}\frac{1}{k_F a_{\rm ac}}\frac{C}{k_F N}\,, \tag{1}$$

where k_F and E_F are the fermi wavenumber and energy, respectively, N is the total number of particles, C is the contact, and $\bar{\omega}$ is the first moment of the detuning from the b to c transition, that is

$$\bar{\omega} = \frac{\int \omega d\omega}{\int d\omega}\,, \tag{2}$$

for $\omega = \omega_{\rm rf} - \omega_{\rm bc}$. We can write the contact as

$$\frac{C}{k_F N} = -\pi k_F a_{\rm ac}\bar{\Delta}\,, \tag{3}$$

with $\bar{\Delta} = \hbar\bar{\omega}/E_F$. So, one can find the contact by finding the first moment of frequency (detuning).

From contact spectra 09-11_E, and dimer spectra 09-22_P we get a first moment of $-5.9(6)\,E_F$, with peak scaled transfer to the dimer state of $\tilde{\Gamma} = 0.00325(5)$. For a single-shot dimer association measurement, we can estimate the first moment by scaling $-5.9(6)$ by the single-shot scaled transfer

$$\frac{C}{k_F N} = \pi k_F a_{\rm ac}\frac{5.9(6)}{0.00325(5)}\tilde{\Gamma}\,, \tag{4}$$

or

$$\frac{C}{k_F N} = \pi k_F a_{\rm ac} 1810(200)\tilde{\Gamma}\,. \tag{5}$$

We can get a rough estimate of $k_F a_{\rm ac}$ by assuming

$$E_d = \frac{\hbar^2}{2m a_{\rm ac}^2}\,, \tag{6}$$

For $E_d = 4\,$MHz, and $k_F = 1.2\times 10^7\,$1/m, we obtain $k_F a_{\rm ac} = 0.066$. Which means for the dimer spectra, we estimate a contact of $C/N = 1.2(1)k_F$ which is quite close to our high-frequency spectra fit of $C/N = 0.98(4)k_F$.

II. HEATING RATE

From Fujii and Nishida PRA 2018 we have the dissipative heating rate density

$$\dot{\mathcal{E}} = d^2(\partial_t a^{-1})^2 a^2\zeta\,, \tag{7}$$

where $d = 3$ is the dimension, a is the scattering length, and ζ is the bulk viscosity. We oscillate the magnetic field at a frequency ω, so that the inverse scattering length a^{-1} oscillates like

$$(k_F a(t))^{-1} = A\sin(\omega t) + (k_F a_0)^{-1}\,, \tag{8}$$

such that $(\partial_t a^{-1})^2 = A^2\omega^2 k_F^2\cos^2(\omega t)$. Integrating the rate equation, and ignoring any time dependence in $a^2\zeta$, we obtain

$$\int_0^t \dot{\mathcal{E}} dt' = 9A^2\omega^2 k_F^2 a^2\zeta\int_0^t \cos^2(\omega t')dt'\,. \tag{9}$$

Next, I average over the integral factor, since we oscillate for many periods. We might have to consider this more carefully if we oscillate slowly for a small number of periods.

$$\mathcal{E}(t) - \mathcal{E}_i = 9A^2\omega^2 k_F^2 a^2\zeta\frac{t}{2}\,. \tag{10}$$

We can then divide out density.

$$\Delta E(t) = \frac{9}{2}A^2\omega^2 t(k_F a)^2\frac{\zeta}{n}\,. \tag{11}$$

Now, we want to fudge the RHS to look like the dimensionless quantity Tilman tabulated for us from Fig. 3a of his 2019 PRL.

$$\Delta E(t) = \frac{9}{2}A^2(\hbar\omega)\omega t\left[\frac{(k_F a)^2\zeta}{\hbar n}\right]\,. \tag{12}$$

Here, I have written the frequency factors to show that the units work out, with energy on both sides. Let us name the quantity in square brackets $\tilde{\zeta}$, and we will explicitely show the dependence on ω

$$\Delta E(t) = \frac{9}{2}A^2(\hbar\omega)\omega t\tilde{\zeta}(\omega)\,. \tag{13}$$

We can write the LHS in terms of T/T_F heating, i.e. in units of Fermi energy E_F, so we obtain

$$\frac{\Delta E(t)}{E_F} = \frac{9}{2}A^2\frac{\hbar\omega}{E_F}\omega t\tilde{\zeta}(\omega)\,. \tag{14}$$

A. Contact from heating rate

At large oscillation frequency, we can determine the contact through Eq. (10) in Tilman's 2019 PRL, which is

$$\zeta(\omega\to\infty) = \frac{\mathcal{C}}{36\pi a^2(m\omega)^{3/2}}\,, \tag{15}$$

where $\mathcal{C}$ is the contact density, and m is the mass. The relation between contact density and our usual contact per atom C/N is

$$\frac{C}{N} = \frac{\mathcal{C}}{n}\,. \tag{16}$$

We proceed by setting $\hbar = 1$

$$\begin{aligned}
\frac{a^2\zeta}{n} &= \frac{1}{36\pi}\frac{C}{k_F N}\left(\frac{E_F}{\omega}\right)^{3/2}\frac{k_F}{E_F^{3/2}m^{3/2}}\,,\\
\frac{k_F^2 a^2\zeta}{n} &= \frac{2^{3/2}}{36\pi}\frac{C}{k_F N}\Delta^{-3/2}\,,\\
\tilde{\zeta} &= \frac{1}{9\sqrt{2}\pi}\frac{C}{k_F N}\Delta^{-3/2}\,,
\end{aligned} \tag{17}$$

which is now a relation between the dimensionless bulk viscosity and the usual form of the contact we use. This can be plugged into Eq. (14) to obtain

$$\frac{\Delta E(t)}{E_F} = \frac{1}{2\sqrt{2}\pi}\frac{C}{k_F N}A^2\sqrt{\frac{\omega E_F}{\hbar}}t\,. \tag{18}$$

rearranging for contact

$$\frac{C}{k_F N} = \frac{2\sqrt{2}\pi}{A^2 t}\sqrt{\frac{\hbar}{\omega E_F}}\frac{\Delta E}{E_F}\,, \tag{19}$$

which is

$$\frac{C}{k_F N} = \sqrt{\frac{2}{\Delta}}\frac{1}{A^2}\frac{h}{t}\frac{\Delta E}{E_F^2}\,, \tag{20}$$

or

$$\frac{C}{k_F N} = \sqrt{\frac{2}{\Delta}}\frac{1}{A^2}\frac{t_F}{t}\frac{\Delta E}{E_F}\,. \tag{21}$$

B. Trap averaging

Tilman gave some tabulated data for trap-integrated bulk viscosity for a Thomas-Fermi distribution versus T/T_F. I believe this was for DC bulk viscosity, but perhaps it is generalizable to dynamic bulk viscosity. For $T/T_F = 0.25$, and 0.58 the ratio between the ζ from Fig. 3a and the trap-averaged ζ are 2.31 and 1.76, respectively. Note the trap integrated ζ for a Gaussian distribution is much less at higher temperatures.