Predicting the recurrence of child abuse

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Funding cuts are forcing protective service agencies to serve only the most serious cases of child abuse among the over 200,000 reported nationally each year. A good indication of the seriousness of a case is the likelihood of repeated abuse. This article describes a study to determine the predictability of repeated abuse by analyzing the relationship of recurrent abuse to case characteristics. The resultant statistical model predicted the recurrence of abuse with 74 percent accuracy. The authors discuss their findings in relation to improving the allocation of resources in protective service agencies.

Each year, over 200,000 cases of the physical abuse of children are reported to child protective service agencies across the United States. Helping all the children who may need protection from abusive parents is becoming more difficult because funding cuts are forcing protective service agencies to reduce services. In response to the cuts in funding, the agencies are coming under increasing pressure to allocate their available resources to the most "serious" cases. A good indication of the seriousness of a case is the likelihood that there will be further abusive incidents after the initial reported incident. Thus, protective service agencies need to establish methods of predicting the repetition of abuse if they are to allocate their shrinking resources appropriately. The study reported in this article was undertaken as a contribution to the so-far infrequent efforts to predict the recurrence of child abuse.

THE LITERATURE

Although predicting the recurrence of child abuse is an important concern to protective service agencies and their

workers, few researchers have studied this problem. A search of the computerized bibliographic data bases of Psychological Abstracts and Sociological Abstracts and manual searches of recent issues of relevant journals for material dealing with the prediction, detection, and measurement of physical, emotional, and sexual abuse revealed only one study that aimed specifically at predicting the recurrence of child abuse by parents already reported to authorities.2 A second study examined one possible predictor of the recurrence of abuse, but the prediction of further incidents of abuse was not its central focus.3 Thus. previous research on the prediction of child abuse has focused primarily on predicting initial instances of abuse rather than on the repetition of abuse.4 Although predictors of both types of abuse may not be identical, the predictors of initial incidents offer the most reasonable source of hypotheses for predicting the recurrence of abuse. Therefore, to identify potential predictors of the recurrence of abuse, the authors looked at the literature on the prediction of first cases of abuse.

Two major themes in the literature

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TABLE 1. Sample Distributions of Variables that Predicted the Recurrence of Abuse in the Analysis and Validation Samples

Variables	N	X	SD
The amount of time that the abused child spends with the abusing adult; rated on a five-point scale from "a little" (1) to "a lot" (5).	135ª	3.20	.98
 The mother figure's parenting skill; rated on a five-point scale from "low" (1) to "high" (5). 	131	2.53	1.02
 The reasonableness of the mother figure's expectations of the abused child; rated on a five-point scale from "very unreasonable" (1) to "very reasonable" (5). 	125	2.69	.92
4. The family's ability to use agency resources; rated on a five-point scale from "low" (1) to "high" (5).	135	3.19	1.05
 The presence of more than one child in the home (dummy variable coded 0 = one child, 1 = more than one child). 	134 ^b		

^{*}Sample sizes vary because of missing information.

have been the relationship of individual differences to abuse and the environmental correlates of abuse.5 Neither individual differences nor environmental factors dominate theoretical conceptions of the causes of abuse, however; current conceptions frequently group putative causal factors according to their association with the parent, the child, or the situational context of the abuse. A large number of correlates or predictors of abuse that appeared to be of value in predicting the recurrence of abuse were included in the current study and were grouped according to the parent-childsituational categories just mentioned.

Factors associated with the parents were unemployment,6 abuse during their childhood,7 the lack of adequate parenting skills,8 unreasonable expectations of the children,9 a poor self-image,10 and substance abuse, arrest records, and psychopathology.11

Age was one factor associated with the abused children. ¹² Another factor was characteristics that set the abused children apart from other children, including retarded social or physical development, retarded educational progress, and the presence of special physical conditions or diseases. ¹³

Factors associated with the situational context of the abuse were inadequate income and inadequate housing;¹⁴ the lack of social supports for the family;¹⁵ the presence of psychosocial crises, such

as the parents' recent divorce, or material crises, such as recent job loss or eviction from a dwelling; ¹⁶ and instability of residence. ¹⁷ On the basis of their assessment, the authors included some measurements of the attitudes of the abuser and the family toward accepting help from an agency, the family's ability to use agency resources (their capacity to cooperate in the receipt of services, such as making and keeping appointments), and the number of children in the family. The final data collection instrument comprised 105 items measuring the aforementioned variables. ¹⁸

SAMPLE AND METHOD

The sample included 120 randomly selected cases of physical abuse from the files of a public social service agency in a large urban California county. Cases were judged to constitute physical abuse if the referral report fitted Gil's definition of physical abuse. Gil defined physical abuse as "intentional, nonaccidental acts or omissions [by] a parent or other caretaker interacting with a child in his care, aimed at hurting, injuring, or destroying the child." Cases of sexual abuse were excluded from the sample unless they involved the use of physical force.

One subgroup of the sample consisted of sixty-five cases referred for physical abuse in which there was no known recurrence of abuse while the case was open and no subsequent referral to the agency for abuse within two years of the initial referral if the case was closed. Cases in which the family was known to have left the county within two years of the initial referral or in which the child was placed out of the home on a long-term basis for the sake of protection were excluded from this subgroup. The other subgroup consisted of fiftyfive cases referred for an initial incident of physical abuse in which a subsequent incident involving the same child or another child in the family had been reported within two years of the initial referral and caused the case to be reopened or continued as open. Case records were examined by ten graduate social work students from the University of California. Information in the records that had been obtained within ninety days of the initial opening of the case and prior to any recurrence of abuse was used to supply data on the potential predictors of recurrent abuse identified in the review of the literature.

After data collection, the total case sample was divided randomly into two new groups. Group 1 consisted of eightyone cases—forty-six single-abuse cases and thirty-five multiple-abuse cases—that were used as an analysis sample for developing a predictive model (hereafter called "the model"). Group 2 consisted of thirty-nine cases—nineteen single-abuse cases and twenty multiple-abuse cases—that were used as a cross-validation sample.

Consistent with Hotelling's arguments on the selection of variates for use in prediction, the authors chose variables to be used in constructing the model that had the highest correlations with repeated abuse in the expected directions in both the analysis and validation samples and for which data were present in almost all cases.20 To construct the model, the recurrence and nonrecurrence cases in Group 1 were compared on the variables in the predictor set using multiple linear discriminant analysis.21 Classification functions derived from using cases in the analysis sample were applied to cases in the validation sample.

In the analysis, treatment effects and case characteristics were confounded in their impact on the outcomes of singleabuse and repeated-abuse cases. The

bNinety of the cases (66.2 percent) had more than one child.

possible role of treatment effects in producing observed relationships between case characteristics and the recurrence of abuse was, therefore, important. To address this problem, data on treatment were collected for a random subsample of ninety cases from immediately after a case was opened until it was closed or an additional incident of abuse was reported, whichever happened first. These data included the number of telephone conversations and in-person interviews between workers and clients, collateral contacts between workers and members of clients' extended families, and collateral contacts between workers and other professionals engaged in providing services to the family. The occurrence of the conversations and interviews was recorded in writing in the case records examined by the researchers. In the ninety-case subsample, nine cases had missing information. Tests for differences between cases for which data were present and those for which data were missing were not conducted because of the small number of cases with missing data.

A set of treatment variables was created, based on the treatment data, to provide statistical control of the effects of treatment in analyzing the relationships between case characteristics and the recurrence of abuse. The four treatment variables were (1) the product of average monthly in-person contacts with clients and the number of months cases were kept open before a recurrence of abuse was reported, (2) the product of average monthly telephone contacts with clients and the number of months cases were kept open prior to the reporting of an additional incident of abuse, (3) the product of average monthly collateral contacts with extended family members and interested professionals and the number of months that cases were kept open before a further incident of abuse was reported, and (4) the number of months that cases were kept open prior to a report of the repetition of abuse. With these data, first-order partial correlation coefficients were computed for the relationship between predictions of abuse (made using the classification functions described previously) and the recurrence of abuse, controlling for each of the treatment variables. A fourthorder partial correlation also was computed between predictions of the repeti-

TABLE 2. Correlations among Variables that Were Related to the Recurrence of Abuse in the Analysis and Validation Samples

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Va	riables	1	2	3	4	5
1.	The amount of time that the abused child spends with the abusing adult.					
2.	The mother figure's parenting skill.	39ª				
3.	The reasonableness of the mother figure's expectations of the abused child.	38ª	.55 *			
4.	The family's ability to use agency resources.	10	.44ª	.33*		
5.	The presence of more than one child in the home.	.10	06	02	~ .08	
6.	The recurrence of abuse (dummy variable coded 0 = no recurrence, 1 = recurrence).	.28 ^b	36ª	33ª	33*	.26b

 $^{^{}a}p = < .0005.$

tion of abuse and the actual recurrence of abuse, controlling for all four treatment variables simultaneously.

In addition, several worker-effort variables were created and studied in relation to the predictor variables in the model. They were (1) the total number of attempted worker-initiated in-person contacts with clients (both unsuccessful and successful), (2) the total number of attempted worker-initiated telephone contacts with clients (both unsuccessful and successful), and (3) the total number of attempted worker-initiated collateral contacts with extended family members of the clients and with professionals involved in case management (both unsuccessful and successful).

The simultaneous availability of treatment data and biographical data made possible a test of the main assumption on which the appropriate allocation of resources among serious and nonserious cases should be based, namely, that the resources to be allocated should have some effect on the outcomes of a case. This assumption was tested by computing partial correlations between the treatment variables and the recurrence of abuse, controlling for case characteristics, represented collectively in the model.

RESULTS

The data in Tables 1 and 2 show the sample frequencies and correlations of

variables in the predictor set with each other and with the recurrence of abuse. The results were consistent with some previous findings on the prediction of initial occurrences of abuse. Variables that appear to measure stress (more than one child in the home and a Likert-scale rating of the amount of time the abuser spends with the child) were positively related to the recurrence of abuse. Variables intended to measure parental coping skills (Likert-scale ratings of the mother's parenting skill and the reasonableness of the mother's expectations of the child) were negatively related to the repetition of abuse, as was a variable intended to measure the client's capacity to use resources (Likertscale ratings of the family's ability to use resources).

The probability that the correlations between the recurrence of abuse and variables in this predictor set are spurious is small, even though the number of items studied was large (105). Each of these variables was correlated with the repetition of abuse at p=.001 or less. In computing 105 correlation coefficients, the authors found that the expected number of spurious coefficients significant at p=.001 or less was .001 \times 105 = .105, or slightly more than one-tenth of one spurious correlation.

The results of the discriminant analysis, including classification function coefficients and univariate F ratios for each of the predictor variables, are

bp = < .001.

TABLE 3. Results of the Discriminant Analysis of the Recurrence of Abuse^a

Variable	<i>F</i> Ratio	Probability	Classification Function Coefficients for Membership in the Recurrent Group	Classification Function Coefficient for Membership in the Nonrecurrent Group
The amount of time that the abused child spends with the abusing adult.	8.27	.005	5.93	5.38
2. The mother figure's parenting skill.	13.51	.0004	3.89	3.85
3. The reasonableness of the mother figure's expectations of the abused child.	6.84	.010	1.42	1.86
The family's ability to use agency resources.	17.89	.0001	1.53	2.36
5. The presence of more than one child in the home.	4.38	.040	3.06	2.15
Classification function constant			- 20.35	~ 21.52

^aThe canonical correlation for the discriminant function was .54 (p = < .01).

shown in Table 3. The classification functions were used to make predictions of the recurrence of abuse for each case in the validation sample. A cross-tabulation of predicted and actual outcomes of abuse and the repetition of abuse in the validation sample is presented in Table 4. As can be seen, the combination of five variables in the model predicted the recurrence of abuse in the validation sample with 74.4 percent accuracy. Predictions of the repetition of abuse correlated r = .50, p = .001, with actual further abusive incidents in the validation sample. Table 5 shows a cross-tabulation of predicted and actual outcomes in the entire sample; 73.3 percent of the cases were classified accurately, and the predicted recurrence of abuse correlated with the actual recurrence, r = .46, p < .0005.

The partial correlations between the predictions of abuse and the recurrence of abuse and actual cases of repeated abuse, controlling for the treatment variables, ranged from r = .47 to r = .52, p < .0005 in each case. The partial correlations showed the accuracy of the predictions made using the model to be independent of the amounts of treatment administered to cases as measured

in the study. In relation to the worker-effort variables, only one of the fifteen correlations of the worker-effort variables with the predictor variables in the model was found to be significant. That is, the total number of worker-initiated telephone contacts with clients was negatively correlated with Likert-scale ratings of the amount of time the abused child spent with the abusive adult (r = -.20, p = .04).

A special analysis was performed to see whether the model was equally accurate with the random subsample for which worker-effort data were collected and with the rest of the sample. First, the researchers constructed a variable that was coded 0 for each case in which the prediction made using the model differed from the actual outcome and coded 1 for each case in which the prediction and the outcome were the same. Second, they constructed a variable that was coded 0 for cases in the subsample for which worker-effort data were collected and coded 1 for all remaining cases in the subsample. By cross-tabulating these two variables, they determined that the model was accurate for 69.2 percent of the cases that were not in the subsample and for 75.3 percent of those in the subsample. A chi-square test of the cross-tabulation was not significant. From this analysis, it is apparent that the model was similarly accurate with the subsample and with the remaining sample. Thus, the subsample seems to be representative of the larger sample and of the population from which it was drawn.

The partial correlations of the treatment variables with repeated abuse, controlling for case characteristics, showed that one treatment variable appeared to affect the actual outcomes. That variable was the amount of time cases were kept open; its partial correlation with the recurrence of abuse was r = -.19, p = .05.

DISCUSSION

The analysis showed that reasonably accurate predictions of the recurrence of abuse can be made using case characteristics. Also, when case characteristics were controlled statistically, one treatment variable appeared to reduce, to some extent, the likelihood of repeated abuse. But the central question is this: Would use of the predictive model improve the ability of agencies to direct their dwindling resources to their most serious cases? The data from this study do not permit a definitive answer to this question, but the tentative answer is yes, for a number of reasons.

First, the research generated considerable data on relationships between case characteristics and the amount of effort that workers exert on each case. In nonrecurrent cases and in recurrent cases during the period proceding the repetition of abuse, workers' decisions about how much effort they will devote to each case can be based only on whatever clinical predictive evaluations they perform on a case. Worker-effort levels, therefore, reflect the workers' evaluations of the information they have for each case. If in deciding how much attention to give to each case, workers recognize and rely on those pieces of information that correctly predict repeated abuse, one would expect to find significant correlations between variables that predict the recurrence of abuse and the differing amounts of effort that workers devote to individual cases. However, as was already discussed, only one of the fifteen correlations was found to be significant. Thus, the findings indicated that workers did not make the best possible use of case characteristics to identify potentially serious cases or pay greater attention to these cases before further incidents of abuse took place.

Other findings also suggested that the model could be used to improve the way agencies allocate their resources. For example, cases in which another abusive incident occurred after the first closing (before the workers knew these cases were "serious") had been kept open a significantly shorter time than those cases in which no further abusive incidents were reported (an average of 2.8 months versus an average of 4.8 months, t = 2.68, p = .009). It is possible that there were no recurrences of abuse in the latter cases simply because they were kept open longer. Such a conclusion must be considered because of the confounding of case characteristics and treatment effects that was discussed earlier.

However, this conclusion seems unlikely for several reasons. First, the recurrent-abuse cases that were closed sooner bore the earmarks of more serious cases from the outset. Specifically, the families had more children-an indication of probable higher stress on the abusers (t = 2.76, p = .009); the children spent more time with the adults who abused them-an indication of probable higher stress on the abusers (t = 2.34, p = .022); the mothers were rated lower on parenting skills (t =2.94, p = .004); the mothers were rated lower on the reasonableness of their expectations of the children (t = 2.92, p= .005); and ratings on the family's ability to use resources were lower (t =3.83, p < .005).

In contrast, 75 percent of the cases that had been kept open longer and in which abuse did not recur were predicted not to be recurrent-abuse cases using the aforementioned variables combined in the model. Another reason to doubt that treatment alone prevented further abuse in these cases is that the treatment effect of maintaining cases in open status, while statistically significant, was not very strong (r = -.19, with case characteristics controlled). It is likely that case characteristics, as dis-

TABLE 4. The Predictive Accuracy of the Statistical Model as Applied to the Validation Sample^a

	Actual Group			
Predicted Group Membership	Nonrecurrent Group	Recurrent-Abuse Group	Total	
Predicted nonrecurrent group	16 (69.6 percent) b	7 (30.4 percent) b	23 (59.0 percent)	
Predicted recurrent group	3 (18.8 percent) ^b	13 (81.3 percent) b	16 (41.0 percent)	
Total	19 (48.7 percent)	20 (51.3 percent)	39 (100 percent)	

 $^{^{8}}N = 39$; the total percentage of cases correctly classified = 74.4 percent. 6 Row conditional percentage.

tinguished from treatment effects, played some role in preventing the recurrence of abuse.

In light of these findings, it is reasonable to consider alternative explanations for the early closing of cases in which abuse subsequently recurred. The data suggest that considerations other than the prevention of abuse may influence workers to close cases or to keep them open. For instance, on a five-point Likert-type scale intended to measure a family's willingness to accept help from the agency, the average scores of repeated abusers whose cases were closed earlier were lower than those of clients whose cases were kept open longer and who did not abuse their child again (t = 2.51, p = .016). This finding indicates that repeated abusers may be less cooperative with workers. However, a clear interpretation of this finding cannot be made because of the confounding of case characteristics and treatment effects, which were controlled, but probably not completely, in the construction of the model. Yet the suggestion remains that cooperative clients who are less likely to abuse their children may receive better services from workers.

CONCLUSIONS

If confirmed by future research, the findings of the current study could have special implications for the allocation of resources and the efficient administration of programs in protective service agencies. Nonrecurrent cases of abuse constituted the larger proportion of the physical abuse caseload at the agency being studied, and they received more treatment resources because they remained open much longer. If the model worked accurately, substantial resources could be withdrawn from such cases, with little risk that the recurrence rate would rise. Because of the proportionate size of this group of cases and the substantial treatment resources that could be taken from each one, a sizable pool of resources would become available. These newly available resources could be applied, as appropriate, to the smaller

TABLE 5. The Predictive Accuracy of the Statistical Model as Applied to All Cases in the Study Sample^a

	Actual Group			
Predicted Group Membership	Nonrecurrent Group	Recurrent-Abuse Group	Total	
Predicted nonrecurrent group	49 (75.4 percent) ^b	16 (24.6 percent) b	65 (54.2 percent)	
Predicted recurrent group	16 (29.1 percent) b	39 (70.9 percent) ^b	55 (45.8 percent)	
Total	65 (54.2 percent)	55 (45.8 percent) ^b	120 (100 percent)	

^aThe total percentage of cases correctly classified = 73.3 percent.

bRow conditional percentage.

group of cases in which recurrent abuse was predicted using the model. This shift in the allocation of resources could lead to a reduction in the rate of recurrent abuse among those persons predicted to be repeated abusers, which could, in turn, cause a drop in the overall rate of recurrent abuse among first-time referrals to protective services agencies. This drop could be sizable, especially if the effectiveness of treatment is increased, and could result in reduced program costs. Another benefit would be a feeling of greater certainty by workers that they are making the right decisions about closing cases or keeping them open. And, what is most important, fewer children would be repeated victims of abuse.

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