CHAPTER 17&18:

17.4 COULOMB'S FORCE LAW

17.5 ELECTRIC POTENTIAL ENERGY

18.2 SKILLS FOR ANALYZING PROCESSES INVOLVING $\vec{E}$ FIELDS

18.3 THE V FIELD: ELECTRIC POTENTIAL

18.4 RELATING $\vec{E}$ FIELD AND THE V FIELD

Review

Electric force between two point charges:

$$F_e = \frac{k_e|q_1||q_2|}{r^2}$$

Potential energy of two-point-charge system

$$U_q = \frac{k_e q_1 q_2}{r}$$

Electric field created by a point charge:

$$E_e = \frac{k_e|q_1|}{r^2}$$

Potential from a point charge:

$$V_q = \frac{k_e q_1}{r}$$

on charge $\vec{\ }$ source charge $\vec{\ }$

$$\begin{tikzcd}
F \arrow[d, "\times \vec{r}"'] & E \arrow[l, "\cdot q"'] \arrow[d, "\times \vec{r}"] \\
U & V \arrow[l, "\cdot q"']
\end{tikzcd}$$

$$\vec{F} = q\vec{E}$$

$$U_q = qV$$

$$|F| = \frac{|\Delta U|}{\Delta x} \qquad E = \frac{|\Delta V|}{\Delta x}$$

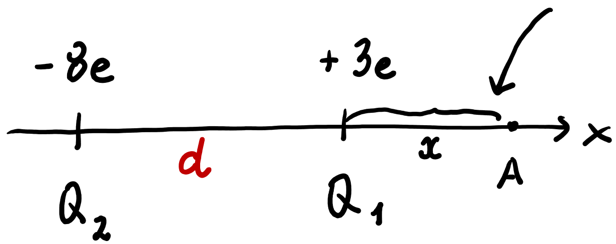
In uniform electric field: $\Delta V = -E\Delta r\cos(\measuredangle(\vec{E},\Delta\vec{r}))$

EXAMPLE A

Two charges, $Q_1 = +3e$ and $Q_2 = -8e$ are placed along the x axis so that Q_1 is at origin and Q_2 is at point $P = (-d, 0)$.
Determine the point(s) along x axis where

a. **Net electric field is zero**
b. Net electric potential is zero.

a)



$$\frac{\cancel{k_e}|Q_1|}{x_1^2} = \frac{\cancel{k_e}|Q_2|}{x_2^2}$$

$$\frac{3\cancel{e}}{x^2} = \frac{8\cancel{e}}{(x+d)^2}$$

$$\frac{3}{x^2} = \frac{8}{(x+d)^2} \rightarrow \frac{\sqrt{3}}{x} = \frac{\sqrt{8}}{x+d}$$

$$\sqrt{3}x + \sqrt{3}d = \sqrt{8}x$$

$$\sqrt{8}x - \sqrt{3}x = \sqrt{3}d$$

$$x = \frac{\sqrt{3}d}{\sqrt{8} - \sqrt{3}} \qquad \frac{\sim 1.7}{\sim 1.1} \sim 1.6d$$

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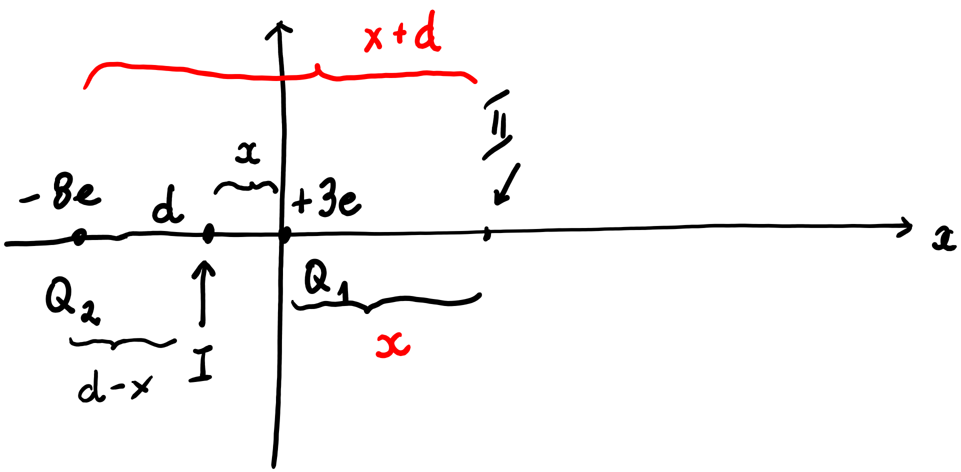


I

$$V_1 + V_2 = 0$$

$$\frac{k_e 3e}{x} - \frac{k_e 8e}{d-x} = 0 \rightarrow \frac{\cancel{k_e} 3\cancel{e}}{x} = \frac{\cancel{k_e} 8\cancel{e}}{d-x}$$

$$\frac{3}{x} = \frac{8}{d-x}$$

$$3d - 3x = 8x \rightarrow 11x = 3d, \quad x = \frac{3}{11} d$$

II

$$V_1 + V_2 = 0$$

$$\frac{\cancel{k_e} 3\cancel{e}}{x} = \frac{\cancel{k_e} 8\cancel{e}}{x+d}$$

$$\frac{3}{x} = \frac{8}{x+d}$$

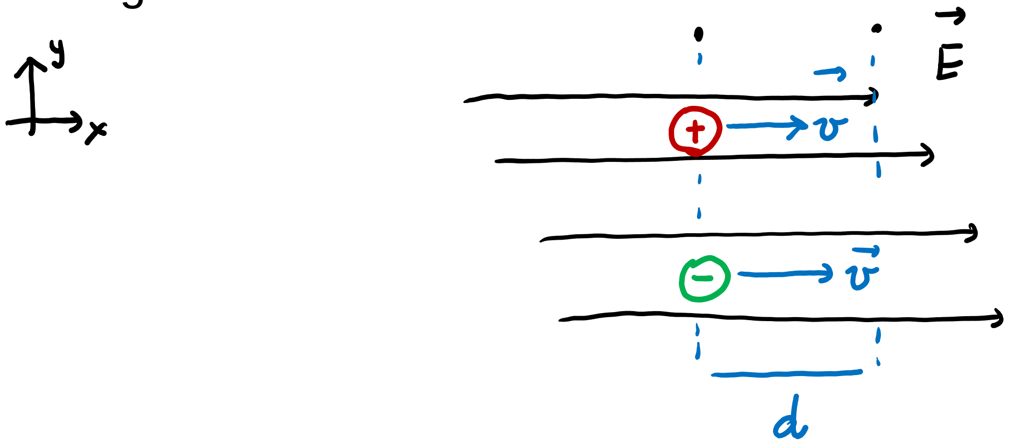
$$3x + 3d = 8x$$

$$5x = 3d$$

$$x = \frac{3}{5} d$$

Example B

Two charges, a proton and the antiproton are launched in the electric field $\vec{E} = 150\ \frac{N}{C}$ in $+x$ direction with the same velocity $\vec{v} = 3.0 \times 10^3\ \frac{\mathrm{m}}{\mathrm{s}}$ in $+x$ direction. Determine the changes in potential energies of a proton $(m_p, +e)$ and an anti-proton $(m_p, -e)$ after they traveled through a distance $d = 0.25\ \mathrm{m}$



a) $\oplus$

$$\Delta V = -E_x d < 0$$

$$\Delta U_+ = q\Delta V = e\Delta V = e(-E_x d) = -eE_x d < 0$$

b) $\ominus$

$$\Delta V = -E_x d < 0$$

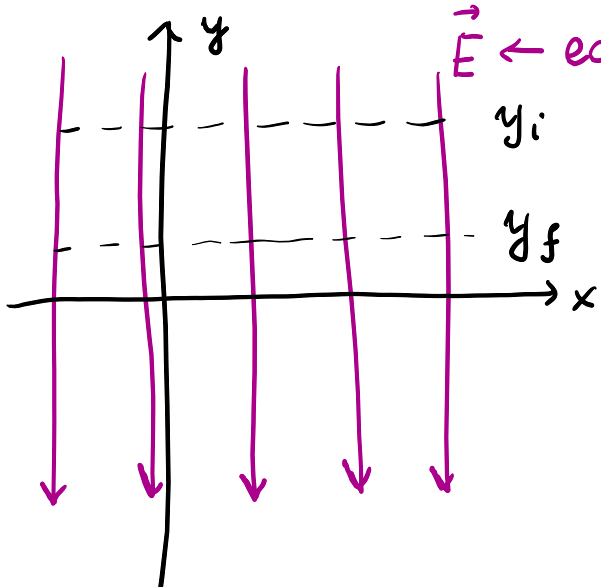
$$\Delta U_- = q\Delta V = (-e)\Delta V = (-e)(-E_x d) = +eE_x d > 0$$

EXAMPLE C

Consider electric field $E_y = -300\,\frac{\mathrm{N}}{\mathrm{C}}$.

a) What is the electric potential difference between points $y_i = +3.0\ \mathrm{m}$ to $y_f = 1.0\ \mathrm{m}$.
b) What is the electric potential difference between points $A = (-1.0, -1.0)\ \mathrm{m}$ and $B = (+2.0, -2.0)\ \mathrm{m}$?
c) If the potential at point A is 400 V, what is the potential at point B?

Always draw



$\vec{E}$ ← equidistant, vertical lines pointing in $-y$ direction

a) 1° method: 2 m move in $E = 300\,\frac{\mathrm{N}}{\mathrm{C}}$ → 600 V difference
moving along $\vec{E}$: $\Delta V = -600\ \mathrm{V}$

2° $|\Delta V| = |E|\,|\Delta y| = 300 \cdot (|1-3|) = 600\mathrm{V}$; along $\vec{E}$ ∴ $\Delta V = -600\ \mathrm{V}$

3° method: $\Delta V = -E_y \Delta y = -(-300)\cdot(1-3) = -(-300)(-2) = -600V$

b. (only 3rd method) $A = (-1.0, -1.0)$
$B = (+2.0, -2.0)$

as E point along y dir only $y_B - y_A$ matters

$$\Delta V_{A\to B} = -E\,\Delta y_{A\to B} = -(-300)\cdot(y_B - y_A) = -(-300)\cdot(-2.0-(-1.0))$$
$$= -(-300)\cdot(-2+1) = +300\cdot(-1) = -300V.$$

c) $V_A = 400V$

$V_B = ?$

1° method: move from $y_A = -1.0\,m$ to $y_B = -2.0\,m$, along $\vec{E}$ field drops 300V $\to V_B = 100V$

3°: $V_B - V_A = \Delta V_{A\to B} = -300V$

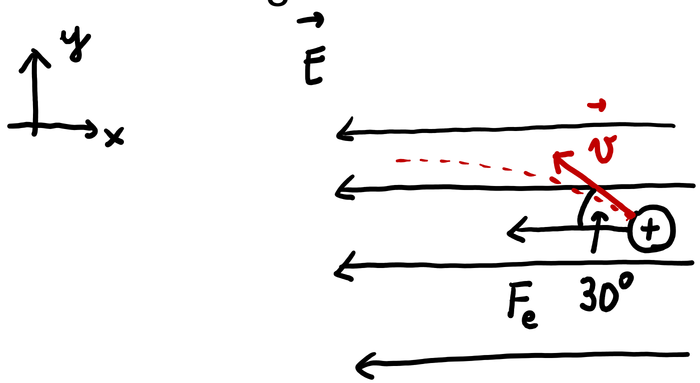
$V_B - 400V = -300V \to V_B = +100V$

We ignore gravity unless specified otherwise $F_g \ll F_e$

EXAMPLE D

A particle of mass $m = 2.0 \times 10^{-6}$ kg and charge $q = 3.0$ mC is launched uniform electric field $E_x = -100$ kN/C so that the its ~~piston~~'s velocity makes an angle of 30 degrees with the direction of the electric field.

a) Sketch the trajectory of the particle
b) What is the acceleration of the particle
c) If the initial speed of the particle is 240 km/s, what is the velocity of the particle after it moves through the field for 1.0 s.



b) $ma = qE$

$a = \dfrac{qE}{m}$ in $-x$ dir

c) $v = 240 \dfrac{km}{s}$

$v_{x_i} = -240 \dfrac{km}{s} \cdot \cos 30$

$v_y = +240 \dfrac{km}{s} \sin 30$

$v_{xf} = V_{xi} + a_x t$

v_y stays constant as there is no accel. in y dir

$v_x = v_{xi} + a_x t$

$q = 3.0 \times 10^{-3} C$

$m = 2.0 \times 10^{-6} kg$

$E_x = -100 \times 10^{3} \frac{N}{C}$

$$a_x = \frac{qE_x}{m}$$

$$a_x = \frac{(3.0 \times 10^{-3} C)(-100 \times 10^{3} \frac{N}{C})}{2.0 \times 10^{-6}\ kg} = -1.5 \times 10^{8} \frac{m}{s^2}$$

$$v_{xi} = -240 \times 10^{3} \frac{m}{s} \cdot \cos 30 = 208 \times 10^{3} \frac{m}{s}$$

$$v_x = -208 \times 10^{3} \frac{m}{s} + (-1.5 \times 10^{8} \frac{m}{s^2}) \cdot 1 s = -1.5 \times 10^{8}\ m/s$$

$$v_y = 120 \times 10^{3} \frac{m}{s}$$

$$v_f \simeq 1.5 \times 10^{8} \frac{m}{s}$$

. it won't accelerate for a long time at that rate

. at this speed mass starts becoming relativistic and acceleration decreases