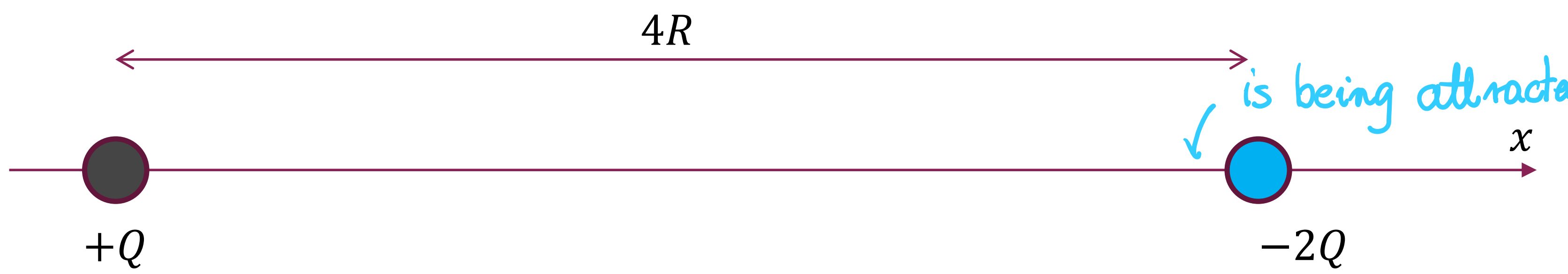
LEC 16: ELECTROSTATICS. COULOMB'S LAW
LEC 17: **ELECTRIC POTENTIAL ENERGY**
LEC 18: ELECTROSTATICS – FORCE AND ENERGY. APPLICATIONS

CHAPTER 17: ELECTRIC CHARGE, FORCE, AND ENERGY

17.1: ELECTROSTATIC INTERACTIONS
17.2: EXPLANATIONS FOR ELECTROSTATIC INTERACTIONS
17.3: CONDUCTORS AND INSULATORS (DIELECTRICS)
17.4: COULOMB'S FORCE LAW
17.5: ELECTRIC POTENTIAL ENERGY
17.6: SKILLS FOR ANALYZING PROCESSES INVOLVING ELECTRIC CHARGES
17.7: CHARGE SEPARATIONS AND PHOTOCOPYING

QUESTION



Two charges, $+Q$ and $-2Q$ are separated by distance $4R$.

What is the electric force exerted by charge $+Q$ on charge $-\cancel{2}Q$?

$$F = \frac{k_e|+Q||-2Q|}{(4R)^2}$$

$\hat{\imath}$ is a **unit vector** defined to point in $+x$ direction

$$A)\ \vec{F}_{+on\,-} = \frac{k_e Q^2}{2R^2}\hat{\imath}$$

$$B)\ \vec{F}_{+on\,-} = -\frac{k_e Q^2}{2R^2}\hat{\imath}$$

$$C)\ \vec{F}_{+on\,-} = \frac{k_e Q^2}{8R^2}\hat{\imath}$$

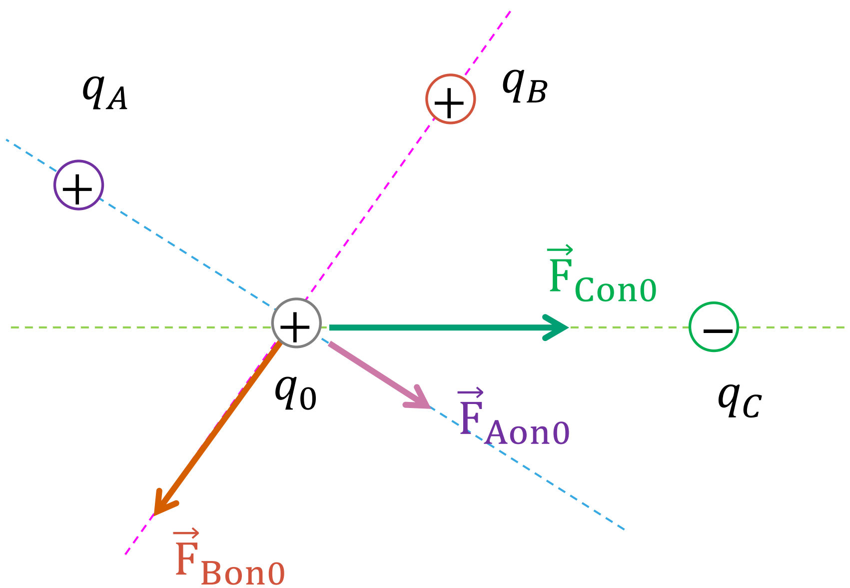
$$D)\ \vec{F}_{+on\,-} = -\frac{k_e Q^2}{8R^2}\hat{\imath}$$

REVIEW

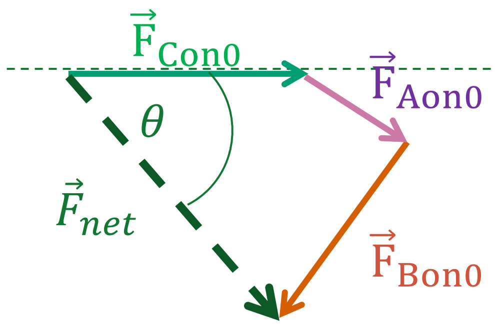
Coulomb's Force

$$F_e = \frac{k_e(|q_1||q_2|)}{r_{12}^2}$$

The direction of **each** force is deduced by looking at the geometry of the charge arrangement.



Resultant force:



$$\vec{F}_{net} = \vec{F}_{Aon0} + \vec{F}_{Bon0} + \vec{F}_{Con0}$$

$$F_{net,x} = F_{Aon0,x} + F_{Bon0,x} + F_{Con0,x}$$
$$F_{net,y} = F_{Aon0,y} + F_{Bon0,y} + F_{Con0,y}$$

$$F_{net} = \sqrt{F_{net,x}^2 + F_{net,y}^2}$$

$$\tan\theta = \frac{|F_{net,y}|}{|F_{net,x}|}$$

REVIEW

Work

$$W = \vec{F} \cdot \Delta\vec{r} = F_x r_x + F_y r_y + F_z r_z$$
$$W = F\Delta r \cos\theta$$

Work – Energy Theorem:

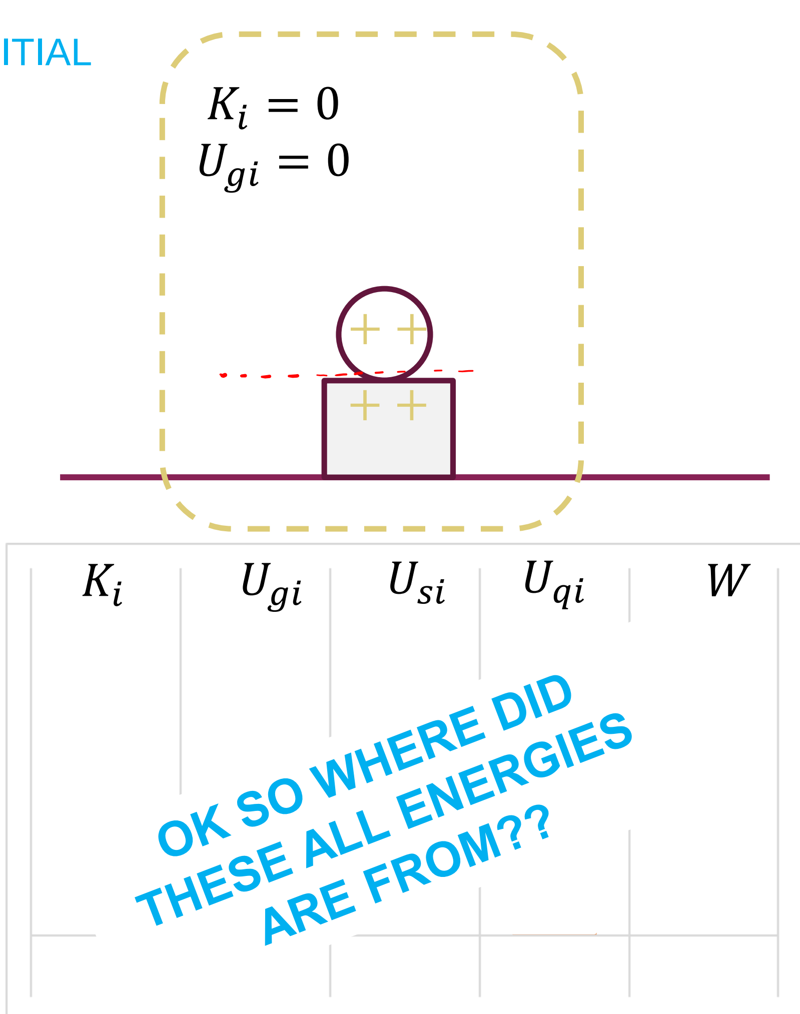
$$K_i + U_{gi} + U_{si} + U_{qi} + W = K_f + U_{gf} + U_{sf} + U_{qf} + \Delta U_{int}$$

$$W = \Delta E_{system}$$

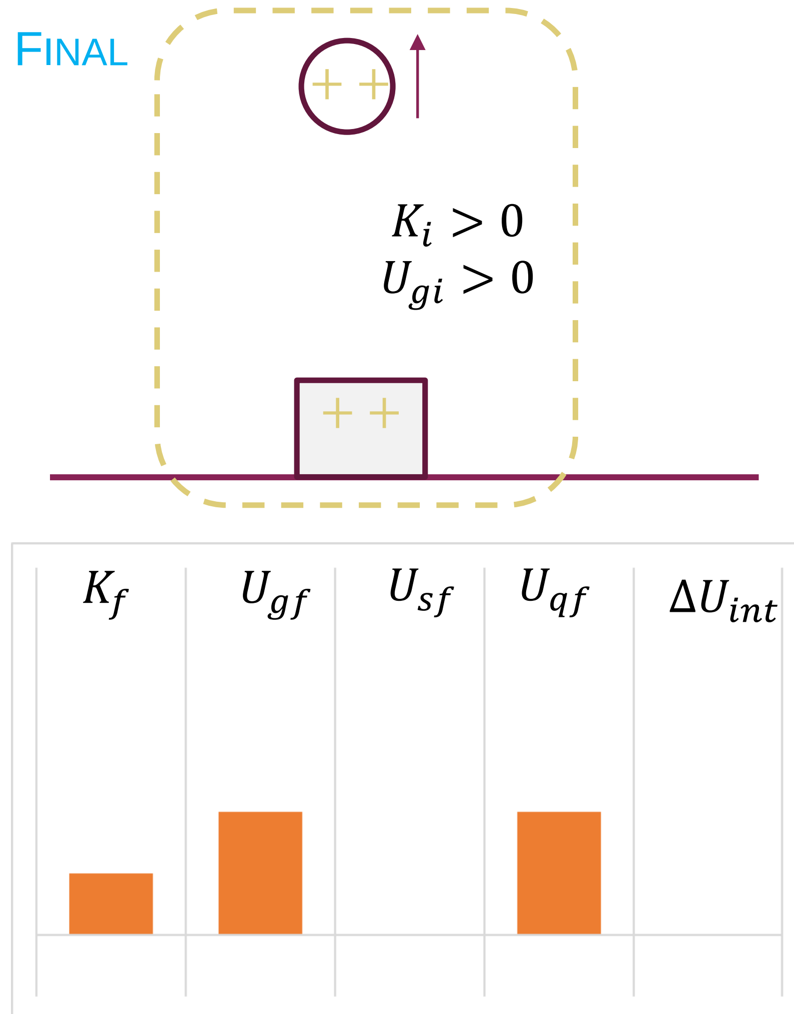
Electric Potential Energy – A qualitative Analysis

Let's look at the electric potential energy of two charged objects:

Initial



Final



ELECTRIC POTENTIAL ENERGY – A QUALITATIVE ANALYSIS

Let’s look at the electric potential energy of two charged objects:

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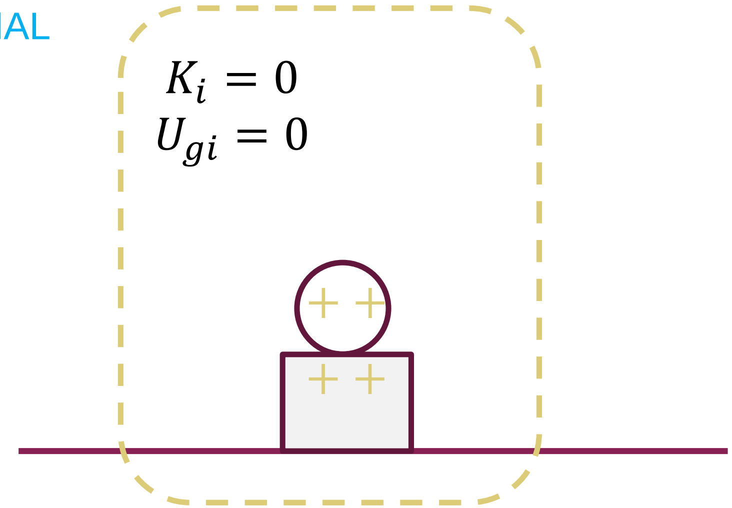
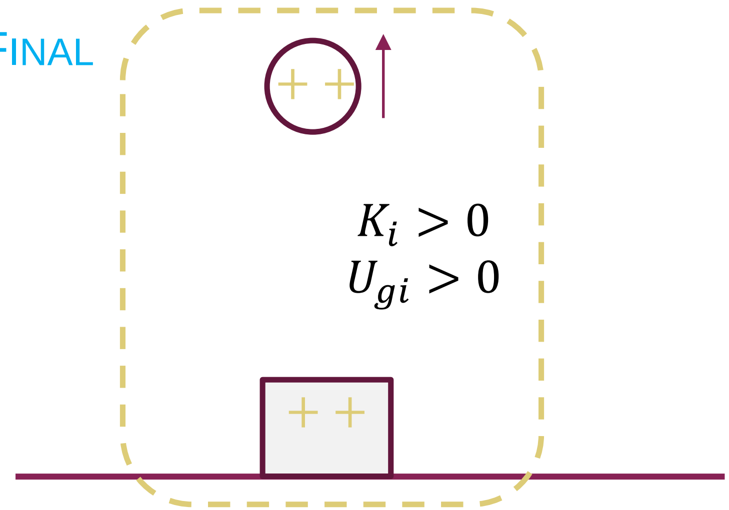
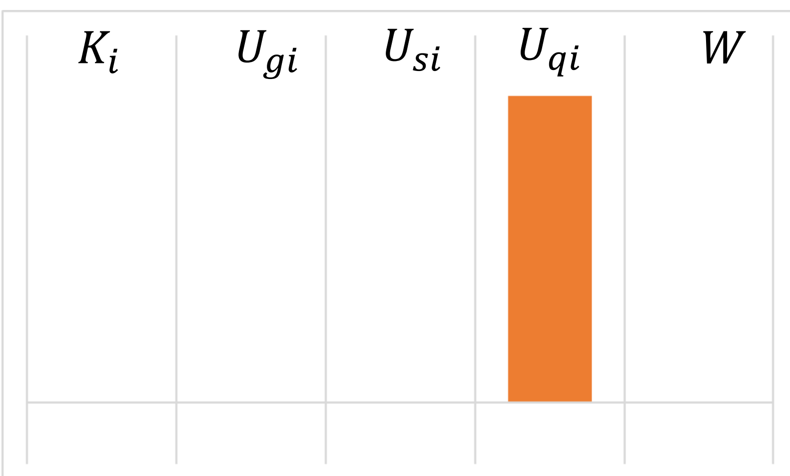
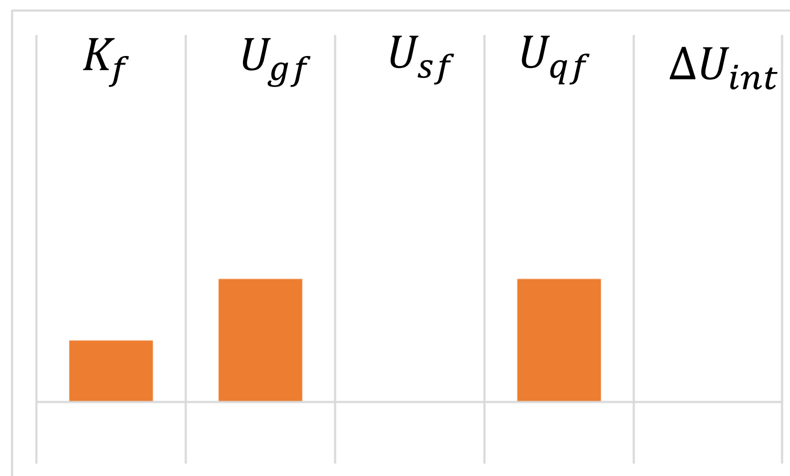


FINAL



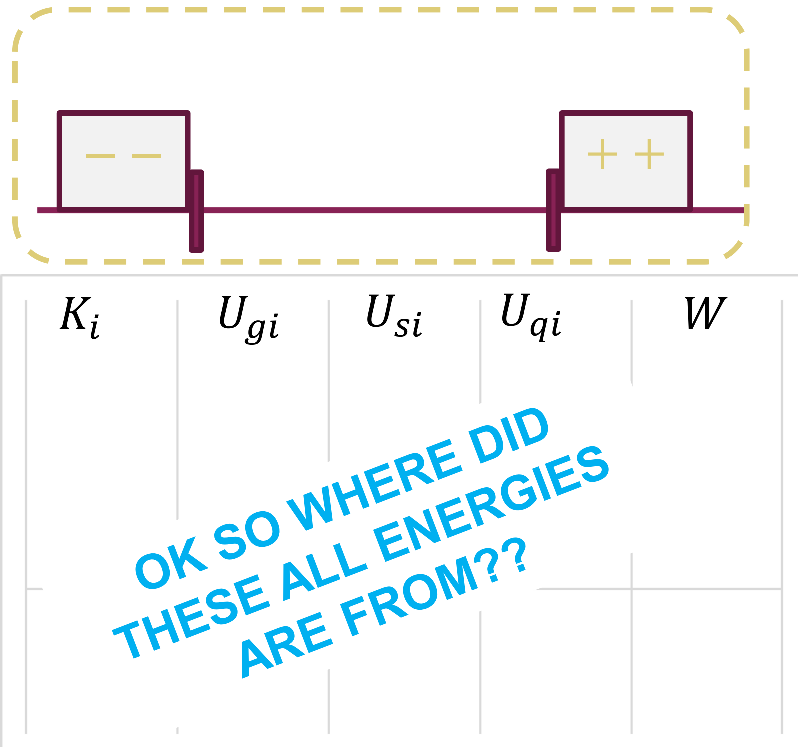




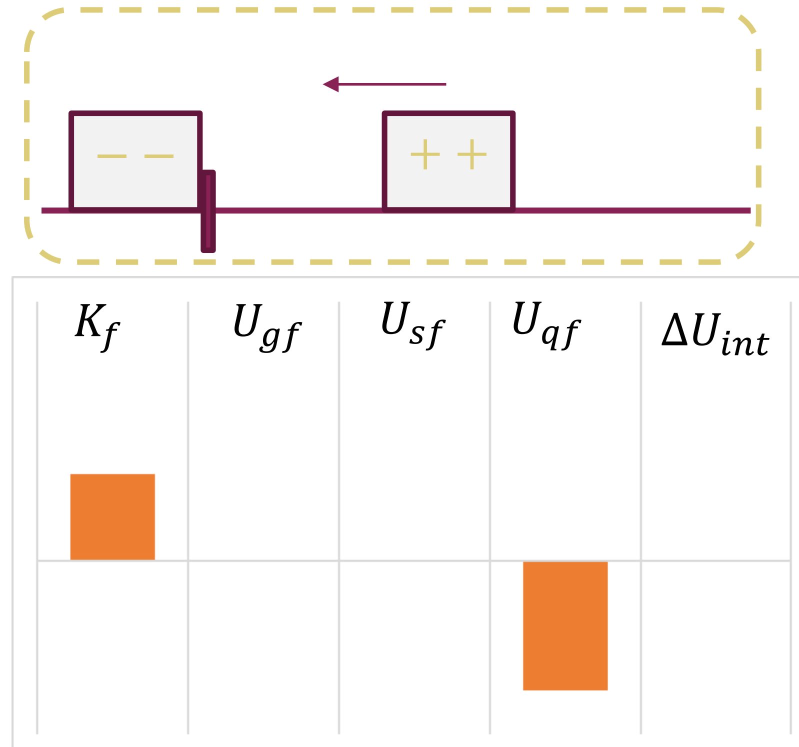
ELECTRIC POTENTIAL ENERGY – A QUALITATIVE ANALYSIS

Let’s look at electric potential energy of two oppositely charged objects

INITIAL



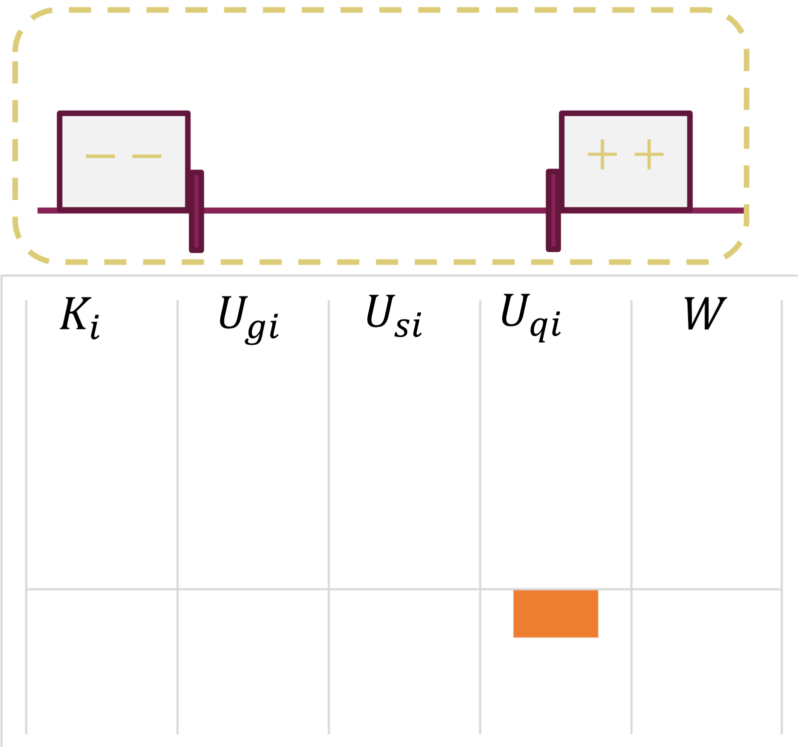
FINAL



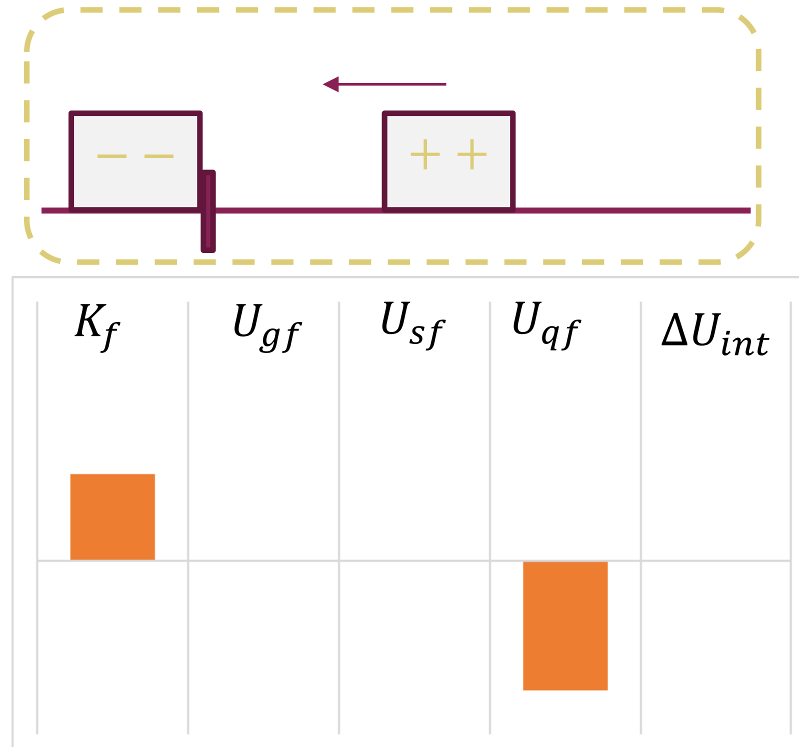
Electric Potential Energy – A qualitative Analysis

Let's look at electric potential energy of two oppositely charged objects

Initial



Final



Electric Potential Energy – A Quantitative Analysis

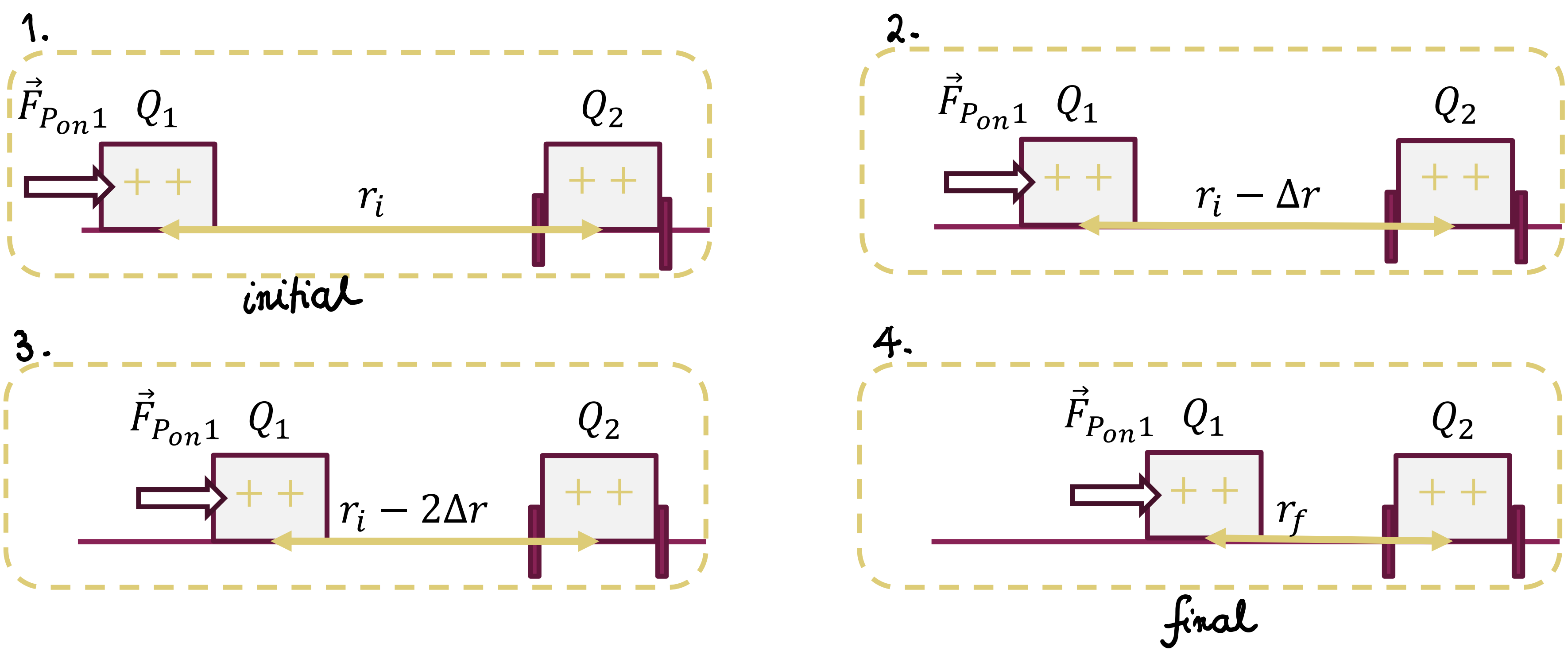
Generalized work energy theorem (assuming no heating, $Q = 0$)

$$W = \Delta E_{SYSTEM}$$

WORK DONE ON THE SYSTEM BY THE OBJECTS IN THE ENVIRONMENT

CHANGE IN THE SYSTEM'S ENERGY

Consider two charges of the same sign that you are trying to bring together:



Force between the charges:
(magnitude)

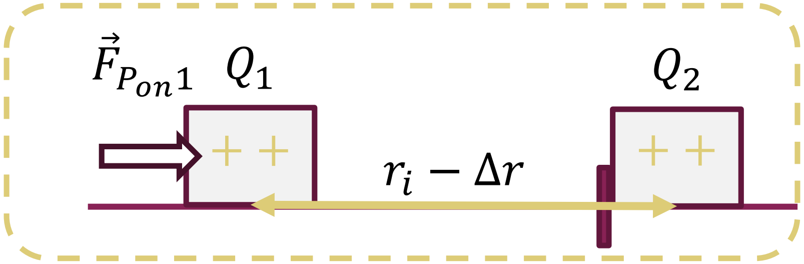
$$F_{q_2\ on\ q_1} = \frac{k_e(|Q_1||Q_2|)}{r^2}$$

If we wish to prevent the first object from flying away we would want to match the electric force with the external force (same way you prevent object from falling by matching its weight).

$$F_{P\ on\ 1} = \frac{k_e(|Q_1||Q_2|)}{r^2}$$

"pusher"

If we match the force and actually bring the object closer together (by, for example, originally giving it a small speed that we then maintain constant), displacing the object by displacement Δr while applying the force $F_{P\ on\ 1}$ would result in person doing work W_1.



$$W_1 = F_{P\ on\ 1}\Delta r = \frac{k_e(Q_1 Q_2)}{r_i^2}\Delta r$$

if $\vec{F}$ & $\vec{\Delta r}$ are not ||

$$W_1 = F_{P\ on\ 1}\Delta r\cos\theta = \frac{k_e(|Q_1||Q_2|)}{r_i^2}|\Delta r|\cos 0$$

really $W = \vec{F}\cdot\vec{\Delta r} = W\Delta r\cos\theta$

The total work equals to the work done in each of those small steps

$$\frac{k_e Q_1 Q_2}{r_i} \cdot \Delta r$$

$$W_{person} = W_1 + W_2 + \cdots + W_n$$

$$\frac{k_e Q_1 Q_2}{r_i - \Delta r} \Delta r$$

$$\boxed{W_{person} = \frac{k_e Q_1 Q_2}{r_f} - \frac{k_e Q_1 Q_2}{r_i}}$$

$$W_{person} = \underline{\Delta K} + \underline{\Delta U_g} + \underline{\Delta U_s} + \boxed{\Delta U_q} + \underline{\Delta U_{int}}$$

$\Delta K = 0$, v is constant

$\Delta U_g = 0$, height is constant

$\Delta U_s = 0$, no springs

$\Delta U_{int} = 0$, no heat / friction

$$W_{person} = \Delta U_q$$

$$\Delta U_q = U_f - U_i = \frac{k_e Q_1 Q_2}{r_f} - \frac{k_e Q_1 Q_2}{r_i}$$

final

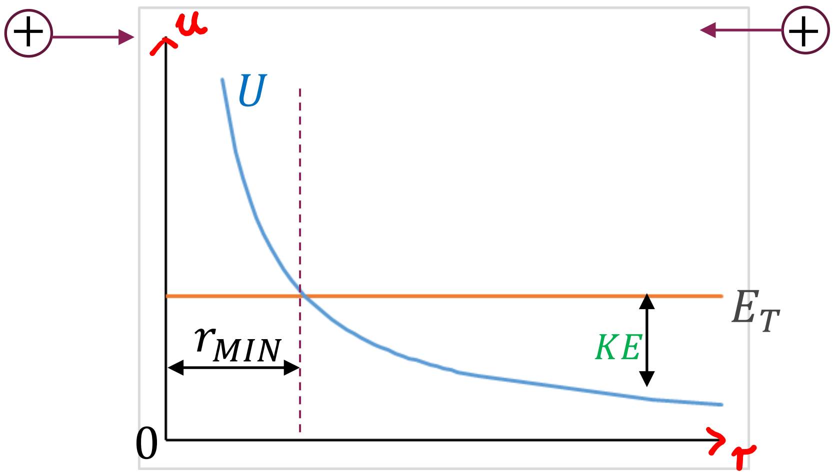
initial

$$U = \frac{k_e Q_1 Q_2}{r}$$

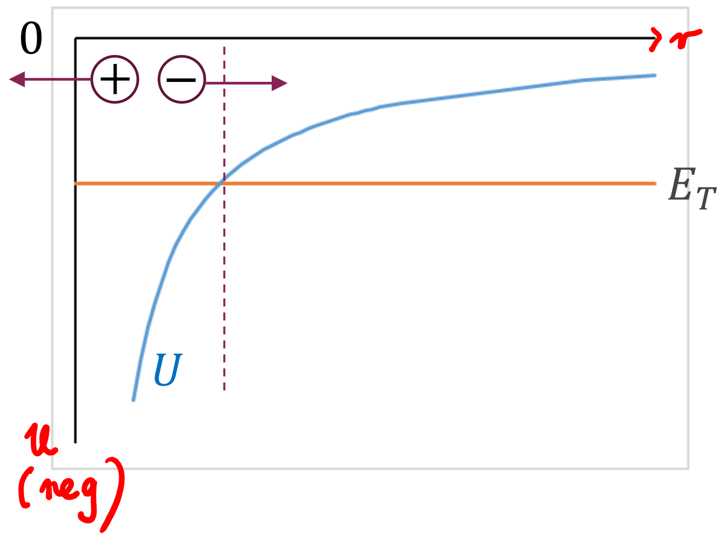
POTENTIAL ENERGY DIAGRAMS

$$U = \frac{k_e Q_1 Q_2}{r} \rightarrow \begin{cases} alike\ charges: U > 0 \\ opposite\ charges: U < 0 \end{cases}$$

Potential energy of two positive charges.



Potential energy of two charges with opposite signs.



A positive charge is held stationary at the origin of a coordinate system. A negative charge is brought **from infinity** towards the positive charge by being moved by an external force **at a constant speed** along **positive *y* axis.**
Which of the following statements is true?

$U_i = 0$

$U_f = ke \dfrac{(\text{positive})(\text{neg})}{\text{distance}\downarrow}$

more & more NEGATIVE number

1. The potential energy of the system increased.

2. The potential energy of the system decreased

3. The total energy of the system has not changed

4. The kinetic energy of the system increased

5. The kinetic energy of the system decreased

constant speed

This LC question is designed to be a bit tricky → you may need to do some reasoning for each answer

Which of the following is **false** about electric potential energy of two charges?

A) Electric potential energy is positive for alike charges and negative for opposite charges.

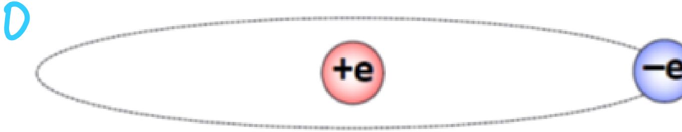
B) Electric potential energy always gets lower with the increasing distance between the charges.

C) If charges were separated by distance $x \to \infty$, the electric potential energy $U \to 0$

D) Change in electric potential energy between two charges is equal to the negative work done by the electric force.

Example 17E

pico = 10^{-12}



The Bohr model of the hydrogen atom consists of an electron orbiting a proton with radius $r_B = 52.9$ pm and an orbital period $\tau = 1.52 \times 10^{-7}$ ns.

nano = 1×10^{-9}

$$k_e = 8.99 \times 10^9 \frac{\mathrm{Nm}^2}{\mathrm{C}^2}$$

$$1\ \mathrm{eV} = 1.6 \times 10^{-19}\mathrm{J}$$

What is the electric potential energy of a hydrogen atom in the model in units of eV?

nucleus

$$U = \frac{k_e (e)(-e)}{r_B} = -4.35 \times 10^{-18}\mathrm{J}$$

electron

$$U = \frac{k_e q_1 q_2}{r}$$

$$1eV = 1.6 \times 10^{-19}\mathrm{J} \rightarrow \mathrm{U} = -27.2\ \mathrm{eV}$$

What is the kinetic energy of the electron in eV?

radius

$$K = \frac{1}{2} m v^2 = \frac{1}{2}(9.11 \times 10^{-31})(2.18 \times 10^6)^2$$

$$\boxed{v = \frac{2\pi r_B}{\tau}}$$

period

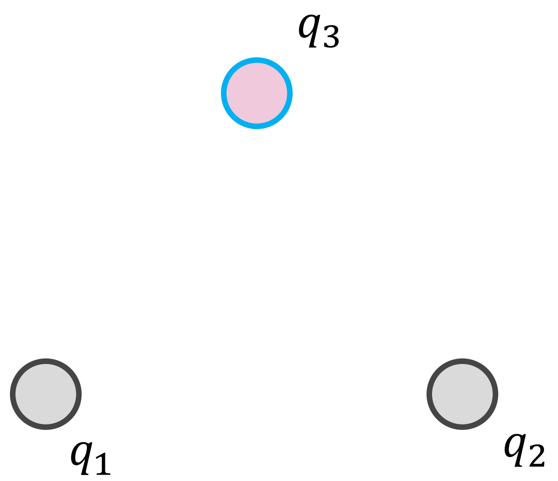
$$K = +13.5\ eV$$

What is the binding energy of atomic hydrogen (H)?

$E_{Total} = K + U \approx -13.6\ eV$ actual value

EXAMPLE 17F

Three point charges, $q_1 = 1.0\ \mu\text{C}$, $q_2 = -2.0\ \mu\text{C}$ and $q_3 = 3.0\ \mu\text{C}$ are assembled into an equilateral triangle as shown in the picture.



Determine the potential energy of this arrangement of charges.
Charge separation : $d = 5.0\ \text{cm}$

If more than two charges are present, the potential energy is the sum of potential energies due to all pairs of charges.

$$U_{total} = U_{12} + U_{13} + U_{23}$$

$$U_{total} = U_{12} + U_{13} + U_{23} = \frac{k_e q_1 q_2}{d} + \frac{k_e q_1 q_3}{d} + \frac{k_e q_2 q_3}{d}$$

$$U_{total} = \frac{k_e}{d}((1.0 \times 10^{-6})(-2.0 \times 10^{-6}) + (1.0 \times 10^{-6})(3.0 \times 10^{-6}) + (-2.0 \times 10^{-6})(3.0 \times 10^{-6}))$$
$$U_{total} = 0.899\ J$$