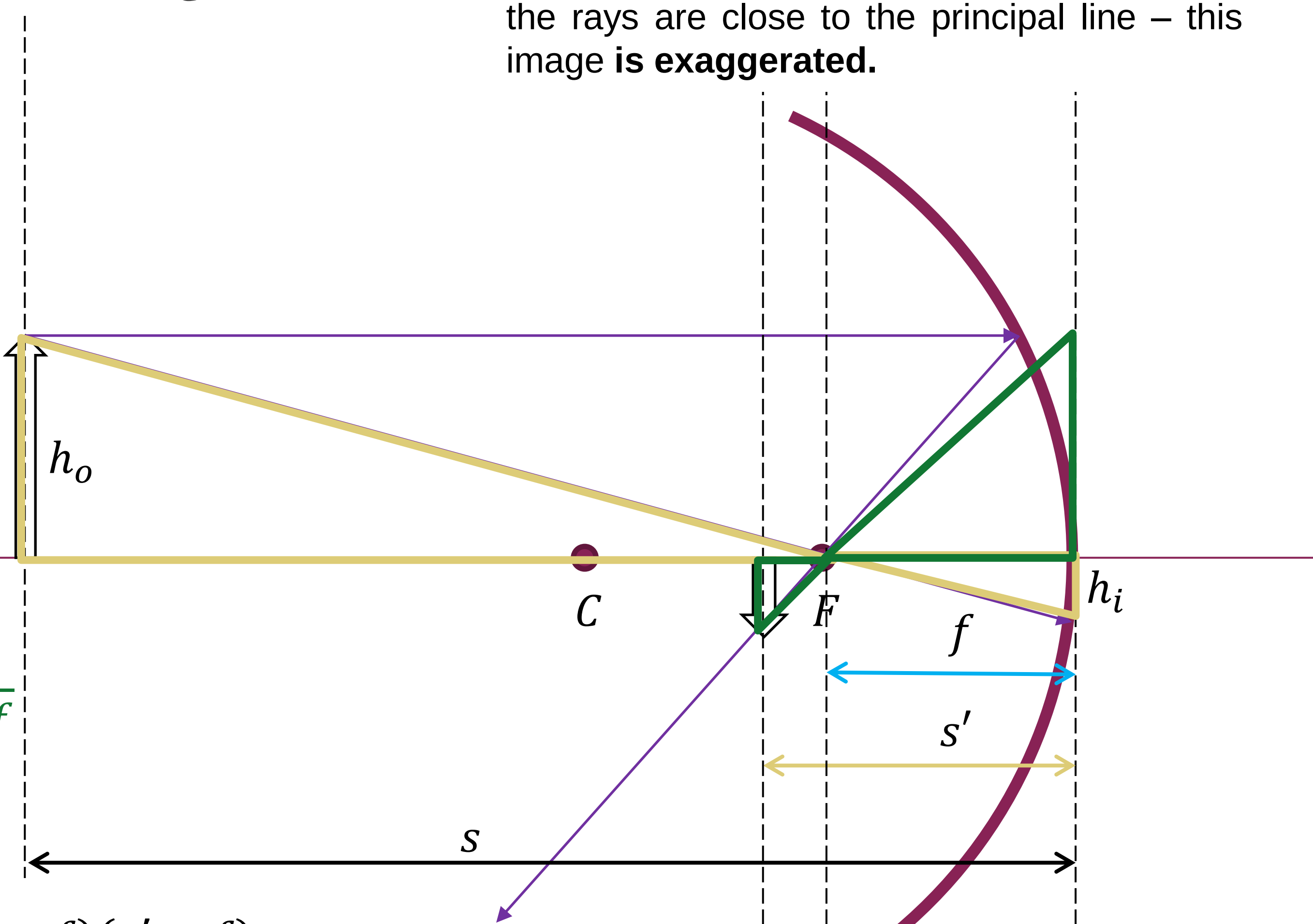
LEC 24: PLANE AND CURVED MIRRORS. MIRROR EQUATION
LEC 25: QUALITATIVE ANALYSIS OF LENSES. THIN LENS EQUATUION
LEC 26: SINGLE LENS OPTICAL SYSTEMS. MAGNIFYING GLASSES
LEC 27: MIRRORS AND LENSES - APPLICATIONS

CHAPTER 23: MIRRORS AND LENSES

23.3 The Mirror Equation

Remember, we assume the curvature is large comparing to the size of the mirror/object and the rays are close to the principal line – this image **is exaggerated.**

$$\frac{h_o}{s-f}=\frac{h_i}{f}\rightarrow\frac{h_o}{h_i}=\frac{s-f}{f}$$



$$\frac{h_o}{f}=\frac{h_i}{s'-f}\rightarrow\frac{h_o}{h_i}=\frac{f}{s'-f}$$

$$\frac{s-f}{f}=\frac{f}{s'-f}\rightarrow f^2=(s-f)(s'-f)$$

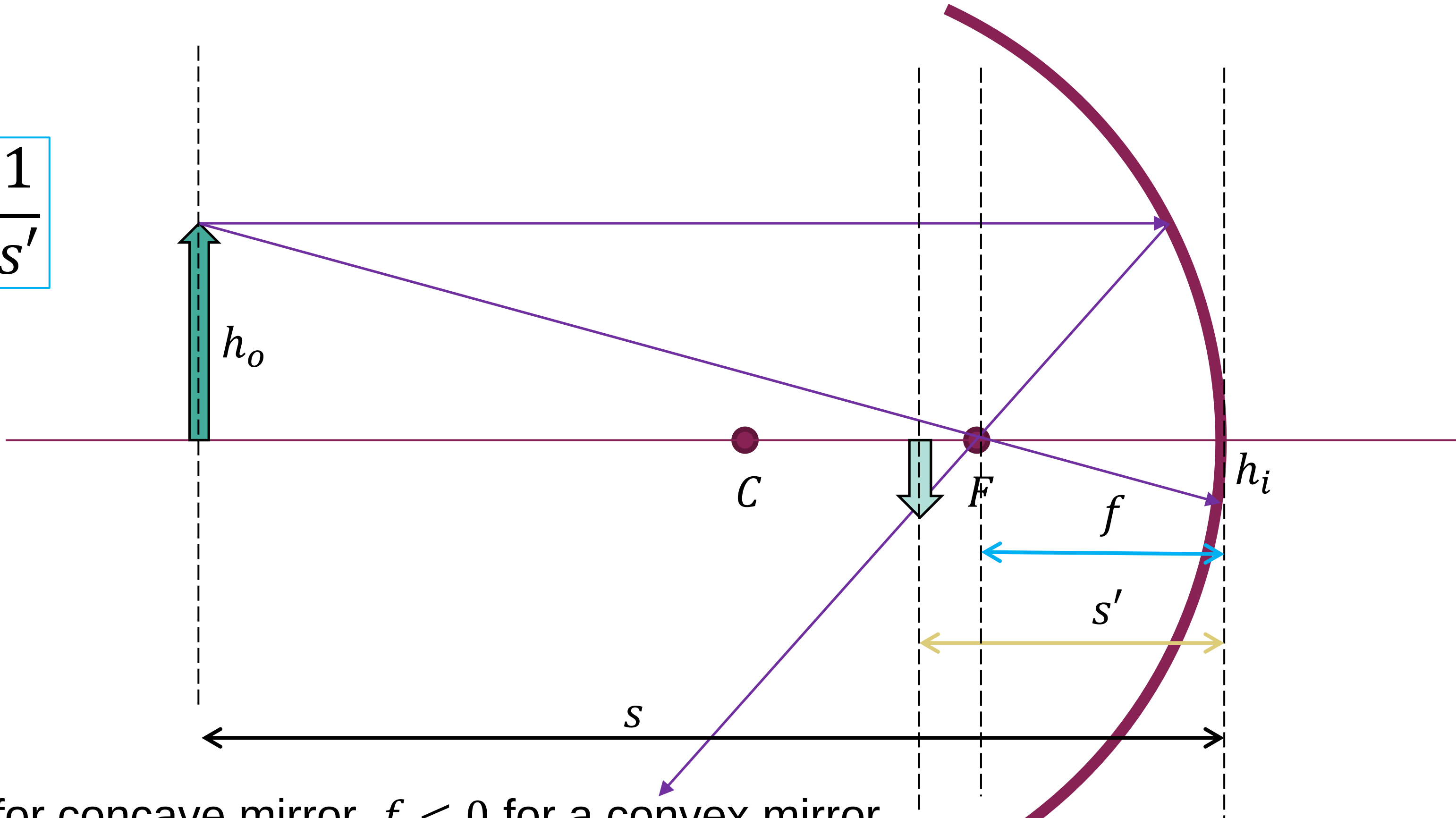
$$f^2=ss'-s'f-sf+f^2\rightarrow ss'-s'f-sf=0\rightarrow ss'=s'f+sf$$

$$\boxed{\frac{1}{f}=\frac{1}{s'}+\frac{1}{s}}$$

23.3 THE MIRROR EQUATION

Remember, we assume the curvature is large comparing to the size of the mirror/object and the rays are close to the principal line – this image **is exaggerated.**

$$\boxed{\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}}$$



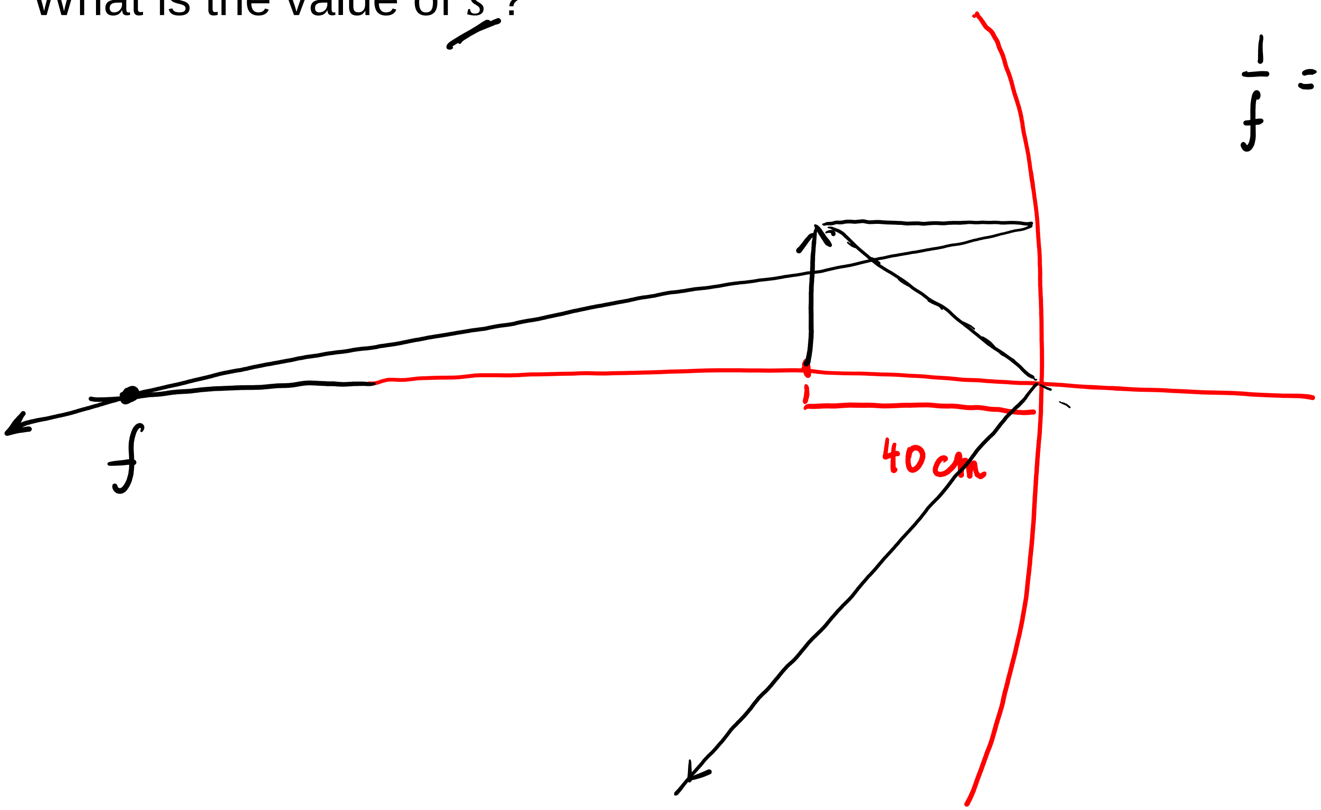
$f > 0$ for concave mirror, $f < 0$ for a convex mirror

$s > 0$

$s' > 0$ for real images, $s' < 0$ for virtual images

Learning Catalytics

You have a concave mirror with the focal length of 2.0 m.
You place an object 40 cm away from the mirror.
What is the value of s'?

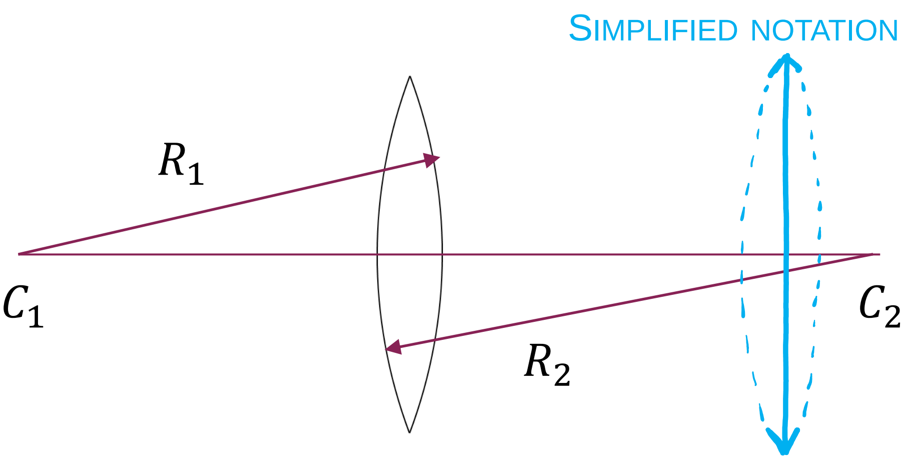


$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

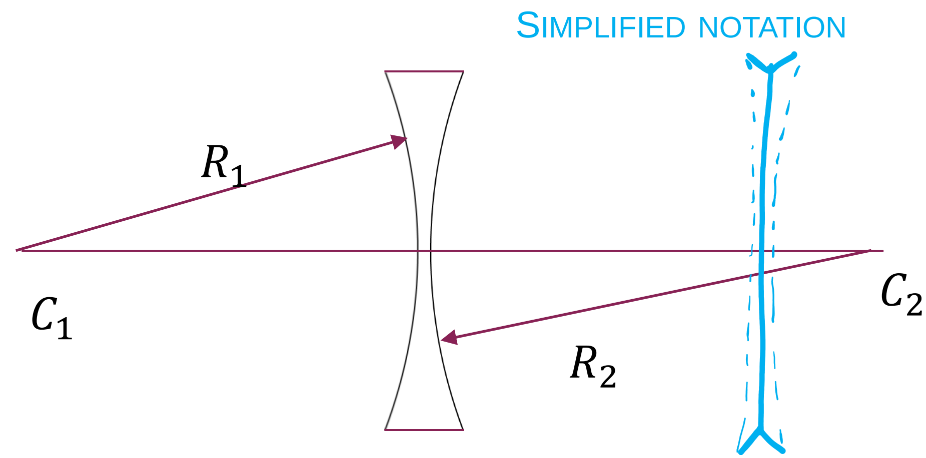
$$\frac{1}{2} = \frac{1}{0.4} + \frac{1}{s'}$$

23.4 Qualitative Analysis of Lenses

Lenses are made from spherical surfaces, which – in principle, could have different radii.



Convex Lens



Concave Lens

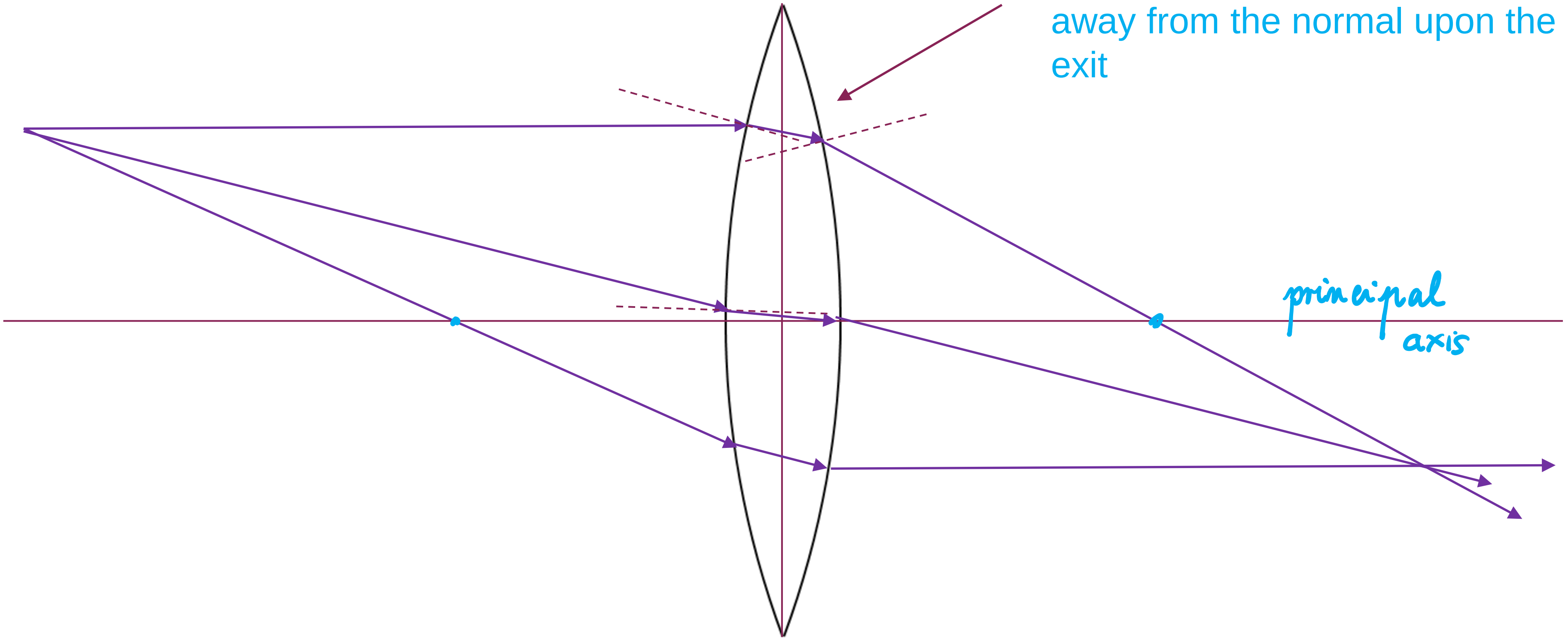
Observations

1. **Light refracts upon entering the lens and upon leaving it.**
2. The focal point of a lens depends **on the curvature** of the lens and **on the material it is made from**.

23.4 QUALITATIVE ANALYSIS OF LENSES – CONVEX LENS

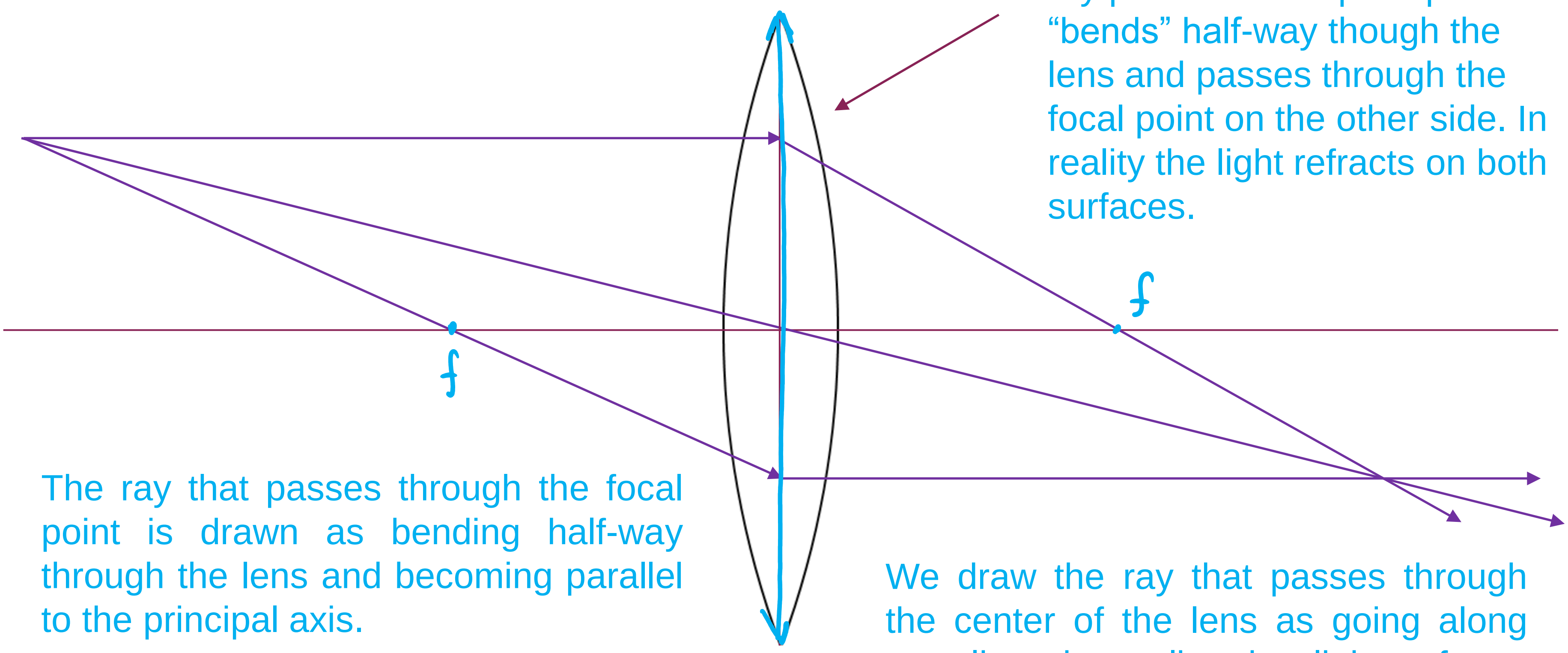
When drawing the rays passing through the lens we will use a simplification:

In reality: the ray refracts toward the normal and then away from the normal upon the exit



23.4 Qualitative Analysis of Lenses – convex Lens

When drawing the rays passing through the lens we will use a simplification:



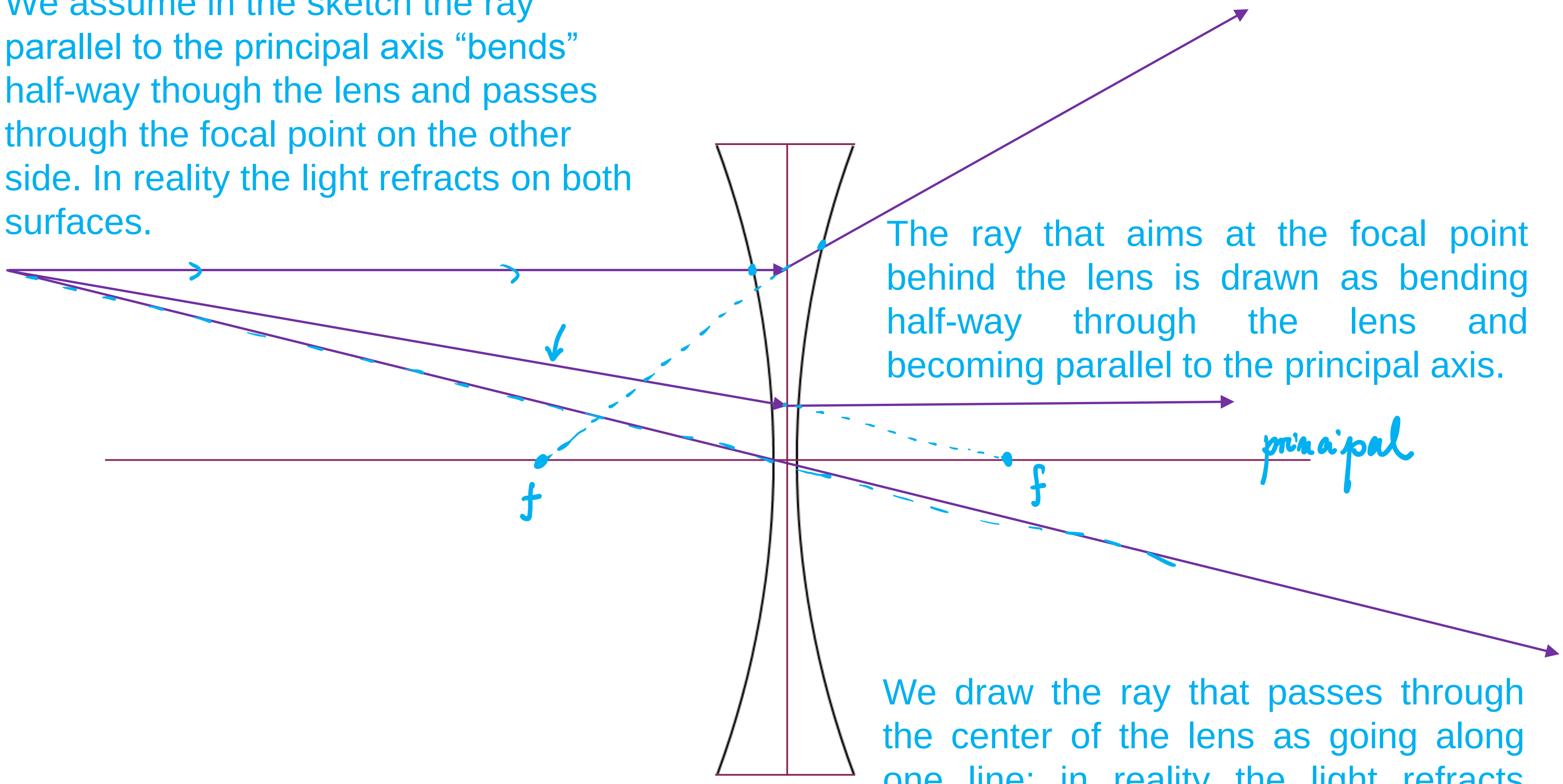
We assume in the sketch the ray parallel to the principal axis "bends" half-way though the lens and passes through the focal point on the other side. In reality the light refracts on both surfaces.

The ray that passes through the focal point is drawn as bending half-way through the lens and becoming parallel to the principal axis.

We draw the ray that passes through the center of the lens as going along one line; in reality the light refracts towards the normal upon entering the lens and away from it upon leaving it.

23.4 Qualitative Analysis of Lenses – concave Lens

We assume in the sketch the ray parallel to the principal axis "bends" half-way though the lens and passes through the focal point on the other side. In reality the light refracts on both surfaces.

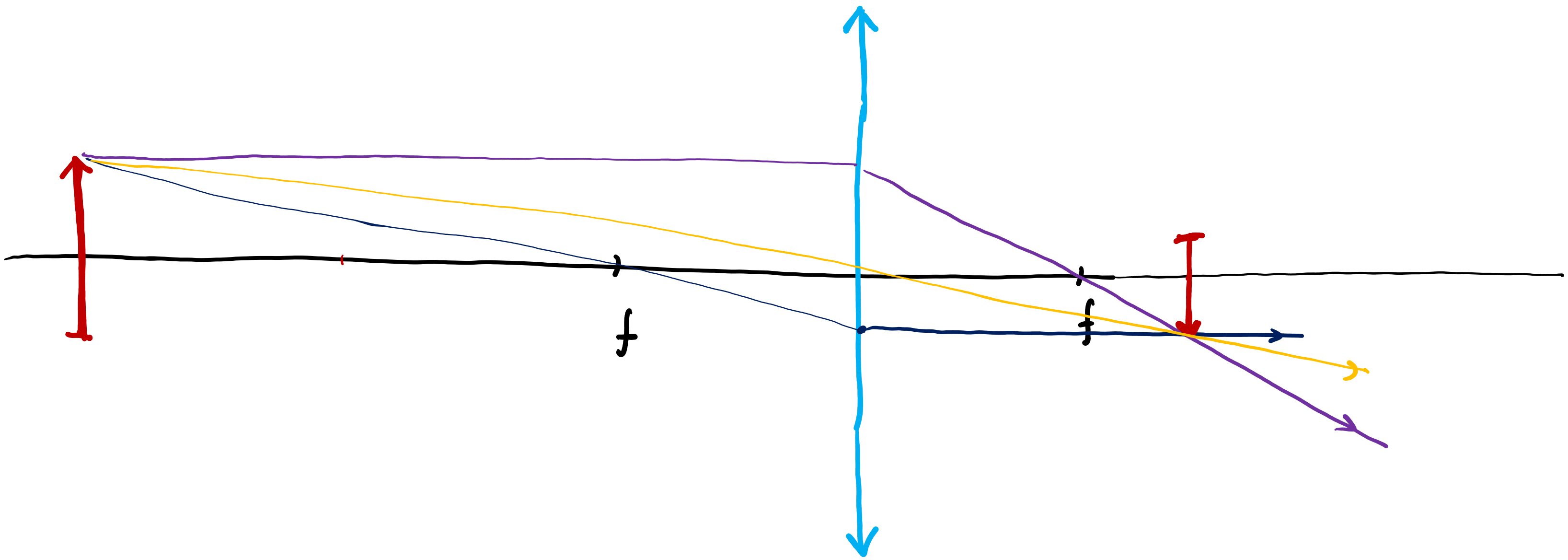


The ray that aims at the focal point behind the lens is drawn as bending half-way through the lens and becoming parallel to the principal axis.

We draw the ray that passes through the center of the lens as going along one line; in reality the light refracts towards the normal upon entering the lens and away from it upon leaving it.

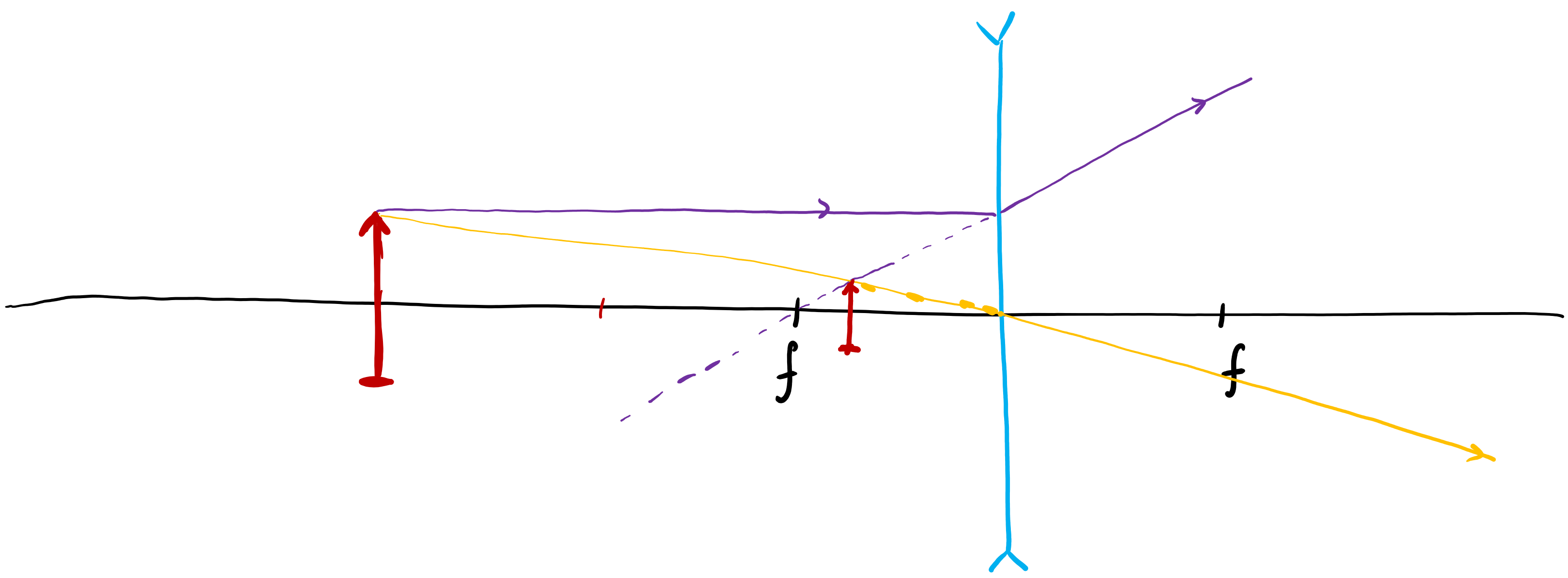
EXAMPLE 23A

Imagine you have an object, located half way above and half way below the principal axis, a **convex** lens and a screen. You place an object at a position $s = 3f$ from the lens. Construct the image you would observe when looking through the lens.



EXAMPLE 23B

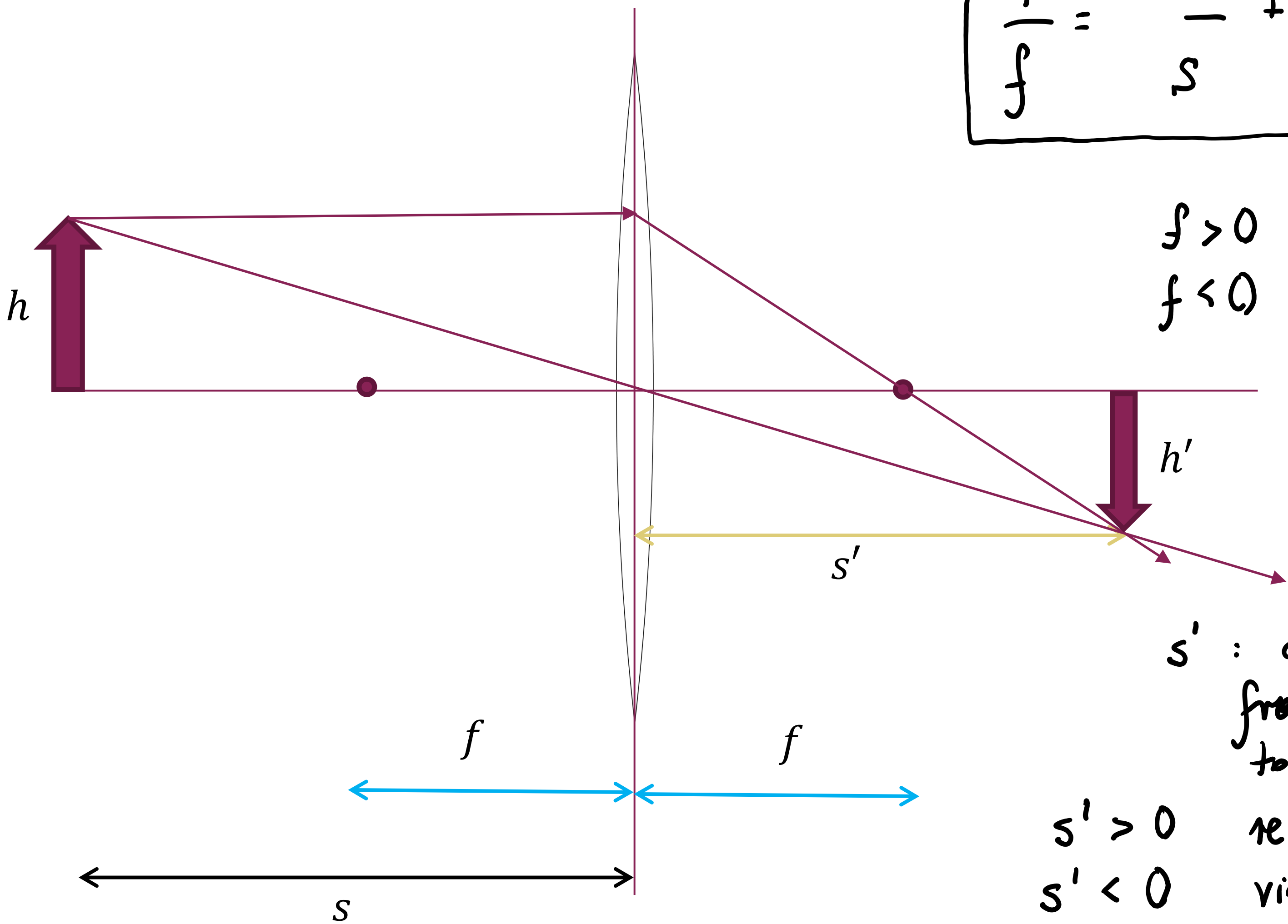
Imagine you have an object, located half way above and half way below the principal axis, a **concave** lens. You place an object at a position $s = 3f$ from the lens. Construct the image you would observe when looking through the lens.



23.5 Thin Lens Equation

Again, we will use a ray diagram to develop a relationship between the location of an object, the image and the focal distance of a lens.

$$\boxed{\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}}$$



$f > 0$ convex

$f < 0$ concave

s - distance from the object to the lens

s' : distance from the image to the lens

$s' > 0$ real

$s' < 0$ virtual

Linear Magnification

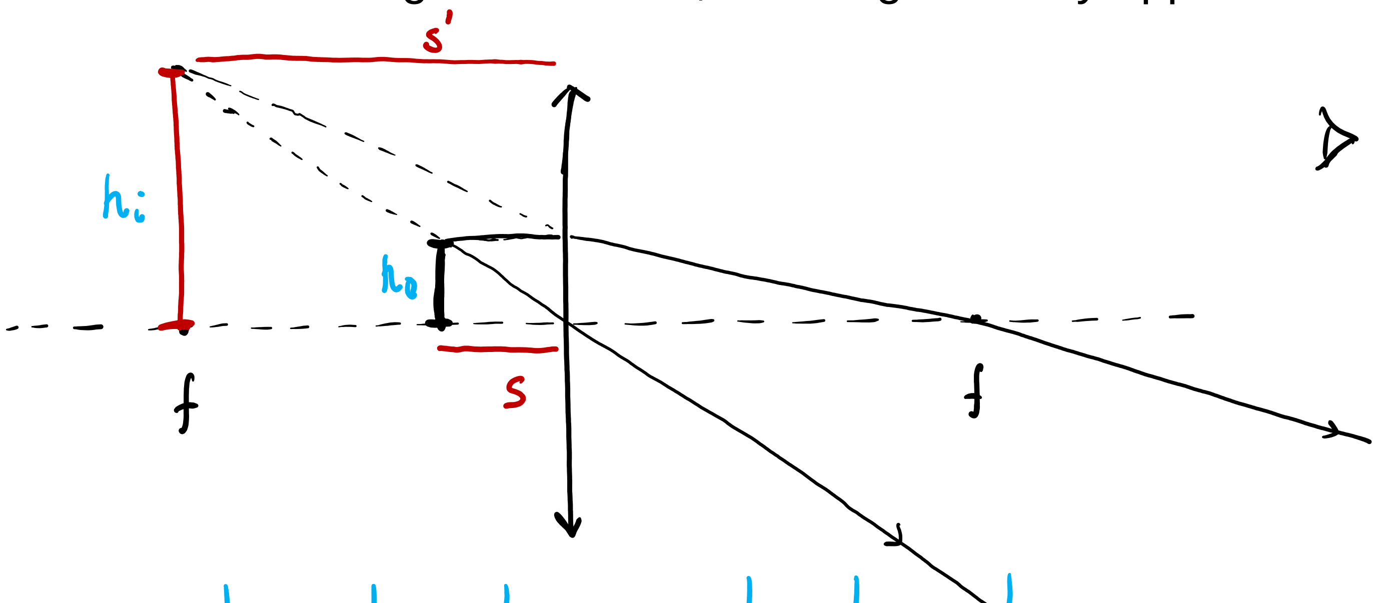
Both lenses and mirrors can produce images of different sizes than the original objects. To characterize the relative size of object and image we use **linear magnification**

$$linear\ magnification = \frac{image\ height}{object\ height} = m = \frac{h'}{h} = -\frac{s'}{s}$$

EXAMPLE 23C – THROUGH THE LOOKING GLASS

Curiouser and curiouser about the writing on an old coin you take a convex lens of focal length $f = +12.0\ \text{cm}$ and bring it close to the letters. Holding a coin $s = 3.0\ \text{cm}$ away from the glass.

If the letters have a height of 2.0 mm, how large do they appear?



$$-\frac{s'}{s} = m = \frac{h_i}{h_o}$$

$$\frac{1}{f} = \frac{1}{s'} + \frac{1}{s} \rightarrow \frac{1}{f} - \frac{1}{s} = \frac{1}{s'}$$

$$\frac{1}{12} - \frac{1}{3} = \frac{1}{s'} \rightarrow \frac{1}{12} - \frac{4}{12} = \frac{1}{s'} \rightarrow \frac{1}{s'} = -\frac{3}{12}$$

$$s' = -4.0\ \text{cm}$$

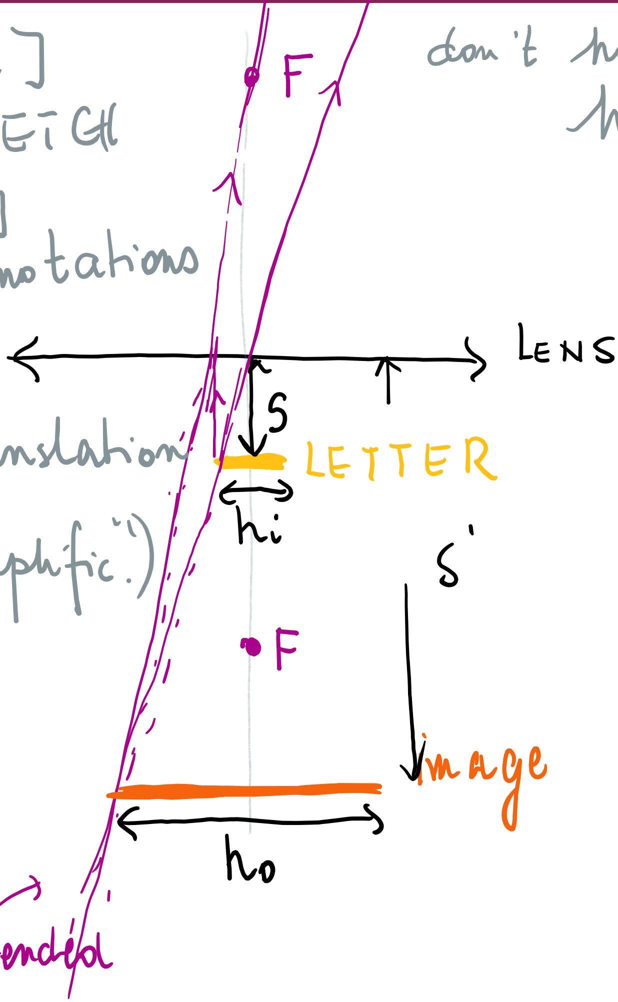
WHAT I WOULD EXPECT TO SEE ON THE EXAM

[1] SKETCH

don't have to be perfect, has to look like an honest attempt

[1] annotations

"translation & simplific.")



$s = 0.03\,m = 3.0\,cm$

$f = +12.0\,cm = 0.12\,m$

↑ part of annotations if cannot fit

[1] Equations

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{fs}$$

(ok to plug in numbers)

$$\frac{1}{s'} = \frac{s-f}{f \cdot s} \rightarrow s' = \frac{fs}{s-f}$$

$$s' = \frac{(12.0\,cm)(3.0\,cm)}{3.0\,cm - 12.0\,cm}$$

$$s' = \frac{36\,cm^2}{-9.0\,cm} = -4.0\,cm$$

[0.5] entering core of values → units can be skipped here as long as worked out somew

$$m = \frac{h_i}{h_o} = \left| \frac{s'}{s} \right|$$

$$m = \frac{|4.0\,cm|}{|3.0\,cm|} = \cancel{\&}\ \frac{4}{3}$$

[0.5] correct values * continuation

$$h_i = h_o \cdot m = \frac{4}{3} \cdot 2.0\,mm = \frac{8}{3}\,mm = 2.67\ mm$$

[1] final answer is clearly shown with units and is reasonable & consistent

* values are correct when they are consistent with the image

IF final answer makes no sense commentary is provided

Mirror / Lens Equation

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

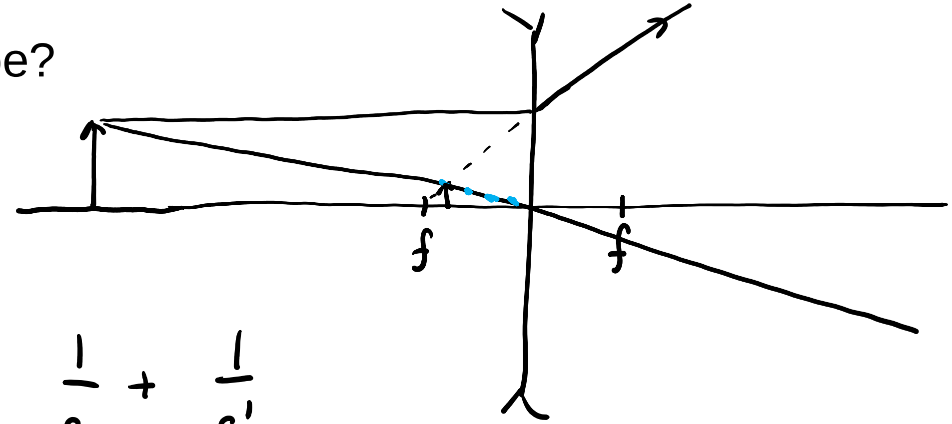
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$M = \frac{h_i}{h_o} = \frac{h'}{h} = -\frac{s'}{s}$$

EXAMPLE 23D – THROUGH THE ~~LOOKING~~ GLASSES

You place an object 40 cm away from a concave lens with focal length $f = -4.0\text{cm}$.

Where does the object appear to be?



$s = 40.0 \text{ cm}$

$f = -4.0 \text{ cm}$

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-4.0} - \frac{1}{40.0}$$

$$\frac{1}{s'} = -\frac{10}{40} - \frac{1}{40} = -\frac{11}{40}$$

$$s' = -\frac{40}{11} = -3.6 \text{ cm}$$

Mirror Equation

Focal length

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$$

Distance from the image to the mirror;
Positive for real images
Negative for virtual images

Distance from the object to the mirror/lens

Focal length:

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$$

s, s' – positive

f – positive

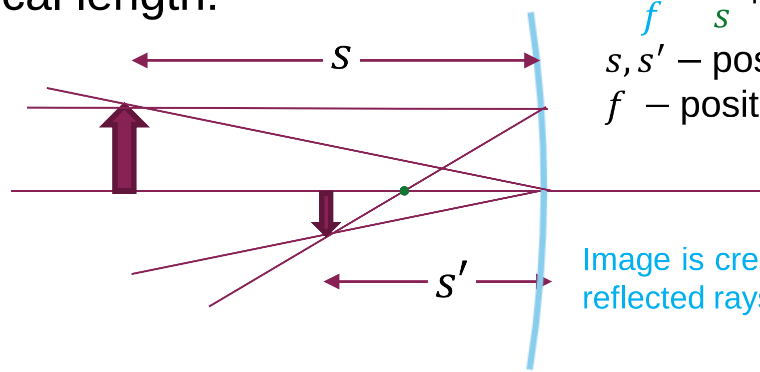


Image is created by reflected rays - **real**

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$$

s – positive

s' – negative, $|s'| > |s|$

f – positive

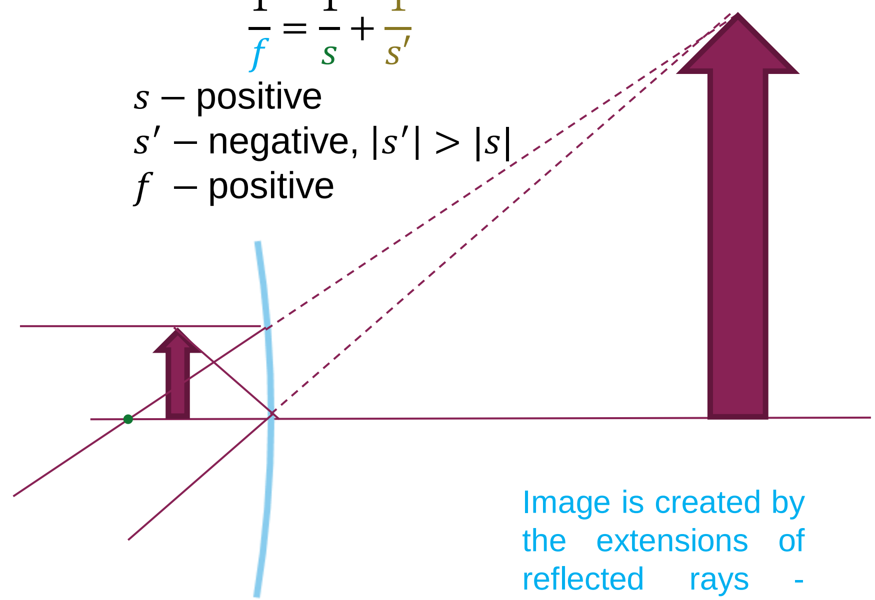


Image is created by the extensions of the reflected rays - **virtual**

Example: A **concave** mirror produces a $\boxed{\text{virtual}}$ image 5.0 cm away from its surface when the object is placed 1.0 cm away from it. What is the focal length of the mirror?

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{1.0\text{cm}}+\frac{1}{(-5.0\text{ cm})}=\frac{5}{5}+\left(-\frac{1}{5}\right)=\frac{4}{5\text{cm}}$$

$$f=\frac{5}{4}\text{ cm}$$

Mirror Equation

Focal length

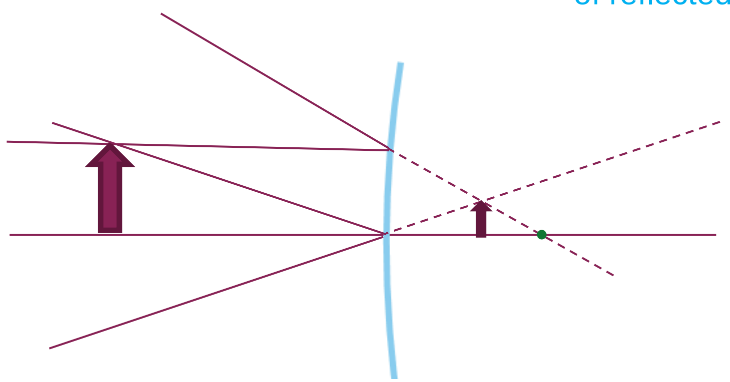
Distance from the image to the mirror;
Positive for real images
Negative for virtual images

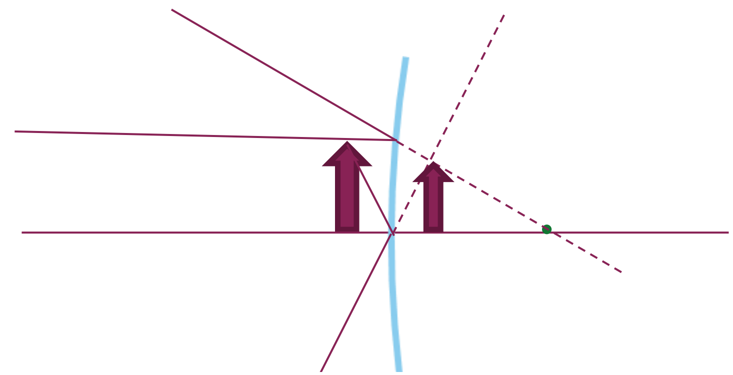
$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

Distance from the object to the mirror/lens

Focal length:

Image is created by the extensions of reflected rays - **virtual**





Example: A **convex** mirror produces a virtual image 1.0 cm away from its surface when the object is placed 5.0 cm away from it. What is the focal length of the mirror?

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{5.0\text{ cm}} + \frac{1}{(-1.0\text{ cm})} = \frac{1}{5} - \frac{5}{5} = -\frac{4}{5\text{cm}}$$

LEARNING CATALYTICS

We know that when an object is placed distance f from the concave mirror.

Consider following questions:

1. Where would you find an image from a concave mirror if $s \to \infty$ (object is very far away)?

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

2. Where would you find an image from a convex mirror if $s \to \infty$ (object is very far away)?

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

Lens Equation

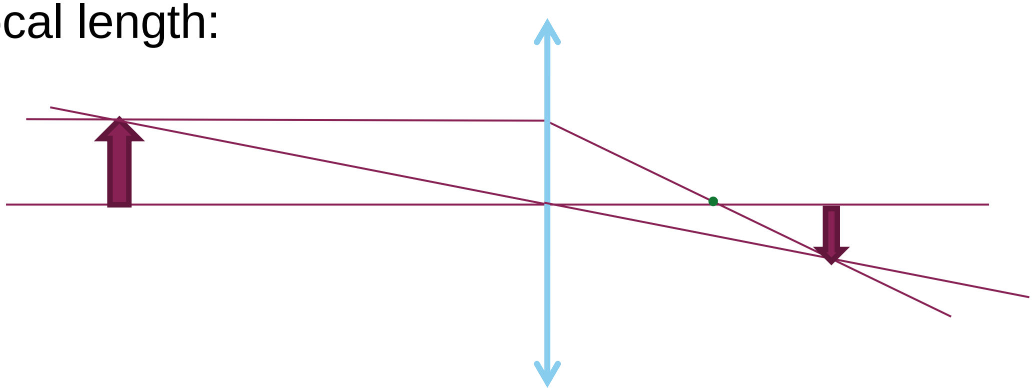
Focal length

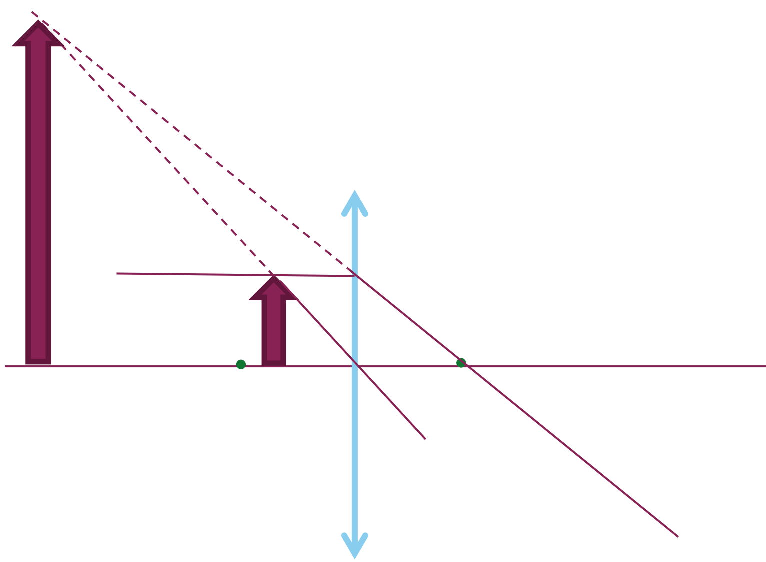
Distance from the image to the mirror/lens;
Positive for real images
Negative for virtual images

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

Distance from the object to the mirror/lens

Focal length:





Lens Equation

Focal length

Distance from the image to the mirror/lens;
Positive for real images
Negative for virtual images

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

Distance from the object to the mirror/lens

Focal length:

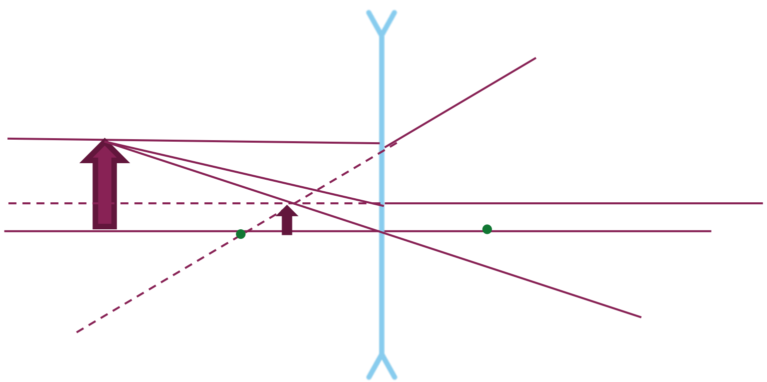
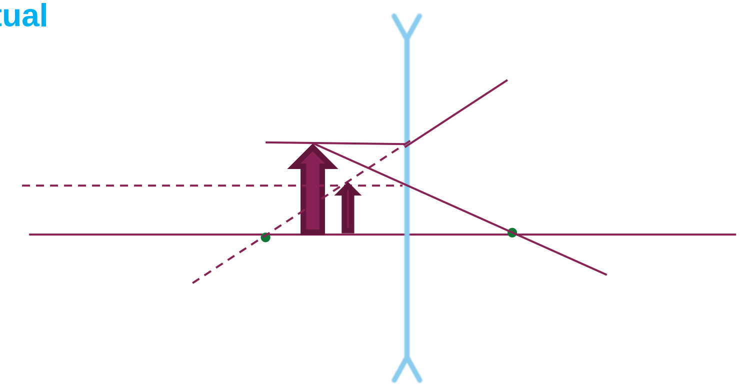
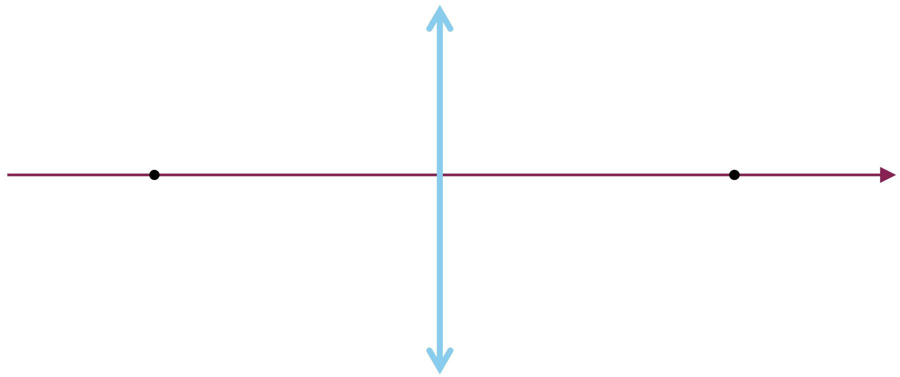


Image is created by the extensions of refracted rays - **virtual**



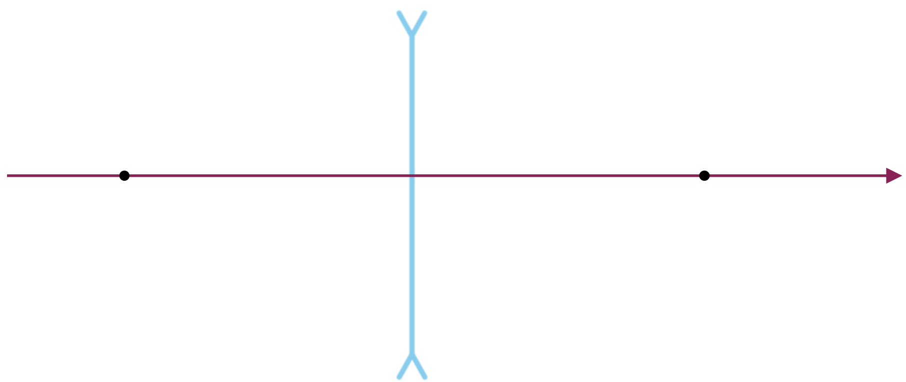
Different way to look at it:



Lenses refract rays

∴ image created by **refracted rays**.
s' is positive

6



∴ image created by **extension** of **refracted rays**
s' is negative

Mirror reflect rays.

Real images are created by **reflected rays** ($s' > 0$).
Virtual images are created by **extensions** of **reflected rays** ($s' < 0$).

relativity

no new material after april 5th

MIDTERM #2
CH. 20, 21, 22, 23
$\boxed{\text{ONLY}}$ LISTED SECTIONS

NO CLASS
7 April

FINAL (Cumulative)
10 T/F, L/R, +/− (1)
20 MC (4 CHOICE) (2)
2 Long Answer (10)

18 APRIL

15/03
22/03
29/03
05/04
12/04

A7 CLOSE
A8 CLOSE
A9 CLOSE
A10 CLOSE

MOVE ?
→ YAY !
→ NAY... (RISKS!)

10 April, Last Class
→ optional review

→ IN PERSON ?
→ ON ZOOM ?

LECTURE WATCHERS #1

A lens with a radius of R =6.0 cm produces **a real, inverted image** distance 4.29 cm from its surface when the object is placed at distance $d_0 > R$

A) What type of lens are we dealing with?

B) What is the value of d_O? Provide your answer in centimeters, rounded to one place after the decimal.

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

$s = d_0$

$$f = \frac{R}{2} = 3.0 \text{ cm}$$

$$\frac{1}{s} = \frac{1}{f} - \frac{1}{s'} = \frac{1}{3.0} - \frac{1}{4.29}$$

2 places after the decimal

EXAMPLE 23E

Nearsightedness causes the image of a distant object to be formed in front of the retina.

To determine the needed focal length we set $s = \infty$ and s' to be equal to the negative of the far point – the farthest point at which an object can be placed along the optical axis of the eye that will focus its image on the retina.

(if you are wearing glasses, you are not looking at the object but at its image formed by your lenses!)

If a patient's far point is equal to 3.6 m, what is the focal length of the lens?

$$\frac{1}{f} = \underbrace{\frac{1}{s}}_{\infty} + \frac{1}{s'} \rightarrow \frac{1}{f} = \frac{1}{s'}$$

$s' = -FP$ (far point)

$f = -3.6\ m$

Learning Catalytics

Optometrists prescribe glasses using the optical power P measured in **diopters,** where

$$P = \frac{1}{f}.$$

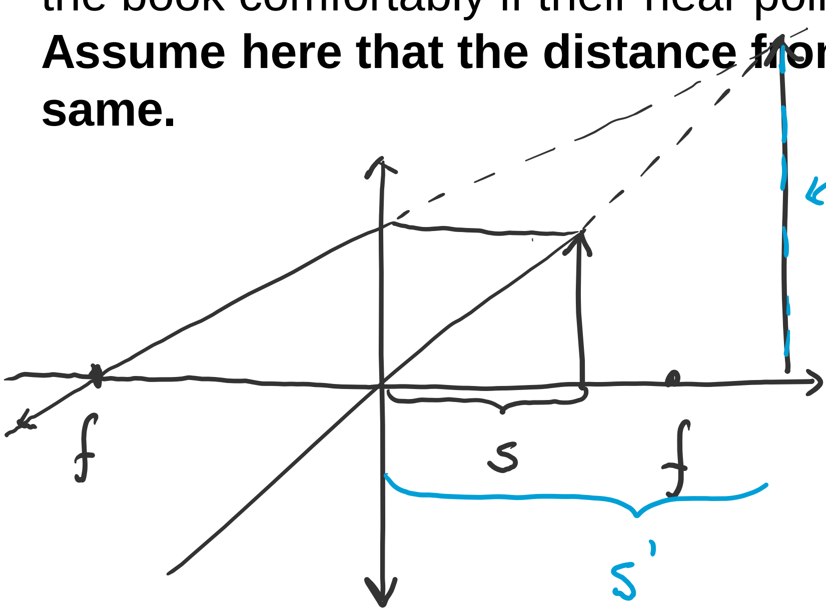
I am slightly nearsighted and my glasses have optical power $P = -0.75$ diopters. What is the focal length of my lenses?

$$f = \frac{1}{P} = \frac{1}{-0.75} = -1.33 \text{ m}$$

EXAMPLE 23F

Farsightedness causes the person’s eye not to be able to form sharp images on the retina for the nearby objects – the image forms behind the retina for object’s closer that the person’s near point.
To determine the focal length of the corrective lens one needs it to create an image at person’s near point.
(We also need to know where the object is).

An average person keeps a book approximately 30 cm in front of their eyes when they read. Determine the focal length of the corrective lenses that will allow them to read the book comfortably if their near point is 90 cm away.
Assume here that the distance from the lens and distance from the eyes is the same.



$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

$$\frac{1}{f} = \frac{1}{30\,\text{cm}} + \frac{1}{(-90\,\text{cm})} = \frac{3}{90} - \frac{1}{90} = \frac{2}{90}$$

$f = 45$ cm (makes sense, $f > s$, virtual)

EXAMPLE 23G – LEARNING CATALYTICS

A lens of focal length $f = -10\ \mathrm{cm}$ creates an image $6.7\ \mathrm{cm}$ from the lens. How far from the lens is the object located?

all images created by concave lens $(f<0)$ ∴ virtual

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{\underbrace{s'}_{-6.7}}$$

$$\frac{1}{-10} = \frac{1}{s} + \frac{1}{-6.7} \rightarrow \frac{1}{s} = -\frac{1}{10} + \frac{1}{6.7}$$

$s \approx$ .....

EXAMPLE 23H – LEARNING CATALYTICS

Angular magnification is defined as

$$M = \frac{\theta'}{\theta}$$

tells you how distorted the object is

Where $\theta' = \frac{h'}{s'}$ and $\theta = \frac{h}{s}$

Recall the example with the coin and the magnifying glass (2.0 mm coin, placed 3.0 cm from the lens appears to be 4.0 cm away and has height of 2.67 mm), What is the angular magnification of the magnifying glass?

$$\theta' = \frac{h'}{s'} = \sim 0.667$$

$$\theta = \frac{h}{s} = \sim 0.667$$

$M \approx 1 \rightarrow$ no distortion, just similar triangles!

	(rays meet) Converging	(rays spread out) diverging
$s = \infty$	$s' = f > 0$	$s' = f < 0$
$s > \lvert f \rvert$	$s' > 0$	$s' < 0$
$s = \lvert f \rvert$	$\frac{1}{s'} = 0 \rightarrow s' \rightarrow \infty$ NO IMAGE	$s' = \frac{f}{2} < 0$
$s < \lvert f \rvert$	$s' < 0$	$s' < 0$
f	$f > 0$	$f < 0$

f converges on 0

f diverges on 0

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

$s = f$:

converge: $\frac{1}{f} = \frac{1}{f} + \frac{1}{s'}$

$\frac{1}{s'} = 0$

diverge $\frac{1}{f} = \frac{1}{-f} + \frac{1}{s'} \rightarrow \frac{1}{s'} = \frac{2}{f}$ $>$