CHAPTER 18:

18.5 CONDUCTORS IN ELECTRIC FIELDS

18.6: DIELECTRIC MATERIALS IN ELECTRIC FIELD

18.7 CAPACITORS

REVIEW

Electric force between two point charges:

$$F_e = \frac{k_e |q_1||q_2|}{r^2}$$

Potential energy of two-point-charge system

$$U_q = \frac{k_e q_1 q_2}{r}$$

Electric field created by a point charge:

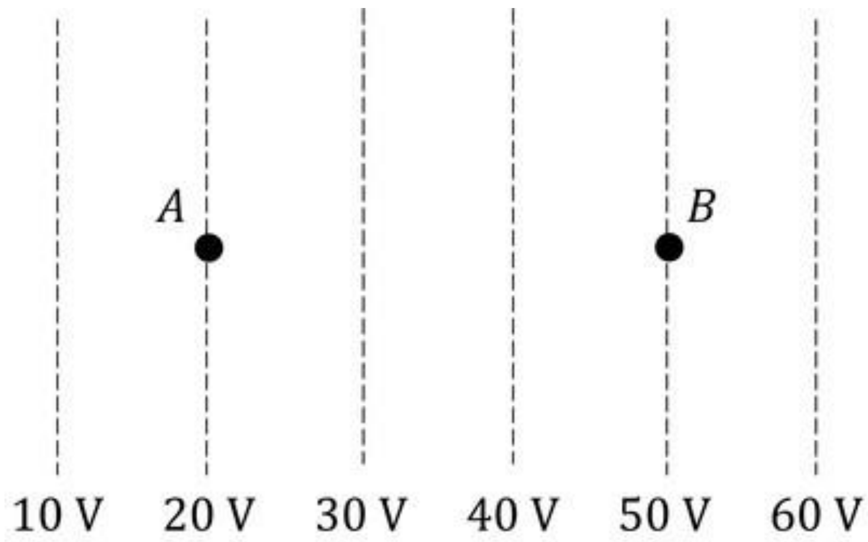
$$E_e = \frac{k_e |q_1|}{r^2}$$

Potential energy of two-charge system

$$V_q = \frac{k_e q_1}{r}$$

In uniform electric field: $\Delta V = -E\Delta r\cos(\measuredangle(\vec{E}, \Delta\vec{r}))$

Based on the information provided in this picture, which way is the electric field pointing?



a) To the right

b) To the left

c) Upwards

d) Downwards

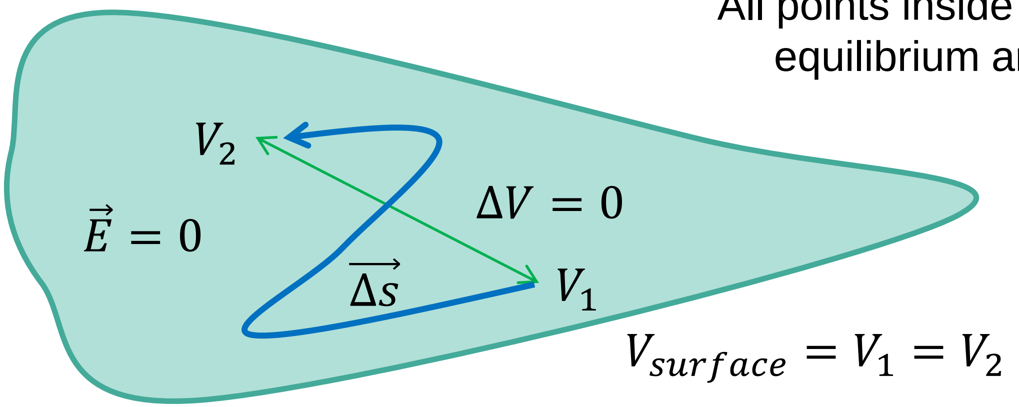
e) Out of the page

f) Into the page

What is the potential difference between points B and A, $\Delta V_{B \to A}$?

A Conductor in Electrostatic Equilibrium

All points inside a conductor is electrostatic equilibrium are at the same potential.



1. All excess charge is on the surface.

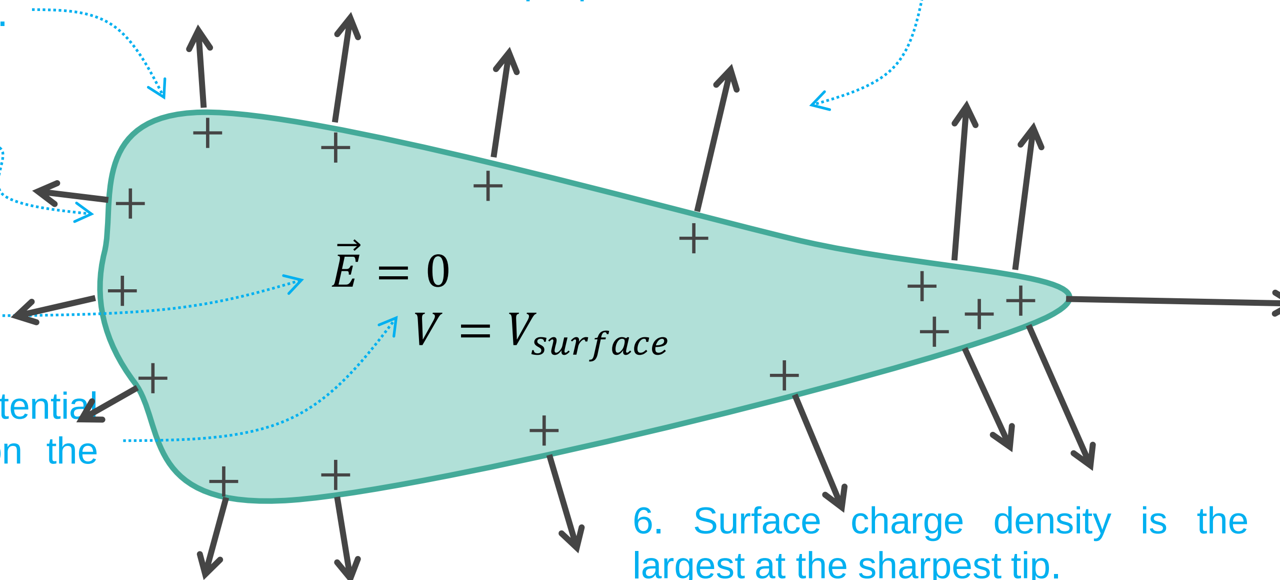
2. The surface is an equipotential.

3. The electric field inside is zero.

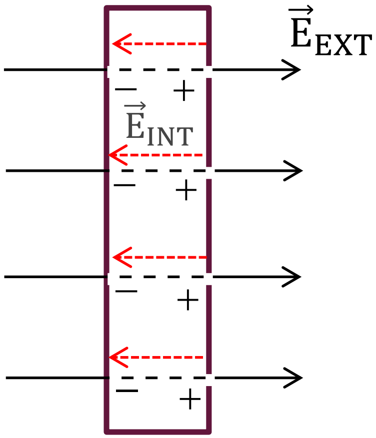
4. The interior is all at the same potential (which is equal to the potential on the surface)..

5. The exterior electric field is perpendicular to the surface.

6. Surface charge density is the largest at the sharpest tip.



$\vec{E}_{net} = 0$ INSIDE THE CONDUCTOR



If the field was non-zero, free charges in the conductor would accelerate under the action of the electric force.

For conductor in external electric field, charges move until $\vec{E}_{INT} = -\vec{E}_{EXT}$ and net electric field inside is equal to zero

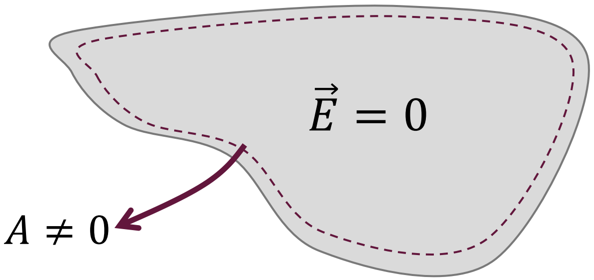
NET CHARGE RESIDES OF THE SURFACE OF A CONDUCTOR IN ELECTROSTATIC EQUILIBRIUM

$\vec{E} = 0$, otherwise the electrons inside the metal would be moving

Gauss's Law is used to prove an important theorem about conductors:

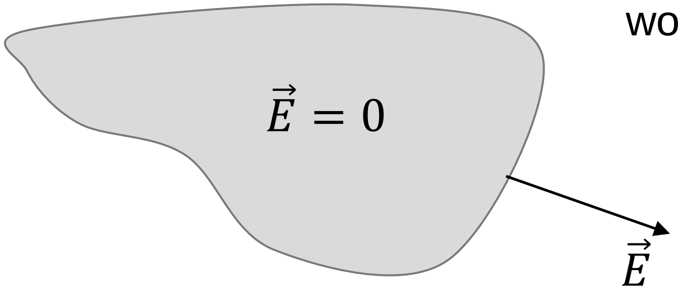
If an excess charge is placed on an isolated conductor in electrostatic equilibrium, that amount of charge will reside exclusively on the surface of the conductor.

None of the excess charge will be found with the body of the conductor.



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} = 0 \rightarrow q_{enc} = 0$$

$\vec{E}$ OUTSIDE THE CONDUCTOR $\vec{E} = E_\perp = \frac{\eta}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0}$



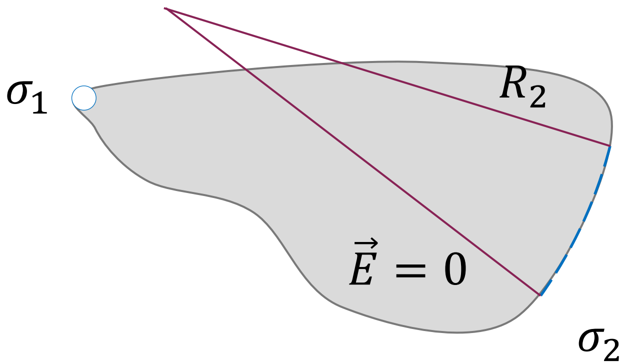
If $\vec{E}$ had a parallel component, charges on the surface would experience acceleration along the surface $\vec{a} = \frac{q\vec{E}_\parallel}{m}$.

The only non-zero flux is through the top surface: $EA = \frac{q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \to E = \frac{\sigma}{\varepsilon_0}$

SURFACE CHARGE DENSITY DEPENDS ON THE SHAPE OF THE SURFACE

If the surface is non-uniform (radius of the curvature varies), then the charge distribution is also non-uniform.

$$\frac{\eta_1}{\eta_2} = \frac{R_2}{R_1}$$



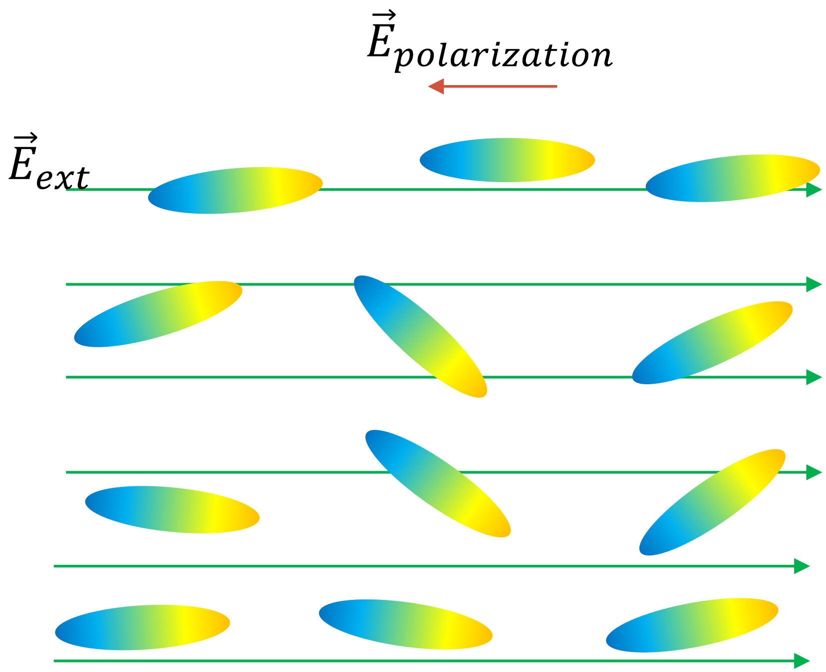
This point is related to the fact that $\Delta V == 0 \to V$ is constant because E is zero

Atomic View of Dielectrics

Polar Dielectrics: electric dipoles tend to line up with an external electric field. Because the molecules are jostling each other due to thermal motion, to complete the alignment increase in the field strength or decrease in temperature may be required.

The alignment produces (smaller) electric field in the direction opposing the external field.

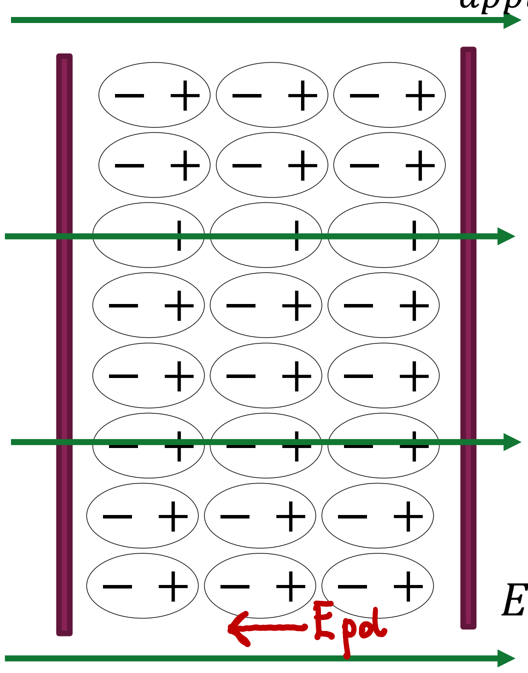
Nonpolar dielectrics: molecules acquire dipole moments due to external field – usually by stretching the molecules and creating separation of positive and negative charges.



DIELECTRIC IN ELECTRIC FIELD

Applied field polarizes the molecules in the insulator. These polarized molecules contribute to the net electric field inside the material

$\vec{E}_{applied}$ – external to dielectric



$E_{polarization}$

$$\vec{E}_{insulator} = \vec{E}_{applied} + \vec{E}_{polarization} = \frac{\vec{E}_{applied}}{\kappa}$$

$$\frac{E_{applied}}{E_{insulator}} = \kappa$$

$$F_{1\ on\ 2-in\ dielectric} = q_2 E_{1 in\ dielectic}$$

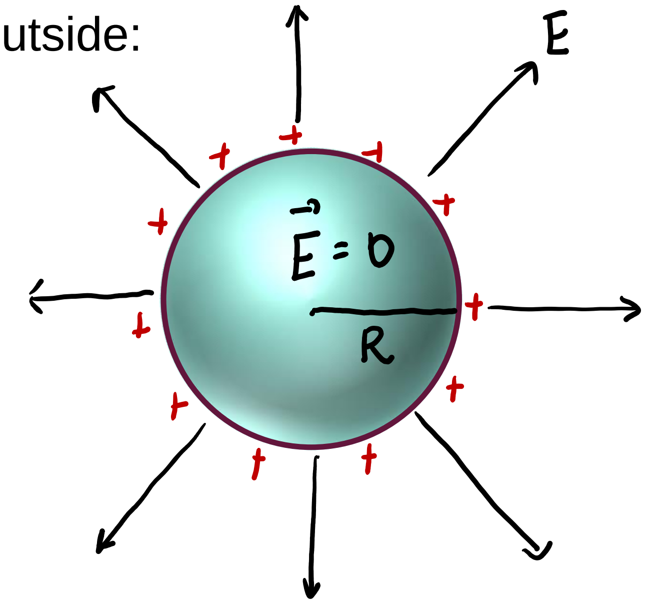
$$F_{1\ on\ 2\ -in\ dielectric} = \frac{k_e|q_1||q_2|}{\kappa r^2}$$

Dielectrics reduce the electric field inside the insulator and decrease the potential difference across the insulator.

$$\Delta V_{insulator} = \frac{\Delta V_{vacuum}}{\kappa}$$

Charged Conducting Sphere

Outside:

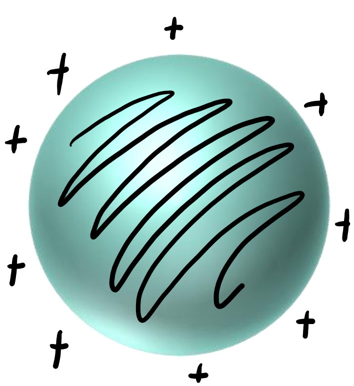


$r > R$

$$E = \frac{k_e Q}{r^2}$$

$$V = \frac{k_e Q}{r}$$

Inside:



$r < R$

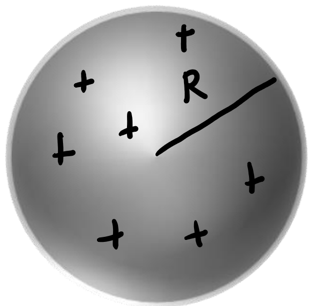
$$E = 0$$

$$V_{inside} = V_{surface}$$

$$\Rightarrow V_{SURF} = \frac{k_e Q}{R}$$

CHARGED INSULATED SPHERE

Outside:

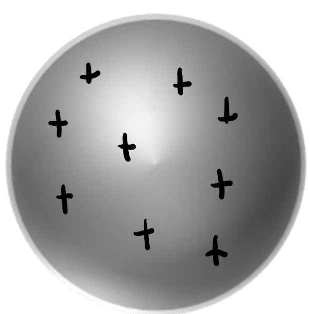


$r \to R$

$$E = \frac{k_e Q}{r^2}$$

$$V = \frac{k_e Q}{r}$$

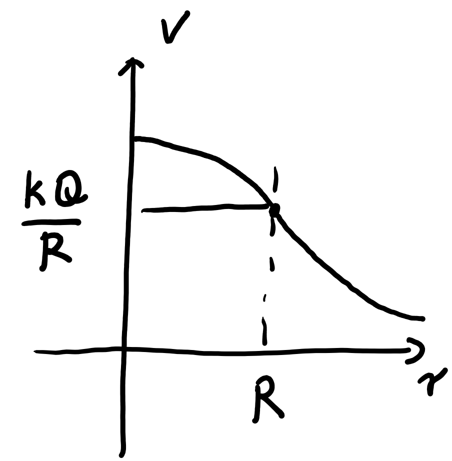
Inside:



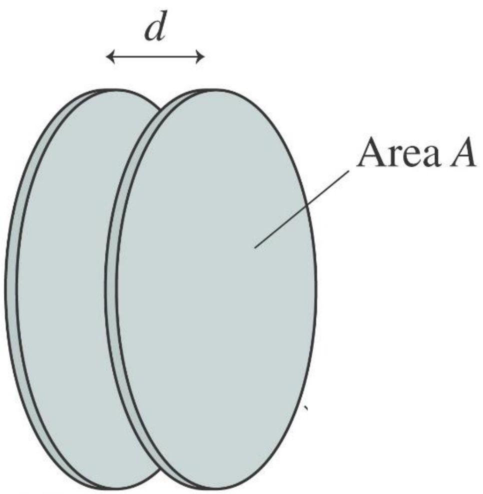
$r < R$

$E \propto r$

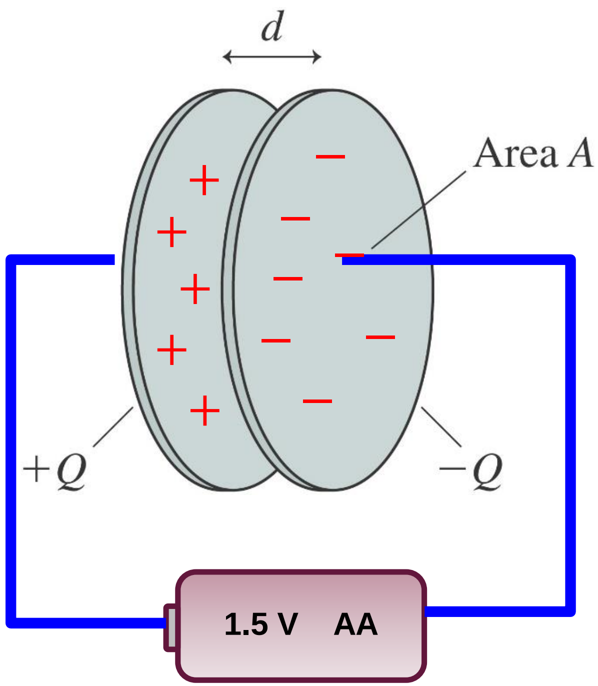
V



CAPACITOR



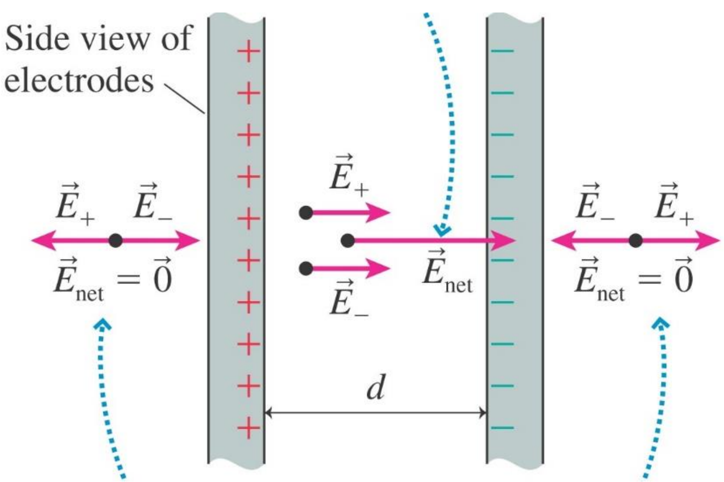
A typical capacitor: Two parallel metal plates



Surface charge densities of equal magnitude and opposite sign.

Each plate creates an electric field of magnitude

$$E = \frac{Q/A}{\varepsilon_0}$$



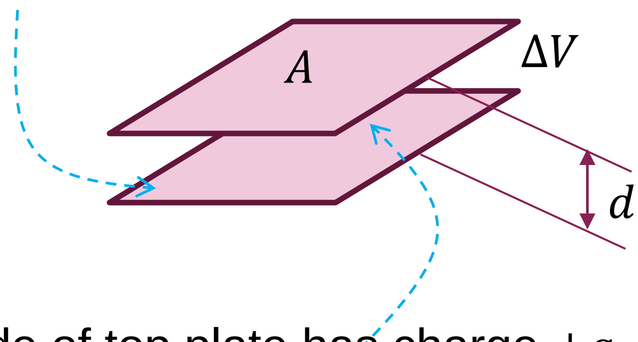
Inside capacitor: $\vec{E}_+$ and $\vec{E}_-$ are parallel; $\vec{E}_{net}$ is large.

Outside capacitor: $\vec{E}_+$ and $\vec{E}_-$ are anti-parallel; $\vec{E}_{net} = 0$

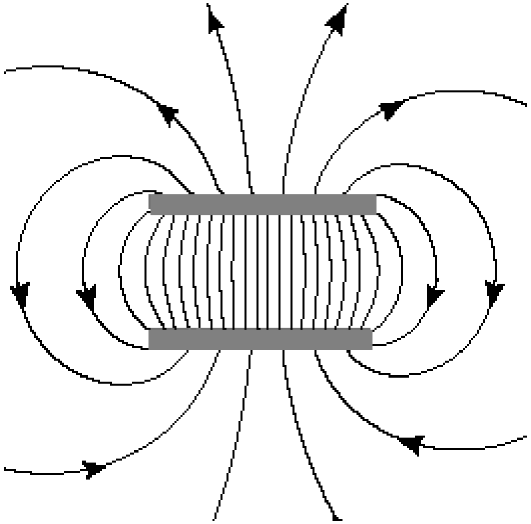
CAPACITANCE

A capacitor consist of two isolated conductors (plates) with charges $+q$ and $-q$ separated by distance d. The charge separation causes $\vec{E}$ field inside the capacitor and, therefore, potential difference ΔV between the plates.

Top side of bottom plate has charge $-q$



Bottom side of top plate has charge $+q$



The charge q and potential difference ΔV are proportional to each other, that is

$$q = C\Delta V$$

$$C = \frac{q}{\Delta V}$$

The proportionality factor C is called the **capacitance**. It is measured in farads (F).

$$1\ \text{farad} = 1\text{F} = 1\text{C/V}$$

What is the capacitance of a capacitor that stores charge $Q = 15\ \mathrm{nC}$ on a positive plate when the potential difference between the plates is $3.0\ \mathrm{V}$?

$$C = \frac{Q}{\Delta V}$$

CHARGE DENSITY

$$Q = \eta \cdot A$$

AREA

Capacitance

$$C = \frac{Q}{\Delta V}$$

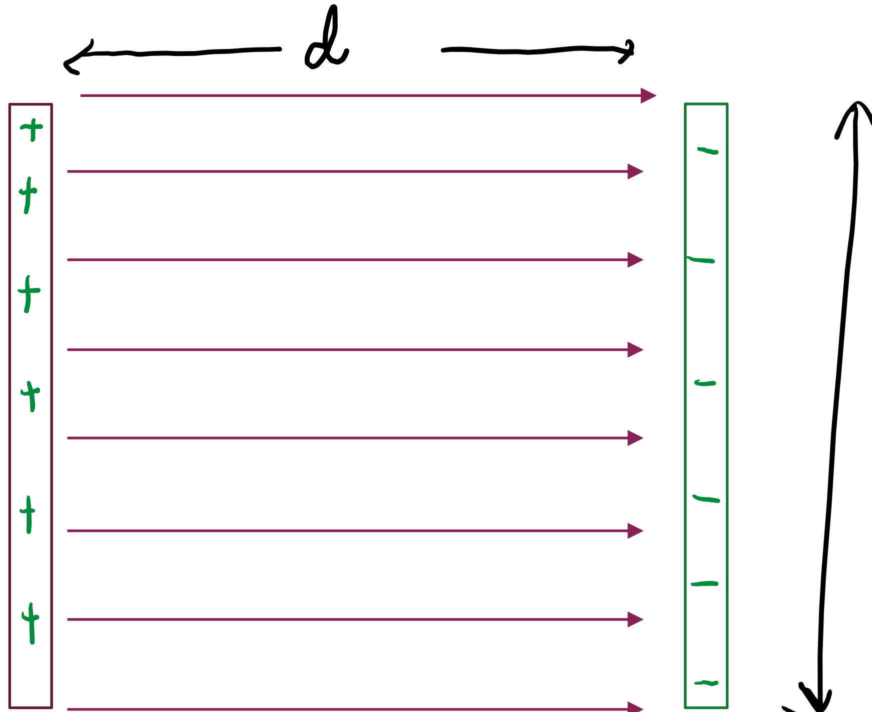
CHARGE

$$|\Delta V| = |E| \cdot |d|$$

$$E = \frac{Q/A}{\varepsilon_0}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{m^2 N}$$

$$C = \frac{Q}{|E||d|} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{A}/\varepsilon_0 \cdot d} = \frac{\varepsilon_0}{\frac{1}{A} \cdot d} = \frac{\varepsilon_0 A}{d}$$



A (area of one plate if plates are uneven → smaller plate; if not aligned, area of overlap

$$C = \frac{Q}{\Delta V}$$ ← charge on one of the plates

↑ potential difference between plates

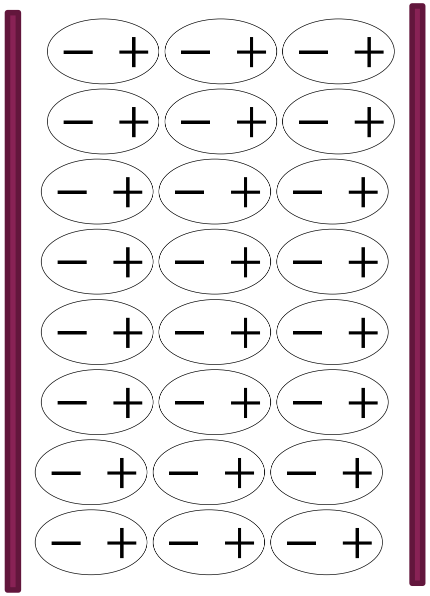
$$|\Delta V| = |E| d$$

$$\rightarrow C = \frac{Q}{Ed} = \frac{Q}{\frac{\eta}{\varepsilon_0} \cdot d} = \frac{\eta A}{\eta / \varepsilon_0 \cdot d} = \frac{A \varepsilon_0}{d}$$

GEOMETRY

Capacitor with a Dielectric

Applied field polarizes the molecules in the insulator. These polarized molecules contribute to the net electric field inside the material



$$\vec{E}_{insulator} = \frac{\vec{E}_{applied}}{\kappa}$$

$$\Delta V_{insulator} = \frac{\Delta V_{vacuum}}{\kappa}$$

Dielectrics decrease the electric field inside the insulator and decrease the potential difference across the insulator.

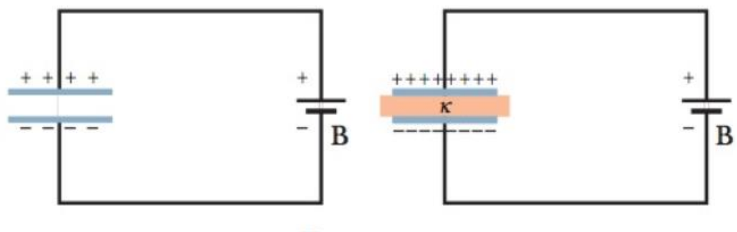
$$C_{dielectric} = \frac{Q}{\Delta \mathrm{V}_{\mathrm{insulator}}} = \frac{Q}{V}\kappa = \kappa C_0$$

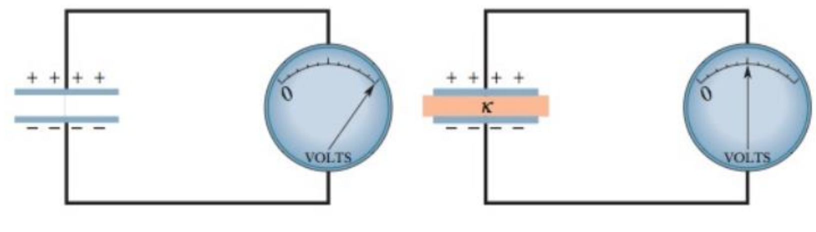
$$C = \kappa \frac{\varepsilon_0 A}{d}$$

We will revisit this ↓

CAPACITOR WITH DIELECTRIC – ELECTRIC FIELD

$$C = \kappa C_0$$





Maintaining potential difference with increasing C means amount of charge on the capacitor increases as $C = \frac{Q}{V} \rightarrow V = \frac{Q}{C}$.

Increasing C by κ means that **the amount of charge stored on the plates can increase κ fold for the same potential difference.**

Keeping constant charge with increasing C means the potential difference between the plates decreases as $C = \frac{Q}{V} \rightarrow Q = CV$.

Increasing C by κ means **the potential difference drops κ times for the same charge.** This also means the electric field in the dielectric decreases