→ deadline tomorrow for ALT midterm time

→ the link is under OCCS recordings on the left-hand side of Quercus page

→ I will be traveling Feb 16 – March 2 for family reasons. Prof. Brian Wilson will teach the classes; I will be answering e-mails / running the course

CHAPTER 19:

Review

Capacitance:

$$C = \frac{Q}{\Delta V}$$

Resistance

$$R = \frac{\Delta V}{I}$$

Work done by uniform electric field:

$$W_e = -\Delta U_q = -\vec{F}_e \cdot \overrightarrow{\Delta r} = -e\vec{E} \cdot \overrightarrow{\Delta r}$$

Change in electric potential

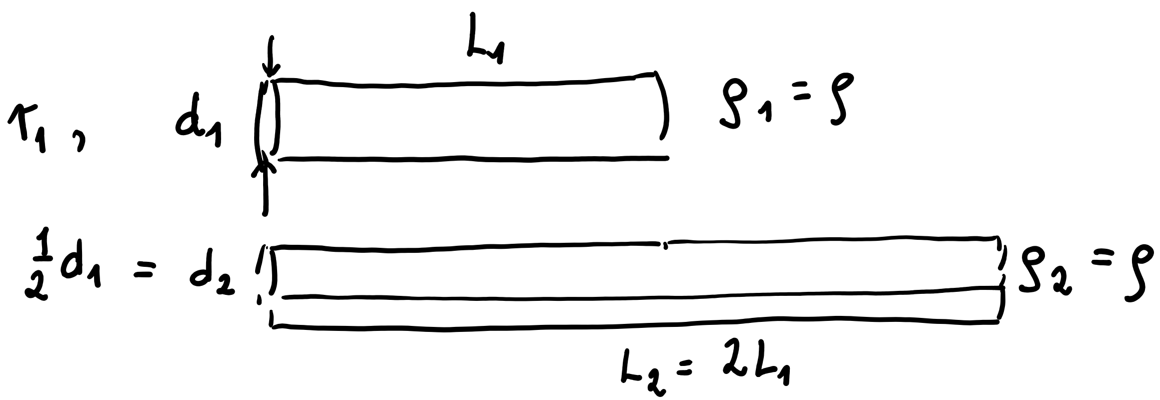
$$\Delta V = -E_s \Delta s$$

If the total displacement is zero $\Delta V = 0$

$\downarrow$ going around the circuit & coming back to the same spot $\rightarrow$ same potential!

EXAMPLE 19A – RESISTANCE AND RESISTIVITY

Two wires are made out of the same material.
Wire **2** has twice the length and half of the diameter of wire **1**
.
What is the ratio of resistances of the two wires?



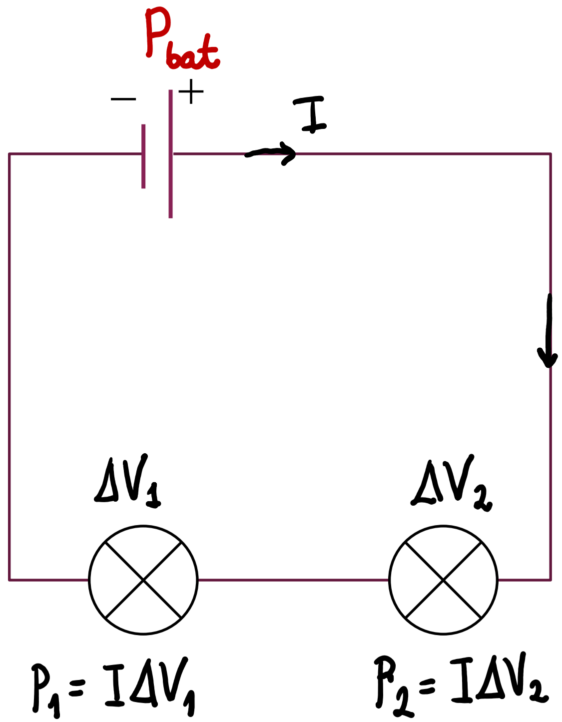
$$R = \rho \frac{L}{A}$$

$$r_1 = 2 r_2$$

$$R_1 = \rho \frac{L_1}{\pi r_1^2}$$

$$R_2 = \rho \frac{L_2}{\pi r_2^2} = \rho \frac{2L_1}{\pi \left(\frac{r_1}{2}\right)^2} = \rho \frac{2L_1}{\pi r_1^2 \frac{1}{4}} = \rho \frac{8L_1}{\pi r_1^2} = \overbrace{\boxed{\rho \frac{L_1}{\pi r_1^2}}}^{R_1} \cdot 8$$

19.6 Joule's Law



Light bulbs (and all other devices) use up energy stored in the battery (E_{chem}).

The rate at which energy is delivered from the battery (and used up by resistors) is called **power.**

$$P_{bat} = \frac{\Delta U}{\Delta t} = \frac{\Delta q \Delta V}{\Delta t} = I\Delta V = I\varepsilon$$

unit: $1W$

$$1W = \frac{1J}{1s}$$

$$P_R = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

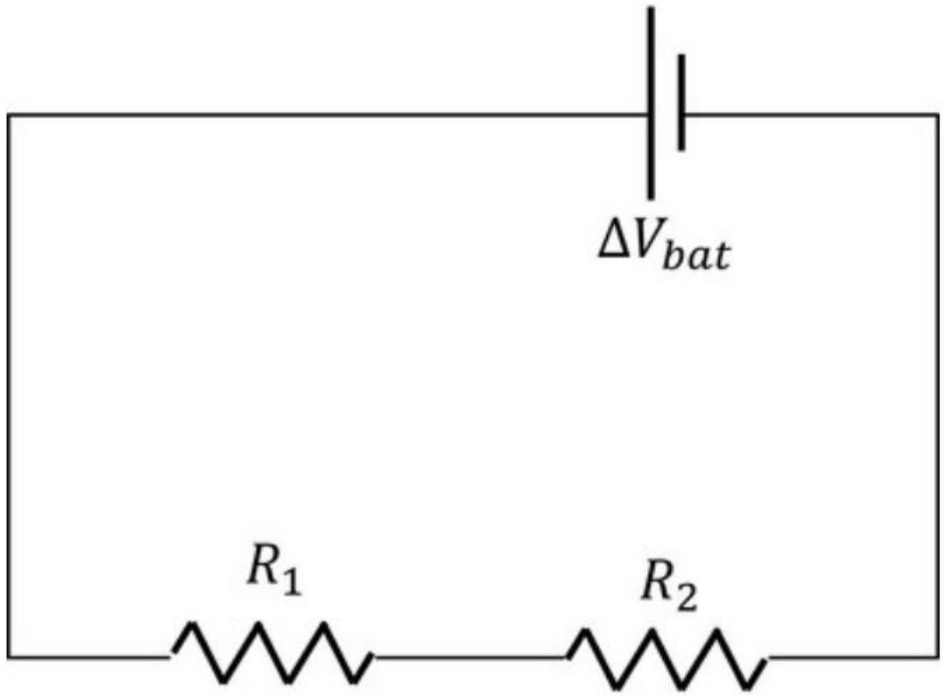
$\Delta V = I.R$

Power provided by the battery is used up by all the devices in the system, no matter how complicated the circuit is:

$$P_{battery} = \sum_i P_i$$

Consider the circuit below.

If the potential differences across the resistors are $\Delta V_1 = 5.0\ \mathrm{V}$ and $\Delta V_2 = 7.0\ \mathrm{V}$ and the current $I = 2.0\ \mathrm{A}$ is measured in the circuit, what is the **total** power dissipated by the resistors?



HOME ECONOMICS – UNDERSTANDING YOUR ENERGY BILL (AND KWH UNITS)

Energy used up by our households is measured in kWh (kilowatt Hours).

Example: if you use a 1500 W hair dryer for 10 minutes you use

$$E_{hairdryer} = P_{hairdryer} \cdot t = (1500W) \cdot \frac{1}{6}h = 250Wh = 0.25kWh$$

In-peak cost of kWh in Ontario (as of Nov 2022) is 15.1 cents.
So using hairdryer for 10 minutes costs you: ~ 3.75¢

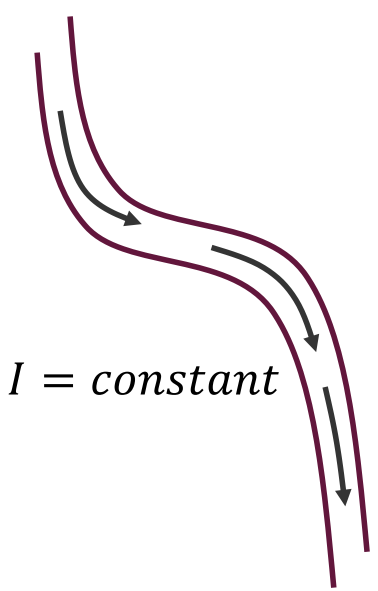
Using it every day for 10 minutes per day: for a month: 30 × 3.75¢

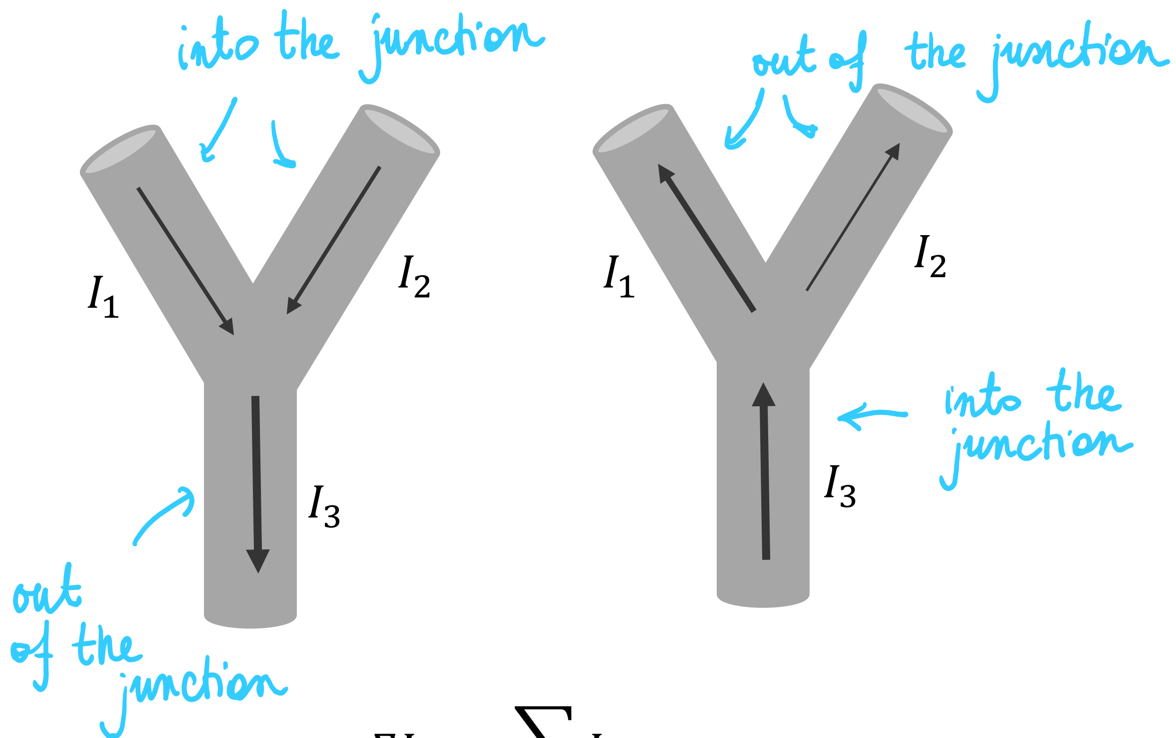
$$1kWh = 1000\ Wh = 1000W * 3600\frac{s}{h} * h = 3.6 \cdot 10^6 J = 3.6MJ$$

19.7 – KIRCHHOFF'S RULES: CONSERVATION OF CURRENT [JUNCTION RULE]

LAW OF CONSERVATION OF CURRENT:

THE CURRENT IS THE SAME AT ALL POINTS IN THE CURRENT CARRYING WIRE.



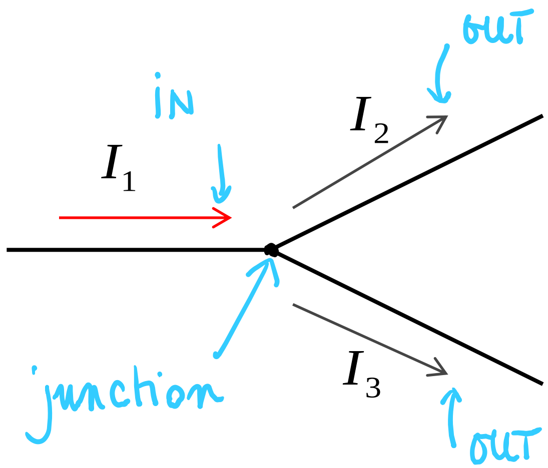


$$\Sigma I_{in} = \sum I_{out}$$

KIRCHHOFF`S RULES – JUNCTION RULE

Junction Rule - At any junction the sum of the currents must equal zero:

$$\sum_{junction} I = 0$$



The amount of charge flowing out of the branches on the right must be equal the amount flowing into the single branch on the left.

This rule is a statement of **conservation of electric charge**. There cannot be any charge buildup at the junction, therefore all charges that enter any given point at the circuit have to leave it.
Currents directed into the junction are considered positive, currents leaving the junction are entered as negative.

$$\sum_{junction} I = I_1 - I_2 - I_3 = 0$$

What is the current in the wire #4?

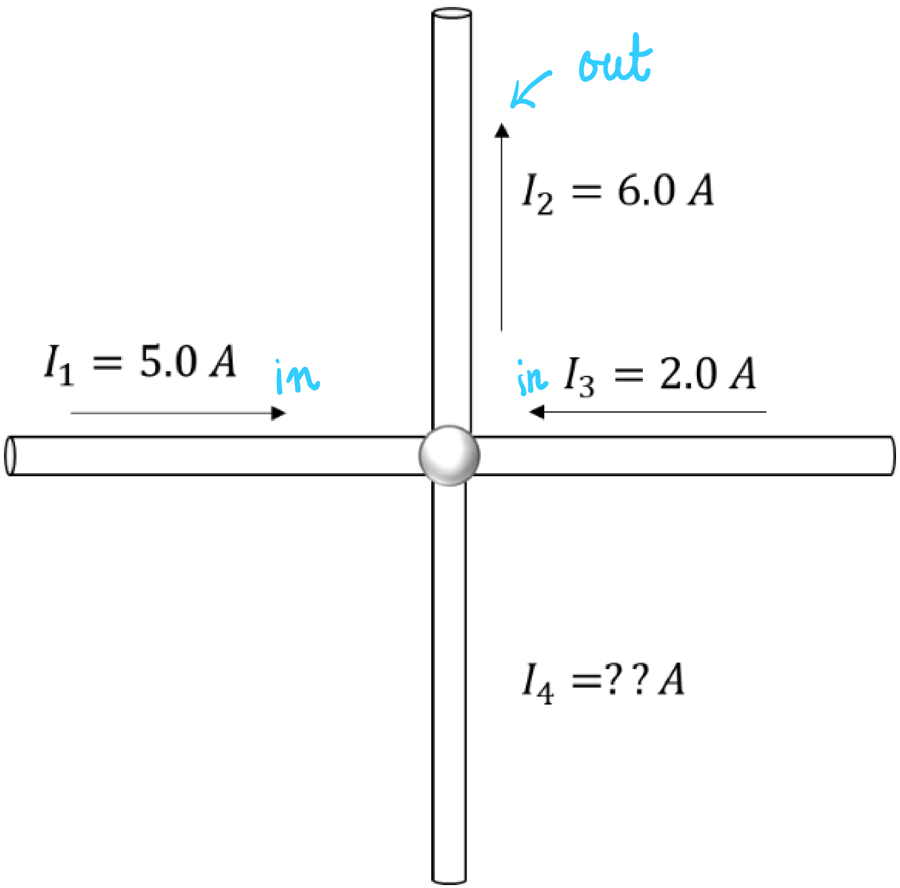


Figure 1. Four-wire junction

7.0 A in

6.0 A out

WHAT'S MISSING?

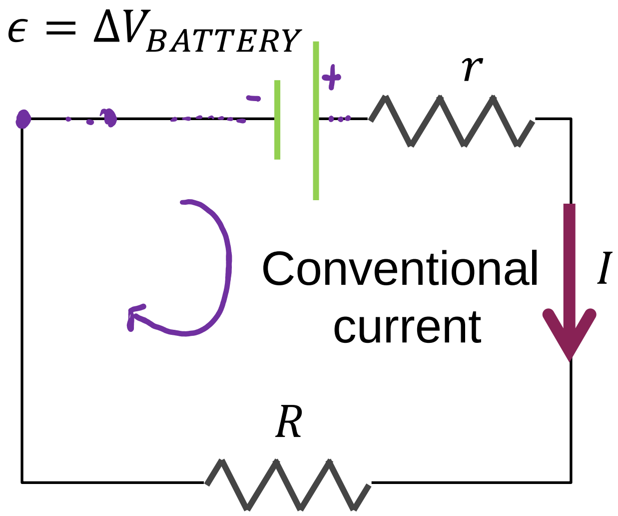
19.7 Kirchhoff's Rules
Conservation of Energy

[Loop Rule]

Law of conservation of Energy

The algebraic sum of the changes in electric potential around any closed circuit loop is zero.

Recall a simple circuit we analyzed when we considered the internal resistance of a battery:

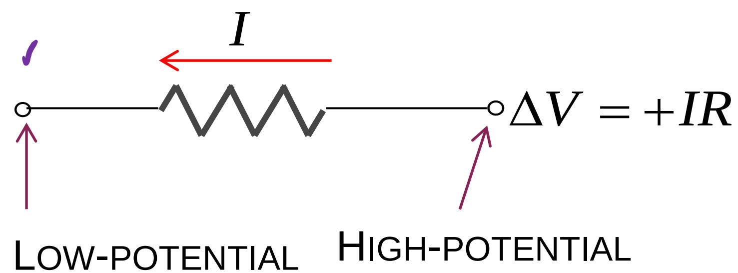


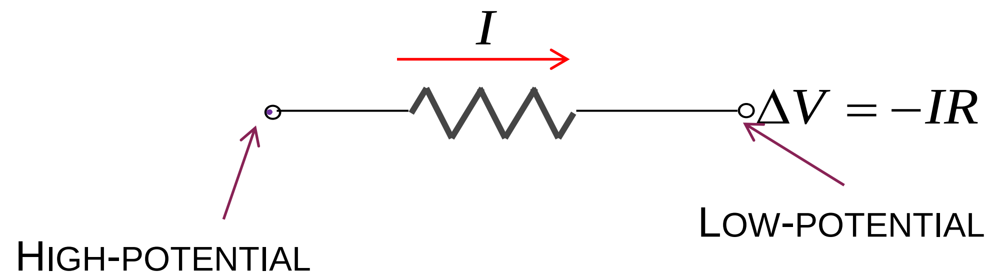
$$+\epsilon - \underbrace{\Delta V_r}_{I \cdot r} - \underbrace{\Delta V_R}_{I \cdot R} = 0$$

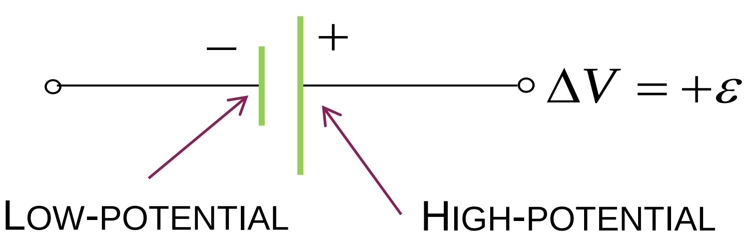
KIRCHHOFF`S RULES – LOOP RULE

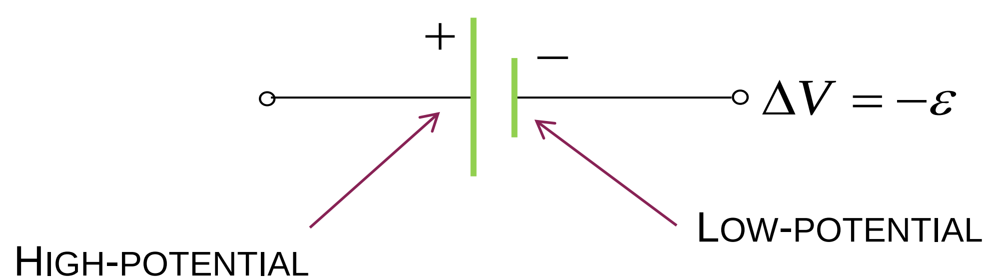
As you move along the circuit, you may pass various elements.

Consider passing through these elements, moving from left to right:









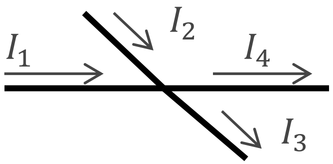
ANALYZING CIRCUITS

To analyze circuit means finding:

1. The potential difference across each circuit component.
2. The current in each circuit component.

Because charge and current are conserved, Kirchhoff's junction law must be satisfied:

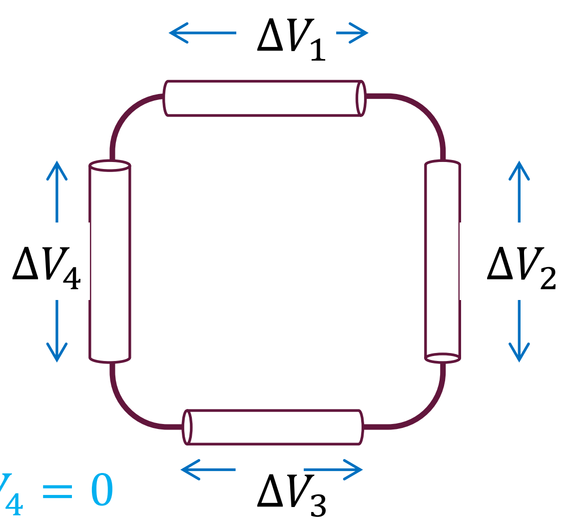
$$\sum I_{in} = \sum I_{out}$$



$$I_1 + I_2 = I_3 + I_4$$

Additionally, the sum of potential differences around any loop (closed path) is zero.

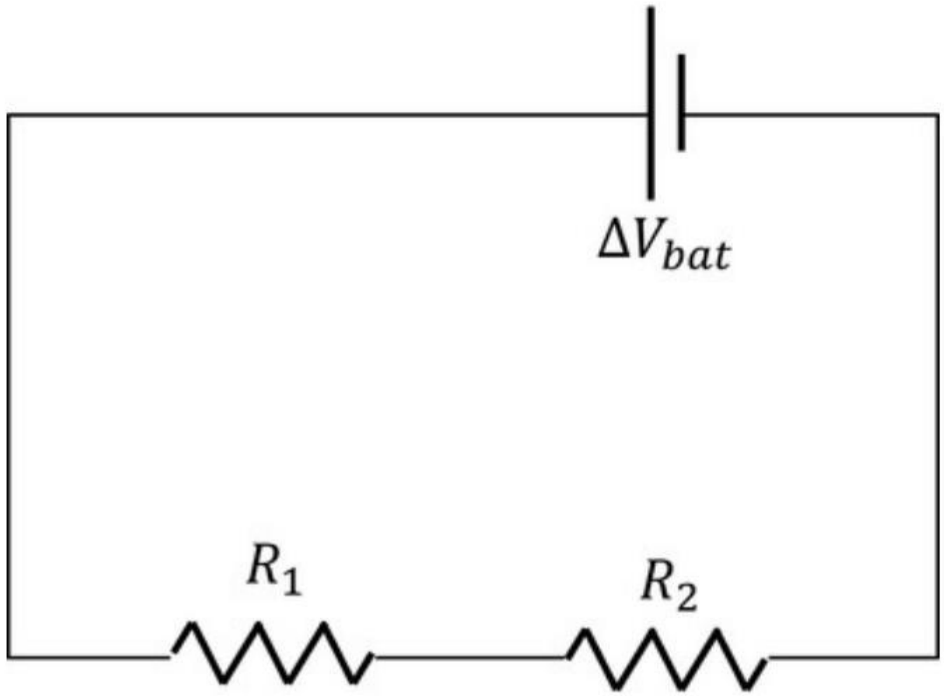
$$\Delta V_{loop} = \sum (\Delta V)_i = 0$$



$$\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$$

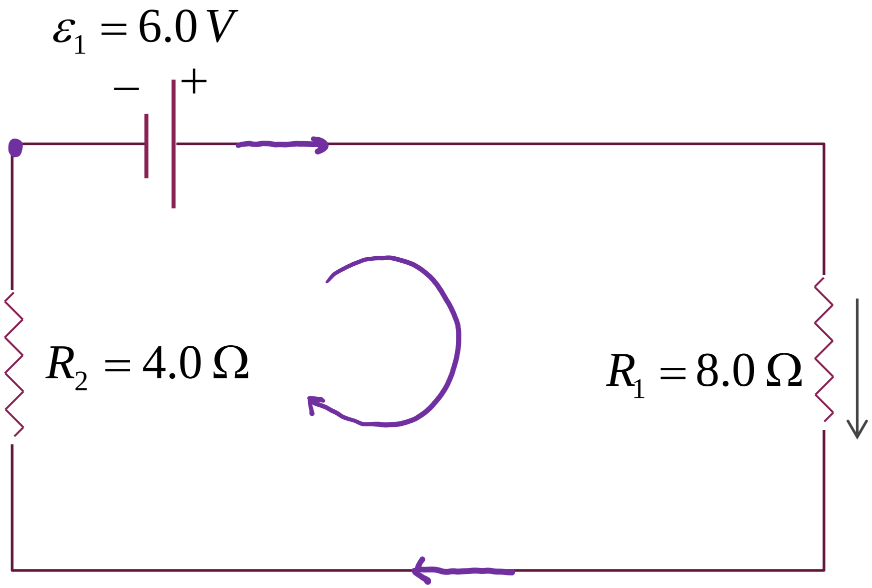
Consider the circuit below.

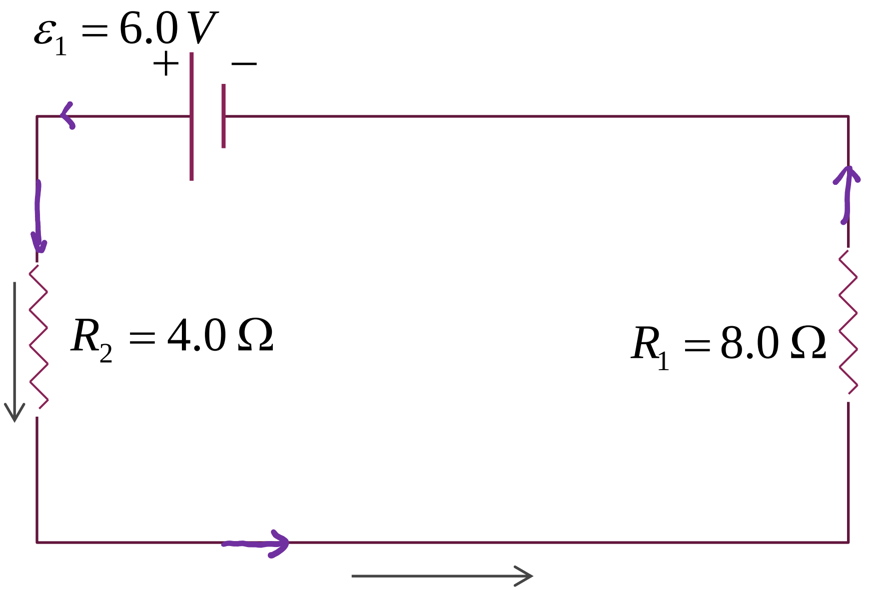
If the potential differences across the resistors are $\Delta V_1 = 5.0\ \mathrm{V}$ and $\Delta V_2 = 7.0\ \mathrm{V}$, what is the total potential provided by the battery?



Assign direction of the current (from positive terminal of the battery) – conventional current

Using Kirchhoff's Loop Law – Example 19b





Start from the top left corner (not mandatory, but makes it easier)

Passing battery from negative to positive:

$$\varepsilon_1 = \Delta V_{bat} = +6.0\text{ V}$$

Passing R_1 "with" the current

$$\Delta V_1 = -IR_1 = -I(8.0\Omega)$$

Passing R_2 "with" the current

$$\Delta V_2 = -IR_2 = -I(4.0\Omega)$$

$$\varepsilon_1 + \Delta V_1 + \Delta V_2 = 0$$

$$6.0 - I(8.0) - I(4.0) = 0 \rightarrow I = 0.5A$$

Passing battery from positive to negative:

$$\varepsilon_1 = -\Delta V_{bat} = -6.0\text{ V}$$

Passing R_1 against the current

$$\Delta V_1 = +IR_1 = +I(8.0\Omega)$$

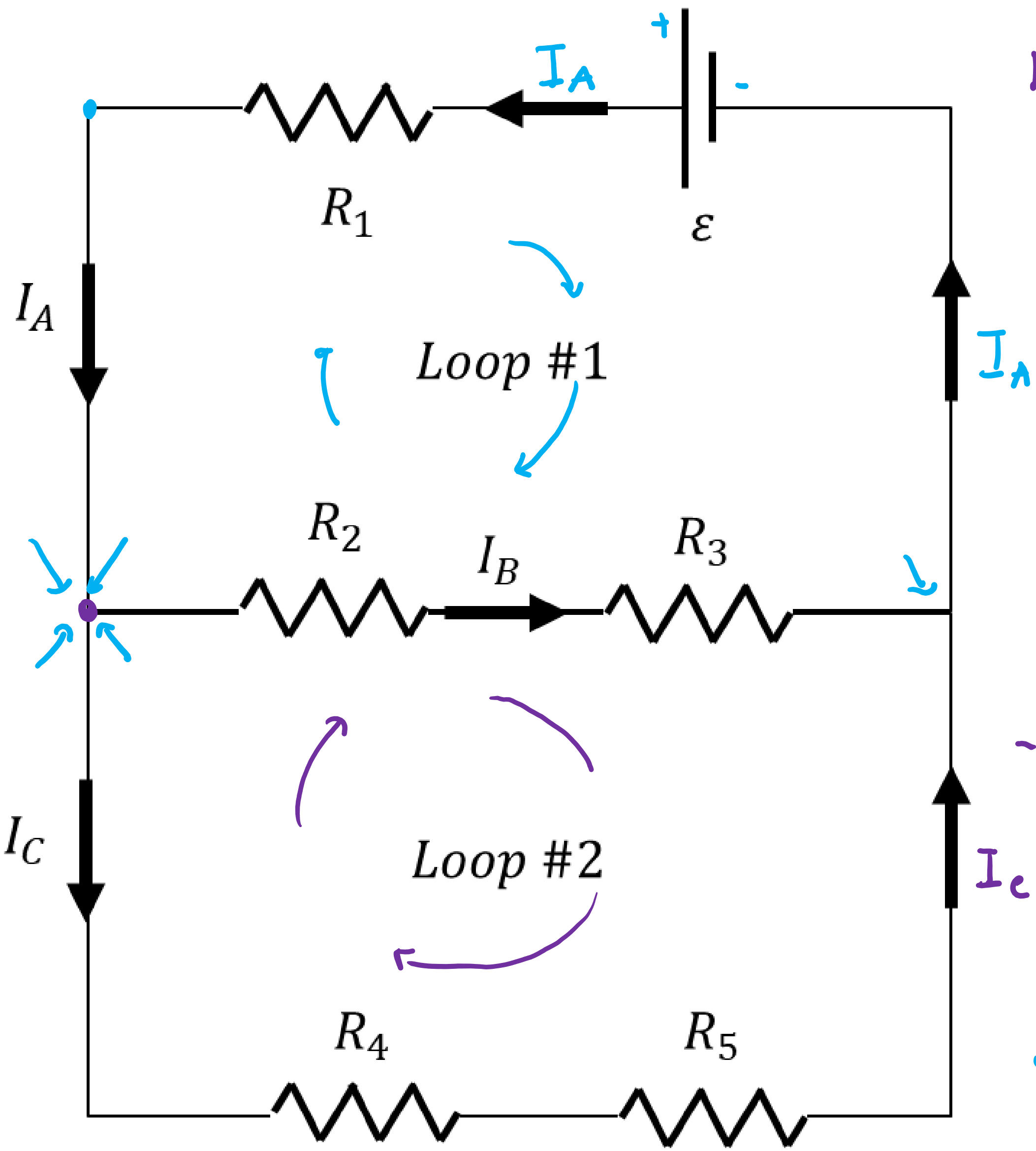
Passing R_2 against the current

$$\Delta V_2 = +IR_2 = +I(4.0\Omega)$$

$$\varepsilon_1 + \Delta V_1 + \Delta V_2 = 0$$

$$-6.0 + I(8.0) + I(4.0) = 0 \rightarrow I = 0.5A$$

Write Kirchhoff's loop law for both loops.



Loop #1

$$I_A \cdot R_1 - \mathcal{E} + I_B R_3 + I_B R_2 = 0$$

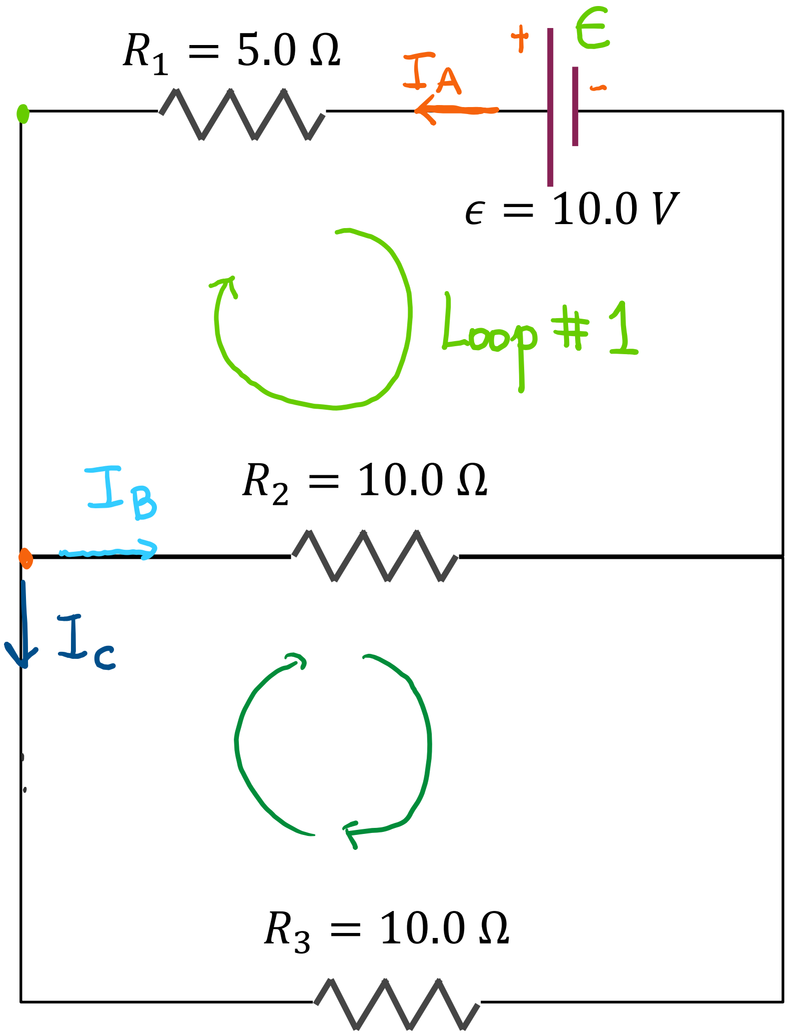
Loop #2

$$-I_B \cdot R_2 - I_B \cdot R_3 + I_c \cdot R_5 + I_c \cdot R_4 = 0$$

Junction: $I_A = I_B + I_c$

Example 19c

Consider the circuit shown in the picture.
Determine the current in each branch of the circuit.



$$\begin{cases} I_A R_1 - \epsilon + I_B R_2 = 0 & \text{Loop \# 1} \\ -I_B R_2 + I_C R_3 = 0 & \text{Loop \# 2} \\ I_A = I_B + I_C & \text{junction} \end{cases}$$

$$\begin{cases} I_A \cdot 5.0 - 10.0 + I_B \cdot 10 = 0 & \# 1 \\ -10.0\, I_B + I_C \cdot 10.0 = 0 & \# 2 \\ I_A = I_B + I_C & J \end{cases}$$

#1. $5.0\, I_A + 10\, I_B - 10.0 = 0$

#2: $10\, I_B = 10\, I_C \rightarrow$ $I_B = I_C$

J: $I_A = I_B + I_C \rightarrow I_A = 2 I_B$

#1: sub $I_A = 2 I_B \rightarrow 5.0 \times (2 I_B) + 10 I_B = 10$

$$10 I_B + 10 I_B = 10$$

$$\underline{I_B = 0.5\,A}$$

$I_C = I_B = 0.5\,A$

$I_A = 1.0\,A$

potential differences: $\Delta V_1 = R_1 I_A = 5.0 \cdot 1.0 = 5.0\,V$

$\Delta V_2 = R_2 I_B = 10.0 \cdot 0.5 = 5.0\,V$

$\Delta V_3 = R_3 I_C = 5.0\,V$

CHECK POWER! $P_{bat} = \underset{\text{total}}{I_A} \cdot \epsilon = 10\,W$

$\underline{P_1} = I_A \Delta V_1 = 1.0 \cdot 5 = \underline{5W}$; $\underline{P_2} = I_B \Delta V_2 = 0.5 \cdot 5.0 = \underline{2.5W}$; $\underline{P_3} = I_C \Delta V_3 = \underline{2.5W}$