CHAPTER 18 & 17

REVIEW

Electric force between two charges:

$$F_e = \frac{k_e|q_1||q_2|}{r^2}$$

Potential energy of two-charge system:

$$W = \Delta \mathrm{E}$$

Work done by **electric force**:

$$W_{F_e} = -\Delta U_q$$

$$U_q = \frac{k_e q_1 q_2}{r}$$

The Field Model

Faraday's idea from looking at magnets:

Space itself around charges is filled with ***some kind of electric influence.***

Mathematically, $\boldsymbol{a\ field}$ is a function that assigns a vector to every point in space.

In Physics, it conveys the idea that **a physical entity exists in every point in space**.

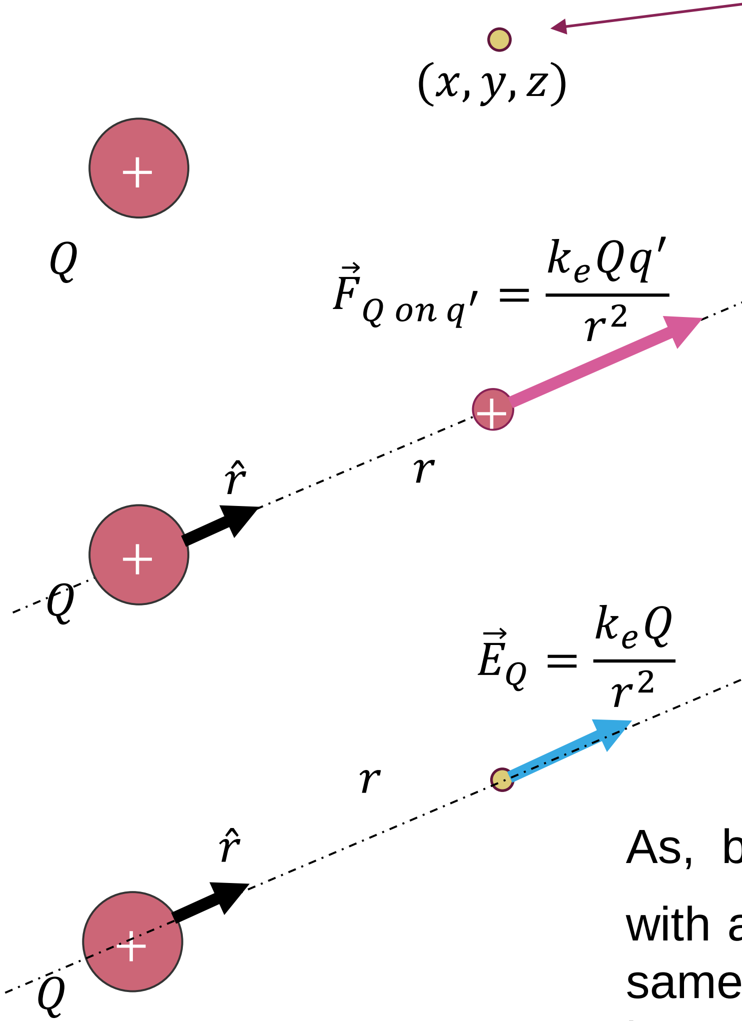
$$F_{Earth\ on\ Mario} = \left(G\frac{M_E}{r^2}\right)m_{Mario}$$

$$F_{Earth\ on\ Luigi} = \left(G\frac{M_E}{r^2}\right)m_{Luigi}$$

$$\vec{F}_{agent\ on\ object} = m_{object}(\text{some vectorial quantity created by the agent})$$

THE ELECTRIC FIELD OF A POINT CHARGE

What exactly is the electric field here?



If we place a small test charge (q’) at this point, it will experience an electric force of magnitude $F = k_e \frac{Qq'}{r^2}$ that points away from the source charge.

As, by definition, $\vec{E}(x, y, z) = \frac{\vec{F}_{on\ q',\ at\ (x,y,z)}}{q'}$, we are left with a vector along the same line as the force, with the same direction (dividing vector by a positive number) but with different magnitude (E).

THE ELECTRIC FIELD

- Source charges alter the space around them, creating *electric field* $\vec{E}$.
- A separate charge q in the electric field $\vec{E}$ experiences force $\vec{F}$ exerted by the field.

The electric field $\vec{E}$ is at a point in space is defined as the electric force $\vec{F}$ acting on a test particle divided by a charge q of the test particle.

$$\vec{E}(x,y,z) = \frac{\vec{\mathrm{F}}_{on\ q} at(x,y,z)}{q}$$

Units: $[E] = \frac{N}{C}$

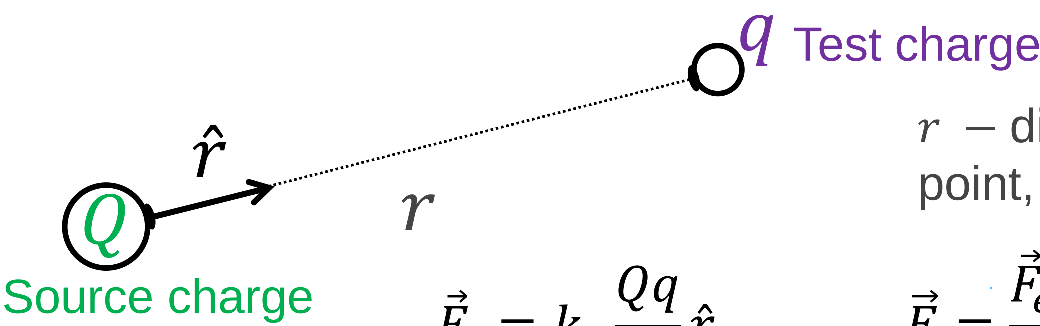
NOTE:

1. Force and electric field **<u>are both vectors</u>**, *therefore they have magnitude **and** direction*

2. Electric charge q creates electric field $\overrightarrow{\boldsymbol{E}}$ in all points in space.

   We have to think about it 3D even if we only draw a 2D representation of the idea

Consider a source charge Q and test charge q

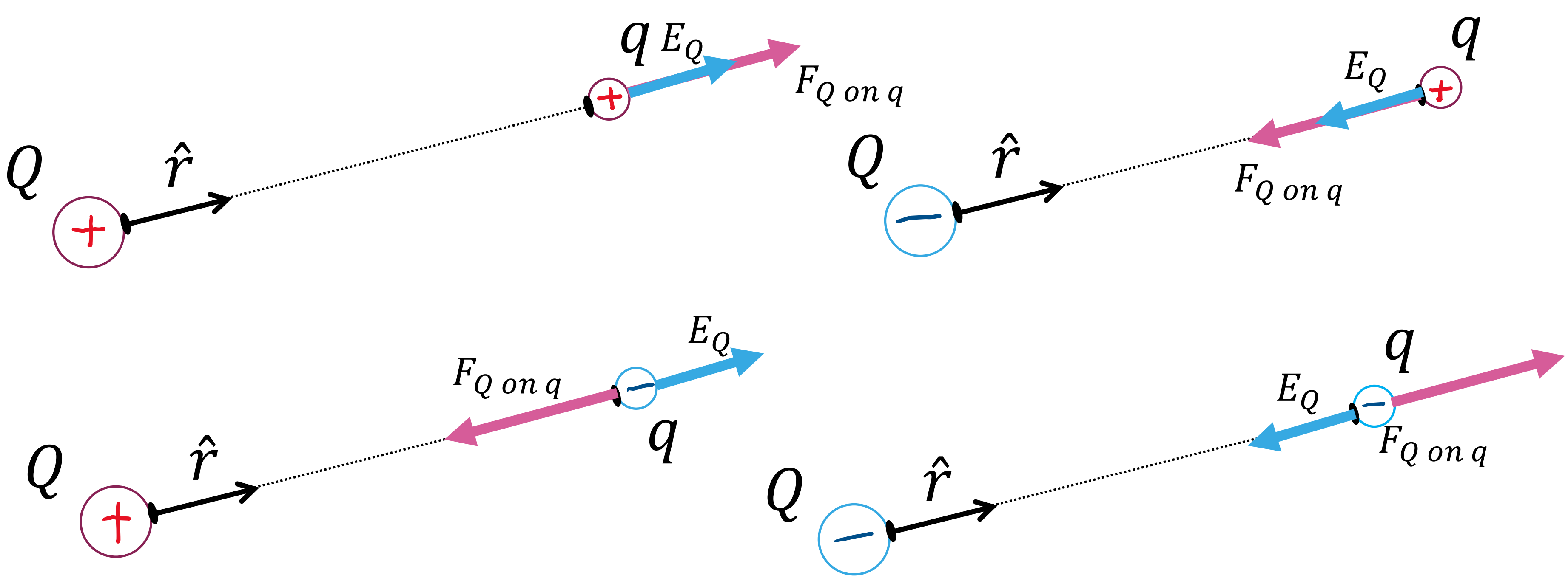


r – distance between source charge and the point, where the test charge would be placed.

$$\vec{F}_e = k_e \frac{Qq}{r^2}\hat{r}, \qquad \vec{E} = \frac{\vec{F}_e}{q} \rightarrow \vec{E} = k_e \frac{Q}{r^2}\hat{r}$$

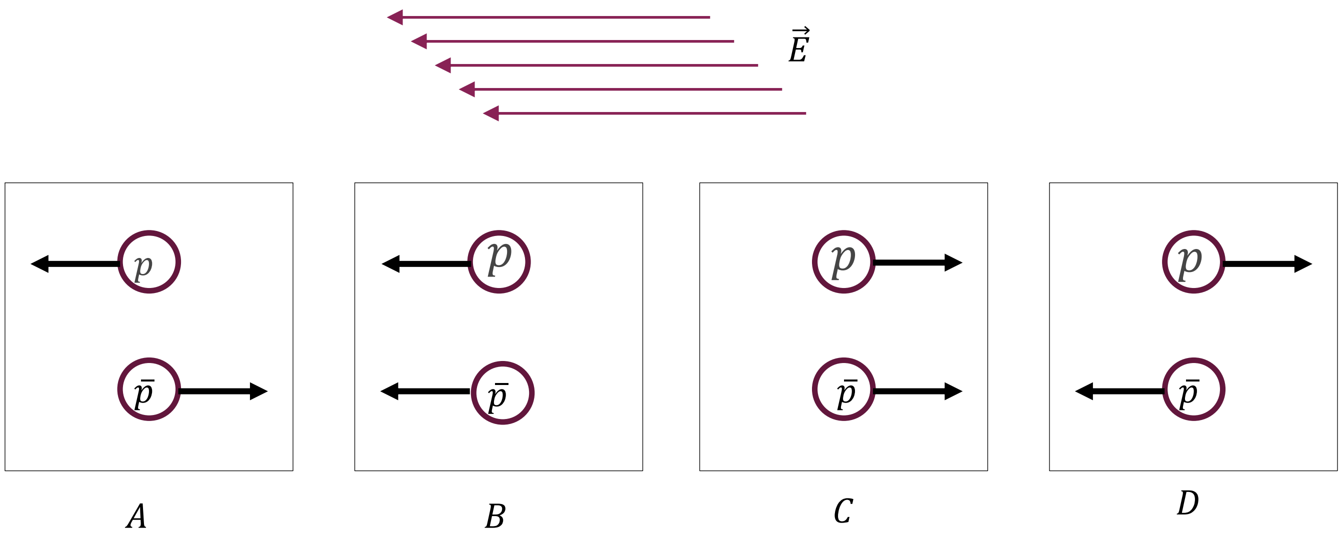
$\vec{E}$ vector points away from a positive charge & toward a negative charge

MAGNITUDE $\rightarrow$ $$E = k_e \frac{|Q|}{r^2}$$



A proton $(+e, m_p)$ and antiproton $(-e, m_p)$ are placed in the electric field shown in the figure.

Which picture correctly represents direction of the acceleration on each charge?



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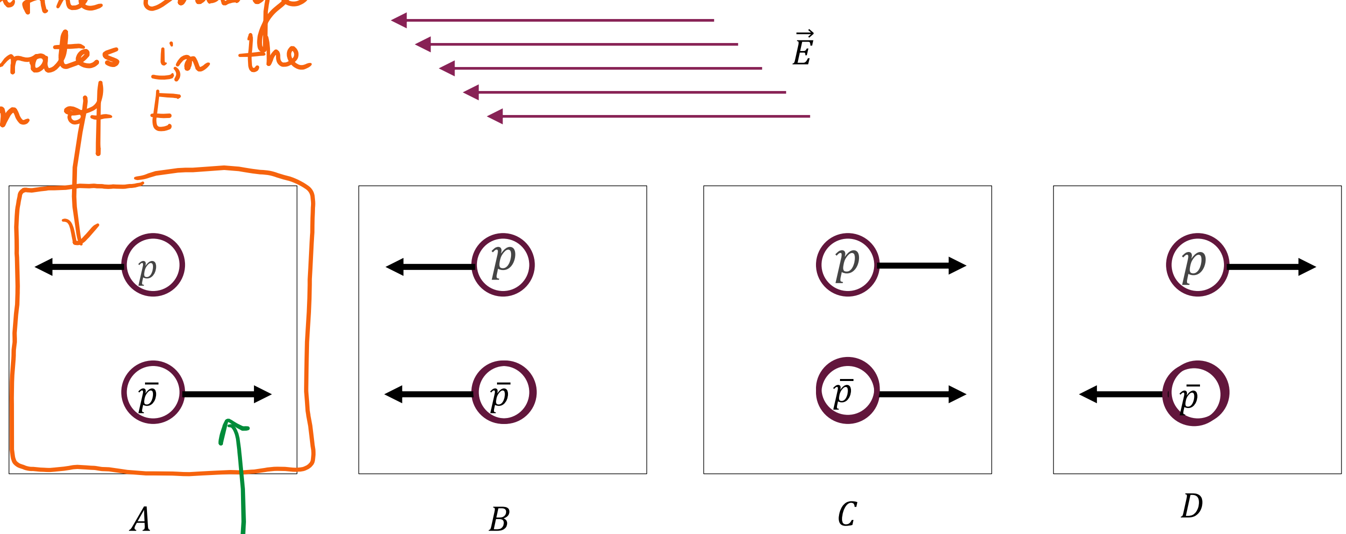
positive charge accelerates in the direction of $\vec{E}$

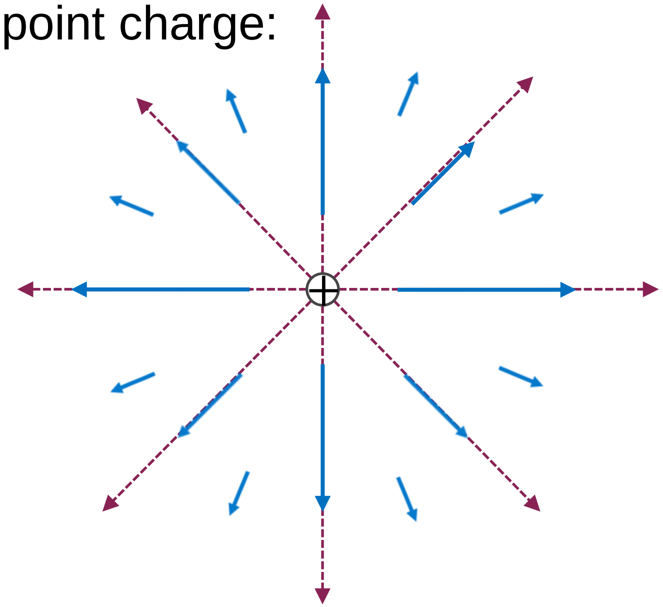


negative charge accelerates against the direction of $\vec{E}$

https://falstad.com/vector3de/

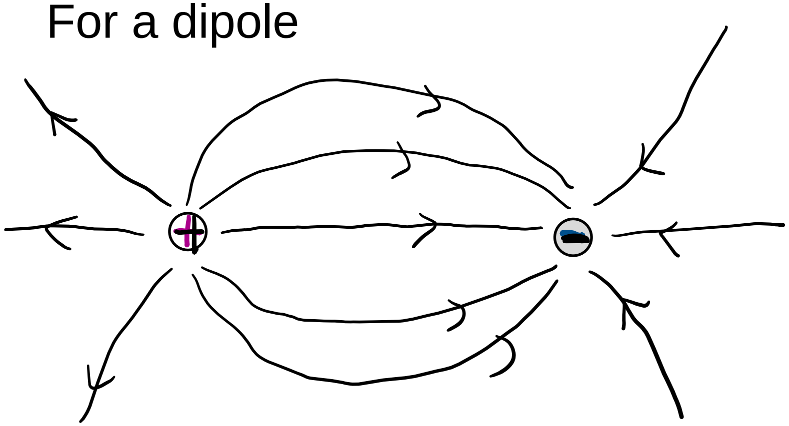
PICTURING THE ELECTRIC FIELD

Recall a point charge:

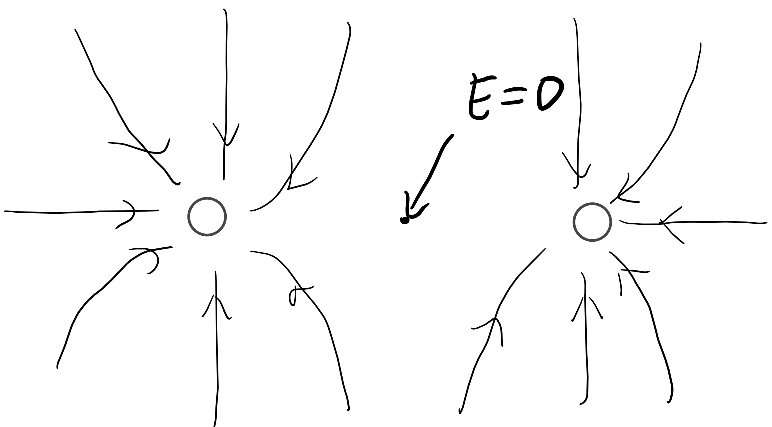


Electric field lines are drawn so they point away from positive charges and towards the negative charges. Electric field **vectors** at a given point are tangential to the electric field lines.

For a dipole



For two identical (negative) charges:



EXAMPLE 18A

Charge $Q = -1.00\ \mu\mathrm{C}$ is placed at the origin. Determine the electric field at points

a) $A = (0, 3.0)$ m

b) $B = (3.0, 3.0)$ m

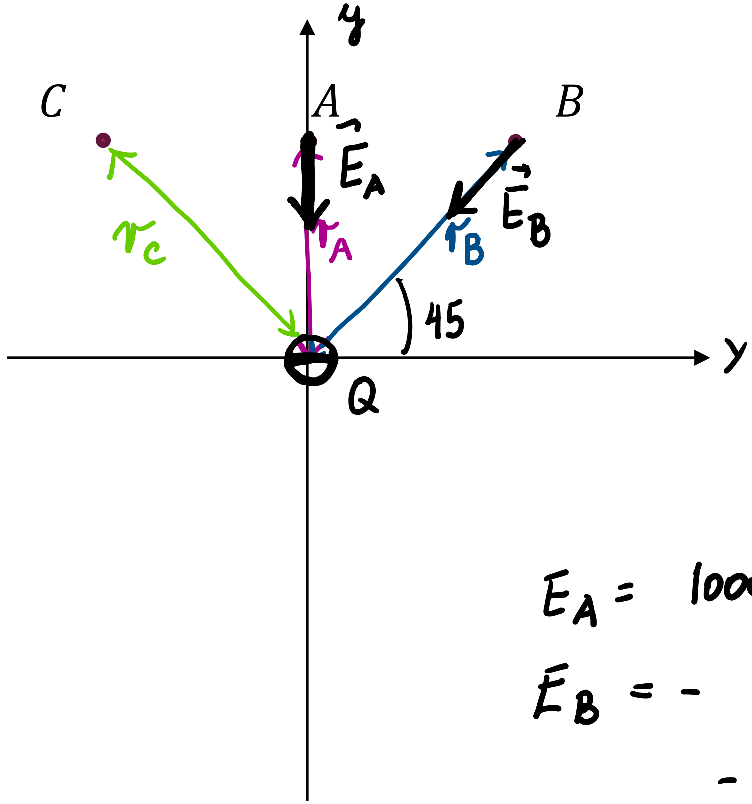
c) $C = (-3.0, 3.0)$ m

$$E_A = \frac{k_e Q}{r_A^2} = \frac{8.99\times10^9 \cdot |1.0\times10^{-6}|}{3.0^2}$$

$$\approx 1000\ \frac{N}{C}$$

$$E_B = \frac{k_e Q}{r_B^2} = \frac{(8.99\times10^9)\cdot|1.0\times10^{-6}|}{18.0}$$

$$\approx 500\ \frac{N}{C}$$



$$E_A = 1000\ \frac{N}{C}\quad -y \text{ dir}$$

$$E_B = -500\ \frac{N}{C}\cos 45\ x \text{ dir}$$

$$-500\ \frac{N}{C}\sin 45\ \ y \text{ dir}$$

ELECTRIC FIELD OF MULTIPLE POINT CHARGES

If the source of electric field is a set up of positive charges, the net electric field is the vector sum (superposition) of the electric fields due to each charge.

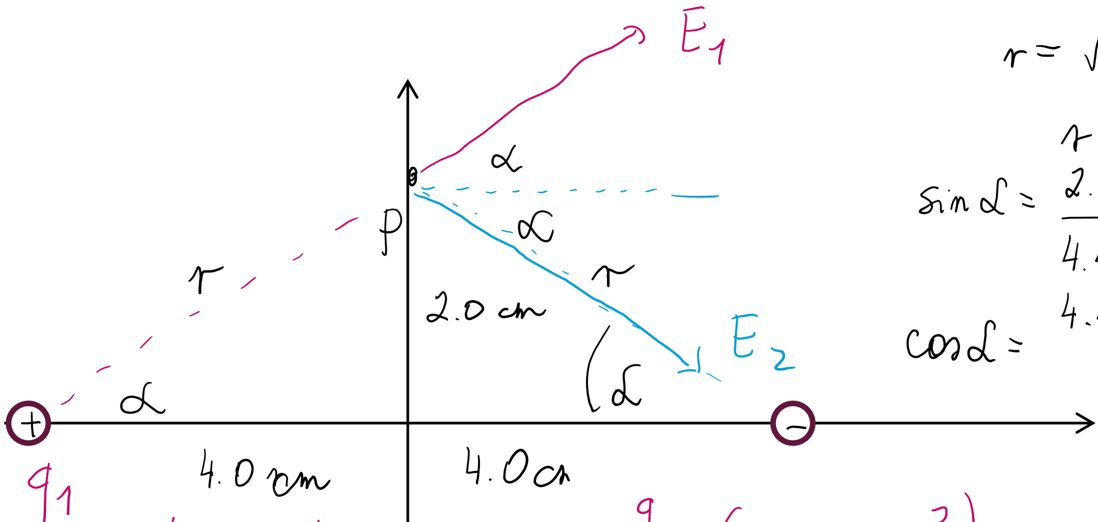
$$E_{net_x} = E_{1x} + E_{2x} + \cdots = \sum E_{ix}$$

$$E_{net_y} = E_{1y} + E_{2y} + \cdots = \sum E_{iy}$$

$$E_{net_z} = E_{1z} + E_{2z} + \cdots = \sum E_{iz}$$

EXAMPLE 18B

Find the electric field at point located at (0,2) cm due to two charges $q_1 = 2.0\ \mathrm{mC}$ and $q_2 = -3.0\ \mathrm{mC}$ located at (-4,0) cm and (4,0) cm, respectively



$r = \sqrt{0.02^2 + 0.04^2}$

$r = 0.0447\ m$

$\sin\alpha = \frac{2.0\,cm}{4.47\,cm} = \frac{2}{4.47}$

$\cos\alpha = \frac{4.0\,cm}{4.47\,cm} = \frac{4}{4.47}$

$E_1 = \frac{k_e |q_1|}{r^2} = \frac{8.99\times10^{9} \cdot (2.0\times10^{-3})}{(0.0447)^2} = 9.0\times10^{9}\ \frac{N}{C}$

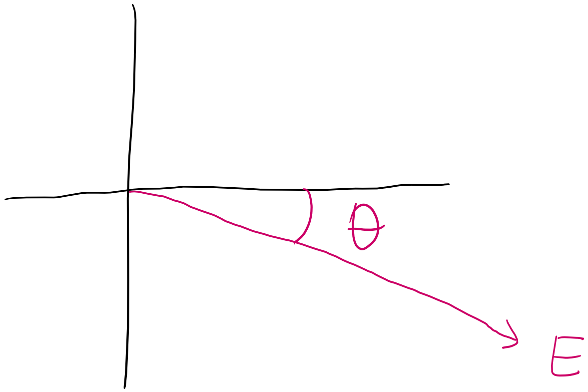
$E_{1x} = E_1 \cos\alpha = 8.05\times10^{9}\ \frac{N}{C}$ $\quad E_{1y} = E_1 \sin\alpha = 4.02\times10^{9}$

$E_2 = \frac{k_e |q_2|}{r^2} = \frac{8.99\times10^{9} \cdot (3.0\times10^{-3})}{(0.0447)^2} = 13.5\times10^{9}\ \frac{N}{C}$

$E_{2x} = E_2 \cos\alpha = 12.1\times10^{9}\ \frac{N}{C}$ $\quad E_{2y} = -E_2 \sin\alpha = -6.04\times10\ \frac{N}{C}$

$$E_{xT} = E_{1x} + E_{2x} = 20.2 \times 10^{9}\ \frac{N}{C}$$

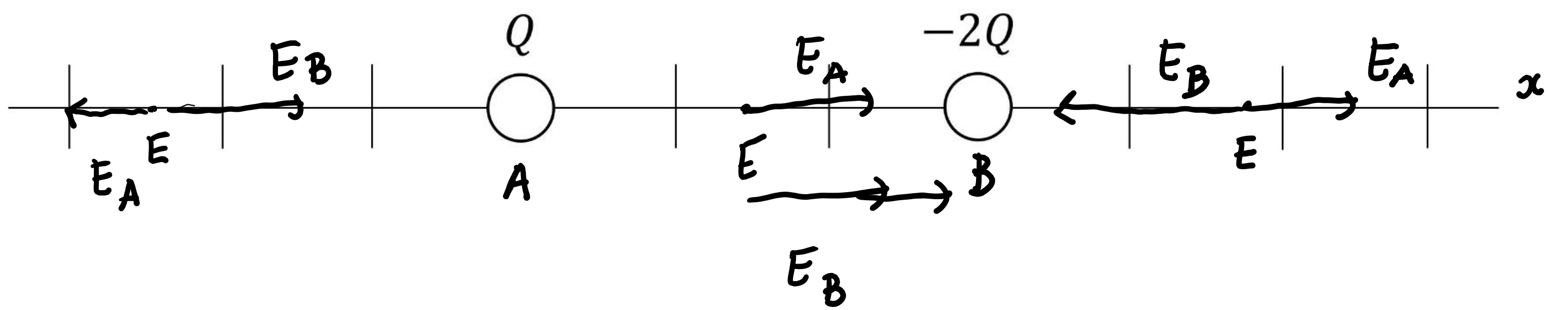
$$E_{yT} = E_{1y} + E_{2y} = -2.02 \times 10^{+9}\ \frac{N}{C}$$



$$\tan\theta = \frac{E_{yT}}{E_{xT}} = 0.1$$

$$\theta = 5.7^\circ$$

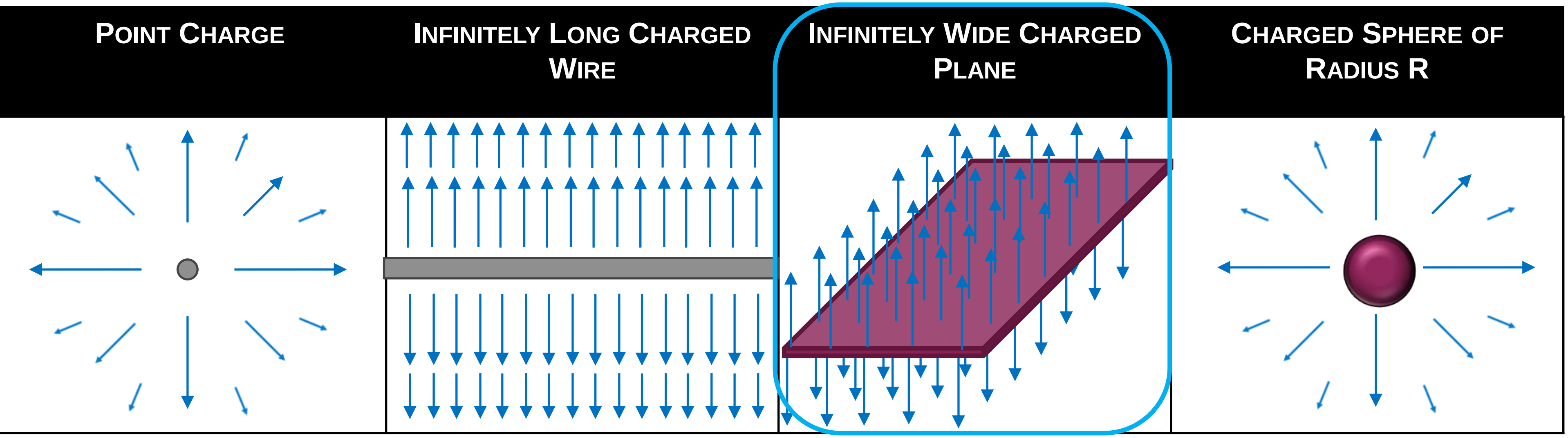
(below +x axis)



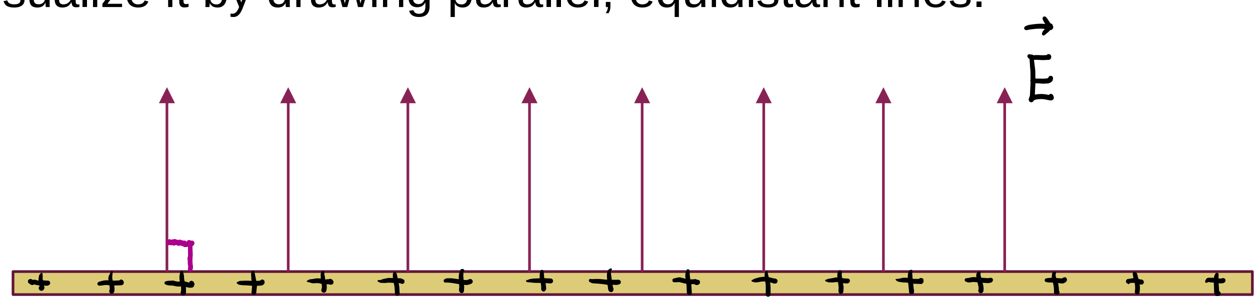
Q
$-2Q$
x
A
B
E_A
E_B
E

Uniform Electric Field

There are four basic electric field models



Uniform electric field is a field that is constant throughout the entire space.
We would visualize it by drawing parallel, equidistant lines.



ELECTRIC POTENTIAL OF A POINT CHARGE

We derived the idea of the field, by taking a force on a charge exerted by some source charge.

Now let's look at the potential energy of a system of two charges – a source charge Q and a test charge q:

$$U = \frac{kQq}{r}$$

The source charge modifies space around it by creating a **potential** for the energy of interactions with other charges.

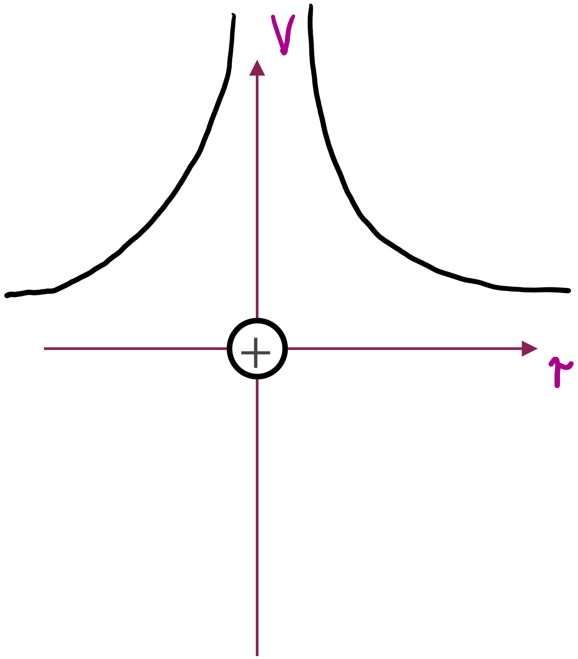
That characteristic of the source charge (any charge)

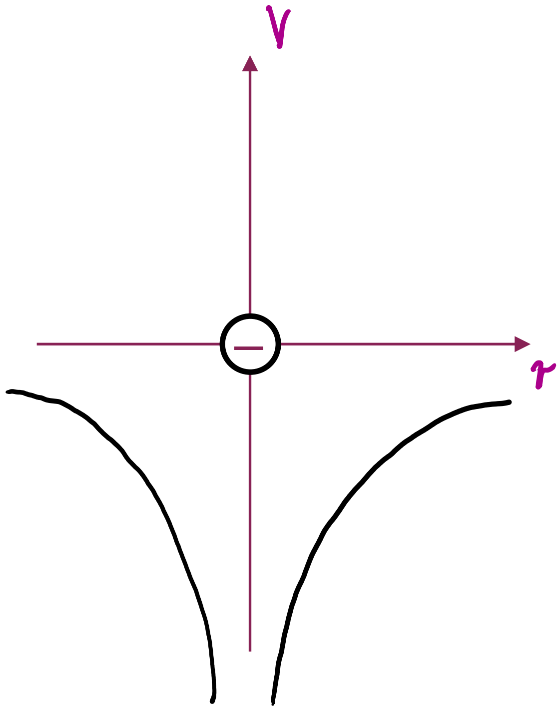
$$V = \frac{U}{q} = \frac{kQ}{r}$$

The unit of electric potential is joule/coulomb $(\mathrm{J/C})$ and it is called the **volt** (V)

ELECTRIC POTENTIAL OF A POINT CHARGE - VISUALIZE

$$V = \frac{kq}{r}$$



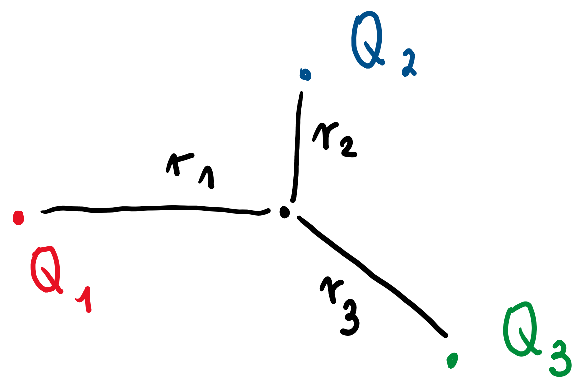



Q
$-2Q$

ELECTRIC POTENTIAL FROM MULTIPLE POINT CHARGES

While potential can be positive or negative, it is a scalar, therefore to find electric potential due to many charges, we simply add potentials sue to each charge in the space:

$$V = V_1 + V_2 + V_3 + \cdots = \frac{k_e Q_1}{r_1} + \frac{k_e Q_2}{r_2} + \frac{k_e Q_3}{r_3} + \cdots$$

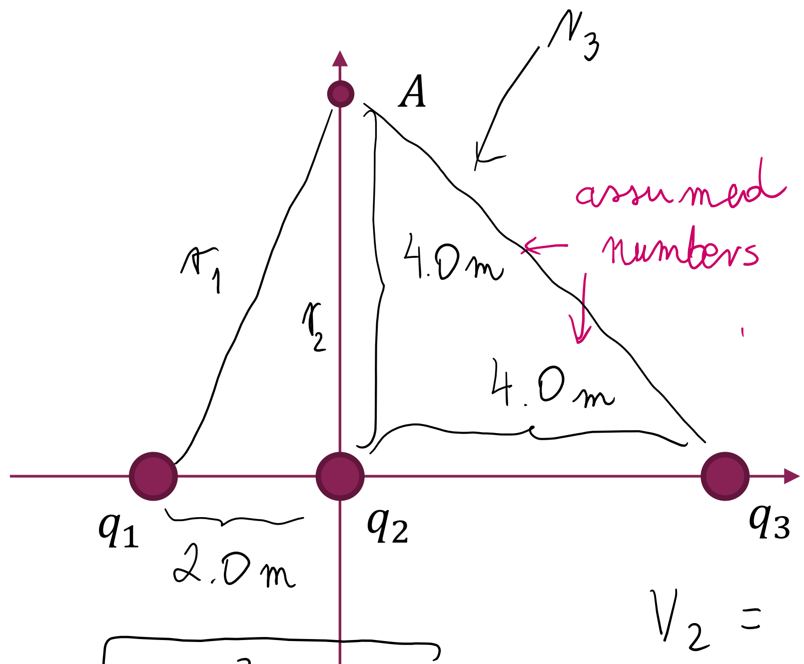


It's really that simple, you just add them!

EXAMPLE 18 C

Note: I doubled the distances (just to not have 1.0 m)

Three point charges, $q_1 = 10\ nC$, $q_2 = -20\ nC$ and $q_3 = 5.0\ nC$ are placed as shown in the figure below.
Determine total electric potential at point A.



$$V_A = V_1 + V_2 + V_3$$

$$V_1 = \frac{k_e q_1}{r_1}$$

$$r_1 = \sqrt{0.02^2 + 0.04^2} =$$

$$r_1 = 0.0447\ m$$

$$V_1 = \frac{8.99 \times 10^9 \cdot 10 \times 10^{-9}}{0.0447} = 2010\ V$$

$$V_2 = \frac{8.99 \times 10^9 \cdot (-20 \times 10^{-9})}{0.04} = -4495\ V$$

$$r_3 = \sqrt{0.04^2 + 0.04^2}$$

$$V_3 = \frac{8.99 \times 10^9 \cdot (5.0 \times 10^{-9})}{0.0566} = 794\ V$$

$$V_T = 1691\ V$$

LOOKING AHEAD: CONNECTING POTENTIAL AND FIELD

The four key ideas:

$\vec{E}, V$: *Affect* charges in surrounding space

IN SPACE

$$\begin{tikzcd} \vec{E} \arrow[d, leftrightarrow] & \vec{F} \arrow[l] \arrow[d, leftrightarrow] \\ V & U \arrow[l] \end{tikzcd}$$

$$\vec{F} = q\vec{E}$$

$$U = qV$$

ON CHARGES

$\vec{F}, U$: *Effect on* charges due to $\vec{E}, V$ in surrounding space

Electric potential and electric field are not the same but the are both related to **how charges ALTER the SPACE AROUND THEM**

Electric force and electric potential energy are related to **what a charge EXPERIENCES** in an electric field/potential created by other (source) charges.