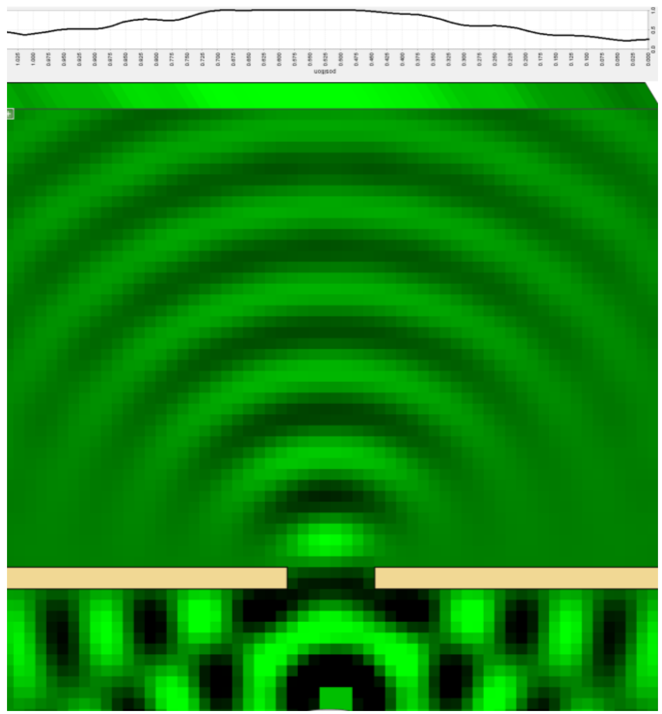
CHAPTER 24: WAVE OPTICS

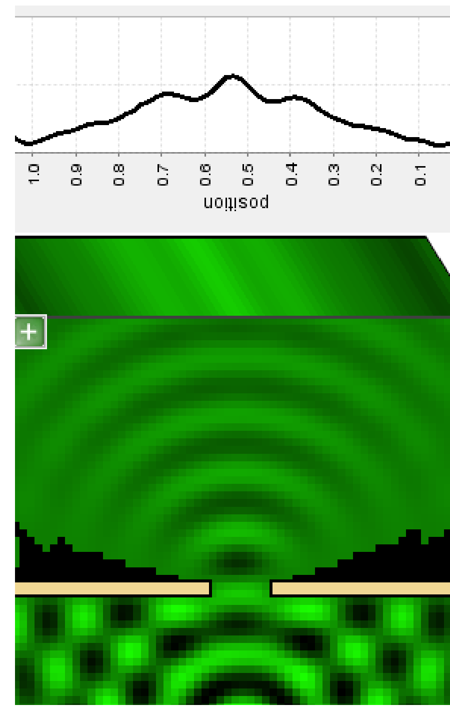
24.5 DIFFRACTION OF LIGHT

Diffraction was defined rather non-rigorously as a “flaring of light as it emerges from a narrow slit”.

Really – a pattern is produced (interference pattern) called a **diffraction pattern.**

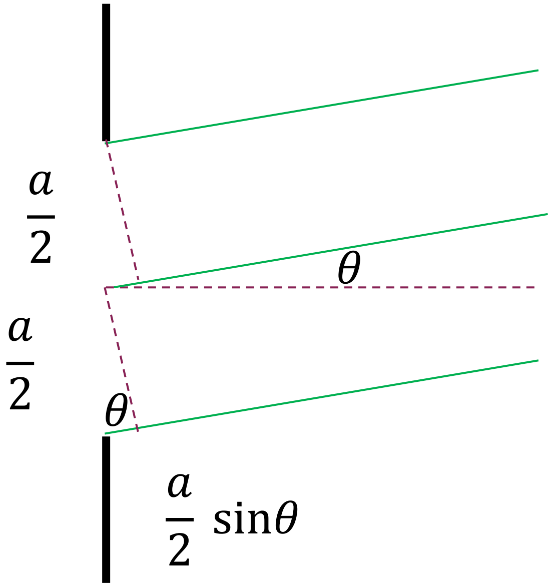
With light normal at incidence, the pattern is symmetrical about the axis of the slit. On a distant screen the path difference between the center (axis) and all the points, $\Delta r = 0$, so a maximum is observed.





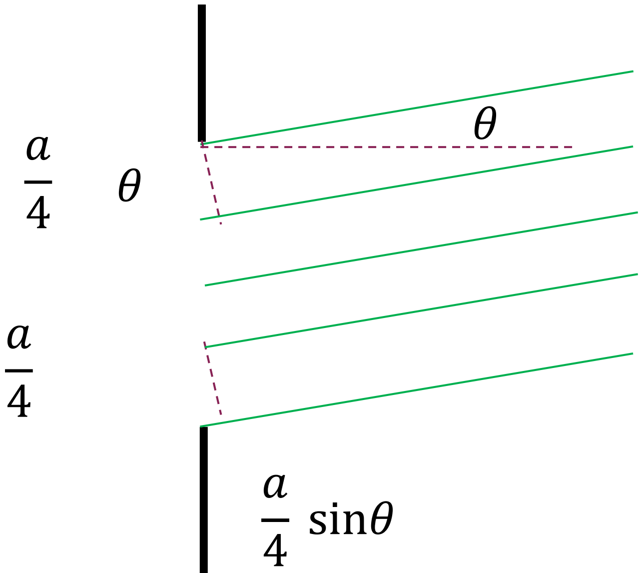
https://phet.colorado.edu/en/simulation/legacy/wave-interference

LOCATING THE MINIMA



Extra distance travelled: $\frac{a}{2}\sin\theta$

If $\frac{a}{2}\sin\theta = \frac{\lambda}{2} \rightarrow a\sin\theta = \lambda$ the rays are out of phase and interfere destructively.



Extra distance travelled: $\frac{a}{4}\sin\theta$

If $\frac{a}{4}\sin\theta = \frac{\lambda}{2} \rightarrow a\sin\theta = 2\lambda$

This could be extended into

$a\sin\theta = m\lambda$

SINGLE SLIT DIFFRACTION SUMMARY

Net result: **cancellation** (dark fringes) given by

$$\sin\theta = \frac{m\lambda}{a} \quad ; m = \pm 1, \pm 2, \ldots$$

Note that $m = 0$ is not included

Careful: don't confuse with 2-slit interference where $d\ sin\theta = m\lambda$ indicates positions of constructive interference.

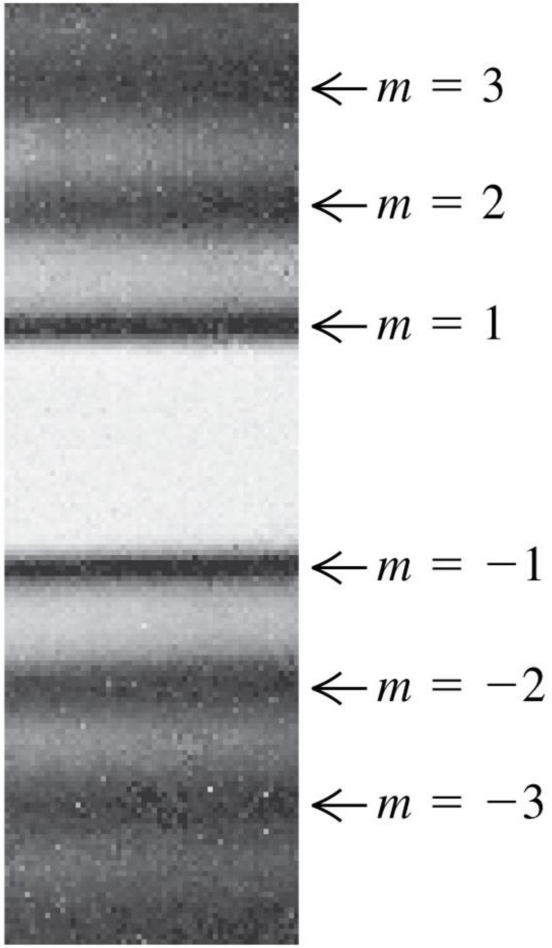


Fig 36.6. Photo of single-slit Fraunhofer diffraction.

INTERFERENCE VS DIFFRACTION

Interference:
$d\ sin\theta = m\lambda$

Diffraction:
$a\ sin\theta = m\lambda$

For **constructive** interference

$m = 0$ is valid

$\rightarrow$bright fringe on-axis

For **destructive** interference

$m = 0$ is not valid

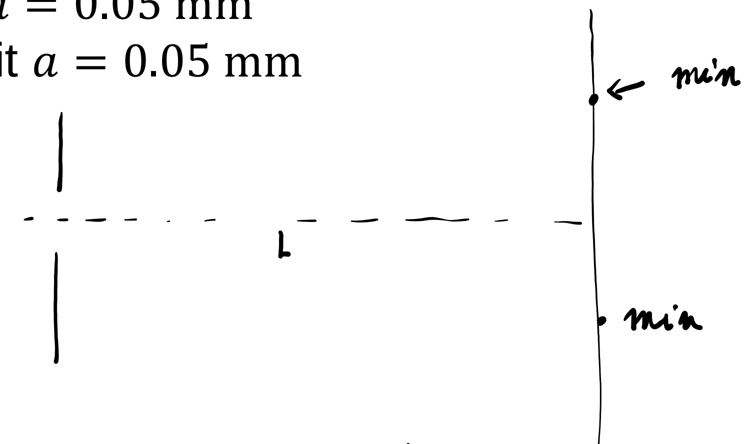
$\rightarrow$bright fringe on-axis
(hence no dark fringe)

EXAMPLE 24D

Determine the angles at which the first minimum occurs when

a) Red light ($\lambda = 750\,nm$) diffracts on a slit $a = 0.05\,\mathrm{mm}$

b) Purple light ($\lambda = 380\,nm$) diffracts on a slit $a = 0.05\,\mathrm{mm}$



a) $a \sin\theta_1 = m\lambda$

$a = 0.05 \times 10^{-3}\,m$

$\lambda = 750 \times 10^{-9}\,m$

$m = 1$

$a \sin\theta_1 = \lambda$

$\sin\theta_1 = \dfrac{\lambda}{a} = \dfrac{750 \times 10^{-9}\,m}{5 \times 10^{-5}\,m} = 150 \times 10^{-4} = 1.5 \times 10^{-2}$

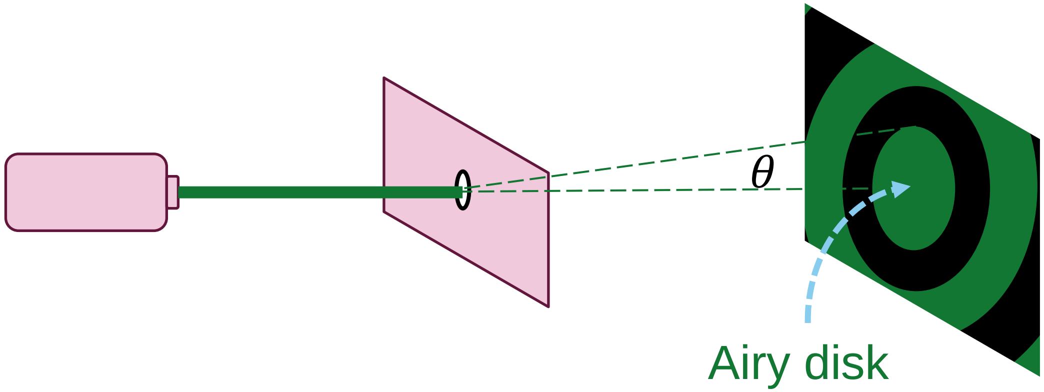
$\theta = 0.86^\circ$

b)

$\sim 0.435^\circ$

24.6 Resolving Power

The wave-like behaviour of light limits our ability to see two distant closely spaced objects as separate objects or to distinguish the details of an individual distant object.

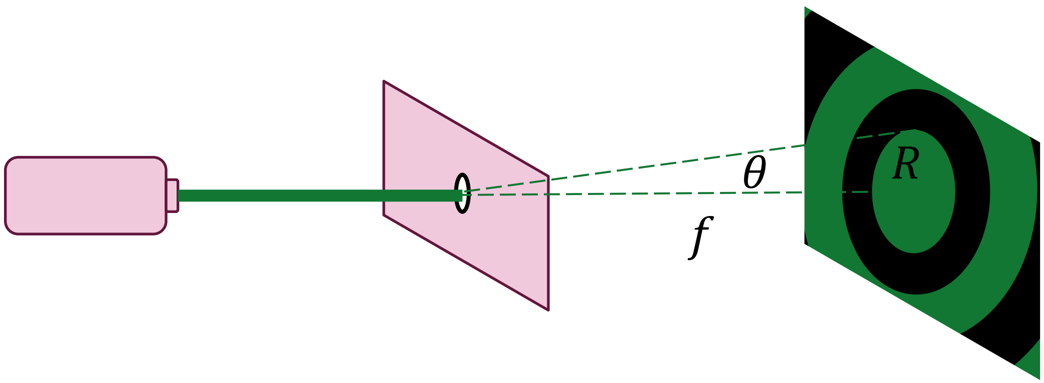


When a light passes through any size aperture (every lens has a finite aperture), diffraction occurs.

The result is a diffraction patter: a bright circular center surrounded by concentric dark and bright circles.

The diffraction pattern due to light passing through a small hole.

24.6 Resolving Power



The Airy disk (minimum size of the spot) on a screen distance f away from the aperture is

$$\tan\theta = \frac{R}{f} \rightarrow R = f\tan\theta$$

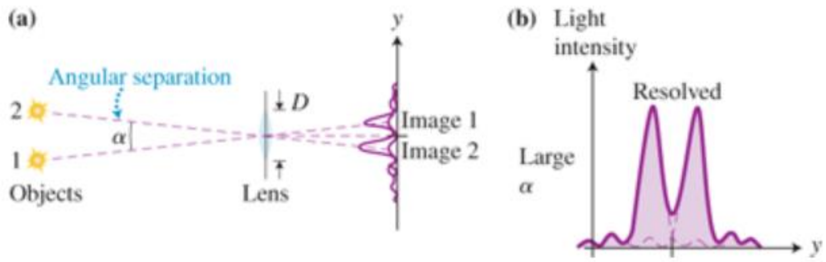
Airy (after whom the disk is named) has shown that

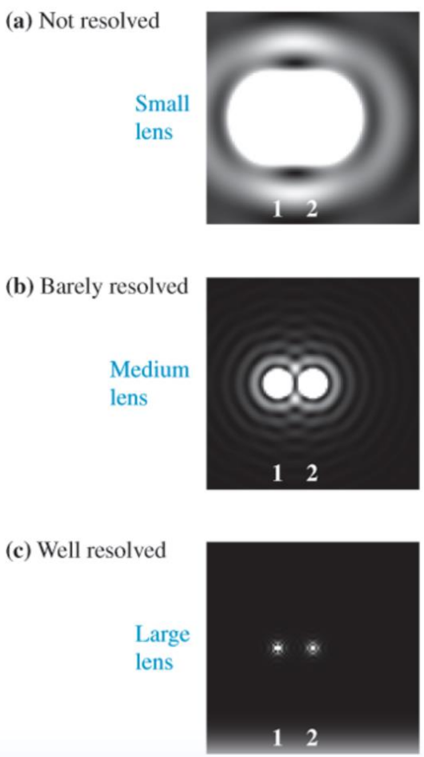
$$\sin\theta = \frac{1.22\lambda}{D}$$

Where D is the diameter of the opening and λ is the wavelength of the light.

The fact that light is a wave makes it **impossible** to form a perfectly sharp image.

Figure 24.28 The angular separation of two sources affects the ability to resolve them.





screen placed at the focal point of the lens ensures max resolution

$$f\ tan\theta = R_{airy}$$

radius of the center disk

$$\sin\theta = \frac{1.22\lambda}{D}$$

diameter of the aperture

$$\alpha_{res} = \frac{1.22\lambda}{D}$$

EXAMPLE 24E

The minimal angular separation of two objects that can be resolved (and perceived as separate) is the limit of resolution α_{res} of the instrument

$$\alpha_{res} = \frac{1.22\lambda}{D}$$

Determine the angular separation of two objects emitting light of wavelength $\lambda = 600\ \mathrm{nm}$ that can be resolved with an aperture of radius $r = 4.0\ \mathrm{cm}$.

$$D = 2r = 8.0 \times 10^{-2}\ m$$

$$\alpha_{res} = \frac{1.22 \cdot 600 \times 10^{-9}}{8.0 \times 10^{-2}} = 9.15 \times 10^{-6}\ \text{rad}$$

→ the value seems small but for example Sun is 150×10^{6} km away

$$\tan\theta \approx \frac{\Delta s}{\text{dist}}$$

→ $\Delta s \,\hat{=}\, 1000$ km

2nd CLOSEST STAR: $4.25\ ly = 4 \times 10^{16}\ m$ → $\Delta s \approx 10^{11}\ m$