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MODELLING RETAIL CENTRE SIZE AND LOCATION

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1. Introduction

The work of Coelho and Wilson (1976) and Harris and Wilson (1978) was primarily concerned with the generation of equilibrium solutions for the location of shopping centres. Much theoretical progress has been made since, and applications have been widened to a range of other urban and regional sub-systems. Most of the main ideas have been illustrated through a number of numerical simulations or experiments. The main objective in this paper is to test the ability of the Harris and Wilson model to reproduce the size and structure of the major retail centres of Leeds. This can largely be accomplished with contemporary data. A second task, to be reported in the light of further research, is to analyse certain types of retail change over time, such as the introduction of new forms of grocery retailing. This will draw upon time-series data far more, and also introduce further types of parameter disaggregation (Clarke, 1986b).

2. The model framework

Much of the work on retail modelling has been based on the production-constrained spatial interaction model of Lakshmanan and Hansen (1965). The model can be stated as

$$S_{ij} = A_i \, e_i \, P_i \, W_j^{\alpha} \, e^{-\beta c_{ij}} \tag{1}$$

where

$$A_i = (\sum_k W_k^{\alpha} \, e^{-\beta c_{ik}})^{-1} \tag{2}$$

to ensure that

$$\sum_j S_{ij} = e_i \, P_i \tag{3}$$

The variables are

S_{ij} - the flow of consumer expenditure from residential zone i to shopping centre j

e_i - the expenditure on retail goods in zone i

P_i - the population of zone i

W_j - the facility size, taken as a measure of shopping centre attraction

c_{ij} - travel cost or distance from residential zone i to shopping centre j

α,β - parameters to be determined

The traditional use of this model in planning terms has typically been to calculate the revenue attracted to each particular centre given exogenously supplied $\{W_j\}$ values. These might be observed values or planned values, or typically a mixture of both. In the latter cases, the purpose of the model run would be to test the impact of the new planned assignment of $\{W_j\}$ values.

Recent research by Harris and Wilson (1978) has placed the emphasis on the supply-side mechanism: to determine what constitutes the equilibrium configuration of $\{W_j\}$ values, now termed the structural variables of the system. This argument is based on the assumption that suppliers of retail facilities will expand their development if revenue exceeds costs or contract if costs exceed revenue. K is defined as the unit cost of providing retail floorspace and D_j as the revenue accruing to zone j. Thus

$$D_j = \sum_i S_{ij} \tag{4}$$

The dynamics of the system work as follows:

If $D_j - KW_j > 0$ then W_j will expand

or If $D_j - KW_j < 0$ then W_j will contract

Hence the equilibrium conditions occur when revenue and costs are exactly balanced:

$$D_j = KW_j \quad \forall_j \tag{5}$$

To satisfy this condition for all zones the following iterative solution procedure can be adopted (see Clarke and Clarke, 1984).

Step 1 : set all the W_js equal to some initial starting value

Step 2 : solve

$$\sum_i S_{ij} = \sum_i [e_i P_i W_j^{\alpha} e^{-\beta c_{ij}} / \sum_k W_k^{\alpha} e^{-\beta c_{ik}}]$$

Step 3 : let $D_j = \sum_i S_{ij}$

Step 4 : if $D_j = KW_j$ $(\forall_j)$ then we have an equilibrium solution. If not set $W_j = D_j/K$ and return to Step 2.

This equilibrium version of the model will be used throughout the paper. However it is important to note that alternative models have been developed since the Harris and Wilson (1978) paper with different hypotheses about both statics and dynamics and involving a number of different sub-systems. (For reviews see Clarke and Wilson, 1983, 1985).

Following the early theoretical work on the model Wilson (1979) concluded:

> "It is important, though difficult, to seek to develop some associated empirical work, partly to test the models and ideas, partly because the use of these models in planning will not be convincing otherwise." (p.12)

Wilson and Clarke (1979) provide the first attempts at implementing the theoretical work through a series of numerical experiments, since when many other papers have followed in similar fashion. However, many of these have concluded in the same vein, emphasising the need for more realistic data sets. For example, Clarke and Wilson (1983) conclude:

> "Much of the theoretical work can only be carried out adequately in the context of more effective empirical work. This is easier said than done given the difficulties of compiling time series data, but it is a vital next step." (p.17)

Similarly, Wilson (1984) simply comments:

> "What is needed now is the empirical justification of the model in a number of real contexts." (p.1427)

It is important to note that some empirical analyses for retailing have begun over the last few years. Lombardo and Rabino (1983, 1986) have attempted to apply the model for their case-study of Rome. Their conclusions still seem somewhat tentative although further work is progressing on both the theoretical and empirical properties of the model. Fotheringham (1983) and Fotheringham and Knudsen (1984, 1985) have undertaken a lot of empirical work in the US, adding a 'competing destinations' term into the basic model framework. This, however, focusses attention more on the individual shopper who is faced with a choice of either visiting isolated outlets or those grouped together in clusters.

However both of these examples are very different to the British urban retail environment which we wish to explore here.

3. Data for the Leeds study

In order to make progress on the empirical work we need two types of data. Aspects of retail change in Leeds since 1960 have been the concern of a much wider study (G. Clarke, 1986). Data has been built up, primarily for some thirty or so shopping centres in Leeds, from a variety of different sources : the data held by Leeds City Planning Office, Goad shopping centre plans, classified trade and street directories and the author's own survey. (More details appear in Clarke, 1986, Clarke and Macgill, 1984).

The second data requirement is for the main variables in the model, namely population and expenditure data and some kind of cost or distance interaction matrix. First, however, we need to focus on the specification of a suitable zoning system.

Much of the numerical experiment work of previous studies has been based on the 27x27 symmetrical grid zoning system, and there are obvious advantages in maintaining such a fine-scale spatial representation. The study area is defined as the whole of the Leeds C.B. and the Census wards of Pudsey, North and Horsforth to the West. The edge of this area is far enough from the city centre, in all directions, to either incorporate the rural-urban fringe or at least a break in the built-up area (ie. to the more densely populated western region).

With such a fine-scale zoning system we are able to make use of the most disaggregated Census data available: that is data collected at the Ed level (enumeration district). This data is readily available in Leeds for 1961, 1971 and 1981. Similarly, and vitally, maps of these Eds are also available for the different Censuses at the same scale of representation.

Although there are boundary changes in the spatial definitions of the Eds over the Census years, these provide no real problems when a grid zoning system is used. The procedure is as follows. We first work with the 1981 Census since this exhibits the most dispersed pattern of population in terms of suburban expansion and hence covers the greatest number of zones. The grid system can now be overlapped onto the Census map for 1981, making certain that the central zone (i.e. number 365 in our case) covers the heart of the Leeds CBD area. This is shown in Figure 1 with the wards, rather than Eds, as background. The remaining areas of the city (in terms of Eds) can then be marked off into their respective grid zones. For the most densely populated areas of the city a large number of Eds (between two and ten) make up one grid zone. From the Ed Census data the population total for each zone can then be calculated by summation. Conversely, at the suburban fringe, one Ed may cover anything up to five grid zones or more. In this latter case the population allocated to each grid zone is carefully worked out for 'built-up' areas on the Census map. Apart from the western and eastern edges of the system (which run into the Bradford CB area and green-field land respectively) there are a large number of grid squares which are 'external' to the Leeds CB area. To help take account of some shoppers travelling into the Leeds CB area these zones were given a very small demand of 20 units each. Exactly the same procedure can be undertaken for 1971 and 1961.

Having plotted population totals for each zone the next task is to ascertain some measure of retail expenditure within each zone. (This in many studies is ignored and population alone is taken as the measure of demand.) For 1981 (and 1971) we can obtain population totals disaggregated into the various socio-economic groups (SEGs), for which expenditure on retail goods can be calculated from the respective Family Expenditure Surveys. [For 1961 we have to use a different status indicator because of the unavailability of SEG data. Population disaggregation by tenure category proves to be the best substitute since expenditure patterns for these groups are also available from the Family Expenditure Survey.]

Clearly the building up of such a fine-scale zoning system is a very time consuming process, yet it provides the opportunity for a far more detailed analysis. For example we can explore facility location across all 729 zones or we can concentrate on allocating facilities between the known shopping centres of Leeds. This reduces the analysis to a more realistic 729 (demand points) x 33 (supply points) zoning system. We shall explore both possibilities for illustration below. However, one disadvantage concerns the formulation of a suitable cost or distance matrix. The only feasible measure of deterrence for a 729 zone system (729x729, or 729x33 matrix) has to be Euclidean straight-line distance. This is easily calculated in a computer program and has already provided the backbone for other numerical experiments. Thus the cost matrix $\{c_{ij}\}$ is given by

$$c_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad \forall_i, \forall_j \tag{6}$$

where x_i, y_i and x_j, y_j are the spatial co-ordinates of two zones i and j. To make this Euclidean distance matrix more realistic for urban travelling, distances to the city centre can be factorised to reflect the fact that most major routes converge on the city and hence travel times to the city centre are generally quicker. Hence we take

$$c\,(j,\,365) \;=\; v \times \text{Euclidean distance} \qquad (7)$$

where v takes a series of values less than one so that the city centre (zone 365) is favoured to varying degrees (Clarke and Wilson, 1983).

4. Parameter exploration

In this section we take each of the main parameters of the model and see how variations affect the model solutions.

4.1 e_i P_i

Having built up such detailed zonal demand data it is interesting first to examine how changes in these values can affect the model solutions. For illustration we can use the 1961, 1971 and 1981 demand data and allow facility location in any of the 729 zones. Figure 2 shows a model solution for each of the three key years. The other parameters of the model have been arbitrarily fixed (for now) as constants, hence the differences between the model solutions can be wholly attributed to variations in the demand data. [Note in all the subsequent figures the peaks (the Z dimension) represent the amount of facilities allocated to a particular zone within the spatial system (the X and Y dimensions)].

The overall outcome of the model runs is very much as expected. As the population expands in the outer suburbs between 1961 and 1981 the model allocates more facilities both to the north and east of the city especially, but also to the west as well. Conversely it is possible to pick up the effects of de-population within the inner-city areas since 1961, especially to the north of the city centre.

4.2 α and β

To explore the impacts of changing other parameters we work mainly with one time period and hence use the 1981 population and expenditure totals (e_iP_is). The effects of β variation are shown in Figure 3 and α-variation in Figure 4. These two figures show that the parameters are operating in exactly the same fashion as in the range of numerical experiments mentioned in section 2. That is, the higher the value of α the more concentrated facilities become (α traditionally representing scale economies) and the higher the value of β the more dispersed facilities become (β representing the ease or willingness of a consumer to travel).

4.3 K (or K_j)

The K parameter was described above as the unit cost of providing retail floorspace. The effect of changing this parameter is shown in Figure 5. A higher value of K (Figure 5b) tends to produce a more concentrated pattern of facilities as smaller centres struggle to compete given higher costs. Conversely a lower K value (Figure 5c) reduces this handicap and smaller centres are more able to compete.

We can build in the effects of space on costs by replacing K with K_j and adding a suitable submodel. In Figure 5d K_j is taken to be inversely proportional to the distance of the zone from the centre, reflecting the broad generalization that faiclities are cheaper to supply away from the centre: note in this case how facilities at the suburban edge become far more dominant. (We shall explore K_j values in more detail in section 7.)

4.4 v

The parameter v was incorporated in equation 7 to reflect cheaper travel to the city centre, and it was noted that it should take a series of values of less than one. In Figures 6a-b the effect of reducing v can be seen whilst Figure 6c shows the actual spread of major retail facilities in Leeds in 1982. It can be seen that a value of 0.75 produces the right sort of scaling between the city centre and the largest suburban centres. (The city centre dominates for a variety of reasons but we shall use this value of v until we look in more detail at defining the attractiveness of centres in a more realistic fashion in section 8.)

5. Refining the spatial system

In the experiments so far, facility location has been allowed in all 729 zones of the city. As we described in section 3, we can build in an element of 'inertia' by specifying the zones which currently house the major supply of facilities in the city (namely the 33 shopping centres shown in the Appendix). Adopting this change we are now faced with the 729x33 spatial system for 1981. Again we can ask how facilities will be allocated between the 33 centres for a range of different parameter values. The parameters themselves are, of course, still working in exactly the same way, so for illustration we concentrate just on β-variation: Figure 7 shows the nature of this variation across the 33 supply centres.

Clearly there are a whole range of intermediate solutions between the four shown in Figure 7. One interesting question which follows then is which set of parameter values gives the 'best' fit compared to the known pattern of facilities in 1982?

6. Calibration and equilibria

The concern to match model solutions to real world centre sizes raises the issue of model calibration. The process of calibration involves estimating the parameters of a model to optimize one or more goodness of fit statistics between the model predictions and real world observations (excellent reviews appear in Wilson 1974, Batty 1976 and Putman 1983). Generally there are two alternative criteria on which models can be calibrated. The first is the so-called 'ij-calibration' procedure which compares real-world interaction flows to those predicted by the model. The second technique involves the 'j-calibration' procedure which compares real world shopping centre size or turnover (for the retail model) with those predicted by the model. It is generally agreed that 'ij-procedure' is the most reliable and indeed Openshaw (1973, 1974) has argued that all calibration procedures are highly suspect when the interaction flow data is not available.

Although a full trip interaction matrix is not available for our case-study of Leeds, it is still interesting to see what values emerge from employing a simple 'j-calibration' procedure such as 'minimum sum of squares'. The idea here is to estimate the α,β combination which minimises the sum of squares function, given fixed values of the remaining parameters: $v=0.75$, $K_j=1$ ($\forall_j$), $e_iP_i=1981$ values

$$\text{Min SS } (\alpha,\beta) = \sum_j (W_j^{obs} - W_j^{mod})^2 \qquad (8)$$

where W_j^{obs} is the known centre size

W_j^{mod} is the centre size predicted by the model

Having to fix the other parameter values shows another problem of calibration in the traditional sense : we are already faced with a number of different parameters in the model which we can anticipate will increase in number as we attempt to include more of the detail present in the real world system. This makes the simultaneous estimation of all the parameters impossible in any conventional sense. As Clarke, M. (1986) explains, this leads us more in the direction of sensitivity analysis rather than formal calibration, where the emphasis is much more on parameter exploration and the qualitative nature of the solutions. Under this procedure the best two combinations of α,β turn out to be $\alpha=0.90$, $\beta=1.75$ and $\alpha=1.00$, $\beta=2.15$. The so-called 'bogus calibration' (cf. Cordey-Hayes, 1968), where $\alpha=1.00$ and $\beta=0.00$, was avoided by only searching for solutions where β was greater than 1.00.

Figure 8 shows the equilibrium solutions for the two combinations of α,β against the real world patterns for 1982. There are clearly some marked differences. In the equilibrium model, under the iterative procedure described in section 2, the most 'attractive' centres in the system expand whilst the least attractive decline. This can be clearly seen in relation to the city centre in Figure 8 for example and the effect this has on surrounding centres nearby. We can see this process more clearly by stopping the iterative procedure after a certain number of steps and looking at the consequent allocation of facilities. This is shown in Figure 9 for one set of the calibrated values, $\alpha=1.00$, $\beta=2.15$. If we start with one iteration, Figure 9a, then we see all centres able to attract facilities although the size differentials are not great. After 5 iterations (Figure 9b) the most 'attractive' centres are beginning to dominate, and after ten iterations the solution is fast approaching equilibrium.

The equilibrium solution is worth looking at more closely. It shows the 'optimal' location of facilities given contemporary population and expenditure patterns and the given values of the parameters. It shows a major focus of facilities at the city centre, with an inner ring of smaller suburban facilities flanked by an outer ring of much larger centres (a pattern increasingly apparent in the USA where less rigorous planning control has allowed such a pattern to develop). Clearly the affluent northern suburbs are especially attractive in the model, a fact borne out in real life by the number of planning applications for retail developments in this area.

However what the model cannot so far account for are the much older, now declining inner-area shopping centres, many of which are the traditional ribbon developments formed along early tramway routes. Even since 1960 these have greatly declined both in size and in quality of outlets (Clarke and Macgill, 1984), yet still survive often by providing a number of unique activities such as antique shops, cut-price stores, junk shops, etc., alongside the basic food products. The question is therefore how can we make these sorts of centres 'viable' in a modelling sense? In reality, retailers may still locate in these smaller centres since it is cheaper than in larger centres where multiple traders are more likely to gather and consequently push up rates and rents. This suggests that we can make considerable progress in making these centres attract facilities in the model by looking at the cost term in far more detail and later on by considering the attractiveness of centres in more detail.

7. Disaggregating the cost function

For most of the previous model runs (Figures 2-9) the value of the cost function K has been taken as one which means that revenue is directly proportional to the size of centre. In real cities the cost of operating at partiuclar sites varies spatially across the city, normally in relation to the size and importance of an individual centre. Rents, for example, are far higher in the main areas of the city centre and major suburban centres and conversely far less in smaller, less attractive centres. Clearly rents are not the only component of costs, but are nevertheless one of the most powerful determinants of retail location. Wilson (1983) provides a richly disaggregated cost function whilst Birkin and Wilson (1985) define the cost of provision at a particular location as (keeping the terminology of this paper)

$$K_j = (C_j + r_j)\ W_j \tag{9}$$

where C_j are assumed to be non-varying costs (buildings, labour, etc.) and r_j is unit land rent.

Although we lack the detailed information on costs it is an interesting exercise to find a set of K_j values, within the equilibrium model, which can reproduce the real world W_j values of the shopping centres in Leeds: that is, we would expect lower overall cost values for the smaller, less attractive centres and higher overall cost values for the most attractive sites.

To achieve this we have designed a very simple iterative procedure for calculating K_j values. It is important to realise that the set of K_js to emerge from this procedure are relative rather than absolute or real values, and worth re-emphasising that the question we are asking is what kind of K_j value does each centre require to attract facilities in proportion to its real-world size and in respect to the size of all other centres?

To obtain the set of K_js we use the following iterative procedure (first described in Clarke, Clarke and Wilson, 1986): using the equilibrium model we wish to find $\{K_j s\}$ so that

$$\sum_i S_{ij}^{Pred} = W_j^{obs} \qquad \forall_j \tag{10}$$

Thus if

$$\sum_i S_{ij}^{Pred} < W_j^{obs} \tag{11}$$

which implies the model is under-allocating facilities to a particular centre, then let

$$K_j = K_j - inc \tag{12}$$

and recalculate $\{S_{ij}\}$

Inc is given by

$$eps \left(\sum_j W_j^{obs} - \sum_i S_{ij}^{Pred}\right) \tag{13}$$

and eps is a suitable constant.

Conversely if

$$\sum_i S_{ij}^{Pred} > W_j^{obs} \tag{14}$$

which implies the model is over-allocating facilities to a particular centre, then let

$$K_j = K_j + inc \tag{15}$$

and re-calculate $\{S_{ij}\}$

Note: W_j^{obs} is the known 1982 facility size of centre j, $\sum_i S_{ij}^{Pred}$ is the amount of facilities allocated by the model to each centre.

It is important to appreciate with this procedure that only the set of K_js are calculated: α,β,ν are input at the outset - α=1.00, β=2.15, ν=o.75. All K_js start the iterative procedure with the value of 1.00.

Figure 10a shows the actual 1982 configuration of retail centres in Leeds to aid comparison with the model outputs. Figure 10b shows the set of K_j values to emerge from operationalizing equations 10-15, whilst Figure 10c shows the resulting equilibrium solution when these K_j values are inputted into the model at the outset (the actual allocation of facilities is shown in Figure 10d to give a flavour for the size differentials).

A comparison of Figures 10a and 10c shows that whilst the 'outer' centres are reproduced reasonably well there are some problems of fit for the 'inner' centres. The explanation is as follows: because the city centre is so attractive relative to the smaller centres close by, the set of K_js to emerge after a set number of iterations (ie. Figure 10b) are forced to be very low for those smaller centres in order to combat the attractiveness of the city centre. When, However, such low K_j values are inputted to the model from the first iteration then it is possible for these centres to compete with the city centre for dominance. Clearly this is what has happened with Kirkstall Road (see Appendix) shown in Figures 10c and 10d. Hence the first run of our iterative procedure has produced a reasonable fit yet some K_j values appear too extreme. One solution to this problem is to input this set of K_j values (Figure 10b) into the iterative scheme from the outset rather than starting with all K_j values equal to one.

Figure 11b shows the new set of K_j values to emerge from the second run of our iterative procedure and these can be inputted back into the equilibrium model to produce the configuration of centres shown in Figure 11a. A comparison of Figure 10a and Figure 11a shows that a much better representation of reality now occurs. However there is still some mismatch between the model solutions and the known pattern of facilities in Leeds, again because of this 'problem' of inputting the set of K_j values to emerge after a set number of iterations into the equilibrium model from the outset. However we can argue that we have created a 'feasibility region' within which it is possible to manipulate the parameters, through trial and error, to get a final set of K_js which produces an almost exact replica. Figures 11d and 11c show the final set of K_j values and the corresponding equilibrium pattern which results from inputting those K_j values. (Compare now Figures 10a and 11c.)

Clearly this final set of K_j values shows that some of the smallest real-world centres do still need to have relatively low values of K_j in order to attract facilities whilst some of the larger centres have much higher values to stop them becoming too dominant. These values would also make sense in terms of real costs in different centres. Rents for example are far lower in smaller, less attractive, centres and much higher in the city centre and larger suburban centres. However, there are clearly some anomolies, all of which are explained by the relative isolation of such centres which forces the model to want to locate facilities in these regions : hence the very high K_j values which these centres attract to stop facilities being allocated. Middleton, to the southern edge of our spatial area, is one such example with a K_j value of over 2!

Middleton is a small centre of some twenty retail outlets in the heart of mainly council housing. It was built up in the inter-war period to act as an 'overspill' area for many of the disadvantaged residents of the city centre. Today it is still a relatively isolated area with little development to the south, east or west, although the latter region is now attracting new private home developments. In terms of retailing, apart from a few isolated parades, there is little development between the small Middleton centre and Beeston and Hunslet to the north. It is not surprising therefore that the models wish to allocate extra facilities to this area. It is also interesting to note that Leeds Planning Department had indeed been trying for some years to attract major retailers to this part of the city, though the retailers themselves have been generally keener on the northern edge of the city where potential demand and wealth seems greatest (see below). Prestos finally agreed to a new site close to the present Middleton District Centre and thus since 1982/3 (and our last retail survey) a new centre has indeed emerged in this area.

This new development can be built into the model and the subsequent set of K_js recalculated. As Figure 12 shows the size of the K_j value for Middleton and the new shopping centre are considerably lower and more realistic. It is interesting to note that the new centre attracts some of the trade previously assigned to the Beestons, Hunslet and Middleton. Hence these centres now have slightly lower K_j values as well. The success of the new Middleton centre will be of considerable interest over the next few years given the results from the model runs. (Birkin and Clarke (1985) also explore some fully dynamic versions of the model to test the stability of retailing in this area as against centres nearby).

Apart from Middleton, we can see that the northern edge of the city also produces high K_j values from the iterative procedure (see again Figure 11d). This again reflects a degree of under-development here despite two new centres at Holt Park and Moor Allerton since 1975. Indeed this has been one of the most sought after areas for retail development with a large number of planning applications for new superstores, from a large number of the major retailing companies. The main attractions here are excellent communications, green-field sites and the wealthiest host population in the whole of the city. The lack of more developments very much reflects the policy of Leeds City Planning Department in curtailing the amount of superstore developments allowed within the city.

At the end of this analysis then we have a set of K_js which can reproduce the known sizes and distributions of centres for 1982 data. It is interesting now briefly to compare this data set with one we can attain from 1961 known retail centre sizes. The argument proceeds exactly as before and so for brevity we can show the final set of K_js to emerge, and the subsequent equilibrium solutions, from our iterative procedure with 1961 e_iP_i data and 1961 known retail centre sizes. It is difficult to compare the two final sets of K_j values directly because of the very different e_iP_i values that influence the final pattern. However some interesting general comparisons are possible.

Figure 13a shows the real pattern of retail facilities in Leeds in 1961; whilst Figure 13c shows the set of K_j values for the 1961 data and Figure 13b shows the resulting equilibrium model solution when these K_j values are inputted to the model. The higher K_j values seen on the northern edge of the city for the 1981 data are now manifest in the earlier suburban ring of development for the 1961 data. This can be seen especially for centres such as Oakwood, Moortown, Hollin Park, Harehills and Chapel Allerton (see Appendix). This emphasizes the pressure on these areas for new facilities as the population was still moving rapidly northwards and outwards.

The second major feature of the 1961 data is the dominance of the city centre; far greater than for 1981. With a more 'compact' population the city centre becomes even more attractive especially with higher e_iP_i values in the inner suburbs to draw upon. Again this is not unrealistic in terms of major retail developments in the city in 1961.

So far we have argued that the introduction of individual K_j values makes sense in the search for more realistic solutions, since smaller centres often need reduced cost terms in order to compete with the larger centres and that this makes sense in terms of real variations in costs such as rents. Clearly however costs are only one determinant of retail location. Although a centre may have much lower rents which therefore attracts some retailers, other retailers will be put off from locating in smaller centres which the public see as 'unattractive'. Hence it is important to offset the attractiveness of a centre against the costs of location. This forms the subject matter of the next section.

8. Defining the attractiveness of centres

So far in our analysis we have taken the attractiveness of a centre as W_j^{α}, that is the size of the centre raised to a power to reflect consumer scale economies. More precisely, the attractiveness is measured by the size of supply : if D_j is the total revenue attracted to j then (for a reminder)

$$D_j = \sum_j S_{ij} \qquad (16)$$

We assume that

$$\text{if } D_j - K_j W_j > 0 \text{ then } W_j \text{ will expand}$$
$$\text{and if } D_j - K_j W_j < 0 \text{ then } W_j \text{ will contract}$$

The equilibrium condition is thus,

$$D_j = K_j W_j \qquad (17)$$

We wish to retain this basic cost/revenue balancing mechanism yet at the same time incorporate the idea that attractiveness can be made up also of components which vary with location or shopping centre (j). The attractiveness of shopping centres has been the focus of a number of previous studies. Kilsby et al (1972) identified seven components including size, quality and environmental attributes. Similarly Timmermans (1981) and Spencer (1978) have looked at alternative attractiveness factors other than simply centre size. In terms of the Harris and Wilson (1978) model, Wilson (1983) has outlined a detailed attractiveness function disaggregated not only by centre (j) but also good type (g) and shop type (h). Fotheringham (1983) and Fotheringham and Knudsen (1984, 1985) have added a term A_j to incorporate competing destinations in the attractiveness function whilst Birkin and Wilson (1985) have recently added a term p_j^{-y} to incorporate prices.

To determine the attractiveness of each retail centre in Leeds we define four basic components. First, a measure of the variety of different functions or goods available at each centre. Clearly the more goods and services available at each centre the more attractive the centre becomes to the general shopper. This ties up most closely with Fotheringham's notion of competing destinations. (Though note we are interested with shopping centres rather than individual stores.) The city centre emerges as providing the most facilities and all other centres are factored according to their score against the city centre total. Second, we add a measure which might reflect the relative importance of a particular centre, in this case the number or percentage of retail 'multiple' groups (defined as owning ten or more retail outlets) in each centre. Again the highest score occurs for the city centre and also the new shopping centre at Hunslet. Each other score is factored accordingly. These two measures are 'objective' in the sense that they can be quantified relatively easily. The data for measure one comes from Clarke and Macgill, 1984 whilst the details in G.Clarke, 1984 provides the data for the second measure on multiples.

The next two components are more 'subjective' in that although they can be measured in quantitative terms the values they are assinged are largely determined on the basis of the author's knowledge of the centres involved. It is hoped that those who know some of the Leeds centres will roughly agree with the judgements made. The third measure aims to capture the 'environmental' aspects of particular centres. Centres are thus scored on the presence of cafes and restaurants, the number of covered walkways, malls and precincts and the number of pedestrianised areas. All of these factors help to improve the safety and comfort of shopping. The last component of the attractiveness term could be labelled the 'transport factor' and includes the availability of car parking facilities (see as particularly important by Timmermans, 1981) and the 'quality' of local public transport facilities. It should be stressed that by using these 4 components we are not claiming to have captured all the attributes that might be deemed to have an influence on shopping behaviour. Clearly each individual shopper is unique and has behaviour patterns very different from their neighbour. However we hope to have captured some of the 'average' attributes of shopping centre attractiveness. The addition of these new elements can be achieved in a number of alternate ways: ie: by incorporating each element through a new set of variables (x_j^1, x_j^2, etc.) or by spatially disaggregating the power function α and subsequently making each α_j a function of the four values recorded for each element. We choose the latter case here to reduce the number of extra variables and list this final point score value in Table 1. The model thus takes the following form:

$$S_{ij} = A_i e_i P_i \, W_j^{\alpha_j} e^{-\beta c_{ij}} \tag{18}$$

with the variables as listed before.

Experience with the parameter space of our equilibrium models suggests that the values of α should be in the region of 0.90 to 1.10. Any value larger than 1.1 for an individual centre would make it far too attractive for nearby centres to compete. Therefore we have devised a 'twenty points' score which is shown in Table 1. The overall total for each centre is then added to 0.90 to obtain a unique α_j term for each centre. These are also shown in Table 1.

Figure 14 shows the equilibrium model run for these α_j values. The other parameters are held constant: β=2.00, K_j=1.00 $\forall_j$, v=0.85. (Note since we have defined the attractiveness of the city centre more precisely we have relaxed the city centre factor back to 0.85; we retain a value less than one to represent the greater accessibility to the city centre.) It is clear from Figure 14a that the larger centres, the most attractive centres, dominate. In Figures 14b and 14c, we then recalculate a set of K_j values, through our iterative procedure in equations 10-15, that are needed to reproduce the real city retail structure (for 1982). To offset the increased attractiveness of the larger centres the smaller centres require increasingly lower K_j values, whilst the majority of the larger centres now have increased cost values. Again this clearly mirrors the real-world trade off between the attractiveness of a centre and the costs of locating in that centre. (Note: also that the more isolated centres still have relatively high K_j values.)

9. Conclusions

In this paper we have attempted to model the existing configuration or retail centres using the type of equilibrium models set up by Harris and Wilson (1978). Our empirical case-study has shown that for real systems we need careful disaggregation of the basic model to reflect locational cost variations and the different attractiveness of individual centres.

Attention has focused primarily on the 1981 Census data with the known retail pattern of 1982. Clearly the next important empirical task is to look more closely at change over time in which new forms of retailing have emerged and in which new forms of parameter disaggregation may be required. This should provide a good comparison with the attempts here to model the retail structure of the major shopping centres.

Figure 1. 729-zone system and Leeds Census wards

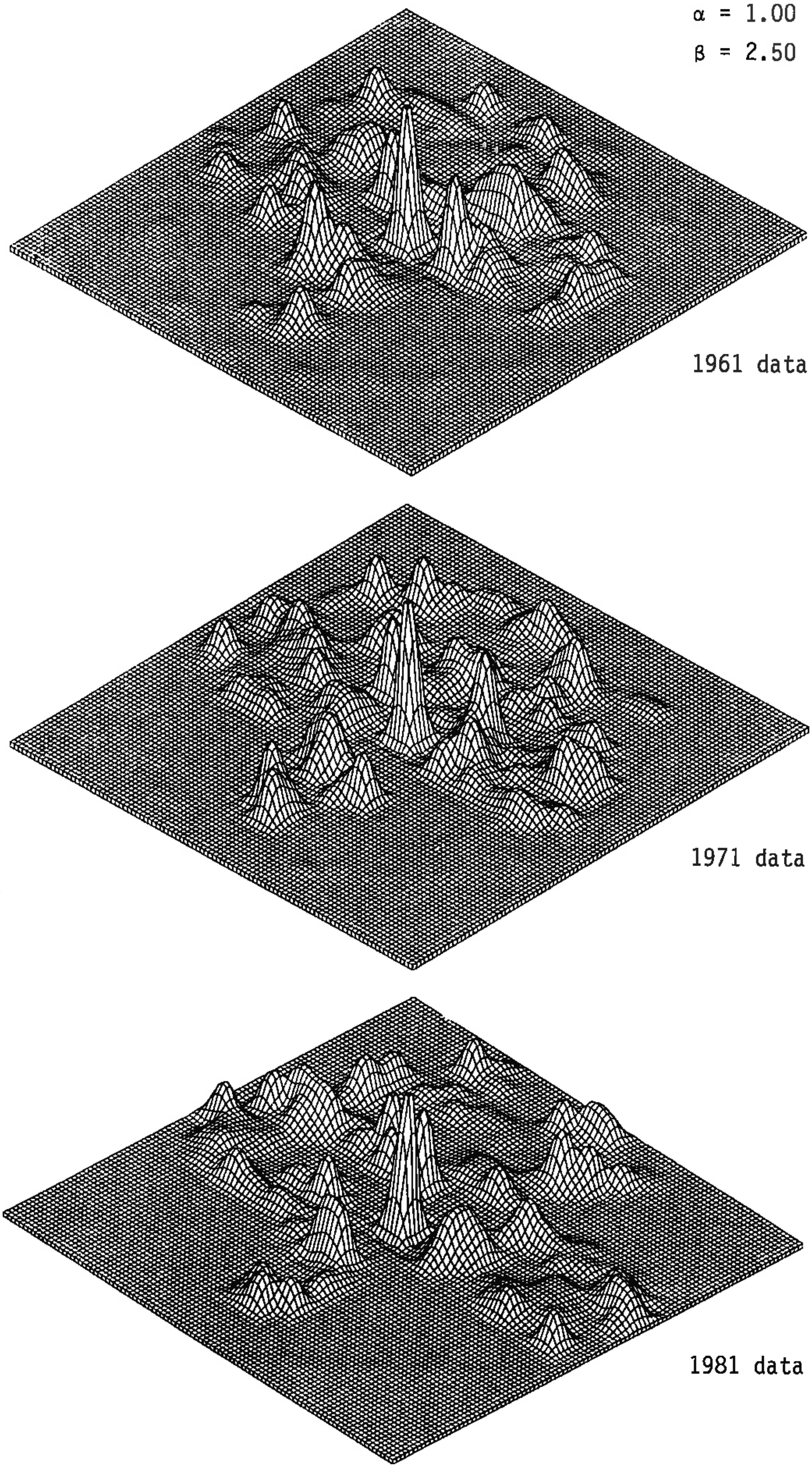


Figure 2. <u>Model solutions with varying e_i P_i values</u>

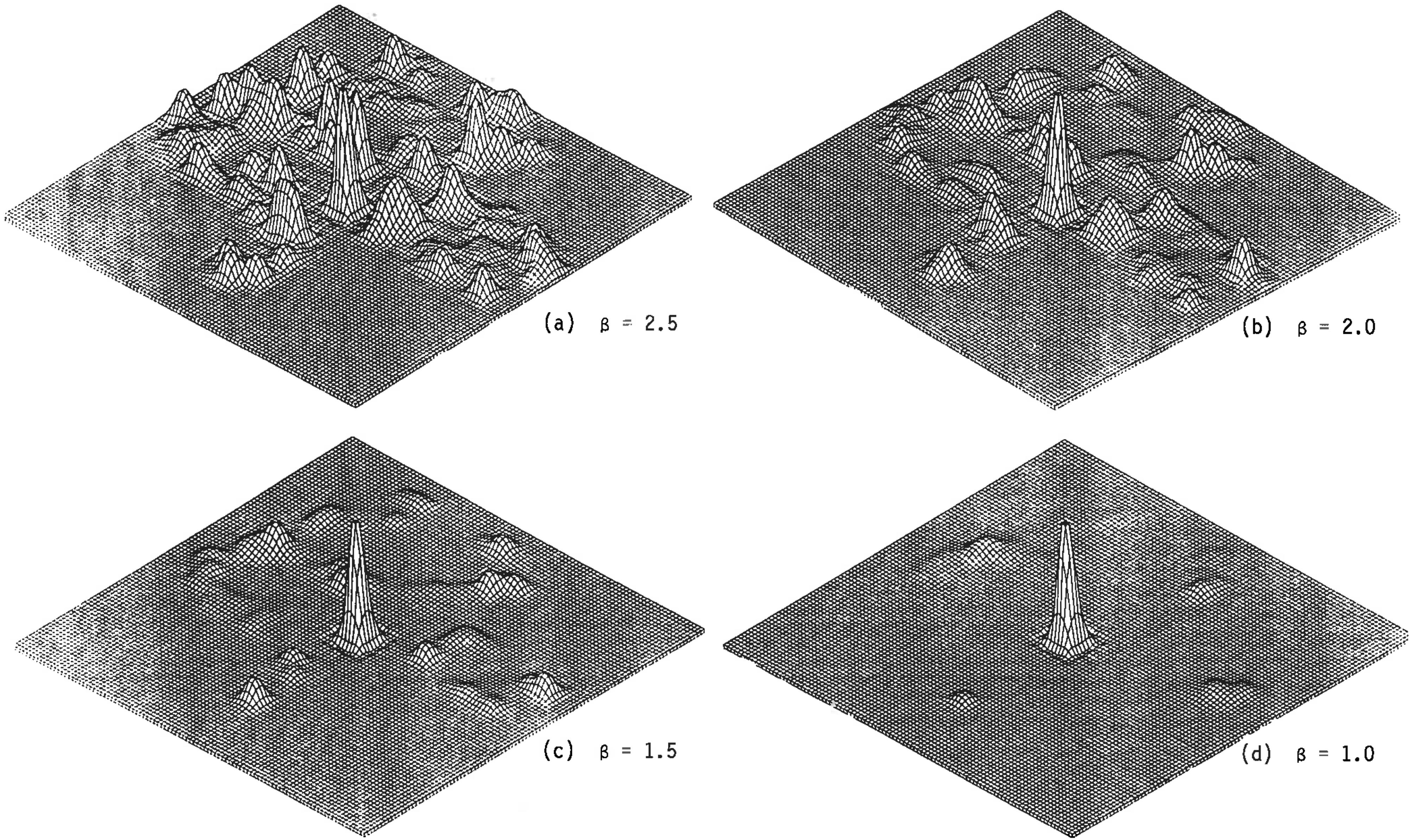
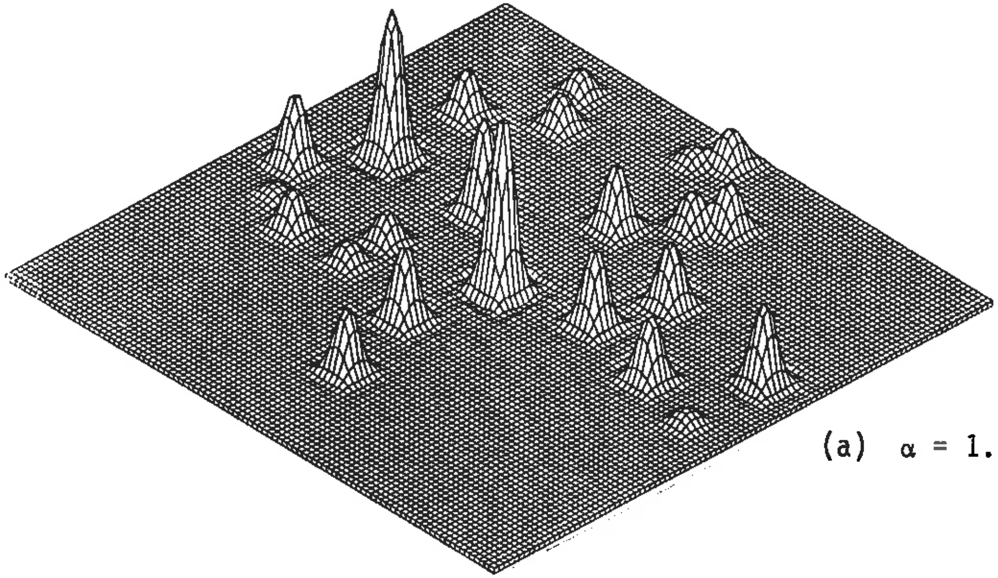
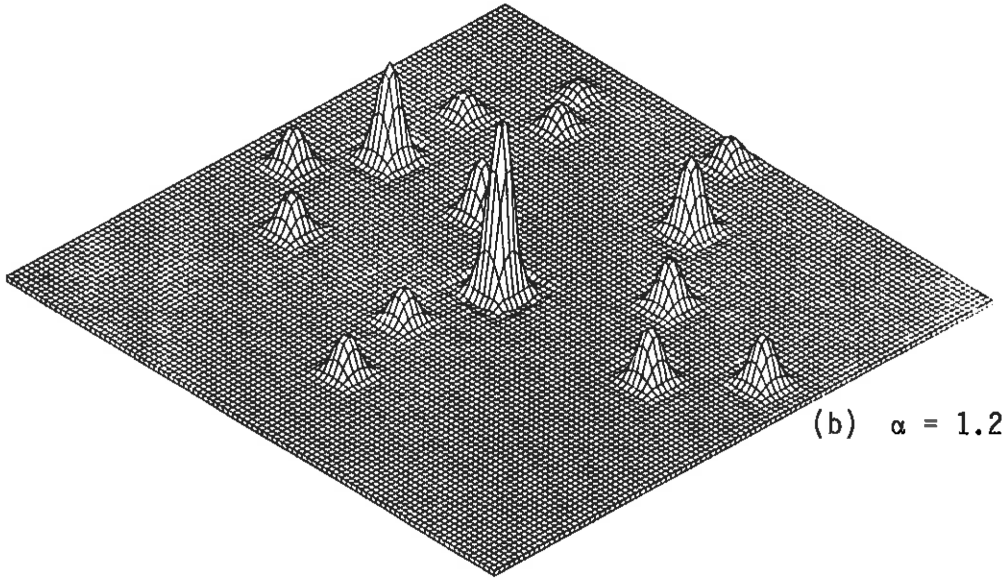
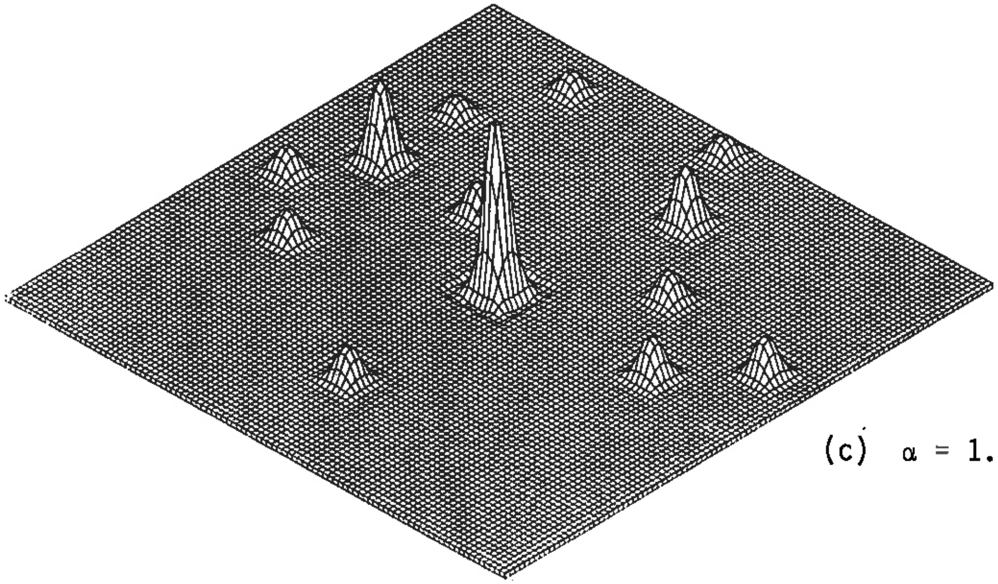


Figure 3. β-variation for 729 possible locations $\alpha = 1.00$


(a) $\alpha = 1.1$


(b) $\alpha = 1.2$


(c) $\alpha = 1.3$

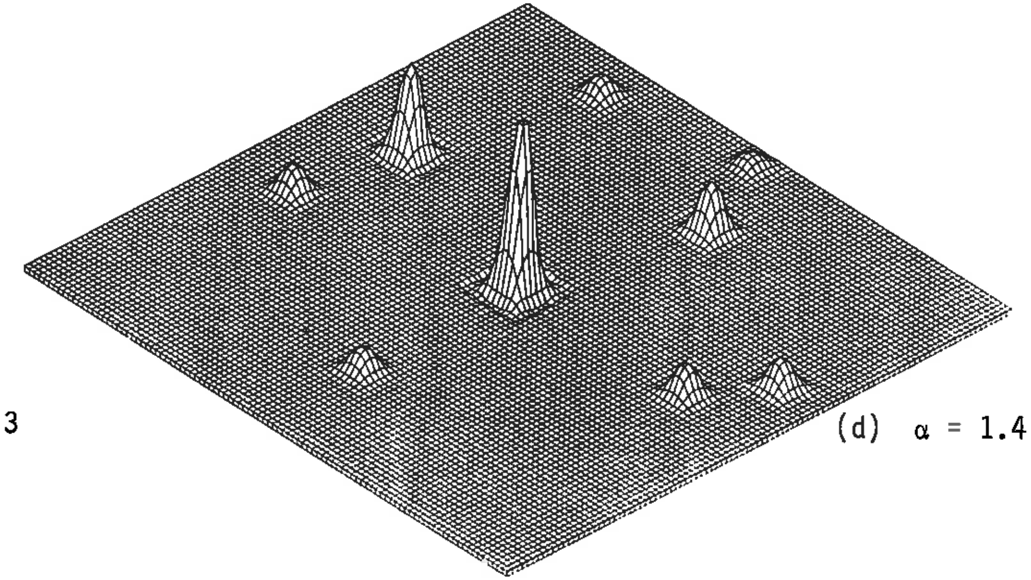

(d) $\alpha = 1.4$

Figure 4. α-variation for 729 possible locations $\beta = 2.0$

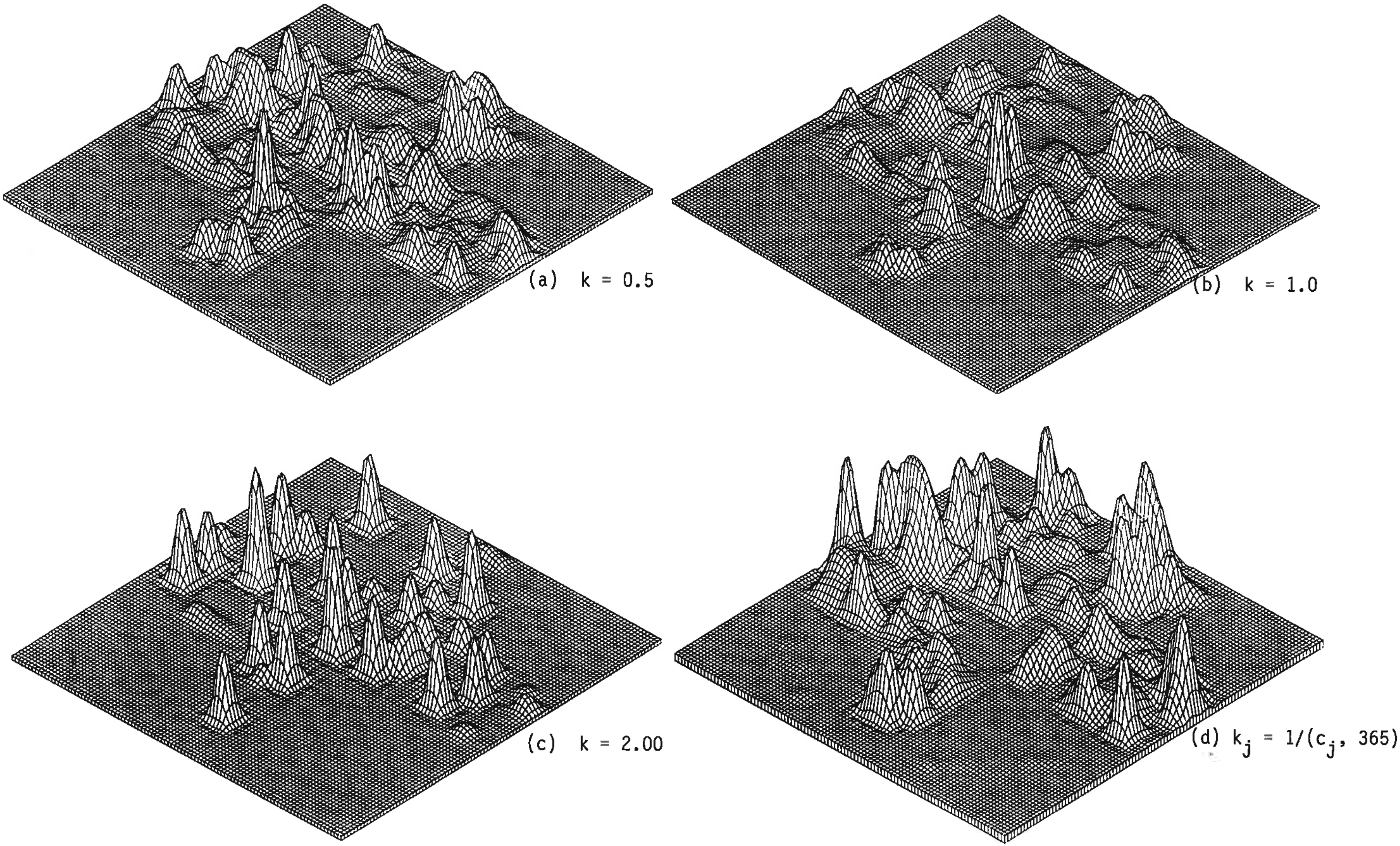
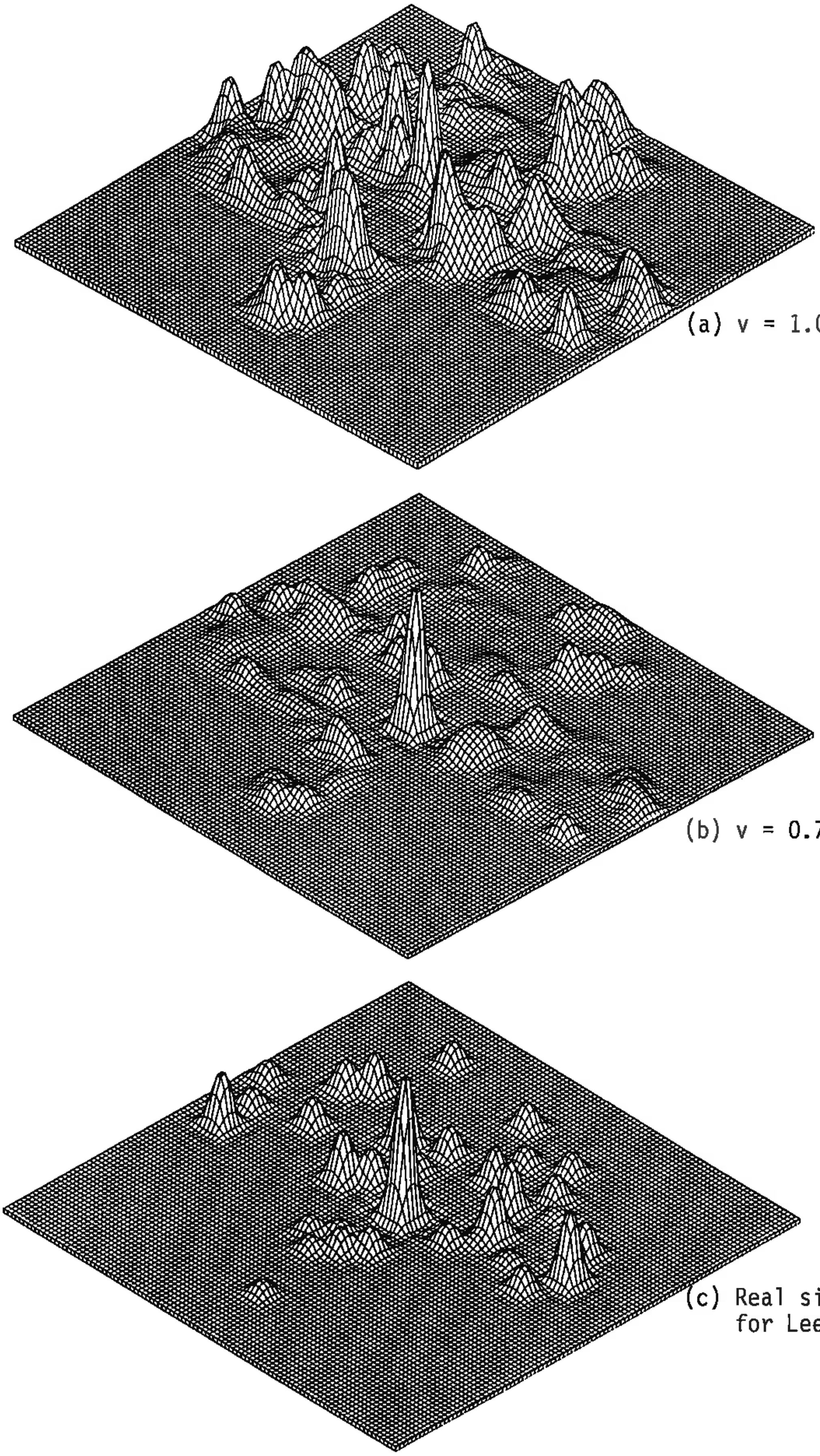


Figure 5. <u>Effects of varying the cost parameter</u>



(a) v = 1.00

(b) v = 0.75

(c) Real situation
for Leeds in 1982

Figure 6. <u>The effects of decreasing v</u>

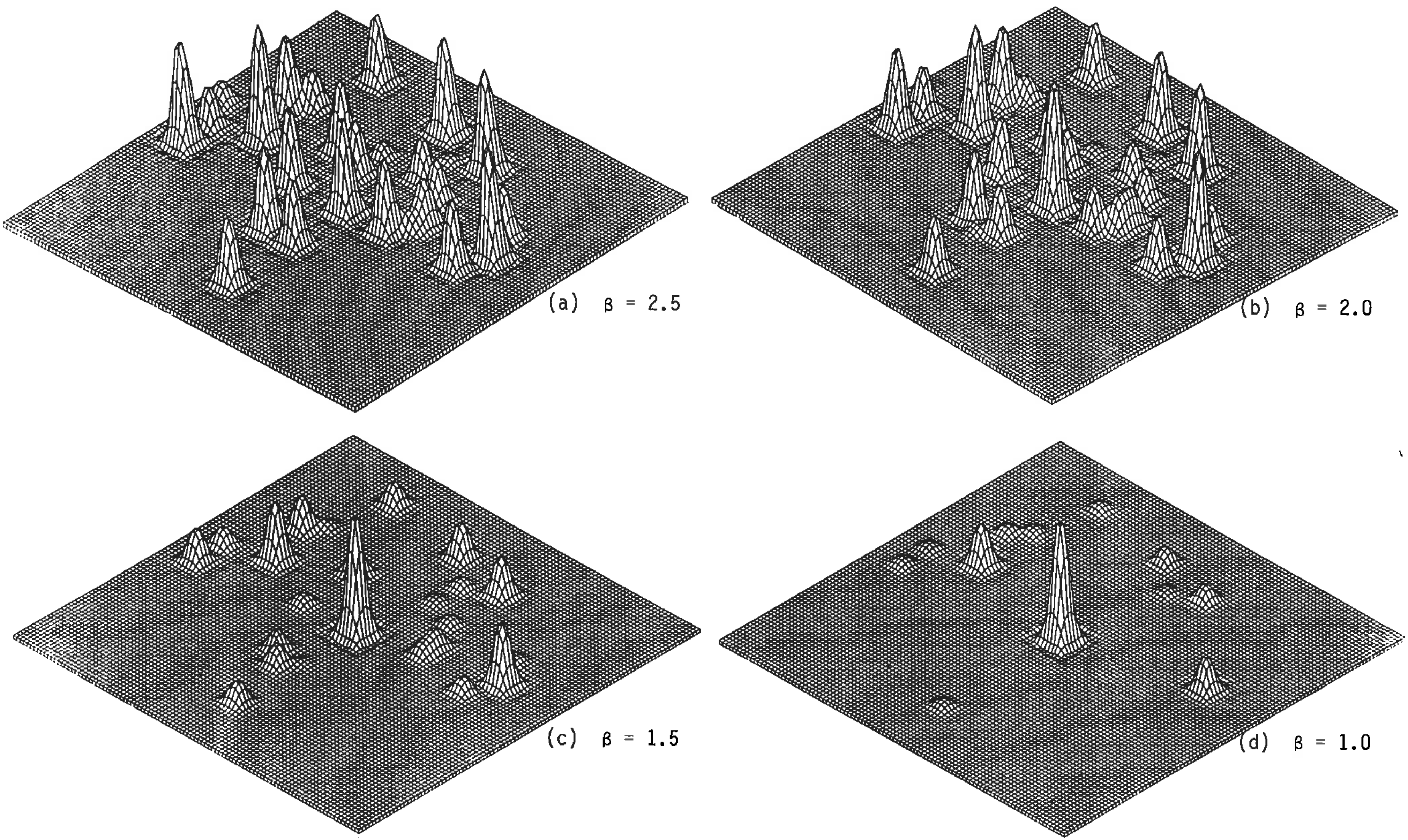
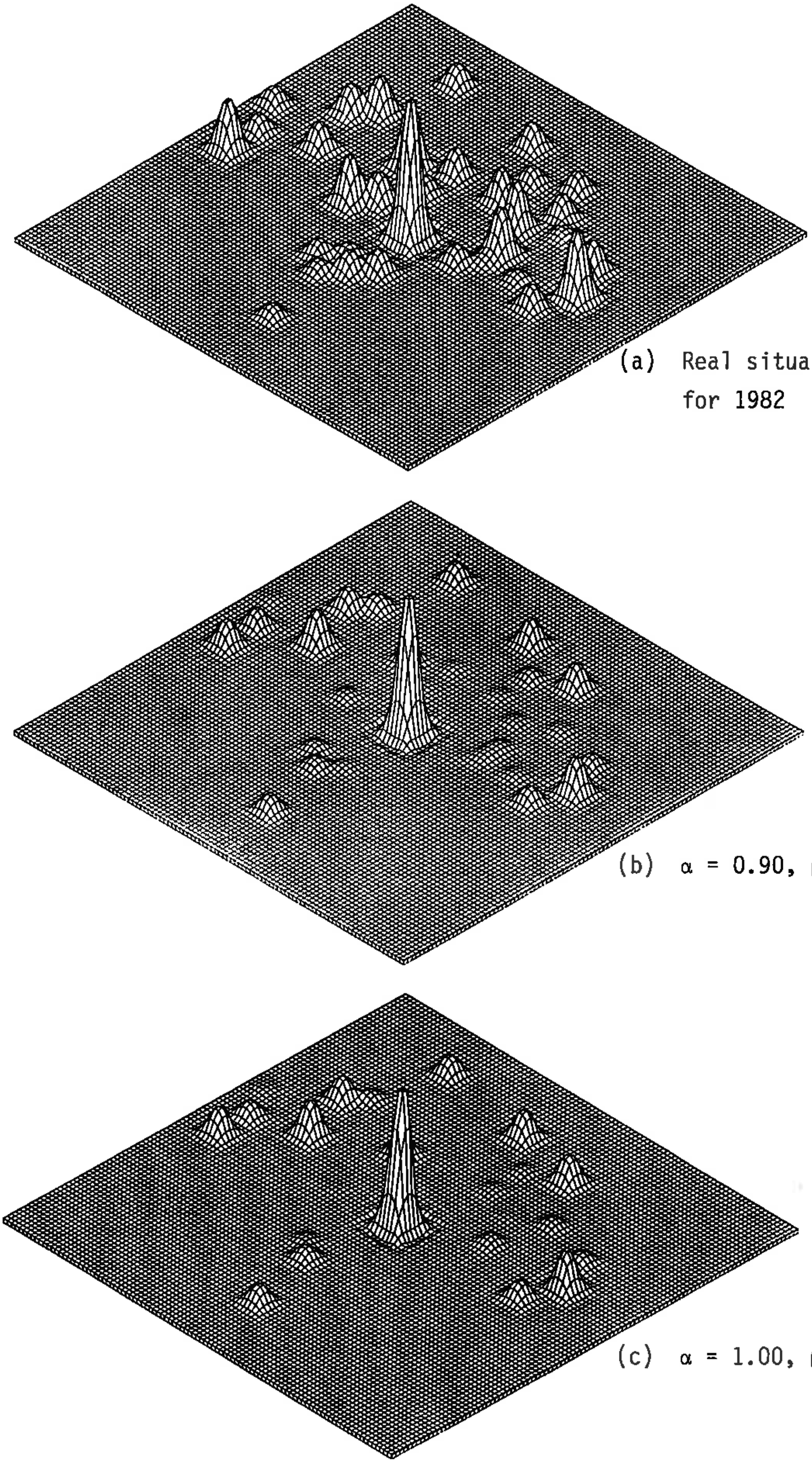


Figure 7. <u>β-variation for 33 possible locations α = 1.00</u>



(a) Real situation for 1982

(b) $\alpha = 0.90$, $\beta = 1.75$

(c) $\alpha = 1.00$, $\beta = 2.15$

Figure 8. <u>Equilibrium solutions for calibrated α, β values</u>

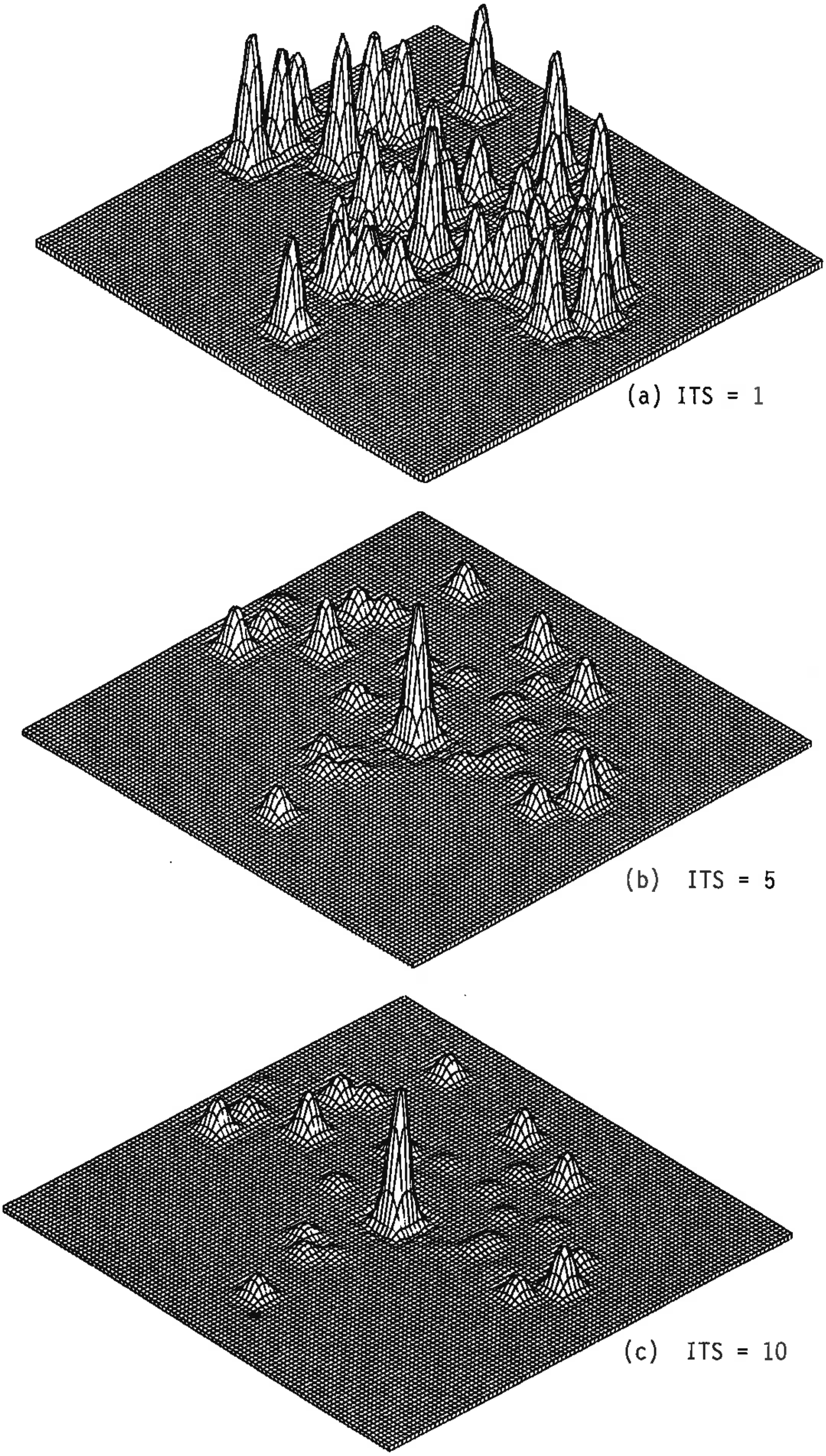


Figure 9. Model solutions for various iteration levels (ITS)

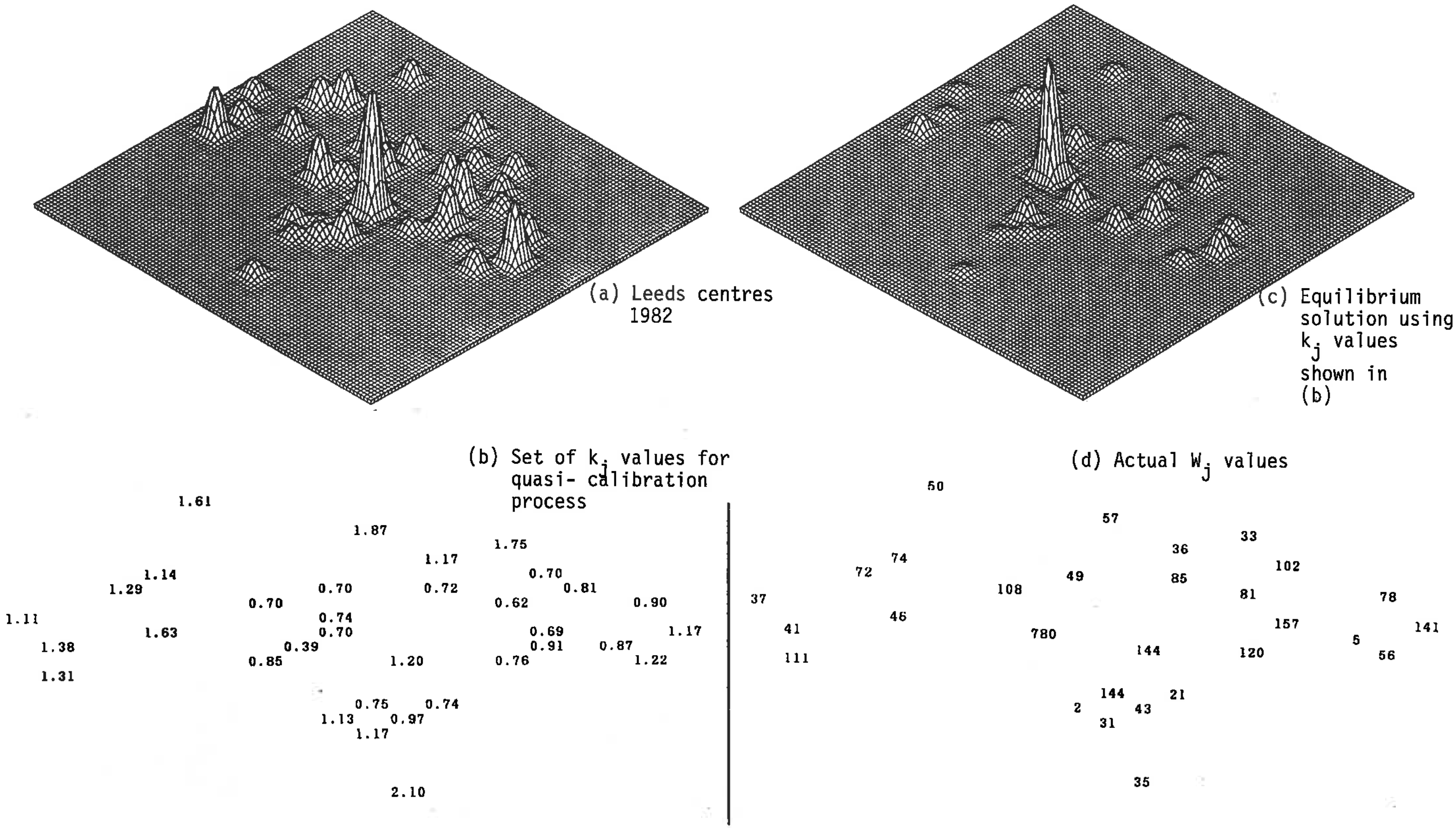
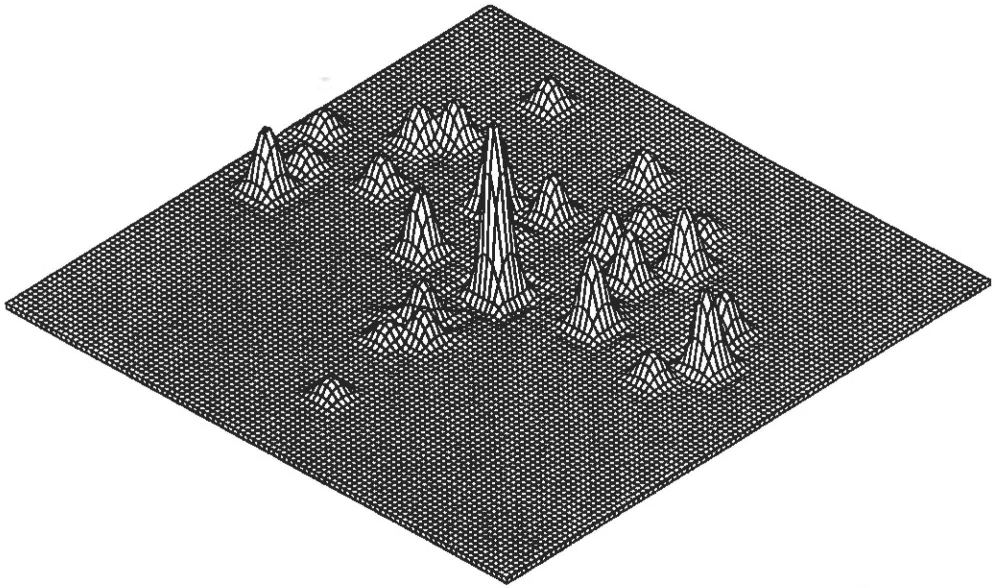
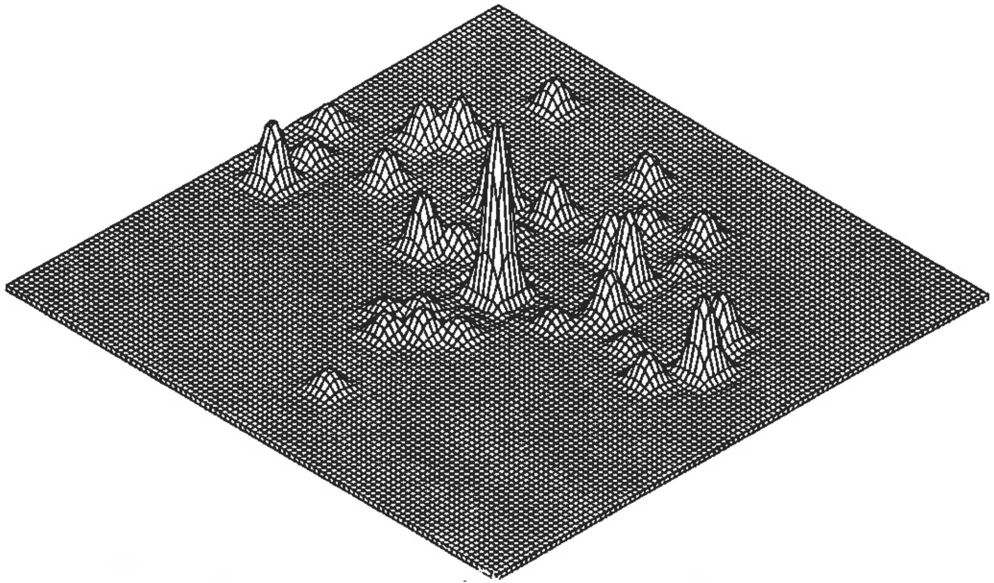


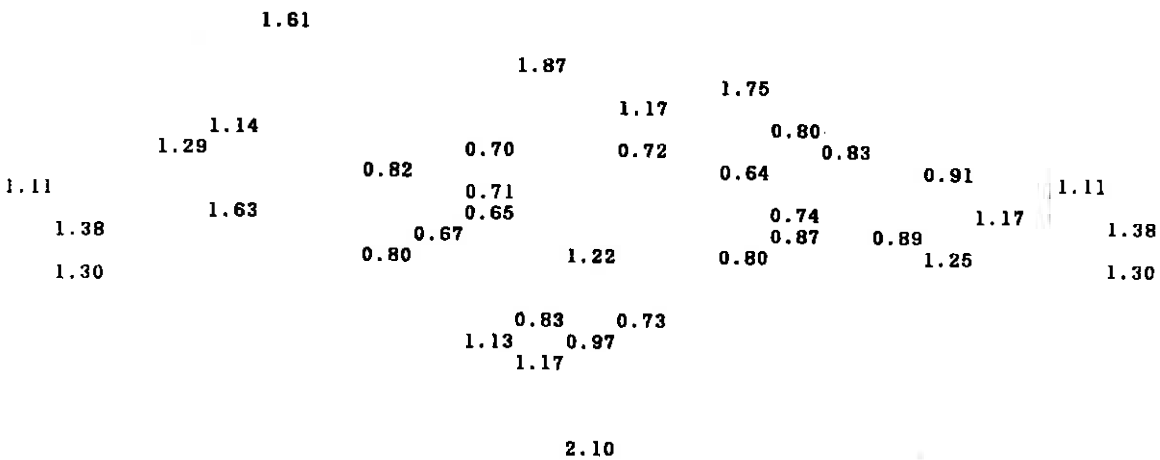
Figure 10. <u>First run of the quasi-calibration procedure for k_j's</u>



(a) Equilibrium solution with k_j values below



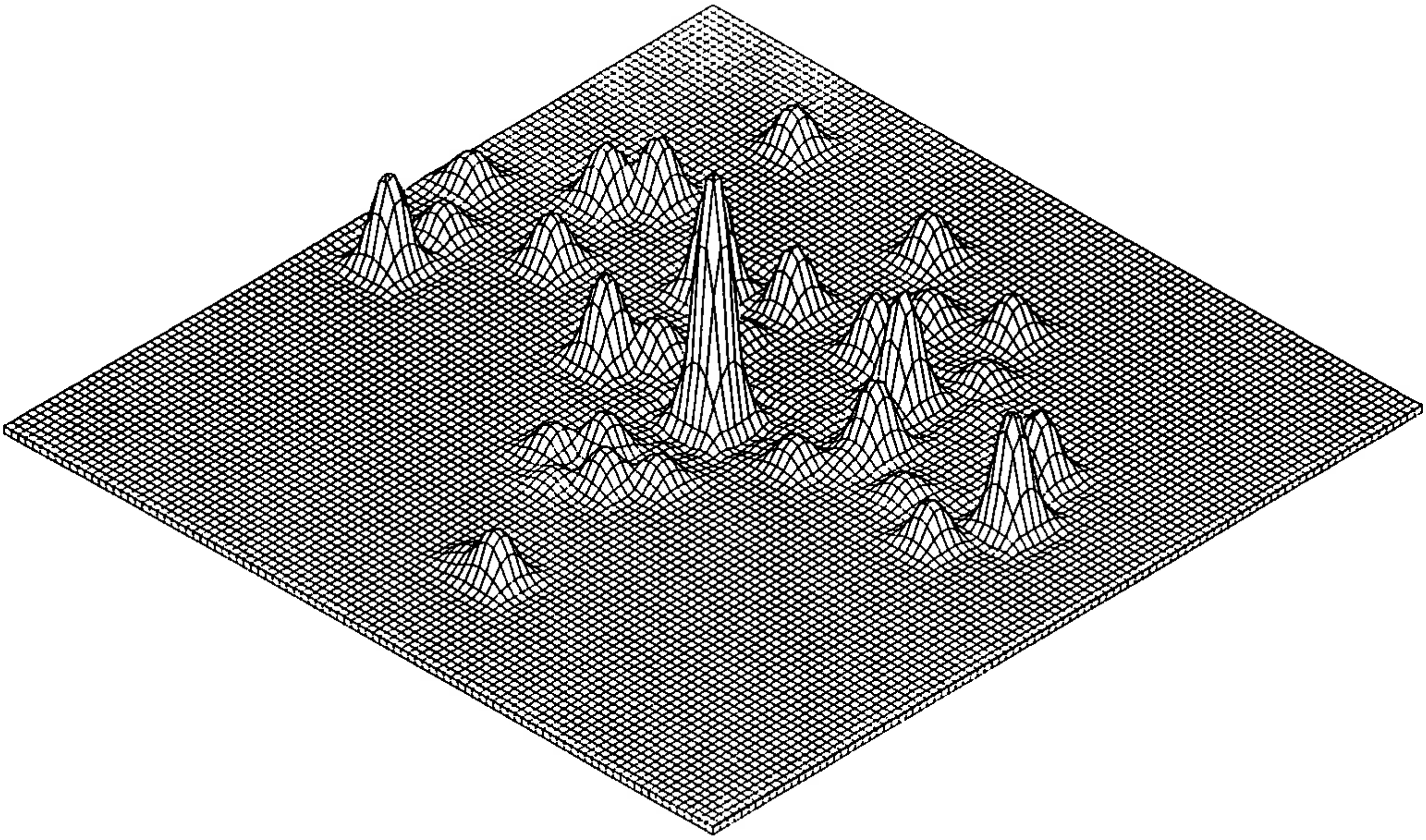
(c) Equilibrium solution with final set of k_j values



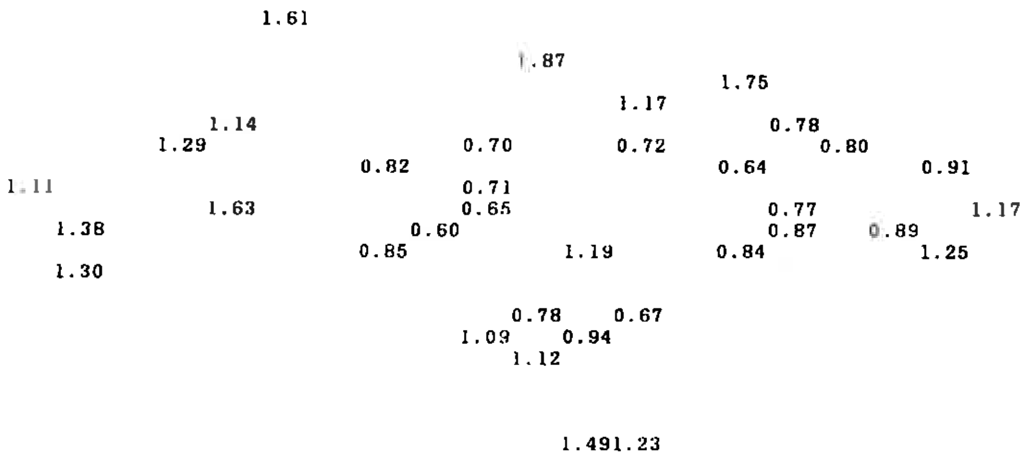
(b)

(d)

Figure 11. <u>Second run of the quasi-calibration procedure and subsequent 'fine tuning'</u>

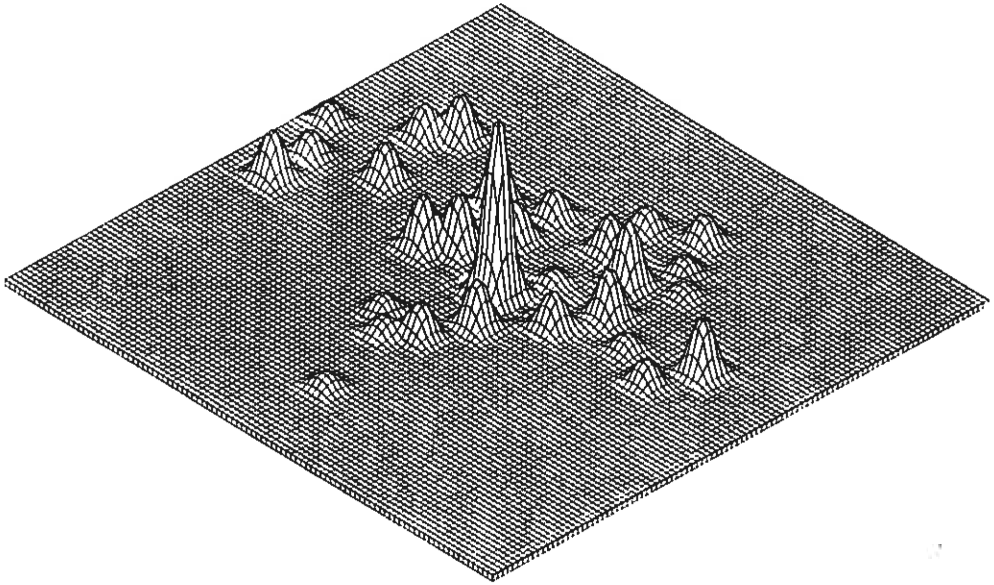


(a) Equilibrium solution using k_j values below

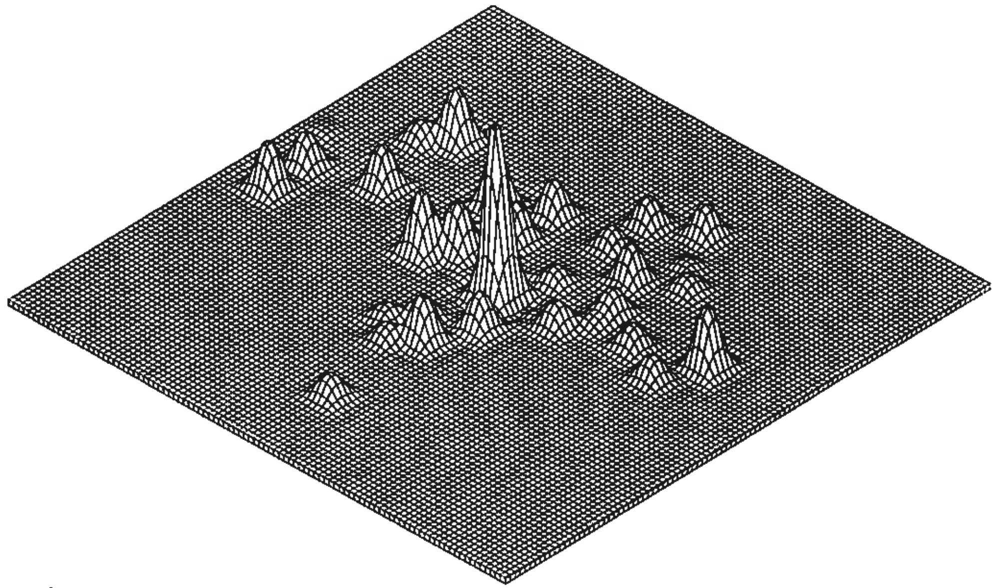


(b) New set of k_j values

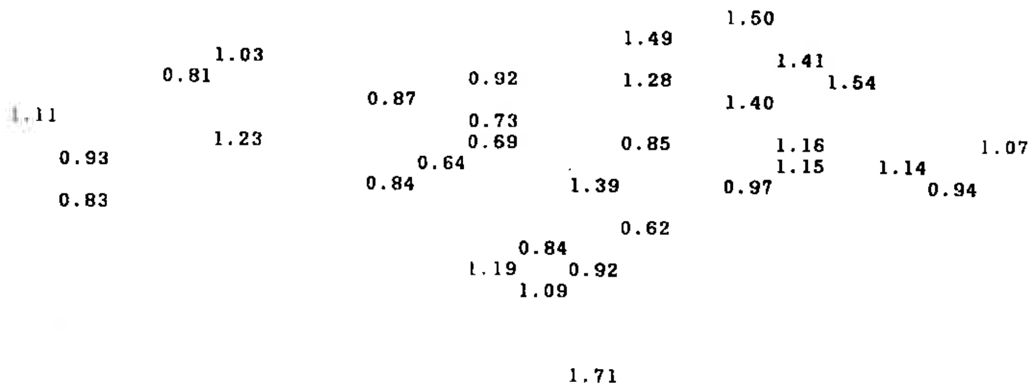
Figure 12. <u>Adding a new centre at Middleton</u>



(a) Actual retail distribution



(b) Equilibrium solution using k_j values below

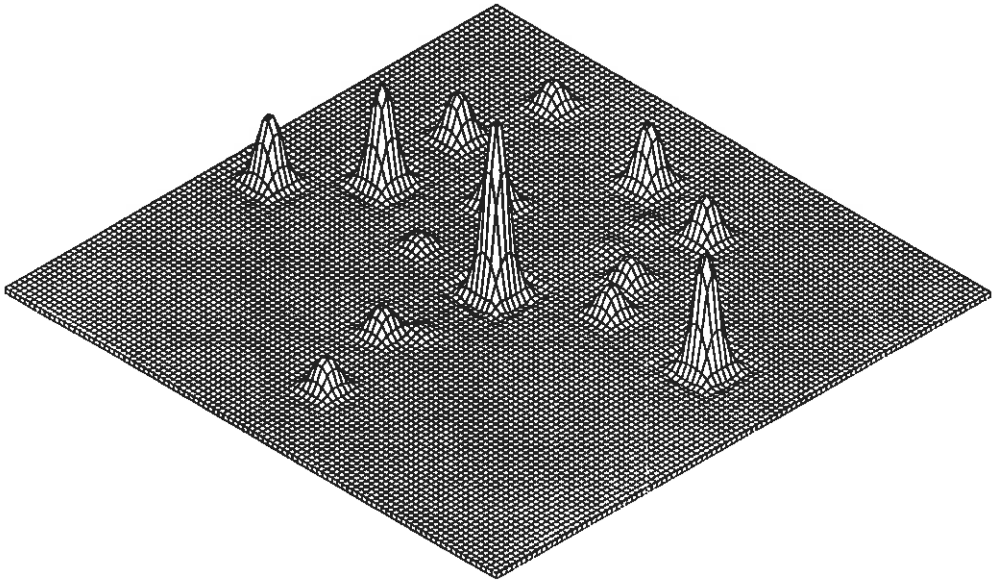


(c)

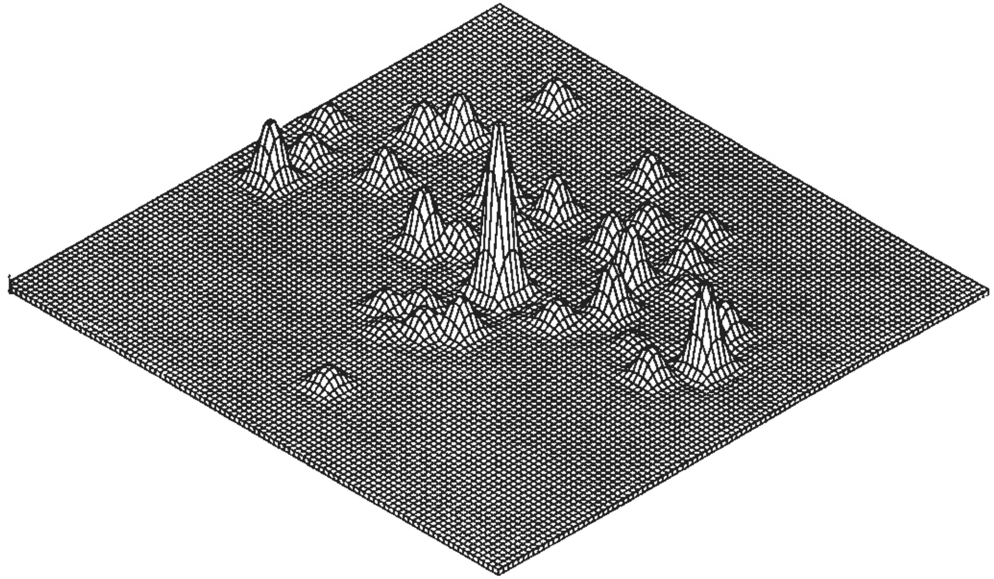
Figure 13. <u>Retail structure for 1961</u>

Table 1. The attractiveness of shopping centres in Leeds

Centre	Functions (max 7.00)	Multiples (max 5.00)	Environ. (max 4.00)	Transport (max 4.00)	Total	α_j
Armley	4.98	2.00	2	2	10.98	1.098
Beeston Hill	1.34	1.83	1	2	6.17	0.9617
Beeston Park	1.08	2.16	1	2	6.24	0.9624
Beeston Road	1.48	0.33	1	2	4.81	0.9481
Bramley	2.60	5.00	3	3	13.60	1.0360
Chapel Allerton	3.10	2.83	2	2	9.93	0.9930
City Centre	7.00	5.00	4	4	20.00	1.100
Crossgates	6.86	3.50	3	3	16.36	1.0636
Dewsbury Road	2.37	2.00	1	2	7.37	0.9737
Farsley	1.61	1.50	2	1	6.11	0.9611
Halton	2.28	2.50	2	2	8.78	0.9878
Harehills	4.66	3.00	2	2	11.66	1.0166
Harehills Lane N.	4.66	2.50	2	2	11.16	1.0116
Harehills Lane S.	1.12	1.50	1	2	5.62	0.9562
Headingley	6.01	2.91	2	2	12.92	1.0292
Hollin Park	0.80	3.00	1	2	6.80	0.9680
Holt Park	0.70	1.66	2	3	7.36	0.9736
Horsforth Town St.	3.59	2.91	3	2	11.50	1.0150
Horsforth N. Rd Side	2.78	2.50	2	2	9.28	0.9928
Hunslet	2.02	5.00	2	3	12.02	1.0202
Hyde Park	2.06	0.50	1	2	5.56	0.9556
Kirkstall Road	1.70	1.83	1	2	6.53	0.9653
Meanwood	2.47	1.83	2	2	8.30	0.9830
Middleton	1.12	1.50	1	2	5.62	0.9562
Moortown	1.61	3.50	2	2	9.11	0.9911
Sainsbury s Moortown	2.28	3.50	3	3	11.78	1.0178
Oakwood	1.93	2.00	2	2	7.93	0.9793
Pudsey	4.93	3.50	3	3	14.43	1.0443
Roundhay	1.92	2.33	2	2	8.25	0.9825
Seacroft	3.00	3.33	2	2	10.33	1.0033
Selby Road	0.58	0.50	1	1	3.08	0.9308
Stanningley	1.75	2.20	1	2	6.95	0.9695
York Road	1.66	2.66	1	2	7.32	0.9732

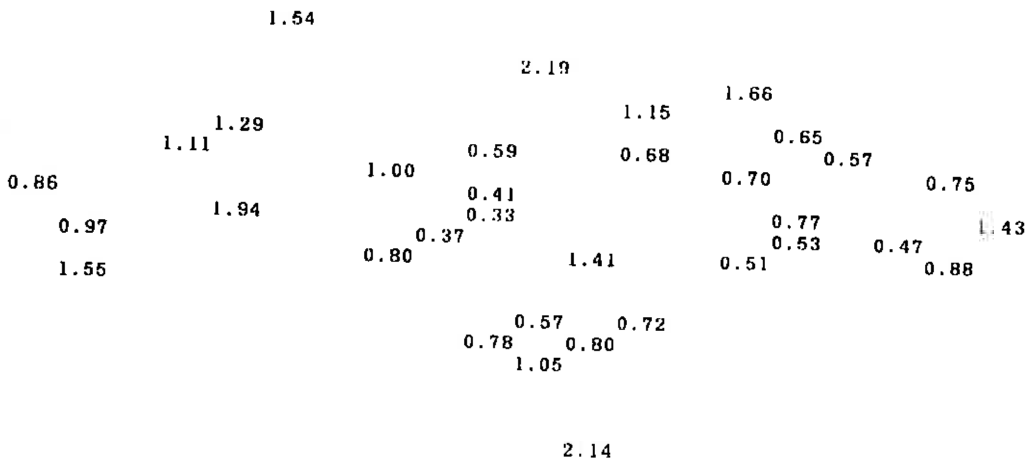


(a) Equilibrium solution for α_j's only

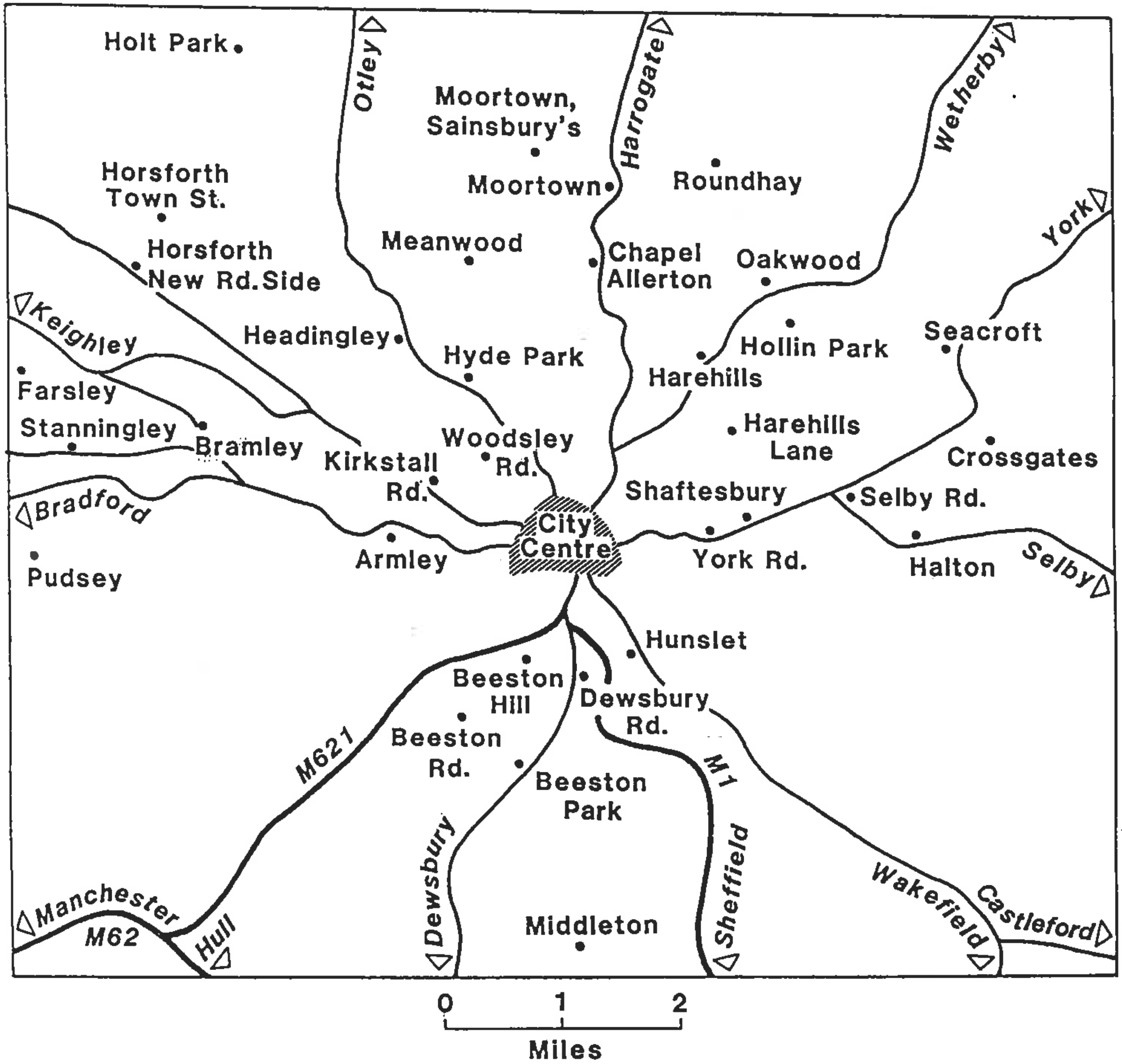


(b) Equilibrium solution using α_j's and k_j's below

Figure 14. <u>Combining costs and attraction-specific parameter values</u>



(c)



Appendix The shopping centres of Leeds

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