WORKING PAPER 385

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March 1984

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* Paper presented to the 80th Annual A.A.G. Meeting, Washington D.C., U.S.A. - April 1984

1. INTRODUCTION

1.1 Nature of the Paper

The analysis of spatial structure is an important and long-established subject area for geographers. Many different approaches have been developed, theories proposed and empirical studies undertaken. In this paper we report on our efforts to apply a spatial interaction and activity (S.I.A.) representation in conjunction with methods from dynamical systems theory (D.S.T.) to address the problem of modelling the dynamics of urban spatial structure. In particular we shall focus on the problem of linking sub-system models into an integrated "interacting fields" model and present some preliminary results from our latest numerical work.

An important point that we hope will emerge from this paper relates to the development and extension of our initial theoretical understanding in relationship to modelling urban structure, as presented in Harris and Wilson (1978). The first step was to gain further insights into one particular sub-system model (of retailing) and to undertake a large number of numerical experiments. Importantly, these themselves threw up a whole new set of theoretical problems that needed to be addressed before further progress could be made. This work is described in Wilson and Clarke (1979) and Clarke and Wilson (1983a). Once we had a sufficient grasp of the problem for the retail model we then needed to extend our analysis to other sub-systems. This has now been achieved, for housing structure and residential location (Clarke and Wilson, 1983b), industrial location (Birkin and Wilson, 1984, Wilson and Birkin, 1983) and agricultural location (Birkin and Wilson, 1984b) and transport network evolution (Wilson, 1983B). The next step is to link these models together in an integrated way and this forms the main substance of this paper. However, we also take the opportunity of briefly describing some of our earlier work and to speculate on future developments, some of which are already underway.

1.2 Relation to previous work

Our interest in spatial structure reflects a geographical legacy handed down from generation to generation by location theorists such as Von Thunen, Weber, Christaller, Losch, Alonso and others. More recently it became apparent that the power and flexibility of a spatial interaction and activity (S.I.A.) representation combined with new methods from dynamical systems theory offered an interesting and potentially exciting model-based approach to the analysis of the dynamics of spatial structure.

Empirical evidence also suggested that there had been some major structural transformations in many Western cities. For example, there has been a massive de-population of inner city areas together with an expansion of suburban population. For those remaining in the inner-city the levels and quality of housing and service provision declined. The location and concentration of employment showed corresponding trends as well as an absolute decline and the end of the domination of the manufacturing sector and the rise of the service sector. The pattern of retail provision has also changed dramatically. Supermarkets have taken over from the corner-shop and we have witnessed the emergence of the out-of-town retail warehouse (Wilson and Oulton, 1983, Clarke, 1984). Public services have also tended towards centralisation and concentration, for example in health care many small hospitals have been closed, compensated for by an expansion of large existing hospitals.

All these changes, and others, have had important ramifications for different groups of the city's population. Poverty is still widespread and for the urban poor the structural changes of the last 20 years have often implied greater problems of accessibility to employment opportunities, retail facilities, public services and recreation. Activity patterns in general have changed. For those who own cars this has resulted in an increased reliance on that form of transportation. For non-car owners however the well documented decline of public transportation has led to its own set of problems.

The response of government and planners has been mixed. In some areas they were responsible for much of the change experienced. This is perhaps best exemplified through slum clearance programmes where families were rehoused in suburban council estates. In other areas they have attempted to control or channel land use development according to some 'strategic plan' or to actively or passively promote economic development in certain areas.

Attempts to construct models of these various processes have, until relatively recently, taken supply-side structures as given and have concentrated on population activities such as journey to work, journey to shop, residential location and so on. The first breakthrough in the move towards modelling structure came with the formulation of a set of mathematical programming problems based on the maximisation of consumer surplus for the retail system (Coelho and Wilson, 1976). In this case the problem focussed on finding an optimal distribution of retail centres subject to a number of constraints. We then became particularly interested in how the solution to these and related models changed in response to change over time in the parameters and exogenous inputs to the models. In other words we were interested in structural dynamics.

The first steps in this research project involved reformulating the retail model as an equilibrium model in which producers balanced revenue against costs (Harris and Wilson, 1978). Using the traditional spatial interaction representation a model of the flows of expenditure to centres can be given as:

$$S_{ij} = e_i P_i \, W_j^{\alpha} e^{-\beta c_{ij}} \;\; / \;\; \sum_j W_k e^{-\beta c_{ik}} \qquad (1.1)$$

where S_{ij} is the flow of expenditure from zone i to j; e_i is per capita expenditure on retail goods in i; P_i is population in i; W_j is size or attractiveness of retail facilities in j; c_{ij} is cost of travel between i and j; and α and β are model parameters.

Revenue, D_j, accruing to a zone is simply the sum over all the flows into a zone:

$$D_j = \sum_i S_{ij} \qquad (1.2)$$

Costs are assumed to be proportional to size, so we have a unit cost parameter k_j, and the total cost is given by $k_j W_j$. For revenue to balance costs we must have

$$D_j = k_j W_j \qquad (1.3)$$

and we define this as the equilibrium of the model. The modelling task is to find the set of W_j's that satisfy this condition in each zone. This appears deceptively simple. The major complication is that zonal revenue, D_j, is a function not only of the attractiveness and accessibility of fa facilities in every other zone. This can be shown by substituting the interaction model for D_j, ie.

$$D_j = \sum_i S_{ij} = \sum_i e_i P_i W_j^{\alpha} e^{-\beta c_{ij}} / \sum_k W_k^{\alpha} e^{-\beta c_{ik}} \qquad (1.4)$$

It can be immediately seen that zonal revenue, D_j, is a highly non-linear function of all the W_k's.

The mechanisms for change in the equilibrium solutions of this model were the subject of two papers, the first by Harris and Wilson (1978), the second by Wilson and Clarke (1979). Harris and Wilson outlined the theoretical mechanisms for change and we refer the reader to that paper for details. Simply stated any of the variables on the right hand side of equation (1.1) could potentially give rise to a change in the equilibrium solution. Furthermore, and perhaps most interestingly in certain instances the change in the solution would be discrete, from, say, a relatively large value of W_j to zero, and this would occur at a critical value of one of the model's parameters or inputs. The identification of such values became a major challenge and taken up in the second paper by Wilson and Clarke.

Whereas the theoretical argument, though rather complicated, appears to be fairly straightforward to implement, in practice, when we undertook a set of numerical experiments, we faced a series of technical problems, some of which still remain unsolved. The most serious of these turned out to be what we have termed the 'backcloth problem': when attention is focussed on a particular zone, j, the analysis is dependent on all other zones, W_k, $k \neq j$. There is an interdependence between the behaviour of all zones with the consequence that it is no longer possible to proceed with a set of numerical experiments in a systematic way that will quickly lead to the discovery of critical parameter values. Instead we have had to exercise judgement and experience in the design of our empirical work.

An important point does emerge though: the value of undertaking numerical experiments in this type of work does not just lie in confirming of rejecting the theoretical concepts that have been derived but also it helps in refining them.

The next step in the research programme was to move from an equilibrium formulation to a disequilibrium one and this lead us to new kinds of bifurcation. Still working with the retail model as an example we can assume the following hypothesis, concerning the dynamics of producer behaviour; if revenue exceeds costs, ie.

$$D_j > kW_j \tag{1.5}$$

the producer will expand; if costs exceed revenue, ie.

$$D_j < kW_j \tag{1.6}$$

the producer will contract, and W_j decrease.

These assumptions can now be built into a set of differential (or difference) equations of the form:

$$\dot{W}_j = \varepsilon\ (D_j - kW_j)\ W_j \tag{1.7}$$

where ε is some response parameter.

It turns out that these differential equations have very interesting dynamic properties. In particular there are a number of bifurcation points at which the generic nature of the dynamic behaviour changes, for example from tending towards an equilibrium solution to a periodic, oscillatory solution. For details of the reasons for this we refer the reader to Wilson (1979) and for numerical illustrations Clarke and Wilson (1983).

The retail system work has been undertaken for hypothetical spatial systems and also real systems, notably for Leeds (Clarke, M. 1984, Clarke, G. 1984). We have illustrated the argument here with the simplest kind of aggregate model, but the same ideas can be applied to disaggregated versions (Wilson, 1983C).

As the next sub-system for analysis we took the housing system and residential location. The same arguments that applied with the retail system also apply here. We have a highly non-linear set of equations and this leads to interesting dynamic behaviour. Once again this implies that there exists critical parameter values at which structural change is found and that there are bifurcations points to be discovered if the system dynamics are represented as a set of differential equations. A further complexity was introduced into the model through the use of a very complicated attractiveness function (in an attempt to capture the realities of the situation) which resulted in an expansion of the number of parameters to 56. Results of our work in this area can be found in Clarke and Wilson (1983).

As we have already mentioned further sub-systems have also by now been examined and these include industrial and agricultural location. We now present a number of reasons why we feel it is important to integrate these different sub-system models into a unified framework.

1.3 The need for integrated models

Models of the major urban sub-systems provide us with a great deal of insight into the dynamics of spatial structure. Nevertheless we are inevitably ignoring some important and potentially powerful interdependencies by considering sub-systems alone. Indeed we have argued elsewhere (Wilson, 1983A) that it may well be at the interface between different sub-systems that many of the most serious urban problems exist. It is simply unrealistic, say in a residential location model, to assume fixed patterns of service supply and vice versa. We therefore not only see the development of comprehensive models as a necessary step but also an important one.

Therefore in Section 2 we outline in some detail both an aggregate and disaggregate interacting fields model. In Section 3 we present a set of numerical results using the disaggregate model. We offer some concluding comments in the final section.

2. INTERACTING FIELDS : A LOWRY-MODEL FRAMEWORK

2.1 Introduction

We shall argue that residential location is related to distance travelled to work (among other factors) and that service jobs, which will often be a large proportion of all jobs, are related to distance from the population who will use the associated facilities. The residential distribution can be seen as a set of fields of influence around job centres; the service job distribution as a set of fields around population centres. But the two kinds of fields interact and the task of this section is to incorporate this explicitly into a modelling framework.

The first person to do this was Lowry (1964) with his *Model of metropolis* and here we use essentially the same mechanism though with improved sub-models. The major addition, however, is the mechanisms that we have derived of supply-side dynamics. First, we develop an aggregate model, which shows the essential procedures without the complexity of the added subscripts and superscripts; and then the diaggregated model.

2.2 The aggregate model

The key distinction is between *basic* jobs (those in sectors, or parts of sectors, not serving the local market) and non-basic, or service, jobs. At this stage, we assume that the spatial distribution of basic jobs is given exogenously. Call this $\{E_j^B\}$. Let $\{E_j^S\}$ be the spatial distribution of service jobs and assume that this can be derived from W_j, the service sector floorspace,by a simple assumption:

$$E_j^S = a^W{}_j \tag{2.1}$$

Total employment is given by

$$E_j = E_j^B + E_j^S \tag{2.2}$$

and we need this as an input to the residential location model.

In the Lowry model mechanism, we begin with E_j^B as *total* employment, generate a residential population, then service employment, then more residential population in relation to the new jobs and so on. Essentially, we have to collect together the equations from both the service and residential models beginning with the aggregate residential model. We will use the simplest possible assumptions initially about attractiveness - that it is proportional to H_i, the amount of housing in zone i. The following model can then be derived:

$$T_{ij} = B_j H_i E_j e^{-\mu c_{ij}} \tag{2.3}$$

$$B_j = 1/\sum_k H_k e^{-\mu c_{ik}} \tag{2.4}$$

$$P_i = \sum_j T_{ij} \tag{2.5}$$

where T_{ij} is the number of people (assuming one per household) who live in zone i and work in zone j; H_i is the amount of housing in zone i; E_j is the number of jobs in zone j; c_{ij} is the usual measure of travel cost; B_j is the balancing factor; and P_i is the population of zone i.

The equilibrium condition for this model is:

$$yP_i = qH_i \tag{2.6}$$

where y is per household expenditure on housing and q is the unit cost of housing. The mechanisms involved will become more interesting in the disaggregate case.

A suitable service sector model was given in Section 1.2 and we repeat this for convenience:

$$S_{ij} = A_i e_i P_i W_j^{\alpha} e^{-\beta c_{ij}} \tag{2.7}$$

$$A_i = 1/\sum_k W_k^{\alpha} e^{-\beta c_{ik}} \tag{2.8}$$

$$D_j = \sum_i S_{ij} \tag{2.9}$$

$$D_j = kW_j \tag{2.10}$$

From this we can derive service sector employment as:

$$E_j^S = aW_j \tag{2.11}$$

with a suitable factor a.

The system can now be solved as follows. Take $E_j = E_j^B$ initially. Then solve equations (2.3)-(2.6) for $\{T_{ij}\}$, (and hence $\{P_i\}$) *and* $\{H_i\}$ simultaneously. The $\{P_i\}$ are then input to equations (2.7)-(2.10) which are solved simultaneously for $\{S_{ij}\}$ and $\{W_j\}$. Service employment is computed from (2.11) and this forms the input to equation (2.2). The cycle is then repeated until convergence is achieved.

We need to add that it would be possible to add land-use accounting equations and density-constraint equations as in the original Lowry model, but we do not do this here: It is useful to examine the differences between the model presented above and the original model and some of its derivatives. There is one sense in which all the models are similar: the balancing conditions (2.6) and - combining (2.10) and (2.11) so that

$$E_j^S = a/k\ D_j \tag{2.12}$$

are Lowry-like in that they seem to assume that housing stock and service facilities follows 'demand'. The difference is that, in the original Lowry model, there were no attractiveness factors at all, and in most subsequent elaborations, the attractiveness factors are fixed at the outset. In the model here they are continually being adjusted in relation to demand and model allocations.

The iterative procedure suggested above is shown diagrammatically in Figure 2.1. In each of the main submodels, the flow-facility size equilibrium is calculated. An interesting question is: how would we proceed if we were using differential (or difference) equations and not making equilibrium assumptions at *each stage* in the outer iteration? In any case, the outer-loop equilibrium assumption may be too strong, since it is only the final equilibrium which counts as such. The differential equations offer an interesting alternative therefore. In fact, we will use difference equations as being more practicable. The balancing equations (2.6) and (2.10) would be replaced by:

$$\Delta H_i = \rho(y^P{}_i - qH_i) \tag{2.13}$$

and

$$\Delta W_j = \epsilon(D_j - kW_j) \tag{2.14}$$

at each stage through the outer iteration with total H_i and W_j being made up cumulatively. It would be interesting to see whether the final equilibrium is the same in the two cases and, if so, whether one is faster than the other in computational terms. Intuition suggests that the second procedure should be faster because there is no balancing at each stage. However, this may be set against the second method needing more iterations in all.

2.3 The disaggregated case

The essence of the disaggregated case is to distinguish jobs by income, and then to let the resulting person-differentiation be transmitted into the residential pattern. We also use a set of constants, a^g, in stead of the single parameter a, to determine employment in each service sector from floorspace:

$$E^g_j = a^g W^g_j \tag{2.15}$$

We define coefficients b^{gw} and b^{Bw} (for each service sector and the basic sector respectively) to represent the proportion of jobs in that sector giving income in group w.

Then, E_j^w, which we need for the residential model, is obtained as:

$$E_j^w = \sum_g b^{gw} E_j^g + b^{Bw} D_j^B \tag{2.16}$$

Once again, we can now assemble the complete model by putting together the two disaggregated models of service and residential location (in reverse order, in fact), in each case making specific assumptions for our present purposes about attractiveness factors. For completeness, we include journey to service from employment as well as from residences.

Following Clarke and Wilson (1983) the residential location model is

$$T_{ij}^{kw} = B_j^w W_i^{res.kw} E_j^w e^{-\mu^w c_{ij}} \tag{2.17}$$

$$B_j^w = 1/\sum_{ik} W_i^{res.kw} e^{-\mu^w c_{ij}} \tag{2.18}$$

to ensure

$$\sum_{ik} T_{ij}^{kw} = E_j^w \tag{2.19}$$

and

$$W_i^{res.kw} = \prod_{\ell} (X_{i\ell}^{kw})^{\alpha_\ell^{kw}} \tag{2.20}$$

In the following an asterisk replacing an index of a variable denotes summation, for example

$$P_i^{kw} = \sum_j T_{ij}^{kw} = T_{i*}^{kw} \tag{2.21}$$

T_{ij}^{kw} is the number of w-income people, working in zone j, who live in type k house in zone i. $W_i^{res.kw}$ is the residential attractiveness of a type k house in i to a member of a w-income group; E_j^w and c_{ij} have already been defined. The $X_{i\ell}^{kw}$'s, ranging over ℓ, are the components of attractiveness and, as shown in equation (2.20) they are assumed to be representable in such a way that they can be combined multiplicatively. The other terms are all parameters with the exception of the balancing factors, B_j^w, which are given by equation (2.18).

To illustrate the ways in which residential attractiveness can be defined, we use five $X_{i\ell}^{kw}$'s as follows. First, it is clear that $W_i^{res.kw}$ must reflect housing supply, and so we take

$$X_{i1}^{k*} = H_i^k \tag{2.22}$$

Note that we use an asterisk to replace w rather as we do with parameters. Secondly, we wish to make zones more attractive if they have a higher accessibility to retail centres, and so

$$X_{i2}^{*w} = \sum_{jm} (W_j^m)^{\alpha^{Rwm}} e^{-\beta^{Rwm} c_{ij}} \tag{2.23}$$

where W_j is a measure of the size of the retail centre of type m at j and α^{Rwm} and β^{Rwm} are parameters for group w which determine the shopping behaviour of that group.

Next, we add two terms which at least crudely describe social aspects of residential location. In this particular example, we assume there are three w groups which can be identified (very roughly) with social class, w=1 being taken as the lowest income (class) group. We define X_{i3}^{*w} and X_{i4}^{*w} to measure the affinity and disaffinity of social groups, their mutual attraction and repulsion. In equation (2.21) we defined P_i^{kw} and we can use this as P_i^{*w} as follows:

$$X_{i3}^{*1} = P_i^{*1} + P_i^{*2} + P_i^{*3} \tag{2.24}$$

This means that class 1 is attracted by the presence of all people, while

$$X_{i3}^{*2} = P_i^{*2} + P_i^{*3} \tag{2.25}$$

and

$$X_{i3}^{*3} = P_i^3 \tag{2.26}$$

so that the others are attracted by people in the same class or higher, and thus a more restricted population.

The disaffinity terms are defined by:

$$X_{i4}^{*1} = 1/(P_i^{*2} + P_i^{*3})^{\alpha_4^{*1}} \tag{2.27}$$

(so that there is some repulsion created by higher groups for group 1)

$$X_{i4}^{*2} = 1/(P_i^{*1})^{\alpha_4^{*2}} \tag{2.28}$$

The second group is only repulsed by the lower group. In these two cases we show the parameters explicitly for convenience in (2.34) below. This is because the parameters in the next equation take an unconventional form:

$$X_{i4}^{*3} = 1/|(P_i^{*1})^{\alpha_4^{'*3}} + (P_i^{*2})^{\alpha_4^{''*3}}| \tag{2.29}$$

The highest group is repulsed by each of the lower ones, but to different extents determined by the parameters $\alpha_4^{'*3}$ and $\alpha_4^{''*3}$ which replace the simple parameter α_4^{*3} in the overall attractiveness given in equation (2.34) below.

The fifth element of attractiveness relates to house price and affordability and is, in effect, the term introduced into the disaggregated model by Wilson (1970). The prices are basically taken as exogenous, but are modified to allow overly-high densities to be controlled. Let p_i^k be the price of a type-k house in zone i, q^{kw} be the average amount available for spending by a w-type household on type-k housing, and let $\bar{c}_i^w$ be the average transport costs associated with a w-type household located in zone i. Then we take

$$X_{i5}^{kw} = e^{-|p_i^k - (q^{kw} - \bar{c}_i^w)|} \tag{2.30}$$

Note that the modulus is taken, so that if p_i^k differs substantially from $(q^{kw} - \bar{c}_i^w)$ in either direction, than the element of attractiveness X_{ij}^{kw} is low. The overall importance of this factor within $W_i^{res.kw}$ is, of course, determined by the parameters α_5^{kw}, which are in fact taken as α_5^{*w} since these are assumed to be independent of k. $\bar{c}_i^w$ is taken as $\theta^w\bar{c}_i$, where $\bar{c}_i$ is measured in relation to i's accessibility and θ^w is a parameter which will be larger for low income groups than for higher ones. It would, of course, be possible to make $W_i^{res.kw}$ j-dependent also and to build jouney to work costs into this term. $\bar{c}_i^w$ could then be replaced by a detailed model of all transport costs. Here, however, we focus mainly on the simplest possible assumption: we take $\bar{c}_i$ as proportional to the distance of i from the city centre. This implies that transport costs (other than the journey-to-work costs of the principal worker which are allowed for in the $e^{-\mu^w c_{ij}}$ term) increase for progressively more suburban zones. We then use the θ^w parameter (making it high for low income groups, low for high) so that this matters for lower income groups relative to higher, and this becomes part of the mechanism which generates social polarization.

To complete the definition of X_{i5}^{kw}, we need to specify p_i^k. The basic prices are taken as given for our present purposes as $p_i^{k(0)}$ - though this is an obvious simplification. We do, however, modify them to handle overcrowding. There are many possible ways of doing this. Here we note three relatively simple ones. In each case, above a certain threshold, we add a component to the price which is proportional to the number of houses.

The thresholds are defined in different ways: first in relation to housing stock by type; secondly in relation to total housing stock; and thirdly in relation to total housing land use. Thus:

$$(1)\quad p_i^k = \begin{cases} p_i^{k(0)} & , H_i^k < D_i^k \\ p_i^{k(0)} + a^k H_i^k & , H_i^k > D_i^k \end{cases} \tag{2.31}$$

$$(2)\quad p_i^k = \begin{cases} p_i^{k(0)} & , \sum_k H_i^k < D_i \\ p_i^{k(0)} + a^k H_i^k & , \sum_k H_i^k > D_i \end{cases} \tag{2.32}$$

$$(3)\quad p_i^k = \begin{cases} p_i^{k(0)} & , \sum_k z^k H_i^k < L_{ii} \\ p_i^{k(0)} + a^k H_i^k & , \sum_k Z^k H_i^k > L_i \end{cases} \tag{2.33}$$

In the first instance, D_i^k is the upper 'non-crowded' limit of type-k houses in i; in the second, D_i is the maximum 'non-crowded' number of houses in total, and in the third case z^k is unit land use by house type, L_i is total available land area in i, and so we have a density constraint, a^k a parameter which determines the 'penalty' increase in house prices in crowded areas. The third kind of constraint is likely to be the best in principle. For our numerical experiments, however, we have concentrated the first two as being simpler and, our records suggest, the second as being the most effective of these. In all cases, the upper limit does not necessarily provide a sharp cut-off: more dwellings (say high rise flats) could be built if the revenue could be attracted, as we will see below.

We can now summarise the overall attractiveness factor, showing the parameters at the level of resolution adopted in each case:

$$W_i^{res.kw} = (X_{i1}^{k*})^{\alpha_1^{k*}} (X_{i2}^{*w})^{\alpha_2^{*w}} (X_{i3}^{*w})^{\alpha_3^{*w}} (X_{i4}^{*w})(X_{i5}^{kw})^{\alpha_5^{*w}} \tag{2.34}$$

The parameters associated with X_{i4}^{*w} were made part of the definition of the terms in equations (2.27) - (2.29) because of their unconventional form. And although the right hand side of (2.30) shows terms varying with k and w, we make the assumption shown in (2.34) that the parameters are dependent on w only.

The equilibrium conditions for this model can be given as

$$\sum_{jw} T_{ij}^{kw} q^{kw} = H_i^k p_i^k \tag{2.35}$$

where we take $\sum_{jw} T_{ij}^{kw} q^{kw}$ as the average 'revenue' attracted to type k housing in i and $H_i^k \cdot p_i^k$ as a measure of the price.

We can now proceed to the disaggregated service model. In this case we also make an allowance for the inclusion of journey to services from workplaces. To do this we add a superscript p to the flow equation to indicate the type of origin of a trip and take p = 1 for residences and p = 2 for workplace. Define

$$Y_i^{pw} = \begin{cases} P_i^w & \text{if } p = 1 \\ E_j^w & \text{if } p = 2 \end{cases} \tag{2.36}$$

A suitable service sector model is

$$S_{ij}^{gp} = A_i^{gp} \left(\sum_w e_i^{gpw} Y_i^{pw}\right) (\hat{W}_j^g)^{\alpha g} e^{-\beta^{gp} c_{ij}} \tag{2.37}$$

where the g superscript denotes good type

$$A_i^g = 1/\sum_k (\hat{W}^g)^{\alpha g} e^{-\beta^g c_{ik}} \tag{2.38}$$

and

$$\hat{W}_j^g = (W_j^*)^{\alpha_1^g} (W_j^g)^{\alpha_2^g} \tag{2.39}$$

$\hat{W}^g$ is the combined attractiveness factor and is designed to account for the situation when the attractiveness of type g facilities in zone j is a function of not only the size of that facility W_j^g but also a function of the presence of other, higher, orders of goods. A variety of different assumptions about this hierarchical arrangement can be made and we refer the interested reader to Clarke (1984).

The equilibrium conditions of this model are

$$D_j^g = k^g W_j^g \tag{2.40}$$

Then, in analogy with the aggregate case, we have

$$E_j^g = a^g W_j^g \tag{2.41}$$

The method of solution would be as follows. Start with $E_j^w = b^{Bw}E_j^B$ initially; solve (2.17)-(2.35) for $\{T_{ij}^{kw}\}$ and $\{H_j^k\}$ simultaneously; solve (2.37)-(2.40) for $\{S_{ij}^{gp}\}$ and $\{W_j^g\}$ simultaneously; calculate E_j^g from (2.41) and substitute into (2.16). Recycle until convergence is achieved, that is until the value of E_j^g does not vary from one iteration to the next.

2.4 Concluding comments

It is clear that in the disaggregate case, at least, we are working at quite a fine level of detail. We need to do this to capture the full variety of factors that influence the development of urban structure. We can now proceed to present some early results that we have obtained. We should re-emphasise that our interests are twofold. First to run the model for a hypothetical spatial system in the first instance, to produce spatial structures that are representative of real world systems. Secondly to identify critical parameter values at which the nature of the equilibrium solution changes.

3. NUMERICAL EXPERIMENTS : FIRST RESULTS

3.1 Introduction

In this section we present some first results obtained using the integrated model outlined in the previous section. Our aim is to demonstrate that the model can produce spatial structures that resemble the types of features found in many cities. In particular, as an example, we address the issue of the influence of the transportation system on the development of urban structure. The results we have obtained are just the first step in the analysis and we outline future experiments which should be undertaken.

3.2 Spatial system for experimentation

In the numerical experiments which are presented in this section we have used a 169 zone hypothetical spatial system. This is based on a regular lattice system of points and is shown in Figure 3.1. Note that we exclude certain zones from the analysis to give the system an approximate circular shape.

The cost matrix, c_{ij}, is calculated as the Euclidean distance between pairs of points, i and j, although suitable transformations of this matrix are possible.

The model has been implemented in the following way:

(i) Make a set of assumtpions concerning the distribution of basic employment, E_j^B.

(ii) Assign values to the parameters of both the residential and retail location sub-models.

(iii) Solve the interacting field models in the manner outlined in Section 2, using an iterative scheme. Convergence is achieved when the values of E_j^* do not change between iterations.

Presentation of the results in graphical form focusses on the following structural variables - H_i^{k*}, P_i^{*w}, W_j^g, E_j^*, and P_i^{kw}.

We are interested in the following range of experiments:

(a) The difference between results obtained from the integrated model and those obtained from the separate sub-system models run independently but for the same parameter values. This will allow us to investigate the importance of the synergetic effect mentioned earlier.

(b) To identify critical parameter values at which the nature of the model solution changes. These are known as bifurcation points.

(c) To run the model in an incremental mode to examine urban development.

Here we focus on (a) and (b) and refer the reader to Clarke, Wilson and Birkin (1984) for results from (c).

3.3 Model results

The model runs we have undertaken have been at the following level of disaggregation. There are three house types (k = 1-3), representing low, medium and high quality housing; three income groups (w = 1-3), low, middle and high incomes; and three service types (g = 1-3) representing low, middle and high order goods. This level of disaggregation is also reflected in the set of parameters which are given in full in Table 3.1, together with their values for the pivotal model run described next. We use a computer produced graphical form of representation where the x-y dimension represents the spatial system and the z-dimension the amount of a particular variable present

Results from the pivotal run are given in Figure 3.2. The parameter values that give rise to these results were assigned on the basis of our previous experience with the sub-models and are given in Table 3.1. The basic-employment distribution that forms the main model input was equally distributed between five 'regions' of the system, one containing five central zones and four satellite 'regions' of five zones each. The distribution of the three housing types mirrors to some extent the distribution of total employment although the highest quality housing is not present in the central area. The income groups show a striking polarisation, with low income households having workplace oriented locations, the middle income group mainly concentrated in an intermediary location between the central and outer industrial location while the highest income group occupies an outer suburban ring. The location of the different orders of services reflects a well known trend. Low order goods are provided in many locations without any marked concentrations, whereas the higher goods show more peaked distributions.

The results given in Figure 3.3 has exactly the same basic employment distributions and parameter values with the exception of μ^W parameters, which are a measure of the effect of journey to work costs and were taken as 1.0, 0.75 and 0.1 respectively. The effect is to reduce the dependence on workplace-based residential locations, particularly for the lowest income group. This feeds through to the service location sector, but only in relation to low-order goods.

It is clear from these two sets of results that the city centre area tends to dominate, especially in terms of service locations. One relatively simple modification allows us to widely represent the effect of congestion experienced in central city areas. This is achieved by factoring all trips into the central area so that travel costs increase. The effects of this can be seen in Figure 3.4 (which is directly comparable with Figure 3.2 except for this factoring). The effect is quite dramatic. The peakedness of the distribution of services disappears and there is no central dominance. Also there are some differences in the housing and income distributions and resultant employment distribution has notably changed. It could be argued, however, that the centrally focussed radial distribution of most urban transport networks in fact improves access to the city centre. To account for this we can factor trips to the central area so that they are in effect cheaper than a corresponding trip elsewhere. This was done to produce the results in Figure 3.5, where a city centre trips were factored by 0.9. Once again the effect is significant. Employment and service distributions are dominated by a central concentration as is the distribution of low and medium quality housing and low income groups. However, high quality housing is not located in the central area and neither are middle and upper income groups. This is probably the effect of firstly the density penalty (given in Equations (2.31)-(2.33) and also the affinity/disaffinity components of the attractiveness function in the residential locations model (Equations (2.24)-(2.29)).

A further trend in urban development which we can focus on is the decentralisation of basic employment. Insight may be obtained here by 'removing' the central city industrial zones and redistributing the employment among the existing satellite centres. Once again taking Figure 3.2 as a base, this redistribution was performed and the model re-run to generate Figure 3.6. As we would expect the suburbanisation of employment appears to drive a decentralisation in the residential sector, although low income groups and poor quality housing are still closely tied to employment centres. Also unsurprising is the out-migration of services of types 1 and 2, but the city centre remains the most accessible location and hence continues to support type 3 services. The overall result bears some similarity to Figure 3.4.

It would be possible to present many more cases similar to those above, but we would hope to have already done enough to suggest that a variety of patterns does exist within the multidimensional parameter space of the interacting fields model presented here and that the static and comparative dynamics of this model may be linked with, and used to gain insights into, real world processes.

As a final pair of experiments, we would like to illustrate the relationship of the interacting fields model to the separate residential and service sub-models which it embraces. For this purpose we have taken the set of parameter values employed in Figure 3.3, and run the two sub-models above. For Figure 3.7 we have assumed a fixed and even distribution of services across the region, and applied the residential location sub-model. The most important effect here is a noticeable 'flattening' in the distribution of housing types and income groups. In the second instance, the effect of fixed and uniform allocation of housing types and income groups on the service sectors is examined.

The results are shown in Figure 3.6. In this case, we see a decentralisation of service facilities in the lower and middle orders relative to Figure 3.3, although services of the highest order maintain a central location.

3.4 Concluding comments

The number of possible combinations of parameter values is very large indeed and the range of possible model runs is correspondingly wide. We can, at best, only search a small subset of the parameter space. The results we have presented represent only the beginning of a research agenda. There are a number of developments, which we outline in the final section of this paper, that warrant attention. The most pressing of these, of course, is to apply these models to real world data. Some preliminary results have been obtained for the retail model (G. Clarke, 1984) but there is clearly a long way to go.

4. CONCLUSIONS

The results presented in this paper are exploratory. Using Webber's (1984) terminology we would argue that the current focus of the model is explanatory but that prediction (via calibration) is an important long-term objective.

In this paper we have presented preliminary results of a two-sector integrated model, which is similar in structure to Lowry's original. As noted earlier, however, it would be possible to go on to build in basic employment as an endogenous subsector (cf. Webber, 1984). Ultimately, with the addition of agricultural location as well, we could begin to answer questions concerning the rural-urban fringe (the expansion of cities), and indeed begin to explore fundamental questions concerning the evolution of the city as a form.

Even at that stage it still remains to build in the transport network as a subsystem too (cf. Putman, 1983), but, in principle, this is relatively straightforward. Hence our results so far are but a modest step forward towards tackling the difficult problem of the evolution of urban spatial structure. There are clearly a number of important extensions to be made and these form the basis of our current research agenda.

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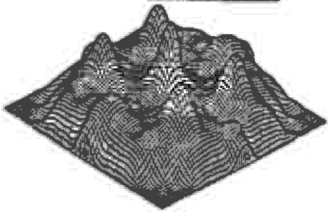
FIGURE 3.1. A 169-ZONE SPATIAL SYSTEM

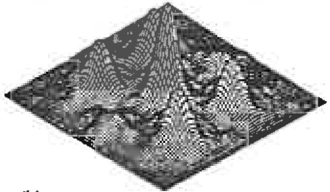
X	X	X	X	X	X	.	X	X	X	X	X	X
X	X	X	.	.	.	.	.	.	.	X	X	X
X	X	.	.	.	.	.	.	.	.	.	X	X
X	.	.	.	.	.	.	.	.	.	.	.	X
X	.	.	.	.	.	.	.	.	.	.	.	X
X	.	.	.	.	.	.	.	.	.	.	.	X
.	.	.	.	.	.	.	.	.	.	.	.	.
X	.	.	.	.	.	.	.	.	.	.	.	X
X	.	.	.	.	.	.	.	.	.	.	.	X
X	.	.	.	.	.	.	.	.	.	.	.	X
X	X	.	.	.	.	.	.	.	.	.	X	X
X	X	X	.	.	.	.	.	.	.	X	X	X
X	X	X	X	X	X	.	X	X	X	X	X	X

X = EXOGENOUS ZONE

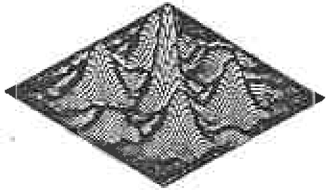
FIGURE 3.2. PIVOTAL CASE (FOR PARAMETER VALUES, SEE TEXT)



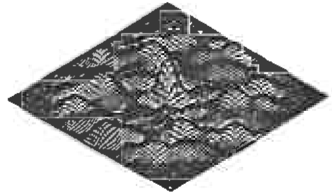
H_i^{1*} - Poor quality housing



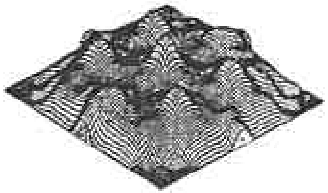
P_i^{*1} - Low income groups



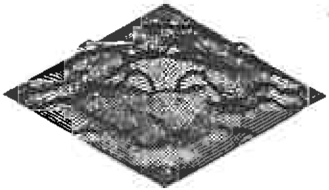
E_j^* - Total employment



W_j^1 - Low order services

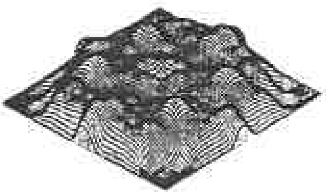


H_i^{2*} - Medium quality housing

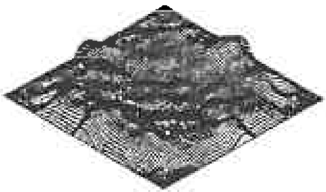


P_i^{*2} - Middle income groups

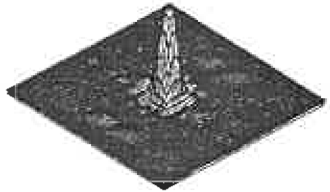
W_j^2 - Intermediate order services



H_i^{3*} - Good quality housing

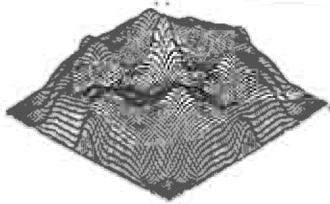


P_i^{*3} - Upper income groups



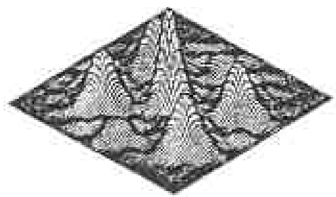
W_j^3 - High order services

FIGURE 3.3. PIVOTAL CASE WITH REDUCED WORKPLACE DEPENDENCE

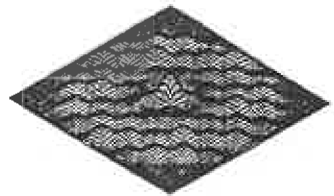


H_i^{1*} - Poor quality housing

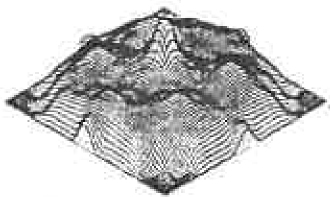
P_i^{*1} - Low income groups



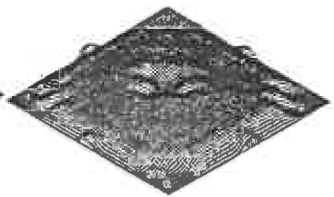
E_j^* - Total employment



W_j^1 - Low order services

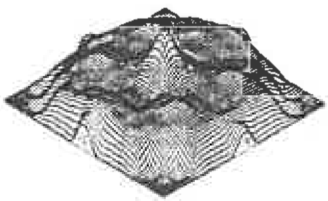


H_i^{2*} - Medium quality housing

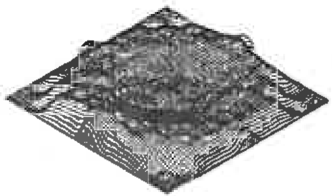


P_i^{*2} - Middle income groups

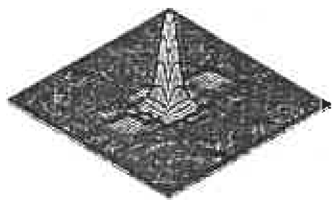
W_j^2 - Intermediate order services



H_i^{3*} - Good quality housing

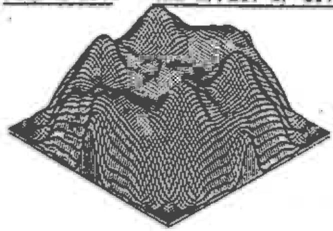


P_i^{*3} - Upper income groups

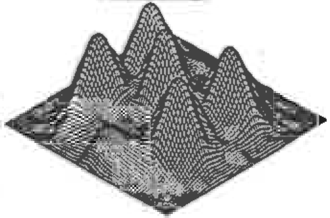


W_j^3 - High order services

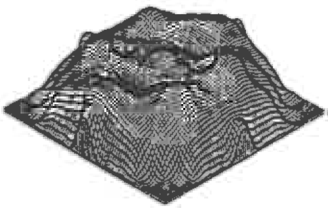
FIGURE 3.4. THE EFFECT OF CITY CENTRE CONGESTION



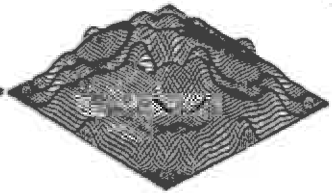
H_i^{1*} - Poor quality housing



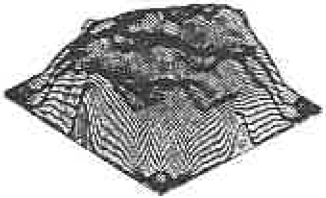
P_i^{*1} - Low income groups



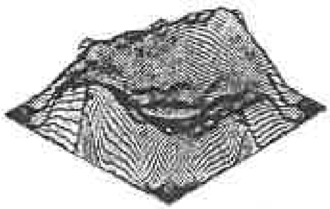
H_i^{2*} - Medium quality housing



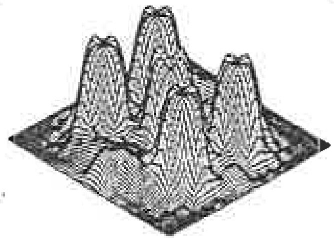
P_i^{*2} - Middle income groups



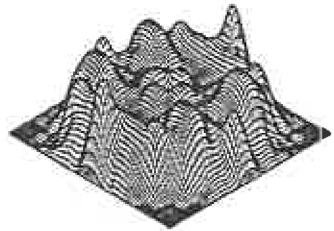
H_i^{3*} - Good quality housing



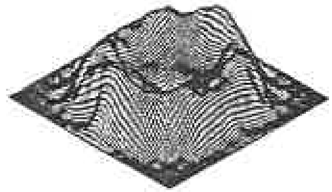
P_i^{*3} - Upper income groups



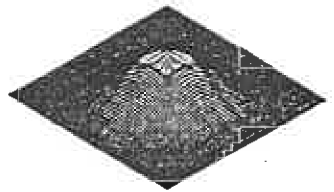
E_j^* - Total employment



W_j^1 - Low order services

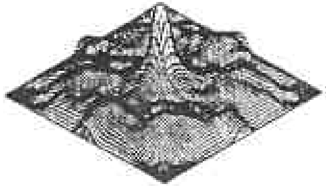


W_j^2 - Intermediate order services

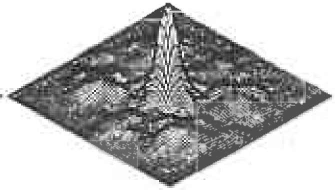


W_j^3 - High order services

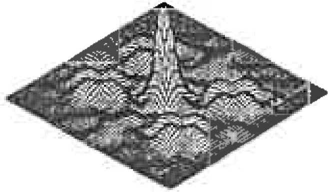
FIGURE 3.5. PIVOTAL CASE WITH INCREASED CITY CENTRE ACCESSIBILITY



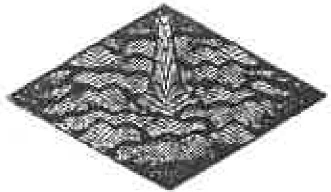
H_i^{1*} - Poor quality housing



P_i^{*1} - Low income groups

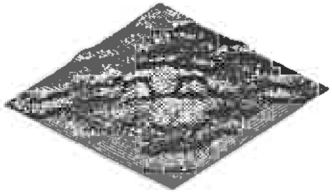


E_j^{*} - Total employment

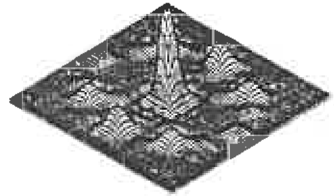


W_j^{1} - Low order services

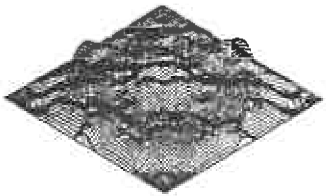
H_i^{2*} - Medium quality housing



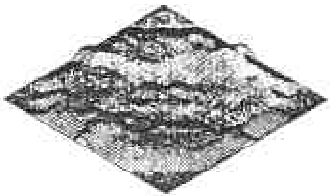
P_i^{*2} - Middle income groups



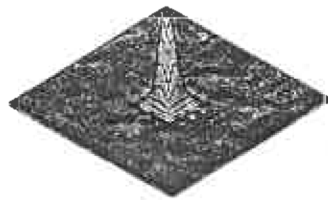
W_j^{2} - Intermediate order services



H_i^{3*} - Good quality housing

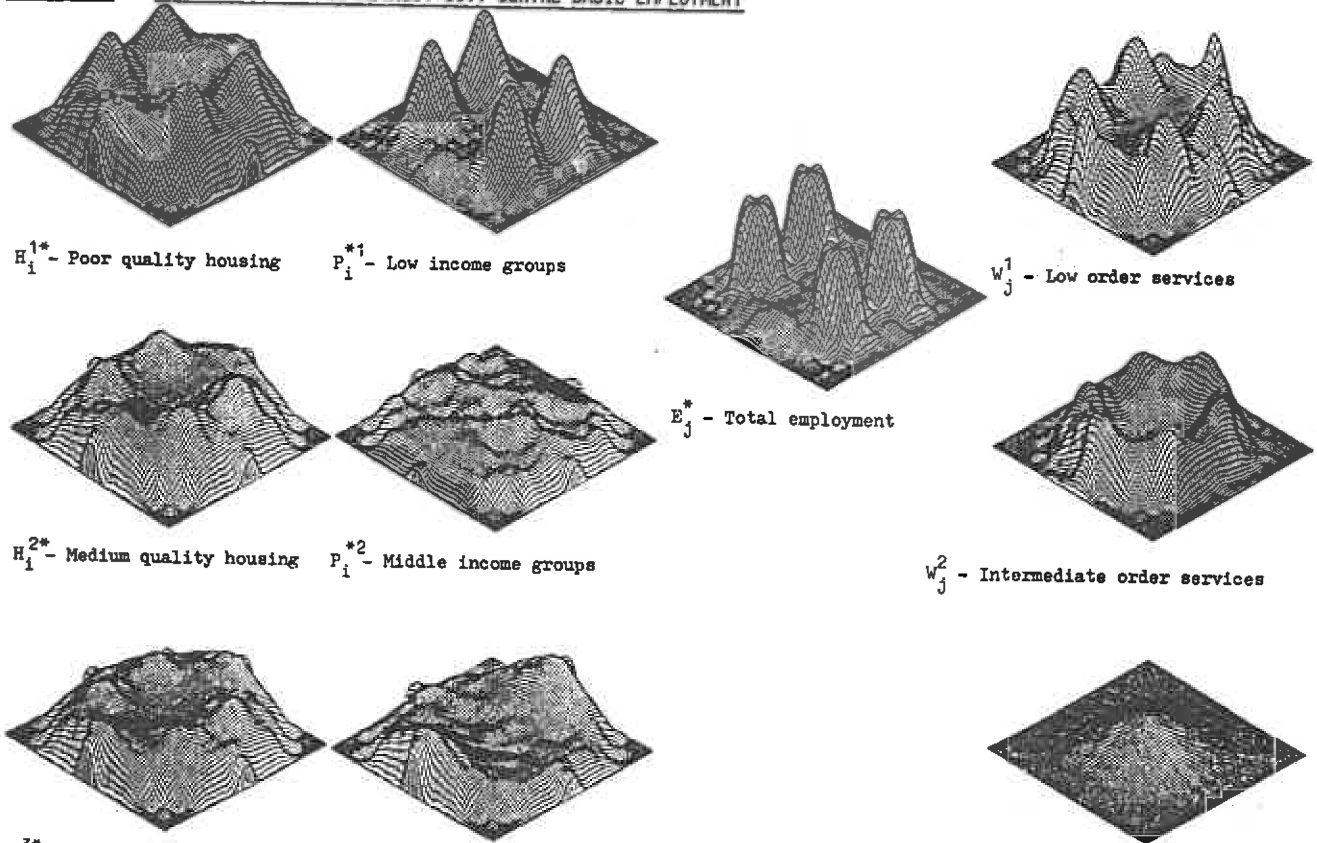


P_i^{*3} - Upper income groups



W_j^{3} - High order services

FIGURE 3.6. EQUILIBRIUM STATES WITHOUT CITY CENTRE BASIC EMPLOYMENT



H_i^{1*} - Poor quality housing

P_i^{*1} - Low income groups

E_j^* - Total employment

W_j^1 - Low order services

H_i^{2*} - Medium quality housing

P_i^{*2} - Middle income groups

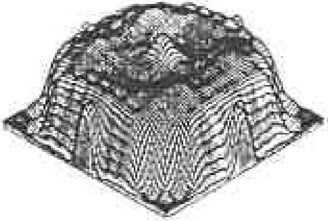
W_j^2 - Intermediate order services

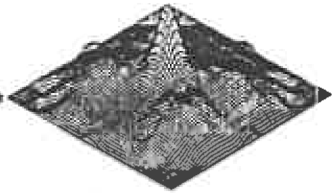
H_i^{3*} - Good quality housing

P_i^{*3} - Upper income groups

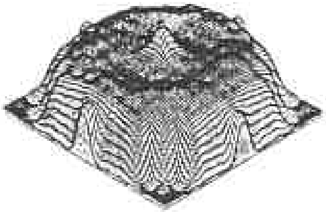
W_j^3 - High order services

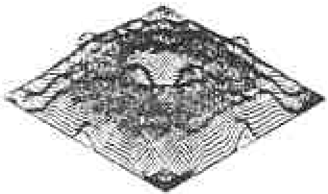
FIGURE 3.7. THE RESIDENTIAL LOCATION SUB-MODEL



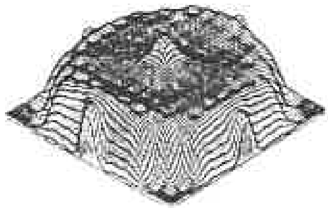


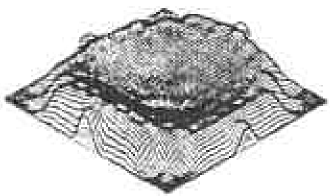
H_i^{1*} - Poor quality housing P_i^{*1} - Low income groups





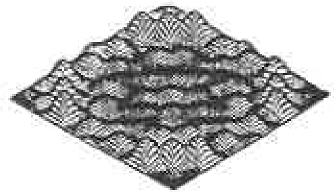
H_i^{2*} - Medium quality housing P_i^{*2} - Middle income groups



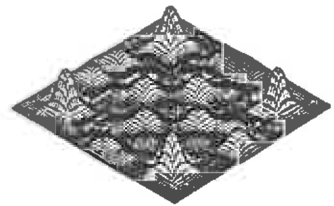


H_i^{3*} - Good quality housing P_i^{*3} - Upper income groups

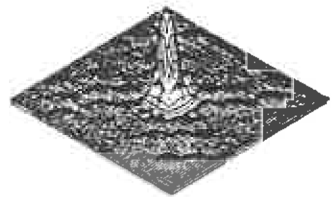
FIGURE 3.8. THE SERVICE SUB-MODEL



W_j^1 - Low order services



W_j^2 - Intermediate order services



W_j^3 - High order services

TABLE 3.1. VALUES OF PARAMETERS IN PIVOTAL RUN

Parameter	Equation	Computer nmenonic	Value
a^1		CGLAB (1)	0.3
a^2	2.15	CGLAB (2)	0.3
a^3		CGLAB (3)	0.3
b^{*1}		CW (1)	0.3
b^{*2}	2.16	CW (2)	0.3
b^{*3}		CW (3)	0.3
μ^1		BETA2 (1)	2.5
μ^2	2.17	BETA2 (2)	1.0
μ^3		BETA2 (3)	0.1
β^{R*1}		BETA1 (1)	2.5
β^{R*2}	2.23	BETA1 (2)	1.0
β^{R*3}		BETA1 (3)	0.1
α^{R*1}		-	1
α^{R*2}	2.23	-	1
α^{R*2}		-	1
α_4^{*1}		ALF41	0.65
α_4^{*2}	2.27	ALF42	0.7
$\alpha_4'^{*3}$		ALF431	0.725
$\alpha_4''^{*3}$	2.29	ALF432	0.85
q^{11}		Q(1,1)	1.0
q^{12}		Q(1,2)	1.2
q^{13}		Q(1,3)	1.3
q^{21}		Q(2,1)	1.1
q^{22}	2.30	Q(2,2)	1.3
q^{23}		Q(2,3)	1.4
q^{31}		Q(3,1)	1.2
q^{32}		Q(3,2)	1.4
q^{33}		Q(3,3)	1.6
θ^1		THETA (1)	0.32
θ^2	2.30	THETA (2)	0.16
θ^3		THETA (3)	0.08
p_i^1		PNORM (1)	1.0
p_i^2	2.30	PNORM (2)	1.1
p_i^3		PNORM (3)	1.2
a^1	2.31	OC (1)	0.3
a^2		OC (2)	0.4
a^3	2.33	OC (3)	0.9
α_1^{1*}		POWER (1,1)	0.1
α_1^{2*}	2.34	POWER (1,2)	0.1
α_1^{3*}		POWER (1,3)	0.1

Parameter	Equation	Computer nmenonic	Value
α_2^{*1}		POWER (2,1)	1.0
α_2^{*2}	2.34	POWER (2,2)	1.0
α_2^{*3}		POWER (2,3)	1.0
α_3^{*1}		POWER (3,1)	0.55
α_3^{*2}	2.34	POWER (3,2)	0.6
α_3^{*3}		POWER (3,3)	0.65
α_5^{*1}		POWER (5,1)	0.95
α_5^{*2}	2.34	POWER (5,2)	0.99
α_5^{*3}		POWER (5,3)	1.0
β^{1*}		BETA1 (1)	2.5
β^{2*}	2.37	BETA1 (2)	1.0
β^{3*2}		BETA1 (3)	0.1
$\alpha_1^{g=1}$		ALPHA2 (1)	0.01
$\alpha_1^{g=2}$	2.39	ALPHA2 (2)	0.005
$\alpha_1^{g=3}$		ALPHA2 (3)	0.0
$\alpha_2^{g=1}$		ALPHA1 (1)	1.01
$\alpha_2^{g=2}$	2.39	ALPHA1 (2)	1.05
$\alpha_3^{g=3}$		ALPHA1 (3)	1.1
$\alpha_{g=1}$		-	1.0
$\alpha_{g=2}$	2.38	-	1.0
$\alpha_{g=3}$		-	1.0

11

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