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THE EVOLUTION OF URBAN SPATIAL STRUCTURE:

THE EVOLUTION OF THEORY

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## 1. Introduction

The theory of the structure of urban areas, and the evolution of that structure, poses one of the most important problems in human geography. In this paper, a very broad review is undertaken of the evolution of this theory of evolution. The argument for doing this is threefold: by examining past contributions to theory within a broad framework, it is possible to see linkages which otherwise might be missed; secondly, we can seek to identify the most important contributions for the present from past work - and we find that some issues raised and partially dealt with in the past are not effectively handled today (and conversely); and thirdly, if a pattern of evolution of theory emerges, this can offer useful pointers for future research.

The argument of the paper is organised as follows. In section 2, some broad concepts are introduced which provide an overall framework. The following three sections contain reviews of theory in three roughly chronological phases: the classical approaches in section 3; the products of the second 'mathematical' phase in section 4; and the rapidly-developing concern with dynamics and system evolution in section 5. Some speculation about future phases are presented in section 6.

## 2. A basic framework

### 2.1 Introduction

It is argued that to approach theory building in relation to urban systems, six 'design' issues are involved. These are concerned with:

- entitation
- scale
- partialness/comprehensiveness
- form of spatial representation
- hypotheses
- methods

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Entitation - Chapman's (1978) notion - is concerned with the definition and articulation of the elements and systems which make up the overall system of interest. Scale relates to the level of resolution at which the system is observed. The approach to theory can then be on a spectrum from very partial relating to particular elements to totally comprehensive. It is argued below that the form of spatial representation is particularly crucial and is not usually considered explicitly. These four steps provide the subject matter and its description about which it is possible to build hypotheses and to select from a variety of methods to build theories and models.

The six steps are outlined briefly in turn.

## 2.2 Entitation

The main types of elements with which we are concerned are:

- people
- organisations
- commodities, goods and services
- land
- physical structures and facilities

Some of the characteristics which are important for describing people are listed in table 1 and some types of organisation are listed in table 2. The latter includes, by implication, a broad description of commodities, goods and services.

Our theoretical questions are concerned with the activities and processes associated with these elements, and in particular the *location* and *spatial interaction* patterns associated with them. A theory of dynamics is concerned with change in such patterns.

## 2.3 Scale

A useful broad division of possible levels of resolution is into micro, meso and macro. At the micro scale, at least some individual elements can be distinguished. The meso scale, in a geographical context, usually implies a sufficiently fine spatial level of resolution that spatial patterns can be identified, but probably deals with elements in groups rather than as individuals. The macro scale also relates to groups and will have at most a very coarse scale of spatial resolution.

The important point to note at this stage is that many hypotheses - about the behaviour of individual people or firms for example - involves a micro-scale representation. However, there are some properties and patterns in urban systems - such as transport flows and densities - which are essentially *meso* in scale, and yet others - such as nationally-determined commodity prices - which are essentially macro.

It is also important to emphasise that the notions of micro, meso and macro are approximate: more subdivisions may be involved. However, there *are* different scales, and one of the most important and difficult tasks of the theoretician is to relate ideas and concepts at different scales: the so-called aggregation problem.

#### 2.4 The partial-comprehensive spectrum

The simplest (at least conceptually) theoretical problem would involve the investigation of the locational behaviour of an individual household or firm with the rest of the overall system being taken as a 'given' environment. This is an important problem in its own right and can generate valuable insights for bigger problems - as we shall see. However, it is impossible to aggregate theories built in this way to add up to a theory of the structure of the system as a whole.

The next step along a spectrum towards total comprehensiveness would be to involve, say, two or more individuals (or firms) of the same type (in the same sector), perhaps because they are in competition. A further step would be to model competing classes or sectors. And so on. We will explore a wide range of examples below.

#### 2.5 Form of spatial representation

It is useful to begin this discussion with a useful, if approximate, distinction made by Paelink and Nijkamp (1978). They define consumers (roughly our 'people', but also including some activities of organisations) and producers (our organisations) and make a distinction between dispersed activities and concentrated ones. For relationships between consumers and producers, this generates the four possible combinations presented in table 3.

Examples of each case are:

- I - a firm and its suppliers (both concentrated)
- II - people (dispersed in residences) and producers of major services, say (concentrated)
- III - markets (concentrated consumers) and farmers (dispersed producers, using a lot of land)
- IV - people (dispersed) and small service facilities (dispersed - like corner shops)

The important argument here is that many activities can be assumed to be concentrated at a *point* in space while others (residing and agriculture in particular) are large land users. (Note that the 'dispersed' category covers the second type of example just mentioned, but also the case where there are a lot of points.)

There are three kinds of element which can make up spatial representation in relation to land use:

- points
- zones
- continuous space.

To these we might add, for completeness

- network links

in relation to spatial interaction.

Points carry the implications of particular spatial addresses and are obviously used in relation to concentrated activities. Continuous-space representations have been much used - for example in theories about densities, which are obviously 'continuous' properties. Zoning systems are interesting in that they can be used in both ways: quantities associated with a zone (such as population) can be notionally considered to be located at the zone centroid, and is thus the basis of a point representation at least approximately; or continuous-space properties (like densities) can be calculated as an average for each zone, and the pattern for all zones then provides an approximate representation of quantities which would more usually be dealt with in a continuous-space representation.

Network links connect vertices which are, of course, points. These may be a subset of real points at which there may also be concentrated activities; or the network may be a notional one, connecting zone centroids (a 'spider network', offering a coarser scale for the network representation).

One further distinction has to be introduced: when point representations are used, the points can be considered fixed - for example on a lattice, or the vertices of a network - or allowed to vary in continuous space. A location question would then be stated in terms of 'at which point?' in the former case, or 'where?' in the latter. If zone centroids are being treated as points, then this representation is of the first type, and 'at which point?' or 'where?' questions are then: 'in which zone?'.

Classical point representations, using a lattice as in central place theory for example, often involve a large number of points together with some undesirable regularities which are imposed by the form of the lattice. Continuous-space representations are usually very difficult to handle in relation to the mathematical tools which are available for them.

It is argued here that zone systems usually offer the best of all worlds, even though approximations are sometimes involved, for the geographical theorist. They include all the benefits of point representations (because they can be interpreted as such if appropriate); they can be regular or not (which may be useful in relation to data availability); and they provide the basis of a mathematical representation which is very powerful - simply by labelling zones consecutively and using the label as a zone subscript. Thus  $P_i$  may be the population of zone  $i$ ,  $T_{ij}$ , the number of trips from zone  $i$  to zone  $j$ .

For present purposes, the main point of this argument is that choice of spatial representation should be explicit; and also that it is useful to scrutinise the corresponding choice by other theorists to see if alternatives would be more fruitful.

## 2.6 Hypotheses

At the micro-scale, the assumptions of neo-classical economics usually dominate theory building. It is assumed in the theory of consumers' behaviour that preferences can be recorded in some way, usually in some form of utility function. The firm is correspondingly assumed to maximise profits in relation to a production function which expresses the technical possibilities. The broad frameworks of these theories contain relatively weak assumptions; what is usually missing is the detail. This is a point to which we will return in the final section.

At the meso- and macro scales, theories can be constructed either by applying micro-type assumptions to groups or sectors, or by using statistical averaging procedures, or through some more ad hoc phenomenological approach.

Further discussion about hypothesis formulation is reserved for the presentation of the various examples below.

## 2.7 Methods

A more detailed account of methods for theory building is given elsewhere (Wilson, 1981-A). Here, we simply present a list and some brief comments and again take the discussion further in relation to the examples below. The main headings include:

- algebraic modelling
- entropy-maximising
- account-based statistical-averaging
- optimisation
- network analysis
- dynamical systems theory (including bifurcation theory).

Essentially, this is a list of some of the more important mathematical techniques which are available to turn hypotheses into theories in the form of operational models. Many of these have become available in relatively recent years and it is important to bear this in mind in assessing the contributions of the classical theorists.

## 2.8 Synthesis

A presentation of the main range of theoretical problems based on the argument of the preceding subsections is offered as table 4.

# 3. Phase 1: the classical approaches

## 3.1 Introduction

In this section, we review some of the major classical contributions to the theory of spatial structure, taking their content as well-known and therefore focussing on the implicit theory-design decisions in relation to the six main headings of the previous section which are repeated here for convenience:



- entitation
- scale
- partial/comprehensive
- spatial representation
- hypothesis
- method

### 3.2 Agricultural land use - von Thunen

von Thunen (1826) was concerned with the intensity and type of agricultural land use around a single point which constituted the market. His analysis generated the well-known concentric rings. The scale for the analysis is basically meso - he does not distinguish individual farms for instance (though in another context, he does apply his method to a single farm). The approach is partial in being concerned with only one sector, and the more so because he does not deal explicitly with competition within that sector. The system being analysed is of the (single) point (consumer) - dispersed (producer) type, and space is treated continuously for producers. This illustrates a feature of geographical theory in such a treatment: that the task is to delimit the *boundaries* of different uses.

One of von Thunen's most important contributions was the basis of his main hypothesis: that land use is determined by the maximisation of economic (sometimes called 'location') rent. This notion of the concept of rent has had implications far beyond von Thunen's use of it. His results can be derived using elementary algebra - and indeed the original presentation is mainly in terms of the arithmetic of a large number of cases. However, the most valuable methodological treatment from a modern point of view is that developed by Stevens (1968) who showed how the analysis could be cast as a mathematical programming problem. It is also possible to use an alternative spatial representation.

### 3.3 Industrial location: Weber

Weber's (1909) main problem was the optimum location of a firm given the locations of its inputs and of its market (assuming all consumption is at a single point). So this is a micro-scale but very partial approach with a spatial representation based on points. There is the element of continuous-space about the treatment as noted earlier, however, in that the position of the firm can vary continuously. The main hypothesis is based on transport cost minimisation

(since it is assumed that all other costs and prices are location-independent). Weber used a mechanical device due to Varignon to 'solve' his problem, or, at least in the special case of two input points, a geometrical construction.

Weber was well aware of the limitations created by his assumptions and showed in a number of ways how his basic problem could be generalised. He used his 'materials index' to make observations about the likely behaviour of a whole industry, thereby making the approach less partial (and at a more meso-scale). He showed how to introduce variation in labour costs (though with some confusion in relation to spatial representation since it is not clear whether he restricts labour to points or not); and he made a valuable contribution about the nature of agglomeration economies. He was always aware of the possibilities of further extension and in his last two chapters writes like a modern systems analyst.

The most direct application of Weberian ideas in the present day is probably to the location of public facilities in a cost-minimising way. And development of mathematical tools have produced more effective formulation and solution of the problem; using mathematical programming and iterative computer algorithms. A simpler programming version of the model can also be obtained through a slight shift in spatial representation by restricting the possible location of the firm (or other facility) to one of a set of fixed points.

#### 3.4 Competition and market areas: Palander, Hoover and Hotelling

The next step in the argument, which was first taken by Palander (1935), is to make the Weber problem less partial by considering two firms competing for consumers. The problem then becomes one of delimiting their respective market areas. This is now a dispersed (consumers) - concentrated (producers) problem with consumers being handled in a continuous space representation and producers again as points. A further and important extension introduced at this time was to use economic theory to model the demand for goods as a function of basic price plus transport costs. Palander considered a variety of cases. Hoover (1937) extended the argument to more than one firm. Note that what is now a market-area problem now has a common feature with von Thunen: the task of delimiting regions in continuous space.

Hotelling (1929) extended the argument further by allowing the two competing firms to change their locations in response to each other and he considered the problem of 'stability under competition'. His famous example is the two ice-cream men on a linear beach. He showed that in the case of inelastic demand, the two would each locate at the centre, sharing the market, rather than at the quantiles, where they would also share the market in locations which would also be optimum for consumers (by minimising their transport costs).

### 3.5 Central place theory: Christaller and Lösch

Central place theory is concerned with two kinds of consumers - a continuously-distributed rural population, and people in settlements. The settlements are considered as points, and incorporate the producers of goods and services. This is, therefore, a coarse meso-scale picture, but it is much more comprehensive in approach than the examples considered previously. The theoretical basis of the model is built on the different kinds of market areas for different kinds of goods. Low-order goods have small market areas, and conversely.

In Christaller's (1933) system, a level is chosen to start the analysis, and an investigation of market areas determines the centre-spacing at that level. Because of the nested nature of the spatial system, this determines the spacing of all other levels, up and down; and, hence, since the range of goods can vary widely, so is the mix of goods sold at different types of centres.

Lösch's system is built from the lowest levels of the hierarchy. Increasingly-large market areas are considered with increasing orders of goods. Many systems are then superposed around a given metropolitan centre to determine a hierarchical system which has, in terms of mixes of goods, more variety than Christaller's system, but which has its own rigidities in other respects.

The theoretical basis of each model is built on the notion of the demand function for different types of good and the transport cost to the consumer of collecting them. As we have seen, the hypotheses determine the location of the settlement points. Relatively simple assumptions are made about the nature of the competition which generates the market area patterns. It can be argued that the form of spatial representation, linked with the hypotheses, creates spatial systems in each case which are too rigid to represent reality.

### 3.6 The urban ecologists: Burgess, Hoyt, Harris and Ullman

The approaches represented under this heading are very different in their disciplinary backgrounds. The overall title comes from the work of Burgess and others such as Park and McKenzie from the 1920s school of Chicago sociologists. The ecological label stems from their use of analogy with plant ecology as the basis for determining the patterns of urban land use. The main emphasis is on residential land use, and a continuous space representation is used. The task then becomes one of delimiting boundaries. Burgess' (1925) model generates concentric rings based on notions of succession and dominance of the outward movement of upwardly mobile people. Hoyt (1930) qualifies this by noting major sectoral differentiation; and Harris and Ullman (1945) by the influence of multiple-nuclei. The three sets of ideas can in principle be combined (Mann, 1965).

The methods involve elementary geometry and the hypotheses are not really sharp enough to make further progress without new tools.

### 3.7 The gravity model: Carey and Ravenstein onwards

The use of the 'gravity model' analogy in the social sciences has a history which goes back to the nineteenth century (for example, Carey, 1858, Ravenstein, 1885). The first applications were to the flow of migration between cities. Interestingly, the applications up to the 1950s were based on point-to-point flows, and in the first applications in retailing, there was a greater concern with the demarcation of market areas than with the flows themselves (Reilly, 1935). Simple terms, usually population and inverse-distance or inverse-distance squared were used for mass terms and impedance functions respectively so that, typically, the model took the form

$$T_{ij} = \frac{K P_i P_j}{d_{ij}^2} \quad (1)$$

4. Phase 2: computers, operational research, mathematical programming and statistical averaging

4.1 The origins of the second phase

It is far beyond the scope of this paper to attempt a detailed analysis of the origins of the current phase of theory in human geography. What will be attempted is a number of broad observations, conjectures really, about the generating impulses.

The beginnings of the transition can be found from about the middle 1950s and the decade of rapid development was the 1960s. At least four different impulses can be identified, although they are closely related. They mostly stem from outside geography. First, we note the development of large electronic computers. It became possible to tackle much bigger problems and to relate theory to observation. This effectively dates from the late 1950s. Preceding this chronologically, but perhaps a second 'cause' in relation to the 'new geography', was the development of operational research during the Second World War. This generated skills of mathematical modelling and provided a tradition within which much theory could be treated more formally and explicitly using mathematical tools. Thirdly, although operational research produced a range of methods, it is worthwhile singling out one which turned out to be particularly important for geographical theory: that is, mathematical programming, both linear and non-linear. The importance of this lies in the *behavioural* optimising processes which underly much geographical theory - utility or 'location rent' maximisation, profit maximisation or consumers' surplus maximisation providing examples. This also connects to an important subsidiary role of geographical theory in providing the analytical basis of much planning, and this involves a different kind of optimisation. Fourthly, it can be argued that geographers learned the techniques for modelling certain types of large system - Weaver's systems of disorganised complexity - either by account-based statistical averaging methods (applied to populations or economics) or by entropy-maximising methods (applied to spatial interaction and location problems).

There are, of course, many other starts to theoretical development. Another, for example, is the push from urban and regional economics from the early 1960s onwards. It is argued that it is useful, though, to focus on the broad background to see the context within which geographical theory made substantial strides forward.

In the rest of this section, we review examples of some of the products of this phase. It is impossible to be comprehensive and we concentrate on showing particularly the benefits of new representations and new methods. It is convenient to deal with spatial interaction first; and then aspects of location theory.

#### 4.2 Spatial interaction

The main influence on the development of spatial interaction models was the work of civil engineers working on large scale transportation studies from the mid-1950s onwards. They conceptualised the structure of transport patterns in terms of generation (production and attraction of trips, from origins to destinations), distribution, modal split and network assignment. Possibly because of the nature of the large scale surveys associated with these studies, the data analysis and modelling were based on discrete zoning systems.

A number of key advances can be identified (see, for example, Wilson, 1974, chapter 9):

- (i) because it was thought that total trip productions and attractions were 'sounder' quantities than flows, trip matrices were 'adjusted' so that origin and destination flows summed to give (or modelled) totals. In effect, this replaced the old constant of proportionality in the gravity model by multiplicative balancing factors.
- (ii) A corollary of this step was to seek more sophisticated models of 'mass' terms.
- (iii) A wide range of distance-decay functions were explored.
- (iv) Trips were categorised by purpose, mode, person type and so on.
- (v) New measures of 'distance' were introduced, culminating in the notion of 'generalised cost'.
- (vi) Network congestion was treated explicitly by coupling certain elements in the generalised cost terms with link costs estimated from network assignment submodels.

A typical spatial interaction model might then be

$$T_{ij}^n = A_i^n B_j^n O_i^n D_j^n f^n(c_{ij}) \quad (2)$$

for trips of category  $n$  between zones  $i$  and  $j$ .  $O_i^n$  and  $D_j^n$  are known origin and destination totals,  $f^n$  is a suitable function,  $c_{ij}$  is generalised cost.  $A_i^n$  and  $B_j^n$  are balancing factors which ensure

$$\sum_j T_{ij}^n = O_i^n \quad (3)$$

$$\sum_i T_{ij}^n = D_j^n \quad (4)$$

and hence are given by

$$A_i^n = 1 / \sum_j B_j^n D_j^n f^n(c_{ij}) \quad (5)$$

$$B_j^n = 1 / \sum_i A_i^n O_i^n f^n(c_{ij}) \quad (6)$$

These ideas were eventually absorbed into a range of other disciplines, including geography. A wealth of empirical experience became available, mainly through the transportation studies and theoretical understanding developed in a number of directions. By the late 1970s, for example, there were a large number of alternative theoretical derivations of spatial interaction models, ranging from entropy maximising (Wilson, 1970) as a form of spatial interaction model to various forms of utility maximising models (Williams, 1977), derived from micro-economic principles. These represent alternative ways of solving the aggregation problem. For a general review, see Wilson (1974, chapter 9).

The basic model was also related to the transportation problem of linear programming and the balancing factors could then be seen as transformations of the dual variables of a non-linear mathematical programme and interpreted in terms of comparative advantage (Evans, 1973, Wilson and Senior, 1974).

The outcome of much research can be seen as the development of an extended *family* of spatial interaction models, some of which have implications for location theory as we will see in the next section (Wilson, 1971).

### 4.3 Location theory

Two broad approaches to location theory will be sketched in this section. The first is, at least initially, essentially phenomenological: likely empirical relationships form the basis of the models; the second is economic. The two approaches were at first based on different spatial representations: using discrete zones and continuous space respectively. But when discrete zoning systems are used in the economic approach, it turns out that the two approaches come very close together in an interesting way.

The phenomenological approach can be seen in the first instance as based on the *singly*-constrained spatial interaction model. This takes the form

$$T_{ij} = A_i O_i W_j f(c_{ij}) \quad (7)$$

where

$$A_i = 1 / \sum_j W_j f(c_{ij}) \quad (8)$$

to ensure that

$$\sum_j T_{ij} = O_i \quad (9)$$

$T_{ij}$  is the flow,  $O_i$  the known origin total,  $W_j$  a measure of the attractiveness of zone  $j$ ,  $c_{ij}$  the inter-zonal cost and  $f$  the impedance function.

The total flow attracted to each zone  $j$ , say  $D_j$ , can then be predicted by the model as

$$D_j = \sum_i T_{ij} \quad (10)$$

Lowry (1964) and Lakshmanan and Hansen (1965) were among early authors who observed that such an interaction model was also a *location* model: the predicted variable  $D_j$  is a locational property of the system. This model is appropriate if it can be argued that such locational distributions are mainly determined by an interaction. Examples are the use of shopping centres, where  $T_{ij}$  is the flow from residences to centres, with  $O_i$  as given expenditure by residential zone and  $W_j$  the attractiveness of the shopping centre at  $j$ ; or residential location, where  $O_i$  would be a given job distribution and  $W_j$  the attractiveness of  $j$  for housing. The 'shopping' example can easily be extended to



other services; and since these services are a substantial source of employment, service models and residential models can be linked as was achieved in Lowry's (1964) *Model of metropolis*.

The economic approach turns on defining utility functions for people, both in relation to residence, purchase of goods and use of services; and profit functions for firms. A turning point in the development of the theory was Alonso's (1960, 1964) use of the concept of 'bid rent' in urban analysis - in some ways an extension of von Thunen's notion of rent. Bid rent is, in effect, a representation of the utility function. It is the maximum amount a consumer would be prepared to bid - not necessarily what he actually pays - for a particular combination of housing, goods and services *at particular locations* (where this is relevant). Most of the development of the theory has used continuous space representations and is well-reviewed by Richardson (1978).

Here we concentrate on the theory as expressed in zoning-system representations. This was achieved early in the Herbert and Stevens (1960) model which was a mathematical programming representation of Alonso's theory. Using the same kinds of variables as above, this can be written as

$$\text{Max } Z = \sum_{ij} T_{ij} (b_{ij} - c_{ij}) \quad (11)$$

subject to

$$\sum_j T_{ij} = O_i \quad (12)$$

This is a linear programming problem.

It turns out that there is the same relationship between the spatial interaction model of residential location as between the doubly-constrained spatial interaction model and the transportation problem of linear programming (Senior and Wilson, 1974). If the spatial interaction model is written in the form

$$T_{ij} = A_i O_j e^{\beta(b_{ij} - c_{ij})} \quad (13)$$

with the constraint (12), then the linear programme is the  $\beta \rightarrow \infty$  limit of (13). In practice, of course, there are more constraints and the model has to be disaggregated to introduce person-type and house-type categories.

There are two major implications of this convergence of modelling styles. First, the attractiveness terms in singly-constrained interaction models can, with suitable transformation, be interpreted in terms of benefits, and these are measures of bid rents, utility or preferences. Secondly, the interaction model can be seen as incorporating the same behavioural *optimising* principles as the economic model, but where the optimum optimum is not achieved. This *dispersion* of behaviour from the optimum is realistic in terms of market imperfections, variations in preferences which are not recorded in the simple functions used, and so on. It can be considered as generated from an entropy-maximising philosophy or from, say, random utility theory. Thus, the phenomenological model can now be reinterpreted as an economic model, and indeed in such a way that the traditional economic model is a special case (achieved when one or more parameters tend to infinity).

An important implication for model development is that great attention should be paid to the design of attractiveness functions. The  $W_j$  in equation (7) can be taken in a function of a number of variables - those which are the components of the appropriate utility function. Formally, we can write

$$W_j = x_{1j}^{\alpha_1} x_{2j}^{\alpha_2} x_{3j}^{\alpha_3} \dots \quad (14)$$

and an important research task is to identify the variables  $x_{kj}$  and the parameters  $\alpha_{kj}$ . To illustrate the potential complexities, we note that these variables themselves may be composite. For example, part of the residential attractiveness function may be related to 'accessibility to shops'. This can be written as

$$x_j = \sum_i W_i^{sh} f(c_{ij}) \quad (15)$$

where  $W_i^{sh}$  is the shopping centre attractiveness function. The term  $x_j$  can now be used in an equation like (14).

The argument so far essentially is that there is a rich set of models available for modelling the distribution of population and population activities. However, many crucial terms in attractiveness (or utility) functions have to be taken as given in the system. The obvious examples are housing supply and shopping centre supply. These represent the physical structures which contain the various activities. The next step in the argument, therefore, is to see how to model these.

In residential location models, it has been common to assume that housing supply 'follows' the population allocated by residential location models. This may be adequate for new development but does not describe the process of change adequately. The turnover of different kinds of people in housing is obviously more rapid than the change in housing stock itself, for example.

In service models, there have been essentially two approaches. The first and simplest involves inputting a *trial* set of exogenous structural variables, like shopping centre locations and size, and to use the activity models to calculate the revenues which would be attracted. Normal adjustments can then be made within a planning process. The alternative is to try to model the supply side. This in turn subdivides into two approaches. If the private sector is being modelled, as with shopping centres, this involves modelling entrepreneurial behaviour. In the case of public sector facilities, a mathematical programming formulation - say minimising costs or maximising consumers' surplus - can be used. This can be done using location-allocation models (as reviewed by Leonardi, 1980, for example) or by *embedding* spatial-interaction type models within an overall programming framework (Coelho and Wilson, 1977, Coelho, Williams and Wilson, 1978). An extensive discussion of many of these models can be found in Wilson, Coelho, Macgill and Williams, 1981). These approaches all involve, in different ways, the study of stability and system dynamics. This takes us into the third phase of modelling the evolution of urban spatial structure and we explore some possible approaches in the next section.

## 5. Phase 3: towards effective dynamic modelling

### 5.1 Origins

Phase 2 models could be, and were, used for forecasting; but in a comparative static mode. It was assumed that the systems of interest were always in equilibrium. In practice, of course, this is unlikely to be the case. The first major break away from this position came with the publication of Forrester's (1968) *Urban dynamics*. His model was a set of simultaneous difference equations which seemed to show quite complicated behaviour. The model was constructed on an ad hoc basis with inadequate theory and even more inadequate data. Nonetheless, it served to remind modellers that it was complacent to assume that strong equilibrium conditions and comparative static methods were always appropriate. However, possibly because of the difficulties of assembling data even for comparative-static modelling let alone anything more complicated, there have been relatively few alternative developments.

The Phase 2 models - particularly the Lowry model - have been developed into difference-equations dynamic models by Batty (1971). Cordey Hayes (1972) has shown how to write 'kinetic' equations - a form of accounting equations - for urban and regional systems. Essentially these take the form

$$\frac{dx_i}{dt} = \sum_j (a_{ji}x_j - a_{ij}x_i) \quad (16)$$

for a set of system-state variables  $\{x_i\}$  and transition coefficients  $\{a_{ij}\}$ . Such equation systems can be large and complicated, first because indices like  $i$  and  $j$  can themselves be lists -  $(i_1, i_2, i_3, \dots, j_1, j_2, j_3, \dots)$  say - and secondly because the transition coefficients can be complicated functions of many other variables, including state variables. The last condition makes the equations non-linear, and this is important in another context as we will see shortly.

Another approach to dynamic modelling has its origins in the work of Orcutt et al. (1961). This involves the micro-simulation of the states of individuals in the system. These states are changed and updated by Monte Carlo methods. The main theoretical

content of the approach is in the specification of the probability functions which determine the possible transitions. This method can be seen as solving numerically large simultaneous equation systems of the type (16). It is likely to be increasingly important, especially as more relevant data becomes available. Current large-scale projects using this method as a basis are described by Clarke, Keys and Williams (1979) and Kain, Apjar and Ginn (1976).

Another current approach is to focus on some basic growth equations and to elaborate the parameters within them to form the basis of dynamic urban models. A family of such equations is

$$\dot{X}_i = \epsilon_i (D_i(X_1, X_2, \dots, X_N) - X_i) X_i^n \quad (17)$$

for parameters  $\epsilon_i$  and  $n$ , and a set of functions  $D_i$  which represent some form of capacity or demand. Allen et al. (1978) have developed large models on this basis, linked to what they call the 'order from fluctuations' approach. A deeper understanding of the possibilities of such an approach - and indeed any approach to dynamic modelling which involves non-linear components to the equations - involves a knowledge of bifurcation theory, and it is to this that we now turn, first in a comparative static context, then in a more fully dynamic one.

## 5.2 The existence and stability of equilibrium states

In this section, we carry through the argument using the shopping model as an example. A phase-2 production-constrained spatial-interaction shopping model takes the form

$$S_{ij} = A_i e_i P_i W_j e^{-\beta c_{ij}} \quad (17)$$

where

$$A_i = 1/W_j e^{-\beta c_{ij}} \quad (18)$$

$S_{ij}$  is the flow of cash from zone  $i$  to zone  $j$ ,  $e_i$  the per capita expenditure in zone  $i$ ,  $P_i$  the population of zone  $i$ ,  $w_j$  the attractiveness of shops in  $j$ ,  $c_{ij}$  the inter-zonal travel cost and  $\alpha$  and  $\beta$  are parameters.  $A_i$  in (18) ensures that

$$\sum_j S_{ij} = e_i P_i$$

and the model can be used as a location model to predict the revenue attracted to each centre,  $D_j$ :

$$D_j = \sum_i S_{ij} = \sum_i \frac{e_i P_i W_j^\alpha e^{-\beta c_{ij}}}{\sum_j W_j^\alpha e^{-\beta c_{ij}}} \quad (20)$$

where we have used (17) and (18) to obtain the version of (20) on the right hand side. This shows how  $D_j$  is a non-linear function of all the  $W_j$ 's.

We explained in section 4 that it was customary to use this model with trial sets of given  $W_j$ 's. Suppose we now add a hypothesis on suppliers behaviour: that a  $W_j$  grows if  $D_j$  exceeds the cost of supply, say  $kW_j$  (where  $k$  is a unit cost and we are assuming that  $W_j$  is measured in size units, say floorspace and vice versa). This can be expressed as

$$\dot{W}_j = \epsilon(D_j - kW_j) \quad (21)$$

where  $D_j$ , recall, is given by (20). The equations (21) can therefore be written in full as

$$\dot{W}_j = \epsilon \left[ \sum_i \frac{e_i P_i W_j^\alpha e^{-\beta c_{ij}}}{\sum_j W_j^\alpha e^{-\beta c_{ij}}} - kW_j \right] \quad (22)$$

We now have a set of equations which model the dynamics of *structural* variables such as  $\{W_j\}$ . In the next section, we focus on the properties of these equations directly. Here, we concentrate on the equilibrium properties. The argument below follows Harris and Wilson (1978).

The equilibrium position is achieved when

$$\dot{W}_j = 0 \quad (23)$$

That is, from (21), when

$$D_j = kW_j \quad (24)$$

or, in full, using (22), when

$$\sum_i \frac{e_i P_i W_j^\alpha e^{-\beta c_{ij}}}{\sum_j W_j^\alpha e^{-\beta c_{ij}}} = kW_j \quad (25)$$

The left and right hand sides of (25) are essentially different ways of calculating revenue at equilibrium. It is convenient to write

$$D_j^{(1)} = \sum_i \frac{e_i P_i W_j^\alpha e^{-\beta c_{ij}}}{\sum_j W_j^\alpha e^{-\beta c_{ij}}} \quad (26)$$

and

$$D_j^{(2)} = kW_j \quad (27)$$

and then the equilibrium condition is

$$D_j^{(1)} = D_j^{(2)} \quad (28)$$

We can now investigate the nature of the equilibrium using a trick. Consider  $D_j^{(1)}$  and  $D_j^{(2)}$  as different functions of  $W_j$ . They can each be plotted ( $D_j^{(2)}$  easily because it is a line of slope  $k$ ). Various combinations for different parameter values are shown in figure 1.

A *bifurcation point* is a point in *parameter space* at which the nature of the solution to the differential equations changes; and here we are focussing on equilibrium solutions only. The parameter values at bifurcation points are *critical* values. Thus, we show for  $\alpha > 1$  that there are critical values of  $k$ , labelled  $k_j^{\text{crit}}$ , at which the non-zero stable equilibrium disappears. Since this *only* happens for  $\alpha > 1$ ,  $\alpha = 1$  is also a critical value. It turns out that the  $D_j^{(1)}$  curves move as functions of  $\alpha$  and  $\beta$ , and so if the system is in a critical state, it is critical with respect to  $\alpha$  and  $\beta$  as well as  $k$  - and indeed all other exogenous variables, like  $\{e_i\}$ ,  $\{P_i\}$  and  $\{c_{ij}\}$ . There is thus a surface in parameter space of critical parameter values. On one 'side' of this surface development in zone  $j$  is possible (a DP-state); on the other, it is not (the NDP-state). This notion is very important for locational analysis. As the whole system develops, zones will 'cross-into' the DP-state, or vice versa. The complexity arises from the mutual interdependence of zones. In particular, at a critical state for zone  $j$ , exogenous 'parameters' include all the  $W_k$ ,  $k \neq j$  - the structural variables for other zones. A numerical example of this argument is given by Wilson and Clarke (1979).

It is highly likely that, at any time, there are multiple stable equilibrium states (by taking different combinations of  $W_j$ 's to be zero). Thus, while this analysis gives a lot of insight into the mechanisms of development, it does not necessarily determine a particular path. This could be influenced in a major way by 'historical accidents' for example. We have the beginnings of a general theory which also demands knowledge of particular circumstances in particular places.

### 5.3 Extensions to non-equilibrium states: difference equations and bifurcation

We now return to equation (21) and explore bifurcation properties of a system which is not in equilibrium as a function of the parameter  $\epsilon$ . Here, we follow the argument of Wilson (1979), as illustrated by Beaumont, Clarke and Wilson (1980-A, 1980-B). The simplest illustration arises when (21) is transferred into difference equation form. This is achieved by writing

$$W_{jt+1} - W_{jt} = \epsilon(D_{jt} - kW_{jt})W_{jt} \quad (29)$$

using an obvious modification of notation. We have added a factor  $W_{jt}$  so that it represents logistic growth. Thus a unit step length is assumed - or that the step length has been absorbed into the parameter  $\epsilon$ . Equation (29) can be written

$$W_{jt+1} = [(1 + \epsilon D_{jt})W_{jt} - \epsilon kW_{jt}^2] \quad (30)$$

May (1970) has shown that a condition for a stable equilibrium point to exist is

$$0 < \epsilon D_{jt} < 2 \quad (31)$$

and that for

$$2 < \epsilon D_{jt} < 3 \quad (32)$$

the equation has various kinds of oscillatory solutions ranging from 2-cycles for  $\epsilon D_{jt}$  near to 2 to 'chaotic' behaviour as it nears 3. Thus  $\epsilon D_{jt} = 2$  is a critical point, and there are other bifurcation points up to  $\epsilon D_{jt} = 3$ . This illustrates a new kind of bifurcation: a transition from a stable equilibrium point to a periodic solution. Examples of this behaviour for a hypothetical case are shown in figure 2.



The bifurcation point arises when  $\epsilon$  becomes so large that the system 'over-responds' to  $D_j - kW_j$  differences. There are the additional complications that  $\epsilon D_{jt}$  itself varies with time and, for a particular  $j$ , is a function of what is happening in other zones - both of these effects arising through  $D_{jt}$ .

This kind of bifurcation has been illustrated in relation to one example. It has wider implications as we will see in the next subsection.

#### 5.4 Towards a comprehensive dynamic model

The example used in the preceding two subsections is obviously partial. Models could be built on similar principles for most sectors and it is clear from the example considered that they would be coupled. For example, the shopping model would be coupled to a residential model (through the  $P_i$ 's) and vice versa. This obviously increases the complexity of the overall model and the range of bifurcation possibilities. Some of the coupling also arises through disaggregation: if retailing is divided into sectors, then attractiveness for lower order goods may be enhanced by the presence of higher order goods. These complexities and couplings often enter through attractiveness terms, and hence into  $D_j$  terms in the differential or difference equation.

Examples of much more complex models are presented elsewhere (Wilson, 1981-B). The main point to make here is that the kind of bifurcation behaviour which has been identified, both in relation to equilibrium points and dynamics, can be expected even when the underlying submodels differ from those used to illustrate the argument here.

### 6. Future phases: towards evolutionary models

#### 6.1 The need for more operational research

Phase 2 has generated an immense variety of models for particular subsystems and for interdependent subsystems. Although most of these models have been 'tested' in some form or other, the quantity of empirical work per model is relatively small (see Wilson, Rees and Leigh, 1977, for examples). The only submodel which has been extensively used is the transport model. This work has been generated

by the large investment in conurbation transport studies all over the world. Its emergence has been the development of the model in a number of directions and a much better understanding of its potential usefulness (as, for example, in Williams and Ortuzar, 1980), though *this* understanding is not yet widely put into practice. Work in other sectors, and with comprehensive models, has suffered from an ill-informed backlash against the use of models in planning. This tide will, in time, turn. Perhaps the most effective operational work which could be carried out to assist this would be on relatively partial problems - such as the location of public facilities such as schools and health services, where the scope seems substantial but is relatively untapped. This may provide the confidence for larger-scale operational research in urban and regional modelling.

There remains the question of empirical work for phase 3 and future-phase models, but these questions are taken up separately below.

#### 6.2 The rewriting of phase 1

The essential argument of section 4 in relation to phase-2 models is that great progress has been possible through a combination of (mainly) two factors: first, a switch to discrete zoning as a form of spatial representation; secondly the availability for the first time of certain powerful mathematical tools, particularly those of mathematical programming. The developments of phase-3, particularly the understanding of possible stability and bifurcation properties of systems, has similarly come about through the availability of new mathematical tools, and these are built solidly onto phase-2 foundations.

It is interesting in the light of this to look back to the classical theories which were identified in section 3. In most instances, their essential contributions can be written, often very powerfully, in the new frameworks. We discuss each briefly in turn.

It is a straightforward matter to rewrite von Thunen's model as a mathematical programming model using discrete zoning. This process has not been taken very far - the seminal contribution being that of Steven's (1968).

Weber's main problem, similarly, can be rewritten as a mathematical programming problem using point locations (which are broadly equivalent to discrete zoning systems - taking the points as zone centroids). It is interesting that its greatest potential

usefulness in a modern context is possibly to the optimal location of public facilities. Indeed the one area which has not been effectively taken up in the phase-2 analysis above is that of industrial location. Much more sophisticated analyses of the relevant factors are now available (as, for example, in Smith, 1971). Some linear programming models have been built - for example by Stevens (1961) but as Smith notes, agglomeration economics and demand factors cannot easily be incorporated in such models. There seems to be no reason in principle why industrial location theory should not leap into phase-3 with models of the form (29) given suitable definitions of the variables.

Palander's and Hoover's work, and to some extent Christaller's and Losch's, were based on theories which involved the demarcation of market areas. These are totally explicit in spatial interaction based models, and the models are no longer tied to the restrictive assumption that market areas do not overlap. However, because the phase-2 models are meso-scale models, individual firms are not represented and so results like Hotelling's are not incorporated. There may be scope for further research here, particularly in sectors where a relatively small number of organisations are involved.

Central place theory, as we saw, is very much at a coarse meso scale. With the phase-2 models, it is possible to offer more precise measures of settlement structure, like  $E_j^r$  - number of employees by zone by sector,  $W_j^g$  - service facilities in sector g by zone, and  $P_i^w$  - the number of type-w people in zone i. And as we have noted, the market areas are all explicit. Further, the phase-3 methods provide the potential for building a dynamic central place theory and of avoiding the rigidities of the original formulations in this respect.

The urban ecologists' theories have been overtaken by factorial ecology - see Rees (1979) for example - again using discrete zoning systems, and by the various approaches to residential location described in section 4. There is one interesting sidelight here: the original work was based on analogies with plant ecology. It turns out that the dynamic models sketched in section 5 above have similar structures to corresponding models in present-day ecology. This is not surprising in that both systems are made up of elements competing with each other for resources.

Finally, we noted in section 4 that the gravity model, based on point-to-point or discrete zoning systems, has been overtaken by a much broader family of models with wider-ranging theoretical properties which can be applied in a variety of circumstances.

### 6.3 Alternative aggregations and submodels

In our review, we have perhaps over-concentrated on particular models. It is worth emphasising explicitly therefore that there is a great variety of possible models for different subsystems, especially at a detailed level. The alternatives often turn on different ways of solving the aggregation problem, relating micro-scale hypotheses to the final models at the meso scale. It can be argued, however, that the basic methods and design choices which form the basis of phase-2 and phase-3 approaches remain the same across most of these alternatives. We have to recall subsection 6.1 above and argue that the differences can only be sorted out in the context of detailed empirical work.

### 6.4 How comprehensive?

Most models are developed for particular subsystems. These can be linked through common elements. Variables which are exogenous in one may be endogenous in another. How important are the links? This raises a research question which has not yet been satisfactorily answered. To what extent are there systemic effects at the whole-system level which can only be picked up by a comprehensive model in which all the links are represented? This question, as with many others, can only be tackled by further empirical work. Intuition and the argument of section 5, however, suggest that there may be more systemic effects that were hitherto thought arising from the non-linearities introduced by the variety of variables in attractiveness functions (or utility functions) and the possible bifurcation points which these can generate.

### 6.5 Developmental dynamics: phase-3 into practice

The methods and models described in section 5 provide us with the potential to reproduce change from known mechanisms. They predict the size of housing developments and organisations given submodels of preference structures, demand functions and technical production possibilities. As in some other areas noted above,

what is now urgently needed is some relevant empirical work. This involves measurement of quantities we are unfamiliar with, like response times (implicit in the  $\epsilon$ -parameters of the illustrative models above) and relaxation times. There is also the hard problem of identifying bifurcation phenomena empirically. It may be possible to do this in a broad way fairly easily - for example in the transition from corner shop retailing to supermarkets - but much more difficult at the zonal level.

#### 6.6 Towards evolutionary models

The headings in this subsection and the preceding one reflect the distinction used in biology between development and evolution. In developmental biology, the concern is with the growth of a known organism through understood (or sought for) mechanisms. Evolution is concerned with the emergence of new species, usually of a 'higher order' in some sense. Is there an analogue in urban studies? It would have to relate to new forms of organisation or new modes of behaviour. The problem in modelling such phenomena is that the 'technical possibilities' are not easily (if at all) known before the form has evolved. So this may pose both an exciting research question and a limit on what can be achieved.

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Age  
Sex  
Education  
Position in a household  
Job description and location  
Income  
Wealth  
House type and residential location  
Shopping frequencies  
Baskets of goods purchased and location  
Recreational activities and locations  
Services used and locations  
Allocation of time amongst different activities

Table 1.    Some characteristics of people

Primary

Manufacturing

Industrial and domestic services:

Utilities

Construction (except housing)

Transport

Communications

Professional and scientific

Financial, etc

Legal

General governmental services to industry and the population:

Administration

Defence

Justice

Other

Housing

Distributive trades:

Wholesaling

Retailing

Governmental personal services:

Education

Health

Social

Fire

Police

Miscellaneous

Cultural and recreational - indoor

Cultural and recreational - outdoor

Table 2.    Types of organisation (by activity)

		PRODUCERS	
		CONCENTRATED	DISPERSED
CONSUMERS	CONCENTRATED	I	III
	DISPERSED	II	IV

Table 3. A spatial classification of activities

SCALE	ELEMENT/SUBSYSTEM	PARTIAL/COMPREHENSIVE	RELEVANT SPATIAL REPRESENTATIONS	STRUCTURE (SYNATIC)	PROCESS (DISPERSED)
MICRO	Households	Single:	Multiple:	Points relating to other points; possibly varying continuously	Location
	Farms	Given	(i) fixed env other sectors	Fields	Flows
	Firms - private (mar) public	Environment	(ii) changing env other sec sectors	Points relating to other points	Birth Death Migration/relocation Change in level of activity
	Services - private public		Points relating to 'markets'		
	Land plots		land units		Objectives/constraints
MESO	Residential	Single sector, given others	Interdept. sectors	Continuous zones	
	Agricultural			"	
	Industrial			"	
	Services - private public			"	Level changes
	Settlement patterns (boundary problems)			"	Location patterns
	Land use mix (networks)			"	Densities
	Transport/commu.		Networks		
				Flows	Accessibilities, etc.
MACRO	Population	Demography	Interdept.	Aggregated	Population totals: urban, demand for goods and services Prices, etc. Level changes Growth/decline processes
	Economy	Economy, given population			Economy: supply of goods and services

Table 4

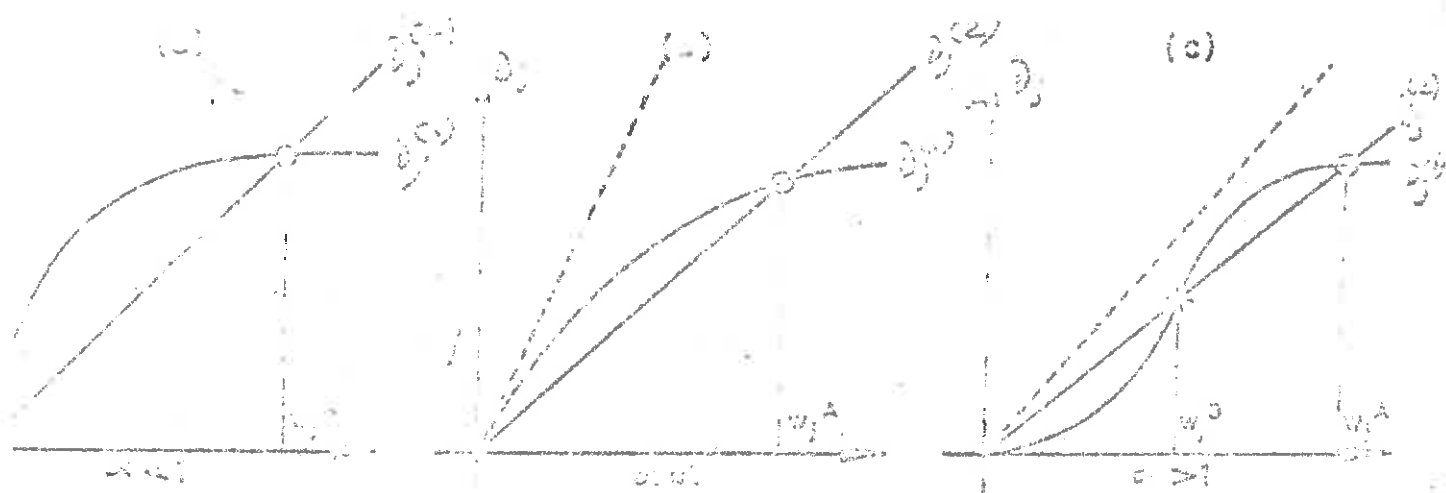


Fig. 1. Revenue-center size relationship

