ACCURACY OF POLISH FORECASTS AND DEMOGRAPHIC PROJECTIONS – SOME METHODOLOGICAL ASPECTS

Marek Kupiszewski

WORKING PAPER 95/8

SCHOOL OF GEOGRAPHY • UNIVERSITY OF LEEDS

WORKING PAPER 95/8

Marek Kupiszewski

School of Geography
The University of Leeds
LEEDS LS2 9JT, UK

Accuracy of Polish forecasts and demographic projections - some methodological aspects

Leeds 1995

Accuracy of Polish forecasts and demographic projections - some methodological aspects[1]

1. Introduction

The demand for more accurate and more disaggregated demographic forecasts and projections[2] arising from the disciplines other than demography, and especially from planning institutions, governmental agencies, health care specialists, education, social security, marketing enterprises or insurance companies, is growing from year to year. This causes a strong pressure acting upon demographers and urging them to construct increasingly accurate projection and forecasting models, that would make it possible to forecast population with more and more details and for smaller and smaller spatial units.

A number of theoretical works published in the '70s tried to satisfy this demand. In particular so called multiregional projection models were invented. These models were pioneered by the early works of Rogers (1968) and Stone (1971), and took their mature shape in the books by Rogers (1975), and Willekens and Rogers (1978), in which the multiregional model of population growth was presented together with the corresponding software package. These together with the book by Rees and Wilson (1977), in which the fundamental precepts of the multiregional population accounting models were presented form the methodological foundation. In later research the multiregional approach has been generalised to form multistate approach (Essays in Multistate..., 1980, Multidimensional Mathematical..., 1982) and multistate, multilevel models have been created (Rees, Stillwell, Convey 1991).

This paper is devoted to a comparison and evaluation of the accuracy of the projections of Polish population prepared using the multiregional methods, with those prepared using traditional methods. The adopted accuracy criterion takes into account the ex-post errors of the projections. Attention should be turned to the fact that in spite of numerous studies of projection accuracy a vast majority of publications refer to the simplest methods, such as extrapolation of population with elementary mathematical functions, while there is a lack of such accuracy studies for more complex methods. In particular, apart from Isserman's (1993) paper, the author is not aware of any attempt to assess the accuracy of multiregional (called interregional by Isserman) or multistate projections.

2. Projections and forecasts subject to comparison

For purposes of this comparison 5 forecasts and projections were taken into account. They have been elaborated in the period of 1978 to 1985. Thus, the accuracy was assessed for: 1. The component method forecast of the Central Statistical Office of 1979 (Prognoza demograficzna..., 1981, Kondrat, Mijakowska 1980). 2. The cohort method forecast of the

[1] I am most grateful to Mr. Marcus Blake who corrected my English.

[2] These two notions are commonly treated as identical (Keyfitz 1972, Stoto 1983, Cohen 1986, Smith 1987), although both for demographers and for representatives of other social sciences the difference between them is obvious. It is motivated by the fact most of the users do not attach much importance to the labelling of calculations made, but treat them always as forecasts, especially in these cases when they are published in the official governmental publications.

Central Statistical Office of 1979 (Prognoza demograficzna..., 1981, Kondrat, Mijakowska 1980). 3. The medium variant of the UN forecast of 1980 (World population prospects..., 1985). 4. Projection obtained with the Rogers model (Willekens, Rogers 1978). 5. Projection obtained with the Leslie model (Józwiak 1985). The latter two projections have been calculated by the Author of this paper.

Projections were calculated either for five-year period of 1978 to 1983 or as close as possible to it. Projections made on the basis of the Rogers and Leslie models were prepared exactly for this period. Two demographic forecasts of the Central Statistical Office from 1979 encompassing the years 1979-1985 based upon the data of 1979, but for purposes of this accuracy study the forecast results for 1984 have been adopted. The UN forecast covers the period of 1980-1985. All the forecasts of the Central Statistical Office (CSO) have been prepared for regions called voievodships with the breakdown according to sex and place of residence (urban-rural areas) and age.

Analysis of accuracy of the age structure results in the regional breakdown was performed for the 5 highly urbanised voievodships (Warsaw, Kraków, Poznan, Katowice and Lódz). Results of the UN forecast are published without spatial breakdown, and therefore in the present comparison they are considered only with respect to the age structure.

Projections assumptions and fundamental information concerning the time period covered and the ways in which data were prepared are presented in Table 1.

3. Conditions for comparability of projection accuracy

Let us consider under what conditions accuracy of population projections can be compared, and to what extent the results of such comparisons allow us to conclude on the quality of forecasting methods.

Within this domain one should consider five issues having essential influence on the results of projections and therefore on the evaluation of their accuracy:

1. Time horizon and starting points of projections.
2. Regional breakdowns.
3. Projection assumptions.
4. Methods of acquisition of data.
5. Benchmark or 'true' population with which projection results are compared.

3.1 Time horizon and starting points of projections

Time horizons of the projections compared should be the same, or, if this is not possible, efforts should be made to make them as close as possible. In the second case, it is necessary to calculate the average error magnitude per year of the forecast period, since the error magnitude increases with the length of the projection period (Isserman, 1977, Stoto, 1983, Smith, 1987). It should be made a condition that in the time between the starting point of the earliest projection and the termination point of the latest projection no essential changes of demographic trends occur.

Table 1: Characterisation of the forecasts and projections

Name of the forecast	Demographic forecast for the years 1980-1985. The component method version. Central Statistical Office, 1980	Demographic forecast for the years 1980-1985. The cohort method version. Central Statistical Office, 1980	Projection with the Rogers model	Projection with the Leslie model	The UN Forecast
Type of forecast	bottom-up	bottom-up	multi-regional	bottom-up	uniregional
Base year, for which data were gathered concerning: a)population stocks b)fertility c)mortality d)internal migration	 1979 1978-1979 see below see below	 1979 1978-1979 see below not applicable	 1978 1978 1978 1978	 1978 1978 1978 1978	 midyear 1980 1980 1980 not applicable
Time horizon of the forecast	1985	1985	1983	1983	1980
Control year for which accuracy was tested	1984	1984	1983	1983	1985
Spatial aggregation level being the basis for a)forecast itself	voievodships, with breakdown according to sex and urban-rural areas	voievodships, with breakdown according to sex and urban/regional areas	13 regions according to Dziewonski, Korcelli (1981)	13 regions after Dziewonski Korcelli (1981)	country as a whole
b)assessment of accuracy	13 regions after Dziewonski, Korcelli (1981)	13 regions after Dziewonski Korcelli (1981)	13 regions after Dziewonski Korcelli (1981)	13 regions after Dziewonski Korcelli (1981)	Country as a whole
Criterion for acquisition of data on a)population stocks b)births c)deaths d)migrations	 de facto de iure de iure de iure	 de facto de iure de iure de iure	 de facto de iure de iure de iure	 de facto de iure de iure de iure	 de facto de iure de iure de iure

Taking into account temporal migration	Yes	Yes	No	No	Not stated
Assumptions concerning changes in intensity of fertility	constant rates at the level observed for 1978-1979	constant rates at the level observed for 1978-1979	constant rates as of 1978	constant rates as of 1978	crude birth rate equal 18.5 for the period 1980-1985
Assumptions concerning changes in intensity of mortality	small decrease of infant and youngest children mortality; mortality of remaining population in 1975-1976 as in 1972-1979	small decrease of infant and youngest children mortality;	constant rates as of 1978	constant rates as of 1978	crude death rate equal 9.0 for the period 1980-1985
Assumptions concerning changes in intensity of internal migration	volume of migration according to the assumption of the Planning Commission; age structure as in 1978-1979, spatial structure as in 1976-1978 home affairs (not published)	not accounted for	constant rates as in 1978	not accounted for	not accounted for
Assumptions concerning changes in intensity of international migration	according to assumptions of the Ministry of home affairs	not accounted for	not accounted for	not accounted for	not accounted for

Source: see section 2.

3.2 Regional breakdowns

It would be advantageous to secure identical regional breakdowns for the projections compared. This condition, however, is not necessary for the projections of total population. In view of experience to date (Rogers 1976, Rees and Willekens 1981) the question whether to aggregate input data or output information has no essential influence on the results of projection. Sauberer (1981) ran Rogers projections for the Austrian population using both the 9-region or the 4-region spatial breakdown. For the 20 year time horizon the difference in the total populations obtained by Sauberer did not exceed 0.002%. A similar experiment was conducted by Soboleva (1980). She concluded that the accuracy of the forecast of population is not sensitive to the spatial breakdown adopted. It can therefore be said that the question of regional division may be of little importance with regard to the forecasted total populations, and this importance shall of course diminish with less spatially diversified population systems.

3.3 Projection assumptions

The most important question in the evaluation of the projection methods is undoubtedly to secure identical assumptions behind projections obtained with various methods. Pittenger considers that " accuracy testing of projection techniques is pointless because what is really being tested are sets of assumptions within limited historical context" (Pittenger, 1978 p. 273). While accepting the opinion of Pittenger one cannot deny the correctness of the following statement by Smith (1987), made in the argument with Pittenger: "I would argue that tests of forecasts' accuracy are essential because projections are so commonly used as forecasts and are so heavily relied upon for planning and building purposes" (Smith, 1987 p. 1002). Let us turn attention to the fact that Pittenger assesses the methodological questions, while arguments of Smith refer to practical problems.

While we would like to assess the quality of projection methods on the basis of the study of their accuracy, it would be advisable to consider to what extent the differences in the assumptions adopted in the projections, concerning future changes in fertility, mortality and migration, would influence the results. In some cases the assumptions may conceal the shortcomings of the projection model (as, for instance, in the comparison of multiregional projections with uniregional ones, adoption of the assumption of lack of migration significantly reduces the fundamental advantages of the multiregional model). Evaluation of the extent to which the projection error results from inadequacy of the projection method, or rather from the inadequacy of the projection assumptions adopted, may be very difficult.

3.4 Methods of data acquisition

In comparing these different methods it is essential to acquire comparable data sets. Rees (1986) notices that among various factors analysed the greatest differentiation of the life expectancy resulted from different manners of acquisition of migration data. Kupiszewski (1987, 1988a) demonstrates the necessity and significance of the ability of working with the comparable data sets when studying multiregional population projections. Specificity of the Polish population statistics with that respect are commented upon in section 4.

3.5 Benchmark or 'true' population with which projection results are compared

It is also important with which benchmark data, forecasts' or projections' results are compared. Will they be the census data or the current registration data, and what shall be their accuracy? The question of accuracy of the benchmark data has fundamental significance, since usually unrealistic assumption is being adopted, according to which the benchmark data represent accurately the size and structure of actual population. The imminent measurement error, usually with no significance in most applications, may play an important role for the assessment of accuracy of a population model.

4. Theoretical assumptions and reality

Let us see to what extent the postulates specified in the previous section are satisfied in reality.

4.1 Time horizon and starting time points of projections

Compared projections have similar starting points and time periods for which they were run: two of them (the Rogers-and Leslie-models'-based ones) are constructed on the basis of the data as of the end of 1978, and testing of accuracy is done for the state as of the end of 1983. In two other forecasts of the Central Statistical Office these two dates are delayed by just one year (1979-1984). The UN forecast encompasses the period of 1980-1985. The differences indicated are not that important as the five-year periods involved: 1978-1983, 1979-1984 and 1980-1985 do not in fact differ much, mainly due to the fact that these periods largely overlap in time. On the other hand, though, the difference between the demographic phenomena observed in 1978 and in 1985 is quite significant.

4.2 Regional breakdowns

Reservations can be raised in view of the incomparability of spatial units for which particular projections were prepared. However, arguments cited in section 3.2. allow us to assume that the differences resulting from varying spatial breakdowns adopted in population projections and forecasts are small enough to have minimal impact on the accuracy of forecasts.

4.3 Projection assumptions

The most significant forecasting assumptions are presented in Table 1. As far as fertility is concerned the forecast of the Central Statistical Office of 1979, and the Rogers and Leslie projections assume a very similar scenario: Over a five year period fertility shall not change in comparison with that observed in 1978 (for projections) and in 1978-1979 (for forecasts). In the other two forecasts the fertility level adopted was lower than the actual one. The assumption of a small decrease of infant mortality made in the forecasts of the Central Statistical Office were correct, while the mortality level in general increased, which was not envisaged in any of the forecasts. Assumptions as to internal and foreign migrations cannot be compared, since the Central Statistical Office did not publish them. More detail discussion of the impact of forecasts' and projections' assumption on their accuracy can be found in section 6.

4.4 Methods of acquisition of data

Preparation of data for all the forecasts and projections was carried out in a similar manner. The stocks of population actually residing in a given territory were calculated to provide the basis for projections. The calculations were based on the results of the National Census of 1978, adjusted with the data from the current registration of births, deaths and migrations. The data on the birth and death rates and migrations of the population, however, were grouped according to the legal criterion used during their acquisition.

In the statistical and forecasting practice in force until the end of 1982, there was an essential lack of consequence. Information related to population stocks was acquired applying as a criterion the actual place of residence, while information concerning all demographic events (births, deaths, migrations) was acquired applying as a criterion permanent place of residence as criterion. Differences between the real stocks of population and the numbers of those having permanent residence resulted from creation of a separate statistical category consisting of persons temporary registered for residence in a location different from the one of permanent residence. These persons increased the numbers of those living in their temporary place of residence, and were not counted in their permanent place of residences. Still, the places of permanent residences of their mothers was adopted as the places of births of children of these persons. Likewise, the death of a person temporarily residing elsewhere than the permanent residence, was counted for in the place of permanent residence, and not in the actual place of residence of the deceased.

The overall number of people living outside of their permanent place of residences was estimated by the Central Statistical Office in a survey made in 1978 as 848 thousand (Kopec, 1982). During the National Census of 1978 this number was evaluated at 978 thousand. The second value seems more reliable. The difference results from the fact that many people do not fulfil the formal duty of registering for temporary residence.

Application of various criteria for qualification of a person as a "migrant" introduced a certain incoherence to migration statistics as well. A person moving temporarily, even for a longer period, e.g. of several years, in connection with education or a job, was not considered a migrant (this person's actual migration was not accounted for in the migration statistics), but increased the number of the temporary residents in the destination. The registration of migration took place only when the change of legal status of this person from temporary to permanent residence occurred. Only in this case is the person in question counted as a migrant, and therefore increase the number of migrations in statistical register, in spite of the fact that in reality (in space) migration had taken place much earlier.

As a consequence of all these practices all the regional data on the fertility, mortality and geographical mobility are biased, and the relevant rates are somewhat inadequate, since numbers in denominators and in numerators refer in fact to various population categories. In order to diminish this bias of the rates used the Central Statistical Office proceeds during preparation of the forecasts through a re-evaluation of the regional stocks of population as well as the age and sex structures of the population, and thereby also the births and death rates (Aleksinska 1973, Mijakowska 1980). Because of the relatively poor statistical information referring to the stocks and structures of population registered for temporary residence (surveys are made every two years and concern only the numbers of temporarily registered in and temporarily absent from a given spatial unit, without any indication of the directions of flows, Kopec 1982), these estimates are not very precise. For more detailed discussion of the quality and availability of the migration data see also Kupiszewski 1992.

The incoherence of the official statistics has an obvious influence on the accuracy of forecasts, especially those carried out outside of the Central Statistical Office. This results from the fact that the Central Statistical Office does not publish the precise data on the age structure of persons temporarily registered outside the place of their permanent residence, and that it is therefore difficult to evaluate the exact birth and death rates.

In the Rogers and Leslie projections principles of the collection of the data were as close as possible to the ones adopted by the forecasters from the Central Statistical Office, in order to preserve the possibly high comparability of results of the Central Statistical Office forecasts with those carried out by the Author. The only difference was that the temporary migration have not been taken into account when the data for these models were prepared.

The UN projection was based upon the data prepared by particular countries (World population..., 1985), so that it can be expected that these data were prepared similarly as the data for the forecasts of the Central Statistical Office.

4.5 Benchmark or 'true' population with which projection results are compared

Projection results were compared with the observed population after 5 years, i.e., depending upon the projection, on December 31st of 1977, 1983, 1984 and 1985. This population was estimated on the basis of results of the National Census of 1978 and the yearly information related to numbers of births, deaths and internal as well as foreign migrations. The accuracy of these estimates is deemed to be very high. Zasepa (1970) who compared the Polish population stocks obtained through current registration and the ones given by the National Census of 1960 and noticed that the difference had not exceeded 0.3%. Attention should be turned to the fact that martial law in force in 1982-1983 in Poland enforced the people to follow the legal procedures properly, including the question of residence registration. It may be expected that this compensated to a large degree for the disorder still prevailing in 1980-1981 with that respect, leading to the possibility of not fulfilling the residence registration duties and therefore to lack of information on the fact of migration.

5. Methods of analysis of the projection error

Application of projections for broadly understood planning purposes, for determination of strategy of various social institutions, for example educational institutions or social security agencies, makes it necessary to estimate the errors of forecasts. Such estimates may be observed *ex post*, through comparison of results of projections with the actual population numbers of a given category, and *ex ante*, through estimation of the error magnitudes already at the time of projection construction.

Estimation of errors *ex ante* is valuable, since it allows to evaluate the forecast accuracy during actual preparation of this forecast. Simultaneously, because of the commonly adopted assumption, fundamental for the forecasting models, stipulating that future population development depends upon its past development, estimation of the forecast error may not give expected results, if the development process lost its continuity. This may occur due to events having catastrophic nature (epidemics, famines, wars) or due to, abrupt changes of population development trends, resulting, for instance, from changes in the economic situation or from changes in demographic policies.

Cohen (1986) classified the methods of confidence interval calculation into two classes: the model based one, in which values of confidence intervals of a projection are calculated as deriving from the model formulation simultaneously with the projection results (e.g. Cohen 1986), and the empirical one, where the error is estimated on the basis of the error distribution observed *ex post* in other forecasts (Keyfitz 1981, Smith and Sincich 1988). An essential drawback of the first method consists in the fact that it is not universal, but depends upon the projection model adopted. It was also criticised as not feasible by Smith and Sincich (1988) who advocated the latter method as delivering good predictions of future forecast accuracy.

Ex post estimation of the projection error made on the basis of comparison of the forecasted and true values allows to rectify the assumptions adopted for the projection, and on the other hand, to give the foundations for evaluation of the quality and adequacy of the accepted forecasting models, as well as assumptions adopted as to the future fertility, mortality and mobility changes, and perhaps also exogenous factors, if they are accounted for in the projection method.

The *ex post* error can be calculated either with regard to scalar values (population stocks in a given population category), vectors (for instance age structure of population in a given region), or matrices (for instance a matrix or age structure and spatial distribution of population in the country).

Comparison of the scalar values can be performed in many ways, starting with the very simple calculation of the magnitude of the average annual forecast error in percentage points (Kupiszewski 1987), or calculation with the formula proposed by Keyfitz (1972):

$$e = (P - c) / (R - c)$$

where: P - forecasted population numbers, R - true population numbers, c - a benchmark, e - the measure of projection accuracy.

Another, method where one calculated the square root of the average difference between the forecasted and the true values for particular years was proposed by Long (1987). More refined technique in which the magnitude of the *ex post* error is measured as the difference between the population growth rate according to projection and observed in reality for a certain period was applied by Stoto (1983).

From the point of view of the needs of users, as well as from the point of view of modern trends in the development of demography is seems that calculation of errors with regard to scalar magnitudes is presently not sufficient, and that it is necessary to calculate errors with regard to vector and matrix values, since the magnitudes projected are usually presented in the form of multidimensional "cubes", or more precisely in the form of multidimensional matrices.

The methods meant for studying the vector values may be easily generalised so as to become useful for the analysis of phenomena described by matrices, for instance - through deployment of the matrix into a vector row by row, wherever it may make sense. Some methods of calculation of the similarity measures and vector differentiation refer to vectors of shares (structure vectors). It is then required that all the vector components be non-negative and that their sum be equal one.

In the econometric literature, there exist quite a number of methods which provide for comparison of phenomena which can be described with vectors, and in particular with vectors describing the structure of a phenomenon. Extensive reviews of these can be found in the literature concerning taxonomy, classification and cluster analysis (see, for instance, Cormack 1971, Anderberg 1973, Rao 1977, and Kukula 1986).

Demographers, though to a lesser degree than econometricians, have been interested in these questions. More attention was devoted to these problems by scholars studying migrations (Sommermeijer 1961, Baydar 1983), who were looking for a tool for comparing changes in migration matrices. Recently, a review of literature concerning applications of methods of determination of similarity (dissimilarity) measures for vectors to estimation of accuracy of demographic forecasts was presented by J. Józwiak (1987).

Let us consider what are the properties of the similarity (dissimilarity) measures for vectors (matrices) which are desired from the point of view of analysis of accuracy of demographic projections. Let x and y be two real, n-dimensional matrices with m rows and n columns with all elements non-negative and bounded (say, matrices representing the forecasted and the observed population in the regional or other dimensions breakdown), let k be the measure of their similarity, that is, when x = y then k takes the maximum value, and d be the dissimilarity measure, that is when x = y, then d takes minimum value.

Let us take measures which meet the following conditions:

1. $k,d \in R$

2. $d(x,y) = d(y,x)$
   $k(x,y) = k(y,x)$

3. for every x,y : $|k(x,y)| < \infty$ and $|d(x,y)| < \infty$

These are very weak conditions, in fact, we are satisfied with any real symmetric and bounded measure.

Table 2 contains a selection of the most frequently encountered similarity (dissimilarity) measures for vectors and matrices. Formulas are given separately for vectors and for matrices, and characteristics are presented together with additional assumptions concerning particular measures, so that they are not commented upon in the text. Once measures which do not satisfy conditions (1) through (3) were eliminated, the following ones remained: the Clark's divergence coefficient, the Canberra metric, the linear correlation coefficient, the Sommermeijer coefficient and the Walesiak coefficient. The Canberra metric and the divergence coefficient of Clark were rejected, since they are very sensitive to small differences between the values considered.

From these the following measures were chosen: the linear correlation coefficient, the Sommermeijer coefficient (Sommermeijer 1961, Baydar 1983), and the Walesiak (1983) measure. The first two measures are used in frequently in the studies of this type. The last measure - the Walesiak coefficient - has been chosen since it emphasises the difference between the largest values, that is, when even a small percentage point error may cause significant consequences. Additionally, in spite of the fact that in this case the theoretical postulates formulated are not satisfied, the value of the χ^2 statistic, normalised with regard to the number of elements, was calculated, because this measure has a fundamental significance in statistical studies.

Table 2: Selected measures of similarity and dissimilarity of vectors and matrices   Notations: $x_i - y_i = d_i$; $x_i + y_i = s_i$; $x_i - \bar{x} = \tilde{x}_i$; $y_i - \bar{y} = \tilde{y}_i$; $x_i y_i = p_i$; $x_i^p - y_i^p = [d_i]^p$;   and analogously for matrices.

Name of measure:	Minkowski metric	Clarke's divergence coefficient	Canberra metric	Linear correlation coefficient	Normalized chi^2 statistics	Sommermeijer coefficient	Walesiak coefficient	Percentage deviation from the absolute average
Formula defining similarity/disimilarity a) for vectors	$\left(\sum_{i=1}^{n} d_i^p\right)^{1/p}$	$\left(\frac{1}{n}\sum_{i=1}^{n}\left(\frac{d_i}{s_i}\right)^2\right)^{1/2}$	$\sum_{i=1}^{n}\frac{\lvert d_i\rvert}{s_i}$	$\frac{\sum_{i=1}^{n}\tilde{x}_i\tilde{y}_i}{\sqrt{\sum_{i=1}^{n}\tilde{x}_i^2\sum_{i=1}^{n}\tilde{y}_i^2}}$	$\frac{1}{n}\sum_{i=1}^{n}\frac{d_i^2}{y_i}$	$\frac{2\sum_{i=1}^{n}p_i}{\sum_{i=1}^{n}x_i^2+\sum_{i=1}^{n}y_i^2}$	$\left(\sum_{i=1}^{n}\lvert [d_i]\rvert^p\right)^{1/p}$	$\frac{100}{n}\frac{\sum_{i=1}^{n}\lvert d_i\rvert}{\sum_{i=1}^{n}x_i}$
b) for matrices	$\left(\sum_{j=1}^{m}\sum_{i=1}^{n} d_{ij}^p\right)^{1/p}$	$\left(\frac{1}{m.n}\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\frac{d_{ij}}{s_{ij}}\right)^2\right)^{1/2}$	$\sum_{j=1}^{m}\sum_{i=1}^{n}\frac{\lvert d_{ij}\rvert}{s_{ij}}$	$\frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\tilde{x}_{ij}\tilde{y}_{ij}}{\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\tilde{x}_{ij}^2\sum_{j=1}^{m}\sum_{i=1}^{n}\tilde{y}_{ij}^2}}$	$\frac{1}{m.n}\sum_{j=1}^{m}\sum_{i=1}^{n}\frac{d_{ij}^2}{y_{ij}}$	$\frac{2\sum_{j=1}^{m}\sum_{i=1}^{n}p_{ij}}{\sum_{j=1}^{m}\sum_{i=1}^{n}x_{ij}^2+\sum_{j=1}^{m}\sum_{i=1}^{n}y_{ij}^2}$	$\left(\sum_{j=1}^{m}\sum_{i=1}^{n}\lvert [d_{ij}]^p\rvert\right)^{1/p}$	$\frac{100}{n.m}\frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\lvert d_{ij}\rvert}{\sum_{j=1}^{m}\sum_{i=1}^{n}x_{ij}}$
Comments	$p \geq 1$. For $p=1$ the taxicab or Manhattan metric obtains, for $p=2$ – Euclidean	This measure gives preference to differences between components of relatively low shares	See comment to Clarke's divergence coefficient	By substituting: $\tilde{x}_i = x_i - \bar{x}$ and $\tilde{y}_i = y_i - \bar{y}$ one gets the formula for cos $(\tilde{x}, \tilde{y})$			$p \geq 1$. This measure gives preference to differences in the components having greatest values. In the work reported square measure was used, i.e. $p = 2$	

Source: Cormack 1971, Anderberg 1973, Rao 1977, Baydar 1983, Walesiak 1984, Kukula 1986.

6. Empirical investigations: an interpretation

The study of accuracy of forecasts and projections, presented in this paper, is composed of two stages: in the first stage the accuracy of forecasting of population stocks is evaluated, while in the second stage the accuracy of forecasting of age structures and spatial distribution of a population is evaluated.

The first stage of the study had been carried out, with regard to all the forecasts and projections presented in the section 3.2, using the Keyfitz (1972) method mentioned before, with the constant c (benchmark value) equal zero. Results of this procedure are presented in Table 3.

Table 3. The accuracy of forecasts and projections of the size of population

	e	1-e
The CSO forecast, 1980		
- component method	1.001083	0.001083
- cohort method	0.999524	0.000476
The UN forecast	1.005745	0.005745
Rogers model, 1978.	0.999992	0.000008
Leslie model , 1978	1.000244	0.000244

Source: own computations

National population could be forecasted with the greatest accuracy using the results of the Rogers model. The component method forecast of the Central Statistical Office of 1979, and the results of the Leslie projection, yielded the values of the accuracy indicator, 1-e, worse by two orders of magnitude than those calculated for the Rogers model, and the other forecasts of the Central Statistical Office, as well as the UN forecast, were even less accurate.

The study of accuracy of forecasting the age structure of total Polish population was carried out with regard to all forecasts, while in the analysis of accuracy of age structures in regions as well as accuracy of forecasting of spatial population distribution the UN forecast was not accounted for since it was elaborated for the whole country

The accuracy of forecasting of the population age structure was the best when the Rogers and the Leslie models were used, and was somewhat worse with the Central Statistical Office forecast of 1979. The differences, however, were quite small. Distinctly less precise, on the other hand, were results of the UN forecast of 1980. Analysis of accuracy of forecasting of spatial distribution indicates that the best results were achieved with the Leslie model, somewhat worse with the 1979 forecast of the Central Statistical Office, while the worst accuracy was achieved with the Rogers model. Representation of the age-spatial structure was the most accurate when results of the 1979 component method forecast of the Central Statistical Office were taken into account, and least accurate for the results obtained with the Rogers model projection (see Table 4).

When comparing the accuracy of various forecasts one can easily see that, whenever age distributions or their aggregations (e.g. total population numbers) are taken into account, projections with the Rogers and Leslie models turned out very precise, while other forecasts were much less accurate. In these cases where the spatial factor entered into play, the Rogers and Leslie model-based projections turned out less accurate than the forecasts of the Central Statistical Office.

Table 4. Accuracy measures of forecasting of the age structures and spatial distribution of the population.

The accuracy of the forecasting of the age structure				
Accuracy measure	1	2	3	4
The Rogers model , 1978	1.0000	0.000006	0.0303	0.9997
The Leslie model , 1978	1.0000	0.000005	0.0296	0.9997
The UN forecast	0.9997	0.000037	0.0523	0.9947
The component method forecast of the CSO, 1980	0.9999	0.000008	0.0316	0.9995
The cohort method forecast (CSO) 1980	0.9999	0.000006	0.0292	0.9997
The accuracy of the forecasting of the spatial distribution of population				
Accuracy measure	1	2	3	4
The Rogers model	0.9997	0.000450	0.0584	0.9984
The Leslie model	1.0000	0.000004	0.0311	0.9999
The component method forecast of the CSO, 1980	0.9999	0.000009	0.0403	0.9997
The cohort method fore cast (CSO) 1980	0.9999	0.000009	0.0385	0.0006
Accuracy of the forecasting of the age structures and spatial distribution of population				
Accuracy measure	1	2	3	4
The Rogers model	0.8961	0.150655	0.0833	0.7414
The Leslie model	0.9990	0.000460	0.0322	0.9970
The component method forecast of the CSO, 1980	0.9997	0.000140	0.0242	0.9991
The cohort method forecast (CSO) 1980	0.9988	0.000685	0.0352	0.9964

CSO = Central Statistical Office
1 - the Sommermeijer coefficient
2 - the normalised chi square coefficient
3 - the Walesiak coefficient
4 - the linear correlation coefficient

Note: Sommermeijer coefficient values equal 1 results from the rounding errors.

Source: own calculations.

Let us consider, what were the factors that could have influenced the accuracy of forecasting of population stocks and age structures, and which were significant for the accuracy of the forecasts in the spatial setting.

Accuracy of forecasting of overall population is influenced by the evaluation of future development of three factors: numbers of births and deaths and net international migrations. Overestimation of population numbers in the UN forecast was caused by adoption of mortality and fertility levels which were too low.

With regard to the period 1978-1984 it turned out relatively effective for the forecasting purposes to adopt the assumption of the constant fertility level, equal to the one observed in 1978, for the whole period of 1978-1984 (this applies to the 1979 Central Statistical Office forecast and to projections based upon the Rogers and Leslie models). Korcelli and Kupiszewski (1988) note, that during the period in question there occurred fluctuations in the fertility level, but not a distinct upward or downward trend, while interregional differentiation was apparently getting smaller. Adoption of the mortality level as for 1978 (the Rogers and Leslie projections) was an assumption expressing better the processes of the turn of the '70s than continuation of the somewhat higher mortality level being the average of the 1972-1979 mortality levels (the 1979 Central Statistical Office forecast).

International migrations deserve a separate comment. Annual inflow to Poland in the '70s and '80s has not been exceeding 2000 persons. Outflow of people for permanent outmigration amounted to approximately twenty thousand persons a year, which means, altogether, a net external migration of the order of a little above 100 thousand people over 5 years (£oboda 1985). It can therefore be concluded that foreign migrations should not have a significant influence on the population numbers and structure in Poland. Statistics mentioned, though, show only the top of the iceberg, since it accounts only for these persons who obtained the permanent outmigration permit before leaving Poland. Such permits were issued in the '70s and '80s only rarely, mainly in cases of necessity of family reunification, and, in other cases, during the period of the martial law. Besides that, the wish of leaving the country for good is treated by the authorities as "antipatriotic" and "antisocial". Because of that foreign migration leading to permanent or very long residence in other countries were carried out in such a way that the potential migrants left the country on tourist passports, and then they remained abroad. Data on this category of migrants are not accessible, but it is estimated that the overall numbers per annum attain the order of tens or even hundreds of thousand per year (Korcelli 1992 Kupiszewski 1993). Movements of this group of people do not influence the evaluation of accuracy of projections and forecasts, since their trips abroad are not registered in the population registers.

Accuracy of the age structure forecasting over such a short time period is first of all influenced by the adopted level and schedule of mortality, which, as has already been said before, was better reflected in the assumption to the Rogers and Leslie projections than in whichever of the other forecasts. Somewhat smaller is the influence of the fertility level adopted (it is for the longer-term forecasts that the importance of this factor increases). It seems that the changes of fertility level and mortality schedule, being a consequence of migration do not play a key role in the 5-year period, although their significance shall increase with the longer time horizons of the forecasts.

When analysing the accuracy of forecasting of age structures one may be surprised by the fact that the forecasts not accounting for internal migrations (the Leslie projection, the cohort forecast of the Central Statistical Office, of 1979) are somewhat more accurate than the results for the Rogers model (with differences being very small), and are significantly more accurate than the remaining forecasts accounting for migrations (the component method forecast of the Central Statistical Office, of 1979, and the UN forecast).

Introduction of the spatial factor to evaluation of accuracy of forecasts and projections causes certain polarisation of results. The worst precision was obtained when the Rogers model was used for projection, both with regard to population stocks in regions and to age structures of population in these regions. This is insofar surprising as the Rogers model, the only one out of the set considered, is designed in such a way that it can account for the consequences of migrations, both with regard to the stocks of population and to the natural increase.

Two problems should be considered here: method of data acquisition and relevance of regional division adopted in the study. Temporary migrations were not accounted for in the migration data prepared for the Rogers model, while in the data prepared for the Central Statistical Office forecasts they were accounted for. The fact that people temporarily registered outside the place of their permanent residence are not treated as such in population forecasts is the main source of forecasting errors in studies of future population distribution. In the Central Statistical Office forecasts there is, for each year of the forecasted period, a recalculation of the population forecasted to account for persons temporarily absent and temporarily registered within every voievodship. Because the number of these people is significant (in 1978 a little more than 1/30 of the total population of Poland), then in spite of the fact that one can expect that the spatial scope of these movements is to a large extent subregional and regional, the ultimate influence exerted by this population category on the structure of population in particular regions - even those relatively big ones, as the spatial units considered in the study here reported - must be significant. This factor was especially important in the study of accuracy of age and spatial distribution of population in 5 highly urbanised regions-voievodships. The cause is the very high number of temporarily registered in these 5 units, attaining in 1980 the level of net balance of 233 thousand (with 147 thousand in the Katowice voievodship alone), while the overall number of temporarily registered in Poland outside of their place of permanent place of residence was 843 thousand. Three voievodships: Katowice, Cracow and Warsaw are the leading ones of those gaining population in this manner. Taking into account the extremely strong selective nature of the temporary migrations with respect to age (much more so than for the permanent migrations), one can envisage high errors in the age groups 15-19 and 20-24 years. Finally, the accuracy of forecasts which took into account shifts of population due to temporal migrations were much better then accuracy of population projections which did not take into account this factor.

The second problem to be considered is the relevance of spatial division adopted in the study. As Long (1987) said the relevance of disaggregation of the subnational model could influence its performance. Certainly the spatial division adopted does not mirror very well the network of flows. Mainly this is due to the lack of rural - urban breakdown (Paradysz 1987). The question to what degree the irrelevance of spatial division influence the accuracy of population projection, however recognised theoretically by J. Long (1988), requires further theoretical reflection as well as empirical investigation.

The key factor influencing the accuracy of forecasts of spatial distribution of population is adequacy of forecasts referring to interregional migrations. Publications of the Central Statistical Office do not indicate how accurate the assumptions as to the changes of migration made during preparation of the forecast of 1979 are. It is only said that the age structure of migrants had to be the same as in the period 1978-1979, and that the sex structure - same as in the years 1976-1979 (Kondrat and Mijakowska, 1980), while the total number of migrants is not given. The work published by Korcelli (1985) indicates that there occurred in the period 1978-1983, besides a reduction in population mobility, important changes of migration magnitude and age structure of migrants, while the general spatial structure of migration connections was preserved. The changes mentioned were not envisaged in any of the analysed projections.

While international migrations did not have essential influence on the number and structure of the total Polish population in the 1970's and early 1980's, they exert, because of the strong concentration of the existing outflow from Katowice and Opole voievodships , a strong influence on the population in these two regions. In order to illustrate this question suffice to say that outmigration abroad in 1982 from the Katowice voievodship equalled to as much as 40.46% of the natural increase and accounted for the 27.43% of real increase in this voievodship, see Kupiszewski, 1988b. Deformations of the age structure are even bigger, because of the selective nature of the migration process. The fact that the factor of foreign migrations was not accounted for in the Rogers and Leslie projections causes that the results concerning the spatial distribution and age structure of the population in some regions are significantly biased.

7. Conclusions

In order to establish which of the forecasts was the most accurate, one has to determine the order of preference over the criteria considered. In my opinion it is equally important to obtain a precise prediction as to the stocks and to the structure of the population. In the light of such an assumption there are no clear premises for distinguishing one forecast as better than the other ones. It seems quite obvious that the quality and complexity of the model used for forecasting has a very limited influence on the accuracy of the forecasts obtained, which is confirmed by the experience of other authors (Keilman, Kucera 1991). All models under consideration were the best in one or another of the accuracy measurement categories considered. The causes of differences in the projection accuracy should rather be looked for in the differences of accuracy and adequacy of data introduced into the models and in the adequacy of the forecasting assumptions adopted with regard to reality.

The fundamental conclusion, which can be drawn from the analysis presented here is confirmation of the opinion expressed by Pittenger (1978), Keilman and Kucera (1991) namely, that comparison of projection accuracy reduces in fact to comparison of the quality of projection assumptions, and is therefore not equal to evaluation of the models themselves and of the projection methods. In the light of the considerations here contained the comparison of projection accuracy means also a comparison of quality and adequacy of data used in projections.

References

Aleksinska J., 1973, *Perspectives of population development until the year 2000*, Statystyka Polski (GUS, Warszawa).

Anderberg M.R., 1973, *Cluster Analysis for Applications* (Academic Press, New York).

Baydar N., 1983. *Analysis of the temporal stability of migration patterns in the context of multiregional forecasting*. Working paper no 38 (NIDI, Voorburg).

Cohen J.E., 1986, "Population forecasts and confidence intervals for Sweden: a comparison of model-based and empirical approaches", *Demography*, 23,1,105-126.

Cormack R.M., 1971., "A Review of Classification", *Journal of the Royal Statistical Society* 134, 321-353.

Dziewonski K., Korcelli P., 1981a. "Migrations in Poland: transformations and policy",in: Dziewonski K., Korcelli P. Eds., *Studies in migrations and transformations of the settlement system in Poland*, Prace Geograficzne 140 (IGiPZ PAN, Wroclaw),10-90..

Dziewonski K., Korcelli P., 1981b. *Migration and Settlement: 11. Poland*. RR-81-20. IIASA, Laxenburg.

Essays in multistate mathematical demography, 1980. A. Rogers (ed.), RR-80-10, (IIASA, Laxenburg).

Gawryszewski A., 1988. *Spatial mobility of Polish population, 1952-1985*. An outline of habilitation thesis. IGIPZ PAN.

Isserman A., 1977, "The accuracy of population projections for subcounty areas", *Journal of the American Institute of Planners* 43, 247-259.

Isserman A., 1993, The right people, the right rates. Making population estimates and forecasts with an interregional cohort-component model. Journal of the American Planning Association, Winter 1993, 45-64.

Józwiak J. 1985. "Mathematical models of population", Monografie i Opracowania SGPiS nr 176 (SGPiS, Warszawa).

Józwiak J. 1987., *On the problem of the accuracy of population projections*. Mimeograph.

Keilman N., Kucera T., 1991. "The impact of forecasting methodology on the accuracy of national population forecasts: Evidence from the Netherlands and Czechoslovakia", *Journal of Forecasting*, 10, 371-398.

Keyfitz N., 1972. "On Future Populations", *Journal of American Statistical Association*, 68, 347-363.

Keyfitz N., 1981, "The limits of population forecasting", *Population and Development Review*, 7, 4, 579-593.

Kondrat W., Mijakowska J., 1980, "Forecasted demographic development of the country in the light of a new forecast", *Wiadomosci Statystyczne*, 25, 11, 1-5.

Kopec D., 1982, "Changes in distribution and structure of temporarily registered population in the years 1970-1980", *Wiadomosci Statystyczne*, 27, 10, 1-5.

Korcelli P., 1985. "Multiregional population projections made on the basis of the Rogers model",Typescript in the archives of the research program 11.5. (SGPiS, Warszawa).

Korcelli P. 1992. "European migration - the Polish example", Paper prepared for the conference: Mass Migration in Europe: Implications in East and West, (IAS-IIASA-IF, Vienna).

Korcelli P., Kupiszewski M., 1988. "Regional population projections. A time series approach", Paper presented at the Polish-GDR seminar, Szymbark.

Kukula K., 1986, "A survey of selected measures of structure coherence", *Przeglad Statystyczny,* 33, 4, 385-401.

Kupiszewski M., 1987. "Measurement of migration in modelling and forecasting of changes in distribution and structure of population", *Dokumentacja Geograficzna,* 5.

Kupiszewski M., 1988a. "Application of two types of migration data to multiregional demographic projections", *Geographia Polonica*. 54.43-61.

Kupiszewski M., 1988b. "Projection of the population number and structure in the Katowice voievodship against the background of current demographic tendencies", *Przeglad Geograficzny,* 60, 1-2, 3-16.

Kupiszewski M., 1992, *Sources and usefulness of information on mobility in Poland.* Working Paper 92/10. (School of Geography. The University of Leeds).

Kupiszewski M., 1993, "Poland as a source of Migration and Travel. A map with an accompanying text", in: P.Jordan, ed. *Atlas of Eastern and South-eastern Europe*, (Östrreichishes Ost- und Südosteuropa Institut, Vienna).

Long J.F. 1987., "The accuracy of population projection methods at the U.S. Census Bureau", Paper prepared for the Annual Meeting of the Population Association of America. Chicago.

£oboda J., 1985. "Regional aspects of external migrations in Poland the years 1950-1983",. Archives KPZK, Typescript no. 4293.

Mijakowska J., 1980, "Demographic forecasting - organization, principles, methodology", *Wiadomosci Statystyczne*, 27, 2, 14-17.

Multidimensional Mathematical Demography, 1982. Land K.C., Rogers A. eds., (Academic Press, New York).

Paradysz J., 1987. "Multidimentional demography in Poland", *Studia Demograficzne,* 88, 131-141.

Pittenger D., 1978. "The role of judgement, assumptions, techniques and confidence limits in forecasting population", *Socioeconomic Planning Sciences*, 12, 271-276.

Prognoza demograficzna na lata 1980-1985 (Demographic forecast for the years 1980-1985). 1981. (GUS, Warszawa).

Rao C.R., 1977. "Cluster Analysis applied to a study of race mixture in human populations", in J. van Ryzin ed., *Classification and clustering*. (Academic Press, New York).

Rees P.H. 1986. "National Case Studies. The United Kingdom". in A.Rogers, F.Willekens ed., *Migration and Settlement. A Multiregional Comparative Study*. (D.Reidel Publishing Company Dordrecht) 265-283.

Rees P.H., Stillwell J., and Convey, A., 1991. *Intra-Community migration and its impact on the demographic structure at the regional level.* International Conference 'Human Resources in Europe at the Dawn of the 21st Century'. Luxembourg, 27-29 November.

Rees P.H., Willekens F. , 1981. *Data Bases and Accounting Frameworks for IIASA's Comparative Migration and Settlement Study*. CP-81-39. (IIASA, Laxenburg).

Rees P.H., Wilson A.G., 1977. *Spatial Population Analysis*. (Arnold, London).

Rocznik demograficzny (Demographic Yearbook) 1976,1980-1984. (GUS, Warszawa).

Rogers A., 1968. *Matrix Analysis of Interregional Population Growth and Distribution*. (University of California Press, Berkeley).

Rogers A., 1975. *Introduction to Multiregional Mathematical Demography*. (Wiley, New York).

Rogers A., 1976. "Shrinking Large-scale Population-projection Models by Aggregation and Decomposition", *Environment and Planning A*, 8, 515-541.

Sauberer M., 1981. *Migration and Settlement: 10. Austria*. RR-81-16, (IIASA, Laxenburg).

Smith S.K., 1987. "Test of Accuracy and Bias of Country Population Projections", *Journal of the American Statistical Association*, 82, 400, 991-1003.

Smith S.K., T. Sincich, 1988. "Stability over time in the distribution of population forecast errors", *Demography*, 25, 3, 461-476.

Soboleva S., 1980. *Migration and Settlement: 8. Soviet Union*. RR-80-36, (IIASA, Laxenburg).

Sommermeijer W.H., 1961. "Een analyse van de binnenlandse migratie in Nederland tot 1947 en van 1957", *Statistische en Economische Onderzoekingen*, 115-174.

Stone R., 1971. *A demographic Accounting and Model-building*. (OECD, Paris).

Stoto M.A., 1983, "The Accuracy of Population Projections", *Journal of the American Statistical Association*, 78, 13-20.

Walesiak M., 1983."A proposition of family of measures of the share structures", *Wiadomosci Statystyczne*, 29, 10, 23-24.

Willekens F., Rogers A., 1978. *Spatial Population Analysis: Methods and Computer Programs*, RR-78-18. (IIASA, Laxenburg).

World population prospects as assessed in 1980. 1985, (New York, United Nations).

Zasepa R., 1970, "Inaccuracies of current countries population estimates on the basis of comparison to the population censuses figures", *Studia Demograficzne*, 22/23, 173-181.