

WORKING PAPER 154

TOWARDS A SIMPLE, PHYSICALLY-BASED,
VARIABLE CONTRIBUTING AREA MODEL
OF CATCHMENT HYDROLOGY

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Towards a simple, physically-based, variable contributing area model of catchment hydrology.

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Abstract

A hydrologic forecasting model is presented that attempts to combine the important distributed effects of channel network topology and dynamic contributing areas with the advantages of simple lumped parameter catchment models. Quick response flow is predicted from a storage/contributing area relationship derived analytically from the topographic structure of a catchment unit. Average soil water response is represented by a variable leakage infiltration store and an exponential subsurface water store. A simple linear routing procedure related to the link frequency distribution of the channel network completes the model and allows distinct catchment subunits, such as headwater and sideslope areas to be modelled separately. The model parameters are physically-based in the sense that they may be determined directly by measurement and the model may be used at ungauged sites. Procedures for applying the model and tests with data from the Crimple Beck catchment are described. The model predicts storm runoff amounts well but reproduction of the hydrograph shape is limited by the lack of an overland flow routing component in the model. However, the modular form of the model should allow application over a range of small and medium sized catchments while retaining the possibility of including more complex model components or optimisation techniques when suitable data are available.

Introduction

There is an undoubted need for a simple, physically-based hydrological forecasting model for medium sized catchments. In particular, a model with parameters that are directly measureable for a given catchment would obviate the need to derive parameter values for ungauged catchments from regional statistical generalisations (as per Nash, 1960; James, 1972; Institute of Hydrology, 1975). However, there is an immediate dilemma facing attempts to formulate such a physically-based model. Every catchment is an exceedingly complex open system with component processes and state variables that may change rapidly over space and time. Even if the processes operating were fully understood then an impossibly large number of parameters would be necessary to the response of the spatially structured system in any but the crudest detail (Stephenson and Freeze, 1974; Beven, 1975). On the other hand exceedingly simple models with only one or two parameters can provide a good empirical fit to the response of a particular catchment (Nash, 1957; Dooge, 1959; Lambert, 1969; Reed et al, 1975; Mandeville and O'Donnell, 1973). This paper presents a model for humid temperate areas that attempts to combine the advantages of simple lumped parameter catchment models with the important distributed effects of variable contributing areas and flow routing through the channel network, while retaining the possibility of deriving parameters by direct measurement within the catchment under study. The model will be presented in the simplest form, so that the principles involved may be clearly seen.

2. Variable contributing area concepts

Runoff may occur in a uniform catchment in at least four major ways:

- (a) Rainfall intensity exceeds soil infiltration or storage capacity resulting in overland flow all over the catchment. This is the classical version of Horton's (1933) model and is thought to have considerable relevance in areas of low vegetation cover and high rainfall intensities. However, in humid temperate areas with a vegetation cover the measured infiltration capacities of soils are generally high in comparison with normal rainfall intensities (Kirkby, 1969; Freeze, 1972). In this case the Horton model of catchment response is not applicable.

(b) Rainfall intensity exceeds soil infiltration or storage capacity on a variable area of near-saturated soils. This is the basis for Detson's (1964) partial area conceptual model of catchment response in which it is recognized that the spatially variable nature of infiltration capacities and differences in moisture status at the soil surface caused by downslope flow of water will result in some parts of the catchment being far more likely to produce infiltration excess overland flow than others. Engman and Rogowski (1974) have produced a relatively simple physically-based model based on this concept.

(c) Rain falling on stream channels and completely saturated soil. Where the latter are adjacent to stream channels (as is common) this source of overland flow contributes directly to the storm hydrograph (Dunne and Black, 1970). The zone of soil saturation may extend completely from bedrock or may build up above a relatively impermeable layer within the soil.

(d) Downslope lateral flow of saturated or unsaturated soil water. Most of this flow will be within the soil ('Subsurface stormflow') but it may locally exceed the soil storage capacity and return to flow over the surface at much higher velocities ('Return flow', Musgrave and Holtan, 1964; Dunne and Black, 1970). Subsurface flow velocities are commonly too slow to contribute appreciably to the peak of the storm hydrographs although in volume terms subsurface flow may dominate the overall response of the catchment in providing the hydrograph tail and low flows (Knisel, 1973)..

In small humid temperate catchments mechanisms 2 and 3 would appear to be the critical sources of storm flow, with subsurface flow making a highly significant contribution in setting up the soil water conditions prior to a further storm rainfall. The nature of these processes are thought to explain the observed non-linearity of catchment response and any simple physically-based hydrograph model must reflect this general conceptual knowledge of the mechanisms involved.

However, a choice is available between an infiltration rate approach to the prediction of overland flow, as in the model of Engman and Rogowski (1974), and a soil storage based approach in which the infiltration rate is essentially considered to be non-limiting such that the prediction of overland flow occurs when storage capacity is exceeded. The latter approach has been adopted here both because it would appear to be more physically realistic in British catchments and because it has operational advantages with respect to moisture accounting.

3. Modelling concepts

A number of physically-based deterministic models of the variable contributing area concept of catchment response are reported in the literature (Knapp, 1973; Calver, Kirkby and Weyman, 1972; Freeze, 1972; Hewlett and Troendle, 1975). These models, of varying degrees of sophistication and methodological rigour, have been essentially based on distributed moisture accounting for soil elements within segments of hillslope. The data and computing requirements of these models are, however, so great as to restrict their practical application to research projects (Stephenson and Freeze, 1974) where economic criteria are less dominant. In formulating a simpler model, the present study attempts to integrate the important distributed effects described above with the simple lumped model of the average response of soil water storage in the catchment. Simplicity is not held to be a virtue in itself but is a pragmatic response to a desire to produce a model that is capable of being applied operationally whilst reflecting the current state of knowledge of hydrological processes.

Kirkby (1975) provides an analysis of simple lumped storage models which forms a basis for the present study. The effect of combining several linear stores of differing time constants in a series chain is particularly noted. It is apparent that the stores with the highest time constants have the greatest effect on the response of the system so that the simplest 2 store approximation will mostly reflect the properties of the two slowest responding stores in a longer chain. If the stores are non-linear, with time parameters taken to represent average residence time in the storage element then this principal holds, as demonstrated by Wooding (1965). Furthermore, the slowest responding store must be most accurately modelled in terms of its non-linearity, because the outflow function is less sensitive to the form of the faster responding store provided an appropriate time parameter is used for it. A linear approximation to this store may commonly be sufficient.

Thus, for quick response flow, such as saturation excess overland flow produced on a variable contributing area, the duration of flow is likely to be critical only in small catchments. Indeed, for larger catchments where channel routing effects become increasingly important, overland flow may be treated as effectively reaching the channel within one time step. It is, however, very important to accurately model the

quantity of quick response flow and the time at which it is produced. This will involve modelling the dynamic response of a variable characteristics of the surface soil layer. The overall timing of infiltration will be particularly important, but the characteristics of the infiltration store will be less important than the subsurface store, which may profoundly affect the overall hydrograph. Non-linear effects of the subsurface store, if apparent, should be modelled carefully. Similar arguments will apply if additional groundwater storage elements become necessary as larger basins are modelled.

The effect of the channel network probably becomes important for catchments larger than about 10 km^2 where the time constants of the network compare with those of the infiltration phase. However, following similar arguments it is suggested that a linear network model may be sufficient in catchments of less than 1000 km^2 routing time becomes more comparable to the subsurface response time. It is also suggested that the important effects of the channel link frequency distribution of a given network on the form of the outflow hydrograph should be taken into account (Kirkby, 1976). In fact it is convenient to use the channel network to subdivide the catchment so that areas of markedly different hydrologic characteristics can be modelled separately. This preliminary study, however, concentrates on modelling the response of one catchment sub-unit. Details of the routing algorithm that completes the model are described by Kirkby (1976).

The exact structure of the model must necessarily reflect the types of hydrologic characteristics that are quick, convenient and economic to measure for a particular catchment. These include the topographic structure of the catchment together with infiltration rates, overland and channel flow velocities, a small number of discharge measurements and some simple measurements of the soil hydrologic characteristics. There will be a number of ways of interpreting such, essentially crude, measurements in terms of simple storage elements consistent with the discussion above. As a first possibility a model has been formulated from the following components (figure 1).

(a) A variable contributing area component related (by the nature of the processes outlined in section 2) to subsurface soil water storage. Rainfall falling on the contributing area, A_c , will immediately become overland flow or

$$q_{of} = iA_c$$

where i is an instantaneous rainfall rate.

(b) A surface interception and depression source, S_1 , with a maximum value S_o , that must be filled before infiltration from this store will take place at the estimated potential rate until it is empty.

(c) A linear infiltration store, S_2 , to model the delay before infiltrated water reaches the subsurface saturated soil water store. A simple form is

$$q_v = i_o \frac{S_2^2}{S_c^2} \quad (1)$$

where q_v is the flow reaching the flow reaching the saturated soil water store, i_o is the infiltration capacity of the surface soil, and S_c is a measure of average maximum near surface storage capacity in excess of field capacity. Under extreme rainfall conditions, water in excess of this maximum value S_c will be treated as overland flow. Unsatisfied potential evaporation may also be allowed from the S_2 store.

(d) A non-linear subsurface saturated soil water store. The simplest form of non-linear store is an exponential store as

$$q_b = q_o e^{S_3/m} \quad (2)$$

where q_b is the flow reaching the channel from the store, q_o is the flow when $S_3 = 0$ and m is a constant.

4. Modelling a variable contributing area

In attempting to model the response of a catchment in simple terms, the dynamic variation of moisture over the catchment must be grossly simplified. For a given average level of moisture storage there will be a wide range of possible spatial distributions, even assuming spatially uniform rainfall conditions. Thus, to try and integrate the distributed effects of a variable contributing area with a lumped model of average soil water response described above, some assumptions must be made about the duration of rainfall inputs. The simplest is to assume a time-independent steady state at a rainfall rate \bar{i} .

For a point in the catchment at which the area drained per unit contour length is a , the local slope angle is B and with a soil water store for which transmissivity at a point may be approximated by an

exponential function of the form

$$q = K_0 e^{S/m} \tan \theta \quad (3)$$

where q is the flow downslope under an assumed hydraulic gradient due to gravity alone and $K_0 \tan \theta$ is the flow when the (relative) storage, S , in rainfall equivalent units (e.g. mm) is zero.

Then under steady state conditions

$$q = \bar{I}a = K_0 e^{S/m} \tan \theta$$

$$\text{or} \quad S = m \log_e \left(\frac{\bar{I}}{K_0} \frac{a}{\tan \theta} \right) \quad (4)$$

The saturated area may then be defined as the area for which $S > S_T$

$$\text{or} \quad \frac{a}{\tan \theta} > \frac{K_0}{\bar{I}} e^{S_T/m} \quad (5)$$

for some local maximum storage values S_T . Over the whole catchment of area A , for constant K_0 and m , mean storage is given by

$$\begin{aligned} \bar{S} &= \frac{1}{A} \int S \, dA \\ &= \frac{1}{A} \int m \log_e \left(\frac{\bar{I}}{K_0} \frac{a}{\tan \theta} \right) dA \\ &= \frac{1}{A} \int m \log_e \left(\frac{\bar{I}}{K_0} \right) dA + m\lambda \end{aligned} \quad (6)$$

where $\lambda = \frac{1}{A} \int \log_e \left(\frac{a}{\tan \theta} \right) dA$
is a constant of the catchment.

Combining equations 5 and 6, the saturated area is that for which

$$\log_e \left(\frac{a}{\tan \theta} \right) > \frac{S_T}{m} - \frac{\bar{S}}{m} + \lambda \quad (7)$$

The spatial distribution of $\log_e \left(\frac{a}{\tan \theta} \right)$ can be readily mapped for a particular catchment unit. Then if S_T can be assumed spatially constant, a single parameter $\lambda + \frac{S_T}{m}$ relates the topographic structure of the catchment and average soil moisture storage to saturated area. In a subsequent rainfall the area obtained in this way may be closely identified with the concept of contributing area so that overland flow may be estimated as

$$q_{of} = i A_c \quad (8)$$

where i is an instantaneous rainfall intensity and A_c is the saturated area calculated from equation calculated from equation 9. Obviously

this procedure does not allow for the mechanism of return flow described in section 2 but does introduce a major source of non-linearity into the model. However, for larger catchments taken as a whole it may be expected that the slope of the contributing area/average storage function will be low at low values of S so that the condition for linearity (constant A) is more closely approximated. Examples of the relationship for small headwater and sideslope areas of the Crimble Beck catchment, near Harrogate, North Yorkshire are given in figure 2.

It should be stressed that these curves do not provide for a direct relationship between topography, average storage and contributing area: since this also depends on the parameters S_m and m . The intention is to provide an indication of the form of the relationship. Comparison of observed saturated areas with the distribution of $\frac{a}{\tan \beta}$ values suggests that this may be a useful indicator (Kirkby and Weyman, 1974). It is interesting to note that the Institute of Hydrology's variable contributing area model uses a functional relationship between soil moisture deficit and runoff/effective rainfall (Douglas, 1974) though without any clear indication of its derivation.

5. A worked example

The model is being applied to the whole of the catchment of Crimble Beck (figure 3) which is a tributary to the River Nidd. The catchment is predominantly under improved pasture with an area of rough pasture above about 220 m OD. The catchment is underlain by mostly sandstones and intercalated shales of the Carboniferous millstone grit series, although these are mainly covered by Pleistocene glacial tills and head deposits of known thicknesses of up to 1.5 m. There are also a number of small lenses of almost pure sand close to the highest divide and at least two thin local aquifers giving rise to small springs. Average annual rainfall (1950-1970) is of the order of 800 mm. The catchment has been subdivided into a number of smaller units on the basis of the channel network. Some of these areas have been instrumented to determine how far they differ in their hydrological characteristics.

This example concentrates on the Lanshaw sub-catchment (figure 4) which is an area of about 0.2 km^2 of rough pasture almost entirely sealed by a layer of clay till. The relief is low with slopes increasing slightly away from the watershed. Notable features of the area are

wet flush areas marked by juncus and flat marshy areas close to the watershed marked by sphagnum and other bog mosses. Drier parts of this small catchment maintain mostly poor heath species with abundant molinia and nardus grasses. Because of the presence of the virtually impervious clay close to the surface, the hydrologically active zone is confined to a shallow peaty layer and the storm response of the catchment is dominated by overland flow. However, measured surface infiltration rates are high and it would appear that the majority of the overland flow is generated by water in excess of that required to saturate the active layer. Areas of soggy surface conditions have been noted to vary over time.

6. The derivation of model parameters

The parameters of the model consist of the $\log \left(\frac{a}{\tan B} \right)$ distribution for the area, and the constants S_0 , S_0 , i_0 , q_0 , m and S_T as defined above. The $\log \left(\frac{a}{\tan B} \right)$ distribution is readily derived from topographic maps or survey of the catchment although it has been found that maps are seldom detailed enough to define small variations in topography, such as convergent hollow areas, which may be important in the correct characterisation of the distribution. Air photographs or an investigation in the field to determine likely flow lines along lines of greatest slope, may be helpful in this respect. Dunne et al (1975) have also reviewed other evidence that may be suggestive as regards areas that are commonly saturated. There is also, of course, scope for introducing impervious areas into the final contributing area relationship. Figure 5 shows this relationship for the Lanshaw catchment.

The parameters of the surface soil/vegetation storage elements (S_1 and S_2), S_0 , S_0 and i_0 may be determined using a sprinkler infiltrometer, producing results similar to those of figure 6. The steady state infiltration capacity is taken as equivalent to the maximum leakage rate, i_0 , from the upper soil store, whilst, if the tests are made after a dry period, the net absorption of water prior to the peak overland flow rate may be taken as a measure of maximum storage in the two stores combined (i.e. $S_0 + S_0$). The values assigned to each may be determined approximately by measurements of soil moisture just below the surface before and after the test. A number of tests should be made and average values determined. A small number of tests on the Lanshaw catchment, unfortunately using only a very high applied sprinkler rate, have, with exceptions, provided relatively stable values of these parameters.

(table 1). Future study will investigate the nature of the distribution of these parameters over space and time.

The parameters of the non-linear subsurface soil store (S_3), q_0 and m may be derived from an analysis of a recession curve for the catchment. It is felt that this is a valid procedure for a physically-based model, since discharge measurements provide the best integrated measure of the nature of storage in the catchment, and, even at ungauged sites, little effort is required to obtain a small number of measurements during a suitable recession period. q_0 is merely a constant of proportionality, being the observed discharge when the (relative) value of S_3 is zero, and m is given by the slope of a discharge/storage curves for the recession period (plotted with a logarithmic discharge axis). Storage values are calculated by an accounting procedure for a period of zero rainfall starting with some arbitrary value of storage. A winter period minimising the effect of evaporation provides the most satisfactory results. Some typical plots for the Lanshaw catchment are given in figure 7 with values of m given in table 1.

A value of maximum subsurface storage S_T can be derived from moisture content and porosity measurements for a number of soil profiles in the catchment at a time when discharge in the stream is known, such that, from equation 2

$$S_3 = m \log_e \left(\frac{Q}{q_0} \right) \quad (9)$$

and S_T , relative to this value of S_3 is given by

$$S = S_3 \text{ (measured maximum storage - measured storage)} \quad (10)$$

Little experience is yet available for the validity of this procedure, which is also the subject of ongoing research. Values derived for the Lanshaw catchment are given in table 1.

Finally, initial values of storage must be specified prior to a run of the model. The simplest case is to assume that the period of interest follows a dry period such that the surface store, S , is dry, the 'unsaturated' store is at 'field capacity' ($S_2 = 0$) and the value of the subsurface store can be derived by substituting a known value of Q for the first time period into equation 9.

7. Test simulations for the Lanshaw subcatchment

The model has been applied to two-monthly periods of data for the Lanshaw catchment using the parameter estimates of table 1. Rainfall records were available from an autographic tilting syphon raingauge installed on the catchment while discharge was calculated from continuous stage measurements upstream of a 90° v-notch weir. Estimates of evaporation were calculated from data collected by the Yorkshire Water Authority at Harlow Hill Waterworks, 4.8 km North East of the Lanshaw area and were arbitrarily distributed within each day.

Figure 8 compares observed and simulated discharges and predicted potential contributing areas for part of the winter period of November-December 1974, and figure 9 for part of the spring period of March-April 1975. Simulation efficiencies were calculated from total discharge volumes as

$$E_1 = 1 - \frac{|\sum Q_{sim} - \sum Q_{obs}|}{\sum Q_{obs}}$$

and

$$E_2 = 1 - \frac{\sum (Q_{sim} - Q_{obs})^2}{\sum (Q_{obs} - \bar{Q}_{obs})^2}$$

where the summation is carried out over all time steps in the two monthly period. It is readily apparent that the storm discharge amounts (as indicated by the high value of E_1) are simulated well but that reproduction of the shape of the hydrograph is poor (low E_2 values). This appears to be directly due to the lack of an overland flow component in the model since, although the response appears to be dominated by overland and return flows the time between cessation of rainfall and peak discharge may be much greater than the half hourly time step on which the simulations are based. Also important in this respect may be the lack of an explicit return flow component in the model, since such flow will normally be delayed by its residence time in the soil. The time delays imposed by flow within the soil are not as yet modelled adequately particularly since the measured parameters as used herein result in effectively zero delay before water reaches the saturated zone.

It is interesting to note that parameter optimisation using the Rosenbrock algorithm presented by Douglas (1974) tends towards parameter values that essentially use the exponential store as a quick response storage component. This is shown by the sensitivity analysis of figure 10, where increasing m and S_T minimises the effect of the

variable contributing area in relation to the exponential store. The result is a marked improvement in the simulated hydrograph shape but suggests that the aims of physically-based modelling may sometimes be at variance with obtaining an optimal fit to the observed discharge record. However, as noted above, as larger basins are simulated the timing of storm flow production in headwater areas may become less important than correct runoff amounts.

8. Conclusion

A simple physically-based hydrograph model has been presented that attempts to combine the advantages of a lumped representation of average soil water response with the distributed effects of a variable contributing area. Procedures for estimating the model parameters are described and the model has some success in predicting winter and spring periods of observed discharge volumes for one small headwater area of the Crimble Beck catchment. Possible conflict between maintaining the physically-based simplifying assumptions of the model and obtaining an optimum fit to the observed records is demonstrated.

In the future it is intended to improve the model structure and apply the complete model to the whole Crimble Beck catchment. A field measurement programme in the catchment has been designed to investigate the spatial and temporal distributions of the parameter values and to enable differences in the hydrologic response of headwater and sideslope areas to be distinguished and modelled. Application to a number of catchments should enable the range of applicability of the model to be evaluated and may generate new procedures for the calibration and validation of hydrograph models.

It is not intended, however, that the model formulation described above should be of complete generality. Rather the modular way in which the subcatchment models and components may be combined has been designed to retain flexibility in the model structure such that the inclusion of more complex components or the use of optimisation techniques is possible when suitable data are available. The aim is towards a physically-based system that is simple to apply over a range of gauged or ungauged, small and medium-sized catchments, but that is firmly based on the intention to use all relevant data that is readily available.

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Table 1.

Estimated values of the model parameters for the
Lanshaw catchment.

Parameter	Average value
S_D	2.5mm
S_C	3.8mm
i_0	17.5mm/hr
m (average of 8 winter storms)	1.26mm
S_T (for $S_3 = 100\text{mm}$ at $Q = 0.07 \text{ l/sec}$)	106.0mm

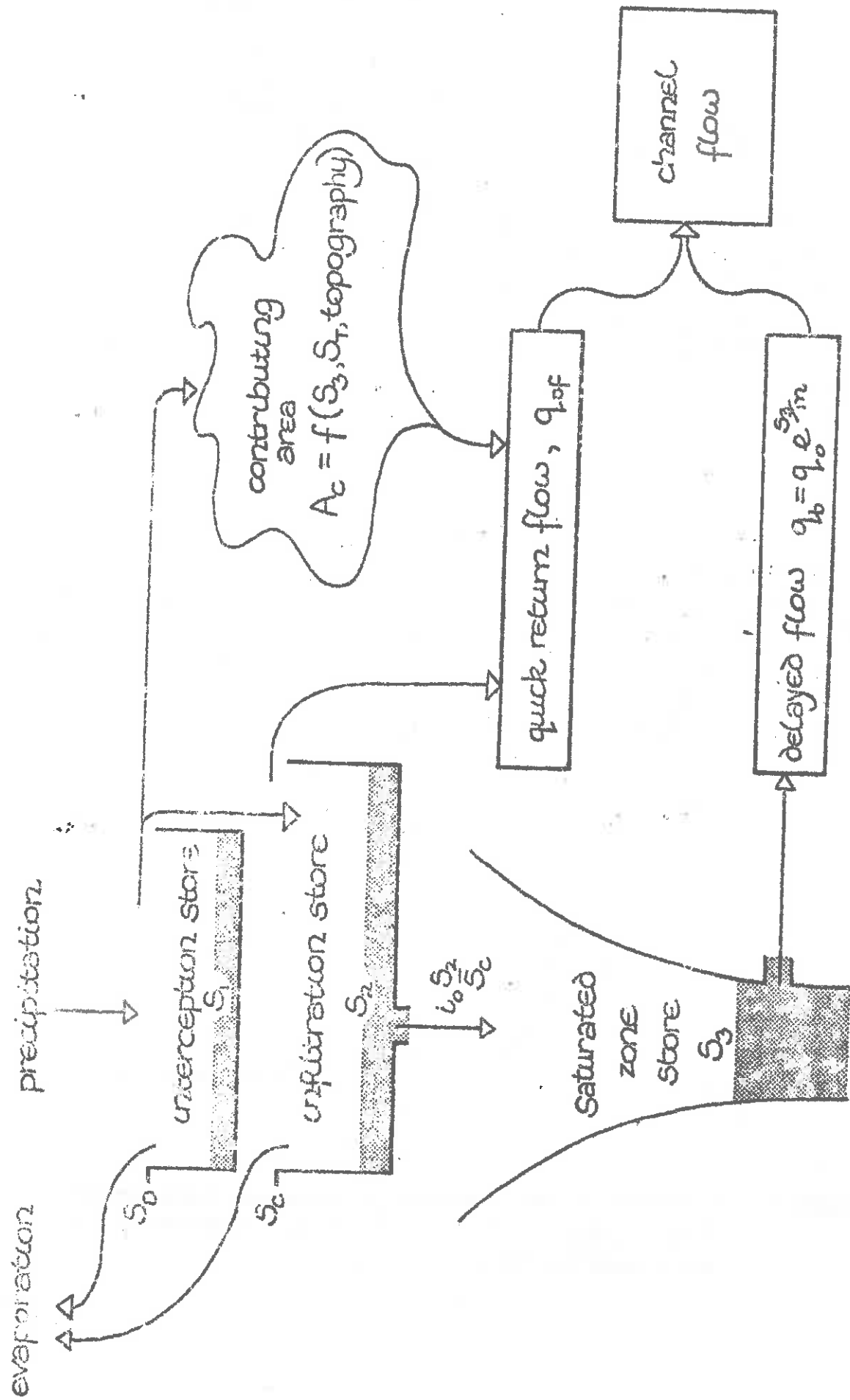


Figure 1. A schematic representation of the model for a single catchment unit (see text for details of the parameters).

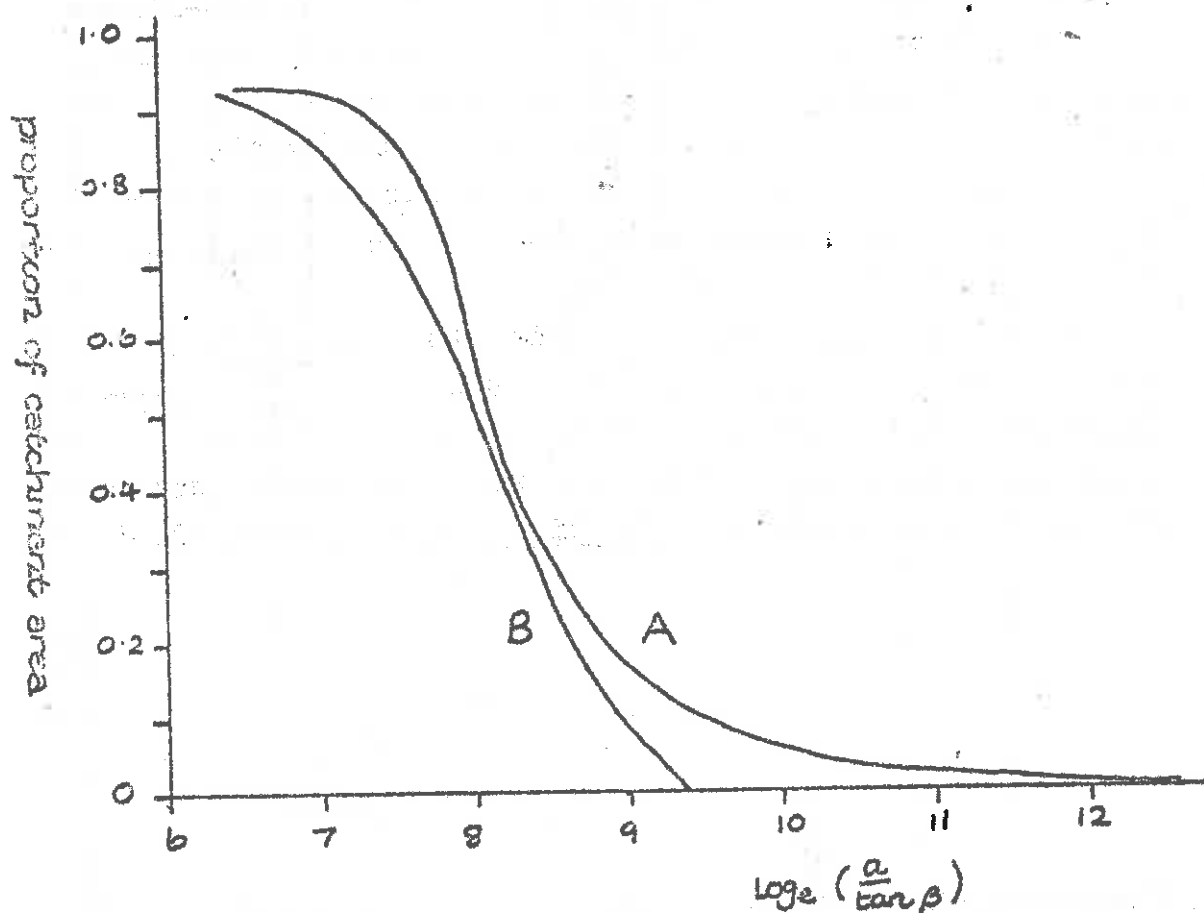
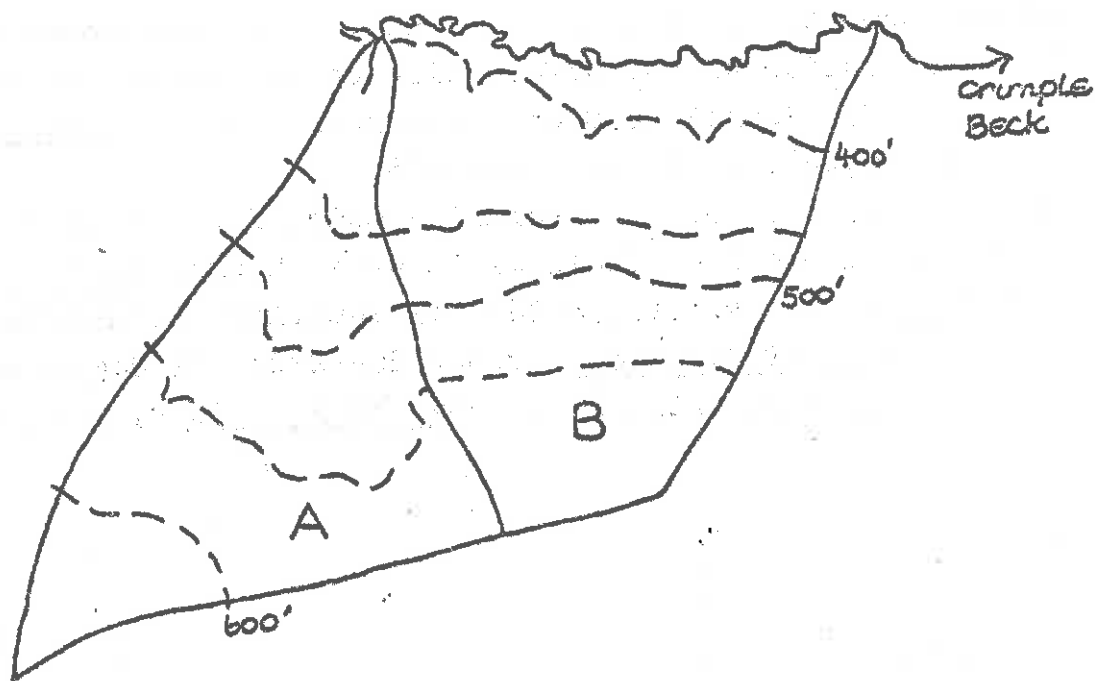


Figure 2. Headwater (A) and sideslope (B) areas and their derived $\log_e \left(\frac{a}{\tan \beta} \right)$ / area distributions. Contours in feet aod., scale 1:10560.

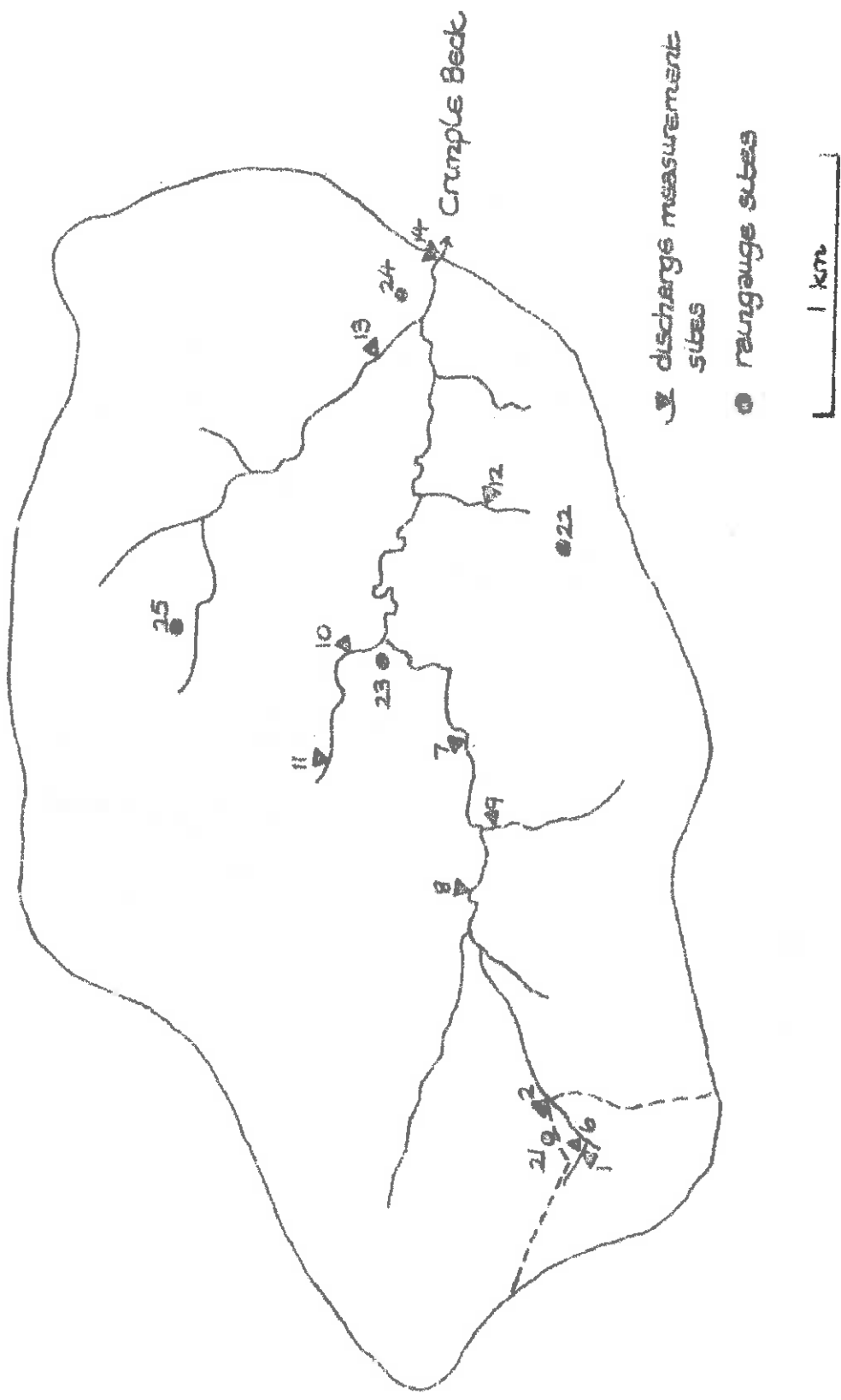


Figure 5. The Crumple Beck catchment and instrument network

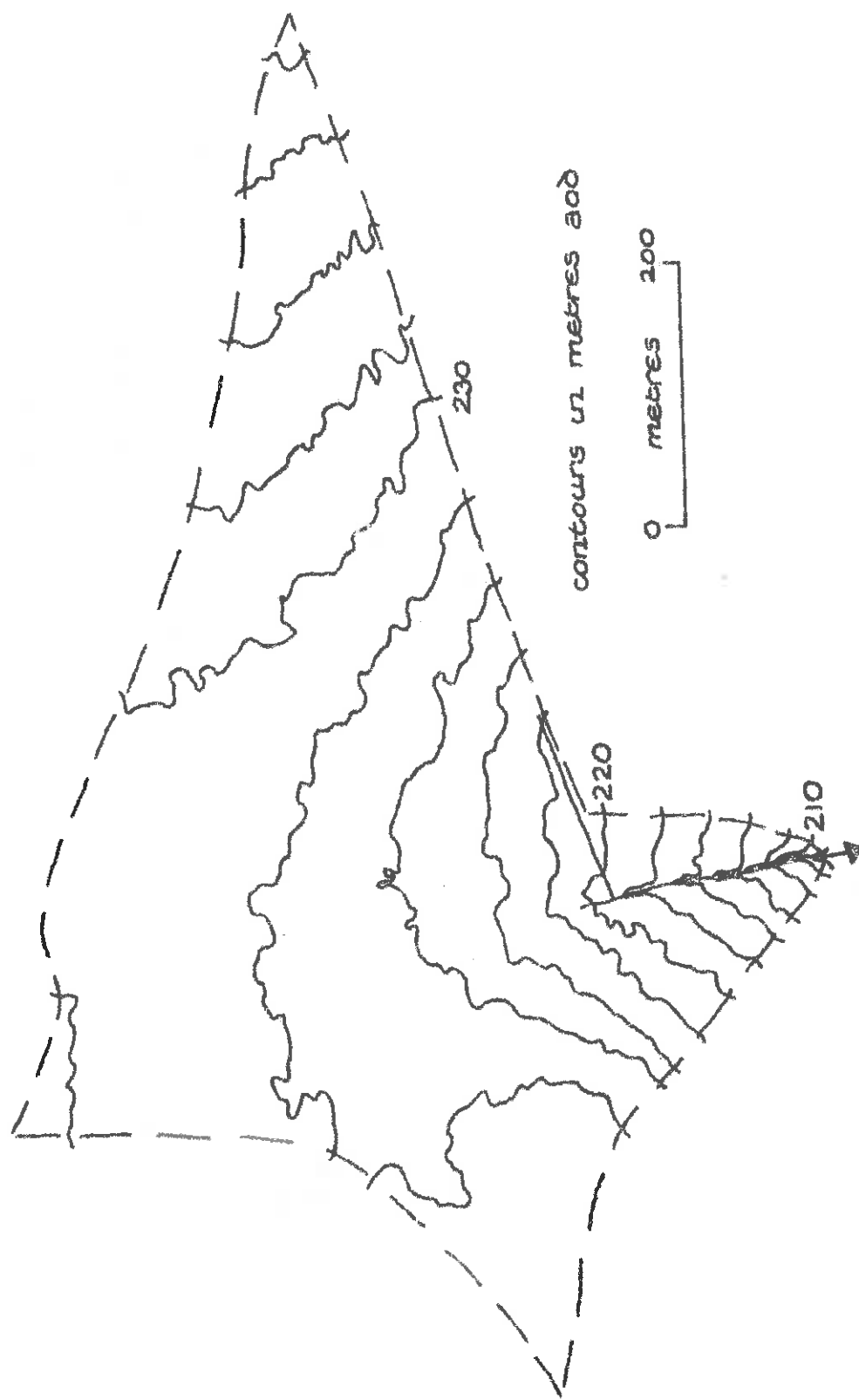


Figure 4. The Lanshaw subcatchment. Contours in metres a.s.l., scale 1:5000

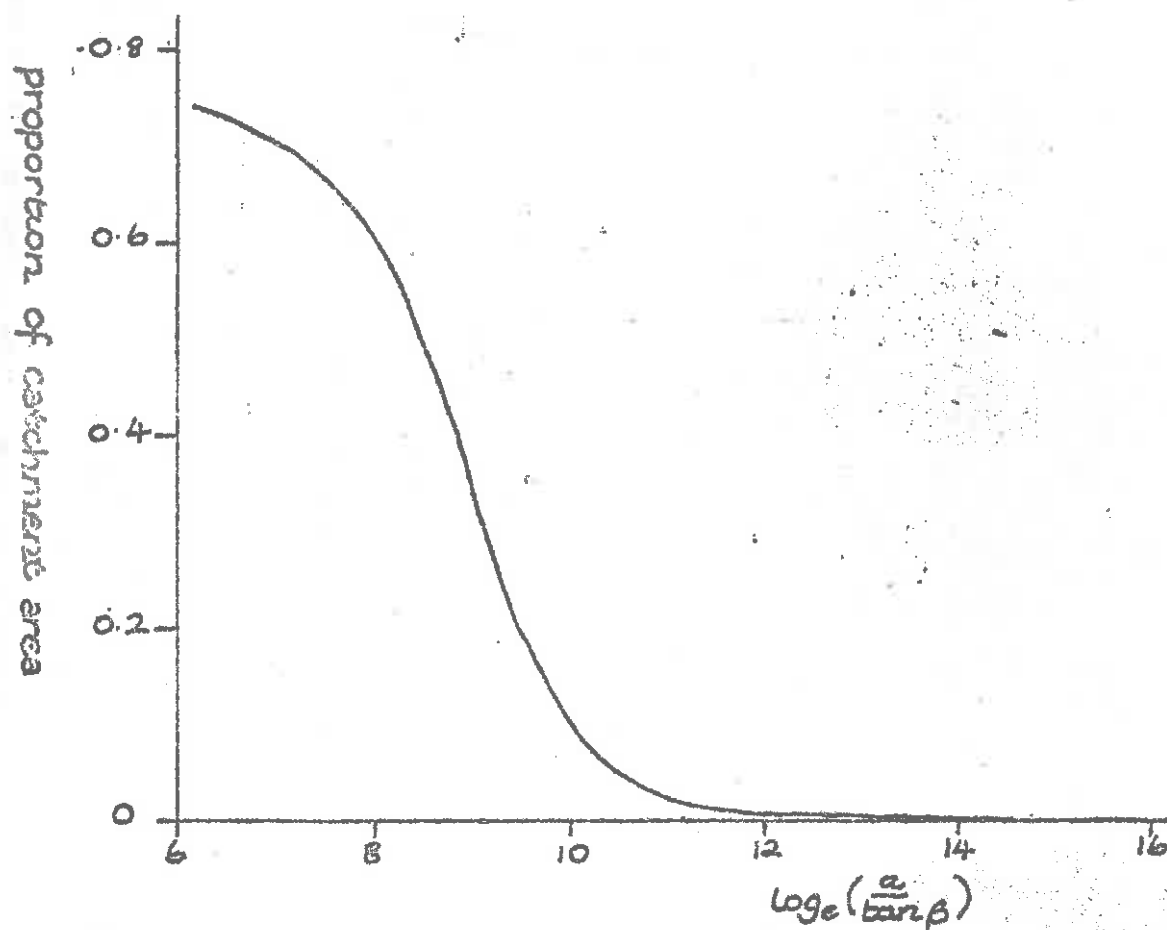


Figure 5. The $\log_e (a / \tan \beta)$ / area distribution for the Lashaw subcatchment.

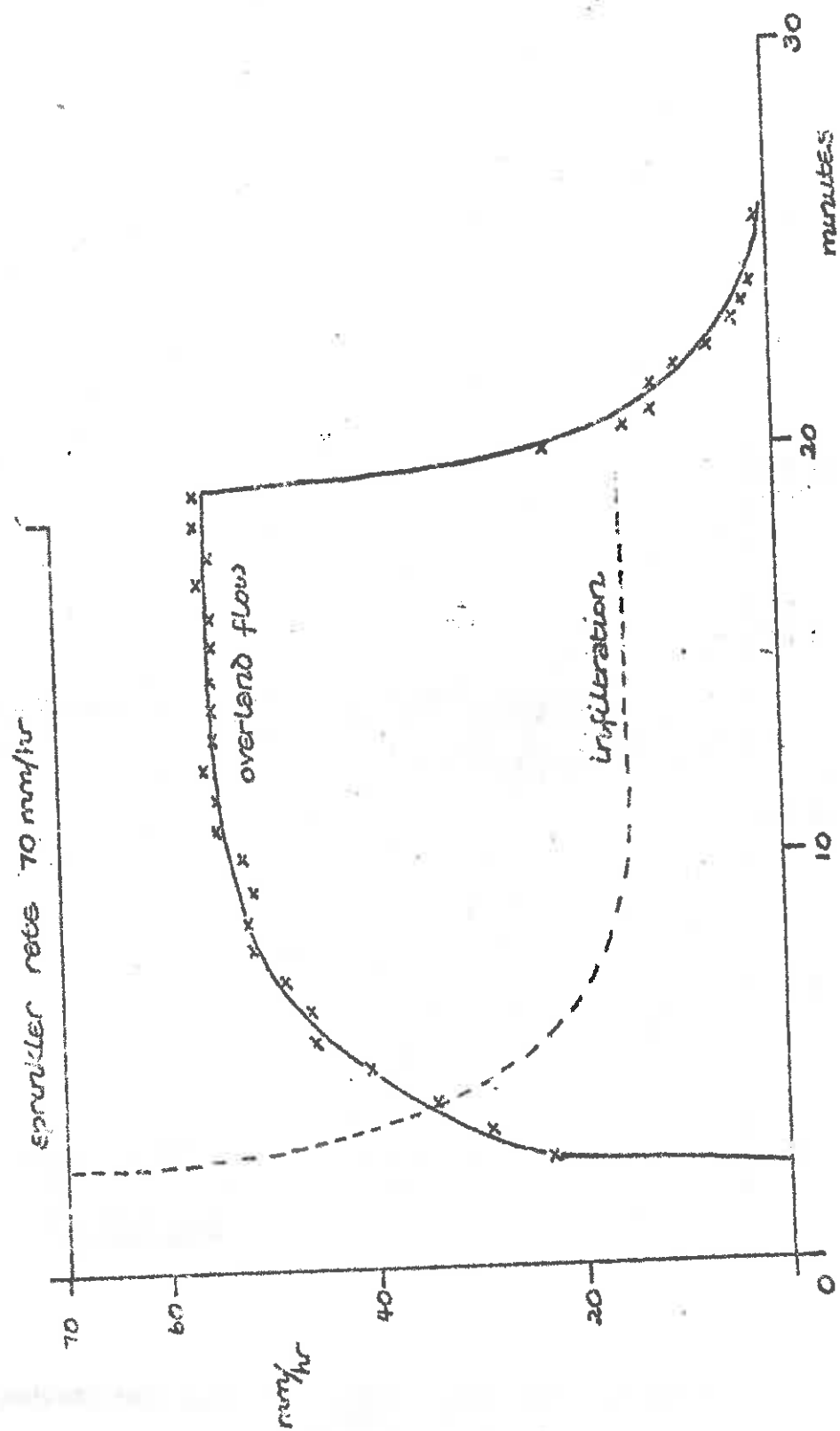


Figure 6. Results from a ram of the sprinkler infiltrometer, area 0.72 m^2 .

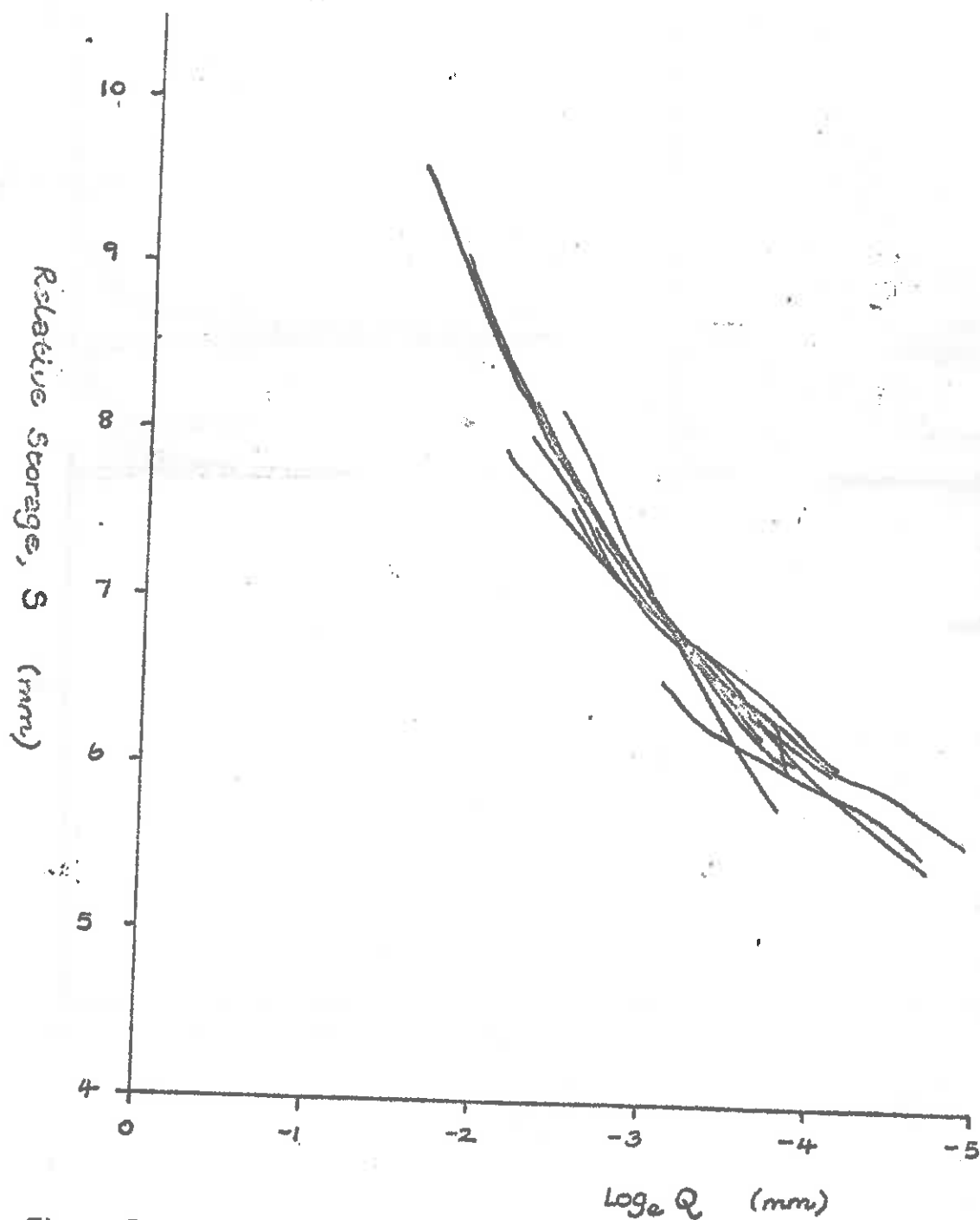


Figure 7. Observed storage/discharge relationships during recession periods for the Lanshaw subcatchment (arbitrary storage axes matched by eye).

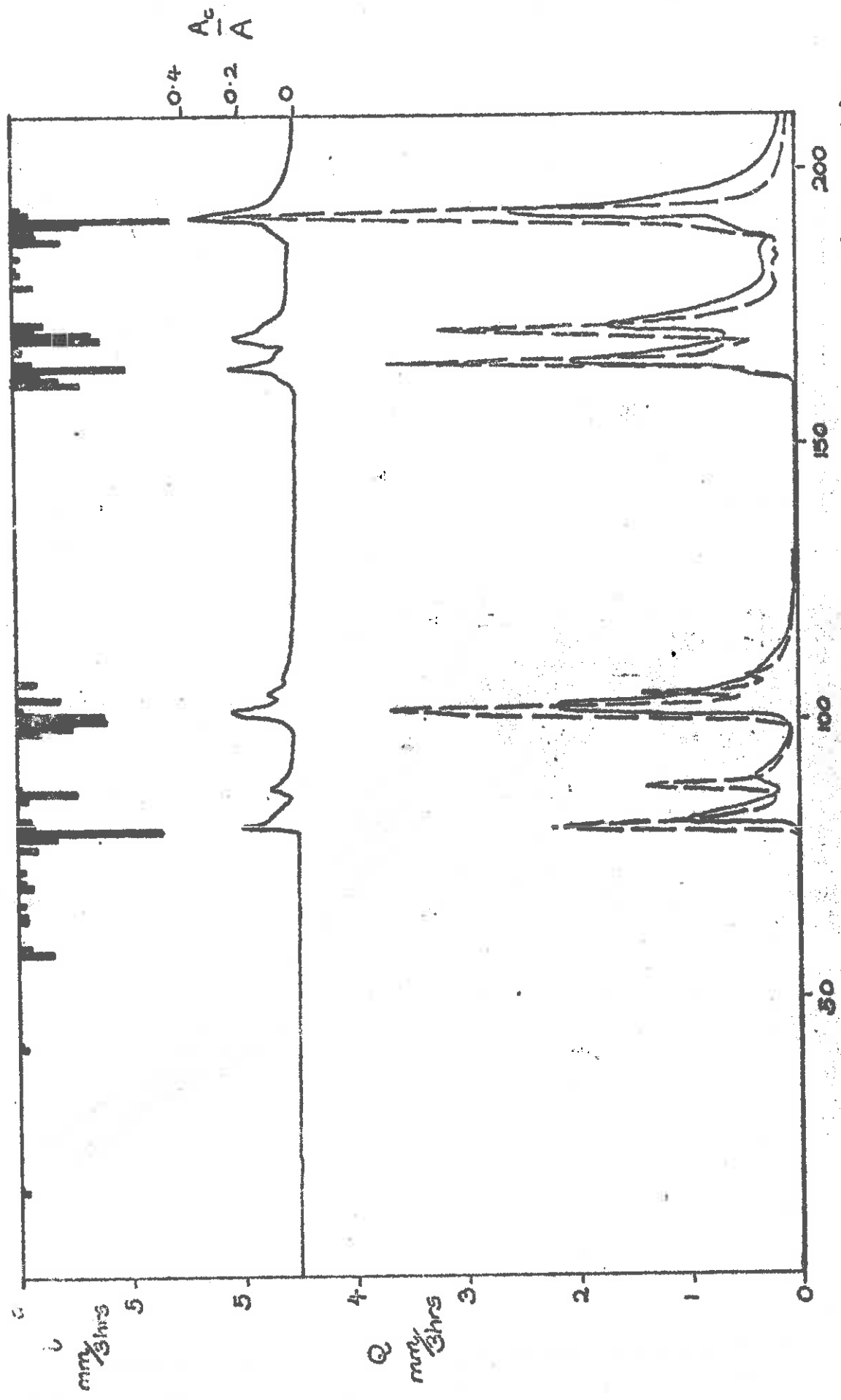


Figure 8. Rainfall, observed discharges (solid line), calculated discharges (broken line) and potential contributing area for the Lambeau watershed during part of November 1974. $E_1 = 0.93$, $E_2 = 0.05$, calculated over 3 hourly time periods for whole of November/December period.

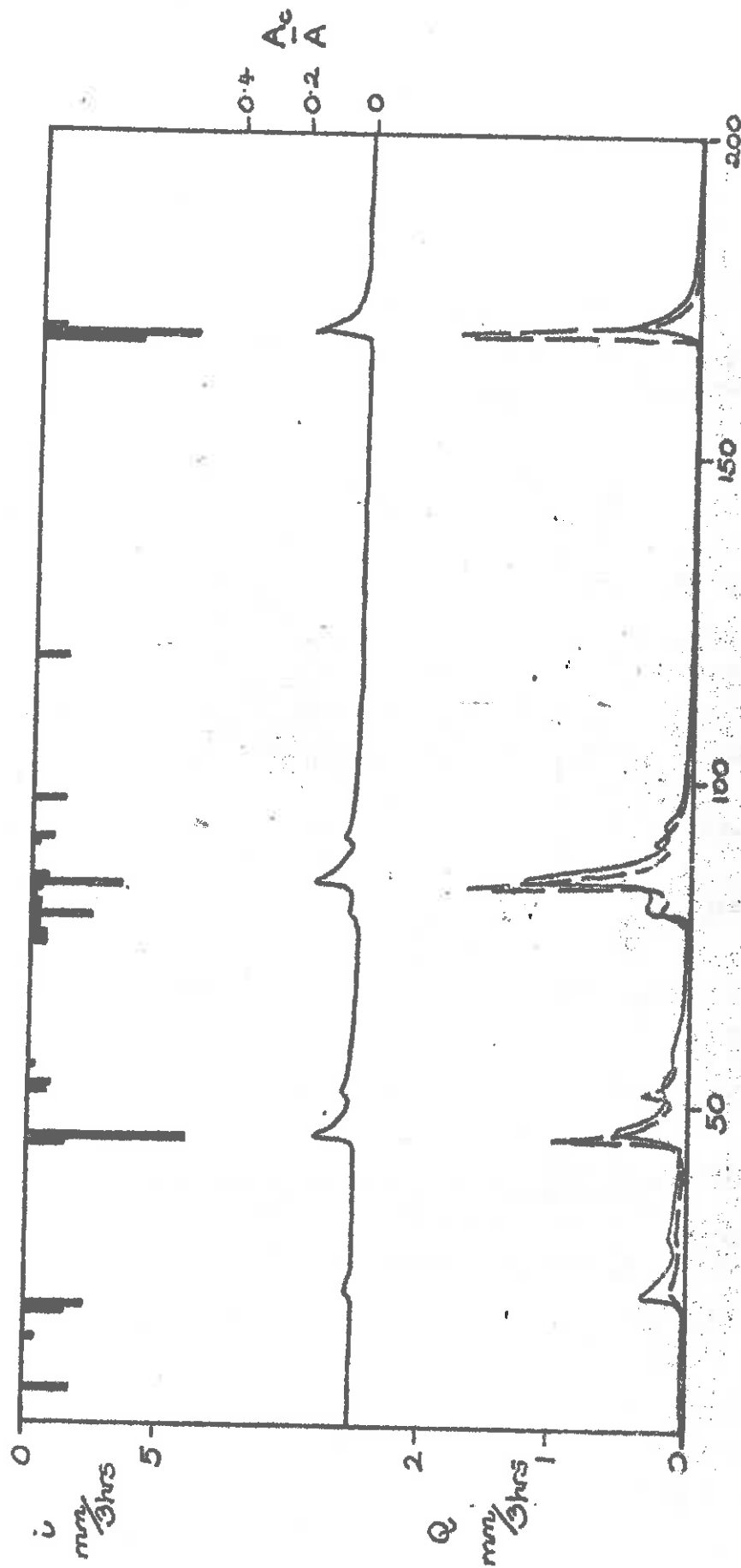


Figure 9. Rainfall, observed discharge (solid line), adjusted discharge (broken line) and potential contributing area for the Lambton catchment during part of March 1975. $E_1 = 0.65$, $E_2 = 0.2$, calculated over 3 hourly time periods for whole of March/April period.

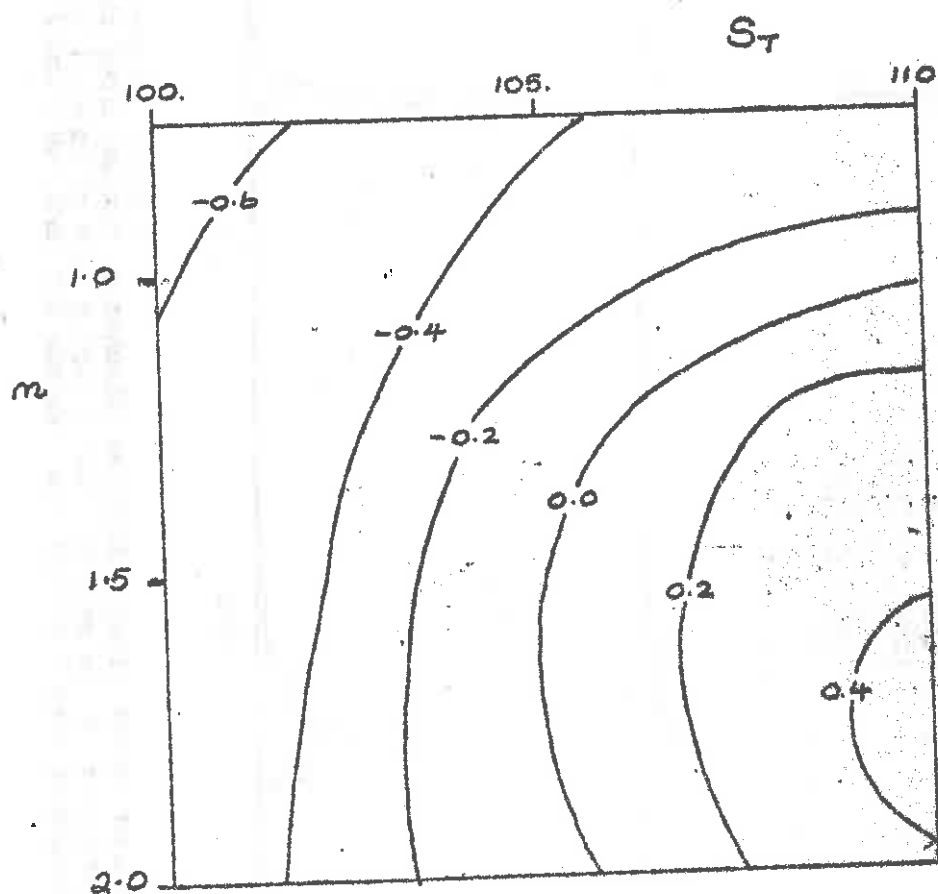


Figure 10. Sensitivity plot for the parameters m and S_T . Contours are values of the E_2 model efficiency measure.