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The evolution of urban spatial structure: a review of progress and research problems using S.I.A. models

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1. Introduction

There are many possible approaches to building dynamical models of urban systems. In this review, we restrict ourselves to the so-called S.I.A. models (Wilson, 1978) based on spatial interaction and activity location concepts and a particular style of models associated with these. In the last few years, a number of threads of argument have come together and these now offer a basis for a reasonable understanding of the evolution of different forms of urban system. It should be emphasised, however, that this understanding, for all the new insights offered, also generates massive new research problems. The purpose of this paper is to review the progress made to date and to assess the research tasks and problems which are now visible. Although the review is restricted in the sense defined, it is hoped that the models used have properties which are sufficiently similar in some broad sense to the alternatives that the conclusions have a more general relevance.

In the next three sections, progress to date is summarised. In section 2, the main concepts are reviewed; in section 3, the overall model system which is needed is investigated; and in section 4, the conceptual importance of 'numerical experiments' is emphasised and some illustrative results are presented. In sections 5-7, ongoing research tasks and problems are discussed. First, a wide variety of problems of analysis and interpretation; secondly, some problems of model design and implementation; and thirdly, the relevance of these approaches to planning and urban policy.

2. The main concepts

2.1 The modelling style: the retail model as an example

The model which serves very effectively as an archetype of the wider class of S.I.A. models is that of retail sales. The consumer part of the model was originally presented by Huff (1964), Harris (1965) and Lakshmanan and Hansen (1965). Various contributions to hypotheses about the supply side are in the work of Harris (1965), Wilson (1976), Coelho and Wilson (1977), White (1974, 1977), Harris and Wilson (1978), Phiri (1980) and Wilson and Clarke (1980) for example. Detailed presentations of this and other related models are available in many places (for example Wilson, 1974) and the treatment here will be as brief as possible.

The initial question is: given a system divided into discrete zones (labelled by indices such as i and j), a set of common demands by residential zone e_iP_i (where e_i is per capita expenditure and P_i the zonal population), a set of shopping centre 'sizes' by zone, W_j (which are also supposed to measure attractiveness), and a transport system specified by inter-zonal costs c_{ij}, then the model of consumer usage (S_{ij}, the flow of money from residents in zone i to shops in zone j) is

$$S_{ij} = A_i e_i P_i W_j^{\alpha} e^{-\beta c_{ij}} \tag{1}$$

where

$$A_i = 1/\sum_k W_k^{\alpha} e^{-\beta c_{ik}} \tag{2}$$

to ensure that

$$\sum_j S_{ij} = e_i P_i \tag{3}$$

α and β are parameters measuring the relative importance for consumers of centre size (W_j) relative to the cost of getting there (c_{ij}). Indeed, it is useful to bear in mind that

$$\frac{\alpha}{\beta} \log W_j - c_{ij} \tag{4}$$

can be interpreted as the average net benefit for a consumer living in i choosing to shop in j.

The model can also be used to predict a locational variable:

$$D_j = \sum_i S_{ij} \tag{5}$$

is the total revenue attracted to the shopping centre at j.

All this assumes that the supply side, $\{W_j\}$, is given. It is possible to add hypotheses about its growth. The most obvious one is something like

$$\dot{W}_j = \varepsilon(D_j - kW_j) \tag{6}$$

where ε is another parameter - measuring the scale of response in a unit time interval to disequilibrium (excess profit or losses). The equilibrium condition is the set of equations

$$D_j = kW_j \tag{7}$$

We will use this model to illustrate various kinds of analysis of system dynamics. It also illustrates what we mean by 'urban spatial structure': the population activity variables (both spatial interaction and location) S_{ij} and D_j, and the supply-variables, W_j - all of which are predicted by the model. Other structural variables, like P_i, are assumed given here, but can, as we will see, be predicted by other submodels.

The ori inal use of the model was for conditional forecasting. A trial set of W_j's, possibly a plan, would form the inputs. S_{ij}'s and D_j's could then be calculated and another trial plan developed in the light of that information and tested. The next step is to add equations (6) and (7) as additional *hypotheses* about suppliers' behaviour. The differential equations (6), or the corresponding difference equations, can be solved numerically. Or they can be used to analyse stability of equilibrium only, and equations (1), (2), (5) and (7) solved simultaneously for $\{S_{ij}\}$, $\{D_j\}$ and $\{W_j\}$ - that is for the equilibrium solution of this part of urban spatial structure. If we substitute for D_j in (7) from (5) and for S_{ij} in (5) from (1) (having already substituted for A_i from (2)), then the equations (7) become N (if there are N spatial zones) simultaneous equations in the W_j's:

$$\sum_i \frac{e_i P_i W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} = kW_j \tag{8}$$

We explore the nature of the solution of these equations in the next subsection.

2.2 The analysis of equilibrium

It is clear from equations (8) that we have a system whose equilibrium state is characterised by nonlinearities and interdependencies. These two conditions often generate multiple equilibrium states for a given set of parameter values and usually in such cases there are *critical* parameter values representing bifurcations of the equilibrium states: a state may disappear, a new one emerge, or, for example, in the non-equilibrium case, there may be a transition to periodic or even 'chaotic' oscillations.

The existence of bifurcation phenomena can be verified by numerical experiments (on which subject, more later). But it is important to gain maximum analytical understanding of the nature of bifurcation. The object of this section is to sketch some techniques for achieving this.

First, we follow the argument of Harris and Wilson (1978) which offers an analysis of the stability of equilibrium for a zone. The trick which is used to gain the insight arises as follows: in equation (8), the left and right hand sides, labelled say as $D_j^{(1)}$ and $D_j^{(2)}$, are each functions of W_j. When they are plotted as such, any stable equilibrium points will be at their intersections. It can be shown that, for $\alpha < 1$, there is always a non-zero stable equilibrium W_j. For $\alpha > 1$, the situation is as shown in figure 1. For $k < k_j^{crit}$, there is a stable non-zero equilibrium; and vice versa. Thus $\alpha = 1$ is a critical parameter value, and so is $k = k_j^{crit}$ for zone j. (This argument needs to be modified slightly in the light of Clarke's recent (1981) paper, but the essence of the argument can be retained.)

The next step in the argument is the observation that when a zone is in a 'critical' state with respect to one parameter, then it is also with respect to any other parameter or indeed any variable which is exogenous to that particular model (Wilson, 1981-A, p.132). Thus for the retail model, when a zone is critical, a small change in α, β, k, e_i, P_i, c_{ij} or W_k, $k \neq j$, could bring about a jump.

This point is particularly serious with respect to W_k, $k \neq j$. To construct the $D_j^{(1)}$ curve in figure 1 it was necessary to assume W_k, $k \neq j$, to be fixed. Wilson and Clarke (1979) showed that this is a particularly difficult problem to resolve and, as we will see in more detail later, it seems to be a new kind of mathematical problem.

It is possible to extend the idea of figure 1 to investigate the relationship of parameters or variables other than k to W_j. For example, we can examine the impact of $Q_i = e_i P_i$ on W_j (across space) - a task begun very sketchily in Wilson (1981-A, p.132) and taken much further in Harris, Choukroun and Wilson (1981). This is all in the spirit of attempting to achieve an analytical understanding of urban system equilibrium states, their stability and their bifurcation properties.

The characteristic bifurcation in a particular zone can be represented by a fold catastrophe. If W_j^* in figure 1 is plotted again k, we get figure 2 which illustrates this. This provides a lead in to two further points. First, while discrete change is obviously important in the evolution of urban spatial structure, it is better to build mechanisms directly into models directly than to try to invoke any general theorems of catastrophe theory. We use the latter theory only to increase our awareness of the possibility of discrete change and as a convenient way of presenting results which have been deduced from the model assumptions as in figure 2.

Secondly, figure 2 emphasises that one of the main properties of the system for each zone is whether it has a non-zero stable equilibrium or not: that is, whether it is in the development-possible (DP) state or the no-development-possible (NDP) state. The final main point to be made about equilibrium, therefore is that the whole-system state is characterised by the number and spatial disposition of zero (or, obviously, non-zero) W_j's. System-wide bifurcation, therefore, is usually concerned with a change in this disposition. It obviously occurs, at its simplest, when there is a change in one zone. A change in one zone, however, will have secondary effects in all other zones (through there version of the W_k, $k \neq j$ dependence), and this may include bifurcations in other zones therefore. Thus, the whole system state may change from one *pattern* to another at a critical value. It is very difficult to characterised and classify such changes and this is a problem to which we return.

2.3 Disequilibrium: differential and difference equations

We noted earlier that the most general dynamical representation of our archetypal system was offered by the differential equations (6) (with the appropriate substitution for D_j analgous to those which produced the left hand side of (8)). If suppliers' responses to disequilibrium are lagged in some way, then a difference equation formulation is more appropriate. If we replace $\dot{W}_j$ by $(W_{jt+\delta t} - W_{jt})/\delta t$ and then, without loss of generality, set $\delta t = 1$, then after some manipulation, (6) becomes

$$W_{jt+1} = \varepsilon D_j - (1 + k)W_{jt} \tag{9}$$

These representations each have a distinctive form of build up from small W_j as the system grows - in this case a very rapid one. If a logistic shape is more appropriate, this can be achieved by adding a factor W_j to (6):

$$\dot{W}_j = \varepsilon(D_j - kW_j)W_j \tag{10}$$

(without, note, changing the nature of any underlying equilibrium). The corresponding difference equations are

$$W_{jt+1} = (1 + \varepsilon D_j)W_{jt} - \varepsilon kW^2_{jt} \tag{11}$$

Using any of these model equation systems as a basis, it is possible to simulate non-equilibrium behaviour by solving the equations numerically. If the system is started off with small initial W_j's, then it is likely simply to 'grow' to an overall equilibrium state. However, this statement needs to be qualified in a number of ways. First, in a difference equation formulation, if ε is sufficiently large, transitions (at the zonal level) from stable equilibria to periodic oscillation are possible. This was first deduced from a comparison with May's (1976) work in ecology and demonstrated by numerical experiments which we return to below (Wilson, 1979; Beaumont, Clarke and Wilson, 1980-B). This can be straightforwardly interpreted: if ε is 'too large', this represents over-reaction on the part of the suppliers, and this generates the oscillations.

Secondly, because of the existence of multiple underlying alternative equilibrium states, the system can follow different evolutionary paths according to starting values (ie., in effect, its previous 'history', including any odd historical 'accidents'), and according to any random fluctuations (cf. Allen, et al., 1978). This leads to a complex problem of interpretation to which we will return in section 5 below. These notions apply to either the differential or difference equation formulations.

2.4 Linked model subsystems

The retail model has a number of simple features which are invaluable as the basis of an experimental device for exploring methods of dynamical analysis and the likely results. Cities as whole systems are obviously more complicated. We now briefly explore the methods available for tackling this complexity and see whether any wholly new kinds of dynamical behaviour are likely to emerge.

First, we consider the disaggregation of the retail model in two respects. The first of these is the disaggregation of the retail model itself, and the second the incorporation of different kinds of service sectors. On the second question, we divide such services which can be treated like retailing - banks for example - from those which may have different behavioural characteristics - health services for instance.

We simply add a superscript label g to represent type of good or retail-like service to the model system, but write the attractiveness function as $\hat{W}^g_j$. We let W^g_j be the floorspace (or whatever measure) for good g. $\hat{W}^g_j$ may be a function of $W^{g'}_j$, $g' \neq g$ (say for higher order goods, as the basis for multiple purpose trips). Thus, equation (1) might become

$$S^g_{ij} = A^g_i e^g_i P_i \hat{W}^g_j e^{-\beta^g c_{ij}} \tag{12}$$

with

$$\hat{W}^g_j = (W^*_j)^{\alpha^g_1} (W^g_j)^{\alpha^g_2} \tag{13}$$

where

$$W^*_j = \sum_g W^g_j \tag{14}$$

represents the effect of centre size as a whole. Note that the job of the parameter in the previous formulation is now done by sets of parameters α^g_1 and α^g_2. Other formulations of (13) are possible of course. The one to be chosen is a matter of detailed empirical investigation.

It seems that although new kinds of patterns may emerge because of the new nonlinearities and interdependencies generated by (13), the types of bifurcation are not fundamentally different from the aggregate case.

If a service is in some basic sense different from retailing, this could be in relation to the consumers' behaviour part of the model, or suppliers' behaviour, or both. The main alternative class of models is provided by the various extensions of location-allocation methods - for a review of recent work, see Leonardi (1980). Again, new kinds of nonlinearities are introduced but we have as yet no knowledge of distinctive bifurcations. For immediate purposes below, therefore, we do not distinguish other service types and assume that 'retailing' of different orders cover all types.

In all forms of the retail model so far discussed, we have taken the distribution of population as given. This is in turn to be supplied by a residential location model coupled with a housing supply model. The principles, within the S.I.A. system, are the same as for the retail model, but there are also some important differences. If at all possible, disaggregation is important at the outset. Let T_{ij}^{kw} be the number of people resident in a type k house in i working in a w-income job in j. (For simplicity, we assume one worker per household, but this can be easily, if messily, relaxed.) The model will take the form

$$T_{ij}^{kw} = B_j^w W_i^{res.kw} E_j^w e^{-\mu^w c_{ij}} \tag{15}$$

B_j^w is a balancing factor to ensure

$$\sum_{ik} T_{ij}^{kw} = E_j^w \tag{16}$$

where E_j^w is the number of type w jobs in j. The main interest is in the residential attractiveness factor $W_i^{res.kw}$. Our knowledge of residential location behaviour suggests that this will be a composite factor:

$$W_i^{res.kw} = (X_{i1}^{kw})^{\alpha_1^w} (X_{i2}^{kw})^{\alpha_2^w} (X_{i3}^{kw})^{\alpha_3^w} \ldots \tag{17}$$

where the $X_{i\ell}^{kw}$'s represent availability of type-k housing at i, price factors in relation to income, measures of accessibility, measures of density, measures of the nearness of different proportions of other income groups (with, broadly, a repulsion factor operating with respect to lower income groups, and attraction for the same or higher). Obviously there is a lot of empirical sorting out to be done here, but it is clear that there are new nonlinearities. It is also clear that the supply model will not be developed in terms of $W_i^{res.kw}$, but of 'primary' factors among the X's - like housing supply H_i^k and developed residential land, L_i^{res}, which will be part of the density term. Thus we have differential (or difference) equations in $\dot{H}_i^k$ and $\dot{L}_i^{res}$ whose equilibrium states and bifurcation properties have to be investigated as for the retail model. However, the principles are the same and we refer the reader elsewhere for the details (Wilson, 1981-A, 198D).

One major point should be stressed, however: a new kind of bifurcation does arise. There will be 'jumps' (or whatever) in H_i^k as for W_j (or W_j^g) in the retail model. But in addition, because of the social attraction and repulsion terms in the residential attractiveness function, there could be rapid changes in the population distributions, T_{ij}^{kw}, also. In other words, as H_i^k's change for example, this will change the w-mixes of zones, and it is highly likely that there will be critical points at which shifts will occur. This is the well-known 'tipping' phenomenon which creates ghettoisation.

We have now covered all the major sectors except the industrial one. This has proved a difficult sector to model for a variety of reasons - mainly because, relative to population, service and housing sectors, it is a sector of disorganised complexity (cf. Weaver, 1958; Wilson, 1977). However, the method for developing differential equations outlined in this section may, with reinterpretation, offer a suitable basis for the development of 'dynamic Weber models'. Let Y_j^s be the amount of industrial sector s activity in zone j. Let F_j^s be the amount of revenue it attracts, and k^s the unit cost of supplying it. Then

$$\dot{Y}_j^s = \varepsilon^s(F_j^s - k^s Y_j^s)Y_j^s \tag{18}$$

might provide a suitable starting point. The main research task would be to formulate the function F_j^s (and possibly k^s) in a Weber-like way: as a function of input and outputs in a space-economy.

The rudiments of how to link these submodels together are already visible. However, we postpone our account of this until section 3 below. We simply remark here that the linkages could cause additional jumps. For example, a jump in P_i will cause secondary jumps in some W_j's when propagated from the residential to the retail model, and possibly consequent bifurcation in the W_j's. This is one of the problems of interpretation to which we will return in section 5.

2.5 Numerical experiments and empirical work

So far, it is evident that we have struggled to keep an analytical grip on the nature of bifurcation in the evolution of urban spatial structure. But there is a limit to this. An immense variety of spatial patterns, and paths of evolution towards those patterns, can be generated by the models we have presented - even with the aggregate retail model. These can be investigated by using idealised data in so-called 'numerical experiments'; and also, of course, by using real data. It is important to emphasise that these numerical experiments contribute an important tool of *theoretical* research. This can be illustrated, perhaps oddly but with insight, using an experimental science analogy. An X-ray crystallographer takes many photographs of a complex molecule and from these can infer its structure. Here, we have a known model structure, but we are interested in the variety of patterns it can generate. Thus, in our numerical experiments, we have to generate this variety of patterns as our equivalent of photographs - a method for dealing with complexity. We return to the resulting problems of pattern recognition in section 4.

3. The overall model system

3.1 An extended Lowry Framework

We noted in subsection 2.4 above that there are linkages between the main submodels. The core of this is the 'interacting fields' notion which implies that if residences are located around workplaces (and hence jobs) - one 'field', and services (and hence service jobs) around residences - another field, then there will be a strong interdependence between the two. This was first fully recognised in urban modelling by Lowry (1964). We have strengthened the interdependence in one way by recognising, through one element of the residential attractiveness factor, that residential location may depend on access to services as well as to jobs. But the most important addition comes from the supply side. Lowry assumed that supply followed demand: he calculated the demand for services at a location, and for people to be located somewhere, and that service facilities and houses would be built. We have shown how to construct explicit models of supply side behaviour and have noted that many of the interesting bifurcation properties of the whole system derive from these submodels. We have also noted that, when house types and person types are distinguished in the residential location model, this is another source of potentially volatile change; and, by implication and with same realism, that people move around the housing stock more rapidly than the stock itself changes.

How are these interdependencies arranged in a comprehensive model? Lowry starts with 'basic sector' jobs distributions (usually industrial, 'export' jobs - at least in relation to the local economy) and locates people to (instantly created) residences around these. Services are then located around these residences. New jobs are thus created; hence new residences are distributed around these, and so on. This structure is shown in figure 3.

We add supply models within the residential location sector (which is also disaggregated) and the service model, and so inner iterations to achieve supply-demand equilibrium have to be carried out within each loop of the outer (Lowry) iteration. Of course there is also the alternative that, within the same overall structure, differential or difference equations are solved numerically. In this case, depending on ε-like parameter values and the extent of other exogenous change, the system will reach equilibrium or not; in the latter case, the behaviour is likely to be governed by the nature of any underlying equilibria in the subsystems. The structure of this extended Lowry framework is shown in figure 4.

3.2 The comprehensive model and bifurcation

Our present feeling is that subsystem linkages do not of themselves, at least in present formulations, bring about new bifurcation phenomena; but the transmission of 'jumps' from one subsystem to another will be an immensely complicating factor in interpretation. The nature of some of the main feedbacks involved can be seen from figure 5.

4. Numerical experiments: the form of the results

4.1 Introduction

In this section, a number of plots are presented to illustrate the kinds of results generated in numerical experiments in our research in Leeds. These are intended to convey the flavour only. Some detailed results are available in Beaumont, Clark and Wilson, 1980-A, 1980-B). A much more detailed presentation is at an advanced stage of preparation (Beaumont, Clarke and Wilson, 1981). The results are presented in two stages: first, to illustrate the range of equilibrium patterns; and recently, to show how the non-equilibrium system works. As a preliminary, we comment on the problem of designing spatial systems.

4.2 Spatial systems

In order to explore the range of equilibrium patterns which can be generated by the models it seemed appropriate to use some regular underlying system. There are difficulties in introducing symmetry-breaking features which bias the results in an unacceptable way, and for this reason we abandoned hexagons and restricted ourselves to 'circles' formed out of square grids by cutting corners off. A typical system is shown in figure 6. We have used both smaller and larger systems than this, and we have also used Leeds wards as the basis for demand side data. We recognise that it is often useful to use an $m \times n$ system with $m \neq n$. In the Leeds case we have taken $m = 30$ (administrative wards) on the population side and $n = 900$ (by imposing a 30×30 grid of possible 'supply' locations. We have also taken $m = 13^2$, $n = 27^2$, for population and supply sides respectively, thus using simultaneously grids of different sizes.

4.3 Some equilibrium patterns

Here, the results are restricted to the retail system to illustrate the principles. The first set, figures 7-10, illustrate several points simultaneously. First, they give an indication of the variety of possible patterns in an idealised situation (because they are based on a *uniform* underlying distribution of population). This is done in figure 7. But this also illustrates a hierarchical disaggregated system ranging from facilities for low order goods in figure 7(a) to the highest of three orders in 7(c).

It did seem that the centre was not being picked out as often as we would expect, and so another test was to factor 'costs' to the central zone by decreasing amounts. Figures 8-10 show the same system with this factor ranging from 0.95 to 0.75. A number of bifurcations are visible for the system as a whole (resulting, of course, from changes in particular zones).

Dominant centres are obviously more likely if a more conventional population density gradient decreasing from the centre is introduced. The underlying assumption is shown in figure 11, and some patterns for different parameter values in figures 12-14.

All these results display one broad feature we expect: that higher α and lower β each generate fewer larger centres, other things being equal.

In figure 15, some results for the Leeds 30 × 900 system are shown for a variety of parameter values, this time using a different form of plot. They also show the same broad features just remarked upon.

4.4 Disequilibrium: numerical integration of difference equations

We noted earlier that different patterns can result from different ϵ-values, all other data and parameters being fixed. This is illustrated with some results for Leeds in figures 16-18. We also illustrate what can happen in particular zones with some stable-to-period transitions in figure 19.

5. Problems of analysis and interpretation

5.1 Introduction

At this point in the paper, there is a shift of focus from a review of what has been achieved to a more informal discussion of ongoing research problems. We begin in this section with problems of analysis and interpretation. This provides an overall basis for discussions of possible advances in model design in section 6 and of planning and policy applications in section 7. In section 8, we attempt to draw the threads together with a discussion of research priorities.

5.2 Development and evolution; hierarchy and complexity

Up to now, the topic of this paper has been taken as the 'evolution of urban spatial structure'. It is useful to elaborate this and to make an imperfect distinction between the ideas of 'development' and 'evolution' based on the obvious biological analogy. A concern with development involves understanding the way in which a known organism or system grows according to (move or less) known rules. Evolution involves the creation of new species. The two ideas are closely linked. A historical analysis of system change will involve both kinds of investigation. The importance of the concept of evolution is that it reminds us to be on the alert for new 'species'. In an urban context, this means new kinds of organisations within the system, or new forms of organisation for the system as a whole. A specific illustration can be given as follows. Suppose for a number of given interdependent industrial sectors, all the production functions and associated 'relations of production' are known (ie., the 'rules'). Then it is possible to predict alternative paths of development for this system in a variety of circumstances: from different initial conditions, different environmental parameter values, and so on. But suppose as a result of some unanticipated invention, the nature of all the production functions changes. Then new 'species' are introduced. At the point we understand this, our analytical problem becomes one of development again. But until we do, it is one of evolution. Perhaps the main distinction we are making here relates to the number of possible growth paths the system can taken in the future.

It is likely that, as research proceeds, we will have much to learn from biologists and ecologists, though they too have similar difficult problems at present. One particular feature of those fields can be noted for their obvious relevance to urban analysis. This is the tendency for systems to become more complex in an overall sense, but for this complexity to be hauled through the evolution of hierarchical forms of organisation.

The last comment illustrates the difficulty of the research problem we face. At present, we are more able to analyse 'development' then 'evolution', and that with with difficulty. And we have not yet understood the nature of hierarchical organisation of structure.

5.3 Portraits of urban system development and evolution

Let us begin by focussing on the comprehensive model system shown in figures 4 and 5. It is necessary at the outset to decide what is endogenous. Parameters like α, β, k and ε will usually be exogenous in our numerical experiments - though not necessarily constant. For example, we would expect α to have increased over time and β and k to have decreased.

The relations (the equations and functions used) represent a specification of the system's development rules. There will be a large number of possible development paths and of possible equilibrium (if reached) states at each stage. These can be explored in numerical experiments. Our previous analysis shows that, because of the nature of system bifurcation properties, there is indeed a great variety of possibilities and that interpretation of the differences between them is complicated. Very different paths can originate from the same starting point and largely the same data and parameter values, differentiated, say, only by a small difference in one exogenous variable or parameter, perhaps introduced by a small random fluctuation, at one time.

5.4 The analysis and interpretation of development and evolution

As a system develops in an experiment along the lines discussed in the previous subsection, ideally we would like to be able to analyse and interpret any major change of form or status, both for particular zones and for the system as a whole. This is very difficult even for particular subsystems. The possibility has been demonstrated, for example in Wilson and Clarke (1979) where a figure reproduced here as figure 20 was generated to illustrate the nature of zonal criticality (and compare figure 1). It would be an immense task to do this for each zone (and for each sector) of a complex system. It could in principle be done, and one task for the future is to develop the appropriate computer programmes to do this. The information generated in this way could then be used as the basis of an analysis of such concepts as resilience for particular systems at particular times.

The complexities of this analysis for whole systems arise because of the number of reasons which can generate a jump in a particular variable. These include:

(i) Bifurcation arising from a parameter or exogenous variable change through a critical point, not triggered by any other such changes. This might be called a primary bifurcation.

(ii) A similar change but brought about by a jump in other variables - a secondary bifurcation.

(iii) A jump which is a knock-on effect from a jump in another variable but which does not involve passing through a critical parameter value for the particular zone.

(iv) If the system is not in equilibrium all the time, then separatrix crossing jumps are possible (cf. Wilson, 1981-A, p. 142).

(v) Any of these possibilities in a zone may be triggered by a transition to periodic behaviour in another zone.

We also need to bear in mind that, at one time, many of these effects are happening virtually simultaneously.

5.5 Patterns in the whole system: recognition and classification

5.5.1 Whole-system patterns

So far, we have focussed mainly on bifurcation at the zonal scale. It is clear from the examples presented in section 4 that distinctive patterns, given the idealised regular base, are generated for the system as a whole. There are elementary forms of classification, as have been used in central place theory, such as number of (non-zero) centres of a particular order (in effect, the DP-NDP pattern), average spacing, and so on. It is a substantial research problem to take this further, particularly for real systems without the regular base. It is then a particularly important research problem to be able to investigate system-pattern bifurcation: when there is a rapid transition from one *form* of pattern to another.

One point can be usefully made here which arises intuitively from scanning many of the results we have. That is, when a relatively small number of parameters is varied, there is a great variety of patterns generated. This suggests that, in particular situations, we may have a good chance of identifying actual parameter combinations which generated them.

5.5.2 Variety and pattern at different levels of resolution

It is clear that the notion of 'pattern' depends on the number of zones in the underlying spatial system. We have experimented with different zonal bases with otherwise the same data and parameters and have confirmed that the models we use do aggregate reasonably well - certainly in terms of the presence and absence of centres. However, the problem to be tackled here is this. At finer scales, more variety in evolution is visible. Can we identify coarse scales at which the pattern is essentially deterministic. As a simple example: for a given city size and parameter values, is there ('usually'?) a definite *number* of centres of a particular order, even though there may be a variety of possible *locations* dependent on alternative starting values.

5.5.3 Zonal-pattern linkages: towards fuller explanation

We conclude this section by reviewing the essential nature of some of the hardest problems of interpretation. We illustrate these by drawing attention to two specific problems. The first is the problem of simultaneous $\{W_j\}$ variation; the second is that of defining regions of parameter space which corresponds to particular types of solution.

As we noted earlier, $\{W_j\}$ variation generates a new kind of mathematical problem. For zonal stability analyses, we have to focus on a particular zone j, and its main structural variable in the retail analysis, W_j. The results of the analysis (on the lines of figures 1 and 2) depends on assumed fixed values for other W_k's, and these values depend on W_j. There is much potential volatility of solutions here. This complicates the analytical problem to be considered next.

We also noted earlier that whatever picture we draw (eg. figure 1), when a zone is in a critical condition, it is critical with respect to all parameters. There is an element of this result which is unsatisfactory and which needs refinement by further research. Our intuition tells us that one parameter may have more influence in 'taking' that zone to criticality than others. So we need to be able to calculate the rates of change of some criticality index with each parameter so that we can compare them. Or the passage through criticality may occur because only one parameter is changing at that time.

Another way of approaching this problem, with which we have made some preliminary progress, is to calculate the surface in parameter space at which the zone is in a 'critical' state. We then assume that for any set of parameter values, the side of the surface which the corresponding point is on determines whether the zone is in the DP-state or the NDP state.

q Suppose we continue to focus on retail centre structure to fix ideas. What we want to do is to progress from individual zonal analyses to pattern-analyses which, in principle at least, can be constructed from these. The nature of the problem we have can be inferred from the following special case. The equilibrium-pattern problem can be formulated, at least in the aggregate case, as a mathematical programming problem. A theorem of mathematical programming can then be used to show that if a pattern exists in which all the W_j's are non-zero, then it is unique. The problems are of the form: for what region of parameter-exogenous-variable space does such a solution exist? Is this a dense and connected region of such a space? We can then go on to take other characteristic patterns and ask the same question. The search for the answers forms a very difficult research problem.

6. Problems of model design

6.1 The adequacy of the model system

There are many alternative model systems, as noted at the outset. One advantage of the S.I.A. system is that it gives results which are intuitively plausible, and there is much empirical experience with static versions of the model which confirms this. A second advantage for theoretical exploration is that it is possible to use reasonably simple structures, such as attractiveness functions, and retain plausibility while being able to keep a semblance of an analytical grip on the problem. It can be argued, therefore, that even if an alternative system is used, many similar kinds of nonlinearities and interdependencies will be incorporated, so that the *bifurcation behaviour will be of the same broad type.* Thus, if the model system turns out, ultimately, to be inadequate, the main results of this approach to dynamics should still hold.

In the rest of this section, we briefly explore some possible extensions of the model system.

6.2 Some detailed model design and development issues

6.2.1 Levels of resolution

We noted earlier that the dynamical properties of the model are to some extent properties of the levels of resolution adopted. This may also be relevant in a more fundamental sense: to incorporate the main processes into the model, appropriate levels of resolution may be necessary. This could then involve model extensions. We illustrate this argument with one example. If the availability of finance for housing, and the associated policies of mortgage lenders, is an important property of a residential location model, then this may have to be incorporated. This could lead to the adoption of micro-simulation techniques in this context (cf. Williams, Clarke and Keys, 1981).

6.2.2 Hypothesis refinement

There are a number of other ways, given appropriate levels of resolution, in which it is likely that the hypotheses represented in the model need to be refined. We illustrate this with a number of examples each of which has been chosen because a corresponding amendment could change, or introduce new, bifurcation properties.

First, it is clear from our brief discussion of residential location models earlier that much work can be carried out on alternative forms of attractiveness functions. This could have a major effect on inter-subsystem linkages and on the form of nonlinearities within the model system. Secondly, also as noted earlier, major developments are needed to add industrial location models, and models of service sectors which do not fit the 'retail' (or even location-allocation) mould. Thirdly, more prices, particularly land rents, should be made endogenous to the model. The 'cost' factor k in the retail model would then beca a function $k_j(W_1, W_2, W_3, \ldots; P_1, P_2, \ldots,$ etc.), including, in effect, an endogenous rent component; and this introduces both new interdependencies and new nonlinearities. Fourthly, attentionneeds to be paid to the $\{c_{ij}\}$ variables, and in particular the relationship of travel costs to the supply and evolution of transport networks. This problem has been tackled in a comparative static way in the transportation field (cf. Boyce, 1980), but has not yet been extended to an analysis of dynamics.

6.2.3 Empirical work

It hardly needs saying that the research outlined in this section could be most effectively carried out in the context of empirical work on real systems. This turns on the supply of, or assembly of (approximate) time series data. This is obviously a difficult task. There are signs in the work of others in other contexts that it can be done (for example, Putman, 1977, on Philadelphia). We can only attempt to progress towards such work, and to hope that theoretical innovation may be a valuable stimulus.

7. Planning and policy applications

7.1 History and prediction

Since the main use of mathematical models in planning in the past has been for conditional forecasting - the 'analysis' part of a 'policy-design-analysis' framework (as described in Wilson, 1974, Chapter 2), it is useful to begin this section with a review of the kinds of new insights which dynamical systems analysis offers. We have seen that, because of the nature of bifurcation phenomenon, there are many possible evolutionary paths for the system as a whole, often distinguished by only small differences in parameter values at particular times. If we add our new-found interpretative capability to this picture, we may often, in the future, be able to offer a convincing explanation of *history*. But it does add a new edge to the notion of 'planning under uncertainty' for the future. This 'uncertainty' has always referred to unanticipated changes in exogenous parameters or variables. We now see that even small changes may have a major impact on the system's future. This means that we have to take a different view, perhaps, of our ability to predict. We explore the implications of this for planning and policy in the next subsection.

7.2 Planning, policy and the future: scenarios and rules

It is useful to distinguish two kinds of planning. First, and most traditionally, there is the notion of conceiving an ideal system *state* - perhaps constrained by obvious resource limits - a portrait of which constitutes the 'plan'. This may be called 'scenario planning'.

Secondly, it is possible to focus on policy in terms of the *rules* under which the system develops. We can then explore the consequences of adopting a particular set of rules. In real planning, both ideas are likely to be present and overlapping of course. What can our ideas about dynamical systems analysis contribute in each of these areas?

While specific forecasting may not be possible at a detailed level of resolution, it may be that broader characteristics are predictable. Such forecasting methods could be used partly to explore how policies might be manipulated to achieve a particular scenario; and also to explore the consequences of adopting particular sets of policies (as rules). It would also be possible to carry out stability analyses of either scenarios or policies, at the whole system scale or for local areas (zones or groups of zones).

The implications of these ideas may be quite radical for the use of models in planning, and much research needs to be done and practical experience gained.

8. The research agenda

Most topics for further research have been identified in sections 5-7 above. Here, by way of a concluding summary, we draw attention to their nature. They fall into essentially two groups. The first consist of ongoing analytical and theoretical problems of interpretation of development and evolution. There are two particular difficulties to be overcome: (a) those associated with major structural variables being dependent on each other (like the W_k's); (b) those associated with the task of defining and classifying whole-system patterns (and ultimately, evolutionary paths) and associating them with regions of parameter space.

Secondly, there is a large group of problems involving practice. Much more experience is needed of dynamical explorations using a wide range of alternative models. This kind of experience will also provide the basis for understanding the nature of the contribution of dynamical systems theory to urban planning practice. (And what can be offered should not be taken as a new panacea - there may be severe limits in our capacity to 'solve' urban problems. See Wilson, 1981-G; for a fuller discussion of this.)

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Figure 1

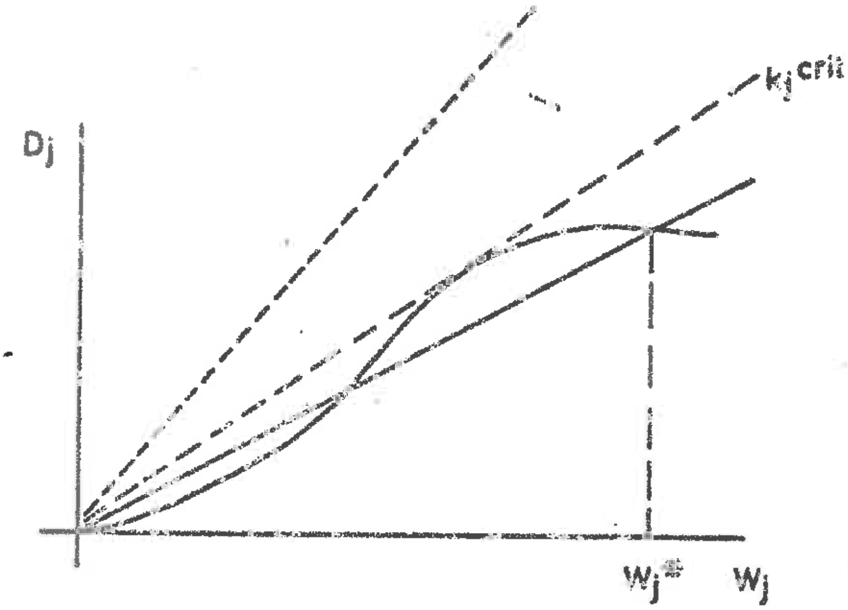
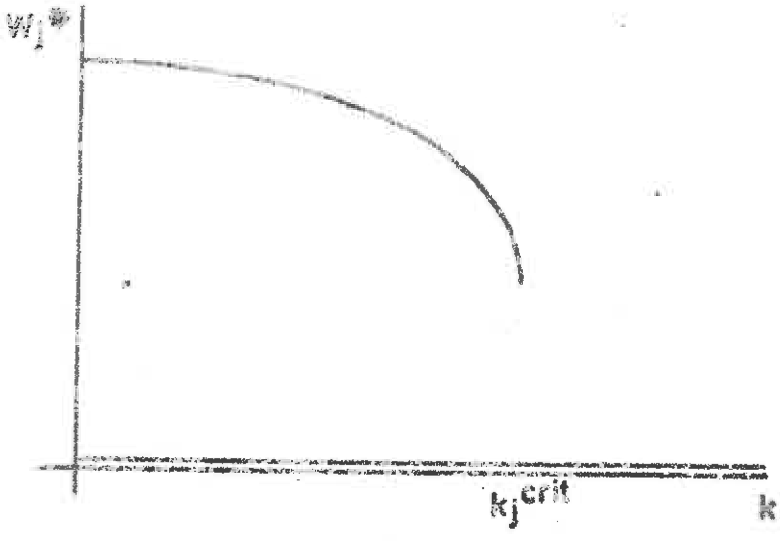


Figure 2



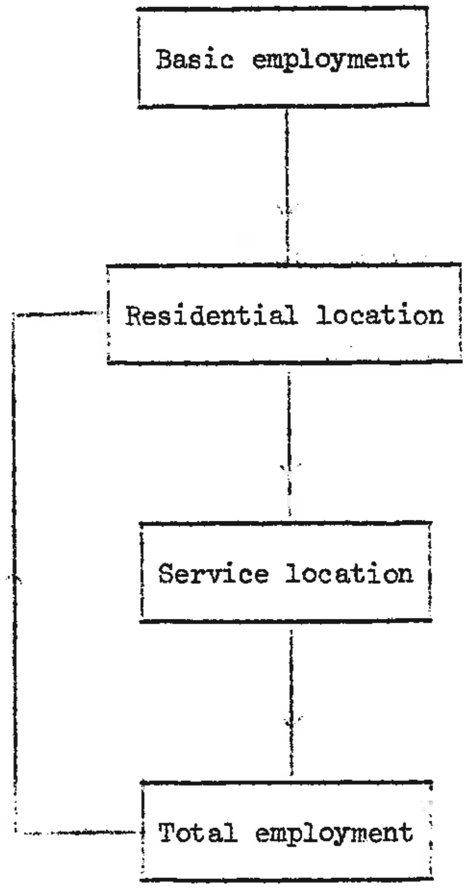


Figure 3. Basic Lowry model structure

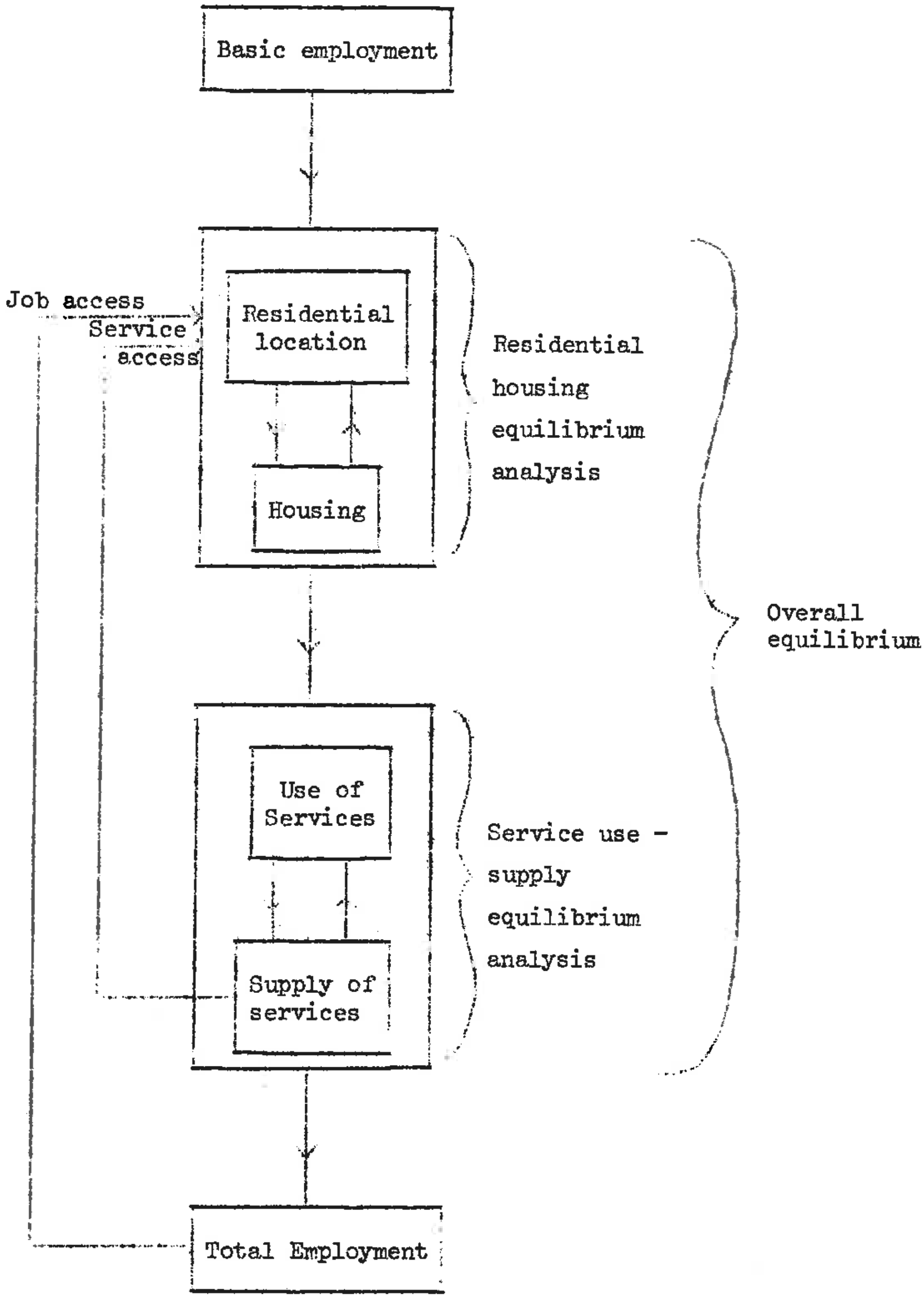


Figure 4. Lowry-type structure, with supply models added

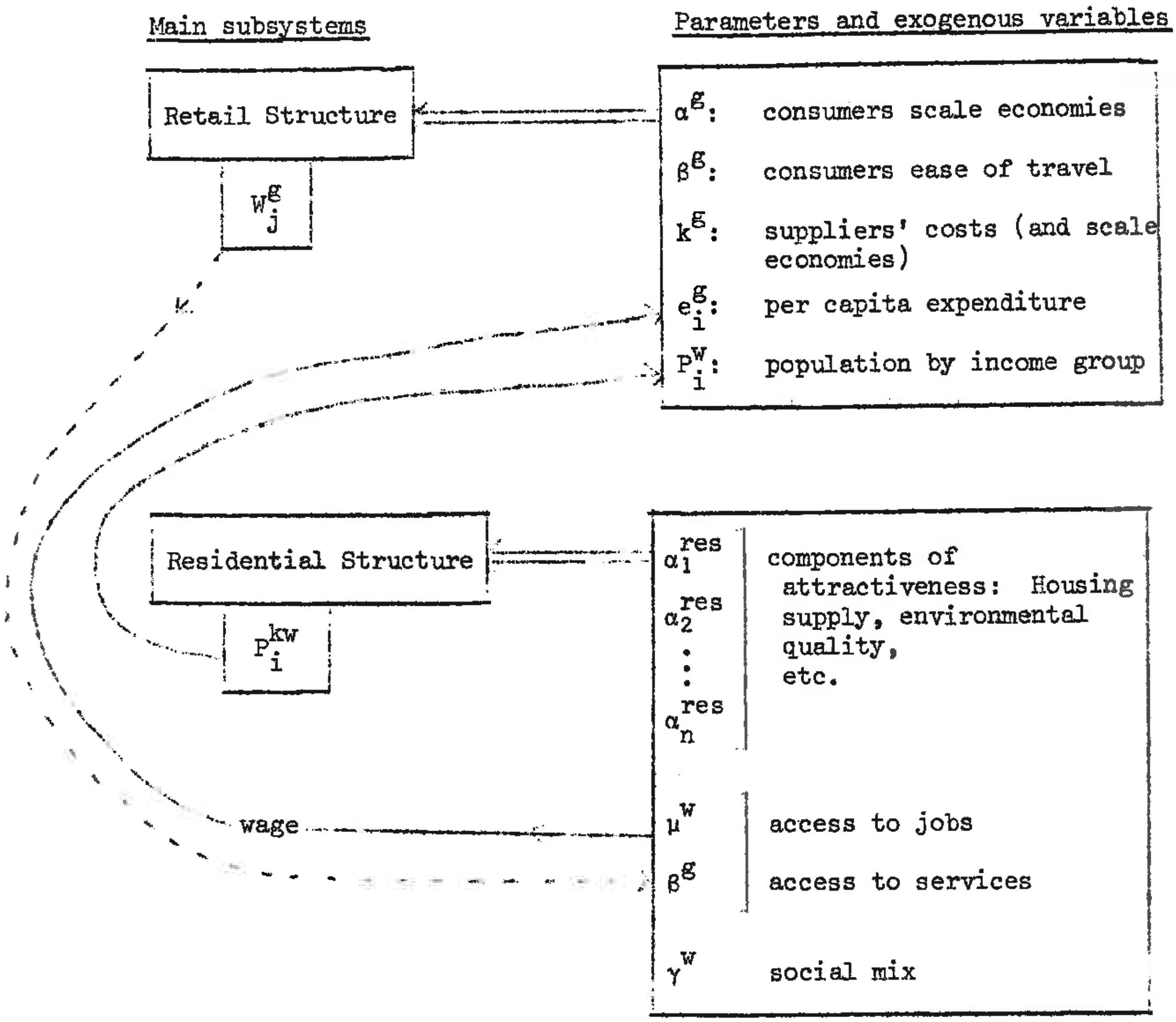
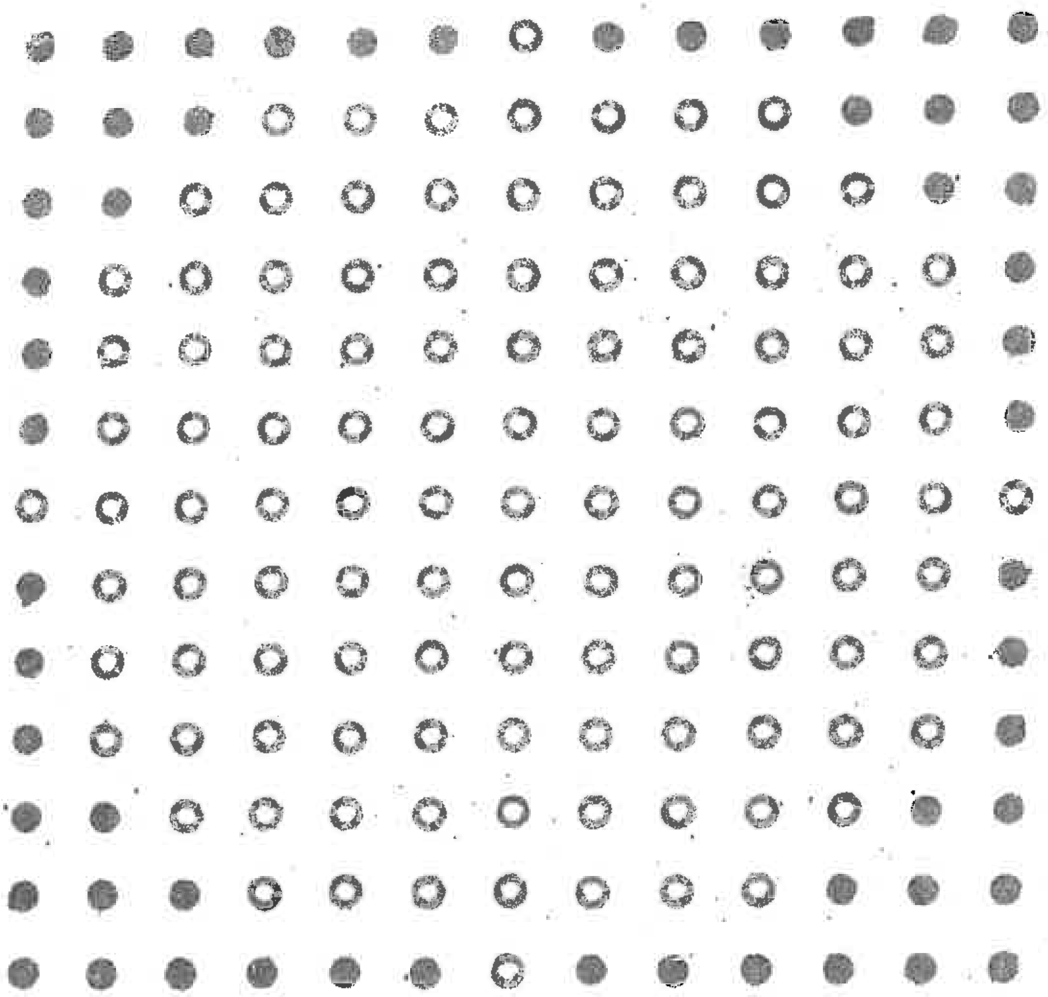

Main subsystems
Parameters and exogenous variables
Retail Structure
W_j^g
k
α^g: consumers scale economies
β^g: consumers ease of travel
k^g: suppliers' costs (and scale economies)
e_i^g: per capita expenditure
P_i^w: population by income group
Residential Structure
P_i^{kw}
α_1^{res}, α_2^{res}, ..., α_n^{res}
components of attractiveness: Housing supply, environmental quality, etc.
wage
μ^w access to jobs
β^g access to services
γ^w social mix

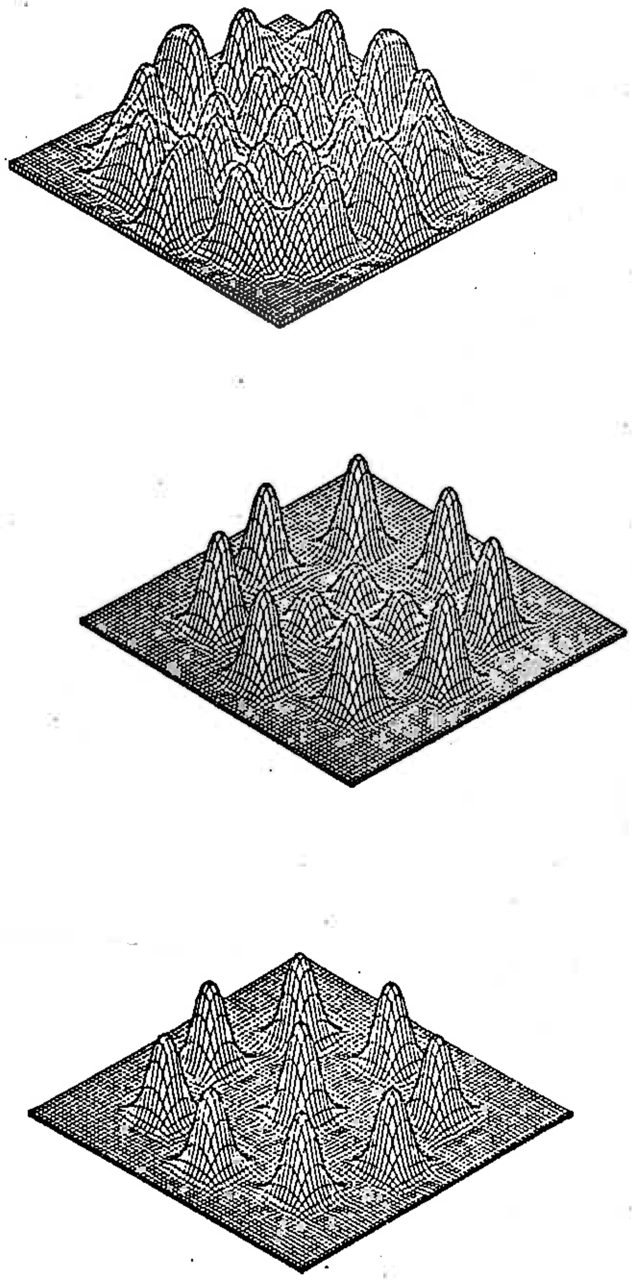
Figure 5.



● Indicates exogenous zone

Fig.6 169 Zone Spatial System

Figure 7.

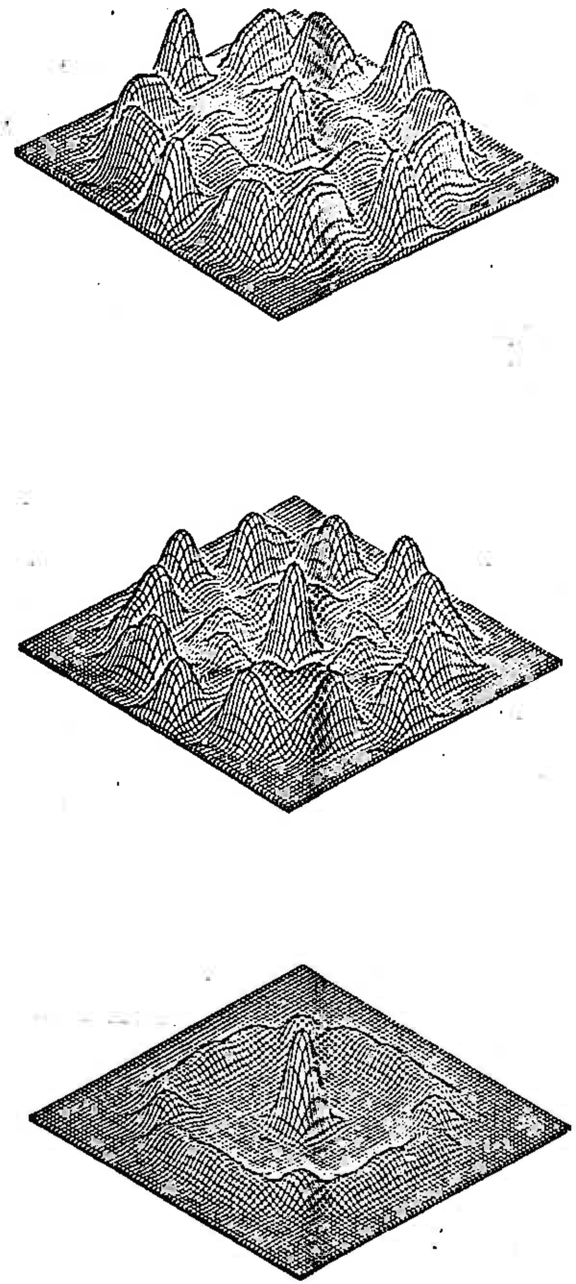


(a)
$\beta = 3.0$
$\alpha = 0.95$

(b)
$\beta = 2.0$
$\alpha = 1.0$

(c)
$\beta = 1.0$
$\alpha = 1.1$

Figure 8



(a)
$\beta = 3.0$
$\alpha = 0.95$

(b)
$\beta = 2.0$
$\alpha = 1.0$

(c)
$\beta = 1.0$
$\alpha = 1.1$

Figure 9.

(a)

$\beta = 3.0$

$\alpha = 0.95$

(b)

$\beta = 2.0$

$\alpha = 1.0$

(c)

$\beta = 1.0$

$\alpha = 1.1$

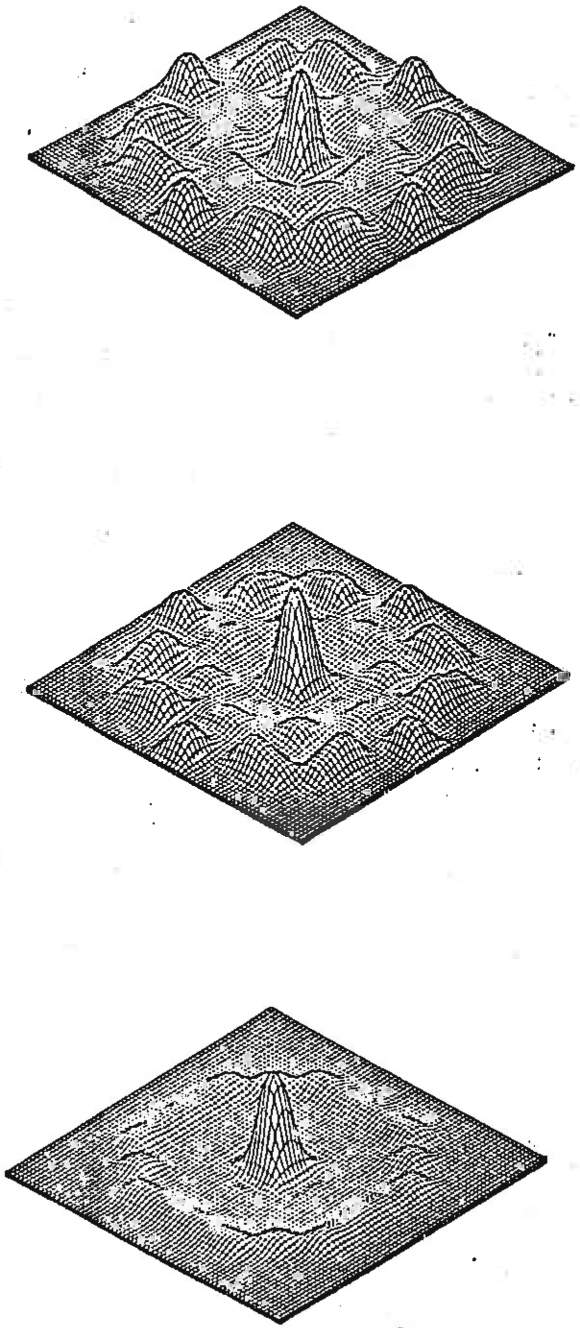


Figure 10

(a)

$\beta = 3.0$

$\alpha = 0.95$

(b)

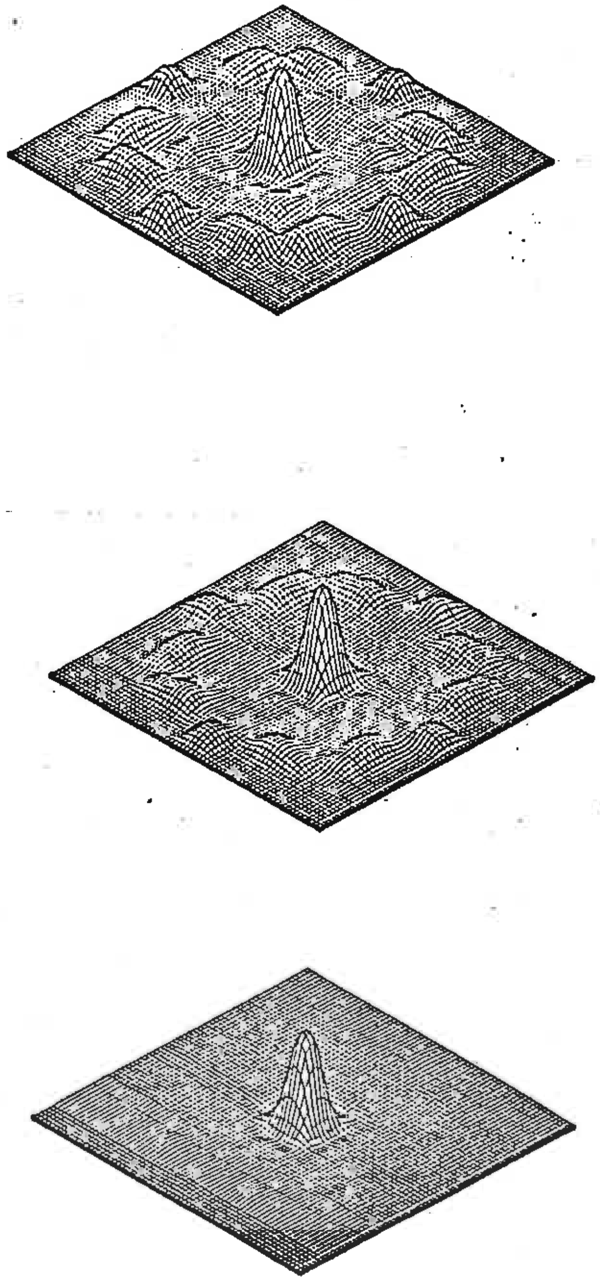
$\beta = 2.0$

$\alpha = 1.0$

(c)

$\beta = 1.0$

$\alpha = 1.1$



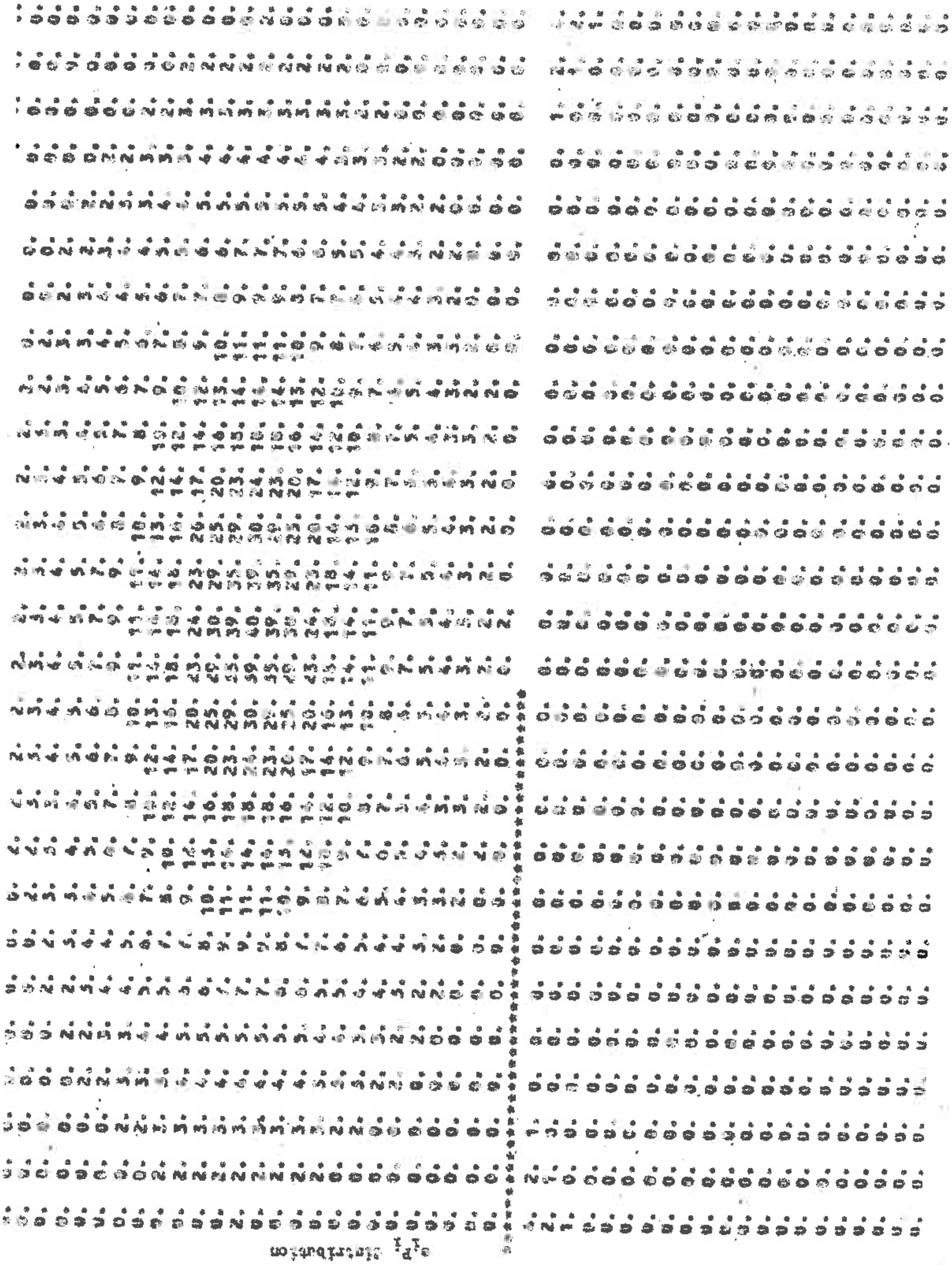


Figure 11

Figure 12. Density decay distribution

$\beta = 3.5$

$\alpha = 1.2$

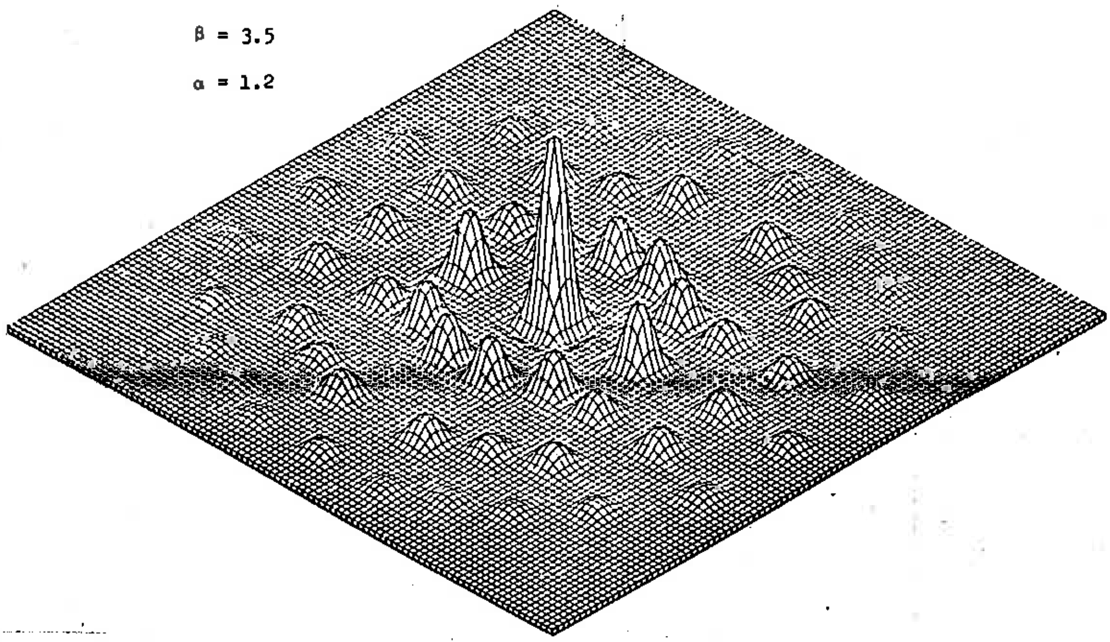


Figure 13. Density decay distribution

$\beta = 2.5$

$\alpha = 1.2$

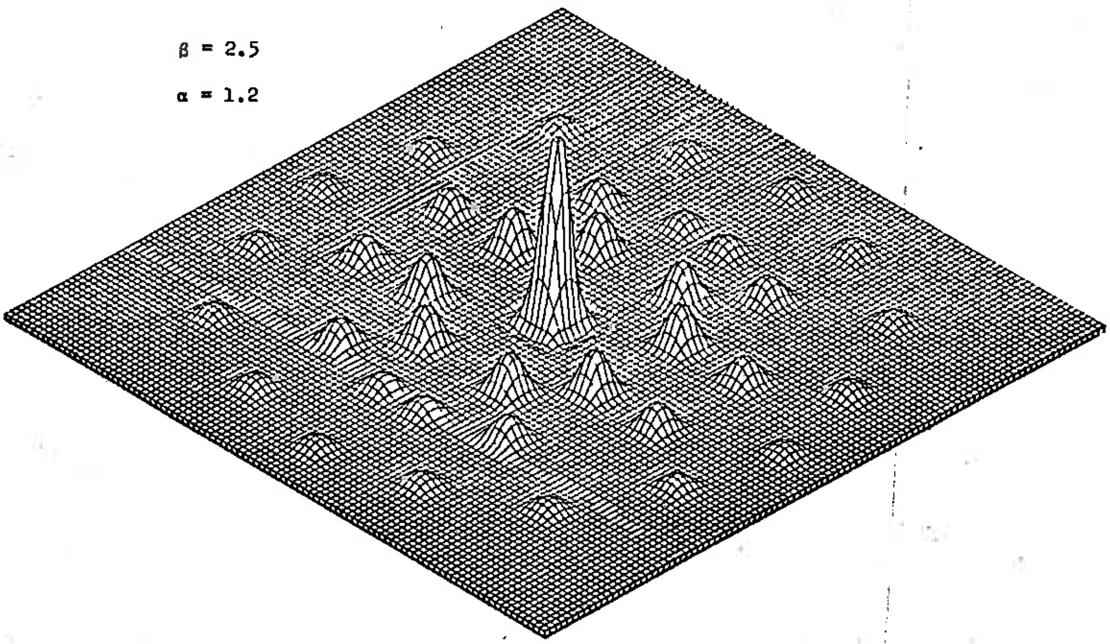


Figure 14. Density decay distributions

$\beta = 1.5$

$\alpha = 1.2$

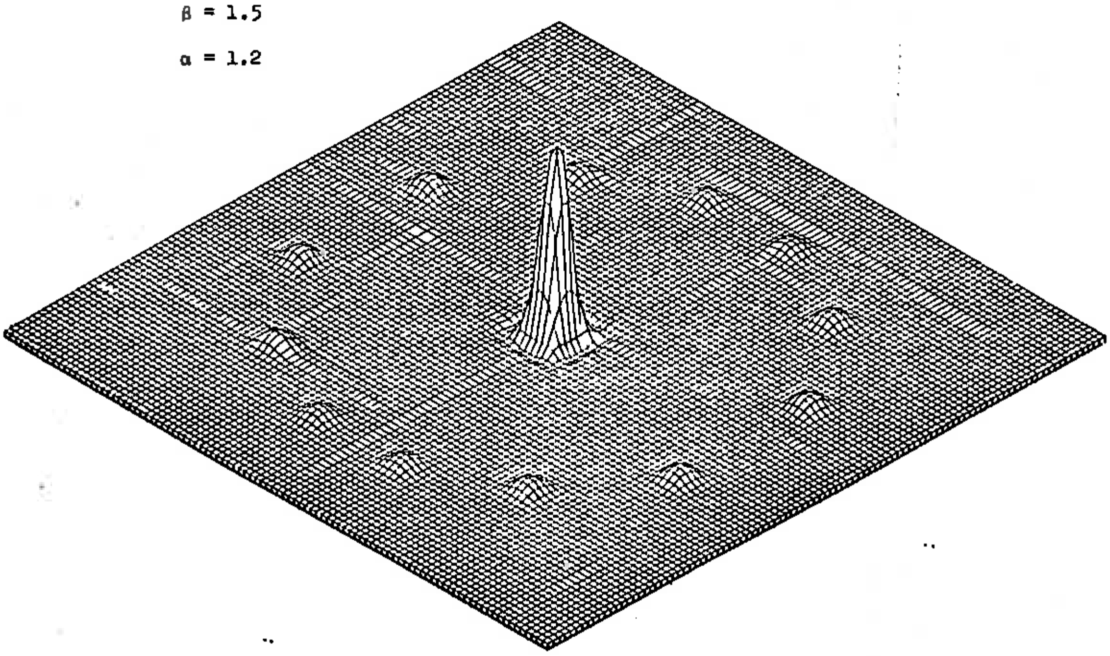


Figure 15.

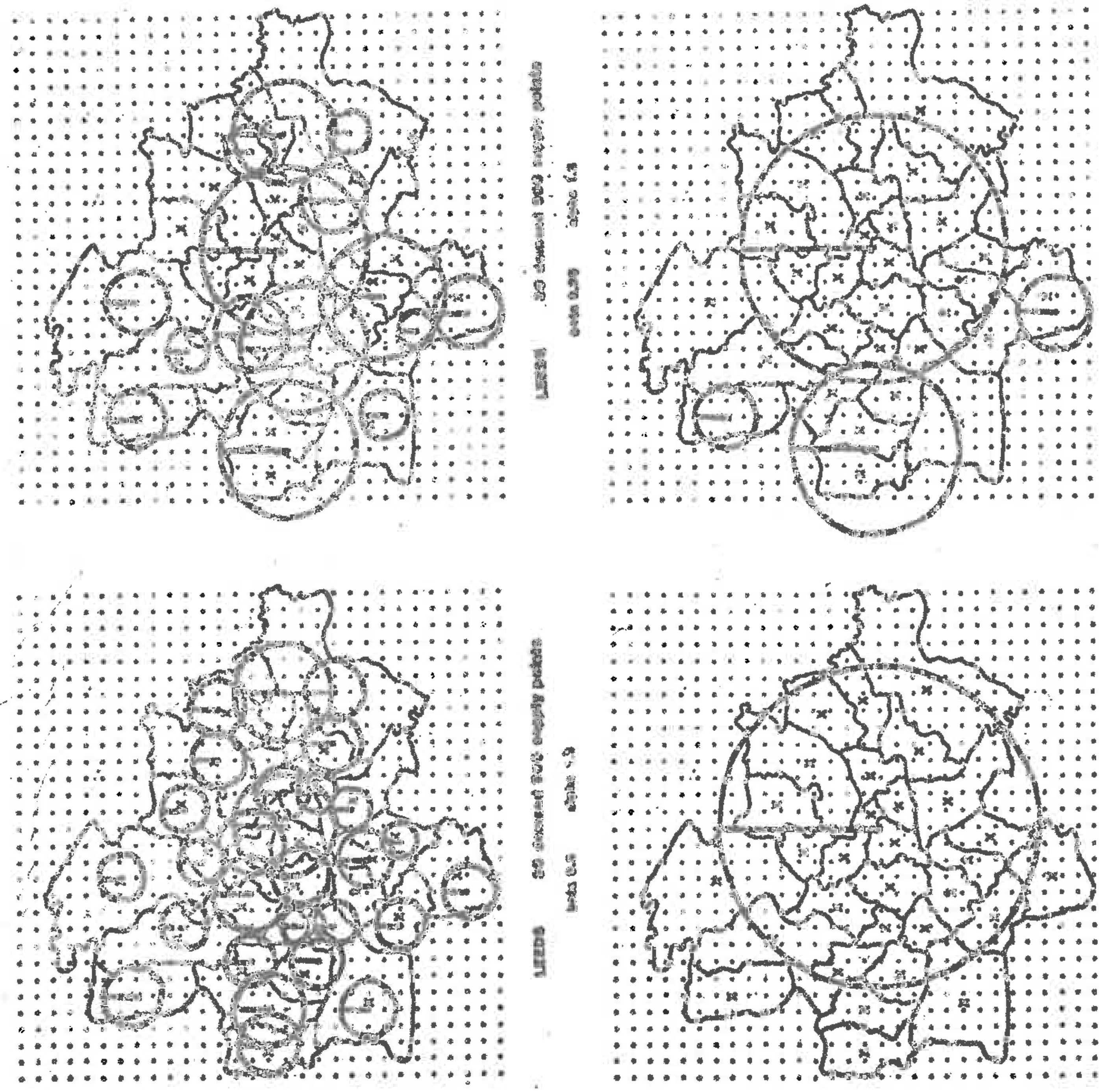


Figure 16.

$\varepsilon = 0.0001$

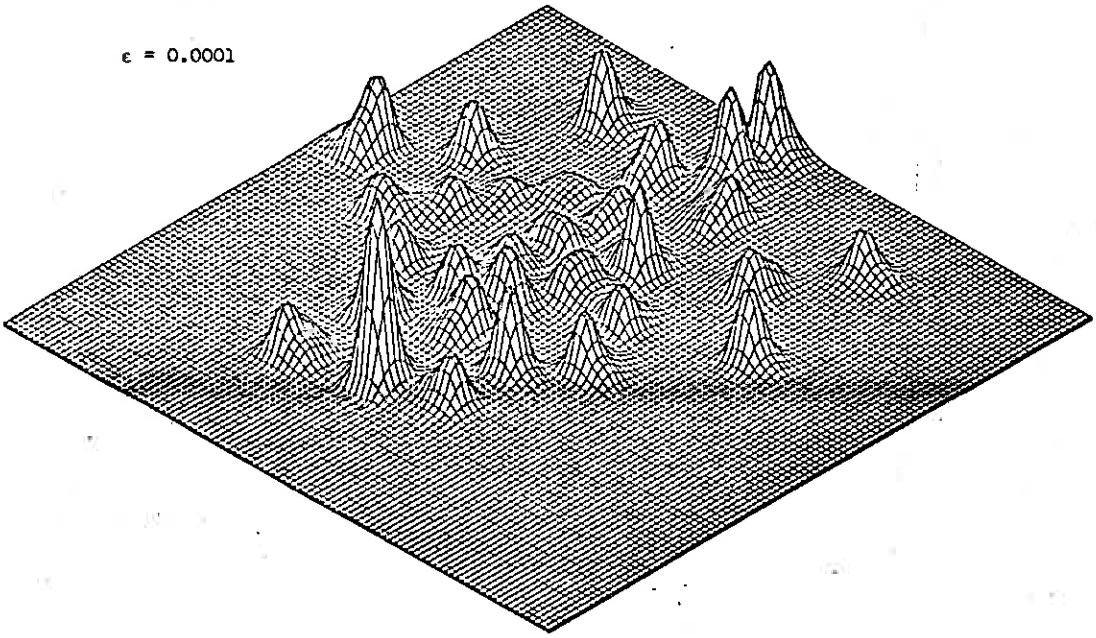


Figure 17.

$\varepsilon = 0.0005$

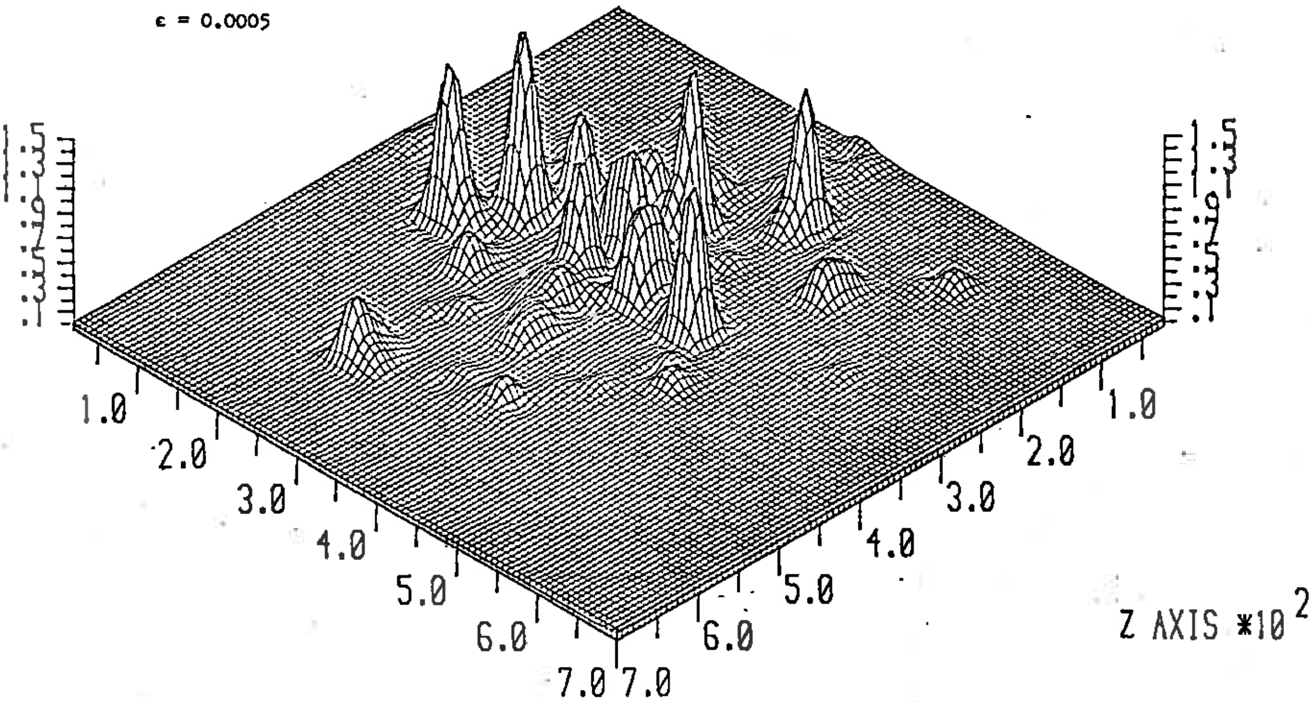
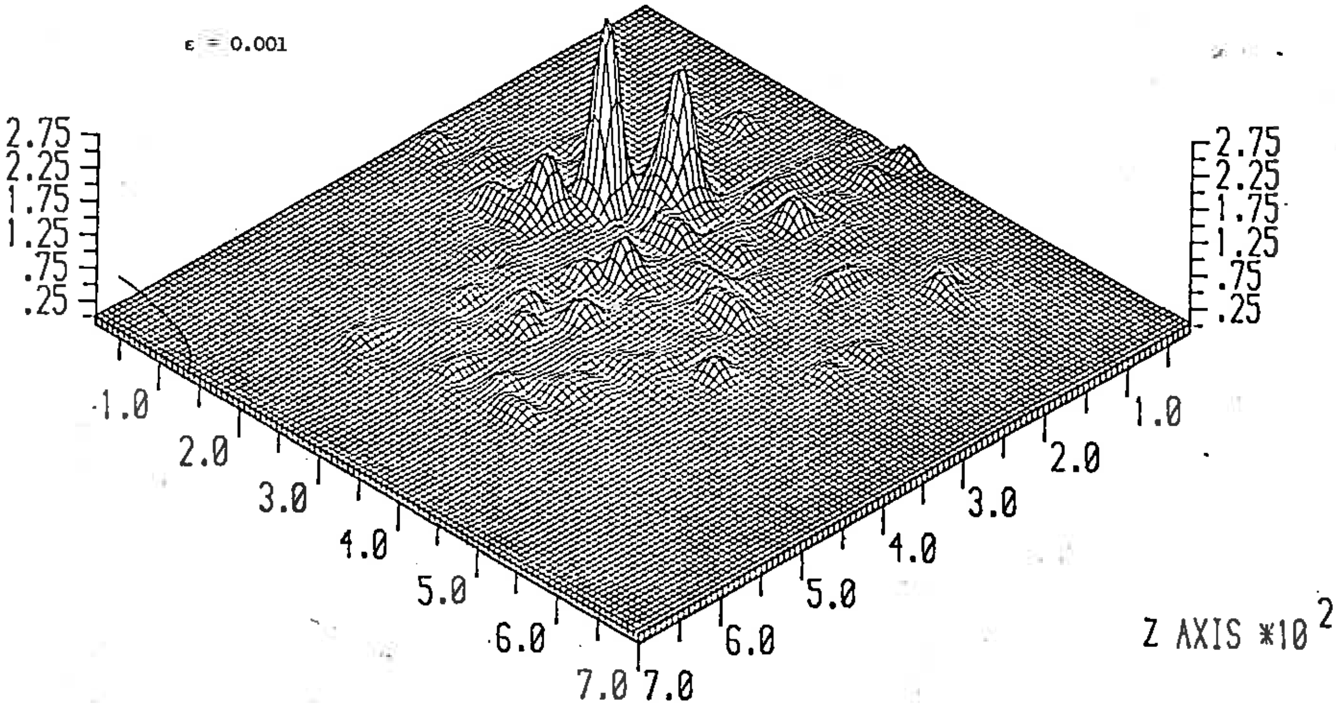


Figure 18.

$\varepsilon = 0.001$



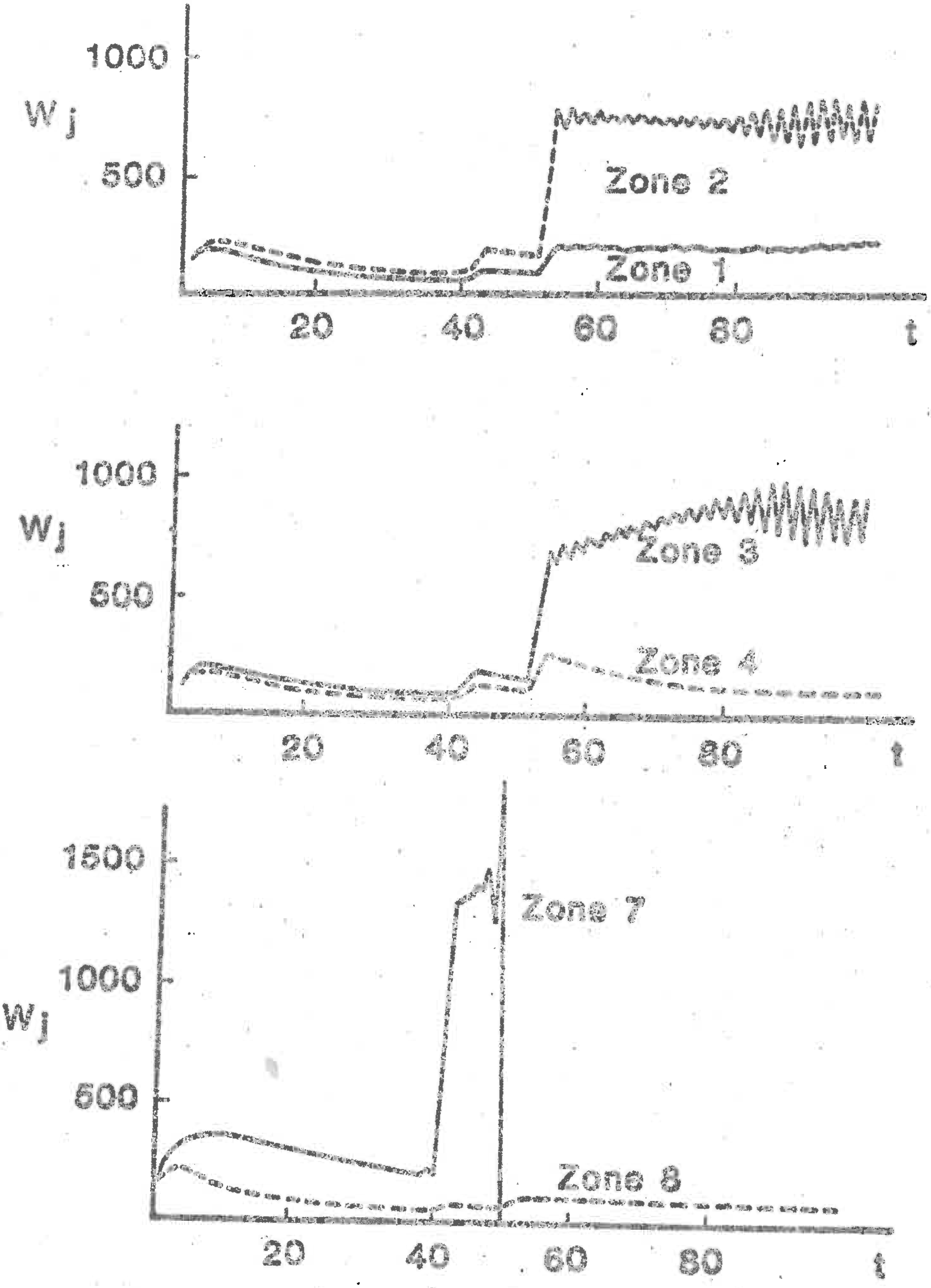


Fig.19 Dynamical Behaviour Retail Model

Selected Zones

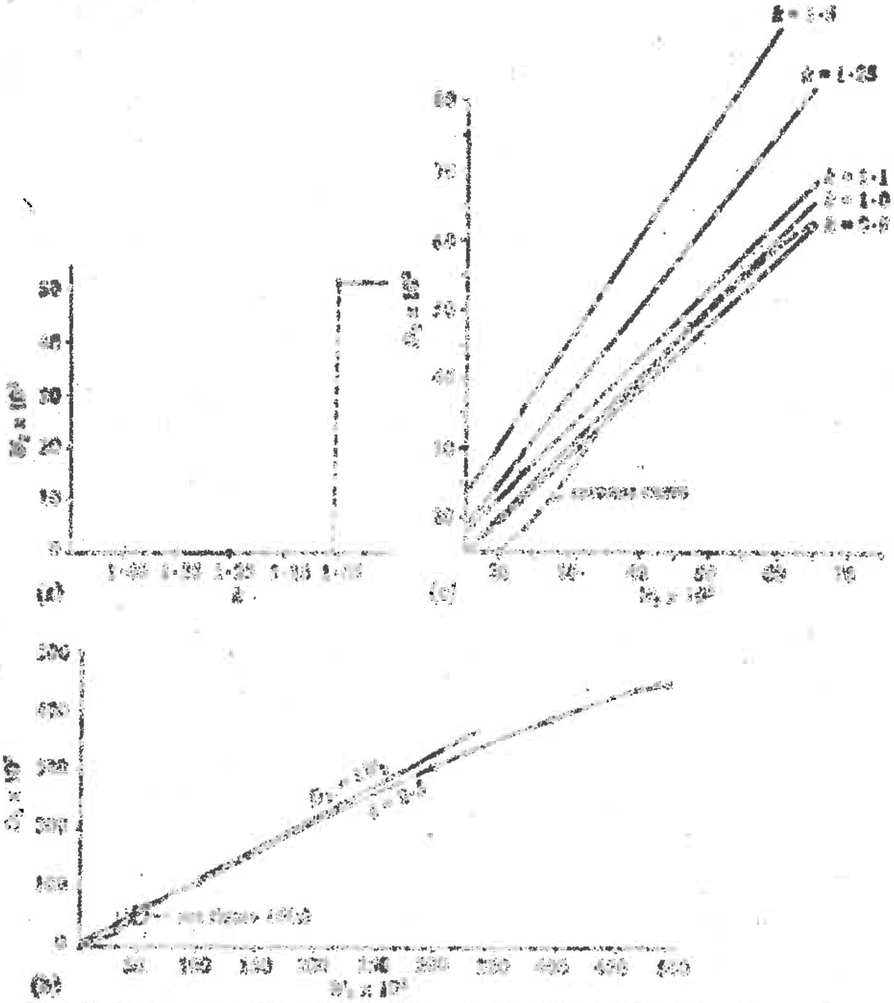


Figure 20. An example of the use of assumption 1', for case 2 with $\beta = 0 \cdot 5$ and $\alpha = 1 \cdot 5$. Note that in (b) $D_1 = kW_1$ is drawn for $W =$ [illegible] and $W_1 =$ [illegible], and therefore appears as a straight line. The straight lines in (c) are the lines $D_2 = kW_2$ for various k.