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A re-appraisal of Q-analysis

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1. Introduction

It is over a decade since the formal introduction of Atkin's methodology of Q-analysis into the social science literature (Atkin 1974). Q-analysis represeted a break from the mainstream of mathematical and statistical methods in social science, emerging out of Atkin's distinctive development of answers to the questions 'What do we mean by "hard" data?', 'How can we represent in formal terms what we intuitively understand by structure?', 'What is the significance of this structure for the system under consideration?', 'How does a structure change and evolve with time?', and 'What are the implications of structural change?'.

These questions reflect Atkin's perception of a need for a new style of 'scientific' analysis of social systems and phenomena. It was to be less a case of bringing already developed mathematical tools and conventions to aid analysis, than of developing a mathematics in the light of a suggested mode of understanding. From the key publication in 1974 "Mathematical structure in human affairs" it was apparent that the perspective being developed (Q-analysis) was to have potential applicability in a very wide range of contexts - architecture, art, chess, transport, retail provision, decision making and organisational structure. The scope for such widespread applicability derived in no small measure from an appreciation that different contexts may appear more different than they may otherwise be if different names and labels are used for what are (qualitatively) the same characteristics. Properties of the mathematics being developed were to be instrumental in laying bare these characteristics.

2. Aims

The purpose of this paper is to evaluate key aspects of the methodology of Q-analysis and in general with mapping the limits of applicability of the approach through examining the effectiveness of different elements in yielding useful insights. Of relevance in this evaluation will be a consideraton of the distinctiveness of Q-analysis vis-a-vis other methods, a consideration of possible alternative means of achieving some of the sought aims of the methodology, a consideration of the importance of these aims and of possible conceptual limitations, and a consideration of the range of contexts in which the methodology has been invoked.

During the decade since the introduction of Q-analysis there has been marked development in methodological critique and philosophical consciousness in social science modelling communities, and increased sophistication in a range of modelling approaches. There have been critiques of individual elements of Q-analysis but still continued interest in invoking selected aspects of the approach on the part of a group of researchers whose imaginations have been noticeably captured by the appproach. Q-analysis has remained something of an enigma to many others, and has also been the subject of some controversy (Macgill 1983). These observations provide both the setting and the justification for the evaluation below. A number of conclusions will emerge: that there are weaknesses in the approach as originally expounded and popularly invoked, which need to be more widely acknowledged within the community who use the approach and, in some cases, rectified; that there are also important strengths that could be more widely appreciated though ultimately, as with so many innovations, in their turn must be surpassed; that there is great scope for reducing the insularity of Q-analysis from other approaches - both because elements of Q-analysis can be fed into other methodological perspectives (the different elements of Q-analysis do not have to be companions only of each other, nor indeed are they necessarily 'natural' partners) and because some elements are evidently already (though not generally recognised as such) special cases or close relatives of other approaches.

3. A summary of key elements

Key elements of Atkin's Q-analysis can be summarised as follows:

I. Description of entities and phenomena of interest in 'hard' (c.f. soft) terms must be a priority in any area of scientific inquiry. This is a non-trivial requirement, to be achieved by observing strict rules of set membership when identifying objects of interest.

II. Sets arising in any empirical context may be related to further sets at finer and at coarser levels of resolution. This leads to the identification of well-defined hierarchical arrangements between sets at different levels of resolution.

III. In a Q-analysis perspective, systems of interest are identified through considering inter-relationships between two sets, these arising either at different levels in a given hierarchy or between sets in different hierarchies. The relation between two sets arises either as a binary or as a weighted array. If weighted, the concept of slicing can be invoked to yield a binary array. Binary arrays are essential for the further elements of Q-analysis summarised here.

IV. Binary relations between sets can be represented by a multidimensional geometry (a simplicial complex). Distinctive aspects of this geometrypotentially correspond to distinctive aspects of the corresponding 'real world' system. The geometry itself reveals what in a Q-analysis perspective is understood to be the structure of the system, and may poignantly reflec its multidimensionality.

V. Numerical indicators are to be used to represent structural features and properties. Such representation is relevant both for analytical purposes and for cases where the geometry is too complex to draw i.e. the numerical indicators should act as signposts or guides as to what the structure actually looks like.

VI. Further insights into the essence, nature, functioning and, possibly, evolution of 'real world' systems could be bound up with the realisation that structure could be viewed as a backcloth capable of supporting certain types of events, activities, observation, measurement, movement and other characteristics, for which Atkin used the term "traffic". Recognition of the duality between struture and traffic was to a prominent feature of the methodology ('if there's no traffic, it is not Q-analysis'), which, as such, could be interpreted as operationalising a particular type of structuralist inquiry. Formal notation and algebra for traffic extends into highly complex mathematics; in this paper we concentrate primarily on that which has been reflected in empirical work.

4. Published illustrations and applications of Q-analysis

The range of contexts in which the methodology of Q-analysis has been invoked is extensive, including transport (Johnson 1976, 1981ab, 1984; see also Macgill and Springer 1985a), retail, market and service structure (Beaumont and Gatrell 1981, Clarke and Macgill, 1983, Johnson and Wanmali, 1981) social area analysis (Gatrell 1981a, Beaumont and Beaumont 1982, Spooner 1980), social network analysis (Doreian 1974, 1981, 1982, Freeman 1980, Spooner and Batty 1981), organisational structure (Atkin 1983), agricultural practice (Gaspar and Gould, 1981), ecosystem structure and dynamics (Gatrell 1981b, Gould 1981, Casti, Kempf, Duckstein and Fogel 1979), geological classification (Griffiths ....) citation indexing (Gatrell 1984a,b), architectural design (Fleming, 1981), design education (Phillips and Johnson, 1981), structural analysis of painting and literature (Atkin, 1974, 1981), football and chess (Gould and Gatrell, 19800, Atkin 1972), medical diagnostics (Chamberlain 1976), psychiatry (Cowley 1985). A range of illustrations is also given in Atkin's (1974, 1977, 1981) own texts. The full range of applications known to the author for which documentation is available is listed in the appendix to this paper.

It will be assumed readers have some familiarity with Q-analysis and a full pedagogic exposition will not be given here. In the appendix the aspects of the methodology utilised in particular contexts are systematically identified. Reference will be made to the appendix in apraising individual ideas below.

5. Ideas in Q-analysis I

Description of entities and phenomena of interest in hard (c.f. soft) terms must be a priority in any area of scientific inquiry. This is a non trivial requirement, to be achieved by observing strict rules of set membership when representing data.

As is well known, a set is a collection of objects or entities for which it is possible to answer either 'yes' or 'no' to the key question 'Does ... belong to the collection?' (where ... can be anything at all). The conventional ways to define a set are either (i) to list all elements or (ii) (often implying or forcing greater cohesion) to specify some property which all elements must possess. In the former case, any article or entity appearing in the list must belong to the set. In the latter case, any article or entity possessing that property must belong to the set. Both ways enable an unambiguous answer to the key question to be given. The lack of ambiguity requires in the former case for the list to be agreed by anyone with an interest in the piece of work - whether author, client or reader - and in the latter case for the property to be unanimously acknowledged. Some examples of sets are listed in Figure 1, indicating whether they are of type (i) or type (ii), and posing the key question about possible members. For further discussion about set membership criteria, see Stewart (1975).

Identification of set membership is not in itself a revolutionary concept.

All data gathering exercises and modelling and statistical work rests (or at least, should rest) implicitly on well defined sets (sets of zones, sets of characteristics...). Intelligible data presentation and mathematical and statistical notation would be impossible if this were not so. The point to recognise, then, is not the novelty of the concept but its power, its applicability in hitherto uncontemplated contexts and its ability to supply a particular meaning to the term "hard".

The virtue of invoking the idea of strict set membership as a fundamental element of Q-analysis is that it can force objects of interest to be described more precisely and unambiguously than might otherwise be the case, - a formal reminder that it may be all too easy to assume that the well-defined sets that are relevant to any particular area of problem solving have been identified when, in fact, this may not be so.

Attempts to defuse minefields of bogus deductions, which can now be seen to have at their root a disregard for the rigour that would be imposed by strict set membership considerations, have been made by the authors of the many excellent books on the theme 'damn lies and statistics'. For example Reichmann (1978) quite correctly recognises the falsity of deducing from the statment: 'Accident statisitics reveal that there are many more injuries to intoxicated pedestrians than there are to intoxicated drivers' that it is safer to be a driver and, if one became inebriated, borrowing someone's car increases one's chances of survival. For by invoking element

I, it may be observed that the deductions do not relate to the sets referred to in the original statement: the statement referring to drivers and pedestrians who have had accidents, and the deductions to all drivers and pedestrians (quite apart from the type of injury incurred). The fault then, is entirely a matter of ambiguity over which sets and relations are being referred to: defective phraseology as can only too readily arise in the attempted compression of descriptions which, perhaps, should not be compressed.

It is inadequate, though, to accept the line of argument embedded in the statement of element I as providing the only appropriate foundations for legitimate social science inquiry. Fuzzy sets and multivalued logics can be counted among lines of response to a recognised illegitimacy of enforcing strict set membership criteria in all contexts. More fundamentally, Cullen (1983) takes issue with the uncompromising rejection of all but one (hard, set-theoretic) way of formalising a description of the social world. He is concerned that no mention is made of the way the analyst, in selecting labels and categories through which to classify the observed world may be adding a layer of interpretive transformation thus separating the data set from the world it is meant to describe. Insight from invoking further elements of Q-analysis, then, must be relative to that which has been chosen to be called data, and does not necessarily have priviliged direct access to the social world itself. Therefore for it to be true that the a Q-analysis perspective really does 'let the data speak for themselves' [a reference to a paper by Gould (1981b)] it must also be true that the only human interpretations that can ever qualify as data bearing upon the issue of social structure are ones expressed in precise set-membership terms. If any other interpretations qualify, then the rule is confounded since they can only be handled after a process of transformation involving the arbitrary establishment of thresholds. In other words (and anticipating material to be discussed in conncetion with further elements below) Q-analysis lets one sort of interpretation of the social world speak for itself about the structure of the world. If other sorts of perceptions and measurements are not to count as data, this must be because they are intrinsically less relevant to the issue of structure. If they are equally or more relevant, then Q-analysis (and any other methodological perspectives for which element I comes into play) is going to miss the point.

The implications of the above discussions of the first idea are twofold: first that the set-membership axiom of Q-analysis (promoted notably by Atkin and Gould) reminds us that it may be all too easy to assume that the basic elements etc that are relevant to any particular area of problem solving have been identified. Proper identification of this could save a considerable amount of sterile debate. Second, having recognised this point it is then necessary to go further, to the recogintion that the depiction of reality in strict set membership terms may, in fact, be a corruption of reality. Each of these two contributions (that by Atkin and

Gould, and that by Cullen) is of value in prompting awareness of different aspects, and better informing the minds of researchers contemplating related choices.

What is interesting to note is how much proposition I has been aired in pedagogic expositions of Q-analysis, and how little it has been in the range of examples as a whole (see Appendix), but how dominant it has been when it has (notably Gould, Johnson and Chapman 1984).

Other authors have projected an understanding of set membership significance in more literary style:

"The first thing to be observed about descriptions given in terms of lists of items (sets?) is so obvious that you have to hold it down or it will drown out every other observation. This is: It is just duller than ditchwater... But if you can hold down that most obvious observation, some other things can be noticed that do not at first appear. First, such information may be impossible to understand unless you alreay know something about the real world system it purporst to represent. Second the objects exist independently of any observer. third, a minimum of value judgements have been expressed. Fourth (and perhaps most powerfully), there is a knife moving about here. A very deadly one; an intellectual scalpel so swift and so sharp that you may not see it moving. You get the illusion that all those parts are just there and are being named as they exist. But they can be named quite differently and organised quite differently depending on how the knife moves... It is important to see the knife for what it is and not to be fooled into thinking objects of inquiry are the way they are just because the knife happened to cut them up that way" Pirsig (1978) pp71-72.

6. Ideas in Q-analysis II

Sets arising in any empirical context may be related to further sets at finer and at coarser levels of resolution. This leads to the identification of well-defined hierarchical arrangements between sets at different levels of resolution.

The very act of identifying a set as a collectio of entities (members) gives rise to a natural hierarchy in which the name of the set is necessarily at a higher (coarser) level of resolution than its members. Moreover, at this higher level, the set name is itself likely to be a member of another set. Thus Atkin (1978) notes that "if X is a set at level N then the subsets of X (groups of members of Z) qualify as elements of a set X' at level (N+1)". In an analogous way particular groups of elements of sets at level N-1 may define elements of sets at level N* (see Figure 2). In the light of these concepts it can be suggested that the 'hardness' of the specification of type (ii) sets at any give level N (cf proposition I) rests either on some suitable specification at level N-1 (and hence, by induction, possibly also at levels N-2, N-3, N-4, ...), or on ground which, in some sense, is not disputed (those greek words for which word and meaning were one, or objectively observable entities, perhaps). Thus to be rigorously defined, hierarchies must be defined from the bottom upwards. Atkin argues further that an element at a given level (level N, say) must relate wholly to at least one and perhaps several elements at a higher level (level N+1). In other words it must always be clear that the higher level can cover the level immediately below it. If this is not the case, the hierarchy is ambiguous (incorrect). In Figure 3 an example of what is strictly an incorrect hierarchy is given.** More generally, it might be instructive to conceive of a cover set hierarchy as a particularly formal structured key word system.

* The technical term for all possible combinations of elements from a given set is the power set. Thus the elements defined at level N are elements of the power set from level N-1. See Atkin (1981a) for an attractive account of these hierarchical notions.

** We cannot answer either 'yes' or 'no' to the question "Is the USSR part of Europe?"

One of the virtues of establishing well-defined hierarchies in empirical contexts is that whatever level in the hierarchy some 'problem area' is identified, the necessary dimensions of it that need to be addressed in analysing the problem are typically depicted at some lower level (eg towns at level N; social-economic and physical characteristics ast level N-1). An awareness of the hierarchical contexts of particular entities of interest and of allowable aggregations and decompositions between levels of resolution within the hierarchy can be crucial to adequate system conceptualisation, but more generally, the notion of hierarchy is, of course, much more widely applicable than the strict cover set hierarchy discussed in the previous paragraph and illustrated in Figure 2.

Having recognised the benefit of clarity that strict observation of the second idea can yield, it is again necessary to look further, in particular whether it is satisfactory to accept the lack of explanation of how higher level entities result from those more primitive (lower levels). Couclelis (1983 p5) argues that it is not, specifically:

"... the construction of hierarchies in Q-analysis proceeds by a step-by-step substitution of new entities for sets of lower order entities, and so on, until a plateua is eventually reached consisting of a few highest-order, or most general entities relevant to a particular study. It is obvious that the potential of Q-analysis to produce meaningful, as opposed to merely formally correct, results out of a data set (usually comprising the elements of lowest order in a hierarchy), must depend crucially upon the legitimacy, from a semantic viewpoint, of the act of substitution. That is, it must be the case that, in any particular problem context, the collection of entities defining a set at level N is for all intents and purposes semantically equivalent to the (N+1) level entity chosen to designate it".

Couclelis (1983) is suggesting that, in as much as a given entity can be described in terms of others, that entity is likely to be a function, or a structured collection of others, i.e. the entities may interact with each other in various ways. If it assumed that such interactions do not exist, or if they are ignored, then Q-analysis may be intrinsically less relevant to the social world that might be deduced from the second idea.

In this argument, Couclelis is lending support to an earlier argument by Pinkava (1981) subsequently refuted unduly swiftly, by Johnson (1981). In elucidating general difficulties arising in the classification of a set of entities (at level N say) in terms of a set of reatures (at level N - 1 say) Pinkava (1981) p231 states, for example, that

'a class is determined by its ruling function'

and this is a logical function of the binary features, not the features themselves. In other words, it is not sufficient to view certain phenomena simply as collections of other things, but as combinations, configurations or functions of other things. And again,

"Whatever we might do with the features we can never arrive at a classification because it is simply not there in terms of the features as such"

Similarly, Couclelis p6,

"Whenever the 'equals' sign relates an individual entit to a class of individual entities, semantic substitution of one for the other is explicitly not allowed".

She suggests, (op cit p7), that it may even be worse than this.

"The entire [conceptualisation of well-defined set theoretic hierarchies] is only valid if one admits an 'atomic' ontology, a world endowed with some 'ultimate furniture' of discrete particulars or individuals (be they objects, concepts, ideas, socio cultural or linguistic entities, or whatever), and their possible combinations. Not every contemporary epistemology would subscribe to this. However, if this assumption is to be seriously questioned, no empirical embodiment of set theory can be defended anymore".

The implications of this discussion are similar to those of that in the previous subsection, namely, that the painstaking exposition of set theoretic hierarchies can be tellingly disciplining, but it is not a uniquely legitimate conceptualisation of thhe elements of sovial systems. Also as with the first, the second idea has been relatively little invoked in empirical applications of Q-analysis (see Appendix) but has been a dominant consideration in a few, notably in an in depth approach to television programme clasification (see Gould, Johnson and Chapman 1984). Again the idea has been dwelt on at length in pedagogic expositions.

7. Ideas in Q-analysis III

Systems of interest are identified through considering relations between sets, these sets arising either at different levels ina given heirarchy or between sets in different hierarchies...

The potential significance of interset relationships can be illustrated in myriad ways. For example, words have meaning because they may be related to other words or to experiences, i.e. elements of given sets may be related to elements of other sets and since different individuals relate given words to their own experiences - which may not be the same as those of others - and they can derive different meanings. Similarly, faithful translation of words from one language to another is often impossible because the wider set of words to which they relate in each language are not the same. However, the basic operation in each case - comprehension and translation - may be represented as one of relating members of sets to other sets. Events or phenomena similarly have meaning because they may be related to other events or phenomena. All explanation involves relating things to things, or, more rigorously, sets of things to sets of things. Substances and materials in the biosphere become significant as resources only in as much as they relate to man's utility (sets of needs and wants). Measurement involves relating (sets of) observations to (sets of) numbers. (As another aside we may note that the depiction of measurement in this way - invoking set membership and inter-set realtions - may further suggest that the roots of Q-analysis are potentially more fundamental than those of traditional quantitative methods. In table 1 a small selection of the many inter-set relations that have been of traditional concern within geography are listed.

Some of these relations are so-called one-to-one relations (eg measurement, speed ...). In such cases the representation in terms of sets may appear to be unnecessarily laborious. It does, however, illustrate the point that such one-to-one relations are a special case of something more general (more highly joined relations). An intermediate stage between one-to-one relations and varying degrees of overlap are so-called partitions (many-to-one relations...). See Figure 4. In this case, the corresponding diagrammatic representation depicts the classic tree structure or dendogram. Finally there are many-many relations. These are typically more difficult to handle in terms of the 'scientific' goal of seeking structure and order, and it is very much the pre-occupation of Q-analysis to yeild crucial insights into how this might be done.

7a. Ideas in Q-analysis IIIa

... The relation between two sets arises either as a binary or as a weighted array. If weighted, the procedure of slicing can be invoked to yield a binary array. Binary arrays are essential for the further elements of Q-analysis.

Slicing is a procedure whereby a weighted or numerical relation is reduced to a binary relation by specifying numerical criteria (either for the whole relation, for each row or each column or for each cell individually) and replacing cell values that meet the criteria by 1 and those that do not by 0. In many studies sensible threshold values for slicing are quite readily identifiable, or perhaps a series of such values, thus giving a succession of 'views' of the original array. In other cases they do not, and here it would be contrary to the philosophy of the approach to use slicing in a mechanical way simply to allow further elements of Q-analysis to be carried out, for it is clear that the procedure can involve a heavy loss of information. (See Macgill 1983b for further discussion of this point).

The possibility of devising further procedures within the framework of Q-analysis for the analysis of weighted relations has led to the development by Johnson (1981) of Q-discrimination analysis (see also Macgill 1982, 1983c). There remain some problems withthe approach, however. In the case of ordinal data, much of the weighted information, paradoxically, cannot be exploited, and for interval data, the approach is technically comparable with an enhanced matching scores clustering analysis method.

Binary array representations are appropriate where systems of inerst can be sensibly depicted by means of a logical relation. What must be given particular consideration, then, is the apprpriateness of such representations. Where slicing is invoked, of course, the appropriateness of ignoring what may be a considerable amount of information. More generally, the typical restriction to a static mode of analysis, and one which, as brought out from consideration of element II, ignores interaction, function, composition.

8. Ideas in Q-analysis IV

Binary relations between sets can be represented by a multi-dimensional geometry (a simplicial complex). Distinctive aspects of this geometry potentially correspond to distinctive aspects of the corresponding 'real world' system. The geimetry itself reveals what in a Q-analysis perspective is understood to be the structure of the system, and may poignantly reflect its multidimensionality.

Stewart (975) remarks that geometry is one of man's most powerful thinking tools. Pictoral representations can often speak much better for themselves than verbal narrative or abstract symbolic notation. The simplicial complex representation of a relation between two sets immediately stresses its multi-dimensionality and its idiosyncracies. Figure 5 is the pictoral (simplicial complex) representation of the data in table 2 (Gatrell 1983) Adjacent simplices in the complex are those which have a degree of commonality, as determined by shared vertices: simplices which are densely embedded have verties in common with several others; those that are isolatedhave few, if any, joins; weakly joined structures may be very brittle and unstable; bridges between two otherwise separate or weakly joined parts may be interpreted as being particularly significant structural features.

In view of the fact that the structure of interset relations can often be rich and complex, Gould (1980) remarks with some concern that in the countless contexts which effectively involve dealing with sets and relations between them, the preference in social science has traditionally been to suppress this multi-dimensionality and to search for or enforce one-to-one relations, or at best, partitions, rather than allowing the naturally richer relations. Thus there has been a tendency towards the rule that if A relates (or, more crudely, belongs) to X it cannot also relate (or belong) to Y. Network analysis (and associated graph-based methods, see for example Roberts, 1976) may be mentioned as notable exceptions, as networks often exhibit characteristically rich structures. However, these are essentially one dimensional structures, composed of a number (perhaps even several hundred in some cases) of one dimensional links between nodes. One of the most distinctive contributions of Q-analysis lies in its emphasis on multi dimensional linkages between entities.

Gould has reiterated Atkin's desire for a fuller recognition of multi-dimensionality through numerous illustrations. Thus, rocks have been unnaturally partitioned into unconnected categories, jobs into distinct types, and disciplines into separated trenches in each case to the detriment of an advancement of knowledge (since natural overlap has bee suppressed. The data is apparently not allowed to speak for itself. Further examples of neglect of multi-dimensionality can be readily added to this list. One individual may see a decision about the development of a new coalfield purely in terms of jobs (rather than, say, wider environmental concerns as well: another may, on the other hand, concentrate only on the latter): a pragmatist may view any theoretical analysis as unconnected to the real world: remarks intended to be innocuous may be taken as insults because the person to whom they are addressed interprets them differently (relating them to different sets) from the person who utters them - sterile or offensive exchanges may be the result. What such illustrations suggest is that the analyst may benefit from exposing what is known of inter-set relations, and view the resulting structures as a whole, rather than from the perspective of a single element. In the field of chess Atkin (1974) argues that the grand master has mastered the art of doing this, whereas the lesser player has not.

Atkin, then, is pleading for social scientists to take fuller recognition of the multi-dimensionality of their worlds of interest. Paraphrasing Atkin (1981): The stage of such a world is not the three dimensional structure that must be inhabited to argue politics, wage wars... Three dimensional space is an important special case, it is one that Einstein found over-restrictive in developing his understanding of the world. He introduced a fourth dimension, time, and theorised about tiny points called Sun, Earth, Mars ..., moving around modest four dimensional space. Atkin is pleading for social scientists to realise that they must typically inhabit a higher dimensional space. The dimension of the 'world' that must be inhabited is the dimension of the structure that may be identified (via the above principles and operations) in any area of study, fully to address the issues that are fundemental to that study. A Q-analysis perspective differs from more widely known multi-dimensional scaling approaches (which also seek to create maps of multi-dimensional social space) through its absence of data transformation in projecting the space, aswell as in its view of the space as an influencng and conditioning 'backloth' for activities, processes and events.

9. Ideas in Q-analysis V

Numerical indicators can be used to represent structural features and properties, and are of vital importance in cases where the geometry is too complex to show. The numerical indicators, then, should act as signposts or guides as to what the structure actually looks like.

There are relatively few applications in which the simplicial complex is presented (see Appendix), making the role of numerical indicators particularly important. The key indicators are:

a) top q and bottom q

b) a listing of components at each dimensional level

c) eccentricities

d) the structure vector

e) the shared face matrix and q-nearness graphs

(a) Top q, depicting the dimension of simplices, has been of particular interest in many applications and illustrations (Doreian 1981, 1982; Gaspar and Gould, 1981; Freeman, 1981, Gatrell, 1981a). It seems to be a case where people's attraction to Q-analysis has led them to see additional significance in a simple indicator - that of ranked frequency -

that would ordinarily not have been of such interest. Some of the most significant interpretations in the paper by Gaspar and Gould, for example, could be re-expressed in more familiar language in terms of rank orders of occurrence, though were evidently inspired by and expressed in the language of Q-anlaysis. The other named indicator, bottom q is used to define eccentricity - see below.

(b) This is the most widely used form of numerical indicator. "Performing a Q-analysis" has typically been synonymous with deriving and then interpreting q-connected components at successive dimensional levels. As explained elsewhere (Macgill 1984) the mechanics of the algorithm used to produce these components are technically the same as those of the single link (or nearest neighbour) method of cluster analysis (generating 'hierarchical' groupings of data), assuming a similarity coefficient calculated by counting common descriptors. For the example given in table 2, the result is given in Fifgure 6

One of the features of this algorithm deemed attractive in the various applications is its capacity to "represent the structure" of a binary relation, without substantively transforming the data. It has been widely remarked that this marks out Q-analysis from all statistical and quasi-statistical techniques which interpose the computation of interset functional relationships. Thus, the forms of the functions themselves must become (usually unstated) components of the definition of structure, and their particularities are, of course, hard to justify on theoretical grounds. The Q-analysis algorithm, by contrast, has been termed "data friendly" (Beaumont and Gatrell 1982). This "data-friendliness" property affords the user particularly close familiarity with his or her data: to a greater extent than with transformational techniques. Listing of q-components are presented in almost all 'applications' (see Appendix).

The operational similarity of the Q-analysis algorithm with single link clustering is of secondary importance to the more fundamental reason as to why this algorith and its product (q-connected component) has been of such interest in a Q-analysis perspective. The significance of an algorithm whose product identifies which simplices are linked to others, at different dimensional levels derived from the deemed importance of inter-simplex connectivity in Q-analysis. Atkin proposed a priori importance to the connectivity structure of simplicial complexes: linkages or connectivities at particular dimensional levels were deemed to be of fundemental importance in providing spaces and channels for 'traffic' (see next section).

Despite the importance of connectivity in Atkin's development of Q-analysis, and its association with ideas associatied with traffic, it is evident from the Appendix that in almost all applications and illustrations of Q-analysis data is fed through the characteristic Q-analysis algorithm, but in relatively few cases is the concept of traffic invoked. Instead, the components generated by the algorithem are typically interpreted in terms of similarity or closeness and discrimination between different simplices, backed up by a belief that the algorithm yielding the components was "data friendly" (cf conventional multivariate statistical approaches). But here we encounter a paradox, namely that despite the (ritual) description of the characteristic algorithm leading to listings of q-components as "data friendly", the components presented are only a broad guide to the appearence of the structure. The structure cannot be constructed from them, and as they stand, little can be inferred about their internal structure. For example, a component given as [A B C D E] could refer to a set of five simplices interconnected either as figure 7a of as figure 7b.

The internal structural detail can be revealed from q-nearness graphs, but these have rarely been presented or alluded to in applications (see again the appendix). (Alternatively, through simultaneous analysis of the conjugate: Johnson (1981) p244 suggests that 'the experienced reader can often tell how the simplices are connected by a simultaneous examination of both the complex and it conjugate'. This may be so, but unless it is reflected in the write-up, (and the point above is that it is generally not) then the reader is none the wiser.

In light of these observations it is instructive to take note of an alternative algorithm adopted by Ho (1982) which can be used to represent the structure of te relationship between two sets. As examined elsewhere (Macgill 1985) this it can do with greater precision and greater transparency than the characteristic algorithm yielding q-connected components. The approach provides a complete representation of the multi-dimensional geometry by producing a definitive representation of the simnilarities of each simplex to any other. The lattice corresponding to Gatrell's example is presented in Figure 8.

The lattice 'output' can be seen to be considerably more informative than that from the Q-analysis; for example, whereas at level 2 in figure 6 (see also table 3) persons 1,3,4, and 10 are lumped in a single component, the lattice output (figure 8) is more particular in only grouping people where there are actual links; nowhere in the lattice are person 1 and 3 grouped together, for they have no activities in common. In a recent paper (Macgill and Springer 1985) the relative performance of the traditional Q-analysis and lattice algorithms has been explored in greater depth, with the conclusion that the lattice approach is superior, given the typical aim of understanding the similarity structure of data (as opposed to its connectivity structure). Moreover, because different interpretations of data are suggested, there may be a need for previous Q-analyses to be re-worked, for as indicators, q-connected components have often been used somewhat blindly and do not necessarily give what has been supposed.

The adoption of the Galois lattice approach is also apparently in line with the re-direction of Q-analysis suggested be Griffiths (1983) and Johnson (1983). Indeed, the galois approach cannot only be promoted as an alternative to the Q-analysis algorithm, but, rather, in its own right, for many contexts where taxonomy and discrimination are of interest, and systems can be sensibly represented as binary arrays, for it produces a complete representation of the multidimensional connectivity (or geometry) of a system identifying all similarities between elements.

c) The concept of eccentricity has been developed as an indicator of the relative distinctiveness of individual simplices (objects) from the complex (collection) as a whole. It has been used somewhat patchily (see Appendix). There would appear to be a significant weakness in the definitions of eccentricity usually used in that it makes no distinction between a simplex that is joined directly to one other simplex at the appropriate dimensional level (q) and a simplex that is joined to many at that dimensional level. In other words, this indicator of eccentricity does not discriminate as to whether an object has a given set of features in common with one or with many other objects. As with q-connected components it may not give what writers have supposed.

Aspects suitable to incorporate in capturing more fully the characteristics of eccentricity for each object appear to be: the number of features it possesses; how many of these are also possessed by other objects; the number of "other" such objects there are. In fact all this information is given in the shared face matrix and, taking Aij to represent the item in row i and column j of this table, a new definition of eccentricity for object i can be given by

$$\mathrm{ecc}_i = \sum_{j \neq i} \frac{1}{n} (A_{ii} - A_{ij})$$

Above, n is the total number of objects. This compares with the previous formula:

$$\mathrm{ecc}_i = \frac{A_{ii} - \sup_{j \neq i} A_{ij}}{\sup_{j \neq i} A_{ij} + 1}$$

It is interesting to note that the new definition of eccentricity could also be used directly in the context of traditional Q-analysis, for it does not require any information that is not readliy available.

d) The structure vector simply gives a count of the number of components at each dimensional level. It was originally developed because of the deemed importance (questioned above and further below) of connectivities of different dimensional strengths.

e) The shared face matrix identifies the dimensional connectivity of individual simplices to one another. Its compilation is an integral part of the characteristic algorithm yielding q-components, and though could be interpreted in its own right this rarely seems to have happened in published illustrations and applications (see Appendix).

10. Ideas in Q-analysis VI

Further insights into the essence, behaviour and activity of 'real world' systems could be bound up with the realisation that structure could be viewed as a backcloth which could suppors certain types of events, activities, observations, movement, and other characteristics, for which Atkin used the term 'traffic'.

The structure that may be defined and represented in ways discussed in earlier sections, then, provides a backcloth - a 'functional space' - which can support further aspects of the system, or against which further aspects can be 'measured'. Paraphrasing Seidman (1983) a fundamental assumption in Q-analysis is that traffic can only exist on the backcloth if permitted by the geometric structure of the backcloth (see, for example Gould 1980, p.181 - 183), and that whereas, in the one hand, the structure of the backcloth constrains or permits traffic, on the other hand, the existence of 'forbidden' traffic can warp the backcloth and force structural change. Seidman comments that as metaphors these statments are reasonable and suggestive, but it is not clear how to translate them into a formalism that will permit analysis and prediction.

Seidman observes that the translation is usually achieved by conceiving of traffic as one or more functions from the set of simplices in a given backcloth to the rational numbers (in the appendix, those cases where this has been done are identified). Once this has been done, it becomes possible to ask questions about the mutual influences that backcloth structure and traffic exert on each other. Seidman "It is reasonable to hope that one could show directly that certain features of backcloth structure might prevent the appearance of certain types of traffic ... but mathematical models effectively describing the influence of backcloth structure on traffic (thus) remain to be proposed".

Johnson (1982) has been more concerned with movement of traffic through a complex. He argues that traffic must be graded, and that traffic of a given grade can only move or circulate within components of the appropriate dimension, and cannot 'reach' entities that do not appear in such components. Here the significance of having identified q-conected components in the characteristic algorithm is evident. Traffic of a given grade may potentially be transmitted through ling tunnels when q-components are large. Johnson (1982) presents a q-transmission theory of the effect of transmission of traffic of one grade on the transmission of traffic of another. However the conceptualisation of traffic movement in this way has not, in fact, led to anything empirically interesting; first, applications to date have as yet apparently encountered only relatively short chains of transmission for traffic; second, and perhaps more fundamentally, the insights achieved by adopting a Q-analysis perspective could just as readily have been achieved from a more conventional graph-theoretic perspective. Specifically, q-transmission for q=1 is one and the same thing as 'feedback' in conventional systems/graph theory. "When I wrote my paper on q-transmission, I found it very difficult to find convincing examples (of q-transmission) with $q \neq 1$" (Johnson 1983 p465).

Part of the difficulty in developing the theory of q-transmission may lie in the restriction that has been (artificially?) imposed that q-transmission can only occur within q-connected components. This may be an articicial restriction because these components do not represent readily interpretable features. Moreover they are not such significant features of the simplicial complex (and therfore of the original data) as the lattice nodes and edges (components being derived, not direct features). As such they (q-connected components) may be less succssful in providing a basis for the development of applicable theory. Also, the (again artificial) restriction of considering traffic transmission grade by grade, rather than freely allowing changes of grade may be a further restriction that can be usefully overcome.

The lack of progress with component-specific traffic is of fundamental significance in reflecting on the utility of structural indicators (section V) within the conception of Q-analysis as a perspective as inevitably involving traffic (ie as conceived to represent backcloth-traffic interrelation). In particular, it casts doubt on the utility of the structure vector, of q-connected components. A reformulation based on galois connections may improve its powers.

Another structural feature that has interested some researchers is the q-hole. Gatrell (1983) for example. suggests that the gap in the structure in Figure 5 would obstruct the movement of traffic (in this case the ability of individuals to reach a consensus about the kind of protest that should be made about the construction of a by-pass near their village. It is not clear, however, why a hole in the backcloth should be more significant than weak connections or complete disconnectedness.

Nonetheless, metaphorically, the concept of traffic has been deemed to have great power - a mental aid, forcing more careful consideration than might otherwise be the case of how the backcloth affects what can be specified on it. Notwithstanding this, conclusions from earlier sections again assert themselves; having recognised great power from this element of Q-analysis, its limitations must also recognised. What is gives at best is a very particular kind of structuralist perspective. There are others. Difficulty (on the part of the author, and apparently by others too) in fully coming to grips with further ideas of traffic that have been suggested in Atkin's work - of time, of force, of dynamics - has made it impossible to offer evaluative comment in this paper on what some - March and Ho 1976, for example - have suggested may be quite fundamentally distinctive and promising ways of looking at the structure and evolution of human systems.

Cullen (19839 takes issue with any view that a formal representation of social structure can reside only in the presence of interset connections, and that there can be nowhere else to look for structure, other than in the interconnection between sets. Further reaches of Q-analysis, then, are "amongst other things, explaining in an openly inductive fashion, the ramifications of a particular definition of structure. If the axioms are wanting, then so too, surely, is the analysis". There are, moreover, other styles of structuralist inquiry, of identification of what might be meant by structure, and of possibilities for which structure may be both medium and outcome. Greater familiarisation with these on the part of researchers attracted by the backcloth-traffic distinction in Q-analysis can be highly informative in setting Q-analysis (or a Galois based reformulation) into characteristic relief.

11. Conclusions

In general there have been weaknesses in the Q-analysis perspective which need to be more widely acknowledged within the community who have used the approach and, in some cases, rectified. There are also important strengths that could be more widely appreciated. Ultimately though, as with all good ideas, the latter in their turn must be surpassed for they are a long way from exhausting a conceptualisation of social systems of interest. More pragmatically, there is great scope for reducing the insularity of Q-analysis from other approaches, both because elements of Q-analysis can be fed into other approaches (the six different elements reviewed above do not have to be companions only of each other, nor indeed are they necessarily 'natural' partners) and because some elements are evidently already (though have not generally been recognised as such) special cases or close relatives of other approaches.

Table 1

1. The relationship between distance and time (speed).

2. The relationship between a line on the ground and mark on a calibrated scale (length).

3. The relationship between people and shopping zones.

4. The relationship between people and activities.

5. The relationship between exported commodities and countries.

6. The relationship between countries and regions.

7. The relationship between students and option courses.

8. The relationship between industries and industries.

Table 2. Incidence matrix; social space in rural England (Gatrell 1983)

		1 Cricket Club	2 Golf Club	3 Village Hall Committee	4 Women's Institute	5 Church	6 Horticultural Society	7 Conservative Club	8 CPRE*
1	Mr Robson	1	1	1	0	0	0	0	1
2	Mrs Francis	0	0	0	1	1	1	0	0
3	Mr Mariner	1	1	0	0	0	0	0	0
4	Mr Thompson	0	0	1	0	1	0	0	1
5	Mr Mills	0	0	1	0	0	1	0	0
6	Miss Butcher	0	0	0	0	1	0	0	0
7	Mr Sansom	0	0	0	0	1	1	0	0
8	Mrs Wilkins	0	0	0	1	1	0	1	0
9	Mrs Coppell	0	0	0	1	1	0	0	0
10	Mr Rix	0	0	1	0	0	0	0	1
11	Mrs Shilton	0	0	0	0	0	0	1	0

*Council for the Protection of Rural England.

Table 3. Dual Q-analyses, Table 2 data (see also Figure 6)

$q = 3$ {Mr Robson}
$q = 2$ {Mr Robson} {Mrs Francis} {Mr Thompson} {Mrs Wilkins}
$q = 1$ {Mr Robson, Mr Mariner, Mr Thompson, Mr Rix} {Mr Mills} {Mrs Francis, Mr Sansom, Mrs Wilkins, Mrs Coppell}
$q = 0$ {all above + Miss Butcher and Mrs Shilton}

$$Q = \{\overset{3}{1} \quad 4 \quad 3 \quad \overset{0}{1}\}$$

$q = 5$ {Church}
$q = 4$ {Church}
$q = 3$ {Village Hall Committee} {Church}
$q = 2$ {Village Hall Committee, CPRE} {WI, Church} {Horticultural Society}
$q = 1$ {Cricket Club, Golf Club} {Village Hall Committee, WI, Church, Horticultural Society, CPRE}
$q = 0$ {all}

$$Q = \{\overset{5}{1} \quad 1 \quad 2 \quad 3 \quad 2 \quad \overset{0}{1}\}$$

Figure 1. Some sets

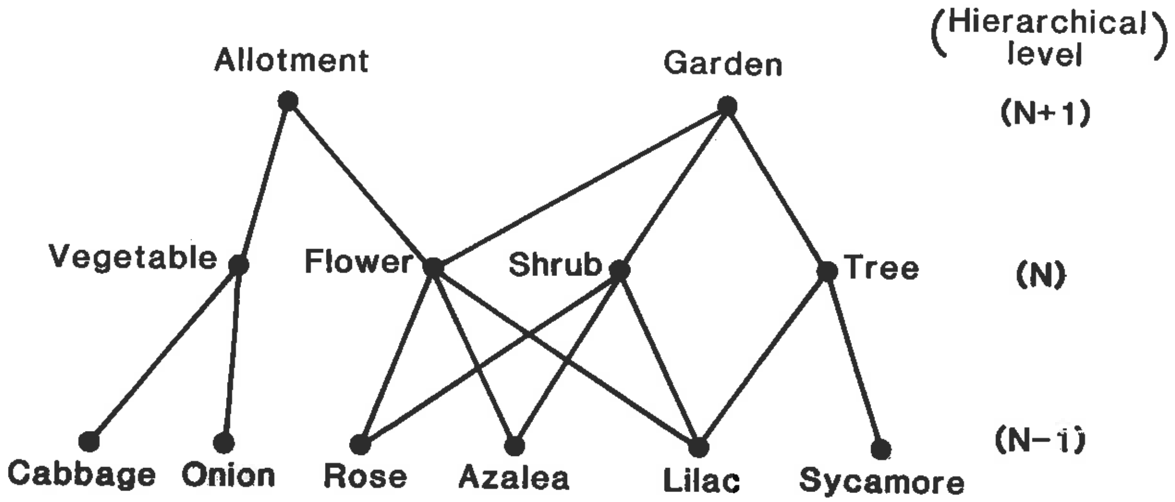
Type			Possible members x	Does x belong to the set?
Set 1	(i)	{cat, pencil, jelly, thunderstorm}	cat dog	Yes No
Set 2	(ii)	{whole numbers}	1 2 101 -3.3	Yes Yes Yes No
Set 3	(ii)	{types of fruit}	apple tomato banana	Yes Yes[1] Yes
Set 4	(i)	{Christaller, Losh, Harton, Haggett, Chorley}	Berry Haggett	No Yes
Set 5	(ii)	{objects topologically equivalent to a solid doughnut}[2]	teacup teapot milk jug ring	Yes No Yes Yes
Set 6	(ii)	{the objects currently on my desk}	telephone paper cat	Yes Yes No
Set 7	(ii)	{local authority district councils in the UK}	Derwentside Dunfermline Duddan	Yes Yes No
Set 8	(ii)	{types of vegetable}	bean tomato carrot banana	Yes Yes[1] Yes No

Notes on Figure 1:

[1] It does not matter that "tomato" has been assigned to two different sets. It is probably more difficult to specify entities and phenomena that are members of only one set than it is to specify those that are members of many.

[2] See, for example, Stewart (1975, p. 146)

Figure 2. Some strict hierarchies (after Atkin 1981 and Chapman 1981)



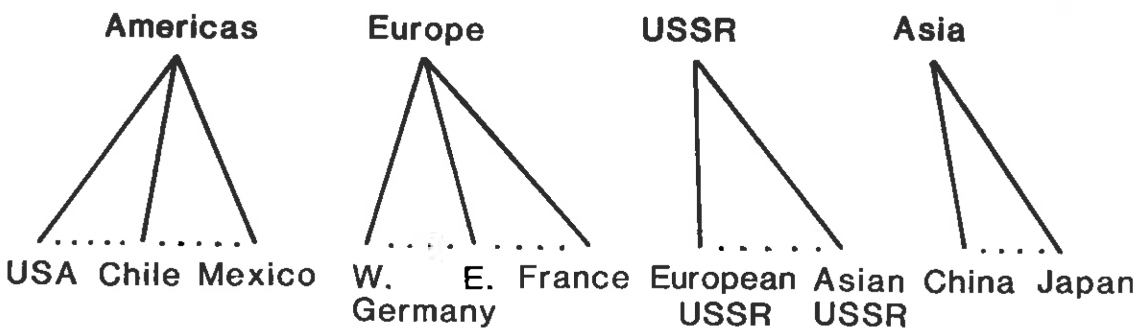


Figure 2 (cont'd). A further strict hierarchy (Gould 1981):the hierarchy of cover sets of physical objects in the electrical power restoration problem

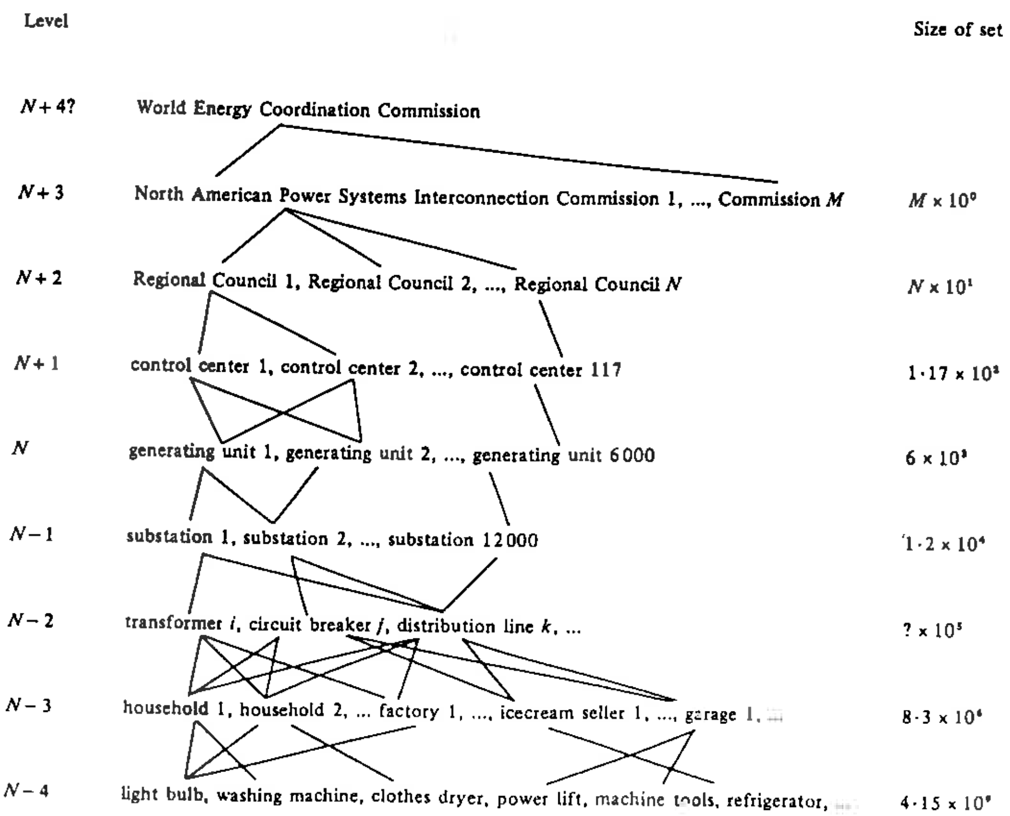


Figure 3 Two hierarchies which are incorrect in strict set-theoretic terms

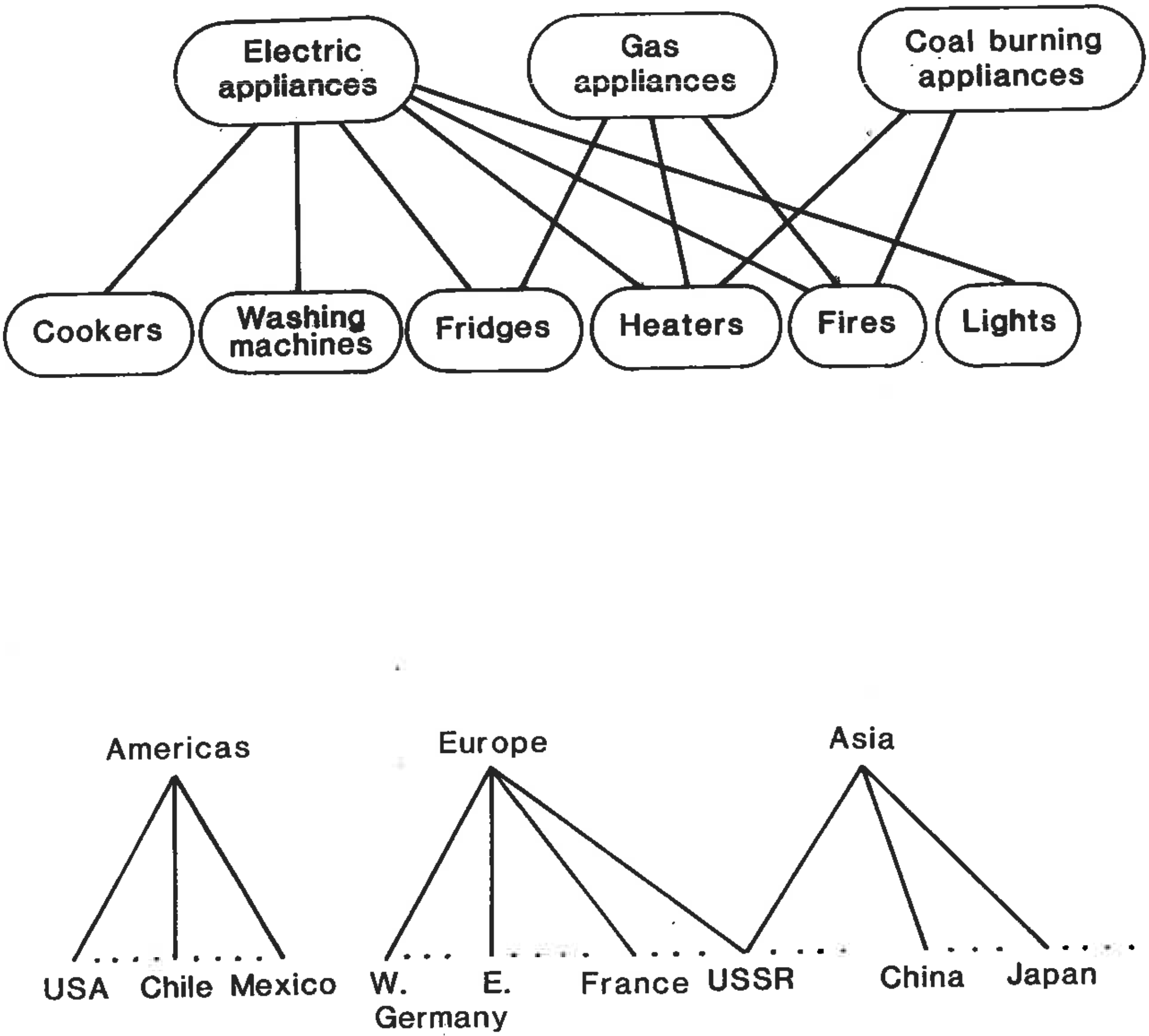


Figure 4a. Tree-type inter-set relations (partitions) between the sets (X,Y) and (A,B,C,D,E)

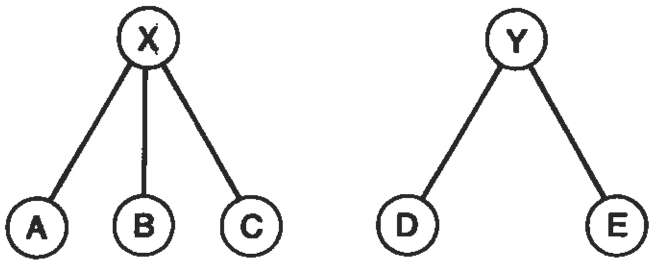


Figure 4b. More general inter-set relations between the sets (X,Y) and (A,B,C,D,E)

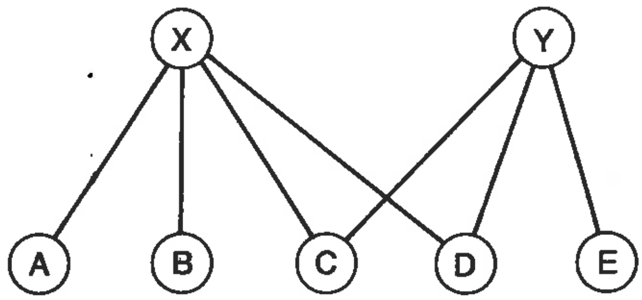


Figure 4c. A latent structure derived from Figure 4b (the two pieces may be joined by glueing at D and C

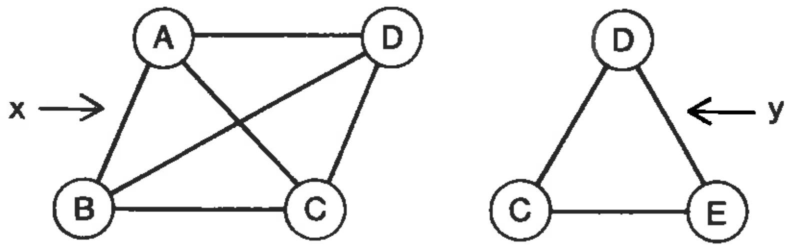
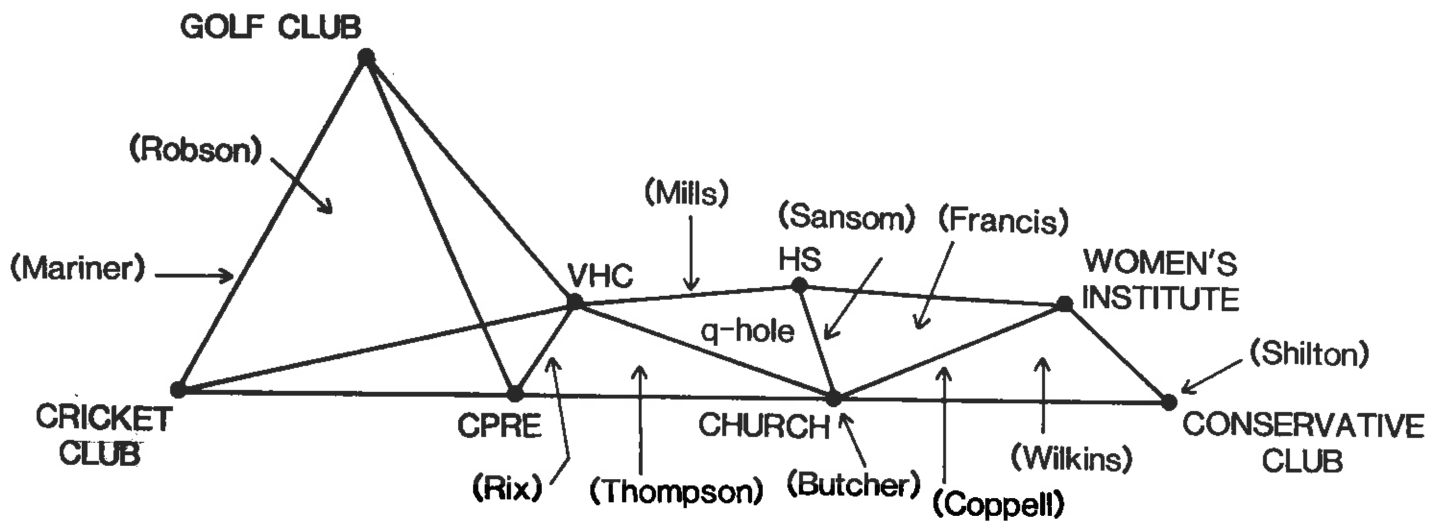


Figure 5. The structure of a social space in rural England (After Galvell 1984)



CPRE Council for the Protection of Rural England
HS Horticultural Society
VHC Village Hall Committee

Figure 6. Tree representation of Q-components from Table 2 data

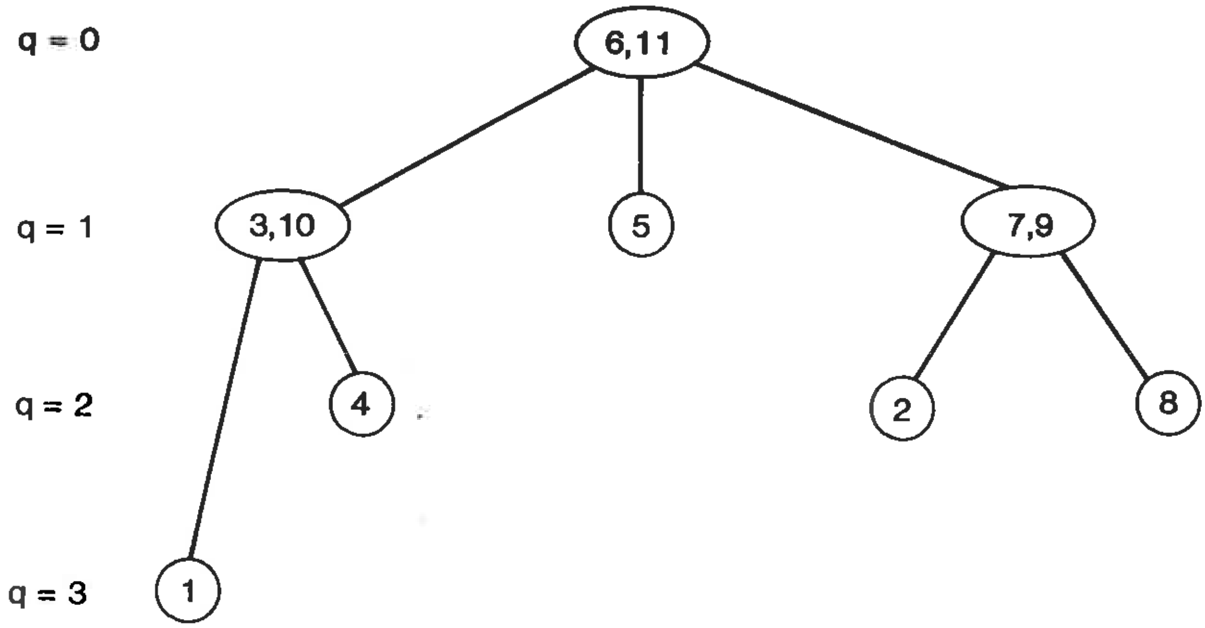
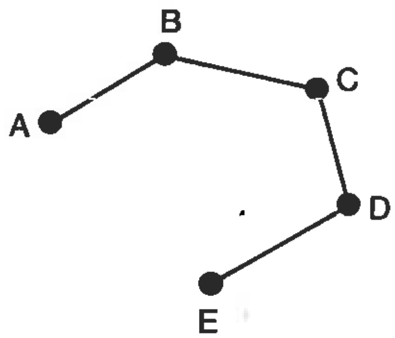


Figure 7. Alternative composition of a simple component (A,B,C,D,E)

7a.



7b.

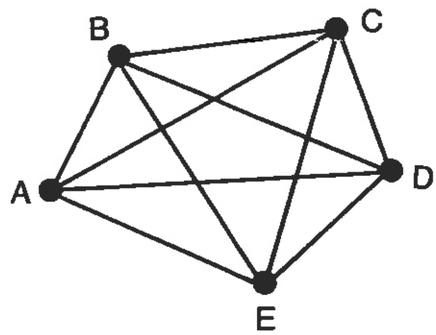
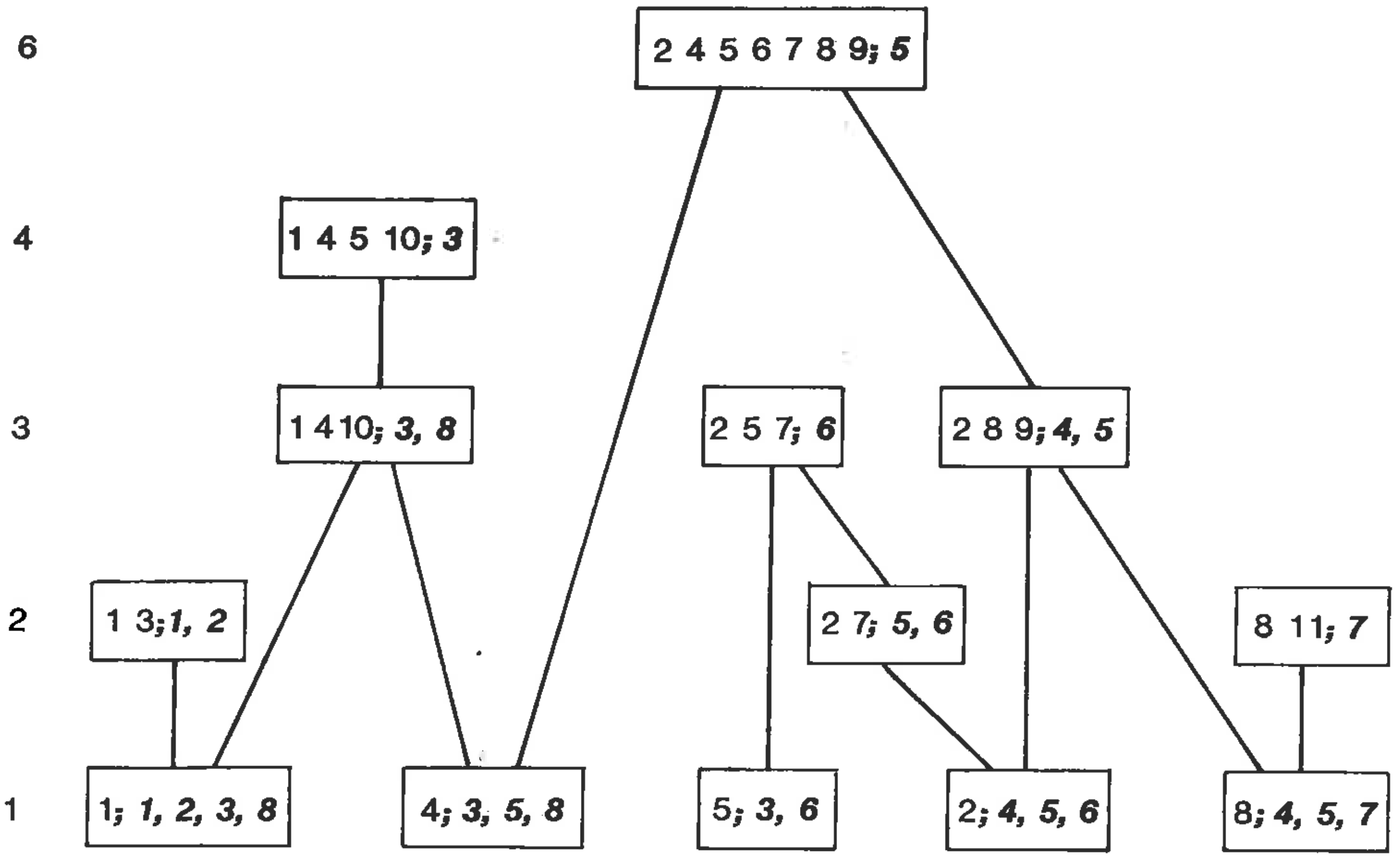


Figure 8. Lattice output from Table 2 data



1	Robson	*1*	Cricket Club
2	Francis	*2*	Golf Club
3	Martiner	*3*	Village Hall Comm.
4	Thompson	*4*	Women's Institute
5	Mills	*5*	Church
6	Butcher	*6*	Horticultural Soc.
7	Sansom	*7*	Conservative Club
8	Wilkins	*8*	CPRE
9	Coppell		
10	Rix		
11	Shilton		

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Appendix

In this appendix, the full range of empirical applications an illustrations of Q-analysis known to the author are cited, systematically identifying which aspects of the methodology have been invoked in each case. The aspects referred to in the column headings can be described more full as follows:

Title and reference: Self explanatory

Sets defined and relations examined: these identify the type and size of the sets defined in each context and which inter-set relations are examined: (key element III)

Set definition considerations: this refers the extent to which consideration of strict set membership rules have had a bearing in particular contexts (key element I)

Hierarchical considerations: this refers to the extent to which consideration of hierarchical relations between sets have had a bearing in particular contexts (key element II)

Slicing: this refers to whether or not the concept of slicing has been invoked to yield a binary relation (or relations) from a weighted relation (or relations). (See element IIIa.)

View of simplicial complex: this refers to whether a pictorial representation of the simplicial complex associated with a binary relation is given (key element IV).

Shared face matrix, top and bottom q, q-nearness graphs, q-components, structure vector, eccentricities: these all refer to various mathematical indicators that can be used to indicate structural properties of interset relations (key element V).

Traffic: various headings to refer to possible ways of invoking the concept of traffic (key element VI), whether in a purely metaphorical sense, or whether, more explicitly, in terms of transmission, t-forces, and q-holes.

Special format for output: to identify whether there are significant innovations in the way the product of a Q-analysis is depicted.

Key ● = prominent
∅ = moderate
0 = some
o = slight

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Mathematical Structure in Human Affairs	Atkin, R.H. (1974)	a) 8 colours 256 geometrical shapes	colour x geometrical shapes	0		●		∅	●		●	●	●	●	∅			
		b) 22 'lozenges' 100 buildings	lozenges x buildings						●		●	●	●	∅				
		c) 18 town activities (A) 7 types of trade (T) 5 types of public administration (P) 5 buildings	A x TP	0	0	●			●		●	●		●	●	●		
		d) 13 university activities 12 university committees 25 people	(i) activities x committees					∅	●		●	●	●	●	●	●	●	
			(ii) people x committees					∅	●		●	●	●	●	●	●	●	
			(iii) committees x committees					∅	●		●	●	●	●	●	●	●	
		e) 64 squares on a chessboard 16 white men 16 black men	(i) squares x white men					0	∅		∅	●	0	∅	∅			
			(ii) squares x black men					0	∅		∅	●	0	∅	∅			
The methodology of Q-analysis. How to study corporations by using concepts of connectivity	Atkin, R.H. (1983)	6 executives 12 responsibilities shared by the executives	executives x responsibilities															
Describing the structure of a research literature: spatial diffusion modelling in Geography	Gatrell, A.C. (1984a, b)	-documents -keywords or terms	documents x keywords or terms	●	●						●	0	∅	o				
A structural language of relations	Gould, P. (1981)	70 grab samples 20 forminifera	grab samples x forminifera		0	●					●		0	o	0	0	0	
Local authorities and energy conservation: the structure of their involvement	Sheldrick, W.S. and Macgill, S.M. (1984)	48 energy conservation activity variables 333 local authorities in England, Scotland and Wal~s	Energy conservation activity variables x local authorities						∅	●	●			o				

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An approach to structure in in architectural and urban design. 3. Illustrative examples.	Atkin, R.H. (1975)	a) 34 lozenge areas (streets) 26 visual features of buildings	(i) 26 visual features x 17 buildings		●	∅			∅		●	●	●	●		●		
			(ii) 26 visual features x 22 buildings		●	∅			∅		●	●	●	●		●		
		b) 21 buildings 121 services at time t_1 96 services at time t_2	Buildings x services at time t_1 and t_2		●				∅		●	●	●	●		●		
		c) 28 committees 200 people 6 catering establishments 4 services 24 houses 45 facilities at times t_1 and t_2	(i) 6 catering establishments x 4 services		●	●					●	●		●		●		
			(ii) 24 houses x 45 facilities at t_1 and t_2		●						∅	∅		●		●	●	
					∅	∅								0		0	0	
			(iii) 200 people x 28 committees		∅	∅								0		0	0	
A comparative study of the multivariate structure of towns	Beaumont, J.R. and Beaumont C.D. (1982)	80 towns 10 socio-economic and political attributes	towns x socio-economic and political attributes			∅			0		●	●	∅					∅
A study of Behcet's disease by Q-analysis	Chamberlein, M.A. (1976)	227 individuals 71 symptons	Individuals x symptons	∅		∅		0	∅		●		∅					
On the evolution of group and network structure	Doreian, P. (1979)	18 individuals 14 events	Individuals x events (added one at a time)				√	∅	●		●			o				
Polyhedrdl dynamics and conflict mobilization in social networks	Doreian, P. (1981)	15 individuals	Individuals x individuals					∅	0		●			0	0			
On the delineation of small group structures	Doreian, P. (1982)	18 individuals 14 events	Individuals x events				√	∅	●		●							
Structure in bungalow plans.	Flemming, U. (1981)	125 constructive descriptions 7 bungalow plans	Constructive descriptions x bungalow plans		0						∅		∅	o				
Q-analysis and the structure of friendship networks	Freeman, L.C. (1981)	29 people 19 linking events between 2 or more people	People x linking events				√		∅	∅	∅			∅			∅	

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Neworks of urban systems analysts	Spooner, R. and Batty, M. (1981)	16 participants 6 factors (affecting the development of systems analysis) 4 fields (defining systems analysis)	(i) participants x fields	0		●	✓				∅		∅	o				●
			(ii) participants x factors	0		●	✓				∅		∅	o				●
			(iii) factors x fields	0		●					∅		∅	o				●
			(iv) participants x (fields and factors)	0		●	✓				∅		∅	o				●
Electrical power failure: reflections on compatible descriptions of human and physical systems	Gould, P. (1981)	Electrical power generation and transmission system		●	●	0								o	o	o		
Q-analysis: some applications in psychometrics	Downs, C.G. (1984)	-psychological attributes -test structure -applications -test tasks -expertise		●	●								o					
A description of structural change in a central place system: a speculation using Q-analysis	Beaumont, J.R. (1984)	24 centres range of goods and services provided in 6 time periods	centres x goods for each time period	0	0	∅			0		●	●*	●	∅	0	∅		0
Q-analysis and social indicator studies	Spooner, R.S. (1980)	17 indices of 'economic disadvantage' 24 local authority districts	indices x districts			∅		∅	∅		●	0	●					
A structural analysis of two farms in Bangladesh	Chapman, G.P. (1984)	139 elements describing Farm 1 120 elements describing Farm 2	(i) 139 elements x 139 elements	●	∅				∅		●		∅	o	o			●
			(ii) 120 elements x 120 elements	●	∅				∅		●		∅	o	o			●

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The changing retail structure in Leeds 1961-1982: initial explorations with the Q-analysis algorithm	Clarke, G.P. and Macgill, S.M. (1984)	130 retail centres and 33 retail activities for 6 time periods	retail centres x retail activities for each time period	0	●				∅	●	●	0	∅					
Exploring the similarities of different risks	Macgill, S.M. (1983)	30 activities or technologies 9 risk descriptions	activities or technologies x risk descriptions	0				∅	●		●		●					
The structure of the teaching of Geography in Cambridge	Chapman, G.P. (1983)	-candidates for examinations in the second and third years -Lecturers -Papers in the three academic years	(i) candidates x papers in two years						●		●	●	●	o				✓
			(ii) lecturers x papers in each of the three years						●		●	●	●	o				✓
			(iii) lecturers x U papers over three years						●		●	●	●	o				✓
			(iv) lecturers x students						●		●	●	●	o				✓
Hierarchies, real and imagined: a Q-analytic note on central places	Chapman, G.P. (1980)	-land areas -central point locations -services	(i) land areas x services	●	●			0	●		●	∅	0					✓
			(ii) central point locations x services	●	●				●		●	∅	0					✓
			(iii) (land areas x central point locations) x (central point locations x services).	●	●				●		●	∅	0					✓
The structure of Television Structure: The world of Television. The structure of Television - Television: The World of Structure	Gould, P.J. Johnson and Chapman, G. (1984)			●	●	o	ϕ	•	o	o	o	o	o	●				
An analysis of urban structure using concepts of algebraic topology	Atkin, R. Johnson, J. and Mancini, V. (1971)	19 activities 4 lozenge areas (streets)	activities x lozenges	0					∅		∅	∅						
A multidimensional approach to positional chess	Atkin, R.H. and Witten, I. (1975) (+ Atkin 1972, Atkin, Hartston, Witten 1976)	64 squares on a chess board 14 white men 14 black men	(i) squares x white men					0	∅		∅	●	0	∅	∅			
			(ii) squares x black men					0	∅		∅	●	0	∅	∅			

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The Cova da Biera: An applied structural analysis of agriculture and communication	Gaspar, J. and Gould, P. (1981)	250 farmers 29 farming characteristics 10 farming activities	(i) Farmers x farming activities V Farming characteristics	0	Ø	Ø	0		●		●	Ø	0	Ø	0	0		
			(ii) Farmers who seek advice x Farmers who give advice	0	Ø	Ø	0		●		●	Ø	0	Ø	0	0		
On the structure of urban social areas: explorations using Q-analysis	Gatrell,A.C. (1981a)	25 community areas 9 occupational groups	community areas x occupational groups	0		●	✓	Ø	●		●	Ø	Ø	o				
On the geometry of man-environment relations with special reference to the tropics: a speculation using Q-analysis	Gatrell,A.C. (1981b)	-geographical locations -soil and soil water variables -plant types and vegetation characteristics -Farmers	(i) Farmers x Farmers		0		✓		0		Ø	0	0	Ø				
			(ii) geographical locations x crops		0		✓		0		Ø	0	0	Ø		Ø		
Geometry in Geography and the geometry of Geography	Gatrell,A.C. (1982)	78 source papers 78 cited papers 13 journals	Source papers x cited papers	0	0			Ø	Ø		●	Ø	Ø	o				
Multidimensional spaces and the structure of relations. Chapter 6 in Distance and Space	Gatrell,A.C. (1983)	a) 8 prey 8 predators	prey x predators			0	✓	Ø	●		●	0	Ø	o				
		b) 7 cities travel time between all pairs	cities x travel time			Ø	✓		0		●	0	0	o				
		c) 34 sectors in U.K. economy	sectors x sectors						0		Ø	0		o				
		d) 26 corporations 49 individuals who are on the boards of at least 2 corporations	corporations x individuals								Ø	0		o				
		e) 11 individuals 8 social activities	individuals x social activities				✓				Ø	0		0				

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A structural analysis of a game: the Liverpool v Manchester United Cup Final of 1977	Gould, P. and Gatrell, A. (1979a,b)	11 Liverpool players (L) 11 Manchester United players (M)	(i) L x L			●	✓		∅		●	0						
			(ii) M x M			●	✓		∅		●	0						
			(iii) No. of times the ball is stolen from a team by the player of the other team			●	✓		●		●	0	0				∅	
Geological similarity by Q-analysis	Griffiths, J.C. (1983)	a) 10 counties and 6 rock types in New Hampshire	counties x rock types				✓	0	∅	0	∅							
		b) 50 states in U.S.A. and Puerto Rico 65 rock types	states x rock types				✓	0	●	●	●		●					
The Q-analysis of road traffic systems	Johnson, J.H. (1981) + other papers	a) 589 routes 135 links	routes x links			∅			0		●	∅	∅	∅	∅	∅		
		b) 4 road junctions links routes	routes x links for each road junction						0		●	●1	●	●	●			
		c) 2 motorway junctions links routes	routes x links for each motorway junction									●1	●	∅	∅	∅		
		d) 2 roundabouts links routes	routes x links for each roundabout									●	●	∅	∅	∅		
		e) zones of land use	trips between zones	●	●				∅		0	●	●	●				
The methodology of Q-analysis in planning and transportation	Johnson, J.H. (1976a)	21 geographical areas (plots of land)			●						●	●		0	∅			
A Q-analysis of periodic markets	Johnson, J.H. and Wanmali, S. (1981)	199 markets	markets x markets	0	0	∅	?		0		●			o	o	o		
A structural investigation into design education	Phillips, E.M. and Johnson, J.H. (1981)	20 children 12 objects 8 constructs	objects x children x constructs	0		●	✓		●	✓	∅	0						

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Q-discrimination analysis	Johnson, J.H. (1981c)	(i) 6 glasses, 7 descriptive features	glasses x descriptive features	0		●	✓		●	✓	∅	0						
		(ii) 12 computer programs, 12 descriptive constructs	computer programs x descriptive constructs	0		●	✓		●	✓	∅	0						
An introduction to the methodology of Q-analysis: the relationship between attitudes to E.C.T. and placebo response	Cowley,P.N. (1985a)	41 patients, 16 attitudes	patients x attitudes						●		●		∅	∅		∅2		
Types of countertransference identified by Q-analysis	Whyte,C.R. Constantopoulos, C. and Bevans, H.G. (1982)	10 patients, 11 nurses with 30 feelings towards each patient	patients x (nurses x feelings)			∅		0	●		●		∅					
Factors associated with the appearance of psychiatric symptons in rheumatoid arthritis	Mindham,R.H.S. Bagshaw, A. James, S.A. & Swannell, A.J. (1981)	28 patients, psychiatric symptoms	patients x symptons			∅			∅		∅	0	0					
Mapping traffic onto a backcloth: a computer programme	Hirst,M.A. (1985)	248 people, 6 disabilities	people x disabilities	∅		0								●		∅		
Lake ecosystems: a polyhedrdl dynamics representation	Casti,J. Kempf, L. Duckenstein and Fogel, M. (1981)	7 environmental inputs and outputs (x), 4 ecosystem components (y)	X x Y	0					●		●	●		o	0	0	0	

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Multidimensional Man Ex1 Language	Atkin, R. (1981)	a) concepts referred to by nouns (N) concepts which provide a general notion of 'lovely' etc (P)	N x P	Ø	Ø		✓		0		Ø	Ø		Ø	Ø	Ø	Ø	
Ex2 Drama		b) play; acts; plots; scenes; characters; sub scenes; speeches	plots x characters	●	●				0		●	0	0	●	●	●	●	
Ex3 Employment Structure		c) 12 local authorities 27 employment categories 10 employment status	local authorities x employment categories	Ø	Ø	Ø			0		●			●	●	●	●	Ø
Classification in medical diagnostics; on some limitations of Q-analysis	Pinkava, V. (1981)	40 individuals 9 features	Individuals x features	0		0			Ø		Ø	0					0	
Combinatorial connectivities in social systems	Atkin, R.H. (1977)	-Visual features (of the fabric of the university) -social amenities -housing types -departments -committees -non-academic staff grades		●	●	Ø			●		●	●	●	●	●	●	●	
An investigation of patients attitudes to E.C.T. by means of Q-analysis	Cowley, P.N. (1985b)	96 patients ("trial-acceptors") 23 patients ("trial-refusers") 16 attitudes	"trial-acceptors" x attitudes "trial-refusers" x attitudes						● ●		● ●		Ø Ø	Ø Ø				
Derivation of clinical syndromes by the application of Q-analysis	Cowley, P.N. (1985c)	139 patients 65 symptoms	patients x symptoms						●	●3	●		Ø	Ø		Ø		

? Cannot be determined from the text
1 flip-over value
2 q-shift
3 q-factor