Working Paper 229

PROBLEMS OF MULTIREGIONAL POPULATION ANALYSIS: DATA COLLECTION AND DEMOGRAPHIC ACCOUNTING

P. H. Rees

Paper prepared for the IIASA conference on the "Analysis of multiregional population systems: techniques and applications", September 18-22,1978 Schloss Laxenburg, Laxenburg, Austria.

School of Geography
University of Leeds
Leeds LS2 9JT

September, 1978

Abstract

The paper presents a summary of the concepts and methods of demographic accounting. Illustraions are drawn from a recently constructed set of accounts for the East Anglia and South East regions of Britain. A simpler version of the model used to construct age classified accounts is described. The framework of multiregional population accounting is compared with that of multiregional population analysis as developed by Rogers and others.

List of figures and tables

Figures

Tables

1. Introduction

The analysis of multiregional population systems requires the assembly of a variety of data sets on regional populations, regional births, regional deaths, interregional migration and external migration to and from the regions. From the data assembled probability matrices of two kinds are derived: the first, called here the survival probability matrix, $\underline{P}$, measures the chances of making region to region transitions (and surviving) between exact ages: the second, called here the survivorship proportions matrix, $\underline{S}$, computes the chances of making region to region transitions (and surviving) between discrete age groups.

This paper outlines how $\underline{P}$ and $\underline{S}$ matrices may be estimated by first organizing the relevant input data into population accounts. Direct estimation of the $\underline{S}$ matrix from population accounts is possible, and the $\underline{P}$ matrix can be interpolated from the $\underline{S}$ matrix. This approach is compared with that of Rogers(1975) and Willekens and Rogers(1976) in which the $\underline{P}$ matrix is estimated from a specially arranged migration and death rates matrix, $\underline{M}$; the $\underline{S}$ matrix is subsequently derived from the $\underline{P}$ matrix. Points of similarity between the approaches are noted, and points of difference.

2. Population accounts defined and illustrated

2.1 Aggregate, age disaggregated and cohort accounts

A full presentation of the concepts and methods involved in multiregional population accounting is given in Rees and Wilson (1977). Here the essentials are described in a slightly extended notation, and a simpler version of the age-sex disaggregated accounts based model is suggested.

Population accounts come in various degrees of aggregation. The aggregate accounts are described in Part 2 of Rees and Wilson (1977). Age disaggregated accounts allow for a double classification of population terms by age, necessary in order to relate the various age disaggregations in which input data comes, and are described in Part 3 of "Spatial population analysis". Intermediate between the aggregate and doubly age classified accounts are what are here called "cohort" accounts, involving only one age classification, and a form of the accounts based model closer to that for the aggregate population than to that for the age disaggregated case.

2.2 The accounts matrix

The classifications which go to make up the states employed in accounts matrices are as follows. A list of the general and particular subscripts used is included.

(1) Sexes: $X = M, F$

X is a general label for sex, M stands for male and F for female.

(2) Life states: $\alpha = \beta, \epsilon$; $\omega = \delta, \sigma$

α is a general label for initial life state, β stands for birth and ϵ for existence. ω is a general label for final life state, δ stands for death and σ for survival.

(3) Regions: $i = 1, 2, \ldots, N$

i is the general label for the initial region or location, of which there are N. It is convenient to have the same set of regions for i and j and essential in constructing the accounts to ensure that the N regions partition the whole world. Usually, this will mean that the Nth region is a rest-of-the-world area covering all places outside the regional set of interest.

(4) Age groups: $r = 1, 2, \ldots, R$; $s = 1, 2, \ldots, R$

r is the general label for initial age groups which are numbered from 1 to R, the last. Age groups can also be defined by the exact age span they refer to in one of two ways: either in terms of exact age at last birthday or in terms of exact age at the limits of the age group. In the former case, age group r would span $x(r)$ to $x(r)+T-1$ exact years; in the latter case, age group r would span ages $x(r)$ to $x(r)+T$, where T is the interval in years between the beginning and end of the age group. We use x to refer to exact age. s is the general label for final or end of period age groups which are numbered from 1 to R, the last. It is convenient to use the same set as for the initial age groups, and to have the age group interval T equal for all age groups, except the last, which will stretch from exact age $x(R)$ to $x(\Omega)$, the oldest age of life.

(5) <u>Time</u>

t is the initial point in time that starts a period. T is the length of the period in years. t+T is the point in time that ends a period. t, t+T when combined, define the limits of a period. These labels are used only when the time to which the variables refer is unclear.

Since, in general, no transitions occur between the sexes, it is customary to treat the sexes separately with the exception that the population at risk of giving birth to male infants is usually taken to be that of women. We can construct quite separate $\underline{K}^{M}$ and $\underline{K}^{F}$ accounts matrices for males and females respectively. For the moment we will drop the sex label: all points apply to both male and female accounts, and to the combined accounts for persons.

The accounts matrix, $\underline{K}$, consists of 4 submatrices:

$$\underline{K} = \left[\begin{array}{c|c} \underline{K}_{ES} & \underline{K}_{ED} \\ \hline \underline{K}_{BS} & \underline{K}_{BD} \end{array}\right] \qquad (1)$$

where E refers to existence, S to survival, D to death and B to birth.

The typical elements of the submatrices involved in the accounts are as follows:

$$\underline{K} = \left[\begin{array}{c|c} \left\{ K_{rs}^{\epsilon(i)\sigma(j)} \right\} & \left\{ K_{rs}^{\epsilon(i)\delta(j)} \right\} \\ \hline \left\{ K_{rs}^{\beta(i)\sigma(j)} \right\} & \left\{ K_{rs}^{\beta(i)\delta(j)} \right\} \end{array}\right] \qquad (2)$$

where $K_{rs}^{\epsilon(i)\sigma(j)}$ are persons in existence in region i at the start of the period in age group r who survive in region j in age group s at the end of the period. $K_{rs}^{\epsilon(i)\delta(j)}$ are persons alive in egion i at the start of the period in age group r who subsequently die in region j in age group s. $K_{rs}^{\beta(i)\sigma(j)}$ are the infants born in region i to mothers in age group r at time of birth. The infants subsequently survive in region j at the end of the period in age group s, usually the first. $K_{rs}^{\beta(i)\delta(j)}$ are the infants born in the time interval in region i, to mothers in age group r at time of maternity. The infants subsequently die in region j in age group s, usually the first.

The number of non-zero age group to age group transitions will vary with the relation of the age group interval to the length of the time period. With all age group intervals equal (bar the last) and equal to the time period length, only age group r to r+1 or s-1 to s transitions are non-zero in the $\underline{K}_{ES}$ submatrix (except the R, R transition). Only r to r and r to r+1 transitions are allowed in the $\underline{K}_{ED}$ submatrix. Only r to 1 transitions for r spanning the fertile age range from age group α to β are non-zero in the $\underline{K}_{BS}$ and $\underline{K}_{BD}$ transitions.

Figure 1 shows the structure of the accounts submatrices for a multiregional system of interest. The general structure of the $\underline{K}_{ES}$, $\underline{K}_{ED}$, $\underline{K}_{BS}$ and $\underline{K}_{BD}$ submatrices is shown in the first in each pair of matrices, and the detail within each $\underline{K}^{\epsilon(i)\sigma(j)}$, $\underline{K}^{\epsilon(i)\delta(j)}$, $\underline{K}^{\beta(i)\sigma(j)}$ and $\underline{K}^{\beta(i)\delta(j)}$ submatrix is shown in the second of each pair in Figure 1. Most of the elements in these latter submatrices are zero.

2.3 Examples of accounts matrices

The full interlocking structure of a multiregional accounts matrix is difficult to present in full. The figures in Chapter 13 of Rees and Wilson (1977) attempt this and Figure 13.32 in that book presents a full accounts table for the West Riding of Yorkshire, 1961-66, for males. This is reproduced as Figure 2.

A more compact illustration in which the diagonals of the full accounts matrix have been presented as columns in a normal table is given in Appendix 1 for a four region system consisting of the standard regions of East Anglia and the South East, the Rest of Britain and the Rest of the World. Appendix 1 is derived from the two sex accounts described in Rees (1977). The death submatrices have been simplified in particular. In the $\underline{K}^{\epsilon(i)\delta(j)}$ submatrix shown in Figure 1 deaths are cross-classified by initial age group in a

Figure 1. The structure of the submatrices making up the accounts

Exist-survive submatrices

$$\underline{K}_{ES} = \begin{bmatrix} \underline{K}^{\epsilon(1)\sigma(1)} & \underline{K}^{\epsilon(1)\sigma(2)} & \cdots & \underline{K}^{\epsilon(1)\sigma(N)} \\ \underline{K}^{\epsilon(2)\sigma(1)} & \underline{K}^{\epsilon(2)\sigma(2)} & \cdots & \underline{K}^{\epsilon(2)\sigma(N)} \\ \vdots & \vdots & & \vdots \\ \underline{K}^{\epsilon(N)\sigma(1)} & \underline{K}^{\epsilon(N)\sigma(2)} & \cdots & \underline{K}^{\epsilon(N)\sigma(N)} \end{bmatrix}$$

$$\underline{K}^{\epsilon(i)\sigma(j)} = \begin{bmatrix} 0 & \underline{K}_{12}^{\epsilon(i)\sigma(j)} & 0 & \cdots & 0 \\ 0 & 0 & \underline{K}_{23}^{\epsilon(i)\sigma(j)} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & \underline{K}_{R-1R}^{\epsilon(i)\sigma(j)} \\ 0 & 0 & 0 & \cdots & \underline{K}_{RR}^{\epsilon(i)\sigma(j)} \end{bmatrix}$$

Exist -die submatrices

$$\underline{K}_{ED} = \begin{bmatrix} \underline{K}^{\epsilon(1)\delta(1)} & \underline{K}^{\epsilon(1)\delta(2)} & \cdots & \underline{K}^{\epsilon(1)\delta(N)} \\ \underline{K}^{\epsilon(2)\delta(1)} & \underline{K}^{\epsilon(2)\delta(2)} & \cdots & \underline{K}^{\epsilon(2)\delta(N)} \\ \vdots & \vdots & & \vdots \\ \underline{K}^{\epsilon(N)\delta(1)} & \underline{K}^{\epsilon(N)\delta(2)} & \cdots & \underline{K}^{\epsilon(N)\delta(N)} \end{bmatrix}$$

$$\underline{K}^{\epsilon(i)\delta(j)} = \begin{bmatrix} \underline{K}_{11}^{\epsilon(i)\delta(j)} & \underline{K}_{12}^{\epsilon(i)\delta(j)} & 0 & \cdots & 0 \\ 0 & \underline{K}_{22}^{\epsilon(i)\delta(j)} & \underline{K}_{23}^{\epsilon(i)\delta(j)} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & \underline{K}_{R-1R}^{\epsilon(i)\delta(j)} \\ 0 & 0 & 0 & \cdots & \underline{K}_{RR}^{\epsilon(i)\delta(j)} \end{bmatrix}$$

Figure 1. Continued

Born-survive submatrices

$$
\underline{K}_{BS} = \begin{bmatrix}
\underline{K}^{\beta(1)\sigma(1)} & \underline{K}^{\beta(1)\sigma(2)} & \cdots & \underline{K}^{\beta(1)\sigma(N)} \\
\underline{K}^{\beta(2)\sigma(1)} & \underline{K}^{\beta(2)\sigma(2)} & \cdots & \underline{K}^{\beta(2)\sigma(N)} \\
\vdots & \vdots & & \vdots \\
\underline{K}^{\beta(N)\sigma(1)} & \underline{K}^{\beta(N)\sigma(2)} & \cdots & \underline{K}^{\beta(N)\sigma(N)}
\end{bmatrix}
$$

$$
\underline{K}^{\beta(i)\sigma(j)} = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
K_{\alpha 1}^{\beta(i)\sigma(j)} & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
K_{\beta 1}^{\beta(i)\sigma(j)} & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
0 & 0 & \cdots & 0
\end{bmatrix}
$$

Born-die submatrices

$$
\underline{K}_{BD} = \begin{bmatrix}
\underline{K}^{\beta(1)\delta(1)} & \underline{K}^{\beta(1)\delta(2)} & \cdots & \underline{K}^{\beta(1)\delta(N)} \\
\underline{K}^{\beta(2)\delta(1)} & \underline{K}^{\beta(2)\delta(2)} & \cdots & \underline{K}^{\beta(2)\delta(N)} \\
\vdots & \vdots & & \vdots \\
\underline{K}^{\beta(N)\delta(1)} & \underline{K}^{\beta(N)\delta(2)} & \cdots & \underline{K}^{\beta(N)\delta(N)}
\end{bmatrix}
$$

$$
\underline{K}^{\beta(i)\delta(j)} = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
K_{\alpha 1}^{\beta(i)\delta(j)} & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
K_{\beta 1}^{\beta(i)\delta(j)} & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots \\
0 & 0 & \cdots & 0
\end{bmatrix}
$$

Figure 13.32 An accounts table for the West Riding of Yorkshire 1961–6 for males

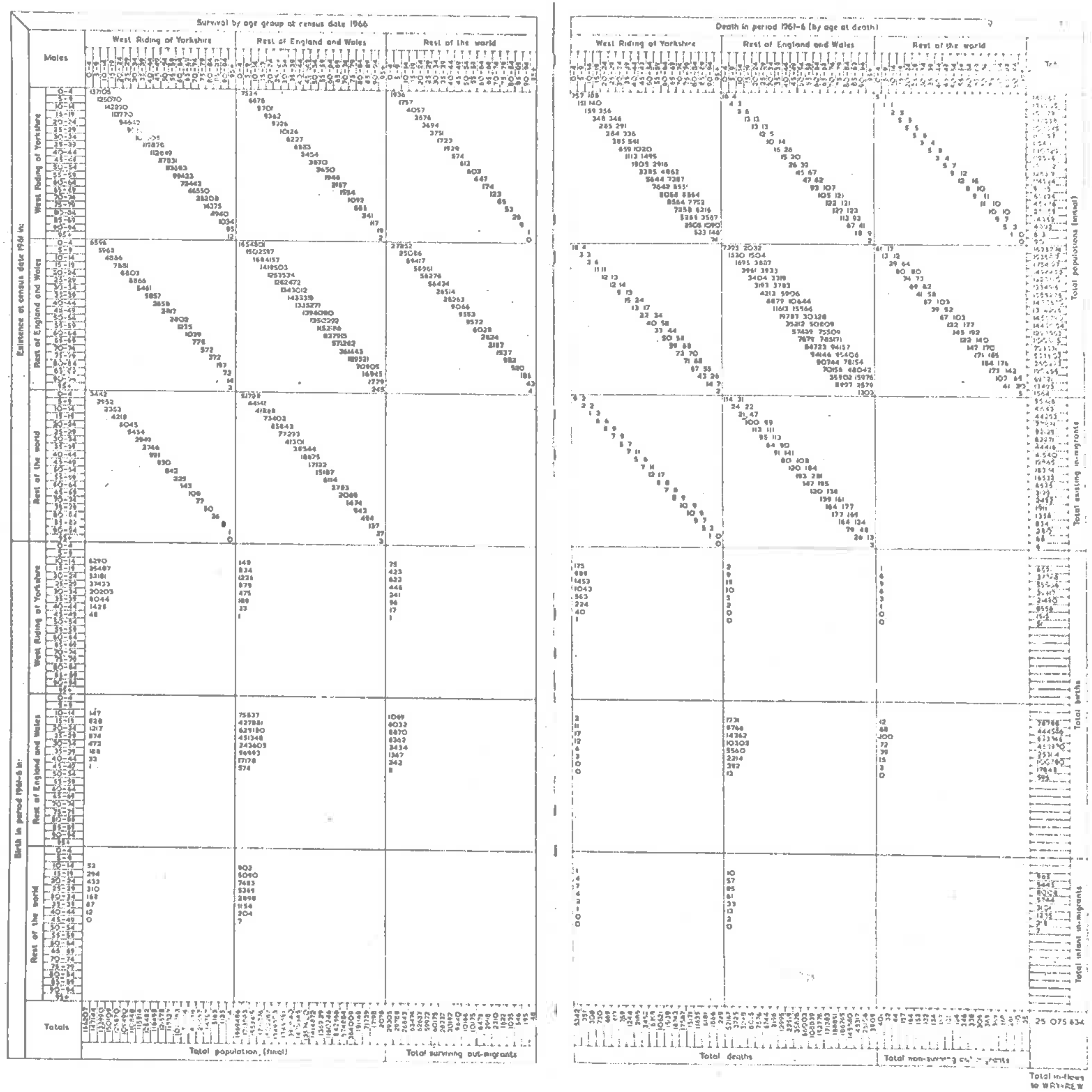

Figure 2

(From Rees and Wilson, 1977)

period and age group at death. Hence the two terms in each row of a $\underline{K}^{\epsilon(i)\delta(j)}$ submatrix. Terms in rows have been summed to yield only one term in the accounts listed in Appendix 1. Table 1 shows a portion of the $\underline{K}_{ED}$ submatrix involving persons originating in and dying in East Anglia. Adoption of this cohort or single age classification leads to considerable simplification in the accounts based model used to estimate the accounts matrix.

The single age classification also makes possible the presentation of accounts for single cohorts at a time, and in Appendix 2 the four region accounts are arranged in the form of 17 cohort tables. The first table refers to persons born in the period and this can be further broken down into the 8 components corresponding to the different ages of the mothers of the infants born in the period 1966-71. Table 2 picks out one of the cohort accounts tables from Appendix 2, that for the cohort that is aged 20-24 at census date 1966 and, given survival, is aged 25-29 at census date 1971, five years later.

How much difference does the accounts framework make to the estimation of death and of survival probabilities? A death probabilility for one of the internal regions is easily calculated as

$$h_{r*}^{\epsilon(i)\delta(*)} = K_{r*}^{\epsilon(i)\delta(*)} / K_{r*}^{\epsilon(i)*(*)} \qquad (3),$$

where $h_{r*}^{\epsilon(i)\delta(*)}$ is a transition rate applying to persons originating in region i in age group r who die, somewhere, in the next period, $K_{r*}^{\epsilon(i)\delta(*)}$ are the relevant deaths formed by summing the i,r row of the $\underline{K}_{ED}$ submatrix, and $K_{r*}^{\epsilon(i)*(*)}$ is the initial population of age group r in region i, the accounts row total.

A conventional death rate, $M_r^{(i)}$, would be calculated using the column total of regional deaths, $K_{r*}^{(*)\delta(\bar{i})}$, and an average population at risk:

$$M_r^{\delta(i)} = K_r^{\epsilon(*)\delta(i)} / \tfrac{1}{2} \left(K_{r*}^{\epsilon(i)*(*)} + K_{*r+1}^{*(*)\sigma(i)} \right) \qquad (4)$$

and from this an estimate would be made of the death probability:

Table 1. A portion of the K_{ED} submatrix involving persons originating in and dying in East Anglia, 1966-71

Initial age group	Age group at death 0- 4	5- 9	10-14	15-19	20-24	25-29	30-34	Totals
0- 4	367	109						476
5- 9		94	77					171
10-14			87	165				252
15-19				214	219			433
20-24					128	146		334
25-29						144	168	312
Totals	-	203	164	379	407	290	361	-

Initial age group	Age group at death 30-34	35-39	40-44	45-49	50-54	55-59	60-64	Totals
30-34	193	232						425
35-39		302	444					746
40-44			539	802				1341
45-49				929	1137			2065
50-54					1471	2105		3576
55-59						2555	3351	5906
60-64							3923	8936
Totals	361	534	983	1731	2608	4660	7274	-

Table 3. Alternative mortality measures

Region	Death probability estimates: Equation (3) Direct transition rate	Equation (4) Conventional rate	Equation (5) Pseudo-life table rate
East Anglia	.003683	.003694	.003687
South East	.003412	.003405	.003399
Rest of Britain	.003689	.003699	.003692

Initial states \ Final states		Survival at c.d. 1971 Age 25–29				Death in 1966–71 Age 20–24 or 25–29				Totals
		East Anglia	South East	Rest of Britain	Rest of World	East Anglia	South East	Rest of Britain	Rest of World	
Existence at c.d. 1966 Age 20–24	East Anglia	79620	7578	8141	5296	334	13	15	10	101007
	South East	14408	984059	79740	101785	25	3691	148	175	1184032
	Rest of Britain	9440	84510	2074448	107473	18	145	8056	208	2284298
	Rest of World	7790	88045	70123	0	14	149	134	0	166255
Totals		111258	1164192	2232452	214554	392	3998	8353	393	3735592

Table 2. <u>An accounts table for the 20–24 cohort</u>

$$q_r^i \quad = \quad 1 - (1 - \tfrac{1}{2}M_r^{\delta(i)}) / (1 + \tfrac{1}{2}M_r^{\delta(i)}) \qquad (5).$$

These three alternative mortality measures are computed from Table 2's data and are compared in Table 3. As noted in Rees (1978a) there is relatively little empirical difference between the methods, although at the extremes of migration only the accounts method guarantees non-negativity of the rates.

2.4 Survivorship rates

Survivorship rates can be defined straightforwardly from accounts matrices

$$\underline{S}(x) \quad = \quad \underline{H}(r_x) \qquad (6)$$

where

$$\underline{H}(r_x) \quad = \quad \tilde{\underline{K}}_E(r_x)\, \underline{K}_{ES}(r_x) \qquad (7).$$

The r_x refers to a typical age group r starting with exact age x. $\underline{H}(r_x)$ is a matrix of transition rates for the cohort starting the period in age group r_x and ending in r_x+1; $\tilde{\underline{K}}_E(r_x)$ is a matrix with terms $(1 / K_{r_x}^{e(i)*(*)})$ in the principal diagonal and zeroes elsewhere; $\underline{K}_{ES}(r_x)$ is the exist-survive submatrix for the r_x cohort. Survivorship rates for the 15-19, 20-24 and 25-29 cohorts are displayed in Table 4. The survivorship rates for the Rest of the World are rather different from the others since they apply to a migrant flow rather than a population stock.

2.5 Survival probabilities

Survival probabilities can then be generated from the survivorship rates by one of a number of procedures. The method suggested in Rees and Wilson (1977) is simple interpolation. Figure 3 shows the logic behind interpolation based on methods used frequently in Pressat (1971). We wish to estimate the probabilities associated with average lifetimes like AB in

Table 4. Survivorship rate, 15-19, 20-24 and 25-29 cohorts

Cohort and initial region	Region of survival: East Anglia	South East	Rest of Britain	Rest of World
15-19				
East Anglia	.818485	.077601	.060175	.039866
South East	.009617	.862148	.054772	.070161
Rest of Britain	.003944	.043799	.913967	.034597
Rest of World	.049491	.589268	.359594	.000000
20-24				
East Anglia	.788262	.075025	.080598	.052432
South East	.012169	.831108	.067346	.085965
Rest of Britain	.004133	.036996	.908134	.047049
Rest of World	.046856	.529578	.421780	.000000
25-29				
East Anglia	.840543	.054557	.061356	.039644
South East	.004193	.856055	.054061	.076689
Rest of Britain	.003416	.024829	.925713	.041771
Rest of World	.054820	.472850	.470247	.000000

Table 5. Survival probabilities, 20 and 25 exact ages

Exact age and initial region	Region of survival: East Anglia	South East	Rest of Britain	Rest of World
20				
East Anglia	.803374	.076313	.070387	.046149
South East	.010893	.846628	.061059	.078063
Rest of Britain	.004039	.040398	.911051	.040823
25				
East Anglia	.814453	.064791	.070977	.046038
South East	.010681	.843582	.060704	.081327
Rest of Britain	.003775	.030913	.916924	.044410

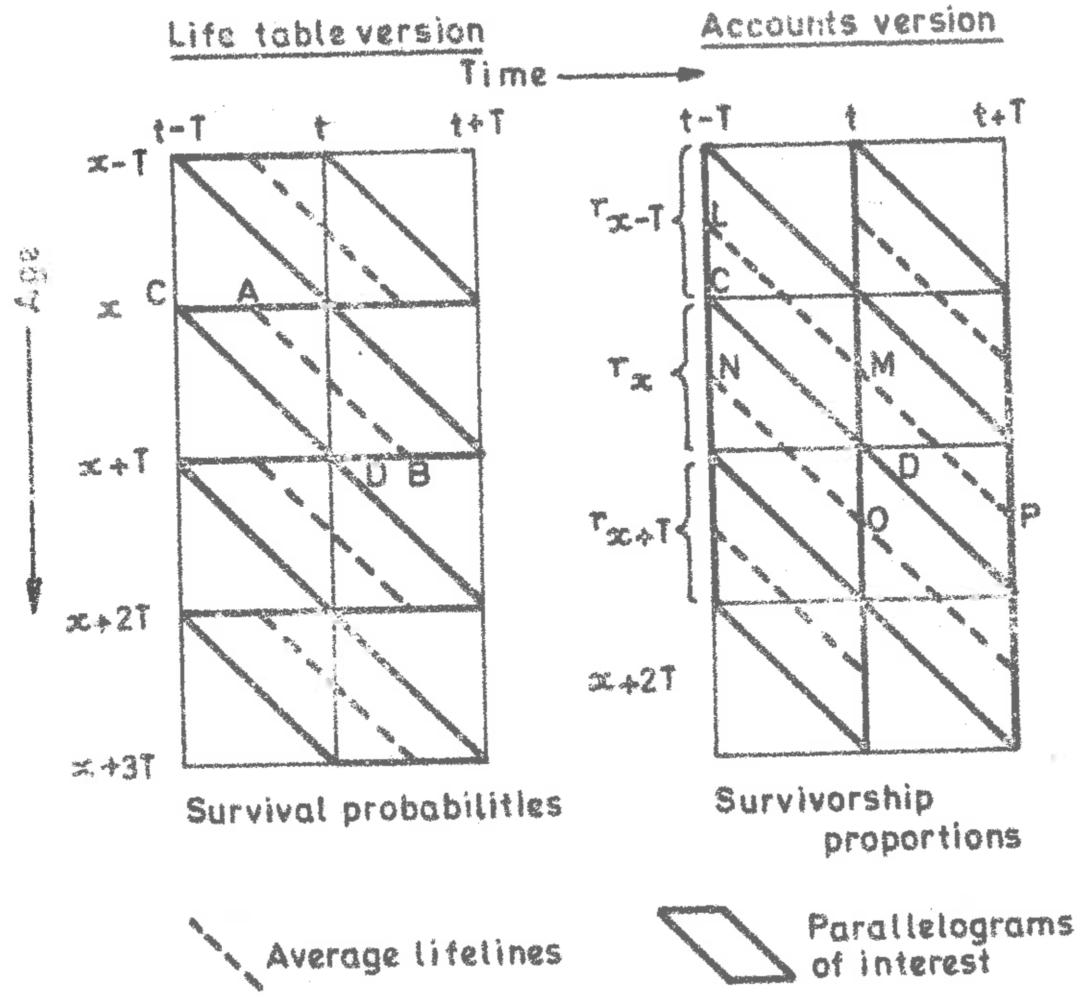

Figure 3. A Lexis diagram showing the method of interpolating survival probabilities from survivorship proportions

the life table version of the Lexis diagram. Ideally, we should interpolate between survivorship proportions from two periods, say between lifelines LM and MP. In practice, we are restricted to one period and interpolate between LM and NO to yield an estimated probability for CD which we assume is equal to the probability for AB. Table 5 shows survival probabilities linearly interpolated between those of Table 4. The method gives rather approximate results for the first age transition and for the oldest ones (see Chapter 16 of Rees and Wilson, 1977).

Alternatively, an "Option 2" method, as described by Rogers (1975) or as improved by Ledent (1978) could be used, though again with approximate results for the first and last age transitions.

2.6 Building blocks for the accounts

To date in this section of the paper, population accounts have been defined and illustrated, and the various rates and probabilities used in population projection and life table construction have been extracted. Something now needs to be said about how data is assembled for the accounts matrix and how the full accounts are estimated using an accounts based model.

Figure 4 classifies the various sections of the accounts table for a typical age group, and for infants (persons born in a period) there exists a similar classification with births substituting for population stocks. The capital letter designation applying to a cell in the table indicates which class of accounts element it belongs to: the shading category indicates the way in which it is estimated. The classes of accounting element are considered briefly in turn. More detail in connection with particular case studies is given in Rees, Smith and King (1977) and in Rees (1977). Some of the subsequent remarks apply only to the British data situation, although equivalent data should be widely available.

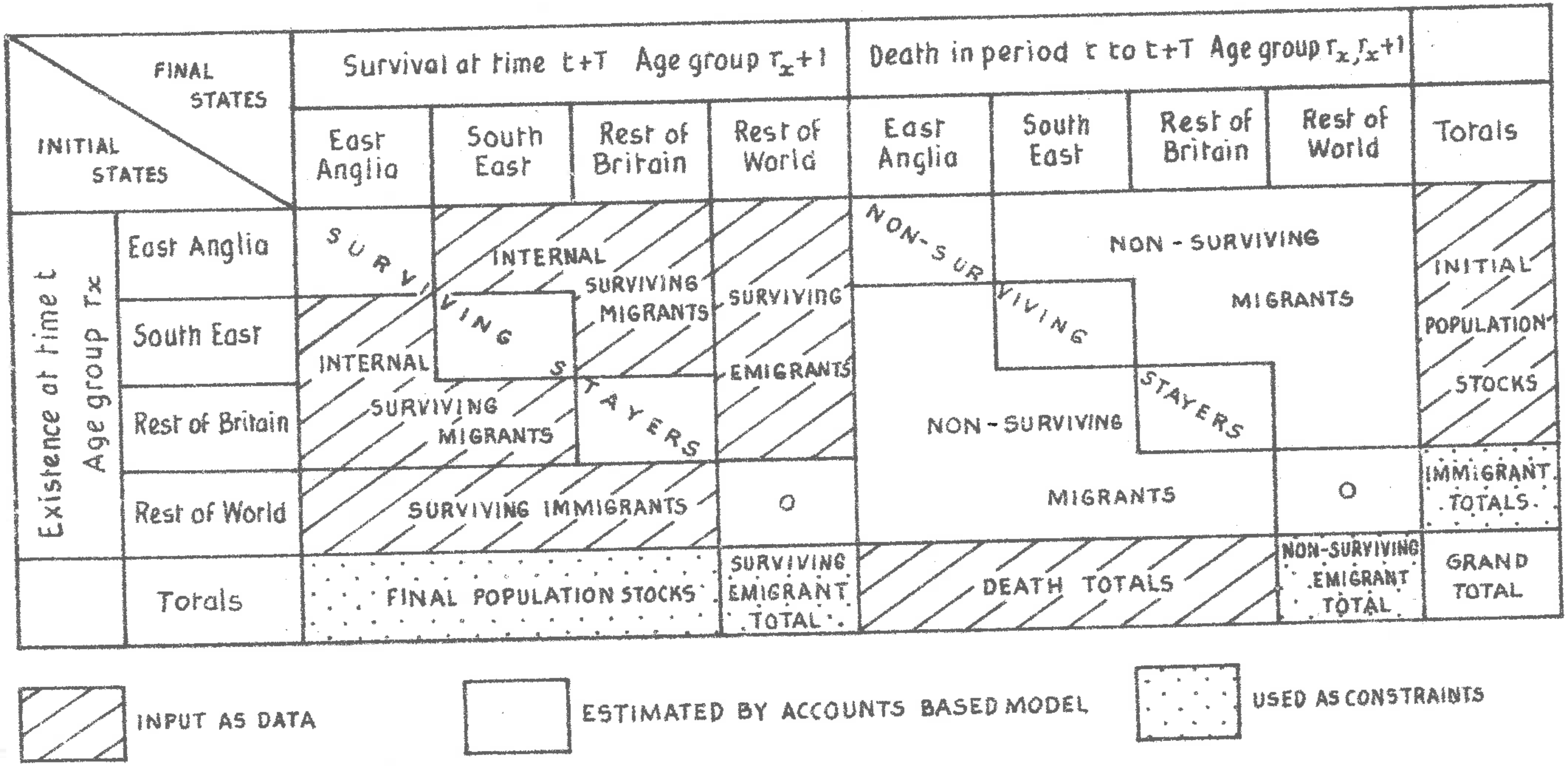

Figure 4. A classification of the elements of an accounts table

2.6.1 Initial population stocks

These are readily available in periodic censuses or in annual estimates, disaggregated by five year age groups (and often by single year age groups). The usually resident population is the most appropriate one to choose from those available in the census tables; the home population concept is the one to employ from the estimates, if only because total population is not given for regions, perhaps for reasons of national security. The initial population stocks are used directly in the model except in the rare case where a back-casting version of the accounts based model must be applied (Thomas and Rees, 1978).

2.6.2 Internal surviving migrants

If available, retrospective data from census migration tables should be used as the definitions involved fit the concepts required in the accounts exactly, although frequently a variety of balancing factor methods must be used to estimate the required age disaggregated interregional flows (Rees, 1978b; Willekens, 1978). If mobility data only are available (giving moves rather than movers or migrations rather than migrants) then some assumptions must be made about the number of "surplus" moves made and a modified form of the accounts based model adopted.

2.6.3 External surviving immigrants

Data on immigrants from outside the country of interest to a region within it are usually available from the same census tables that contain the internal migrant data. International migrations are also counted on a continuing basis and these moves data may be used in the absence of census tabulations. However, in the case of Great Britain the two sources fail to agree very closely even for a period as short as a year.

2.6.4 External surviving emigrants

No tables for emigrants exist in the censuses of the sending country, and counts of international migrations must be used. Corrections must be made, however, if census data have been used for immigration from abroad.

2.6.5 Deaths

Deaths in a region classified by age are available from vital registration sources although some further deconsolidation is often required, for example, from ten year age groups to five year.

2.6.6 Surviving stayers, non-surviving stayers and non-surviving migrants

These are items which are estimated by the accounts based model.

2.6.7 Final population stocks

These data may be used within certain versions of the accounts based model, for example, to estimate the numbers of surviving stayers. Or the end of period populations may be used, along with surviving emigrant totals, non-surviving emigrant totals, immigrant totals, and initial population stocks as row and column constraints to which initial estimates of the accounts matrix are balanced.

2.6.8 Equivalent data for the "infant" accounts

Equivalent data should exist in the same kind of sources for the accounts table involving infants (the first in Appendix 2). Births take the place of initial population stocks and derive from vital registration sources. Infants deaths figures can be estimated from the normal infant mortality figures as can infant immigration and emigration figures. The final population stocks are in this case the populations of the regions in the first age group and they play the same role as in the "non-infant" accounts. Difficult terms to estimate are, however, the internal infant migrants. Despite many lobbying efforts by British population researchers the Office of Population Censuses and Surveys (England and Wales) has yet to acknowledge that infant migrants should be counted and tabulated in the regular census migration tables. Resort must be made to estimating equations that are far from satisfactory.

2.7 The accounts based model

This model is used to estimate the missing items in the accounts matrix. The model is said to be run, in that context, in "historical" mode. The accounts based model can also be used, with relatively minor modification, as a projection model, though the results differ only marginally from those obtained using the accounts matrix to generate survivorship rates (as in section 2.4) for use in a multiregional cohort survival model.

The accounts based model is described in Rees and Wilson (1977) at two levels of aggregation: all age and sex, and disaggregated by age (doubly classified) and sex. Here an intermediate version for cohorts (single age classification) is described. The assumption is made that all input data have been converted to the required cohort form and apply to the accounts version parallelograms shown in Figure 3's Lexis diagram. The age group interval is assumed equal to the time period length. The model is descibed in a series of steps, which are as follows:

Step 1: assemble known input data
Step 2: calculate populations at risk of death and death rates
Step 3: calculate the unknown minor flows
Step 4: calculate the unknown major flows and final populations
Steps 2-4 are iterated until convergence is achieved.
Step 5: consistent row and column constraints are derived
Step 6: the accounts matrix is adjusted to satisfy the constraints.

The notation used is as described earlier in section 2.2.

2.7.1 Step 1: assemble known data

Assemble the variables

$K_{r*}^{\epsilon(i)*(*)}$, initial population stocks, for regions i within the country

$K_{r*}^{*(*)\delta(i)}$, deaths, for regions i within the country

$K_{r*}^{\epsilon(i)\sigma(j)}$, internal and external migrants, for all regions i,j

for age cohorts $r = 1, 2, \ldots, R$ and the variables

$K_{r*}^{\beta(i)*(*)}$, births classified by cohort of mother, for regions within the country

$K_{r*}^{\beta(*)\delta(i)}$, infant deaths classified by cohort r of mother, for regions i of death within the country

$K_{r*}^{\beta(i)\sigma(j)}$, infant internal and external migrants, for all regions i,j

for age cohorts $r = \alpha, \ldots, \beta$ that represent the cohorts spanning the reproductive ages.

2.7.2 Step 2: calculate populations at risk of death

The at risk population is designated $\hat{K}_{r*}^{D\epsilon(*)i}$ to signify that it refers to death (D) in region i (at death) of persons in cohort r.

This is in general, for all cohorts $r = 1, 2, \ldots, R$

$$\hat{K}_{r*}^{D\epsilon(*)i} = \sum_{k,j} {}_{r}^{i}\Theta_{r*}^{D\epsilon(k)\sigma(j)} K_{r*}^{\epsilon(k)\sigma(j)} + \sum_{k,j} {}_{r}^{i}\Theta_{r*}^{D\epsilon(k)\delta(j)} K_{r*}^{\epsilon(k)\delta(j)} \quad (8)$$

where ${}_{r}^{i}\Theta_{r*}^{D\epsilon(k)\sigma(j)}$ and ${}_{r}^{i}\Theta_{r*}^{D\epsilon(k)\delta(j)}$ are weights that express the the risk that $K_{r*}^{\epsilon(k)\sigma(j)}$ and $K_{r*}^{\epsilon(k)\delta(j)}$ population flows suffer of dying in region i in cohort r.

In the cohort case the weights used in the all age and sex equations (Rees and Wilson, 1977, Chapter 4) can be used:

$$\hat{K}_{r*}^{D*i} = (1)\, K_{r*}^{\epsilon(i)\sigma(i)} + (0.5) \sum_{j\neq i} K_{r*}^{\epsilon(i)\sigma(j)} + (0.5)\, K_{r*}^{\epsilon(i)\delta(i)}$$
$$+ (0.25) \sum_{j\neq i} K_{r*}^{\epsilon(i)\delta(j)} + (0.5) \sum_{j\neq i} K_{r*}^{\epsilon(j)\sigma(i)} + (0.25) \sum_{j\neq i} K_{r*}^{\epsilon(i)\delta(} \quad (9).$$

Initially, the $K_{r*}^{\varepsilon(i)\sigma(i)}$, $K_{r*}^{\varepsilon(i)\delta(i)}$, $K_{r*}^{\varepsilon(i)\delta(j)}$ and $K_{r*}^{\varepsilon(j)\delta(i)}$ terms will not have been estimated so to speed convergence the population at risk can be set to the initial population

$$\hat{K}_{r*}^{D*i} = K_{r*}^{\varepsilon(i)*(*)} \tag{10}$$

for the first iteration.

For infants, the population at risk of dying is designated $\hat{K}_{r*}^{D\beta(*)i}$ and is estimated as

$$\hat{K}_{r*}^{D\beta(*)i} = (0.5)\, K_{r*}^{\beta(i)\sigma(i)} + (0.25) \sum_{j\neq i} K_{r*}^{\beta(i)\sigma(j)}$$
$$+ (0.25)\, K_{r*}^{\beta(i)\delta(i)} + (0.125) \sum_{j\neq i} K_{r*}^{\beta(i)\delta(j)}$$
$$+ (0.125) \sum_{j\neq i} K_{r*}^{\beta(j)\sigma(i)} + (0.125) \sum_{j\neq i} K_{r*}^{\beta(j)\delta(i)} \tag{11}$$

for mothers' cohorts $r = \alpha, \ldots, \beta$. For the first iteration this could be approximated as

$$\hat{K}_{r*}^{D\beta(*)i} = (0.5)\, K_{r*}^{\beta(i)*(*)} \tag{12}$$.

Death rates are then calculated using observed deaths in a region in the denominator:

$$d_{r*}^{\varepsilon(*)i} = K_{r*}^{\varepsilon(*)\delta(i)} \;/\; \hat{K}_{r*}^{D\varepsilon(*)i} \tag{13}$$

for age cohorts $r = 1, 2, \ldots, R$, and

$$d_{r*}^{\beta(*)i} = K_{r*}^{\beta(*)\delta(i)} \;/\; \hat{K}_{r*}^{D\beta(*)i} \tag{14}$$

for infants and age of mother cohorts $r = \alpha, \ldots, \beta$.

2.7.3 Step 3: calculate the unknown minor flows

The unknown minor flows are the off-diagonal terms of the $\underline{K}_{ED}$ and $\underline{K}_{BD}$

submatrices. The flows are called minor because they are much smaller than those in the principal diagonal or in the $\underline{K}_{BS}$ and $\underline{K}_{BS}$ submatrices.

The estimating equations use the death rates computed from step 2 and the migrant data from step 1 (see Chapter 5 in Rees and Wilson, 1977 for derivations)

$$K_{r*}^{\epsilon(i)\delta(j)} = \frac{(0.5)\ d_{r*}^{\epsilon(*)j}}{1 - (0.25)\ d_{r*}^{\epsilon(*)j}} \qquad K_{r*}^{\epsilon(i)\sigma(j)} \tag{15}$$

for r = 1,2, ... , R, and

$$K_{r*}^{\beta(i)\delta(j)} = \frac{(0.25)\ d_{r*}^{\beta(*)j}}{1 - (0.125)\ d_{r*}^{\beta(*)j}} \qquad K_{r*}^{\beta(i)\sigma(j)} \tag{16}$$

for mothers' cohorts $r = \alpha, \ldots, \beta$. The assumption underlying these equations is that migrants will die at the rate characteristic of the region of destination.

2.7.4 Step 4: calculate the unknown major flows and final population

Non-surviving and surviving stayers (the unknown major flows) can then be computed as residuals:

$$K_{r*}^{\epsilon(i)\delta(i)} = K_{r*}^{\epsilon(*)\delta(i)} - \sum_{j\neq i} K_{r*}^{\epsilon(j)\delta(i)} \tag{17}$$

$$K_{r*}^{\beta(i)\delta(i)} = K_{r*}^{\beta(*)\delta(i)} - \sum_{j\neq i} K_{r*}^{\beta(j)\delta(i)} \tag{18}$$

$$K_{r*}^{\epsilon(i)\sigma(i)} = K_{r*}^{\epsilon(i)*(*)} - \sum_{j\neq i} K_{r*}^{\epsilon(i)\sigma(j)} - K_{r*}^{\epsilon(i)\delta(i)} - \sum_{j\neq i} K_{r*}^{\epsilon(i)\delta(j)} \tag{19}$$

and

$$K_{r*}^{\beta(i)\sigma(i)} = K_{r*}^{\beta(i)*(*)} - \sum_{j\neq i} K_{r*}^{\beta(i)\sigma(j)} - K_{r*}^{\beta(i)\delta(i)} - \sum_{j\neq i} K_{r*}^{\beta(i)\delta(j)} \qquad (20)$$

for all appropriate age groups.

Final populations can then be computed as

$$K_{r*}^{\epsilon(*)\sigma(i)} = K_{r*}^{\epsilon(i)\sigma(i)} + \sum_{j\neq i} K_{r*}^{\epsilon(j)\sigma(i)} \qquad (21)$$

for $r = 1, 2, \ldots, R$, and

$$K_{r*}^{\beta(*)\sigma(i)} = K_{r*}^{\beta(i)\sigma(i)} + \sum_{j\neq i} K_{r*}^{\epsilon(j)\sigma(i)} \qquad (22).$$

The terms $K_{r*}^{\epsilon(*)\sigma(i)}$ and $K_{r*}^{\beta(*)\sigma(i)}$ are equivalent to $K_{*r+1}^{*(*)\sigma(i)}$ and $K_{r1}^{*(*)\sigma(i)}$ respectively. The unknown minor and major flows have now been estimated and may be fed back into the population at risk equations, Equations (9) and (11). Steps 2 through 4 are repeated until convergence is achieved. Convergence can be tested for on the accounts elements, the populations at risk or final populations, and is achieved fairly rapidly.

2.7.5 <u>Step 5: consistent row and column constraints derived</u>

In steps 2 through 4 of the model certain information was ignored, in particular, the final populations which are known for a historical base period.

Alternative equations are possible for the unknown major flows if this information is used. Surviving stayers can be estimated thus:

$$K_{r*}^{\epsilon(i)\sigma(i)} = K_{*r+1}^{*(*)\sigma(i)} - \sum_{j\neq i} K_{r*}^{\epsilon(j)\sigma(i)} \qquad (23)$$

and surviving infant stayers thus:

$$K_{r*}^{\beta(i)\sigma(i)} = K_{ri}^{*(*)\sigma(i)} - \sum_{j\neq i} K_{r*}^{\beta(j)\sigma(i)} \qquad (24).$$

Non-surviving stayers and non-surviving infant stayers can then be estimated from the row accounting equations:

$$K_{r*}^{\epsilon(i)\delta(i)} = K_{r*}^{\epsilon(i)*(*)} - K_{r*}^{\epsilon(i)\sigma(i)} - \sum_{j\neq i} K_{r*}^{\epsilon(i)\sigma(j)} - \sum_{j\neq i} K_{r*}^{\epsilon(i)\delta(j)} \qquad (25)$$

and

$$K_{r*}^{\beta(i)\delta(i)} = K_{r*}^{\beta(i)*(*)} - K_{r*}^{\beta(i)\sigma(i)} - \sum_{j\neq i} K_{r*}^{\beta(i)\sigma(j)} - \sum_{j\neq i} K_{r*}^{\beta(i)\delta(j)} \qquad (26).$$

What will then happen is that the deaths column totals from the accounts matrix will not necessarily equal to the observed regional deaths total, just as in the original accounts estimated in steps 2-4 the surviving column totals of the accounts matrix will not necessarily equal the observed final populations.

The solution to this problem of over-determination is to decide on a consistent set of row and column totals for the accounts and to adjust the initially estimated accounts matrix to be consistent with these totals using a balancing factor model.

But which accounts based model version should be used to generate the initial accounts matrix to be adjusted? Should it be that including Equations (17), (18), (19) and (20), or the alternative version substituting Equations (23), (24), (25) and (26)? Experience suggests that estimation of surviving stayers using the row accounting equation is more reliable because deaths, upon which the term partially depends, are more reliably counted than are migrants upon which the alternative calculation wholly depends. In the case of the four region accounts of Appendices 1 and 2, the migrant flow terms are based on a 10 per cent sample of enumeration forms in the census and errors in small flow terms in the migration matrix can be large relatively. Deaths, on the other hand, are counted in a 100 per cent enumeration. The most unreliable population projections presented in Rees (1977) were those for the counties of East Anglia which depended on $\underline{K}_{ES}$ submatrices generated from Equations (23) through (26).

In using a balancing factor model it is essential that the row and column constraints are consistent, that is, that they add up to exactly the same total. In the following equations the error term E_r should equal zero:

$$\sum_{i\in I} K_{r*}^{\epsilon(i)*(*)} + K_{r*}^{\epsilon(R)*(*)} + E_r^{\epsilon} = \sum_{i\in I} K_{r*}^{\epsilon(*)\sigma(i)} + K_{r*}^{\epsilon(*)\sigma(R)} + \sum_{i\in I} K_{r*}^{\epsilon(*)\delta(i)} + K_{r*}^{\epsilon(*)\delta(R)} \tag{27}$$

$$\sum_{i\in I} K_{r*}^{\beta(i)*(*)} + K_{r*}^{\beta(R)*(*)} + E_r^{\beta} = \sum_{i\in I} K_{r*}^{\beta(*)\sigma(i)} + K_{r*}^{\beta(*)\sigma(R)} + \sum_{i\in I} K_{r*}^{\beta(*)\delta(i)} + K_{r*}^{\beta(*)\delta(R)} \tag{28}$$

where by I is meant the set of regions internal to the country of interest and by R is meant the rest of the world.

The error term must be distributed among the other terms in the equation before accounts adjustment can begin. The decision as to which terms should be loaded with the error will depend on which terms are regarded as the most unreliable. In the British case the emigrant terms are most suspect and the error is apportioned to them:

$$\hat{K}_{r*}^{\epsilon(*)\sigma(R)} = K_{r*}^{\epsilon(*)\sigma(R)} + E_r^{\epsilon} \left[\frac{K_{r*}^{\epsilon(*)\sigma(R)}}{K_{r*}^{\epsilon(*)\sigma(R)} + K_{r*}^{\epsilon(*)\delta(R)}} \right] \tag{29}$$

$$\hat{K}_{r*}^{\epsilon(*)\delta(R)} = K_{r*}^{\epsilon(*)\delta(R)} + E_r^{\epsilon} \left[\frac{K_{r*}^{\epsilon(*)\delta(R)}}{K_{r*}^{\epsilon(*)\sigma(R)} + K_{r*}^{\epsilon(*)\delta(R)}} \right] \tag{30}$$

with similar equations for infants. The $\wedge$ refers to an adjusted estimate.

2.7.6 Step 6: the accounts matrix is adjusted to satisfy the constraints

Simple two balancing factor models are used. Let us simplify the accounts notation for each age cohort to K^{mn} as the typical element, and call the row constraints, R^m, and the column constraints, C^n, irrespective of whether they are populations, births, deaths, emigrants or immigrants in nature.

Then the adjusted estimates are derived as follows

$$\hat{K}^{mn} = A^m B^n K^{mn} \tag{31}$$

subject to

$$\sum_n \hat{K}^{mn} = R^m \tag{32}$$

and

$$\sum_m \hat{K}^{mn} = C^n \tag{33}$$

Substituting the right hand side of Equation (31) in the left hand sides of Equations (32) and (33), and rearranging, we obtain

$$A^m = R^m \;/\; \sum_n B^n K^{mn} \tag{34}$$

$$B^n = C^n \;/\; \sum_m A^m K^{mn} \tag{35}$$

These equations must be solved iteratively in the usual fashion (Wilson, 1974; Willekens, 1977). Experience suggests that, because the row constraints are almost equal or are exactly equal to the row totals of the initially estimated accounts matrix, convergence can take a long time.

2.8 A projection version of the accounts based model

The accounts based model described above can be converted into a projection model by substituting migration and births submodels for the migrant and births terms in the step 1 list of known data, and by substituting projected death rates for deaths totals. Equations (13) and (14) are re-arranged thus

$$K_{r*}^{\epsilon(*)\delta(i)} = d_{r*}^{\epsilon(*)i} \; {}^{\Delta}K_{r*}^{D\epsilon(*)i} \qquad (36)$$

and

$$K_{r*}^{\beta(*)\delta(i)} = d_{r*}^{\beta(*)i} \; {}^{\Delta}K_{r*}^{D\beta(*)i} \qquad (37).$$

Deaths totals are re-estimated on each iteration of the model rather than death rates. The form of the population at risk equations may need adjustment to match the population at risk definitions used in deriving the projected rates series.

However, it should be emphasized that there are a great many ways in which a population projection model could be constructed employing an accounts base. The range of choices over and above those implied in section 2.4 and here needs thorough exploration, though not in this paper.

The paper continues with a brief discussion of the similarities and differences between the multiregional life table and the multiregional accounts frameworks.

3. The multiregional life table and multiregional accounts frameworks compared

3.1 Age-time relations

Figure 3 outlined, in passing, for a system in which the age group intervals are equal to the time period lengths, the age-time relations characteristic of a life table approach to population modelling and of a projection approach to population modelling (the approach from which accounts stem). The life table approach uses transitions between exact ages; the projection/accounts approach employs transitions between discrete age groups. As age and time intervals tend to zero the approaches become more and more similar, and they coincide in the continuous model.

Data are sometimes collected in such a way that events or transitions occurring in the life table or projection parallelograms of Figure 3 are counted, but often they are not. Very simple changes in existing procedures would make exact matching possible, but for the moment in many countries data have to be massaged to the right age-time relations.

3.2 Regions

Both frameworks adopt a multiregional viewpoint. The emphasis to date in the former approach has been in partitioning a country into an exhaustive set of regions (Rogers, 1976); in the latter approach this partitioning is extended to the whole of the world in order to close the system effectively. This concern with system closure stems form the national accounting approach pioneered by Stone (1971, 1975).

How important is it to include a rest of the world region or regions in the system set of regions? For some countries, such as the USSR or China, it is, for practical purposes, unnecessary, given the miniscule size of external migration flows. However, in other countries, such as Canada, Israel or the UK, the importance of external flows makes it essential to include such an external region. Such considerations have long been recognized for projection purposes via inclusion of a net migration flow or rate in national population projections (see, for example, O.P.C.S., 1978). A rest of the world region should perhaps be included as well in multiregional life table analysis, although this would necessitate the estimation of the age and sex breakdown of the population of the whole world. If the possibility of spending part of a lifetime outside the country of birth is not recognized (as it is not in Willekens and Rogers, 1976), then the method of estimation of the $\underline{P}$ matrix means that the time spent outside the country is added on in the program calculations to that spent in the region of birth or current residence, thus overestimating those terms.

This point can be demonstrated using transition rates derived from accounts matrices. If we divide through Equation (19) by the initial population, separate out internal and external surviving migrant terms and consolidate the terms involving death, we obtain:

$$h_{r^*}^{\epsilon(i)\sigma(i)} = 1 - \sum_{\substack{j \neq i \\ j \in I}} h_{r^*}^{\epsilon(i)\sigma(j)} - h_{r^*}^{\epsilon(i)\sigma(R)} - \sum_{j} h_{r^*}^{\epsilon(i)\delta(j)} \quad (38)$$

where h refers to transition rate. The equivalent equation in multiregional life table analysis can be represented in this notation as:

$$\begin{aligned} \text{rate of staying and survival} &= 1 - \text{death rate} - \text{total internal migration rate} \\ &\doteq 1 - \sum_{\substack{j \neq i \\ j \in I}} h_{r^*}^{\epsilon(i)\sigma(j)} - \sum_{j} h_{r^*}^{\epsilon(i)\delta(j)} \\ &= h_{r^*}^{\epsilon(i)\sigma(i)} + h_{r^*}^{\epsilon(i)\sigma(R)} \quad (39). \end{aligned}$$

In the simulation version of the Willekens and Rogers (1976) programs this rate is correctly adjusted for by addition of a net external migration rate

$$g_{r^*}^{\epsilon(i)\sigma(i)} = h_{r^*}^{\epsilon(i)\sigma(i)} + h_{r^*}^{\epsilon(i)\sigma(R)} + h_{r^*}^{\epsilon(i)\sigma(R)} + \left(h_{r^*}^{\epsilon(R)\sigma(i)} - h_{r^*}^{\epsilon(i)\sigma(R)} \right) \quad (40)$$

where $g_{r^*}^{\epsilon(i)\sigma(i)}$ is the term that is enetered in the diagonal of the rates matrix used in the projection model and $h_{r^*}^{\epsilon(R)\sigma(i)}$ is an immigration rate. If the rate of staying and survival is specified as it is in the accounts framework, a gross immigration rate (not a net) should be used.

3.3 Migration and death

In the multiregional life table probability matrix two types of events are recognized as possible: migration or death (and their complements staying and survival, of course). You either migrate or stay put, or you die. Death takes place in the region in which you were last counted (previous exact age). This is the logic of the continuous population model.

However, in discrete population models combinations of events become possible. Thus when the $\underline{P}$ matrix has been applied many times as it is in the life table model, the possibility of migrating and then dying is apparent. You can be born in one region and die in another within two age intervals. This logic is extended to the single period or age interval in the multiregional accounting approach. The types of events now possible are

(i) staying and surviving
(ii) migration and survival
(iii) staying and dying
(iv) migration and death.

As we have already seen, non-surviving migrant flows are estimated as one of the steps of the accounts based model.

How important are such events likely to be? In the four region accounts for Great Britain presented in Appendices 1 and 2 migrants who die make up an estimated 1.2 per cent of all persons dying in the five year period 1966-71, so that it would be unwise to ignore such combined events in computing a multiregional life table. If such events were accounted for then there would be a slight increase in off-diagonal life expectancies compared with those generated by the Willekens and Rogers (1976) program. This point is recognized in Ledent (1978, p.114).

A more important advantage of accepting the possibility of migration followed by death within a time period or age interval is that proper probabilities of survivorship and non-survivorship can be computed directly from an accounts matrix. Use of accounts based survivorship proportions would probably improve on the performance of the Option 2 method of life table probability estimation (Ledent, 1978, pp.114ff).

3.4 The Markovian assumption and return migration probabilities

Underlying both the multiregional life table model and the derived population projection model is the Markovian assumption that the probability of making a state to state transition in an age interval or time period is independent of a person's prior history of duration in particular states.

This assumption is clearly true in the single region case since everyone has the same history of state occupations: everyone who reaches 80 years will have been 0, 5, 10, ... , 75 in that order only!

However, when the states are regions, the question becomes an empirical one. Do people aged 25 living in the South East, for instance, have a different probability of moving to East Anglia if they were born there than if they were not? The weight of evidence is that such probabilities will differ substantially and that return migration is an important phenomenon (Long and Hanson, 1975). Long and Hansen report that the rate of migration of the Southern born from the non-South of the USA to the South is 4.8 times that of the non-Southern born, and that the rate of migration of the non-Southern born from the South to the non-South is 6.1 times that of the Southern born in the 1965-70 period (Long and Hansen, 1975, Tables A-1 and A-2).

Of course, construction of a multiregional life table with probabilities dependent on region of birth demands that data are available on interregional migrants cross-classified by region of birth. No tables equivalent to those used by Long and Hansen exist in the Great Britain censuses, although the right questions are asked (on place of birth and on place of previous residence) are asked, and some tables for persons born outside Great Britain are produced. In the absence of substantial and successful lobbying efforts concerning census processing the improvement of multiregional life tables in this respect remains a theoretical possibility only.

Fortunately, the Markovian assumption matters less in population projection since classification of future populations by region of birth is not usually required whereas such a classification is of crucial significance in the evolution of the regional cohorts of the multiregional life table.

3.5 The one year/five year problem

The methods used in the Willekens and Rogers (1976) programs assume data input in which stocks and flows are disaggregated into five year age groups, and the population flows refer to a one year period. Unfortunately, the method by which these data are converted to a five year period basis through multiplication of the migration and death rates matrix by five leads to substantial overestimation of off-diagonal probabilities in the P matrix as Ledent (1978) has pointed out.

Similar problems have characterized the accounts approach. Both Stone (1975) and Rees and Wilson (1977) show how accounts with a one year time period and five year age intervals can be constructed. The implication is that the transition rates matrix from such accounts can be used in population modelling (as in Rees and King, 1974). However, what happens as the model rolls forward is that persons remain in age group states long past their rightful time.

The only solutions to such awkward problems would seem to be to deconsolidate to single year age intervals while retaining an annual period or to increase the length of the period to five years while retaining five year age groups. Both solutions have their penalties in terms of data requirements and/or estimation methods.

4. Conclusions

In this paper the concepts and methods of demographic accounting have been reviewed. The multiregional accounts framework has been shown to be consistent, give or take a few details, with the framework of multiregional population analysis developed by Rogers and his collaborators. One might therefore ask the following question. Why go to all the bother of following the route of demographic accounting with all its myriad pernicketinesses about data collection and data arrangement? The answer must be that the accounting methods described here represent a way of making sure that the base period data in multiregional population analysis is correctly estimated and correctly reproduces the observed population change. Demographic accounting represents the way in which multiregional population analysis can be "calibrated".

References

Ledent, J. (1978). Some methodological and empirical considerations in the construction of increment-decrement life tables. Research Memorandum, RM-78-25, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Long, L.H. and Hansen, K.A. (1975). Trends in return migration to the South. Demography, 12, 4, 601-614.

Office of Population Censuses and Surveys (O.P.C.S.) (1978). Population projections 1976-2016. Series PP2, No.8. H.M.S.O., London.

Pressat, R. (1972). Demographic analysis. Edward Arnold, London.

Rees, P.H. (1977). The future population of East Anglia and its constituent counties (Cambridge, Norfolk and Suffolk). A report prepared for the East Anglian Economic Planning Council, University of Leeds Industrial Services Ltd., Leeds.

Rees, P.H. (1978a). Increment-decrement life tables: some further comments from a demographic accounting point of view. Environment and Planning A, 10, 705-726.

Rees, P.H. (1978b). Migration and settlement in the U.K.: dynamics and policy. Internal Working Paper, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Rees, P.H. and King, J.R. (1974). A simple model for population projections applied to ethnic group and small area populations. Working Paper 76, School of Geography, University of Leeds.

Rees, P.H., Smith, A.P. and King,J.R. (1977). Population models. Chapter 3 in Wilson, A.G., Rees, P.H. and Leigh, C.M. (ed.s) Models of cities and regions. John Wiley, Chicester.

Rees, P.H. and Wilson, A.G. (1977). Spatial population analysis. Edward Arnold, London.

Rogers, A. (1975). Introduction to multiregional mathematical demography. John Wiley, London.

Rogers, A. (1976). Shrinking large scale population projection models by aggregation and decomposition. Environment and Planning, A, 8, 515-541.

Stone, R. (1971). Demographic accounting and model building. O.E.C.D., Paris.

Stone, R. (1975). Towards a system of social and demographic statistics. United Nations, New York. Department of Economic and Social Affairs, Statistical Office, Studies in Methods, Series F, No. 18.

Thomas, D. and Rees, P.H. (1978). The analysis of population change in the republics of the Soviet Union using multiregional accounting and projection methods. Working Paper, School of Geography, University of Leeds, forthcoming.

Willekens, F. (1977). The recovery of detailed migration patterns from aggregate data: an entropy maximizing approach. Research Memorandum, RM-77-58, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Willekens, F. and Rogers, A. (1976). Computer programs for spatial demographic analysis. Research Memorandum, RM-76-58, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Wilson, A.G. (1974). <u>Urban and regional models in geography and planning</u>. John Wiley, London.

APPENDIX 1. East Anglia and South East region accounts, 1966-71, persons: by region to region transition

EAST ANGLIA TO

Initial age group	Survival in EA	Survival in SE	Survival in RB	Survival in RW	Death in EA	Death in SE	Death in RB	Death in RW	Totals
0- 4	108108	5739	6880	4366	476	11	16	9	125605
5- 9	100217	3972	4578	4011	171	4	4	4	112961
10-14	96606	3893	3673	3334	252	5	5	5	107773
15-19	99103	9396	7286	4827	433	14	14	8	121081
20-24	79620	7578	8141	5296	334	13	15	10	101007
25-29	75425	4895	5505	3557	312	10	12	7	89723
30-34	78257	3697	4445	2544	425	11	14	6	89399
35-39	85217	2859	3237	2133	746	14	18	9	94233
40-44	91858	2640	2416	1497	1341	20	22	11	99805
45-49	85541	1964	1763	991	2065	28	27	12	92391
50-54	90042	1274	1031	649	3576	28	27	12	96639
55-59	88193	1003	967	435	5906	33	39	14	96590
60-64	77460	794	1284	325	8936	33	99	17	88948
65-69	60783	652	1105	177	11580	44	128	15	74484
70-74	41145	492	876	87	13852	63	165	13	56693
75+	42545	549	754	86	36338	176	311	29	80788
Totals	1300120	51397	53941	34315	86743	507	916	181	1528120

SOUTH EAST TO

Initial age group	Survival in EA	Survival in SE	Survival in RB	Survival in RW	Death in EA	Death in SE	Death in RB	Death in RW	Totals
0- 4	11171	1249290	58934	84055	22	5489	142	171	1409274
5- 9	7513	1112234	39793	77256	6	1720	33	56	1238611
10-14	5710	1015938	31756	63870	7	2399	37	72	1119789
15-19	12590	1128627	71702	91847	24	4019	131	148	1309088
20-24	14408	984059	79740	101785	26	3691	148	175	1184032
25-29	9799	912460	57623	81742	19	3957	125	164	1065889
30-34	7366	905165	41016	55390	18	5372	125	154	1014606
35-39	6039	961140	32917	41027	25	9267	173	189	1050777
40-44	5512	1031932	27662	28717	38	16024	261	212	1110358
45-49	4737	973153	23445	19024	56	26602	353	249	1047619
50-54	4612	1026101	23250	12506	87	45369	566	258	1112749
55-59	6428	975196	25097	8539	204	71524	998	276	1088262
60-64	4638	806332	27368	6014	291	99003	1882	333	945861
65-69	3379	595172	13168	3183	298	118223	1439	286	735148
70-74	2008	408765	6800	1385	286	141859	1146	219	562468
75+	2123	420930	6384	1782	702	366750	2509	615	801795
Totals	108033	14506494	566655	678122	2109	921268	10068	3577	16796326

APPENDIX 1. Continued

REST OF BRITAIN TO

Initial age group	Survival in EA	SE	RB	RW	Death in EA	SE	RB	RW	Totals
0- 4	8869	52833	2892220	89577	18	109	14227	215	3058068
5- 9	6129	37861	2651944	83011	5	28	4459	69	2783506
10-14	5814	46077	2394405	68014	8	51	5785	83	2520237
15-19	10725	119097	2485220	94074	20	192	9651	177	2719156
20-24	9440	84510	2074448	107473	18	145	8056	208	2284298
25-29	7129	51616	1931915	87173	13	105	8606	191	2086948
30-34	4864	37201	1926934	59043	12	105	12083	184	2040381
35-39	4422	31037	2061548	43673	19	144	21953	231	2163027
40-44	5550	26522	2227808	30207	36	200	42591	286	2333200
45-49	5017	19553	2054382	19954	51	260	62885	306	2162408
50-54	4800	14992	2101598	13023	77	319	107926	328	2243063
55-59	1819	12220	2008920	8685	57	405	170133	343	2202582
60-64	2824	11886	1703828	6178	152	684	241293	422	1967267
65-69	2480	7955	1268768	3238	178	745	289654	352	1573370
70-74	1710	5950	831111	1661	209	948	309352	283	1151224
75+	1729	6083	713197	1501	538	2150	712844	593	1438635
Totals	83321	565593	31328246	716485	1411	6590	2021453	4271	34727370

REST OF THE WORLD TO

Initial age group	Survival in EA	SE	RB	RW	Death in EA	SE	RB	RW	Totals
0- 4	6900	48268	58283	0	14	99	140	0	113704
5- 9	5840	45949	53146	0	5	34	45	0	105019
10-14	3800	49389	48969	0	5	55	60	0	102278
15-19	8830	105136	64158	0	18	160	116	0	178418
20-24	7790	88045	70123	0	14	149	134	0	166255
25-29	6340	54686	54385	0	12	110	119	0	115652
30-34	4920	37919	35351	0	13	107	108	0	78418
35-39	2730	28287	26848	0	11	131	140	0	58147
40-44	1440	20178	18812	0	10	153	179	0	40772
45-49	1080	14026	11906	0	13	189	183	0	27397
50-54	590	8983	8022	0	12	194	206	0	18007
55-59	310	5966	5401	0	10	204	216	0	12107
60-64	190	4344	4844	0	12	247	331	0	9968
65-69	140	2536	3021	0	13	227	321	0	6258
70-74	79	1222	1571	0	12	194	258	0	3336
75+	81	1254	1421	0	28	441	554	0	3779
Totals	51060	516188	466261	0	202	2694	3110	0	1039515

APPENDIX 1. Continued

BORN IN EAST ANGLIA

Mother's initial age group	Survival in EA	SE	RB	RW	Death in EA	SE	RB	RW	Totals
10-14	3297	123	107	69	56	1	1	0	3654
15-19	29249	1091	951	605	493	10	9	5	32413
20-24	40517	1512	1316	839	683	14	12	7	44900
25-29	23833	889	775	493	402	8	7	4	26411
30-34	10692	399	347	221	180	4	3	2	11848
35-39	4171	156	135	86	71	2	2	0	4623
40-44	791	29	26	17	13	0	0	0	876
45-49	42	2	1	0	0	0	0	0	45
Totals	112592	4201	3658	2330	1898	39	34	18	124770

BORN IN SOUTH EAST

Mother's in. age grp	Survival in EA	SE	RB	RW	Death in EA	SE	RB	RW	Totals
10-14	162	30914	826	1102	2	585	8	10	33609
15-19	1509	287912	7689	10261	12	5444	75	93	312995
20-24	2289	436624	11659	15562	17	8257	114	141	474663
25-29	1505	287004	7664	10229	12	5427	75	93	312009
30-34	718	137060	3660	4885	5	2592	35	44	148999
35-39	282	53830	1438	1919	2	1018	14	17	58520
40-44	51	9883	264	352	0	187	3	4	10744
45-49	3	467	13	17	0	9	0	0	509
Totals	6519	1243694	33213	44327	50	23519	324	402	1352048

BORN IN THE REST OF BRITAIN

Mother's in. age grp	Survival in EA	SE	RB	RW	Death in EA	SE	RB	RW	Totals
10-14	270	915	85905	1426	2	8	1656	13	90195
15-19	2221	7542	707288	11738	18	69	13632	111	742619
20-24	3055	10373	972788	16145	25	95	18749	154	1021384
25-29	1923	6529	612344	10162	16	60	11802	96	642932
30-34	936	3178	302043	4947	7	29	5746	46	316932
35-39	401	1362	127690	2119	3	13	2461	20	134069
40-44	79	267	25054	416	0	2	485	4	26305
45-49	4	12	1143	19	0	0	22	0	1200
Totals	8889	30178	2834255	46972	71	276	54551	444	2975636

BORN IN REST OF THE WORLD

Mother's in. age grp	Survival in EA	SE	RB	RW	Death in EA	SE	RB	RW	Totals
10-14	100	612	955	0	1	6	9	0	1683
15-19	889	5697	7865	0	7	52	75	0	14585
20-24	1233	8640	10818	0	10	79	102	0	20882
25-29	725	5679	6809	0	6	51	64	0	13334
30-34	326	2712	3314	0	3	25	31	0	6411
35-39	127	1065	1420	0	1	10	14	0	2637
40-44	24	195	278	0	0	2	3	0	502
45-49	2	10	12	0	0	0	0	0	24
Totals	3426	24610	31471	0	28	225	298	0	60058

APPENDIX 1. Continued

COLUMN TOTALS OF THE ACCOUNTS

Age group Ini-	Final	Survivors in EA	SE	RB	RW	Deaths in EA	SE	RB	RW	Totals
Birth	0- 4	131426	1302683	2902597	93629	2047	24059	55207	864	4512512
0- 4	5- 9	135048	1356130	3015317	177998	530	5708	14525	395	4706651
5- 9	10-14	119699	1200016	2740461	164278	187	1786	4541	129	4240097
10-14	15-19	111930	1115297	2478803	135218	272	2510	5827	160	3850077
15-19	20-24	131248	1362256	2628366	190748	495	4385	9912	333	4327743
20-24	25-29	111258	1164192	2232452	214554	392	3998	8353	393	3735592
25-29	30-34	98693	1023857	2049428	172472	356	4182	8862	362	3358212
30-34	35-39	95407	983982	2007746	116977	468	5595	12285	344	3222804
35-39	40-44	98408	1023323	2124550	86833	801	9556	22284	429	3366184
40-44	45-49	104360	1081272	2276698	60421	1425	16397	43053	509	3584135
45-49	50-54	96375	1008696	2091496	39969	2185	27079	63448	567	3329815
50-54	55-59	100044	1051350	2133901	26178	3752	45910	108725	598	3470458
55-59	60-64	96750	994385	2040385	17659	6177	72166	171386	633	3399541
60-64	65-69	85112	823356	1737124	12517	9391	99967	243605	772	3012044
65-69	70-74	66782	606315	1286062	6598	12069	119239	291542	653	2389260
70-74	75+	44942	416429	840358	3133	14359	143064	319921	515	1773721
75+	80+	46478	428816	721756	3369	37606	369517	716218	1237	2324997
Totals		1673960	16942355	35317700	1522551	92512	955118	2090754	3893	58603843

Notes

EA East Anglia

SE South East

RB Rest of Britain

RW Rest of World

APPENDIX 2. East Anglia and South East region accounts, 1966-71, persons: by cohort

BIRTH TO 0-4

0-4 / Birth	Survival aged 0-4 EA	SE	RB	RW	Death 0-4 EA	SE	RB	RW	Totals
Existence: East Anglia	112592	4201	3658	2330	1898	39	34	18	124770
Existence: South East	6519	1243694	33213	44327	50	23519	324	402	1352048
Existence: Rest of Britain	8889	30178	2834255	46972	71	276	54551	444	2975636
Existence: Rest of World	3426	24610	31471	0	28	225	298	0	60058
Totals	131426	1302683	2902597	93629	2047	24059	55207	864	4512512

0-4 TO 5-9

5-9 / 0-4	Survival aged 5-9 EA	SE	RB	RW	Death 0-4, 5-9 EA	SE	RB	RW	Totals
Existence: East Anglia	108108	5739	6880	4366	476	11	16	9	125605
Existence: South East	11171	1249290	98934	34055	22	5489	142	171	1409274
Existence: Rest of Britain	8869	52833	2892220	89577	18	109	14227	215	3058068
Existence: Rest of World	6900	48268	58283	0	14	99	140	0	113704
Totals	135048	1356130	3016317	177998	530	5708	14525	395	4706651

5-9 TO 10-14

10-14 / 5-9	Survival aged 10-14 EA	SE	RB	RW	Death 5-9, 10-14 EA	SE	RB	RW	Totals
Existence: East Anglia	100217	3972	4578	4011	171	4	4	4	112961
Existence: South East	7513	1112234	39793	77256	6	1720	33	56	1238611
Existence: Rest of Britain	6219	37861	2651944	83011	5	28	4459	69	2783506
Existence: Rest of World	5840	45949	53146	0	5	34	45	0	105019
Totals	119699	1200016	2749461	164278	187	1786	4541	129	4240097

10-14 TO 15-19

15-19 / 10-14	Survival aged 15-19 EA	SE	RB	RW	Death 10-14, 15-19 EA	SE	RB	RW	Totals
Existence: East Anglia	96606	3893	3673	3334	252	5	5	5	107773
Existence: South East	5710	1015938	31756	63870	7	2399	37	72	1119789
Existence: Rest of Britain	5814	46077	2394405	68014	8	51	5785	83	2520237
Existence: Rest of World	3800	49389	48969	0	5	55	60	0	102278
Totals	111930	1115297	2478803	135218	272	2510	5887	160	3850077

15-19 TO 20-24

20-24 / 15-19	Survival aged 20-24 EA	SE	RB	RW	Death 15-19, 20-24 EA	SE	RB	RW	Totals
Existence: East Anglia	99103	9396	7286	4827	433	14	14	8	121081
Existence: South East	12590	1128627	71702	91847	24	4019	131	148	1309088
Existence: Rest of Britain	10725	119097	2485220	94074	20	192	9651	177	2719156
Existence: Rest of World	8830	105136	64158	0	18	160	116	0	178418
Totals	131248	1362256	2628366	190748	495	4385	9912	333	4327743

APPENDIX 2. <u>Continued</u>

20-24 TO 25-29

20-24 \ 25-29		Survival aged 25-29				Death 20-24,25-29				
		EA	SE	RB	RW	EA	SE	RB	RW	Totals
Exist-ence	East Anglia	79620	7578	8141	5296	334	13	15	10	101007
	South East	14408	984059	79740	101785	26	3691	148	175	1184032
	Rest of Britain	9440	84510	2074448	107473	18	145	8056	208	2284298
	Rest of World	7790	88045	70123	0	14	149	134	0	166255
Totals		111258	1164192	2232452	214554	392	3998	8353	393	3735592

25-29 TO 30-34

25-29 \ 30-34		Survival aged 30-34				Death 25-29,30-34				
		EA	SE	RB	RW	EA	SE	RB	RW	Totals
Exist-ence	East Anglia	75425	4895	5505	3557	312	10	12	7	89723
	South East	9799	912460	57623	81742	19	3957	125	164	1065889
	Rest of Britain	7129	51816	1931915	87173	13	105	8606	191	2086948
	Rest of World	6340	54686	54385	0	12	110	119	0	115652
Totals		98693	1023857	2049428	172472	356	4182	8862	362	3358212

30-34 TO 35-39

30-34 \ 35-39		Survival aged 35-39				Death 30-34,35-39				
		EA	SE	RB	RW	EA	SE	RB	RW	Totals
Exist-ence	East Anglia	78257	3697	4445	2544	425	11	14	6	89399
	South East	7366	905165	41016	55390	18	5372	125	154	1014606
	Rest of Britain	4864	37201	1926934	59043	12	105	12038	184	2040381
	Rest of World	4920	37919	35351	0	13	107	108	0	78418
Totals		95407	983982	2007746	116977	468	5595	12285	344	3222804

35-39 TO 40-44

35-39 \ 40-44		Survival aged 40-44								
		EA	SE	RB	RW	EA	SE	RB	RW	Totals
Exist-ence	East Anglia	85217	2859	3237	2133	746	14	18	9	94233
	South East	6039	961140	32917	41027	25	9267	173	189	1050777
	Rest of Britain	4422	31037	2061548	43673	19	144	21953	231	2163027
	Rest of World	2730	28287	26848	0	11	131	140	0	58147
Totals		98408	1023323	2124550	86833	801	9556	22284	429	3366184

40-44 TO 45-49

40-44 \ 45-49		Survival aged 45-49								
		EA	SE	RB	RW	EA	SE	RB	RW	Totals
Exist-ence	East Anglia	91858	2640	2416	1497	1341	20	22	11	99805
	South East	5512	1031932	27662	23717	38	16024	261	212	1110358
	Rest of Britain	5550	26522	2227808	30207	36	200	42591	286	2333200
	Rest of World	1440	20178	18812	0	10	153	179	0	40772
Totals		104360	1081272	2276698	60421	1425	16397	43053	509	3584135

APPENDIX 2. Continued

45-49 TO 50-54

50-54 / 45-49		Survival aged 50-54 EA	SE	RB	RW	Death 45-49,50-54 EA	SE	RB	RW	Totals
Exist-ence	East Anglia	85541	1964	1763	991	2065	28	27	12	92391
	South East	4737	973153	23445	19024	56	26602	353	249	1047619
	Rest of Britain	5017	19553	2054382	19954	51	260	62835	306	2162408
	Rest of World	1080	14026	11906	0	13	189	185	0	27397
Totals		96375	1008696	2091496	39969	2185	27079	63448	567	3329815

50-54 TO 55-59

55-59 / 50-54		Survival aged 55-59 EA	SE	RB	RW	Death 50-54,55-59 EA	SE	RB	RW	Totals
Exist-ence	East Anglia	90042	1274	1031	649	3576	28	27	12	96639
	South East	4612	1026101	23250	12506	87	45369	566	258	1112749
	Rest of Britain	4800	14992	2101598	13023	77	319	107926	328	2243063
	Rest of World	590	8983	8022	0	12	194	206	0	18007
Totals		100044	1051350	2133901	26178	3752	45910	108725	598	3470458

55-59 TO 60-64

60-64 / 55-59		Survival aged 60-64 EA	SE	RB	RW	Death aged 55-59,60-64 EA	SE	RB	RW	Totals
Exist-ence	EA	88193	1003	967	435	5906	53	39	14	96590
	SE	6428	975196	25097	8539	204	71524	998	276	1088262
	RB	1819	12220	2008920	8685	57	405	170133	343	2202582
	RW	310	5966	5401	0	10	204	216	0	12107
Totals		96750	994385	2040385	17659	6177	72166	171386	633	3399541

60-64 TO 65-69

65-69 / 60-64		Survival aged 65-69 EA	SE	RB	RW	Death aged 60-64,65-69 EA	SE	RB	RW	Totals
Exist-ence	EA	77460	794	1284	325	8936	33	99	17	88948
	SE	4638	806332	27368	6014	291	99005	1882	333	945861
	RB	2824	11886	1703828	6178	152	684	241293	422	1967267
	RW	190	4344	4844	0	12	247	331	0	9968
Totals		85112	823356	1737324	12517	9391	99967	243605	772	3012044

65-69 TO 70-74

70-74 / 65-69		Survival aged 70-74 EA	SE	RB	RW	Death aged 65-69,70-74 EA	SE	RB	RW	Totals
Exist-ence	EA	60783	652	1105	177	11580	44	128	15	74484
	SE	3379	595172	13168	3183	298	118223	1439	286	735148
	RB	2480	7955	1268768	3238	178	745	289654	352	1573370
	RW	140	2536	3021	0	13	227	321	0	6258
Totals		66782	606315	1286062	6598	12069	119239	291542	653	2389260

APPENDIX 2. Continued

70-74 TO 75

70-74 \ 75-79		Survival aged 75-79 EA	SE	RB	RW	Death aged 70-74, 75-79 EA	SE	RB	RW	Totals
Existence	EA	41145	492	876	87	13952	63	165	13	56693
	SE	2008	408765	6800	1385	286	141959	1146	219	562468
	RB	1710	5950	831111	1661	209	948	309352	283	1151224
	RW	79	1222	1571	0	12	194	258	0	3336
Totals		44942	416429	840358	3133	14359	143064	310921	515	1773721

75+ TO 80+

75+ \ 80+		Survival aged 80+ EA	SE	RB	RW	Death aged 75+,80+ EA	SE	RB	RW	Totals
Existence	EA	42545	549	754	96	36328	176	311	29	80788
	SE	2123	420930	6384	1772	702	366750	2509	615	801795
	RB	1729	5083	713197	1501	538	2150	712844	593	1438635
	RW	81	1254	1421	0	28	441	554	0	3779
Totals		46478	428816	721756	3369	37606	369517	716218	1237	2324997

ALL AGE ACCOUNTS

Initial states \ Final states		Survival at c.d. 1971 EA	SE	RB	RW	Death 1966-71 EA	SE	RB	RW	Totals
Existence	EA	1300120	51397	53941	34315	86743	507	916	181	1528120
	SE	108033	14506494	566655	678122	2109	921268	10068	3577	16796326
	RB	83321	565593	31328246	716485	1411	6590	2021453	4271	34727370
	RW	51060	516188	466261	0	202	2694	3110	0	1039515
Birth	EA	112592	4201	3658	2330	1898	39	34	18	124770
	SE	6519	1243694	33213	44327	50	23519	324	402	1352048
	RB	8889	30178	2834255	46972	71	276	54551	444	2975636
	RW	3426	24610	31471	0	28	225	298	0	60058
Totals		1673960	16942355	35317700	1522551	92512	955118	2090754	8893	58603843