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# A FAMILY OF DEMOGRAPHIC ACCOUNTS

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Table of contents

## Abstract

Three forms of demographic accounts, closed, open and balance sheet, are described in two alternative notations, that of Stone and that of Rees and Wilson. The relationship of each member of this family of accounts to each other member is established. The nature of the migration entries in each kind of account is identified and a set of migration accounts which link multiple migrations to the migration entries of the family of demographic accounts is developed. Finally, the open demographic matrix of Stone is expanded to form an integrated demographic accounting system.

## 1. Aims

Demographic accounts are methods of recording the changes that people experience in their lives. By studying the changes that have occurred in the past we can construct and test out models of population change that will help us forecast and plan the future.

There are several ways in which demographic accounts may be constructed. It is the purpose of this paper to attempt a systematic description of the variety of demographic accounts that have been proposed. The existence of a family of demographic accounts will be demonstrated: that is, it will be shown that each type of demographic account is explicitly related to each other type. To accomplish this we draw heavily on the work of Richard Stone (Stone, 1965, 1966, 1971a, 1971b, 1972a, 1972b, 1973a, 1973b; Stone, Stone and Gunton 1968) who has pioneered accounting in both the economic and demographic spheres, and on previous work by Alan Wilson and the present author (Wilson 1972, 1974; Rees 1972a, 1972b, 1973; Rees and Wilson 1973a 1973b, 1974a, 1974b; Wilson and Rees 1973, 1974). We use a common notation and example to show the interrelatedness of the different kinds of accounts.

## 2. The family

Stone (1971b) presents three different kinds of accounts in numerical form:

(1) accounts laid out in the form of a company's profit and loss accounts, which we will call balance sheet accounts;

(2) accounts which link the opening and closing population stocks of a period which we will call, with Stone (1971a), closed demographic accounts; and

(3) accounts which show the inflows and outflows from a period and those from one period to the next, which we will call, with Stone (1971a), open demographic accounts.

Accounts may be in the form of a table or in the form simply of a matrix in which the marginal totals of the table form are omitted.

## 3. Closed demographic accounts

We begin by considering the second type of accounts defined in the list above, closed demographic accounts, as there are perhaps the most fundamental. The other types, we will show can be derived from them.

Stone has presented these accounts in the following form (Stone,1971b, 1972a, 1973a, 1973b):

Table 1. Stone's closed demographic accounts matrix

| State at New Year $\theta+1$ \ State at New Year 0 | Outside World | Our country: Opening states | Closing stocks |
|---|---|---|---|
| Outside world | $\alpha$ | d' | |
| Our country: closing states | b | S | $\Lambda$ n |
| Opening stocks | | n' | |

The following definitions are attached to each of the symbols in the table by Stone (1972a, pp.513-514)

"$\alpha$, a scalar, denotes the total number of individuals who both enter and leave 'our country' in the course of year 0 and so are not recorded in either the opening or closing stock of that year...

b, a column vector, denotes the new entrants into 'our country', namely the births and immigrants of year $\theta$, who survive to the end of the year...

d', a row vector (the prime superscript indicates transposition), denotes the leavers from 'our country', namely the deaths and emigrants of year $\theta$...

S, a square matrix, denotes the survivors in 'our country' through year $\theta$, and these are recorded in both the opening and closing stock. They are classified by their opening states in the columns and by their closing states in the rows.

n', a row vector, denotes the opening stock in each state...

$\Lambda$n a column vector, denotes the closing stock in each state.

This presentation of the closed demographic accounts matrix is quite general - the states referred to can be year of birth or age group. Sex can be distinguished by drawing up matrices and tables separately for males and females. The states which people leave at the start of the year and to which they move at the end of the year could also be regions. The form of the closed dempgraphic accounts matrix developed by Rees and Wilson (1973a) gives to regional location a more prominent place than in Table 1 and also distinguishes birth entries from entries via immigration, and death exits from exits via emigration. Table 2 shows the resultant accounts table for two regions equivalent to the ones ('our country' and 'the outside world') contained in table 1. The equivalence of Table 1 and Table 2 forms of the closed demographic accounts matrix was originally outlined in Rees and Wilson (1973b).

Table 2 Rees and Wilson's closed demographic accounts table

| State at time t → <br> State at time t+T ↓ | | Birth in: The rest of this world R | Birth in: The region of interest i | Opening states/Existence: The rest of this world R | Opening states/Existence: The region of interest i | TOTAL |
|---|---|---|---|---|---|---|
| Death in: | The rest of this world R | $\underline{K}^{\beta(R)\delta(R)}$ | $\underline{K}^{\beta(i)\delta(R)}$ | $\underline{K}^{R\delta(R)}$ | $\underline{K}^{i\delta(R)}$ | $\underline{K}^{*\delta(R)}$ |
| | The region of interest i | $\underline{K}^{\beta(R)\delta(i)}$ | $\underline{K}^{\beta(i)\delta(i)}$ | $\underline{K}^{R\delta(i)}$ | $\underline{K}^{i\delta(i)}$ | $\underline{K}^{*\delta(i)}$ |
| Closing states in/ survival in | The rest of this world R | $\underline{K}^{\beta(R)R}$ | $\underline{K}^{\beta(i)R}$ | $\underline{K}^{RR}$ | $\underline{K}^{iR}$ | $\underline{K}^{*R}$ |
| | The region of interest i | $\underline{K}^{\beta(R)i}$ | $\underline{K}^{\beta(i)i}$ | $\underline{K}^{Ri}$ | $\underline{K}^{ii}$ | $\underline{K}^{*i}$ |
| TOTALS | | $\underline{K}^{\beta(R)*}$ | $\underline{K}^{\beta(i)*}$ | $\underline{K}^{R*}$ | $\underline{K}^{i*}$ | $\underline{K}^{**}$ |

The terms in Table 2 have the following meanings. Note that K is the general variable that refers to population. The superscripts refer to the various 'life states" (existence, birth, death and survival) and regional locations (the region of interest, i, and the rest of this world, R) involved. The underlining means that each term represents a submatrix

$\underline{K}^{ii}$ denotes the people who begin the period alive in region i and the end of period in region i. $K^{RR}$ is the similar term for people surviving in the rest of this world.

$\underline{K}^{iR}$ denotes the people who begin the period alive in region i, who emigrate from region i to region R, and end the period alive in the rest of this world. These are the surviving emigrants from the region of interest. $\underline{K}^{Ri}$ is the equivalent immigration term.

$\underline{K}^{i\delta(i)}$ refers to the people who begin the period in region i and who die there. $\underline{K}^{R\delta(R)}$ is the equivalent term for the rest of the world.

$\underline{K}^{\beta(i)i}$ denotes infants born in region i during the period and surviving there at the end of the period. $\underline{K}^{\beta(R)R}$ is the corresponding item for region R.

$\underline{K}^{\beta(i)R}$ are infants born in region i who migrate to the rest of the world and survive there at the end of the period. $\underline{K}^{\beta(R)i}$ are the surviving infant migrants born in the region R who migrate to region i.

$\underline{K}^{\beta(i)\delta(i)}$ are infants born in region i who die there in the same period. $\underline{K}^{\beta(R)\delta(R)}$ are the equivalent infant deaths in the rest of the world.

$\underline{K}^{\beta(i)\delta(R)}$ denote infants born in region i, who migrate to region R and die there during the period. $\underline{K}^{\beta(R)\delta(i)}$ are the surviving infant imigrants who move from region R to region i.

The Table 2 marginals refer to the opening and closing stocks of population in the two regions, and to the total birth and death occurrences in the regions. The terms are defined as follows.

$\underline{K}^{i*}$ denotes the population of region i at the beginning of the period (in the various states, e.g. sex/age groups).

$\underline{K}^{\beta(i)*}$ denotes the total number of births (live) that occur in region i of interest during the period.

$\underline{K}^{*\delta(i)}$ denotes the total number of deaths recorded in region i during the period.

$\underline{K}^{*i}$ refers to the population of region i at the end of the period

$\underline{K}^{R*}$, $\underline{K}^{\beta(R)*}$, $\underline{K}^{*\delta(R)}$ and $\underline{K}^{*R}$ are the equivalent totals for the rest of the world, region R.

Compared with previous expositions of these life state/region generalized closed demographic accounts, the present form is a transposed version re-arranged so as to match the structure of Stone's closed demographic matrix

(Table 1). The original ordering of the subscripts is retained, however, so that the first one in the Table 2 form of the closed demographic matrix refers to the column and the second one to the row.

Careful comparison of Table 1 and Table 2 and the acompanying definitions reveals the following matching of Stone's terms and those of Rees and Wilson, regarding region i as "our country".

| <u>Stones's terms</u> | <u>Equivalent Rees and Wilson terms</u> |
|---|---|
| S | $\underline{K}^{ii}$ |
| d' | $\underline{K}^{i\delta(R)}$ |
| | $\underline{K}^{i\delta(i)}$ |
| | $\underline{K}^{iR}$ |
| b | $\underline{K}^{\beta(R)i}$ $\underline{K}^{\beta(i)i}$ $\underline{K}^{Ri}$ |
| α | $\underline{K}^{\beta(i)\delta(i)}$ |

Terms $\underline{K}^{R\delta(R)}$, $\underline{K}^{R\delta(i)}$, $\underline{K}^{RR}$, $\underline{K}^{\beta(i)\delta(R)}$, $\underline{K}^{\beta(i)R}$, $\underline{K}^{\beta(R)\delta(R)}$, $\underline{K}^{\beta(R)\delta(i)}$ and $\underline{K}^{\beta(R)R}$ are not explicitly considered in Stone's framework. Table 3 shows how the symbols used byStone are matched with those used by Rees and Wilson.

Table 3 <u>The correspondence of the Stone and the Rees and Wilson closed demographic accounts notations</u>

| State at time t + T \ State at time t | | Birth in: The rest of this world R | Birth in: The region of interest i | Opening states in: The rest of this world R | Opening states in: The region of interest i | TOTALS |
|---|---|---|---|---|---|---|
| Death in: | The rest of this world R | - | $\underline{K}^{\beta(i)\delta(R)}$ | - | $\underline{K}^{i\delta(R)}$ | - |
| | The region of interest i | $\underline{K}^{\beta(R)i}$ | $\underline{K}^{\beta(i)\delta(i)}$ | $\underline{K}^{i\delta(i)}$ | $\underline{K}^{i\delta(i)}$ | $\underline{K}^{*\delta(i)}$ |
| Closing states in: | The rest of this world R | - | $\underline{K}^{\beta(i)R}$ | α - | $\underline{K}^{iR}$ d' | - |
| | The region of interest i | $\underline{K}^{\beta(R)i}$ | $\underline{K}^{\beta(i)i}$ | $\underline{K}^{Ri}$ b | $\underline{K}^{ii}$ S | $\underline{K}^{*i}$ An |
| TOTALS | | - | $\underline{K}^{\beta(i)*}$ | - | $\underline{K}^{i*}$ n | - |

The notation used for time is matched as follows

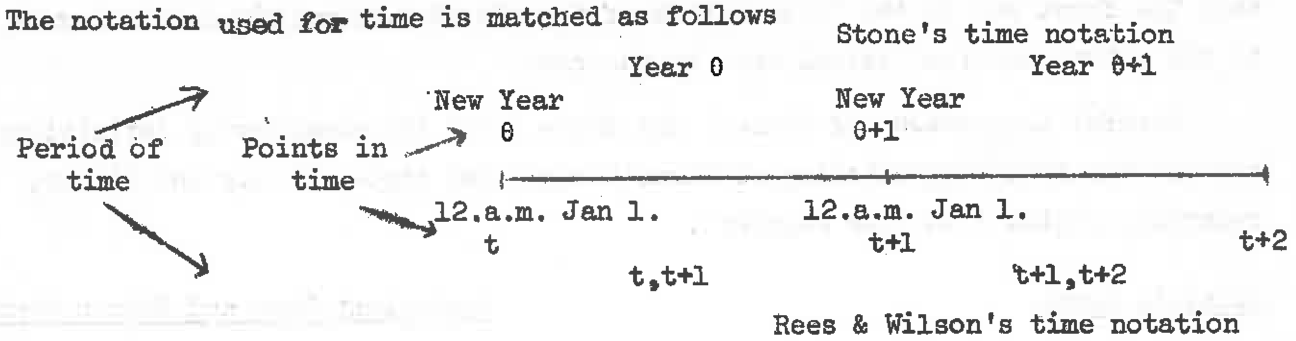


Figure 1. Time notation

The transition flows of population take place over periods θ,θ+1, ... in Stone's notation or periods (t,t+1), (t+1,t+2) in Rees and Wilson's notation. Generally, however, time labels are not attached to closed demographic accounts terms as it is usually clear which single period of time between two particular points in time is being referred to.

An example of a closed demographic accounts table is given in Table 4. The table is a condensed version of that presented in Smith and Rees (1974).

Table 4. A closed demographic accounts table for England and Wales, 1961-1966 intercensal period

| State at Census date 1966 \ State at Census date 1961 | | | Births in: The rest of this world RTW | Births in: England and Wales EW | Opening states in: The rest of this world RTW | Opening states in: England and Wales EW | Totals |
|---|---|---|---|---|---|---|---|
| Death in: | The rest of this world | RTW | - | 586 | - | 7,315 | - |
| | England & Wales | EW | 483 | 84,970 | 8,096 | 2,670,671 | 2,764,220 |
| Closing States in: | The rest of this world | RTW | - | 57,574 | - | 829,346 | - |
| | England & Wales | EW | 47,533 | 4,115,430 | 1,071,258 | 42,597,211 | 47,831,432 |
| Totals | | | - | 4,258,560 | - | 46,104,543 | - |

It differs somewhat therefore, because of its age/sex disaggregated basis, from that presented in Rees and Wilson (1973a) or Rees and Wilson (1974a, chapter 7, Table 7.9) The methods involved in filling these accounts are described there.

Table 4 reveals that, in the intercensal five years period 1961 - 1966 (April 23/24 to April 24/25) some 42,597,211 persons* survive in England and Wales of the initial population. Some 829,346 of the initial population leave the country to settle and survive in the rest of the world. Rather more, 2,670,671 leave for the heavenly regions while still living in England and Wales, and some 7,315 depart to the rest of this world first before journeying to the other worlds. These four population flows account for the behaviour of the initial England and Wales population over the period, and sum to the column total of 46,104,543 the census date 1961 population of England and Wales. England and Wales receives population flows from the rest of this world, the 1,071,258 surviving immigrants, from births to mothers in England and Wales, some 4,115,430 and from births to mothers in the rest of the world, some 47,533, the infants subsequently migrating to and surviving in England and Wales. These three population flows together with the survivors in England and Wales add up to the population of 47,831,432 in England and Wales at census date 1966. The other terms recorded in the table do not contribute to either the opening or the closing population stock of England and Wales, but those in the second column do contribute to the total of clients recorded in England and Wales, and those in the second row do contribute to the total of deaths recorded in England and Wales.

The flows, $K^{R\,\delta(R)}$, $K^{RR}$, $K^{\beta(R)\,\delta(R)}$ and $K^{\beta(R)R}$ are unrecorded because of difficulty in estimation and lack of direct involvement in the England and Wales accounts. We shall show that persons in these flows may have had indirect involvement in the England and Wales accounts.

The relationships inherent in a closed demographic accounts table can be stated formally as a set of accounting identities referring to the columns and the rows of Table 2.

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*This figure is stated rather more exactly than the likely error of estimation would suggest is likely. The accounting framework demands such "precision" though correction to the nearest 100, or possibly 1000, would be more representative of the accuracy table.

The initial population vector is given by

$$\underline{K}^{i\delta(R)} + \underline{K}^{i\delta(i)} + \underline{K}^{iR} + \underline{K}^{ii} \equiv \underline{K}^{i*} \tag{1}$$

which in our simple all age and sex example reads

$$7{,}315 + 2{,}670{,}671 + 829{,}346 + 42{,}597{,}211 \equiv 46{,}104{,}543 \tag{2}$$

The total number live born in the region is represented by the identity.

$$\underline{K}^{\beta(i)\delta(R)} + \underline{K}^{\beta(i)\delta(i)} + \underline{K}^{\beta(i)R} + \underline{K}^{\beta(i)i} \equiv \underline{K}^{\beta(i)*} \tag{3}$$

This identity for England and Wales is, not distinguishing age and sex,

$$586 + 84{,}970 + 57{,}574 + 4{,}115{,}430 \equiv 4{,}258{,}560 \tag{4}$$

The identity referring to the total number dying in a region is

$$\underline{K}^{\beta(R)\delta(i)} + \underline{K}^{\beta(i)\delta(i)} + \underline{K}^{R\delta(i)} + \underline{K}^{i\delta(i)} \equiv \underline{K}^{*\delta(i)} \tag{5}$$

which for our Table 3 example for England and Wales is

$$483 + 84{,}970 + 8{,}096 + 2{,}670{,}671 \equiv 2{,}764{,}220 \tag{6}$$

The closing stock or end of period population identity is

$$\underline{K}^{\beta(R)i} + \underline{K}^{\beta(i)i} + \underline{K}^{Ri} + \underline{K}^{ii} \equiv K^{*i} \tag{7}$$

with the Table 3 example for England and Wales being

$$47{,}533 + 4{,}115{,}430 + 1{,}071{,}258 + 42{,}597{,}211 = 47{,}831{,}432 \tag{8}$$

Each of the identities represented by equations (1), (3), (5) and (7) can be made explicit for many regions and for individual age-sex groups of various kinds (Rees and Wilson, 1973a, Wilson and Rees, 1974). Disaggregation by age-sex groups is, of course, essential for any study of population change and for forecasting.

## 4. Balance sheet accounts

Closed demographic accounts are very demanding of our existing population statistics and extensive estimation methods have had to be devised to fill in the accounts from available information (Rees and Wilson, 1973a, Wilson and Rees, 1974). Conventional population accounts are less demanding. These accounts derive from the accounting equations linking opening and closing stock populations which may be verbally stated as

| | | | |
|---|---|---|---|
| Population at the end of the period | ≡ | Population at the start of the period. | |
| | | plus births<br>plus in-migrants | less deaths<br>less out-migrants |

We can view this equation as representing the inputs into and outputs from a "region-period" in a time-space diagram (Figure 2).

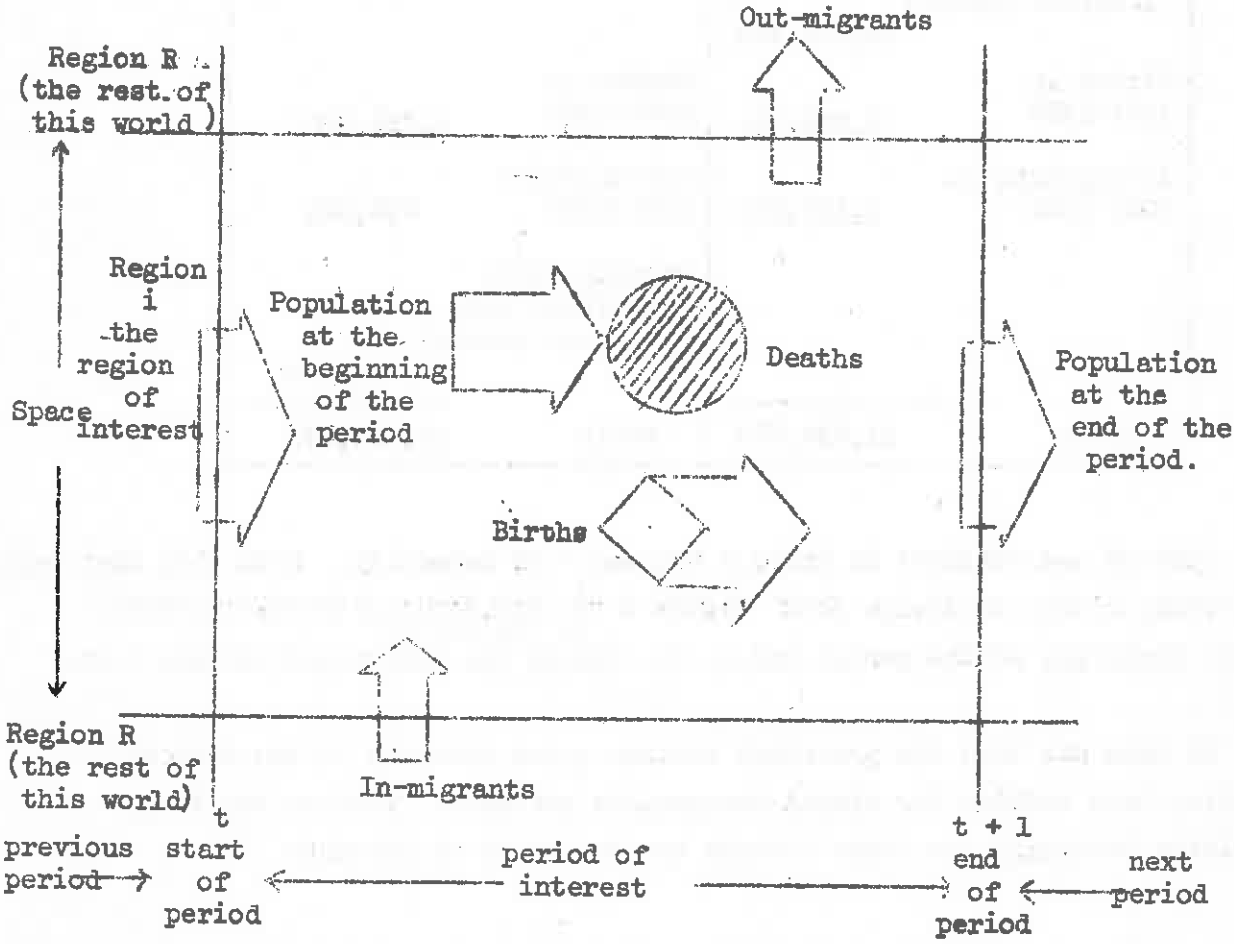



Figure 2 A time-space diagram representation of balance sheet demographic accounts

Inputs to the "region-period" are the population stock crossing time t (the population of the region at the start of the period), births that occur in the region and in the period t,t+1 and in-migrants from the rest of the world. Outputs from the "region-period" are deaths occuring in the region and period, out-migrants to the rest of the world, and the population stock crossing time t + 1 (the population of the region at the end of the period).

These inputs and outputs can be arranged in the form of a balance sheet following the structure of Table 3.1 in Stone, 1971b. This is done for England and Wales aggregate accounts for intercensal period 1961-1966 in Table 5.

Table 5. Balance sheet accounts for England and Wales population (both sexes) for intercensal period 1961-1966

| Inflows | | Outflows | |
|---|---|---|---|
| Opening stock (survivors from previous period) | 46,104,543 | | |
| Births in 1961-1966 | 4,258,560 | Deaths in 1961-1966 | 2,764,220 |
| In-migrants in 1961-1966 | 1,127,370 | Out-migrants 1961-1966 | 894,821 |
| | | Closing stock (survivors into the next period) | 47,831,432 |
| TOTAL | 51,490,473 | TOTAL | 51,490,473 |

Inflows and outflows in Table 5 balance - of necessity. Note also that we gain relatively little knowledge from figure 2 or from Table 5 about the states at the beginning of the period and at the end of the period of the inflow and outflow terms.

We have not thus far presented balance sheet accounts in the mathematical notation used earlier for closed demographic accounts. This we can do by examining carefully the links between the two kinds of accounts.

## 5. The links between closed demographic accounts and balance sheet accounts

The verbal terms in equation (9) can be replaced by terms in the notation of Table 2.

$$K^{*i}(t+1) = K^{i*}(t) + K^{\beta(i)*}(t,t+1) + M^{Ri}(t,t+1) - K^{*\delta(i)}(t,t+1) - M^{iR}(t,t+1) \quad (10)$$

and Table 5 shows these terms in an accounts table.

Table 5. Balance sheet accounts for region i of interest: in K notation and Stone's open demographic accounts notation

| Inflows | | Outflows | |
|---|---|---|---|
| Opening stock (survivors from previous period) | $K^{i*}(t)$ $(A^{-1}s)$ | | |
| Births in period | $K^{\beta(i)*}(t,t+1)$ | Deaths in period | $K^{*\delta(i)}(t,t+1)$ |
| In-migrants in period | $M^{Ri}(t,t+1)$ (b') | Out-migrants in period | $M^{iR}(t,t+1)$ (d) |
| | | Closing stock (survivors into the next period) | $K^{*i}(t+1)$ (s) |
| Total population flows in | (p') | Total population flows out | (p) |

NOTE: Stone's notation is in parentheses. Stone's open demographic notation is explained in section 7 of the paper.

The in-migrant and out-migrant terms have been recorded in a different notation from most of the flows in Table 2. They may be defined as of persons migrating between region i and region R ($M^{Ri}$) or between region R and region i, irrespective of location at the beginning or end of the period.

For example, the term $M^{iR}$ may include migrants who survive the period, $K^{iR}$, migrants who don't, $K^{i\delta(R)}$, persons born in region i who migrate to region R, $K^{\beta(i)R}$, or persons born in regions i who migrate to region R and die there, $K^{(i)(R)}$.

We can ascertain what terms from the closed demographic accounts $M^{iR}$ and $M^{Ri}$ are made up of by substituting for $K^{*i}$, $K^{i*}$, $K^{\beta(i)*}$ and $K^{*\delta(i)}$ from the accounting identities set out earlier. First, we rearrange equation (10) to give an equation for $(M^{iR}-M^{iR})$ dropping the time notation.

$$M^{Ri} - M^{iR} = K^{*i} - K^{i*} - K^{\beta(i)*} + K^{*\delta(i)} \tag{11}$$

and then we substitute for $K^{*i}$ from equation (7), for $K^{i*}$ from equation (1), for $K^{\beta(i)*}$ from equation (3) and for $K^{*\delta(i)}$ from equation (5).

$$\begin{aligned} M^{Ri} - M^{iR} = &\ (K^{\beta(R)i} + K^{\beta(i)i} + K^{Ri} + K^{ii}) \\ - &\ (K^{i\delta(R)} + K^{i\delta(i)} + K^{iR} + K^{ii}) \\ - &\ (K^{\beta(i)\delta(R)} + K^{\beta(i)\delta(i)} + K^{\beta(i)R} + K^{\beta(i)i}) \\ + &\ (K^{\beta(R)\delta(i)} + K^{\beta(i)\delta(i)} + K^{R\delta(i)} + K^{i\delta(i)}) \end{aligned} \tag{12}$$

After cancelling, equation (12) becomes

$$\begin{aligned} M^{Ri} - M^{iR} = &\ (K^{\beta(R)i} + K^{Ri}) \\ &- (K^{i\delta(R)} + K^{iR}) \\ &- (K^{\beta(i)\delta(R)} + K^{\beta(i)R}) \\ &+ (K^{\beta(R)\delta(i)} + K^{R\delta(i)}) \end{aligned} \tag{13}$$

which can be rearranged as

$$\begin{aligned} M^{Ri} - M^{iR} = &\ (K^{Ri} - K^{iR}) \\ &+ (K^{R\delta(i)} - K^{i\delta(R)}) \\ &+ (K^{\beta(R)i} - K^{\beta(i)R}) \\ &+ (K^{\beta(R)\delta(i)} - K^{\beta(i)\delta(R)}) \end{aligned} \tag{14}$$

Verbally, this final rearrangement of the equation for the net balance of in-migrants and out-migrants can be expressed as a sum of net balance of migration =

the net balance of surviving migrants alive at times t and t+1

+

the net balance of non-surviving migrants alive at time t, dying in period t to t+1

+

the net balance of surviving infant migrants born in t to t+1, alive at time t.

+

the net balance of non-surviving infant migrants born in period t to t+1, and dying in the period.

(15)

Equations (14) and (15) have an important implication for the way in which balance sheet accounts are constructed. It is clear that use of the net balance of surviving migrants alone as an estimate of net migration is not an adequate practice though a common one.

Is it possible to extend the finding of equation (14) and split up the left and right hand sides into two separate equations? Those equations might read as follows

$$M^{Ri} = K^{Ri} + K^{R\delta(i)} + K^{\beta(R)i} + K^{\beta(R)\delta(i)} \qquad (16)$$

and

$$M^{iR} = K^{iR} + K^{i\delta(R)} + K^{\beta(i)R} + K^{\beta(i)\delta(R)} \qquad (17)$$

The answer is, in fact, no and we shall see why if we examine carefully the phenomenon of multiple migration in a period.

## 6. Multiple migration in a period and associated accounts

Consider the lifelines represented on the time-space diagram in Figure 3. A selection of the lifelines associated with the populations of a closed demographic accounts matrix have been plotted.
Lifeline A is that of a person who is counted in the $K^{Ri}$ flow, but who makes two migrations from the rest of this world to the region of interest, and one from the region of interest to the rest of this world. The second, lifeline B, shows that it is quite possible for a person who survives the period within the region of interest to have migrated abroad and back again within the period*

*For example, between the censuses of 1966 and 1971 in the U.K. in both of which the author was recorded as living (and therefore "surviving") within the U.K., he spent four out of five years of the intercensal period in the U.S.A.

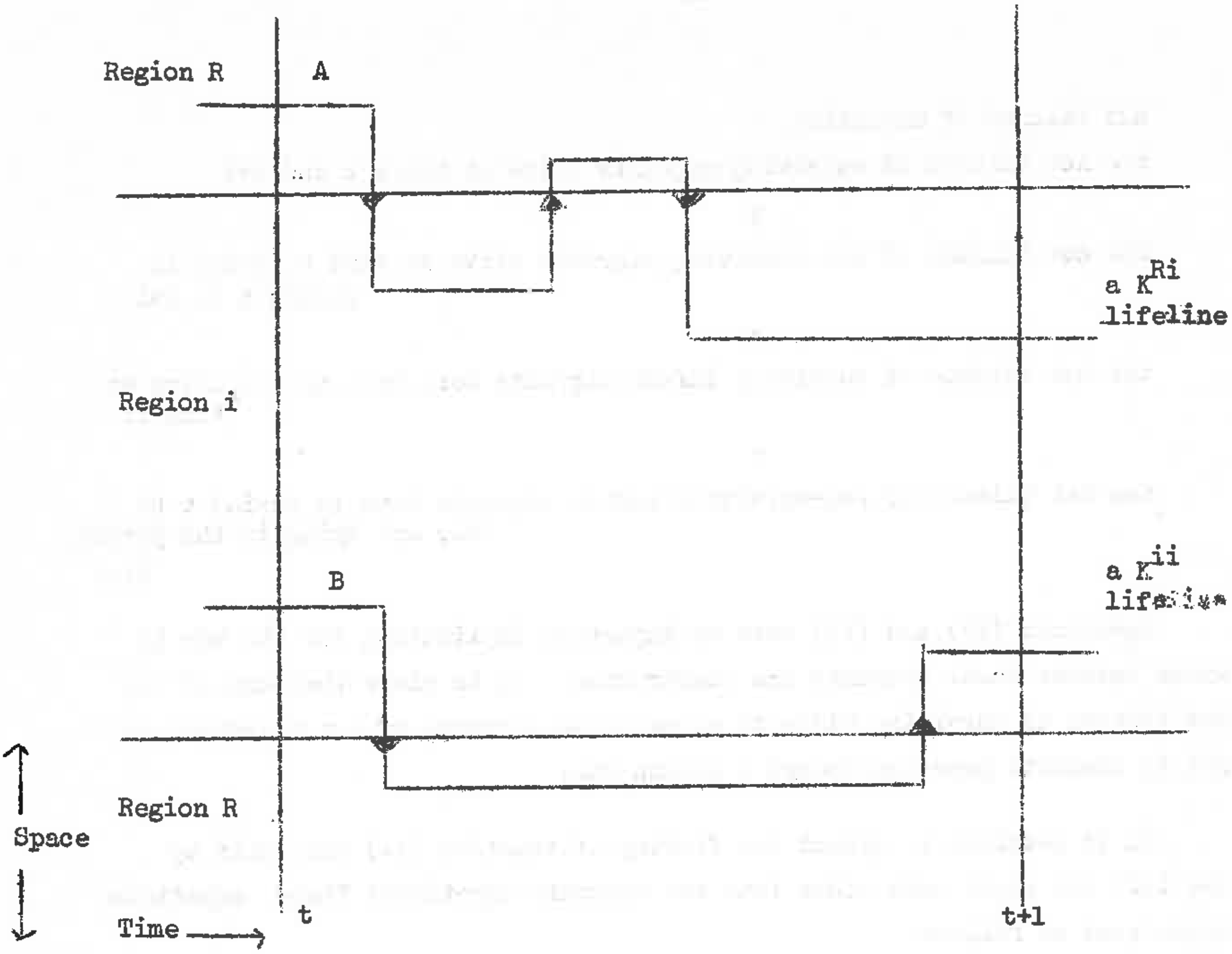



Figure 3. A time-space diagram showing multiple migration in a period

So we must include in our count of migrants from region i to region R or from region R to region i the additional migrations of persons recorded in migrating population flows in the closed demographic accounts and the migrations of persons recorded in the non-migrating population flows in the closed demographic accounts. This means that equations (16) and (17) are not valid counts of $M^{Ri}$ and $M^{iR}$ respectively. However, equation (14) which expresses the net balances of migration as a sum of the population flow terms is valid numerically as the additional migrations between region i and region R and between region R and region i must always balance exactly and therefore cancel each other out numerically in equation (14).

Why must these additional migrants always balance? Consider persons in the $K^{ii}$ flow or survivors in region i. If a person counted in this population flow makes one migration from region i to region R, he or she must necessarily make a return migration from region R to region i. Otherwise the person would not be included in the $K^{ii}$ term. If a $K^{ii}$ person had made two migrations from region i to the rest of the world, he or she would have to make two return migrations. Similar cosiderations hold for persons in one of the migrating population flows. Consider persons in the $K^{iR}$ flow who make two migrations between region i and region R, one of which is recorded in the (i,R) label and one of which is additional. The additional one must be balanced by a return migration from the rest of the world to region i.

These arguments can be generalized as follows. If a person in a (i,i) flow makes n migrations between region i and region R, n migrations must have been made between region R and region i. If a person in a (i,R) flow makes n migrations between region i and regionR, n-1 migrations must have been made in the reverse direction*. Similar arguments apply to (R,R) and (R,i) flows.

We can classify persons in any population flow according to the number of migrations made from region i to region R and from region R to region i. We define, in general,

| | |
|---|---|
| $K^{kl}(i,j;n)$ | as the number of persons who start the period in state k, and finish it in state l who make n migrations from region i to region j. |

Table 7 shows for the population flows involved in our closed demographic accounts matrix (Table 2) what migration accounts derived from such an extended notation might look like. The rows of the table designate the population flow involved and the columns the number of migrations from region i to region R and vice versa. The rows sum to the number in the population flow for the two migrations distinguished.

*Note that these relationships apply only to the two regions i (of interest) and R (the rest of the world), and not between any pair of regions that do not completely partition the world.

Table 7. An accounts table linking population flows in a closed demographic accounts matrix and migrations between regions.

| Population flow | Migrations from region i to region R : number | | | Total |
|---|---|---|---|---|
| | 0 | 1 | 2 ... | |
| (i,i) | $K^{ii}(i,R,0)$ | $K^{ii}(i,R,1)$ | $K^{ii}(i,R,2)$ | $K^{ii}$ |
| (i,R) | $K^{iR}(i,R,0)$ | $K^{iR}(i,R,1)$ | $K^{iR}(i,R,2)$ | $K^{iR}$ |
| (i,δ(i)) | . | . | . | $K^{i\delta(i)}$ |
| (i,δ(i)) | . | . | . | $K^{i\delta(R)}$ |
| (R,i) | . | . | . | $K^{Ri}$ |
| (R,R) | . | . | . | $K^{RR}$ |
| (R,δ(i)) | . | . | . | $K^{R\delta(i)}$ |
| (R,δ(R)) | . | . | . | $K^{R\delta(R)}$ |
| (β(i),i) | . | . | . | $K^{\beta(i)i}$ |
| (β(i),R) | . | . | . | $K^{\beta(i)R}$ |
| (β(i),δ(i)) | . | . | . | $K^{\beta(i)\delta(i)}$ |
| (β(i),δ(R) ) | . | . | . | $K^{\beta(i)\delta(R)}$ |
| (β(R),i) | . | . | . | $K^{\beta(R)i}$ |
| (β(R),R) | . | . | . | $K^{\beta(R)R}$ |
| (β(R),δ(i)) | . | . | . | $K^{\beta(R)\delta(i)}$ |
| (β(R),δ(R)) | $K^{\beta(R)\delta(R)}(i,R,0)$ | $K^{\beta(R)\delta(R)}(i,R,1)$ | $K^{\beta(R)\delta(R)}(i,R,2)$ | $K^{\beta(R)\delta(R)}$ |

| Population flow | Migrations from region R to region i : number | | | Total |
|---|---|---|---|---|
| | 0 | 1 | 2 ... | |
| (i,i) | $K^{ii}(R,i,0)$ | $K^{ii}(R,i,1)$ | $K^{ii}(R,i,2)$ | $K^{ii}$ |
| (i,R) | $K^{iR}(R,i,0)$ | $K^{iR}(R,i,1)$ | $K^{iR}(R,i,2)$ | $K^{iR}$ |
| (i,δ(i)) | . | . | . | $K^{i\delta(i)}$ |
| (i,δ(R)) | . | . | . | $K^{i\delta(R)}$ |
| (R,i) | . | . | . | $K^{Ri}$ |
| (R,R) | . | . | . | $K^{RR}$ |
| (R,δ(i)) | . | . | . | $K^{R\delta(i)}$ |
| (R,δ(R)) | . | . | . | $K^{R\delta(R)}$ |
| (β(i),i) | . | . | . | $K^{\beta(i)i}$ |
| (β(i),R)) | . | . | . | $K^{\beta(i)R}$ |
| (β(i),δ(i) | . | . | . | $K^{\beta(i)\delta(i)}$ |
| (β(i),δ(R)) | . | . | . | $K^{\beta(i)\delta(R)}$ |
| (β(R),i) | . | . | . | $K^{\beta(R)i}$ |
| (β(R),R) | . | . | . | $K^{\beta(R)R}$ |
| (β(R)δ(i)) | . | . | . | $K^{\beta(R)\delta(i)}$ |
| (β(R),δ(R)) | $K^{\beta(R)\delta(R)}(R,i,0)$ | $K^{\beta(R)\delta(R)}(R,i,1)$ | $K^{\beta(R)\delta(R)}(R,i,2)$ | $K^{\beta(R)\delta(R)}$ |

Certain of the entries in the table do not occur, by definition. People in flows involving (i,R) transitions must make at least one (i,R) migration. The terms $K^{iR}(i,R,0)$, $K^{i\delta(R)}(i,R,0)$ and so on, and similar $K^{Ri}(R,i,0)$, $K^{R\delta(i)}(R,i,0)$ terms have numerical values of zero. Note, however, that terms such as $K^{iR}(R,i,0)$ are very likely to contain substantial numbers of persons. The necessary balancing relationships referred to above will occur in the table as equations like

$$K^{ii}(i,R,0) = K^{ii}(R,i,0) \tag{18}$$
$$K^{ii}(i,R,1) = K^{ii}(R,i,1) \tag{19}$$
$$K^{ii}(i,R,2) = K^{ii}(R,i,2) \tag{20}$$

and

$$K^{iR}(i,R,1) = K^{iR}(R,i,0) \tag{21}$$
$$K^{iR}(i,R,2) = K^{iR}(R,i,1) \tag{22}$$

or in general,

$$K^{ii}(i,R,n) = K^{ii}(R,i,n) \tag{23}$$

and

$$K^{iR}(i,R,n) = K^{iR}(R,i,n-1) \tag{24}$$

No empirical example can be offered to illustrate Table 6 and its multiple relationships as very detailed information on migration histories would be required. Such a table could, however, be generated fairly easily in a country with a population register.

The number of migrations between region i and region R can be linked to each term in the Table 7 accounts as follows

$$M^{iR}(K^{kl}) = \sum_n nK^{kl}(i,R,n) \tag{25}$$

where $M^{iR}(K^{kl})$ is the number of migrations between region i and region R made by persons in the $K^{kl}$ population flow.
Equation (25) can be summarized as

$$M^{iR} = \sum_k\sum_l M^{iR}(K^{kl}) = \sum_k\sum_l\sum_n nK^{kl}(i,R,n) \tag{26}$$

A similar equation can be stated for $M^{Ri}$.

We are now in a position to specify correct versions of equations (16) and (17). Each term on the right hand side of the equations (16) and (17) can be stated in the new notation a little differently as, for example,

$$K^{Ri} = \sum_n (1) K^{Ri}(R,i,n) \tag{27}$$

we are counting just one migration per person here. The additional migrations are $\sum_n (n-1)K^{Ri}(R,i,n)$ in this case. We can therefore revise equation (16) to read

$$\begin{aligned} M^{Ri} = \; & K^{Ri} + K^{R\delta(i)} + K^{\beta(R)i} + K^{\beta(R)\delta(i)} \\ & + \sum_n (n-1)K^{Ri}(R,i,n) + \sum_n (n-1)K^{R\delta(i)}(R,i,n) \\ & + \sum_n (n-1)K^{\beta(R)i}(R,i,n) + \sum_n (n-1)K^{\beta(R)\delta(i)}(R,i,n) \\ & + \sum_{k\neq R}\sum_{l\neq i} n\, K^{kl}(R,i,n) \end{aligned} \tag{28}$$

and equation (17) to read

$$\begin{aligned} M^{iR} = \; & K^{iR} + K^{i\delta(R)} + K^{\beta(i)R} + K^{\beta(i)\delta(R)} \\ & + \sum_n (n-1)K^{iR}(i,R,n) + \sum_n (n-1)K^{i\delta(R)}(i,R,n) \\ & + \sum_n (n-1)K^{\beta(i)R}(i,R,n) + \sum_n (n-1)K^{\beta(i)\delta(R)}(i,R,n) \\ & + \sum_{k\neq i}\sum_{l\neq i} n\, K^{kl}(i,R,n) \end{aligned} \tag{29}$$

For a two region world, as we have said before, the following equality holds

$$\begin{aligned} & \sum_n (n-1)K^{Ri}(R,i,n) + \sum_n (n-1)K^{R\delta(i)}(R,i,n) \\ & + \sum_n (n-1)K^{\beta(R)i}(R,i,n) + \sum_n (n-1)K^{\beta(R)\delta(i)}(R,i,n) \\ & + \sum_{k\neq R}\sum_{l\neq i} nK^{kl}(R,i,n) = \sum_n (n-1)K^{iR}(i,R,n) + \sum_n (n-1)K^{i\delta(R)}(i,R,n) \\ & \qquad + \sum_n (n-1)K^{\beta(i)R}(i,R,n) + \sum_n (n-1)K^{\beta(i)\delta(R)}(i,R,n) \\ & \qquad + \sum_{k\neq i}\sum_{l\neq R} nK^{kl}(i,R,n) \end{aligned} \tag{30}$$

The counts of migration between region i and region R can be assembled into a simple migration table (Table 8) which indicates how many crossings of the (i,R) border took place in either direction and how many internal migrations took place.

Table 8. A migration table : conceptual form

| From \ To | | Region of destination in period | | |
|---|---|---|---|---|
| | | Region i | Region R | Totals |
| Region of origin in period | Region i | $M^{ii}$ | $M^{iR}$ | $M^{i*}$ |
| | Region R | $M^{Ri}$ | $M^{RR}$ | $M^{R*}$ |
| TOTALS | | $M^{*i}$ | $M^{*R}$ | $M^{**}$ |

The term $M^{ii}$ in Table 8 consists of the migrations within a region i of interest that occur between places of permanent residence. Internal migrations of this kind are represented in the time-space diagram (Figure 4) as shifts of lifelines in the vertical direction where the shifts do not cross regional boundaries

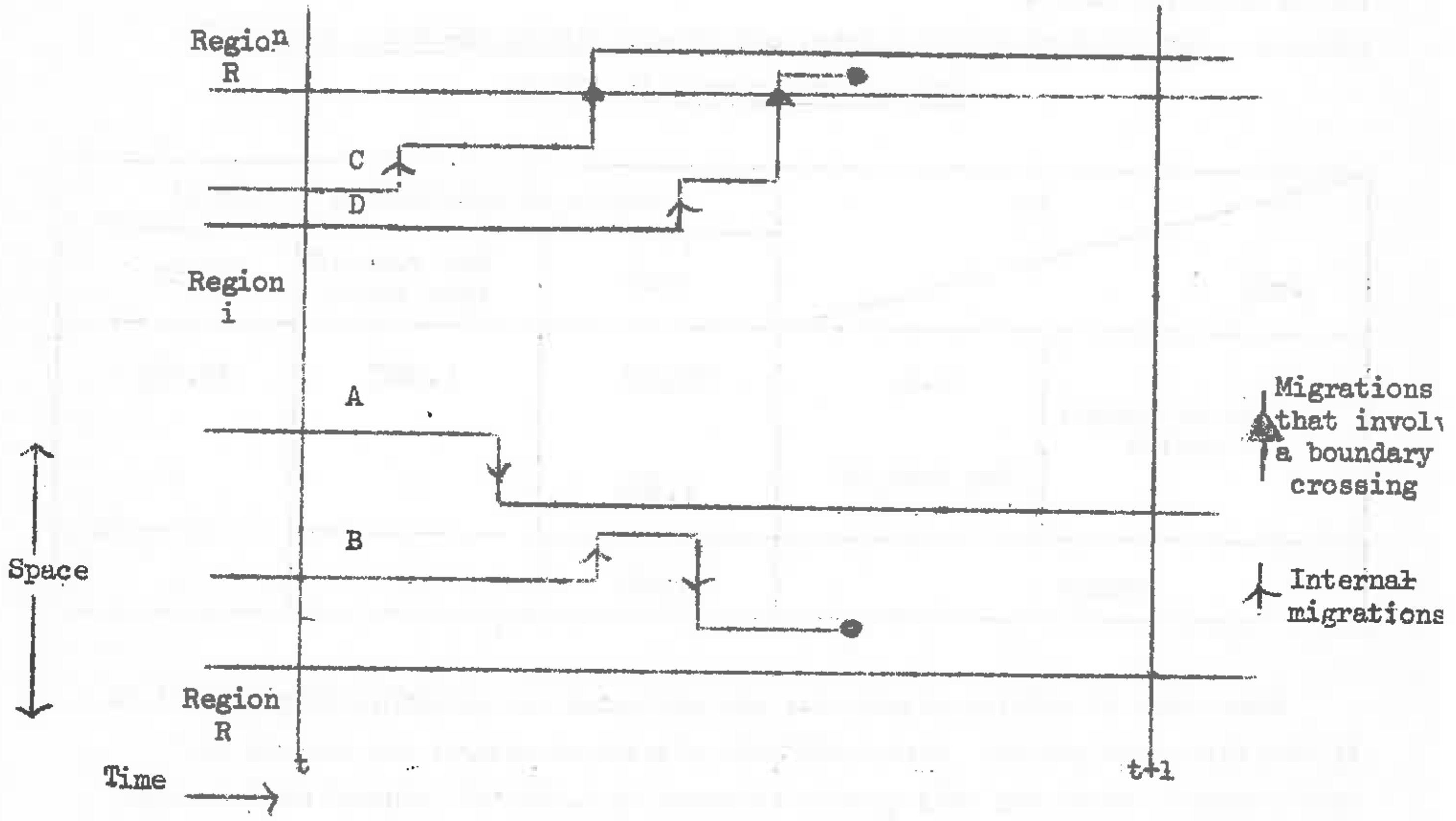



Figure 4. A time-space diagram showing internal migration

Whereas regional boundary crossing migrations are normally recorded in some fashion or other (see Table 9), internal migrations are not fully recorded except in population registers. Census migration tables usually give information about the number of surviving migrants ($K^{ij}$ type information). Even if full migration histories are collected in censuses or by sample survey (by general household survey, for example) they fail to record the migrations of persons who die, out-migrate, or outmigrate and die (lifelines B,C, and D in Figure 4).

An approximate estimate of the number of migrations taking place into, out of and within the U.K. in the intercensal period 1966-1971 is given in Table 9. The figures are based on counts derived from the International Passenger Survey and reported in the Central Statistical Office's Social Trends No.2, 1971, Table 10, page 54, and Social Trends, No,3, 1973 together with estimates of internal migration obtained by multiplying the 1971 census U.K. population by the proportions reported in various move categories and the number of moves in each category (cf. equation (25)) given in Table 5.52, page 155 of O.P.C.S. (1973).

Table 9. Estimated migrations into, out of and within the U.K., 1966-1971 inter-censal period in '000's

| From \ To | | Region of destination in period | | |
|---|---|---|---|---|
| | | U.K. | The rest of this world | TOTALS |
| Regions of origin in period | U.K. | 32,091 | 1,607 | 33,715 |
| | The rest of this world | 1,393 | - | - |
| TOTALS | | 33,477 | - | - |

Just over 32 million migrations are estimated as occurring within the U.K. in the five year period. Some 1,607,000 migrations abroad are thought to have occured; these are only partly balanced by 1,393,000 migrations into the country.

As a result of this discussion of migration in relation to the closed demographic and balance sheet accounts we can list a series of alternative definitions of migration into and out of a region. Each definition is related to the terms introduced in Table 2 and Table 6.

In the first three definitions the count of migrations is an enumeration of persons with no double counting.

(1) The first definition is that of <u>surviving migrants</u> or persons who are alive at the beginning of the period, who migrate from one region to another, and survive there (that is, are alive there at the end of the period)

In-migration into region i is $\underline{K}^{Ri}$
Out-migration from region i is $\underline{K}^{iR}$
This is the definition commonly adopted in national censuses.

(2) The second definition includes <u>surviving migrants</u>, as above, and <u>surviving infant migrants</u>, or persons born in the period in a region who migrate from it to another region and survive there.

In-migration into region i is $\underline{K}^{Ri} + \underline{K}^{\beta(R)i}$
Out-migration from region i is $\underline{K}^{iR} + \underline{K}^{\beta(i)R}$

This definition has been used in some demographic work (Gilje and Campbell, 1972).

(3) The third definition includes <u>surviving migrants</u>, <u>surviving infant migrants</u>, as in the second definition and adds to these <u>non-surviving migrants</u> and <u>non-surviving infant migrants</u>. Non-surviving migrants are persons who start the period alive in a region, who migrate to another region and die there. Non-surviving infant migrants are persons born in a period in a region who migrate to another region and die there.

In-migration to region i is $\underline{K}^{Ri} + \underline{K}^{\beta(R)i} + \underline{K}^{R\,\delta(i)} + \underline{K}^{\beta(R)\delta(i)}$

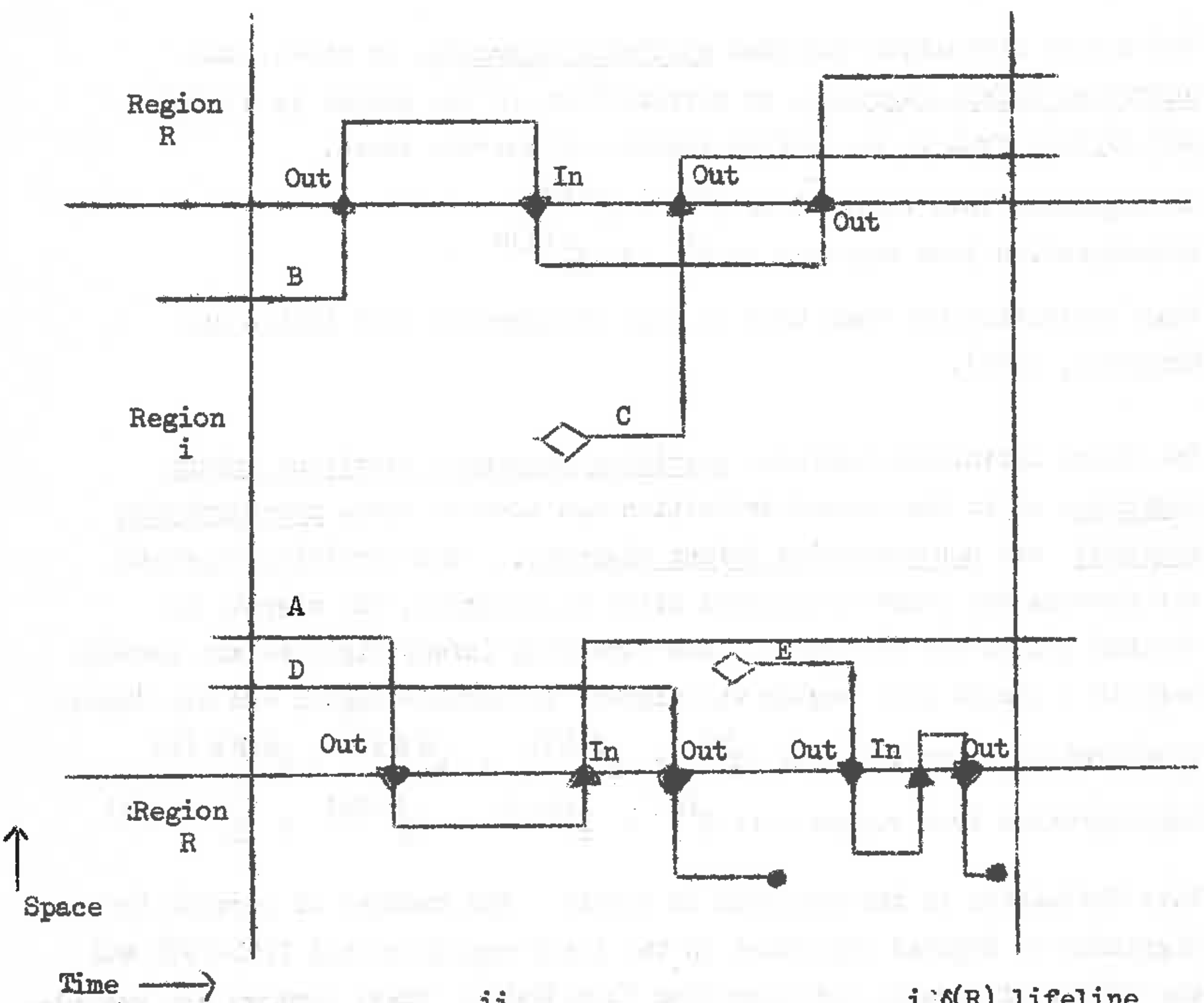
Out-migration from region i is $\underline{K}^{iR} + \underline{K}^{\beta(i)R} + \underline{K}^{R\,\delta(R)} + \underline{K}^{\beta(i)\,\delta(R)}$.

This definition is the one used to obtain the numbers of persons in-migrating to England and Wales in the inter-censal period 1961-1966 and the number of persons out-migrating from there. These numbers are recorded in Table 5.

The final definition counts migrations as moves between regions. A single person may make several moves from one region to another.

(4) The fourth definition of migration is that the migration into or out of a region is <u>the number of moves made by persons into or out of the region</u> In-migration into region i is $M^{Ri}$, which is broken down into the sum of person flows multiplied by number of move terms in equation (28). Out-migration from region i is $M^{iR}$ for which the corresponding equation is equation (29) .
This is the definition of migration adopted in international migration surveys such as the International Passenger Survey conducted by the Office of Population Censuses and Surveys (O.P.C.S., 1971)

Figure 5 illustrates the different numerical results which may be obtained using each of these definitions of migration. Some five lifelines corresponding to some five population flows in the closed demographic accounts have been plotted in a time-space diagram.




A a $K^{ii}$ lifeline
B a $K^{iR}$ lifeline
C a $K^{\beta(i)R}$ lifeline
D a $K^{i\delta(R)}$ lifeline
E a $K^{\beta(i)\delta(R)}$ lifeline

Figure 5. <u>A time-space diagram illustratinf the different concepts of migration</u>

The migration from region of interest i to the rest of the world is according to the respective definitions

| | | | |
|---|---|---|---|
| (1) | 1 | (3) | 4 |
| (2) | 2 | (4) | 7 |

We note that there are 3 returning in-migrations so that

Migration according to definition (3) = Migration according to definition (4) _ return migrations (31)

or

4 = 7 - 3 (32)

These findings concerning the definition of migration have implications for the use of balance sheet accounts. Conventionally, balance sheet accounts are laid out in forms that correspond to equations (9) and (10). Table 10 arranges the demographic statistics for England and Wales given in Tables 4 and 5 in such a conventional accounts table. In the table we use the third definition set out above. If we use definitions (1) or (2) of migration we obtain different and incorrect totals for the 1966 census date population of England and Wales (Table 11). Note, however, that the accounts still balance population flows into the region/period with population flows out. This would be true for any set of in-migration and out-migration figures.

Table 10. A conventional demographic accounts table for England and Wales, inter-censal period 1961-1966

| Population item | Sign in equation (10) | Number | Composite term | Sign | Number | Population flows total |
|---|---|---|---|---|---|---|
| Population at Census date 1961 | + | 46,104,543 | | + | 46,104,543 | In |
| Births | + | 4,258,560 | inflows from "outside world" | + | 5,385,930 | 51,490,473 |
| In-migrations (Definition (3)) | + | 1,127,370 | | | | |
| Deaths | - | 2,764,220 | outflows to "outside world" | - | 3,659,041 | Out |
| Out migrations; (Definition (3)) | - | 894,821 | | | | 51,490,473 |
| Population at Census date 1966 | = | 47,831,432 | | = | 47,831,432 | |

Table 11. <u>Conventional demographic accounts tables for England and Wales inter-censal period 1961-1966, noting alternative definitions of migration.</u>

| Population item | Sign in equation (10) | Number | Population flows total | Number | Population flows total |
|---|---|---|---|---|---|
| Population at Census date 1961 | + | 46,104,543 | 51,434,361 | 46,104,543 | 51,481,899 |
| Births | + | 4,258,560 | | 4,258,560 | |
| In-migration<br>Definition (1)<br>Definition (2) | + | 1,071,258 | | 1,118,791 | |
| Deaths | - | 2,764,220 | 51,434,361 | 2,764,220 | 51,481,899 |
| Out-migrations<br>Definition (1)<br>Definition (2) | - | 829,346, | | 886,920 | |
| Population at Census date 1966 | = | 47,840,795 | | 47,830,759 | |

The net balance of migration under definition (2) involved in the accounting equation is + 231,671 which yields a population for England and Wales at census date 1966 fairly close to that of definition (3). The first definition not involving infant migration yields a higher net balance of + 241,912 and a higher population. Note that the net balance of migration under definition (4) is the same as that under definition (3), that is, + 232,549 because the additional moves cancel out. If the migrations out of region i, counted as moves are,

$$M^{iR} = 894{,}821 + x \tag{33}$$

and into region i are

$$M^{Ri} = 1{,}127{,}370 + y \tag{34}$$

where x is the number of additional moves out and y the number of moves in and

$$x = y \tag{35}$$

then
$$\begin{aligned}(M^{Ri} - M^{iR}) &= (1{,}127{,}370 + x)-(894{,}821 + x) \\ &= 232{,}549 \end{aligned} \tag{36}$$

The lesson we can draw from this detailed examination of migration is that if balance sheet accounts and associated accounting equations are to be used to estimate , and in model form, to predict populations, then the correct definition of migration (definition (3) or (4)) must be used. The calculation of migration probabilities is made easier if definition (3) is adopted.

We return, now, from this digression on the nature of migration statistics to consider the final member of the family of demographic accounts proposed by Richard Stone - open demographic accounts.

## 7. Open demographic accounts

Closed demographic accounts are concerned principally with transitions between states within a period. Open demographic accounts are involved, on the other hand, with the transfers that occur into and out of a period, in particular from and to other periods. Stone has described the formal structure of open demographic accounts most fully in his 1966 Minerva paper (Stone,1966) and used such accounts most extensively in his O.E.C.D. monograph (Stone,1971a).

The symbolic version of Stone's open demographic accounts is reproduced in Table 12 from Stone, Stone and Gurton, 1968.

Table 12. An open demographic accounts matrix (after Stone)

| From \ To | | Outside world | Our country: Last year | Our country: This year | Total flows |
|---|---|---|---|---|---|
| Outside world | | | | $b'$ | |
| Our country | Last year | $\Lambda^{-1}d$ | | $\Lambda^{-1}s$ | $\Lambda^{-1}p$ |
| | This year | | | | |
| Total flows | | | | $p'$ | |

Source: Stone, Stone and Gunton, 1968, Table 1.

Stone assigns the symbols in the table the following meaning

"p = the vector of population flows;

b = the vector of births and immigrations

d = the vector of deaths and emigrations

S = the matrix of survivors

$\Lambda$ = The lag operator which shifts in time the variable to which it is applied; thus if p = this year's population flows, $\Lambda^{-1}p$ = last years population flows and $\Lambda p$ = next year's population flows.

The above vectors are defined as column vectors and the addition of a prime (') indicates the corresponding row of vectors" (Stone, Stone and Gunton, 1968, notes to Table 1).

One important point to note about these set of definitions is that the S matrix of the open demographic accounts matrix is not the same as the S matrix of the closed demographic accounts matrix defined earlier and located in Table 1. The closed accounts matrix consisted of population flows from the beginning of the period to the end of the period. The open accounts S matrix consists of transfers from the end of one period to the beginning of the next. The open S matrix contains the closing stock vector of the closed accounts as a diagonal and shows how it becomes the opening stock vector of the next period. Occasionally the open S matrix may contain transition elements or flows akin to those of the closed S matrix when transfers between states occur at the end/beginning of periods as they do as persons progress through the various classes of the educational system. In Table 7 of Stone, Stone and Gunton (1968), for example, pupils move at a year end/beginning from an "at school state" to a "not at school state".

The same point applies to the definition of the b and d vectors. These are not defined in the same way in Stone's open demographic accounts as they are in his closed accounts. In the open demographic accounts, they refer to the total births and in-migrations, and total deaths and out migrations respectively: the items, in fact, that figured in the balance sheet accounts The meaning of the S, b and d in the open demographic accounts context are perhaps best established by a table of correspondences (Table 13) between the Rees and Wilson notation and Stone's open demographic accounts notation. The reader can compare Table 13 with Table 3 and see how S, b and d are differently defined in the closed and open frameworks.

Table 13. The correspondence of the Stone open demographic accounts notation and the Rees and Wilson notation

| State at time t+T \ State at time t | | Birth in: The rest of this world R | Birth in: The region of interest i | Opening states in: The rest of this world R | Opening states in: The region of interest i | TOTALS |
|---|---|---|---|---|---|---|
| Death in: | The rest of this world R | – | $\underline{K}^{\beta(i)\delta(R)}$ | – | $\underline{K}^{i\delta(R)}$ | – |
| | The region of interest i | $\underline{K}^{\beta(R)\delta(i)}$ | $\underline{K}^{\beta(i)\delta(i)}$ | $\underline{K}^{R\delta(i)}$ | $\underline{K}^{i\delta(i)}$ | $\underline{K}^{*\delta(i)}$ |
| Closing States in: | The rest of this world R | – | $\underline{K}^{\beta(i)R}$ | – | $\underline{K}^{iR}$ d | – |
| | The region of interest i | $\underline{K}^{\beta(R)i}$ | $\underline{K}^{\beta(i)i}$ | $\underline{K}^{Ri}$ b' | $\underline{K}^{ii}$ | $\underline{K}^{*i}$ $\Lambda^{-1}S$ |
| TOTALS | | – | $\underline{K}^{\beta(i)*}$ | – | $\underline{K}^{i*}$ S | |

Note that d may be regarded as the sum of the terms picked out in the interior of the table (within the accounts matrix) or as the sum of $\underline{K}^{*\delta(i)}$, total deaths in region i, and the out-migrations from the region,

$$d = \underline{K}^{*\delta(i)} + \underline{K}^{\beta(i)\delta(R)} + \underline{K}^{i\delta(R)} + \underline{K}^{\beta(i)R} + \underline{K}^{iR} \qquad (37)$$

adopting the definition of migration which ignores multiple moves. Similarly b may be regarded as the sum of $\underline{K}^{\beta(i)*}$, total births in the region, and the in-migrations to the region.

$$b = \underline{K}^{\beta(i)*} + \underline{K}^{\beta(R)\delta(i)} + \underline{K}^{R\delta(i)} + \underline{K}^{Ri} \qquad (38)$$

The stock transfer terms $\Lambda^{-1}s$ and s correspond to $\underline{K}^{i*}$ and $K^{*i}$ respectively.

Since last year's row in Table 12 and this year's column add up to the total flows in the margin, $\Lambda^{-1}p$ and $p^{+}$ (see equations (40) and (46) below), we can identify p with all the terms in the closed demographic accounts (neglecting (R,R)type flows)

$$\begin{aligned} p = {} & \underline{K}^{\beta(i)\delta(R)} + \underline{K}^{i\delta(R)} \\ & + \underline{K}^{\beta(R)\delta(i)} + \underline{K}^{\beta(i)\delta(i)} + \underline{K}^{R\delta(i)} + \underline{K}^{i\delta(i)} \\ & + \underline{K}^{\beta(i)R} + \underline{K}^{iR} \\ & + \underline{K}^{\beta(R)i} + \underline{K}^{\beta(i)i} + \underline{K}^{Ri} + K^{ii} \end{aligned} \qquad (39)$$

Table 12 considers only a single period to period transition. This framework is extended by Stone to show the flows into this year and out of it, and the flows out of and into last year.

We can look at all the flows into and out of this year by expanding Table 12 into Table 14.

Table 14. An open demographic accounts matrix showing both inflows and outflows from this year (after Stone)

| From \ To | | Outside world | Our country: Last year | Our country: This year | Our country: Next year | Total flows |
|---|---|---|---|---|---|---|
| Outside world | | | | $b'$ | $\Lambda b'$ | |
| Our country | Last year | $\Lambda^{-1}d$ | | $\Lambda^{-1}s$ | | $\Lambda^{-1}p$ |
| | This year | $d$ | | | $S$ | $p$ |
| | Next year | | | | | |
| Total flows | | | | $p'$ | $\Lambda p'$ | |

Source: Stone, Stone and Gutton, 1968, Table 2.

Two sets of accounting identities characterize the Table 14 form of the open demographic accounts. The first concern the population flows out of "region-period", for example, "our country-this year":

$$p \equiv Si + d \tag{40}$$

where i denotes the unit column vector, that is,

$$i = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{41}$$

and thus Si denotes the row sums of S. A similar equation applies to "our-country/last year"

$$\Lambda^{-1}p \equiv \Lambda^{-1}Si + \Lambda^{-1}d \tag{42}$$

or in general, adopting θ as a period label

$$p(\theta) \equiv S(\theta)i + d(\theta) \tag{43}$$

The second set of accounting identities concern the population flows into a "region-period":

$$p' \equiv i' \Lambda^{-1}S + b' \tag{44}$$

or if the variables are transposed

$$p \equiv \Lambda^{-1}S'i + b \tag{45}$$

or in general,

$$p(\theta) \equiv \Lambda^{-1}S'(\theta)i + b(\theta) \tag{46}$$

These accounting identities also characterize the balance sheet accounts (Tables 5 and 6). In order to clarify the concepts involved in open demographic accounts we have represented Table 14's accounts in a time-space diagram in figure 5.

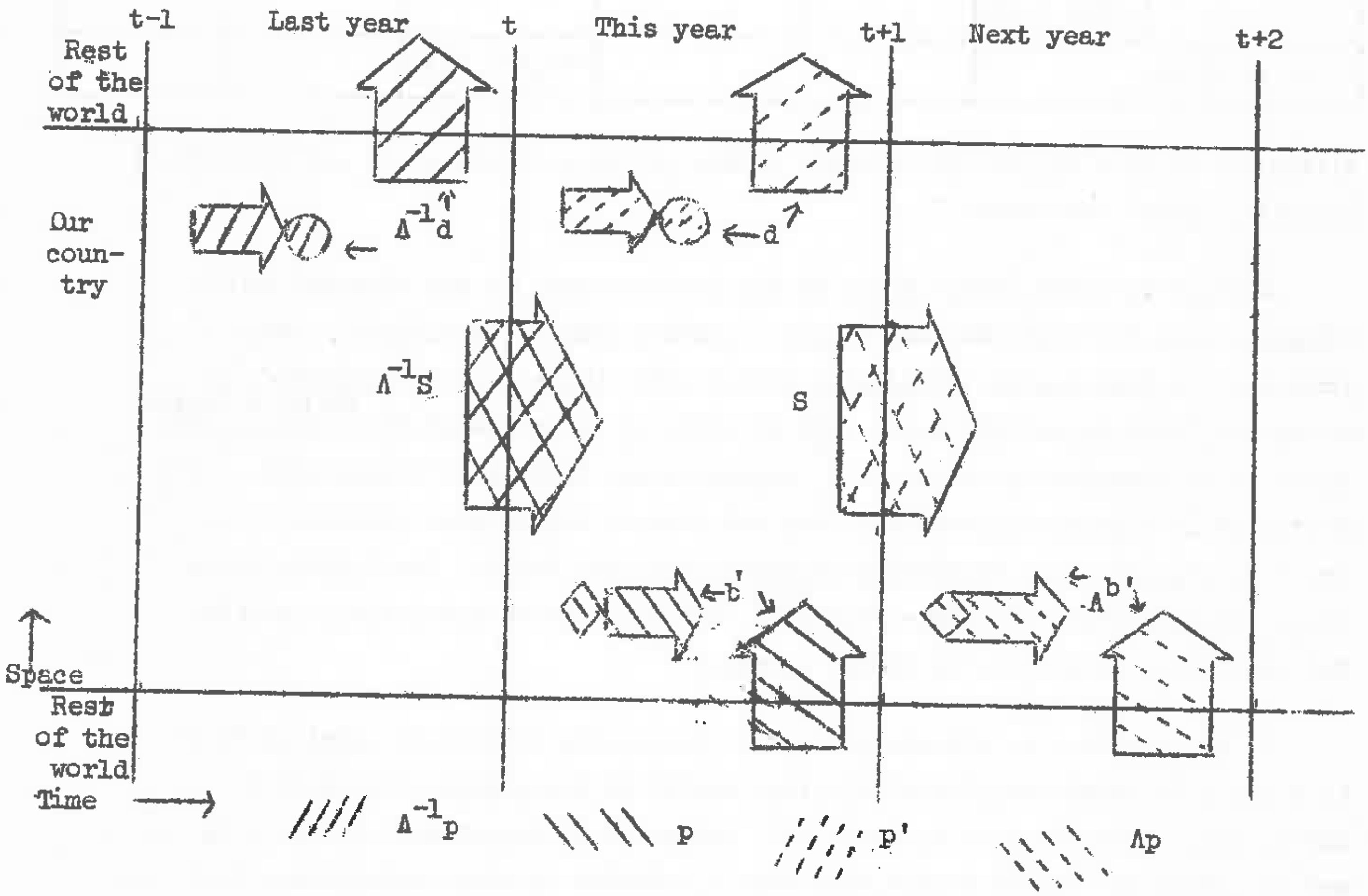



Figure 5. <u>The terms in an open demographic matrix represented in a time-space diagram</u>

The diagram shows clearly that the $\Lambda^{-1}S$ and S matrices are transfers from one period to another of population stocks. They are shaded twice to indicate that they appear in both the previous year's population flows and in the next year's.

We can flesh out at least part of Table 14 and Figure 6 with observations drawn from the England and Wales example. In Table 15 the items d, b', $\Lambda^{-1}S$, S, p and p' have been filled in from the closed demographic accounts of Table 4 and the balance sheet accounts of Table 5. The table is

Table 15. An open demographic accounts table for England and Wales in recent five year intercensal periods

| From \ To | | Outside world | England and Wales 1956 - 61 | England and Wales 1961 - 66 | England and Wales 1966 - 71 | Total flows |
|---|---|---|---|---|---|---|
| Outside world | | | | 5,385,930 | | |
| England and Wales | 1956 - 61 | | | 46,104,543 | | |
| | 1961 - 66 | 3,659,041 | | | 47,831,432 | 51,490,473 |
| | 1966 - 71 | | | | | |
| Total flows | | | | 51,490,473 | | |

difficult to fill in further because of the problems involved in estimating the migration flows concerned.

Neither in Table 14 nor Table 15 are entries made in the diagonal cells labelled with the same row and column period heading. Stone(1966, Table 5) presented "a demographic accounting system with intra-year transitions" in which the "this year-this year" cell of Table 14 (reproduced from Stone & Gunton, 1968 Table 2) is occupied by a matrix $S^*$ representing intra-year transitions. The matrix $S^*$ is defined differently from the closed demographic accounts S in order to preserve the accounting identities in the table. In a later paper Stone (1971b, Table 2.2) has shown that the standard closed demographic matrices for successive years can be nested together.

It is possible to combine these two accounting frameworks used by Stone in a way that shows explicitly how the family of demographic accounts is integrated. This is done in Table 16. Table 16 is structured in the same way as Tables 14 and 15 except that the K notation is used throughout, that the diagonal boxes in the table, representing within period transitions, are now occupied by closed demographic account matrices appropriate to each period(minature versions of Tables 3 and 4), and that in-migrations are distinguished from births and out-migrations from deaths.

Table 16. An integrated demographic accounting system

| From \ To | Out-Migration (Definition (4)) | Out-Migration (Definition (3)) | Deaths | Regional | | | |
|---|---|---|---|---|---|---|---|
| | | | | Period θ-1 | | | |
| In-migration (Definition (4)) | | | | $\underline{M}^{Ri}_{(\theta-1)}$ | | | |
| In-migration (Definition (3)) | | | | $\underline{K}^{\beta(R)*(i)}_{(\theta-1)}$ | | | $\underline{K}^{R*(i)}_{(\theta-1)}$ |
| Deaths | | | | $\underline{K}^{\beta(i)*}_{(\theta-1)}$ | | | |
| Period (θ-1) | $\underline{M}^{iR}_{(\theta-1)}$ | $\underline{K}^{*(i)\delta(R)}_{(\theta-1)}$<br>$\underline{K}^{*(i)R}_{(\theta-1)}$ | $\underline{K}^{*\delta(i)}_{(\theta-1)}$ | -<br>$\underline{K}^{\beta(R)\delta(i)}_{(\theta-1)}$<br>-<br>$\underline{K}^{\beta(R)i}_{(\theta-1)}$ | $\underline{K}^{\beta(i)\delta(R)}_{(\theta-1)}$<br>$\underline{K}^{\beta(i)\delta(i)}_{(\theta-1)}$<br>$\underline{K}^{\beta(i)R}_{(\theta-1)}$<br>$\underline{K}^{\beta(i)i}_{(\theta-1)}$ | -<br>$\underline{K}^{R\delta(i)}_{(\theta-1)}$<br>-<br>$\underline{K}^{Ri}_{(\theta-1)}$ | $\underline{K}^{i\delta(R)}_{(\theta-1)}$<br>$\underline{K}^{i\delta(i)}_{(\theta-1)}$<br>$\underline{K}^{iR}_{(\theta-1)}$<br>$\underline{K}^{ii}_{(\theta-1)}$ |
| Period (θ) | $\underline{M}^{iR}_{(\theta)}$ | $\underline{K}^{*(i)\delta(R)}_{(\theta)}$<br>$\underline{K}^{*(i)R}_{(\theta)}$ | $\underline{K}^{*\delta(i)}_{(\theta)}$ | | | | |
| Period (θ+1) | $\underline{M}^{iR}_{(\theta+1)}$ | $\underline{K}^{*(i)\delta(R)}_{(\theta+1)}$<br>$\underline{K}^{*(i)R}_{(\theta+1)}$ | $\underline{K}^{*\delta(i)}_{(\theta+1)}$ | | | | |
| Total flows | | | | $\underbrace{\underline{K}^{*(i)\delta(R)}_{(\theta-1)} \quad \underline{K}^{*(R)i}_{(\theta-1)} \quad \underline{K}^{*(i)\delta(i)}_{(\theta-1)}}_{P'(\theta-1)}$ | | | |

| system of interest | | Total flows |
|---|---|---|
| Period<br>$(\theta)$ | Period<br>$(\theta+1)$ | |
| $\underbrace{\underline{M}^{Ri}_{(\theta)}}$ | $\underbrace{\underline{M}^{Ri}_{(\theta+1)}}$ | |
| $\underline{K}^{\beta(R)*(i)}_{(\theta)}$ $\underline{K}^{R*(i)}_{(\theta)}$ | $\underline{K}^{\beta(R)*(i)}_{(\theta+1)}$ $\underline{K}^{R*(i)}_{(\theta+1)}$ | |
| $\underline{K}^{\beta(i)*}_{(\theta)}$ | $\underline{K}^{\beta(i)*}_{(\theta+1)}$ | |
| | | $\left.\begin{array}{l}\underline{K}^{*(i)*(R)}_{(\theta-1)}\\ \underline{K}^{*(R)*(i)}_{(\theta-1)}\\ \underline{K}^{*(i)*(i)}_{(\theta-1)}\end{array}\right\} p(\theta-1)$ |
| $\begin{array}{cccc} - & \underline{K}^{\beta(i)\delta(R)}_{(\theta)} & - & \underline{K}^{i\delta(R)}_{(\theta)}\\ \underline{K}^{\beta(R)\delta(i)}_{(\theta)} & \underline{K}^{\beta(i)\delta(i)}_{(\theta)} & \underline{K}^{R\delta(i)}_{(\theta)} & \underline{K}^{i\delta(i)}_{(\theta)}\\ - & \underline{K}^{\beta(i)R}_{(\theta)} & - & \underline{K}^{iR}_{(\theta)}\\ \underline{K}^{\beta(R)i}_{(\theta)} & \underline{K}^{\beta(i)i}_{(\theta)} & \underline{K}^{Ri}_{(\theta)} & \underline{K}^{ii}_{(\theta)}\end{array}$ | | $\left.\begin{array}{l}\underline{K}^{*(i)*(R)}_{(\theta)}\\ \underline{K}^{*(R)*(i)}_{(\theta)}\\ \underline{K}^{*(i)*(i)}_{(\theta)}\end{array}\right\} p(\theta)$ |
| | $\begin{array}{cccc} - & \underline{K}^{\beta(i)\delta(R)}_{(\theta+1)} & - & \underline{K}^{i\delta(R)}_{(\theta+1)}\\ \underline{K}^{\beta(R)\delta(i)}_{(\theta+1)} & \underline{K}^{\beta(i)\delta(i)}_{(\theta+1)} & \underline{K}^{R\delta(i)}_{(\theta+1)} & \underline{K}^{i\delta(i)}_{(\theta+1)}\\ - & \underline{K}^{\beta(i)R}_{(\theta+1)} & - & \underline{K}^{iR}_{(\theta+1)}\\ \underline{K}^{\beta(R)i}_{(\theta+1)} & \underline{K}^{\beta(i)i}_{(\theta+1)} & \underline{K}^{Ri}_{(\theta+1)} & \underline{K}^{ii}_{(\theta+1)}\end{array}$ | $\left.\begin{array}{l}\underline{K}^{*(i)*(R)}_{(\theta+1)}\\ \underline{K}^{*(R)*(i)}_{(\theta+1)}\\ \underline{K}^{*(i)*(i)}\end{array}\right\} p(\theta+1)$ |
| $\underbrace{\underline{K}^{*(i)*(R)}_{(\theta)}\ \underline{K}^{*(R)*(i)}_{(\theta)}\ \underline{K}^{*(i)*(i)}_{(\theta)}}_{p^{*}(\theta)}$ | $\underbrace{\underline{K}^{*(i)*(R)}_{(\theta+1)}\ \underline{K}^{*(R)*(i)}_{(\theta+1)}\ \underline{K}^{*(i)*(i)}_{(\theta+1)}}_{p^{*}(\theta+1)}$ | |

To display the accounting relationships implied in the table properly would require the terms to be lighted up in coloured neon tubes but most of these should be familiar. The closed demographic account identities for the initial population (equation (1)), births (equation (3)), deaths (equation (5)) and the final population (equation (7))are displayed. To these can be added identities for migration into the region of interest.

$$\underline{K}^{\beta(R)*(i)}_{(\theta)} \equiv \underline{K}^{\beta(R)\delta(i)}_{(\theta)} + \underline{K}^{\beta(R)i}_{(\theta)} \tag{47}$$

and

$$\underline{K}^{R*(i)}_{(\theta)} \equiv \underline{K}^{R\delta(i)}_{(\theta)} + \underline{K}^{Ri}_{(\theta)} \tag{48}$$

which are consequent on omitting those terms involved only in the rest of the world. The notation $*(i)$ indicates summation over all possible states with location in region i: at the beginning of the superscript list this means $*(i) = \beta(i) + i$ and at the end of the superscript list this means $*(i) = \delta(i) + i$. The $\theta$ label in equations (47) and (48) and in Table 16 identifies the period to which the terms refer. Similar identities hold for out-migrations.

$$\underline{K}^{*(i)\delta(R)}_{(\theta)} \equiv \underline{K}^{\beta(i)\delta(R)}_{(\theta)} + \underline{K}^{i\delta(R)}_{(\theta)} \tag{49}$$

and

$$\underline{K}^{*(i)R}_{(\theta)} \equiv \underline{K}^{\beta(i)R}_{(\theta)} + \underline{K}^{iR}_{(\theta)} \tag{50}$$

These in-migrations and out-migrations correspond to definition (3) of migration. Also viable is definition (4) represented by $\underline{M}^{Ri}_{(\theta)}$ and $\underline{M}^{iR}_{(\theta)}$ which are functions of all the terms in the closed demographic accounts.

The term transfered from one period to another is $\underline{K}^{*i}_{(\theta)}$ which becomes $\underline{K}^{i*}_{(\theta+1)}$ when looked at from a column point of view. The open demographic accounts identities are formed by ignoring the within-period transitions and adding one of the two migration definitions to deaths (row identity) or births (column identity) to give a population flow total recorded in the right most column and the bottom most row of Table 16. The concept of total population flow through a region/period (either in or out) is difficult to represent in the K notation but essentially it refers to the total of all persons who either start the period or end the period or do both in the region of interest if we have adopted the third definition of migration. If we have adopted the fourth definition it also includes anyone who appears in the accounts as an (R,R) flow who has migrated into and out of the region i and also any "surplus" moves of anybody in an (i,i), (i,R) or (R,i) flow.

## 8. Multiregional and age/sex disaggregated accounts, populations at risk, time spent accounts, demographic models and other matters.

The paper, it is hoped, has served at least two functions. It has provided a guide to the accounting frameworks underlying the recent work of Richard Stone in demographic accounting and modelling for those not familiar with it, and it has shown how these frameworks may be made more general in a regional sense, more integrated as family, and more closely related to conceptual findings on migrations.

There are, however, a good many other aspects of demographic accounting which it has not proved possible to cover in the paper. Among these are the following questions.

(1) How do the essentially two-region (i,R) accounts convert into truly multiregional accounts?

(2) What are the internal structures of the matrices and vectors contained in the accounting tables? How are thay structured when disaggregated by age/and sex and social class, for example?

(3) How does the family of accounts relate to the concept of population at risk? Can accounting principles be used to improve upon conventional notions of populations at risk of giving birth or of dying?

(4) Can we relate the family of accounts to accounts of time spent in various states and regions?

(5) What sorts of demographic models can be constructed on the basis of the accounts presented in this paper? How can these models and the associated accounting framework be used in demographic projections?

The reader is referred to the works cited in the opening section of the paper where these and other issues are examined further.

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