

Working Paper 402

Spatial dynamics: classical problems, an  
integrated modelling approach and system performance

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Prepared for the Twenty-Fourth European Congress,  
Regional Science Association,  
Milan, August, 1984.

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August 1984

SPATIAL DYNAMICS: CLASSICAL PROBLEMS, AN  
INTEGRATED MODELLING APPROACH AND SYSTEM PERFORMANCE

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ABSTRACT

The common nature of a range of 'classical' geographical problems is described. By adopting a particular system description and set of methods, it is shown that it is possible to develop an integrated framework within which the traditional models can be seen as special cases and whose assumptions can be relaxed. It is the ideas of dynamical systems theory, developed in a specific way for geographical systems, which enables such an integrated model system to be built. The framework is then extended by introducing and developing the concept of performance indicators which have roles both in model development and planning.

1. INTRODUCTION

A main objective of this paper is to explain and interpret in broad terms the development of a style of dynamical modelling that was initiated in Harris and Wilson (1978) mainly in the context of the retail model but which has now been applied to all the major sectors of an urban and regional system. These developments have been articulated in a series of papers which will be referred to at appropriate points in the argument. The detail will not be repeated.

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A further objective is to extend the framework in two ways: first, to broaden the description of the decision making which shapes urban and regional development and to use the associated ideas to begin to construct performance indicators for cities; secondly, to carry the argument through to confront contemporary urban problems and to explore the possible use of this model system in a planning context.

The first objective is tackled in five steps. In Section 2, we summarise 'classical' problems and the corresponding models and this provides the context for showing how the new methods achieve a unification of approach. The methods and framework are sketched in Section 3 and the range of application in Section 4. The heart of this paper - in the sense that earlier sections mainly provide an 'index' to earlier papers - begins in Section 5: we focus on the interpretations of the models which can be developed in the new framework and begin to lay the foundations for further extensions. These are presented in Section 6 and they reflect the second objective of the paper. We build on the notion of performance indicators and the contribution of these ideas to both modelling and planning. The framework is set up in Section 6 and the applications are explored in Section 7.

## 2. CLASSICAL PROBLEMS AND MODELS

We take as examples of classical problems and models those shown in Table 1. Their chronology ranges from 1826 to the 1960s, the end-point being there because this is a good place to fit in Alonso's (1964) extension of the use of bid rent in the residential location context.

The agricultural and residential sectors represent the main land-using sectors for urban and regional systems; the rest can be considered as point locators. The scales and comprehensiveness of approach differ. The von Thunen and Alonso models are comprehensive, while Weber is largely partial by focussing on a single firm in relation to its inputs and products. Reilly represents an early example of spatial interaction modelling in relation to services and hence the beginnings of a 'modern' approach. Christaller and Losch are comprehensive in trying to combine sectors but inevitably work at a coarse level of resolution.

These authors indicate the essence of the modelling problems for the main types of sector in urban and regional systems. A major omission is perhaps the transport subsystem. However, they are all restricted by certain decisions which form the basis of the treatment of their problems together with the unavailability of certain mathematical tools. All work with continuous space. This forces simplifying assumptions: a single market (or city centre workplace) in the von Thunen (or Alonso) model; restriction to a single firm for Weber; a failure to analyse competition among more than two centres by Reilly; landscapes with overly rigid geometries in the case of Christaller and Losch.

It is argued in detail elsewhere (Wilson, 1983-A) that these difficulties can be overcome if a discrete zone spatial representation is used and that problems equivalent to the classical ones, especially with newer mathematical tools, can then be solved in this framework. However, this does involve a shift of resolution: the focus is on

the total quantity of each activity in a zone rather than the individual units such as firms which generate this. These particular shifts guarantee release from the restrictive assumptions which have to be made to enable the classical problems to be solved in their own frameworks.

### 3. A UNIFYING FRAMEWORK

The research which generated this framework was initiated in Harris and Wilson (1978). Consider some activity distributed across space and let  $Z_j$  be the quantity of that activity in zone  $j$ . If  $Z_j$  is a product which can be marketed and  $D_j(Z_j)$  is the revenue attracted to  $j$  when  $Z_j$  is sold at  $j$ , and  $C_j(Z_j)$  is the cost of supplying this quantity, then a reasonable hypothesis about dynamics might be

$$\dot{Z}_j = \epsilon [D_j(Z_j) - C_j(Z_j)] f(Z_j) \quad (1)$$

where  $f(Z_j)$  is a function which determines the shape of the growth curve of  $Z_j$  against time  $t$  for small  $t$ . Typically  $f(Z_j)$  is taken as  $Z_j$  which produces logistic growth near the origin so that

$$\dot{Z}_j = \epsilon [D_j(Z_j) - C_j(Z_j)] Z_j \quad (2)$$

is a representative form of the set of equations. Note that if the system of interest is in equilibrium then  $\dot{Z}_j = 0$  so that

$$D_j(Z_j) = C_j(Z_j) \quad (3)$$

constitute the set of simultaneous equations to be solved for  $\{Z_j\}$ .

The model building task now is one of specifying the functions  $D_j(Z_j)$  and  $C_j(Z_j)$  - in part, the production function of the corresponding process. Spatial interaction models will often be relevant at this stage both to represent the flows of consumers who contribute to the revenue,  $D_j$ , and the flows of inputs to the production process which will be reflected in the costs,  $C_j$ . The flows contributing to revenue at  $j$  will also reflect the competition of other available sources for the good. Typically, therefore, each  $j$ -equation in (1) to (3) will also have many  $Z_k$ ,  $k \neq j$ , appearing within  $D_j$  on the right hand side. Usually also such appearances will be in nonlinear form. So the differential equations (1) [or (2)] and the equilibrium equations (3) will usually each have to be solved as sets of nonlinear simultaneous equations. These characteristics virtually guarantee the existence of critical values of the parameters of the models at which bifurcations take place. This gives a new character to many of the models of urban and regional analysis and turns out to reflect important features of real systems.

So far, we have focussed on the production and consumption of a single good in a market economy. It is relatively straightforward in principle to disaggregate by sector ( $n$ , say) and type of good ( $g$ , say) and to focus on  $Z_j^{ng}$  as the production of  $g$  by sector  $n$  in  $j$ , attracting revenue  $D_j^{ng}$  at cost  $C_j^{ng}$ . Obvious extensions of (1) to (3) follow. This, as we will see later, leads to representations of interdependence between sectors to add further complexity to the notions of spatial competition mentioned earlier. What is slightly more difficult is to extend the mechanism beyond a market situation. Usually, the  $C_j$  function can be handled as before; but the  $D_j$  term

will have to be measured as something like 'benefits' or as we will argue in Sections 6 and 7 later - using other broader performance indicators. In all these cases, there are some difficulties of interpretation which we take up in Section 5 below.

The general theory is presented in detail in Wilson (1983-A).

#### 4. THE RANGE OF APPLICATION

The systems to which these ideas have been applied are listed in Table 2, along with references to key papers which spell out the mechanisms used in each case in detail. Because of the shift in spatial representation, there are some corresponding changes in the way model outputs are presented. In the classical 'continuous space' approaches, in the land-intensive models (agriculture, residential location), the task is one of identifying boundaries which separate different land uses. In the discrete zone representation, there can in principle be a mix of land uses in each zone (which is predicted by the model). Provided the zones are reasonably small, however, it is still possible to discern clearly appropriate patterns of land use. In the case of point patterns (industry, services), in the new representation these are located more coarsely by zone but can be considered to be located at zone centroids. Alternatively, it can be taken as a second order problem to locate activity more precisely within a zone. The transport subsystem is of a more distinctive kind since it is dominated by a physical network which carries flows between zones and the supply-side modelling problem is the representation of the evolution of such networks.

The comprehensive model involves stitching together all the major submodels and ultimately represents a replacement for something like central place theory in the location theory canon and for the Lowry (1964) model in more recent history.

For most of these examples it has been possible to carry out numerical experiments which leads to a deeper understanding of what the model equations can represent and provides a foundation for the start of proper empirical work. The beginnings of empirical work have been executed with different studies of retailing in Leeds (M. Clarke, 1984; G. Clarke, 1983, 1984-A, 1984-B). A useful feature of the numerical experiments on idealised (eg. symmetrical) systems is that they can be made to illustrate quite clearly the connections (and power) of these models to the classical models. It is usually possible to set the initial conditions to reproduce the classical results and then to show how easy it is to deal with much more general situations.

##### 5. INTERPRETATIONS

It is useful to begin with an observation on the historical development of modelling and the role of models in planning. The emphasis of the new methods summarised here is on the supply side - the  $Z_j$ s in (1) to (3). Before these ideas were available, the appropriate spatial interaction models had been developed and they were run with exogenously-supplied values of  $Z_j$ -variables. These might be observed values or planned values. In the latter case, the purpose of the model run would be to test the impact of that planned assignment of  $\{Z_j\}$ -values.



Given the availability of the new methods, we must now continue to distinguish these two cases very clearly. In the first case, we are assuming that we can model directly the behaviour of the supply side; and such hypotheses should obviously be tested. This is the kind of research which is being pursued with retail models. In the second case, it may still be appropriate to run a model of the form (1) to (3) to give an indication of the form of the  $\{Z_j\}$ -pattern which might result in a particular set of circumstances. In this case where the  $Z_j$ -variables can be planned and controlled, the model outputs can then be seen as idealised designs and the model system as a design tool.

These two cases represent the poles of a more complicated reality. In the first kind of case it may be desirable to fix a subset of the supply variables according to a plan - for example a small number of planned shopping centres. We should also bear in mind that type-1 models may also be used in a planning context to test the impact of other controlled changes. For example, they could be used to test the impact on retail centres of planned transport network changes.

The second main observation to be made at the outset relates to the nature of the models outputs given the presence of bifurcations. The nonlinearities guarantee the presence of multiple equilibrium solutions, and, not least because of market imperfections and the like, the system of interest may well not be in the state representing the global optimum. The trajectory of the system through time is

likely to depend heavily on the initial conditions (including 'historical accidents') and the nature of the perturbations ('noise') acting on it over time. It will be one of a very large number of possible trajectories. The task of analysis is made even more difficult because at critical values of parameters, the system state can make a discrete jump to another state which may be of a structurally-different type. There are also bifurcations in the temporal domain associated with parameters like  $c$  in (1) and (2).

This is likely to make it very difficult to make forecasts - or at least forecasts of a simple kind - with models, and this obviously complicates their use in planning. What should be possible is always to give a historical account and interpretation of a trajectory after the event; and to chart a range of possible kinds of futures given conjectures about the exogenous variables which drive the system (population change, technological change, relative prices and so on).

It is worth remarking, however, that at a zonal scale of analysis, notwithstanding what has just been argued, there should be more stability than at a more micro scale involving individual units.

#### 6. EXTENSIONS: A PERFORMANCE INDICATORS FRAMEWORK

In this section, we explore further the implicit overall picture of an urban and regional system which has been developed with a view to confronting more directly the problems of such systems; we use the ideas which then develop to extend the models; and begin to formulate a more explicit basis for the use of these ideas in planning.

If the whole system is considered to be dominated by the market economy with any sort of planning having relatively little influence, then the obvious directions of extension involve the integration of the modelling ideas presented with those of more conventional neo-classical economies, and in some contexts this is appropriate and we have attempted to pursue such developments (Wilson, 1983-A; Wilson and Birkin, 1983, 1984; Birkin and Wilson, 1984-A, 1984-B). Here, however, we adopt a different focus. We assume that the underpinning economy is fundamentally mixed and that the development of the public side of it (and to some extent the private side through planning controls) is determined by explicit policy. What is needed then is a framework within which problems can be articulated and investigated and alternative policy options can be explored. The way we pursue this here is through batteries of performance indicators associated with each sector which change both with the evolution of the system on its 'internal' dynamic and through the (thus measured) effects of policy intervention. The framework used here is an extension of ideas developed in Britain in the context of the National Health Service (D.H.S.S., 1984; Wilson, 1984-A). But it also relies on adopting a particular perspective on urban and regional systems: that they are functioning for their inhabitants and that policy goals should be directed accordingly.

Some of the main flows involved in this view of the city are indicated in Figure 1. Households in residential areas supply labour to the job market. They are sustained by income received from employment together with a variety of public and private services.

The income is partly used to purchase the products of the agricultural and industrial sectors, mainly marketed through retail sectors. This, of course, is a substantial oversimplification but it suffices to begin an attack on the problem. Even at the coarse scale, the figure also shows most of the interdependencies necessarily reflected in any comprehensive model, notably those developed from the Lowry (1964) model.

It should be emphasised that there are at least three different sets of performance indicators, the first relating to households by residential location, the second to other units of the economy by facility-location and the third to the system as a whole. The spatial interrelationships of the first two sets of indicators provide both interesting research questions and new insights, for example in relation to the equity-efficiency problem.

The first step in the argument is the generation of a system description which connects to the models and which provides the basis for the calculation of performance indicators. We will use  $i$  for residential zone,  $j$  for other zones;  $m$  as a list  $\{m_1, m_2, \dots\}$  of population categories;  $g$  as characterising goods or services. These indices then give the basis of a description of a complete system which will suffice to produce an illustrative argument.

#### System description

Let  $P_i^m$  be the number of type- $m$  people in zone  $i$ , and let  $\hat{e}_i^{mg}$  be an estimate of their demand for goods or services of type  $g$  (and this may, of course, be a function of accessibility to the sources of  $g$ ).

Let  $Z_j^g$  measure the scale of supply of  $g$  at  $j$  in suitable units and let the total cost of this provision be  $C_j^g$ . Let  $N_{ij}^{mg}$  be the consumption, in suitable units, of  $g$  from  $j$  by type  $m$  people resident in  $i$ . If there is little demand for  $g$ , then the numbers in this array will be small. It is also useful to define an array  $P_{ij}^{mg}$  which is a partition of  $P_i^m$  which represents the numbers of the  $P_i^m$ -population who are 'served' at  $j$  for  $g$ . In this case

$$\sum_{jg} P_{ij}^{mg} = P_i^m \quad (4)$$

and so the magnitudes of the elements of  $P_{ij}^{mg}$  are of the same order as those of  $P_i^m$  even if the numbers in the  $N_{ij}^{mg}$  array are small. If we then calculate

$$\pi_j^g = \sum_{im} P_{ij}^{mg} \quad (5)$$

then  $\pi_j^g$  can be seen to be a catchment population for  $g$  in  $j$ , and this is a useful concept for the construction of performance indicators. In setting it up in this way, we follow the argument of Cottrell (1983). An algebraic method of calculating  $\{P_{ij}^{mg}\}$  from  $\{N_{ij}^{mg}\}$  and  $\{P_i^m\}$ , again following Cottrell, is given in Wilson (1984-B).

Given  $\{N_{ij}^{mg}\}$ , it is also possible to calculate

$$e_i^{mg} P_i^m = \sum_j N_{ij}^{mg} \quad (6)$$

and

$$D_j^g = \sum_{im} N_{ij}^{mg}. \quad (7)$$

Equation (6) can be taken as defining  $e_i^{mg}$  as the actual per capita take up of g by the (i,m) population, and  $D_j^g$  in Equation (7) is the total 'revenue' attracted to g in j - and we can assume that money units are used for  $N_{ij}^{mg}$  where convenient, so that  $D_j^g$  can be compared to  $C_j^g$  as in the usual equilibrium condition of our underpinning models.

The main variables which constitute this system description are  $\{P_i^m\}$ ,  $\{Z_j^g\}$  and  $\{N_{ij}^{mg}\}$  and these are precisely the arrays which are generated in the model system outlined earlier - for appropriate definitions of m and g. We now proceed to show how they can also be used as the basis for the construction of performance indicators - taking residential and 'production' zones in turn - and then indicate in the next sections how these ideas provide a basis for a more effective integration of the model system into planning.

#### Residence-zone indicators

Some of the most obvious indicators which can now be constructed are listed and defined in Table 3. A particularly powerful, if simple idea, is now evident in the case where g represents a public service. The budget,  $C_j^g$ , can be 'allocated' to residents through the construction of  $R_i^{mg}$  as shown in the Table and  $R_i^{mg}/P_i^m$  can be calculated. Thus, when  $C_j^g$ s are manipulated as part of a planning procedure, the impact on residential populations can be calculated. Due account is taken of cross-boundary flows whereas in many cases in the past, a particular  $C_j^g$  is related only to the local population in j.

Production-zone indicators

Examples of indicators which can be constructed are given in Table 4. Of particular interest is  $D_j^g/C_j^g$  which, in effect, forms the basis of the equilibrium condition in 'market' models where we assume that the condition

$$D_j^g = C_j^g \quad (8)$$

determines the spatial pattern  $\{Z_j^g\}$ . But the Table shows that a variety of indicators can be constructed which have a bearing on the 'efficiency' of the supply  $Z_j^g$  at  $j$ .

System-wide indicators

It is also useful to recognise that some indicators can be constructed for the system as a whole and examples are given in Table 5 - the most common one from past experience being total travel cost for each  $g$ -purpose'.

In the next section, we now work out the implications of these kinds of performance indicators for (a) model development, (b) problem analysis and (c) planning. We conclude this one with the observation that it is obviously a substantial research problem to articulate in detail appropriate sets of performance indicators from Tables 3 to 5 for individual classes,  $m$ , and sectors,  $g$ . These should relate to housing, income and employment, retailing, individual public services such as health and education, and so on. The Tables as presented are illustrative only.

## 7. APPLICATIONS OF A 'PERFORMANCE INDICATORS' FRAMEWORK

### Model developments

The first point to note is that these ideas extend the basic concept of the  $D_j$ - $C_j$  mechanism by offering alternative indicators which can be used in place of the revenue-based  $D_j$  in existing models.

Secondly, we gain an interesting new perspective on the equity-efficiency problem by calculating residence-based and production-based indicators. For residents, the focus is on a vector of indicators which, though difficult to weight relative to each other, offer a picture of the quality of life. In policy terms, there will be an interest in some kind of equitable distribution of these vectors across space (and across population classes). For production, the focus is on efficiency. Interestingly, there are two interacting fields of influence involved in this: those fanning out from residential areas on the one hand and the catchment populations of facilities on the other. The study of both and their interdependence will be crucial to planning and policy.

Thirdly, it is important to examine the residential indicator-vectors in more detail. We cannot simply consider individual elements of such vectors as there will be trades-off to take into account - for example between decreasing accessibility and increasing land consumption, or vice versa. However, there will be elements of the vector which are much more important than others, the most



obvious example being income from employment: if income is high, it can be assumed that many of the other trades-off are determined by individual or household choice; this is not so easy to assume for those whose income is low and/or are unemployed. Public services also become more significant in this last case.

This last point means that particular attention needs to be paid in modelling to the way jobs and income are acquired in relation to other factors - in effect a dynamic model of  $P_{*}^m$  to include transitions between  $m$  categories. This would build in the inertia of class structures, for example; perhaps modified to some extent by educational policy.

The analysis also reflects the importance of interdependence between sectors, both because of the existence of a vector of residence-based indicators and because of flows between different kinds of  $j$ -facilities. This means that it will be important in model development to base the whole system on interlocking sets of accounts so that changes in one sector are appropriately reflected in others and their corresponding indicators.

#### Problem analysis

The use of an indicators approach gives a simple way of describing the social geography of an urban and regional system. The advantage of carrying out such a study within a framework of dynamic models is that trends can be more clearly identified along with an account of the possibilities and limits of what can be achieved by different kinds of policy intervention.

### Planning

We have to begin by reiterating the point made earlier that the significance of understanding the nature of dynamic models - and by implication and with hindsight, aspects of the real world - is that the old use of models for conditional forecasting or impact assessment is too simplistic. There are too many possibilities of relatively discrete, rapid, structural change at critical parameter values. The emphasis has to turn therefore to seeking to understand stability coupled with the processes of structural change.

We also continually need to assess and to evaluate the system state, and hence the emphasis on performance indicators. It is necessary to decide whether to maintain (through planning control) an equilibrium or near-equilibrium because the performance indicators are 'satisfactory', or to try to change it. In the latter case, it may well be that the only way to bring about change successfully is through what Beer (1972) calls 'riding the dynamic'. That is: understanding the processes of structural change and influencing parameters to go through critical values to achieve desirable effects. Other attempts to bring about planned change are likely to fail. The shift to a 'performance indicators' perspective also helps system modelling in cases where it is known that behaviour is influenced by a knowledge of such indicators.

What emerges, therefore, is a need to develop the whole set of subsystem models and to learn to use each as design tools. It is also necessary to develop and use a comprehensive model in the same

way. We have shown how some of the relevant ideas can be extended in the direction of public services, particularly using the concept of performance indicators. 'Design', in this context, we reiterate, means inventing alternative and productive ways of 'riding the dynamic', not manufacturing a master plan in the old sense. In fact Beer (1972) emphasises that a corporate plan for an enterprise should be a plan of how to respond to change rather than a map of an idealised system state.

These ideas can be put together as in Figure 2. This shows the main feedbacks involved in model use in planning when coupled to the notion of performance indicators. It is also significant that we might try to develop artificial-intelligence procedures to improve the level of assistance offered by the system to planners. The scheme implied by the figure also has the effect of putting human intelligence at the centre of the planning operation. This is the advantage of working with performance indicators and allowing judgements about them to be made in relation to policy options rather than formulating grand mathematical 'black-box' optimization procedures.

Finally, we noted the crucial distinction between the focus on performance indicators relating to the residents of particular areas and those relating to the production of goods and services. These perspectives represent the interaction of access fields and catchment fields. A crucial consequence of the fact that this interaction takes place in a complicated nonlinear way is that multiple solutions exist. This provides the note on which to conclude: we need to integrate research which focusses on attempting to understand

complexity with more direct ways of assessing performance. This integration should generate more effective planning methods of a new type. The connections back to the classical problem of locational analysis show regional science and its related disciplines to be at the centre of this enterprise.

#### References

- Alonso, W. 1964. Location and land use. Cambridge, Mass: Harvard University Press.
- Beer, S. 1972. Brain of the firm. Harmondsworth: Allen Lane; second edition, 1981, Chichester: John Wiley.
- Birkin, M., Clarke, M. and Wilson, A.G. 1984. Interacting fields: comprehensive models for the dynamical analysis of urban spatial structure. University of Leeds, School of Geography, Working Paper 385; Paper delivered at the Annual Conference, Association of American Geographers, Washington D.C.
- Birkin, M. and Wilson, A.G. 1984-A. Industrial location models I: a review and an integrating framework. University of Leeds, School of Geography, Working Paper (forthcoming).
- Birkin, M. and Wilson, A.G. 1984-B. Industrial location models II: application of a new approach. University of Leeds, School of Geography, Working Paper (forthcoming).
- Christaller, W. 1933. Die centralen orte in Suddeutschland. Jena: Gustav Fischer; English translation by Baskin, C.W. Central places in Southern Germany. Englewood Cliffs, N.J.: Prentice-Hall.
- Clarke, G.P. 1984-A. The expansion of service outlets across the city: a review. University of Leeds, School of Geography, Working Paper 379.

- Clarke, G.P. 1984-B. The changing morphology of urban retailing in the recession. University of Leeds, School of Geography, Working Paper 382.
- Clarke, G.P. 1984-C. The diffusion of retail multiples across the city: a review. University of Leeds, School of Geography, Working Paper 388.
- Clarke, M. 1984. Integrating dynamic models of urban and regional systems: an application to urban retailing models. Ph.D. thesis (forthcoming), School of Geography, University of Leeds.
- Clarke, M. and Prentice, R. 1982. Exploring decisions in public policy making: strategic allocation, individual allocation and simulation. Environment and Planning, A 14:400-524.
- Clarke, M. and Wilson, A.G. 1983-A. Exploring the dynamics of urban housing structure: a 56-parameter residential location and housing model. University of Leeds, School of Geography, Working Paper 363; paper delivered at the European Meeting of the Regional Science Association, Poitiers, France.
- Clarke, M. and Wilson, A.G. 1983-B. Dynamics of urban spatial structure: progress and problems. Journal of Regional Science 21:1-18.
- Clarke, M. and Wilson, A.G. 1983-C. Modelling for health services planning: an outline and an example. In London Papers in Regional Science 13. Planning and analysis in health care systems, ed. M. Clarke, pp. 22-56. London: Pion.
- Cottrell, K. 1983. Determining the population served by health service institutions. North Western Regional Health Authority, mimeographed.

- D.H.S.S. 1984. Performance indicators: national summary for 1981.  
London: D.H.S.S. Regional Liaison Division.
- Harris, B. and Wilson, A.G. 1978. Equilibrium values and dynamics  
of attractiveness terms in production-constrained spatial-  
interaction models. Environment and Planning, A 10:371-388.
- Losch, A. 1940. Die räumliche ordnung der wirtschaft. Jena:  
Gustav Fischer; English translation by Stolper, W.H. 1954.  
The economics of location. New Haven, C.T.: Yale University  
Press.
- Lowry, I.S. 1964. A model of metropolis. Santa Monica: Rand  
Corporation, RM-4035-RC.
- Reilly, W.J. 1931. The law of retail gravitation. New York:  
G.P. Putman and Son.
- von Thunen, J.H. 1826. Der isolierte staat in beziehung auf  
landwirtschaft und nationalökonomie. Stuttgart: Gustav Fischer;  
English translation by Wartenburg, C.M. 1966. The isolated  
state. Oxford: Oxford University Press.
- Weber, A. 1909. Über den standort der industrien. Tübingen;  
English translation by Friedrich, C.J. Theory of the location  
of industries. Chicago, I.L.: University of Chicago Press.
- Wilson, A.G. 1983-A. Location theory: a unified approach.  
University of Leeds, School of Geography, Working Paper 355;  
paper delivered at the British Section Meeting of the Regional  
Science Association, Leeds.
- Wilson, A.G. 1983-B. Transport and the evolution of urban spatial  
structure. Atti delle Giornate di Lavoro, A.I.R.O., Naples:  
7-27.

- Wilson, A.G. 1984-A. Performance indicators and health services planning. University of Leeds, School of Geography, Working Paper (forthcoming).
- Wilson, A.G. 1984-B. Catchment population and performance indicators. University of Leeds, School of Geography, Working Paper (forthcoming).
- Wilson, A.G. and Birkin, M. 1983. Industrial location theory: explorations of a new approach. University of Leeds, School of Geography, Working Paper 361.
- Wilson, A.G. and Birkin, M. 1984. Agricultural location models: a review and exploration of a new approach. University of Leeds, School of Geography, Working Paper (forthcoming).
- Wilson, A.G. and Clarke, M. 1979. Some illustrations of catastrophe theory applied to urban retailing structures. In London Papers in Regional Science 10. Developments in urban and regional analysis, ed. M.J. Breheny, pp. 5-27. London: Pion.
- Wilson, A.G. and Crouchley, R. 1983. The optimum sizes and location of schools. University of Leeds, School of Geography, Working Paper 369.

TABLE 1. Classical problems

<u>Sector</u>	<u>Author and reference</u>
Agriculture	von Thunen (1826)
Residential location	Alonso (1964)
Industry	Weber (1909)
Services	Reilly (1931)
Comprehensive/ Central place theory	Christaller (1933) Losch (1940)

TABLE 2. Elements for an integrated framework

<u>Sector</u>	<u>Reference</u>
Agriculture	Wilson and Birkin (1984)
Residential and housing	Clarke and Wilson (1983-A)
Industrial	Wilson and Birkin (1983); Birkin and Wilson (1984-A; 1984-B)
Retailing and private services	Wilson and Clarke (1979) Clarke and Wilson (1983-B)
Transport	Wilson (1983-B)
Public services	Clarke and Prentice (1982) Wilson (1984) Wilson and Crouchley (1983)
Comprehensive	Birkin, Clarke and Wilson (1984)



TABLE 3. Residence-zone performance indicators

$e_i^{mg}/\hat{e}_i^{mg}$	Proportion of need met
$R_i^{mg} = \sum_j \frac{N_{ij}^{mg}}{N_{*j}^{mg}} c_j^g$	Expenditure on g allocated to m-type residents of zone i
$R_i^{mg}/p_i^m$	Per capita expenditure on m-type residents of zone i for g
$\sum_j N_{ij}^{mg} c_{ij}^g$	Transport expenditure by m-residents in i to obtain g*
$\sum_j N_{ij}^{mg} (\frac{\alpha}{\beta} \log Z_j^g - c_{ij}^g)$	Consumers surplus for m-residents of i in relation to g
* $c_{ij}^g$ is the transport 'cost' involved in obtaining g.	

TABLE 4. Production-zone performance indicators

$y_j^{zg}, g_j^{zg}$	The resource inputs, $z$ and the unit cost of these, needed to produce a unit of g at j. These should be as low as possible consistent with the attainment of an appropriate standard.
$Z_j^g/\pi_j^g, C_j^g/\pi_j^g, D_j^g/\pi_j^g$	Total production, total cost and total revenue attracted per head of <u>catchment</u> population.
$D_j^g/Z_j^g, D_j^g/C_j^g$	Revenue attracted per unit of product and per unit of cost. It is the second of these which is the usual basis of the dynamic retailing equilibrium model.

TABLE 5. System-wide performance indicators

$\sum_j C_j^g$	Total expenditure on production of g.
$\sum_{ijm} N_{ij}^m C_{ij}^g$	Total transport costs associated with the supply of g.

FIGURE 1

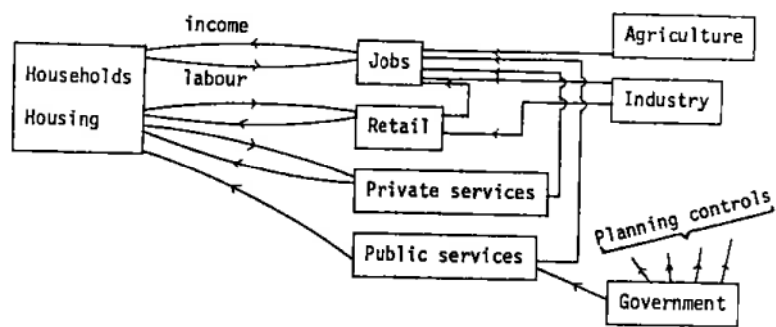


FIGURE 2

