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# COMMENTS ON ALONSO'S "THEORY OF MOVEMENT"

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## Comments on Alonso's 'Theory of Movement'

Alonso (1973, 1974, 1978) has proposed and developed a set of equations as the basis for a 'Theory of movement'. The comments in this paper are based on the exposition and exploration of Alonso's model by Hua (1979). The model can be stated as follows:

$$N_{i*} = V_i D_i^{\alpha} \tag{1}$$

$$N_{*j} = W_j C_j^{\beta} \tag{2}$$

$$D_i = \sum_k W_k C_k^{\beta-1} F_{ik} \tag{3}$$

$$C_j = \sum_k V_k D_k^{\alpha-1} F_{kj} \tag{4}$$

$$N_{ij} = V_i D_i^{\alpha-1} W_j C_j^{\beta-1} F_{ij} \tag{5}$$

An asterisk replacing a subscript denotes summation. $N_{ij}$ is the number of moves, $N_{i*}$ and $N_{*j}$, origin and destination totals respectively; $V_i$ and $W_j$ are factors which represent zonal characteristics related to outflow and inflow; $D_i$ and $C_j$ are "two endogenous factors summarising the external conditions of i and j affecting the outflow and the inflow" (Hua, 1979); $F_{ij}$ is a measure of something like travel impedence in the usual way; $\alpha$ and $\beta$ are parameters. The definitions are deliberately loose so that the model can be used in a variety of ways.

It is well known that, by choosing the parameters $\alpha$ and $\beta$ appropriately, and perhaps $V_i$ and $W_j$, members of the well known family of spatial interaction models (see, for example, Wilson, 1971) can be generated. The purpose of this paper is to argue that the Alonso system of equations can virtually wholly be identified with such a family. This may be helpful in a research context since at present they are viewed as related, with the family of spatial interaction models being a special case of the Alonso system. The argument here, therefore, is that since they are to all intents and purposes identical, everything which could be achieved with Alonso's system can be achieved with the more familiar system. It may also be helpful to spell out the nature of the correspondence.

The family of spatial interaction models can be taken as:

(i) Unconstrained.

$$t_{ij} = kv_i w_j F_{ij} \tag{6}$$

where k may be determined as

$$k = t / \sum_{ij} v_i w_j F_{ij} \tag{7}$$

to ensure that

$$\sum_{ij} t_{ij} = t \tag{8}$$

say.

(ii) Production constrained.

$$t_{ij} = a_i o_i w_j F_{ij} \tag{9}$$

where

$$a_i = 1 / \sum_k w_k F_{ik} \tag{10}$$

to ensure that

$$\sum_j t_{ij} = o_i \tag{11}$$

(iii) Attraction constrained.

$$t_{ij} = b_j v_i d_j F_{ij} \tag{12}$$

where

$$b_j = 1 / \sum_k v_k F_{kj} \tag{13}$$

to ensure that

$$\sum_i t_{ij} = d_j \tag{14}$$

(iv) Doubly-constrained.

$$t_{ij} = a_i b_j o_i d_j F_{ij} \tag{15}$$

where

$$a_i = 1/\sum_k b_k d_k F_{ik} \qquad (16)$$

to ensure that

$$\sum_j t_{ij} = o_i \qquad (17)$$

and

$$b_j = 1/\sum_k a_k o_k F_{kj} \qquad (18)$$

to ensure that

$$\sum_i t_{ij} = d_j \qquad (19)$$

Lower case letters have been used instead of the standard notation to ensure that there is no clash with Alonso's notation. $t_{ij}$ is the flow from i to j, $o_i$ the total inflow to j when known; $v_i$ and $w_j$ are weights to be used when outflow or inflow totals (respectively) are not known; the $a_i$'s and $b_j$'s are balancing factors; $F_{ij}$ is the usual impedance factor and a capital letter is used because it can be taken as the same as Alonso's. No particular functional form is specified to facilitate comparison between the two models.

We first catalogue the obvious senses in which the family given by (6)-(19) is a special case of (1)-(5). Setting one of the parameters $\alpha$ and $\beta$ to zero essentially makes the Alonso model production or attraction constrained respectively. Setting both to zero makes it doubly constrained. For convenience and clarity, we set first any non-zero parameters to one, but we will see later that this is not necessary. One this basis, we take $\alpha$=1, $\beta$=1 to generate the unconstrained model. We can now examine the four standard cases.

(i) Unconstrained: $\alpha$=1, $\beta$=1.

Alonso's terms $V_i$ and $D_i$ both contribute to an explanation of total outflow at i, $W_j$ and $C_j$ to inflow at j. Take

$$v_i = V_i \qquad (20)$$

$$w_j = W_j \qquad (21)$$

Then equation (5) becomes

$$N_{ij} = v_i w_j F_{ij} \tag{22}$$

which is the same as (6) without the constant of proportionality. This can be considered to be absorbed into the products $v_i w_j$ without any difficulty.

$D_i$ and $C_j$ play no direct role in the model. Alonso's equations (3) and (4) give

$$D_i = \sum_k w_k F_{ik} \tag{23}$$

$$C_j = \sum_k v_k F_{kj} \tag{24}$$

which can be interpreted as accessibilities.

His equations (1) and (2) are

$$N_{i*} = V_i D_i = v_i \sum_k w_k F_{ik} \tag{25}$$

as

$$N_{*j} = W_j C_j = w_j \sum_k v_k F_{ki} \tag{26}$$

which follow directly from (22) and add nothing new. We will see later that this is more interesting when $\alpha$ and $\beta$ take values other than one.

(ii) Production constrained: $\alpha=0$, $\beta=1$.

$t_{i*}$ (or $N_{i*}$) is now assumed to be known, by definition, as $o_i$. Equation (1) shows that $V_i$ must be taken as $o_i$ in this case:

$$V_i = o_i \tag{27}$$

Take

$$w_j = W_j \tag{28}$$

Alonso's equations (3) and (4) give

$$a_i = D_i^{-1} = 1/\sum_k w_k F_{ik} \tag{29}$$

which coincides with (3) and (10) and ensures that (1) holds. Then equation (5) becomes

$$N_{ij} = a_i o_i w_j F_{ij} \tag{30}$$

which coincides with (9).

We have not used Alonso's equations (2) and (4). The latter gives

$$C_j = \sum_k a_k o_k F_{kj} \tag{31}$$

which is another measure of accessibility, this time modified by the weights $a_k$, and again this plays no direct role in the model. Equation (2), using (31), gives

$$N_{*j} = w_j \sum_k a_k o_k F_{kj} \tag{32}$$

which could have been computed directly from (30).

(iii) Attraction constrained: $\alpha=1$, $\beta=0$.

This is the mirror image of case (ii) and will not be spelled out in detail. Essentially, take

$$V_i = v_i \tag{33}$$

and

$$W_j = d_j \tag{34}$$

$$b_j = C_j^{-1} \tag{35}$$

and the argument procedes as before.

(iv) Doubly constrained: $\alpha=0$, $\beta=0$.

We take

$$V_i = o_i \tag{36}$$

$$W_j = d_j \tag{37}$$

$$a_i = D_i^{-1} \tag{38}$$

$$b_j = C_j^{-1} \tag{39}$$

and then equation (5) becomes

$$N_{ij} = a_i b_j o_i d_j F_{ij} \tag{40}$$

which coincides with (15). Equation (3) and (4) become

$$a_i^{-1} = \sum_k b_k d_k F_{ik} \tag{41}$$

$$b_j^{-1} = \sum_k a_k o_k F_{kj} \tag{42}$$

which coincides with (16) and (18) respectively. Equations (1) and (2) are simply the usual constraints equations (17) and (19) respectively.

Next, consider what happens if we drop the restriction that the non-zero parameters, $\alpha$ or $\beta$, have to be one. This is obviously important because it provides the case for the Alonso model going beyond the standard family of spatial interaction models given by (16)-(19). We will continue to use the names of the four cases for reasons which will become apparent and we again take each in turn.

(i) Unconstrained: $\alpha$ and $\beta$ can take any value except zero.

We take

$$v_i = V_i D_i^{\alpha-1} \tag{43}$$

$$w_j = W_j C_j^{\beta-1} \tag{44}$$

This generates equation (22) as before. Essentially, therefore, what we have is an unconstrained model with certain hypothesised forms, (43) and (44), for the weights $v_i$ and $w_j$. Equation (3) and (4) now give

$$D_i = \sum_k w_k F_{ik} \tag{45}$$

and

$$C_j = \sum_k v_k F_{kj} \tag{46}$$

and so are still measures of accessibility as before. When $\alpha$ and/or $\beta$ are not equal to one, therefore, equations (43) and (44) represent hypotheses that accessibility terms appear multiplicatively in the weights raised to the powers $\alpha-1$ and $\beta-1$ respectively. It is perfectly possible to achieve this within the standard unconstrained model by *hypothesising* (43) and (44). Indeed, as Hua (1979) notes in a similar context which we take up below, arriving at the same argument by another route, $v_i$ and $w_j$ could be taken as more general functional forms

$$v_i = v_i(V_i, D_i) \tag{47}$$

and

$$w_j = w_j(W_j, C_j) \tag{48}$$

Equations (1) and (2) are derivable directly from (27) by summation.

(ii) Production constrained: $\alpha=0$, $\beta$ non zero.

Again, take

$$V_i = o_i \tag{49}$$

$$a_i = D_i^{-1} \tag{50}$$

and now

$$w_j = W_j C_j^{\beta-1} \tag{51}$$

This generates (30) as before. (3) gives the balancing factor:

$$a_i^{-1} = D_i = \sum_k w_k F_{ik} \tag{52}$$

as before. The unused equation is (4) which gives

$$C_j = \sum_k o_k a_k F_{kj} \tag{53}$$

which is a weighted-accessibility term and this is now *hypothesised* to appear in $w_j$ in (51) raised to the power $\beta-1$. Again, we could now suggest a more general relationship

$$w_j = w_j(W_j, C_j) \tag{54}$$

if this was thought to be appropriate.

In this model, we have used Alonso's equation (1) as a constraint equation and equation (2) is a restatement of what can be derived from the model equation (30) by summing over i. However, a parameter which plays the same role as Alonso's $\alpha$ (which we have taken as zero to generate this case), can be re-introduced as follows, say as $\alpha'$. Suppose we hypothesise that, while $o_i$ is 'known' for the movement model, it is separately modelled as

$$o_i = V_i'(a_i^{-1})^{\alpha'} \qquad (55)$$

for a suitable variable $V_i'$ and a parameter $\alpha'$. In other words, $o_i$ depends on accessibility raised to a power $\alpha'$. Then equation (55) could be added to the scheme for this model. This presentation, however, clarifies the role which accessibility plays instead of having it looking like a pseudo-balancing factor as in Alonso's formulation. It would also be possible to generalise (as in Hua, 1979) and take

$$o_i = o_i(V_i', a_i) \qquad (56)$$

(iii) Attraction constrained case: $\alpha$, non zero, $\beta=0$.

This is the mirror image of case (ii) above and so is not spelled out in detail here.

(iv) Doubly constrained: $\alpha=0$, $\beta=0$.

Clearly, since the parameters are fixed at zero, there is no direct generalisation of the previous treatment of this case in equations (36)-(42). However, we can introduce parameters $\alpha'$ and $\beta'$ along the lines of $\alpha'$ in in case (ii) above if we wish to model $o_i$ and $d_j$ separately from the movement model. The same line of argument suggests

$$o_i = V_i'(a_i^{-1})^{\alpha'} \qquad (57)$$

and

$$d_j = W_j'(b_j^{-1})^{\beta'} \qquad (58)$$

for suitable explanatory variables $V_i'$ and $W_j'$; or, more generally,

$$o_i = o_i(V_i', a_i) \qquad (59)$$

and

$$d_j = d_j(W_j', b_j) \qquad (60)$$

The results set out above can be summarised as follows. It can be argued that the essence of the task of deciding the form of a spatial interaction model is the assumption at the outset about which total outflows and inflows are known. They can be 'known' in the sense of given as numbers or as determined by a separate generation model, as in equations (56), (59) or (60) above. Alonso's formulation has outflows and inflows as $N_{i*}$ and $N_{*j}$ respectively and these terms can be interpreted *either* as given *or* as calculated within the model. The essence of the above argument is that it may be better to make the particular assumption adopted explicit in the notation used and that this then generates one of the standard family of spatial interaction models. The member of the family which arises is determined by which of the $\alpha$ and $\beta$ parameters are zero.

In the last analysis, however, it may be a matter of taste as to which formulation is used: if each is used carefully, then a correctly formulated model should be obtained. However, it is important that the essential equivalence of the two systems should be understood.

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