WORKING PAPER 243

TRAVEL DEMAND FORECASTING - AN OVERVIEW OF THEORETICAL DEVELOPMENTS.

H.C.W.L. WILLIAMS

School of Geography
University of Leeds
LEEDS LS2 9JT.

March 1979

TRAVEL DEMAND FORECASTING - AN OVERVIEW OF THEORETICAL DEVELOPMENTS.

H.C.W.L. Williams

School of Geography

University of Leeds

LEEDS, LS2 9JT.

CONTENTS

1. Introduction

It is commonly remarked that the approach to travel demand forecasting inherited from the pioneering Detroit and Chicago studies, which was to become institutionalised in the early 1960's, was essentially descriptive. It was sought to *describe* the variability in travel behaviour rather than *account for* observable patterns. Indeed, it was not until the mid-1960's that specific theoretical perspectives began to assert themselves. Whether it be considered a science or an art, travel forecasting has, since that time, been the subject of invasion by theoretical constructs from disciplines traditionally associated with other paradigms. The ideas borrowed from information theory, statistical mechanics, economics and psychology come immediately to mind. These have provided frameworks within which formal problems have been posed, data collected, and solutions sought. The exponents of each particular perpsective have sought to provide insights into the nature of topical issues, and have in turn by the very nature of their 'world view' been subject to limitations in the interpretation and analysis of behavioural response.

A very considerable number of models have now been proposed and implemented in various studies, and several classification schemes have been put forward (see, for example, Manheim, 1973; Brand, 1973). The classes themselves are often the product of theoretical innovations with the result that what is deemed to be a significant classification is a function of the progress made in problem solving. Many of these theoretical issues with which the travel forecaster becomes pre-occupied are, as Alan Wilson has pointed out, often associated with different manifestations of the aggregation problem. I think this is particularly true of recent times in which the prime distinctions between models have been associated with the way in which a model interfaces with survey data - at an aggregate (grouped) or disaggregate (individual) level - and with the process of aggregation over travel related substitutes. The debate over the adoption of "simultaneous" or "sequential" model structures relates to this latter issue and is fundamentally associated with the relationship between behavioural hypotheses and the analytic structure of the model itself.

The purpose of this paper is to review the theoretical development of travel forecasting models, *with particular emphasis on British experiences*, to examine the problems which have been overcome and those which currently confront us, and finally to offer some views on future prospects.

Many of the criticisms addressing models relate to very basic issues concerned with the manipulation of data, and the generation of forecasting assumptions in conjunction with the analysis of statistical variance in travel patterns. In section 2 some such aspects relating to the aggregation problem will therefore be discussed. Ultimately *theories* of travel demand, as distinct from the technical process of analysis of variance, are concerned with the *interpretation* of this trip dispersion as perceived by an observer - the modeller. Theories of dispersion are thus the subject of section 3. The extension of simple dispersion theories to accommodate choice contexts involving many 'complex' alternatives is examined in section 4. We discuss how an interpretation of the distinction between the traditional 'simultaneous' and 'sequential' travel demand model structures may be shown to relate to the perceived similarity between travel substitutes.

An important line of theoretical enquiry addresses the implementation and refinement of specific demand models, and in particular those currently adopted in transportation studies. In section 5, some theoretical issues concerned with the requirements of internal consistency and the 'goodness-of-fit' of models are considered.

In section 6 a number of topics relating to the evaluation of transport/land-use strategies are reviewed. We discuss the search for mutual compatibility between the demand model, as a predictor of *response*, and corresponding measures of welfare change. This search has shed light on both the interpretation of benefit expressions, and on the derivation of alternative model structures.

In a final section a general overview is given of issues relating to the state-of-the-art, and problems and prospects are discussed.

It is not easy to present in a few thousand words a well balanced view of the many theoretical issues which have emerged in recent years and indeed are currently emerging, and at the same time provide a non-parochial view of British research. Limited by space, and, as ever, by an author's individual perspectives, the material is inevitably selective. The work may be seen as complementary to Neuberger's (1973) review and to the paper by Wilson (1973a).

2. Information, models and the aggregation issue

If the classification process tends to emphasise the *distinctions* between travel forecasting models, it is important to emphasise at the outset that the overwhelming commonality of those implemented in Transport Studies has been their construction with data collected at one snapshot moment of time - the base year. The reliance on cross-sectional information alone has rather significant implications for theory development - the equifinality issue - as we shall note in later sections.

The very possibility of forecasting in the cross-sectional approach - with no other information available than that obtained from traditional surveys - ultimately depends on the ability to associate with a policy a set of (objective) variables which may be shown to account statistically for observed variability in behaviour, and the will to attribute to this *correlation* a *causal* association. Having specified and estimated travel demand models on the basis of statistical criteria in conjunction with hypotheses and a miscellany of assumptions, the focus of attention changes to the elasticity parameters - essentially the first derivatives of the demand function with respect to policy variables. The model is converted into a sitmulus-response relation.

With the restrictions on available data, it is not perhaps surprising that modellers have resorted so readily to the classical notion of equilibrium in the transport and activity systems. Although many dynamic models have been proposed which may be calibrated with two or more cross-sectional data sets (see for example, Wilson, 1974) very few structures have reached an operational stage. The dynamic disequilibrium models developed at Cambridge

(Echenique and de la Barra, 1977) and Leeds (Mackett, 1977) which incorporate the mutual interaction between the land use and transport systems are two exceptions. In this representation the causal structure of the model may be unambiguously defined.

The cross-section may be 'probed' for information on the attributes of 'spatial actors' - individuals, households and firms - and for their *association* with activity-transport options. The variability in behaviour, manifested in a data set, is crucially dependent on the definition of states and categories. In turn, the series of transformation on data between the raw state and travel forecasts - which constitutes the modelling process itself - inevitably involves many simplifications, and it is a matter of current concern that the approximations currently adopted prevent the process from exceeding a threshold of relevance. These aspects are closely related to the aggregation issue itself, and we can point to: the conventional treatment of individual decision units within wider organisational units (individuals within households); the treatment of linked trips; the *definition* of routes, modes, location, etc., as increasingly studied areas in which grouping may, and usually does, imply a loss of information crucial to the understanding of behaviour. It cannot be emphasised too strongly how basic data processing and state definition (together of course with specific modelling assumptions) immediately presents limitations on the range of behaviour which a model can address. Indeed, at the preliminary stage of state specification possibly more than any other, the observer (modeller) can impose his perceived model of behaviour on the interpretation and analysis of response. We shall return to these issues in section 7.

Much of the (current?) euphoria which has been associated with the 'second generation' modelling framework relates to the successful marriage of micro-economic theory with an efficient process of information analysis, advances in sampling theory and econometric methods of model estimation. (Later it will be important to distinguish carefully between these aspects in dealing with criticisms raised against the state-of-the-art.) In the early 1960s there was concern that the aggregate approach simply did not exploit the full range of statistical variability in a data set and was, at best, inefficient. The specification of appropriate units of analysis has long been the subject of interest both from the viewpoint of analysis of variance and with the problem of interfacing with the representation specific behavioural or forecasting assumptions. Wootton and Pick (1967) and Fleet and Robertson (1968) clearly showed, in the context of trip generation, how to exploit a greater range of variability in a data set by avoiding an initial aggregation to the zonal level. The work reported by the Coventry Transportation Study (1973), and by Lawson and Mullen (1976), (see also Dale, 1977) on trip generation models based on the individual can be seen as a natural extension of these earlier studies both from the viewpoint of statistical efficiency and in coming to grips with the analysis of household interactions, which are now the subject of so much interest (see Section 7).

Ultimately *theories* of travel demand are concerned with an explanation of the influence of policy variables on both the 'natural' processes of development of decision units and the dynamics of their association with activity/travel options. In the case of households, in contrast to many businesses and firms, transport and location variables are not considered to have a significant direct influence on life-cycle dynamics, and theories have traditionally addressed the problem of association. Now, *current* patterns of association may be attributed to individual decisions made at some previous point in time, and the cross-section itself may be viewed in terms of a compression of the processes giving rise to observable aggregate patterns. In section 3 we shall examine two theoretical approaches to the problem of aggregation over the decisions made at a micro-level and the treatment of association.

Before proceeding to these aspects it is useful to comment briefly on some general characteristics of model specification. In the formulation of a model three types of formal entity are encountered: *endogeneous variables* which will, in general, be implicit or explicit functions of time *and* policy variables; *exogenous variables* which will be a function of time but not policy variables; and *parameters* or *constants* which are assumed to be dependent on neither. Models differ very significantly according to the classes in which particular entities are to be found. A whole range of criticisms directed at models may be traced to these distinction. In table 1 we note the typical classification involved in strategic Transportation Studies. Those studies which embrace models of the activity system clearly include certain land use variables in the endogenous set.

It would be desirable in a number of study contexts to specify additional endogenous variables. These might include, for example: car availability; car ownership; car occupancy; trip rates; peak hour factors; stock supply functions, etc. The subtle relations between many of these variables and transport system attributes have long resisted *satisfactory* identification. In the last five years considerable progress has been made in this task by using disaggregated data sets. Indeed dissatisfaction with the use of car occupancy factors and the requirement to test car pooling schemes has greatly contributed in the United States to the acceptability of the micro-analytical approach (Spear, 1977), in which multiple occupancy has been interpreted in terms of a choice process.

ENDOGENOUS VARIABLES	EXOGENOUS VARIABLES	PARAMETERS OR CONSTANTS
Demand for Travel over space (distribution function)	Planning variables	Peak hour factors
	Zonal population, employment, etc.	Trip rates
	Income	"Elasticity Parameters"
Mode Shares	Value-of-Time	β, λ, δ etc.
Route Shares	Car Ownership	

Table 1: Classification of Selected Variables in Conventional Transport Studies.

3. Theories of Dispersion and the policy-model interface

In the absence of specific theoretical considerations it was perhaps inevitable that the traditional components of trip modelling - generation, distribution, modal split, and assignment - should be seen as such distinct areas of study. One of the tasks of the theorist is to present a unified account of the dispersion existing in observed patterns of movement. This dispersion is as we have stressed identified in terms of the occupancy of *pre-specified* states or categories. Both Entropy Maximising and Random Utility maximising methods which embody the notions of probability distribution likelihood and choice between discrete alternatives, respectively, may be abstracted from specific choice contexts (frequency, location, mode and route) and can thus address all these contexts (though of course this does not guarantee satisfactory models).

It may in fact be partly due to the simultaneous emergence of the much heralded generalised cost concept (developed in the micro-statistical studies of Warner, 1962 and Quarmby, 1967) and its incorporation into the Entropy maximising models introduced in the SELNEC study (Wilson, *et al.*, 1969) that the methodology which generated these models (Wilson, 1967, 1969) had such an immediate impact and profound influence on British Transportation research and practice. Here was a *statistical* interpretation of trip dispersion, a flexible, easily applied and systematic way of generating models, which were *typically* expressed or converted into multinomial logit form, consistent with any relevant information available to the analyst. The method was deemed to generate models with the minimum number of *a priori* assumptions. It brought internal consistency to more highly disaggregated models (which involved improved market segmentation on 'trip-end' variables) and allowed new structures to be proposed. Indeed, its role in model generation has been nothing short of spectacular in the wider context of urban reserach in which complex phenomena and 'inter-locking' systems demand that models be underpinned by a rigorous accounting framework (for recent references see Macgill and Wilson, 1979; Wilson and Macgill, 1979).

While there are at least four interpretations of the Entropy Maximising method (Wilson, 1970) it is perhaps unfortunate, though natural, that the physical analogy should be primarily recalled. Recently the generalisation of the approach to include Kulback's information measure (Kulback, 1959), has found application in minimum information adding contexts (Batty and March, 1977; Snickars and Weibull, 1977).

Wilson has expressed the view that aggregation over preference relations at the micro-level may be performed only under unacceptably restrictive assumptions. He has however envisaged the formulation of models, using Entropy Maximising methods in conjunction with utility maximising principles (Wilson, 1973b). Southworth (1977) has recently implemented such a model which involves elastic trip frequencies and expenditures, and which provides a link between the trip generation and distribution stages of the strategic transport model.

The micro-approach beckoned not only because of the potential for greater statistical efficiency, the *possibility* of increased policy sensitivity and the greater likelihood of parameter transferability, but also because of the possibility of combining these facets with a 'first principles' attack on the aggregation problem which the 'behavioural school' finds a desireable prerequisite to the satisfactory *explanation* of macro-behaviour. It is most important in a consideration of random utility theory of probabilistic choice models to distinguish between those assumptions which express the basic philosophy of the approach with those 'secondary' assumptions which are invoked to produce 'workable' models.

Among these 'secondary' assumptions are included the following:

(i) That the multiple objectives of each individual may be resolved in the formation of (net) utility functions, linear in attributes, which are used to record preferences.

(ii) Interpersonal variation of characteristics (eg. values of travel attributes and *unobserved* attributes) may be introduced by means of random parameters or 'stochastic' residuals in this linear function, the distributions of which are pre-specified by the modeller.

(iii) Each individual decision maker scrutinises all alternatives and selects that which offers the highest net utility.

(iv) Under changed conditions individuals re-assess their choice contexts.

It should be stressed that the uncertainty which requires the introduction of random variables is a characteristic of the observer (the modeller) who invokes this device to account for dispersion - the fact that some people with identical *observable* characteristics select different alternatives. The decision maker himself is, in this theory of rational choice, considered to choose optimally and consistently within his own utility frame of reference.

The probability that an individual with particular characteristics (drawn randomly from a population with similar *observable* characteristics) selects a given alternative, may now be formally expressed as the probability that the net utility associated with that option is the maximum associated with the set of alternatives. An analytic expression for the choice probabilities in terms of the choice and individual characteristics will result if certain specialised distributions are assumed for the random utilities. We have:

(v) The multinomial logit model may be derived from the assumption of identical and independently distributed Weibull functions. Simple probit models are similarly generated from normal distributions.

Policies are directly interpreted in terms of changes in the attributes incorporated in generalised cost functions which are essentially the linear utility functions discussed above. One of the merits of the random utility approach was that it provided a unifying behavioural context for earlier studies on the value of travel attributes (see the discussion by Daly, 1978). In British Studies it has become the widespread practice to *input* a *given* generalised cost function estimated from disaggregate data external to the model(as for example, that recommended by McIntosh and Quarmby, 1970) with possible 'tuning' of the coefficients. The generalised cost function is scaled in spatially aggregated expressions by parameters (β and λ in conventional notations) and their estimation, together with such factors as modal penalties, forms the basis for model calibration. (A more rigorous and consistent approach is available to the aggregation and estimation procedures as for example outlined by Domencich and McFadden, 1975; and Ben Akiva, *et al.*, 1977).

The generation of identical model functions from alternative theoretical standpoints - the equifinality issue - has been recognised for some considerable time. The interpretation of such a model - in this case the multinomial logit model - does however depend on the theoretical notions assumed to underpin it, and this is particularly true of the embedded parameters. Take, for example, the λ and β parameters which appear in association with generalised and composite costs in the modal split (choice) and distribution (location) models. Now, we can seek to interpret these parameters through an examination of the model properties, and indeed they may be seen to be prominent in dispersion, moment (eg. mean cost) and elasticity (response) relations (Thrift and Williams, 1980). On the other hand each has an additional and distinct interpretation dependent on the underpinning theory. In Entropy Maximising models the parameters correspond to Lagrange multipliers associated with the change in likelihood of observing a given allocation (share) pattern (probability distribution) with respect to incremental changes in system trip cost measures. In random utility theory on the other hand the parameters are inversely related to the standard deviation of the utility distributions from which the choice model is generated.

These general comments are equally true for the interpretation of the 'balancing factors' or 'shadow prices' which are associated with constrained gravity models. Indeed, one of the interesting bby-products of the equifinality issue is the elaborate correspondence between the partition (generator) and potential functions associated with the model in its statistical mechanical interpretation and the various cost related value indicators in the economic interpretation (Williams and Senior, 1978; Leonardi, 1978).

4. Model structures and the treatment of substitutes

Did the theoretical constructs above extend to an arbitrary number of substitutes of differing character? Here there was certainly need for clarification. In a multiple-mode context involving, for example, car, bus and rail, should a three movement multinomial logit structure be adopted or should a hierarchical structuring be developed which entailed an initial division of appropriate trips between private transport and a public 'composite' mode, followed by a sub-split between bus and rail. These 'simultaneous' (or 'joint choice') and 'sequential' (or 'recursive') representations can be pictorally represented as in figure 1.

Many such dilemmas presented themselves in different modelling contexts which included mixed-modes, multiple routes and mode-route combinations. Perhaps the most obvious example of structural ambiguity, however, related to combinations of choice 'dimensions' involving frequency (G), location (D), mode (MS), and route (A), which were employed in the strategic transport planning models. With regard to the location and modal combinations in particular, should 'sequential' or 'simultaneous' models be developed as in the traditional pre-distribution (MS/D), post-distribution (D/MS) and joint distribution-modal split (D-MS) structures represented in figure 2. How were the D and MS 'sub-models' to be interfaced in the MS/D and D/MS structures, and what was the significance of the composite costs used for this purpose? Were they subject to restrictive properties? Could we appeal to theory to ratify the general model structures, and indeed to discriminate between them?

On the question of interfacing the distribution (location and modal-split models the entropy maximising method could not give unambiguous guidance because it did not involve any unifying statements or hypotheses linking these segments. Nevertheless Wilson discussed in detail the simultaneous and sequential (nested logit) structures and proposed a series of possible composite costs to transmit modal information to the distribution model in the D/MS system (Wilson, 1969). This important paper introduced several innovations which were to be implemented in the SELNEC model (Wilson, *et al.*, 1969) and more recent transportation studies.

In the early 1970's the question of model structure came into sharper focus. Three American studies by Manheim (1973), Charles Rivers Associates (1972) (see also Domencich and McFadden, 1975) and Ben-Akiva (1974) were to have considerable influence on theoretical and practical developments. The developments of Charles Rivers Associates (1972) and Ben-Akiva (1974) were concerned with the implementation of micro-models for shopping trips involving the combination of frequency, location and mode, and with the rationalisation of model structures from behavioural principles of utility maximisation.

Ben-Akiva strongly recommended the simultaneous structure - and the multinomial logit model in particular - both because the latter was assumed to faithfully embody a simultaneous consideration by decision makers of the mode-destination combinations, and because there was, it was argued, no *a priori* reason to choose between the alternative orderings D/MS and MS/D which were demonstrated to yield different values for the elasticity parameters.

In other modelling contexts however, particularly those involving multiple modes and mode-route combinations it was becoming increasingly clear that the multinomial logit model could not be indiscriminately applied. The model suffered a well-known restrictive property of cross substitution, quaintly termed the Independence from Irrelevant Alternatives characteristic (IIA), rendering it highly suspect in those case in which a subset of options were more 'similar' than others. The dangers implied by the IIA property were epitomised by the red bus-blue bus conundrum (Mayberry,1970) which was a whimsical but important example, characterising a whole class of problems, to be resolved in theoretical developments designed to account for the 'similarity' between alternatives of a choice set.

In random utility theory the notion of similarity between substitutes could be directly accommodated in terms of *correlation* between the utility distributions associated with different alternatives. It was soon recognised that the multinomial logit model, which results from the assumption of equal and uncorrelated Weibull distributions, was often applied in conjunction with utility functions which themselves manifested natural correlation. In other words the model was, in general, inconsistent with the form of utility function assumed to generate it.

The injection of correlation between the random components of utility functions took a number of forms. Williams (1977a) showed that the introduction of stochastic components which corresponded to the form of certain additive separable utility functions, resulted in the natural formation of 'structured' or 'hierarchical' models. Further, just as Strotz (1957) had shown in his treatment of commodity grouping in classical theory, here in the random utility theory of choice between discrete alternatives, composite options and suitably defined composite costs, or index prices could be defined which allowed the consistent integration of partial models in hierarchical structures. One special member of this class of models - the nested or hierarchical logit form - was independently shown by Williams (1977a) and by Daly and Zachary (1978) to be generated from random utility maximising behaviour. The latter implemented the structure in the Huddersfield Bus study. McFadden (1978) in turn showed that the nested logit model was a special case of a class of general extreme value (GEV) models.

In essence this line of theoretical research has indicated the conditions under which the nested logit and similar structured models can be generated from random utility theory, and more specifically has shown that:

(i) one interpretation of the distinction between the 'sequential' and 'simultaneous' forms shown in figure 1 can be traced to the existence of correlation or similarity between alternatives. In the absence of this similarity one structure will transform into the other, D/MS, MS/D → D-MS.

(ii) in the structured logit model the underpinning requirement of rational choice behaviour imposes two particular restrictions on the resulting model relating to:

<u>A</u>: the form of the composite cost (utility), which is unique (up to an additive constant)

<u>B</u>: a relationship between certain elasticity parameters characterising the partial shares of the hierarchy.

If either of these conditions is violated the model will not be consistent with utility maximisation.

It is disturbing to note that in those British Transport Studies employing the (aggregate) nested model structure G/D/MS/A, one or other, and usually both, of these conditions has been violated, and this can be shown to result in highly unrealistic response characteristics in common policy testing contexts (Senior and Williams, 1977; Williams and Senior, 1977). The source of these pathological results for the calibrated D/MS structure has recently been attributed by Hawkins (1978) to the coarse market segmentation and the treatment of short trips usually adopted in Transport Studies (again aspects of the aggregation problem).

An alternative attack on the correlation problem has been made by adopting multinomial probit models (derived from normal distributions for the random utility components) for the multi-modal problem both in particular, and in a general form with arbitrary variance-covariance matrix (Domencich adn McFadden, 1975; Langdon, 1976; Haussman and Wise, 1978). The recent implementation of the Clark approximation (Clark, 1961) by Daganzo, Bouthelier and Sheffi (1977) has extended the possibility of applying this very general model in arbitrary choice contexts beyond the few alternatives (four or five) achieved by direct numerical integration.

In spite of advances in the generalised probit field there appears to be a need for a compromise between the generality afforded by this model and the computational tractibility of simpler models, for example, the nested logit model which incorporates a more restricted form of 'similarity' between alternatives. McFadden's general class of extreme values models (McFadden, 1978) is one such possible research direction. Another *ad hoc* model is the cross correlated logit model D*MS, proposed by Williams (1977a) which contains the two structured logit (D/MS, MS/D) and multinomial logit (D-MS) forms as special cases.

A further line of enquiry, which is increasingly adopted for model implementation and in testing the theoretical accuracy of models involving approximations in their derivation, is the use of Monte Carlo simulation. Here sampled 'individuals' are confronted by choice contexts (Albright, Lerman and Manski, 1977; Ortuzar, 1978, 1979; Robertson, 1977; Robertson and Kennedy, 1979). Using this method Williams and Ortuzar (1979) have recently examined the accuracy of the multinomial, nested logit and cross-correlated logit models in two dimensional contexts (say D and MS). Their results suggest that the practice suggested by Williams and Senior (1977) and Ben-Akiva (1974, 1977a) of employing the multinomial logit model and alternative forms of hierarchical structure (and be guided by the size of the 'similarity coefficient') is not likely to result in substantial error.

5. Issues in model development and implementation

The development of the subject is closely associated with the more efficient and satisfactory implementation of particular demand models chosen for trip forecasting. Perennial concerns of both theorists and practitioners are with the appropriate level of parameter classification, estimation procedures, the production of acceptable goodness-of-fit measures, and with the achievement of consistency between the measures of transport service (times and costs) incorporated in different parts of the model. With the exception of a very few models, developed in a research context (see the reviews by Wigan, 1977) the approaches adopted in British studies for the achievement of equilibrium have been *indirect* - the submodels are calibrated independently and the output of one fed into the next, with a limited degree of 'feedback' between the segments.

The recognition that certain common models corresponded to the optimal solution(s) of particular mathematical programs has had a number of practical implications. Firstly, it allowed certain algorithms and solution procedures to be subject to formal scrutiny and their convergence properties assessed. Secondly, in those cases for which the programs may be underpinned by some theory of behaviour at the macro- or micro-levels, the dual variables have usually had some very relevant significance to the evaluation problem and could furthermore be exploited in model solution. While primal variables tend to have a 'stock' or 'flow' interpretation, duals could be interpreted in terms of 'prices' or 'value differentials'. Thirdly, the properties of the models themselves could be illucidated through an examination of the structure of the mathematical program. Fourthly it has allowed demand models to be integrated into consistent frameworks of analysis including those embodying demand-supply equilibria.

That the doubly constrained gravity (distribution) model could be generated from extremal principles was recognised by Murchland (1966) and Wilson (1967), and the dual properties of this program have been examined by Evans (1973a) and Wilson and Senior (1974). It has been shown that its limiting form corresponds to the optimal solution of the corresponding transportation problem of linear programming (Evans, 1973). Murchland (1977a) has formally examined the convergence properties of the doubly constrained gravity model balancing equations and proposed a termination criterion, while Champernowne, *et al.* (1976) have solved the dual program through direct minimisation to produce simultaneously the 'deterrence parameter' and balancing factors. Evans and Kirby (1974) have furthermore indicated how the conventional iterative approach to the balancing of the model with an empirically derived deterrence function could be formally examined in terms of a three dimensional Furness procedure.

It has long been known that the Kuhn-Tucker conditions (Kuhn and Tucker, 1951) of a program introduced by Beckmann, *et al.* (1956) corresponded to the Wardrop network equilibrium conditions for congested assignment (Wardrop, 1952). Murchland's elegant exposition of Rockafellar theory (Murchland, 1969) has provided the basis for the construction of algorithms for computing solutions for congested assignment models with elastic demand and the rigorous analysis of convergence. This work has been extended and elaborated by Evans (1976) and others in combining distribution and assignment models and more recently incorporating modal split (Florian, 1976). The comparison of heuristic approaches to the achievement of equilibrium with rigorous methods remains an important line of research (see for example, Murchland, 1977b; Van Vliet, 1977).

The work described above is concerned with the properties and implementation of models which can be shown to have a programming derivation. But what of their ability to adequately correspond, in a statistical sense, with base year travel patterns? Some recent British concerns include studies on the design of appropriate zoning systems for the analysis of interaction data (see the collection of papers in the special issue of *Environment and Planning, A.*, 10, 1978), the fine disaggregation of distribution and modal split model parameters by Southworth (1977) using the relative likelihood approach developed by Hathaway (1973, 1975) and Ian Williams (1976) to investigate the *significance* of segmentation; and the influence on modal split parameter values of alternative methods of estimation (Senior and Williams, 1977). Recent work at University College London, has examined in considerable detail the statistical issues involved in calibrating gravity models (Kirby, 1974; Leese, 1977; Kirby and Lees, 1978) and a new test for its acceptance or rejection has been proposed.

We shall return in section 7 to reconsider the role which the analysis of variability and 'goodness-of-fit' plays in the concerns of model validity.

6. The demand model - evaluation interface

Although forecasts of travel demand are seldom ends in themselves, they have traditionally dominated those parts of the planning process implemented in transport studies. While the scope of transport strategy appraisal has broadened considerably since the early 1960's (Dalvi and Martin, 1973; Bayliss, 1976) the computation of so-called 'user benefits' associated with a policy remains a central feature of scheme evaluation.

In this section I shall comment briefly on certain aspects of the relation between the demand model as an instrument of *response prediction* and corresponding *measures of welfare change* implied by that response. Five specific areas have recently attracted attention:

(i) The possibility of obtaining exact nonmarginal measures of user benefit

(ii) The generation of mutually consistent models and evaluation measures within the framework provided by random utility theory.

(iii) The accuracy of the so-called 'rule-of-a-half' consumer surplus measure

(iv) The disaggregation and interpretation of spatial benefit measures

(v) The broadening of the evaluation framework to embrace arbitrary changes in the land use (activity)-transport system, and in particular the establishment of links with urban economic theory.

Since the mid-1960's the change in consumer surplus arising from the introduction of a scheme has become a standard measure of user benefit, and the 'rule-of-a-half' (ROH) introduced in the London Transport Study Phase III (Tressider, *et al.*, 1968) has served to relate such changes in welfare to the demand response. This marginal expression was initially proposed as an *ad hoc* extension of the Marshallian (area under a demand curve) trapezium equation and, as such, it was recognised that there were certain problems in accounting for the 'shifting' demand curve phenomenon accompanying simultaneous changes in the perceived generalised cost of travel substitutes. Elaborate arguments were put forward to suggest that such effects were small and could for all intents and purposes be safely ignored.

By appealing to the generalised measure of consumer surplus proposed by Hotelling (1938) - a natural extension of the Marshallian framework which embraced the full range of substitution accompanying demand response - Neuberger (1971) noted that for the multinomial logit model (in fact the singly constrained gravity model) an exact measure of benefit could be obtained. This result was to be reconstructed within the framework of random utility theory by Koenig (1975) and Cochrane (1975). It was further recognised (Williams, 1976; Champernowne, *et al.*, 1976) that the ROH itself resulted from a particular method of evaluating the line integrals which were central to Hotelling's scheme of welfare measurement.

It was natural to enquire whether exact measures could be generated using Hotelling's method for the more complex models involved in Transport Studies, and under what conditions were such measures compatible with the models derived within the framework of random utility theory. These questions were addressed by Harris and Tanner (1974), Williams (1977a, 1977b) and by Daly and Zachary (1978). In contrast to the common method of extracting user benefits *after* the forecasting process, it was found that the condition of optimality which underpins the behavioural models allowed the economic measures to be extracted directly from the calculations performed during model implementation. For *suitably specified* urban transport planning models this distinction is between the assessment of benefits at the trip generation stage rather than *after* the assignment submodel.

With the provision of exact measures of consumer surplus it has been possible to test the accuracy of the 'rule-of-a-half' in policy analysis. Except in those cases for which the ROH is known to be inapplicable - as, for example, may happen in the introduction of new options or the suppression of existing ones - the marginal approximation has been found to be in error typically by less than 5% (Neuberger, 1971; Williams and Senior, 1977) and does not affect the ranking of schemes.

It should be emphasised that the main value of these exact measures is not therefore in their numerical or computational significance, but in the provision of new perspectives on the evaluation problem. In particular, by focussing on the choice contexts of spatial actors it allows a meaningful analysis of the spatial disaggregation of benefits, and a rigorous - though not necessarily unique - interpretation of such concepts as 'accessibility' in terms of consumer surplus (Neuberger, 1971; Koenig, 1975; Cochrane, 1975; Williams, 1977a; Williams and Senior, 1978; Ben-Akiva and Lerman, 1979).

It has been shown that these exact measures may be extended to embrace arbitrary changes in the transport *and* land use (activity) system, in cases for which theresultant (derived) demand for travel may be considered to result from rational choice behaviour (Neuberger, 1971; Williams, 1976; Williams and Senior, 1978). This provides a basis for the generation of activity patterns themselves from programming frameworks which embrace the transport related options available to producers and consumers (Coelho and Wilson, 1976; Coelho, 1977; Coelho and Williams, 1978), and thus establishes a link with urban economic theory in the tradition of Alonso (1964) and Herbert and Stevens (1960). To derive meaningful measures of locational benefit in cases for which the 'stock' of houses and employment locations changes slowly, it is necessary to underpin appropriate spatial interaction models with an economic rationale (Beckmann and Wallace, 1969; Cochrane, 1975; Champernowne, *et al.*, 1976; Williams and Senior, 1978; and Brotchie, 1979). Total benefit is now apportioned between the demand *and* supply sides in housing, job and land 'markets'. The nature and determination of urban rent is now central to the computation and distribution of benefit.

The results of these analyses on the mutual interaction of the transport and activity systems are, I believe, not solely confined to academic interest. Who *does* benefit from the introduction of a transport policy? What portion of the benefit ultimately will percolate into the activity system and manifest itself in changes of rent differentials? What are the distributional implications of these changes? The answers to these questions will ultimately depend on the model of the urban economic system proposed and the mechanisms assumed to be embedded in it (Scott, 1978; Williams and Senior, 1978).

7. Problems and prospects

It is not infrequently argued that the modelling experience over the past twenty years has contributed significantly to our understanding of traveller behaviour. Puristsmight well argue that this rather optimistic perspective is at best misplaced, and would point to a desireable distinction between our knowledge of current patterns of movement (a question of information, which, though still limited, few would deny has greatly increased) and our understanding of, and ability to predict realistically, the *response* of travellers to proposed changes in the activity-transport system. They might indeed question what *can* be known about some future state from observations on cross-sectional information alone.

Pragmatists on the other hand, who view the holding of absolute standards as an ultimate luxury, accept incomplete knowledge as a natural state of affairs and look to the evolutionary upgrading of the technical process for improvement in the forecasting method. They would point to the conventional stage of the problem solving paradigm - generalised sensitivity analysis - as the appropriate means for handling the many uncertaintities involved in modelling response. That the practitioner is invariably placed under severe implementation constraints of costs, times, and existing software is widely appreciated. Where practicalities come into conflict with strict theoretical integrity, the compromise is not surprisingly in favour of the former as the more candid technical reports reveal.

It is against such a background of conflicting attitudes and changing requirements of the planning process, that innovations and the state of the art are judged. Inevitably *theoretical* debate however focusses on three generations of model development: the first involving the so-called aggregate models; the second, the disaggregate perspective based on analytic probabilistic choice models; and the third, the activity frameworks in which social roles emerge as a theme of prime significance. In this context it is worth remarking that for many years the models in Britain have been of hybrid form and if a classification were demanded it would be somewhere between first and second generations. By incorporating the household based category analysis, and embedding generalised and composite costs (with micro-parameters) within the framework provided by Entropy Maximising models, the SELNEC and later studies (though deficient in many known respects) could I believe be defended from the more severe forms of censure reserved for American Transportation Study practice in the late 1960's and early 1970's, as for example discussed by Barker (1973), Ben Akiva (1973) and Domencich and McFadden (1975).

So, what of the contribution of theory? Did theoretical innovations consolidate, or provide a post hoc rationalisation of, existing practice? Did they deal hammer-blows to widely held beliefs? What new interpretations, concepts, or suggestions emerged? Some of these questions have been addressed already in Sections 2-6. A few more comments are in order.

We noted above the dominance of the Entropy Maximising models (and therefore presumably the acceptability of the method) in British practise. If we take the SELNEC model (Wilson, *et al.*, 1969) as the first practical expression of the approach it clearly built on and consolidated existing G/D/MS/A models. The method did in turn inspire the detailed specification of many subsequent models including those applied in the most recent West Yorkshire (G/D/MS/A) and Greater London (G/D-MS/A) Transportation Studies.

Random Utility theory too provided a rationalisation of existing model structures, modifying them to be consistent with the notion of travel as a derived demand. By showing how alternate structures could be derived, it uncovered an interpretation of the theoretical distinction between them. It further indicated how general structures could be formulated which embodied many existing models (eg. D/MS, MS/D, D-MS) as special cases. A solution was found to the red-bus/blue bus, and related, problems which in turn allowed insight into the nature of travel options themselves. Indeed the hypernetwork concept discussed by Sheffi (1978) and others may be seen in terms of a generalisation of the abstract mode concept introduced in the mid-1960s. The framework also provided a basis for the construction of consistent evaluation measures.

The theory of course is not without its critics (I have myself heard it described by a seasoned North American transport planner as "a load of statistical junk"!) and some of these have been anticipated in Sections 2 and 3. As with all 'first principles' approaches the formalism does provide a basis for its own self examination, and most importantly reveals the extent to which the modeller imposes *a priori* assumptions on the analysis.

It is argued that the current geneartion of models 'pay lip-service to behaviour' (Burnett and Thrift, 1979) and involves "More of a characature than a true description" (Heggie, 1978). How can the subtleties of adaptive behaviour involving complex trip linkages be portrayed by choice models? How can the framework accommodate the detailed constraints in time and space under which individuals find themselves, and how can the detailed interactions within and between households be incorporated? How do we account for limited and often heterogenous information in choice contexts and the how do we incorporate habit, learning and satisficing behaviour on the part of the decision maker? (Heggie, 1978; Banister, 1978; Heggie and Jones, 1978; Burnett and Hanson, 1979).

With an appropriate definition of states and market segmentation, and the relaxation of the 'secondary restrictions' involved in the formation of random utility models, many of the above facets of choice contexts could in principle be accommodated within the existing theoretical framework, and some progress has been made in these direction' (see for example, Adler and Ben-Akiva, 1979; Goodwin, 1977; Hensher, 1975; Ben-Akiva, 1977b; Gaudry and Dagenais, 1979; Williams and Ortuzar, 1979). Of more profound importance however is whether a satisfactory empirical basis for the implementation and validation of such models is available!

Such fundamental questioning of the tenets of choice models based on random utility theory has been accompanied by the call for new approaches to the study of activities within a space-time framework originally conceived by Hagerstrand (see, for example, Jones, 1977; Carlstein *et al.*, 1978; Burnett and Hanson, 1979). Heggie (1978), in advocating the use of gaming techniques, has expressed doubts whether behaviour can ever be successfully examined within the conventional mathematical paradigm. Such a gaming approach has found expression in the HATS technique developed at the Oxford Transport Studies Unit in which the individual, within a family context, may determine preferable courses of action and resolve any conflicts which these imply (Jones, 1979).

The extension of this approach to formal modelling on a computer (if indeed it is sought) will be aided by the development of an appropriate state-attribute representation. It may well be that the list processing device employed by Orcutt, *et al.*(1961) and Wilson and Pownall (1976) in conjunction with Monte Carlo simulation will prove a useful step in this direction. A similar type of micro-simulation study has been developed in a car-pooling model by Bonsall (1979) in which transfer pricing methods were adopted in the estimation of model parameters. This procedure of processing individual decision units within an extended micro-representation using Monte Carlo methods is currently being applied in Leeds in the examination of a broader range of urban phenomena (Wilson and Pownall, 1976; Clark, Keys and Williams, 1979, 1980).

Although considerable progress may be claimed in the development of models since the mid-1960s, particularly in relation to the use of micro-data, a variety of problems remain as the weight of the above criticism implies. What successes may be claimed in relation to *solved* problems have taken typically five to ten years to bring to fruition. It is sobering to think, for example, of the alternative and possibly more realistic explanations of model structure based on satisficing behaviour which awaits full development (Brand, 1973; McFadden, 1975; Williams and Ortuzar, 1979; Richardson, 1978). What then of the status of generalized cost and separability?

Ultimately we seek the invariants in traveller and location behaviour, and this was of course one of the prime incentives for the development of the disaggregate approach. The issue of transferability of model parameters in space and time appears now however to be a much more controversial issue than it was five years ago (Southworth, 1978, Kirshner and Talvitie, 1977). It remains to be seen how successful hypotheses based on time budgets will prove. Hypothesis generation and testing remains a most fundamental concern, and progress here is hampered by the lack of available data, and the recognition that the cross section alone will often prevent discrimination between competitors.

I believe that one of the most significant classifications of formal models will in future be not on the basis of their structure but on their calibration procedures - whether they will be implemented on the basis of stated or revealed preferences. That is, whether some indication of response is sought (through, for example, transfer pricing methods) or whether currently observed behaviour is thought to be the most reliable indicator of future actions. In the case of "new modes", for example, the "abstract mode" concept was indeed attractive, but to the extent that success in the public transport field is sought in unconventional concepts, it will be necessary to rely more heavily on attitudinal data and stated preferences. Finally, the much called for distinction between demand and need must here be fully recognized. in examining the implications and possible biases of models calibrated on the basis of observed behaviour.

But what now of the practitioner, who usually works in an environment of competing models and theories? He must be guided by the theorist's understanding of their differences and the circumstances under which such differences would be important (if these are known). He might take some comfort in the equifinality issue which would imply that the ultimate validity of a model (to be judged post hoc) is not necessarily dependent on a discredited or "unrealistic" theory. Sensitivity analysis of parameters, structures, aggregation procedures, etc. will remain his indispensable tool (Bonsall, et. al., 1977; Ben Akiva, 1973; Senior and Williams, 1977; Southworth, 1978). As ever the choice of model and indeed method of forecasting will depend on the questions asked and the nature of the answers sought.

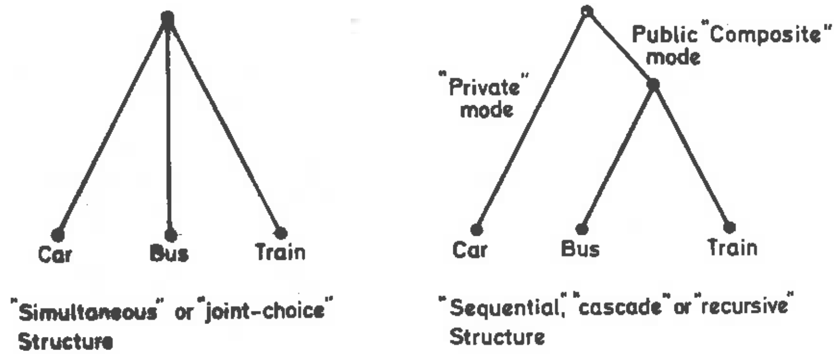

Figure 1: A representation of alternative demand model structures for the choice between car, bus and train

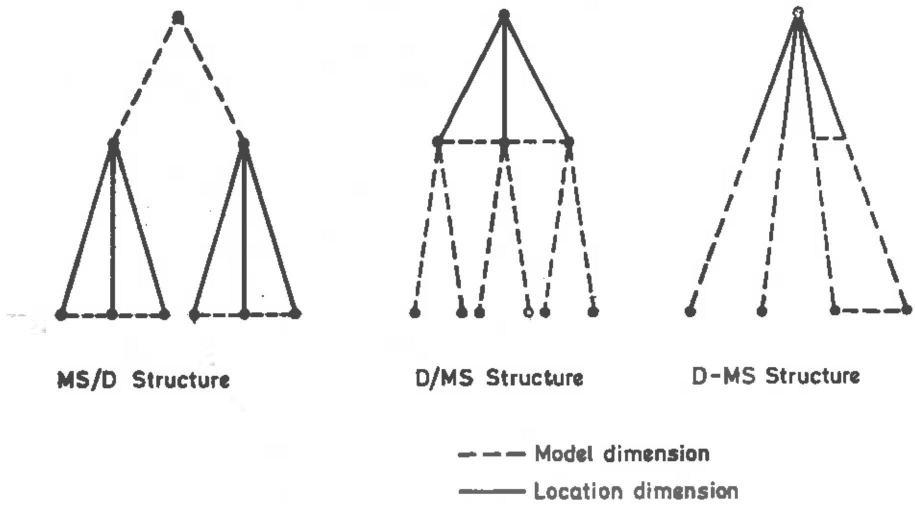

Figure 2: A representation of alternative model structures for the choice between mode-location combinations

References.

Adler, T. and Ben-Akiva, M.E. (1979) A theoretical and empirical model of trip chaining behaviour. Transportation Research 13B, pp. 243-257.

Albright, R.L., Lerman, S.R. and Manski, C.F. (1977) Report on the development of an estimation program for the multinomial probit models. Prepared for the Federal Highway Administration, by Cambridge Systematics, Inc.

Alonso, W. (1964) Location and Land Use. Harvard University Press, Cambridge, Mass.

Banister, D.J. (1978) Decision-making, habit formation and a heuristic modal split model based on these concepts. Transportation, (1978) 7, pp. 5-18.

Barker, W.G. (1973) The use of models in urban transportation planning. Report prepared for the U.S. Department of Transportation.

Batty, M. and March, L. (1977) Dymanic urban models based on information-minimising. In Dynamic Analysis of Spatial Systems Eds. R.L. Martin, R.J. Bennett and N.J. Thrift, Pion Press, London.

Bayliss, D.A. (1977) Priorities in urban transport research. In Urban Transportation Planning: current themes and future prospects. (Eds. P. Bonsall, Q. Dalvi and P.J. Hills), Abacus Press, Tunbridge Wells, pp. 349-362.

Beckmann, M.J., McGuire, C.B. and Winsten, C.B. (1956) Studies in the Economics of Transportation. Cowles Commission monograph, Yale University Press, U.S.A.

Beckmann, M.J. and Wallace, J.P. (1969) Evaluation of user benefits arising from changes in transportation systems. Transportation Science 4, pp. 344-351.

Ben-Akiva, M.E. (1973) Structure of passenger travel demand models. PhD dissertation, Department of Civil Engineering, M.I.T. Cambridge, Massachusetts.

Ben-Akiva, M.E. (1974) Structure of passenger travel demand models. Transportation Research Record, (1974) 526, pp. 26-42.

Ben-Akiva, M.E. (1977a) Passenger travel demand forecasting: Applications of disaggregate models and directions for research. Presented at the World Conference on Transport Research, Rotterdam.

Ben-Akiva, M.E. (1977b) Choice models with simple choice set generating processes. Working Paper, Department of Civil Engineering, Massachusetts Institute of Technology, U.S.A.

Ben-Akiva, M., Adler, T.J., Jacobson, J. and Manheim, M. (1977) Experiments to clarify priorities in urban travel demand forecasting research and development. Summary report, U.S. Department of Transportation.

Ben-Akiva, M. and Lerman, S. (1979) Disaggregate travel and mobility choice models and measures of accessibility. In Behavioural Travel Modelling. (Eds. D.A. Hensher and P.R. Stopher), Croom Helm, London.

Bonsall, P.W. (1979) Microsimulation of mode choice: a model of organised car sharing. P.T.R.C. Proceedings, Summer Annual meeting, University of Warwick.

Bonsall, P.W., Champernowne, A.F., Mason, A.C. and Wilson, A.G. (1977) Transport modelling: sensitivity analysis and policy testing. Progress in Planning, Vol 7 Part 3, pp. 153-237. Pergamon Press, Oxford.

Brand, D. (1973) Travel demand forecasting, some foundations and a review. In Urban travel demand forecasting, (1973), Special Report 143, Highway Research Board, Washington, D.C.

Brotchie, J.F. (1979) A model based on non-homogeneity in allocation problems. In Behavioural Travel Modelling (Eds. D.A. Hensher and P.R. Stopher), Croom Helm, London.

Burnett, P. and Hanson, S. (1979) A Rationale for an alternative mathematical paradigm for movement as complex human behaviour. Paper presented at the Transportation Research Board Mettings, Washington, D.C., January, 1979.

Burnett, P. and Thrift, N. (1979) New approaches to understanding traveller behaviour. In Behavioural Travel Modelling (Eds. D.A. Hensher and P.R. Stopher), Croom Helm, London.

Carlstein, T. Parkes, D. and Thrift, N. (1978) (Eds.) Timing space and spacing time, Vol.2: Human Activity and Time Geography, Edward Arnold.

Champernowne, A.F., Williams, H.C.W.L. and Coelho, J.D. (1976) Some comments on urban travel demand analysis, model calibration, and the economic evaluation of transport plans. Journal of Transport Economics and Policy, (1976) 10(3), pp. 267-285.

Charles River Associates (1972) A Disaggregate behavioural model of urban travel demand. Federal Highway Administration, U.S. Department of Transportation, Washington, D.C.

Clark, C.E. (1961) The greatest of a finite set of ransom variables. Operations Research, (1961) 9, pp. 145-162.

Clarke, M., Keys, P. and Williams, H.C.W.L. (1979) Household dynamics and economic forecasting. Paper presented at the European Congress of the Regional Science Association, London. Also as Working Paper 257, School of Geography, University of Leeds.

Clarke, M., Keys, P. and Williams, H.C.W.L. (1980) Micro-analysis and simulation of sicio-economic systems: Progress and prospects. In Quantative Geography in Britain: Retrospect and Prospect. (Eds. Bennett, R.J. and Wrigley, N.) Routledge and Kegan Paul, London. (To be published).

Cochrane, R.A. (1975) A Possible economic basis for the gravity model. Journal of Transport Economics and Policy, 9, pp. 34-49.

Coelho, J.D. (1977) The use of mathematical optimization methods in model based land use planning. PhD Thesis, School of Geography, University of Leeds.

Coelho, J.D. and Williams, H.C.W.L. (1978) On the design of land use plans through locational surplus maximisation. Papers of the Regional Science Association 40, pp. 71-85.

Coelho, J.D. and Wilson, A.G. (1976) The optimal location and size of shopping centres. Regional Studies 10, pp. 413-421.

Coventry City Council (1973) Coventry Transportation Study, Report on Phase Two: Technical report, Parts i and ii.

Daganzo, C.F., Bouthelier, F. and Sheffi, Y. (1977) Multinomial probit and qualitative choice: A computationally efficient algorithm. Transportation Science, 11, pp. 338-358.

Dale, H.M. (1977) Trip Generation: What should we be modelling? In Urban Transportation Planning: Current themes and future prospects. Eds. P. Bonsall, Q. Dalvi and P.J. Hills, Abacus Press, Tunbridge Wells.

Dalvi, M.Q. and Martin, K.M. (1973) Urban transport evlauation procedures. Working Paper 23, Institute for Transport Studies, University of Leeds.

Daly, A.J. (1978) Issues in the estimation of journey attribute values. In Determinants of Travel Choice, Eds. D.A. Hensher and Q. Dalvi, Saxon House.

Daly, A.J. and Zachary, S. (1978) Improved multiple choice models. In Determinants of Travel Choice, Eds. D.A. Hensher and Q. Dalvi, Saxon House.

Domencich, T. and McFadden, D. (1975) Urban travel demand - A behavioural analysis, North Holland, Amsterdam.

Echenique, M. and Barra, T. de la. (1977) Compact land-use transportation models. In Urban Transportation Planning: Current themes and future prospects. Abacus Press, Tunbridge Wells.

Evans, S. (1973a) The use of optimization in transportation planning. PhD dissertation, University College, London.

Evans, S. (1973b) A relationship between the gravity model for trip distribution and the transportation problem in linear programming. Transportation Research 7, pp. 39-61.

Evans, S.P. (1976) Derivation and analysis of some models for combining trip distribution and assignment. Transportation Research 10, pp. 37-57.

Evans, S.P. and Kirby, H.R. (1974) A three dimensional Furness proceedure for calibrating Gravity models. Transportation Research 8(2), pp. 105-122.

Fleet, C.R. and Robertson, S.R. (1968) Trip generation in the transport planning process. Highway Research Record 240, Highway Research Board, Washington, D.C.

Florian, M. (1976) A traffic equilibrium model of travel by car and public transport modes. Report No. 32, Centre de Recherche sur les Transports, Universite de Montreal, Canada.

Gaudry, M.J.I. and Dagenais, M.G. (1979) The DOGIT model. Transportation Research 13B(2), pp. 105-112.

Goodwin, P.B. (1977) Habit and hysterisis in mode choice. Urban Studies 14, pp. 95-98.

Harris, A.J. and Tanner, J.C. (1974) Transport demand models based on personal characteristics. Transport and Road Research Laboratory, Supplementary Report 65 UC.

Hathaway, P.J. (1973) Some statistical problems associated with trip distribution models. Proceedings of P.T.R.C. Summer Annual Meeting, University of Sussex.

Hathaway, P.J. (1975) Trip distribution and disaggregation. Environment and Planning A, 7, pp. 71-97.

Hausman, J.A. and Wise, D.A. (1978) A Conditional probit model for qualitative choice: discret decisions recognizing inter-dependence and heterogeneous preferences. Econometrica, 46, pp. 403-426.

Hawkins, A.F. (1978) Some anomalies due to misspecification in transport models. P.T.R.C. Proceedings, Summer Annual Meeting, University of Warwick.

Heggie, I.G. (1978) Behavioural dimensions of travel choice. In Determinants of Travel Choice (Eds. D.A. Hensher and Q. Dalvi) Saxon House Press, pp. 100-125.

Heggie, I.G. and Jones, P.M. (1978) Defining domains for models of demand. Transportation 7, pp. 119-125.

Hensher, D.A. (1975) Perception and commuter modal choice: an hypothesis. Urban Studies 12, pp. 101-104.

Herbert, D.J. and Stevens, B.H. (1960) A Model for the distribution of residential activity in urban areas. Journal of Regional Science 2, pp. 21-36.

Hotelling, H. (1938) The general welfare in relation to taxation and of railway and utility rates. Econometrica, 6, pp. 242-269.

Jones, P.M. (1977) Travel as a manifestation of activity choice: trip generation re-visited. In Urban Transportation Planning: current themes and future prospects. (Eds. P. Bonsall, Q. Dalvi, P.J. Hills) Abacus Press, Tunbridge Wells. pp. 31-49.

Jones, P.M. (1979) 'HATS': a technique for investigating household decisions. Environment and Planning A, 11, pp. 59-70.

Kirby, H.R. (1974) Theoretical requirements for calibrating gravity models. Transportation Research, 8(2), 97-104.

Kirby, H.R. and Leese, M.N. (1978) Trip distribution calculations and sampling error: some theoretical aspects. Environment and Planning A 10, pp. 837-851.

Kirshner, D. and Talvitie, A. (1977) Specification, transferability and the effect of data outliers in modelling the choice of mode in urban travel. Working Paper SL-7701, Department of Economics, University of California, Berkeley, California.

Koenig, J.G. (1975) A Theory of urban accessibility: a new working tool for the urban planner. In Urban Traffic Models, (1975) P.T.R.C. Summer Annual Meeting, University of Warwick, Coventry, England.

Kuhn, H.W. and Tucker, A.E. (1951) Non-linear programming. Proceedings of the 2nd Berkeley Symposium on Mathematical Statistics and Probability, University of California Press, Berkeley, pp. 481-492.

Kullback, S. (1959) Information theory and statistics. John Wiley, New York.

Langdon, M. (1976) Modal split models for more than two modes. In Urban Traffic Models, P.T.R.C. Summer Annual Meeting, University of Warwick.

Lawson, G.P. and Mullen, P. (1976) The use of disaggregate modelling techniques in the Telford Transportation Study. P.T.R.C. Proceedings, Summer Meeting, University of Warwick.

Leese, M.N. (1977) A Statistical test for trip distribution models. Paper presented at the 9th conference of the Universities Transport Studies Group, January 1977.

Leonardi, G. (1978) Optimum facility location by accessibility maximising. Environment and Planning, A, 10, pp. 1287-1305.

Macgill, S.M. and Wilson, A.G. (1979) Equivalences and similarities between some alternative urban and regional models. To be published in Urbano Systemi. Also Working Paper 239, School of Geography, University of Leeds.

Mackett, R.L. (1977) A Dynamic activity allocation transportation model. In Urban Transportation Planning: Current themes and future prospects. (Eds. P. Bonsall, Q. Dalvi and P.J. Hills) Abacus Press, Tunbridge Wells.

Manheim, M.L. (1973) Practical implications of some fundamental properties of travel demand models. Highway Research Record, (1973) 422, pp. 21-38. Highway Research Board, Washington, D.C.

Mayberry, J.P. (1970) Structural requirements for abstract-mode models of passenger transportation. In R.E. Quandt (Ed.) The Demand for Travel: Theory and measurement. Heath Lexington Books, Lexington, Massachusetts. pp. 103-125.

McFadden, D. (1975) Economic applications of psychological choice models. Working Paper 7519, Travel Demand Forecasting Project, Institute of Transportation and Traffic Engineering, University of California at Berkeley.

McFadden, D. (1978) Modelling the choice of residential location. In Spatial Interaction Theory and Planning Models (Eds. A. Karlqvist, L. Lundqvist, F. Snickars and J.W. Weibull) North Holland, Amsterdam.

McIntosh, P.T. and Quarmby, D.A. (1970) Generalised cost, and the estimation of movement costs and benefit in transport planning. MAU note 179, Department of the Environment, London, England.

Murchland, J.D. (1966) Some remarks on the gravity model of trip distribution and an equivalent maximising procedure. Mimeo, LSE-TNT-38, London School of Economics, London.

Murchland, J.D. (1969) Road traffic distribution in equilibrium. In Mathematical Methods in the Economic Sciences. (Mathematisches Forehungsinstitut Oberwolfach, West Germany).

Murchland, J.D. (1977a) Convergence of gravity model balancing operations. P.T.R.C. Summer Annual Meeting Proceedings, University of Warwick.

Murchland, J.D. (1977b) Constructing congested assignment test problems with a known solution. In Urban Transport Planning: Current themes and future prospects. (Eds. P. Bonsall, Q. Dalvi and P.J. Hills) Abacus Press.

Neuberger, H.L.I. (1971) User benefit in the evaluation of transport and land use plans. Journal of Transport Economics and Policy (1971) 5, pp. 52-75.

Neuberger, H.L.I. (1973) Travel demand models. Department of the Environment, Economic and Statistical Notes, No. 14.

Orcutt, G., Greenberger, M., Rivlin and Korbel, J. (1961). Microanalysis of Socio-Economic Systems: A simulation study, Harper and Row, New York.

Ortuzar, J.D. (1978) Mixed mode demand forecasting techniques: an assessment of current practice. P.T.R.C. Proceedings, Summer Annual Meeting, University of Warwick.

Ortuzar, J.D. (1979) Testing the theoretical accuracy of travel choice models using Monte Carlo simulation, Working Paper (Forthcoming), Institute for Transport Studies, University of Leeds.

Quarmby, D.A. (1967) Choice of travel mode for the journey to work, some findings. Journal of Transport Economics and Policy (1967) 1,(3). pp. 273-314.

Richardson, A.J. (1978) A Comparative analysis of transport choice models. Civil Engineering Department Working Paper 78/11, Monash University, Australia.

Robertson, D.I. (1977) A Deterministic method of assigning traffic to multiple routes of known cost. Department of the Environment, Department of Transport, TRRL Report 757, Crowthorne.

Robertson, D.I. and Kennedy, J.V. (1979) The choice of route, mode, origin and destination by calculation and simulation. Department of the Environment, Department of Transport, TRRL Report 877, Crowthorne.

Scott, A.J. (1978) Urban Transport and economic surplus: notes towards a distributional theory. In Spatial Interaction Theory and Planning Models. (Eds. A. Karlqvist, L. Lundqvist, F. Snickars and J.W. Weibull) North Holland, Amsterdam.

Senior, M.L. and Williams, H.C.W.L. (1977) Model based transport policy assessment, 1: the use of alternative forecasting models. Traffic Engineering and Control, 18, pp. 402-406.

Snickars, F. and Weibull, J. (1977) A Minimum information principle - theory and practice. Regional Science and Urban Economics 7, pp. 137-168.

Southworth, F. (1977) The calibration of disaggregated trip distribution and modal split models: some comparisons of macro- and micro-analytic approaches. PhD Thesis, School of Geography, University of Leeds.

Southworth, F. (1978) Modelling the changing relationship between travel demand and land use. Working Paper 225, School of Geography, University of Leeds.

Spear, B.D. (1977) Application of new travel demand forecasting techniques to transportation planning: a study of individual choice models. U.S. Department of Transport, FHWA, Office of Highway Planning.

Strotz, R.H. (1957) The empirical implications of a utility tree. Econometrica, (1957), 25, pp. 269-280.

Thrift, N.J. and Williams, H.C.W.L. (1980) On the development of behavioural location models. Economic Geography (In press).

Tressider, J.O., Meyers, D.A., Burrell, J.E. and Powell, T.J. (1968) The London transportation study: methods and techniques. Proceedings of the Institute of Civil Engineers, 39, pp. 433-464.

Van Vleit, D. (1977) An application of mathematical programming to network assignment. In Urban Transportation Planning: Current themes and future prospects. (Eds. P. Bonsall, Q. Dalvi and P.J. Hills) Abacus Press, Tunbridge Wells. pp. 147-158.

Wardrop, J.G. (1952) Some theoretical aspects of road traffic research. Proceedings of the Institute of Civil Engineers, 39, pp. 433-464.

Warner, S.L. (1962) Stochastic choice of mode in Urban Travel: A study in Binary Choice. Northwestern University Press, Evanston, Illinois.

Wigan, M.R. (Ed.) (1977) New Techniques for Transport Systems Analysis. Special Report Number 10, sponsored by the Australian Road Research Board and the Bureau of Transport Economics.

Williams, H.C.W.L. (1976) Travel demand models, duality relations and user benefit measures. Journal of Regional Science 16(2), pp. 147-166.

Williams, H.C.W.L. (1977a) On the formation of travel demand models and economic evaluation measures of user benefit. Environment and Planning A, 9, pp. 285-344.

Williams, H.C.W.L. (1977b) Generation of consistent travel-demand models and user-benefit measures. In Urban Transport Planning: Current themes and future prospects. (Eds. P. Bonsall, Q. Dalvi and P.J. Hills) Abacus Press, Tunbridge Wells.

Williams, H.C.W.L. and Ortuzar, J.D. (1979) Behavioural travel theories, model specification and the response error problem. Presented at the P.T.R.C. Annual Summer Meeting, University of Warwick.

Williams, H.C.W.L. and Senior, M.L. (1977) Model based transport policy assessment, 2: Removing fundamental inconsistencies from the models. Traffic Engineering and Control 18, pp. 464-469.

Williams, H.C.W.L. and Senior, M.L. (1978) Accessibility, spatial interaction and the spatial benefit analysis of land use - transportation plans. In Spatial Interaction Theory and Planning Models. (Eds. A. Karlqvist, L. Lundqvist, F. Snickars and J.W. Weibull) North Holland, Amsterdam.

Williams, I. (1976) An assessment of calibration techniques for doubly constrained trip distribution entropy models. Transportation Research 10(2).

Wilson, A.G. (1967) A Statistical theory of spatial distribution models Transportation Research, Volume 1, pp. 253-269.

Wilson, A.G. (1969) Entropy maximising models in the theory of trip distribution, mode split and route split. Journal of Transport Economics and Policy, Volume 3, pp. 108-126.

Wilson, A.G. (1970) Entropy in Urban and Regional Modelling. Pion, London.

Wilson, A.G. (1973a) Travel demand forecasting: achievements and problems. In Urban Travel Demand Forecasting, Special Report 143, Highway Research Board, Washington, D.C.

Wilson, A.G. (1973b) Further developments of entropy maximising transport models. Transport Planning and Technology, 1, pp. 183-193.

Wilson, A.G. (1974) Urban and Regional Models in Geography and Planning. John Wiley and Sons, London.

Wilson, A.G., Hawkins, A.F., Hill, G.J. and Wagon, D.J. (1969) Calibrating and testing the SELNEC transport model. Regional Studies, 2, pp. 337-350.

Wilson, A.G. and Macgill, S.M. (1979) A systems analytic framework of comprehensive urban and regional modelling. To be published in Geographica Polonica. Also Working Paper 209, School of Geography, University of Leeds.

Wilson, A.G. and Pownall, C.E. (1976) A new representation of the urban system for modelling and for the study of micro-level interdependence. Area 8, pp. 246-254.

Wilson, A.G. and Senior, M.L. (1974) Some relationships between entropy maximising models, mathematical programming models and their duals. Journal of Regional Science, 14, pp. 207-215.

Wootton, H.J. and Pick, G.W. (1967) A model of trips generated by households. Journal of Transport Economics and Policy, 1, pp. 137-153.