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COMPREHENSIVE DYNAMIC URBAN MODELS : INTEGRATING MACRO- AND MICRO-APPROACHES

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CONTENTS

1. INTRODUCTION

Over the last ten years the most important single development in urban modelling has been the construction and operationalisation of dynamic models. Progress, on many fronts, has been swift, often involving elegant mathematics and novel insights into the dynamic properties of models that have been used for some time have been presented, as well as new models being proposed. The field has also become fashionable, attracting researchers from other disciplines to try their hand at this new and growing subject area. As a consequence there now exists an impressive body of literature concerning the development of dynamic urban models (for example, Wilson (1981), Griffith and Lea (1983) and Dendrinos and Mullally (1985)).

Yet despite all the progress made there do remain some unanswered questions which largely concern the applicability and relevance of dynamic models to understanding and helping solve problems of contemporary urban society. In this paper we aim to raise some of these issues and attempt to respond to them in a number of ways. What we shall argue is that the dynamic models that have been developed so far need to be extended into a framework that incorporates two different but complementary approaches to dynamics and at the same time allows for a more detailed assessment of policy issues. We do not pretend that this extension is an easy one or that we have solved all the problems associated with it. Instead we present some early results as well as some topics for further research.

The majority of dynamic urban models that have been constructed have been done so at the aggregate (or meso-) scale. In addition attention has largely focussed on spatial structure - such as the distribution of service centres, housing, industry and so on. These models have attempted to examine how changes in stock variables by location takes place over time in response to a number of factors. Most importantly it has been effectively demonstrated that discrete changes in the structural variables (such as shopping centre size and location) can occur in response to a small and smooth change in one of the control parameters. This concept of critical points of parameter values, either side of which the model solution takes a significantly different qualitative form, is of tremendous significance in urban analysis.

However, concentration of attention on the 'supply side' of urban systems has been at the expense of consideration of the demand and activity aspects of these systems. For example, in retailing it is equally important to understand and model how the demand for different types of goods changes over time and also how consumers' activity patterns respond to changes in the structure of the supply side. We shall argue that models of these separate sub-systems should also incorporate dynamics and then be integrated within a comprehensive framework that includes all elements of the demand- supply-activity system. The methodology we propose for modelling the demand-activity component of this system is termed micro-simulation and will be described in a little detail in sections 3 and 5 of this paper. The reason for adopting this approach we hope will become evident as the paper progresses but largely relates to the fact that in terms of the demand and activity sub-systems we are often faced with the problem of a population that is characterised by heterogeneity and interdependence, and in such situations an aggregate representation, in the form of an occupancy matrix,becomes both unwieldy and computationally unmanageable. A benefit of adopting a micro-level approach is that it allows a consistent treatment of accounting relationships because of some fairly straightforward conservation rules being applied. This will all be described in more detail below.

The biggest potential advantage of the micro-simulation approach, and we emphasise at this stage the word "potential", is the possibility of examining a wide range of policy related issues, particularly when these involve assessing the distributional impacts of plans on various sectors of a community or a city's population. This can be achieved because of the high level of information retention in micro-simulation models due to the absence of any prior aggregation scheme. Not that we can avoid the issue of aggregation and this is a topic to be discussed in section 3.

The rest of the paper is structured as follows. In section 2 we briefly outline some of the main developments in comprehensive dynamic modelling that have occurred over the last 10 years or so. Section 3 will present a framework that allows for the integration of macro- and micro-approaches to urban dynamics. Section 4 examines some models of spatial structure that take account of some recent developments in relation to handling rents and prices explicitly. Section 5 contains the specification of a micro-simulation model of an urban system focussing on household activity dynamics. Section 6 presents some first results from an attempt to use the framework we have proposed, in this case related to retailing systems. Section 7 outlines how the range of models discussed thus far can be extended into a comprehensive framework and also explains how list-processing can be used as an efficient accounting scheme. Finally, in section 8, we highlight some outstanding research tasks and offer some concluding comments.

2. DEVELOPMENTS IN COMPREHENSIVE DYNAMIC URBAN MODELS

In this section we briefly review some of the more important approaches to dynamic modelling, with obvious emphasis on applications in urban and regional analysis. In particular we shall focus on the developments associated with the application of dynamical systems theory and demonstrate how both of the methodologies developed later in this paper fall into this category of modelling.

Perhaps one of the most surprising aspects of the early development of dynamic modelling in the mid-1970s was that it came at a time when conventional urban modelling was coming under a critical backlash that was of both a philosophical (eg. Sayer, 1976) and practical (eg. Lee, 1973) nature. While much of the criticism had a degree of validity it also sparked a counter-response (see for example Bennett and Wrigley 1981, Hay 1979, Wilson 1977 and for a more general view, Giddens 1979) which at the same time as defending the position of urban models in general, also helped to nurture the development of new methods based around dynamical analysis.

Of course a concern with dynamics is not exclusively a recent phenomenon but it can convincingly be argued that the publication of Renee Thom's *Structural Stability and Morphogenesis* in 1975 and the popularisation of the ideas from catastrophe theory by Chris Zeeman, Ian Stewart and Tim Poston, amongst others, in the social sciences stimulated a number of modellers to take dynamics seriously again. An irony, that has emerged as work in this field progresses, is that while catastrophe theory stimulated many to investigate dynamic models, it has really had little impact on the nature of the models that have been developed. Urban systems simply cannot be represented in the restricted form that is imposed by C.T., despite many novel attempts to do so. Of more relevance is the general approach of dynamical systems theory which is concerned with the properties and solution of differential equations. What makes this approach interesting is the focus on non-linearities, which are a characteristic feature of most urban models. This commonly implies the existence of multiple equilibrium solutions to dynamic models and it is in the understanding of how change from one equilibrium solution to another takes place that has been the centre of a considerable amount of work in recent years.

We shall briefly review some of this work before discussing dynamic models developed at the micro-level.

Two main developments in dynamic modelling can be singled out for attention. The first is the work built upon the approach outlined in Harris and Wilson (1978), the second is the research by the Brussels school particularly that of Peter Allen. Harris and Wilson elegantly demonstrated how a conventional retail trade model could be reformulated into an equilibrium model of retail size and location and how dynamics could be introduced through changes in exogenous variables and parameters. It was shown that for small and smooth change in any of these variables discrete change in the endogenous variables (retail centre size) could occur. This was due to a shift from one equilibrium solution to another. This behaviour had analogies with the fold catastrophe but is generated independently of C.T. Early numerical experiments which demonstrated not only the subtlety of the mechanisms of dynamic change but also the analytic intractability of these models were reported in Wilson and Clarke (1979). A particularly important concept was described which is known as the 'backcloth problem'. In simple terms this concerns the problem that the dynamic behaviour of any one structural variable is dependent not only on the factors directly influencing that variable but also on the behaviour of every other structural variable in the model. This leads to some horrendous mathematical complications and resort to numerical experiments is essential. This in itself raises some interesting points concerning the role of numerical experiments in non-linear analysis. As Campbell *et al* (1985) point out in an extremely interesting paper "in the past *non-linear* was nearly synonymous with *nonsolvable*" (p.374). It is through the combination of theoretical *and* numerical work that most progress has and will continue to be made in the field of dynamic urban models.

The Harris and Wilson formulation of the retail model was further developed and modified and results of the work can be found in Clarke and Wilson (1983a, 1985), Clarke (1981, 1984, 1985). Extensions of the Harris-Wilson argument for retail systems to other urban sub-systems include: housing and residential location (Clarke and Wilson 1983b), industrial location (Birkin and Wilson 1984) and agricultural location (Wilson and Birkin 1984).

In addition to the equilibrium model a set of disequilibrium models based on differential or difference equations have also been constructed and tested. These have interesting properties in that there exist a number of bifurcation points at which the qualitative nature of the dynamics change, for example from smooth to oscillatory behaviour. Accounts of this strand of work can be found in Beaumont, Clarke and Wilson (1981), Clarke (1985) and Wilson (1981).

The final step in the argument is of course to develop comprehensive models where the important relationships and inter-dependencies between subsystems are modelled explicitly, and in addition to the components mentioned above a separate transportation model is also incorporated. Preliminary results using this type of framework are presented in Birkin, Clarke and Wilson (1984) and a more detailed analysis can be found in Clarke, Wilson and Birkin (1985).

There are a number of shortcomings with the models that have so far been developed, notably the explicit representation of prices and rents. Steps have been taken to remedy this situation and we shall outline the approach we have adopted for the retail system in section 4.

A related and complementary approach to that already discussed can be found in the work of Peter Allen and his colleagues in Brussels. Their work is a development of ideas in physical chemistry that were pioneered by Ilya Prigogine, a Nobel laureate. He discovered that the dynamic evolution of certain chemical reactions could be strongly influenced by small and random fluctuations. These ideas have been transferred into dynamic modelling of socio-economic processes in the following way. Take a set of deterministic difference equations for a spatial system and add to these equations small, random terms. Then use numerical simulation to iterate forward over time. Repeat this exercise several times, for different random number strings and it may well be that rather different dynamic trajectories are generated. This occurs because of the non-linear nature of the difference equations and the existence of bifurcation points. Small changes in the random terms can result in a different branch of the possible trajectories being selected at these bifurcation points.

Applications of these principles have ranged across intra- and inter-urban evolution, population dynamics and energy modelling.

One of the main problems faced in using any of the above set of models is the adequate representation of the heterogeneity and variability that is found in many socio-economic systems. To represent this heterogeneity in conventional urban models requires substantial disaggregation of the state variables. This can be both cumbersome and computationally inefficient and resort to a micro-level representation is often attractive to the model builder. This forms the basis of the micro-simulation approach we describe in more detail later. Micro-simulation involves the solving of a set of dynamical stock-flow equations through numerical simulation using a smallest-unit representation. Underpinning the methodology is the updating of individual and household attributes stored in the form of lists. This list processing is undertaken using Monte-Carlo simulation to determine if events such as birth, death, migration, job-loss, etc. take place. The method therefore relies upon obtaining conditional probabilities for the range of transitions that are to be considered. We present a more detailed exposition of the main features of the approach through an example in section 5.

Micro-simulation has a somewhat chequered history having first been promoted through the work of Guy Orcutt in the early 1960s (Orcutt *et al* 1961) in economics. Indeed the most recent developments have been in economics (Orcutt *et al* 1976, Havemann and Hollenbeck 1980) but there is growing evidence that its application in spatial analysis is being pursued by a number of workers (eg. Bonsall, 1979, Clarke *et al* 1981, Kain and Apgar, 1978, Wegener, 1983). We shall suggest in later sections of this paper that for addressing many contemporary issues of urban policy and planning micro-simulation has much to offer. We are also convinced that an integration of the two main approaches described in this section is a potentially rewarding step forward. We now outline a framework within which this can be achieved.

3. A FRAMEWORK FOR THE INTEGRATION OF MACRO- AND MICRO-APPROACHES

In this section we outline a general framework for integrating dynamic urban models specified at different levels of aggregation. This framework will be embellished in the next four sections through both example and further analysis.

The combination of macro- and micro-models within the same overall framework poses some difficult problems, particularly in relation to aggregation. The question of aggregation has, of course, been a central theme of economic theory for a long time (eg. Fisher, 1922) and continues to attract the attention of urban modellers (eg. Couclelis, 1985). In general terms the aggregation problem relates, on the one hand, to how micro-relations or behaviour can be transformed into aggregate relations, and, on the other to how, say, allocations of resources to aggregate groups are allocated to individuals and households. The problem is particularly acute when strong interdependencies exist between micro- and macro-processes. For example, the price of a given product is determined by the interaction of demand and supply, both of which are themselves a function of price. Thus the demand for a good at a given price cannot be simply taken as an aggregate of individual decisions since the resultant aggregation may yield a change in the price that the producer sets. This micro-macro interdependence presents crucial problems for equilibrium theory. A classic example is Alonso's (1964) model of the housing market, which is essentially driven by the need to simultaneously solve the micro- and macro-level problems unambiguously. However, the problems are equally pervasive in aspatial economic theory, eg. the Walras-Wald model of general economic equilibrium is an approach to the same kind of problem. One of the key features of our own style of research is the argument that reality does not operate in this way. Thus systems are typically not in equilibrium, and if they are this is only as a result of some combination of dynamic processes reaching a steady state. The explicit recognition of the dynamic nature of spatial-economic processes provides an alternative route of attack to these difficult problems, while also providing new kinds of insight into system behaviour (eg. Wilson, 1981).

The framework we propose is set out in simplified form in Figure 3.1 and we now describe each part in turn.

We start with a micro-level specified population of individual and household attributes. We discuss how this may be obtained later but for the moment assume it is available in the requisite form. It is possible to generate a measure of the 'demand' for certain goods and services by aggregating over appropriate individual and household attributes and applying probabilistic models to determine whether, in a given time period, an individual will make use of a certain service or purchase a particular good. To take a health care example, an individual with a given set of age, sex and socio-economic attributes will have a certain probability of becoming a hospital in-patient in a year. By using Monte Carlo sampling and testing every individual in the population we can generate the total sub-population that requires in-patient care in a year. This process can then be repeated to produce an estimate demand for any of the other sub-systems being studied. The technical aspects of this approach are discussed further in section 5.

The aggregate demand generated in this way then forms the input into the supply side models which are specified at the meso-scale. For example, the Harris and Wilson retail location model required the zonal distribution of demand for different types of goods as a model input (and of course the model solution is sensitive to changes in the distribution of demand). The appropriate supply-side models can now be solved to produce the new spatial distribution of (say) facilities as well as associated prices and rents. We explain in detail how this is done in the next section.

The next step in the argument is to determine how these changes in spatial structure, prices, rents, etc. influence the activities of individuals and households in the population. This can be achieved in a number of ways. First the resultant interaction matrices from the supply model can be converted into probabilities. This would then allow, say, the shopping location of each household for particular types of goods to be determined, again using Monte Carlo sampling. The effects of changing prices may be examined by making changes to the household budget, along the lines presented in Wilson and Pownall (1976). It may be possible to account for constraints on activities - for example, to ensure no non-car owning households are allocated to out of town shopping centres - but this is not always straightforward and is a topic we return to later. Some example applications are discussed further in section 6.

The household and individual attributes can then be updated through list processing to account for demographic and other change. This completes the cycle and we can once more aggregate over the new set of attributes to provide the next estimate of demand.

This is the framework in its simplest form and we hope to elaborate the argument in subsequent sections of the paper, building up to the specification of a comprehensive or multi-subsystem model in section 7.

The approach is exploratory but also flexible. For example it might be thought that the "activity" side of the model has a faster dynamic than the spatial structure component and this could be reflected by suitable alteration to the above scheme. We now, however, progress to examining each of the sub-models in more detail and presenting results of their use.

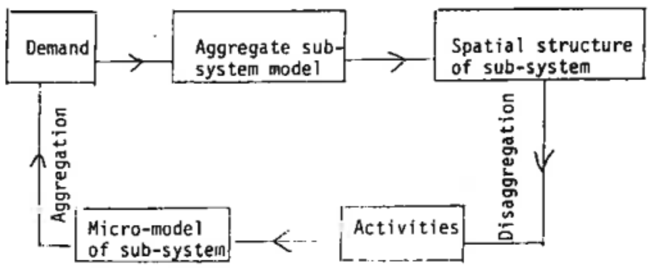


FIGURE 3.1

Structure of the Integrated Framework

4. MODELS OF SPATIAL STRUCTURE

In this section we attempt to summarise recent progress in modelling the supply of urban activities. This is done in four stages. First of all the Harris-Wilson version of the retail trade model is discussed and its implications for comparative statics are drawn out. Disaggregate specifications which enhance the richness and explanatory power of the model are discussed. Under certain circumstances, the equilibrium conditions to the Harris-Wilson model may be seen as the steady state of a dynamic process. The dynamic process provides a useful basis for the economic interpretation of the model solution, and the possibility of alternative mechanisms arises, involving the introduction to the model of rents and prices. This is Step Two.

An alternative interpretation of the Harris-Wilson equilibrium is as the optimality conditions to an appropriate mathematical programme. A flexible approach to this programming problem introduces the possibility of new kinds of equilibria, which may have greater empirical relevance.

Finally, the ideas of the retail model may be applied to other subsystems of the urban and regional economy. The relationships are such that further insights into the basic model structure are provided.

4.1 Comparative statics and disaggregation

Assume that a region is divided into zones labelled i and j, such that the residential populations are given as P_i, demanding a quantity of retail goods per capita, e_i, from a set of producers with output W_j. If S_{ij} is the amount of goods consumed in i which originate at j, and c_{ij} the cost of obtaining those goods, then the equilibrium version of the retail model may be written, following Harris and Wilson (1978) as:

$$S_{ij} = e_i P_i \frac{W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} \tag{4.1}$$

$$\sum_i S_{ij} = kW_j \tag{4.2}$$

where k is an appropriately scaled measure of the cost of provision of facilities.

The optimum number and size of centres may be shown to vary with α, a parameter representing consumer economics of scale, and β, a measure of the friction of distance (Figure 4.1). Specifically, for high α and low β, the size of centres increases and they become less numerous. In principle this analysis could be extended to k, e_i, P_i and c_{ij}. We return to this in section 4.2 below.

One of the most important features of these variations of spatial structure with respect to α and β in Figure 4.1 is that it is not continuous. It is possible to show analytically that individual zones may "jump" between zero and non-zero statis at critical parameter values (Harris and Wilson, 1978; see also Rijk and Vorst, 1983, Lombardo and Rabino, 1984). This feature may also be demonstrated numerically (eg. Wilson and Clarke, 1979) and arises because of the non-linearities implicit in (2) through the interaction terms. Non-trivial dynamic behaviour is also generated through the interdependence in the state variables, $\{W_j\}$, in (1), although these properties are less well understood.

The relationship between the individual plots of Figure 4.1 may be viewed as a temporal transition in which α and β are varying (the "cornershop to supermarket transition" eg. Wilson and Oulton, 1983); or, alternatively, as a series of cases for goods with different characteristics. The latter can be made explicit through a disaggregate specification of the model (1), (2):

$$S^g_{ij} = e^g_i P_i \frac{(W^g_j)^{\alpha^g} e^{-\beta^g c^g_{ij}}}{\sum_k (W^g_k)^{\alpha^g} e^{-\beta^g c^g_{ik}}} \tag{4.3}$$

$$\sum_i S^g_{ij} = k^g W^g_j \tag{4.4}$$

where the superscript g now represents the type of goods involved. High α^g and low β^g would now be associated with goods of a higher order (and likewise high k^g and low e^g_i). All of these features are conducive to a concentration of activities.

To explain the evolution of shopping *centres* we need to take some account of the interdependence between different activities within the retail sector. We can begin to tackle this problem through the concept of the *combined attractiveness factor* (Clarke, 1985). The procedure here involves the new assumption that the attractiveness of a location j for good g which we will now call $\hat{W}_j^g$, depends not only on the level of provision of that good, but also on the levels of provision of all other goods at that location. Thus in general:

$$\hat{W}_j^g = \hat{W}_j^g \,(W_j^1, W_j^2, \ldots, W_j^G) \tag{4.5}$$

In fact a common hierarchical assumption is that:

$$\hat{W}_j^g = (W_j^g)^{\alpha_1^g} \,(\sum_{h \in G'} W_j^h)^{\alpha_2^g} \tag{4.6}$$

where G' is the set of activities of a higher order than g. Thus (4.6) states that the attractiveness of a centre j to good g is multiplicitively related to the attractiveness due to that good itself, but also to the attraction of higher order goods. The modification to the basic interaction equations is then simply the replacement of the basic attractiveness term by the appropriate combined version:

$$S_{ij}^g = e_i^g P_i \frac{\hat{W}_j^g \, e^{-\beta^g c_{ij}^g}}{\sum_k \hat{W}_k^g \, e^{-\beta^g c_{ij}^g}} \tag{4.7}$$

Notice that combined attractiveness provides the basis for flexible hierarchies since the presence of higher order activities will now generate an attraction for lower order goods, but centres of a given order need not accommodate all (or even any) activities of a lower order.

To a certain extent, the combined attractiveness concept provides an inexplicit representation of the basic processes which underpin it. One force which is at work here is "agglomeration economies" to which we return in section 4.4. The other is multi-purpose trip-making, where shoppers visiting a centre may be attracted by a particular oder of activity in the purchase of a different kind of good. Thus one possible avenue for further disaggregation is via the dominant purpose or combination of purposes of a trip (eg. O'Kelly, 1981).

There exist many further possibilities for disaggregation on the demand-side. For example, it will often be desirable to differentiate between trips which originate at home, and those which start at the place of employment. A lot of this detail can be added through a micro-level treatment of a demand-side, as we argue elsewhere in the paper.

Although most of the work conducted with this class of models to date has been of a theoretical nature, empirical work is now beginning. This research is essentially of two types. The first method, which is quasi-theoretical, is based on the longitudinal identification of patterns within the urban service system. One can then hope to identify the kinds of parameter value which give rise to these structures (ie. to locate the system within "parameter space") and thus infer likely patterns of evolution, while also providing a novel interpretation of the historical development of the system (G. Clarke, 1984a,b; 1985a,b). To be more explicitly empirical we need to attempt model calibration, at least with respect to α, β and k. Comparative static forecasts can then be generated on the basis of future projections of these parameters. However one can also identify supply-demand disequilibrium which generates an internal system dynamic for change (using the methods outlined in the next section). Work of this type has begun for Leeds (Birkin and G. Clarke, 1985) and also for Rome (Rabino and Lombardo, 1984).

4.2 From dynamics to economics

It is possible to add a dynamic interpretation to the model of Section 4.1 by allowing floorspace provision to vary with time such that:

$$S_{ij}(t) = e_i P_i \frac{W_j(t)\, e^{-\beta c_{ij}}}{\sum_k W_k(t) e^{-\beta c_{ik}}} \tag{4.8}$$

If we define the suppliers costs, C_j, and revenues D_j, explicitly (cf.(2)) as:

$$C_j(t) = KW_j(t) \tag{4.9}$$

$$D_j(t) = \sum_i S_{ij}(t) \tag{4.10}$$

then it is logical to assume that the change in floorspace through time is related to $D_j(t)$, $C_j(t)$ and $W_j(t)$, plus other relevant parameters:

$$\Delta W_j(t) = f(W_j(t), D_j(t), C_j(t), \ldots) \qquad (4.11)$$

A particularly interesting functional form (eg. Wilson, 1979) is logistic growth:

$$\Delta W_j(t) = \varepsilon(D_j(t)-C_j(t))W_j(t) \qquad (4.12)$$

Notice first of all in this case that growth becomes zero when revenue and cost are balanced, hence the equilibrium of (4.2) is also a steadystate to this set of equations. The second interesting feature of this equation system is that growth can become periodic or chaotic according to the value of epsilon (Wilson, 1979; Clarke and Wilson, 1983).

Equation (4.8) demonstrates an explicit assumption that W_j is the state variable (it is time-dependent), while in effect everything else may be considered as a parameter of the model. In practice, however, the world does not operate like this. Although W_j may be varying relatively quickly, all the variables will in fact be changing simultaneously, at speeds determined by parameters like ε in (4.12). In the case of equation (4.8), W_j is referred to as a fast variable; all the others are slow. This is a crucial concept with respect to subsystem comprehensiveness, since when linked subsystems are considered, the state variables of one model become the endogenous variables of another. We explore these issues further in section 4.7.

It is necessary now to provide some kind of economic interpretation to equations (4.2) and (4.12). Suppose we define:

$$E_j(t) = D_j(t)-C_j(t) \qquad (4.13)$$

to be the economic surplus generated by production at j. The implication now is that in situations when a surplus is available ($E_j(t)>0$) output will expand until that surplus disappears (equation (4.12)). Thus the equilibrium activity patterns will reflect comparative advantage through accessibility. There are certain peculiarities to this treatment - in particular, there is no mechanism by which output can be restricted to maintain an excess profit. We return to this in section 4 below. For the present, however, we are concerned with the method of "distribution" of the surplus, $C_j(t)$. If we define P_j to be the price of goods, and r_j to be a location rent for operation at j, then an alternative mechanism might be:

$$\Delta W_j(t) = \varepsilon^1 E_j(t)\ f^1(W_j(t)) \tag{4.14}$$

$$\Delta P_j(t) = \varepsilon^2 E_j(t)\ f^2(W_j(t)) \tag{4.15}$$

$$\Delta r_j(t) = \varepsilon^3 E_j(t)\ f^3(W_j(t)) \tag{4.16}$$

These equations now imply that spatial advantage may be reflected in differential rents and prices, as well as output levels. The relative importance of the three factors, and the stability of the system, will be determined by the size of the ε parameters, and the nature of the functions f^1, f^2 and f^3. Further discussion is provided by Birkin and Wilson (1985), whence Figure 4.2 is derived.

4.3 Mathematical programming approaches

The importance of Wilson's (1967, 1970) derivation of the gravity model is that the interaction formula (4) is implied as an equilibrium condition to an appropriate mathematical programming problem in which the trip-end constraints are embedded within an entropy-maximising framework. Suitable modifications to the constraint set may then be applied to generate the auxiliary conditions (2). There are many possible ways to do this (Wilson *et al*, 1981), but one of the most appealing is to impose a constraint (W) on total floorspace (Clarke, 1981) so the Lagrangian becomes:

$$\underset{\{S_{ij},W_j\}}{\text{Max}}\ Z = \sum_{ij} S_{ij}(\log S_{ij}-1) + \sum_i \mu_i(e_iP_i-\sum_j S_{ij}) + \gamma(W-\sum_j W_j)$$
$$+ \alpha(\sum_{ij} S_{ij}\log W_j-B) + \beta(C-\sum_{ij} S_{ij}c_{ij}) \tag{4.17}$$

when partial differentiation with respect to S_{ij} $(\frac{\partial Z}{\partial S_{ij}}=0)$ yields the interaction model (4.1), and differentiating with respect to W_j gives:

$$\frac{\partial Z}{\partial W_j} = -\gamma + \alpha\sum_i S_{ij}/W_j = 0$$

$$= \sum_i S_{ij} = \frac{\gamma}{\alpha} W_j \tag{4.18}$$

which is condition (4.2), where $k = \frac{\gamma}{\alpha}$. An interpretation is offered by Clarke (1981).

Alternative models may be derived in the optimisation framework simply by associating new kinds of constraint with the γ term. It was argued by Birkin and Wilson (1985) that the distribution of facilities implied by (4.17) is typically too concentrated for the real world, and that more general versions of the model could be derived by embedding a constraint like (4.2) directly, in place of the total floorspace constraint. A suitable assumption might be that:

$$\sum_j (C_j - \sum_i S_{ij})^2 = \phi \tag{4.19}$$

when the presence of the S_{ij} terms in (4.19) gives a modified interaction model for S_{ij} with the effect of introducing the desired dispersion. In the case of (4.19) one has (Birkin and Wilson, 1985):

$$\frac{\partial Z}{\partial S_{ij}} = \log S_{ij} - \mu_i + \alpha \log W_j - \beta c_{ij} - 2\gamma(C_j - \sum_i S_{ij}) = 0$$

$$S_{ij} = A_i E_i e^{-\gamma(D_j - C_j)} W_j^{\alpha} e^{-\beta c_{ij}} \tag{4.20}$$

Note that many other kinds of assumption could be incorporated in this manner. The possibility exists for building an economic assumptions which are stronger, not weaker, than the original (4.2).

4.4 Extensions to other subsystems

The framework provided by the basic model system (4.1)-(4.2) has a very general theoretical relevance, as argued by Wilson (1983).

In fact many locational problems can be defined in terms of the spatial interactions (cf. 4.1) between a set of location actors whose behaviour is conditioned accordingly (equation (4.2)). The exercise in a given context is then to spell out the nature of the interactions, and the locational dynamics or equilibrium characteristics for the problem at hand.

A good example of this is Weber's (1909) well-known industrial location model, where the problem is to <u>locate</u> production to minimise the cost of the <u>interactions</u> with suppliers and markets. In this specific case modelling the interactions is a trivial exercise, since only one producer is involved, and the interest turns on the specification of the cost function. However extensions to situations with many producers in competition, which Weber found difficult to handle, can easily be tackled within the kind of framework advocated here.

To be a little more explicit, we may define a multi-sector industrial location model as:

$$Y_{ij}^{mn} = \hat{X}_i^{mn} \frac{f(Z_j^n)e^{-\beta^n c_{ij}^n}}{\sum_k f(Z_k^n)e^{-\beta^n c_{ik}^n}} \tag{4.21}$$

where $\hat{X}_i^{mn}$ is the demand of industrial sector m at i for the product of sector n; Z_j^n is production of n at j. The equilibrium conditions would then be something like:

$$\sum_{im} p_i^m Y_{ij}^{mn} = C_j^n \tag{4.22}$$

where prices have now been introduced explicitly into the revenue function on the left hand side.

There are many possible ways of expressing the cost function. One might allow that:

$$C_j^n = F_j^n + K_j^n Z_j^n + \sum_{im} (P_j^m + c_{ij}^m)\, Y_{ij}^{mn} \tag{4.23}$$

Here F_j^n is some kind of fixed operating cost, which usually accounts for the presentce of internal or external economies of scale; K_j^n is a generalised variable cost. The simple Weber problem falls out of (4.21)-(4.23) as a special case with one producer and $F_j^n = K_j^n = P_j^m = 0$ (Birkin and Wilson, 1984B). The retail model of section 4.1 is also a special case in which there is a single producing and consuming sector, prices are standardised, and $F_j^n = 0$ with freight costs absorbed into K_j^n in (4.23).

These ideas can be extended so that it is possible to define a wide range of industrial location models as special cases of the framework (4.21)-(4.23) (Birkin and Wilson, 1984A,B). By varying the mix of assumptions it is possible to create new kinds of model, and many extensions to the basic model form are facilitated, in particular the incorporation of prices and non-linear cost terms.

The problem of industrial location has the same general features as the retail model in the sense that the objective is to locate a set of punctiform supply facilities with respect to a series of demand nodes. A slightly different kind of problem arises when one set of actors is explicitly space-consuming. Residential and agricultural location are both good examples of such a problem.

Agricultural location is rather interesting because of the explicit focus on rent, passed down from von Thunen's (1826) early approach to the problem. This treatment not only opens up the possibility for new kinds of dynamic agricultural land-use model, but can also yield insights into the type of surplus distribution problem discussed in section 4.2.

The problem of residential location is essentially similar to retailing in that a distribution of trips may be generated in relation to a fixed stock of housing which may imply a location disequilibrium between supply and demand, to which the housing market responds. Thus if:

$$T_{ij} = B_j \bar{W}_j E_i e^{-\mu b_{ij}} \tag{4.24}$$

then

$$\Delta H_j = f(\sum_i T_{ij}, H_j, \ldots) \tag{4.25}$$

where H_j, the stock of housing at j, is generally supposed to feed back into the interaction terms through $\tilde{W}_j$, a compound attractiveness factor representing consumer preferences for locating in j; while b_{ij} is a bid rent term, and E_i employment opportunities at i. Much of the interest here arises through the specification of $\tilde{W}_j$. This has been explored numerically by Clarke and Wilson (1983), while Birkin (1985A,B) has endeavoured to demonstrate the links to classical theories of residential location.

5. MICRO-ANALYTIC SIMULATION MODELS

5.1 Introduction

We have briefly alluded to the potential advantages of using micro-analytic simulation models in situations where a system is characterised by a high degree of heterogeneity and interdependence amongst the attribute set under consideration. In this section we discuss the main features of the approach through example - by constructing a dynamic model of retail demand in a metropolitan system. The presentation is necessarily brief. For a full description we refer the reader to Clarke (1985a). The model has two main components: first a model that generates on initial population for a base year; secondly a model that updates the attributes of each household and individual for each year of the simulation period. We describe each in turn.

5.2 The generation of an initial population

One of the main advantages of adopting a micro-simulation approach in modelling complex systems is the efficiency of the micro-level representation. A conventional aggregate occupancy matrix approach tends to become unwieldy and inefficient as the number of attributes under consideration increases. The size of the occupancy matrix will be dependent upon the number of attributes and the number of classes assigned to each attribute. Formally this number, N^1, is given by:

$$N^1 = \prod_{\mu=1}^{M} \alpha^{\mu} \tag{5.1}$$

where α^{μ} is the number of categories associated with the μ^{th} attribute. Typically N^1 will be very large for even a moderate number of attributes and categories. Moreover the occupancy matrix will be very sparse - a large proportion of states will not be occupied by <u>any</u> individual or household.

The micro-level representation will simply consist of the number of individuals and households in the population or sample, multiplied by the number of attributes considered, that is:

$$N^2 = (S^i x\ X_I) + (S^h x\ X_H) \tag{5.2}$$

where S^i and S^h refer to the number of individual and household attributes respectively and X_I and X_H are the number of individuals and households in the sample. Furthermore, for many purposes the sample size needed for a satisfactory representation of the whole population is relatively small and therefore, typically:

$$N^1 >> N^2 \tag{5.3}$$

One possibility to improve the efficiency of the occupancy matrix representation is to decompose the matrix into smaller sub-matrices. It is commonly remarked that considerable information loss is a result of this process.

The storage efficiency and information loss arguments have been used for a strong justification of the micro-level approach. There are, however, a number of crucial considerations that need to be examined before this argument can be fully accepted. These relate to the nature of the joint distribution of characteristics over the population, in particular the amount of interdependence that exists. (for a full discussion of this issues see Clarke 1985a, chapter 5).

However let us assume for present purposes that it is deemed appropriate to use a micro-level representation. The first step in the modelling process is to obtain an initial population or sample of individuals and households specified by an appropriate set of attributes. Ideally, this should take the form of a real sample of micro-units, but for a variety of reasons this may not be available or not available with the required attribute set. In this case the production of a synthetic sample from contingency tables may prove a suitable solution to the problem. The theoretical basis for generating entries to a full contingency table consistent with available conditional and marginal probability distributions is long established (see, for example, McFadden et al (1977)). The method in principle is straightforward, but in practice, due to the lack of detailed conditional probabilities leads to some problems of inconsistency. The basic idea is to build up household and individual attribute lists on the basis of conditional probabilities obtained from published information such as the Census, F.E.S., G.H.S., and so on.

The basic procedure takes the form of producing a joint probability distribution $p(\underline{x})$ as a product of conditionals, as

$$p(\underline{x}) = p(x_1)\, p(x_2|x_1)\, p(x_3|x_2,x_1) \; \ldots\ldots \\ p(x_M|x_{m-1}, \ldots, x_1) \qquad (5.4)$$

and adopting approximations which impose a simplified structure on the conditional dependencies, because of missing information. The estimation of missing information is of course the province of information theory and several well-known techniques are available to assist us in this task, notably entropy maximising and information minimising (Shannon, 1948). These techniques can be used for matrix infilling where row and column totals are known and where a matrix configuration may have to satisfy certain criteria. The scheme we have adopted is based on Feinberg (1970) and is fully described in Clarke (1985a).

For the application discussed in this paper we need to generate the following attributes: for the household - household size, location, age of head, total household income, tenure type, number of children in the household and a household identification label; for each individual - age, sex, race, marital status, occupation, earned income, education status, weeks worked in present year and an identification label. The conditional probabilities that enable the joint distribution of these attributes to be generated are obtained from a number of sources: the 1981 Census, the Family Expenditure Survey, the General Household Survey and so on. To produce our synthetic population we use Monte Carlo sampling and create, on the computer, an appropriate number of linked households and individuals. Perhaps the most important attributes to generate are those relating to income. This is done in the following way. For all individuals in occupation we need to derive a weekly wage and this is calculated on the basis of age, sex and occupation. Unfortunately, a complete age, occupation and sex wage distribution is not available for the 18 occupation classes at the regional or national level in the U.K. What is available consists of age and sex distributions for manual and non-manual workers. The approach we adopted therefore consisted of fitting a non-linear curve to these distributions and then modifying these curves by an appropriate occupation multiplier. That is, we assume the curve takes the same general shape for all the occupations in the manual or non-manual class, for a particular sex. What we do then is to adjust the y-intercept and the maximum value of the curve on the y-axis by an appropriate occupation multiplier that is easily calculated for each occupation/sex category. We can then sample from the resultant adjusted distribution to obtain an age, sex, occupation specific wage. This, however, is only the first stage of the procedure. We also have to take account of the dispersion that is clearly evident around an age, sex, occupation specific wage. This is done in the following way. The New Earnings Survey publish occupation, sex, and region specific wage distributions in the form of deciles. What we do when we have obtained an age, sex occupation specific wage, say W_i^{so}, is to assume there is an equal probability of the wage dispersion lying between the lowest decile and W_i^{so} and between W_i^{so} and the highest decile. Thus we calculate an appropriate cumulative probability distribution between the lowest and highest decile and sample from this in the usual way. So, suppose W_i^{so} was found to be £65 and the highest and lowest decile for that occupation were £40 and £150 respectively, there would be a 0.5 chance that the wage lay in the interval £65-£150.

Total household income can now be obtained by summing over individual members' wages and adding any state benefits they are entitled to. Because we have details of individual attributes most of these are relatively easy to calculate.

To calculate the zonal demand for different types of retail goods we use expenditure multipliers from the Family Expenditure Survey. These give the proportion of household income spent on different goods by household income and type. Again each household is assigned an annual expenditure total based on these multipliers. Aggregation over each household in a given spatial unit provides an estimate of zonal demand for retail goods. These can be used as an input into the aggregate models described in the previous section. A full set of comparative results from this first model are given in Clarke (1985a).

5.3 Household dynamics and retail expenditure

To complement the dynamic nature of the supply-side model we develop a model of household dynamics based on list processing. This simply involves examining each household and individual on the list and determining if they are eligible for certain events or transitions (such as birth, death, migration, redundancy, and so on). If they are, the appropriate conditional probability of the event occurring is determined and Monte Carlo sampling once again invoked.

The effect of a transition from one state to another may have secondary effects. For example, if a woman is deemed to have given birth in a time period not only do we have to "create" a new individual but in addition the female will be taken out of the labour force for a period of time, and this itself may change the total household income which will instead be eligible for certain welfare payments. Household size will also change and this may increase the likelihood of moving house.

In modelling both the housing and employment sub-systems we have to face up to the difficult problem of "chaining". This occurs when an individual or household is both a 'supplier' and 'demander' of a particular good. An individual who moves job occupies a vacancy but, usually, also creates a vacancy on his or her departure. To model these types of systems in a consistent manner we have to aggregate linked supply and demand pools and usually solve an appropriate mathematical program. A full discussion of this family of models can be found in Williams, Keys and Clarke (1985).

In the model developed for retail demand four main components of change are identified: demographics, employment, housing and income/expenditure. The model has been constructed for the Leeds City system, disaggregated into 30 Census wards. As illustration the structure of the demographic model and associated interdependencies is given in Figure 5.1. The model has been run for a 10-year period - 1975-84 and some results are presented in Tables 5.1 and 5.2. Table 5.1 gives the model produced zonal income and expenditure distribution (on 4 good types for 1975) and Table 5.2 gives the same distributions produced by the model for 1984. Further output will be presented in the next section when we describe the integrated model.

To conclude this section it is worth noting that despite the rather brief description of micro-simulation models they are rather complex in terms of the computer programmes that have to be written and the amount of data that is required for their operationalisation. In addition there are a number of technical issues that require further discussion. We refer the reader to Clarke and Williams (1985) which contains a detailed analysis of these topics.

6. AN EXAMPLE OF A COMBINED MACRO-MICRO FRAMEWORK

6.1 Introduction

In this section we describe an example of the use of the integrated framework we proposed in section 3. Essentially it combines the aggregate structural models of section 4 with the micro-model described in section 5, once again using retailing as our example. An important point to emphasise at the outset is that, typically, the use of micro-models in this context is justified for reasons of efficiency and practicality and not on 'philosophical' grounds. In most cases micro-models will exist as an alternative solution method for a set of dynamic equations and the merits or otherwise of the equivalent macro-model should also be examined. Only in cases where a model is specified and estimated at the micro-level can we realistically speak of a separate class of model, and in some situations these may be equivalent to a related aggregate model (see Clarke and Williams, 1985 and Anas, 1983 for further details).

Nevertheless as we have pointed out earlier we believe that there are potentially a wide-ranging number of applications where the integrated framework will have much to offer.

6.2 A dynamic retail supply and demand model for Leeds 1975-1984

The structure of the model is shown in Figure 6.1. This is simply a modification of diagram 3.1 in a retailing context. The time period we consider is from 1975 to 1984. No data was available for the distribution of retail facilities in Leeds when the model was constructed, but this is now becoming available (G. Clarke, 1985b) and will be duly incorporated shortly.

The first results we present are for 1979 using the micro-model derived retail demand and solving the structural model for a three order good system. Because we have no data on retail supply it is not possible to calibrate the structural model. Instead we solve the model under three sets of β^g values (β^1=0.5, β^2=0.4, β^3=0.3), (β^1=0.4, β^2=0.3, β^3=0.2) and (β^1=0.3, β^2=0.2, β^3=0.1). In each case the value of α^g were α^1=1.05, α^2=1.1 and α^3=1.15 and the value of the combined attractiveness function parameter α^{*g} were α^{*1}=0.1, α^{*2}=0.05 and α^{*3}=0.02. The results from the equilibrium model are shown in Figures 6.2, 6.3 and 6.4 respectively. As we would anticipate, lower β^g values give a more concentrated pattern of facility distribution (Clarke and Wilson, 1983). The spatial system consists of 30 demand zones (corresponding to 1971 census wards) and, potentially, 900 supply zones which consist of a 30 x 30 regular lattice of zone centroids superimposed over the city. In the computer produced graphical output the amount of facilities present at a point is indicated by the height of the peak, although due to scaling factors the plots are not strictly comparable in an absolute sense.

In addition we have the output from the associated micro-model. For illustration we present the following tables, while emphasising that many more cross-tabulations are available for reasons mentioned earlier.

(i) The average cost incurred by all households in a zone in travelling to each of the three orders.

(ii) The average cost for each of 10 different household types, in the city for each of the three goods.

(iii) The average cost incurred in travel by different age groups in the city for each order.

Tables 6.1, 6.2 and 6.3 present this information for each of the three sets of parameter values. The most striking yet expected pattern amongst these results is that the average distance travelled by households in different zones increases rapidly as the value β^g decreases, by 277%, 317% and 296% between the highest and lowest values for each order respectively. More specifically the range of distances travelled varies enormously as β^g decreases, and this is illustrated by the increase in variance and standard deviation as shown on each table.

The distributions of costs between different households and between different age groups emphasises the same expected trend - trip lengths become longer as β decreases. There is not a great deal of variation between the different groups in these distributions - it would require a more detailed analysis of activity patterns and constraints to account for mobility differences between these groups. Some suggestions on this score are made in Section 8.

We can now switch attention to looking at how planners may intervene in our system of interest and to examine how they may measure the impact of their policies. The task we set the planner is how to encourage retail development in an area of the city which is badly served. The policy instrument that is available to the planner is to subsidise development costs in a zone. This may allow a zone to move from a no-development-possible state to a development-possible state. The zone we select for analysis is supply zone 294 which is located within demand zone 12, and is an outer area zone. The equilibrium model was firstly run with no variation in k_j, with β^g values of 0.35, 0.3 and 0.25 and α values of 1.1, 1.15 and 1.25 for the three orders respectively. The plots for this run are shown in Figure 6.5, and it can be observed that only the lowest order centre is present in zone 294. A range of successive k_j values were then tested for this zone; these were factors of 0.95, 0.9, 0.8 and 0.5 of the previous value. The resultant plots for these runs are shown in Figures 6.6, 6.7, 6.8 and 6.9. It can be observed that a second order centre emerges when the k_j^2 factor is 0.95 and a third order centre emerges when k_j^3 factor is 0.8. This suggests that higher order centres (of which there are naturally fewer) require a greater amount of "persuasion" to establish facilities in a zone that does not already contain some facilities of that order. As the k_j factor becomes much smaller zone 294 begins to dominate the pattern of structural activity, as can be observed from Figure 6.10. Perhaps a more important feature to observe from these plots is that when a centre is "encouraged" to locate in a certain area this almost certainly has ramifications for the provision of that facility in the rest of the system. As can be seen from the plots when a centre does emerge and grow in zone 294 it does so at the expense of already existing centres which either shrink or disappear completely. This arises simply through the fact that demand is fixed and our hypothesis stating that only normal profits can be made prevents an expansion of supply.

If we now move our attention to the spatial impacts of these policies in terms of average costs we will note a similar set of events arising. Tables 6.4, 6.5 and 6.6 give the average costs for each of the 30 zones for a selected number of k_j factorings, including the case where there is no factoring ($k_j = 1.0$), for each of the three orders. For order I there are no significant changes for each of the runs. This is largely to be expected, as in the "no policy" case order 1 facilities do exist in zone 294 (remember it is changes in zone 12 we should focus on). For order 2 a significant reduction in costs for zone 12 can be observed, changing from 11.47 to 5.92 just for a 0.95 factoring of supply costs, and continues to fall. However this reduction is offset by an increase in the costs of other zones, particularly zone 25 which already had fairly high costs. For order 3 the situation is more complex. Little change occurs for k_j factoring of 0.9 and 0.95 (what change does occur can be attributed to sampling error), and as we would expect it is not until 0.8, when a centre emerges in zone 294, that a shift in costs occurs. Zone 12 has a reduction from 8.722 and 1.742, but this is offset by increases in zones 22 and 19. However, other zones also experience a reduction in costs, most notably zones 11, 21 and 25. This once again illustrates the complexity of spatial equity and distributional issues and when the planner attempts to intervene in a market system to affect a re-distribution, the outcome of intervention can be difficult to predict. On a more positive note these methods do allow the analyst to investigate the decision or parameter space that exists and whereas they do not guarantee some globally optimal plan to be designed they present the opportunity for a wide range of potential plans to be tested.

These preliminary results are mainly designed to show the potential of the approach rather than to justify it. As we noted earlier many of the tables produced here could be generated by a corresponding aggregate model. When we proceed to examine more complex systems the benefits of using this approach should become more self-evident. We now proceed to examine how the framework can be developed for a comprehensive urban system.

7. A COMPREHENSIVE FRAMEWORK

In Sections 4 and 5, we reviewed certain developments in model building at the meso- and micro-scales respectively. An example was then provided demonstrating the method of integration for the retail sub-model. In this section we wish to examine some of the implications of applying this type of approach within a comprehensive modelling framework. The discussion is structured as an attempt to explain the significance of Figure 7.1, wherein a schematic illustration of the type of system we propose is outlined.

The first stage in the exercise of model construction is the identification of a set of initial conditions, which is typically some kind of uniform distribution of activities for theoretical work, or an existing situation for applied work. In the case of the supply-side, this is a methodologically trivial but often practically demanding task, since the data does not need to be processed, but is often not collected at the appropriate levels of spatial and sectoral resolution. On the other hand, much more data is generally available (from various censuses) in relation to demand-side characteristics, but initial populations will usually need to be constructed using appropriate sampling procedures to obtain full joint probability distributions (see Section 5).

For the purposes of this illustration we have assumed a model with four subsystems which are reflected in the supply-side structure variables. $Z_j^k(t)$ represents the distribution of industrial activities producing goods type k in j at time t; $W_j^g(t)$ is retail and service floorspace for good g; $H_i^l(t)$ the stock of housing type l in zone i; and $c_{ij}^{\theta}(t)$ is the transport cost between i and j where the superscript θ represents the union of all supply-side activities which interface with one another through the transport network. We have assumed the demand-side to be represented by a single array of variables, $P_i^{\phi}(t)$, being the population in zone i at time t, with characteristics in the set ϕ, embracing attributes such as age, sex, occupation, marital status and so on to an appropriate degree of detail.

Both the supply- and demand-side structure variables are assumed to be "slow", and form the inputs to a series of "fast" interaction models. These interactions are thus taken to be generated instantaneously with respect to the structure variables, and are of three kinds. The classical journey-to-work and shopping models arise directly at the interface between population distributions and the suppliers; we therefore refer to these as "supply-demand interactions". At the same time, many interdependencies exist between the supply sectors, eg. there are many product flows within the industrial subsystem, while all suppliers must compete for scarce resources such as land and capital - these are "supply-side interactions". Similar processes are operative on the demand-side, for example to determine labour supply and the demand for goods (to satisfy various utilities in consumption) - these are "demand-side interactions".

For a given distribution of activities, the instantaneous allocation of interactions allows a meso-scale dynamic to be generated. Thus summing the flows from a retail model at the supply-side generates a measure of revenue attracted, and this may be coupled with the cost of supply to generate a slow dynamic in which the structure variables respond to disequilibrium through processes like (4.11).

With respect to the demand-side, the first observation which can be made is that the interaction matrices to be generated can be very large and sparse. For example, even a coarsely specified journey-to-work array would be characterised as $\{T_{ij}^{kmn}\}$, where k is the type of industry in which the individual works, m is the mode of the trip, and n is the occupation of the individual. With 100 origin and distribution zones, 10 occupation groups, 10 industry types and 5 modes of transport, an array with 5 million elements is implied. The techniques of micro-simulation and list-processing allow us to sidestep the need for storing and manipulating such huge arrays. Rather the interactions may be modelled, and then converted to conditional trip-making probabilities for input to the simulation process. As we have observed above, this allows us to bring in much finer levels of detail at the demand-side.

The micro-simulation framework is efficient not only in terms of storage, but also with respect to accounting. Since the characteristics of each individual are treated explicitly, the appropriate conservation laws must be obeyed, and global transitions may always be traced back to their original components. Furthermore by processing the individual lists in a variety of ways, numerous types of aggregate variable may be examined for the same system. This may be particularly useful in the calculation of appropriate performance indicators to examine various policy issues (Clarke and Wilson, 1985).

Once the structure variables have been updated through the slow dynamic, the whole process is repeated for the next time period or iteration. Notice that in respect of the structure variables, increments area always applied at the end of the time period, so the sub-models are always solved with respect to lagged variables. In this case a crucial determinant of system evolution is the relative speeds of change (ε) operating with respect to the different subsystem activity levels.

However certain problems may arise in relation to the order of solution of the "fast" interaction models, since the outcome of one submodel may be dependent on the outcomes of one or more others. To take a simple example, the expenditure on goods might be calculated as a residual once costs of tripmaking are deducted from disposable income. Thus inputs to a retail submodel would clearly depend on journey-to-work expenditures. Once again, however, the obvious solution is to adopt an appropriate system of lags, eg:

$$S^g_{ij}(t) = A^m_i(t)\, e^{gm}_i(t-1)\, P^m_i(t)[W_j(t)]^{\alpha^g(t)}\, e^{-\beta^g(t)c^g_{ij}(t)} \tag{7.1}$$

$$e^{gm}_i(t) = f\,(S^g_{ij}(t),\, T^{km}_{ij}(t),\, \dots\,) \tag{7.2}$$

The parameters of the model (7.1) (α^g and β^g) could be projected completely independently, eg. through a time series extrapolation.

8. RESEARCH TASKS AND CONCLUSIONS

In this paper we have attempted to summarise some recent developments in modelling at the meso-scale (Section 4) and the micro-scale (Section 5). We have tried to concentrate our attention on significant theoretical developments, although there reamin many non-trivial technical problems to be resolved. On the activity side this involves consideration of factors such as disaggregation (by mode, by trip purpose); incorporation of more sophisticated measures of the cost matrix; and the treatment of problems of closure. In relation to the supply-side, we need to concern ourselves again with more spatially disaggregate specifications taking into account store size, and the way in which choices are perceived by consumers, whence trip patterns are generated. These problems are discussed in more detail by M. Clarke (1985, Chapter 6).

The kind of integrated framework introduced in this paper represents a potentially important route into the linkage of such consumer choice (or discrete choice) problems within the spatial interaction-activity-structure framework. However, it should be emphasised, as we observed in Section 6, that the main use of the micro-simulation device at present is as a solution technique, which allows efficient storage and manipulation of microdata which is essentially synthetic. Thus one outstanding research task is the problem of interfacing the behaviouralist and activity oriented work (eg. Lenntorp, 1976; Jones, 1979) with the micro-simulation methodology (cf. Thrift, 1981).

In this context it may be argued that one of the weakest features of behavioural geography is its lack of a powerful analytic base. Apart from the development of the logit set of models, themselves taken directly from discrete choice theory in economics, and often applied in a highly contentious manner (see Clarke and Williams, 1982, for reasons why) the behaviouralist tradition has been dominated by empirical analysis, usually in the form of specific case studies. What the newer trends in this field offer, particularly in the area of travel behaviour is a greater emphasis on the choices and constraints that face individuals and households in the decision making process. As we noted earlier, not enough weight is attached to these factors in the micro-simulation models that we have used so far. There is no denying that if the marriage between these two approaches was to be successful there would be a need for a substantial research effort, although the potential rewards are high.

The second major theme in applied research in this area is the extent to which the methods being developed are taken up by planners. Arguably the record in the past is not very encouraging. There typically exists a significant lag between model development and application and in a large number of examples it is simply a case of the models not being used at all. There are a large and diverse number of reasons for this and as much blame can probably be attached to the model builder as to the potential user. Model builders must effectively demonstrate the use of their products, and this often requires the construction of a "package" that is widely transportable and highly flexible. It is believed that the construction of a micro-simulation package that would serve a wide variety of needs, ranging from demographic and household forecasting, labour market analysis, housing system analysis to the kind of appplication seen here, would be a useful addition to the small range of models in practical use at the moment.

What all of this implies is that the logical next step is the construction of a set of models with a practical focus. In view of the synthesis of simulation techniques with location theory (Section 4.4) and the integration of micro- and macro-level approaches, together with ongoing developments in information storage and processing, we believe the potential is now greater than ever for the breakthrough from model-building in theory to model use in actual planning practice.

REFERENCES

Alonso, W. (1964) Location and Land-use. Harvard University Press, Cambridge, Mass.

Anas, A. (1983) Discrete choice theory, information theory and the multinomial logit and gravity models, Transportation Research, B, 17, 13-23.

Beaumont, J.R., Clarke, M., and Wilson, A.G. (1981) Changing energy parameters and the evolution of urban spatial structure, Regional Science and Urban Economics, 11, 287-315.

Bennett, R.J. and Wrigley, N. (Eds) (1981) Quantitative Geography in Britain: Retrospect and Prospect. Routledge and Kegan Paul, London.

Birkin, M. (1985a) The development and application of comprehensive frameworks in modelling the dynamics and evolution of urban spatial structure, forthcoming Ph.D. thesis, School of Geography, University of Leeds.

Birkin, M. (1985b) Models of spatial interaction and urban land use: an application to residential location problems, forthcoming Working Paper, School of Geography, University of Leeds.

Birkin, M. and Clarke, G. (1985) On the design and implementation of an information system for urban modelling and spatial planning, forthcoming Working Paper, School of Geography, University of Leeds.

Birkin, M., Clarke, M. and Wilson, A.G. (1984) Interacting fields: Comprehensive models for the dynamical analysis of urban spatial structure, Working Paper 385, School of Geography, University of Leeds.

Birkin, M. and Wilson, A.G. (1984a) Industrial location models I: A review and an integrating framework, Working Paper 400, School of Geography, University of Leeds.

Birkin, M. and Wilson, A.G. (1984b) Industrial location models II: Weber, Palander, Hotelling and extensions within a new framework, Working Paper 401, School of Geography, University of Leeds.

Bonsall, P. (1979) Micro-simulation of mode choice: a model of organised car sharing, PTRC Proceedings, Summer Annual Meeting, University of Warwick, July 1979.

Campbell, D., Crutchfield, J., Fermer, D., and Jen, E. (1985) Experimental mathematics: the role of computation in non-linear science, Communications of the Association for Computing Machinery, 28, 374-384.

Clarke, G. (1984a) The expansion of service outlets across the city, Working Paper 379, School of Geography, University of Leeds.

Clarke, G. (1984b) The diffusion of retail multiples across the city: a review, Working Paper 388, School of Geography, University of Leeds.

Clarke, G. (1985a) The changing expression of small-scale retailing in the city, 1955-1983, Working Paper 418, School of Geography, University of Leeds.

Clarke, G. (1985b) Retail centre usage and structure: Empirical, theoretical and dynamic explorations, Ph. D. thesis (forthcoming), School of Geography, University of Leeds.

Clarke, G. and Wilson, A.G. (1985) A review of indicators, planning and system performance, forthcoming Working Paper, School of Geography, University of Leeds.

Clarke, M. (1981) A note on the stability of equilibrium solutions of production-constrained spatial interaction models, Environment and Planning A, 13, 601-604.

Clarke, M. (1985a) Integrating dynamical models of urban structure and activity: an application to urban retailing systems, Ph. D. thesis (forthcoming), School of Geography, University of Leeds.

Clarke, M. (1985b) The role of attractiveness functions in the determination of equilibrium solutions to production-constrained spatial interaction models, Environment and Planning A, 17, 175-183.

Clarke, M. and Williams, H.C.W.L. (1982) Spatial organisation of activities and the location of public facilities, Paper presented to a workshop on spatial choice models in housing, transportation and land use, IIASA, Austria.

Clarke, M. and Williams, H.C.W.L. (1985) Micro-Analysis in Urban and Regional Systems. Cambridge University Press, Cambridge (forthcoming).

Clarke, M. and Wilson, A.G. (1983a) The dynamics of urban spatial structure: progress and problems, Journal of Regional Science, 23, 1-18.

Clarke, M. and Wilson, A.G. (1983b) Exploring the dynamics of urban housing structure: a 56 parameter residential location and housing model, Working Paper 363, School of Geography, University of Leeds.

Clarke, M. and Wilson, A.G. (1985) The dynamics of urban spatial structure: the progress of a research programme, Transactions of the Institute of British Geographers, forthcoming.

Clarke, M., Wilson, A.G., and Birkin, M. (1985) The Dynamics and Evolution of Urban Spatial Structure. In preparation.

Couclelis, H. (1985) Cellular worlds: a framework for modelling micro-macro dynamics, Environment and Planning A, 17, 585-596.

Dendrinos, D.S. and Mullally, H.A. (1985) Urban Evolution: Studies in the Mathematical Ecology of Cities. Oxford University Press, Oxford.

Feinberg, S.E. (1970) An iterative procedure for estimation in contingency tables, Annals of Mathematical Statistics, 41, 907-917.

Fisher, I. (1922) The Making of Index Numbers. Boston.

Giddens, A. (1979) Central Problems in Social Theory: Action, Structure and Contradiction in Social Analysis. Macmillan, London.

Griffith, D. and Lea, A.C. (Eds) (1983) Evolving Geographical Structures. Martinus Nijhoff, The Hague.

Harris, B. and Wilson, A.G. (1978) Equilibrium values and dynamics of attractiveness values in production-constrained spatial interaction models, Environment and Planning A, 10, 371-388.

Haveman, R.H. and Hollenbeck, K. (Eds) (1980) Micro-economic Simulation Models for Public Policy Analysis, Academic Press, New York.

Hay, A. (1979) Positivism in human geography: a response to critics, in D.T. Herbert and R.J. Johnston (Eds) Geography and the Urban Environment: progress in Research and Application, John Wiley, Chichester.

Jones, P.M. (1979) HATS: a technique for investigating household decisions, Environment and Planning A, 11, 59-70.

Kain, J.F. and Apgar, W.C. (1977) Simulation of the market effects of housing allowances, Department of City and Regional Planning, Harvard University.

Lee, D.B. (1973) Requiem for large-scale models, Journal of the American Institute of Planners, 39, 163-178.

Lenntorp, B. (1976) Paths in space-time environments, Lund Studies in Geography. Series B, Human Geography, 44.

Lombardo, S. and Rabino, G. (1984) Non-linear dynamic models for spatial interaction: the results of some empirical experiments, Papers of the Regional Science Association, 54, forthcoming.

McFadden, D., Cosslett, S., Duguay, G. and Jung, W. (1977) Demographic data for policy analysis: the urban travel demand forecasting project, Phase 1, Final Report Series, Vol 8, Institute of Transportation Studies, University of California, Berkeley.

O'Kelly, M.E. (1981) Impacts of multi-purpose trip-making on spatial interaction and retail facility size, unpublished Ph.D. thesis, McMaster University.

Orcutt, G.H., Caldwell, S. and Wertheimer, R. (1976) Policy Exploration through Micro-Simulation. The Urban Institute, Washington D.C.

Orcutt, G.H., Greenberger, M., Ruslin, A. and Korbel, J. (1961) Micro-Analysis of Socio-Economic Systems: A Simulation Study. Harper and Row, New York.

Rabino, G.A. and Lombardo, S.T. (1984) On the identification of a non-linear dynamic SIA model and related stability considerations, Paper presented at the Workshop on Dynamics of Metropolitan Areas - Second World Congress of the RSA.

Rijk, F. and Vorst, A. (1983) Equilibrium points in an urban retail model and their connection with dynamic systems, Regional Science and Urban Economics, 13, 383-399.

Sayer, R.A. (1976) A Critique of Urban Modelling: From Regional Science to Urban and Regional Political Economy, Pergamon, Oxford.

Shannon, C.E. (1948) A mathematical theory of communication, Bell System Technical Journal, 27, 379-423 and 623-656.

Thom, R. (1975) Structural Stability and Morphogenesis. Benjamin, Reading, Mass.

Thrift, N. (1981) Behavioural geography: a paradigm in search of a paradigm, in Wrigley and Bennett (Eds) op cit.

von Thunen, J.H. (1826) Der Isolierte Staat in Beziehung auf Landwirtschaft und Nationalekonomie. Gustav Fischer, Stuttgart; English translation by C.M. Wartenburg, The Isolated State. Oxford University Press, Oxford, 1966.

Weber, A. (1909) Uber den Standort der Industrien. Translated by C.J. Friedrich (1929) Theory of the Location of Industries. University of Chicago Press, Chicago.

Wegener, M. (1983) A simulation study of movement in the Dortmund housing market, Tijdschrift voor Economische en Sociale Geografie, 74, 267-281.

Williams, H.C.W.L., Keys, P. and Clarke, M. (1985) Vacancy chain models for housing and employment systems, Environment and Planning A, forthcoming.

Wilson, A.G. (1967) A statistical theory of spatial distribution models, Transportation Research, 1, 253-269.

Wilson, A.G. (1970) Entropy in Urban and Regional Modelling. Pion, London.

Wilson, A.G. (1978) Review of R.A. Sayer (1976), Environment and Planning A, 10, 1085-1086.

Wilson, A.G. (1979) Some new sources of instability and oscillation in dynamic models of shopping centres and other urban structures, Working Paper 267, School of Geography, University of Leeds.

Wilson, A.G. (1981-A) Catastrophe Theory and Bifurcation: Applications to Urban and Regional Systems. Croom Helm, London.

Wilson, A.G. (1983) Location theory: a unified approach, Working Paper 355, School of Geography, University of Leeds.

Wilson, A.G. and Birkin, M. (1984) Models of agricultural land-use: Some numerical experiments, Working Paper 399, School of Geography, University of Leeds.

Wilson, A.G. and Clarke, M. (1979) Some illustrations of catastrophe theory applied to urban retailing structures, in Breheny, M.J. (Ed) Developments in Urban and Regional Analysis. Pion, London.

Wilson, A.G., Coelho, J.D., Macgill, S.M., and Williams, H.C.W.L. (1981) Optimisation in Location and Transport Analysis. John Wiley, London.

Wilson, A.G. and Oulton, M. (1981) The cornershop to supermarket transition in retailing: the beginnings of empirical evidence, Environment and Planning A, 15, 265-274.

Wilson, A.G. and Pownall, C. (1976) A new representation of the urban system for modelling and for the study of micro-level interdependence, Area, 8, 246-254.

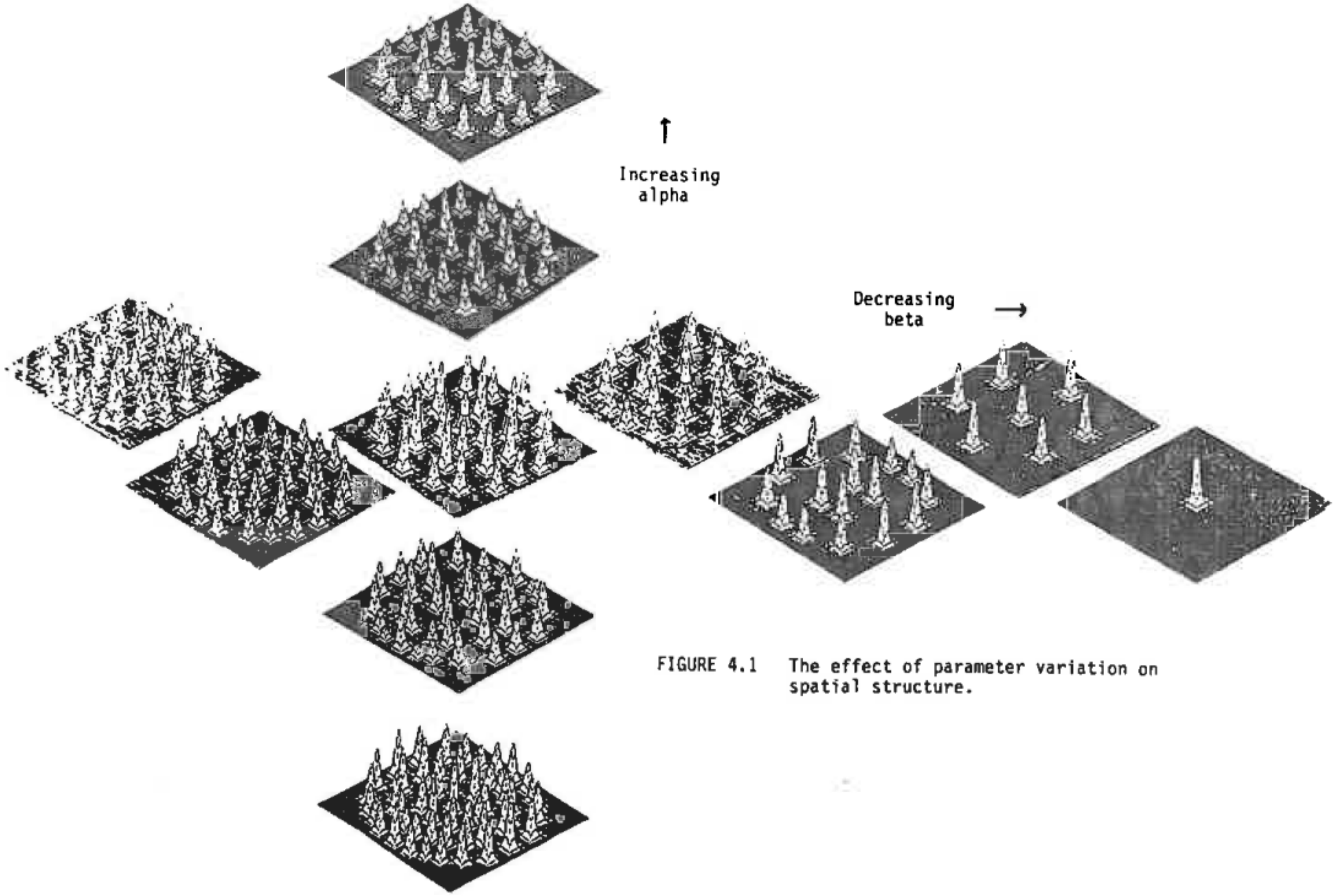


FIGURE 4.1 The effect of parameter variation on spatial structure.

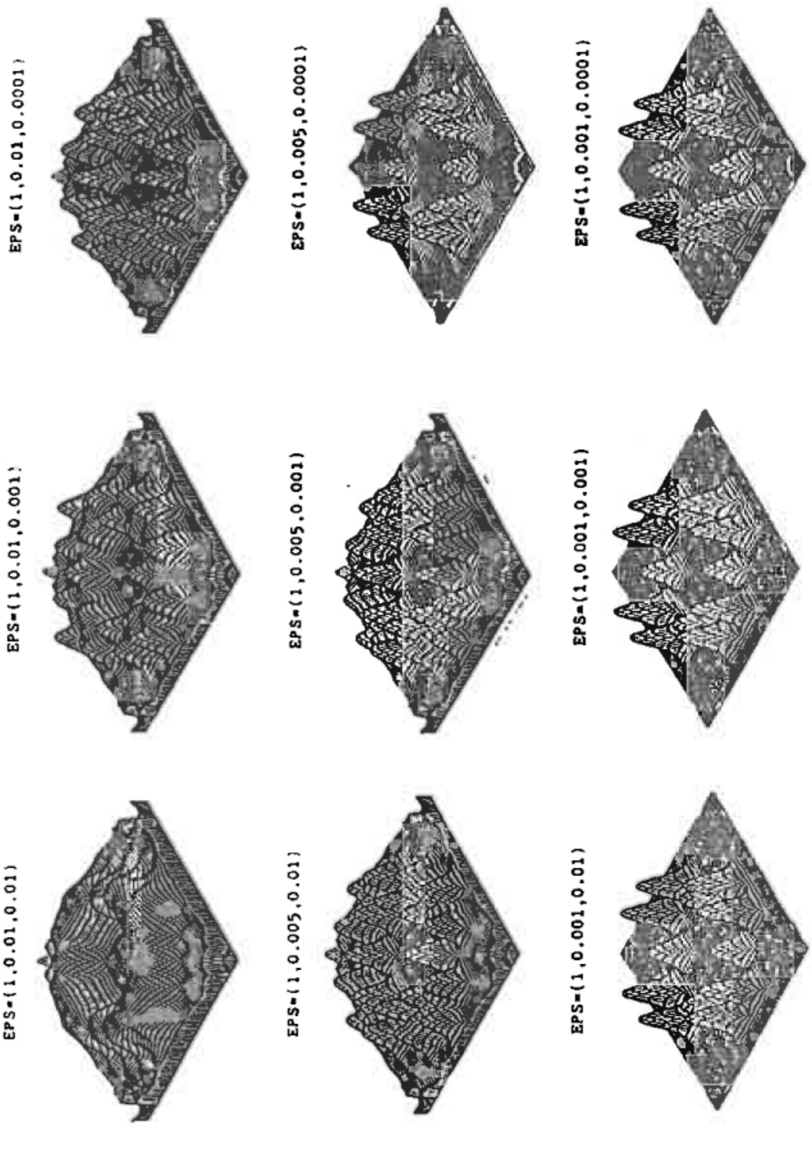


Figure 4.2 The effect of alternative surplus distributions on spatial structure

Zone	Income	Good 1	Good 2	Good 3	Good 4
1 I	34390688.	5448633.	1825499.	1819357.	1834619.
2 I	35608112.	5340428.	1865343.	1750383.	1758946.
3 I	40893344.	6516041.	2152156.	2033474.	2102 94.
4 I	34048032.	5802845.	1949597.	1880359.	1863239.
5 I	24414656.	4018076.	1231805.	1134159.	1272239.
6 I	23618160.	4164103.	1325145.	1252005.	1314757.
7 I	30606528.	5084389.	1617274.	1541200.	1641010.
8 I	20095888.	3542082.	1141620.	1103429.	1142106.
9 I	37559584.	5483711.	1928950.	1933485.	1848383.
10 I	28273760.	4750602.	1603217.	1520631.	1586398.
11 I	36819856.	5678602.	1953057.	1875062.	1868183.
12 I	40324336.	5876273.	2294315.	2130376.	2073222.
13 I	29698816.	4705244.	1571706.	1458677.	1540174.
14 I	47056592.	7173418.	2392871.	2227387.	2193574.
15 I	20955488.	3673021.	1208109.	1133470.	1130051.
16 I	40618416.	6378682.	2199647.	2152870.	2132656.
17 I	43119328.	6677955.	2276379.	2215413.	2173322.
18 I	35313120.	5349153.	2107782.	1950889.	2012002.
19 I	34668080.	5559772.	1886487.	1725234.	1759983.
20 I	31023472.	5139684.	1753326.	1651230.	1646559.
21 I	46453024.	7077341.	2535837.	2480633.	2435080.
22 I	47638080.	7308692.	2535725.	2424401.	2436039.
23 I	32226256.	5050244.	1739637.	1639736.	1633521.
24 I	40677024.	6116850.	2118732.	2007294.	2023593.
25 I	45879456.	6417217.	2396121.	2221581.	2232165.
26 I	35185616.	5858334.	2098438.	1983769.	1970635.
27 I	26425600.	4305441.	1472347.	1360612.	1450630.
28 I	35085600.	5283952.	1784816.	1788762.	1738703.
29 I	28532352.	4338685.	1431197.	1475806.	1399956.
30 I	40903360.	8708043.	4819165.	4280425.	2165920.

Table 5.1 : 1975 Income and expenditure distribution

Zone	Income	Good 1	Good 2	Good 3	Good 4
1 I	52857920.	8691694.	2854650.	2780149.	2768770.
2 I	54217728.	8516846.	2850177.	2757946.	2745397.
3 I	66839120.	10751463.	3583421.	3453993.	3493085.
4 I	53820080.	7946023.	2717669.	2461297.	2554575.
5 I	45108752.	7723700.	2553750.	237739 .	2511663.
6 I	31633152.	5666618.	1694118.	1632396.	1686608.
7 I	52892496.	7939966.	2724810.	2643824.	2639024.
8 I	36390816.	6181840.	2029539.	1947044.	2011043.
9 I	56329072.	8574931.	3134031.	2956497.	2937099.
10 I	45000880.	7792088.	2656640.	2370103.	2605647.
11 I	57722144.	9732448.	3350347.	3067439.	3135851.
12 I	67114912.	9238426.	3117146.	3142279.	3235013.
13 I	47357216.	7369809.	2523487.	2238243.	2329330.
14 I	71202368.	12065476.	4192193.	3838946.	4039057.
15 I	35048976.	5733623.	1883751.	1808950.	1815987.
16 I	62946624.	9421412.	3398452.	3191980.	3225634.
17 I	63943072.	92109522.	3141702.	2922667.	3038996.
18 I	66379600.	9646900.	3510564.	3359722.	3363484.
19 I	56439744.	8589959.	2856314.	2788960.	2756383.
20 I	46045728.	7352905.	2531710.	2306042.	2405086.
21 I	69824512.	10115402.	3784705.	3659074.	35184 4.
22 I	70165344.	9828024.	3405569.	3669582.	3473540.
23 I	49206368.	7929203.	2798666.	2525428.	2621369.
24 I	52122304.	8857340.	2991640.	2783486.	2855542.
25 I	66486800.	10600374.	3673951.	3484527.	3532286.
26 I	54594224.	8299455.	3020888.	2817881.	2858392.
27 I	42526736.	6843499.	2218205.	2049649.	2169085.
28 I	49020176.	8295221.	2853017.	2867361.	2757270.
29 I	44203712.	6948670.	2375117.	2342989.	2305250.
30 I	5945224.	9670293.	4146761.	3956856.	3583524.

Table 5.2 : 1984 Income and expenditure distribution

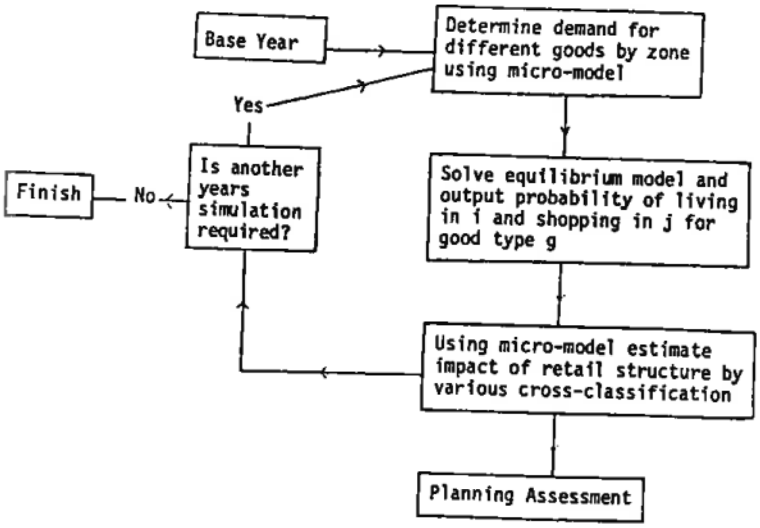
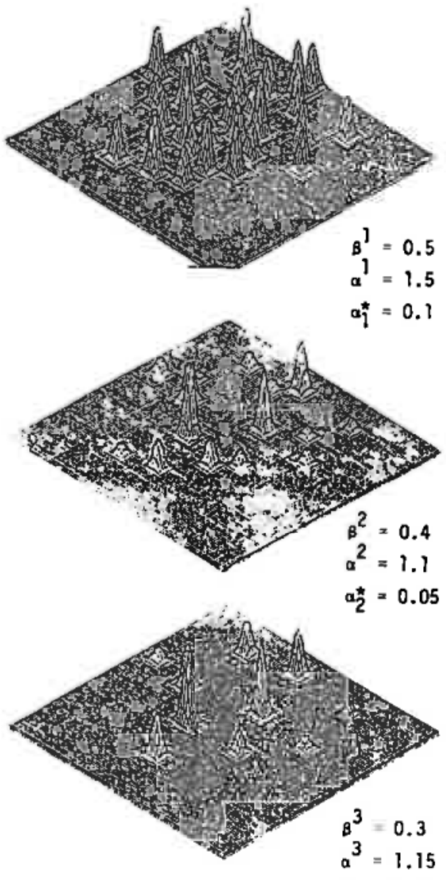
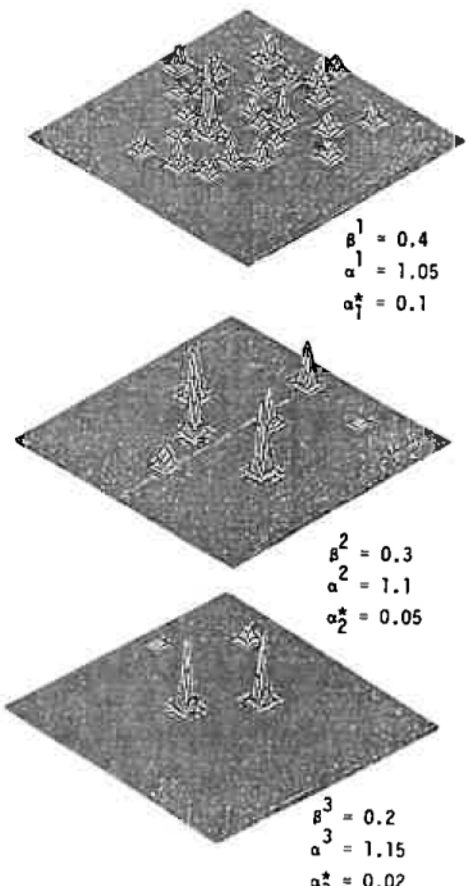


Figure 6.1 : Structure of Integrated Model



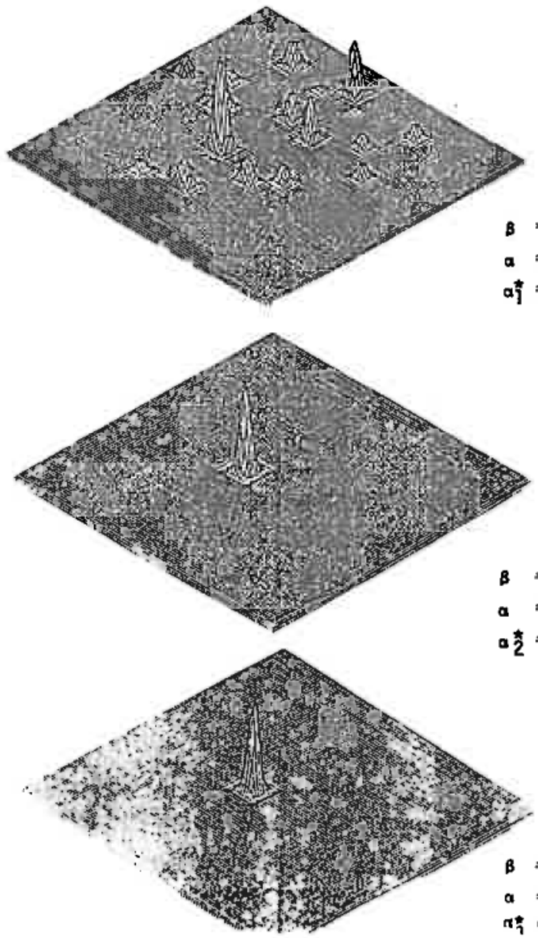
$\beta^1 = 0.5$
$\alpha^1 = 1.5$
$\alpha_1^* = 0.1$
$\beta^2 = 0.4$
$\alpha^2 = 1.1$
$\alpha_2^* = 0.05$
$\beta^3 = 0.3$
$\alpha^3 = 1.15$
$\alpha_3^* = 0.02$

Fig. 6.2



$\beta^1 = 0.4$
$\alpha^1 = 1.05$
$\alpha_1^* = 0.1$
$\beta^2 = 0.3$
$\alpha^2 = 1.1$
$\alpha_2^* = 0.05$
$\beta^3 = 0.2$
$\alpha^3 = 1.15$
$\alpha_3^* = 0.02$

Fig. 6.3



β = 0.3
α = 1.05
α_1^* = 0.1

β = 0.2
α = 1.1
α_2^* = 0.05

β = 0.1
α = 1.15
α_1^* = 0.02

Fig. 6.4

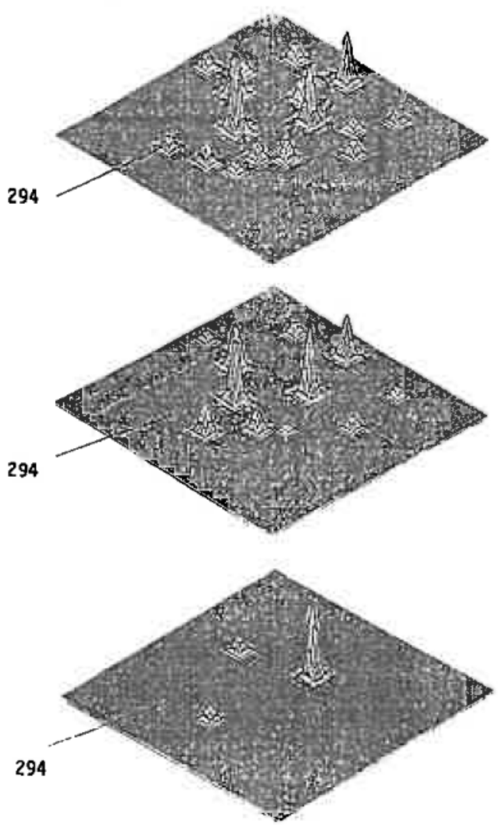


Fig. 6.5 $k_{294} = 1.0$

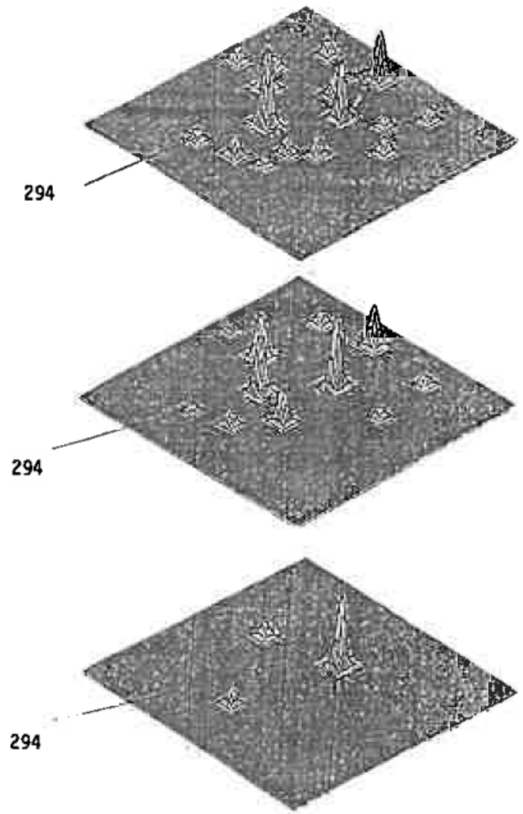


Fig. 6.6 $k_{294} = 0.95$

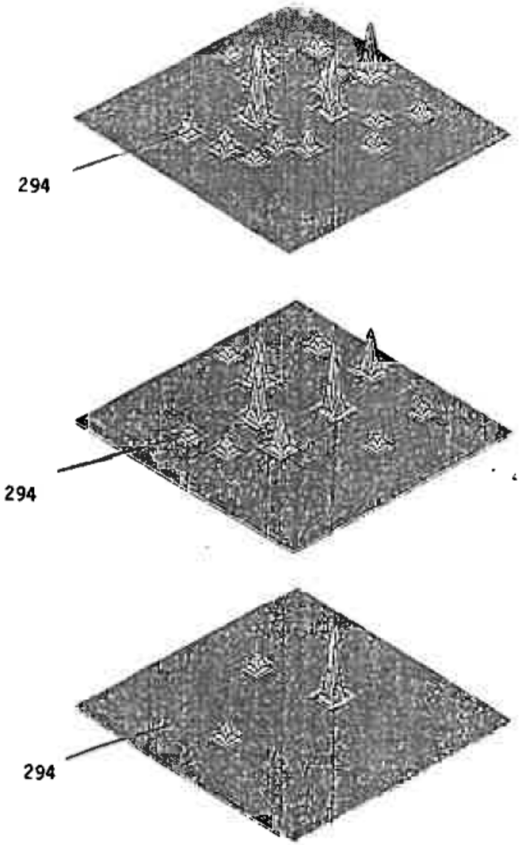


Fig. 6.7 $k_{294} = 0.9$

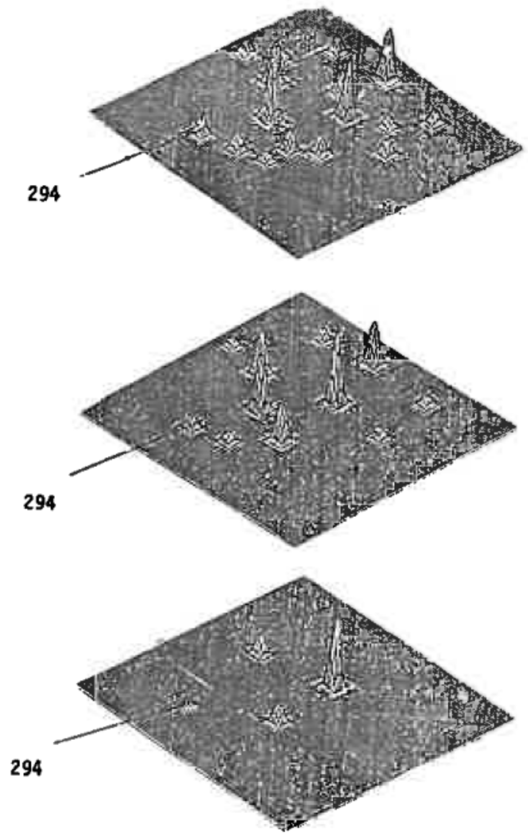


Fig. 6.8 $k_{294} = 0.8$

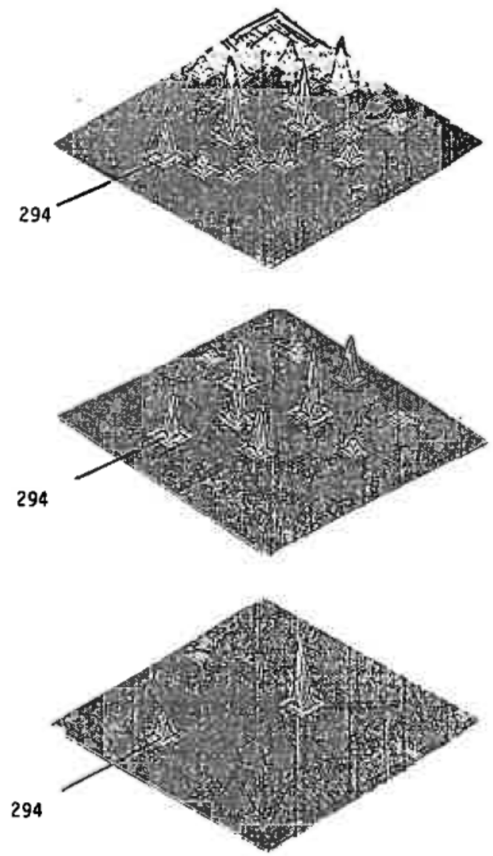


Fig. 6.9 $k_{294} = 0.5$

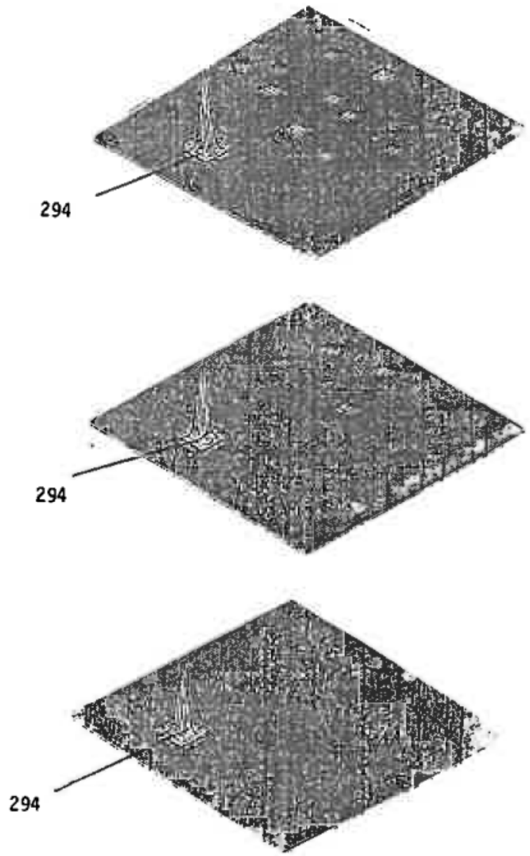


Fig. 6.10 $k_{294} = 0.1$

[illegible] RETAIL MODEL -- AVERAGE COST OF TRAVEL
BY ZONE

[illegible]	ORDER 1	ORDER 2	ORDER 3
1	1.92	7.90	5.92
[illegible]	0.70	8.72	5.87
[illegible]	2.12	1.35	1.57
[illegible]	1.34	0.67	1.60
[illegible]	3.34	8.93	9.46
6	2.75	11.94	12.14
7	2.86	8.04	9.33
8	3.53	7.01	3.63
[illegible]	0.60	0.60	1.80
10	2.14	5.54	6.84
11	2.94	3.24	3.55
[illegible]	1.76	3.11	4.77
[illegible]	2.71	5.10	6.22
14	1.23	0.91	1.31
15	4.30	4.70	6.26
16	1.75	5.35	8.89
17	1.46	1.93	2.71
[illegible]	1.22	1.15	2.21
[illegible]	2.07	7.92	7.99
[illegible]	1.62	[illegible]	8.17
[illegible]	[illegible]	8.17	2.71
[illegible]	1.41	1.05	2.04
[illegible]	3.17	6.86	7.73
[illegible]	2.01	5.43	5.92
[illegible]	[illegible]	1.27	3.05
[illegible]	[illegible]	[illegible]	5.45
[illegible]	[illegible]	[illegible]	1.12
	1.23	[illegible]	[illegible]
	2.23	[illegible]	10.23
	1.37	1.37	4.11
[illegible]	[illegible]	[illegible]	5.58
[illegible]	0.87	3.53	3.04
[illegible]	2.73	12.03	3.91

Table 6.1(a)

IMPACT OF RETAIL MODEL ON DIFFERENT HOUSEHOLD TYPES

[illegible] TRANSPORT COSTS BY HOUSEHOLD

[illegible]	ORDER 1	ORDER 2	ORDER 3
	1.[illegible]	5.2[illegible]	5.27
	1.[illegible]	4.76	5.20
	2.04	5.41	5.17
	2.1[illegible]	5.[illegible]4	5.89
	1.63	5.32	4.97
	1.53	6.21	5.12
	1.7[illegible]	5.47	5.33
	N.A.	N.A.	N.A.
	1.57	5.33	5.05
	1.[illegible]	5.42	5.52

Table 6.1(b)

[illegible] DIFFERENT AGE GROUPS

[illegible]	[illegible]	25-34	35-44	45-54	55-64	65+
	1.[illegible]	1.63	1.	1.[illegible]7	1.[illegible]4	2.0[illegible]
	[illegible]	5.[illegible]	[illegible].44	5.1[illegible]	5.41	5.35
	[illegible]	5.1[illegible]	5.3[illegible]	5.10	5.30	5.33

Table 6.1(c)

IMPACT ANALYSIS FROM RETAIL MODEL — AVERAGE COST OF TRAVEL BY ZONE

[illegible]	ORDER 1	ORDER 2	ORDER 3
1	2.93	13.22	9.80
2	1.50	12.23	10.927
3	2.68	16.08	2.04
4	0.88	0.67	3.02
5	9.82	8.26	10.32
6	3.88	19.13	12.16
7	4.24	8.26	9.27
	7.08	4.77	[illegible]
	0.66	20.03	[illegible]
[illegible]	3.83	13.71	7.90
11	5.79	9.19	8.83
12	2.45	5.14	6.85
13	5.77	5.60	2.19
14	1.35	0.92	2.11
[illegible]	5.72	15.80	7.13
[illegible]	2.54	11.14	1.71
17	1.88	17.32	13.65
[illegible]	1.63	17.32	7.28
[illegible]	4.21	7.17	11.02
[illegible]	4.26	7.91	9.32
[illegible]	1.92	14.15	4.75
[illegible]	1.75	14.34	5.70
[illegible]	5.47	[illegible]	8.50
[illegible]	2.61	19.65	7.21
[illegible]	2.02	15.91	12.24
[illegible]	2.19	10.[illegible]	12.97
[illegible]	1.52	13.84	1.81
[illegible]	2.16	15.95	9.91
[illegible]	3.15	9.22	10.23
[illegible]	1.75	1.4[illegible]	13.16
[illegible]	1.07	10.52	8.51
[illegible]	1.66	5.55	3.39
[illegible]	5.65	2[illegible].1[illegible]	11.12

Table 6.2(a)

IMPACT OF RETAIL MODEL ON DIFFERENT HOUSEHOLD TYPES

TRANSPORT COSTS BY HOUSEHOLD

[illegible]	ORDER 1	ORDER 2	ORDER 3
[illegible]	2.99	10.65	8.12
[illegible]	3.00	10.10	8.34
[illegible]	3.01	11.33	8.70
[illegible]	3.15	10.18	8.95
[illegible]	2.68	11.30	8.55
[illegible]	2.79	10.62	11.70
[illegible]	2.75	11.56	8.77
[illegible]	N.A.	N.A.	N.A.
[illegible]	2.75	11.02	8.22
[illegible]	2.70	11.11	8.60

Table 6.2(b)

AVERAGE TRANSPORTATION COSTS FOR DIFFERENT AGE GROUPS

[illegible]	25-34	35-44	45-54	55-64	65+
[illegible]	2.47	2.[illegible]1	2.[illegible]6	2.87	3.13
[illegible]	11.[illegible]	11.40	11.59	11.19	10.73
[illegible]	[illegible].[illegible]6	[illegible].77	[illegible].90	[illegible].58	8.39

Table 6.2(c)

IMPACT ANALYSIS FROM RETAIL MODEL -- AVERAGE COST OF TRAVEL
BY ZONE

ZONE	ORDER 1	ORDER 2	ORDER 3
1	7.41	24.67	22.42
2	3.92	21.71	21.48
3	3.78	30.08	27.50
4	6.79	[illegible]	[illegible]
5	4.48	17.78	15.83
6	9.80	10.31	10.97
7	10.00	15.09	15.23
[illegible]	7.82	6.13	[illegible]
[illegible]	1.42	36.30	33.24
[illegible]	7.16	14.27	15.26
[illegible]	5.75	9.85	9.46
[illegible]	4.14	15.93	10.75
[illegible]	6.15	4.81	2.03
[illegible]	1.75	16.95	11.54
[illegible]	6.63	15.42	15.74
[illegible]	6.82	26.07	23.26
[illegible]	3.00	24.49	25.40
[illegible]	2.30	28.50	25.75
[illegible]	8.08	7.01	10.01
[illegible]	8.14	7.55	9.65
[illegible]	3.04	12.93	12.30
[illegible]	2.82	14.07	15.86
[illegible]	5.58	11.09	8.02
[illegible]	5.06	36.01	33.47
[illegible]	3.33	16.71	14.79
[illegible]	4.04	26.60	23.59
[illegible]	2.17	13.23	13.57
[illegible]	7.81	15.82	16.52
[illegible]	8.02	12.21	9.49
[illegible]	2.53	30.05	29.54
MEAN	5.19	17.51	16.56
S.D.	2.71	9.20	8.30
VAR	7.12	83.27	69.76

Table 6.3(a)

IMPACT OF RETAIL MODEL ON DIFFERENT HOUSEHOLD TYPES

AVERAGE TRANSPORT COSTS BY HOUSEHOLD

HOUSEHOLD TYPE	ORDER 1	ORDER 2	ORDER 3
1	5.03	17.77	16.90
2	4.88	17.22	17.52
3	5.12	18.34	17.52
4	5.35	16.96	16.86
5	4.62	18.48	17.38
6	3.83	16.69	16.33
7	4.88	18.41	17.61
8	N.A.	N.A.	N.A.
9	4.66	18.60	17.71
10	4.89	17.97	17.16

Table 6.3(b)

AVERAGE TRANSPORTATION COSTS FOR DIFFERENT AGE GROUPS

AGE GROUP	< 25	25-34	35-44	45-54	55-64	65+
ORDER 1	5.14	4.64	4.88	4.99	4.96	5.15
ORDER 2	17.66	19.10	18.33	18.55	18.59	17.62
ORDER 3	16.69	18.22	17.55	17.72	17.72	16.30

Table 6.3(c)

IMPACT ANALYSIS FROM RETAIL MODEL -- AVERAGE COST OF TRAVEL BY ZONE

ANALYSIS FOR ORDER 1 WITH ZONAL FACTORING

ZONE	K FACTOR 1.0	0.95	0.90	[illegible]
1	6.22	6.52	6.56	[illegible]
2	3.86	[illegible]	2.83	3.32
3	1.68	1.61	1.42	1.42
4	1.81	0.99	0.93	0.86
5	2.45	3.94	2.78	3.18
6	1.94	0.71	3.78	0.45
7	0.82	9.04	[illegible]	0.64
8	7.54	7.45	7.07	7.35
9	0.84	1.17	[illegible]	0.93
10	6.71	5.85	6.20	6.28
11	5.32	5.33	5.56	5.59
12	2.81	2.36	2.37	2.40
13	5.48	5.98	5.84	5.79
14	1.17	1.12	1.30	1.25
15	5.39	5.61	5.68	5.21
16	6.35	8.37	8.37	8.43
17	2.50	2.71	3.05	2.26
18	1.37	1.60	1.64	1.66
19	7.71	7.64	7.87	7.72
20	7.95	8.11	8.05	8.14
21	2.93	3.13	3.37	3.07
22	2.57	2.24	2.12	2.43
23	7.29	7.73	7.97	7.60
24	5.77	6.29	5.85	5.73
25	2.27	2.14	2.24	2.06
26	3.99	4.39	4.16	4.11
27	1.20	1.23	1.64	1.36
28	4.05	3.15	2.93	3.06
29	5.97	8.73	9.09	3.87
30	2.38	2.00	2.13	2.43

Table 6.4

IMPACT ANALYSIS FROM RETAIL MODEL -- AVERAGE COST OF TRAVEL BY ZONE

ANALYSIS FOR ORDER 2 WITH ZONAL FACTORING

ZONE	K FACTOR 1.0	0.95	[illegible]	0.80
1	7.38	[illegible]	[illegible]	9.70
2	7.91	[illegible]	[illegible]	[illegible]
3	2.06	2.04	1.95	1.85
4	1.55	1.23	[illegible]	2.20
5	7.40	5.79	7.20	9.05
6	13.03	[illegible]	[illegible]	12.94
7	9.38	[illegible]	[illegible]	[illegible]
8	8.76	[illegible]	[illegible]	[illegible]
9	3.38	2.95	[illegible]	3.18
10	7.48	6.79	6.53	7.61
11	6.20	6.16	6.78	6.91
12	11.47	5.92	5.75	4.40
13	6.49	7.16	7.22	6.63
14	1.57	1.77	1.52	1.23
15	7.29	6.41	5.96	6.65
16	9.20	9.34	9.25	9.42
17	5.23	6.17	5.51	6.16
18	3.74	3.64	3.99	4.88
19	8.25	8.32	8.47	8.45
20	8.64	8.58	8.27	8.65
21	4.05	2.97	2.85	2.31
22	2.96	3.81	9.42	5.46
23	7.57	7.47	7.54	7.60
24	6.27	5.87	6.33	6.77
25	3.11	11.00	10.66	10.52
26	12.02	11.42	11.56	11.60
27	2.23	1.49	1.87	1.55
28	8.56	8.44	8.67	8.51
29	9.78	9.93	9.73	9.92
30	5.08	5.05	5.17	4.48

Table 6.5

IMPACT ANALYSIS FROM RETAIL MODEL — AVERAGE COST OF TRAVEL BY ZONE

ANALYSIS FOR ORDER 3 WITH ZONAL FACTORING

ZONE	K FACTOR [illegible]	0.95	[illegible]	[illegible]
1	14.25	14.09	[illegible]	14.23
2	[illegible]	9.40	[illegible]	9.13
3	16.32	16.22	[illegible]	16.35
4	15.21	15.13	15.16	14.47
5	8.05	8.07	8.07	5.17
6	12.59	12.62	12.50	12.17
7	7.49	7.49	[illegible]	7.76
8	11.57	[illegible]	[illegible]	11.69
9	19.92	[illegible]	19.72	20.82
10	5.74	5.63	5.97	5.84
11	13.61	14.21	13.45	9.99
12	8.60	8.65	8.72	1.74
13	12.96	12.78	[illegible]	12.88
14	8.83	[illegible]	8.83	8.92
15	5.99	[illegible]	[illegible]	5.06
16	10.68	10.68	10.74	10.68
17	12.71	13.86	12.84	12.35
18	17.16	17.16	17.16	17.39
19	12.12	12.08	12.24	14.14
20	11.19	10.73	11.28	10.80
21	15.45	15.37	15.64	1.36
22	1.08	0.83	0.96	11.28
23	6.02	6.13	6.28	6.32
24	22.25	22.12	22.13	22.30
25	15.46	15.61	15.64	10.05
26	10.64	10.64	10.64	10.81
27	0.99	1.04	0.89	1.15
28	7.34	7.42	7.35	5.01
29	8.38	8.42	8.25	8.37
30	22.29	21.87	22.05	21.73

Table 6.6

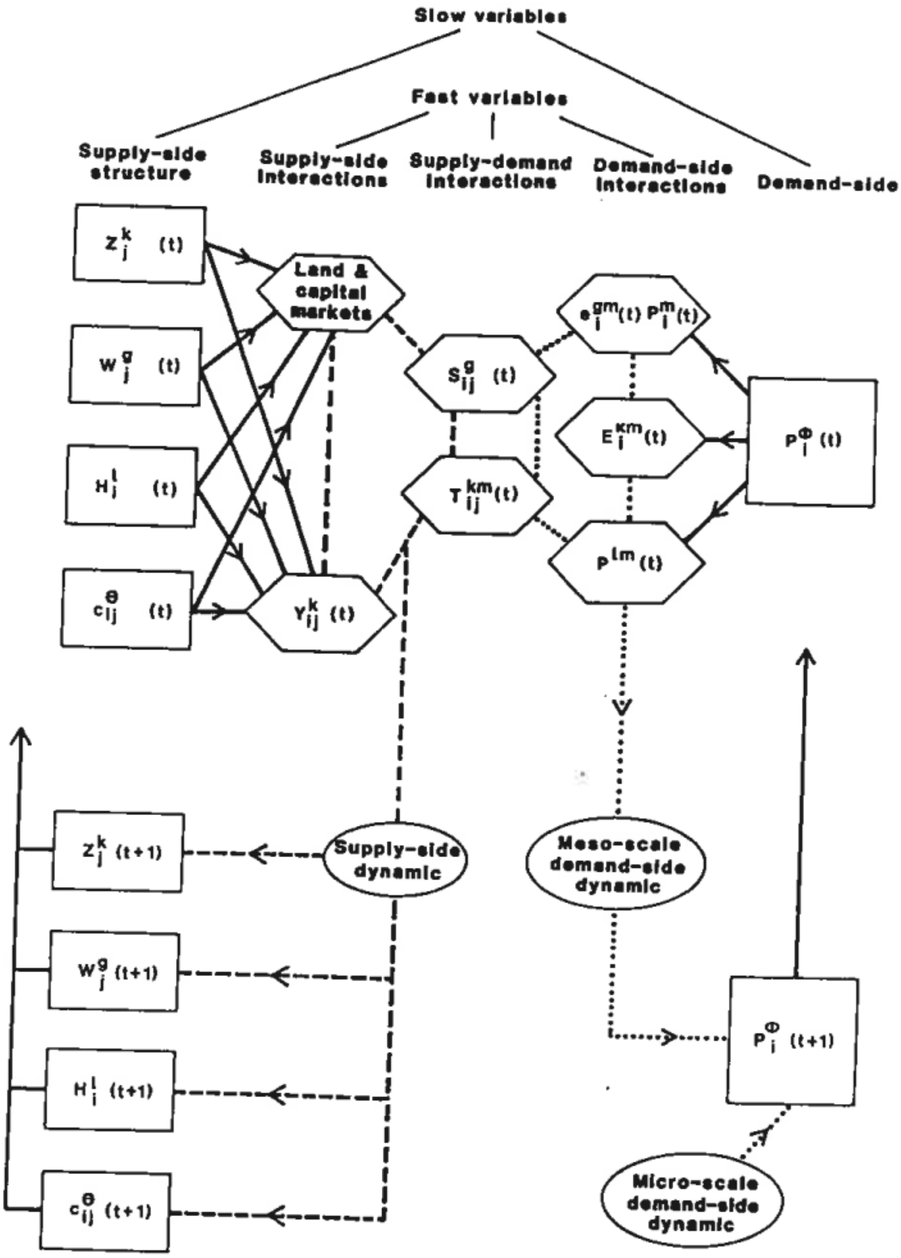


Figure 7.1 Structure of a comprehensive and integrated model