Working Paper 400

INDUSTRIAL LOCATION MODELS I:
A REVIEW AND AN INTEGRATING FRAMEWORK

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CONTENTS

ABSTRACT

The paper applies a unified theoretical framework to a broad range of alternative approaches to industrial location modelling. This framework provides a basis for the comparison of alternative assumptions employed by various location theorists. In a 'twin' paper (Working Paper 401), these observations are used to construct a more general industrial location model, from which many others may be derived as 'special cases'.

TABLES

FIGURES

1. INTRODUCTION.

Spatial interaction models have long been used as a basis for modelling activity location (and hence may be called SIA models, or models of Spatial Interaction and Activity). Since the late 1970s it has been possible, through the addition of further hypotheses and new analytical methods, to model simultaneously the location of structural variables previously taken as exogenous. These may now be referred to, therefore, as 'SIAS' models, standing for models of Spatial Interaction, Activity and Structure. The technique rests on the integration of the now well-known entropy-maximising spatial interaction models as models of consumer behaviour (or alternative models at a similar level of resolution) with some kind of producers' optimisation criterion.

The theory was largely developed in the context of retail models by Coelho and Wilson (1976) and Harris and Wilson (1978) and has received fairly extensive numerical tests in the work of Wilson and Clarke (1979), Beaumont, Clarke and Wilson (1981), Clarke and Wilson (1983-A) and Clarke (1984). Work on residential patterns has been undertaken by Clarke and Wilson (1983-B) and exploratory attempts to calibrate such models in relation to real world data are currently underway.

However a vastly rich field of application for these models which so far remains untapped is in the realm of conventional agricultural and industrial location theory. Explorations into agricultural location theory are conducted elsewhere (Wilson and Birkin, 1984); here we wish to focus on industry. Our aim is to try and pick up the flavour of the field by sampling through the vast range of model-based approaches to the industrial location problem, and to demonstrate the compatibility of any and all of these approaches with the SIAS framework. Thus we hope to be able to bring together the whole richness of industrial location theory within a single all-embracing structure.

The paper is divided into two parts. In the first, our main objective is to review a range of models with a consistent notation, which helps us to gain new insights into their structure. We begin, in Section 2, by outlining our approach to the classification of theory, and by defining the notation which may be used in the formal algebraic presentation and comparison of model structure. In Section 3 we broach the classification question by sampling selectively across the whole range of industrial location theory. The analysis is taken a step further in the second part of the paper, where we present a model framework to unify all that has gone before.

Finally, it is necessary to note that no approach to such a huge field could ever attempt to be comprehensive. Thus to the extent that Section 3 may be considered a 'literature review' it is by necessity selective and somewhat arbitrarily so. At the same time, we do not concern ourselves with non-model based approaches such as those of systems analysis ('the geography of enterprise'), behaviouralism or critical theory (the structural approach). In due course, we expect that the most valuable insights from these approaches will be incorporated in models and we are confident that such further developments could be incorporated within the frameworks we present.

2. THE STRUCTURE OF INDUSTRIAL LOCATION THEORY.

Following Wilson (1981-A, 1983) it may be argued that the model-builder must take decisions in relation to the following dimensions:-

(i) Entitation - the enumeration of individuals, organisations and other components of the (sub-)system of interest.

(ii) Levels of resolution - sectoral, spatial and temporal.

(iii) Partialness/ comprehensiveness - whether to model a single unit or actor within a given environment; or whether to try and handle a number of (interdependent) units.

(iv) Spatial representation - discrete or continuous.

(v) Elements of theory -

a) Processes shaping the environment

b) Processes associated with consumer behaviour (the 'demand' side)

c) Processes associated with producer behaviour (the 'supply' side)

d) Any whole system criteria: maximise consumers' surplus, producers' profits or whatever

e) The representation of interdependence

f) The development of explanatory concepts

(vi) Methods - relevant methods include mathematical programming/ optimisation, simulation and geometric techniques.

By examining the way in which these issues are dealt with (either explicitly or implicitly) by different authors, we obtain a useful basis for the classification of theory. However first it is useful to elaborate a little on the general points outlined above in relation to our specific system of interest.

Begin by defining the following sets:

$G = \{1,2,...,n_G\}$ - products

$H = \{1,2,...,n_H\}$ - organisations

$I = \{1,2,...,n_I\}$ - locations

Products might be seen as comprising the following:

Raw materials
Utilities
Local government services
Central government services
Goods and other services
Capital
Labour
Land
Natural resources
Transport services
Managerial skills
. .
. .
etc etc

All products are simultaneously inputs to some organisations and outputs from others. Different types of organisation will use different mixes of inputs to produce different mixes of output. While recognising the uniqueness of the operations of any individual organisation, we may identify certain families at various levels. Three example classifications are shown in Table 1.

At the 'coarse' level, the different classes of activity may be seen as using a range of input mixes to produce a certain range of outputs. For example, primary sector activities typically apply capital and labour (inter alia) to 'natural resources'[1] to produce raw materials. The secondary sectors again apply labour and capital to

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1. Note that what we here refer to as natural resources is broadly equivalent to the economist's notion of 'land'. Land is used in a more limited sense as 'building land'.

these raw materials to produce 'goods'. Individuals may be seen as utilising transport services to gain access to goods and services, producing labour as an output.

Ofcourse these subdivisions are rather arbitrary, but the aggregation question must inevitably be tackled successfully if an effective model is to be constructed.

In respect of the spatial dimension, we may firstly observe that the friction of distance and, to a lesser extent, competition for space, leads to a dispersion of activities. Thus the accumulation of inputs and the distribution of outputs has a strong spatial dimension, which ofcourse gives rise to the location element of the problem.

Spatial resolution here refers to the geographical scale of the problem under study. Our argument is that spatial scale certainly affects the roles of the various processes, and therefore the articulation of theory, but need not affect basic model structure. In the literature this has typically not been the case - programming models have fared well at the interregional scale, but at the international scale the geography of enterprise is now the dominant theme, while at the micro-scale behavioural models apparently have more sway.

The question of spatial representation is so important as to stand out alone as an issue. One of the present authors has argued persistently that the use of a discrete zone spatial system greatly enhances the scope of the mathematical tools at our disposal in modelling urban and regional systems (e.g. Wilson 1974; 1981-A). Support for this view must be tempered, however, by the observation that the two types of spatial framework are not always compatible in different problem contexts. For example, analytical solutions to the

Moses and related problems would not be possible in a discrete zone spatial framework.

The notation used in this paper follows closely that of Wilson (1983). All the flows in the system are described as activity-commodity variables (cf. Cripps, MacGill and Wilson, 1974; MacGill, 1977) as follows:

$$Y_{ij}^{mng} \qquad g \in G;\ m,n \in H;\ i,j \in I$$

- a flow of goods of type g from organisations of type m in i to organisations of type n in j.

In addition we can describe Z_j^{ng} as production of g by organisations type n in j; X_i^{mg} as demand for r by organisations of type m at i; D_j^n is revenue for organisations of type n at j; C_j^n the associated expenditure; c_{ij}^g is the cost of transporting a unit of g from i to j; and p_i^g the price of g at i. Auxiliary variables are described as they are required.

Our isolation of flows into activity-commodity variables implies an assumption - that some division of activities into 'organisations', and of their outputs into 'products' can adequately represent the flows in the industrial system. As we shall see below, industrial location models will rarely attain such a degree of sectoral refinement in practice. In the real world, however, we also need to deal with different 'processes' employed by the organisations, which would imply an extended interaction variable of the form $Y_{ij}^{m(r)n(s)g}$ where r and s are now sets of processes employed by organisations m and n respectively. The way in which this type of variable is compacted will determine the style of analysis being conducted. Typically in the past spatial interaction models have tended to abstract from the supply-side and are therefore concerned with variables like $Y_{ij}^{*(*)*(*)g}$. 'Process analysis' [e.g. Isard and Schooler (1959); Isard, Schooler and Vietorisz (1959); Vietorisz and Manne (1963)] typically focuses on the mix of outputs that may be produced - $Y_{**}^{*(r)*(*)g}$ - and activity analysis on the mix of inputs required - $Y_{**}^{*(*)*(s)g}$. A 'geography of enterprise' would ideally be concerned with the fully disaggregate variable $Y_{ij}^{m(r)n(s)g}$.

The kinds of model with which we are here concerned will usually deal with variables of the form $Y_{ij}^{m(*)n(*)*}$, where the level of sectoral resolution is such that there is now an assumed one-to-one correspondence between products and organisations, that is sectors of type m produce a single output, also type m. Given such an aggregation, there are two types of industrial location theory, described as 'partial' or 'general' as follows (e.g. Paelinck and Nijkamp, 1975, p93 et seq.). Partial theories are concerned with the location of a small number of production centres in relation to a given distribution of other centres and of markets. General location theory, on the other hand, aims to determine simultaneously an optimum distribution of all production centres and markets.

This idea of general location theory accords broadly with our notion of a 'comprehensive' theory. Such a 'partial - comprehensive' distinction provides the main axis for our classification of industrial location theory, and we deal in turn with: single firms; duopoly models; multi-firm, single sector situations; and multi-firm, multi-sector models.

A second issue which is intimately linked to the question of sectoral aggregation concerns the 'problem-focus' of the theory. Typically, problems defined at different levels on the partial/comprehensive axis will be attempting to answer different questions. For example, single firm models are usually concerned with problems of optimal location, duopoly models with optimal allocation and stability. The multi-firm models attempt a more integrated approach, but only at the expense of simplifications in the theory. 'Comprehensive' models could obviously be much improved if they could incorporate the full richness of partial theory.

Once the problem has been defined to this level, a set of critical questions remain to be answered to facilitate the problem solution. These are the 'elements of theory', consisting of a set of assumptions about the deteminants of the system's behaviour, and giving rise to questions of both selection ('which variables?'), and of operation ('in what way?'). The key elements are:

1). Producers' behaviour - profit maximising, cost minimising, satisficing....

2). Consumer demand - elastic, inelastic, sensitive with respect to distance.....

3). Interdependence between activities - competition for markets, raw materials; interdependence through location, pricing, output;....

4). Technology - of transportation, production....

5). Whole system objectives - maximise production, profits, consumers' surplus; minimise costs of transportation, production....

Finally, a variety of techniques may be used in the development of models to represent the theory. These include (as noted earlier) algebraic and geometric methods, mathematical programming/ optimisation and simulation.

3. MODELS OF INDUSTRIAL LOCATION: FROM PARTIAL TO COMPREHENSIVE

We have already indicated that a range of industrial location models can be identified which broach the problem with varying degrees of comprehensiveness. In this section we work systematically through from partial to comprehensive models. At each stage we try to identify one key model and to conceptualise it in terms of the framework outlined in Section 2. A variety of models with a similar problem focus can be isolated, and these may be seen as locations on a hyperplane of theory generated by the conjunction of different assumptions and elements of theory. From this survey we then attempt to determine alternative models which may also have interesting properties.

3.1 Single firm models

An appropriate starting point is with the work of Alfred Weber whose Theory of the Location of Industries (1909) has had a huge influence on the subsequent development of industrial location theory, being among the first to attack the problem in a coherent and systematic way. The key assumption is that it is possible to tackle the general (i.e. multi-sectoral) location problem through an examination of the behaviour of individual firms. It is postulated that the geography of industrial activity is important because of cost advantages - all locational factors may be expressed in such terms. For example, proximity to markets is viewed as an advantage in product distribution costs, rather than an advantage in terms of potential revenue. In turn, locational cost differentials may arise from only three causes -

costs of transportation, real labour costs, and agglomerative/ deglomerative factors. It is assumed that transport is the dominant force, but the 'transport orientation' of the system may be distorted by wage variations, or by agglomeration.

All activities are 'concentrated' (i.e. they take place at a point). The loci of consumption, labour supply and raw materials production are all given; the locus of production is to be determined. All the action takes place in continuous space.

The basic transport orientation is deduced easily. Suppose there are n_R inputs to the production process. The problem is then to minimise the sum costs of accumulation of raw materials and distribution of the product. Since only one market is supplied, the problem for a firm in sector n trying to find a location (j) is:

$$\min_{\{j\}} \sum_{im} Y_{ij}^{mn} c_{ij}^{m} + Z_j^n c_{jk}^n \tag{1}$$

where:

Y_{ij}^{mn} is the flow of goods from sector m at i to sector n at j;

Z_j^n is the output of sector n at j;

c_{ij}^m is the cost of transporting a unit of m from i to j.

The location of the inputs, i, are assumed to be given and fixed, and k is the location of the single market. The solution turns on the number of inputs, n_R, and their importance relative to one another and relative to the output. A set of constants $\{a_{mn} / m=1,\ldots,n_R\}$ may be defined as the weight of a raw material

(m) used to produce a unit of n (in effect, an input-output coefficient). We can write:

$$Y^{mn}_{ij} = a^{mn} Z^{n}_{j} \qquad (2)$$

hence the problem reduces to:

$$\underset{(j)}{\mathrm{Min}} \; [\sum_{im} a^{mn} c^{m}_{ij} + c^{n}_{jk}] \; Z^{n}_{jk} \qquad (3)$$

The output term is irrelevant since it is a constant. Thus it is clear that the model solution depends on the selection of a location such that the travel distances weighted by the I-O coefficients are minimised. Weber was able to solve the problem geometrically for simple cases, or by attaching physical weights representing the coefficients to a 'Varignon frame' having the raw materials sources and market as vertices (Weber, 1909, pp227-239). The mechanical solution to a problem with three inputs is illustrated in Figure 1.

Weber attempted a preliminary classification of industries and their locational tendencies according to the balance of pulls in the locational figure. The total weight of products which has to be moved per unit of output is called the 'locational weight'. Thus:

$$LW^{n} = \sum_{m} a^{mn} + 1 \qquad (4)$$

In particular the following rules may be deduced:

1). Industries with a high locational weight are attracted towards the materials sources; those with a low locational weight towards the point of consumption. All industries with LW < 2 lie at the point of consumption.

2). Pure materials[2] can never bind production to their deposits. However weight-losing materials may do this if their weight is equal to or greater than the weight of the product plus the weight of the rest of the localised materials. [In fact this latter result is slightly inaccurate. It only holds for situations in which n_R is less than 3 - cf. Khalili, Mathur and Bodenhorn, 1974, and Miller and Jensen, 1978.]

The basic Weberian transport minimisation problem can be reduced to the elements set out in Table 2. The degree of sectoral resolution is described as fine because the model could be applied to any activity - no matter how specialised - given its productive technology and the location of markets and inputs. On the other hand, the spatial scale is not really important. Typically the problem has been applied at the regional scale - a good example of such an application is Smith (1955) who examines the locational tendencies of a number of British industries within a Weberian materials index framework [others are discussed by Lloyd and Dicken, 1977, Chapter 4; and by Smith, 1981, Chapter 13]. However it is now a fashionable view that the Weber problem may be appropriately applied to deal with comparative advantage at an international scale (e.g. Dicken, 1977, quoted in Smith, 1981, pp134-5). A third point to notice is the characterisation of producer's behaviour as 'profit maximising'. It would be inexact to describe the approach as cost minimising in view of Weber's explicit transformation of all pecuniary elements to cost

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2. Materials which lose no weight in the production process, hence the I-O coefficients are always less than 1.

factors, as described above.

It is worth noting that the work reviewed above is a far from balanced presentation of Weber's original treatise. The author was in reality far more concerned with the way in which this simple model could be extended and applied deductively to real world situations. For present purposes this work is somewhat tangential since it involves a rather large shift in problem focus to a consideration of interdependence within a multi-firm environment.

Beginning from the basic model of Table 2, Moses (1958) set out to investigate the dependence of the Weber solution on the simple production assumptions employed. Moses posited a 'neoclassical production function' in place of the linear fixed-coefficients of Weber. This implies a generalised production function of the form:

$$Z^n_j = Z^n_j\,(Y^{1n}_{*j}, Y^{2n}_{*j}, \ldots, Y^{mn}_{*j}, \ldots, Y^{N_R n}_{*j}) \qquad (5)$$

for an activity r at i with N_R inputs. In particular the neoclassical assumptions are taken to imply that continuous substitution between inputs is possible to define a smooth surface of equal production in 'input-space' - such a surface is called an 'isoquant'. Secondly, the isoquants are assumed to be concave downwards i.e. we have 'diminishing returns to a factor' for all inputs.

Now for a given level of output there are two elements to the problem - to determine a) the location of the firm; and b) the factor mix employed. The two are ,however, simultaneously defined. This is because the factor mix only varies in relation to relative factor prices, and factor prices vary only as a function of location. In fact the optimal location/ factor mix occurs where "the ratio of [the factors'] marginal productivities is equal to the ratio of their delivered prices" (Moses, 1958, p265). The further conclusion that it would be "purely by chance if total transportation costs were a minimum at this point" (Moses, 1958, p265) is at best irrelevant though. As was later shown (Emerson, 1973) the Moses problem is equivalent to minimisation of transport costs after the fact i.e. once the input mix is known. If this input mix is not given, there is no way the Moses and Weber problems may be directly compared. This result simply points up the obvious - that the basic Weber problem is a special case of Moses' formulation, with an homogeneous production function of degree one.

The most important result of Moses work was that "If the production function is not homogeneous of the first degree, there is no single optimum location...The optimum location varies with the level of output" (Moses, 1958, p265). This situation arises because the ratio of the marginal productivities of the factors (to one another) may then change. In fact the exact condition for a single stable location is that the expansion path of the production function be a 'ray' (Khalili et al, 1974) - in this case the marginal productivity ratios do not change, even though increasing or decreasing returns to scale may be in operation.

There is the possibility of interesting dynamic behaviour within the Moses model framework if temporal change in the technology of production can be built in explicitly. This is illustrated in Figure 2. Here we see two inputs with sources of which one (R1) has a constant price, the second (R2) being gradually available more cheaply. Not surprisingly, a gradual shift in the optimal production location from R1 to R2 occurs.

The same kind of analysis is applicable to situations of increasing output rather than changing price. Khalili et al (1974) deduced that for a production function homogeneous of degree h, then for h>1 the firm moves toward its market (with increasing output) and away with h<1. As Brown (1979) has pointed out, this represents a testable hypothesis - that firms with increasing returns to scale will tend to move toward their markets as the scale of activity expands.

The power of the Weberian cost minimisation approach was enhanced by Smith (1966), who introduced the concept of a 'space cost curve'. Taking the basic transportation problem, this involves calculating the aggregate transport costs at different points in the locational figure (see Figure 3). The importance of the space cost curve as a conceptual device is in allowing an easy extension to incorporate other cost factors, such as labour and the benefits of agglomeration, without resorting to the theoretically elegant but practically clumsy isodapane techniques of Weber. In fact, the idea had already been used by Greenhut (1956), whence Figure 4 is derived. Perhaps the main contribution of Smith was the notion of 'spatial margins to profitability'. Assuming, in the Weberian tradition, that all locational factors may be expressed as costs, then revenue will not vary in space and the least cost location will clearly be the maximum profit location as before. However at any point on the space cost curve we can now identify a profit level as revenue minus costs. Where costs and revenue become equal are the spatial margins to profitability, and Smith argues that any location within these margins (i.e. with a non-negative profit level) will be a satisfactory location for an entrepreneur who lacks perfect information, or does not adhere rigidly to the goal of profit maximisation. Smith's work thus provides an important interface to the behavioural school of location theory e.g. Pred (1967) [this relationship is explored explicitly by Smith, 1981, pp117-120]. It may also provide connections to model-based approaches building dispersion into economic models such as entropy-maximising and random utility theory.

The concept of the space cost curve has an obvious twin in the 'space revenue curve'. Lacking competition for markets, variations in gross revenue will only arise in these single firm models when demand is less than perfectly inelastic. Clearly the least-cost location is no longer necessarily the optimum and a more interesting configuration of spatial margins to profitabilty is probable (e.g. Figure 5).

Elasticity of demand is taken into account by Alonso (1967) within a different spatial framework. In his model a number of markets are catered for, each having an elastic but straight-line demand function. The treatment of this more complex problem is facilitated by the adoption of a discrete zone spatial system.

Alonso considers a number of whole system objectives. The first assumption is one of imperfect profit maximisation, the imperfection arising because the objective is sought by varying only a single fob price for the product. In our notation the problem may be written as follows:

$$\max_{\{p_j^s\}} \; \sum_j^n p_j^n - \sum_{im} Y_{ij}^{mn} \left(p_i^m + c_{ij}^m \right) \tag{6}$$

s.t.

$$\sum_i Y_{ij}^{mn} = a^{mn} \sum_j^n \tag{7}$$

$$X_k^m = \sum_{jn} Y_{jk}^{nm} = \left(\bar{p}_k^m - k_k^1 \right) / k_k^2 \tag{8}$$

$$\bar{p}_k^m = h_{jk} \cdot \left(p_j^m + c_{jk}^m \right) \tag{9}$$

$$h_{jk} = \begin{cases} 1 \text{ if } k \text{ supplied by } j \\ 0 \text{ otherwise} \end{cases} \tag{10}$$

where (6) gives profit as net revenue minus input costs; (7) is the technology of the production process; (8) is the demand function at market j $(\{k_j^1\},\{k_j^2\}$ = constants); and (9) defines delivered prices, $\bar{p}_k^m$, under the f.o.b. assumption, in terms of the binary variables (10).

Alonso shows that this maximisation problem is NOT equivalent to transport minimisation. This is because firms no longer bear the costs of distribution and so the market pull is due not to transport costs but to their 'relative demand elasticities'. The solution balances this new market pull against the classical Weberian pull of the materials.

The same is not true of perfect monopoly (discriminatory) pricing. When prices are set separately in each market we find that the optimum location IS equivalent to minimisation of transport cost after the fact, as in the Moses model.

A third case which is considered is the maximisation of consumer surplus. The price in the individual markets is set at the average production cost plus delivery costs. The problem is then to find a level of output and a location which maximise the aggregate consumer surplus. Here we find ex-post minimisation of transport cost, but the location will not (in general) be the same as that of the discriminating monopolist since the output and its allocation to markets are both different.

However the logical fourth case, in which consumer surplus[3] is to be maximised with respect to prices in each market, is not considered.

The ideas of Alonso and Moses have recently been tested empirically by Karlson (1983). He sets out to establish, in relation to US steel plants, the relative importance of supply and demand factors. The essential conclusion is that producers do not locate with regard to the elasicities of their demand markets, but aim to minimise transportation costs. This minimisation depends on the technology of production, but this is not subject to well-defined elasticities of substitution as in the Moses model. Rather there exist a variety of production methods within which the potential for substitution is limited, but between which input requirements may vary markedly. This

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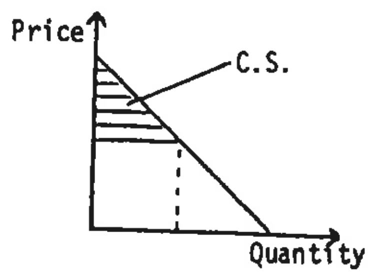
3. In the strict economic sense: consumer surplus at k is the area under the local demand curve above the price, p.



reinforces the (potential) importance of process analysis as a tool in location economics.

Ofcourse the iron and steel industry is one of the more transport oriented activities anyway. One might reasonably expect the balance of demand versus supply pulls to be rather different in an industry with a lower materials index. This kind of analysis might be seen as throwing up a number of interesting research questions. For example Taylor (1970) has argued that the Weberian cost surface for the metal trades in the UK is virtually flat, and thus that Weberian considerations are not of critical importance to location decisions in that industry. That being the case one might ask at what spatial scale such factors do become important. Restating our above argument: for what industries are locational factors still important (and at what spatial scales)? Historically, how have changes in technology, particularly in relation to transport perhaps, affected the relative balance of the different locational factors both to one another, and to non-locational factors? What does that imply about the spatial organisation of industries? In short, the single source Weber problem and its derivatives still have value as inductive methods in relation to the (changing) locational processes which shape the industrial environment. Ultimately however, the search for a truly deductive theory of the location of the firm cannot be approached simply in respect of the influence on the firm of the environment. As Weber was the first to recognise (1909, Ch. 7) one must also account for the influence on the environment of the firm. Some of the more elementary models of the feedbacks between firms are examined next.

3.2 Duopoly Models

Moving on to duopoly models involves a shift in both the number of firms - from one to two - and in the 'problem focus'. Duopoly situations are (by definition) those involving competition between the two firms. The market areas of each firm may vary with respect to the price charged, and the location of the firm. Assuming locations as fixed we arrive at the market area models of Palander (1935). Given fixed f.o.b. prices and that consumers will buy from the firm offering the lowest delivered price, three basic situations may be identified (see Figure 6):

1) $p_1 = p_2$) i.e. prices and transport costs are the same
   $t_1 = t_2$) for both firms, giving rise to a rectilinear market area boundary.

2) $p_1 = p_2$) equal prices, higher unit transport costs for
   $t_1 > t_2$) firm 1 giving rise to a circular market area boundary around firm 1 (though note - the firm is not at its centre).

3) $p_1 > p_2$) higher prices for firm 1 gives rise to a
   $t_1 = t_2$) hyperbolic market area boundary.

The main features of Palander's approach are summarised in Table 3. In terms of our 'elements of theory', the shift in approach is related to the treatment of interdependence since the market area of the one firm is now crucially dependent on the location of its competitor. It is not without reason, therefore, that this style of analysis is frequently dubbed 'locational interdependence'.

A more complex situation arises when prices are allowed to become variable. Each firm now tries to maximise profits with respect to its f.o.b. price. The important thing to notice about this situation is that the solution now depends on the strategies of the two firms. Since the locations are fixed, the tools of neoclassical economics are now available (see for example Paelinck and Nijkamp, 1975, pp55-58). Again it was Palander who was the first to cast this problem in an explicitly spatial framework.

Suppose now that firms may attempt to maximise profits by varying both price and location. Assuming for simplicity a linear market, this is the problem of Hotelling (1929). If a Palander Type 1 situation applies with both firms charging the same f.o.b. price and facing equal uniform transport costs, the market boundary of the two firms will fall halfway between them. It follows that if the two firms are located non-centrally, either one will be able to maximise its market area by moving towards the centre of the market (Figure 7a). Under the further assumption of zero relocation costs, stability with respect to location will only occur with both firms clustered together at the centre of the market (Figure 7b).

The firms may still try to extend their markets through the determination of price. But because we assume f.o.b. pricing, if one firm lowers its price slightly, it may attract some of the trade from the other firm's market only at the expense of lost revenue in its own market. The equilibrium price occurs when a marginal reduction in price by either firm would not affect its profit levels. It is true that profits could be maintained at higher levels with a price agreement between the firms, however such a situation would be inherently unstable since profits could be increased by simple price cutting.

Equally the stable locational solution is sub-optimal. To minimise transport costs, the firms should locate at the quartiles in a linear market (Figure 7c) - an inherently unstable situation.

The properties of the solution to continuous linear demand problems has been further explored by Serck-Hanssen (1970, Part IV). He concludes that local optima with respect to both production volumes and locations occur when all factories are of equal size and locate at the centre of their market areas. The global optimum is found by varying the total number of firms. In a sense, therefore, this is a generalisation of the transport minimisation case of the Hotelling problem (Figure 7c). However a conflict arises here between supply- and demand- side optimality, since the situation described above is non-Pareto optimal: if firms set f.o.b. prices equal to the marginal cost of production, they will normally be running at a loss.

3.3 Multi-firm, Single-sector Models.

In practice, the shift to models incorporating the activities of a relatively large number of firms, goes hand-in-hand with a fundamental shift in the methodology of the solution. The exhaustive analytical solutions to single or dual firm problems are now replaced by mathematical programming approaches. In effect, this implies an attempt to 'simulate' the behaviour of a number of activities, and from this try to either deduce or induce certain rules and patterns to that behaviour.

In this section we discuss and compare two particularly interesting families of approach to the single sector problem - 'transportation models' and 'location-allocation models'. In the case of transport models, the methodological shift is accompanied by an important change in spatial representation to a discrete zone framework. This allows a much larger number of actors to be handled, and for this reason the discrete zone framework is becoming increasing popular in location-allocation modelling (see below). A certain degree of loss of spatial resolution may be incurred in this shift, but possibly more important is the implication that individual production units must now be replaced by zonal activity levels, and hence a loss of sectoral resolution.

In a third sub-section we turn to focus attention on the important class of models which pay explicit attention to the role of pricing in the allocation process.

A. TRANSPORTATION MODELS:

A simple transportation problem may be written down as:

$$\min_{\{Y_{ij}\}} Z = \sum_{ij} c_{ij} Y_{ij} \tag{11}$$

s.t.

$$\sum_i Y_{ij} \leq Z_j \tag{12}$$

$$\sum_j Y_{ij} \geq X_i \tag{13}$$

$$Y_{ij} \geq 0 \quad \forall\, i,j \tag{14}$$

Here Y_{ij} and c_{ij} are respectively the flow of and unit cost of transport of a good from a production location at j to a market at i. Equation (12) states that the amount shipped from any location may not exceed production; equation (13) that the amount shipped to any location must exceed the demand. A stronger assumption sometimes used is that total capacity and aggregate consumption must balance exactly i.e. we have a further constraint:

$$\sum_j Z_j = \sum_i X_i \tag{15}$$

This implies, ofcourse, that equality must hold in (12) and (13).

The essential features of the transport model given by equations (11)-(15) are outlined in Table 4. The model may be described as a 'linear market area allocation model'. What it effectively does is to carve up a given regime of demand into markets to be supplied by a known locus of producers according to the criterion of minimising aggregate transport cost. Note that there is an implicit assumption of spatial price homogeneity - the allocation of consumers to producers is based purely on physical distance. Clearly there are close links to the Palander model, notwithstanding a change in focus from producers' to consumers' behaviour.

Because of the linear allocation procedure, we are guaranteed a rather small number of non-zero flows. In fact the 'basic theorem of linear programming' (e.g. Baumol, 1977, p81) tells us that there may not be more non-zero flows than constraints - in this case a total of (i+j) from (12) and (13). Each consumer will be served by the nearest supplier, subject to the capacity constraints, and market areas will not overlap.

While this is necessary for the absolute minimisation of transport costs, many people, particularly geographers, have argued that a realistic pattern should be more dispersed. Such models can be derived as analogues to the transport model by embedding the objective function (11), and constraints (12) - (15), in an entropy-maximising framework (Wilson, 1970). This gives a mathematical programme to find the 'most likely' allocation of trips given the aggregate expenditure on transport (C) viz.

$$\max_{\{Y_{ij}\}} Z = \sum_{ij} [Y_{ij} \log Y_{ij} - Y_{ij}] \qquad (16)$$

s.t.

$$\sum_{ij} c_{ij} Y_{ij} = C \qquad (11)'$$

$$\sum_{i} Y_{ij} = Z_j \qquad (12)'$$

$$\sum_j Y_{ij} = X_i \qquad (13)'$$

$$Y_{ij} \geq 0 \qquad (14)$$

The analytical solution to this programme yields a spatial interaction model for Y_{ij}:

$$Y_{ij} = A_i B_j X_i Z_j e^{-\beta c_{ij}} \qquad (17)$$

where;

$$A_i = [B_j Z_j e^{-\beta c_{ij}}]^{-1} \qquad (18)$$

$$B_j = [A_i X_i e^{-\beta c_{ij}}]^{-1} \qquad (19)$$

Here β is the Lagrange multiplier associated with the constraint (11)', and may be interpreted as a distance deterrence parameter. So here we have an 'overlapping market area allocation model'. As β increases, the degree of overlap is reduced, and in the limit, as $\beta \rightarrow \infty$, the solution converges to that of the equivalent linear transportation model (cf. Evans, 1973; Senior and Wilson, 1974).

The models we have looked at above are said to be 'doubly constrained' - both the production totals [equation (12)'] and the consumption totals [equation (13)'] are known. Alternative models may be generated which are singly-constrained, when only one of the totals is known, or unconstrained, in the case that neither is known.

The question now arises as to how to build locational factors into these allocation models. One way of doing this is to calculate the producers' activity levels endogenously using the embedding theorem of Coelho and Wilson (1977) (and see Coelho, Williams and Wilson, 1978). The first thing we need to do is to find a new optimisation criterion for the whole system. The flow equation (17) can then be embedded as a constraint to ensure that the optimal solution is consistent with the spatial interaction model which defines the behaviour of the actors. The original problem of Coelho and Wilson was to maximise a consumers' welfare criterion for a production-constrained shopping model:

$$\max_{\{Z_j\}} Z = \sum_{ij} Y_{ij} \left(\alpha/\beta \ln W_j - c_{ij} \right) \tag{20}$$

s.t.

$$Y_{ij} = A_i e_i P_i W_j^{\alpha} e^{-\beta c_{ij}} \tag{21}$$

$$A_i = \left[\sum_k W_k e^{-\beta c_{ij}} \right]^{-1} \tag{22}$$

It was shown that there is an equivalent and more tractable problem:

$$\max_{\{Z_j\},\{Y_{ij}\}} Z = -1/\beta \sum_{ij} Y_{ij} \log Y_{ij} + \alpha \left(\sum_{ij} Y_{ij} \log W_j - \right) - \sum_{ij} Y_{ij} c_{ij} \tag{23}$$

s.t.

$$\sum_j Y_{ij} = e_i P_i \tag{24}$$

The major point of this set-up is that locational costs can now be tacked onto the objective function while ensuring that the allocation problem defined by (21) and (22) is still obeyed. Suppose, for example, that we were to assume a cost per unit of floorspace

provision varying spatially, but constant with output - call this k_j.
The objective function now becomes:

$$Z' = Z^{23} - k_j W_j \qquad (25)$$

where Z^{23} is the objective function of equation (23).

Now since:

$$dZ/dW_j = \alpha \sum_i Y_{ij} / W_j - k_j \qquad (26)$$

the first order Kuhn-Tucker conditions tell us that for optimality:

$$\sum_i Y_{ij} = (k_j/\alpha)\, W_j \qquad (27)$$

The problem then becomes amenable to comparative static analysis in the style of Harris and Wilson (1978).

An alternative approach involves the explicit fusion of a Weberian input framework with a simple transportation model. The resulting structure is referred to as a 'generalised transportation model' by Paelinck and Nijkamp (1975). For an activity n, the objective is now to find the pattern of inputs from other sectors, $\{Y_{ij}^{mn}\}$, and the pattern of outputs to final demand, $\{Y_{jk}^{n*}\}$, which maximises aggregate profits:

$$\underset{\{Y_{ij}^{mn}\},\{Y_{jk}^{n*}\}}{\text{Max}} \; Z = \sum_{jk} (p_k^n - c_{jk}^n) \cdot Y_{jk}^{n*} - \sum_{ijm} (p_i^m + c_{ij}^m)\, Y_{ij}^{mn}$$

$$i,j,k \in I; \quad m,n \in G \qquad (28)$$

Input requirements may easily be deduced in the framework of linear activity analysis, hence a first constraint is that:

$$a^{mn} Z^{n}_{j} \leq \sum_{i} Y^{m*}_{ij} \tag{29}$$

where $\{a^{mn}\}$ are constant I-O coefficients, i.e total input requirements may not exceed the total amount of inputs acquired.

Secondly, we have capacity restrictions on supply:

$$\sum_{j} Y^{mn}_{ij} \leq b^{m}_{i} \tag{30}$$

The production and consumption constraints for sector n hold as before:

$$\sum_{k} Y^{n*}_{jk} \leq Z^{n}_{j} \qquad (31) \text{ cf } (12)$$

$$\sum_{j} Y^{n*}_{jk} \geq X^{n}_{k} \qquad (32) \text{ cf } (13)$$

plus the non-negativity conditions:

$$Y^{n*}_{jk} \geq 0 \quad \forall\, j,k,n \tag{33}$$

$$Y^{mn}_{ij} \geq 0 \quad \forall\, i,j,m,n \tag{34}$$

Such models were first introduced by Koopmans and his associates (Koopmans, 1951 - especially chapters 3 and 14) and further developed by Beckmann and Marschak (1955). The principal concern of these authors was, however, with the analytical examination of the role of the price system and how to achieve an optimal (e.g. a welfare optimal) distribution of production by price manipulation. This is an interesting example of how similar model frameworks may be effectively used to combat a wide variety of problems.

It is also particularly clear from the notation used here that there is a ready generalisation to a multi-sector model simply by optimising over all sectors rather than a single one. This line of attack has been pursued by Lefeber (1958), whose work is discussed further below.

B. LOCATION-ALLOCATION MODELS.

We begin this analysis with a return to a continuous space problem - the so-called 'p-median problem'. This may be written (e.g. Scott, 1971) as:

$$\underset{\{\underline{s}, h_{ij}\}}{\text{Min}} \quad Z = \sum_{ij} X_i h_{ij} c_{ij} \tag{35}$$

s.t.

$$\sum_j h_{ij} = 1 \tag{36}$$

$$h_{ij} = \begin{cases} 1 \\ 0 \end{cases} \tag{37}$$

where;

X_i is the quantity demanded at location i, with coordinates (x_i, y_i);

$\underline{s}$ is the vector of supply locations (x_j, y_j) and c_{ij} is the distance or transport cost (assumed proportional to distance) between demand point i and supply point j;

h_{ij} is a binary variable taking the value 1 if a demand location at i is supplied by j, 0 otherwise.

The general problem defined above aims to find the optimal location of p supply facilities which minimises the cost of supplying n demand points. Total consumption at any one demand point (X_i) is assumed known and fixed. A continuous space representation is usually assumed - thus we have coordinates rather than zone numbers for supply and demand locations - but discrete zone equivalents are easily derived.

The structure of the p-median is summarised in Table 5. Clearly there are certain similarities to the simple transportation problem. The objective function is the same since Y_{ij} and the product $X_i h_{ij}$ are equivalent. However in the transportation problem the capacities of the supply locations are fixed, in the p-median only the total number of facilities (p) is given - they may be located anywhere. It is in this sense that both location and allocation elements are involved.

As with the transport models, it is possible to derive spatial interaction versions of the p-median problem. This is done by changing the nature of h_{ij} from a zero-one to a stochastic variable, i.e. the probability that an individual at i patronises a supply facility at j:

$$h_{ij} = W_j e^{-\beta c_{ij}} / \sum_k W_k e^{-\beta c_{ij}} \qquad (38)$$

where;

W_j is some measure of the attractiveness of centres at j, and might be a function of size, as in the typical entropy-maximising versions of the transport model, or related to any other appropriate parameters; as before, β, is an elasticity /dispersion parameter, such that when β is infinite, consumers always travel to the nearest facility; when β is zero, distance has no effect and the qualitative spatial choice attributes reflected by the W_j's become dominant (cf Beaumont, 1981).

Similarly alternative objective functions may be optimised, for example maximising locational surplus or accessibility as alternative welfare criteria. A more radical alternative attempt to increase equity in the system is to minimise the maximum distance of any 'consumer' from the nearest supply centre - the so-called 'Centre

Problems', which are widely used in the theoretical location of emergency services. Again Beaumont (1981) reviews these and other possible extensions of the basic p-median problem.

Typically location-allocation models may be used to address one of two rather different kinds of problem. On the one hand is a rather large and rapidly growing literature on the location of public facilities (e.g. Leonardi, 1981, for a review of this material); on the other a slightly less substantial body of work concerning private sector problems (an early overview is provided by Revelle, Marks and Liebman, 1970). What the two strands have in common is a reliance on the same basic solution algorithms for optimising locations and carving up markets. The basic techniques are reviewed by Scott (1971) (but see also Ross and Soland, 1977, for some recent advances); a suite of computer programs are presented in Rushton et al. (1973).

The main difference between the two lies in the choice of an optimisation criterion. Typically the public facility problems focus on the maximisation of suitably defined consumers surplus; private facility problems with the maximisation of producers surplus. Note that in the latter case it is just as easy (conceptually) to build in cost factors to the objective function as it was in the Coelho-Wilson example presented above.

A typical example of a private sector problem is the 'uncapacitated (simple) facility location problem' which may be written (e.g. Ross and Soland, 1977) as:

$$\underset{\{s,h_{ij}\}}{\mathrm{Min}} \; \sum_j f_j H_j\left(\sum_i X_i h_{ij}\right) + k_j \sum_i (X_i h_{ij}) + \sum_{ij} (c_{ij} X_i h_{ij}) \qquad (39)$$

s.t.

$$h_{ij} = 1 \qquad (40)$$

$$h_{ij} >= 0 \quad \forall\, i,j \qquad (41)$$

$$H(y) = 1 \text{ if } y > 0, = 0 \text{ if } y <= 0 \qquad (42)$$

with;

X_i, c_{ij}, h_{ij} as above;
f_j = fixed cost of establishing a supply centre at j;
v_j = variable cost of production at j.

Note that in this formulation the h_{ij}'s need not be specified as binary variables; this is a necessary condition for global optimisation of the objective function anyway.

Verbally, the problem is to minimise supply costs with two elements - an aggregate transport or distribution cost; and a cost of facility provision. The latter in turn has two elements - a fixed cost of supply, and a variable cost (assumed to vary linearly with output). The assumption of a split into fixed and variable costs is both enduring and characteristic with location- allocation problems (public or private). It is based on the theory of diminishing returns to scale in the production process (cf. Vietorisz and Manne, 1963). The aggregate cost function is of the form:

$$C_i = Q_i (Z_i)^{a_i} \qquad (43)$$

with the assumption that:

$$0 <= a_i <= 1 \qquad (44)$$

for diminishing returns, where a_i and Q_i are appropriate parameters of the production process at i.

Between certain bounds the function (43) may be approximated (see Fig. 8) as:

$$C_i = f_i + v_i Z_i \qquad (45)$$

with f_i, v_i both constants. This derivation also explains the usual adoption of capacity restrictions within location-allocation models (see, for example, Section 3.4B below) viz:

$$L_B <= h_{ij} Z_i <= L_T \quad \forall\ i \qquad (46)$$

where L_B and L_T are respectively the assumed lower and upper bounds to capacity.

The approximation (45) is invaluable in facilitating the 'linearisation' of the problem. However the introduction of a fixed cost element also implies the need to solve a new combinational element to the problem. In principle, this means that for location-allocation models in which the optimal number of centres is to be determined, a separate problem must be solved for each possible value of p. The difficulty is manageable for single sector models, and with a limited choice of alternatives, but is rather more problematic in multi-sector cases (see Section 3.4C below).

C. PRICE-DEPENDENT ALLOCATION MODELS

One of the outstanding features of the models discussed in the last two sub-sections is that the allocation mechanisms around which they revolve are intrinsically spatial - that is, the allocation of 'consumers' to 'producers' is determined by the physical distances that separate the two sets of actors.

In this sub-section we add a further dimension, by considering the way in which space affects patterns of consumption through the price mechanism.

As a preliminary, consider again the simple transportation model presented in sub-section A:

$$\min_{\{Y_{ij}\}} \sum_{ij} c_{ij} Y_{ij} \tag{11}$$

s.t.

$$\sum_j Y_{ij} <= Z_i \tag{12}$$

$$\sum_i Y_{ij} >= X_j \tag{13}$$

$$Y_{ij} >= 0 \quad \forall\ i,j \tag{14}$$

The Lagrangean of this optimisation problem may be written as:

$$L(Y,\gamma,\lambda) = \sum_{ij} c_{ij} Y_{ij} - \sum_j \gamma_j (X_j - \sum_i Y_{ij}) - \sum_i \lambda_i (Y_{ij} - Z_i) \tag{47}$$

The Lagrange multipliers $\{\gamma_i\}$ and $\{\lambda_i\}$ may be interpreted as shadow supply and demand prices respectively (e.g. Baumol, 1977). Following Takayama and Judge (1972, Chapter 3) we may use the first order Kuhn-Tucker conditions to elucidate further the price relationships which must hold for equilibrium. The Kuhn-Tucker conditions are:

$$\frac{dL}{dY_{ij}} = c_{ij} + Y_j - \lambda_i >= 0 \quad \text{and} \quad \left|\frac{dL}{dY_{ij}}\right| Y_{ij} = 0 \qquad (48)$$

so if there is a non-zero flow of goods from i to j, then $c_{ij} = Y_j - \lambda_i$, i.e. the market price in the receiving region exceeds the market price in the supplying region by the transport cost between the two. Secondly:

$$\frac{dL}{d_{j}} = \sum_i Y_{ij} - X_j >= 0 \quad \text{and} \quad Y_j \frac{dL}{dY_j} = 0 \qquad (49)$$

implies that for the regional supply price to be non-zero, we must have an equality of demand with quantity supplied. Finally:

$$\frac{dL}{d\lambda_i} = Z_i - \sum_j Y_{ij} <= 0 \quad \text{and} \quad \lambda_i \frac{dL}{d\lambda_i} = 0 \qquad (50)$$

implies that for the regional demand price to be positive, the quantity demanded must exactly balance supply.

This problem can be modified by defining a regional 'quasi-welfare' function as:

$$QW_i = QW_i(X_i, Z_i) =$$

$$= \int_0^{X_i} f_i^d(\theta_i)\, d\theta_i - \int_0^{Z_i} f_i^s(\theta_i)\, d\theta_i \qquad (51)$$

where

$$p_i^d = f_i^d(X_i) \qquad (52)$$

is a regional demand function, and:

$$p^s_j = f^s_j (Z_j) \qquad (53)$$

is a regional supply function. Assuming community quasi-welfare to be additive:

$$QW = \sum_i QW_i (X_i, Z_i) \qquad (54)$$

implies a net quasi-welfare function of the form:

$$NW = F^w (X,Z,Y) = \sum_i QW_i (X_i, Z_i) - \sum_j c_{ij} Y_{ij} \qquad (55)$$

This function can be substituted for the simple aggregate transport cost objective function of equation (11) and maximised subject to the original constraints to yield the Lagrangean:

$$L = \sum_i QW_i (X_i, Z_i) - \sum_j c_{ij} Y_{ij} + \sum_i \gamma_i (Y_{ij} - Z_i)$$
$$- \sum_j \lambda_j (X_j - Y_{ij}) \qquad (56)$$

The optimality conditions are:

$$\frac{dL}{dZ_i} = \frac{dQW}{dZ_i} - \lambda_i = p^d_i - \lambda_i <= 0 \qquad (57)$$

$$\frac{dL}{dZ_i} Z_i = (p^d_i - \lambda_i) Z_i = 0 \qquad (58)$$

-for non-zero output, the imputed or market supply price λ_i is exactly equal to the regional supply price p^d_i;

$$\frac{dL}{dX_j} = \frac{dQW}{dX_j} + \gamma_j = -p^s_j + \gamma_j <= 0 \qquad (59)$$

$$\frac{dL}{dX_j} X_j = (\gamma_j - p^s_j) X_j = 0 \qquad (60)$$

- for non-zero demand, the imputed or market demand price, γ_j, is exactly equal to the regional demand price, p^s_j;

$$\frac{dL}{dY_{ij}} = -c_{ij} + p^s_j - \gamma_i <= 0 \qquad (61)$$

$$\frac{dL}{dY_{ij}} Y_{ij} = (-c_{ij} + p^s_j - \gamma_i) Y_{ij} = 0 \qquad (62)$$

- for there to be a non-zero flow from i to j, the market demand price in j must exceed the market supply price in j by the transport cost between the two locations.

This problem is illustrated for a two region case in Figure 9 below. Verbally, the aim is to set prices in the two regions so that the gain through trade in aggregate quasi-welfare for the importing region, exceeds the loss (through trade) in aggregate quasi-welfare for the exporting region by the greatest possible amount, subject to the constraint that the price in the importing region exceeds the price in the exporting region by the transport cost between the two.

The structure of this model is summarised in Table 6. The problem structure is similar to the simple transportation models discussed in section A in terms of level of resolution and spatial scale. The difference in problem-focus is purely one of emphasis - it is just as easy to look at activity levels as prices in this kind of model.

At the same time, these models do introduce different kinds of theoretical assumption. In particular, we have already encountered the idea of interdependence between producers through competition for markets, but now we also have interdependence between consumers. Thus, for example, an area with a high regional demand for goods can attract productive resources and force down the quantities demanded elsewhere.

3.4 Multi-firm, Multi-sector models

In this sub-section, we move on to consider a variety of more complex models which tackle problems involving a number of (interrelated) activities. It is worth noting at the outset that many of these models arise as direct extensions of the types of approach discussed in sub-section 3.3. Thus Lefeber's model may be considered as a generalised transportation model, Dokmeci presents a hierarchical location-allocation model, while Takayama and Judge present a whole variety of extensions to the basic model discussed above. However, other models may be developed which supersede these somewhat arbitrary typologies and Gunnarsson's work may be taken as a good example of this. Finally we offer some comments on the Loesch and Christaller frameworks, for although these works use intuitive concepts rather than the more formal mathematical techniques on which we have concentrated elsewhere in the paper, their work is simply too influential to ignore.

A. LEFEBER (1958)

Lefeber presents a version of the generalised transportation model extended to many sectors. It is assumed that there is a given demand for goods at fixed market prices. A distinction is made between 'goods' and 'factors'. The latter are used only as inputs to manufacturing processes and not sold direct to the final demand sector. The goods produced by the secondary sector are sold only to final demand, and not used in the production of either factors or other goods. To handle this distinction we need to expand to a full

Y^{mng}_{ij}, notation, where g represents the type of commodity under consideration, so g=1 means a factor, g=2 is a good. The superscripts m and n may be seen as representing either organisations or products as the two are still coincident.

Since space is treated in terms of discrete zones, the objective - to maximise the total value of production - may be written as:

$$\underset{\{Y^{mn1}_{jk}\},\{Y^{n*2}_{jk}\}}{\text{Max}} \quad Z = \sum_{jk} \{ p^{n}_{k} \quad Y^{n*}_{jk} \} \tag{63}$$

where:

p^{n}_{k} is the price of good n at market k;

Y^{mn1}_{jk} is the flow of factors type m produced at location j to goods- producing sectors type n at k;

Y^{n*2}_{jk} is the amount of good n produced at location j and consumed at k.

The first constraint is that there be no excess demand for any good - this situation is known as 'Quasi-Walrasian equilibrium':

$$\sum_{k} Y^{n*2}_{jk} <= Z^{n2}_{j} \tag{64}$$

The production function is expressed in the general form:

$$Z^{n}_{j} = f^{n} (Y^{1n1}_{*j} , Y^{2n1}_{*j} , . . . , Y^{mn1}_{*j} , . . .) \tag{65}$$

In practice a Leontief technology (fixed coefficients) is assumed.

An important element of Lefeber's formulation is the assumption of an endogenous transport sector. This comes in through the constraint:

$$T(\{Y_{ij}^{n*1}\},\{Y_{ij}^{mn2}\}) <= f^{T}(Y_{i*}^{1T1}, Y_{i*}^{2T1}, \ldots, Y_{i*}^{mT1}, \ldots) \quad (66)$$

Here T is a function relating the quantities of factors and goods transferred to the amount of transport services required to enable the transfer. Note that the production function for transportation (f^{T}) once again uses only factors, and no goods.

Fourthly, we have a set of constraints on total factor allocations:

$$\sum_{j} Y_{ij}^{mn1} + Y_{i*}^{mT} <= Z_{i}^{m1} \quad (67)$$

where Z_{i}^{m1} is the total amount of m available at i.

Finally we have the non-negativity conditions:

$$Y_{jk}^{mn1} >= 0 \;\forall\; j,k,m,n \;; \quad Y_{jk}^{n*2} >= 0 \;\forall\; j,k,n \quad (68)$$

Lefeber's model might be expanded as a still more general spatial equilibrium model if the price vectors were to be determined endogenously. This aspect is pursued further in section D below.

B. TINBERGEN MODELS

We consider here the work of Gunnarsson (1977) as an advanced programming application of the ideas of Tinbergen (1961) and Bos (1965). There are two assumptions which define a 'Tinbergen system':

1. In each centre with an industry of rank h, all industries of a lower rank are also located;

2. There is only one production unit of the highest ranking industry in a centre; this is the only one which exports to other centres or

the hinterland of the centre.

Gunnarsson's aim was to develop a model to investigate under what conditions these assumptions become an accurate reflection of reality. We start off, like Lefeber, by assuming a fixed regime of demand which is just large enough to support a single plant of the highest order activity.' These orders are defined ex ante through the use of maximum size restrictions on plant activity levels.

In the Gunnarsson hierarchy, two types of sector are again defined - 'restricted' and 'footloose'. However while these are, by analogy with the Lefeber model, producers of 'factors' and 'goods' respectively, transactions are here allowed between the restricted sector and the 'final demand' sector, and the outputs of footloose industries may be used as inputs to other footloose industries. The notation is now g=1 for a restricted sector, g=2 for a footloose sector.

The problem is to find the pattern of footloose plants which minimises total distribution costs i.e:

$$\underset{\{Y_{ij}^{mng}\},\{Y_{ij}^{m*2}\}}{\mathrm{Min}} \quad F = \sum_{ijmng} Y_{ij}^{mng} \; d_{ij} \; t^{m} \qquad (69)$$

where d_{ij} is the distance from i to j; t^{m} is the unit cost of transporting a good type m.

The first set of constraints are the supply-distribution relationships:

$$Y_{j*}^{m**} + \sum_k Y_{kj}^{*m*} - \sum_k Y_{jk}^{m**} - \sum_n a^{mn} \; Y_{j*}^{n**} \;) = 0 \qquad (70)$$

These terms are (reading from the left) outputs, imports, exports and local consumption of m in j. Thus the condition states that total supply must always exceed total demand - the quasi-Walrasian equilibrium conditions again.

Output of the restricted sectors is given:

$$Y_{i*}^{m*1} = u_i^m \bar{Y}^{m1} \qquad (71)$$

where $\bar{Y}^{m1}$ is the total output of locationally restricted sector m, u_i^m is the proportion produced at i.

For the footloose sectors, we have minimum and maximum size restrictions..

$$\left.\begin{array}{l} Y_{i*}^{m*2} + (h_i^m - 1)\, L_T^m >= 0 \\ -Y_{i*}^{m*2} - (h_i^m - 1)\, L_B^m >= 0 \end{array}\right\} \qquad (72)$$

with:

$$h_i^m = \begin{cases} 0 & \text{if } Y_{i*}^{m*2} > 0 \\ 1 & \text{if } Y_{i*}^{m*2} = 0 \end{cases} \qquad (73)$$

where L_T^m is the maximum , L_B^m the minimum size.

Finally the non-negativity of the decision variables is stipulated:

$$Y_{ij}^{mng} >= 0 \quad \forall \; i,j,m,n,g \; ; \quad Y_{i*}^{m*2} \quad \forall \; i,j,m \qquad (74)$$

C. DOKMECI (1973)

Dokmeci presents a hierarchical location-allocation model in which twelve population centres demand services of three different types. The problem is to locate service centres in a continuous space to minimise the total cost of provision such that this this demand is satisfied. There are two elements to the total cost - production and transport.

Aggregate transport cost is given by:

$$T = \sum_{ijmn} h_{ij}^{mn} X_i^{mn} d_{ij} t \qquad (75)$$

where X_i^{mn} is the demand of centre type m at i for a service type n, measured as the number of trips required, t is the unit transport cost, d_{ij} the distance from i to j, and h_{ij}^{mn} an interaction variable determining the pattern of supply of goods from n in j to m in i.

Note that t is independent of service type because consumers are assumed to travel to the markets.

The cost of a facility at j is given as a function of its capacity, Z_j^m ...

$$C_j^m = f^m + v^m Z_j^m \qquad (76)$$

where f^m = fixed cost) of a facility
v^m = variable cost) type m.

These costs are specified s.t.

$$f^0 = 0 < f^1 < f^2 < f^3$$
$$v^0 = 0 < v^1 < v^2 < v^3 \qquad (77)$$

where m=0 denotes the demand sector.

Thus the overall problem is:

$$\min_{\{Z_j^m\},\{h_{ij}^{mn}\}} \quad F = T + \sum_{jm} C_j^m \tag{78}$$

s.t.

$$Z_j^m = \sum_i X_i^{mn} h_{ij}^{mn} \tag{79}$$

- the outputs exactly balance the demand sums (a full Walrasian equilibrium);

and the non-negativity of the decision variables:

$$Z_j^m >= 0 \;\forall\; j,m \;; \quad h_{ij}^{mn} >= 0 \;\forall\; i,j,m,n \tag{80}$$

The hierarchical ordering of facilities is defined by (77). The interaction variables are defined such that any order of facility demands services as inputs from both the other two facilities, but not from those of a similar order.

The main characteristics of the Lefeber, Gunnarsson and Dokmeci models are outlined in the usual way in Table 7. Ofcourse the essence of the three models is the same in terms of the entitation of the problem and the scales of resolution at which it is broached. Although Dokmeci expresses her system in terms of 'services', these activities have the same properties as the 'goods' of the other models. Similarly the distinction between 'restricted' and 'footloose' sectors in the Lefeber and Gunnarsson models is really only a means of reducing the conceptual complexity of the model frameworks (e.g. they could be introduced into the Dokmeci system by fixing the locations in any one of the sectors).

In relation to elements of theory, one of the most striking features of Table 7 is the similarity between the Gunnarsson and Dokmeci models which tends not to come out so strongly in the algebraic presentation. In part this is due to Dokmeci's service element (especially the definition of an average number of trips, which is effectively an input-output coefficient), and partly because of the spatial representation which necessitates a different solution method involving the substitution of binary variables for gross flows.

The adoption of a continuous space representation in the Dokmeci model is a severe restriction on the problem dimensions. A second difficulty is the combinational problem referred to in Section 3.3B above. In the original paper it was necessary to restrict the number of demand points and facilities to 12 in each sector.

Rather disappointing in the Gunnarsson model is its ultimate use to allocate four footloose sectors to only seven centres. This model also loses out to Dokmeci's version in its treatment of hierarchical relationships. Although economies and diseconomies of scale can be built in through (72), these capacity restrictions imply atleast a pseudo-fixed cost element (see above, p35). It will then be advantageous in terms of production costs to have a smaller number of maximum size facilities. This effect is not accounted for in the objective function.

If Gunnarsson's hierarchy is inferior, there is a strong case that Lefeber's is actually invalid, since 'goods' are not required in the production of either factors or other goods, and factors are not included in final demand. At the same time Lefeber's inclusion of an endogenous transport sector is a truly original contribution. It is important to note that it is this feature which facilitates the adoption of an alternative objective function. Since the total factor outputs and the technical coefficients are all fixed, the maximisation of the value of production must imply the minimisation of factors 'wasted' on transportation, which is a competing activity. Thus the most efficient transport system is guaranteed, as in the other two models.

D. TAKAYAMA AND JUDGE (1972)

In this sub-section, we deal with the model presented in Chapter 14 of the Takayama-Judge book, under the heading of a 'Spatial Price Equilibrium Activity Analysis Model'. The model is more complex than those discussed above not only in respect of the determination of equilibrium prices, but also in view of the fact that different processes are considered in the production of goods. In relation to our extended Section 2 notation, we are dealing with variables of the form $Y_{ij}^{m(r)*(s)g}$, which may be described as a flow of a good type g from the rth process in i to the sth process in j. The organisation-type index, m, is now used to define the 'order' of the good g, and hence the hierarchy of production and consumption interrelationships.

Four types of commodity are identified: final commodities (m=1); intermediate commodities (m=2); primary (mobile) commodities (m=3); and immobile commodities or resources (m=4).

By analogy with Section 3.3C, a multiregional net benefit function may be defined as:

$$Z = QW - \sum_{ijrgm} t_{ij}^{m(r)g} Y_{ij}^{m(r)*(r)g} \tag{82}$$

where QW is, as before, the sum of the regional total benefit functions each being defined as the line integral of individual demand relations.

Notice out description of the flow variables here as $Y_{ij}^{m(r)*(r)g}$. This is because the assumption of Takayama and Judge is that it is a process rather than a good which flows from region i to region j. We return to the implications of this rather unusual idea below.

The hierarchical ordering of the production and consumption sectors necessitates the articulation of separate constraints for each type of good. The final commodity production-consumption constraints are:

$$Z_i^{1(*)g} + \sum_{rj} Y_{ij}^{1(r)*(r)g} - X_i^{1(*)g} >= 0 \tag{83}$$

Intermediate commodity production and allocation constraints:

$$Z_i^{2(*)g} + \sum_{rj} Y_{ji}^{2(r)*(r)g} - \sum_{urj} q_i^{2(*)g1(r)u} Y_{ij}^{1(r)*(r)u} >= 0 \tag{84}$$

Mobile primary commodity allocation and flow constraints:

$$Z_i^{3(*)g} - \sum_{rj} \{ Y_{ij}^{3(r)*(r)g} - Y_{ji}^{3(r)*(r)g} \} - \tag{85}$$

$$\sum_{urj} q_i^{3(*)g1(r)u} Y_{ij}^{1(r)*(r)u} - \sum_{urj} q_i^{3(*)g2(r)u} Y_{ij}^{2(r)*(r)u} >= 0$$

Immobile primary commodity allocation constraints:

$$Z_i^{*g.} - \sum_{urj} q_i^{4(*)g2(r)u} Y_{ij}^{2(r)*(r)u}$$

$$- \sum_{urj} q_i^{4(*)g1(r)u} Y_{ij}^{1(r)*(r)u} >= 0 \tag{86}$$

Finally, we have the non-negativity conditions:

$$Y_{ij}^{m(r)*(r)g} >= 0 \;\forall\; i,j,m,r,g; \quad X_i^{m(*)g} >= 0 \;\forall\; i,m,g \tag{87}$$

The problem is now to maximise the benefit function (82) with respect to the allocation variables $\{Y_{ij}^{m(r)*(r)g}\}$, and to the final demand levels $\{X_i^{m(*)g}\}$, which are in turn uniquely related to the price vectors, subject to constraints (83) - (86). There is little point in specifying the elements of the problem in tabular form since these are generally the same as in Table 6. However there are two further points of interest arising from the multi-sector model. Firstly, the hierarchy adopted here is similar in nature to that of Lefeber and hence, as we have already argued, unnecessarily restrictive. Although there are four types of good here, as opposed to Lefeber's two, goods are only transferred 'up' the hierarchy, so 'lower order' goods are only used in 'higher order' processes.

In the second place, and as noted above, we have this unusual concept of the flow of a process between zones. This is emphasised in the transport term on the right hand side of the benefit function [equation (82)] which is a constant transport cost applied to a unit of flow of a process, rather than a unit of flow of a good. However, since the input requirements are determined at the origin point of the process [by the $q_i^{m(r)gn(r)u}$ multipliers in equations (84), (85) and (86)], and since the nature of the process at its destination is never significant, it would be possible to rewrite the flow terms as $\{Y_{ij}^{m(r)g}\}$ i.e. goods g of type m produced by process r at i and transferred to j. This model may now be seen as a more complex type of transportation model in which processes and hierarchies are explicitly defined.

E. LOESCH (1944)

The approach of Loesch differs from those we have looked at above in that it begins from an assumption of an undifferentiated demand environment - specifically a rural population evenly distributed across a plain. The implicit formulation of the problem is as a mathematical programme from which the equilibrium conditions are derived intuitively. The equilibrium system is then built up assuming these conditions. There is no single objective function. The system is assumed to be determined by the opposition of two fundamental tendencies: the individual chooses his location in such a way as to maximise his profit, but the system as a whole acts to maximise the number of economic units (and eliminate surplus profits).

There are five sets of unknowns; each is associated with a set of equilibrium conditions :

UNKNOWN	EQUILIBRIUM CONDITION
1. Coordinates of production locations.	The location for an individual unit must be as advantageous as possible.
2. Number of production locations.	The locations must be so numerous that the entire space is occupied.
3. F.O.B. price of goods.	Abnormal profits must disappear.
4. Sales areas of products.	The areas of supply, production and sales must be as small as possible.
5. Equations of market area boundaries.	At the boundaries of market areas it must be a matter of indifference as to which of two neighbouring locations are patronised.

Consider the production of a single commodity. With constant negative sloping demand curves and strict f.o.b. pricing, demand for the product will fall off steadily away from the production point until at some distance the demand falls to zero - this is the extreme sales radius. A circular market area is therefore defined in which a product may be sold. Overall demand is in fact given by a cone in this area (see Figure 10).

For different f.o.b. prices the extreme sales radius and the area of the cone will vary. Thus we can construct a total demand curve as a function of price. Assuming we also know the supply curve at P, we can see whether or not production at P is feasible.

However while a circle is the most efficient market area boundary, it is clear that equilibrium condition 2 cannot be satisfied by circular market areas. It is not difficult to show that a hexagon is the nearest to the ideal form that can tesselate a plane. Thus it is argued that equilibrium conditions 1 and 2 are mutually satisfied by a hexagonal market area network.

Now assume a discontinuous population distribution in a honeycomb of small hamlets or farmsteads of even size. It is obvious that the size of the market area for a good will vary with the elasticity of demand, and with supply costs for its production. This simple observation is the basis for a hierarchy of production activities.

The problem then is how to carve up the honeycomb into hexagonal market areas so that different activities will take place within networks with appropriate market area sizes. This is the idea behind the derivation of a large number of 'variable k' tesselations. As Loesch notes (1954, p120), this leads to something of an indivisibility problem - market areas will tend to be too large and surplus profits will appear.

Now suppose we consider the ten smallest market areas (k=3, 4, ..., 25). If we assume that some point which is the centre of the largest market area also markets products of the other nine, we can define the market area meshes—centred on this point. It turns out that for market areas 3, 6 and 8 there are two possible alignments of the mesh. By assuming that the centres of these areas locate as near as possible to the other large centres, it is possible to develop the well-known 'city-rich, city-poor' sectoral alignments.

F. CHRISTALLER (1933)

The Christaller framework need only be commented on briefly since it is very similar to Loesch's. Both arrive at the conclusion that the hexagonal market area is the most efficient way of filling the space. Christaller defines the ideal range of a good, which is equivalent to Loesch's extreme sales radius. However rather than defining a supply curve for production, the threshold is given explicitly as the minimum area that is needed to support a production activity. This must be less than the real range of the production activity i.e. its actual market radius/area. Just as with Loesch, variation in the threshold provides the basis for a hierarchy of centres. Once again a wide variation in market area sizes is effectively suppressed, in this case even more so as only one k-system is generally allowed. In practise, for the k=3 system, this means that market area sizes increase progressively by a factor of $\sqrt{3}$ all the way up the hierarchy (cf. Wilson, 1979).

The other interesting thing about the Christaller model in relation to Loesch is that here any centre of a rank h contains activities of all lower ranks, giving rise to a rather less diverse economic environment. This is compatible with the assumptions of Tinbergen, but not with the Loesch system.

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TABLE 1: A THREE LEVEL CLASSIFICATION OF 'ORGANISATIONS'

COARSE	SEMI-COARSE	FINE
Public Utilities	Water	⋮
	Gas	
	Electricity	
	Telecommunications	
	⋮	⋮
Primary Sector	Mining	Smith & Sons Ltd.
	Agriculture	⋮
	⋮	Jones & Co.
Secondary Sector	Vehicles	⋮
	Engineering	
	Food manufacture	
	⋮	
Tertiary Sector	Retailing	
	Banking	
	⋮	
Quaternary Sector	Schools	
	Hospitals	
	⋮	
Transport Sector		
Infrastructure	Railways	
	Roads	
Rolling Stock	Haulage companies	
	⋮	⋮
Individuals	Skilled manual	Joe Bloggs
	Semi-skilled manual	⋮
	Professional	
	⋮	

TABLE 2: ELEMENTS OF THE WEBER MODEL.

1. <u>Problem focus</u>: a single producing activity locates in relation to a single market and a number of input sources.

2. <u>Spatial representation</u>: continuous.

3. <u>Levels of resolution</u>

   3.1 Sectoral - fine scale.

   3.2 Spatial - indeterminate.

   3.3 Temporal - the model is static.

4. <u>Elements of theory</u>

   4.1 Producers behaviour is to set f.o.b. prices

   4.2 Consumer demand is inelastic.

   4.3 There is no interdependence between activities.

   4.4 The technology is constant transport costs and Leontief production functions.

   4.5 The whole system objective is to minimise transport costs.

5. <u>Solution procedure</u>: geometric

TABLE 3: ELEMENTS OF PALANDER'S APPROACH.

1. Problem focus: two firms are located in a plain with continuous density of demand. Find the market areas of the two firms.

2. Spatial representation: continuous.

3. Levels of resolution

   3.1 Sectoral - fine scale.

   3.2 Spatial - indeterminate.

   3.3 Temporal - the model is static.

4. Elements of theory

   4.1 Producers behaviour is to set f.o.b. prices arbitrarily e.g. to production costs plus normal profits.

   4.2 Consumer demand is spatially constant and inelastic. Consumers buy from the centre offering the lowest priced good.

   4.3 There is a certain amount of interdependence in that the sales of each firm are determined by both the price it charges, and the price charged by the other firm.

   4.4 The technology is constant transport costs and constant average production costs.

   4.5 There are no overall system criteria.

5. Solution procedure: graphical/ geometric

TABLE 4: STRUCTURE OF THE HITCHCOCK-KOOPMANS TRANSPORTATION PROBLEM.

1. Problem focus: given the locus of production and consumption for a good or activity, find the optimal pattern of flows linking demand and supply.

2. Spatial representation: discrete zones.

3. Levels of resolution

   3.1 Sectoral - fine scale.

   3.2 Spatial - indeterminate.

   3.3 Temporal - the model is static.

4. Elements of theory

   4.1 Producers behaviour is unimportant as prices and production levels are fixed.

   4.2 Consumer demand is spatially dispersed and inelastic.Consumers buy from the nearest centre.

   4.3 There is interdependence between producers in that the ability to supply goods is dependent on the absence of other producers in a more favourable position.

   4.4 The technology is constant transport costs. All other technology is irrelevant.

   4.5 The whole system objective is to minimise transport costs.

5. Solution procedure: mathematical programming.

TABLE 5: ELEMENTS OF THE P-MEDIAN PROBLEM.

1. Problem focus: locate p facilities to serve n demand points.

2. Spatial representation: continuous.

3. Levels of resolution

   3.1 Sectoral - fine scale.

   3.2 Spatial - indeterminate.

   3.3 Temporal - the model is static.

4. Elements of theory

   4.1 Producers behaviour is not relevant.

   4.2 Consumers travel to the nearest centre.

   4.3 The location and output of each facility is dependent on the locations of all other facilities.

   4.4 Unit transport costs are constant. Production costs have a fixed and a variable element.

   4.5 The whole system objective is to minimise the sum of transport and production costs.

5. Solution procedure: heuristic search.

TABLE 6: ELEMENTS OF A SINGLE-SECTOR PRICE MODEL.

1. Problem focus: find a set of regional supply and demand prices to generate an optimal distribution of activities.

2. Spatial representation: discrete zones.

3. Levels of resolution

   3.1 Sectoral - fine scale.

   3.2 Spatial - indeterminate.

   3.3 Temporal - the model is static.

4. Elements of theory

   4.1 Producers have given supply functions: the lower the supply price of a good, the less they will produce.

   4.2 Consumers have given demand functions: the lower the market price, the more they will buy.

   4.3 There is a high degree of interdependence between both producers and consumers.

   4.4 The technology is constant transport costs with productive technology accounted for implicitly in the supply function.

   4.5 The whole system objective is to maximise net quasi-welfare.

5. Solution procedure: geometric

TABLE 7: FEATURES OF THREE MULTI-SECTOR LOCATION MODELS.

	LEFEBER	GUNNARSSON	DOKMECI
1. Problem focus:	- -	locate optimally a set of interdependent activities of different types given the locus of consumption.	- -
2. Spatial representation:	discrete	discrete	continuous
3. Levels of resolution			
3.1 Sectoral	- - - - -	coarse	- - - - - - -
3.2 Spatial	- - - - -	indeterminate	- - - - - -
3.3 Temporal	- - - - -	static	- - - - - - -
4. Elements of theory			
4.1 Producers behaviour	Only 'factors' used to produce 'goods'	Produce 'goods' using 'factors' and other 'goods'	Produce goods using all other goods as inputs
4.2 Consumer behaviour	Goods sold to final demand sector only	- - Goods sold to final demand and to other sectors	- - -
4.3 Treatment of inter-dependence	- -	There is a high degree of interdependence in competition for both markets and suppliers	- - - - -
4.4 Technology	Leontief prod. fns. Endogenous tpt. sector	Fixed + vble cost (implied). Const. unit tpt. costs	Explicit fixed + vble costs. Const. unit tpt. costs
4.5 Whole system criteria	Maximise value of production	Minimise total distribution costs	Minimise total cost of production and transportation
5. Solution procedure	Mathematical programming	Mathematical programming	Heuristic search

FIGURE 1: A Mechanical Solution to a Simple Weber Problem (After Lloyd and Dicken, 1977; Figure 4.3)

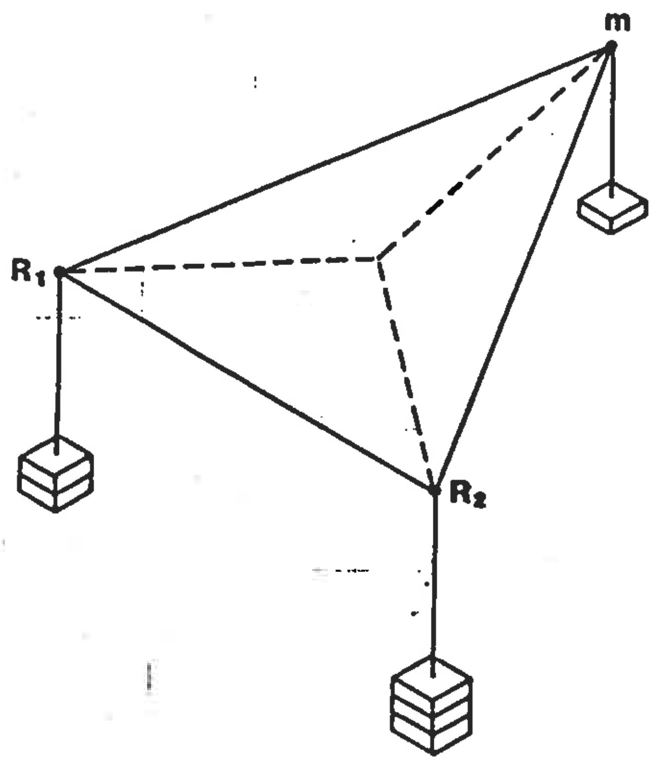


Figure 2: Dynamic Tendencies with Input Substitution.

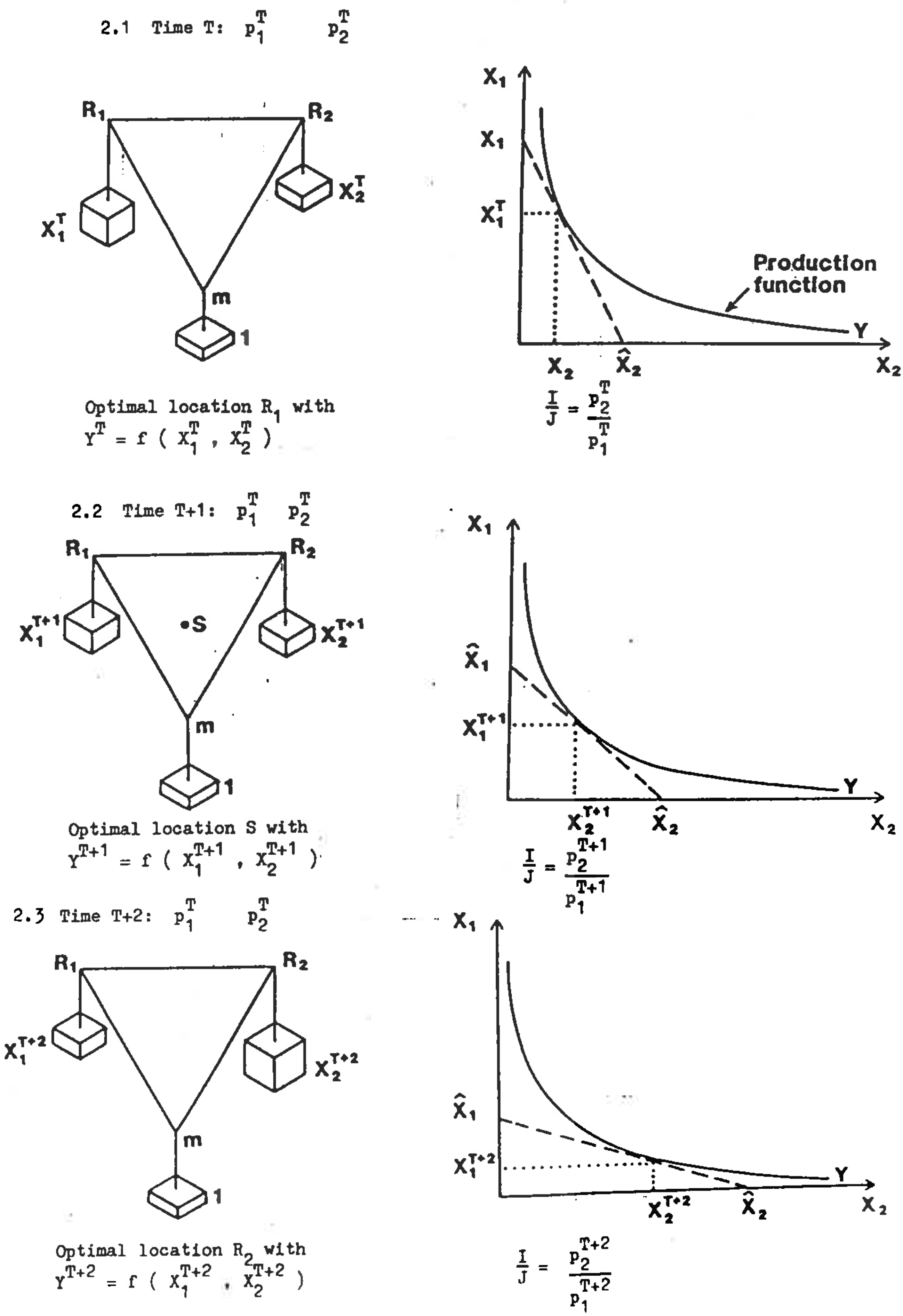


FIGURE 3: Derivation of the Space Cost Curve

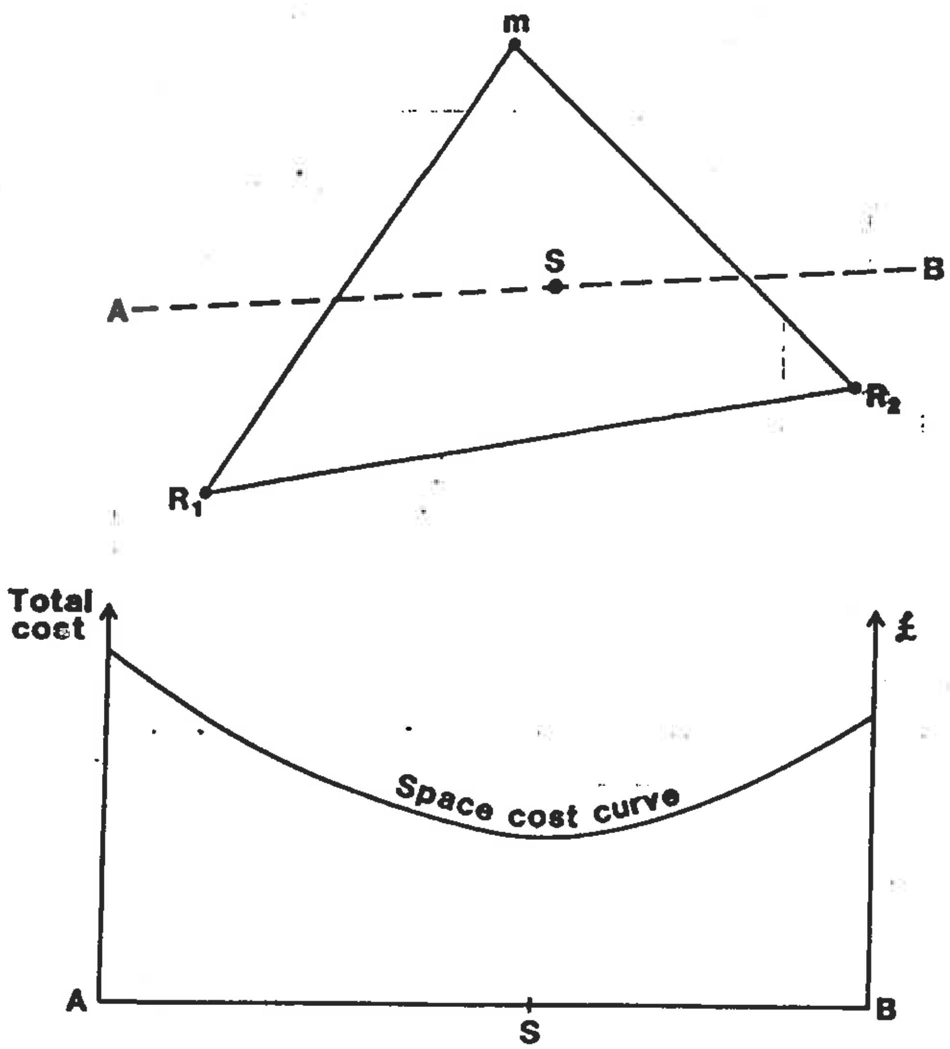


FIGURE 4: Individual Components and Total Unit Costs in the Greenhut model. (Adapted from Greenhut, 1956, Figures 14 and 15).

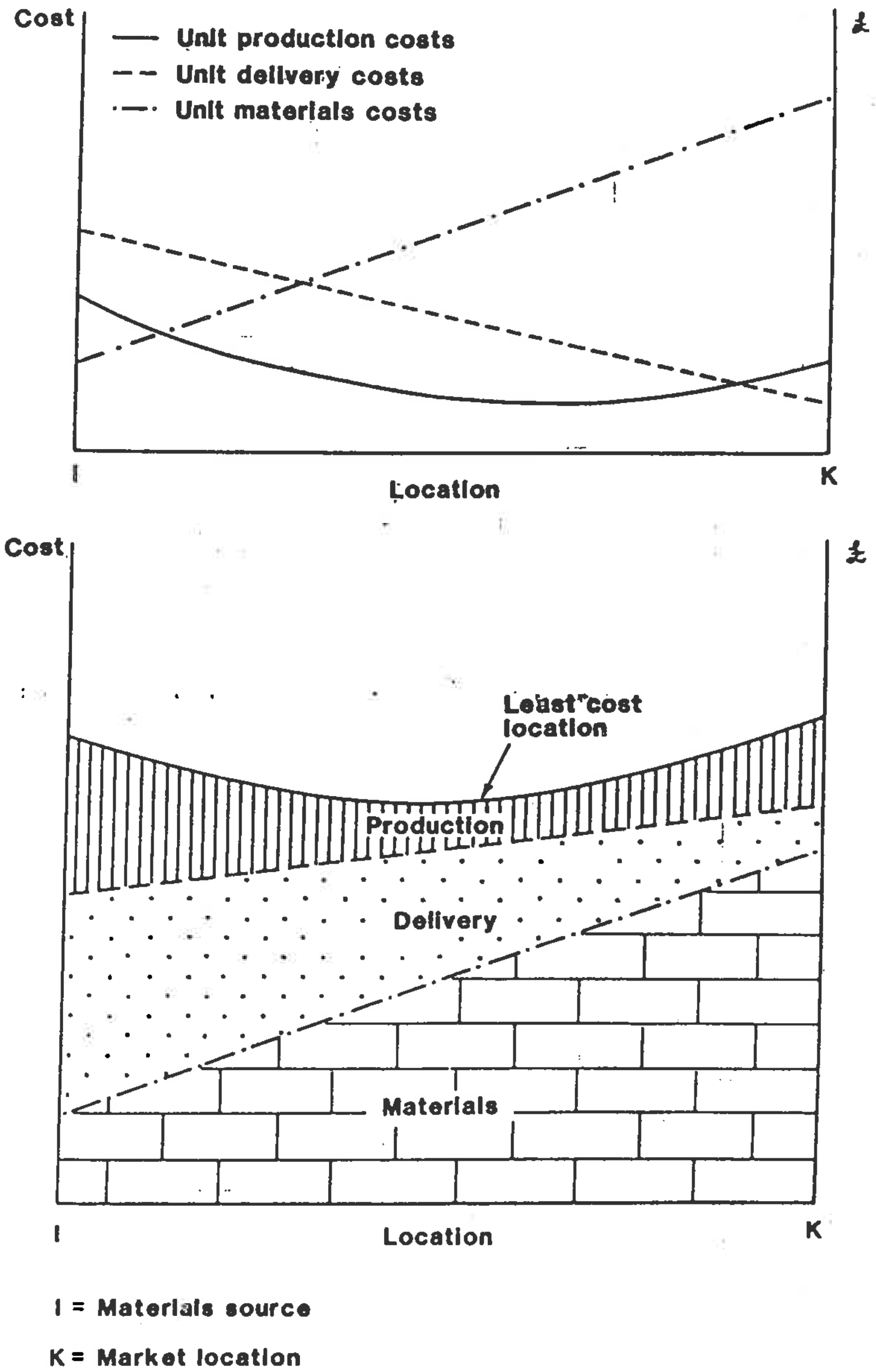


FIGURE 5: Spatial Margins to Profitability
(After Lloyd and Dicken, 1977; Figure 7.2b)

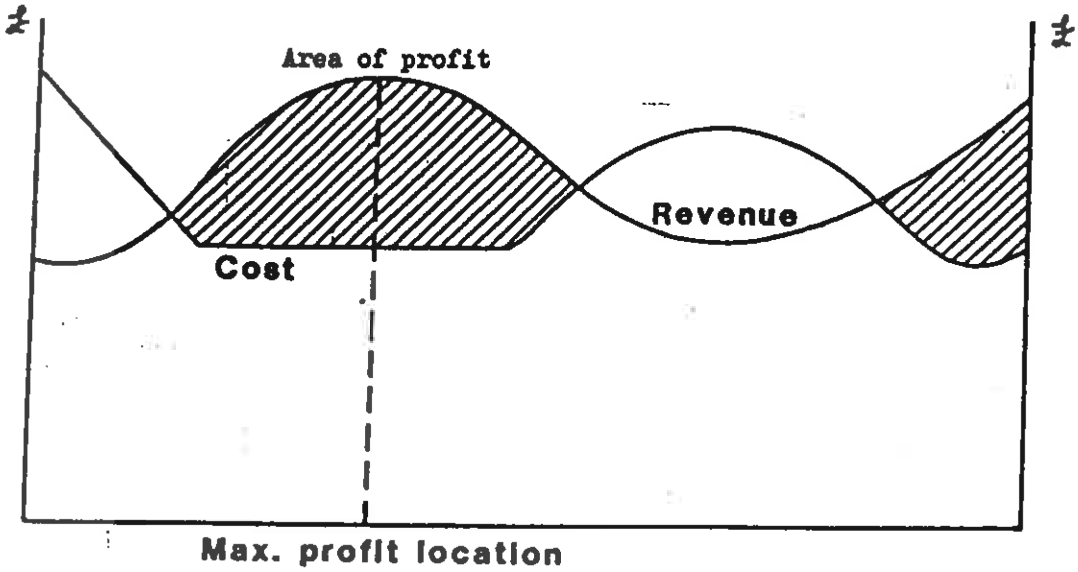


FIGURE 6: Market Areas in the Palander Problem

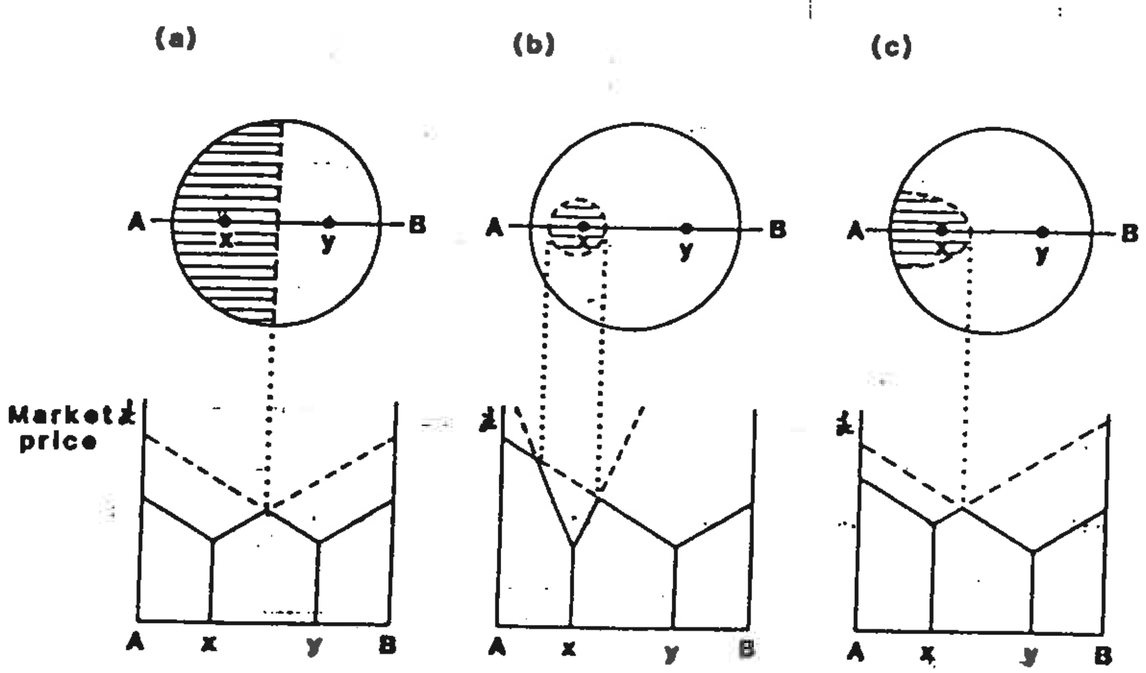
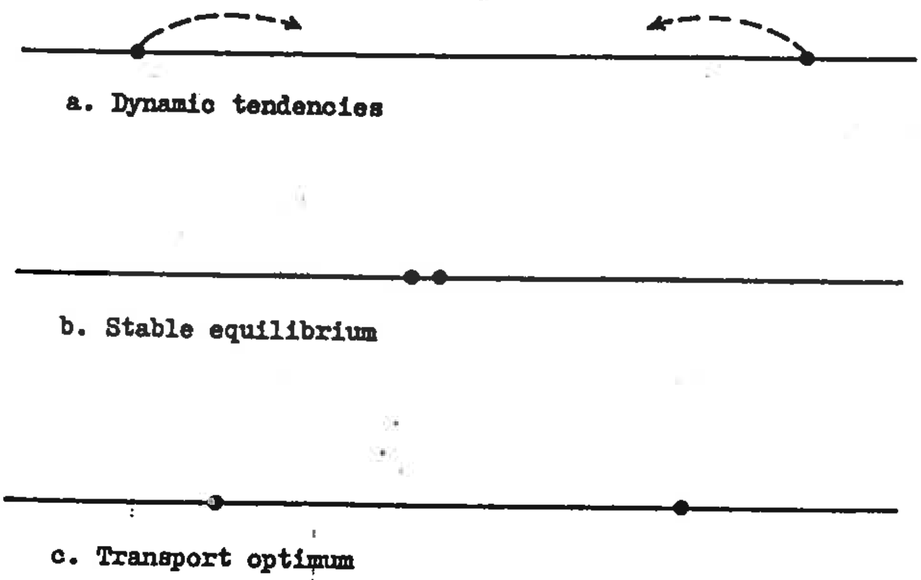


FIGURE 7: Location of Firms in the Hotelling Model



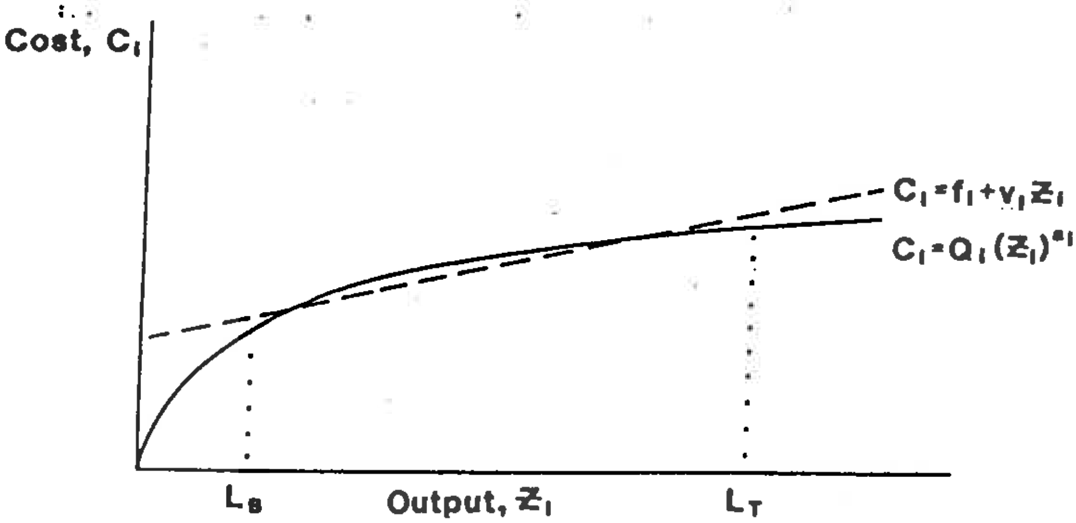


FIGURE 8: Linear Approximation to a Production Process with Diminishing Returns to Scale. (After Vietorisz and Manne, 1963; Figure 3)

FIGURE 9: Trade and Welfare in a Two-region System. (Adapted from Takayama and Judge, 1972; Figure 6.1)

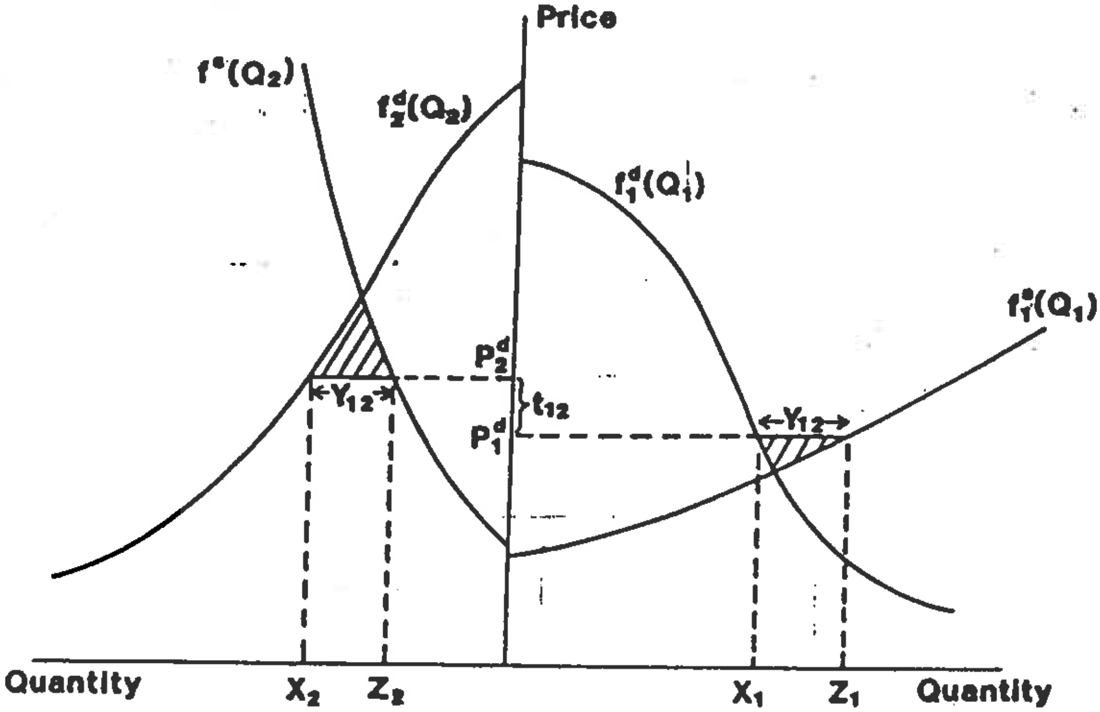


FIGURE 10: Derivation of Market Areas and Demand Cones in the Loesch System. (After Loesch, 1954; Figures 20 and 21)