Working Paper 321

GRAVEL IN MOTION

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NOTE

This paper originally began as that quoted in Working Paper 314. However, following discussions with Mike Leeder and John Allen during and after the Fluvial Sedimentology Conference at Keele (September 1981), the analysis of the paths of particle motion and the conclusions concerning grain size variation have been modified from those quoted on page 4 of Working Paper 314.

Tables

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I. Introduction and statement of problem

The problem of gravel movement - its speed and path of travel, contact with the bed and a criterion for deposition - is very different from and far more demanding than the question of the initiation of particle motion. Two distinct types of motion have been identified. The first is suspended motion in which the excess weight of the solid is entirely supported by the fluid. In this type of motion, the particle remains out of contact with the bed indefinitely. Secondly, there is unsuspended motion where the only upward impulses imparted to the solid take place at contacts with the bed. This second type of motion includes sliding, rolling and saltation. In fact, sliding and rolling may be simply viewed as incipient saltation (c.f. Bagnold 1973; page 477). In the case of gravel sizes particles ($D \geqslant 4$ mm), the large excess weight of the particles rules out the possibility of the suspension in most flows. Consequently, this paper aims simply to review and where possible to analyse unsuspended motion.

The more specific questions to which answers are sought are as follows:

(1) What is the downstream speed of the grain?

(2) What are the dimensions (downstream and vertical) of the trajectories traced out by the grains?

(3) What is the nature of the particles' contact with the bed and what, therefore, is the probability of deposition?

Theoretically based answers to these questions may only be obtained by whole or partial solutions to the equations of motion for particle trajectories. This paper illustrates how intractable such an approach is and instead offers some semi-empirical relationships for specifying particle motion. These relationships are based entirely on the flume experiments of Francis (1973), Meland and Norrman (1966) and Abbot and Francis (1977) although, where possible, they are backed up by some sort of theoretical rationale.

II. A semi-theoretical approach based on flume experiments reported in the literature

(1) Downstream speed of grain

Provided that the fluid drag force can be considered to be the dominant force acting in the downstream direction of movement, a sort of slip-stick motion akin to sliding may be used to simplify both rolling and saltating modes of motion. Under this assumption, the force equations for a steady sliding motion can be expressed as follows.

The time-mean horizontal fluid drag force on the particle, F_x, may be given according to Yalin (1977; page 144) and Francis (1973; page 458) as:

$$F_x = 0.157\ \rho D^2\ (\bar{u} - \bar{U})^2 \qquad (1)$$

where ρ is water density
D is particle diameter
$\bar{u}$ is time-mean water velocity at average particle centre
$\bar{U}$ is time-mean particle velocity

This downstream drag force ignores any spin on the grain and any extra drag due to vertical forces (Bagnold 1973; page 503) but may be satisfactory as a first approximation. The downstream force is resisted by an upstream friction force, FF, proportional to the submerged weight of the particle:

$$FF = (\rho_s - \rho)\ g\ \frac{\pi}{6}\ D^3 \tan \alpha \qquad (2)$$

where ρ_s is the density of the particle
α is the angle of dynamic friction
g is the acceleration due to gravity

For steady motion, F_x = FF i.e.

$$0.157\ \rho\ D^2\ (\bar{u} - \bar{U})^2 = (\rho_s - \rho)\ g\ \frac{\pi}{6}\ D^3 \tan \alpha$$

$$\rightarrow \bar{U} = \bar{u} - \sqrt{\left(\frac{(\rho_s - \rho)}{\rho}\ g\ \frac{\pi}{6}\ \frac{D \tan \alpha}{0.157}\right)} \qquad (3)$$

Substituting the logarithmic flow law, it is possible to express the water velocity at the particle centre as a function of the bed shear stress, u_*:

$$\bar{U} = 5.75\ u_* \log_{10}\left(\frac{30.1}{D}\ y\right) - \sqrt{\left(\frac{\rho_s - \rho}{\rho}\ g\ \frac{\pi}{6}\ \frac{D \tan \alpha}{0.157}\right)} \qquad (4)$$

where y is the average height of the particle centre above the bed in m.

Now, the settling velocity of a large grain in still water is given by:

$$\frac{\pi}{6} D^3 (\rho_s - \rho)\, g \;=\; 0.157\, \rho\, D^2\, V_g^{\,2}$$

where V_g is particle settling velocity in m/s.

$$\rightarrow \; \frac{(\rho_s - \rho)}{\rho} D \;=\; 0.031\, V_g^{\,2} \qquad (5)$$

Substituting equation (5) into equation (4) gives an alternative version of the expression for mean particle velocity:

$$\bar{U} \;=\; 5.75\, u_* \log_{10} \left(\frac{30.1\, y}{D} \right) \;-\; V_g \sqrt{\tan \alpha} \qquad (6)$$

Equations (4) and (6) express the steady downstream motion of a single particle. All that requires to be specified is the mean height of the "slide", y, and an appropriate angle of friction, α. The lower limit of y should be of the order of 0.7 D above the theoretical bed (0.5 D above a surface tangential to the bed grains). However, it is also expected that both y and α will vary with the shear velocity. For instance, at higher excess shear velocities, the saltations will be higher and, therefore, the mean height of movement will be greater. Similarly, if the friction angle expresses the ratio between downstream and gravity forces, it too should increase with an increase in excess shear velocity. Abbot and Francis (1977; Table 2, page 235) give a number of values of α as related to stage and they do follow this trend. However, in view of the small sample size, wide scatter (Figure 10, page 235) and Abbott and Francis's own caveat (page 244) on the problems of measuring α accurately, even a tentative relation cannot be put forward. Bagnold (1973; page 481) indeed seems to be of the opinion that α is a constant at about 18^{o} - somewhat lower than any of the values quoted by Abbott and Francis. In fact, as far as the prediction of downstream motion goes, Francis (1973) and Abbott and Francis (1977) seem to suggest that the equation should be very simply of the form:

$$\bar{U} \;=\; b u_* \;+\; a \qquad (7)$$

where a and b are constants

This suggests that both y and α should be treated as constants. In the case of the mean height of thrust, y, consideration of the logarithmic flow law suggests that this assumption leads to very little error. As far as the friction angle goes, there is so much uncertainty around its true value, and such a relatively small trend with stage, that it too could be assumed constant. Thus, in the absence of hard evidence to the contrary, a and b in equation (7) are considered to be constant, and their values sought from the empirical data.

Table I gives the available data as presented by Meland and Norrman (1966; Figure 4, page 174 and Table I, page 168) and Francis (1973; Appendix 2, page 470). The eight separate sets of data are plotted in Figure 1 and the regression results given in Table II. In order to pool the data and so increase the number of observations to gain a little more confidence in the results, a regression analysis of $\frac{U}{V_g}$ against $\frac{u_*}{V_g}$ has also been carried out and the scatter shown in Figure 2.

The resultant regression line is:

$$\frac{U}{V_g} = 11.83 \frac{u_*}{V_g} - 0.52 \pm 0.29 \qquad (8)$$

$$N = 52 \; ; \; R = 0.94$$

This would appear to be a very adequate compromise solution although the regression coefficients cannot be strictly interpreted in terms of y and α. Figure 3 shows the predicted particle velocities using equation (8) plotted against the observed particle velocities. There is reasonable agreement in all cases except the second data set taken from Meland and Norrman (1967). The reason for this discrepancy, however, can be found by looking at the velocity distributions in the vertical given by Meland and Norrman (1966; figure 2, page 171). In the series A data, the standard logarithmic flow law shows a very good degree of fit in both the slope and the intercept. In the series B data, however, from which the second data set comes, the intercept is very different from that expected - 0.07 cm against 0.03 cm. This entirely accounts for the discrepancy found in Figure 3. Reanalysing the data, omitting these errant data points gives a revised version of equation (8) as:

$$\frac{U}{V_g} = 11.8 \frac{u_*}{V_g} - 0.44 \pm 0.20 \qquad (8a)$$

$$N = 46 \; ; \; R = 0.98$$

Given Abbott and Francis's observation (1977; Table 3, page 236) that rolling speeds are approximately 0.75 times saltation speeds, equation (8a) agrees very well with Francis's analysis (1973; Table 4, page 459) of grain speed on cessation of motion using data of White (1940) and Shields (1936).

Thus, it can be concluded that there is some theoretical rationale for equation (8a) as an analogy with steady sliding motion and that, furthermore, the majority of empirical evidence is in support of this approach as an initial assumption for predicting particle speeds.

In extending this equation to mixed grain sizes, however, some interpretation of the constants a and b in equation (7) must be offered. At face value, the constants of equation (8a) seen in conjunction with equation (6), imply values of $y = 3.75$ D and $\alpha = 11^{\circ}$. This value of y is clearly not an average representation of trajectory heights (c.f. Abbott and Francis, 1977; Figure 3, page 232 and discussion below) and will, therefore, be dropped from the discussion. The formulation of $a = -V_g \tan \alpha$ from equation (6), however, can be used to throw additional light on the case of mixed grain sizes. Simplifying the present concern to grain size ratios of 1 : 2, Meland and Norrman (1966; Figure 4, page 174) present two sets of data - one for grains twice the diameter of the bed grains and one for those half the size of the bed grains. The regression results for these two data sets are given in Table III. Again the values associated with Meland and Norrman's series B data do not match expectations - the value of α is even greater than that for static friction. However, adjusting the value of the settling velocity to be consistent with the logarithmic flow law intercept, yields a far more reasonable value of α. In the absence of further evidence, the friction angles shown in Table IV have been adopted along with equation (8a).

(2) Path of motion

In considering the details of particle motion - the interaction between particles on collision & the likely deposition of particles - the path of motion of each particle needs to be specified as well as its downstream velocity. Following Leeder (1979, page 232), it is assumed that at a given transport stage, there is a characteristic or mean saltation trajectory. All that remains to be determined is the dimensions of the trajectories and the likely spread about the mean value. Again, a tentative theoretical statement might be made.

Unsuspended motion has been defined as that in which the only upward impulse on the grain occurs at the contact with the bed. Applying the impulse - momentum theorem to define this upwards acceleration (Krishnappan 1976, page 17-3; Yalin 1977, page 145-150) gives:

$$\frac{mdV}{dt} = -R_y - G + F_y \tag{9}$$

where m is the mass of the grain
$\frac{dV}{dt}$ is the upwards acceleration of the grain
R_y is the vertical drag force
G is the gravitational force
F_y is the vertical lift force

For a spherical grain of diameter, D, and density, ρ_s, within a flat bed of equally sized spheres, these forces may be given as:

$$R_y = 0.157 \rho D^2 V^2 \tag{10}$$

$$G = \frac{\pi}{6} D^3 (\rho_s - \rho) g \tag{11}$$

$$F_y = 0.07 \rho D^2 u^2_y \tag{12}$$

where ρ is density of water
ρ_s is density of grain
g is acceleration due to gravity
D is grain diameter
V is vertical grain velocity
u_y is the "<u>instantaneous</u>" downstream water velocity at a height of $y = 0.35$ D above the theoretical bed
0.157 and 0.07 are drag and lift coefficients respectively - for derivation see Naden (1981a)

Now, the maximum vertical velocity of the grain is given when $\frac{dV}{dt} = 0$. Assuming that this occurs near the ned, instantaneously, and under conditions of constant acceleration, equations (9) to (12) yield:

$$0.157 \rho D^2 V^2_{max} = 0.07 \rho D^2 u^2_y - \frac{\pi}{6} D^3 (\rho_s - \rho) g$$

$$\rightarrow V^2_{max} = 0.45 u_y^2 - 32.7 \frac{(\rho_s - \rho)}{\rho} D \tag{13}$$

Yalin (1977, page 150) goes on to give a simplified derivation for the maximum saltation height by assuming that the kinetic energy possessed by the grain at the instant of detachment is spent on work against the downward directed forces - grain submerged weight + average fluid resistance force i.e.

$$\frac{1}{2} m V^2_{max} = (G + \overline{R_y}) H \tag{14}$$

where V_{max} is instantaneously imparted maximum upwards grain velocity

H is the height of the grain's upward movement

$\overline{R_y}$ is average fluid resistance

i.e. $\dfrac{0.157 \rho D^2 V^2_{max}}{2}$ approx.

$$\rightarrow \quad H = \frac{0.5 \rho_s \frac{\pi}{6} D^3 V^2_{max}}{\left(g \frac{\pi}{6} D^3 (\rho_s - \rho) + \frac{0.157 \rho D^2 V^2_{max}}{2}\right)}$$

$$= \frac{V^2_{max}}{19.6 \left(\frac{\rho_s - \rho}{\rho_s}\right) + 0.30 \frac{\rho}{\rho_s} \frac{V^2_{max}}{D}} \tag{15}$$

Substituting for V_{max} from equation (13) gives:

$$H = \frac{0.45 u^2_y - \frac{(\rho_s - \rho)}{\rho} D\, 32.7}{9.8 \frac{(\rho_s - \rho)}{\rho_s} + 0.135 \frac{\rho}{\rho_s} \frac{u^2_y}{D}} \tag{16}$$

or

$$H = \frac{0.45 u^2_y - \frac{(\rho_s - \rho)}{\rho} D\, 32.7}{0.135 u^2_y + 9.8 \frac{(\rho_s - \rho)}{\rho} D \quad \frac{\rho}{\rho_s}} \tag{16a}$$

The question to be asked now is whether this theory is supported by the experimental evidence. The only published source of trajectory data known is the work of Abbott and Francis (1977). As shown in Figure 4a, taken from Abbott and Francis (1977; figure 3, page 232), the results are expressed in terms of the mean transport stage of the flow i.e. the ratio of the time-mean shear velocity to the threshold shear velocity at a Shields' value of 0.06. A further complication is that the height data are expressed as mean maximum heights <u>ignoring rolling trajectories</u>. This implies that there is some bias towards higher values of trajectory jumps which must decrease as stage increases. However, this may not be a very serious bias beyond a stage of 1.0 when over 60% of grain motions are saltations (Abbott and Francis 1977; figure 1, page 230). Grain size is constant at D = 0.00828 m. but grain density varies between 1240 kg/m³ and 2570 kg/m³. There are thus a number of problems associated with testing the theoretical statement given above against the data and in the analysis which follows a number of assumptions have to be made.

First, in order to test the theory, the data points have been extracted from Figure 1a and expressed with their particle densities in Table Va. Secondly, the logarithmic flow law has been assumed (although there is no direct evidence for this):

$$\bar{u} = 5.75\, u_* \log (30.1 \times 0.35) \qquad (17)$$

where $\bar{u}$ is time mean velocity at depth y = 0.35 D
u_* is shear velocity

Coupled with the Shields' criterion (according to Abbott and Francis (1977; page 228))

$$u_{*o} = \sqrt{\left(0.06\, \frac{(\rho_s - \rho)}{\rho}\, g\, D\right)} \qquad (18)$$

this gives the definition of stage as:

$$\frac{u_*}{u_{*o}} = \frac{\bar{u}}{5.75 \log (30.1 \times 0.35)} \cdot \frac{1}{\sqrt{\left(0.06\, \frac{(\rho_s - \rho)}{\rho}\, g\, D\right)}} \qquad (19)$$

$$\rightarrow \quad \bar{u}_{(y = 0.35\, D)} = 4.5 \sqrt{\frac{(\rho_s - \rho)}{\rho}\, D}\quad \frac{u_*}{u_{*o}} \qquad (20)$$

Furthermore, using the relation between instantaneous velocity fluctuations and the mean velocity derived from the data of McQuivey (1973b)(Naden, 1981a), this gives a standard deviation also related to stage:

$$\sigma_u = 0.16\left(\frac{y}{D}\right)^{-0.65} \bar{u}_y$$

where $y = 0.35\ D$

$$\rightarrow \quad \sigma_u = 0.32\ \bar{u}_y$$

$$\rightarrow \quad \sigma_u = 1.45 \sqrt{\left(\frac{\rho_s - \rho}{\rho}\right) D} \quad \frac{u_*}{u_{*o}} \qquad (21)$$

Using equations (20) and (21), it is now possible to convert the value of stage for each of the data points given in Table Va into a mean water velocity at y = 0.35 D and a velocity fluctuation component. These are shown in Table Vb. It is now necessary to work out the mean height of the saltations which occur for any given stage and grain. For this, the mean instantaneous velocity which produces a saltation is required. This can be calculated knowing the normal distribution of instantaneous velocities as given in equation (12) and the threshold for a jump (Naden, 1981a):

$$u \geqslant \sqrt{\left(\frac{\rho_s - \rho}{\rho}\ g\ \frac{\pi}{6} D \ .\ \frac{1}{0.07}\right)} \qquad (22)$$

This threshold value of u for each of the data points concerned is given in Table Vb. In order to calculate the mean saltation height, the area under the normal curve for velocities exceeding this threshold is calculated, halved to find the average and converted back into a mean instantaneous velocity. This is the velocity which has been substituted into equation (16) to give the predicted mean maximum jump height, also given in Table Vb.

Figure 5 shows a plot of the predicted height of saltation as calculated from the above theory against the observed height of saltation in Abbott and Francis's (1977) experiments. It reveals a considerable discrepancy between theory and observation, with the theory grossly under-estimating saltation heights. Two questions must now be considered: why should there be this discrepancy between theory and observation and what strategy should be pursued in order to make some progress towards predicting grain motion.

A number of possible reasons for the discrepancy between theory and observation can be put forward. Firstly, the particles in the experiments did not start from rest or within the bed of similar sized spheres. However, data presented by Abbott and Francis (1977; Table 5, page 239) comparing the trajectories from rest with those from motion suggest that this should not have affected the results substantially and in more general terms their conclusion is that trajectories are independent of the previous history of grain movement, i.e. there is no effective elastic rebound on collision (see below). In terms of the original bed geometry of the grain to be moved, if the grain centre is above the theoretical bed, the lift force is much reduced. Consequently, in the choice of starting conditions, predicted saltation trajectories have been given the best advantage. Thus, in terms of the discrepancy between the experimental and theoretical starting conditions, predictions remain unaltered or become slightly worse.

A further discrepancy, however, is revealed in Figure 4b taken from Abbott and Francis (1977; Figure 7, page 233). At low transport stages, the above reasoning would anticipate a truncated normal curve as that describing the distribution of saltation heights. Figure 4b reveals a normal curve with a slight positive skew.

The only conclusion to be drawn from these comments is that the theoretical framework discussed above is inappropriate for the problem in hand. In particular, it can be suggested that the main reason for the failure of the theory is that the impulse-momentum approach does not specify all the forces concerned in the motion nor can the lift on impact be considered to be the dominant force. Gordon, Carmichael and Isackson (1972) and Bagnold (1973) both stress the conservation of tangential momentum on collision with the bed. However, this does not assist in the case where particles start from rest and the reliance on an initial or previous velocity does not square with Abbott and Francis's observation (1977; pages 243-244) that trajectories are independent of the grain's previous history. A number of other contributory forces, however, are mentioned by Abbott and Francis (1977; pages 239-243) - in particular, the effect of enhanced drag due to the downstream particle and fluid velocities and the possibility of a shear drift force adding spin to the grains, thus enabling them to rise higher. In fact, in the vertical dimension, the picture becomes one in which a multitude of small forces produce the observed trajectory rather than one in which the hydrodynamic lift is predominant. This suggests that a rather formidable overhaul of the theory is required which, at this stage of understanding is impossible.

The alternative strategy to adopting the theoretical type of analysis outlined above is to take the experimental data of Abbott and Francis (1977; Figure 3) for saltating grains and develop the argument from an empirical standpoint. For values of transport stage less than 2.3, a regression analysis of the data given in Table Va shows that:

$$H_{max} = 0.0055 \frac{u_*}{u_{*_o}} + 0.0021 \pm 0.006 \qquad (23)$$

$$N = 8 \quad ; \quad R = 0.974$$

Converting this equation into one relating jump height to the bed shear velocity gives:

$$H_{max} = 0.0042 \frac{u_*}{\sqrt{\frac{(\rho_s - \rho)D}{\rho}}} + 0.0021 \qquad (24)$$

and in terms of the flow velocities near the bed ($y = 0.35$ D):

$$H_{max} = 0.0012 \frac{\bar{u}_y}{\sqrt{\frac{(\rho_s - \rho)D}{\rho}}} + 0.0021 \qquad (25)$$

For an individual reajectory, while the lift force is not the dominant force operating, it is the only force which fluctuates through time, and the instantaneous velocity at $y = 0.35$ D should be substituted for the time-mean velocity in equation (25). Working through the data presented in Figure 1b (Abbott and Francis 1977; Figure 7) using the relation for standard deviation of velocity developed from McQuivey (1973b) (see Naden 1981a, page 10):

$$\sigma_u = 0.16 \left(\frac{y}{D}\right)^{-0.65} \bar{u}_y \qquad (26)$$

where σ_u is standard deviation of velocity fluctuations
u_y is time-mean velocity at height y above the theoretical bed

and equation (25) reveals a large measure of agreement. Furthermore, at the same transport stage for the case of heavier grains, a larger standard deviation is expected. This is in agreement with Figure 4b and arises because the downstream fluid velocity for moving heavier grains is greater and, hence, from equation (26) so is the spread of velocity fluctuations. The slight degree of skew shown in Figure 4b, however, is not accounted for. Another question might be posed.

In order to extrapolate the relations given in equations (23) to (25) to situations beyond the experimental evidence, some assumption as to the behaviour of this relation with changing grain size has to be made. There is no evidence for such an assumption in any of the known published literature on grain transport. Abbott and Francis themselves conclude by saying that it is "highly likely that this method of non-dimensionalizing (i.e. expressing H_{max} as H_{max}/D) would be effective with a wide range of data" (1977, page 232). The result of this substitution into equation (24) above for the grain size D = 0.00828 m is:

$$\frac{H_{max}}{D} = 0.51 \frac{u_*}{\sqrt{\frac{\rho_s - \rho}{\rho} D}} + 0.254 \qquad (27)$$

$$\rightarrow \quad H_{max} = \frac{0.51\, u_* \sqrt{D}}{\sqrt{\frac{(\rho_s - \rho)}{\rho}}} + 0.254\, D \qquad (27a)$$

If this is a reasonable representation of what happens in the case of different sizes of grains then height of jump increases proportionally to the square root of the grain diameter and begins at a height proportional to the grain diameter, i.e. a graph of H_{max} versus stage should show increases both in the slope of the line and in the intercept proportional to grain size. Now, whereas it is reasonable to suppose that the intercept of the graph increases with grain size - the centre of the larger grain is higher in the flow and, therefore, subject to a greater downstream velocity; the increase in the height of jump over and above this is not so easy to understand. It rather implies that for a given shear stress, a greater force is imparted to the larger of two particles. The possible candidates for supplying this force are the shear drift force quoted by Taylor, a Magnus spin effect (Abbott and Francis 1977; page 242), or a greater lift force (Bagnold 1973; page 503). It is true that these forces are ones dependent on the surface area of the grain and hence do increase with the diameter squared but they must be balanced against the fluid drag on the grain and the inertia to rotation of the grain. These are also dependent on the square of the diameter. Consequently, I remain unconvinced that equation (27) expresses the time relationship between relative jump height and shear velocity for different grain sizes.

Further light, however, is shed on this problem by an undergraduate project conducted in the Earth Science Department, University of Leeds by Pickup (1980). This experiment used glass beads of diameter, D = 1.4 mm and thus provides some evidence for the variation in grain size, although too much weight cannot be placed upon the accuracy of these data. The figures are presented in Table VI.

The values of transport stage quoted go far above the limit of 2.3 set by Abbott and Francis (1977; Figure 3) in their definition of saltation. However, Leeder (1979, page 233) is of the opinion that Abbott and Francis's suspension trajectories are not true suspensions and should be treated along with the saltation data. They are after all simply saltation jumps with slight upwards deviations on the trajectories (see Abbott and Francis 1977; Figure 16, page 248). The relation found for Pickup's data is:

$$\frac{H}{D}_{max} = 1.197 \frac{u_*}{u_{*o}} - 0.676 \pm 0.615 \qquad (28)$$

$$N = 16 \; ; \; R = 0.747$$

This relation implies that the two observations made in the previous paragraph regarding the regression coefficients do not hold.

However, Figure 6 on which both sets of data are plotted as <u>relative</u> jump height versus transport stage suggests the possibility of a single quadratic curve fitted through the data and shown in Figure 7 viz:

$$\frac{H}{D}_{max} = 0.251 \left(\frac{u_*}{u_{*o}}\right)^2 + 0.663 \pm 0.509 \qquad (29)$$

$$N = 24 \; ; \; R = 0.804$$

<u>or</u> $$\frac{H}{D}_{max} = 0.43 \frac{u_*^2}{\left(\frac{\rho_s - \rho}{\rho}\right) D} + 0.663 \qquad (30)$$

<u>or</u> $$\frac{H}{D}_{max} = 0.012 \frac{u_y^2}{\left(\frac{\rho_s - \rho}{\rho}\right) D} + 0.663 \qquad (31)$$

In this instance, plotting actual jump height against shear velocity, only the intercept is grain size dependent. Furthermore, the variation in jump height is dependent on the square of the instantaneous velocity fluctuation thus providing the slight skew seen in Figure 4b. This velocity fluctuation argument, also supports the observed increase in scatter with increasing stage for a single grain size in Figures 6 and 7.

Consequently, in the absence of further information, equation (30) has been adopted to describe the mean jump height and equation (31) coupled with equation (26) to calculate the variance of the jump height.

It is now necessary to go a stage further in describing particle trajectories and to comment on the length of saltation jumps. Abbott and Francis (1977) provide a number of observations relating to jump length and transport stage (Figure 4, page 232; Figure 5, page 233) and to the geometrical relation between jump length and jump height (Figure 8, page 234). These results are reproduced here as Figure 8 a, b, c, d. As a whole, the data appear somewhat equivocal as to the type of relation needed to predict trajectory lengths. In the absence of further information, an approach similar to that developed for jump height has been followed through to its consequences. Data abstracted from Figure 8b (Abbott and Francis 1977; Figure 4, page 232) are presented in Table VII and the following regression line fitted:

$$\frac{L}{D} = 2.5\left(\frac{u_{*}}{u_{*o}}\right)^2 + 3.1 \pm 1.1 \qquad (32)$$

$$N = 10 \quad ; \quad R = 0.99$$

This implies the following relations:

$$\frac{L}{D} = 4.3 \frac{u_*^2}{\frac{(\rho_s - \rho)}{\rho}D} + 3.1 \qquad (33)$$

$$\frac{L}{D} = 0.12 \frac{u_y^2}{\frac{(\rho_s - \rho)}{\rho}D} + 3.1 \qquad (34)$$

and

$$\frac{L}{D} = 10.0 \frac{H}{D} - 3.5 \qquad (35)$$

Equation (35) agrees remarkably well with Abbott and Francis's data (1977; Figure 8b - reproduced here as Figure 8d) for saltant trajectories at a transport stage of 1.53, with most of the "suspensive" trajectories lying to the right of this line. Figure 8c shows a similar plot for a transport stage of 2.52. Here most of the data relate to "suspensive" trajectories and given the small number of data points, it is not impossible to suggest that even here equation (35) might form an envelope curve to true saltations. Consequently, if it is assumed that for truly saltating trajectories geometric similarity is preserved as in equation (35), it simply remains to incorporate the extra variance in trajectory length via the probability of a disturbance in the trajectory as shown in Figure 8e and f (Abbott and Francis 1977; Figure 16, page 248).

Essentially disturbances to a ballistic trajectory occur when the flight of the grain is interrupted by the "instantaneous" lift force momentarily supporting the grain in motion. Discussions in the previous section, however, have revealed a large number of problems associated with defining this lift force. An alternative approach has, therefore, been adopted using the data for percentage suspensive trajectories versus stage given by Abbott and Francis (1977; Figure 1, page 230) - here reproduced as Figure 9.

Consider a grain of diameter 0.00828 m and specific weight 1240 kg/m^3 at a transport stage of $\frac{u_*}{u_{*o}} = 2$. The standard path of motion for this grain is given by equations (29) and (32) as $H_{max} = 1.7$ D and $L = 13.1$ D. Equation (8a) gives a grain speed of $U = 1.8$ m/s. Instantaneous velocity distributions on which discussions of lift are based operate on a time scale of 0.01 seconds (McQuivey, 1973a; page A41). There is, therefore, the opportunity for a maximum of 6 uplifts during the characteristic trajectory. The probability of at least one uplift is given by:

$$8 = (1 - (1 - p)^6) \qquad (36)$$

where p is the probability of an uplift occurring

Referring to Figure 9, at a transport stage of 2 some 45% of trajectories are suspensive in Abbott and Francis's terminology, i.e. 45% trajectories contain at least one uplift. Thus, in equation (36) $8 = 0.45$ which implies $p = 0.09$. In referring this probability of uplift back to the velocity distribution, it has been decided to use the distribution at height $y = 0.35$ D above the theoretical bed. This ties up with the use of bed shear velocity in the trajectory and speed equations. The standard deviation of vertical velocity fluctuations at this height is given by Naden (1981a, equations 23 and 32) as:

$$\sigma_v = 0.24\ \bar{u}_y \quad \underline{\text{or}} \quad \sigma_v = 1.4\ u_* \tag{37}$$

For the above probability $p = 0.09$ and the given u_* of 0.068 m/s, a vertical velocity of 0.13 m/s is, therefore, required to produce an uplift. This should be balanced against the gravity force by analogy with equation (13) as follows:

$$v = \sqrt{\left(k\ \frac{(\rho_s - \rho)}{\rho}\ D\right)} \tag{38}$$

where v is the vertical velocity of the water
and k is a constant

This means that $k = 8.5$. The validity of this value of k can be checked by working backwards through the equations (36) to (38) for a transport stage of one and comparing this with Figure 9. A velocity of 0.13 m/s is required for uplift and, for a transport stage of 1, this occurs with a probability of 0.0035. The characteristic trajectory at this stage can be described as $H_{max} = 0.9\ D$; $L = 5.6\ D$; $U = 0.29$ m/s. There is, therefore, the opportunity for 5 uplift events and the probability of at least one occurring is 0.02. This suggests that some 2% of trajectories are suspensive; a value at least consistent with Figure 9.

This approach to correcting for suspensive trajectories would also appear to be consistent with the meagre data of Figure 8c. The range of length values should follow a binomial distribution with the probability of m uplifts in a total of n possible uplifts being given by $p^m\ q^{(n-m)}$ where $q = (1 - p)$. The expression for trajectory length thus becomes:

$$\frac{L}{D} = 10.0\ \frac{H}{D} - 3.5 + mU \tag{39}$$

where m is number of uplifts
U is grain speed

(3) Probability of deposition

The probability of deposition of a grain depends upon the frequency of contact with the bed and upon the nature of that contact. The question of frequency of contact has been dealt with in the previous section on trajectory dimensions in conjunction with the work on downstream grain speeds. It, therefore, only remains to comment on the nature of the bed contact.

Previous discussions on the nature of saltation (Bagnold 1973; page 477 and Gordon, Carmichael and Isackson, 1972) have centred around the question of angle of contact with the bed and consequently the conservation of tangential momentum. However, the more recent work of Abbott and Francis (1977) using more detailed recording of trajectories reveals two observations which contradict this interpretation viz:

> "it would appear that the trajectory velocities and heights are insensitive to the previous impact history, and it can be inferred that there is no effective elastic rebound between the bed and a moving grain impinging upon it ...... It follows that momentum is not conserved at such collisions : the external (fluid) forces acting on the moving grain must be large compared to the internal, impulsive forces developed by the collision, the latter being effectively damped." (page 244)

> "previous ideas on saltation have always assumed, from eye observation only, that a grain on impacting a bed immediately starts a new trajectory. The photographic evidence shows otherwise." (page 244)

Thus, it would appear that it is appropriate simply to assume the momentary deposition of a moving particle at the end of a trajectory and simply to refer to the equations for the initiation of particle motion to determine whether or not the particle remains in motion (Naden 1981a).

III. Conclusion

A strategy for the prediction of individual grain motions has been outlined above. The relations put forward require the specification of just a single parameter of the flow - the bed shear velocity - and two grain parameters - the diameter of bed and mobile grains and their specific weight. This is obviously a very simplified view of sediment movement but one which is supported by some theoretical statements and much of the evidence from single grain experiments. Two points, however, bear brief mention.

Firstly, there is the question as to the applicability of this work to gravel bed rivers in the field. Abbott and Francis's (1977) experiments are not scale models of a particular field situation and, as they acknowledge, although the relations may be transferred, the actual magnitudes of the predictions may not. Indeed, sediment tracing experiments such as those of Laronne and Carson (1976) do support the general form of the above analysis when taken in conjunction with thresholds of motion (Naden, 1981a). In particular, distance travelled shows an exponential decline with grain size (Figure 12, page 83) but is also dependent on the bed geometry where the grains come to rest (Figure 11, page 82). Precise testing of the magnitude of the predicted grain motions, however, requires a far more rigorous attempt at field experiments than has hitherto been achieved.

The second problem in using the individual grain data refers to the extension of this work to multiple grains in motion and the development of a general relation for total bed load transport. This must incorporate the ideas on grain collisions developed by Leeder (1979) as a mechanism for generating Bagnold's (1956) dispersive stress.

An analytical approach to both the problem of total sediment transport and the development of gravel bedforms does not look promising. Consequently, in order to take the work a stage further the relations developed above for individual grain movements are being incorporated into a computer simulation model of sediment transport. This can then be used to analyse the sensitivity of the results to particular parameters and derive some specific hypotheses for field testing.

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Table I : Data on downstream particle velocities taken from Meland and Norrman (1966) and Francis (1973)

	Observed particle velocity $\bar{U}$ (m/s)	Bed shear velocity u_* (m/s)	Fall velocity V_g (m/s)
Francis (1973)			
1	0.1709	0.0532	0.896
	0.2780	0.0627	0.896
	0.4170	0.0733	0.896
	0.5300	0.0815	0.896
	0.6250	0.0939	0.896
2	0.3430	0.0532	0.628
	0.459	0.0627	0.628
	0.590	0.0733	0.628
	0.680	0.0815	0.628
	0.761	0.0939	0.628
	0.181	0.0434	0.628
3	0.528	0.0532	0.289
	0.600	0.0627	0.289
	0.701	0.0733	0.289
	0.783	0.0815	0.289
	0.842	0.0939	0.289
	0.403	0.0434	0.289
	0.270	0.0322	0.289
	0.519	0.0521	0.289
	0.478	0.0495	0.289
4	0.1043	0.0245	0.582
	0.172	0.0337	0.582
	0.254	0.0394	0.582
	0.546	0.0611	0.582
	0.477	0.0542	0.582
	0.385	0.0471	0.582
	0.069	0.0263	0.582
5	0.309	0.0245	0.187
	0.363	0.0337	0.187
	0.415	0.0394	0.187
	0.703	0.0611	0.187
	0.609	0.0542	0.187
6	0.293	0.0337	0.303
	0.377	0.0394	0.303
	0.664	0.0611	0.303
	0.578	0.0542	0.303
	0.187	0.0263	0.303
Meland and Norrman (1966)			
7	0.542	0.0689	0.324
	0.440	0.0571	0.324
	0.367	0.0488	0.324
	0.300	0.0448	0.324
	0.260	0.0413	0.324
	0.211	0.0368	0.324
	0.125	0.0282	0.324
	0.052	0.0202	0.324
	0.021	0.0172	0.324

Table I (continued)

	Observed particle velocity $\bar{U}$ (m/s)	Bed shear velocity u_* (m/s)	Fall velocity Vg (m/s)
8	0.510	0.1108	0.615
	0.395	0.0928	0.615
	0.273	0.0815	0.615
	0.224	0.0748	0.615
	0.186	0.0696	0.615
	0.092	0.0636	0.615

Table II : Statistical analysis of particle velocity data

Francis (1973) Appendix 2, page 470
Meland and Norrman (1966) Figure 4, page 174 and Table I, page 168

Independent variable is u_* ; dependent variable is $\bar{U}$

Data set	N	R	b	a	S.E. est.
1. Francis (M) Vg = 0.896 D = 0.0159	5	0.995	11.52	- 0.44	± 0.02
2. Francis (M) Vg = 0.628 D = 0.0159	6	0.990	11.57	- 0.28	± 0.03
3. Francis (M) Vg = 0.289 D = 0.0159	9	0.991	9.27	+ 0.01	± 0.03
4. Francis (N) Vg = 0.582 D = 0.00633	7	0.993	13.30	- 0.26	± 0.02
5. Francis (N) Vg = 0.187 D = 0.00633	5	0.989	11.15	+ 0.01	± 0.03
6. Francis (N) Vg = 0.303 D = 0.00633	5	1.000	13.73	- 0.17	± 0.01
7. Meland and Norrman Vg = 0.324 D = 0.00209	9	0.998	10.34	- 0.16	± 0.01
* 8. Meland and Norrman Vg = 0.615 D = 0.00776 (Vg' = 1.01)	6	0.992	8.65	- 0.43	± 0.02

* Data set which does not appear to fit in Figure 3

Table III : Speed of motion in mixed grain sizes - data of Meland and Norrman (1966)

Data set	N	R	b	a	S.E. est.	Calc α
Series A	9	0.997	10.1	- 0.135	± 0.015	5^{o}
grains twice the size of bed grains						
Vg = 0.44 Dg = 0.00393 Db = 0.00209						
Series B	6	0.983	8.99	- 0.511	± 0.032	53^{o}
grains half the size of bed grains						
Vg = 0.44 Dg = 0.00393 Db = 0.00776						
Series B adjusted for log flow law						
Dg^{1} = 0.0211 Vg^{1} = 0.72						27^{o}

Dg is size of bed grain; Db is size of moving grain

Table IV : Dynamic friction angles adopted

Ratio of moving grain size to bed grain size	Dynamic angle of friction
1 : 2	27^{o}
1 : 1	11^{o}
2 : 1	5^{o}

Table V : Data on particle trajectories - observed and calculated

Table Va : Data extracted from Figure 1a
D = 0.00828

Observed height of jump (m)	ρ_s (kg/m^3)	Stage $\frac{u_{*}}{u_{*o}}$
0.0064	1800	0.9
0.0080	1240	1.0
0.0084	2570	1.0
0.0088	1800	1.3
0.0096	1430	1.4
0.0102	1240	1.6
0.0126	1240	1.9
0.0134	1430	2.0

Table Vb : Calculated information

$\bar{u}_y (y = 0.35\ D)$ (m/s)	σ_u (m/s)	Threshold u (m/s)	Mean u for jump (m/s)	Predicted H (m)
0.33	0.11	0.69	0.70	0.0004
0.20	0.06	0.38	0.39	0.0009
0.51	0.16	0.97	1.01	0.0027
0.48	0.15	0.69	0.74	0.0032
0.38	0.12	0.51	0.55	0.0031
0.32	0.10	0.38	0.43	0.0042
0.38	0.12	0.38	0.46	0.0065
0.54	0.17	0.51	0.64	0.0089

Table VI : Data on saltation heights taken from Pickup (1980)
D = 0.0014 mm

Jump height (m)	$\frac{H}{D}_{max}$	Stage $\frac{u_*}{u_{*o}}$
0.005	3.6	2.52
0.004	2.9	2.63
0.0025	1.8	2.34
0.003	2.1	3.30
0.0025	1.8	1.93
0.0055	3.9	3.19
0.002	1.4	1.67
0.003	2.1	2.34
0.0035	2.5	2.11
0.0018	1.3	1.71
0.0015	1.1	2.15
0.0013	0.9	1.22
0.002	1.4	2.00
0.0035	2.5	2.71
0.0015	1.1	1.67
0.002	1.4	2.11

Table VII : Data from Abbott and Francis (1977; Figure 5, page 233) appertaining to jump length for a grain size of
D = 8.28 mm

Observed jump length (L) (m)	Relative jump length (L/D)	Transport stage (u_*/u_{*o})	Specific gravity $\rho_s (kg/m^3)$
0.048	5.8	0.9	1800
0.032	3.9	1.0	1240
0.056	6.8	1.1	2570
0.060	7.2	1.3	1800
0.068	8.2	1.4	1430
0.076	9.2	1.5	1240
0.096	11.6	1.9	1240
0.160	19.3	2.5	1240
0.182	22.0	2.7	1240
0.138	16.7	2.5	1240

Figure 1 : Regression analysis of particle velocity versus bed shear velocity - data of Meland and Norrman (1966) and Francis (1973)

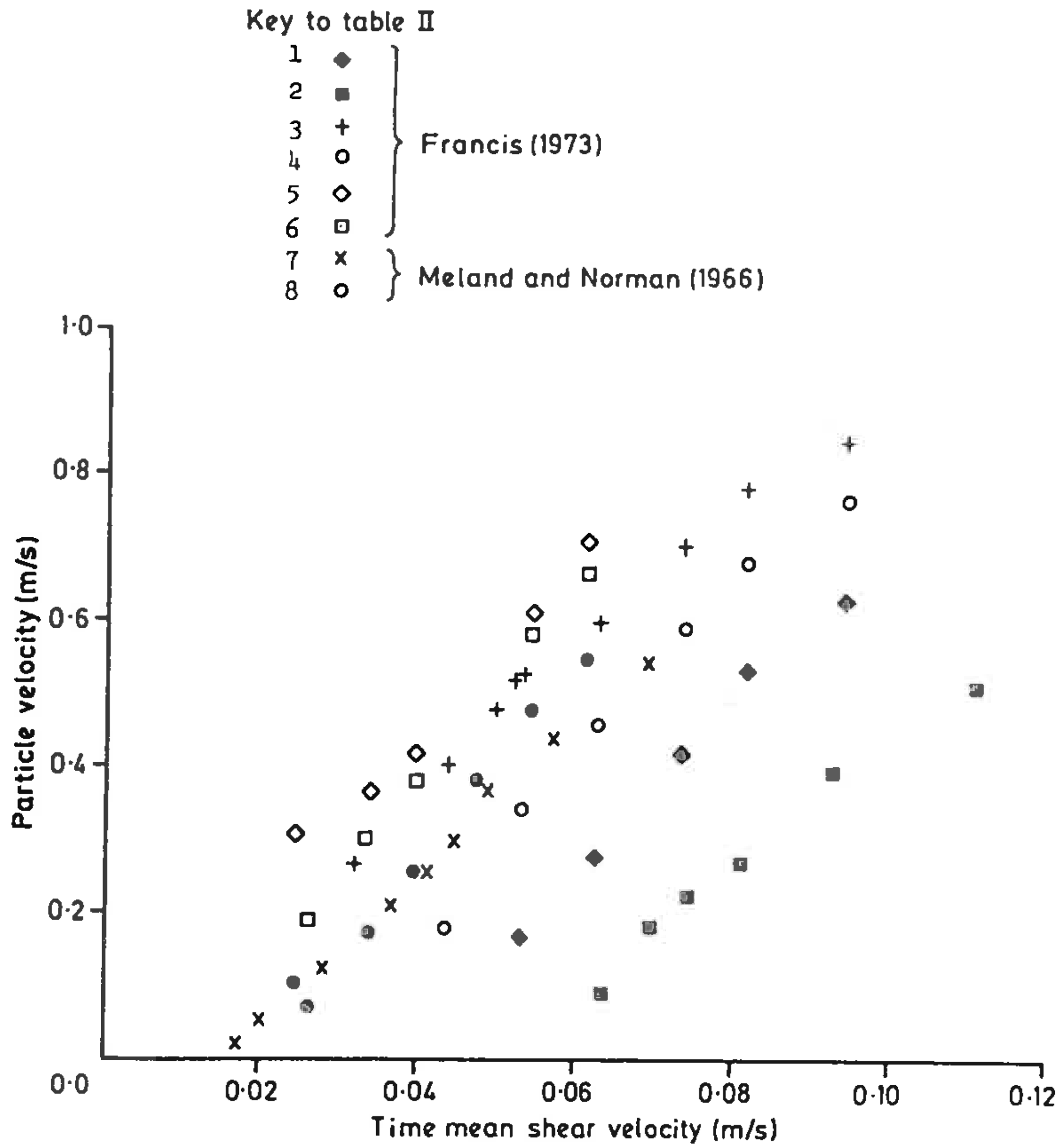


Figure 2 : Regression analysis of $\frac{U}{V_g}$ against $\frac{u_*}{V_g}$

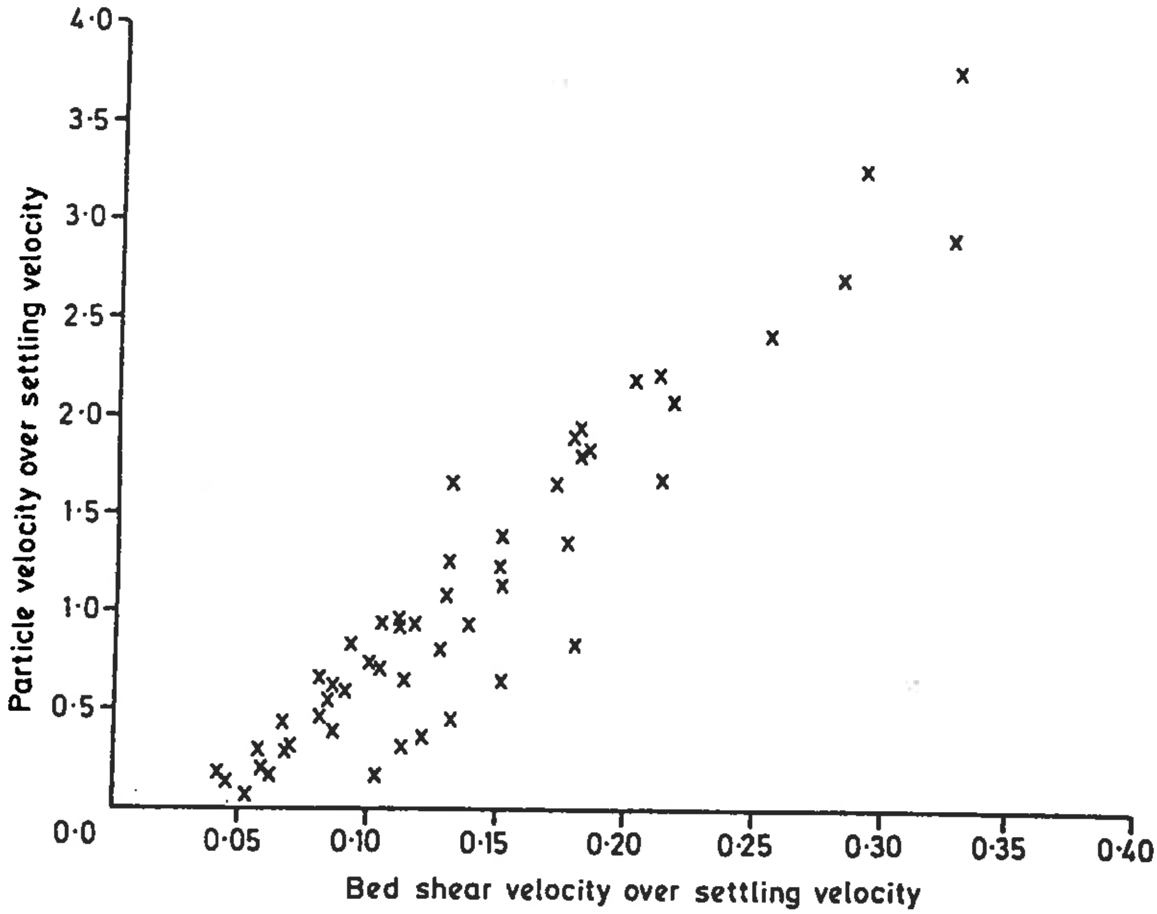


Figure 3 : Predicted velocities (equation 8) versus observed velocities

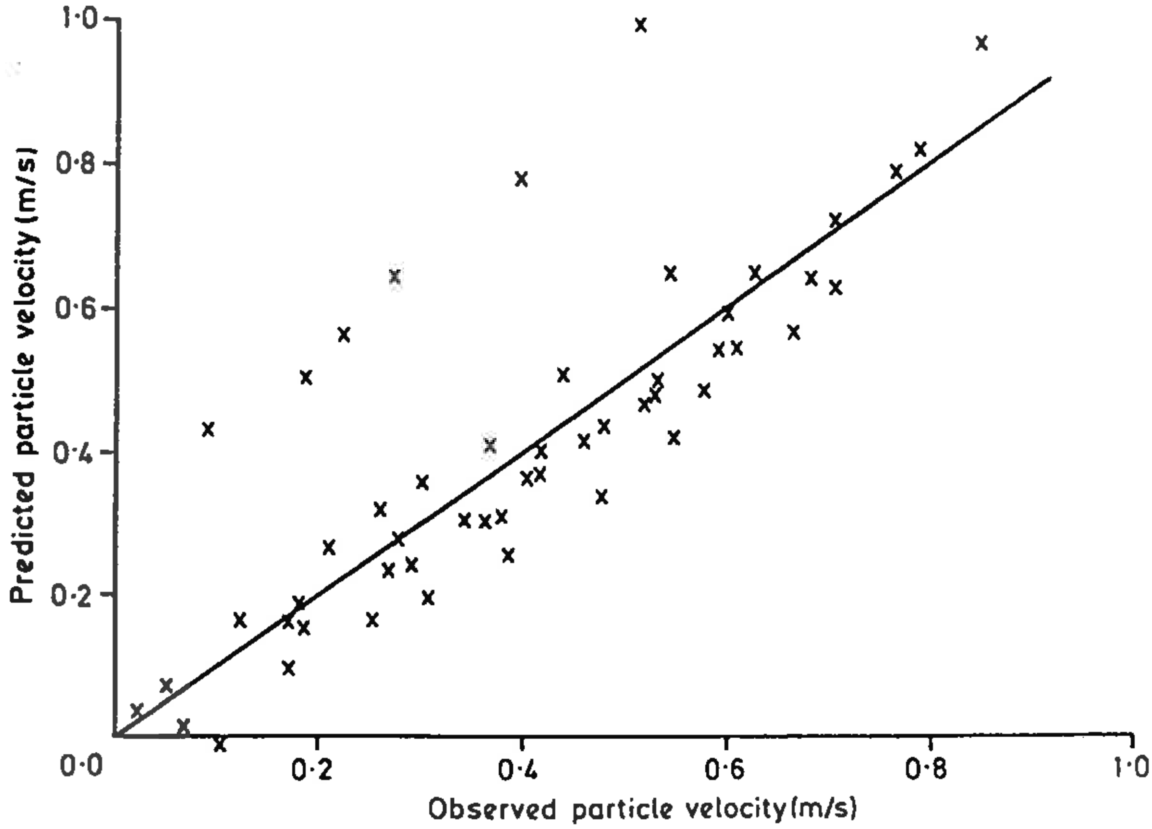


Figure 4 : Empirical data on particle trajectories - Abbott and Francis (1977)

Fig. 4a

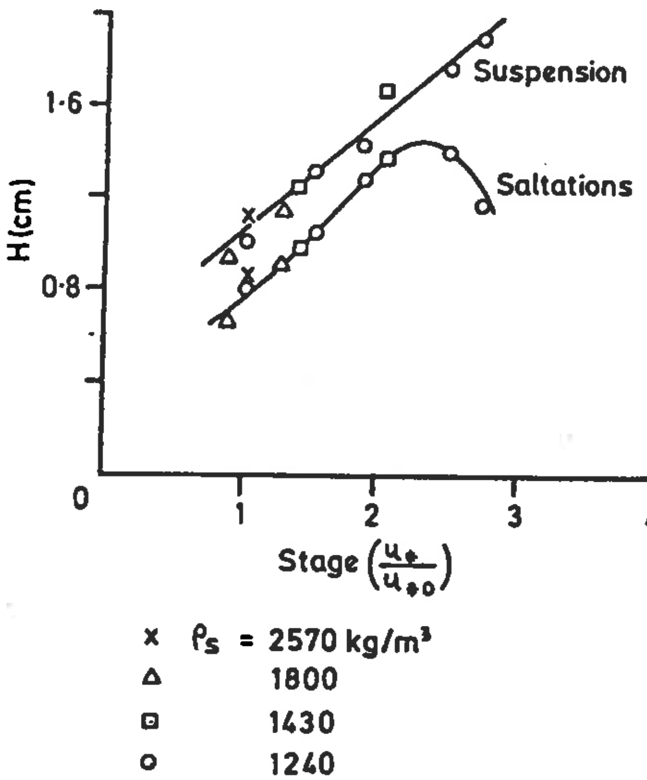
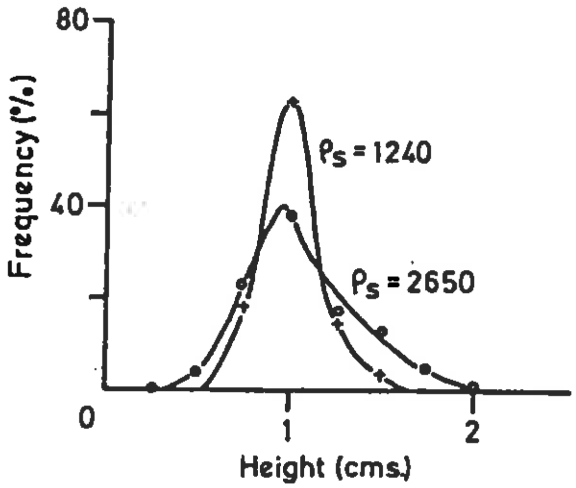


Fig. 4b



ρ_s is particle density in kg/m³
(At a low value of stage)

Figure 5 : Predicted versus observed height of saltation

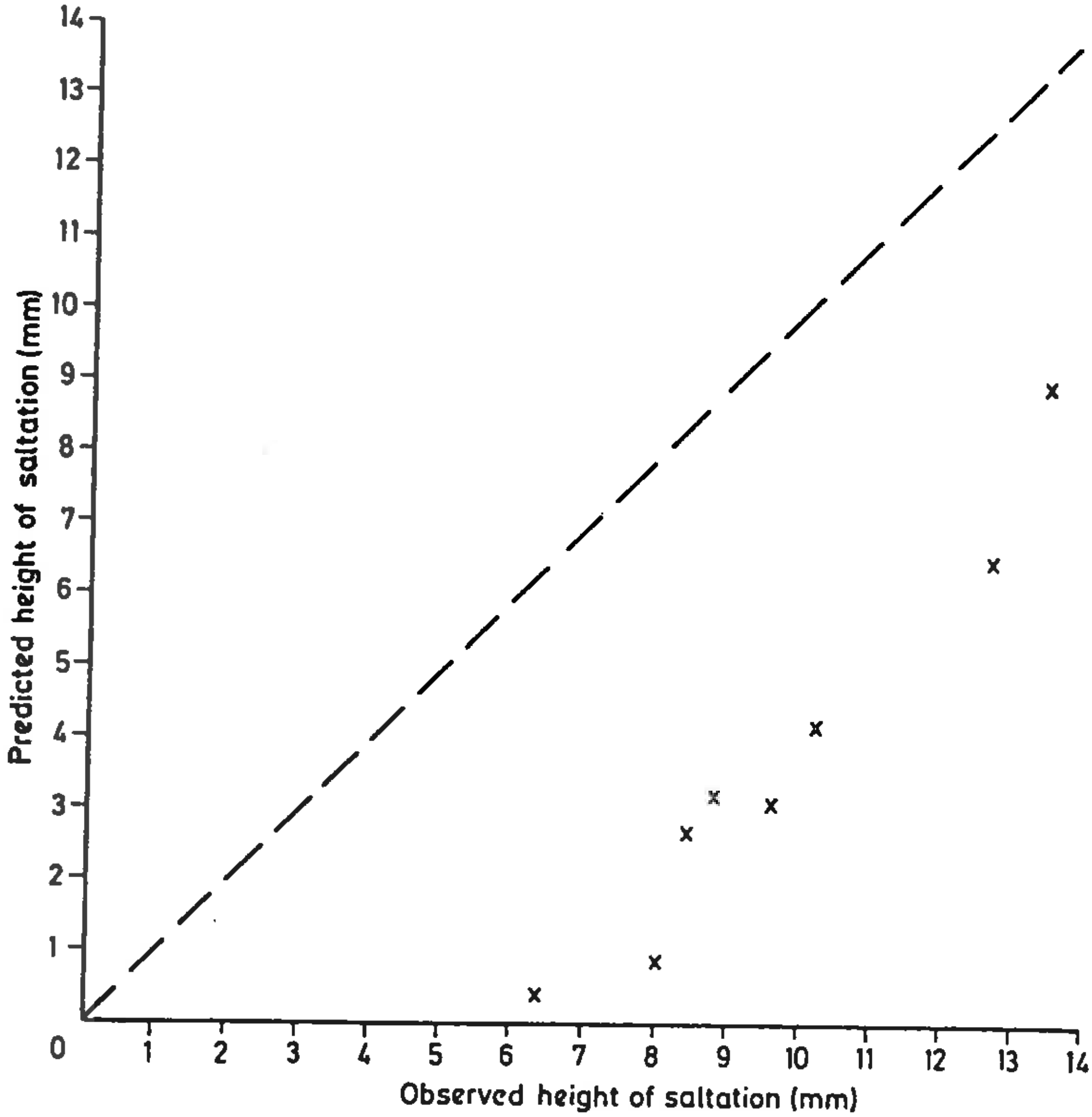


Figure 6 : Saltation trajectory heights versus transport stage

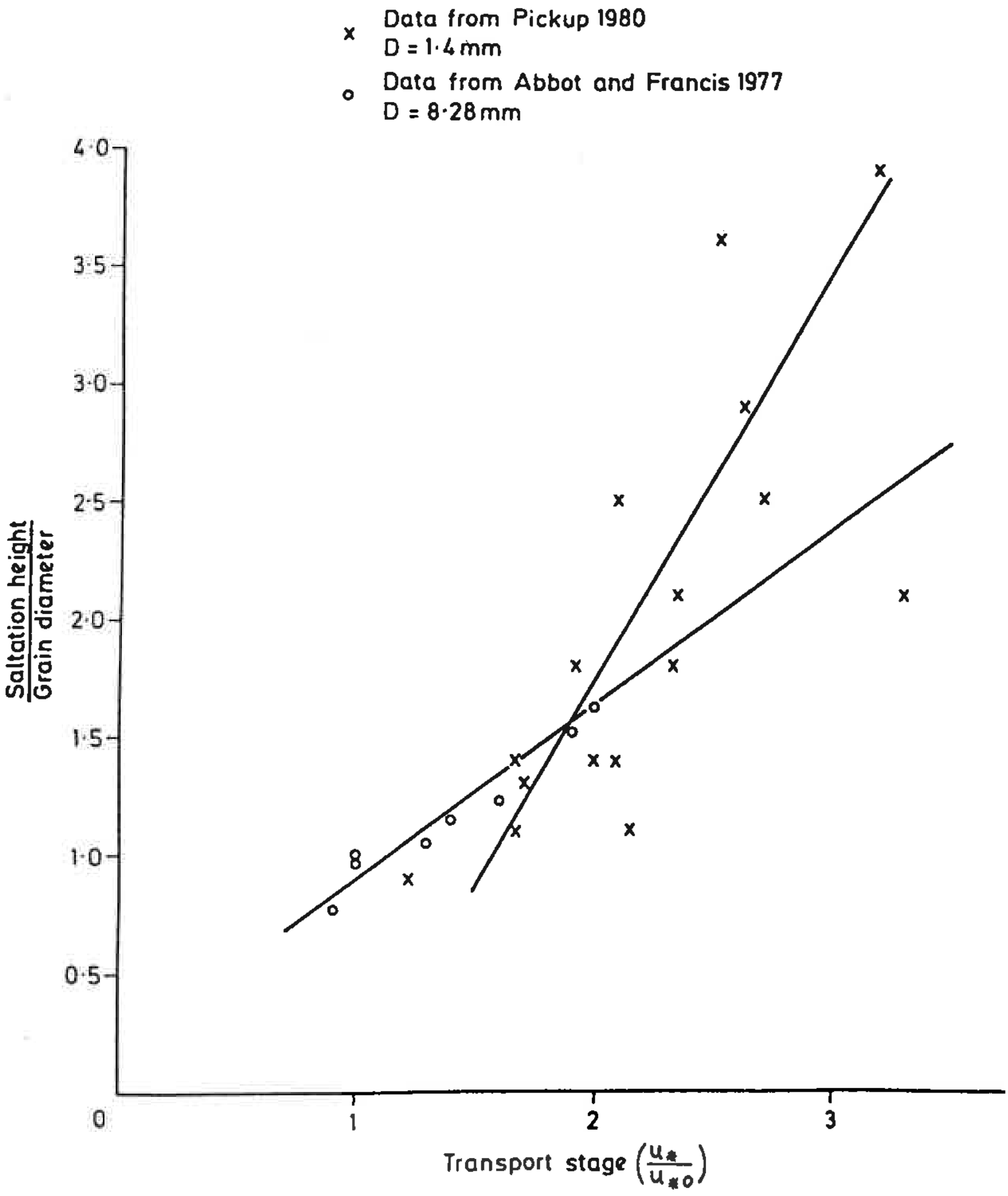


Figure 7 : Relative jump height as a function of the square of transport stage - data from Abbott and Francis (1977) and Pickup (1980)

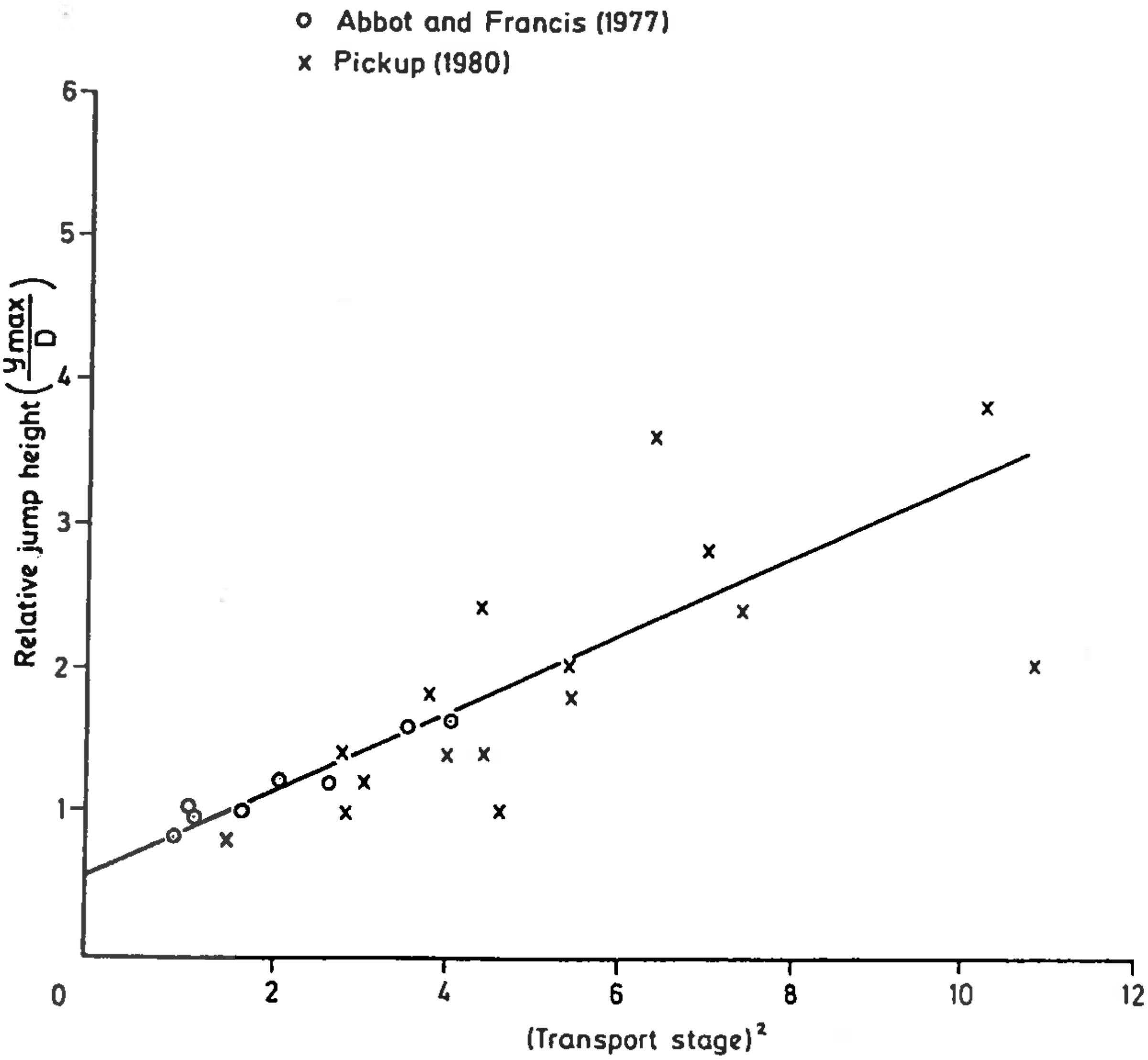
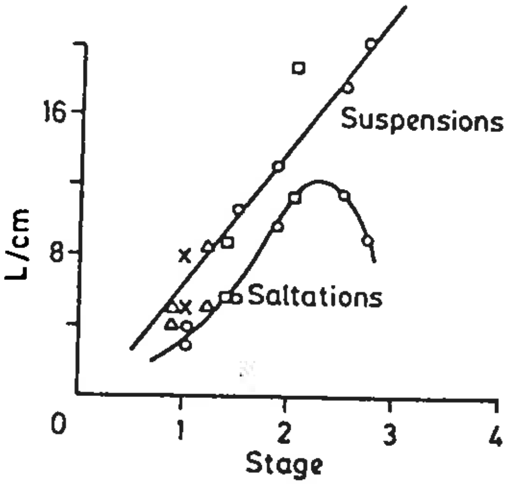
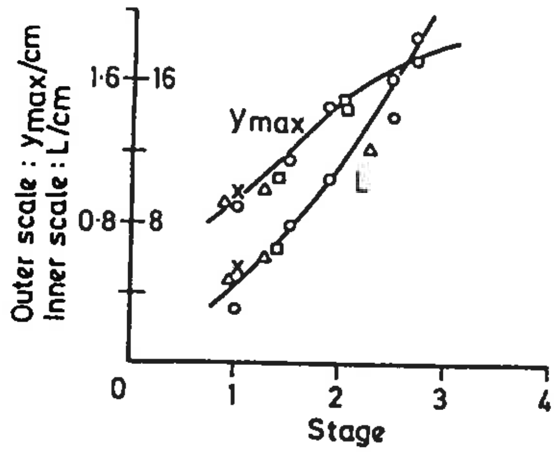


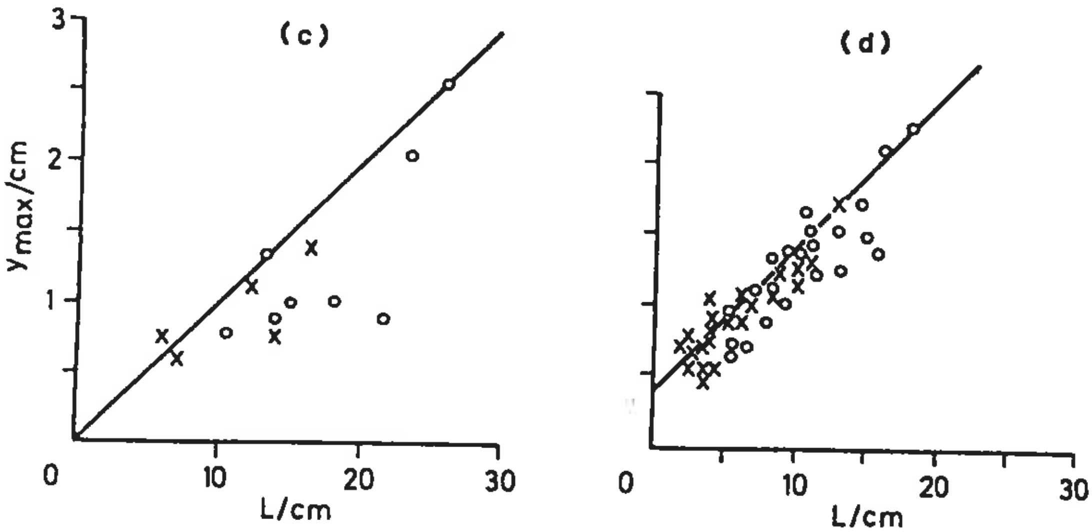
Figure 8 : Data from Abbott and Francis (1977) referring to the length of trajectory jumps



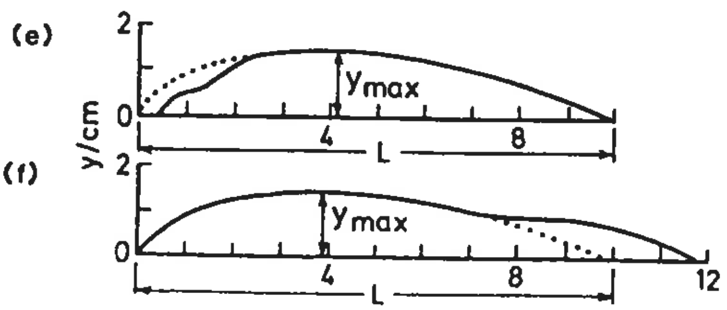
(a) The mean length $\bar{L}$ of trajectories subdivided into saltations(lower curve)+ suspended motions. Water depth 4.8cm; grains of set 2 only. Information from 385 photographs, 371 saltations & 171 suspensive trajectories



(b) Average heights y_{max} and lengths L of all trajectories from 385 photographs including 716 trajectories for height + 542 for length. Stream depth 4.8cm; set 2 only



Trajectory dimensions at two different stages for set 2 grains, circles being suspensions, crosses being saltations. (c) u_*/u_{*0} 2.52 , Information from 14 trajectories; (d) $u_*/u_{*0} = 1.53$; 45 trajectories



Comparison of two types of suspensive trajectories. (e) Impulse received before the crest. (f) Impulse received after the crest in each case the dotted line indicates a typical saltation of the same y_{max} which would have occured if the impulse had not been present

Figure 9 : Modes of motion versus stage - Abbott and Francis (1977)