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AGGREGATE POPULATION ACCOUNTS FOR THE DISTRICTS OF THE WEST YORKSHIRE METROPOLITAN COUNTY, PART I, 1961-1966.

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ABSTRACT

This two part paper describes the construction of four sets of aggregate closed demographic accounts. Two regional systems are used. The main set of regions consists of eight areas, these being the five metropolitan districts of the West Yorkshire Metropolitan County, plus three residual regions. (such as the rest of the world for example). The other set of regions contains fifty eight areas, these being basically the set of preexisting local authorities which can be aggregated to form the metropolitan districts, plus Harrogate MB and Knaresborough UD, and three residual regions. For each of the two regional systems used, aggregate accounts are constructed for two intercensal periods, 1961-1966, and 1966-1971. Full sets of accounts cannot be presented for the second set of regions, as these would occupy too much space. Some results for the second set of areas are nevertheless presented. A prerequisite of the accounts for the period 1966-1971 was the estimation of migrant flows from England and Wales to the rest of the world. The methods by which these estimates were obtained are described, see Part II.

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Part I (1961-1966)

1. INTRODUCTION

1.1 Purposes and Aims

This paper describes the construction of sets of aggregate closed demographic accounts for two sets of regions over two intercensal periods. Part I deals with the first period, and Part II the second. Part II will be bound in separate covers. The method of accounting used, and some of the estimation techniques employed have been described in greater detail elsewhere, in Rees and Wilson (1973 and 1975).

The main set of regions that the accounts concern are the five metropolitan districts of the West Yorkshire Metropolitan County. Accounts are also constructed for the set of old authorities which fall either wholly or partly into the new county. These authorities form the second main set of regions. Various other regions are included with the main sets, or are used incidentally. Section 1.2 contains a full description of the main sets, and of all the other regions used, so reference should be made to that.

Accounts are constructed for each main set of regions for two periods. Thus four sets of accounts will be estimated. The periods are the two between the last three census nights. These periods are from midnight 23/24th April 1961 to midnight 24/25th April 1966, and from the latter to midnight 25/26th April 1971. Section 2 discusses the construction of accounts for the first period, 1961-66, and Section 3 in Part II, the second, 1966-71. Both sections include a selection of the results.

The set of accounts for the period 1961-66 for the five metropolitan districts have already been used in another paper, Illingworth (1975B). In this paper they were disaggregated by sex, and then the males were disaggregated by socioeconomic group, both processes being performed by a balancing factor estimation technique. It is hoped that the accounts presented here will be used in predicting accounts for the period 1971 to 1976 and for further five year periods. One of the more important outputs of such a set of accounts would be the 1976 populations for the two main sets of regions. These may be of interest to planners, especially as no census will be undertaken in that year.

1.2 Regions used

Two main sets of regions were used in this paper. To define the first main set, we need to name three other areas. These are:

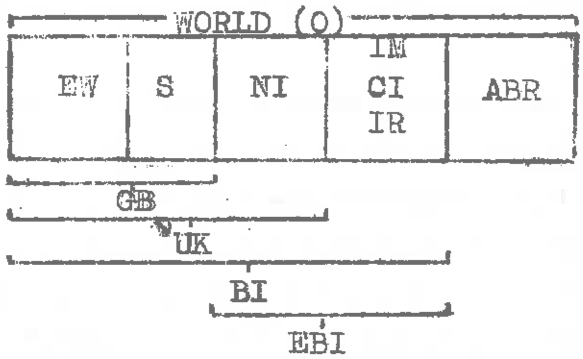
Notation name used for this area	Name of the area
WR	The West Riding of Yorkshire, AC+CB's Note that this area undergoes boundary changes in 1967 and 1968, when various areas are added to it. The accounts for the second period, 1966-71, use this enlarged area. This affects the definitions of the rest of the West Riding regions, RWR and WR-55 used. The definition of the rest of England and Wales, REW also alters. See later in this section for details.
EW	England and Wales
O	The world (globe)

The main set contains the eight areas listed in Figure 1. This set contains the five metropolitan districts of the new West Yorkshire metropolitan county, and three 'residual' regions. An exact definition of the new metropolitan districts in terms of the pre existing local authorities and a map of the areas is given in Illingworth Smith and Rees (1974, Table 4 and Figure 4).

The second main set of areas studied contains the areas listed in Figure 2. This set contains 55 local authorities from the West Riding, and three residual regions. The 55 local authorities include all authorities which now fall either wholly or partly into the new metropolitan county. Two other authorities, Harrogate (19) and Knaresborough (28) are included because they were included in the West Yorkshire Study Area, widely used in research at the School of Geography, Leeds. A map of all these 55 authorities is available in Illingworth Smith and Rees (1974, Figure 4). Three residual regions are also included in the second main set of regions studied. Two of these, REW and RW are identical to those used in the first main set, described in Figure 1. The remaining residual, WR-55, is the West Riding minus the 55 local

Figure 1 The areas used in the five metropolitan districts accounts

Notation name used for this area.	Name of the area
	The metropolitan districts of the new West Yorkshire metropolitan county
W	Wakefield
K	Kirklees (contains Dewsbury and Huddersfield CB's)
C	Calderdale (contains Halifax CB)
L	Leeds
B	Bradford
	Three residual regions
RWR	The rest of the West Riding WR-(W+K+C+L+B)
REW	The rest of England and Wales EW-WR
RW	The rest of the World O-EW



authorities listed in Figure 2. This area is not therefore the same as the RWR region used in Figure 1. In notation terms this area can be expressed as follows: 55LAS = LA1 + LA2 + LA3.....+LA55

WR-55 = WR-55LAS

Various other areas will be used in estimating the data for the amounts. These are:

Notation name used for this area	Name of the area
S	Scotland
GB	Great Britain GB=EW+S
NI	Northern Ireland
UK	United Kingdom UK=EW+S+NI
IR	Irish Republic
IM	Isle of Man
CI	Channel Islands
EBI	Elsewhere in the British Isles EBI=NI+IM+CI+IR
BI	British Isles BI=EW+S+EBI
ABR	Abroad O- BI

(SEE P. 3) FOOT OF

1.2.1 Boundary changes

The problem of boundary changes is one commonly experienced during the study of areas over a time period. This section deals with this problem in the regions that have just been listed and defined. Only changes with affect the borders of these regions need to be studied, changes in internal boundaries can be ignored. Also, as we are concerned with population, only those changes which reallocate population need concern us. As an example of changes that can be ignored, part of the York order of 1967 transfers two unpopulated a cre s of H e worth ward, York, to Flaxton RD in the N. Riding. This is in fact a transfer from the rest of the W. Riding regions, RWR and WR-55, to the rest of Endland and Wales, REW. However, as the change is not significant in population terms, as the area transferred was unpopulated, so it can be ignored.

During the ten year period studied here there are relatively few changes in the boundaries of the areas used. In fact, the only area affected by significant change was the West Riding region, WR. The changes in the boundary of this area took place in the second period, 1966-71. Details of these changes are given in OPCS (1973, Table 4). The information is also repeated in GRO (Annual, Appendix A) and OPCS (Annual, Appendix A) in a more digestible form. The significant changes were as follows:

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Figure 2. The areas used in the 55 local authorities accounts

Notation Name	Name of Area	Notation Name	Name of Area
LA1	Bradford CB	LA31	Mirfield UD
LA2	Dewsbury CB	LA32	Morley MB
LA3	Halifax CB	LA33	Normanton UD
LA4	Huddersfield CB	LA34	Ossett MB
LA5	Leeds CB	LA35	Otley UD
LA6	Wakefield CB	LA36	Pontefract MB
LA7	Aireborough UD	LA37	Pudsey MB
LA8	Baildon UD	LA38	Queensbury and Shelf UD
LA9	Batley MB	LA39	Ripponden UD
LA10	Bingley UD	LA40	Rothwell UD
LA11	Brighouse MB	LA41	Shipley UD
LA12	Castleford MB	LA42	Sowerby Bridge UD
LA13	Colne Valley UD	LA43	Spenborough MB
LA14	Denby Dale UD	LA44	Stanley UD
LA15	Denholme UD	LA45	Todmorden MB
LA16	Elland UD	LA46	Hepton RD
LA17	Featherstone UD	LA47	Osgoldcross RD
LA18	Garforth UD	LA48	Tadcaster RD
LA19	Harrogate MB	LA49	Wakefield RD
LA20	Hebden Royd UD	LA50	Wetherby RD
LA21	Heckmondwike UD	LA51	Wharfedale RD
LA22	Holmfirth UD	LA52	Hemsworth UD
LA23	Horbury UD	LA53	Silsden UD
LA24	Horsforth UD	LA54	Hemsworth RD
LA25	Ilkley UD	LA55	Skipton RD
LA26	Keighley MB		
LA27	Kirkburton UD	Three residual areas	
LA28	Knaresborough UD	WR-55	West Riding - the 55 LA's
LA29	Knottingley UD	REW	Rest of Endland and Wales
LA30	Meltham UD	RW	Rest of the World

The Sheffield Order, 1967 (Date of change 1.4.67)

Added to WR –	Enumerated population of area added, at 1961 Census
Parts of Beighton, Eckington, Holmesfield and Killarmarsh CP's from Chesterfield RD, Derbyshire	32224

The York Order, 1968 (Date of change 1.4.68)

Added to WR –	
Part of Derwent RD (part of Heslington CP) from Yorkshire E. Riding.	782
Part of Flaxton RD (Part of Clifton (without) Heworth (without), Huntington and Osbaldwick CP's) from Yorkshire N. Riding	2959
Total 1961 Enumerated populations of additions	35965

All the areas added are added to local authorities which fall into RWR and WR-SS. Most of the additions are made to Sheffield and York CB'S. The areas added therefore fell in REW before 1967 and in RWR and WR-55 after 1968. Thus the boundaries of the regions RWR, WR-SS and REW in the main sets of regions altered during 1966-71.

It might be possible to ignore these boundary changes, and hold the definition of the WR used constant throughout the whole ten years. Here however, it is easier to use the enlarged definition of the WR in the whole of the second time period, because this will minimise the amount of data correction that has to be done. Some of the statistics for the second period will still have to be corrected though. These corrections will be dealt with as the statistics are estimated, in Section 3 of this paper.

The definition of the WR used for the second period is not therefore the same as the one used for the first period, 1961-66. The residuals RWR and WR-55 in the second period refer to the areas called this in the first period, plus the areas added in the 1967 and 1968 boundary changes. The residual REW in the second period refers to the area called this in the first period, minus the areas involved in the boundary change.

2. 1961-1966

2.1 Data needed by the model

Aggregate demographic accounting is fully described in Rees and Wilson (1973). The estimation techniques needed to construct this type of accounts are also discussed. These estimation techniques do however assume that certain data is available for the given set of regions. This paper is mainly concerned with the problems involved in obtaining this data for the two main sets of regions just described in Section 1.2. Sections 2.2 and 2.4 show how the data was obtained for the two main sets of regions , for the period 1961-1966. The remaining parts of Section 2 relate to the accounts that can then be produced using Rees and Wilson's estimation methods.

The same estimation methods will be used in constructing accounts for the main sets of regions for 1966 to 1971. Similar data will therefore be required for this period. Estimation problems encountered in the second period are different, however, so these are dealt with separately, in Section 3.

Using the usual notation, the accounts to be estimated here for either period consist of the flows:

$$K^{ij},\ K^{i\delta(j)},\ K^{\beta(i)j} \text{ and } K^{\beta(i)\delta(j)}$$

where i and j are any regions in the main set used, and β and δ represent the states 'born in' and 'died in'. The precise way in which the notation will be used in this paper should be noted here. The use of i or j (or any letter) as a subscript without the β or δ terms, implies existence in that region at the beginning or end of the period. Thus for example, the term K^{ij} refers to surviving migrants, and surviving non migrants where i≠j. The K^{ij} term does not therefore refer to the whole of the accounts matrix, only the 'existed throughout the period' quadrant of it. An extended form of notation has recently been developed for the accounts. In this, the variables have the form:

$K^{\alpha(i)w(j)}$ in general, where

α, the initial states, are either ε- existence at t
or β- to be born in the period

w, the final states, are either σ - existence at t+T
or δ - died in the period

and i, j are the regions used, i=1,...., N; j=1, ..., N

This enhanced notation is valuable, but it tends to be tedious when used continually. In this paper therefore, it is only used where it simplifies matters, and it may only be used partially in some cases. For example, K^{ij}, as defined above and used in this paper, could be written as $K^{\varepsilon(i)\sigma(j)}$, using enhanced notations. As an example of partial use of the enhanced notation, $K^{i*(j)}$ will be used in this paper to represent the sum of $K^{i\delta(j)}$ and K^{ij} (the sum of $K^{\varepsilon(i)\delta(j)}$ and $K^{\varepsilon(i)\sigma(j)}$ using full enhanced notation).

The data required for the estimation procedure suggested by Rees and Wilson is as follows. Where i and j are any regions in the appropriate main set;

K^{i*}	-	The initial population of each area, this being its 1961 population in Section 2. This information is not required for the rest of the world region, RW.
K^{ij}, $i \neq j$	-	The number of migrants, who existed in i at the beginning of the period and who existed in j at the end of the period. These will be called the 'surviving migrants'. $i \neq j$
$K^{\beta(i)j}$, $i \neq j$	-	The number of persons who were born in i during the period who survived to the end of the period, and existed in j at that time. These will be called the 'infant surviving migrants'. $i \neq j$
$K^{\beta(i)*}$	-	The total births that occur in each area during the period. This information is not required for RW.
$K^{*\delta(j)}$	-	The total deaths that occur in each area during the period. This is not required for RW.

Once this data has been assembled, it can be processed using an existing computer programme, to produce the appropriate set of accounts. This programme is described in Rees and Wilson (1974).

2.2 Obtaining the data, 1961-66, for the 55 local authorities accounts

This section shows how the data for the fifty five local authorities (55LA's) set of accounts is obtained, for the period 1961-66. The 55LA's set of accounts is for the second main set of areas, as described in Section 1.2. This main set contains the fifty five local authorities, plus three residual regions, WR-55, REW and RW. The data for the second main set is collected first because when this data is aggregated is provides virtually all the data needed for the five metropolitan districts, or main set, of regions. This aggregation is described in Section 2.4.

An existing set of accounts is relevant to the estimation procedures here. These accounts deal with three regions, WR, REW and RW, for the same period, 1961-66. Two of these three regions, REW and RW, correspond exactly to those used in both of the two main sets here. These three region accounts can be obtained by aggregating the age and sex disaggregated accounts presented in Smith and Rees (1974). They are shown in Figure 3. These accounts are likely to be reliable because they are based on a disaggregated model, which tends to give better results. The data for the accounts to be estimated here will therefore be based upon these 'reliable' estimates.

Using the order established in Section 2.1, the data was estimated as follows:

2.2.1 Initial populations in 1961, for the 55 local authorities accounts (K^{i*})

The usually resident populations of the 55LA's at the 1961 census can be obtained from GRO(1964, Table 1). These figures can then be summed, and then deducted from the total population of the West Riding, WR. The latter can be obtained from the same source, or from Figure 3. This gives the population of WR minus the 55 authorities, WR-55. Hence:

$$K^{WR-55*} = K^{WR*} - \sum_{x=1}^{55} K^{x*}$$

$$= 1510994 \qquad (1)$$

Where x is used as a label for one of the 55 LA's.

Figure 3 Aggregate accounts for West Riding, rest of England and Wales and rest of the World, 1961-66

Initial state \ Final state		Survive in at census date 1966			Died 1961-66 in			Total
		WR	REW	RW	WR	REW	RW	
Exist in at census date 1961	WR	3210763	168201	51713	215962	3490	457	3650586
	REW	137190	39081057	777633	2472	2448755	6858	42453965
	RW	58792	1012457	0	434	7662	0	1079345
Born 1961 - 66 in	WR	314286	7392	3820	7703	75	45	333321
	REW	7326	3786423	53754	88	77104	541	3925236
	RW	2654	44879	0	32	451	0	48016
TOTAL		3731011	44100409	886920	226691	2537537	7901	51490469

Source - aggregation of accounts in Smith and Rees (1974)

Similarly, using the total population of England and Wales, EW, from GRO(1964) or Figure 3:

$$K^{REW*} = K^{EW*} - K^{WR*}$$

$$= 42453965 \qquad (2)$$

The population of the only remaining area, the rest of the World, or RW, is not required by the Rees and Wilson model. It can therefore be set to zero. In practice it would be extremely difficult to get a good estimate of it, if it were needed.

In all the accounts here though, population events which only affect areas outside EW are ignored, so RW to RW flows are all set to zero. Thus in the row containing people who were in RW in 1961, the flows $K^{RW,RW}$ and $K^{RW\delta(RW)}$ are set to zero. All the other elements in the RW row may have a value greater than zero in these accounts though. Now in order to calculate rates of migration in Section 2.2.3, the sum of these elements is required. The value of this sum is in fact $K^{RW*(EW)}$, and this is given in the Figure 3 accounts as 1079345. Thus because the RW to RW flows are omitted, K^{RW*} is equal to $K^{RW*(EW)}$, and the value of the latter from Figure 3 is 1079345. Note that the enhanced notation is used here for the first time, though only partially. $K^{RW*(EW)}$ represents the sum of $K^{RW,EW}$ and $K^{RW\delta(EW)}$, using the old notation, or the sum of $K^{E(RW)\epsilon(EW)}$ and $K^{E(RW)\delta(EW)}$, using the enhanced notation fully. Running the Rees and Wilson model does in fact produce another estimate of this total, $K^{RW*(EW)}$, but this will probably be less reliable than the one used from Figure 3, so the above figure will suffice for the rates calculations.

2.2.2 Surviving migrants, 1961-66 for the 55 local authorities accounts (K^{ij})

Some of these flows are obtainable immediately, hence from Figure 3 we know that -

$K^{REW,RW}$ is 777633 and $K^{RW,REW}$ is 1012457

These can be fitted into the table directly.

Next, information on migration between each authority must be collected. This was obtained from the sources mentioned later. The three residual areas were then dealt with. Flows to and from WR-55 can be found by deducting the sum of all authorities flows from the totals for WR, for example. The final steps in the procedure are to correct the flows so that they agree with those given in Figure 3. Inflows to each authority from the three residual regions will be dealt with first.

2.2.2.1 Inflows to each authority

For each of the 55 LA's, the following information can be obtained:
For authority y -

1. In migration from each of the 54 other authorities, K^{xy}, $x=1,55; x \neq y$

2. The total inmigration from all other authorities in WR, $K^{WR-y,y}$

3. The inmigration from Scotland to that authority K^{Sy}

4. The inmigration from Elsewhere in the British Isles K^{EBIy}

5. The inmigration from Abroad, K^{ABRy}

6. The total in migration from all areas, $K^{E(*)y}$

Parts of this data, for the larger authorities, can be obtained from GR(1968A, Table 7; 1968C, Table 10). However, large parts of it were obtained from other sources, see Acknowledgements.

All this date, and indeed all 1966 Census data, must be corrected by multiplying by 10 (as the census was only a 10% sample) and by 1.0142143, to correct for underenumeration, see Smith and Rees (1974). The required flows can now be obtained, for each authority y, as follows -

$$K^{WR-55,y} = K^{WR-y,y} - \sum_{\substack{x=1 \\ x \neq y}}^{55} K^{xy} \tag{3}$$

where $x \neq y$, and x are therefore the 54 other authorities

$$K^{REWy} = K^{E(*)y} - (K^{Sy} + K^{EBIy} + K^{ABRy}) - K^{WR-y,y} \tag{4}$$

$$K^{RWy} = K^{Sy} + K^{EBIy} + K^{ABRy} \tag{5}$$

For each local authority y, the total inflow of surviving migrants can be expressed as follows:

$$\sum_{\substack{i=1 \\ i \neq y}}^{58} K^{iy} = \sum_{\substack{x=1 \\ x \neq y}}^{55} K^{xy} + K^{WR-55,y} + K^{REWy} + K^{RWy} \tag{6}$$

Where i refers to all other regions except y
and x refers to all other local authorities except y.

The expressions for $K^{WR-55,y}$, K^{REWy} and K^{RWy} given by equations 3,4 and 5 can be substituted into the right hand side of equation 6. If this is done, then terms on the right hand side of equation 6 cancel out, leaving only $K^{E(*)y}$, the total inmigrants to y, as it should.

2.2.2.2 Inflows to the area West Riding minus the 55 local authorities (WR-55)

If the sum of the terms for K^{REWy} and K^{RWy} are obtained, this gives :

$$\sum_{y=1}^{55} K^{REWy} = 71016 \tag{7}$$

$$\sum_{y=1}^{55} K^{RWy} = 40630 \tag{8}$$

Now from the amounts in Figure 3 we know that:

$$K^{REW,WR} = 137190 \tag{9}$$

$$K^{RW,WR} = 58792 \tag{10}$$

These pieces of information allow us to calculate the two remaining unknown inflows in the migration and survival matrix, as by subtraction:

$$K^{REW,WR-55} = K^{REW,WR} - \sum_{y=1}^{55} K^{REWy} = 66174 \qquad (11)$$

and

$$K^{RW,WR-55} = K^{RW,WR} - \sum_{y=1}^{55} K^{RWy} = 18162 \qquad (12)$$

2.2.2.3 Outflows for each area

For each of the 55 LA's, the following information can be obtained:

For authority x -

1. Out migration to each of the 54 other authorities, K^{xy}, $y=1,55$, $y \neq x$
(The same information as was used for inflows)

2. The total out migration to all other authorities in WR, $K^{x,WR-x}$

3. The total out migration to Scotland, K^{xS}

4. The total out migration to all other areas in GB, $K^{x,GB-x}$

Parts of this data can be obtained from the references mentioned in Section 2.2.2.1, GRO (1968A, Table 7; 1968C, Table 1B). These sources only cover the larger local authorities though. Other sources were tapped to complete the data sets therefore, see acknowledgements. All the data was corrected for underenumeration and because it only refers to a 10% sample of the population, as described in the section on in migration. (2.2.2.1)

Some of the required flows can now be calculated:

$$K^{x,WR-55} = K^{x,WR-x} - \sum_{\substack{y=1 \\ y \neq x}}^{55} K^{xy} \qquad (13)$$

Where $y \neq x$, and y are therefore the 54 other authorities.

$$K^{xREW} = K^{x,GB-x} - K^{xS} - K^{x,WR-x} \tag{14}$$

Summing the second term for each authority gives:

$$\sum_{x=1}^{55} K^{xREW} = 83442 \tag{15}$$

From Figure 3 we know that $K^{WR,REW}$ is 168201 so for WR-55:

$$K^{WR-55,REW} = K^{WR,REW} - \sum_{x=1}^{55} K^{xREW} = 84759 \tag{16}$$

To obtain the remaining flows, K^{xRW}, where x=1, 55, and $K^{WR-55,REW}$, an estimation technique will be used. From Figure 3 we know that $K^{WR,RW} = 51713$. From the Census, GRO(1968A, Table 7) it can be found that $K^{WR,S} = 6085$. Now as:

$$O-GB = O-EW - S \tag{17}$$

So

$$K^{WR,O-GB} = K^{WR,O-EW} - K^{WR,S} \tag{18}$$

$$= K^{WR,RW} - K^{WR,S}$$

$$= 45628$$

Now for each local authority, we know the value of K^{xS}, and a similar equation can be written:

$$K^{x,O-GB} = K^{xRW} - K^{xS} \tag{19}$$

So to obtain K^{xRW} flows we can use[5]:

$$K^{xRW} = K^{xS} + K^{x,O-GB} \tag{20}$$

An estimate of the $K^{x,O-GB}$ flow is needed therefore. This can be obtained by allocating the known $K^{WR,O-GB}$ flow to the local authorities, using the proportion of the population of WR that the authority contained in 1961. Thus a constant propensity to migrate to O-GB is assumed in WR. The number of migrants from any part of WR will therefore be proportionate to that area[5] population in 1961. So:

$$K^{x,O-GB} = \frac{K^{x*}}{K^{WR*}} \quad K^{WR,O-GB} \tag{21}$$

Where x is any of the 55 authorities.

The final estimation equation is therefore:

$$K^{xRW} = K^{xS} + \frac{K^{x*}}{K^{WR*}} \quad K^{WR,O-GB} \tag{22}$$

$$= K^{xS} + 0.0125\ K^{x*}$$

If we then sum K^{xRW} for each authority we obtain

$$\sum_{x=1}^{55} K^{xRW} = 30371 \tag{23}$$

From Figure 3 we know that $K^{WR,RW} = 51713$, so for WR-55:

$$K^{WR-55,RW} = K^{WR,RW} - \sum_{x=1}^{55} K^{xRW} \tag{24}$$

$$= 21342$$

This completes the procedure for estimating the surviving migrants.

Gray and Gee (1972, p104), who tested the results of the 1966 Sample Census, took a sample of 5417 people, of which 1985 said they were 5 year migrants. The responses made to the migration questions in the 1966 Census by all these people were checked against the 1961 Census forms. From this check, the fact emerged that in this sample, the number of five year migrants was probably a 4% underestimate. Of the 3432 people who said they were non migrants, 91 were, in fact, probably migrants. Of the 1985 persons who were shown on the census forms as five year migrants, 3 did not seem to be migrants at all, and 158 had an address in 1961 different to the one given on their 1966 Census forms. The 91 migrants who were stated to be non migrants were probably misclassified because of confusion over the wording of the questions and notes dealing with migration on the 1966 Census form. Thus the five year migration data is probably an underestimate and there are also errors in peoples initial addresses. The counts of five year migrants form an important part of the input to the demographic accounts. These problems should be borne in mind when considering the results presented here. If required, the data used could be inflated, in an attempt to deal with this problem. This was not however done here.

2.2.3 Infant surviving migrants 1961-66, for the 55 local authorities accounts ($K^{\beta(i)j}$)

For all areas except RW, these are simply estimated using the surviving migrants flows calculated above, K^{ij} ($i \neq j$; $i=1,57$; $j=1,57$), the total population of each area K^{i*}, and the total births in each area, $K^{(i)*}$, as follows:

$$K^{\beta(i)j} = 0.5\ K^{\beta(i)*}\ \frac{K^{ij}}{K^{i*}} \tag{25}$$

For i and j equal to all areas in turn, except $i \neq RW$, and of course $i \neq j$. First, the rate of migration from i to j is calculated, K^{ij}/K^{i*}. This is then multiplied by the total births in the areas $K^{(i)*}$. This value must then be multiplied by 0.5. The multiplication by 0.5 is carried out because it is assumed that the average birth occurs half way through the period. Thus persons who are born are only at risk of migrating for half of the period.

A similar formula is used to calculate the flows from RW. In soing so though it should be remembered that the accounts ignore RW to RW flows. The result of this is that the values of K^{RWRW}, $K^{RW\delta(RW)}$, $K^{\beta(RW)RW}$ and $K^{\beta(RW)\delta(RW)}$ are all set to zero. Thus the sum of the two rows dealing with RW do not give either the total population of RW, or the total number of births in RW. These row sums do in fact contain the values $K^{RW*(EW)}$ and $K^{\beta(RW)*(EW)}$. These values are obtainable from Figure 3, and are mentioned in the subsections on initial populations and births, Sections 2.2.1 and 2.2.4 respectively. So, since:

$$K^{RWRW} = K^{RW\delta(RW)} = K^{\beta(RW)RW} = K^{\beta(RW)\delta(RW)} = 0 \tag{26}$$

$$K^{RW*} = K^{RW*(EW)} \tag{27}$$

and

$$K^{\beta(RW)*} = K^{\beta(RW)*(EW)} \tag{28}$$

If the equation used for the 57 other regions is applied Equation 25 this gives:

$$K^{\beta(RW)j} = 0.5\ K^{\beta(RW)*(EW)}\ \frac{K^{RWj}}{K^{RW*(EW)}} \tag{29}$$

Where j is any of the 57 other regions.

However, if this is rearranged, it gives:

$$\frac{{}_K\beta^{(RW)j}}{{}_K^{RWj}} = 0.5 \; \frac{{}_K\beta^{(RW)*(EW)}}{{}_K^{RW*(EW)}} \qquad (30)$$

This is unlikely to be the case, so we remove the value 0.5, and assume that

$$\frac{{}_K\beta^{(RW)j}}{{}_K^{RWj}} = \frac{{}_K\beta^{(RW)*(EW)}}{{}_K^{RW*(EW)}} \qquad (31)$$

So to obtain ${}_K\beta^{(RW)j}$ we can use:

$$ {}_K\beta^{(RW)j} = \frac{{}_K\beta^{(RW)*(EW)}}{{}_K^{RW*(EW)}} \; {}_K^{RW}{}_j \qquad (32)$$

Where j is any of the 57 other areas, and $j \neq RW$

value Returning to Figure 3, the reliable accounts, it will be noted that the 2 of \of \the infant surviving migrant flows are known. These are:

$${}_K\beta^{(REW)RW} = 53754 \text{ and } {}_K\beta^{(RW)REW} = 44879 \qquad (33)$$

The values estimated by the model given here were:

$${}^w_K\ (REW)RW = 35949 \text{ and } {}^w_K\beta^{(RW)REW} = 45040 \qquad (34)$$

Thus the model here has obtained different estimates for these two flows, compared to the more reliable Figure 3 flows. It was therefore decided to adjust the value of these two flows to those given in the 'reliable' set of accounts in Figure 3. The reliable estimates were therefore taken as correct.

Also obtainable from the estimates, by aggregation, are the four flows ${}^w_K\beta(WR)REW$, ${}^w_K\beta(WR),RW$, ${}^w_K\beta(REW)WR$ and ${}^w_K\beta(RW)WR$. These can be compared to those which have been accepted as being more reliable, in Figure 3. Generally, the disaggregated flows forecast by the simple model are too small, again. The estimated flows were therefore corrected, by calculating a correction factor $C^{\beta(i)j}$ for each of the four flows. This is obtained as follows:

$$ {}_C\beta^{(i)j} = {}^{W}_{K}\beta^{(i)j} / {}_K\beta^{(i)j} \quad \text{(Figure 3)} \tag{35} $$

If i=WR, j=REW or RW, If j=WR. i=REW or RW.

The factors used were as follows:

$$ {}_C\beta^{(WR)REW} = .966 \qquad {}_C\beta^{(WR)RW} = 1.619 \tag{36} $$
$$ {}_C\beta^{(REW)WR} = 1.155 $$
$$ {}_C\beta^{(RW)WR} = 1.015 $$

Obviously there are differences in the estimates obtained for these flows as the C values are not equal to 1. The final estimates have however been corrected to agree with the known, and probably more reliable set of accounts. This is done by applying the correction factors as follows:

For flows from WR to REW and RW -

$$ {}_K\beta^{(WR)k} = {}_C\beta^{(WR)i}\ {}_K(WR)k \tag{37} $$

Where k = one of the 55 LA's, or WR-55
i = REW or RW.

For flows from REW and RW to WR -

$$ {}_K\beta^{(k)WR} = {}_C\beta^{(i)WR}\ {}_K\beta^{(k)WR} \tag{38} $$

Where k = one of the 55 LA's or WR-55
i = REW or RW

This completes the estimation procedure adopted for this set of flows.

2.2.4 Births, 1961-66, for the 55 local authorities accounts $({}_K\beta^{(i)*})$

Births in each of the 55 LA's for each year can be obtained from GRO (Annual, Table E), and OPCS (Annual, Table E). Here, however, we require the number of births in each area in the intercensal period, 1961-66. The number of births in the period 1962 to 1965 inclusive of both years, can obviously be obtained by simple aggregation. Estimates of the number of births after census night in 1961 and before census night in 1966 must be added to this total though. It is estimated, for any area in EW, that .684744 of the births in 1961 occurred after census night in that year, and that .321253 of the births in 1966 occurred before census night in that year. These estimates are made from the quarterly live births records given for the whole of England and Wales in OPCS (Quarterly, Table 1A). These proportions are known as BTCF's, Birth time correction factors, and are described and used by Rees and Wilson (1975, Section 7.7 and 7.8). To show how they are estimated, an example will be given. There was a total of 811281 births in EW in 1961. The births in the last three quarters of 1961 in EW, were 207540, 204067 and 196368, in that order. The census night occurred in the second quarter, and 68 out of the 91 days in that quarter fell after this point. Thus the estimate of the proportion of births occuring in 1961 after the census night for any area in EW is

$$\text{BTCF 1961 (after census)} = \left(\frac{68}{91}.207540 + 204067 + 196368\right)/811281$$

$$= .684744 \tag{39}$$

And in a similar fashion, we can calculate:

$$\text{BTCF (1966, before census)} = .321253 \tag{40}$$

The final estimation equation is therefore

$$_K\beta(i)^* = \text{BTCF 1961 (after census) } E^{1961i} + E^{1962i} + E^{1963i} + E^{1964i} + E^{1965i} + \text{BTCF 1966 (before census) } E^{1966i} \tag{41}$$

Where E^{pq} are the births in year p in area i, i is any area in EW. The aggregation of this data for the 55LA's was performed by Jacqueline Dods, see Acknowledgements.

The total birts in WR and EW are given in Figure 3, or can be obtained from similar sources. By using the standard relationships as used in equation (1) and (2) for example, the births in WR-55 and REW can be calculated. The accounts will disregard RW to RW flows so the value of cells $_K\beta(RW)RW$ and $_K\beta(RW)\delta(RW)$ are set equal to zero. Thus the total for the last row, for births in RW, is given by $_K\beta(RW)*(EW)$. This value can be obtained from Figure 3, and equals 48016. This figure is not needed as data for the model, but it is used in calculating rates, see the subsection 2.2.3.

2.2.5 Deaths, 1961-66, for the 55 local authorities accounts ($K^{*\delta(i)}$)

The required total deaths for each area can be calculated in exactly the same fashion as for the births. Instead of BTCF's, DTCF's (Deaths time correction factors) are used. The DTCF's used were:

$$\text{DTCF 1961 (after census)} = .621777 \qquad (42)$$
$$\text{DTCF 1966 (before census)} = .369327 \qquad (43)$$

The same source, OPCS (Quarterly, Table 1B) and methods are used to estimate DTCF's as were used to estimate the BTCF's. An equation similar to Equation (39) is used to estimate each DTCF. An equation similar to equation (41) is then used to obtain the required $K^{*\delta(i)}$. Most of the data processing required for this section was performed by Jacqueline Dods, see Acknowledgements. It should be noted that the BTCF (equation (40)) and DTCF (equation (43)) for 1966 are for the part of that year before the census date. The BTCF and DTCF for after the census date are used in Section 3 (Part II). These are of course obtainable by deducting the BTCF or DTCF for before the census date from 1.

2.3 Output of the Model, 1961-66, for the 55 local authorities accounts

The data assembled in Section 2.2 can be input to the computer programme described in Rees and Wilson (1974). This produces the required set of aggregate accounts. The programme has been slightly modified, to cope with the increased number of regions, and to simplify the output. It has been decided not to present the full set of accounts as produced by the model, because this would require too much space. Copies of the output are available from the School of Geography, Leeds, on demand. To test the accuracy of the model however, the estimated 1966 population of the 57 areas involved, K^{*i}, have been presented, together with the totals given in the 1966 Census, in Figure 4. The figures shown for the 55 LA's only were compared using r squared and the value of the index of dissimilarity. The value of r squared obtained was .9993, and the value of the index was .[illegible]26%. Thus both measures indicate that the estimates made are near to the known census population.

Figure 4 1966 Populations for the 55 local authorities. Census date and estimates from the 1961-66 55 LA's accounts

Authority	1966 census pop.	55 LA's accounts pop.
Bradford CB	295197	299087
Dewsbury CB	51603	52499
Halifax CB	95874	94579
Huddersfield CB	132122	132849
Leeds CB	509125	506386
Wakefield CB	59920	58573
Aireborough UD	28388	29201
Baildon UD	13276	13851
Batley MB	42100	42253
Bingley UD	25082	25578
Brighouse MB	33074	33112
Castleford MB	40254	38671
Colne Valley UD	21948	21449
Denby Dale UD	10426	9870
Denholme UD	2434	2711
Elland UD	19128	19100
Featherstone UD	15315	15378
Garforth UD	19544	19404
Harrogate MB	60336	61400
Hebden Royd UD	9412	8995
Heckmondwike UD	9574	8603
Homfirth UD	19300	18865
Horbury UD	9047	8624
Horsforth UD	16359	16790
Ilkley UD	20163	19585
Keighley MB	56512	55799
Kirkburton UD	18337	18960
Knaresborough UD	10791	9766
Knottingley UD	13935	13528
Meltham UD	6035	6122
Mirfield UD	14169	13716
Morley MB	43398	43418
Normanton UD	18692	18006
Ossett MB	17191	16456
Otley UD	13094	12088
Pontefract MB	28814	28637
Pudsey MB	36897	37713
Q & S UD	9828	9540
Ripponden UD	4503	4954
Rothwell UD	27201	27229
Shipley UD	26461	28782
Sowerby Br. UD	17252	17357
Spenborough UD	39281	39450
Stanley UD	18540	19406
Todmorden MB	16582	16952
Hepton RD	3722	4172
Osgoldcross RD	8925	9053
Tadcaster RD	30639	31834
Wakefield RD	23236	22632
Wetherby RD	26785	26986
Wharfedale RD	7282	8035
Hemsworth UD	14676	14796
Silsden UD	5964	6034
Hemsworth RD	52830	52820
Skipton RD	23226	24312
WR-55	1535372	1535840
REW	44031956	44106327
TOTAL	47811127	47837833

Source of Census population, see Section 3.2.1

It should be noticed that the 'estimates' made for the 1966 populations did use data from the 1966 census, in the estimates of migration and survival. Also, the data for births and deaths for the period 1961-66 were used in making them. Thus the 'estimates' are not strictly predictions, or completely blindfold guesses of the future, as they were obtained with partial knowledge of what population events were going to occur. The word estimate is only used in this sense here.

It should be noted that the index of dissimilarity only compares the distribution of population estimated. The index is only a measure of whether each cell contains the correct proportion of the population, and not a measure of how accurately the total population has been estimated. Thus in using the index, we should also look at whether the total population of all the 55 areas that has been estimated is accurate. In this case, the population of the 55 areas given by the census was 2197799 The models forecast was 2195666. This was quite close to the census population, the difference being 1867 persons.

Generally, therefore, the agreement between the census and the estimates is good. Thus the model is estimating the 1966 populations of the areas well. This statement is of course less true for some of the authorities.

2.4 Obtaining the data, 1961-66 for the 5 metropolitan districts accounts

This section shows how the data for the five metropolitan districts (5MD's) set of accounts is obtained, for the period 1961-66. This set of areas contains the five metropolitan districts, plus three residual regions, RWR, REW AND RW. Much of this data can be obtained by aggregation from data for the 55LA's set of accounts for the same period. The sources of data for the 55 LA's and the estimation methods used for that set of areas, are described in Section 2.2

The existing set of accounts given in Figure 3 is again relevant to the estimation process described here. The data produced by this section will be corrected to correspond to these accounts where possible, as was done for the 55LA's accounts data in Section 2.2.

A major problem in this section is how the data for 55LA's can be aggregated to give values for the 5 MD's. Illingworth, Smith and Rees (1974), describe a procedure whereby information for the 55 LA's used here can be aggregated to provide values for the five metropolitan districts. This is done using a matrix multiplication procedure.

The first requirement is a suitable aggregation matrix, and this can be obtained from that given in Illingworth, Smith and Rees (1974, Figure 5). This matrix shows which new metropolitan district each of the 55 LA's is allocated to. For example, Bradford CB(1) goes into Bradford metropolitan district, see Figure 5. For the seven authorities which are divided between districts, or between a disrrict and the rest of the West Riding, weights are used. These weights will be used to divide the total population of or the events in each of these authorities between the two new areas. These weights are estimated from parish population data, and are the proportion of the enumerated population of the dividend area that falls into each metropolitan district. Thus for example, Figure 5 contains the set of weights for the female population in 1966. Hence it can be seen from Figure 5 that .3790323 of the female population of Queensbury and Shelf UD(38) fell in Calderdale in 1966, and the remaining .620977 fell into Bradford metropolitan district. The other divided authorities are Osgoldcross RD (47), Tadcaster RD (48), Wetherby RD (50), Wharfedale RD (51), Hemsworth RD (54) and Skipton RD (55).

Illingworth, Smith and Rees (1974, Figures 6,7 and 8) give several sets of weights, some of which are used here. The weights were calculated for three census years, 1961, 66 and 71. For each of these three census years, weights for the whole population, and for the male and female population separately, were calculated.

The data that is to be aggregated here refers to a set of 58 regions in fact, and includes three residual areas as well as the 55 LA's dealt with by Illingworth Smith and Rees (1974). Also, we are dealing with eight 'new' regions, that is the 5MD's already dealt with, plus three new residual ones. Thus the aggregation matrix described has to be extended slightly. This is easily done. Another two columns must be added to the matrix given in Illingworth Smith and Rees (1974, Figure 5), on the left hand side. These columns will be called, from the left, RW AND REW. Also, the column originally called Rest of England and Wales

FIGURE 5 THE NEW AGGREGATION MATRIX

CELLS CONTAINING ONES OR ZERO'S DO NOT ALTER IN VALUE

THE FRACTIONAL VALUES SHOWN ARE WEIGHTS CALCULATED FROM THE 1966 FEMALE POPULATIONS OF PARISHES

	RW	REW	*RWR	WAKEFIELD	KIRKLEES	CALDERDALE	LEEDS	BRADFORD
BRADFORD CB	0	0	0	0	0	0	0	1
DEWSBURY CB	0	0	0	0	1	0	0	0
HALIFAX CB	0	0	0	0	0	1	0	0
HUDDERSFIELD CB	0	0	0	0	1	0	0	0
LEEDS CB	0	0	0	0	0	0	1	0
WAKEFIELD CB	0	0	0	1	0	0	0	0
AIREBOROUGH UD	0	0	0	0	0	0	1	0
BAILDON UD	0	0	0	0	0	0	0	1
BATLEY MB	0	0	0	0	1	0	0	0
BINGLEY UD	0	0	0	0	0	0	0	1
BRIGHOUSE MB	0	0	0	0	0	1	0	0
CASTLEFORD MB	0	0	0	1	0	0	0	0
COLNE VALLEY UD	0	0	0	0	1	0	0	0
DENBY DALE UD	0	0	0	0	1	0	0	0
DENHOLME UD	0	0	0	0	0	0	0	1
ELLAND UD	0	0	0	0	0	1	0	0
FEATHERSTONE UD	0	0	0	1	0	0	0	0
GARFORTH UD	0	0	0	0	0	0	1	0
HARROGATE MB	0	0	1	0	0	0	0	0
HEBDEN ROYD UD	0	0	0	0	0	1	0	0
HECKMONDWIKE UD	0	0	0	0	1	0	0	0
HOLMFIRTH UD	0	0	0	0	1	0	0	0
HORBURY UD	0	0	0	1	0	0	0	0
HORSFORTH UD	0	0	0	0	0	0	1	0
ILKLEY UD	0	0	0	0	0	0	0	1
KEIGHLEY MB	0	0	0	0	0	0	0	1
KIRKBURTON UD	0	0	0	0	1	0	0	0
KNARESBOROUGH UD	0	0	1	0	0	0	0	0
KNOTTINGLEY UD	0	0	0	1	0	0	0	0
MELTHAM UD	0	0	0	0	1	0	0	0
MIRFIELD UD	0	0	0	0	1	0	0	0
MORLEY MB	0	0	0	0	0	0	1	0
NORMANTON UD	0	0	0	1	0	0	0	0
OSSETT MB	0	0	0	1	0	0	0	0
OTLEY UD	0	0	0	0	0	0	1	0
PONTEFRACT MB	0	0	0	1	0	0	0	0
PUDSEY MB	0	0	0	0	0	0	1	0
QUEENS & SHELF UD	0	0	0	0	0	.3790323	0	.620977
RIPPONDEN UD	0	0	0	0	0	1	0	0
ROTHWELL UD	0	0	0	0	0	0	1	0
SHIPLEY UD	0	0	0	0	0	0	0	1
SOWERBY BR. UD	0	0	0	0	0	1	0	0
SPENBOROUGH MB	0	0	0	0	1	0	0	0
STANLEY UD	0	0	0	1	0	0	0	0
TODMORDEN MB	0	0	0	0	0	1	0	0
HEPTON RD	0	0	0	0	0	1	0	0
OSGOLDCROSS RD	0	0	.8738095	.1261905	0	0	0	0
TADCASTER RD	0	0	.6083727	0	0	0	.3916273	0
WAKEFIELD RD	0	0	0	1	0	0	0	0
WETHERBY RD	0	0	.2104446	0	0	0	.7895554	0
WHARFEDALE RD	0	0	.3010471	0	0	0	.7071097	0
HEMSWORTH UD	0	0	0	1	0	0	0	0
SILSDEN UD	0	0	0	0	0	0	0	1
HEMSWORTH RD	0	0	.2928903	.7071097	0	0	0	0
SKIPTON RD	0	0	.8143908	0	0	0	0	.1856092
W.RIDING − 55 LA'S	0	0	1	0	0	0	0	0
REST OF ENG & WALES	0	1	0	0	0	0	0	0
REST OF WORLD	1	0	0	0	0	0	0	0

MATRIX AS PRESENTED BY ILLINGWORTH, SMITH AND REES (1974, FIGURE 5)

* RE-NAMED – WAS REST OF ENGLAND AND WALES IN ILLINGWORTH, SMITH AND REES (1974)

⊛ NEW SECTIONS ADDED TO THE MATRIX

OTHER WEIGHTS ARE OBTAINABLE FROM ILLINGWORTH, SMITH & REES (1974, FIGURES will be renamed RWR. There are now eight columns, each of which is one of the regions used in the 5 MD's set of accounts. Finally, three rows must be added, and these are called WR-55, REW and RW. These are the three remaining regions from the 55 LA's set of accounts. There are now 58 rows, each of which relates to a region used in the 55 LA's set of accounts. The new aggregation matrix is shown in Figure 5. The values which are given to the new cells which have been added are also shown. All the new cells are in fact zero in value except for one in each of the new rows. These non zero cells show that the old residual WR-55 is now allocated to RWR; REW is allocated to REW; and RW to RW. Thus the matrix shows the complete relationship between the old set of regions used in the 55 LA's accounts (rows) and the new, used in the 5 MD's accounts. (columns)

Notation is required to show that this aggregation matrix is being used. This must also distinguish between the different possible sets of weights that are available. The following notation will be used -

<u>W</u>	Indicates that the aggregation matrix given in Figure 5 is being used.
<u>61</u>,<u>66</u>,<u>71</u>	Shows the census year which the weights contained in W were calculated for.
<u>T</u>,<u>M</u>,<u>F</u>	Shows whether the wights refer to the total population, or just males or females.

Hence

<u>W66F</u> (58x8)	is the aggregation matrix shown in Figure 5 plus the weights from Illingworth, Smith and Rees (1974) calculated from the 1966 female populations of the divided authorities. This is the matrix shown in Figure 5.

Using the order established in Section 2.1, the data was estimated as follows:

2.4.1 Initial populations in 1961, for the 5 metropolitan districts accounts(K^{i*})

From Section 2.2.1, on the Initial populations in 1961 for the 55 LA's accounts, a vector containing the initial population of the 58 areas can be obtained. This vector will be called IP61(55LA). The vector can be aggregated by multiplying it by the aggregation matrix already described, to give a vector of initial populations for the eight areas that is required here. The required vector is therefore called IP61 (SMD). Hence:

$$\underset{(1\text{x}8)}{\underline{\text{IP61(5MD)}}} = \underset{(1\text{x}55)}{\underline{\text{IP61(55LA)}}} \times \underset{(55\text{x}8)}{\underline{\text{W61T}}} \qquad (44)$$

Note that the weights for 1961, for all persons (T) are used in the W matrix.

2.4.2 Surviving Migrants, 1961-66, for the 5 metropolitan districts accounts (K^{ij})

From Section 2.2.2, on the surviving migrants for 1961-66 for the 55 LA's accounts, a migration and survival matrix for the 58 areas used in the 55 LA's accounts is obtainable. This matrix will be called MS6166(55LA). This matrix can be aggregated using the aggregation matrices already described. This produces a first estimate of the matrix required here, the migration and survival matrix for the same period, for the eight regions used in the SMD's accounts. The required matrix will be called MS6166(SMD). Note that this is only a first estimate of this, as 'internal migrants' have to be added to this before it is complete, this will be done later in this section. The initial estimate is therefore:

$$\underset{(8\text{x}8)}{\underline{\text{MS6166(SMD)}}} = \underset{(8\text{x}58)}{\underline{\text{W61T}^{t}}} \times \underset{(58\text{x}58)}{\underline{\text{MS6166(55LA)}}} \times \underset{(58\text{x}8)}{\underline{\text{W66T}}} \qquad (45)$$

Note that W61T is transposed. Thus where an authority is divided, each outmigration flow from it to the other 57 areas is divided using the 1961 weight for all persons. For example, if half of a particular authority (using the 1961 weight) falls in a particular metropolitan district, then half of its outmigration flows to the other 57 areas are counted as having originated from that district. This is rather a simplistic assumption to make, but there seems to be no alternative.

Most of the better estimation techniques that can be suggested will require more data than is available here. Similarly, immigration into the authorities that are divided uses the 1966 total person weight. Thus if LAx is the origin authority and falls wholly or partly into MDa, and if LAy is the destination authority and falls wholly or partly into MDb. Then if:

LAx is wholly in MDa
and LAy is wholly in MDb

$$- \; K^{xy} \text{ is added to } K^{ab}$$

or if: LAx is partly in MDa
and K^{x*} is the population of x initially
and K^{p*} is the population of the part of x initially (the part following into MDa)
and LAy is wholly in MDb

$$- \; K^{xy} \frac{K^{p*}}{K^{x*}} \text{ is added to } K^{ab}$$

or if: LAx is partly in MDa as before
and LAy is partly in MDb
and K^{*y} is the population of y at the end of the period
and K^{*q} is the population of the part of y at the end of the period (the part falling into MDb)

$$K^{xy} \frac{K^{p*}}{K^{x*}} \cdot \frac{K^{*q}}{K^{*y}} \text{ is added to } K^{ab}$$

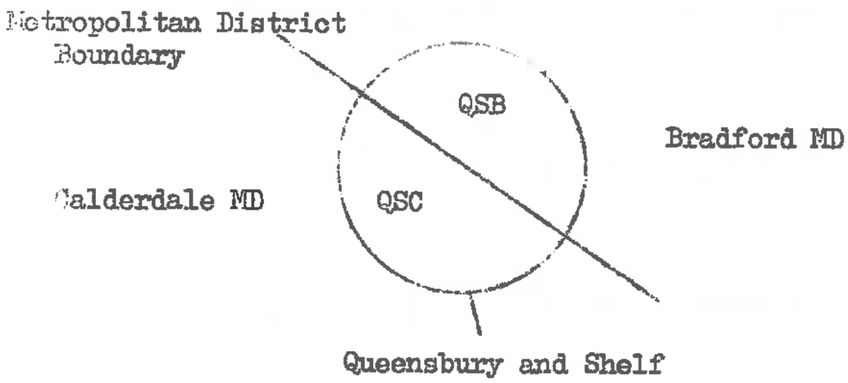
This process is continued for each authority and metropolitan district.

The above aggregation process produces an 8 x 8 migration and survival matrix MS6166(SMD). This matrix does not however contain all the flows yet, as some flows which were internal to the local authorities which were divided have to be calculated, and added to this.

Thus, out of the fifty five local authorities, forty eight fall either wholly in or wholly outside one of the metropolitan districts. The internal migration in these authorities can therefore be disregarded, as in the aggregated migration matrix it becomes internal to the area into which the authority falls.

Seven of the authorities are split by the aggregation process however. One effect of this is that some of the migration which was internal to these authorities now crosses a metropolitan district boundary. These flows must be estimated, and added to the existing migration estimates. Thus the aggregation process as described so far allocates the migration between authorities correctly, but fails to divide up internal migration in the seve n divided authorities.

For example, the local authority Queensbury and Shelf is split between two metropolitan districts, Calderdale and Bradford. This is shown diagramatically:



Queensbury and Shelf (QS) has been divided into two parts, QSC (in Calderdale) and QSB (in Bradford), so:

$$QS = QSB + QSC \tag{46}$$

Now some of the internal migration is QS must involve crossing the new metropolitan district boundary, from QSB to QSC or vice versa. These flows must be estimated, and added to the existing migration data. We know the total internal migration, $K^{QS,QS}$. (Note: This is not persons existing in QS in 1961 and 1966 for the purposes of this section). We require a partition of th's to obtain $K^{QSB,QSC}$ and $K^{QSC,QSB}$, as follows:

$$\begin{array}{cc|c|c|}
 & & \multicolumn{2}{c}{QS} \\
 & & QSB & QSC \\
\hline
QS & QSB & & K^{QSB,QSC} \\
\hline
 & QSC & K^{QSC,QSB} & \\
\hline
\end{array}$$

Total internal migration, $K^{QS,QS}$

This partition will be done using the proportion of the total population of QS in QSB or QSC. These are the same factors that are used in aggregating the migration matrix and other data. They are given in Illingworth, Smith and Rees (1974). We therefore assume that:

$$K^{QSB,QSC} = \frac{K^{QSB*}}{K^{QS*}} \cdot \frac{K^{*QSC}}{K^{*QS}} \cdot K^{QS,QS} \qquad (47)$$

This must then be added to the $K^{B,C}$ flow already estimated. Similarly:

$$K^{QSC,QSB} = \frac{K^{QSC*}}{K^{QS*}} \cdot \frac{K^{*QSB}}{K^{*QS}} \cdot K^{QS,QS} \qquad (48)$$

Which must be added to the $K^{C,B}$ flow.

This estimation procedure is by no means ideal. More accurate forecasts would however require more data, which was not easily available. Better estimates could of course be made and inserted here, if more accurate results were needed.

This process can be continued to deal with the remaining six authorities. These authorities are just divided between one of the metropolitan districts and the rest of the West Riding, RWR. For example, Skipton RD is split between Bradford metropolitan distrct, B, and RWR. Skipton therefore has two parts, Skipton B and Skipton RWR. Thus Skipton = Skipton B + Skipton RWR. Partition of the internal migration of Skipton will give two flows to be added to the migration estimates,

$K^{\text{SKIPTON B, SKIPTON RWR}}$ added to $K^{B,RWR}$

and

$K^{\text{SKIPTON RWR, SKIPTON B}}$ added to $K^{RWR,B}$

The allocation of internal migrants for 1961-66 is as follows:

Name of divided Authority	Internal Migration of that Authority	Migr.& Survival flow which is increased	Amount of the increase
Queensbury & Shelf UD	1542	$K^{B,C}$	377=.6551x.3736x1542
		$K^{C,B}$	333=.3449x.6264x1542
Osgoldcross RD	1288	$K^{W,RWR}$	102=.0911x.8716x1288
		$K^{RWR,W}$	150=.9089x.1284x1288
Tadcaster RD	4189	$K^{L,RWR}$	976=.3859x.6037x4189
		$K^{RWR,L}$	1020=.6141x.3963x4189
Wetherby RD	3935	$K^{L,RWR}$	616=.7756x.2017x3935
		$K^{RWR,L}$	705=.2244x.7983x3935
Wharfedale RD	446	$K^{L,RWR}$	97=.7024x.3106x446
		$K^{RWR,L}$	91=.2976x.6894x446
Hemsworth RD	10031	$K^{W,RWR}$	2057=.7203x.2847x10031
		$K^{RWR,W}$	2007=.2797x.7153x10031
Skipton RD	3600	$K^{B,RWR}$	548=.1869x.8149x3600
		$K^{RWR,B}$	542=.8131x.1851x3600

Internal migration data was obtained from GRO (1968C, Table 1B). Adding together all these flows produces another 8x8 migration and survival matrix, which must be added to the estimates already obtained. This matrix consists of zeros, except for the following flows:

	W	K	C	L	B	RWR
Wakefield	-	-	-		-	2159
Kirklees	-	-	-	-	-	-
Calderdale	-	-	-	-	333	-
Leeds	-	-	-	-	-	1689
Bradford	-	-	377	-	-	548
Rest of W.R.	2157	-	[illegible]	1816	542	-

Adding this matrix to the estimates produced earlier by aggregation completes the estimation process for surviving migrant flows. The final estimates are shown in Figure 6. It can be seen that adding the internal migrant flows makes a considerable difference to the size of some of the estimates. The $K^{W,WR}$ flow is a good example of this. Thus it is important that accurate estimates are made. However, it should be noticed that the balance of migration into and out of any particular area is not significantly affected. Thus large outflows are balances by large inflows. This is a result of the method of estimation used, and it is probably not unrealistic. The end of period populations may not therefore be greatly affected by the addition of the 'Internal' migrants just presented therefore.

2.4.3 Infant surviving migrants, 1961-66, for the five metropolitan districts accounts ($_K\beta^{(i)j\cdot}$)

These flows are estimated from the surviving migrant flows estimated in Section 2.4.2. The formulae developed and used in Section 2.2.3 are used in this procedure. Equation (25) from 2.2.3 is used in the same way as it is in that Section to estimate most of these flows. Equation (32) can then be used to calculate the remaining unknown flows, from RW. Finally, the estimates are corrected to agree with those given in the more reliable set of accounts, Figure 3. This is done in the same way as was done in Section 2.2.3. The dimensions of the differences between these estimates and the Figure 3 flows were similar to those experienced in the earlier section.

2.4.4 Births, 1961-66, for the 5 Metropolitan districts accounts ($_K\beta^{(i)*}$)

From Section 2.2.4, on the Births in 1961-66 for the 55 LA's accounts, a vector of the births in the 58 areas during the period can be obtained. This vector will be called B6166(55LA). This vector can be aggregated using the aggregation matrix already described, see Figure 5. The only problem arises when deciding which set of weights should be used. The difficulty is that the weights calculated refer to a point in time, that is a census day, and not to the period between them, as do the total births. It was therefore decided that two sets of weights would be used, those for the beginning of the period, W61, and those for the end, W66. This produced two estimates of the births in each of the eight areas, and the average of these two was taken. The weights for females were used, as they are 'at risk' of giving birth. The vector required is called B6166(SMD). The estimation equation used was as follows:

$$\underset{(1\times8)}{\underline{B6166(SMD)}} = \underset{(1\times1)}{0.5} \times \underset{(1\times58)}{\underline{B6166(55LA)}} \times \underset{(58\times8)}{\underline{W61F}} + \underset{(1\times1)}{0.5} \times \underset{(1\times58)}{\underline{B6166(55LA)}} \times \underset{(58\times8)}{\underline{W66F}} \qquad (49)$$

Where the + sign refers to addition of each element in each (1x8) vector to the corresponding element in the other (1x8) vector. The estimates made are shown in Figure 6.

2.4.5 Deaths, 1961-66, for the 5 Metropolitan districts accounts $(K^{*\delta(i)})$

From Section 2.2.5, on the Deaths in 1961-66 for the 55 LA's accounts, a vector of the deaths in the 58 areas during the period can be obtained. This vector will be called D6166(55LA). This vector was aggregated in a similar manner as was the vector for births, in Section 2.4.4, Equation (49). This produces the required vector, containing the deaths in the eight areas for the appropriate period, and this will be called D6166(SMD). Different weights to those used in equation (49) are required, as the total population was at risk of death. The weights Wb1T and WbbT are therefore used instead of Wb1F and WbbF. The estimation equation is therefore:

$$\underset{(1x8)}{D6166(SMD)} = \underset{(1x1)}{0.5} x \underset{(1x58)}{D6166(55LA)} x \underset{(58x8)}{Wb1T} + \underset{(1x1)}{0.5} x \underset{(1x58)}{D6166(55LA)} x \underset{(58x8)}{WbbT} \qquad (50)$$

Where the + sign denotes vector addition, see Section 2.4.4.
The estimates made are shown in Figure 6.
Note that the Rees and Wilson model does not require that a value of $K^{*(EW)\delta(RW)}$ (or indeed $K^{\beta(RW)*(EW)}$) is input as data. The model does however estimate these terms, and these are shown in Figure 6. The estimate for $K^{*(EW)\delta(RW)}$ differs substantially to that given in Figure 3. This flow total is therefore slightly suspect.

2.5 Output of the model, 1961-66 for the 5 metropolitan districts accounts

The data collected in Section 2.4 can be input to the Rees and Wilson programme. This produces the set of accounts shown in Figure 6. Figure 6 also contains the data which was input to the programme, for example, the total births.

It was again decided to test the accuracy of the model by comparing the end of period populations estimated for the seven areas, to the populations given by the 1966 census. The 1966 census populations given in Figure 4 must first be aggregated to estimate the populations of the 5MD's. This is done by multiplying the vector of census population by the aggregation matrix, WbbT, as in equation (44). The populations estimated for the authorities by the 55LA's model given in Figure 4 were also aggregated. The following results were obtained:

Areas	From Figure 6: Pop.est. by the 5MD's, 1961-66 model	Aggregated from Figure 4: Census Population	Aggregated from Figure 4: Pop.est. by the 55LA's 1961-66 model
Wakefield	293622	298557	293653
Kirklees	364635	364895	364636
Calderdale	202331	203219	202485
Leeds	731533	732552	731928
Bradford	462090	455545	461903
Rest of WR	1677285	1724403	1676901
Rest of EW	44106334	44031956	44106327
Total	47837830	47811127	47837833

The three sets of figures for the metropolitan districts only were compared, using the value of r squared, and the index of dissimilarity as discussed in Section 2.3. The following results were obtained:

<u>R Squared</u>

	Census	5MD est.	55LA est.
Census populations	–	.9996	.9996
5MD forecast	–	–	.9999

<u>Modified index of dissimilarity</u> (%)

	Census	5MD est	55LA est
Census populations	–	.3246	.3142
5MD forecast	–	–	.0180

It can therefore be seen for example that when the census figures are compared to the output of the 5 MD's accounts, a value of r squared of .9996 is obtained, the value of the index being .3246%. Similarly the census figures compared to the aggregated output of the 55LA's accounts gives a value for r squared of .9996. and for the index of .3142 %. The calculations were only done for the five metropolitan districts because if RWR and REW are included the value of r squared becomes meaningless, because these values are too extreme.

		Wakefield	Kirklees	Calder-dale	Leeds	Bradford	Rest of W. Riding	Rest of E & W	Rest of World	Die 1961 to 1966 in W	K	C	L	B	RWR	REW	RW	TOTAL
	W	244279	[illegible]	249	3993	595	[illegible]	8574	3952	14985	55	9	123	21	215	256	110	256505
	K	1475	304444	1599	2350	1552	3005	11929	5042	42	22465	57	72	55	68	355	175	354659
	C	257	1695	177327	555	2324	1475	7954	2795	7	58	13621	26	81	43	256	[illegible]	[illegible]
	L	3460	3871	[illegible]	605663	5539	11235	29764	10539	99	131	93	39633	192	329	805	373	[illegible]
	B	639	2562	[illegible]	6915	464527	4664	16104	4690	20	39	106	212	23604	137	479	252	[illegible]
	RWR	7435	2606	1218	3850	3707	1406387	95976	23296	913	59	45	272	129	92311	2791	653	[illegible]
	REW	7236	9480	[illegible]	27277	11397	77496	39122575	777535	997	304	153	939	325	2273	3339241	23119	[illegible]
	RW	3021	9124	3026	12072	11946	21602	1012457	0	87	282	113	371	414	604	30132	0	[illegible]
Born 1961 to 1966 in	W	24626	87	12	186	26	343	395	297	785	1	0	2	0	5	6	1	[illegible]
	K	57	29603	72	105	59	136	518	369	1	1051	1	2	1	2	3	6	[illegible]
	C	11	72	15746	36	34	63	327	133	0	1	604	1	2	1	5	4	[illegible]
	L	160	179	89	60525	255	518	1323	739	2	3	1	1908	4	8	20	11	[illegible]
	B	32	119	132	321	39747	216	709	502	0	2	2	5	1380	3	11	9	65605
	RWR	339	119	55	403	175	138265	4120	1717	5	2	1	6	3	4157	61	25	[illegible]
	REW	397	453	264	1456	608	4139	7750830	53754	6	3	3	22	10	60	112422	795	[illegible]
	RW	136	367	136	546	540	929	44979	0	2	6	[illegible]	8	9	14	658	0	[illegible]
TOTAL		293622	364655	302331	731533	462090	1677295	44106334	886321	16403	24556	14791	43691	31300	95950	2557537	23592	[illegible]

Small errors in the row and column totals are due to rounding

Figure 6 Accounts for the Metropolitan Districts, 1961–66

Thus the estimates that have been made a very similar, and they are quite good estimates of the census population. Of the two, the estimate by the 55LA's accounts is slightly better. This is probably because it was a more disaggregated set of accounts.

The total population of the Metropolitan districts estimated from the 1966 census was 2054768 The 5 MD's model forecasted a total of 2054211 (557 persons less than the census), and the 55LA's model, when aggregated, forecasted a total of 2054605 (163 persons less than the census). These facts should be born in mind when considering the values of the index given, as the index only measures differences in distribution, as discussed in Section 2.3

NOTE: THE SECOND PART OF THIS PAPER, DEALING WITH THE PERIOD 1966-71, HAS BEEN BOUND IN SEPARATE COVERS. THE SECOND PART ALSO CONTAINS THE REFERENCES.

References.

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OPCS	Office of Population Censuses and Surveys.
CSO	Central Statistical Office.
GNI	Government of Northern Ireland.
HMSO	Her Majesty's Stationary Office, London. (Unless otherwise specified).

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		Wakefield	Kirklees	Calder-dale	Leeds	Bradford	Rest of W. Riding	Rest of E & W	Rest of World	Die 1966 to 1971 in: W	K	C	L	B	RWR	REW	RW	TOTAL
	W	247601	1867	353	4018	718	7715	10697	9225	15334	64	13	123	25	225	317	263	298557
	K	2527	305102	2065	2566	1557	2705	14010	10782	72	22442	74	78	54	79	415	368	364895
	C	250	2260	166565	966	3003	1321	9071	6042	7	77	13001	30	103	39	268	216	203219
	L	5169	4571	634	605549	5583	11355	34647	22220	147	156	23	40269	192	332	1025	680	732552
	B	840	3032	2874	5942	373611	4753	21472	13339	24	103	103	182	28037	139	635	459	455545
	RWR	8772	2950	1328	9246	4229	1443136	106433	51669	250	101	48	283	146	91155	3149	1510	1724403
	REW	9967	9640	5879	30158	13698	78817	40055158	1333505	284	329	211	922	472	2503	2451156	39457	44031956
	RW	2929	7470	2614	12194	13148	19625	1082500	0	83	255	94	373	453	574	32030	0	1174342
Born 1966 to 1971 in	W	24560	84	16	181	32	347	481	415	722	1	0	3	1	5	7	6	26261
	K	117	30929	95	119	72	125	647	498	2	1083	2	2	1	2	10	8	33711
	C	10	94	15447	40	126	55	379	253	0	2	571	1	2	1	6	4	16990
	L	221	195	27	57176	239	465	1481	950	3	3	0	1804	4	7	22	14	62632
	B	38	138	130	270	37612	216	975	605	1	2	2	4	1336	3	14	10	41357
	RWR	374	126	57	394	180	135010	4538	2203	5	2	1	6	3	4061	67	32	147059
	REW	423	410	250	1281	582	3349	3571345	56661	6	7	4	19	10	49	106611	832	3741840
	RW	197	501	176	818	882	1316	72603	0	3	8	3	12	15	19	1066	0	77620
	TOTAL	303995	369369	198510	730198	455271	1710329	44986437	1508367	16944	24635	14149	44111	30853	98992	2596798	43885	

Small errors in the row and column totals are due to rounding

Figure 10 Accounts for the Metropolitan Districts 1966-71