Working Paper 363

Exploring the Dynamics of Urban Housing Structure : a 56 parameter residential location and housing model.*

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The authors wish to acknowledge the financial support of the Social Science Research Council.

* To be presented at the European Congress of the Regional Science Association, Poitiers, September 1983

1. Introduction

In the field of urban modelling, it has proved easier to represent the location of population activities and spatial interaction than to model the location of economic activity. Thus, given a spatial distribution of shopping centres and a distribution of population, it is possible to predict the spatial pattern of trips and hence the revenue attracted to each centre (cf. Lakshmanan and Hansen, 1965, and much work since then). Since 1978, it has also been possible to model the supply of shopping centres (Harris and Wilson, 1978) and this work proved to have interesting connections to catastrophe theory and bifurcation theory. The purpose of this paper is to show how these ideas can be used to extend disaggregated residential location models to be connected to a submodel of housing supply. This problem is much more complicated than that offered by the retail system. Houses are of different types; so are the people who live in them; and much of the interest arises from operating at a level of disaggregation which makes this explicit. We operate at the level of disaggregation used by Wilson (1970) and first tested by Senior and Wilson (1973). We make the usual simplifying assumptions of one worker per household. An important point to emphasise at the outset, however, is that the kinds of results reported here will in character be independent of the particular form of model used and of simplifying assumptions of the type mentioned above. The bifurcation properties in particular will be dependent only on the general kinds of nonlinearities and interdependencies which are present.

A further point to be made in this introductory section relates to the context of the model developments described in this paper. The main aim of our work is to demonstrate the types of processes that operate in a metropolitan housing system. In particular we are interested in the interdependencies between these different processes and different groups of households. In this respect we are not in the position, as yet, to model the housing system in a particular urban system or indeed to calibrate our set of models for a city, although ultimately we would see this as the long term goal of our work. We therefore concentrate on demonstrating the combined effects of the interdependencies that we define and illustrate how, through parameter variation, these can give rise to very different types of structures and how these relate to the different types of housing systems found in various types of cities. In addition we examine how the mechanisms that give rise to the changes between different structures operate and how bifurcations emerge.

The main variables to be predicted in the model represent residential location (in relation to workplace) and housing supply. These are T_{ij}^{kw}, the number of w-income people, working in zone j, who live in type k houses in zone i; and H_i^k, the number of type k houses in zone i.

In the next section, we present the form of model to be used for illustrative purposes and then in Section 3, we report and interpret a range of numerical experiments with this model. Some concluding comments are offered in Section 4.

2. The Form of Model Used

A suitable and flexible spatial interaction model of residential location takes the following form:

$$T_{ij}^{kw} = B_j^w W_i^{res.kw} E_j^w e^{-\mu^w c_{ij}} \tag{1}$$

where

$$B_j^w = 1/\sum_{ik} W_i^{res.kw} e^{-\mu^w c_{ij}} \tag{2}$$

to ensure that

$$\sum_{ik} T_{ij}^{kw} = E_j^w \tag{3}$$

and

$$W_i^{res.kw} = \prod_{\ell} (X_{i\ell}^{kw})^{\alpha_\ell^{kw}} \tag{4}$$

In the following, an asterisk replacing an index of a variable denotes summation - for example

$$P_i^{kw} = \sum_j T_{ij}^{kw} = T_{i*}^{kw} \tag{5}$$

whereas an asterisk replacing an index in a parameter simply indicates that a more aggregate form of parameter is required - for example α_ℓ^{k*} instead of the set of parameters $\{\alpha_\ell^{kw}\}$.

T_{ij}^{kw} was defined earlier. The other variables are defined as follows: $W_i^{res.kw}$ is the residential attractiveness of a type k house in i to a member of a w-income group; E_j^w is the (given) number of type-w jobs in j; c_{ij} is the generalised cost of travel from i to j. The $X_{i\ell}^{kw}$'s, ranging over ℓ, are the components of attractiveness and, as shown in (4), they are assumed to be representable in such a way that they can be combined multiplicatively. The other terms are all parameters with the exception of the balancing factors, B_j^w, which are given by equation (2).

Equations (1) - (4) provide the framework of a model which is only completed when the definitions of the $X_{i\ell}^{kw}$ terms are added. We do this shortly. The implicit assumptions of the model can easily be seen. The spatial distribution of jobs by income, $\{E_j^w\}$, is given and these people are allocated to housing in relation to its attractiveness, $W_i^{res.kw}$ and the distance from the workplace, c_{ij}. (A possible modification is to incorporate the 'distance from workplace' effect into the attractiveness function, but we do not pursue that any further here.)

To illustrate the ways in which residential attractiveness can be defined, we use five $X_{i\ell}^{kw}$'s as follows. First, it is clear that $W_i^{res.kw}$ must reflect housing supply, and so we take

$$X_{i1}^{k*} = H_i^k \tag{6}$$

Note that we use an asterisk to replace w rather as we do with parameters. Secondly, we wish to make zones more attractive if they have a higher accessibility to retail centres, and so

$$X_{i2}^{*w} = \sum_{jm} (W_j^m)^{\alpha^{Rwm}} e^{-\beta^{Rwm} c_{ij}} \tag{7}$$

where W_j is a measure of the size of the retail centre of type m at j and α^{Rwm} and β^{Rwm} are parameters for group w which determine the shopping behaviour of that group.

Next, we add two terms which at least crudely describe social aspects of residential location. In this particular example, we assume there are three w groups which can be identified (very roughly) with social class, w=1 being taken as the lowest income (class) group. We define X_{i3}^{*w} and X_{i4}^{*w} to measure the affinity and disaffinity of social groups, their mutual attraction and repulsion. In equation (5), we defined P_i^{kw} and we can use this as P_i^{*w} as follows:

$$X_{i3}^{*1} = P_i^{*1} + P_i^{*2} + P_i^{*3} \tag{8}$$

This means that class 1 is attracted by the presence of all people, while

$$X_{i3}^{*2} = P_i^{*2} + P_i^{*3} \qquad (9)$$

and

$$X_{i3}^{*3} = P_i^{3} \qquad (10)$$

so that the others are attracted by people in the same class or higher, and thus a more restricted population.

The disaffinity terms are defined by:

$$X_{i4}^{*1} = 1/(P_i^{*2} + P_i^{*3})^{\alpha_4^{*1}} \qquad (11)$$

(so that there is some repulsion created by higher groups for group 1)

$$X_{i4}^{*2} = 1/(P_i^{*1})^{\alpha_4^{*2}} \qquad (12)$$

The second group is only repulsed by the lower group. In these two cases we show the parameters explicity for convenience in (18) below. This is because the parameters in the next equation take an unconventional form:

$$X_{i4}^{*3} = 1/[(P_i^{*1})^{\alpha_4^{'*3}} + (P_i^{*2})^{\alpha_4^{''*3}}] \qquad (13)$$

The highest group is repulsed by each of the lower ones, but to different extents determined by the parameters $\alpha_4^{'*3}$ and $\alpha_4^{''*3}$ which replace the simple parameter α_4^{*3} in the overall attractiveness given in equation (18) below.

The fifth element of attractiveness relates to house price and affordability and is, in effect, the term introduced into the disaggregated model by Wilson (1970). The prices are basically taken as exogenous, but are modified to allow overly-high densities to be controlled. Let p_i^k be the price of a type-k house in zone i, q^{kw} be the average amount available for spending by a w-type household on type-k housing, and let $\overline{c}_i^w$ be the average transport costs associated with a w-type household located in zone i. Then we take

$$X_{i5}^{kw} = e^{-|p_i^k - (q^{kw} - \overline{c}_i^w)|} \qquad (14)$$

Note that the modulus is taken, so that if p_i^k differes substantially from $(q^{kw} - \overline{c}_i^w)$ in either direction, than the element of attractiveness X_{ij}^{kw} is low. The overall importance of this factor within $W_i^{res.kw}$ is, of course, determined by the parameters α_5^{kw}, which are in fact taken as α_5^{*w} since they are assumed to be independent of k. $\overline{c}_i^w$ is taken as $\theta^w\overline{c}_i$, where $\overline{c}_i$ is measured in relation to i's accessibility and θ^w is a parameter which will be larger for low income groups than for higher ones. It would, of course, be possible to make $W_i^{res.kw}$ j-dependent also and to build journey to work costs into this term. $\bar{c}_i^w$ could then be replaced by a detailed model of all transport costs. Here, however, we focus mainly on the simplest possible assumption: we take $\bar{c}_i$ as proportioned to the distance of i from the city centre. This implies that transport costs (other than the journey-to-work costs of the principal worker which are allowed for in the $e^{-\mu^w}c_{ij}$ term) increase for progressively more suburban zones. We then use the θ^w parameter (making it high for low income groups, low for high) so that this matters for lower income groups relative to higher, and this becomes part of the mechanism which generates social polarization.

To complete the definition of X_{i5}^{kw}, we need to specify p_i^k. The basic prices are taken as given for our present purposes as $p_i^{k(0)}$ - though we return to this issue in Section 4 below. We do, however, modify them to handle overcrowding. There are many possible ways of doing this. Here we note three relatively simple ones. In each case, above a certain threshold, we add a component to the price which is proportional to the number of houses. The thresholds are defined in different ways: first in relation to housing stock by type; secondly in relation to total housing stock; and thirdly in relation to total housing land use. Thus:

$$(1)\quad p_i^k = \begin{cases} p_i^{k(0)} & , H_i^k < D_i^k \\ p_i^{k(0)} + a^k H_i^k & , H_i^k \geqslant D_i^k \end{cases} \tag{15}$$

$$(2)\quad p_i^k = \begin{cases} p_i^{k(0)} & , \sum_k H_i^k < D_i \\ p_i^{k(0)} + a^k H_i^k & , \sum_k H_i^k \geqslant D_i \end{cases} \tag{16}$$

$$(3)\quad p_i^k = \begin{cases} p_i^{k(0)} & , \sum_k z^k H_i^k < L_i \\ p_i^{k(0)} + a^k H_i^k & , \sum_k z^k H_i^k \geqslant L_i \end{cases} \tag{17}$$

In the first instance, D_i^k is the upper 'non-crowded' limit of type-k houses in i; in the second, D_i is the maximum 'non-crowded' number of houses in total; and in the third case, z^k is unit land use by house type, L_i is the total available land area in i, and so we have a density constraint. a^k is a parameter which determines the 'penalty' increase in house prices in crowded areas. The third kind of constraint is likely to be the best in principle. For our numerical experiemnts, however, we have concentrated on the first two as being simpler and, as we report below, the second as being the most effective of these. In all cases, the upper limit does not necessarily provide a sharp cut-off: more dwellings (say high rise flats) could be built if the revenue could be attracted, as we will see below.

We can now summarise the overall attractiveness factor, showing the parameters at the level of resolution adopted in each case:

$$W_i^{res.kw} = (X_{i1}^{k*})^{\alpha_1^{k*}} (X_{i2}^{*w})^{\alpha_2^{*w}} (X_{i3}^{*w})^{\alpha_3^{*w}} (X_{i4}^{*w})(X_{i5}^{kw})^{\alpha_5^{*w}} \quad (18)$$

The parameters associated with X_{i4}^{*w} were made part of the definition of those terms in equations (11) - (13) because of their unconventional form. And, although the right hand side of (14) shows terms varying with k and w, we make the assumption shown in (18) that the parameters are dependent on w only.

We have now finally specified a particular residential location model. It is obviously oversimplified and is for illustrative purposes only; however, it does have some useful properties. It is job-based and takes account of the distance from the workplace through the parameters μ^w. The attractiveness of a particular house type at a particular location then depends on the five factors in (18), each of which is potentially interesting. The affinity and disaffinity terms, for example, could produce bifurcations - the so-called 'tipping' phenomena - which will add to those produced as changes in the housing pattern. To examine these we need to look at the $\{P_i^{kw}\}$ array defined in (5) above.

The next major stage in the argument is to develop a suitable supply side model for test purposes. We work by analogy with the retail model (cf. Harris and Wilson, 1978) but with an appropriately modified formulation. We can take $\sum_{jw} T_{ij}^{kw} q^{kw}$ as the average revenue attracted to type k housing in i and $H_i^k p_i^k$ as a measure of the price. We could therefore expect H_i^k to grow according to something like the following:

$$\dot{H}_i^k = \varepsilon^k \left[\sum_{jw} T_{ij}^{kw} q^{kw} - H_i^k p_i^k\right] H_i^k \tag{19}$$

and this obviously has as an equilibrium condition

$$\sum_{jw} T_{ij}^{kw} q^{kw} = H_i^k p_i^k \tag{20}$$

It would be possible to solve the differential equations (19) - or equivalent difference equations - numerically; but in the rest of this paper we focus on the equilibrium patterns which are the solutions of (20). It is easy to see that these equations are simultaneous equations in $\{H_i^k\}$ and that, after making all the relevant substitutions, starting with T_{ij}^{kw} from (1), they are highly nonlinear. This guarantees the existence of bifurcation phenomena: that the nature of the equilibrium pattern changes at critical values of the parameters.

One difficulty in taking the analysis further is that there is such a large number of parameters. They are summarised in Table 1 along with their computer mnemonics. As we have found in our study of other sectors, the only way to make analytical progress is through numerical experiments. We describe these in the next section. If we work with 3 house types, 3 income groups, and 3 retail types, then the table shows that the model has 44 parameters. Naturally enough, we are forced into a position where we can consider the effects of varying relatively few of these, and then not very systematically.

We run the model in two modes. The first is holistic: we can simply solve (20) for the $\{H_i^k\}$ pattern, and this gives much insight on the way different combinations of parameters generate particular types of pattern. The second is incremental. In this case, we recognise first that various parameters and exogenous variables which form the backcloth to the model change over time; and secondly that the pattern at one time will be to some extent a function of structures at previous times. The model can then be run in the following steps:

(i) Derive an initial value of exogenous variables and parameters and $\{H_i^k\}$. These may well reflect some known historical situation.

(ii) Update parameter values and exogenous variables.

(iii) Run the model and calculate provisional values of $\{H_i^k\}$, say $\{H_i^{kprov}\}$.

(iv) Set

$$H_i^k = H_i^{kprov} \tag{21}$$

$$g'H_i^{k(prov)} < H_i^{kprov} < g''H_i^{k(prov)} \tag{22}$$

where $H_i^{k(prov)}$ is the value from the previous iteration, or set

$$H_i^k = \begin{cases} g'H_i^{k(prov)} & \text{if } H_i^{kprov} < g'H_i^{k(prov)} \\ g''H_i^{k(prov)} & \text{if } H_i^{kprov} > g''H_i^{k(prov)} \end{cases} \tag{23}$$

In words: take H_i^k from the model unless it infringes the constraint (22); otherwise set it to the appropriate upper or lower bound. The constraints g' and g'' measure the amount of inertia in the system. If g' is near to zero and g'' is large, there is no inertia; g' and g'' each close to 1 means almost total inertion. It would also be possible to extend this idea further and apply it to the $\{T_{ij}^{kw}\}$ variables to add a degree of inertia to them but we do not do that for these particular numerical experiments.

Most of our results below are for straightforward equilibrium patterns and are thus the outcome of step (iii). In the incremental mode, each outcome of step (iv) is an element of the sequence.

3. Numerical Experiments

3.1 Introduction

We have found from previous experience that different combinations of parameter values produce a great variety of spatial structures. The only way at present to explore these is through numerical experiments. This means that we use an idealised spatial system - in this case of 169 zones on a grid basis - and hypothetical data and we examine the spatial structures which are generated by different sets of parameter values. Experience shows that the easiest results to produce look rather nonsensical. Much prior experimentation, therefore, goes into producing the kinds of results presented here to ensure that they look like examples of possible real structures.

We have already mentioned the difficulties which arise from having such a high-dimensional parameter space. This makes it difficult to organise the presentation of the results. It is also appropriate to mention at this stage that much time is consumed on the computer and its associated graph plotters. The results presented here were obtained on an

AMDAHL VM470 and our experience emphasises the need for access to large fast machines for this kind of research.

With three house types and three person groups, and with variations in exogenous variables such as the employment distribution being an important determinant of the results, a full presentation of the results of a run involved fifteen plots in this case: the underlying employment distribution, the distributions of total housing stock, housing stock by type (H_i^k - three plots) and the allocation of people by type to the housing stock (T_{ij}^{kw} - nine plots). In some cases, for economy, we do not present the full range of outputs.

One of our early tests was on the form of density constraint to use - from equations (15) - (16). As noted earlier, it turned out, as we would expect, to be more satisfactory to use the 'total' constraint (16). To see this, and to take the opportunity for a preliminary presentation of the form of our results, consider Figure 1. In Figure 1(a), the density constraints are applied to each type separately and the result, for these parameters, is a central orientation for each house type. When the constraint is applied to total housing (with the same parameter values), we get Figure 1(b). There is now only sufficient capacity in the centre for type 1 housing and so the spatial structure is very different. This particular structure arises because the poorer people need to live in the centre to minimise the transport cost element within the bid rent function.

The rest of our numerical experiments are presented in Section 3.2 below. We focus on the following variations:

(a) Figure 2. μ - changes, which in effect alter the importance of the access to employment.

(b) Figure 3. Here we vary β^w which, as seen in the analysis following equation (14), measures the significance of reducing bid rents in relation to average transport costs for different groups.

(c) Figure 4. We vary the cost of housing, $P_i^{k/d}$; and at this stage it is useful to present spatial distribution of population by house type as well as housing stock.

(d) Figure 5. We explore the effects of the affinity and disaffinity parameters.

(e) Figure 6. We add a rudimentary structure to the underlying transport system by allowing for cheaper trips to the city centre and we assess the effects of this in relation to different underlying employment distributions.

(f) Figure 7. Here, we change the proportions in the different employment groups.

(g) Figure 8. We present a number of incremental runs.

3.2 Results of Numerical Experiments

(a) Figure 2(a) shows structures and population distributions for a set of high μ-values; Figure 2(b) for low. In each case, the parameter takes a relatively higher value for lower income groups. The elements of Figure 2(a)i are: the given employment distribution (E_j^*), the resulting total housing distribution (H_i^*) and the distribution of housing by type (H_i^k). In Figure 2(a)ii, we present p_i^{kw}-distributions. The results are as one would expect but quite striking. In the first case, the high μ's force housing stock to be located mainly near to employment and the population distribution reflects this. In the second case, the lowest housing class is largely concentrated in the centre, probably through a combination of the attraction of the main employment centre together with the working of the affinity and disaffinity parameters. Figure 2(b)ii shows more concentration of the lowest income group and greater dispersion of the higher two (the highest more than the middle) relative to Figure 2(a)ii.

(b) In Figure 3, we explore the effect of varying θ^w which scales the transport cost element of the bid rent function. In the first example, the two lowest income grops, with high θ^w, are concentrated in the centre and there is increasing dispersion in parts (b) and (c) of the figure as the θ^w parameters are lowered.

(c) In Figure 4, we vary housing costs, $p_i^{k(0)}$, higher in part (a), lower in part (b). This seems to have little effect on the cheapest housing and the distribution of the lowest income group. When prices are higher, there is more spatial dispersion of the upper two house types, probably because with higher p_i^k, the bid rent function plays a smaller role. The distribution of the populations changes in different ways for the middle and high groups. In the former case, there is more dispersion for lower prices; in the latter, vice versa.

It is difficult to disentangle exactly which elements of the attractiveness function produce these effects.

(d) The first point to note about Figure 5 is the obvious working of the disaffinity factors. If the (1,1) and (3,3) positions on the two population distributions are compared, they are seen to be almost complementary. Figure 5(a) has much higher affinity parameters than 5(b); and slightly higher disaffinity parameters. An examination of the figures suggests that the slight change in disaffinity in this case has a greater affect than the change in affinity.

(e) In Figure 6, we explore the effects of cheapening trips to and from the city centre to represent the effect, say, of a radial road system. These experiments are conducted for a number of different employment distributions. In parts (a) and (b), we start with a uniform distribution of employment, without and with factoring (by a factor of 0.7). The change is clear: from a non-centrally oriented structure to a central one. In parts (c) - (e), we return to the non-uniform distribution used earlier with no-factoring, and factors of 0.9 and 0.7 respectively. Because of the high μ values for the two lowest income groups, the effect on the two lowest housing types is minimal. In the case of the highest income group and housing type, presumably because of disaffinity, there is a 'hole' in the centre until the factor reaches 0.7 when even the best housing is found in the centre. In parts (f) and (g), we stick with the non-uniform distribution but remove employment from the central zone. A factor of 0.7 still produces central zone housing because of the relative cheapness of access to employment from there.

(f) In Figure 7, we show the effects of changing the overall mix of employment types, starting with dominance of the lowest income group in part (a), a uniform distribution in (b) and a majority in the highest income group in (c). The type 1 housing distribution changes little in the sequence and follows the employment distribution. Type 2 housing becomes more dispersed and type 3 housing more concentrated as higher income groups increase numerically. This presumably arises from the effect of affinity and disaffinity factors.

(g) Parts (a) - (d) of Figure 8 show a preliminary incremental run.

μ-values are assumed to decrease, affinity and disaffinity parameters to increase and the employment distribution to spread out over time. We solve the equilibrium problem for successive time periods, but we also constrain the possible rates of change using the mechanism set out in equation (22). The g' and g'' parameters are 0.7 and 1.3 respectively. (There is one exception: where no housing exists in a zone in a previous period, there is no 'inertial' constraint.) There are some striking effects. First, type 1 housing becomes progressively more concentrated, while the central elements of types 2 and 3 housing are reduced.

4. Concluding Comments

We hope that the results presented demonstrate the rich variety of patterns, for both housing and population distributions, which can be generated. We have only tackled a small fraction of the possible range and an important research topic for the larger run will be the development of pattern recognition algorithms so that the range of structures can be 'searched' more systematically. So far, it seems that when a single parameter is varied, the results can be interpreted and are in the expected direction. However, when more than one is changed simultaneously, as we have already seen, interpretation becomes more difficult. We also need to expand our analysis so that the methods of Harris and Wilson (1978) can be used. Again, this is a difficult research problem since this model is obviously more complicated than the retail case.

There are other extensions to consider also. In a later paper, we will report on the incorporation of this model into a more comprehensive 'interacting fields' framework. This would have the effect of making the retail variables, for example, endogenous. We are also aware that we should seek to make other parameters and variables endogenous, especially the house prices, p_i^k. The ideas of this kind of modelling could then be linked with those of urban economists.

Finally, of course, we note that a major problem is to find ways of testing these kinds of models empirically. This has been begun for the retail case (Clarke, 1983) but not yet for residential location and housing.

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TABLE 1: Table of Parameters

Parameter	Relevant Equation No.	No of parameter when there are 3k's and 3w's	Computer mnemonic
μ^{w}	1	3	BETA
α_1^{k*}	18	3	GM
α_2^{*w}	18	3	GM1
α_3^{*w}	18	3	AF
α_4^{*w}	11,12	2 (NB $\alpha_4^3=1$)	SG
$\alpha_4^{'*w}$	13	1	SG
$\alpha_4^{''*w}$	13	1	SG
α_5^{*w}	18	3	XMV
α^{Rwm}	7	3 (NB taken as α^{R*m})	ALF
β^{Rwm}	7	3 (NB taken as μ^{w})	BETA
D_i	16	1	-
a^{k}	16	3	XLF
$p_i^{k(0)}$	16	3 (assumed independent of i)	XLB
q^{kw}	14	9	G
θ^{w}	14	3	BRF
g', g''	22	2	GL,GU
λ (City centre factor)	1	1	LAM
ρ^{k} (proportion in different employment groups)	3	3	RO

TABLE 2: Values of Parameters for Incremental Runs

Parameter (mnemonic - see Table 1)	RUN 1				RUN 2				RUN 3				RUN 4			
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
BETA	1.75	1.5	0.75		1.5	1.25	0.6		1.25	1.0	0.5		1.0	0.75	0.4	
GN	0.95	0.975	1.1		-	-	-		-	-	-		-	-	-	
GN1	1.0	1.0	1.0		-	-	-		-	-	-		-	-	-	
AF	0.55	0.6	0.65		0.65	0.7	0.75		0.75	0.8	0.85		-	-	-	
SG	0.65	0.7	0.725	0.85	0.75	0.8	0.825	0.95	0.85	0.9	0.925	0.975	-	-	-	
SMV	0.95	0.99	1.0		-	-	-		-	-	-		-	-	-	
ALF	1.15	1.0	0.8		-	-	-		-	-	-		-	-	-	
XLF	0.3	0.4	0.9		-	-	-		-	-	-		-	-	-	
XLB	1.0	1.2	1.4		-	-	-		-	-	-		-	-	-	
BRF	1.0	0.5	0.25		-	-	-		-	-	-		0.75	0.4	0.2	
GL	0.7	0.7	0.7		-	-	-		-	-	-		-	-	-	
GU	1.3	1.3	1.3		-	-	-		-	-	-		-	-	-	

A dash indicates no change from previous run.

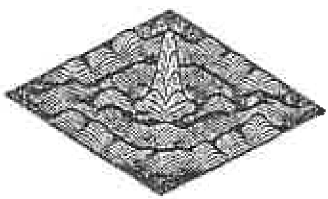
$\beta^1 = 3.5$
$\gamma^1 = 0.9$
$\alpha_1^w = 1.0$
$a_1 = 0.05$

H_i^1

$\beta^2 = 3.0$
$\gamma^2 = 0.95$
$\alpha_2^w = 1.0$
$a_2 = 0.05$

H_i^2

$\beta^3 = 2.5$
$\beta^3 = 0.99$
$\alpha_3^w = 1.0$
$a_3 = 0.05$

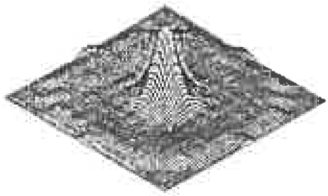


H_i^3

$D_i^k = 10{,}000$

FIGURE 1(a): Housing Distribution with Type I Density Factor.

$\beta^1 = 3.5$
$\gamma^1 = 0.9$
$\alpha_1^w = 1.0$
$a_1 = 0.05$

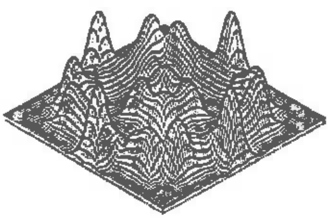


H_1^1

$\beta^2 = 3.0$
$\gamma^2 = 0.95$
$\alpha_2^w = 1.0$
$a_2 = 0.05$

H_1^2

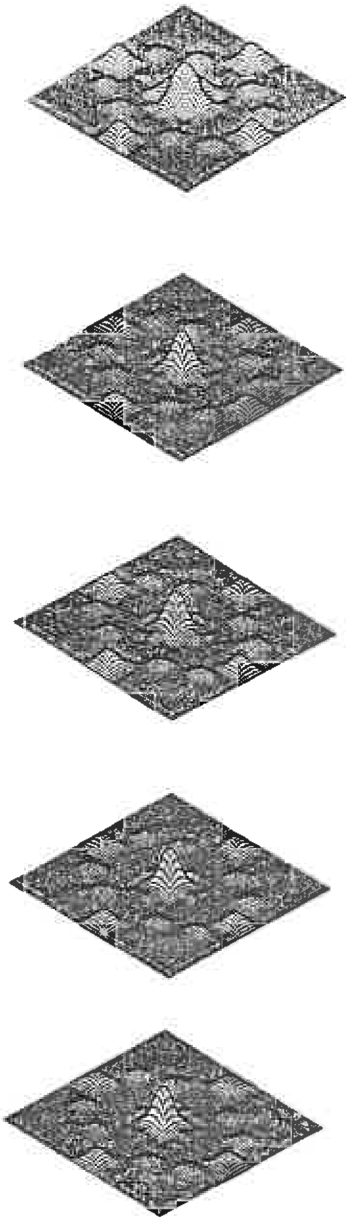
$\beta^3 = 2.5$
$\gamma^3 = 0.99$
$\alpha_3^w = 1.0$
$a_3 = 0.05$



H_1^3

$D_1 = 10,000$

<u>FIGURE 1(b)</u>: Housing Distribution with Type II Density Factor.



E_j^*

H_i^*

H_i^1

$\beta^1 = 3.75$
$\beta^2 = 3.5$
$\beta^3 = 2.5$

H_i^2

H_i^3

FIGURE 2(a)i: Housing Distribution with High β^W.

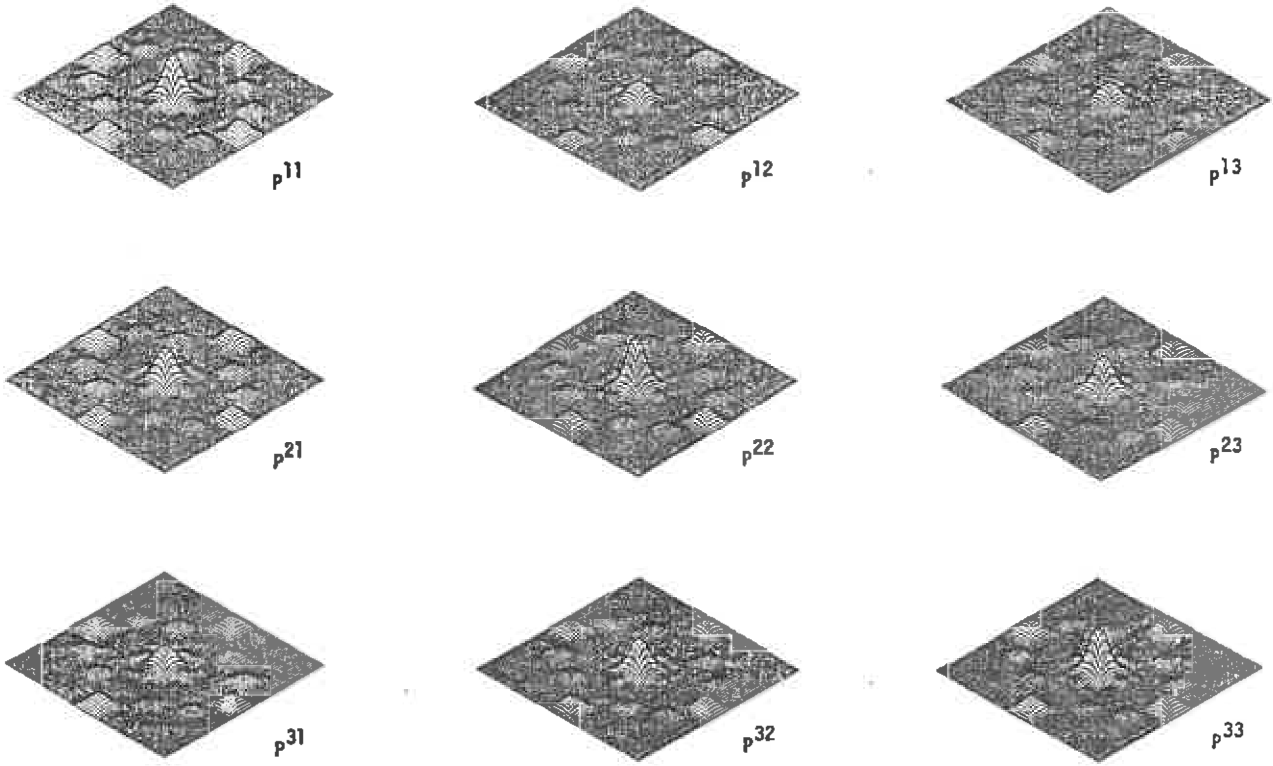


FIGURE 2(a)ii: Population Group By Housing Type with High β^W.

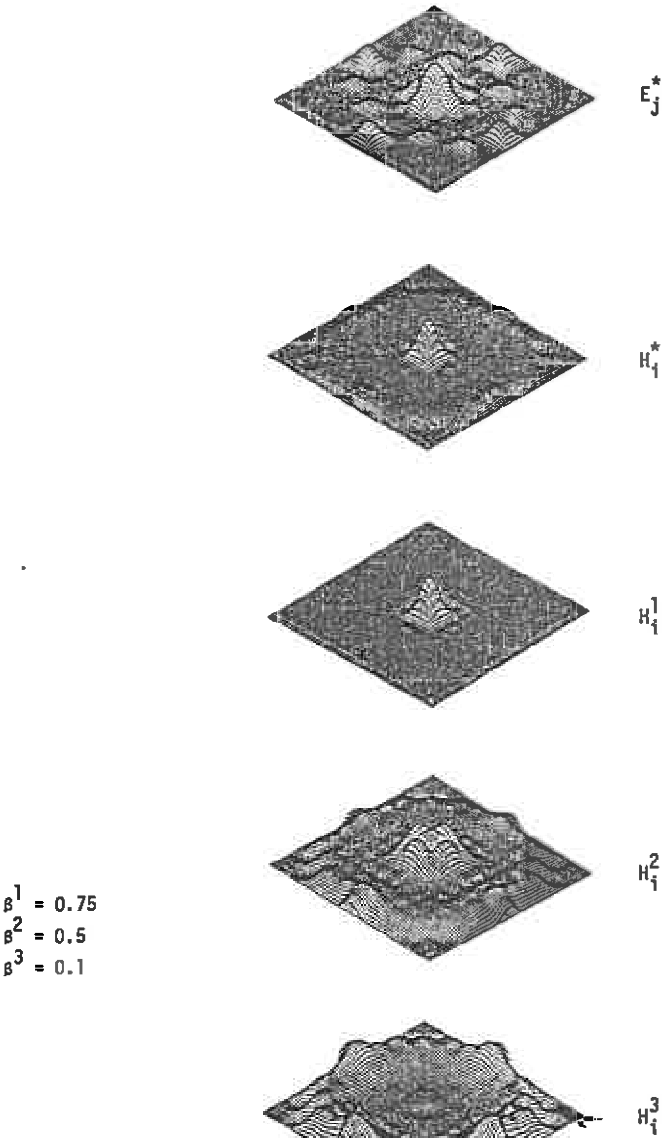


FIGURE 2(b)i: Housing Distribution with Low β^W.

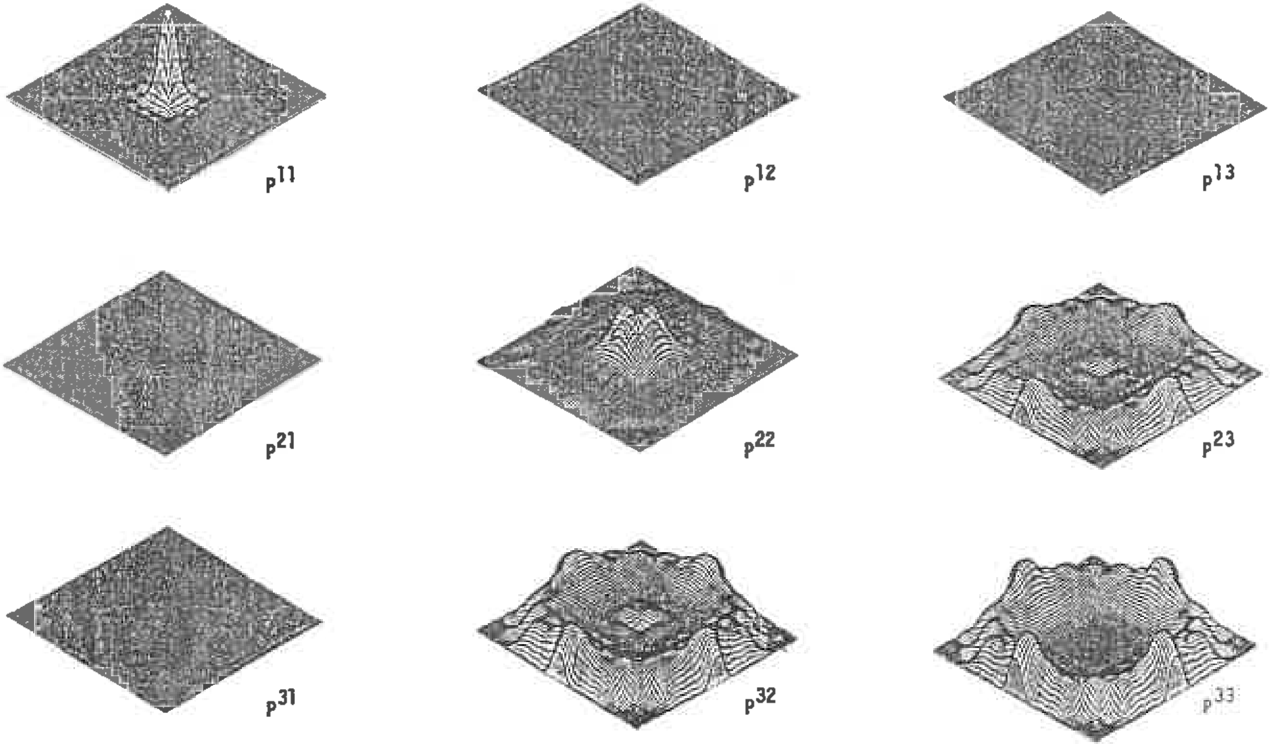


FIGURE 2(b)ii: **Population Group by Housing Type with Low β^W.**

H_i^1

H_i^2

H_i^3

$\theta^1 = 3.5$
$\theta^2 = 2.5$
$\theta^3 = 0.05$

FIGURE 3(a): Housing Distribution with High θ^W for Population Groups 1 and 2.

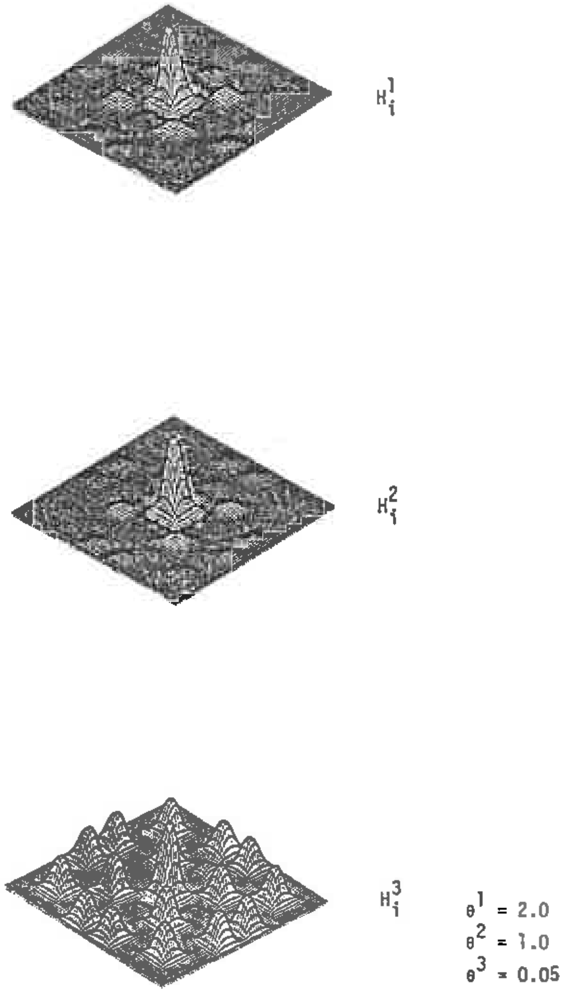
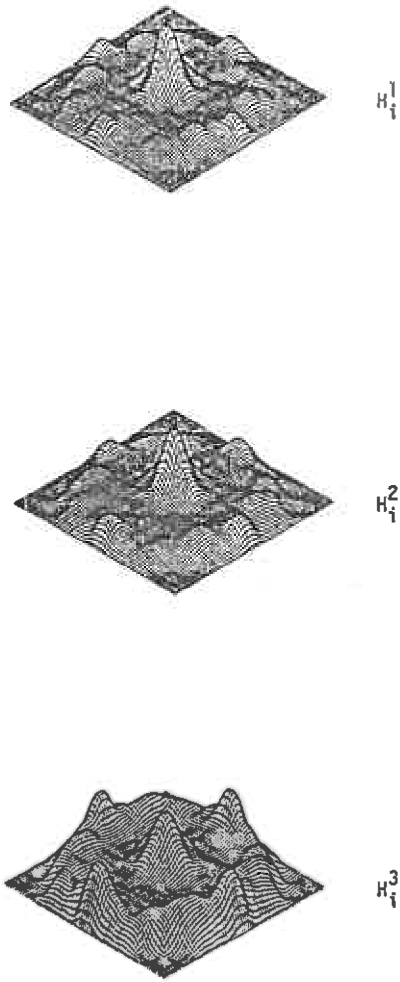


FIGURE 3(b): Housing Distribution with Lower Values of θ^w for Population Groups 1 and 2.



$\theta^1 = 0.3$
$\theta^2 = 0.2$
$\theta^3 = 0.05$

<u>FIGURE 3(c)</u>: Housing Distribution Under Much Lower Values of θ^W for Population Groups 1 and 2.

E_j^*

$\beta^1 = 1.75$
$\beta^2 = 1.25$
$\beta^3 = 0.75$

H_i^*

$p_i^1 = 1.3$ H_i^1

$p_i^2 = 1.5$ H_i^2

$p_i^3 = 1.7$ H_i^3

FIGURE 4(a)i: Distribution of Housing with High p_i^k Values.

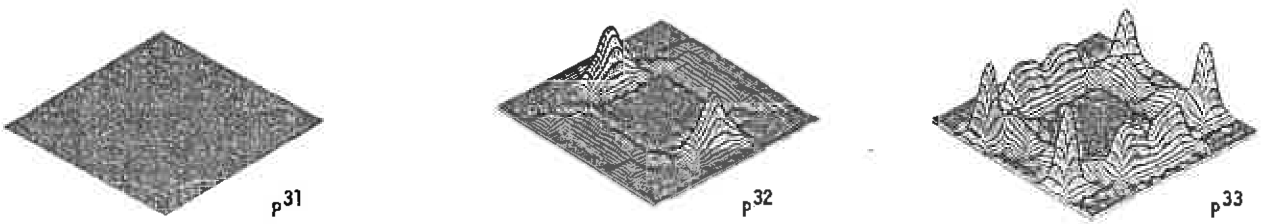
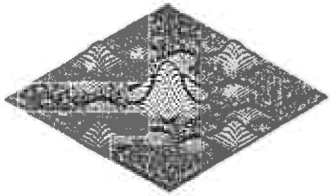
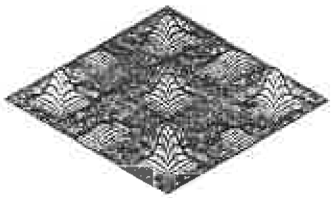


FIGURE 4(a)ii: Population Group by Housing Type with High p_k^i Values.

$\beta^1 = 1.75$
$\beta^2 = 1.25$
$\beta^3 = 0.75$



E_j^*



H_i^*

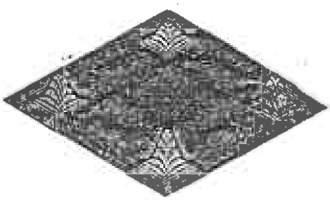
$p_i^1 = 1.1$

H_i^1

$p_i^2 = 1.2$

H_i^2

$p_i^3 = 1.3$



H_i^3

FIGURE 4(b)i: Distribution of Housing with Low p_i^k Values.

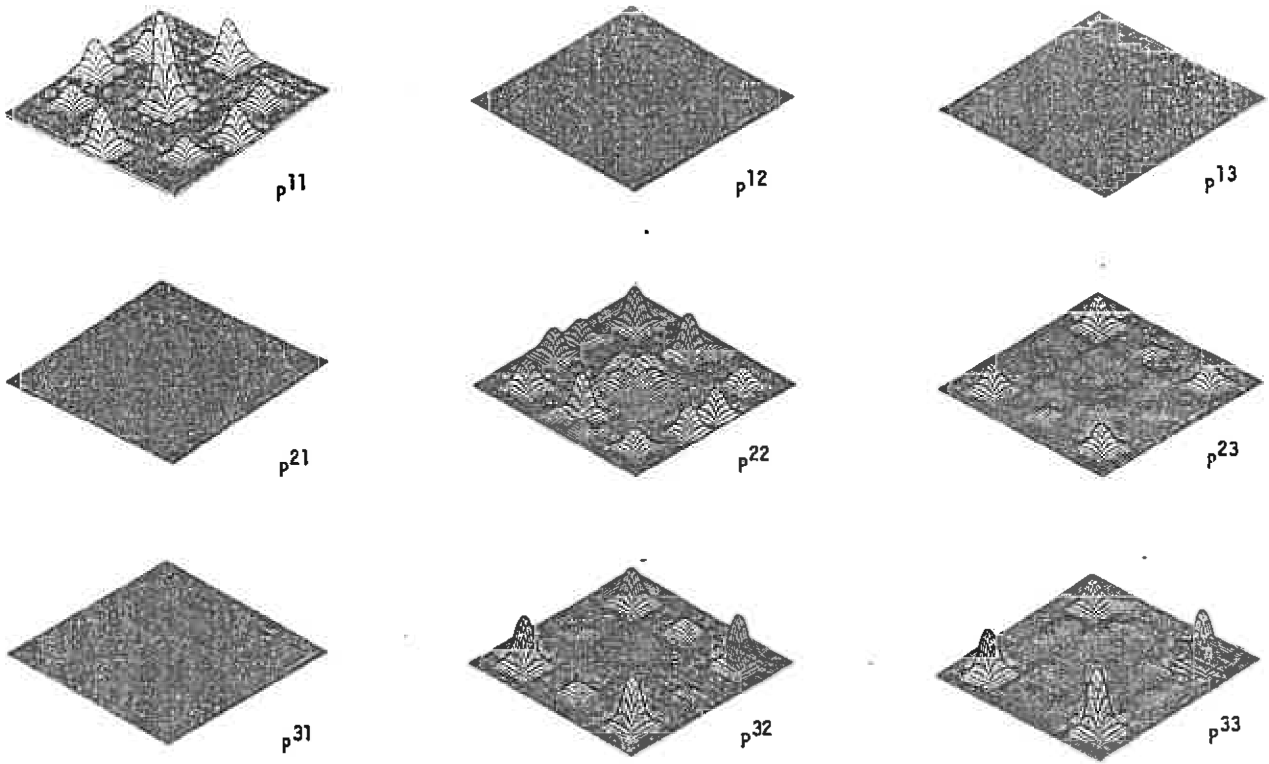
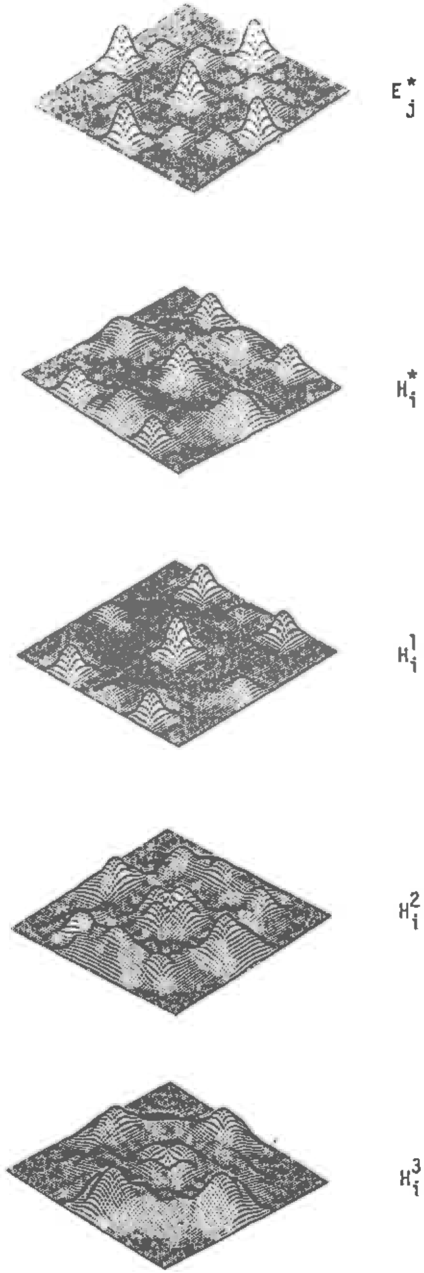


FIGURE 4(b)ii: Population Group by Housing Type with Low p_i^k Values.



$\beta^1 = 1.75$
$\beta^2 = 1.5$
$\beta^3 = 1.0$

$\alpha_3^{*1} = 0.7$
$\alpha_3^{*2} = 0.7$
$\alpha_3^{*3} = 0.7$
$\alpha_4^1 = 0.8$
$\alpha_4^2 = 0.8$
$\alpha_4'^3 = 0.84$
$\alpha_4''^3 = 0.86$

<u>FIGURE 5(a)i:</u> Housing Distribution with High α_3^{*w} and α_4^{*w} Parameters.

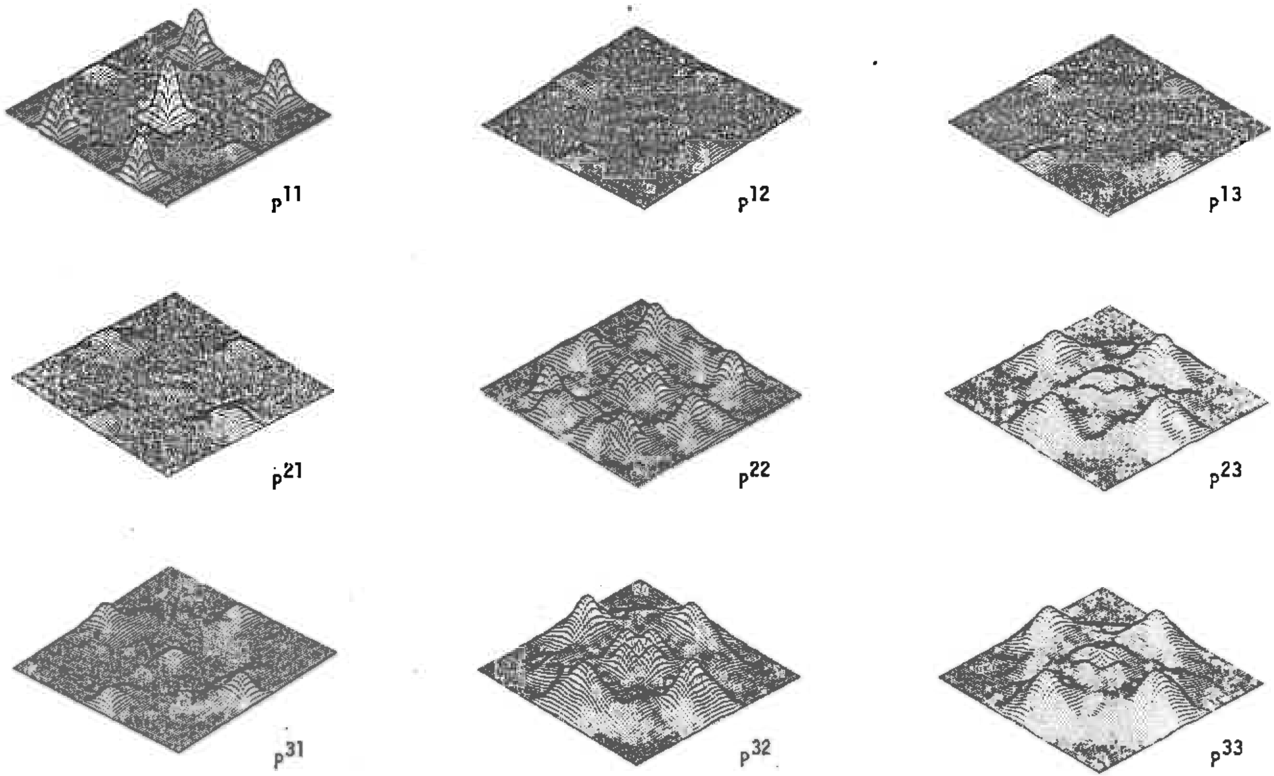
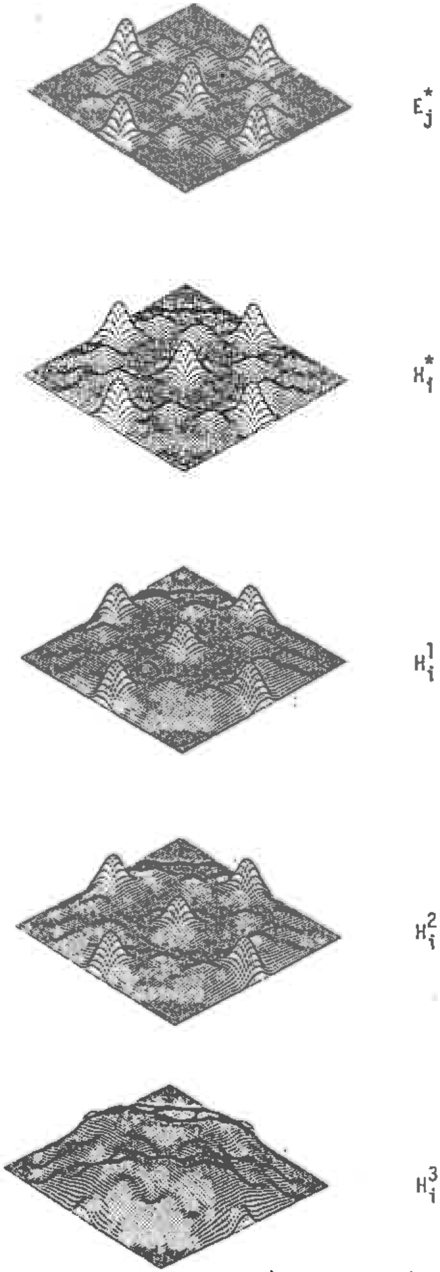


FIGURE 5(a)ii: Population Group by Housing Type with High α_3^{*w} and α_4^{*w} Parameters



$\beta^1 = 1.75$
$\beta^2 = 1.5$
$\beta^3 = 1.0$
$\alpha_3^{*1} = 0.001$
$\alpha_3^{*2} = 0.001$
$\alpha_3^{*3} = 0.001$
$\alpha_4^1 = 0.70$
$\alpha_4^2 = 0.75$
$\alpha_4^{'3} = 0.75$
$\alpha_4^{''3} = 0.8$

FIGURE 5(b)i: Housing Distribution with Low α_3^{*w} and High α_4^{*w} Parameters.

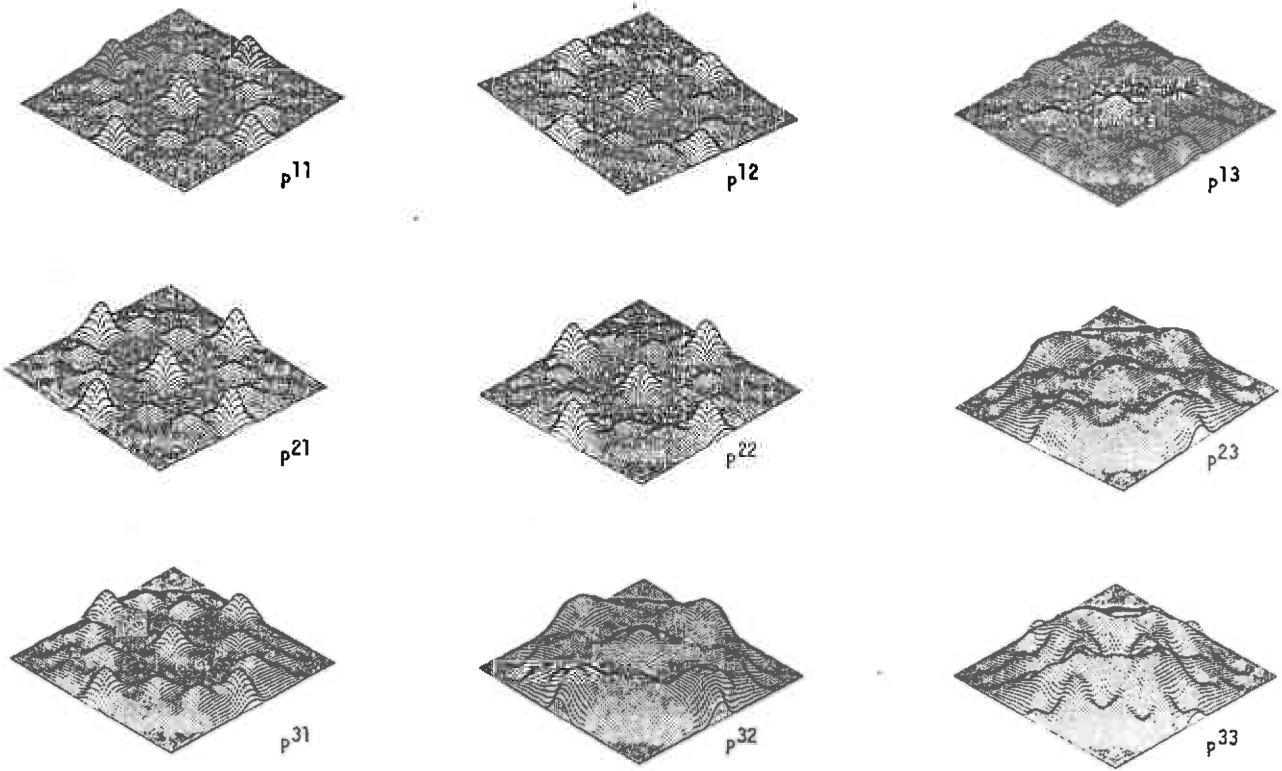
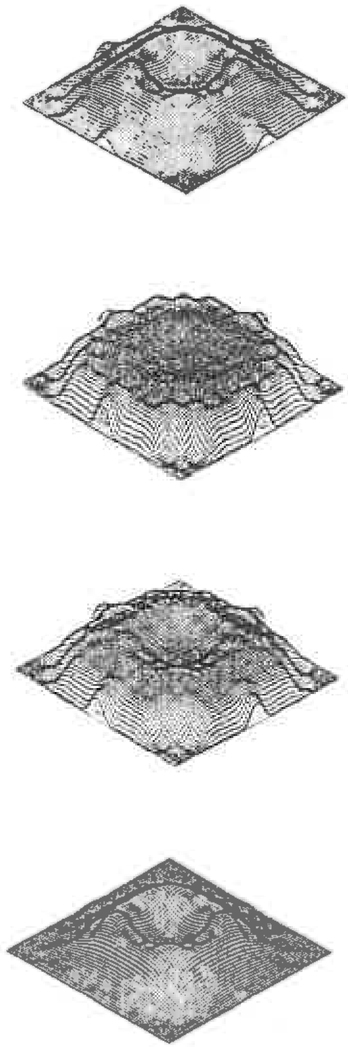


FIGURE 5(b)ii: Population Group by Housing Type with Low α_3^{*w} and High α_4^{*w} Parameters.



H_i^*

H_i^1

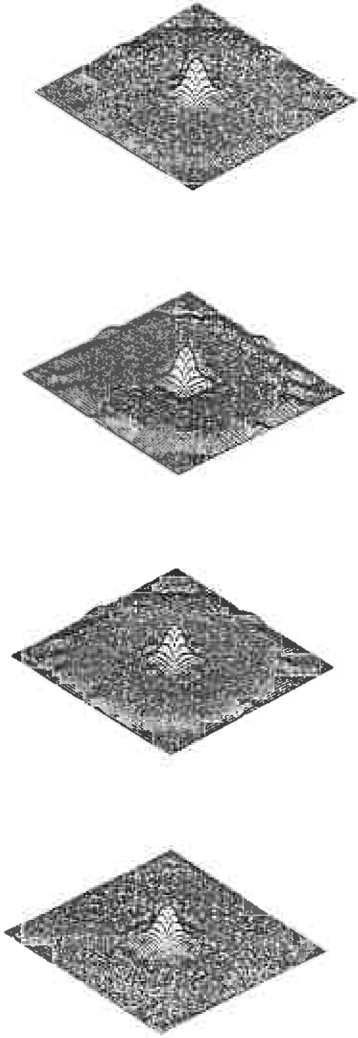
H_i^2

H_i^3

$\beta^1 = 3.5$
$\beta^2 = 3.25$
$\beta^3 = 0.75$

$\alpha_3^{*1} = 0.4$
$\alpha_3^{*2} = 0.33$
$\alpha_3^{*3} = 0.45$
$\alpha_4^1 = 0.7$
$\alpha_4^2 = 0.75$
$\alpha_4^{'3} = 0.75$
$\alpha_4^{''3} = 0.8$

<u>FIGURE 6(a):</u> Housing Distribution with Uniform Distribution of E_j and no City Centre Factoring



H_i^*

H_i^1

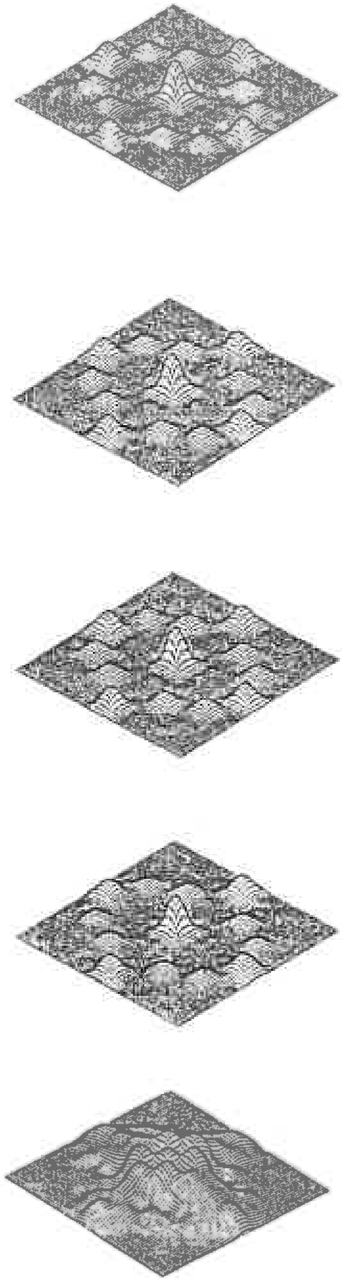
H_i^2

H_i^3

$\beta^1 = 3.5$
$\beta^2 = 3.25$
$\beta^3 = 0.75$

$\alpha_3^{*1} = 0.4$
$\alpha_3^{*2} = 0.33$
$\alpha_3^{*3} = 0.45$
$\alpha_4^1 = 0.7$
$\alpha_4^2 = 0.75$
$\alpha_4^{'3} = 0.75$
$\alpha_4^{''3} = 0.8$

FIGURE 6(b): Housing Distribution with Uniform Distribution of E_j and City Centre Factoring of 0.7.



E_j^*

H_i^*

H_i^1

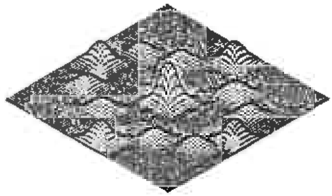
H_i^2

H_i^3

$\beta^1 = 3.5$
$\beta^2 = 3.25$
$\beta^3 = 0.75$

$\alpha_3^{*1} = 0.4$
$\alpha_3^{*2} = 0.33$
$\alpha_3^{*3} = 0.45$
$\alpha_4^1 = 0.7$
$\alpha_4^2 = 0.75$
$\alpha_4^{'3} = 0.75$
$\alpha_4^{''3} = 0.8$

<u>FIGURE 6(c)</u>: Non Uniform Distribution of E_j No City Centre Factoring



E_j^*

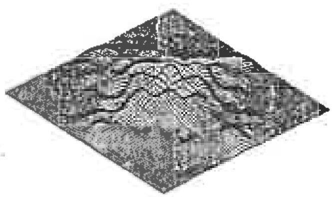
H_i^*

H_i^1

$\beta^1 = 3.5$
$\beta^2 = 3.25$
$\beta^3 = 0.75$

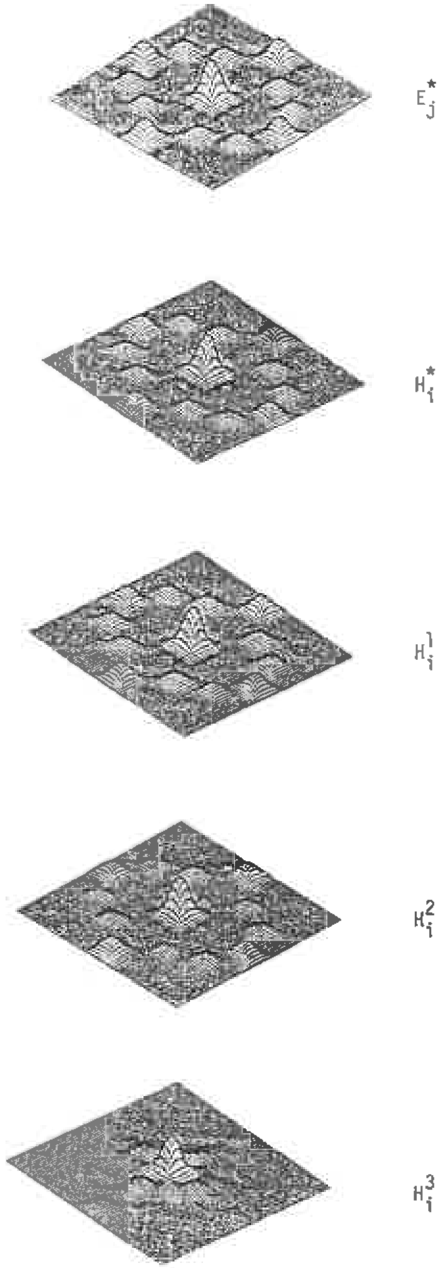
$\alpha_3^{*1} = 0.4$
$\alpha_3^{*2} = 0.33$
$\alpha_3^{*3} = 0.45$
$\alpha_4^1 = 0.7$
$\alpha_4^2 = 0.75$
$\alpha_4^{'3} = 0.75$
$\alpha_4^{''3} = 0.8$

H_i^2



H_i^3

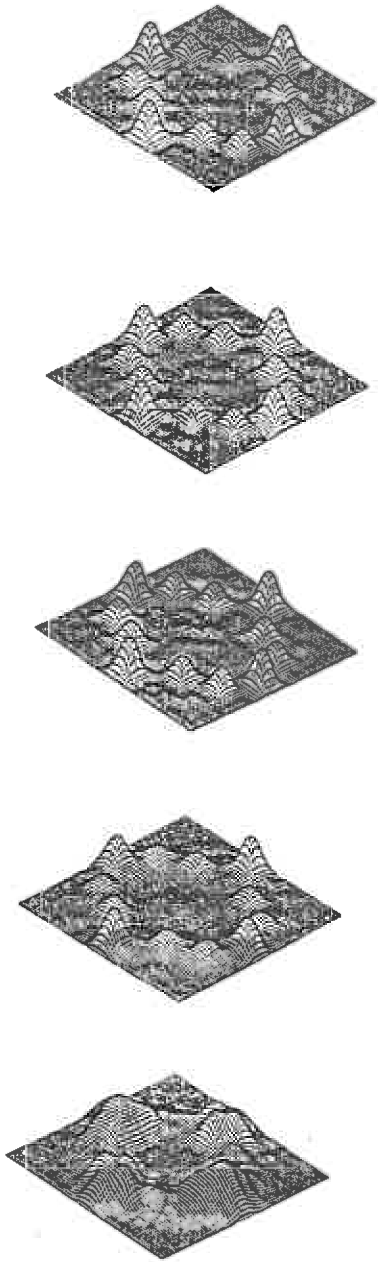
FIGURE 6(d): Non Uniform Distribution of E_j City Centre Factoring 0.9.



$\beta^1 = 3.5$
$\beta^2 = 3.25$
$\beta^3 = 0.75$

$\alpha_3^{*1} = 0.4$
$\alpha_3^{*2} = 0.33$
$\alpha_3^{*3} = 0.45$
$\alpha_4^{1} = 0.7$
$\alpha_4^{2} = 0.75$
$\alpha_4^{'3} = 0.75$
$\alpha_4^{''3} = 0.8$

<u>FIGURE 6(e)</u>: Non Uniform Distribution of E_j - City Centre Factoring of 0.7.



E_j^*

H_i^*

H_i^1

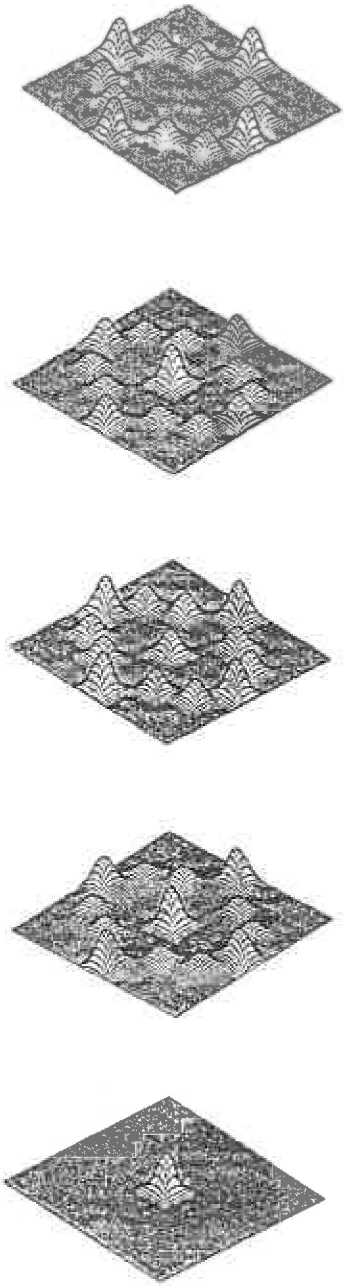
H_i^2

H_i^3

$\beta^1 = 3.5$
$\beta^2 = 3.25$
$\beta^3 = 0.75$

$\alpha_3^{*1} = 0.4$
$\alpha_3^{*2} = 0.33$
$\alpha_3^{*3} = 0.45$
$\alpha_4^{1} = 0.7$
$\alpha_4^{2} = 0.75$
$\alpha_4^{'3} = 0.75$
$\alpha_4^{''3} = 0.8$

<u>FIGURE 6(f)</u>: Non-Uniform No City Centre E_j - No City Centre Factoring.



E_j^*

H_i^*

H_i^1

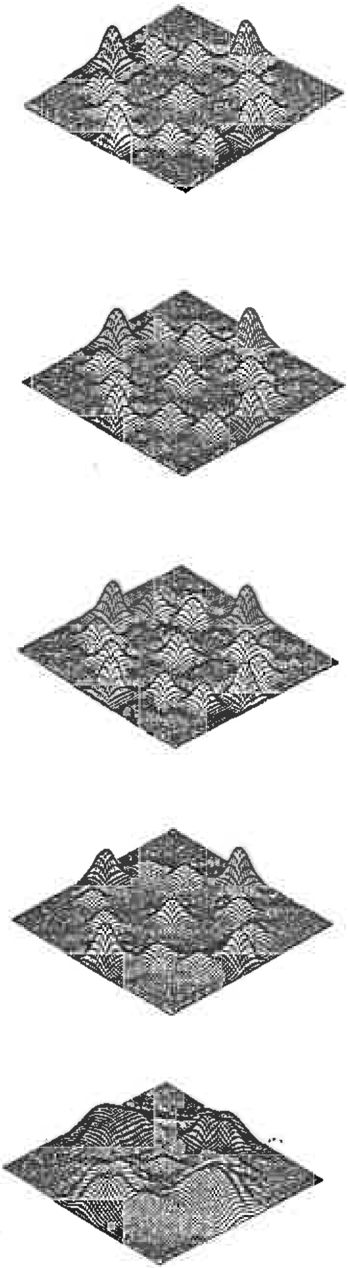
H_i^2

H_i^3

$\beta^1 = 3.5$
$\beta^2 = 3.25$
$\beta^3 = 0.75$

$\alpha_3^{*1} = 0.4$
$\alpha_3^{*2} = 0.33$
$\alpha_3^{*3} = 0.45$
$\alpha_4^1 = 0.7$
$\alpha_4^2 = 0.75$
$\alpha_4^{'3} = 0.75$
$\alpha_4^{''3} = 0.8$

FIGURE 6(g): Non Uniform No City Centre E_j - City Centre Factoring 0.7.



E_j^*

H_i^*

H_i^1

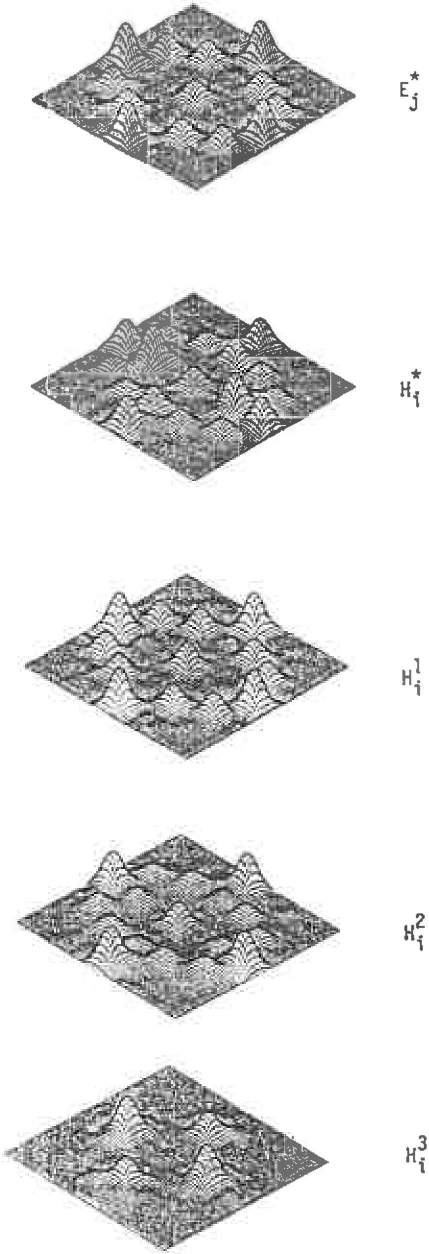
H_i^2

H_i^3

$\beta^1 = 3.5$
$\beta^2 = 3.25$
$\beta^3 = 0.75$

$\alpha_3^{*1} = 0.4$
$\alpha_3^{*2} = 0.33$
$\alpha_3^{*3} = 0.45$
$\alpha_4^1 = 0.7$
$\alpha_4^2 = 0.75$
$\alpha_4^{'3} = 0.75$
$\alpha_4^{''3} = 0.8$

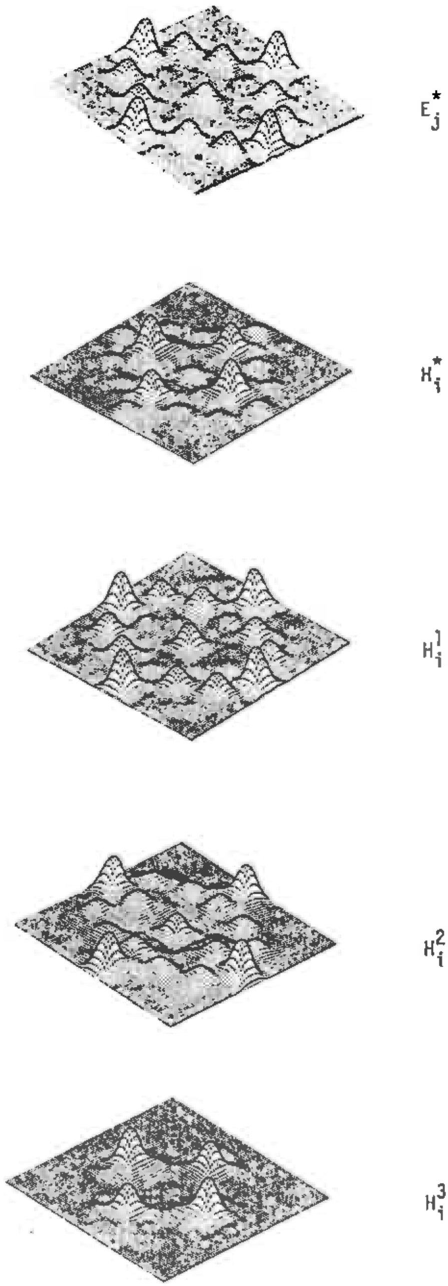
FIGURE 7(a): Housing Distribution Under Employment Split Assumtion 1
Ratio of $E_j^1 : E_j^2 : E_j^3$ 200-100-50



$\beta^1 = 3.5$
$\beta^2 = 3.25$
$\beta^3 = 0.75$

$\alpha_3^{*1} = 0.4$
$\alpha_3^{*2} = 0.33$
$\alpha_3^{*3} = 0.45$
$\alpha_4^1 = 0.7$
$\alpha_4^2 = 0.75$
$\alpha_4^{'3} = 0.75$
$\alpha_4^{''3} = 0.8$

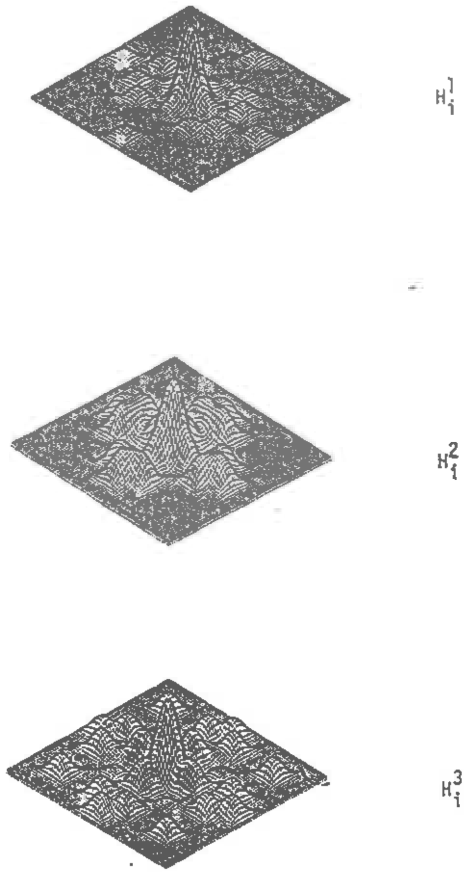
FIGURE 7(b): Housing Distribution Under Employment Split Assumption 2
Ratio of $E_j^1:E_j^2:E_j^3$ 100-100-100



$\beta^1 = 3.5$
$\beta^2 = 3.25$
$\beta^3 = 0.75$

$\alpha_3^{*1} = 0.4$
$\alpha_3^{*2} = 0.33$
$\alpha_3^{*3} = 0.45$
$\alpha_4^1 = 0.7$
$\alpha_4^2 = 0.75$
$\alpha_4^{'3} = 0.75$
$\alpha_4^{''3} = 0.8$

FIGURE 7(c): Housing Distribution Under Employment Split Assumtion 3
Ratio of $E_j^1:E_j^2:E_j^3$ 50:100:200



<u>FIGURE 8(a)</u>: Incremental Run 1
For parameter values see Table 1.

H_i^1

H_i^2

H_i^3

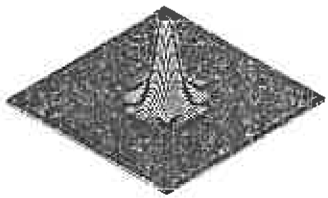
FIGURE 8(b): Incremental Run 2.

H_i^1

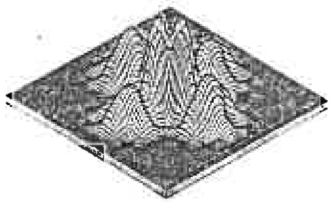
H_i^2

H_i^3

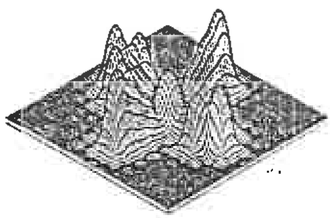
FIGURE 8(c): Incremental Run 3.



H_i^1



H_i^2



H_i^3

FIGURE 8(d): Incremental Run 4.