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A multi-attribute decision-making module for the evaluation of alternative land consolidation plans

Demetris Demetriou[1], John Stillwell[1] and Linda See[1,2]

[1] School of Geography
University of Leeds
Leeds LST 9JT

[2] International Institute of Applied Systems Analysis (IIASA)
Schlossplatz 1
Laxenburg, A-2361 Austria

Email: [1]demdeme@cytanet.com.cy;
[1]j.c.h.stillwell@leeds.ac.uk;
[2]see@iiasa.ac.at

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Abstract

This paper explains the ‘Evaluation module’ of a computerised land consolidation planning support system. It is a multi-attribute decision-making approach that evaluates alternative land redistribution solutions generated by a Design module. The method introduces the idea of a ‘parcel concentration coefficient’ for measuring the dispersion of holdings and a ‘landowner satisfaction rate’ for estimating the acceptance of the new land redistribution plan by the landowners involved. The paper explains how the evaluation criteria have been selected, weighted, standardised and used for ranking the alternative solutions. Non-linear value functions for each evaluation criterion are introduced through the involvement of five land consolidation experts. A sensitivity analysis is reported and a case study application of the whole ‘Evaluation module’ is presented together with screenshots of the user interface.

Keywords

Land; consolidation; evaluation; criteria; parcels; landowners; standardisation; weights; sensitivity

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1 Introduction

The problem of land fragmentation is tackled in many rural areas through programmes of land consolidation In Cyprus, 74 land consolidation projects have been completed with 14 projects currently underway and a further 33 are under consideration by the Land Consolidation Department (LCD). In each case, the process of reallocating land between owners is both complex and time consuming. We have outlined the case for using sophisticated computational methods and tools in order to support the planner in the decision-making process in a previous paper (Demetris *et al.,* 2011b) and we have proposed a new framework for developing an integrated planning and decision support system (IPDSS) for land consolidation called *LACONISS* (LAnd CONsolidation Integrated Support System) whose structure is illustrated in Figure 1 and explained in detail in Demetriou *et al.* (2010; 2011c).

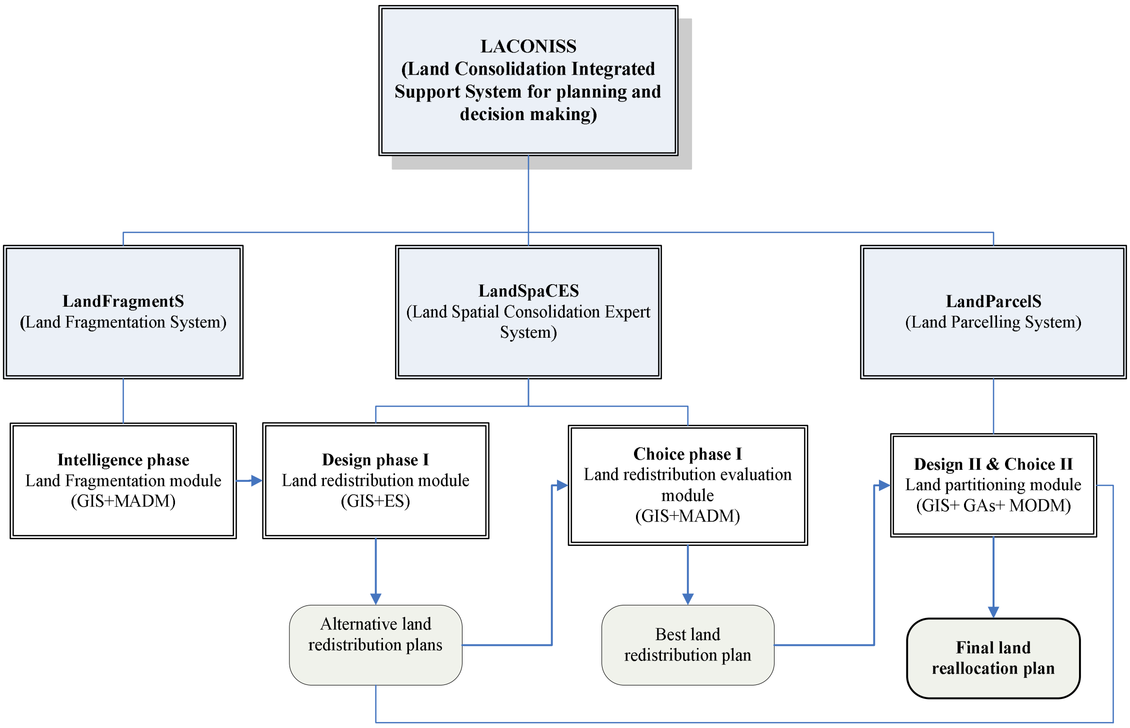


Figure 1: The operational framework of *LACONISS*

Three sub-systems are required in order to produce a land reallocation plan for a case study area. The first of these, *LandFragmentS* (Land Fragmentation System), involves building an appropriate geographical information system (GIS) model for the case study area and scanning the current land tenure system to measure the extent of land fragmentation. The second, *LandSpaCES* (Land Spatial Consolidation Expert System), contains (i) a ‘Design module’ that integrates the GIS with an expert system and generates a set of alternative land redistributions, and (ii) an ‘Evaluation module’ that uses multi-attribute decision-making (MADM) methods to evaluate the alternative distributions and identify the optimal configuration of parcel centroids (together with their land value and ownership attributes), which are then transferred to the third sub-system, *LandParcelS* (Land Parcelling System), in order to create the optimum set of boundaries for the land parcels around each of the centroids by integrating GIS with a genetic algorithm (GA) and multi-objective decision-making (MODM) methods.

The Design module of the *LandSpacES* sub-system has been explained and illustrated elsewhere (Demetris *et al.,* 2010; 2011c). This paper explains the development of the *LandSpacES* Evaluation module which follows a multi-attribute decision-making (MADM) approach to assess the alternative land redistribution solutions generated by the Design module and determines the best solution for input to the land partitioning module.

The structure of the paper is as follows. Section 2 set outs the problem and the model structure for selecting the evaluation criteria and introduces new concepts of (i) the parcel concentration coefficient (PCC) for measuring the dispersion of holdings; and (ii) the landowner satisfaction rate (LSR) for estimating the acceptance of the new land redistribution plan by the landowners in terms of the location(s) of the new parcels they receive. Thereafter, sections 3 to 7 focus on each of the basic elements of the module in terms of the theory underpinning the methods adopted and in terms of practice by presenting the module interface. More specifically, section 3 explains how the 'Impact table' is generated and section 4 deals with the crucial issue of assigning weights to the evaluation criteria and introduces a new 'qualitative rating' method. The standardisation process used for the creation of the decision table is discussed in section 5 focussing on the creation of specific non-linear value functions for each evaluation criterion through the involvement of five land consolidation experts. Thereafter, section 6 defines the 'decision rule' approach utilised for ranking alternatives and section 7 outlines a methodology for analysing the sensitivity of both the weights of the criteria and the performance scores of the alternative solutions. Section 8 reports an application using case study alternatives. Two different scenarios are tested based on changing the weights of the criteria. Finally, some conclusions are drawn in section 9.

2 Model structure

2.1 Problem definition

The Evaluation module of *LandSpaCES* follows a MADM process where a planner or a decision maker is confronted with a discrete number of alternative solutions but it is not clear, *a priori,* which solution is the best, i.e. one solution does not dominate all the other alternatives across all the evaluation criteria. In this case, the aim is to find the best alternative land redistribution plan among those generated by the Design module and to rank alternatives based on their ability to achieve the objectives.

This MADM problem can be defined as having i = 1,2,3…, N criteria and j = 1,2,3…, M alternatives. Alternatives and criteria are combined in a table with the former as rows and the latter as columns to create an Impact table (or effect or analysis table) of N x M dimensions. The preference of the planner at this stage of the process is incorporated by assigning a weight (or scaling constant), $w_{i,}$ to each criterion, C_i, representing the relative importance of that criterion for the problem concerned, where the sum of the weights always equals 1. Each element α_{ij} of the Impact table represents a score which indicates the performance, i.e. the outcome of alternative j for criterion i. The typical form of an Impact table is illustrated in Figure 2 and the aggregate performance of each alternative across all weighted criteria defines the ranking of the alternative.

	Alternatives						
Criteria	A_1	A_2	A_3	.	A_j	.	A_M
C_1 (w_1)	α_{11}	α_{12}	α_{13}	.	α_{1j}	.	α_{1M}
C_2 (w_2)	α_{21}	α_{22}	α_{23}	.	α_{2j}	.	α_{2M}
C_3 (w_3)	α_{31}	α_{32}	α_{33}	.	α_{3j}	.	α_{3M}
.	.	.	.	.	.	.	.
C_i (w_3)	α_{i1}	α_{i2}	α_{i3}		α_{ij}	.	α_{iM}
.	.	.	.	.	.	.	.
C_N (w_N)	α_{N1}	α_{N2}	α_{N3}	.	α_{Nj}	.	α_{NM}

Figure 2: The typical form of an Impact table

Once the problem has been articulated, the alternative land redistributions plans must be determined and the evaluation criteria must be identified.

2.2 Structure the objective tree

The land consolidation objective tree

The land consolidation process requires the specification of a hierarchical objective tree with goal, aims and objectives as indicated in Figure 3. The goal is an expression of the reason to take action and what needs to be achieved, i.e. sustainable rural development (Malczewski, 1999; Sharifi *et al*., 2004).

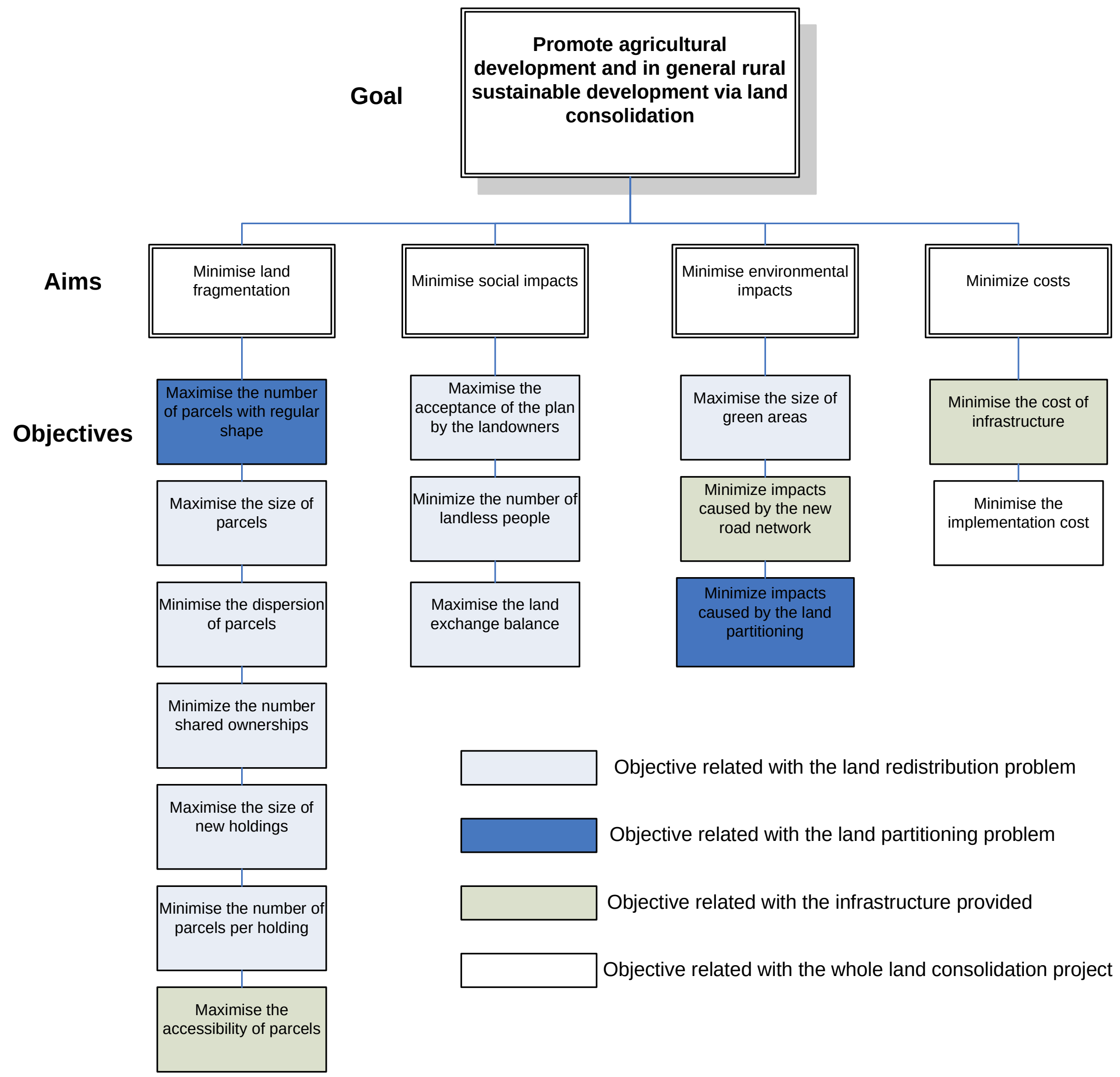


Figure 3: The objective tree for land consolidation

The aims are the broader changes to be achieved for a specific aspect of the problem and the specific objectives set out how each aim will be fulfilled. A further level includes the criteria/attributes shown in Figure 4 and discussed later. The objective tree classifies objectives in groups that correspond to the critical issues, i.e. land fragmentation, social concerns, environmental concerns and costs. This research has broken down land consolidation into two sub-processes: land reallocation and the planning/provision of infrastructure, and the former is then split into land redistribution and partitioning. Thus, each objective is classified (using different colours) in the objective tree based on the problem concerned.

The land redistribution objective tree

The objectives of land redistribution cover the main aims of land consolidation: to minimise land fragmentation, both the social and environmental impacts. Five objectives are classified under the first of these aims:

- maximise the size of parcels;
- minimise the dispersion of parcels;
- minimise the number of shared ownerships;
- maximise the size of new holdings; and
- minimise the number of parcels per holding.

Three objectives are grouped under the second aim:

- maximise the acceptance of the plan by the landowners;
- minimise the number of landless people due to the implementation of land consolidation; and
- maximise the land exchange balance, i.e. the optimum percentage of landowners who will not receive property.

This first group of landowners, who generally have small properties that are not worth agricultural exploitation will benefit through these arrangements because they will receive adequate compensation for their land and are therefore in full agreement with this arrangement. A second set of affected landowners are those who receive this land and thereby increase the size of their properties, especially those who will complete their ownership in the minimum area limits. If this percentage is not optimum, one or other of these two groups of landowners will be disadvantaged. Thus, the optimum percentage represents the level at which both groups of landowners are satisfied.

The third aim includes one objective:

- to maximise the size of the 'green areas', i.e. areas such as streams, rivers or other areas that need protection.

The objective tree for the land redistribution problem showing the objectives and the corresponding criteria/attributes is illustrated in Figure 4. The number of evaluation criteria depends on the characteristics of the problem. Since land redistribution is a very narrow and focused process, only a small number of criteria are required to reflect the differences between the alternatives.

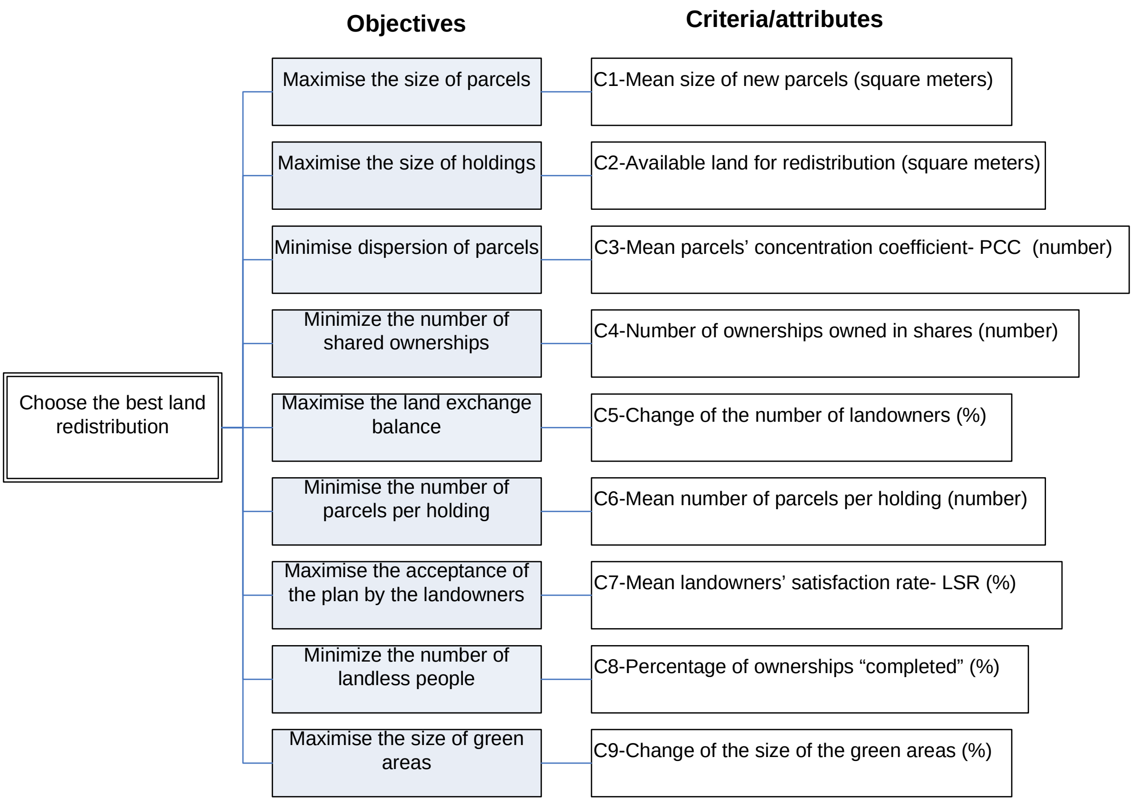


Figure 4: The objective tree for the land redistribution problem

Although the definition and association of objectives and criteria/attributes shown in Figure 4 is straightforward for a land consolidation planner, some further clarification is necessary. The mean size of new parcels criterion (C1) is straightforward since it is a basic statistical measure referring to parcels. Criterion C2 is the available land for redistribution and refers to the private land belonging to landowners who will not receive property under the new plan; they will be offered compensation instead and hence their land will be redistributed to others and to public land which is granted by the Government via purchase and redistributed to other landowners. Criterion C3, the mean *parcel concentration coefficient* (PCC), is a new index initially introduced in this research project and will be explained in a later section. Criterion C4 refers to shared ownership when land belongs to more than one landowner. The change in the number of landowners (C5), used as a proxy indicator of the land exchange balance, is a concept that has already been explained. Criterion C6 is a straightforward statistical measure.

The landowner satisfaction rate, LSR (C7), is also a new index developed in this research and is explained in full in a later section. An ownership is referred to as 'completed' (C8) if land is added to a property to create a new unique parcel so as to reach the minimum area limit as defined by legislation. Green areas (C9) are public land as noted earlier.

According to Keeney (1992), natural criteria are derived directly from the context of the objectives, constructed criteria subjectively measure the degree to which an objective has been satisfied and proxy criteria are indirectly associated with an objective. Based on this classification, C1, C2, C3, C4, C6 and C9 are natural criteria and C5, C7 and C8 are proxy criteria. There are no constructed criteria associated with this problem. Beinat (1997) notes that the employment of proxy attributes involves potential distortion in the evaluation process because of their inherited property of not being directly involved with the decision; hence special awareness of this should be addressed via the independence tests. Despite these criteria adequately reflecting the land redistribution decision-making problem, the selection of criteria used in the Evaluation module is a critical task which is addressed in the next section.

2.3 Evaluation criteria

Assessment of the appropriateness of the selected criteria is crucial since they define the quality of the outcome of the MADM process. The selection of criteria involves two stages: a higher level selection stage and a further filtering stage. The relevant literature (Malczewski, 1999; Sharifi *et al*., 2004; Keeney and Raiffa, 1993) suggests that a number of requirements need to be fulfilled by each criterion and by the whole set of criteria, which will drive the initial selection of the criteria in the higher level stage. In particular, each criterion should be *comprehensive* in terms of clearly representing the associated objective. In addition, each criterion needs to be *measurable*, i.e. it can be objectively estimated. On the other hand, a set of criteria must be *complete* since they should cover all aspects of the decision problem, i.e. the efficiency of the plan and the social and environmental impacts. Furthermore, they must be *operational* because they must have clear content, i.e. they can be easily understood by planners and decision makers in terms of the consequences of each alternative. Moreover, they must be *decomposable* so the decision problem can be split into smaller parts by grouping criteria based on their content, i.e. economic, environmental, social, etc. A further property of the final set of criteria is that they must be *parsimonious,* i.e. the number of criteria should be kept as small as possible but they should provide adequate and reliable representation of the decision problem, quantifying the decision makers' preferences.

In the second stage, the final criteria are chosen from the initial set based on ensuring lack of redundancy or independence. In other words, criteria should be defined in such a way as to avoid duplication of the consequences of the decision since this may act in favour of some alternatives and the outcome may be misleading. In addition, this double counting or duplication must be avoided in case the aggregated performance of each alternative results from an additive value function model, which is used in this research. If the correlation coefficient of a pair of criteria approximates to zero, then the two criteria are independent and hence non-redundant (Easton, 1973). Malczewski (1999, p.109) notes that *"this situation is an extremely unlikely occurrence in the context of spatial decision making*".

Furthermore, Collins and Glysson (1980) and Keeney and Raiffa (1993) propose a more practical term representing the association between a pair of criteria which is called 'preferential independence'. A criterion X is said to be preferentially independent of criterion Y if preferences for specific outcomes of X do not depend on the level of criterion Y. If X is preferentially independent of Y and Y is preferentially independent of X, then these criteria are 'mutually preferentially independent'. For example, criterion X might represent the cost of a project with values £1,000 or £2,000 and Y criterion might represent the time to complete the project of, say, 5 or 10 days. Thus, if the 5 day option is preferred regardless of whether the cost is £1,000 or £2,000, then the Y is preferentially independent of X. Similarly, if the lower cost of the project is preferred regardless of the time it takes to complete, then X is preferentially independent of Y and X and Y are mutually preferentially independent. As Beinat (1997) notes, preferential independence does not mean difference (or cardinal) independence, i.e. that the correlation coefficient between this pair of criteria equals or approaches zero. Therefore, cardinal independence is not a precondition for mutual preferential independence between a pair of criteria. Moreover, if two criteria are mutually preferential independent, then they are included in the evaluation module regardless of difference or cardinal independence as explained below.

If a criterion does not vary among alternatives, then there is no need to keep it in the final list for evaluation. In particular, Figure 4 indicates that there are three criteria that should not be maintained in the Evaluation module: 'the available land for redistribution' (C2), which refers to public land that is granted via purchase; the 'number of ownerships held in shares' (C4); and the 'change in the size of the green areas' (C9). This is because the Design module*,* which generates the alternative land redistribution solutions, does not take them into account since the relevant decisions are taken directly by the planner (in the case of C4 and C9) or the governmental authorities (in the case of C2). In particular, the issue of which ownerships should be in shares after the completion of a land consolidation project is a matter of negotiation between the planner and the landowners. The size of green areas depends on the local topography and the need, for example, to extend the width of a stream or a river, decisions that are technical rather than land redistribution decisions. In addition, for criterion C2, the private land available for redistribution is directly related to the change in the number of landowners (including those who will not receive land), i.e. criterion C5. This is clearly double counting or duplication of criteria and hence should be avoided.

Based on the above considerations, the following six criteria are retained for further independence testing: mean size of new parcels (C1); mean parcel concentration coefficient (C3); change in the number of landowners (C5); mean number of parcels per holding (C6); mean landowner satisfaction rate (C7); and percentage of ownerships ‘completed’ (C8).

Testing for independence

The independence test results are shown in Table 1 which involves cross-checking pairs of criteria for independence. Check marks in Table 1 indicate mutual preferential independence and the criteria will therefore be retained regardless of the value of the correlation coefficient, which appears in brackets in each cell. The only cells in Table 1 that require consideration are therefore the ones with crosses, i.e. criterion C6 in relation to criteria C1 and C3. The correlation coefficients indicate strong relationships between these criteria and therefore C6 should be eliminated from the list. Therefore, the criteria that are to be used further in the evaluation module are C1, C3, C5, C7 and C8.

Table 1: A pairwise independence test for the criteria

	C1	C3	C5	C6	C7	C8
C1		✓ (-0.97)	✓ (0.06)	× (-0.99)	✓ (0.71)	✓ (-0.51)
C3			✓ (0.06)	× (0.97)	✓ (-0.72)	✓ (0.42)
C5				✓ (0.08)	✓ (-0.23)	✓ (0.17)
C6					✓ (0.74)	✓ (-0.54)
C7						✓ (0.25)
C8						

The calculation of the scores of C1, C5 and C8 is straightforward since they involve simple statistical measures directly extracted from the output table of the Design module. The next two sections explain criteria C3, the mean parcel concentration coefficient (PCC) and C7, the mean landowner satisfaction rate (LSR), which are new criteria introduced in this research.

2.4 Parcel concentration coefficient (PCC)

A basic measure of spatial dispersion is standard distance (Ebdon, 1985; Wong and Lee, 2005), the spatial equivalent of the standard deviation, showing how locations or points are scattered around the spatial mean (Wong and Lee, 2005). The spatial mean or mean centre of gravity is also an important spatial statistical measure of central tendency which indicates the average location of a set of points defined in a Cartesian coordinate system. Thus, standard distance measures the degree to which parcels (or more precisely the centroids of each of the parcels) are concentrated or dispersed around their geometric mean. Although, in practice, the dispersion of holdings is dependent on the location of the farmstead or the village where the farmer resides, the extra information needed is usually not available, so the mean centre of parcels of a holding is a proxy criterion that gives an adequate representation of the dispersion before and after land consolidation.

An extension of standard distance is the weighted standard distance where centroids may have different attribute values representing the different sizes or land values of each parcel. For instance, if the largest parcels of a holding are very dispersed in terms of location, this may have more negative effects on productivity than if the smaller parcels are dispersed. Tourino *et al*. (2003) suggest weighting the score of each parcel using its agronomic value, i.e. taking into account the productivity and soil quality of a parcel. Wong and Lee (2005) note that the weighted mean centre and the weighted distance should also be utilised when the point locations under study have varying frequencies or occurrences.

Both spatial statistics, i.e. the mean centre and standard distance, are rephrased in this research as the mean centre of the parcels (CoP) and the dispersion of parcels (DoP) belonging to a holding, respectively. The mean centre of the parcels of a holding can be found by calculating the mean of the x co-ordinates (eastings) and the mean of the y co-ordinates (northings) of the centroids of the parcels that belong to a holding. The two coordinate means define the location of the mean centre of a holding as:

$$\text{CoP} = \left(\bar{x}_{hmc}, \bar{y}_{hmc}\right) = \left(\frac{\sum_{i=1}^{n} x_i}{n}, \frac{\sum_{i=1}^{n} y_i}{n} \right) \tag{1}$$

where $\bar{x}_{hmc}$ and $\bar{y}_{hmc}$ are the co-ordinates of the holding's mean centre; x_i and y_i are the co-ordinates of the centroid of parcel i; and n is the number of parcels belonging to a holding.

The larger the size of a parcel, the greater its importance in terms of its contribution to production, productivity, labour and hence the income of a farmer. Similarly, the land value of a parcel could be also used as a weight instead. Thus, the weighted mean centre of a holding is a better indicator than the simple mean centre because it reflects not only the spatial dispersion of parcels but also the agricultural importance of each parcel. The weighted mean centre of a holding can be found by multiplying the *x* and *y* coordinates of the centroid of each parcel by a weight:

$$\text{CoP} = \left(\bar{x}_{whmc}, \bar{y}_{whmc}\right) = \left(\frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}, \frac{\sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i} \right) \tag{2}$$

where $\bar{x}_{whmc}$ and $\bar{y}_{whmc}$ are the co-ordinates of the holding's weighted mean centre and w_i is the weight of each parcel *i*. Then the dispersion of parcels (DoP) for the holding can be calculated by as:

$$\text{DoP} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - x_{hmc})^2 + \sum_{i=1}^{n} (y_i - y_{hmc})^2}{n}} \tag{3}$$

and the weighted DoP as:

$$\text{DoP} = \sqrt{\frac{\sum_{i=1}^{n} w_i (x_i - x_{hwmc})^2 + \sum_{i=1}^{n} w_i (y_i - y_{hwmc})^2}{\sum_{i=1}^{n} w_i}} \tag{4}$$

Both simple and weighted measures of parcel dispersion were utilised by Tourino *et al*. (2003). Although they both constitute a classic measure of spatial dispersion, the disadvantage is that they may result in an unlimited range of values with no explicit extreme values. In this research, a new indicator is developed called the *parcel concentration coefficient (PCC)* which is measured on a scale between 1 to -1. A zero value means there is no change in the dispersion of a holding's parcel before and after land consolidation. The value of +1 refers to the situation of 'perfect concentration' and -1 represents 'worst concentration'. The dispersion of parcels can be calculated for each holding twice, i.e. before (DoP_b) and after (DoP_a) land consolidation and then combined to calculate the PCC such that:

(i) If $DoP_b = DoP_a$ then PCC = 0 (for both equations 5 and 6 that follow) means that the dispersion of parcels has not changed and the land consolidation has not achieved any concentration of parcels for the holding concerned. This occurs independently for either the number of new parcels allocated to a landowner or the number of original parcels owned by the landowner because the purpose of PCC is to compare, i.e. calculate the difference in the dispersion of parcels and not to compare the difference in the number of parcels (before and after a project) which is involved in another criterion.

(ii) If $DoP_b > DoP_a$ then $$\mathrm{PCC} = \frac{\left(\frac{\mathrm{DoP_b} - \mathrm{DoP_a}}{\mathrm{DoP_b}}\right)}{\mathrm{n'}} \qquad (5)$$

where n' is the number of new parcels allocated to a holding after land consolidation.

In this situation, an improvement in the dispersion of parcels occurred and therefore the PCC may take values between 0 and 1. The extreme value of 1 means that after land consolidation, there is just one single parcel, i.e. n' =1 and hence perfect concentration has been achieved. This happens when the DoP_a equals 0 and consequently n' =1. The numerator in equation 5 represents the change of dispersion before and after land consolidation for a holding. The denominator, i.e. the number of parcels allocated to a holding after land consolidation, adjusts the proportional change in dispersion, i.e. the level of concentration, since the PCC increases as n' decreases. In other words, the more parcels that are allocated after land consolidation, the less the concentration of new parcels and hence PCC reduces towards a value of zero.

(iii) If $DoP_b < DoP_a$, then $PCC = -\dfrac{\left(\dfrac{DoP_a - DoP_b}{DoP_a}\right)}{n}$ (6)

where n is the original number of parcels in a holding before land consolidation. In this situation, a deterioration has occurred in the dispersion of parcels and thus PCC may take values between 0 and -1. Actually, this may occur either when the number of parcels allocated to a holding after land consolidation is greater than the original number of parcels and/or when the parcels have been allocated at greater distances. The extreme value of -1 means that the concentration of parcels after land consolidation is the worst configuration, independent of the number of new parcels allocated because, in this case, the basic aim of parcel concentration via land consolidation completely fails. This happens when the DoP_b equals 0 and consequently n =1.

The numerator in equation 6 represents the change of dispersion before and after land consolidation for a holding. The denominator, i.e. the number of original parcels that belonged to a holding, adjusts the proportional change in dispersion, i.e. the level of concentration, since the PCC reduces as n increases. In other words, the greater the number of original parcels owned by a landowner, the smaller the difference (before and after a project) in parcel concentration; hence the PCC tends towards zero because the dispersion was already poor. For example, it is a worse situation if a landowner had only one parcel, i.e. perfect dispersion and then was allocated more than one parcel rather than having say three parcels initially (which means that the holding was already dispersed) and was then allocated four parcels, causing even worse dispersion.

However, it should be noted that allocating a landowner more parcels than those originally owned is a very rare case in land consolidation projects, i.e. the above considerations are clarified by utilising an example which applies equations 5 and 6 for various ranges of values (Table 2).

Table 2: Calculation of PCC for various set of values

DoP_b	DoP_a	n'	n	PCC
Applying equation 5 (If $DoP_b \geq DoP_a$)				
500	500	1		0
1,000	1,000	2		0
1,500	1,500	3		0
2,000	1,500	1		0.25
2,000	1,500	2		0.13
2,000	1,500	3		0.08
5,000	1,000	1		0.80
5,000	1,000	2		0.40
5,000	1,000	3		0.27
2,000	0	1		1
1,000	0	1		1
500	0	1		1
Applying equation 6 (If $DoP_b \leq DoP_a$)				
500	500		1	0
1,000	1,000		2	0
1,500	1,500		3	0
1,500	2,000		1	-0.25
1,500	2,000		2	-0.13
1,500	2,000		3	-0.08
1,000	5,000		1	-0.80
1,000	5,000		2	-0.40
1,000	5,000		3	-0.27
0	2,000		1	-1
0	1,000		1	-1
0	500		1	-1

2.5 Landowner satisfaction rate (LSR)

The landowner satisfaction rate (LSR) is an indicator showing the satisfaction of the landowners' preferences for the whole consolidation project in terms of the location of their new parcels. It is based on the idea of a parcel priority index (PPI) which ranks the preferences of landowners regarding the locations of the new parcels they wish to receive. For instance, a landowner with five parcels is assigned (via the Design module as explained in Demetris *et al.,* 2010) a PPI index for each parcel which defines the rank order of parcels representing the preference of the landowner, i.e. first, second, third, fourth and fifth in terms of the new parcel locations. Thus, the LSR searches for the solutions in which the preferences of each landowner have been satisfied and assigns a proportional percentage of satisfaction depending on the ranking of the preference satisfaction, with a maximum of 100%.

In particular, each new parcel is assigned a partial satisfaction rate (PSR), with a maximum of 100%, based on the rank order of the preferences satisfied. A critical point in this process is that the original parcels (n) of a landowner which are already in a preference rank order (based on PPIs) are divided into two parts. The first covers the situation up to $n = n'$ (where n is the original number of parcels owned by a landowner and n' is the number of new parcels held by that landowner) whilst the other part covers the situation for the rest of the parcels, i.e. from $n'+1$ to n. Then each new parcel is examined to determine in which part, in terms of preference, it falls. Thus, if it falls in the first part, then the PSR of the landowner will be 100%, but if it falls in the second part, then the partial satisfaction is assigned proportionally, namely reduced, depending on the number of original parcels and the number of new parcels. This can be expressed mathematically as follows:

If $n \geq n'$, then the PSR for each new parcel i allocated to a landowner can be calculated as follows:

$$PSR_i = m_i P \tag{7}$$

where m_i is a variable that takes into account the number of parcels originally owned by a landowner (n) and the ranking order of the preference of each original parcel i ($\mathrm{RO_i}$) and P is a linear function that expresses decreasing satisfaction for each landowner. The two variables, m_i and P are computed as follows:

$$m_i = n - RO_i + 1 \tag{8}$$

$Maxm_i$ is the m_i value assigned to those new parcels that falls in the first part of the original parcels as explained earlier. In this case, the parameter RO_i in equation 8 is replaced by the number of new parcels (n') as follows:

$$Maxm_i = n - n' + 1 \tag{9}$$

P, which is a constant percentage for the redistribution of each holding, is calculated based on the two parts mentioned earlier. In particular, the parcels belong in the count in the first part as one sub-part whilst the parcels belong in the count in the second part individually as a separate sub-part. Thus, P is determined by dividing 100% by the total number of sub-parts, which always equals n-n'+1. Therefore, P can be computed as:

$$P = \frac{100}{n - n^{'} + 1} \tag{10}$$

Combining equations 7 and 8 yields:

$$PSR_i = \frac{100(n - RO_i + 1)}{n - n^{'} + 1} \tag{11}$$

The total LSR for each landowner j is then calculated as the mean value of the PSR:

$$\bar{LSR}_j = \sum_{i=1}^{n^{'}} \frac{PSR_i}{n^{'}} \tag{12}$$

Similarly, the average LSR for the whole land consolidation area, i.e. the whole project, can be calculated as the mean LSR of all landowners, l, who received property in the plan as follows:

$$LSR = \sum_{j=1}^{l} \frac{LSR_j}{l} \tag{13}$$

The above assumptions become clearer by utilising an example for calculating PSR and LSR. Table 3 involves a landowner who originally had five parcels (i.e. n = 5) and after land consolidation receives either 1, 2 or 3 parcels (i.e. $n^{'}$ = 1, $n^{'}$ = 2, $n^{'}$ = 3). Each cell of the table contains the PSR value for each combination of n and n'.

Table 3: An example for the calculation of the partial satisfaction rate

	Number of new parcels ($n^{'}$) allocated to the landowner		
n	1	2	3
1	maxM * P = 5*20 = 100%	maxM * P = 4*25 = 100%	maxM * P = 3*33.33 = 100%
2	M_2 * P =(5-2+1)*20 = 80%	maxM * P = 4*25 = 100%	maxM * P = 3*33.33 = 100%
3	M_3 * P = (5-3+1)*20 = 60%	M_3 * P = (5-3+1)*25 = 75%	maxM * P = 3*33.33 = 100%
4	M_4 * P = (5-4+1)*20 = 40%	M_4 * P = (5-4+1)*25 = 50%	M_4 * P = (5-4+1)*33.33 = 66.66%
5	M_5 * P = (5-5+1)*20 = 20%	M_5 * P = (5-5+1)*25 = 25%	M_5 * P = (5-5+1)*33.33 = 33.33%

The interpretation of the PSR values shown in Table 3 is straightforward. In particular, if the landowner who originally had five parcels has been allocated one parcel (i.e. the first column) in the same location as its fourth preference (i.e. fourth row), then he/she will be partially satisfied by 40%. Similarly, if the landowner has been allocated two parcels (i.e. the second column), say in the same location as the first (i.e. the first row) and the fourth preference (i.e. the fourth row), then he/she will be satisfied partially by 100% for the location of the first parcel and by 50% for the location of the second parcel. As a result, the average LSR can be calculated as 75%. In the same vein, if the landowner has been allocated three parcels (i.e. the third column), say in the same location as the second (i.e. the second row), third (i.e. the third row) and fifth preference (i.e. the fifth row), then he/she will be satisfied partially by 100% for the location of the first parcel, by 100% for the location of the second parcel and by 33.33% for the location of the third parcel. As a result, the average LSR will be 77.78%.

Based on the above, it is clear that if $n' \geq n$, it is first determined if the n' up to n new parcels fall in the first part in which case the PSR is estimated as above. The remainder of the new parcels, i.e. the number n'-n, fall outside of the ranked preferences of the landowners because the preferences coincide exactly with the number of original parcels, i.e. n; hence the PSR is 0% for those new parcels.

3 Using the model interface to generate the Impact table

3.1 The module toolbar

The toolbar of the Evaluation module illustrated in Figure 5 consists of seven menu items: Alternatives; Criteria; Impact Table; Value Functions; Decision Table, Ranking Alternatives; and Sensitivity Analysis. Each menu item represents a stage of the MADM process and launches a separate window with one or more functionalities. The menu items are organised in the order in which they must be executed in the process. The functionality of each menu item will be described separately in the sections that follow.

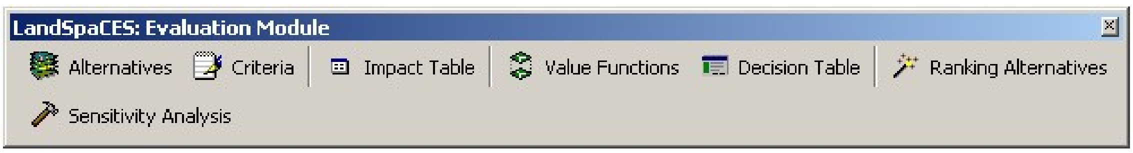


Figure 5: The toolbar of the Evaluation module of *LandSpaCES*

3.2 Selecting alternatives

The Alternatives menu item allows the user to select alternative land redistribution solutions and launches a window entitled 'Set alternative land redistributions' (Figure

6). The user first loads all the alternative land redistributions generated by the Design module of *LandSpaCES* by pressing the button ‘Load all alternatives’. Then the user selects which alternative solutions to use in the evaluation process by checking the appropriate boxes, or all alternatives can be selected at once. Clicking the OK button completes this process.

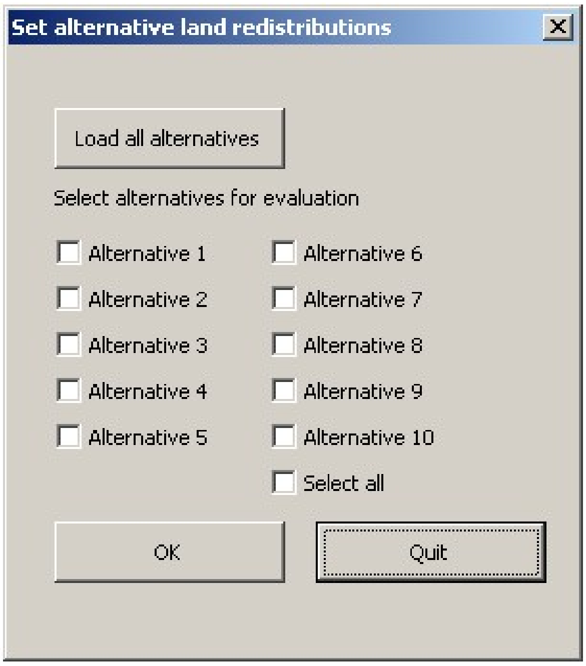


Figure 6: The ‘Set alternative land redistributions’ window

3.3 Selecting criteria

The user then selects which criteria to include via the ‘Criteria’ menu item. The selection of the criteria works in the same way as selecting the alternatives (Figure 7). This window also allows the assignment of weights to criteria as discussed in a later section.

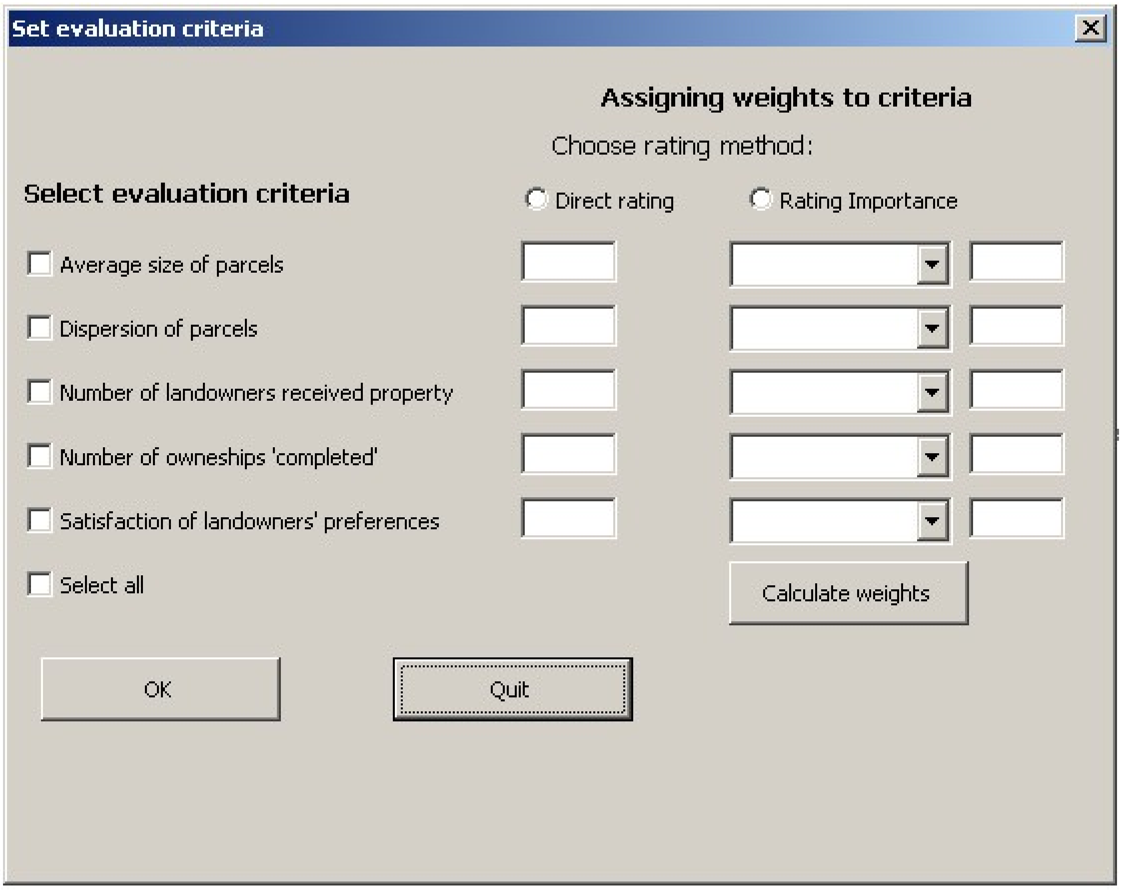


Figure 7: The ‘Set evaluation criteria’ window

Impact table

Once the alternatives and criteria are selected, this creates the structure of the ‘Impact table’ as shown in Figure 2. A score represents the performance of an alternative associated with a particular criterion. If the user selects the Impact table menu item, then the dialogue box is shown (Figure 8). This window provides four main functions (with corresponding buttons): the appearance of the structure of the Impact table (Figure 9); ‘pre-calculations’ regarding the dispersion of parcels and the satisfaction of the landowners’ preferences, which are not provided by the Design; calculation of the scores α_{ij} of the impact table (via the ‘Calculate Scores’ button); and the final table with the scores (via the ‘Show Impact Table’ button).

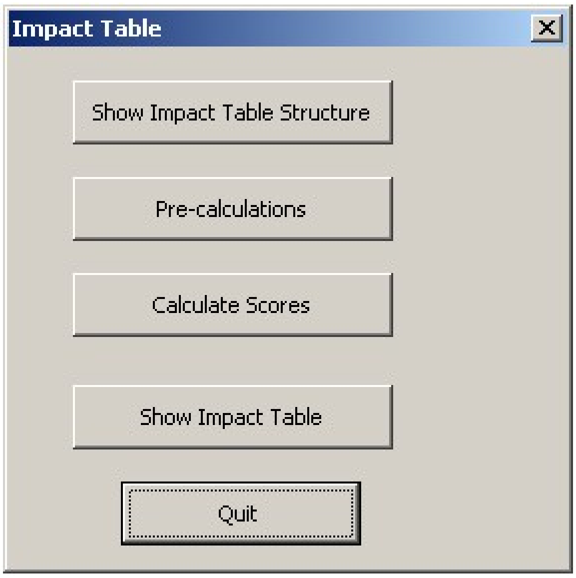
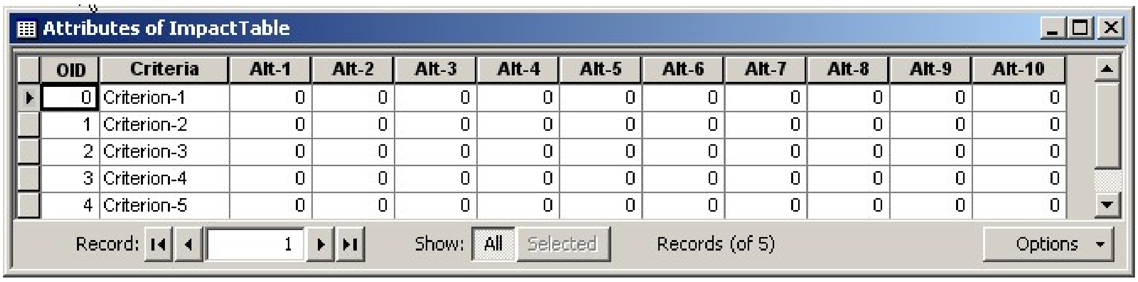


Figure 8: The 'Impact Table' window



OID	Criteria	Alt-1	Alt-2	Alt-3	Alt-4	Alt-5	Alt-6	Alt-7	Alt-8	Alt-9	Alt-10
0	Criterion-1	0	0	0	0	0	0	0	0	0	0
1	Criterion-2	0	0	0	0	0	0	0	0	0	0
2	Criterion-3	0	0	0	0	0	0	0	0	0	0
3	Criterion-4	0	0	0	0	0	0	0	0	0	0
4	Criterion-5	0	0	0	0	0	0	0	0	0	0

Figure 9: An example of the structure of the Impact table

4 Weighting criteria

4.1 The scope of weighting

The purpose of giving different priorities to the evaluation criteria by assigning a weight to each is to represent the relative importance of each criterion in the evaluation process (Malczewski, 1999; Triantaphyllou, 1997; Sharifi *et al*., 2004). Although this role of weighting criteria is generally accepted, Beinat (1997, p.42) considers that it is not 'completely appropriate' because there is no clear association between the measurement scales and ranges of the performance scores, which are themselves weighted, i.e. it is not possible to compare criteria with different ranges. In fact, the weights, which sum up to one, represent the trade-offs between the performance scores achieved by the alternatives for each evaluation criterion, so there is a strong relationship between the ranges and the weights. To make this relationship understandable, the following example is provided in Table 4.

Table 4: An example for demonstrating the relationship between weights and score ranges

	Route 1		Route 2	
Criterion	Best	Worst	Best	Worst
C1 Loss of vegetation (hectares)	2,300	2,500	500	4,500
C2 Travel time (hours)	1	2	1	2

Let us assume that there are two potential routes for a motorway construction. Route 1 passes across a flat area where there is no way to avoid the destruction of vegetation whilst Route 2 passes across a hilly area that may be constructed by building tunnels and bridges to avoid deep excavations and embankments, respectively. As shown in Table 4, for Route 1, the difference between the worst and best range for the 'loss of vegetation' criterion is only 200 hectares (or 8%) while for the criterion travel time it is 1 hour (or 100%), which is relatively much greater. In contrast, for Route 2, the difference between the worst and best ranges for loss of vegetation is huge, i.e. 4,000 hectares (or 88.9%) because the first (best) case involves the construction of tunnels and bridges, i.e. it will dramatically reduce the loss of vegetation, while the second (worst) case involves no construction of tunnels and bridges and thus a huge area of vegetation will be lost. Therefore, it is clear that for Route 1, the most important criterion is travel time while for Route 2 the most important criterion is loss of vegetation. Thus, if a planner had initially only Route 1 in the evaluation process, then criterion C2 will be assigned a higher weight than C1. On the other hand, if Route 2 joined the evaluation as an extra alternative, then the planner would need to readjust the weights since C1 should be assigned a higher weight than C2; otherwise, the ranking order will be biased and unreliable. Also, this relationship needs to be taken into account since it can arise if the standardisation method changes, e.g. when using linear or non-linear functions.

Nitzsch and Weber (1993) investigated this relationship and found that weighting methods which do not take into account score ranges may lead to biased weighting, especially when planners do not know the ranges *a priori*. Therefore, changes in the ranges imply readjustment of the weight values. The existence of this problem is emphasised in a case study carried out by Costa and Vansnick (1997) and the issue has also been recognised by Malczewski (1999) and Sharifi *et al*. (2004). However, in this model, score ranges are known before assigning weights and the value functions utilised for standardisation (as discussed later) do not vary according to the planner or the project, so the planner may check the potential biases via the sensitivity tests provided by the model.

The above discussion suggests that weighting is a very critical task in decision making because it involves controversy and uncertainty (Chen *et al*., 2009) and it influences the final outcome, the ranking of alternatives. Several methods have been developed for this purpose, which are reported in the literature: swing weights; ranking; rating; pairwise comparison; trade off analysis; qualitative translation, etc. Reviews of these methods are provided by Beinat (1997), Malczewski (1999) and Sharifi *et al*. (2004). Crucial factors for selecting the most appropriate method for assigning weights to criteria for a certain decision problem are the number of criteria and the grade of uniqueness between them. Two factors were taken into account when making the decision to choose methods for the Evaluation module. First, the number of criteria involved in the evaluation process carried out in this model is quite small, i.e. five. This falls within the so called 'seven plus or minus two' range that is considered as the maximum number of entities that can be simultaneously processed by the human brain (Miller, 1956). Second, given that certain evaluation criteria are explicit in terms of their context and meaning, it was judged that two of the most straightforward and popular methods should be utilised and provided by the Evaluation module: direct ranking and qualitative rating. The direct ranking method allows the user to enter weights when they are known *a priori* or developed using another method while the qualitative ranking method developed here offers users a way of developing the weights within the process. These two methods are explained in more detail below.

4.2 Direct ranking

Direct ranking (or direct estimation) is the most straightforward method for assigning values to criteria (that sum up to 1) when the number of criteria is small and manageable. However, even for such a small number of criteria, it is not straightforward when weighting values have two or more decimals. For instance, sometimes is not easy to justify why a criterion has a weight of 0.2 and another criterion has 0.18; it is even more difficult to differentiate a criterion from another by assigning weights of 0.125 and 0.120. Thus, weighting with this method can be reliable and accurate when values have one decimal, i.e. 0.1, 0.2 or two decimals with the last digit being 5, i.e. 0.15 or 0.25. Because these preconditions cannot always be fulfilled, we provide the planner with a modified rating method called 'qualitative rating', which has been proposed in this research and is explained in the next section.

4.3 Qualitative rating method

Ranking methods involve the ordering of criteria to identify the most important to the least important criteria or *vice versa*. Several procedures (e.g. rank sum, rank reciprocal and rank exponent method) are then utilized for estimating a numerical value of weights based on that rank order (Malczewski, 1999). Although these methods are simple, they involve a great disadvantage since they do not provide the potential to rank two or more criteria with equal importance, a fact that is obviously not reasonable in practice.

Similar to ranking methods, rating methods and in particular the point allocation approach, involve allocation across the evaluation criteria of a number of points, e.g. ranging from 0 to 100. The higher the number of points assigned to a criterion, the greater its importance. These scores can then be easily standardized on a scale of 0 to 1. This method is actually very similar to direct ranking so it has the same disadvantages and hence is not accurate. In particular, how does one justify assigning say 20 points to a criterion and not 22? The ratio estimation procedure combines the ranking of criteria as employed in ranking methods and the scoring of points carried out in the point allocation method. The difference is that a score of 100 is assigned to the most important criterion and then proportionally smaller scores are assigned to other criteria. Weights are then normalized to a scale from 0 to 1. This method inherits the disadvantages of both methods.

Taking into account that a simple method is still needed for assigning weights to a small number of criteria but overcoming at least some of the disadvantages referred to above, a modified version of the ratio estimation procedure is introduced here. In particular, this method overcomes the problem of assigning either direct numeric values as weights or scores, which are then transformed into weights, by adopting a similar qualitative scale to that used in the pairwise comparison method. In particular, criteria are classified in the following seven classes of importance: extremely high; very high; high; intermediate; moderate; low; and very low. Experience shows that it is easier to ask a decision maker or a planner to intuitively describe the category of importance of a criterion rather than assigning a number or a score. Comparison of the importance of criteria is also easier with this approach. In addition, criteria may have the same importance, which is a reasonable fact in practice.

Similarly with the pairwise comparison method, each class on a scale has a predefined rate, i.e. a score. In particular, the scale is divided into two parts: the upper and the lower range, and the sub-division point is then the middle class. Each part involves three levels of importance and a differential increase in the scores. More specifically, the rate of increase in the lower part is 10 points whilst in the upper part it is double, i.e. 20 points. This represents an imposed weighting in favour of the upper part and against the lower part. Although this scoring seems arbitrary, in practice this discrimination is realistic since the weight of the classes belonging to the upper range should be more than that of the lower range because planners and decision makers tend to degrade or even ignore the less important criteria in the decision-making process. The scale of importance of each criterion and the corresponding scores are presented in Table 5.

Table 5: The scale of importance and the relevant scores utilized by the qualitative rating method

Rank order	Scale of importance	Score	Classes
1	Extremely high	100	Upper
2	Very high	80	
3	High	60	
4	Intermediate	40	Middle
5	Moderate	30	Lower
6	Low	20	
7	Very low	10	

After selecting the appropriate scale of importance for each criterion, the weights are standardized based on the score assigned to each criterion so that the weights sum to 1. As an example, suppose we have seven criteria and each criterion corresponds to a different scale of importance. Table 6 shows the values of the weights that result, which cannot be defined directly by a planner in the same way.

Table 6: An example of the actual weight values with seven criteria where each criterion corresponds to a different scale of importance

Criterion	Scale of importance	Score	Weight
C1	Extremely high	100	0.294
C2	Very high	80	0.235
C3	High	60	0.176
C4	Intermediate	40	0.118
C5	Moderate	30	0.090
C6	Low	20	0.059
C7	Very low	10	0.029
Total		340	1.000

It is clear that if all the criteria have the same importance, then all of the criteria will have same weight. Weighting is carried out in the ‘Set evaluation criteria’ window

(Figure 7). The user should select the method of weighting, i.e. direct rating, and then enter the values directly into this window, or the qualitative rating method, which helps the user to define the weights. It is acknowledged that the land consolidation planner will select the most appropriate method of the two depending on his/her experience, the nature of the project and the ease of defining the weights of the five criteria either directly or indirectly.

5 Standardisation

Standardization (or normalization) is the process of transforming the scores of the evaluation criteria into the same scale, which is commonly a dimensionless scale of values from 0 to 1. As a result, the measurement unit is uniform and hence the criteria can be combined and compared. Several standardization methods have been developed and the selection of which one to use depends on the problem concerned and the type of evaluation criteria involved. These methods are classified into two broad categories: linear scale transformation and value/utility function approaches. The former are utilized for deterministic problems whilst the latter are used for both deterministic and probabilistic problems (Sharifi *et al*., 2004).

The most popular linear scale transformation methods are: maximum, interval and goal standardization. The first two methods use the highest and the lowest values of a dataset for the transformation (proportional or not) into a scale which results in values between 0 and 1. Conversely, the third method utilizes reference points that reflect an ideal point (e.g. a desired value to be achieved) and a minimum point that defines the range of standardization; thus they are independent of the dataset. Malczewski (1999) and Sharifi *et al*. (2004) discuss these methods. Although these methods have the advantage of simplicity and a predefined behaviour, they have two significant disadvantages: firstly, they assume a linear association between the original values and the standardised values when, in practice, this relationship is more complex; and secondly, they ignore the judgements of the decision makers as they have no input in the development of the simple linear standardisation function that is commonly used. Both of these limitations are overcome by the use of value functions.

5.1 Value functions

Value functions reflect, in a mathematical form, human judgement regarding what is desired to be achieved for a certain decision-making problem. In particular, value functions are associated with factual information, human judgement and multiple criteria. They translate the performance score of an alternative for a criterion into a value score taking values between 0 and 1, representing the degree to which a certain decision objective is achieved. Normally, a value of 1 indicates the best available performance while 0 indicates the worst performance.

The creation of value functions for a problem is a difficult task and it is crucial for the whole process since it affects the ranking of alternatives. Beinat (1997, p11) notes that *"value functions have to be estimated through a specially designed interviewing process in which the relevant judgments for the decision are organised and represented analytically",* and is the approach followed here.

There are linear and non-linear value functions. In addition, they can be classified as cost functions, benefit functions and mixed functions, i.e. both cost and benefit. For a cost function, the higher the value of a criterion, the worse it is, whereas for a benefit function, the higher the value, the better it is. For example, on a slope map, we may consider two cases: finding an area suitable for agriculture and an area suitable for skiing. In the first case, the lower the slope, the better the area, i.e. slope is a cost criterion, whereas in the second case, the higher the slope the better the area, so slope is a benefit criterion. A typical form of a linear cost and benefit value function is illustrated in Figure 10 while Figure 11 shows examples of three different variables: salary (11a), noise (11b) and temperature (11c), representing benefit, cost and mixed non-linear value functions respectively. Explanation of the shapes of the value functions in Figure 11 will appear in the next section.

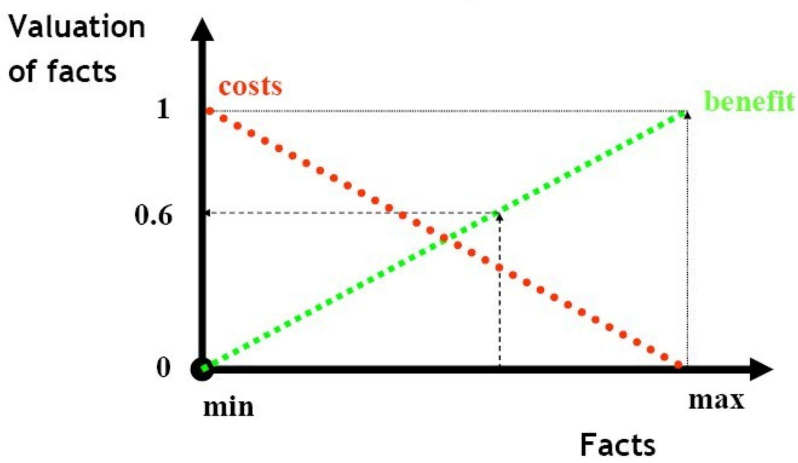


Figure 10: A linear benefit and a linear cost value function (Sharifi *et al*., 2007)

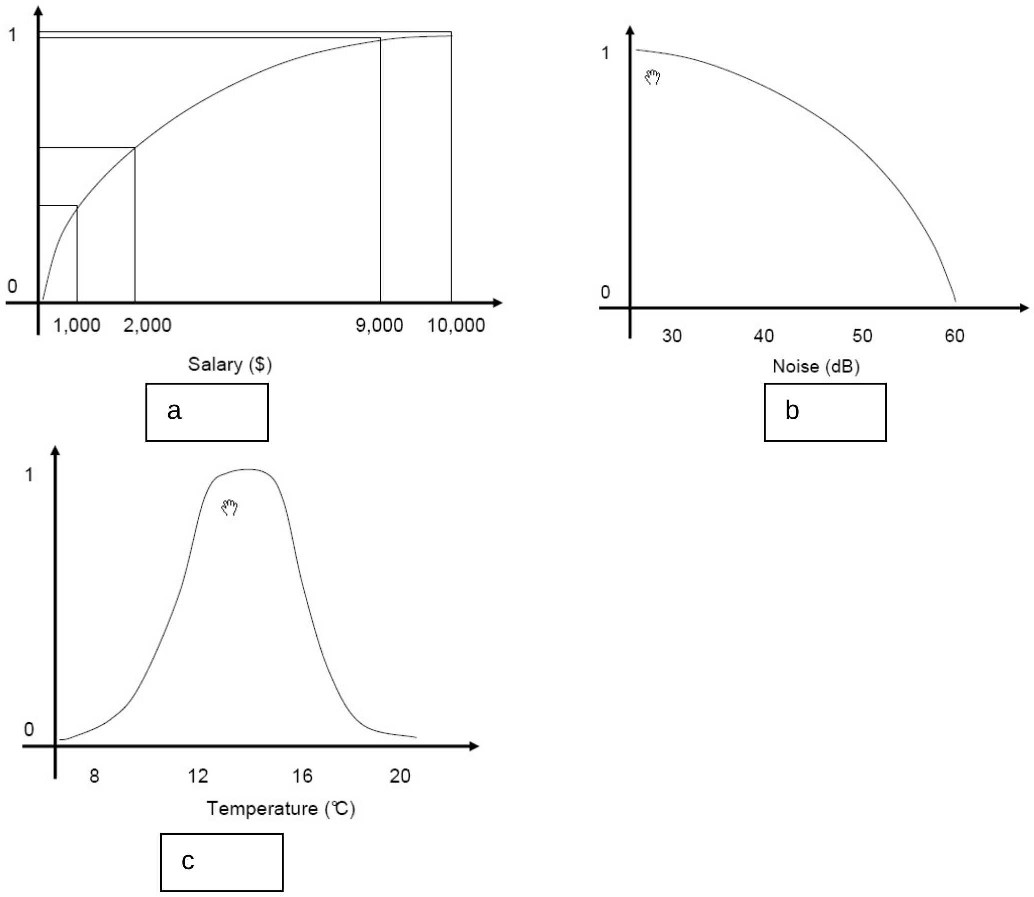


Figure 11: Benefit (a), cost (b) and mixed (c) value functions (Sharifi *et al*., 2007)

A number of methods have been developed for the creation of value functions. The most common are the midvalue method which has been proposed by Bodily (1985), the Evalue method (that combines the assessment of the range of scores and the weights of the evaluation criteria), which has been developed by Beinat (1997), and the direct value rating method (Beinat, 1997). The latter has been utilised for the purposes of this research because of its simplicity and flexibility in terms of assigning values not in predefined performance scores (as required in the midvalue method) but depending on the criterion concerned, taking into account that five land consolidation experts (not having knowledge of value functions) have been involved in the process as noted later.

5.2 Modelling value functions for land redistribution criteria

Direct value rating involves the following five steps for each criterion:

1) Selection of the score range for a criterion, i.e. the ideal or goal value (i.e. maximum value) and the minimum value which corresponds to values of 1

(best) and 0 (worst) respectively. The ideal values for each criterion have been defined based on the 40-year statistical records provided by the LCD for 74 land consolidation projects. In particular, the maximum 40-year figures for the ‘mean percentage change of size of parcels’, ‘percentage change of the number of landowners’, and ‘percentage of the ownerships completed’ were considered as the perfect achievement of the relevant objective. The other two attributes, i.e. the mean LSR and the mean PCC involve, by definition, a maximum value, i.e. 100% and 1, respectively. Minimum values are all zero. Beinat (1997) emphasises the significance of the end points of the value functions, hence the need to be interpreted and defined as accurately as possible.

2) Definition of the qualitative characteristics of the value function, i.e. monotonicity, shape, *etc*.
3) Assignment of values for selected criterion scores that have been defined by dividing the attribute range into 3 to 6 equal intervals resulting in 4 to 7 points, respectively. This task was carried out separately by five land consolidation experts after they were explained the concept and the aim of value functions by the author.
4) Curve fitting using appropriate software, which results in an explicit mathematical equation.
5) Consistency checks to confirm the validity of the functions as representations of preference. This involves examining the intermediate scores given by the five experts in step 3. This process actually finalises the value functions which are shown graphically and mathematically in Figures 12-17 and equations 14-18 respectively.

In all functions, scores lower than x_{min} are standardised at 0, while scores higher than x_{max} are standardised to 1. Figure 12 shows a concave benefit value function represented by:

$$V(x_i) = \frac{x_i}{13.754 + 0.882x_i + 2.290\sqrt{x_i}} \tag{14}$$

The value function increases sharply from 0 to 0.8 since the latter value corresponds to 100%, a score considered by experts as easily achievable, and then gradually increases up to the maximum score of 600% which is the highest ever achieved in land consolidation projects.

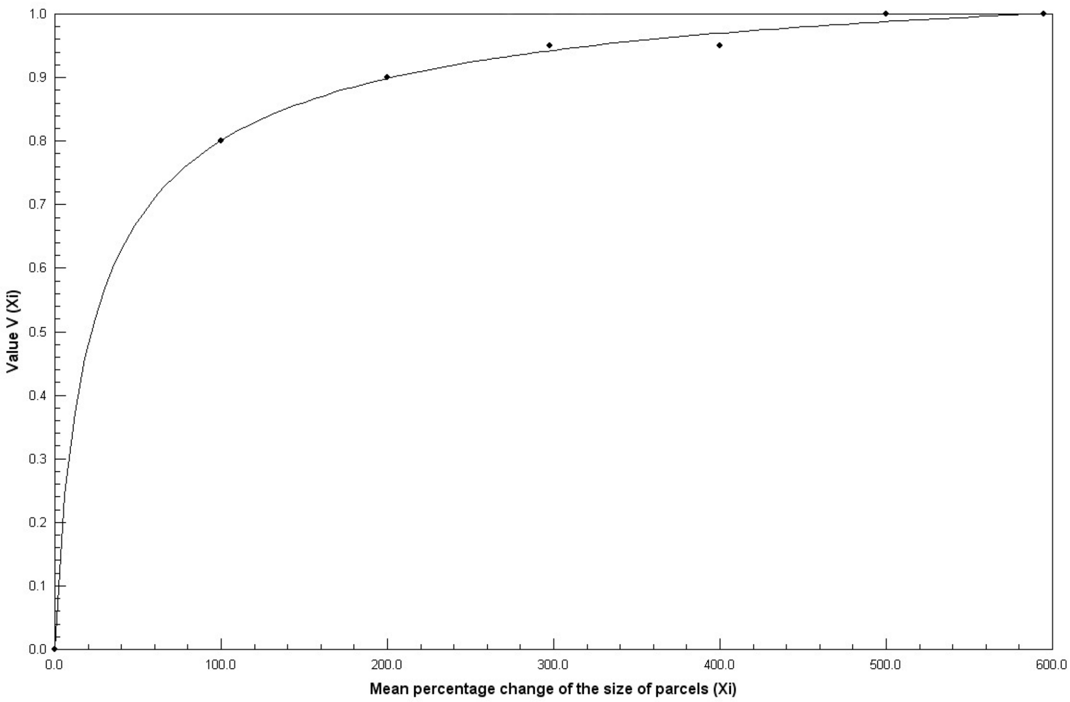


Figure 12: The value function for the criterion 'Mean percentage change of the size of parcels'

Figure 13 presents another concave benefit value function (represented by equation 15) which involves a dramatic upward movement from a score of 0 to 0.25 and then increases at a more gradual pace:

$$V(x_i) = \frac{x_i}{0.181 + 0.975x_i - 0.153{x_i}^2} \qquad (15)$$

The highest PCC score is 1 which means a perfect concentration of parcels, i.e. all the parcels belonging in a holding have been joined after land consolidation into just one parcel. On the other hand, the lowest PCC score, i.e. 0, means that nothing changed in terms of parcel concentration after land consolidation. A mean PCC of more than 0.5 (which corresponds to a value of 0.8 or 80%) is highly satisfactory for the experts since on most occasions it is not always possible to join all the parcels of a holding into a single parcel.

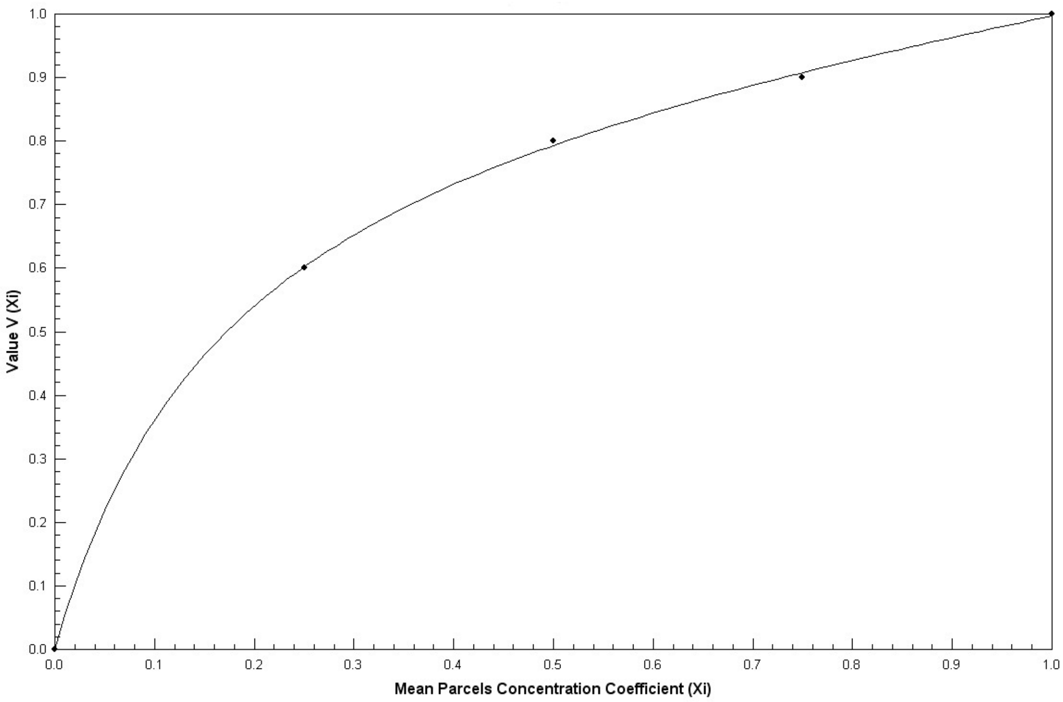


Figure 13: The value function for the criterion 'mean PCC'

Figure 14 shows a mixed bell-shaped benefit-cost value function represented by:

$$V(x_i) = 7.914\times10^{-6} x_i^{\ 4} - 6.368\times10^{-4} x_i^{\ 3} + 1.361\times10^{-2} x_i^{\ 2} - 3.208\times10^{-2} x_i \ + 1.332\times10^{-3} \tag{16}$$

The first half of the function, the rapid growth from 0 to the peak, i.e. 20%, means that the more the score extends, the better it is, since this range includes the holdings that are much smaller in size than the minimum limits provided by legislation, hence these properties will be redistributed to the other landowners either mainly to 'complete' their property at the minimum size or increase the size of other properties, e.g. the property that belongs to a farmer. Therefore, in this half of the function, more or less everybody is happy since, on the one hand, the landowners with very low property sizes will receive compensation to leave their property (and practice shows that 20% of them fully agree with this action) whereas, on the other hand, other landowners with greater than the minimum limits will be happy because they will be completed, which favours the efficiency of the land redistribution plan.

The other part of the function, i.e. which falls steeply from a score of 20% to 40%, represents a negative effect since a high percentage of landless landowners is generated with many not agreeing to leave their property for compensation. Thus, whilst there are negative social effects, there are also positive plan efficiency effects since land fragmentation is reduced.

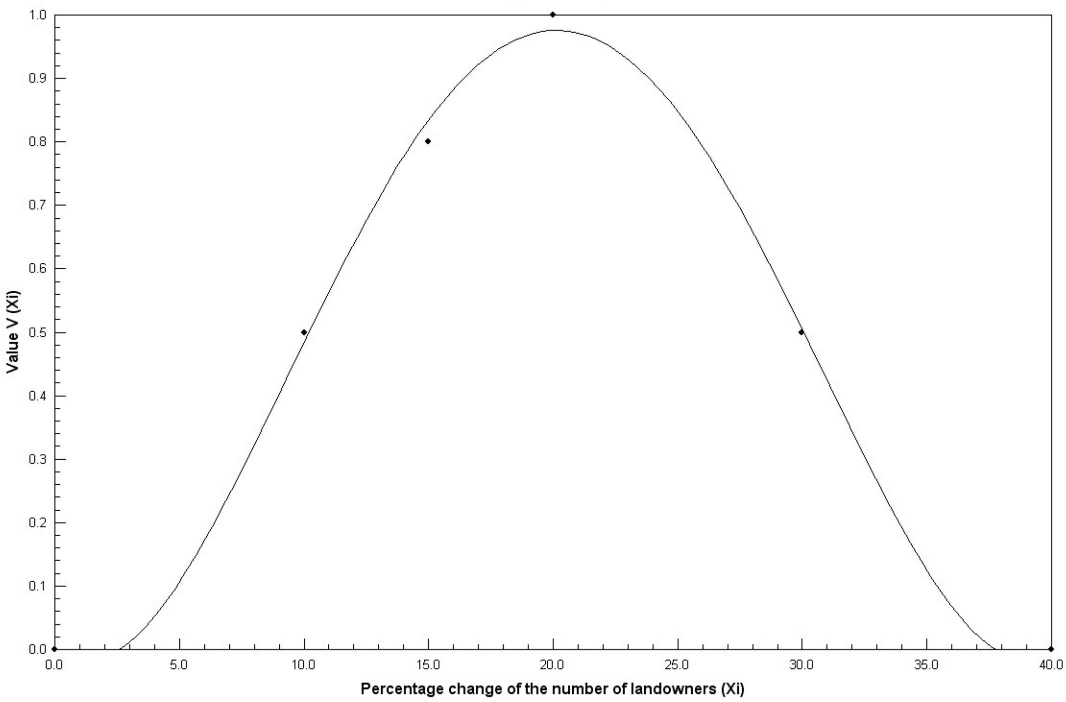


Figure 14: The value function for the criterion 'percentage change of the number of landowners'

Figure 15 shows an s-shaped benefit value function represented by:

$$V(x_i) = \frac{x_i}{420.714 + 7.681x_i - 106.064\sqrt{x_i}} \quad (17)$$

This function can be divided into three parts: from 0% to around 25%, from 25% to around 40% and from around 40% to 60%, represented by a convex curve, a straight line and a concave curve, respectively. The first part represents a low satisfaction rate of the objective concerned from 0 to 0.3 for a score distance of 25%; the second part presents a significant increase in the objective satisfaction from 0.3 to 0.7 for a score distance of 15%. Many land redistribution alternatives are expected to fall within this part. The third part is the most desirable in terms of alternative performance but is the most difficult to achieve with a value from 0.7 to 1 for a score distance of 20%.

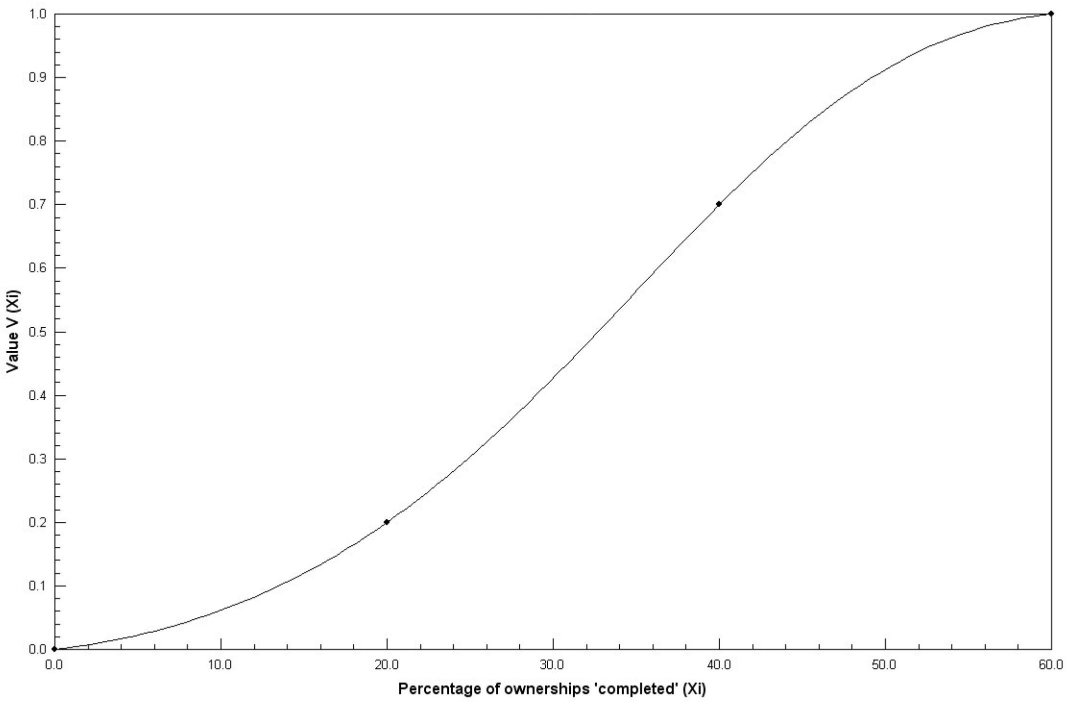


Figure 15: The value function for the criterion 'percentage of ownerships completed'

Figure 16 presents a convex benefit value function represented by:

$$V(x_i) = -3.906\times10^{-9}x_i^4 + 8.623\times10^{-7}x_i^{3} + 6.441\times10^{-5}x_i^2 - 1.161\times10^{-3}x_i + 1.984\times10^{-4} \quad (18)$$

This function increases slightly from 0% to 60% with values 0 to 0.3, respectively, representing low importance and then steeply increases from 60% to 90% with values 0.3 and 1 respectively, representing the most significant part of the function. This function reveals that the objective 'to maximise the acceptance of the plan by the landowners' is achieved only in high scores, i.e. from 80% and above. Even for the score of 80%, the value of the experts' satisfaction is just around 0.65. The reason is that the 40-year statistical records show that the objections submitted against the land reallocation plan are always (and in most cases considerably) less than 10%. The reason is that there is a continuous direct participation of the landowners during the entire process. However, it should be noted that the process of submitting objections against the plan is an official process which some landowners may avoid regardless of whether they are satisfied or not with the plan.

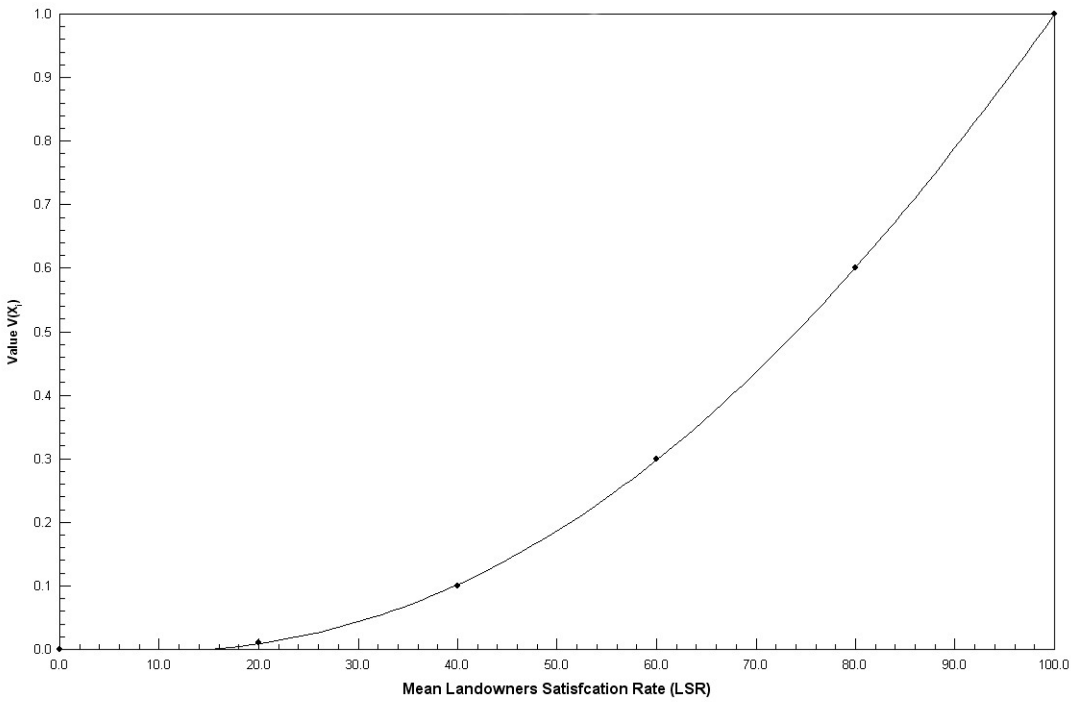


Figure 16: The value function for the criterion 'mean LSR'

Value functions are not modifiable by the users so they can only be illustrated graphically through the 'Value functions' window (Figure 17).

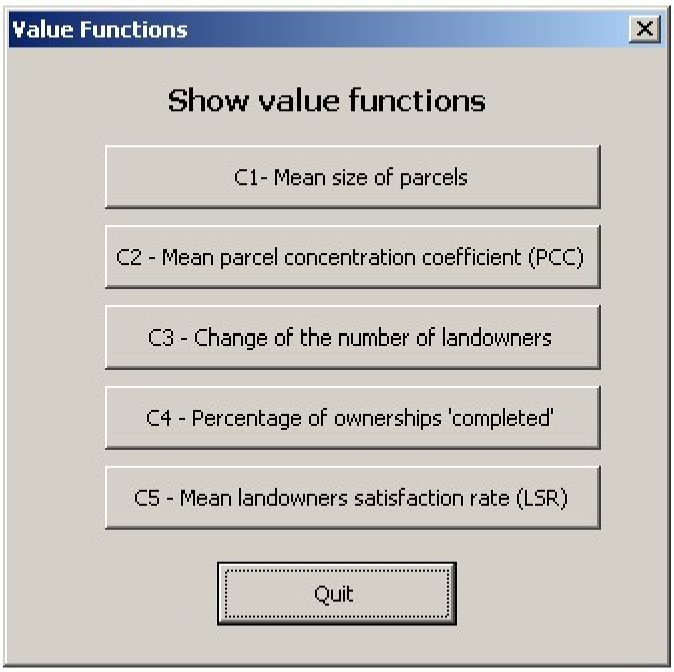


Figure 17: The 'Value Functions' window

6 Ranking alternatives

6.1 Decision rules

The outcome of the MADM is a ranking of alternatives which actually identifies the best alternative for the decision problem concerned. This process involves utilising an appropriate method which is commonly called a 'decision rule'. The most prominent decision rules are classified by Sharifi *et al*. (2004) into three main categories: compensatory methods, outranking methods and non-compensatory approaches.

Compensatory methods assume that a weak performance in one criterion may be compensated by a high performance of an alternative in another criterion. Thus, these methods involve aggregation of the performance scores of all criteria concerned. However, this additive representation is appropriate only if the evaluation criteria are independent of each other or, more precisely, are preferentially independent as explained earlier. The most well known compensatory approaches are: the simple additive weighting method; the value/utility function approach; the analytic hierarchy process (AHP); and the ideal point method. Malczewski (1999) notes that compensatory methods are the most popular for spatial decision-making problems. An interesting finding reported by Triantaphyllou (1997) regarding the first three methods noted earlier is that the choice of a method has little impact on the sensitivity of the results.

Outranking methods are partially compensatory since in practice some compensation is acceptable whilst others are not. These methods are based on pairwise comparisons between the alternatives and their outranking relations. The most popular series of such methods is called ELECTRE. Non-compensatory methods assume no compensation between the criteria at all. An example is the dominance method. For a comprehensive overview of decision rules, see Keeney and Raiffa (1993), Triantaphyllou (2000) and Sharifi *et al.* (2004).

6.2 The value function approach

As already noted, the method utilises a value function approach for ordering alternative land redistributions. Both value function and utility function approaches are based on multi-attribute utility theory. The difference between these approaches is that the former is applicable in decision problems under certainty (i.e. deterministic problems) whilst the latter is appropriate for decision problems under uncertainty (i.e. probabilistic problems). The problem concerned in this paper is clearly deterministic since the scores of the attributes are generated in a certain and straightforward way and not from a probabilistic distribution.

Apart from the problem being deterministic, the value function approach is preferred because of the following reasons: i) it incorporates the decision makers' preferences in the process through the development of the value functions (which have a series of advantages over other standardisation methods as discussed earlier); ii) evaluation criteria are preferentially independent (as shown and discussed earlier) and they can be expressed in the same value (via value functions) and thus a major requirement of the approach is met; and iii) the simplicity of the method.

The value function approach is actually the weighted average of the single attribute utilities (values) as shown in:

$$V_j = \sum_{i=1}^{N} w_i v_{ij} \tag{19}$$

where V_j is the overall value (or performance score) of the j^{th} alternative (j = 1 to M), v_{ij} is the standardised value of the score α_{ij} in the of the j^{th} alternative with respect to the ith criterion/attribute (i =1 to N) measured by utilising an appropriate value function, and w_i is the normalised weight for criterion/attribute i so that:

$$\sum_{i=1}^{N} w_i = 1 \tag{20}$$

The alternative that results in the highest V_i is characterised as the best alternative compared with the other competitive alternatives solutions. This approach is similar to the simple additive weighting method (SAW) or weighted summation, which instead of v_{ij} utilises x_{ij}, which is the standardised score of the j^{th} alternative with respect to the i^{th} criterion/attribute using linear transformation methods and not value functions.

7 Sensitivity analysis

Sensitivity analysis (SA) is a process involving investigation of the impacts in the decision outcomes of even potentially slight changes and errors in the problem inputs, i.e. data and parameters. It is a very critical task which must be carried out in decision-making processes since it reveals how reliable the final decisions are (Pannell, 1997).

In contrast, a study carried out by Delgato and Sendra (2010), which reviewed how SA has been involved in spatial multi-criteria decision problems, revealed that it is not a common practice. In the case of MADM, two important elements need to be examined in the context of SA: the weights of the evaluation criteria and the criterion scores (or performance measures) (Triantaphyllou, 1997; Malczewski, 1999).

Work by Delgato and Sendra (2010) showed that most models involve a SA only on the former element. It should be noted that SA is a wide issue for which there is a huge literature (Panell, 1997) and specific software packages have been developed such as DEFINITE (Janssen *et al*., 2001), Expert Choice (Malczewski *et a*l. 1997) and Best Choice (Jankowski, 1995). Thus, this research does not aim at developing a new method of SA; it adopts the SA methodology used by Triantaphyllou (1997, 2000) which is reliable and has been incorporated into other SDSS (e.g. MULINO; Mysiak, 2004). It provides several useful SA parameters and focuses on both the weights of the evaluation criteria and criterion scores.

7.1 Sensitivity of the weights of the criteria

The sensitivity of criterion weights is crucial because the process of assigning weights is subjective and hence it may demonstrate significant variation between the decision makers' perceptions and preferences. In addition, the available methods for defining weights may lead to different results. The decision maker can take better decisions if he/she is aware of how critical each criterion is.

Thus, Triantaphyllou (1997, 2000) developed a methodology to determine the most critical criterion in a twofold way: in the first instance, the focus is on identifying the criterion for which the smallest change of a current weight may alter the best alternative; and in the second case, the aim is to find out for which criterion the smallest change in a current weight may change the ranking of any alternative. In this vein, Triantaphyllou introduced the terms 'top critical criterion' and 'any critical criterion', respectively. Each term may be associated with two concepts with respect to changes in the weights: in relative terms and absolute terms. However, the latter may be misleading since, e.g. a change of 0.02 is very different in terms of influence if the original values of the weights are 0.08 and 0.8. Thus, this research adopts the former approach since expressing the change in relative terms, i.e. as a percentage, is more meaningful. Therefore, the relevant terms are called 'percent-top critical criterion' (PTCC) and 'percent-any critical criterion' (PACC), respectively.

Triantaphyllou (1997, 2000) defined the PTCC as the criterion which has the smallest $\delta'_{k,i,j}$ quantity, which expresses changes in weights in relative terms. Thus, the ranking of alternatives will be reversed as follows:

$$\delta'_{k,i,j} < \frac{(p_j - p_i)}{(a_{jk} - a_{ik})}\frac{100}{w_k} \quad \text{if } (a_{jk} > a_{ik}) \tag{21}$$

or

$$\delta'_{k,i,j} > \frac{(P_j - P_i)}{(a_{jk} - a_{ik})}\frac{100}{w_k} \quad \text{if } (a_{jk} < a_{ik}) \tag{22}$$

Furthermore the following condition should be met for the value $\delta'_{k,i,j}$ to be feasible:

$$\frac{(P_j - P_i)}{(a_{jk} - a_{ik})} \leq W_k \tag{23}$$

In particular, if the above condition is not satisfied, it means that it is impossible to reverse the existing ranking of the alternatives A_i and A_j by making changes to the current weight of criterion C_k, i.e. the examined criterion, where $\delta'_{k,i,j}$ is the minimum change in relative terms (i.e. percentage) in the current weight W_k of criterion C_k such that the ranking of alternatives A_i and A_j will be reversed, i is the number of the alternative, j is the number of the criterion and k is the criterion for which the weight change is calculated. a_{ij} is the standardised score of alternative i for criterion j and P_i and P_j are the ranking order of alternatives i and j, respectively.

Thus, in order to find the most critical criterion, all possible $\delta'_{k,i,j}$ should be calculated for all combinations of alternatives (M) and all criteria (N) which equal to $(N \times M(M-1))/2$. The process is shown in Table 7.

Table 7: The process for finding the most critical criterion

	Criteria		
Pairs of alternatives	C_1	.	C_n
$A_1 - A_2$	$\delta'_{1,1,2} = \frac{(P_2 - P_1)}{(a_{21} - a_{11})} \frac{100}{w_1}$	.	$\delta'_{1,1,2} = \frac{(P_2 - P_1)}{(a_{2N} - a_{1N})} \frac{100}{w_1}$
$A_1 - A_3$	$\delta'_{1,1,3} = \frac{(P_3 - P_1)}{(a_{31} - a_{11})} \frac{100}{w_1}$	.	$\delta'_{1,1,3} = \frac{(P_3 - P_1)}{(a_{3N} - a_{1N})} \frac{100}{w_1}$
$A_2 - A_3$	$\delta'_{1,2,3} = \frac{(P_3 - P_2)}{(a_{31} - a_{11})} \frac{100}{w_1}$	.	$\delta'_{1,2,3} = \frac{(P_3 - P_2)}{(a_{3N} - a_{1N})} \frac{100}{w_1}$
…	…	.	…
$A_{M-1} - A_M$	$\delta'_{1,M-1,M} = \frac{(P_M - P_{M-1})}{(a_{M,1} - a_{M-1,1})} \frac{100}{w_1}$	.	$\delta'_{N,M-1,M} = \frac{(P_M - P_{M-1})}{(a_{M,N} - a_{M-1,M})} \frac{100}{w_1}$

The PTCC is that with the minimum $\delta'_{k,i,j}$ absolute value between the best ranked alternative and any other alternative whilst the PACC is that with the minimum $\delta'_{k,i,j}$ absolute value between any pair of alternatives. A *robust criterion* is that for which all its associated $\delta'_{k,i,j}$ quantities are infeasible.

Triantaphyllou (1997, 2000) also defines some more terms as follows: the criticality degree of criterion C_k denoted as D'_k is the smallest percentage by which the current value of weight W_k should change such that the existing ranking of alternatives will change:

$$D'_k = \min_{1 \le i < j \le M} \left| \delta'_{k,i,j} \right| \text{, for all } N \ge k \ge 1 \tag{24}$$

Based on this, the sensitivity coefficient of criterion $\mathrm{C_k}$ denoted as $\mathrm{sens(C_k)}$ has been defined and is equivalent to the reciprocal of the criticality degree, i.e.:

$$sens(C_k) = \frac{1}{D'_k} \text{, for all } N \ge k \ge 1 \tag{25}$$

If the criticality degree is infeasible, i.e. no weight change alters the order of the alternatives, then the sensitivity coefficient of that criterion is set equal to zero. This measure is very useful since it represents how sensitive each criterion is.

All of the above SA measures defined by Triantaphyllou (1997, 2000) can be calculated in the Evaluation module using the window that appears when clicking on the 'Sensitivity Analysis' menu item. The relevant functions are provided in the first part of this window illustrated in Figure 18.

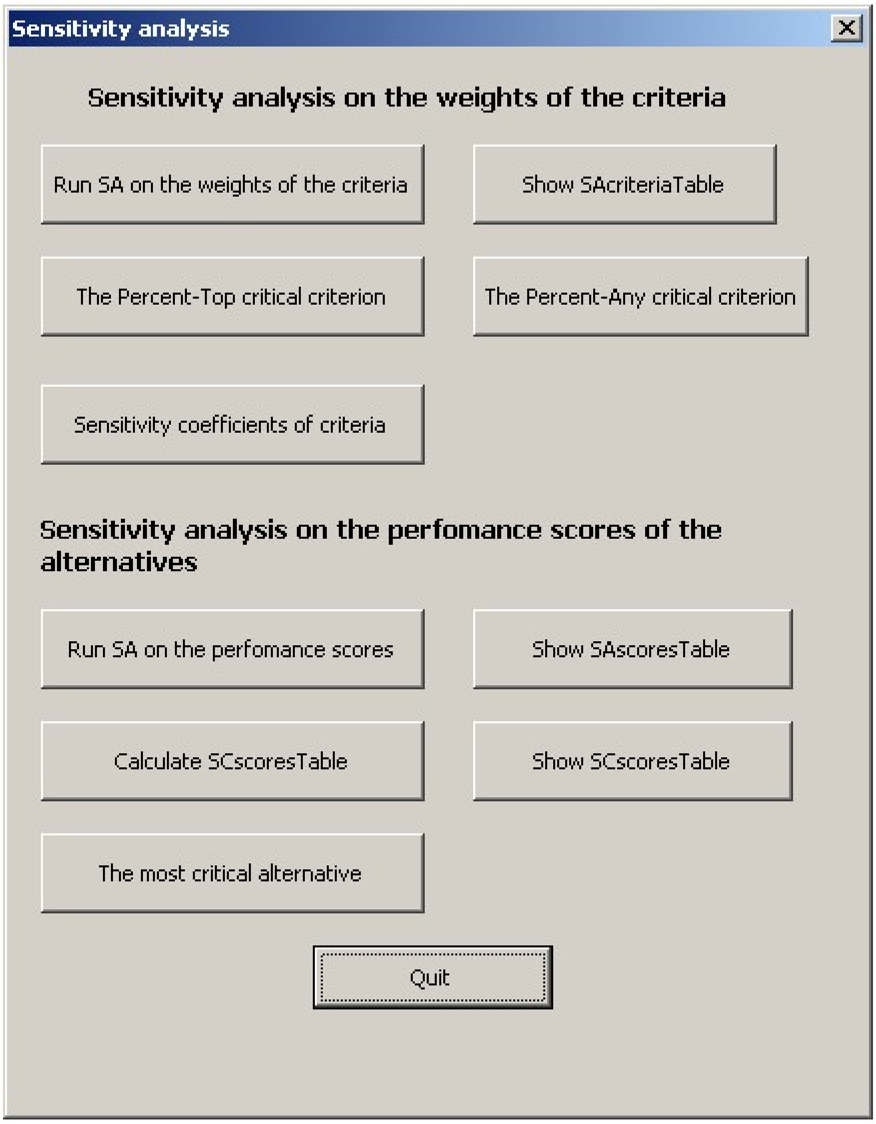


Figure 18: The 'Sensitivity Analysis' window

The most important table calculated for the SA of the weights of the criteria is the so called SAcriteriaTable based on the table illustrated in Figure 5. An example is illustrated in Figure 19.

Attributes of SAcriteriaTable

OID	Alt(i)	Alt(j)	C1	C2	C3	C4	C5
23	3	10	-89.82	-232.56	11111	81.13	1111111
24	4	5	11111	111111	11111	11111	1111111
25	4	6	11111	111111	11111	11111	-1132.44
26	4	7	11111	111111	11111	11111	-1006.44
27	4	8	11111	111111	11111	11111	-5990.06
28	4	9	35.791	111111	11111	11111	-55.7429
29	4	10	-47.27	-132.46	11111	43.91	94.94031
30	5	6	11111	111111	11111	11111	-260.983
31	5	7	11111	111111	11111	11111	-246.618
32	5	8	11111	111111	11111	11111	-963.893
33	5	9	11111	111111	11111	11111	1111111
34	5	10	18.514	33.5271	-78.93	-18.71	-129.338
35	6	7	-32.19	39.1945	-10.10	9.363	1111111
36	6	8	11111	111111	11111	11111	1111111
37	6	9	11111	111111	11111	11111	-1286.72
38	6	10	77.228	111111	-612.7	-321.3	-230.733
39	7	8	11111	111111	-1067.	11111	1111111
40	7	9	11111	111111	11111	11111	-1127.33
41	7	10	73.065	111111	11111	-201.8	-220.439
42	8	9	11111	111111	11111	11111	-8357.47
43	8	10	11111	111111	-1669.	11111	-738.621
44	9	10	-44.45	-128.94	11111	42.75	88.41496

Record: 28 Show: All Selected

Figure 19: Example output: part of the SAcriteriaTable

For each combination of pair of alternatives and for each criterion that is involved in the evaluation process, the $\delta'_{k,i,j}$ is calculated, which expresses the percentage change in a given criterion weight that will inverse the ranking order of a given pair of alternatives. If the value is negative, this means an increase change and *vice versa.* The values containing ones means that the ranking order of the pair of alternatives concerned cannot be changed under any change of the weights of the criterion concerned. As noted above, the PTCC, the PACC and the $sens(C_k)$ are calculated from this table. These are very vital measures for assessing the reliability of the defined weights.

7.2 Sensitivity of the performance scores

Although the process of calculating the scores of attributes for each criterion is more certain than assigning weights to criteria, the standardisation process of utilising value functions involves a considerable subjectivity after the calculation of the scores, since value functions have been defined by experts and the process of assessing them is inherently prone to uncertainties. Therefore, carrying out a SA for the scores of attributes is also important in this model.

In the same vein as for the sensitivity of the weights of the criteria, Triantaphyllou (1997, 2000) defined the following concepts for the sensitivity of the performance scores:

The 'most sensitive alternative' is the alternative which is associated with the smallest threshold value called $\sigma'_{i,j,k}$. This value denotes the threshold value of the performance score a_{ij}, i.e. the minimum change that must occur in order to change the current ranking of alternatives A_i and A_j. Thus, the weighted summation methods (including the value function method) can be defined as follows:

The $\sigma'_{i,j,k}$ in relative terms (i.e. %) by which the performance score of alternative A_i with regards to criterion C_j denoted as a_{ij} needs to be modified so that the ranking of alternatives A_i and A_k will be reversed if:

$\sigma'_{i,j,k} < R$ when $i < k$

or

$\sigma'_{i,j,k} > R$ when $i > k$ where R defined as:

$$R = \frac{(P_i - P_k)}{W_k} \times \frac{100}{a_{ij}} \tag{26}$$

The threshold value $\sigma'_{i,j,k}$ should be ≤ 100 to be feasible.

Following the same notion as in the SA of criterion weights, Triantaphyllou (1997, 2000) defined the following terms:

The 'criticality degree of alternative', A_i, is the minimum threshold value $\sigma'_{i,j,k}$ associated with that alternative and any other alternative. In other words, the smaller the criticality degree is, the easier the ranking of an alternative can change. Based on this, the smallest criticality degree between all alternatives gives the 'most critical alternative'. In addition, a sensitivity coefficient of alternative A_i in terms of criterion C_j denoted as $sens(a_{ij})$ is the reciprocal of its criticality degree. It can be concluded that the most sensitive alternative is the one with the highest sensitivity coefficient. These SA concepts are calculated by utilising the functions provided in the second part of the window shown in Figure 17. In particular, the SAscoresTable, which is an example output illustrated in Figure 20, involves in each of its elements the $\sigma'_{i,j,k}$ value for each combination of pair of alternatives and criteria involved in the process. Each value means that a given performance score should be changed by this percentage from its current value in order to switch the ranking of a pair of alternatives. The smallest values for each alternative and each criterion, which are called criticality degrees as mentioned earlier, are output to a table called the SCscoresTable an example of which is shown in Figure 21. The criticality degrees define the most competitive alterative of each alternative for each criterion which may reverse the ranking. This is also a very critical measure for our case since a potential slight change in the definition of a value function and in general of the standardisation method may considerably alter the ranking order of the alternatives.

Attributes of SAscoresTable

OID	Alt(i)	Alt(j)	C1	C2	C3	C4	C5
0	1	2	-10.718635	-8.881943	-6.54342	-4.858953	-5.446717
1	1	3	-12.855529	-10.652669	-7.847933	-5.827646	-6.532589
2	1	4	-1.034141	-0.856936	-0.631314	-0.468795	-0.525503
3	1	5	21.276097	17.63033	12.988449	9.644844	10.811534
4	1	6	66.362081	54.990604	40.512154	30.083145	33.722158
5	1	7	65.508927	54.283643	39.991328	29.696395	33.288625
6	1	8	1111111	1111111	95.013228	70.554055	79.088639
7	1	9	-0.618366	-0.512406	-0.377495	-0.280316	-0.314225
8	1	10	14.610101	12.106587	8.919049	6.623026	7.424181
9	2	1	10.550715	8.881943	6.266317	4.823883	5.43899
10	2	3	-2.103417	-1.770727	-1.249269	-0.961701	-1.084331
11	2	4	9.532775	8.025007	5.661739	4.358471	4.914233
12	2	5	31.493497	26.512273	18.704728	14.399113	16.235187
13	2	6	75.873156	63.872547	45.06285	34.689896	39.113309
14	2	7	75.033369	63.165586	44.56408	34.305937	38.680391
15	2	8	1111111	1111111	97.255897	74.868698	84.415432

Record: 1 Show: All Selected Records (of 90)

Figure 20: Example output: part of the SAscoresTable

Attributes of SCscoresTable

OID	Alt(i)	C1	Alt-C1	C2	Alt-C2	C3	Alt-C3	C4	Alt-C4	C5	Alt-C5
0	1	0.618366	9	0.512406	9	0.377495	9	0.280316	9	0.314225	9
1	2	2.103417	3	1.770727	3	1.249269	3	0.961701	3	1.084331	3
2	3	2.06366	2	1.770727	2	1.233412	2	0.976525	2	1.076934	2
3	4	0.41152	9	0.34453	9	0.253819	9	0.188479	9	0.211278	9
4	5	6.793695	10	5.919721	10	4.0694	10	3.021818	10	3.594175	10
5	6	1.2712	7	0.846661	7	0.520826	7	0.424285	7	0.420807	7
6	7	1.323463	6	0.828758	6	0.549137	6	0.405892	6	0.419441	6
7	8	193.497422	6	112.823942	6	54.501075	6	56.59338	6	44.771422	6
8	9	0.416306	4	0.34453	4	0.253819	4	0.188479	4	0.21048	4
9	10	4.969975	5	5.031358	5	4.29061	5	3.603758	5	3.696908	5

Record: 1 Show: All Selected Records (of 10) Options

Figure 21: Example output: SCscoresTable

8 Case study: evaluating alternatives

The system ran for ten different sets of facts generating ten alternative land redistributions. The facts, i.e. decision variables, for each alternative are shown in Table 8. Each alternative is described briefly in Table 8 by comparing its facts with those of alternative 1, which it is the solution given by the experts:

Table 8: The description of the 10 alternative land redistributions

Alternative No	Description
1	The expert's solution (Irrigated project)
2	Medium area and land value minimum limits
3	High area and land value minimum limits
4	Unequal PPI weights for area and land value
5	Low small-medium-large holdings sizes
6	High minimum area of new parcels with high area and land value minimum limits
7	Low minimum area of new parcels with high area and land value minimum limits
8	Low area and land value minimum limits with low small-medium-large holdings sizes
9	Inverse unequal PPI weights for area and land value (comparing to alt-4)
10	Arid project

These alternative solutions were input into the Evaluation module for assessment based on two different basic scenarios 1 and 2. The former involves changing the weights of all criteria involved in the process including four different case scenarios and the latter focuses on different project objectives including two case scenarios.

8.1 Changing the weights of the criteria

Ranking alternatives

Ranking alternatives is carried out using four scenarios. In scenario 1, all five criteria have the same weight. In scenario 2, weights were assigned to each of the five criteria in the following descending order of importance: extremely high, very high, high, intermediate and moderate. In contrast, the weights in scenario 3 have been assigned in ascending order of importance, whilst in scenario 4, they were assigned based on the judgement of the principal author as: extremely high, high, high, intermediate and very high, respectively. The performance score and the rank order of each alternative for each scenario are shown in Table 9 while the critical criteria and most critical alternative for each scenario are shown in Table 10. A graphical representation of the ranking of alternatives per scenario is also illustrated in Figure 22.

Table 9: The performance score and the ranking order of each alternative for four weighting scenarios

	Scenario-1		Scenario-2		Scenario-3		Scenario-4	
Ranking	**Alternative**	**Score**	**Alternative**	**Score**	**Alternative**	**Score**	**Alternative**	**Score**
1	Alt-3	0.823	Alt-10	0.791	Alt-3	0.875	Alt-10	0.797
2	Alt-2	0.820	Alt-3	0.765	Alt-2	0.873	Alt-3	0.789
3	Alt-4	0.809	Alt-2	0.761	Alt-9	0.863	Alt-2	0.784
4	Alt-9	0.809	Alt-4	0.751	Alt-4	0.863	Alt-4	0.775
5	Alt-1	0.808	Alt-9	0.749	Alt-1	0.862	Alt-9	0.774
6	Alt-10	0.804	Alt-1	0.749	Alt-5	0.839	Alt-1	0.773
7	Alt-5	0.787	Alt-5	0.729	Alt-7	0.818	Alt-5	0.750
8	Alt-6	0.737	Alt-6	0.652	Alt-6	0.816	Alt-6	0.695
9	Alt-7	0.735	Alt-7	0.646	Alt-10	0.815	Alt-7	0.690
10	Alt-8	0.647	Alt-8	0.555	Alt-8	0.734	Alt-8	0.612

Table 10: Critical criteria and alternatives for each scenario

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Percent top critical criterion	C1	C1	C4	C1
Percent any critical criterion	C1	C5	C1	C1
Most critical alternative	A9	A9	A4	A1

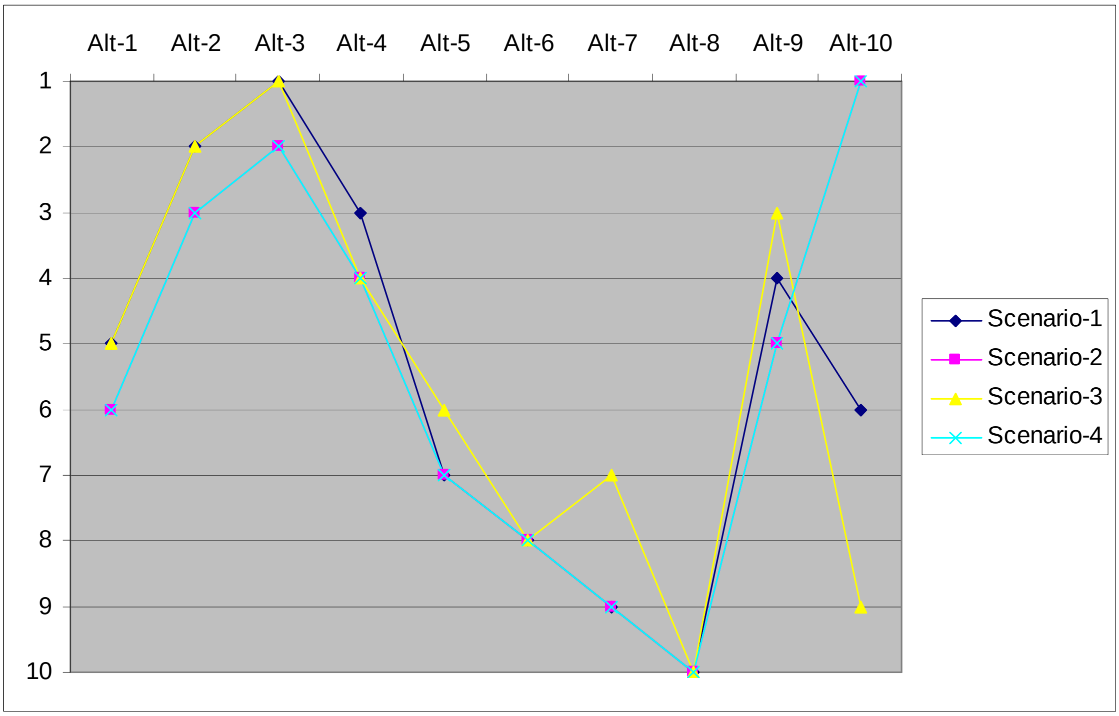


Figure 22: Ranking of alternatives for four different criteria weighting scenarios

Some interesting findings are the following: no one alternative is best in all scenarios. In particular, alternatives 3 and 10 are ranked as best in scenarios 1, 3 and 2, 4 respectively. It is also remarkable that alternative 10 ranks third and seventh in scenarios 1 and 3 respectively while alternative 3 is ranked first. In contrast, alternative 3 is ranked second in the case of the scenarios in which alternative 10 is ranked first, revealing that alterative 3 is more reliable in terms of performance, presenting stable behaviour in all scenarios. Hence this is the best alterative in the eye of the expert. The unreliability of alternative 10 is also revealed by the fact that it is the only alternative that presents so much distance in ranking positions (i.e. first, sixth and ninth) while all the other alternatives change at worst by two positions in terms of ranking. It also remarkable that the variability of the performance scores of the best and worst alternatives varies greatly per scenario, i.e. 21.4%, 29.8%, 16.1% and 23.2% for the four scenarios respectively, which means that different facts and different weight schemes may produce considerably varying alternatives.

Furthermore, the ranking of alternative 1 (which represents the solution given by the human experts in the case study), i.e. fifth or sixth in the four scenarios, indicates that it underperforms compared to alternatives 2, 3, 4 and 9 with which it is comparable in terms of facts. This proves that the system may produce better solutions than the experts. Moreover, it is clear that alternative 8 ranks last in all scenarios. A general finding is that the ranking of alternatives is very sensitive to the alteration of the weights of the criteria. Therefore, planners should be aware both of the weights assigned to each criterion and hence the weighting method utilised.

Performance of alternatives per criterion and scenario

Figures 23-26 show the performance of each alternative for each criterion in the four scenarios revealing some more in depth findings associated with the ranking of the alternatives. In particular, while alternative 10 achieves the highest performance (standardised values) for criteria 1 and 2 in all scenarios (Figures 19-22) and is ranked first in scenarios 2 and 4 overall, it significantly underperformed in scenarios 1 and 3 (ranked sixth and ninth, respectively) because criteria 1 and 2 were given lower weights in scenarios 1 and 3 so had significantly less impact on the overall results. More specifically, the weights are very high for both criteria in the former scenarios while in contrast they are very low in the latter scenarios. In addition, alternative 10 presents the worst performance values for criteria 3 and 5 in all the scenarios. In contrast, alternative 3 is actually the best and more balanced as noted earlier. Regardless, it only achieves the best performance (among all alternatives) in criterion 3. In addition, as the aggregated ranking showed earlier, Figures 23-26 illustrate more analytically that no alternative is best in all of the criteria in any scenario. It is also clear that alternative 8, which is ranked last for all scenarios, gives the worst performance in criteria 1, 2 and 4. Another general finding from the figures concerned is that criteria 1, 2 and 4 indicate a high variability in the values of the alternatives while in contrast, criteria 3 and 5 present a low variability.

If the above findings are associated with the facts of alternatives, the results are reasonable. In particular, alternative 10, which involves the highest values in facts F1-F8 and F11, was expected to achieve the best performance in the aim 'to minimise land fragmentation', which is linked with criterion C1, the mean size of parcels and criterion C2, the dispersion of parcels. In contrast, alternative 10 achieves the worst or poorer performance in terms of the aim 'to minimise social impacts', which is associated with criterion C3, the land exchange balance, criterion C5, the LSR and criterion C4, the number of ownerships 'completed', respectively. On the other hand, alternative 3 involves facts that try to balance a trade-off between all the criteria so as to achieve a good performance as much as possible across all criteria. In addition, alternatives 6, 7 and 8, which involve the lowest value of the fact F1, i.e. the minimum area limit of the new parcels, achieves the worst outcomes in the criteria that are associated with minimising land fragmentation, i.e. C1, C2 and C4. Hence they rank as the last three alternatives in scenarios 1, 2 and 4 and in the last four in scenario 3.

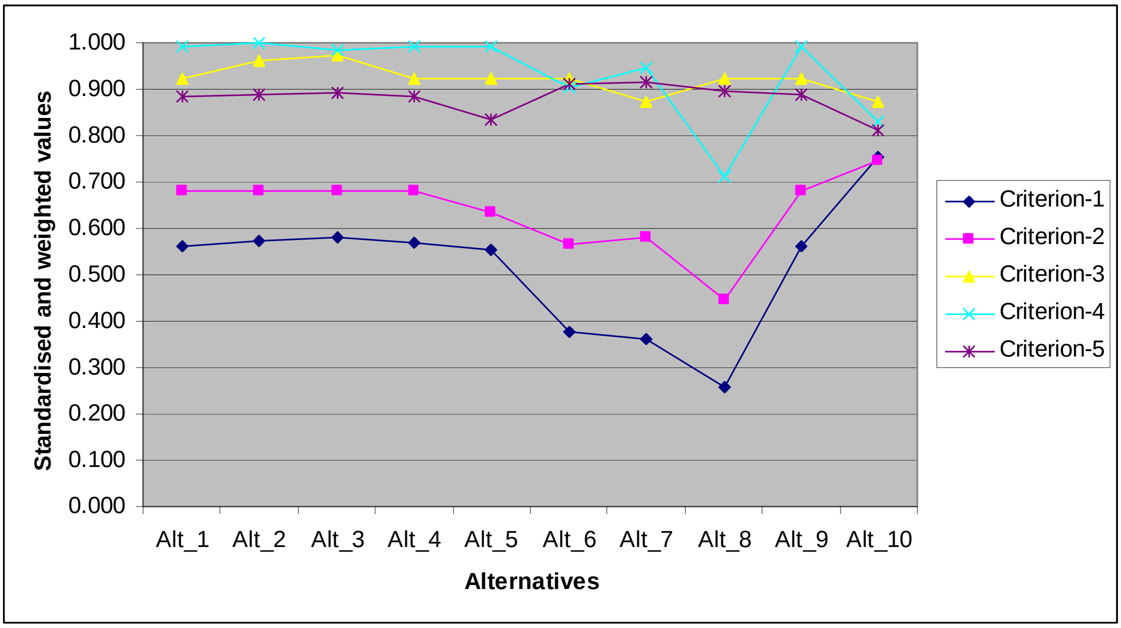


Figure 23: Performance of alternatives for all criteria in scenario 1

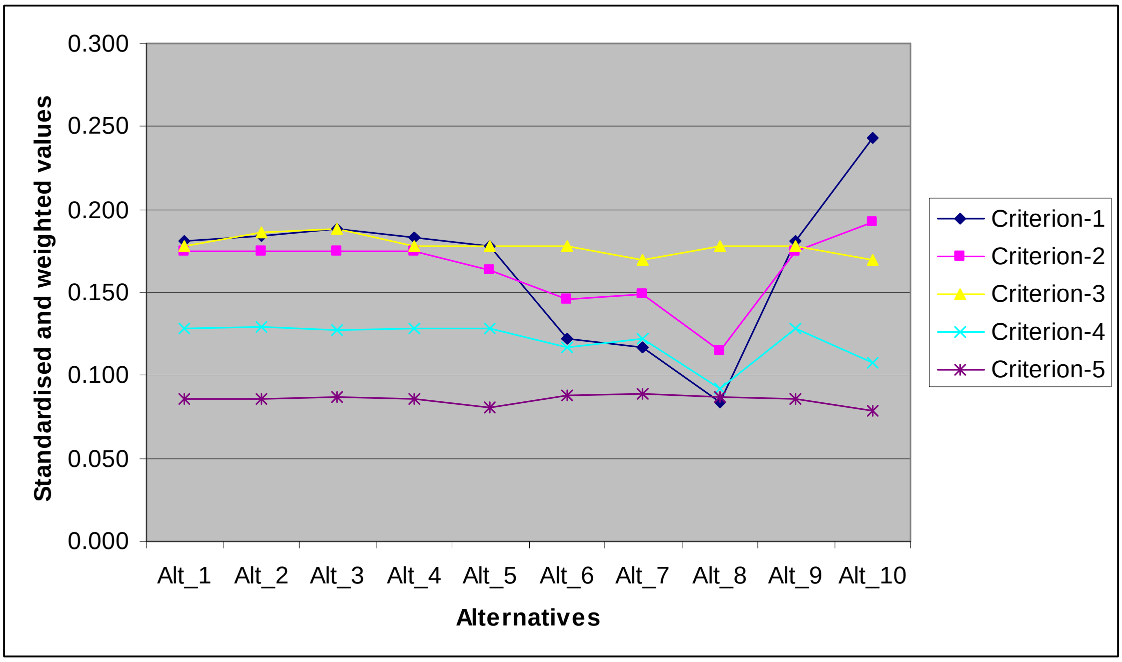


Figure 24: Performance of alternatives for all criteria in scenario 2

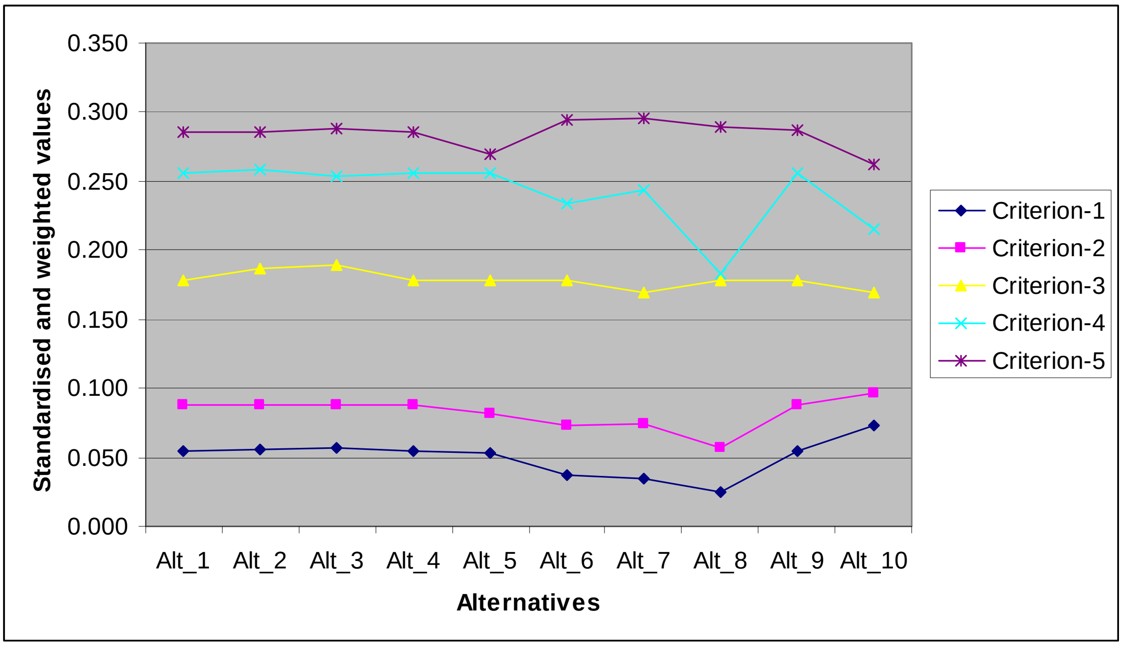


Figure 25: Performance of alternatives for all criteria in scenario 3

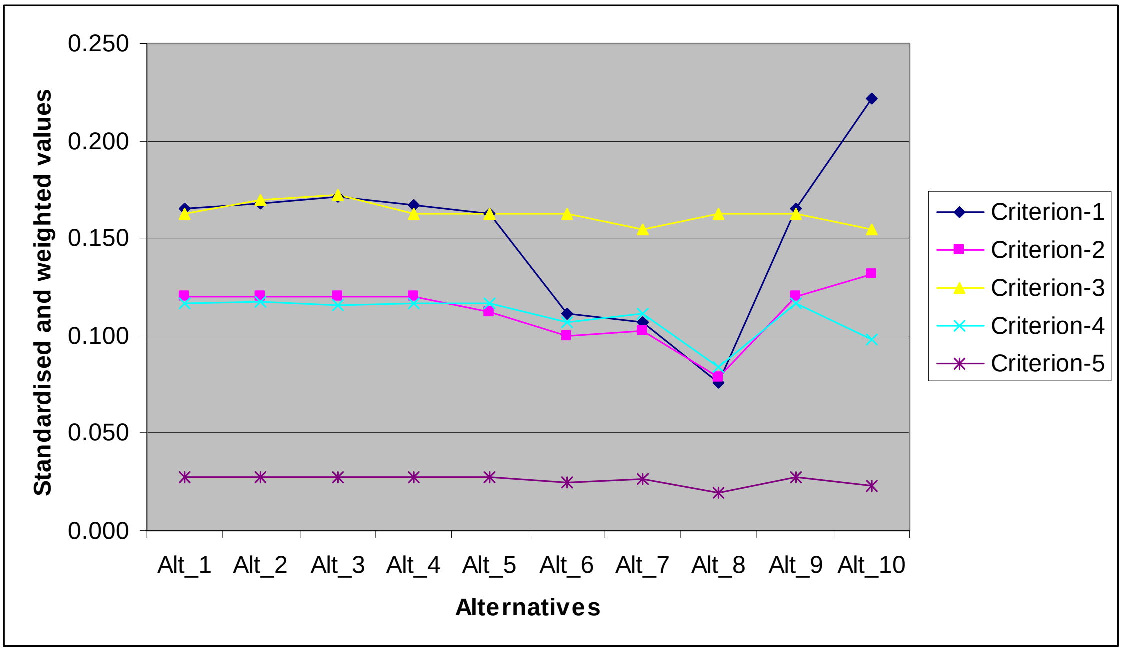


Figure 26: Performance of alternatives for all criteria in scenario 4

Sensitivity analysis

As mentioned earlier, the module utilises the approach of Triantaphyllou (1997, 2000) in undertaking a SA of both the weights of the criteria and the performance scores. Table 11 and Figure 27 show the sensitivity coefficient variability for all criteria for each scenario; the higher the sensitivity coefficient, the more sensitive is that criterion in terms of changing the rank of the best alternative or any pair of alternatives. It is apparent that all the criteria are very sensitive in scenario 3. The reason is that the weighting scheme in scenario 3 can be considered as paradox in terms of the logical importance of criteria that could be assigned by land consolidation experts. As a result, a slight change of weights towards a more reasonable scheme causes a change in the rank order of the alternatives. In contrast, the criteria are much less sensitive for the other three scenarios because they involve a ‘sensible’ weighting pattern in terms of practice.

Table 12 shows the most critical criteria and alternatives. The ‘percent top critical criterion’ (PTCC) is C1 for scenarios 1, 2 and 4. That is, if the weight for C1 changes by 55.84%, 46.15% and 14.74%, the ranking of best alternatives will alter, i.e. alternatives 3, 10 and 4 for the relevant scenarios will change. It is noted that the ‘qualitative rating’ method involves a change of 90% from best (i.e. extremely high importance) to worst (i.e. very low importance) (Table 8), hence it is not impossible having the percentage changes mentioned earlier. Criterion C1 is the most critical for three out of four scenarios since it presents the highest range of values for the former and a low range of values for the latter scenario. Similarly, the ‘percent any critical criterion’ (PACT) is C1 for scenarios 1, 3 and 4 if the weights of C1 change by 12.28%, 2.62% and 14.74% respectively, i.e. any ranking may change.

In addition, the most sensitive alternative in terms of changing ranking is alternative 9 (because of C4 and C1, respectively) for scenarios 1 and 2, alternative 4 (because of C5) for scenario 3, and alternative 1 (because of C5) for scenario 4. This finding is illustrated in Table 11 where each alternative pair, i.e. 9-1, 9-1, 9-4, and 9-1 which correspond to the four scenarios respectively, has almost the same performance scores. Thus, even a slight change in weighting will change ranking. Another interesting finding extracted from Table 11 is that there is no association between the sensitivity coefficient and the weights for each criterion for the three first scenarios since the correlation coefficient (R) was calculated as 0, -0.24, -0.09, respectively. However, there is a relationship (R = 0.79) in the case of scenario 4 perhaps because this scenario involves weights assigned by the expert and they have not been randomly defined as in the first three scenarios. In addition, the most critical criterion is that with the highest weight, a result that confirms the finding of Triantaphyllou (1997).

Table 11: Sensitivity coefficient and weight for criteria for the four scenarios

Criteria	Scenario-1		Scenario-2		Scenario-3		Scenario-4	
	SensC	Weight	SensC	Weight	SensC	Weight	SensC	Weight
C1	0.081	0.200	0.025	0.323	0.382	0.097	0.068	0.294
C2	0.028	0.200	0.006	0.258	0.241	0.129	0.016	0.176
C3	0.077	0.200	0.018	0.194	0.096	0.194	0.024	0.176
C4	0.068	0.200	0.010	0.129	0.196	0.258	0.024	0.118
C5	0.032	0.200	0.035	0.097	0.341	0.323	0.026	0.235

Table 12: Critical criteria and alternatives for each scenario

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Percent top critical criterion	C1	C1	C4	C1
Percent any critical criterion	C1	C5	C1	C1
Most critical alternative	A9	A9	A4	A1

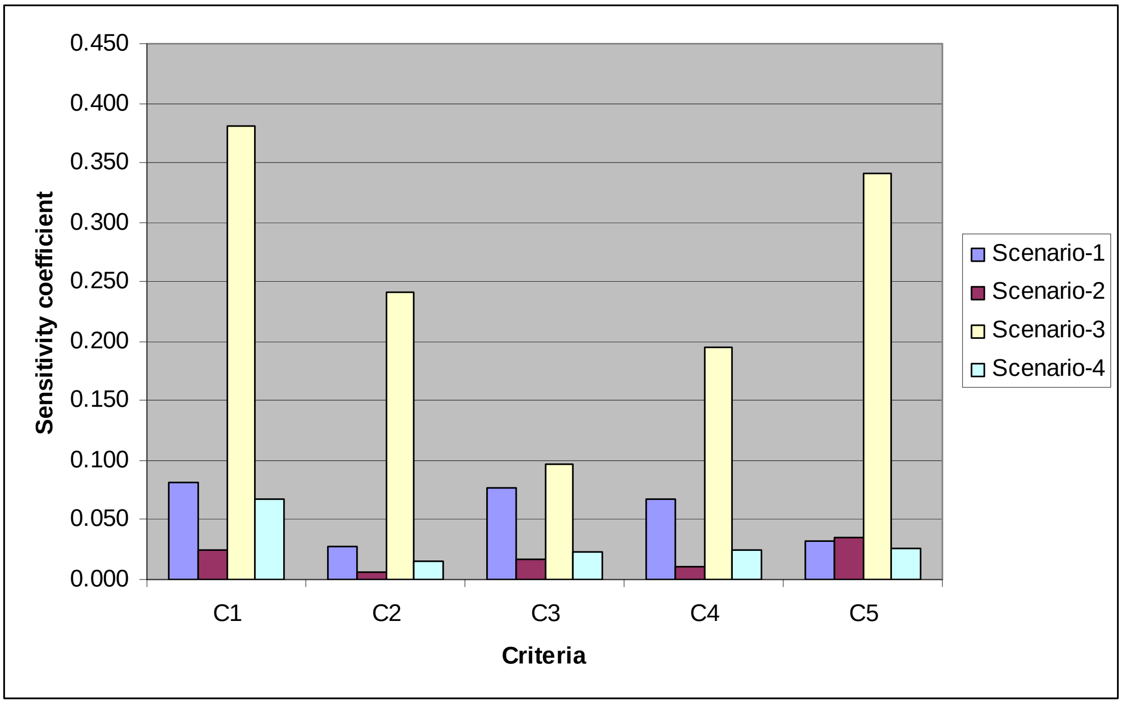


Figure 27: Variability of the sensitivity coefficient for each criterion for four scenarios

8.2 Changing project objectives

Ranking alternatives

In this case, the ranking of alternatives is carried out using scenarios 1 and 2. In scenario 1, the objective of the project focuses only on minimising land fragmentation, i.e. they are involved in the evaluation of only two criteria, i.e. C1: Mean size of new parcels and C2: Mean PCC, and they ignore the other three criteria that refer to the objective 'minimising social impacts', i.e. C3: Change in the number of landowners; C4: Percentage of ownerships 'completed'; and C5: Mean LSR. In contrast, in scenario 2, the objective of the project focuses only on minimising the social impacts and ignores the objective 'minimising land fragmentation'; thus, only criteria C3, C4, and C5 are involved in the evaluation process. The ranking of alternatives for each scenario is shown in Table 8.

As shown in Table 13 and Figure 28, alternative 10 is ranked first in scenario 1 while alternative 3 is ranked best in scenario 2. In other words, alternative 10 is best to minimise land fragmentation and in contrast it is the worst in minimising social impacts. In contrast, alternative 3 is best at minimising social impact but is also ranked second in scenario 1, i.e. minimising land fragmentation, revealing again a stability in performance. In particular, Figures 29 and 30 indicate that alternative 10 is best in both land fragmentation criteria, i.e. C1 and C2, and worst in C3 and C5. This finding reveals that the selection of the criteria will be involved in the evaluation process. This clearly illustrates that the objectives of a project play a crucial role in the ranking order in addition to the weight of the criteria.

Table 13: The performance score and the ranking order of each alternative for the two scenarios

	Scenario-1		Scenario-2	
Ranking	**Alternative**	**Score**	**Alternative**	**Score**
1	Alt-10	0.750	Alt-3	0.951
2	Alt-3	0.631	Alt-2	0.950
3	Alt-2	0.625	Alt-9	0.934
4	Alt-4	0.624	Alt-1	0.933
5	Alt-1	0.621	Alt-4	0.933
6	Alt-9	0.620	Alt-5	0.916
7	Alt-5	0.593	Alt-6	0.913
8	Alt-6	0.472	Alt-7	0.912
9	Alt-7	0.471	Alt-8	0.843
10	Alt-8	0.352	Alt-10	0.839

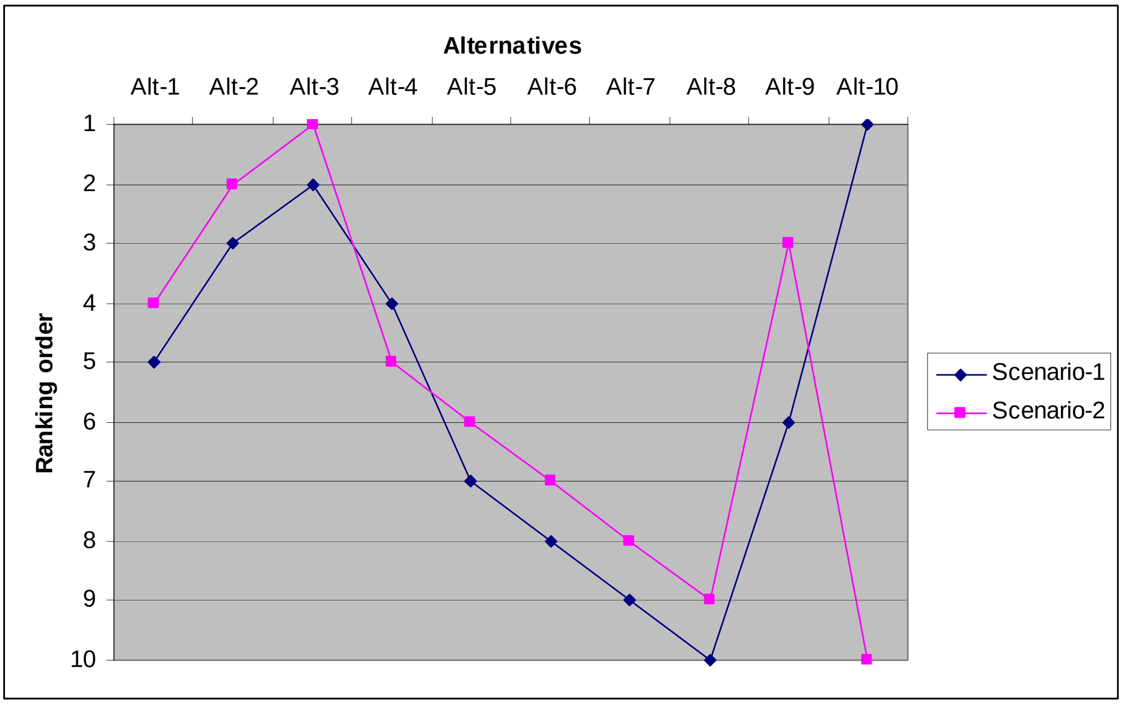


Figure 28: Ranking of alternatives for the two scenarios

It is also remarkable that the variability of performance scores of alternatives ranked best and worst for scenario 1 is extremely high (53.07%) whilst it is low for scenario 2 (11.78%). This indicates that the input facts in the Design module strongly influence the outcome solutions regarding minimising land fragmentation and, in contrast, slightly influence the outcomes regarding minimising social impacts. As a result, this finding suggests a flexibility for the planner in the former case and limitations for the planner in the latter case because of the strict provisions in the legislation.

Performance of alternatives per criterion

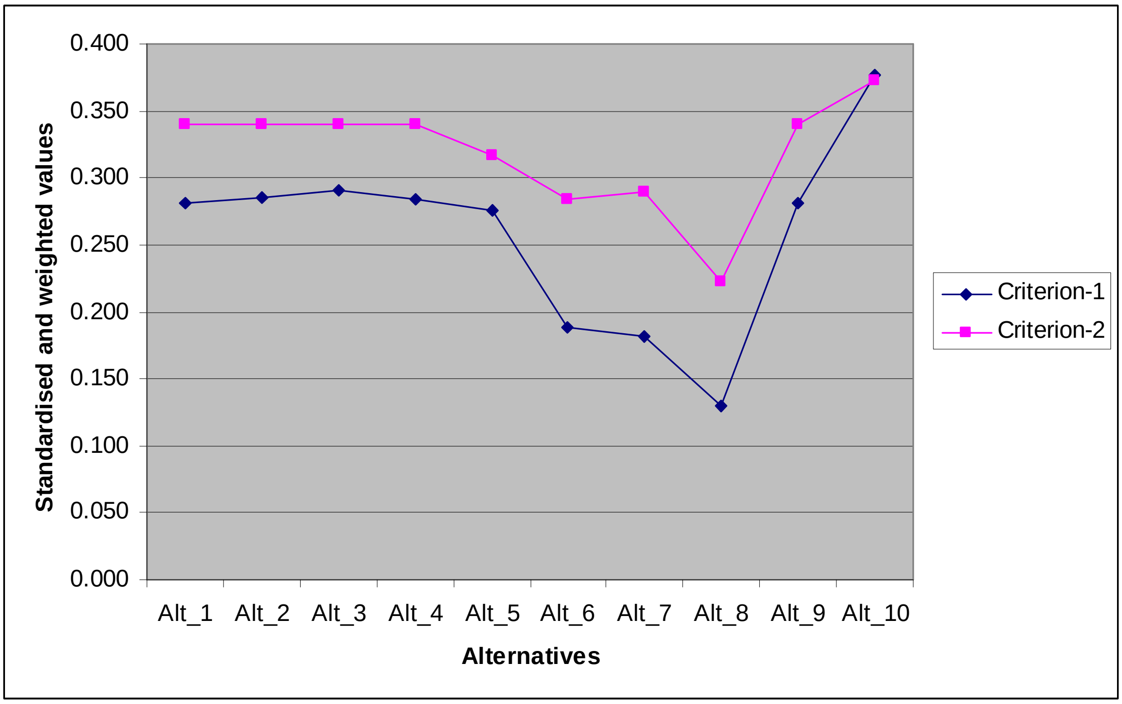


Figure 29: Performance of alternatives for criteria 1 and 2

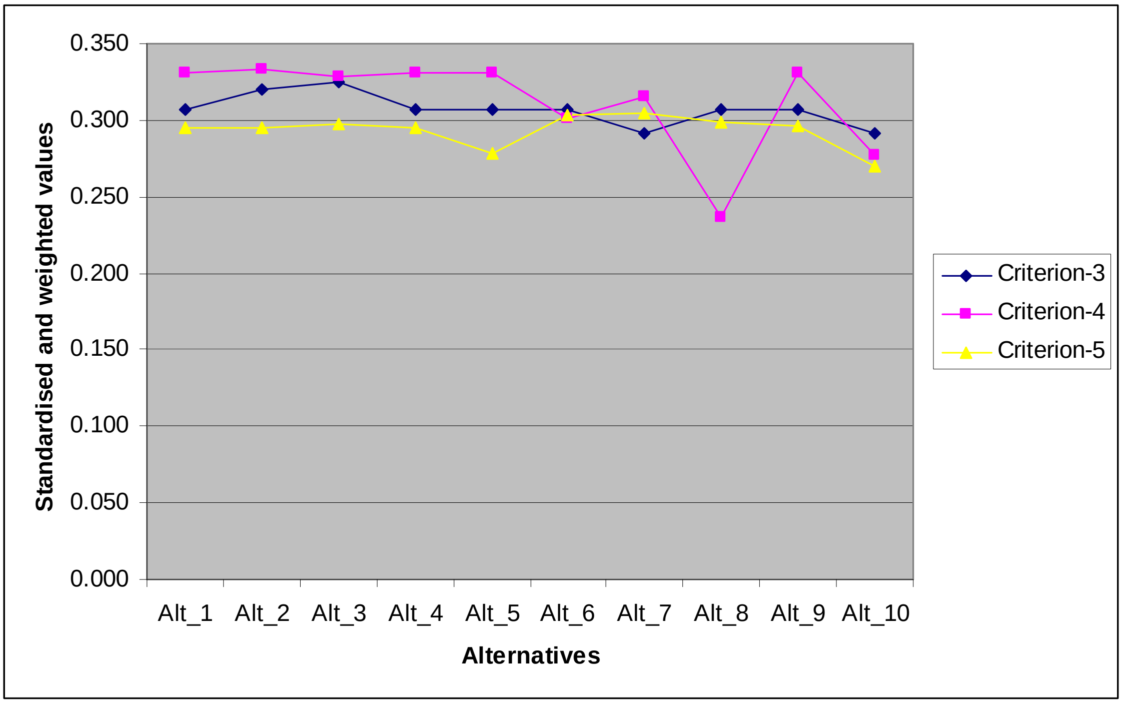


Figure 30: Performance of alternatives for criterion 3, 4 and 5

Sensitivity analysis

Figure 31 shows that the most sensitive criteria are those associated with scenario 2. In particular, the criteria for scenario 2 are more sensitive than those of scenario 1 regardless of the higher variability in the values in the former case. This is a controversial finding compared with that for scenario 1. This finding reveals that the selection of the criteria involved in the evaluation process, and hence the objective of a project, play a crucial role in the ranking in addition to the weight of the criteria.

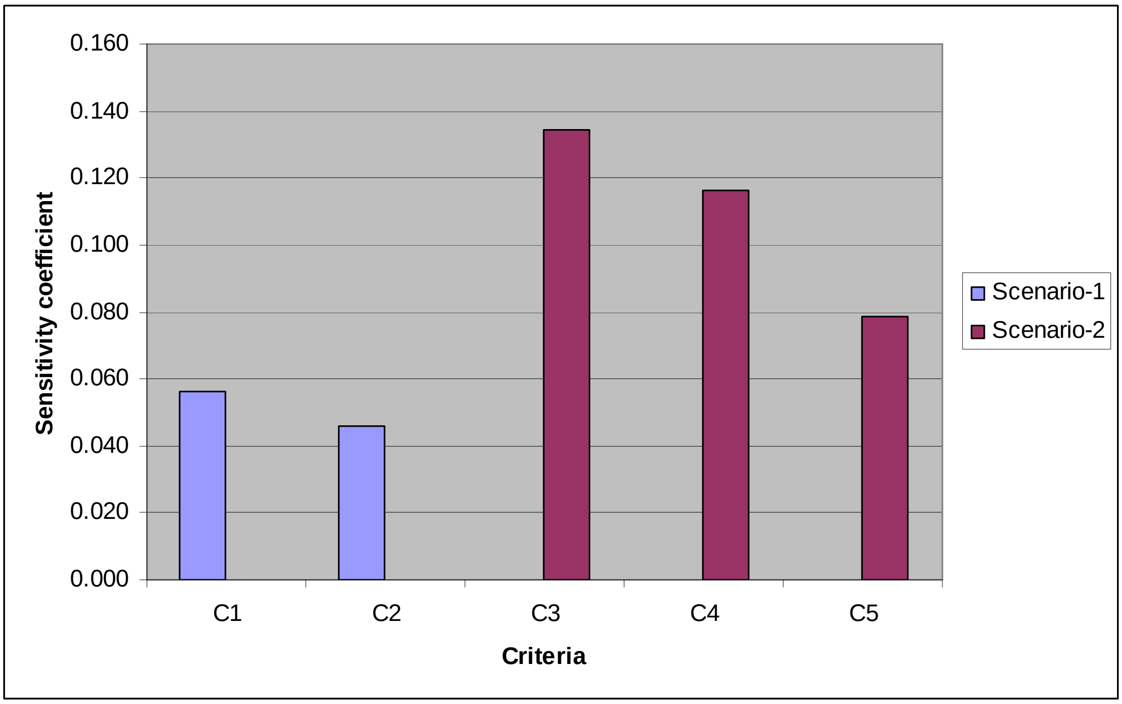


Figure 31: Variability of the sensitivity coefficient for each criterion for the two scenarios

9 Conclusions

MADM is currently a widely applied approach for assessing a discrete number of alternative solutions for a decision problem including those with a spatial context. Although the process is straightforward it involves several crucial tasks that should be carefully customised for the problem concerned in order to obtain robust outcomes. In particular, these tasks are: set out evaluation criteria; weighting of criteria; standardisation of the performance scores of alternatives; aggregation of performance scores via a 'decision rule' and sensitivity analysis tests. This paper has dealt with these issues in the context of the land redistribution problem and as a result four research innovations have been generated. These include the introduction of a new index called the PCC (parcel concentration coefficient) for measuring the dispersion of parcels; the introduction of a measure called LSR (the landowner satisfaction rate) for predicting the acceptance of the land redistribution plan by the landowners in terms of the location of their new parcels; an approach called the 'qualitative rating method' for assigning weights to the evaluation criteria; and a set of value functions for standardising the performance scores of alternatives for five evaluation criteria. The PCC measures the dispersion of each holding separately on a scale from -1 to 1 by comparing the situation before and after land consolidation based on which an average is estimated for all holdings of the consolidated area. The LSR measures the satisfaction of landowners regarding the location(s) at which they received their new parcels on a percentage scale with maximum 100%. Similarly with the PCC, the LSR is also calculated for each landowner and then an average is calculated for all the landowners who received property in the new plan. The qualitative rating method, which is a slightly modified version of the ratio estimation procedure, overcomes the problem of assigning either direct numerical values as weights or scores, which are then transformed as weights by adopting a similar qualitative seven level scale of importance like that used in the pairwise comparison method. Value functions have been defined by five land consolidation experts for this purpose and the forty year statistical records for 74 land consolidation projects. None of the functions created is linear.

This paper has shown that the Evaluation module of *LandSpaCES* is a powerful tool for a comprehensive evaluation of alternative land redistributions. The outcome of alternatives, i.e. the best solution, will be passed to the land partitioning module for automatically generating the new parcels in terms of shape, size, land value and location.

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