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Some properties of spatial-structural-economic-dynamic models

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Abstract

In this paper, further explorations are presented of spatial-structural-dynamic models taking into account extensions of these which incorporate prices of goods and land rents on a dynamic basis. These recent developments generate a great variety of alternative model structures obtained by permuting the designations of the main variable types as "fast" or "slow"; and combining this adjustment with the possibility of exogenous backcloth changes. The properties of this array of models are explored with a view to deepening understanding of which particular models are appropriate to different circumstances. A platform for further research is offered through the ability of this model-based approach to reproduce phenomena which are structurally similar to the real world.

1. Introduction

In this paper, we explore the properties of an increasingly complex family of dynamic urban models. We demonstrate the importance of being as explicit and as clear as possible about the particular hypotheses which are to be represented in particular situations given the variety of possibilities available. Our approach is applicable to many different urban models, but we employ the ever-useful retail model as an archetypal example.

The argument is presented as follows. In section 2, we outline the retail model which includes hypotheses on stock dynamics in both comparative static and disequilibrium frameworks. We show in section 3 how this can be expanded to incorporate locational price indexes of retail goods and land rents for retail developers. In section 4, we then show that there are many ways in which a particular dynamic model can be specified in relation to additional hypotheses about the mechanisms of change. And this variety of possibilities is the central focus of the paper. We explore the properties of models based on different hypotheses, both to show the consequences of adopting particular hypotheses and to make some preliminary deductions from the results on the appropriateness of particular hypotheses in different circumstances. In section 5, we briefly list a range of further extensions which can be made to the models. A selection of results are presented in section 6 and some concluding comments in section 7.

2. The background: retail stock dynamics

Our starting point is the model stated in Harris and Wilson (1978). Let S_{ij} be the flow of expenditure from residents of i to shops in j; E_i the expenditure by residents of i; W_j, a measure of the attractiveness of j (usually taken as proportional to size for illustrative purposes); and c_{ij} as the cost of travel from i to j. Then the flow model is (for suitable parameters α and β)

$$S_{ij} = A_i E_i W_j^{\alpha} e^{-\beta c_{ij}} \tag{1}$$

with

$$A_i = 1/\sum_k W_k^\alpha e^{-\beta c_{ik}} \quad (2)$$

The total revenue attracted to j is

$$D_j = \sum_i S_{ij} \quad (3)$$

and the cost of providing facilities of 'scale' W_j is

$$C_j = k_j W_j \quad (4)$$

for suitable constants k_j. Given α, β, $\{E_i\}$ and $\{c_{ij}\}$ and initial values of $\{W_j\}$, say $\{W_j^0\}$, then an obvious hypothesis for the dynamics of $\{W_j\}$ is:

$$\dot{W}_j = \varepsilon(D_j - C_j)W_j \quad (5)$$

for a suitable ε, with equilibrium condition

$$D_j = C_j \quad (6)$$

By making the appropriate substitutions, equations (5) and (6) can be written explicitly in terms of $\{W_j\}$ as

$$\dot{W}_j = \varepsilon\left[\sum_i \frac{E_i W_j^\alpha e^{-\beta c_{ij}}}{\sum_k W_k^\alpha e^{-\beta c_{ik}}} - k_j W_j\right] \quad (7)$$

and

$$\sum_i \frac{E_i W_j^\alpha e^{-\beta c_{ij}}}{\sum_k W_k^\alpha e^{-\beta c_{ik}}} = k_j W_j \quad (8)$$

It was shown in Harris and Wilson (1978) that the nonlinear equations (8) had interesting bifurcation patterns when parameters like α and β vary; and in Wilson (1981-A), building on May (1976), that (7) has interesting bifurcation properties in relation to ε. The variety of patterns and trajectories which arise for different parameter values (together with a discussion of some of the computational difficulties) can be found in Wilson and Clarke (1979) and Clarke and Wilson (1983). The model has also been explored in a number of other papers: see, for example, Phiri (1980), Leonardi (1981-A, 1981-B), Harris, Choukroun and Wilson (1982), Lombardo and Rabino (1983), Rijk and Vorst (1983-A, 1983-B), Kaashoek and Vorst (1984), Chudzynska and Slodkowski (1984), Clarke (1984), Pumain, Saint-Julien and Sanders (1984), Roy and Brotchie (1984), Fotheringham (1985) and G Clarke (1985).

3. Extensions to include prices and land rents

The model presented above is a simple representation of the process of competition across space by retailers. Those with a particular locational advantage are likely to press this home with price reductions; though this will be tempered by land owners at those sites being able to demand higher land 'rents'. (We assume that all monetary variables are adjusted to refer to a suitable common time or time period.) In this section, we follow the argument of Wilson (1985) and extend the 1978 model to include prices and rents.

As a background to this, it can be argued that it is best to set out hypotheses in relation to the main agents involved: consumers, retailers, developers, land owners and manufacturers of retail goods - treating them all as separate for convenience, though in many cases a single person may wear several hats. The manufacturers play a role in the prices they charge to retailers in balancing consumer demand and supply (mediated by retailers) at an aggregate scale. However, we neglect this relationship as it is not our central concern here. We also assume that consumers move instantly to a new equilibrium following a change. Again, this assumption is easy to relax (as in Haag, 1985 for instance). Then, for simplicity, we assume that developers determine the scale and pattern of provision, $\{W_j\}$; retailers, the prices, $\{p_j\}$ and land owners, the land rents, $\{r_j\}$. We assume for our present purposes that demand $\{E_i\}$ is fixed, though again, this assumption can be easily relaxed.

An extended model is then given by amended versions of equations (1) and (2) for consumers' behaviour as follows:

$$S_{ij} = A_i E_i W_j^{\alpha} p_j^{-\gamma} e^{-\beta c_{ij}} \tag{9}$$

with

$$A_i = 1/\sum_k W_k^{\alpha} p_k^{-\gamma} e^{-\beta c_{ik}} \tag{10}$$

The main change is that a factor $p_k^{-\gamma}$ is added to the attractiveness term, so that destinations are more attractive to consumers when the price index is lower. γ is a suitable parameter.

Revenue is now partly determined by the price index:

$$D_j = p_j \sum_i S_{ij} \tag{11}$$

If $p_j < 1$, this can be interpreted as a benefit to the consumer which can be spent in other ways; and vice versa. The cost of provision is

$$C_j = (k_j + r_j)W_j \tag{12}$$

where k_j are the assumed non-varying costs, including buildings (annualised, say), labour, purchase of goods from manufacturers, while r_j is unit land rent.

The dynamics are then given by a version of equation (6) extended to include price and land rent adjustments:

$$\dot{W}_j = \varepsilon_1 (D_j - C_j)W_j \tag{13}$$

$$\dot{p}_j = \varepsilon_2 (C_j - D_j)p_j \tag{14}$$

$$\dot{r}_j = \varepsilon_3 (D_j - C_j)r_j \tag{15}$$

4. The mechanisms of change

In practice, equations (13)-(15) are solved as difference equations. For reasons which will become clear shortly, we need a fairly complicated notation to describe the solution procedure. Let t and t + 1 be successive points in time, so $(t, t + 1)$ represents a time period; let n be an iteration number for when the equations have to be solved iteratively within a time period. The argument to be presented here partly follows that in Leonardi and Wilson (1985) and also relates to Wilson and Birkin (1985).

$$\Delta W_j^{n+1}(t) = \epsilon_1[D_j^n(t) - C_j^n(t)]W_j^n(t) \tag{16}$$

$$\Delta p_j^{n+1}(t) = \epsilon_2[C_j^n(t) - D_j^n(t)]p_j^n(t) \tag{17}$$

$$\Delta r_j^{n+1}(t) = \epsilon_3[D_j^n(t) - C_j^n(t)]r_j^n(t) \tag{18}$$

$$W_j^{n+1}(t) = W_j^n(t) + \sigma\Delta W_j^{n+1}(t) \tag{19}$$

$$p_j^{n+1}(t) = p_j^n(t) + \sigma\Delta p_j^{n+1}(t) \tag{20}$$

$$r_j^{n+1}(t) = r_j^n(t) + \sigma\Delta r_j^{n+1}(t) \tag{21}$$

σ is a 'step length' - a constant between 0 and 1.

$$W_j(t + 1) = W_j^{n_1^{max}}(t) \tag{22}$$

$$p_j(t + 1) = p_j^{n_2^{max}}(t) \tag{23}$$

$$r_j(t + 1) = r_j^{n_3^{max}}(t) \tag{24}$$

n_1^{max}, n_2^{max}, and n_3^{max} are the maximum number of iterations to be carried out (at time t, or within a $(t, t + 1)$-period) for W, p and r-variables respectively. Equations (16)-(21) are solved iteratively (if necessary), starting with $n = 0$ and

$$W_j^0(t) = W_j(t - 1) \tag{25}$$

$$p_j^0(t) = p_j(t - 1) \tag{26}$$

$$r_j^0(t) = r_j(t - 1) \tag{27}$$

(or, for $t = 1$, whatever initial values are appropriate). At the end of the iteration, equations (22)-(24) give the $t + 1$ values of the W, p and r-variables.

Different combinations of ε-parameters and n^{max}-values generate a great variety of dynamic models, many of which can be interpreted as representing interesting real processes for one kind of system or another. This, therefore, is the heart of the paper: we want to show what this variety is and to explore the properties of some of the more interesting models which can be generated.

For convenience, let us adopt the convention that $n_k^{max} = 0$ means that there is no adjustment of the k-th variable type. (This is equivalent to setting the associated ε_k parameter to zero, which is the more conventional way of expressing this). Then the following table shows how a variety of interesting cases can be generated.

$$n^{max}\text{-values} \begin{cases} W & p & r \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ \text{high} & \text{high} & \text{high} \end{cases}$$

A model is any combination of three possible n^{max} values. Interesting combinations are

$$(n_1^{max}, n_2^{max}, n_3^{max}) = \begin{cases} (\text{high}, 0, 0) & (28) \\ (\text{high}, \text{high}, \text{high}) & (29) \\ (1, \text{high}, \text{high}) & (30) \\ (1, 1, 1) & (31) \end{cases}$$

The case (28) is essentially the Harris-Wilson model: W_j's only are adjusted until equilibrium is achieved (if it is possible, or by choosing sufficiently small σ) at each time. Indeed, if $\varepsilon_1 = 1$, the iterative process is precisely the usual iterative scheme for the Harris-Wilson model (see Wilson, 1985). The process is repeated, following any adjustment of exogenous variables or parameters, such as the E_i's or α or β, at $t + 1$.

Case (29) is comparative statics with W's, p's and r's adjusted simultaneously. The relative influence of stock, price or rent adjustment on the final outcome is determined by the relative magnitudes of ε_1, ε_2 and ε_3.

Case (30) represents a structure often used by economists (cf. Anas, 1985). From an initial disequilibrium state, there is an adjustment in stock values for one step in the direction of equilibrium, and the price and rent variables are iterated until equilibrium is forced at that time.

Case (31) represents constant disequilibrium with adjustments of one step being made to all variables and the possibility of changing exogenous variables at each time period. Of course, if there are no exogenous changes, then after a long period, a state will be reached which is the same as that represented by case (29).

We present results of numerical experiments for these cases in section 6 and reserve further interpretation until then.

5. Further extensions

We begin by noting four points of (possibly important) detail which have to be tackled in any particular case. Then we consider one more substantial extension.

(1) At present, in equations like (16)-(18), we have factors W_j, p_j and r_j respectively on the right hand side. These are particularly important in determining the 'pick up' of ΔW_j (say) for small W_j. For a more detailed exploration of this point, see Wilson (1981-B).

(2) The ε-parameters are assumed constant. In some cases, it would be more enterprising to discover how different agents would 'learn' in different circumstances and thus how ε's would vary over time. (For instance, after a big 'overshoot', ε may be reduced.)

(3) Demand should be made elastic in relation to price. One way to do this (cf. Wilson, 1985) is to take

$$\hat{p}_i = \frac{\sum_j \dot{p}_j e^{-\beta c_{ij}}}{\sum_j e^{-\beta c_{ij}}} \tag{32}$$

as an exponentially-weighted average of p_j's to give a perceived price at i, and

$$E_i = E_i^0 \left(\frac{\hat{p}_i(t)}{\hat{p}_i(t-1)} \right)^{\hat{\gamma}} \tag{33}$$

say.

(4) At present, the increments, say (16)-(18), are calculated simultaneously. It would be possible to generate more variants of the model by recalculating the flow, revenue and cost variables between calculations of increments. For example, in model (30) above, it may be appropriate to calculate $W_j(t+1)$ from its single step increment, and then to recalculate $\{S_{ij}\}$, $\{D_j\}$, and $\{C_j\}$ before solving for $\{p_j\}$ and $\{r_j\}$ iteratively.

The more substantial extension is to modify the mechanism for calculating the W_j's. At present, it can be argued (as in Wilson, 1985) that they are 'optimal' in the sense that the T_{ij} terms in the transportation problem of linear programming are optimal (cf. Evans, 1973, Wilson and Senior, 1974). We can find a more general, and typically more dispersed, $\{W_j\}$ solution by incorporating an entropy term, $-\sum_j W_j(\log W_j - 1)$. We show how to do this by taking the (high, 0, 0) model - case (28) above: that is, we neglect price and rent adjustments. The mathematical programming version of the Harris and Wilson (1978) model is (cf. Wilson, 1981-B):

$$\begin{aligned} \underset{\{S_{ij},\, W_j\}}{\text{Max}} \quad Z = & - \sum_{ij} S_{ij}(\log S_{ij} - 1) \\ & + \sum_i a_i\Big(E_i - \sum_j S_{ij}\Big) \\ & + \sum_j \lambda_j\Big(W_j - \sum_i S_{ij}\Big) \end{aligned}$$

$$+ \alpha(\sum_{ij} S_{ij} \log W_j - H)$$

$$+ \beta(C - \sum_{ij} S_{ij} c_{ij}) \qquad (34)$$

We need to add the term $- \sum_j W_j(\log W_j - 1)$ into the objective function and relax the constraint

$$\sum_i S_{ij} = W_j = C_j \qquad (35)$$

(where we have taken k_j as 1 for simplicity) which appears as $\sum_j \lambda_j(W_j - \sum_i S_{ij})$ in the Lagrangian. Given the kind of market imperfections which generate dispersion, we would expect $W_j > \sum_i S_{ij}$ and one way to express this is by taking

$$\sum_j (C_j - D_j)^2 = B \qquad (36)$$

for some $B > 0$.

A term $\lambda[\sum_j (C_j - D_j)^2 - B]$ then replaces $\sum_j \lambda_j(W_j - \sum_i S_{ij})$ in the mathematical programme (34), with $C_j = W_j$ and $D_j = \sum_i S_{ij}$. After some manipulation, it is possible in this case to obtain an explicit equation for W_j (rather as for T_{ij} in the doubly-constrained spatial interation model relative to the transportation problem of linear programming) - and also a modified flow model:

$$S_{ij} = A_i E_i e^{-\lambda(D_j - W_j)} W_j^{\alpha} e^{-\beta cij} \qquad (37)$$

$$A_i = 1/ \sum_k e^{-\lambda(D_k - W_k)} W_k^{\alpha} e^{-\beta cik} \qquad (38)$$

$$W_j = e^{\lambda(D_j - W_j)} e^{\alpha W_j / F_j} \qquad (39)$$

Some preliminary results using this model are presented by Wilson (1985).

6. Numerical experiments

The arguments presented in section 2-5 have been theoretical and somewhat abstract. As usual with this class of models, it is useful to undertake numerical experiments which help to fix ideas, and also point the way to new kinds of theoretical extensions (see Clarke and Wilson, 1983; also Campbell *et al*, 1985).

We proceed in the following way. Initially a set of comparative static cases are presented which are a special case of the framework of section 4 corresponding to either $\underline{n}^{max}$ = (high, 0, 0), or the (1, 0, 0) case extrapolated over several time periods without exogenous change. By extracting a single example from the comparative static analysis, it is possible to show how the introduction of prices and rents greatly enriches the variety of spatial structures which arise. These models are basically of the (high, high, high) type in which the various actors may be seen as competing for a locational surplus, with each individual's economic strength represented by the relative magnitude of the associated ε parameter.

This is the kind of speed of change which has been discussed in the past, in relation to equations like (5) (e.g. Wilson, 1981-A; Clarke and Wilson, 1983). However the introduction of different actors in the system allows us to consider a second kind of speed of change, which relates essentially to cross-sectional adjustment as a response to disequilibrium. Thus if an actor is associated with a high n^{max}, he is assumed to respond instantaneously to seek a new equilibrium. In a third set of numerical experiments we therefore present a scenario in which a fixed kind of exogenous change is taking place, and explore how the mode of response to disequilibrium affects the spatial structure of the state variables - prices, rents and floorspace.

6.1 Comparative statics with the stock variable only

Comparative static cases, in which structural equilibria are contrasted over a variety of parameter values, are special cases of the extended approach offered here: and first, prices and rents are excluded. This type of analysis provides a convenient starting point, not least in the sense that it directs us towards the areas of parameter space which are likely to prove worthy of further investigation.

Figure 1 shows the structure of our basic network, which takes the form of a 129-zone grid. It is assumed that each grid point generates an identical demand $\{E_i\}$, and initially that the friction of distance is uniform, although at a later stage this assumption is relaxed to allow preferential access to the city centre. Under these circumstances, and with an even distribution of initial floorspace levels $\{W_j^0\}$, Figure 2 demonstrates the equilibrium floorspace configuration for a plausible array of parameter values. Where the attractiveness of centres is strong relative to their size ($\alpha = 1.5$) and travel is easy ($\beta = 0.25$), it is reasonable that a single centre should emerge (Figure 2.1.1). However, as differential attractiveness is reduced, or the cost of travel increases, more dispersed patterns begin to arise, until a rather more balanced pattern is attained for $\alpha = 1.05$, $\beta = 1.50$ (Figure 2.3.3).

6.2 Comparative statics with more than one variable

Once non-zero values are introduced for the parameters ε_2 and ε_3, a competitive process is introduced in which patterns of spatial advantage may be exploited not only through the expansion of activity levels (via ε_1), but also through price reductions (ε_2), or the extraction of rents by landowners (ε_3). The objective of this section is to show how the relative strengths of the respective competitors influences the outcome of the spatial development process. In achieving this, we also take another trip through "parameter space", and hence further insights are attained into the operation of the models.

In the numerical simulations presented here, we assume the functional forms:

$$\dot{W}_j = \varepsilon_1(D_j - C_j) \qquad (40)$$

$$\dot{p}_j = -\varepsilon_2(D_j - C_j)p_j \qquad (41)$$

$$\dot{r}_j = \varepsilon_3(D_j - C_j) \qquad (42)$$

While these forms could be varied, the important point to note here is that the ε's also function as scaling parameters: thus the values of ε_1 we deal with below are usually at least two orders of magnitude greater than ε_2 and ε_3 because it scales on the dimensions of floorspace, rather than price or rent indices.

Initially, we focus on the variation of the ε_2 and ε_3 parameters individually with respect to ε_1. First of all, let us consider the rent parameter, ε_3. Since this parameter diverts surplus revenue from reinvestment in spatial structure (through (40)) to an inactive rent stockpile (through (42)), we would expect increases in ε_3 to generate a dispersion of activities. As a pivotal case, we have therefore selected a fairly concentrated case from figure 2, with $\alpha = 1.2$ and $\beta = 0.75$ (figure 2.2.2). Figure 3 then shows the effect on spatial structure of increasing ε_3 from 0.0001 through three orders of magnitude to 0.1.

For a low enough value of ε_3, we would expect its effect to be negligible, so that the structure of figure 2.2.2 would be reproduced. $\varepsilon_3 = 0.0001$ is evidently a sufficiently small value (figure 3.1.1). As the adjustment parameter increases, we can see the expected dispersion as the four initial centres multiply to 8 (figure 3.1.2) and then 20 (figure 3.1.3). Finally, for a rather large ε_3 of 0.1, the spatial order of activities loses this clear definition and becomes extremely fuzzy (figure 3.1.4). In effect, all the locational advantage generated in the system is transferred to the landowners, so there is still some differentiation in the rent surface. The rents are shown, in a new graphical form, in figure 3.2. Notice that although the rent surface of figure 3.2.4 is more differentiated than the activity surface (figure 3.1.4), it is still much less peaked than the corresponding distribution of figure 3.2.3.

Thus one might argue that $\varepsilon_3 = 0.1$ represents an extreme, but unrealistic, representation of the rent accumulation process. Values of $\varepsilon_3 = 0.01$ and 0.001, on the other hand, show plausible outcomes to such a process, with corresponding implications for spatial structure, with $\varepsilon_3 = 0.0001$ as a limiting case in which rents are effectively eliminated.

It is possible to repeat this kind of exercise with respect to ε_2 and price variation, with one complication. Once non-zero ε_2 is introduced, the price indices $\{p_j\}$ can move from their initial values of unity, and one therefore needs to consider the role of γ in the attractiveness factors (equations (9) and (10)). For high values of gamma the introduction of ε_2 yields a strong positive feedback between reducing prices and increased attractiveness, which in turn implies that large centres are able to sustain their own growth by introducing massive price reductions. This mechanism is illustrated in figure 4 where a value of $\gamma = 1$ has been considered against $\varepsilon_2 = 0.001$ (figure 4.1) and $\varepsilon_2 = 0.025$ (figure 4.2). In the former case we see that the original distribution is maintained with significant price reductions at the non-zero centres. Notice here that the prices which are in effect for centres of no activity operate as some kind of dual or shadow prices. Their relationship to the active prices in figure 4.2 becomes extreme. One of the less sensible aspects of this situation is that the dominant activities also become rather small (figure 4.21). This is because we have a total stock constraint of the form:

$$\bar{W} = \sum_j K_j W_j = \sum_j p_j D_j \tag{43}$$

Since both D_j and K_j are externally scaled here, $\bar{W}$ falls in proportion to reductions in p_j.

Although one might conceive of situations for which figure 4.2 is not a wholly unrealistic representation (for high order goods and services), we choose to focus here on less extreme cases where price competition is another mechanism for diffusion. Figure 5 illustrates a situation in which ε_2 is varied for $\gamma = 0.25$. We can see here a rather interesting effect in which one pattern of activities is stable across a very wide range of parameter values, eventually yielding to dispersion for sufficiently high values of ε_2.

Having varied rents and prices separately, it is now possible to assess their combined effects. The results of figure 3 suggested sensible values of ε_3 to range from 0.01 through to 0.0001, while for $\gamma = 0.25$, ε_2 might go from 0.01 to 0.001 (see figure 5). For $\varepsilon_1 = 1.0$, $\alpha = 1.2$ and $\beta = 0.75$, a uniform starting distribution was projected forward for 30 time periods using a combination of ε_2 and ε_3 values as shown in figure 6. Of course the two parameters now have a cumulative influence, so even low parameter values can generate relatively dispersed patterns (compare figure 6.3.3 to figures 3.1 and 5.5). One of the encouraging things here is that the pattern is sensitive to changes in both parameters, but as in figure 5 there appear to be large areas of parameter space over which spatial structure is fairly stable, e.g. for $\varepsilon_2 = 0.001$ as ε_3 varies from 0.01 to 0.0001 (figures 6.3.1 - 6.3.3); $\varepsilon_2 = 0.005$ for $\varepsilon_3 = 0.01$ to 0.001 (figures 6.2.1 and 6.2.2).

6.3 Fast and slow dynamics

In section 6.2 we focused on the temporal evolution of our spatial system against a fixed backcloth. It was possible to show how a variety of structures might be generated under different speeds of adjustment of the state variables. As an approximation to reality this situation is oversimplified, as one would expect the backcloth to be constantly changing. In terms of dynamical systems theory, this changing backcloth is the "slow manifold". It was argued in section 4 above that three types of adjustment in the state variables are possible: a single step in the direction of the equilibrium; an iterative adjustment process to return the system to equilibrium; or no adjustments at all. In sections 6.1 and 6.2 we have covered cases involving single step or zero adjustments. We now extend this to a consideration of iterative adjustment procedures.

For the changing backcloth we focus on beta, which decreases in steps of 0.25 from its "high" of 1.5 down to 0.25 with $\alpha = 1.2$. The $\underline{n}^{max}$ = (high, 0, 0) case (equation (28)) now represents structural adjustment in the style of Harris and Wilson, but to a changing backcloth. Further interest is generated if we consider exactly what is meant by "high", as a certain amount of disequilibrium may be maintained in the system by keeping this parameter finite and not forcing the system all the way to stability at each time period.

Figure 7 illustrates our basic scenario for $\underline{n}^{max} = (10, 0, 0)$ and $\varepsilon_1 = 1$. We can see two kinds of process at work here: first, the adjustment of the system to its equilibrium; and secondly, the changing nature of that equilibrium. Because the adjustments in $\{W_j\}$ are fast relative to the backcloth change, a stable solution is eventually reached. Observe, however, that the stable solution is not the same as the comparative static equilibrium which we derived for $\alpha = 1.2$, $\beta = 0.25$ in figure 3.1.2. This is a function of the evolutionary dynamic, in which the centre is eliminated at high beta and cannot then force its way back as travel becomes cheaper.

While such an outcome is internally consistent, it is rather unrepresentative of real patterns, because we are failing to pick up the true advantage of the "city centre" in terms of its accessibility. One way round this is to factor the cost of trips to the city centre. Thus if one assumes that it is 5 per cent cheaper to make a trip of a given length if that trip terminates at the city centre, we can attain a situation, as in figure 8, in which the city centre becomes the focus of activity at the high beta extreme, and gradually forces out even the centres which compete from the periphery as beta is reduced.

A city centre factor of 0.95 is used throughout the remainder of this section. The principal challenge to the dominance of the city centre is now through the price and rent mechanisms as in section 6.2. In figure 9, we consider the case where $\underline{n}^{max} = (10, 10, 1)$ so that rents vary as a single step adjustment to equilibrium, while a trade-off between floorspace expansion and price cutting determines the iterative shift towards equilibrium. Even for a low value of $\varepsilon_2 = 0.001$, large price differentials are seen to arise, where typically prices are forced up as declining centres endeavour to maintain competitiveness, while more advantageously placed producers reinforce their dominance through price cutting.

Figure 10 takes the case of figure 9.2 with $\underline{\varepsilon} = (1, 0.01, 0.001)$ and increases the number of cross-sectional rent iterations to 10 (hence $\underline{n}^{max} = (10, 10, 10)$). Peripheral development is now slightly more well-defined throughout and we obtain the beginnings of a peaked rent surface. However in terms of real world pattern, again one would perhaps like to see still greater differentiation in the rent surface, and a little more uniformity of price.

In figure 11 we have assumed rent adjustment to be the dominant equilibriating mechanism with $\underline{\epsilon} = (1, 0.001, 0.01)$, while floorspace adjustment has been restricted to a single step towards equilibrium ($\underline{n}^{max} = (1, 10, 10)$. For the first time period only, an iteration vector of (10, 0, 0) was applied to obtain a non-uniform, disequilibrium initial allocation of $\{W_j\}$. An intuitively pleasing pattern of development results, with central growth which is tempered not only by the slow reaction of activity levels to disequilibrium, but also by the formation of an increasingly peaked rent surface. The rents fall most rapidly in the inner city ring, rising to minor peaks in the periphery. The rent surface is now much more differentiated than the price surface, which nonetheless reflects the same pattern of comparative advantage.

Conclusions

In this paper we have demonstrated, both theoretically and numerically, how the variety of spatial-structural-dynamic urban models can be extended through the introduction of new kinds of economic agent. Further refinement is still desirable, however, in the ways discussed in section 5. Thus we might concern ourselves in future with alternative surplus allocation mechanisms, different kinds of dynamic adjustment, the methods in which the various procedures are nested, and especially the introduction of entropic dispersion. In the new, and more explicitly dynamic models we also need to consider carefully the possibility of instability and multiple equilibria. This paper has only been illustrative, therefore, of the potential of economic-dynamic extensions to models of urban spatial structure.

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FIGURE 1. The hypothetical grid system

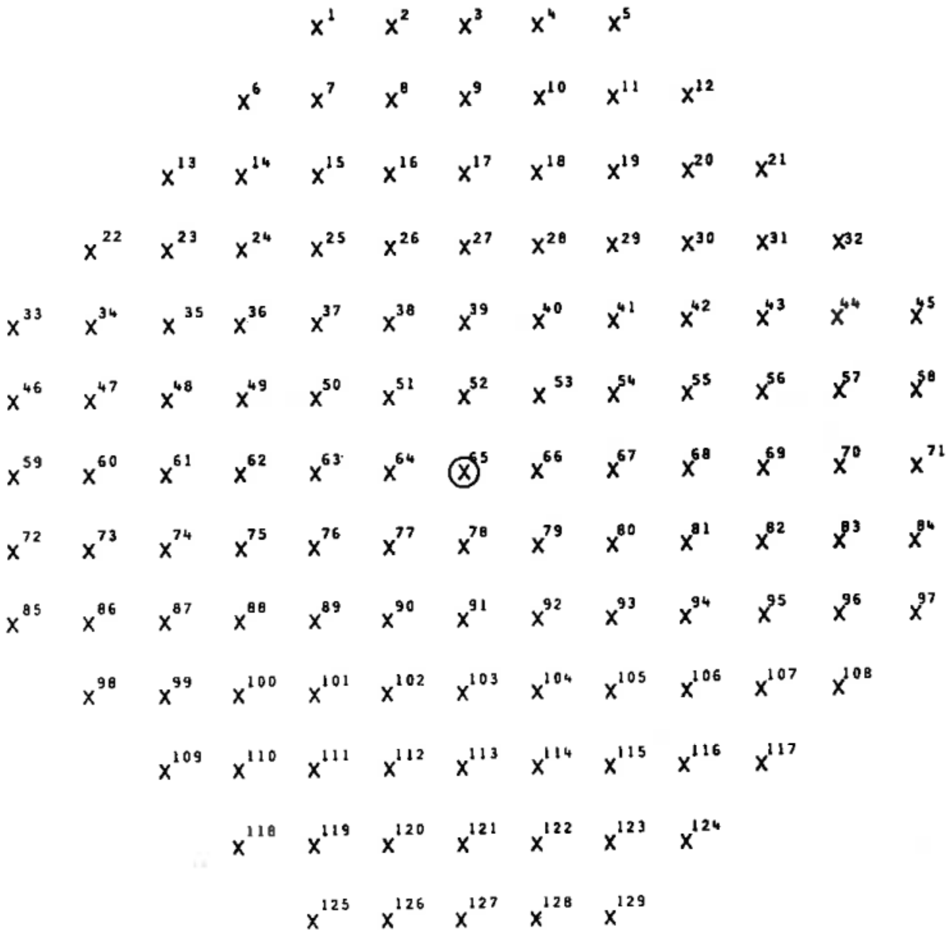


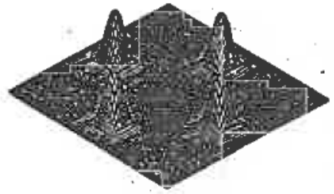
FIGURE 2: COMPARATIVE STATICS WITH THE STOCK VARIABLE ONLY

2.1.1 alpha=1.5, beta=0.25

2.1.2 alpha=1.5, beta=0.75

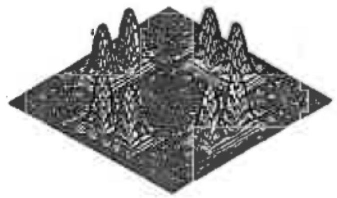
2.1.3 alpha=1.5, beta=1.5

2.2.1 alpha=1.2, beta=0.25



2.2.2 alpha=1.2, beta=0.75

2.2.3 alpha=1.2, beta=1.5



2.3.1 alpha=1.05, beta=0.25

2.3.2 alpha=1.05, beta=0.75

2.3.3 alpha=1.05, beta=1.5

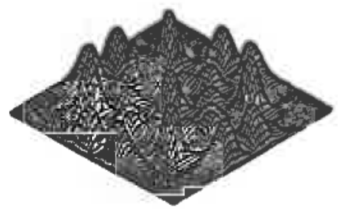


FIGURE 3: COMPARATIVE STATICS. ACTIVITY LEVELS VERSUS RENT

3.1.1 ACTIVITIES

EPS=(1,0,0.0001)

3.2.1 RENTS

				0.50	0.50	0.50	0.50	0.50				
			0.50	0.50	0.50	0.50	0.50	0.50	0.50			
		0.50	0.50	0.50	0.50	0.52	0.50	0.50	0.50	0.50		
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.52	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.52	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
		0.50	0.50	0.50	0.50	0.52	0.50	0.50	0.50	0.50		
			0.50	0.50	0.50	0.50	0.50	0.50	0.50			
				0.50	0.50	0.50	0.50	0.50				

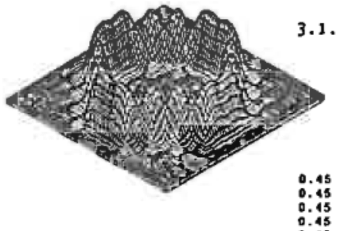
3.1.2 ACTIVITIES

EPS=(1,0,0.001)

3.2.2 RENTS

				0.50	0.50	0.50	0.50	0.50				
			0.50	0.50	0.50	0.50	0.50	0.50	0.50			
		0.50	0.50	0.50	0.58	0.49	0.57	0.50	0.50	0.50		
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.57	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.58	0.50	0.50
0.50	0.50	0.49	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.49	0.50	0.50
0.50	0.50	0.58	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.57	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
		0.50	0.50	0.50	0.57	0.49	0.58	0.50	0.50	0.50		
			0.50	0.50	0.50	0.50	0.50	0.50	0.50			
				0.50	0.50	0.50	0.50	0.50				

3.1.3 ACTIVITIES



EPS=(1,0,0.01)

3.2.3 RENTS

				0.45	0.45	0.45	0.45	0.45				
			0.45	0.45	0.45	0.45	0.45	0.45	0.45			
		0.45	0.45	0.45	0.75	0.74	0.75	0.45	0.45	0.45		
	0.45	0.45	0.45	0.76	0.45	0.45	0.45	0.76	0.45	0.45	0.45	
0.45	0.45	0.45	0.76	0.45	0.45	0.45	0.45	0.45	0.76	0.45	0.45	0.45
0.45	0.45	0.75	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.75	0.45	0.45
0.45	0.45	0.74	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.74	0.45	0.45
0.45	0.45	0.75	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.75	0.45	0.45
0.45	0.45	0.45	0.76	0.45	0.45	0.45	0.45	0.45	0.76	0.45	0.45	0.45
	0.45	0.45	0.45	0.76	0.45	0.45	0.45	0.76	0.45	0.45	0.45	
		0.45	0.45	0.45	0.75	0.74	0.75	0.45	0.45	0.45		
			0.45	0.45	0.45	0.45	0.45	0.45	0.45			
				0.45	0.45	0.45	0.45	0.45				

3.1.4 ACTIVITIES

EPS=(1,0,0.1)

3.2.4 RENTS

				0.21	0.32	0.34	0.32	0.21				
			0.29	0.46	0.53	0.54	0.53	0.46	[illegible]			
		0.29	0.47	0.56	0.59	0.60	0.59	0.56	0.47	0.29		
	0.29	0.47	0.56	0.59	0.59	0.59	0.59	0.59	0.56	0.47	0.29	
0.21	0.46	0.56	0.59	0.58	0.57	0.57	0.57	0.58	0.59	0.56	0.46	0.21
0.32	0.53	0.59	0.59	0.57	0.56	0.55	0.56	0.57	0.59	0.59	0.53	0.32
0.34	0.54	0.60	0.59	0.57	0.55	0.55	0.55	0.57	0.59	0.60	0.54	0.34
0.32	0.53	0.59	0.59	0.57	0.56	0.55	0.56	0.57	0.59	0.59	0.53	0.32
0.21	0.46	0.56	0.59	0.58	0.57	0.57	0.57	0.58	0.59	0.56	0.46	0.21
	0.29	0.47	0.56	0.59	0.59	0.59	0.59	0.59	0.56	0.47	0.29	
		0.29	0.47	0.56	0.59	0.60	0.59	0.56	0.47	0.29		
			0.29	0.46	0.53	0.54	0.53	0.46	0.29			
				0.21	0.32	0.34	0.32	0.21				

FIGURE 4: COMPARATIVE STATICS OF ACTIVITY SIZE VERSUS PRICE: GAMMA=1.0

4.1.1 EPS2=0.001 - ACTIVITY SURFACE

				0.00	0.00	0.00	0.00	0.00				
			0.00	0.00	0.00	0.00	0.00	0.00	0.00			
		0.00	0.00	0.00	0.00252.77		0.00	0.00	0.00	0.00		
	0.00	0.00-	0.00	0.00	0.00.	0.00	0.00	0.00	0.00	0.00	0.00	
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00252.77		0.00	0.00	0.00	0.00	0.00	0.00	0.00252.77		0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
		0.00	0.00	0.00	0.00252.77		0.00	0.00	0.00	0.00		
			0.00	0.00	0.00	0.00	0.00	0.00	0.00			
				0.00	0.00	0.00	0.00	0.00				

4.1.2 EPS=0.001 - PRICE SURFACE

				1.01	1.01	1.01	1.01	1.01				
			1.01	1.01	1.01	1.01	1.01	1.01	1.01			
		1.01	1.01	1.01	1.01	0.78	1.01	1.01	1.01	1.01		
	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
1.01	1.01	0.78	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.78	1.01	1.01
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	
		1.01	1.01	1.01	1.01	0.78	1.01	1.01	1.01	1.01		
			1.01	1.01	1.01	1.01	1.01	1.01	1.01			
				1.01	1.01	1.01	1.01	1.01				

4.2.1 EPS=0.025 - ACTIVITY SURFACE

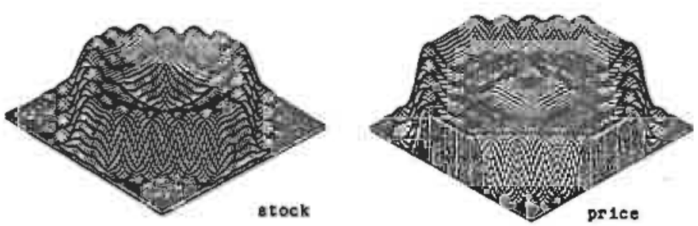
				0.50	0.50	0.50	0.50	0.50				
			0.50	0.50	0.50	0.50	0.50	0.50	0.50			
		0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
		0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
			0.50	0.50	0.50	0.50	0.50	0.50	0.50			
				0.50	0.50	0.50	0.50	0.50				

4.2.2 EPS=0.025 - PRICE SURFACE

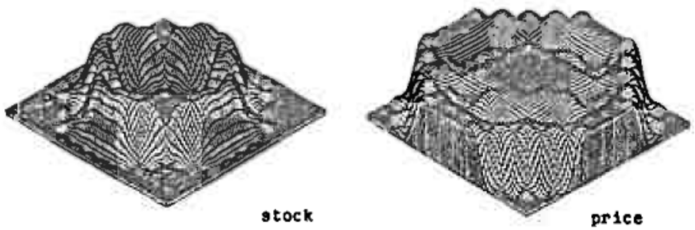
				0.00	0.00	0.00	0.00	0.00				
			0.00	0.00	0.00	0.00	0.00	0.00	0.00			
		0.00	0.00	0.00	27.93	0.00	27.93	0.00	0.00	0.00		
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	27.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00	27.93	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	27.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00	27.93	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
		0.00	0.00	0.00	27.93	0.00	27.93	0.00	0.00	0.00		
			0.00	0.00	0.00	0.00	0.00	0.00	0.00			
				0.00	0.00	0.00	0.00	0.00				

FIGURE 5 COMPARATIVE STATICS: ACTIVITY LEVELS VERSUS PRICE FOR GAMMA = 0.25

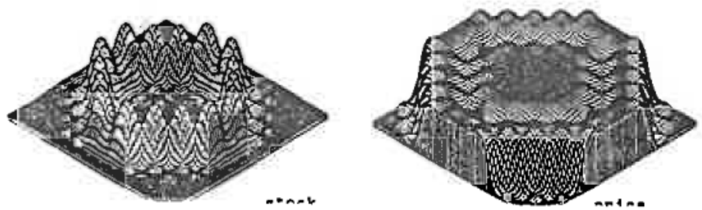
5.1 EPS = (1,0.025,0)



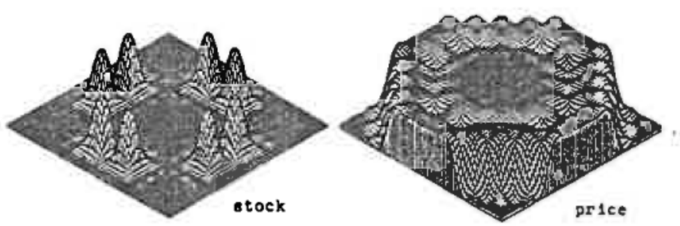
5.2 EPS = (1,0.01,0)



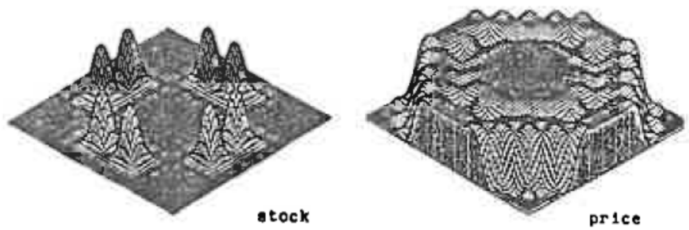
5.3 EPS = (1,0.005,0)



5.4 EPS = (1,0.002,0)



5.5 EPS = (1,0.0001,0)



5.6 EPS = (1,0.00001,0)

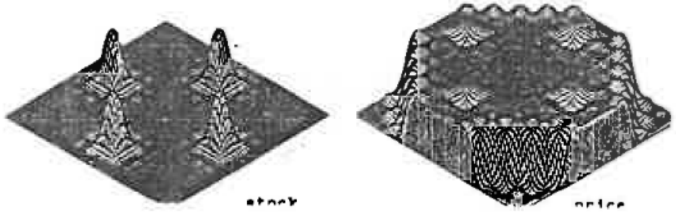


FIGURE 6: COMPARATIVE STATICS WITH THREE ACTORS

6.1.1 EPS=(1,0.01,0.01)

6.1.2 EPS=(1,0.01,0.001)

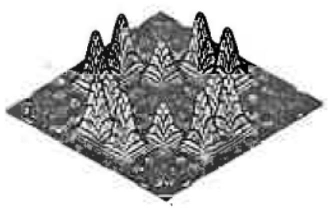
6.1.3 EPS=(1,0.01,0.0001)

6.2.1 EPS=(1,0.005,0.01)

6.2.2 EPS=(1,0.005,0.001)

6.2.3 EPS=(1,0.005,0.0001)

6.3.1 EPS=(1,0.001,0.01)

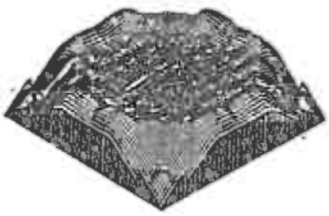


6.3.2 EPS=(1,0.001,0.001)

6.3.3 EPS=(1,0.001,0.0001)

FIGURE 7 STOCK DYNAMICS WITH BACKCLOTH CHANGE

7.1 BETA = 1.5



7.2 BETA = 1.25

7.3 BETA = 1.0

7.4 BETA = 0.75

7.5 BETA = 0.50

7.6 BETA = 0.25

FIGURE 8 STOCK DYNAMICS WITH BACKCLOTH CHANGE AND CITY CENTRE FACTORING

8.1 BETA = 1.5

8.2 BETA = 1.25

8.3 BETA = 1.0

8.4 BETA = 0.75

8.5 BETA = 0.5

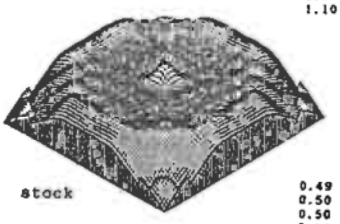
8.6 BETA = 0.25

FIGURE 9.1   DYNAMIC ADJUSTMENT: NMAX = (10,10,1); EPS = (1,0.01,0.01)

9.1.1 BETA = 1.5

price

				1.1[illegible]	1.07	1.07	1.07	1.10				
			1.10	0.91	0.91	0.93	0.91	0.91	1.10			
		1.10	0.9[illegible]	0.96	1.00	1.01	1.00	0.96	0.95	1.10		
	1.10	0.95	0.97	1.01	1.02	1.02	1.02	1.01	0.97	0.95	1.10	
1.10	0.91	0.96	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.96	0.91	1.1
1.07	0.91	1.00	1.02	1.01	1.02	1.03	1.02	1.01	1.02	1.00	0.91	1.0
1.07	0.93	1.01	1.02	1.01	1.03	0.79	1.03	1.01	1.02	1.01	0.93	1.0
1.07	0.91	1.00	1.02	1.01	1.02	1.03	1.02	1.01	1.02	1.00	0.91	1.0
1.10	0.91	0.96	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.96	0.91	1.1
	1.10	0.95	0.97	1.01	1.02	1.02	1.02	1.01	0.97	0.95	1.10	
		1.10	0.95	0.96	1.00	1.01	1.00	0.96	0.95	1.10		
			1.10	0.91	0.91	0.93	0.91	0.91	1.10			
				1.10	1.07	1.07	1.07	1.10				



stock

				0.49	0.50	0.50	0.50	0.49				
			0.49	0.50	0.50	0.50	0.50	0.50	0.49			
		0.49	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.49		
	0.49	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.49	
0.49	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.4
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.5
0.50	0.50	0.50	0.50	0.50	0.50	0.51	0.50	0.50	0.50	0.50	0.50	0.5
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.5
0.49	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.4
	0.49	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.49	
		0.49	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.49		
			0.49	0.50	0.50	0.50	0.50	0.50	0.49			
				0.49	0.50	0.50	0.50	0.49				

rent

9.1.2 BETA = 0.75

price

				1.10	1.10	1.10	1.10	1.10				
			1.10	1.05	0.83	0.85	0.83	1.05	1.10			
		1.10	1.10	0.82	1.10	1.10	1.10	0.82	1.10	1.10		
	1.10	1.10	0.77	1.10	1.10	1.10	1.10	1.10	0.77	1.10	1.10	
1.10	1.05	0.82	1.10	1.00	0.90	1.01	0.90	1.00	1.10	0.82	1.05	1.1
1.10	0.83	1.10	1.10	0.90	1.10	1.10	1.10	0.90	1.10	1.10	0.83	1.1
1.10	0.85	1.10	1.10	1.01	1.10	0.62	1.10	1.01	1.10	1.10	0.85	1.1
1.10	0.83	1.10	1.10	0.90	1.10	1.10	1.10	0.90	1.10	1.10	0.83	1.1
1.10	1.05	0.82	1.10	1.00	0.90	1.01	0.90	1.00	1.10	0.82	1.05	1.1
	1.10	1.10	0.77	1.10	1.10	1.10	1.10	1.10	0.77	1.10	1.10	
		1.10	1.10	0.82	1.10	1.10	1.10	0.82	1.10	1.10		
			1.10	1.05	0.83	0.85	0.83	1.06	1.10			
				1.10	1.10	1.10	1.10	1.10				

stock

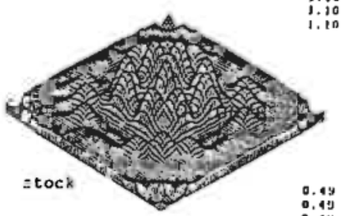
				0.49	0.49	0.49	0.49	0.49				
			0.49	0.50	0.51	0.51	0.51	0.50	0.49			
		0.49	0.49	0.51	0.50	0.50	0.50	0.51	0.49	0.49		
	0.49	0.49	0.51	0.50	0.50	0.50	0.50	0.50	0.51	0.49	0.49	
0.49	0.50	0.51	0.50	0.50	0.51	0.50	0.51	0.50	0.50	0.51	0.50	[illegible]
0.49	0.51	0.50	0.50	0.51	0.50	0.50	0.50	0.51	0.50	0.50	0.51	[illegible]
0.49	0.51	0.50	0.50	0.50	0.50	0.57	0.50	0.50	0.50	0.50	0.51	[illegible]
0.49	0.51	0.50	0.50	0.51	0.50	0.50	0.50	0.51	0.53	0.50	0.51	[illegible]
0.49	0.50	0.51	0.50	0.50	0.51	0.50	0.51	0.50	0.50	0.51	0.50	0 4
	0.49	0.49	0.51	0.50	0.50	0.50	0.50	0.50	0.51	0.49	0.49	
		0.49	0.49	0.51	0.50	0.50	0.50	0.51	0.49	0.49		
			0.49	0.50	0.51	0.51	0.51	0.50	0.49			
				0.49	0.49	0.49	0.49	0.48				

rent

9.1.3 BETA = 0.25

price

				1.10	1.10	1.10	1.10	1.10				
			1.10	1.10	1.06	1.05	1.06	1.10	1.10			
		1.10	1.10	0.91	1.10	1.10	1.10	0.91	1.10	1.10		
	1.10	1.10	0.85	1.10	1.10	1.10	1.10	1.10	0.85	1.10	1.10	
1.10	1.10	0.91	1.10	0.92	0.64	0.92	0.64	0.92	1.10	0.91	1.10	1.10
1.10	1.06	1.10	1.10	0.64	1.10	1.10	1.10	0.64	1.10	1.10	1.06	1.10
1.10	1.05	1.10	1.10	0.92	1.10	0.61	1.10	0.92	1.10	1.10	1.05	1.10
1.10	1.06	1.10	1.10	0.64	1.10	1.10	1.10	0.64	1.10	1.10	1.06	1.10
1.10	1.10	0.91	1.10	0.92	0.64	0.92	0.64	0.92	1.10	0.91	1.10	1.10
	1.10	1.10	0.85	1.10	1.10	1.10	1.10	1.10	0.85	1.10	1.10	
		1.10	1.10	0.91	1.10	1.10	1.10	0.91	1.10	1.10		
			1.10	1.10	1.06	1.05	1.06	1.10	1.10			
				1.10	1.10	1.10	1.10	1.10				



stock

				0.49	0.49	0.49	0.49	0.49				
			0.49	0.49	0.49	0.50	0.49	0.49	0.49			
		0.49	0.49	0.50	0.50	0.50	0.50	0.50	0.49	0.49		
	0.49	0.49	0.51	0.50	0.50	0.50	0.50	0.50	0.51	0.49	0.49	
0.49	0.49	0.50	0.50	0.51	0.53	0.51	0.53	0.51	0.50	0.50	0.49	0.49
0.49	0.49	0.50	0.50	0.53	0.50	0.50	0.50	0.53	0.50	0.50	0.49	[illegible]
0.49	0.50	0.50	0.50	0.51	0.50	0.61	0.50	0.51	0.50	0.50	0.50	[illegible]
0.49	0.49	0.50	0.50	0.53	0.50	0.50	0.50	0.53	0.50	0.50	0.49	[illegible]
0.49	0.49	0.50	0.50	0.51	0.53	0.51	0.53	0.51	0.50	0.50	0.49	[illegible]
	0.49	0.49	0.51	0.50	0.50	0.50	0.50	0.50	0.51	0.49	0.49	
		0.49	0.49	0.50	0.50	0.5[illegible]	[illegible].50	0.50	0.49	0.49		
			0.49	0.49	0.49	0.50	0.48	0.49	0.49			
				0.49	0.49	0.49	0.49	0.49				

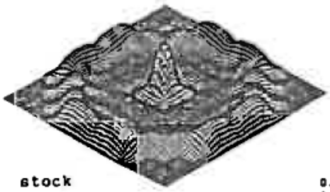
rent

FIGURE 9.2 DYNAMIC ADJUSTMENT: NMAX = (10,10,1); EPS = (1,0.001,0.01)

9.2.1 BETA = 1.5

price

				1.01	1.01	1.01	1.01	1.01				
			1.01	0.99	0.99	0.99	0.99	0.99	1.01			
		1.01	0.99	1.00	1.00	1.00	1.00	1.00	0.99	1.01		
	1.01	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01	
1.01	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01
1.01	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01
1.01	0.99	1.00	1.00	1.00	1.00	0.95	1.00	1.00	1.00	1.00	0.99	1.01
1.01	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01
1.01	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01
	1.01	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01	
		1.01	0.99	1.00	1.00	1.00	1.00	1.00	0.99	1.01		
			1.01	0.99	0.99	0.99	0.99	0.99	1.01			
				1.01	1.01	1.01	1.01	1.01				



stock

				0.50	0.50	0.50	0.50	0.50				
			0.50	0.50	0.50	0.50	0.50	0.50	0.50			
		0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
		0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
			0.50	0.50	0.50	0.50	0.50	0.50	0.50			
				0.50	0.50	0.50	0.50	0.50				

rent

9.2.2 BETA = 0.75

price

				1.01	1.01	1.01	1.01	1.01				
			1.01	1.01	0.96	1.01	0.96	1.01	1.01			
		1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01		
	1.01	1.01	0.91	1.01	1.01	1.01	1.01	1.01	0.91	1.01	1.01	
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
1.01	0.96	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.96	1.01
1.01	1.01	1.01	1.01	1.01	1.01	0.76	1.01	1.01	1.01	1.01	1.01	1.01
1.01	0.96	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.96	1.01
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
	1.01	1.01	0.91	1.01	1.01	1.01	1.01	1.01	0.91	1.01	1.01	
		1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01		
			1.01	1.01	0.96	1.01	0.96	1.01	1.01			
				1.01	1.01	1.01	1.01	1.01				

stock

				0.50	0.50	0.50	0.50	0.50				
			0.50	0.50	0.50	0.50	0.50	0.50	0.50			
		0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.54	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
		0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
			0.50	0.50	0.50	0.50	0.50	0.50	0.50			
				0.50	0.50	0.50	0.50	0.50				

rent

9.2.3 BETA = 0.25

price

				1.01	1.01	1.01	1.01	1.01				
			1.01	1.01	1.01	1.01	1.01	1.01	1.01			
		1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01		
	1.01	1.01	0.99	1.01	1.01	1.01	1.01	1.01	0.99	1.01	1.01	
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
1.01	1.01	1.01	1.01	1.01	1.01	0.54	1.01	1.01	1.01	1.01	1.01	1.01
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
	1.01	1.01	0.99	1.01	1.01	1.01	1.01	1.01	0.99	1.01	1.01	
		1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01		
			1.01	1.01	1.01	1.01	1.01	1.01	1.01			
				1.01	1.01	1.01	1.01	1.01				

stock

				0.50	0.50	0.50	0.50	0.50				
			0.50	0.50	0.50	0.50	0.50	0.50	0.50			
		0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
	[illegible]	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	[illegible]	[illegible]	[illegible]	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
		[illegible]	[illegible]	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
			[illegible]	[illegible]	0.50	0.50	0.50	0.50	0.50			
				[illegible]	[illegible]	0.50	0.50	0.50				

rent

FIGURE 10 DYNAMIC ADJUSTMENT: NMAX = (10,10,10); EPS = (1,0.001,0.01)

10.1 BETA = 1.5

stock

price

				1.01	1.01	1.01	1.01	1.01				
			1.01	0.99	0.99	0.99	0.99	0.99	1.01			
		1.01	0.99	1.00	1.00	1.00	1.00	1.00	0.99	1.01		
	1.01	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01	
1.01	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.0
1.01	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.0
1.01	0.99	1.00	1.00	1.00	1.00	0.95	1.00	1.00	1.00	1.00	0.99	1.0
1.01	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.0
1.01	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.0
	1.01	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.01	
		1.01	0.99	1.00	1.00	1.00	1.00	1.00	0.99	1.01		
			1.01	0.99	0.99	0.99	0.99	0.99	1.01			
				1.01	1.01	1.01	1.01	1.01				

				0.50	0.50	0.50	0.50	0.50				
			0.50	0.51	0.51	0.50	0.51	0.51	0.50			
		0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
0.50	0.51	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.51	0.50
0.50	0.51	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.51	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.53	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.51	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.51	0.50
0.50	0.51	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.51	0.50
	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	
		0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
			0.50	0.51	0.51	0.50	0.51	0.51	0.50			
				0.50	0.50	0.50	0.50	0.50				

rent

10.2 BETA = 0.75

stock

price

				1.01	1.01	1.01	1.01	1.01				
			1.01	1.01	0.96	1.01	0.96	1.01	1.01			
		1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01		
	1.01	1.01	0.91	1.01	1.01	1.01	1.01	1.01	0.91	1.01	1.01	
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
1.01	0.96	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.96	1.01
1.01	1.01	1.01	1.01	1.01	1.01	0.80	1.01	1.01	1.01	1.01	1.01	1.01
1.01	0.96	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.96	1.01
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.0
	1.01	1.01	0.91	1.01	1.01	1.01	1.01	1.01	0.91	1.01	1.01	
		1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01		
			1.01	1.01	0.96	1.01	0.96	1.01	1.01			
				1.01	1.01	1.01	1.01	1.01				

				0.50	0.50	0.50	0.50	0.50				
			0.50	0.50	0.52	0.50	0.52	0.50	0.50			
		0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
	0.50	0.50	0.55	0.50	0.50	0.50	0.50	0.50	0.55	0.50	0.50	
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.52	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.52	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.63	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.52	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.52	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	0.50	0.50	0.55	0.50	0.50	0.50	0.50	0.50	0.55	0.50	0.50	
		0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
			0.50	0.50	0.52	0.50	0.52	0.50	0.50			
				0.50	0.50	0.50	0.50	0.50				

rent

10.3 BETA = 0.25

stock

price

				1.01	1.01	1.01	1.01	1.01				
			1.01	1.01	1.01	1.01	1.01	1.01	1.01			
		1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01		
	1.01	1.01	0.93	1.01	1.01	1.01	1.01	1.01	0.93	1.01	1.01	
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
1.01	1.01	1.01	1.01	1.01	1.01	0.63	1.01	1.01	1.01	1.01	1.01	1.01
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
	1.01	1.01	0.93	1.01	1.01	1.01	1.01	1.01	0.93	1.01	1.01	
		1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01		
			1.01	1.01	1.01	1.01	1.01	1.01	1.01			
				1.01	1.01	1.01	1.01	1.01				

				0.50	0.50	0.50	0.50	0.50				
			0.50	0.50	0.49	0.50	0.49	0.50	0.50			
		0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
	0.50	0.50	0.54	0.50	0.50	0.50	0.50	0.50	0.54	0.50	0.50	
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.49	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.49	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.78	0.50	0.50	0.50	0.50	0.50	0.50
0.50	0.49	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.49	0.50
0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	0.50	0.50	0.54	0.50	0.50	0.50	0.50	0.50	0.54	0.50	0.50	
		0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50		
			0.50	0.50	0.49	0.50	0.49	0.50	0.50			
				0.50	0.50	0.50	0.50	0.50				

rent

FIGURE 11 DYNAMIC ADJUSTMENT: NMAX=(1,10,10); EPS=(1,0.001,0.01)