WORKING PAPER 477

CONDITIONS FOR VALLEY ASYMMETRY DERIVED FROM A SLOPE EVOLUTION MODEL

M.J. KIRKBY

School of Geography
University of Leeds
Leeds LS2 9JT

November 1986

Conditions for Valley Asymmetry derived from a Slope Evolution Model.

Mike Kirkby
School of Geography,
University of Leeds, England.

Abstract.

Valley asymmetry may be due to an initially off-centre stream (auto-asymmetry), or to asymmetries in either slope process rates (due to aspect or lithology) or in rates of lateral stream migration (*e.g.* ingrowing meanders). A computer model is described which simulates the evolution of a valley cross-section, subjected to soil creep/ rainsplash/ solifluction, wash and a simplified from of landslides. Divides are either allowed to act as reflectors or allowed to roll laterally, simulating a repeated valley form. The stream is treated as a linear feature, with an incremental sediment transport capacity in the modelled cross-section which is proportional to its elevation above base level. For creep, rainsplash and solifluction, process asymmetry is introduced by making the rate coefficient of gradient itself gradient-dependent. Stream asymmetry is introduced by setting a constant ratio between sediment transport from left and right banks (1.0 where there is no asymmetry in stream migration).

The model shows that auto-asymmetry (due to initial stream position alone) increases over time only where wash processes are significant and streams are actively down-cutting. As a rough guide, asymmetry increases where the longer slope is long enough to support tributary streams. Process or stream-led asymmetries invariably produce asymmetry in valley form where the process imbalance can be maintained, and may be amplified by auto-asymmetry. Landslides tend to slow the rate of lateral stream movement once their threshold gradient is exceeded. There is no simple relationship between rates of lateral migration and rates of downcutting.

Introduction.

Asymmetrical valleys have been described in several contexts and for many areas. The commonest causes given for consistent asymmetry in an area are process rate differences due to either aspect or lithology (e.g. Crampton, 1977; Dohrenwend, 1978; French, 1979; Williams, 1980 and Budel, 1982), and stream migration (e.g. Melton, 1960) ultimately leading to ingrown meanders where the whole valley profile broadly mimics the asymmetry of the stream cross-section. An additional possible cause of asymmetry, proposed by Band (in press), is that initial chance asymmetry in stream position will be accentuated through the operation of processes which are not themselves asymmetric. This form of asymmetry is here called auto-asymmetry. In most cases it should not lead to a persistent direction or pattern of asymmetry in an area, but areas which favour auto-asymmetry should show a high proportion of markedly asymmetric valleys. This paper explores these possibilities for asymmetry through a slope evolution model, with some theoretical analysis of equilibrium cases and some runs of a computer implementation.

Process asymmetry has been ascribed to microclimatic differences due to aspect; that is to slope gradient, orientation and position. Differences are mainly due to the variations in solar radiation with aspect, particularly for direct beam radiation. The largest single influence is usually the difference in angle between the slope surface and the solar beam, effects being integrated over the year. Subsidiary effects are due to shading by local skylines and reduction in the total solid angle of visible sky. Both of these factors depend on slope position and overall valley form. Where cloud frequently obscures the sun, the last factor may predominate. In cold climates, aspect differences have a strong influence on freeze-thaw frequency. Under very cold conditions the warmer slopes may thaw more often; and under less cold conditions the cooler slopes may freeze more often. In both cases higher freeze-thaw frequencies are thought to be associated with more rapid slope sediment transport, especially by creep or solifluction processes. In semi-arid climates warmer slopes dry out more rapidly, giving a more xerophytic vegetation, resulting in a thinner ground cover and less litter. In consequence, overland flow runoff, rainsplash and wash erosion all tend to be greater on the warmer slopes.

Lithological effects are simple and strong for inclined strata of differing resistance, for example in sandstone and shale sequences. They are associated with dip and scarp slope topography and with monoclinal shifting in the down-dip direction. The greatest contrast between layers is thought to lie in their threshold angles with respect to landslides, though downslope mixing tends to reduce any differences in the overlying soil. There may also however be significant differences in percolation rates, leading to differing intensities of overland flow and higher pore water pressures on dip slopes. Such lithological effects can be simulated through these differences in process rates, although the variation behaves as a step function rather than with the gradual variations due to aspect.

Ingrown meanders are a form of valley asymmetry which have been described from many areas. The whole valley cross-section is steepened during incision on the outside of meander bends, and becomes gentler on the inside of the bend. It is inferred that meander amplitude has increased steadily during incision; in distinction from the case of incised meanders, for which amplitude has remained more or less constant during downcutting. In the absence of any asymmetry in stream migration, it has been assumed here that sediment loads from the left and right banks of the stream remain equal. This criterion has been proposed by Smith and Bretherton (1973), and some such criterion is needed to determine stream lateral position within its valley. If the stream as a whole (*i.e.* ignoring meanders for the moment) moves too much into one bank, it dislodges additional sediment into its course, while the opposite bank provides little or none. In consequence the stream course tends to shift back towards a position of equal sediment contribution which is a least work location. In a meander bend, the outside of the bend is able to carry and erode more sediment by virtue of its curvature. It is proposed here that this tendency can be described by setting the sediment contribution from the two banks in constant ratio (not equal to 1.0) related to bend curvature.

The operational definition of asymmetry is somewhat ambiguous except in the simplest cases. Here it is examined in the context of profiles from one divide (at the left) down to the stream, and up again to the right hand divide. It could, with almost equal validity be examined in a profile from stream to divide to stream. Where the two divides are at equal heights, then asymmetry can most conveniently be described as the ratio of horizontal slope lengths to left and right of the stream, and this measure is the one used most frequently here. Where the valley form is considered to 'roll', so that its form is repeated to left and right of the valley studied, then the condition of equal divide heights must be met (though it will be seen that 'rolling' may also take place where divide heights differ). Where divide heights are unequal, then at least five types of asymmetry can occur separately or together. Figure 1 shows definitions of width, height, mean gradient, maximum gradient and form asymmetries. Substantial deviations from a value of 1.0 in any of these ratios is recognised as a valley asymmetry, and all but mean gradient asymmetry can deviate while the other indices remain unity. The final two indices, for maximum gradient and form asymmetry, are the most difficult to define operationally, relying on simply convexo-concave profiles with an unambiguous maximum gradient segment. In all cases these simplified cross-sectional analysis assumes parallel contours on opposite sides of a valley, whereas most valley sides, particularly in headwater areas, are inclined towards the downstream direction. Profiles should be taken normal to the contours, at least for the main part of the slope. Contrasts in aspect and lithology may then be less than if opposite slopes were diametrically opposed.

Model Formulation

To simulate the development of valley asymmetry, a standard slope profile model for transport limited removal has been modified to simulate a valley cross-profile from divide to divide. The main changes required are that distance from the divide must be related to its appropriate divide, and that boundary conditions for the divides and the stream banks must be re-examined. The model is analysed for some equilibrium cases, and set up in finite difference form as a BASIC computer program.

As in other standard slope development models (*e.g.* Kirkby, 1971), the model is based on the mass balance equation:

$$S/\ x = -\ z/\ t \qquad (1)$$

where S is the sediment transport from left to right,
x is distance from the left hand boundary,
z is elevation above base level
and t is elapsed time.

For the case of transport limited removal, sediment transport S is taken to be equal to transporting capacity, C, which is related to slope process mechanisms. Three processes are included here. The first process is creep or solifluction or rainsplash, which is assumed to transport material downslope at a rate directly proportional to tangent gradient. The second process is wash by overland flow. It is modelled as transporting material at a rate directly proportional to gradient, and proportional to (distance from divide)^2, reflecting an assumption of uniform overland flow production which is thought to be most appropriate for sparsely vegetated slopes with thin soils. The third process modelled is a simplified landslide model, in which transport is proportional to gradient in excess of a threshold value. Modelling landslides as a transport limited process is an adequate approximation provided that no slopes exceed the threshold appreciably and that the majority of the profile is at gradients below the threshold. The purpose of this 'process law' is to eliminate gradients above the threshold value, and this is achieved by using a rate constant several orders of magnitude greater than that associated with creep etc. Where slopes are generally above the threshold, a more complete landslide formulation is required. Where slopes contain segments at or above the threshold, asymmetry effects are weak because of the high sensitivity of sediment yield to gradient, so that the approximation is thought adequate for present purposes.

The process law used for creep/ rainsplash/ solifluction combined with wash is thus:

$$S = C = kg[1 + (y/u)^2] \qquad (2)$$

where k is a rate 'constant' which vary with gradient to allow for aspect effects,
g is gradient, taking due account of sign,
y is distance measured from the relevant divide
and u is the distance beyond which wash exceeds creep etc (*i.e.* the rate constant for wash is ku^{-2}).

The process law used for landslides is:

$$S = C = I(g - g_{\emptyset}) \qquad (3)$$

where I is a rate constant which is usually larger than k
and $g_{\emptyset}$ is a threshold gradient, which may vary with lithological layer.

The form of equation (3) is consistent with a more general model of landsliding (Kirkby, 1984) at gradients well below the angle of repose, at which a transporting capacity is defined. Some appropriate example values for I, $g_{\emptyset}$ are 0.06 m^2y^{-1} and 0.4 for Old Red Sandstone (including some marl and shale) in South Wales; and 100 m^2y^{-1} and 0.1 respectively for London Clay. Lithological differences are represented by a series of up to ten layers, specified by their common dip, their thicknesses and the threshold gradient for each. The series of layers is indefinitely repeated, although forms are constrained by the need to repeat the sequence an integral number of times within the valley width if divides are required to roll. Bedrock level is obtained as the minimum elevation at each point during a model run, and a transition from bedrock threshold, $g_{\emptyset R}$ to soil threshold $g_{\emptyset S}$ is estimated from the soil thickness (*i.e.* elevation less bedrock elevation), using the plausible expression:

$$g_{\emptyset} = g_{\emptyset S} + (g_{\emptyset R} - g_{\emptyset S})\ (1 + z_S/z_*)\ \exp(-z_S/z_*) \qquad (4)$$

where z_S is the soil thickness
and z_* is a scale depth, set to 1.0 m.

The total rate of sediment transport is obtained as the sum of these two process contributions.

Two types of divide are considered in the model: rolling and reflecting divides. At reflecting divides, sediment transport at the divide is set to zero, so that equal and opposite amounts of sediment must leave the divide grid cell in both directions. Only in this way can divide symmetry be maintained, and divide migration through erosion prevented. At a rolling divide, sediment transport from the extreme right hand grid cell is fed into the extreme left hand cell, maintaining sediment continuity in a loop. In this case divides may and do migrate. In finite difference form, the exact position of the divide is interpolated on an inverted upright parabola through the highest elevation cell and its two neighbours. In cases where there are persistent causes of asymmetry like aspect or lithology, the rolling divide appears to be the more appropriate.

At the stream, sediment transport from each hillside is initially calculated in relation to the lowest elevation cell. The criterion of constant sediment ratios is then applied to interpolate the exact stream position. The total stream lowering is then divided between the lowest elevation cell and one of its neighhbours according to the exact stream position. For instance, if the stream is exactly on the lowest cell, the full lowering is be applied to this cell; and if the exact stream position lies half way between the lowest cell and one of its neighbours, the lowering is equally divided between these two cells.

If the calculated sediment transport from the base of the left slope, S_L is modified to allow for a basal increment increased from dx to dx(1+) and similarly S_R from the right slope modified for a basal increment of dx(1−), then the condition that the ratio of modified left to right bank sediment yields should be in ratio is, ignoring second order effects:

$$= (S_L - S_R)/\{S_L + S_R + g_0 l[(g_L > g_0) + (g_R > g_0)]\} \qquad (5)$$

where S_L, S_R refer to the unmodified estimates of sediment transport from left and right hand slopes,
  is the required ratio of modified left:right hand sediment transports,
l, g_0 are the rate constant and threshold for landslides in equation (3) above,
$(g_L > g_0)$, $(g_R > g_0)$ take the value 1.0 if the condition is met and zero otherwise,
and the proportion can take values in the range −1 to 1.

Several simple alternatives could be used for the rate of lowering of the stream axis. It may be specified in terms of rate of stream lowering or as the increment in stream sediment transport in passing through the valley cross-section. Either of these may be simply specified as a constant, as a function of time, or as a function of elevation. The choice arbitrarily adopted here in computer simulations is to specify the increment of sediment transport as proportional to elevation above a base level. This is equivalent to an assumption that mean denudation in the valley cross-section is proportional to stream elevation, which may be compared with Ahnert's (1970) empirical proportionality to mean elevation, and is similar to the basal downcutting routine used in his SLOP3D models (e.g. Ahnert, 1976). At the same time this basal condition gives a realistic dynamic interaction between stream and slope proceses. In analysing equilibrium forms, constant rates of downcutting have also been assumed where appropriate.

The incremental sediment transport is an additional rate of lowering to be applied to the stream cell, superimposed on the net erosion or aggradation already computed from slope process transfers. It is allocated to the cells on either side of the computed exact stream position. The total is divided in proportion to the distance of each cell centre from the stream position. In computation, the stream position is seen to fluctuate across a single cell for several iterations before migrating into the next cell, but clear longer term trends may be seen. The fluctuations are not unlike those to be found in real streams in response to large sediment inflow events from one slope or the other, followed by accelerated undercutting of the opposite bank.

Aspect effects due to gradient have been incorporated into the model in a simple form which is strictly appropriate only for moderate slopes in extra-tropical latitudes, where shading is unimportant, and there is a continuous change in microclimatic influence as gradient varies from maximum positive to maximum negative values. For symmetry of the expression, the rate constant k in equation (2) has been replaced by the gradient dependent expression:

$$k = k_0 \exp(\ ) \tag{6}$$

where is a measure of the slope process asymmetry.

Stream asymmetry effects have similarly been included by setting the ratio of left to right bank sediment transport equal to exp( ), where is thought to be proportional to meander curvature (1/radius). In simulations presented, has been held constant, although there is scope for simulating more complex interactions. As meanders become more ingrown, their amplitude and curvature increase, but three-dimensional effects also come into play. Sediment inflows converge towards the outside of the bend and diverge on the inside of the bend. Increases in the sediment ratio, due to increased curvature are therefore compensated by an amplified ratio of sediment inputs which is not represented in a profile model. By retaining a constant sediment ratio, , it is implicitly assumed that these opposing effects are of similar magnitudes.

This model has provided the algorithms for the BASIC program 'SlopeV', for which the core listing is given in the Appendix. Iteration times have been chosen to allow gradients to change by a maximum of 30% at any point. This condition has been relaxed where height differences between neighbouring cell are less than 1mm, to allow for divide migration. Runs have generally been based on a 32 point division of the profile length, as the minimum to give reasonable resolution for stream migration.

Analysis of equilibrium forms

If it is assumed that downcutting and lateral cutting proceed indefinitely at uniform rates, rather than at rates related to elevation above a base level, then equilibrium forms may readily be obtained from eqaution (1) to (3) above. If the rate of vertical cutting is set to T, and of lateral cutting to V, then the continuity equation (1) requires that, in equilibrium:

$$S = T\,x + V\,z \qquad (7)$$

where S and x are measured from left to right
and z is measured positive in the upward direction.
Setting this expression equal to the total sediment transport summed from equations (2) and (3), an ordinary differential equation is obtained for gradient $g = - z/ x$. In all cases of interest, this equation must be solved computationally, by iteration to obtain the value of gradient and then by numerical integration over a small increment of distance, working down from the divide. The only exceptional case is at the divide itself, near which z is approximately obtained by direct integration as:

$$z = -\,(T/2k)\,x^2 \qquad (8)$$

Figures 2 to 4 illustrate some of the equilibrium solutions obtained in this way. The Appendix gives a brief outline BASIC program, 'Asymm' to compute the relevant slope forms.

Figure 2 shows some examples of equilibrium profiles, obtained for creep etc processes alone (*i.e.* u very large and landslides negligible), assuming no lateral shifting of divides ($V = 0$) and varying degrees of process asymmetry, expressed through the parameter in equation (6). As the degree of process asymmetry is increased, other things being held constant, the slope with more rapid processes becomes progressively less steep, and the less active slope becomes steeper. The contrast between opposing slopes is minimal close to the divide, and increases progressively downslope. Since total sediment transport is, by equation (7) proportional to distance from the divide (since $V = 0$), sediment transport at equal horizontal distances from the divide are equal, so that these profiles may be abutted, as in figure 2(c), to give an asymmetric landform which might represent a section across a series of tributaries cut into a long spur. This example has been defined as one with no lateral migration of streams and divides, but this is an exceptional special case, and not a general one.

In figures 3 and 4, sets of cross-profiles have been drawn which meet several conditions, which are again not general, even in equilibrium. In each case the slope to the right of the divide is 100 metres long and about 20 metres high. The slope on the left of the divide has been set to the same height as the right hand slope, and its sediment yield is the same (figure 3) or in constant ratio (figure 4). Thus repeated forms can be abutted, as in figure 2(c) giving width asymmetry, but no height asymmetry. This is similar to the use of 'rolling divides' in the simulation model. The rates of constant vertical and lateral movement have been chosen to meet these boundary conditions and provide comparable example profiles.

Figure 3 shows slopes without landslides and with varying degrees of process asymmetry, combined with different degrees of wash (described by u in eqaution 2). It can be seen that both the degree of width asymmetry and the overall direction of migration (the vector combining vertical and lateral movements T and V) increase as either the slope asymmetry, increases or the importance of wash (reduced u) increases. Figure 4 shows, for slopes with no process asymmetry, profiles in equilibrium with a constant ratio of sediment removal at left: right of 2 for (a), (c) and (d); and of 5 for (b). It may be seen that width asymmetry is again much stronger where wash is significant, and that the rate of lateral relative to vertical migration is larger than in figure 3 for the same degree of slope asymmetry. Figure 4 is related to the case of ingrowing meanders, and it should be noted that the necessary assumption here of rolling divides, implying meanders in phase in neighbouring valleys, is not plausible, so that slopes are unlikely to approach equlibrium forms. If landslides are added to the examples of figure 3 and 4, their effect is to eliminate slopes steeper than the threshold gradient, somewhat reducing the width asymmetry in the most extreme cases.

Simulation results

Even without valley asymmetry, a slope model has too many parameters to illustrate its complete set of outcomes economically. In presenting example results here, most parameters have been held constant to show the effect of asymmetry as influenced by two parameters which have proved to be particularly relevant, the relative rate of stream downcutting and the balance between creep etc and wash processes. Incremental sediment transport per unit length downstream is set equal to z where z denotes the stream elevation. The balance between creep etc and wash is parameterised by u, the distance beyond which wash is dominant (equation 2 above). Most of the remaining process and initial form parameters have been held constant, and the results obtained are, in general dependent on the particular values selected, which are set out in Table 1. The rate for creep etc has been set at a low value appropriate for soil creep. For solifluction, and for rainsplash in favourable environments this rate might be increased by up to ten times, mainly resulting in a corresponding tenfold decrease in times taken to reach each stage of development. The landslide threshold gradient has been set to the artificially high value of 2.0 (63.4°) to minimise the influence of slides which generally slow the development of asymmetry, especially where rates of downcutting are high. The valley has been set to 100m width, with divide and stream elevations initially at 58m and 23m above the base level to which downcutting tends.

For runs investigating auto-asymmetry, the stream has initially been set off-centre, so that left and right hand slopes have the same relief (35m) but lengths of 31.25 and 68.75 m respectively. Divides have arbitrarily been allowed to roll, and this choice has a significant impact after long time periods. Figure 5 illustrates a run in which the initial asymmetry has been reinforced (positive auto-asymmetry) under conditions of strong wash (u = 10m) and some net downcutting. At times of 30,000 years and more, the left hand slope is controlled by landsliding. The disparity of slopes has allowed some divide migration. If divides are reflecting rather than rolling barriers, a substantial elevation asymmetry develops instead after about 20,000 years.

Figure 6 illustrates a valley profile with strong process asymmetry so that creep and wash move more material down the right hand slope than the left for any given gradient. With the asymmetry adopted (=1), gradients of 45°(1.0) left, zero and 45° right have creep etc rate constants of $k = 3.67\times10^{-4}$, 10^{-3} and 2.72×10^{-3} respectively. These contrasts are thought to be greater than those ususally experienced due to aspect differences. For the example combination of little wash (u=1000m) and low stream removal ($=10^{-7}$ y^{-1}), the valley shows net aggradation, since removal is slower than valley side erosion. The stream shows definite migration towards the face with lower process rates, and the width asymmetry is further increased by divide migration. As a result the valley side with slower rates is steepened to maintain equality of slope base sediment outputs.

In figure 7 asymmetry results from unequal sediment removal from the two valley sides. In the example, sediment is taken from the right hand slope at $= e^2$ 7.4 times the rate from the left hand slope, representing the action of a meander bend. Wash is again insignificant (u=1000m), but the rate of downcutting is slightly greater than in the previous example ($=10^{-6}$), resulting in almost no net change in stream elevation. Without process or gradient asymmetry near the divides, they have not shown a tendency to become unequal (with reflecting divides) or migrate (with rolling divides). Stream migration is clear, producing an ingrown meander form, even though stream elevation has not, in this case, changed so that the 'slip off slope' shows considerable colluvial fill near its base.

The particular features of figures 5 to 7 cannot be generalised in any very simple way, but the set of outcomes as and u are varied is shown more fully in figure 8 and 9.

Figure 8 shows the initial rates of vertical and lateral stream migration in response to initial (auto), process and stream asymmetry; and figure 9 shows the position of the stream after 50,000 years of evolution.

Figures 8(a) and 8(b) show the initial rates of vertical (a) and lateral (b) migration in response to an initial width asymmetry, from the starting profile shown in figure 5. Rates of vertical movement are not strongly dependent on the form of asymmetry, and (a) is not repeated for the process or stream asymmetric cases. As wash becomes more important (*i.e.* as u is decreased) greater rates of stream transport (measured by) are required to remove the slope sediment flux, so that the curve of zero vertical movement shows a trade-off between and u values. For any given wash intensity, rates of stream lowering increase with stream sediment increment above and below the zero curve. Lateral migration in (b) shows almost total independence of vertical movement, with curves running almost orthogonal to those in (a). It may be seen that lateral migration may also be in either direction. Migration which tends to diminish the initial width asymmetry (negative auto-asymmetry) occurs most strongly where wash is insignificant and stream removal weak, but it may be seen that it is never pronounced. The only strong auto-asymmetry is positive, and is favoured by strong wash and high incremental rates of stream sediment transport, in the top left hand corner of the figure. Figure 5 represents the point A in (a) and (b), and may be seen as an example of pronounced positive auto-asymmetry. It is provisionally concluded that strong auto-asymmetry only occurs where wash is the dominant process before the slope base is reached (*i.e.* u is less than the slope length), and where the base of the slopes are actively downcutting. These conditions are essentially those required for instability (Smith and Bretherton, 1973), which should be exhibited in the incision of the longer slope by tributary streams. Thus initial inequalities in slope length should only grow significantly where the longer slope is able to support tributaries (and it may be that its sediment load is mainly brought to the main stream along those tributaries).

Figure 8 (c) and (d) show the rates of lateral migration in response to process asymmetry and stream asymmetry respectively. In each case the rates have been derived for a fixed asymmetry value, and presented after normalising by dividing by the asymmetry parameter (for process; for stream). It may seen that the form of the curves is substantially different from those in (b), although increases in and/or decreases in u produce increases in migration rate in all three cases. Both process asymmetry and, to a less extent stream asymmetry show a greater sensitivity of migration rate to than for auto-asymmetry; and a reduced sensitivity to u. It is commonly thought that asymmetry is reduced by high rates of downcutting, but no support is given to this simple view from the simulated rates of lateral migration.

Figure 9 shows the migration of the valley bottom stream over a 50,000 year span. Superimposed on the initial form is a curvilinear grid which shows the final position of the stream for any combination of the parameters u and . Thus for example, the evolution shown in figure 5, for u=1000m and $=10^{-5}$, is shown by the migration of the stream from its initial position P to its final position Q, shown in figure 9(a) and figure 5. The positions shown are not simply extrapolations of rates from figure 8, because of effects which only operate over longer time spans: for example, control by landslides, approach to base level, and interactions with divide behaviour. Another significant factor in large lateral migrations is the influence of increasing auto-asymmetry as the valley becomes more asymmetric. Thus asymmetry which is initiated by process differences for example, may be amplified by auto-asymmetry in suitable cases. Over a long period therefore , parameter values which favour auto-asymmetry ultimately accelerate all forms of asymmetric development.

The three parts of figure 9 again emphasise the rather similar responses of vertical migration in all three cases, and the very different lateral migration pattern, although responding in the same direction to the two factors considered. Although auto-asymmetry magnifies the extreme migrations, it is perhaps more significant that there are appreciable migrations due to stream or process imbalance, even without strong auto-asymmetry.

Lateral migration in response to dipping strata of varying resistance shows a less clear pattern than for the previous cases. This lack of pattern may reflect the imperfections in the modelling assumptions, which rely on differences in landslide threshold angles rather than on rates of removal at gradients above this threshold, to parameterise lithological differences. Figure 10 (a) and (b) illustrates the forecast slope evolution in two cases. In each the more resistant band has a threshold gradient of 1.0, and the less resistant of 0.2, and the layers dip at gradients between these thresholds. The main difference between the two cases lies in the relative rates of landslides to other slope processes. In (a) landslides are rapid and wash etc slow; whereas in (b) wash processes are more rapid and landslides slower.

In both of the examples shown and more generally, the model runs forecast the development of asymmetry with the formation of scarp and dip slope topography, but the directions of migration are not consistent, and it appears that both stream and divide migration play significant roles. Divide migration is generally in a down-dip direction within the resistant strata and up-dip in the weaker strata. Stream migration responds both to the rates of channel incision and to the balance of landslides to other processes. When stream incision is rapid, as initially in figure 10b, because stream removal exceeds slope sediment supply, there is little lateral stream migration. When downcutting slows stream migration is mainly up-dip under conditions of rapid landsliding, as in (a), and mainly down-dip where wash is relatively important, as in (b). In the latter case, auto-asymmetry amplifies the initial tendency to migration.

For major escarpments, with long dip slopes supporting substantial streams, wash is relatively important, so that escarpments in resistant rock are forecast to migrate in the down-dip direction, tending towards the pattern of monoclinal shifting. In simple valleys however, where slopes are not long enough to support substantial wash, and particularly in rocks which slide rapidly down towards their threshold gradient, migration of the scarp form can take place mainly in the up-dip direction. More work is needed however to establish the limiting conditions.

Conclusions

Where wash is strong and downcutting effective, a random occurence of initially non-central streams should, through auto-asymmetric migration give strongly marked and randomly oriented valley asymmetry in an area. Typically the longer valley side should then itself be long enough to support tributary valleys. Band (in press) has suggested that this mechanism may be able to reduce tributary junction angles through migration towards the tributary divide on the up-mainstream side.

Strong microclimatic contrasts due to aspect may be sufficient to produce substantial contrasts in process rate on a systematic basis of slope orientation. This should lead in all cases to asymmetric slope forms, with greatest contrasts usually between north and south; or north-east and south-west facing slopes. It is suggested from the simulations here that some migration should occur, and that its direction should be independent of whether the valley bottom is aggrading or degrading. The only case in which asymmetry does not result is where two pressures towards asymmetric development are in opposition. For example, in an initially asymmetric valley, auto-asymmetry may be opposed by process asymmetry, resulting either in no net lateral migration or in an anomalous rate.

Similarly stream meanders are forecast to produce asymmetry invariably, except where migration tendencies are in opposition. The development of ingrown meanders is thus seen to depend mainly on the maintenance of the meander in plan, so that inequalities of sediment removal are maintained; and not primarily on whether or not the stream is actively downcutting. Meander maintenance is favoured by incision to the extent that downstream migration of the meander is inhibited, but incision is usually associated with steep channel gradients which are not typical of functioning meanders.

In dipping layered strata, differences in landslide thresholds lead to the development of asymmetric valleys with longer dip slopes and shorter, steeper scarp slopes. Where the ratio of landslide to wash rates is of the order 1:1 at the slope base, stream migration is predominantly down-dip giving an approximation to monoclinal shifting provided that stream incision is not too rapid. Where the process ratio is about 1000:1 in favour of landslides, stream migration is mainly up-dip, even though there is net denudation everywhere. In both cases divide migration plays an important role in maintaining the scarp form.

The possible causes of valley asymmetry, and the responses in valley and hillslope form are a good deal more complex than was originally envisaged, and factors may work together or in opposition. It is clear however that there are many circumstances in which the possibility of stream or divide migration cannot safely be ignored, as in much unidimensional slope modelling, and that the effects of migration on slope form are substantial. As well as providing a framework for field studies and testing, the results reported ask a number of intriguing questions. For example, it implicitly asks why ingrown meanders are not more common than they are; and about the detailed mechanisms of superimposition through a series of strata.

References cited

F. Ahnert, 1970. Functional relationships between denudation, relief and uplift in large, mid-latitude basins. *American J. Science*, 268, 243-263.

F. Ahnert, 1976. Quantitative slope models. *Zeitschrift fir Geomorphologie, Supp.* 25, 168pp.

L. Band, 1987 (in press) A model for asymmetric valleys, *Catena Supplement* on COMTAG meeting, Aachen, 1986.

J. Budel, 1980 (English Edition). *Climatic Geomorphology*. (translated from 1977 German edition by L. Fisher and D. Busche) Princeton Univ. Press, N.J. 443pp.

C.B. Crampton, 1977. A note on asymmetric valleys in the central McKenzie River catchment, Canada. *Earth Surface Processes*, 2(4), 427-429.

J.C. Dohrenwend, 1978. Systematic valley asymmetry in the central California Coast Ranges. *Geological Society of America, Bulletin*, 89(6), 891-900.

H.M. French, 1979. Periglacial geomorphology, *Progress in Physical Geography*, 3(2), 264-273.

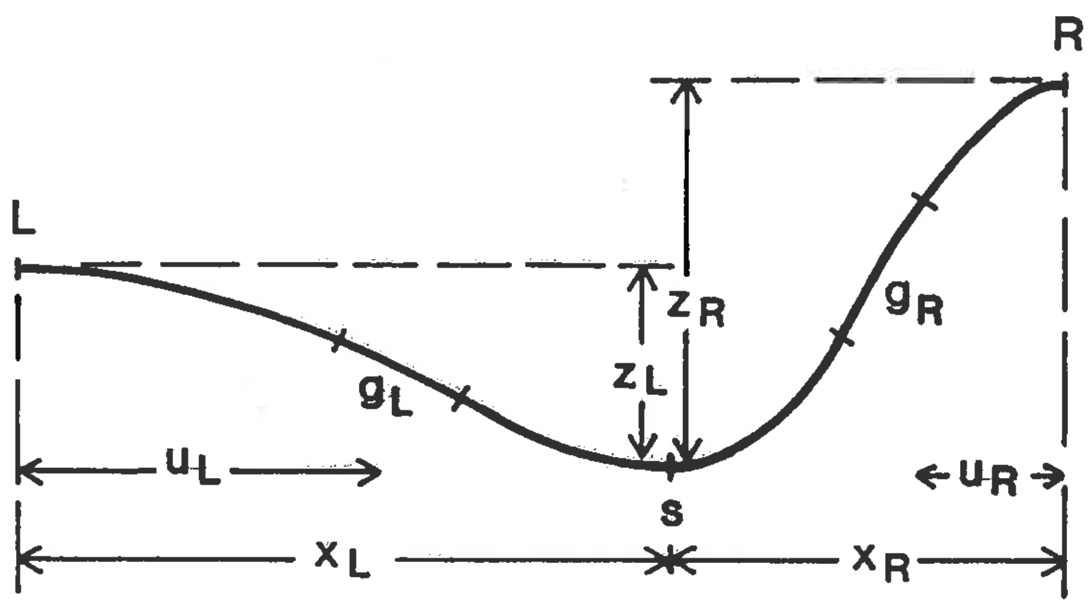
M.J. Kirkby, 1971. Hillslope process response models based on the continuity equation. *Institute of British Geographers, Special Publication* 3, 15-30.
M.J. Kirkby, 1984. Landslide models in the context of cliff profiles in South Wales. *Zeitschrift fur Geomorphologie,* 28(4), 405.
M.A. Melton, 1960. Intravalley variation in slope angles related to microclimate and erosional environment, *Geological Society of America, Bulletin* 71, 134-144.
T.R. Smith and F.P. Bretherton, 1972. Stability and conservation of mass in drainage basin evolution. *Water Resources Research*, 8(6), 1506-1529.
R.B.G. Williams, 1980. The weathering and erosion of chalk under periglacial conditions. *Institute of British Geographers, Special Publication* 11, 225-248.

Table 1. Standard parameter values used in model runs.

Valley width = 100m
Divide elevations = 58m
Valley elevation = 23m

Creep etc rate $=10^{-3}$ m^2 y^{-1}
for runs not involving geology:
Landslide threshold gradient = 2 (63.4°)
Landslide rate over threshold =1 m^2 y^{-1}

	Stream position (m)	Divide type
Auto-asymmetry runs	31.25	Rolling
Process asymmetry runs	50.	Rolling
Stream asymmetry runs	50.	Reflecting



1. Definitions of valley asymmetry.

g_L, g_R are maximum valley side slopes on left and right.
u_L, u_R are distances to points of inflexion in gradient.

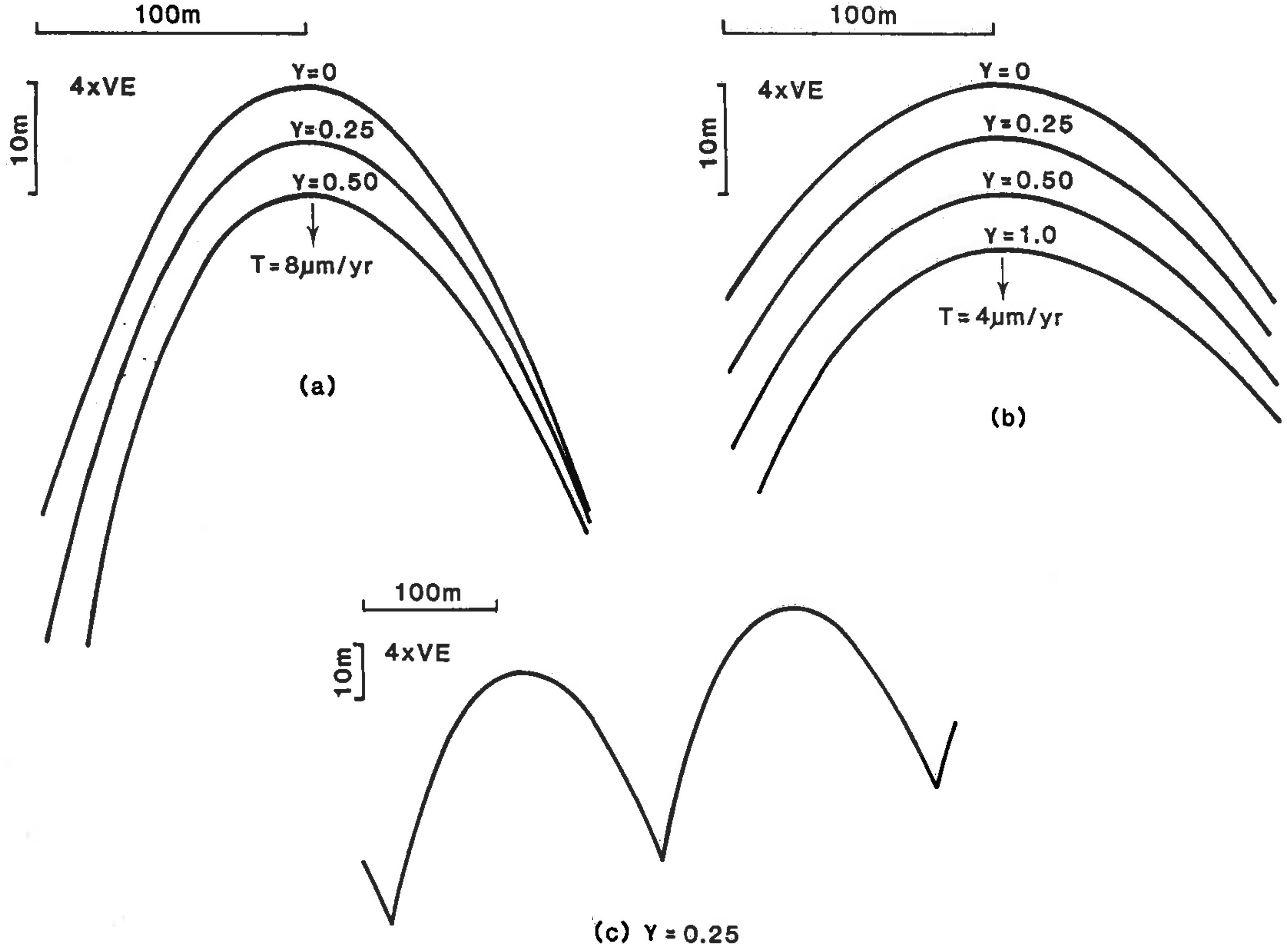
(i) Width asymmetry = x_L/x_R

(ii) Height asymmetry = z_L/z_R

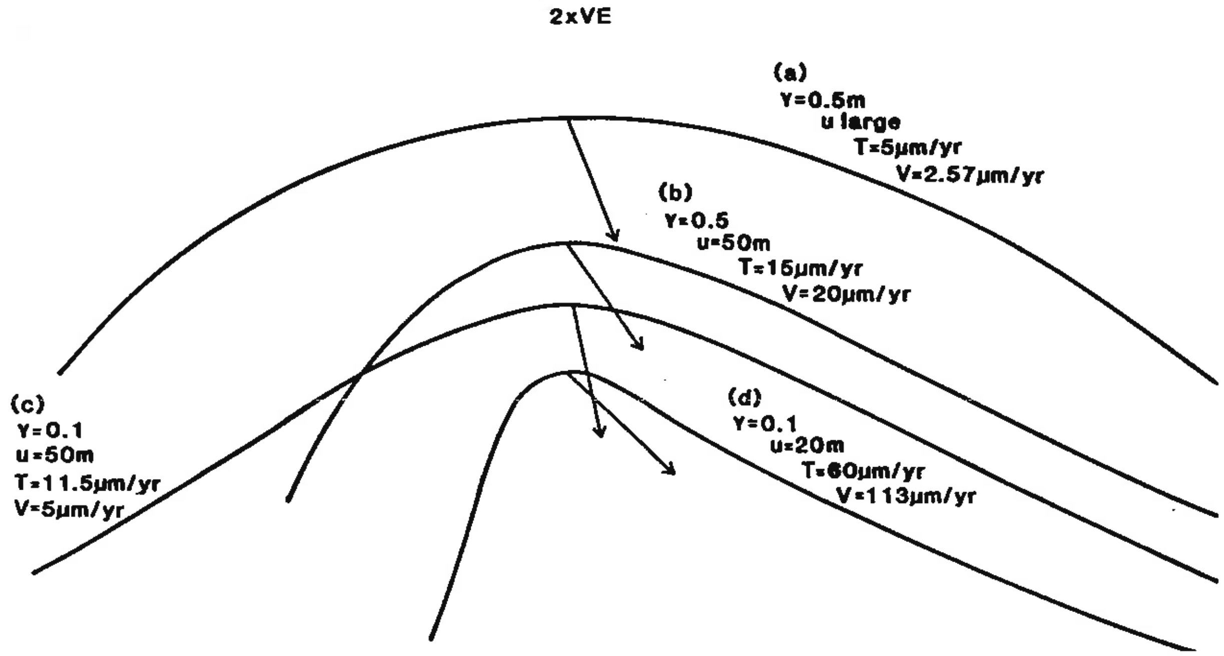
(iii) Mean gradient asymmetry = $(z_L/x_L)/(z_R/x_R)$

(iv) Maximum gradient asymmetry = g_L/g_R

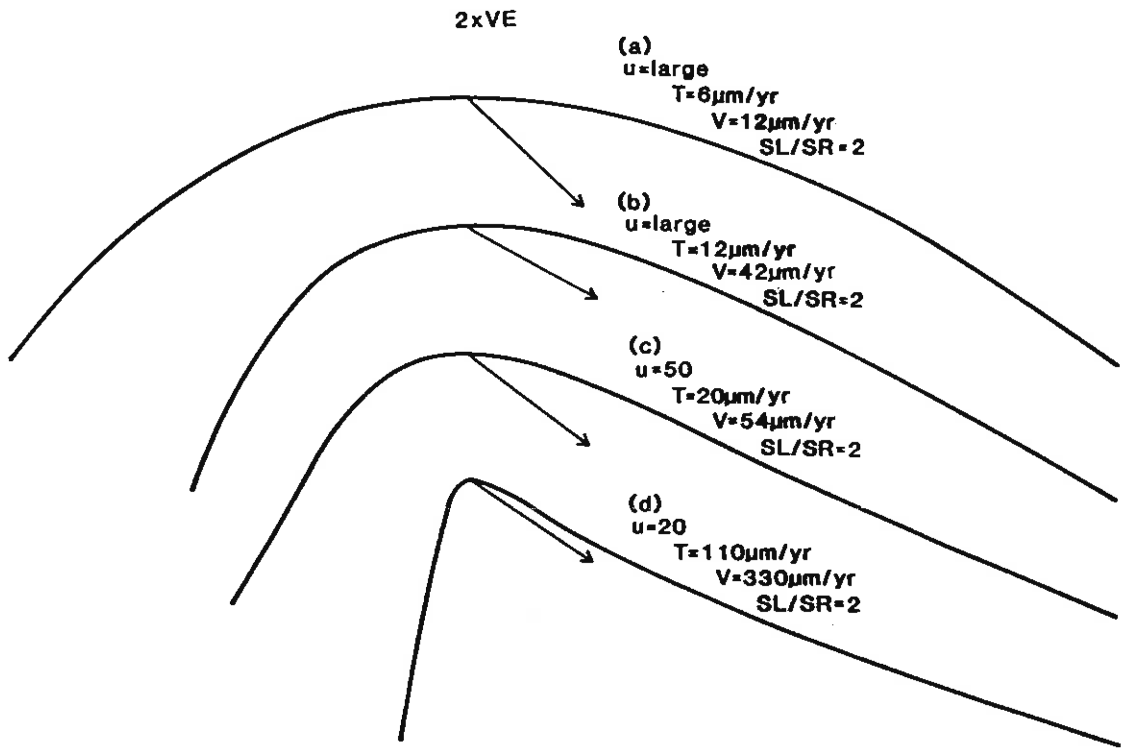
(v) Form asymmetry = u_L/u_R



2. Equilibrium slope profiles for creep only.
Y indicates degree of process asymmetry (process rates more rapid to right of divide).
$k_{\infty} = 0.001 \ m^2 \ y^{-1}$.
4x vertical exaggeration in all profiles.
(a) Lowering at 8 µm y^{-1}
(b) Lowering at 4 µm y^{-1}
(c) Example of height asymmetry with vertical downcutting, using values as for mid curve in (a).

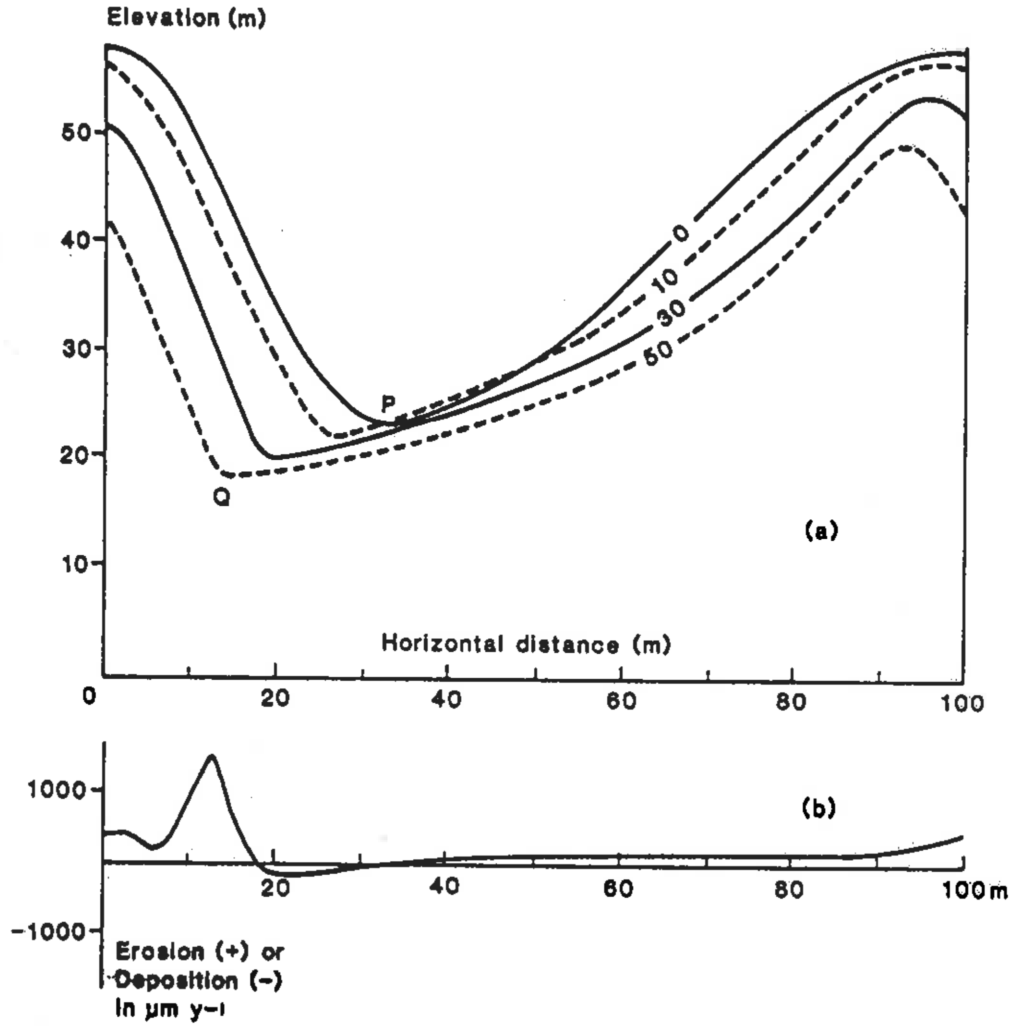


3. Example equilibrium profiles with rolling divides and no height asymmetry.
2x vertical exaggeration. Arrows show direction but not magnitude of migration vector.
Comparisons between (a) and (b); or between (c) and (d) show effect of increasing wash for constant process asymmetry.
Comparison between (b) and (c) shows effect of increasing process asymmetry for constant wash.

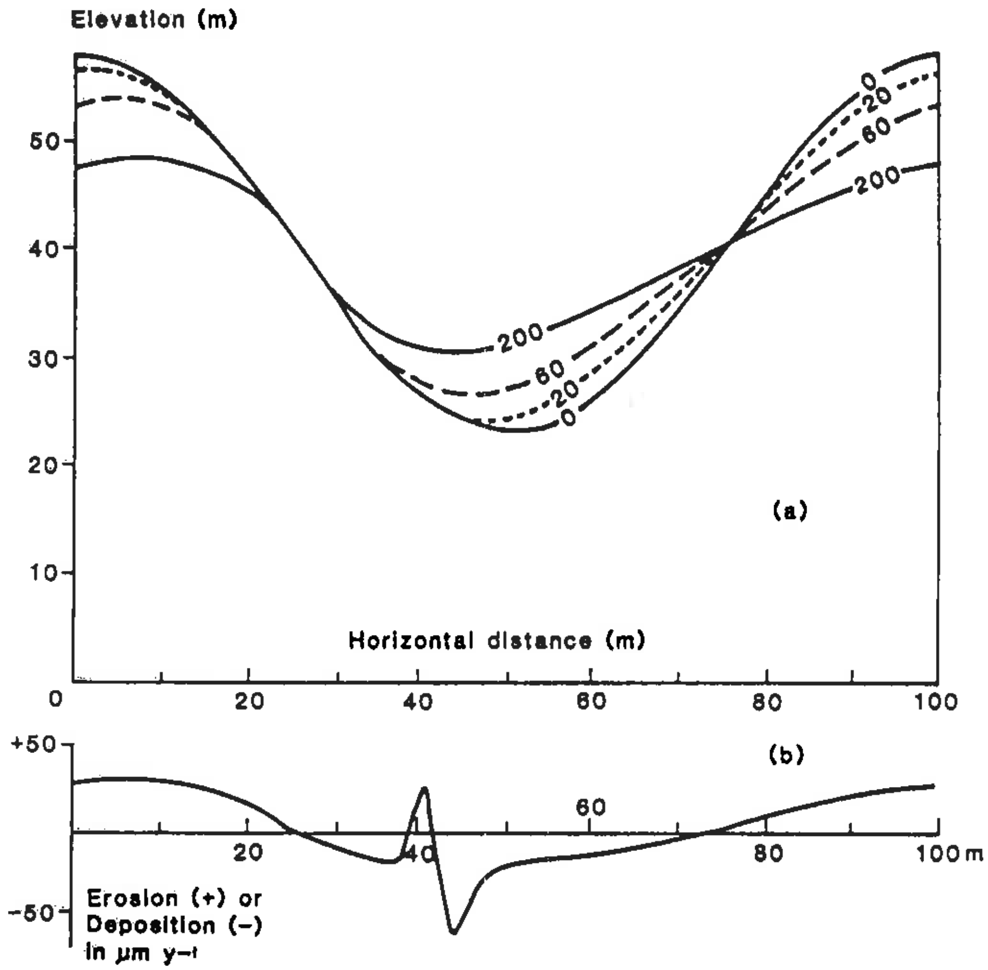


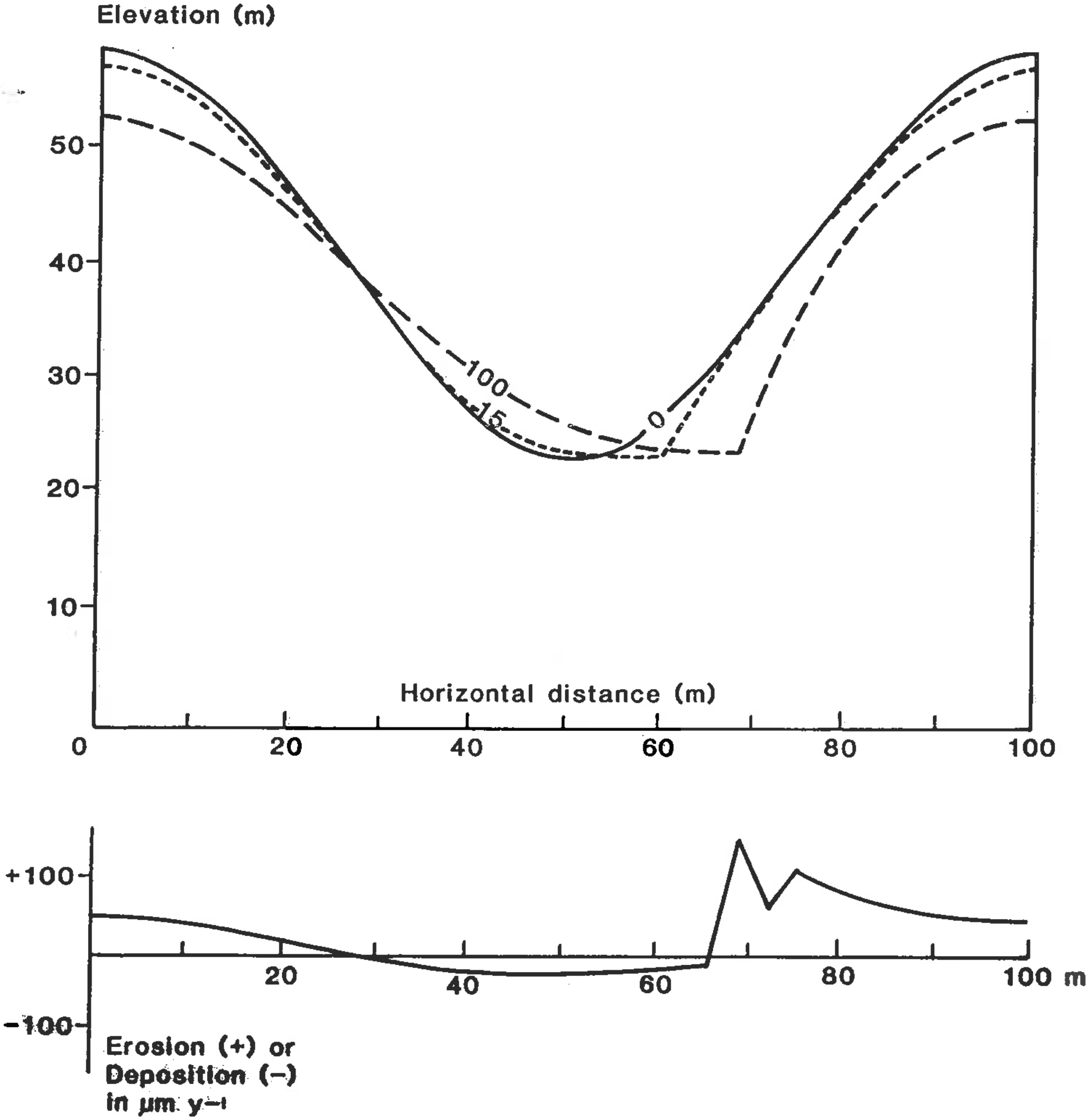
4. Example equilibrium profiles with rolling divides and no height or process asymmetry. Asymmetry is due to differences in stream sedimentt removal, given by ratio *SL/SR*.
2x vertical exaggeration.

5. Example run showing strong auto-asymmetry.
$u=10$ m. $\lambda=10^{-5}$ y^{-1}. Rolling divides.
(a) Slope profile evolution. Curves give times in 1000's of years.
(b) Rates of erosion or deposition at 50,000 years.

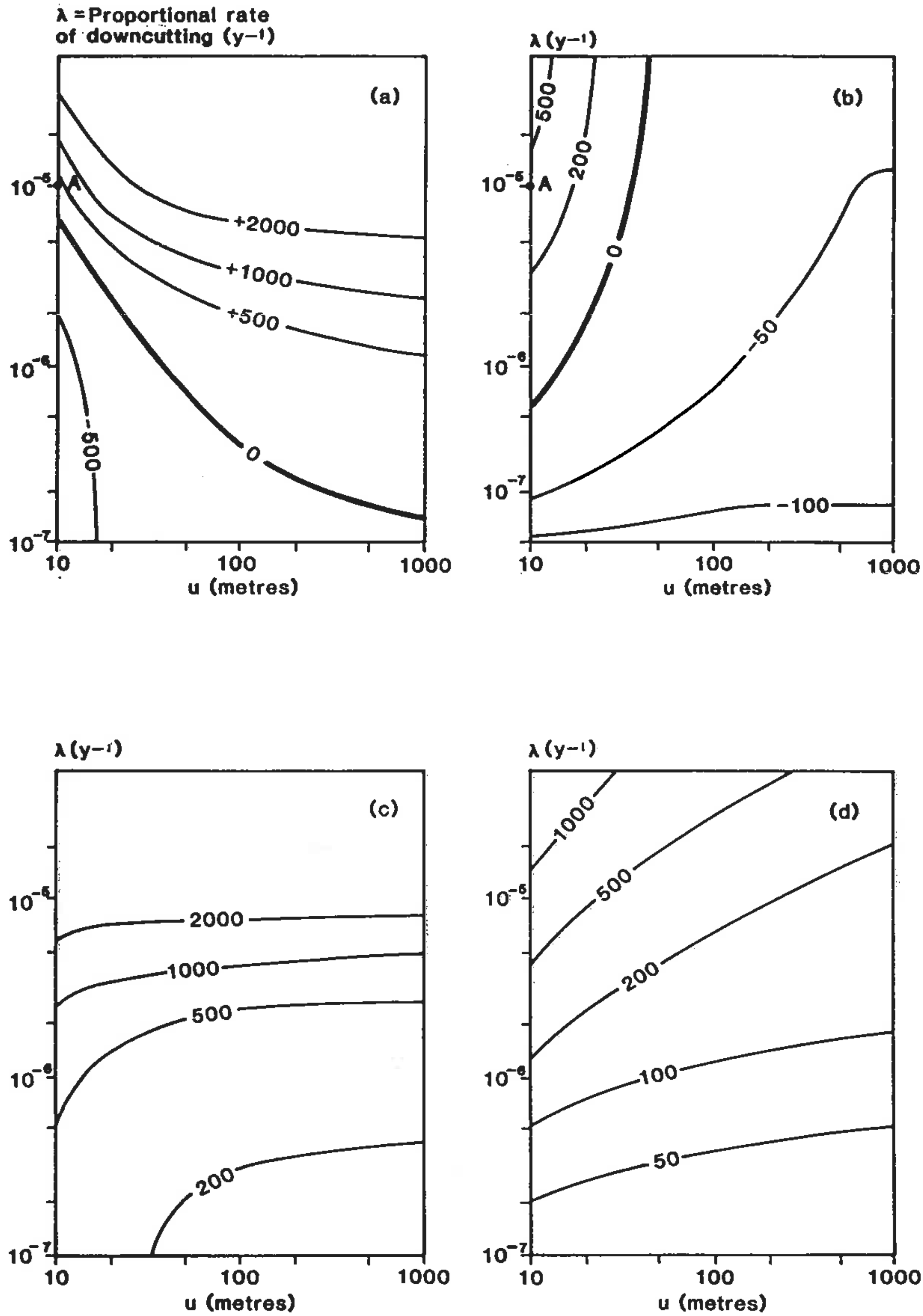


6. Example run showing process asymmetry.
$\gamma=1$. $u=1000$m. $\lambda=10^{-7}$ y^{-1}.
(a) Slope profile evolution. Curves give times in 1000's of years.
(b) Rates of erosion or deposition at 200,000 years.





7. Example run showing stream asymmetry.
$a=2$: $e^{*} = SL/SR = 7.4$. $u=1000$m. $\lambda=10^{-6}$ y^{-1}.
(a) Slope profile evolution. Curves give times in 1000's of years.
(b) Rates of erosion or deposition at 100,000 years.



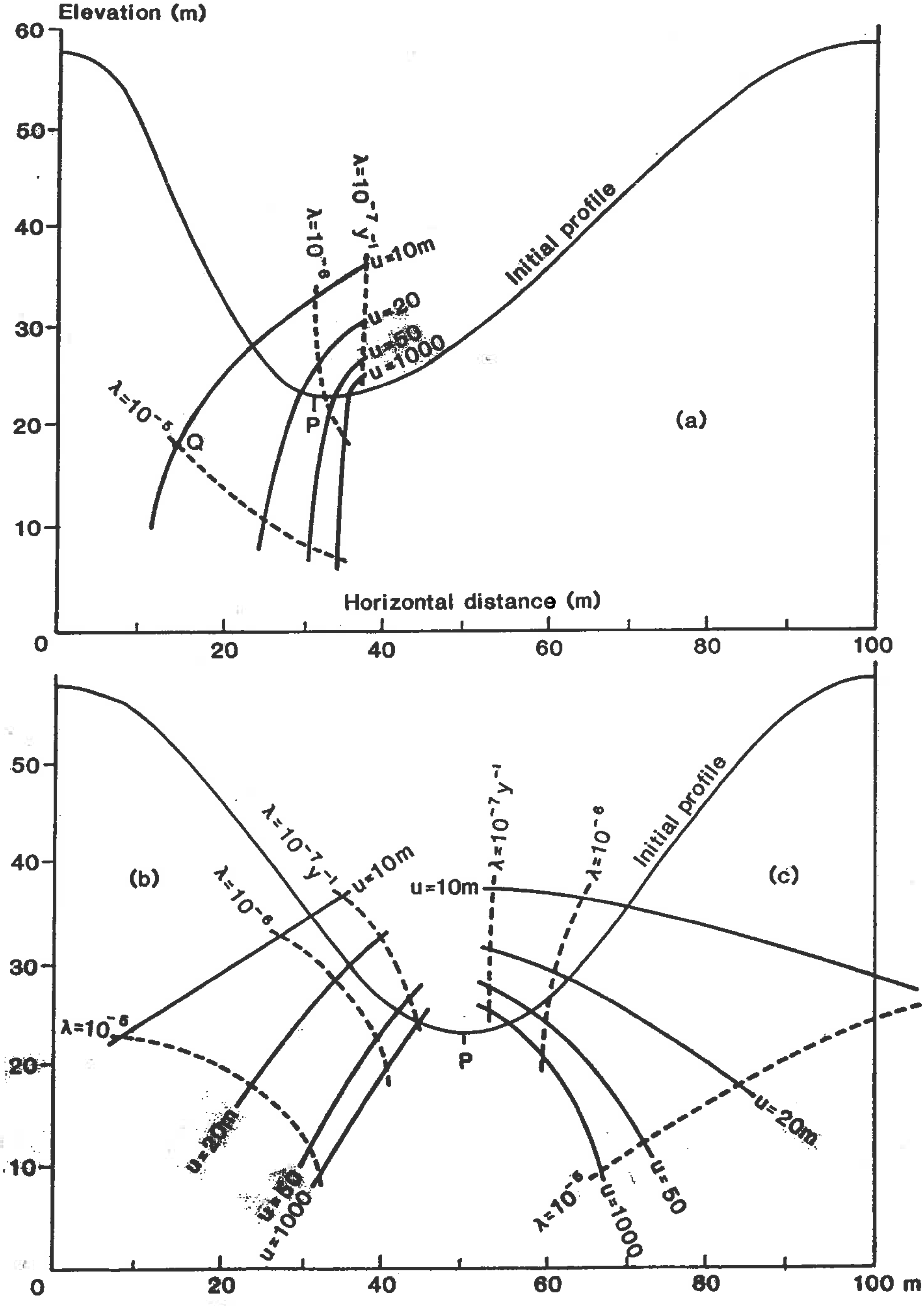
8. Initial rates of change in the devlopment of asymmetry in model runs, as proportional rates of downcutting (λ) and distance to wash dominance (u) are varied.

(a) Rates of downcutting ($\mu m\ y^{-1}$)

(b) Rates of lateral migration accentuating auto-asymmetry ($\mu m\ y^{-1}$)

(c) Rates of lateral migration per unit process asymmetry ($\mu m\ y^{-1}$)

(d) Rates of lateral migration per unit stream asymmetry ($\mu m\ y^{-1}$)

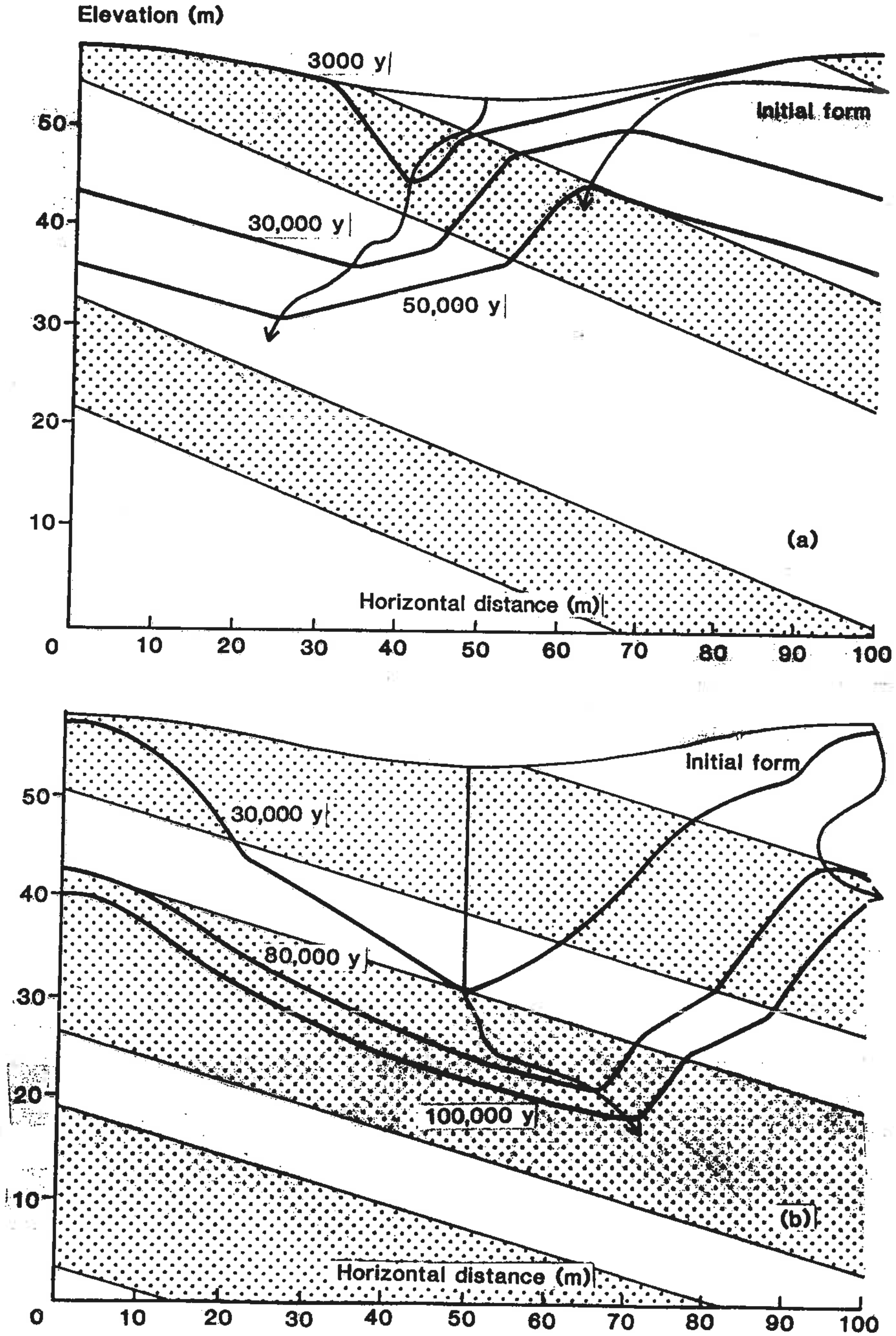


9. Positions of valley bottom after 50,000 years for a range of values of λ, u.

(a) for auto-asymmetry

(b) for process asymmetry (γ=1)

(c) for stream asymmetry (a=2).



10. Example runs showing the effect of lithological layering. Stippled areas represent more resistant rocks, with threshold gradient =2. Threshold on less resistant rocks of 0.2. No other sources of asymmetry present.
(a) $\lambda=10^{-5}$ y^{-1}. $u=1000$m.
(b) $\lambda=10^{-5}$ y^{-1}. $u=10$m.

APPENDIX: PROGRAM LISTINGS

LISTING FOR 'SlopeVG' EXCLUDING I/O PROCEDURES

```
100 RESTORE : READ length,n,tend,bdy_type: READ k, u, l0, soil_g0, scale_depth, ass_g,
   lamda: READ lateral,cb,top_L,bottom, top_R
110 DIM outcrop(10,2): READ n_layers, dip,outcrop(10,0): FOR i=0 TO n_layers-1
120   READ outcrop(i,0), outcrop(i,1)
130   END FOR i
140 REMark ====================================
150 REMark data for slope length(m) and No of points along it
160 DATA 100,32
170 REMark data for end time (y); and 0 for reflecting divides/ 1 for rolling
180 DATA 50000, 1
190 REMark data for process rates, k (creep m^2/y),u( m where wash=creep), landslide
   rate const, L0, threshold for soils, soil_g0 and scale depth of soil for reducing
   to soil thr, scale_depth: aspect asymmetry for gradient, ass_g: Sediment function
   at Line 1880
200 DATA 1E-3, 10,.1,.2,1,.2
210 REMark : Data for stream down and lateral cutting
220 REMark : lamda for mean rate of downcutting (yr^-1: cf Ahnert's 1E-7)
230 REMark : lateral (+/-5: + to R) rate of meander cutting (? prop. to curvature)
240 DATA 1E-6, 0
250 REMark Data for init_form (m):% length for valley pos; elev for top_L, valley,
   top_R
260 DATA 50,58,33,58
270 REMark Data for number of strata, dip in degrees (+ dip to right) and base of
   layer #0
280 DATA 10,30,-5
290 REMark Data for each layer from bottom/left: thickness in m and threshold gradient
300 DATA 10,1,10,1,10,1,10,1,10,1,10,1,10,1,10,1,10,1,10,1
310 REMark ====================================
320  ed_win: PRINT 'Check printer is switched ON'
330 REMark :::::::::::::  Initialising
340 REPeat forever
350 OPEN #3, ser: PRINT #3, CHR$(27);'2';:  DIM n$(15): n$=FILL$(' ',15)
360 ed_win: PAPER 0: AT 0,2:q$='Run Name/ Number: ':INPUT (q$);title$:PRINT #3,
   q$;title$;' started at ';DATE$
370 AT 2,2:q$='Enter slope length (m): ':PRINT q$;:a$=length: length=led$(1,2,26,a$):
   PRINT #3, q$;length
380 AT 3,2:q$='Enter number of points: ': PRINT q$;:a$=n:n=led$(1,3,26,a$): PRINT #3,
   q$;n: dx=length/n
390 AT 5,2:q$='Enter End time (years): ': PRINT q$;:a$=tend:tend=led$(1,5,26,a$):
   PRINT #3, q$;tend
400 REPeat loop:AT 7,2:q$='0=Reflecting: 1=Rolling boundary: ':PRINT
   q$;:a$=bdy_type:bdy_type=led$(1,7,36,a$): IF bdy_type=0 OR bdy_type=1: EXIT loop
410 PRINT #3, q$;bdy_type:AT 8,2:q$='Proportional rate of stream downcutting (/yr): ':
   PRINT q$;:a$=lamda:lamda=led$(1,8,50,a$): PRINT #3, q$;lamda
420 AT 10,2:q$='Creep/ solifluction rate (m.sq/yr): ': PRINT
   q$;:a$=k:k=led$(1,10,40,a$): PRINT #3, q$;k
430 AT 11,2:q$='Distance over which wash>creep (m): ': PRINT
   q$;:a$=u:u=led$(1,11,40,a$): PRINT #3, q$;u
```

```
440 AT 13,2:q$='Landslide rate over threshold (msq/yr): ': PRINT
   q$;:a$=10:10=led$(1,13,40,a$): PRINT #3, q$;10
450 AT 14,2:q$=' and threshold grad for soil: ': PRINT
   q$;:a$=soil_g0:soil_g0=led$(1,14,40,a$): PRINT #3, q$;soil_g0
460 AT 15,2:q$='Soil depth needed to reach threshold: ': PRINT
   q$;:scale_depth=led$(1,15,40,scale_depth): PRINT #3, q$;scale_depth
470 AT 17,2:q$='Gradient assymmetry (+ higher creep rates on left): ': PRINT
   q$;:a$=ass_g:ass_g=led$(1,17,55,a$)
480 PRINT #3, q$;ass_g:AT 18,2:q$='Preferential stream migration (+/-5: + to right):
   ': PRINT q$;:a$=lateral:lateral=led$(1,18,55,a$): lat_factor=EXP(-lateral): PRINT
   #3, q$;lateral
490 AT 19,2:q$='% of distance from left for stream: ': PRINT
   q$;:a$=cb:cb=led$(1,19,40,a$): cc=INT(cb/100*length/dx+.5):PRINT
   #3,q$;cb:cb=cc*dx/length*100
500 AT 20,2:q$='Elevation of top left divide (m): ': PRINT
   q$;:a$=top_L:top_L=led$(1,20,40,a$): ro=21:PRINT #3,q$;top_L:IF bdy_type=0
510 AT ro,2: q$='Top right divide elevation(m): ': PRINT q$;:a$=top_R:
   top_R=led$(1,ro,40,a$): ro=22: PRINT #3,q$;top_R
520 ELSE top_R=top_L:END IF
530 AT ro,2: q$='Stream elevation(m): ': PRINT q$;:a$=bottom: bottom=led$(1,ro,25,a$):
   PRINT #3, q$;bottom
540 AT 23,2:PRINT '"A" to accept: "R" to revise';: a$=INKEY$(-1): x= a$ INSTR
   CHR$(255)&'AR': ON x+1 GO TO 540,540,550,370
550 DIM za(n),dz(n),bedrock(n)
560 WINDOW #1,512,206,0,50: PAPER 0: INK 7: BORDER 10,2
570 OPEN#2,scr_512x10a0x0: INK#2, 0: PAPER#2, 7: CLS#2
580 WINDOW #0,512,40,0,10: INK#0, 7: PAPER#0, 2: CLS#0: CLS
590 OPEN #4,scr_126x10a20x50: INK #4,7: PAPER #4,0: CLS #4
600 OPEN #6,scr_250x10a146x50: INK #6,7: PAPER #6,0: CLS #6
610 OPEN #7,scr_96x10a396x50: INK #7,7: PAPER #7,0: CLS #7
620 OPEN #5,scr_472x10a20x246: f_label
630 sc=length*1.2*1.37*186/472: SCALE sc,-length/10,-sc/10
640 LINE length,-sc/155 TO -length/393,-sc/155 TO -length/393,sc
650 FOR x=0 TO length STEP length/10: LINE x,0 TO x,sc/50: CURSOR x,0,0,2:PRINT x;
660 FOR y=length/10 TO sc STEP length/10: LINE -sc/50,y TO 0,y: a$=y:CURSOR
   0,y,-6*LEN(a$)-5,0:PRINT y;
670 SELect bdy_type = 0: a$='Reflecting': =1: a$='Rolling'
680 a$=a$&' boundaries': CLOCK #4,'$d %d $m %y   %h:%m': AT #6,0,8: PRINT #6,a$;
690 init_form: nt=0: ot=0:plot 1,0,1,0:SAVE_SCR
700 DIM sa$(1),sb$(6): REPeat sloop
710 PAPER #0,0:CLS #0: REPeat sinner: AT #0,0,0: PRINT #0,"Enter Number of strata
   (1-10): ";:sx=led$(0,0,35,n_layers): IF sx>0 AND sx<11: EXIT sinner
720 n_layers=sx:REPeat sother: AT #0,1,0: PRINT #0,'       dip degrees (+ to right):';:
   sx=led$(0,1,35,dip): IF sx>=-90 AND sx<=90: EXIT sother
730 ssp%=INT(60/n_layers): IF ssp%>10: ssp%=10
740 dip=sx: REPeat sother:AT #0,2,0: PRINT #0,' Thickness (m) of layer at (0,0) BELOW
   corner';:sx=-led$(0,2,50,(-outcrop(10,0))): IF sx<=0: EXIT sother
750   gbase=sx:CLS #0: PRINT #0, 'Layer';:FOR sy=0 TO n_layers -1:PRINT #0,TO
   16+ssp%*sy,sy;
760   AT #0,1,0:PRINT #0,'Thickness (m)';:FOR sy=0 TO n_layers-1:PRINT #0,TO
   16+sy*ssp%,form$(outcrop(sy,0),5);
770   AT #0,2,0: PRINT #0,'Thresh grad';:FOR sy=0 TO n_layers-1: PRINT #0, TO
   16+sy*ssp%,form$(outcrop(sy,1),5);
780   AT #0,3,0: PRINT #0,"Edit value, '<' '>' '^' 'v' to select or 'Q' to Quit";
```

```
790   sb$=CHR$(192)&CHR$(200)&CHR$(208)&CHR$(216)&'Qq':sxpos%=0: sypos%=0
800   REPeat sinner
810     hilite 0:sa$=INKEY$(#0,-1): sx=1:REPeat six
820       IF CODE(sa$) = CODE(sb$(sx)): EXIT six
830       sx=sx+1: IF sx=7: sx=0: EXIT six
840       END REPeat six: hilite 1
850     SELect ON sx
860       =0:
   outcrop(sxpos%,sypos%)=led$(0,1+sypos%,16+ssp%*sxpos%,sa$&outcrop(sxpos%,sypos%)):
   NEXT sinner
870       =1: sxpos%=sxpos%-1+n_layers*(sxpos%=0): NEXT sinner
880       =2: sxpos%=sxpos%+1-n_layers*(sxpos%=n_layers-1): NEXT sinner
890       =3,4: sypos%=1-sypos%: NEXT sinner
900       =5,6: EXIT sinner
910       END SELect
920     END REPeat sinner
930   x=0:outcrop(0,2)=0:FOR sy=1 TO n_layers:x=x+outcrop(sy-1,0):outcrop(sy,2)=x
932   grep=x: sindip=SIN(RAD(dip)): cosdip=COS(RAD(dip)):x=1
933   IF (NOT(cosdip==1) AND (bdy_type=1))
934     x=length*sindip/grep: x=-INT(-x)/x: grep=grep/x
936     FOR sy=1 TO n_layers: outcrop(sy,2)=outcrop(sy,2)/x
938     END IF
940   outcrop(10,0)=gbase:gbase=gbase/x: INK 2: SHOW_SCR: x=gbase: flag=0
950   REPeat sinner
960     sind=0: REPeat sother
970       z=x+outcrop(sind,2): IF NOT((sindip==1)OR (sindip==-1))
980         sy=z/cosdip: ty=sy-length*sindip/cosdip
990         IF ty>0 AND sind=0 AND flag=0: x=x-grep: NEXT sinner: ELSE flag=1
1000          IF ty<length*.6 OR sy<length*.6: LINE 0,sy TO length,ty: ELSE EXIT sinner
1010          ELSE
1020           IF z>0 AND sind=0 AND flag=0: x=x-grep: NEXT sinner: ELSE flag=1
1030           IF z<length: LINE z,-length*.2 TO z, length*.6: ELSE EXIT sinner
1040          END IF
1050        sind=sind+1: IF sind=n_layers: x=x+grep: NEXT sinner
1060        END REPeat sother
1070     END REPeat sinner
1080   AT #0,3,0: PRINT #0,'"R" to Revise: "X" to exit';FILL$(' ',40);
1090   REPeat sinner: sa$=INKEY$(#0,-1): sx=sa$ INSTR CHR$(255)&'RX': IF sx>1: EXIT
   sinner
1100   IF sx=3: EXIT sloop
1110 END REPeat sloop
1120 PRINT #3,n_layers;' strata dipping to the right at gradient ';dip: PRINT #3,'Base
   of bottom layer ';-gbase;'m. below/left of (0,0): total repeat = ';grep;'m.': PRINT
   #3
1130 PRINT #3,'LAYER #';TO 16;:FOR sy=0 TO n_layers-1:PRINT #3,TO 16+ssp%*sy;sy;
1140 PRINT #3,\'DEPTH (m)';TO 16;:FOR sy=0 TO n_layers-1:PRINT #3,TO
   16+ssp%*sy;outcrop(sy,0);
1150 PRINT #3,\'THR GRAD';TO 16;:FOR sy=0 TO n_layers-1:PRINT #3,TO
   16+ssp%*sy;outcrop(sy,1);
1160 FILL 1:INK 0:plot 1,0,1,0:LINE TO length*1.2,za(n) TO length*1.2,length*.7 TO
   -length*.2,length*.7 TO -length*.2,za(0): FILL 0
1170 INK #0,7: PAPER #0,2: CLS #0
1180 PRINT #3: SAVE_SCR: outcrop(n_layers,0)=outcrop(0,0)+100:
   outcrop(n_layers,1)=outcrop(0,1)
```

```
1190 header 2: header 3
1200 tst=tend/1000: loop=0: pf=2: dd=0: dt=0
1210 REMark ++++++++++++++++++++++++++++
1220 REMark Beginning of main program loop
1230 FOR it=.5,1,2 TO 10 STEP 2,15 TO 50 STEP 5,60 TO 100 STEP
  10,150,200,300,400,500,600,800,1000
1240   t=it*tst:tplot=t-ot: nt=0:REPeat nt
1250     output: centre=n: dz(0)=0
1260     IF bdy_type=0: li=1: ELSE li=n-1
1270     FOR index=0 TO n
1280       grad=(za(li)-za(index))/dx: z=(za(li)+za(index))/2
1282       IF index=0
1284         direction=1: x=divide+.5:IF grad<0: direction=-1
1285         x=-x*direction:x=dx*(x+n*(x<0))
1286       ELSE
1290         IF grad<0
1300           IF direction=1: direction=-1: centre=li
1310           grad=-grad:x=divide-index+.5: x=dx*(x+n*(x<0))
1320         ELSE
1330           IF direction=-1: direction=1:new_div=li
1340           x=index-divide-.5:x=dx*(x+n*(x<0))
1350         END IF
1355       END IF
1360       sed_LtoR=sed(x,grad,index,z)/dx*direction
1370       IF li=centre
1380         sr=-sed_LtoR:gr=-grad:gl=ogl:g0l=og0:g0r=g0: sl=osl
1390         END IF
1400       IF index>0: dz(index)=sed_LtoR: dz(li)=dz(li)-sed_LtoR
1410       bdy_cond (bdy_type)
1420       osl=sed_LtoR: ogl=grad: og0=g0: li=index: q=KEYROW(0): IF q<>0: ftest
1430       END FOR index
1440
  dc=(sl-lat_factor*sr)/(sl+lat_factor*sr+10*(gl*(gl>g0l)+lat_factor*gr*(gr>g0r))):
  oc=centre: zc=za(oc): vert_factor=lamda*length/dx*zc
1450     dz(oc-1+n*(oc=0))=dz(oc-1+n*(oc=0))+vert_factor*dc*(dc<0): IF
  oc=1:dz(n)=dz(0)
1460     dz(oc)=dz(oc)-vert_factor*(1-ABS(dc)): IF oc=0: dz(n)=dz(0)
1470     dz(oc+1)=dz(oc+1)-vert_factor*dc*(dc>0): IF oc=n-1:dz(0)=dz(n)
1480     centre=oc+dc: IF centre<0: centre=centre-n
1490     max=0: FOR index=0 TO n-1
1500       diff=ABS(za(index+1)-za(index)): IF diff>1E-3
1510         test=ABS((dz(index+1)-dz(index))/diff)
1520         IF test>max: max=test
1530         END IF
1540       END FOR index: q=KEYROW(0): IF q<>0: ftest
1550     dt=.3/max: pf=0:IF nt+dt>tplot: dt=tplot-nt: pf=1
1560     FOR i=0 TO n
1570       za(i)=za(i)+dz(i)*dt
1580       IF za(i)<bedrock(i): bedrock(i)=za(i)
1590       END FOR i
1600     q=KEYROW(0): IF q<>0: ftest
1610     divide=new_div: IF bdy_type=1
1620       h0=divide-1+n*(divide=0):h0=za(h0)
1630       h1=za(divide): h2=divide+1-n*(divide=n): h2=za(h2)
```

```
1640        dd=INT((h0-h2)/(h0-2*h1+h2)*500+.5)/1000: IF divide+dd>n: dd=dd-n
1650        IF divide+dd<0: dd=dd+n
1660        END IF
1670      IF pf=1:SHOW_SCR: f_title: AT #0,1,0
1680      loop=loop+1: nt=nt+dt: IF pf=1: elev_flag=0: n$(4 TO 5)='  ': f_title: EXIT
   nt
1685      SELect ON elev_flag
1690        =1: n$(4 TO 5)='  ': f_title: elev_plot
1692        =2: EXIT it
1694        =3: CLOSE #4: NEXT forever
1696        END SELect : elev_flag=0
1700      END REPeat nt
1710    ot=t: END FOR it
1720 ot=t-tplot: output: sdump
1730 CLOSE #4:END REPeat forever
1740 REMark End of main program section
1750 REMark ++++++++++++++++++++++++++++++
1760 REMark Procedures to support main program
1770 REMark : type = 0 for reflecting boundaries: 1 for rolling edges together
1780 DEFine PROCedure bdy_cond (type)
1790   LOCal h: SELect ON type
1800     =0: IF li=0: dz(li)=dz(li)-sed_LtoR
1810       IF index=n: dz(index)=dz(index)+sed_LtoR
1820     =1: IF index=n: h=dz(0)+dz(n): dz(0)=h: dz(n)=h
1830     END SELect
1840   END DEFine bdy_cond
1850 REMark ------------------------------
1860 DEFine FuNction py(y,base,factor): RETurn base+factor*y: END DEFine
1870 REMark ------------------------------
1880 DEFine FuNction sed(x,g,index,z)
1890  LOCal sdz,sdloop,sdx,z_rock:sdz=x/u:
   z_rock=z-(bedrock(index-1+n*(index=0))+bedrock(index))/2:z_rock=z_rock*(z_rock>0)
1900 sdx=(index-.5)*dx*sindip+(z-z_rock)*cosdip-gbase
1910  REPeat sdloop: IF sdx>grep:sdx=sdx-grep: ELSE EXIT sdloop
1920  REPeat sdloop: IF sdx<=0: sdx=sdx+grep: ELSE EXIT sdloop
1930  FOR sdloop=0 TO n_layers:IF sdx<=outcrop(sdloop+1,2):g0=outcrop(sdloop,1):EXIT
   sdloop
1940  g0=(g0-soil_g0)*(1+z_rock/scale_depth)*EXP(-z_rock/scale_depth)+soil_g0
1950  RETurn k*g*(1+sdz*sdz)*EXP(g*direction*ass_g)+10*(g-g0)*(g>g0)
1960  END DEFine sed
1970 REMark ------------------------------
1980 DEFine PROCedure init_form
1990 FOR index=0 TO cc:
   za(index)=(top_L+bottom)/2+(top_L-bottom)/2*COS(RAD(index/cc*180)):
   bedrock(index)=za(index)
2000 FOR index=n TO cc STEP -1:
   za(index)=(bottom+top_R)/2+(top_R-bottom)/2*COS(RAD((n-index)/(n-cc)*180)):
   bedrock(index)=za(index)
2010 centre=INT(cc): IF za(centre+1)<za(centre): centre=centre+1
2020 divide=0: oc=centre: zc=za(centre): flip_pos=length/5: f_flag=0
2030 max=top_L:IF top_R>max: max=top_R
2040 max=1.15*length/2-max: IF bottom>1.5*max : er_base=.1*length: ELSE
   er_base=.475*length
2050 elev_flag=0: END DEFine init_form
```

```
2060 REMark ++++++++++++++++++++++++++++++++++++
5000 REMark Input and Output procedures
5010 DEFine PROCedure output: REM to print and plot as needed
5150 DEFine PROOCedure header (channel): REM Print table headings
5240 DEFine PROCedure line_print(channel): REM Print line of data in table
5320 DEFine PROCedure ftest: REM Test for F-key presses and set flags or act
5520 DEFine PROCedure f_title: REM Show which F-keys have been pressed
5540 DEFine PROCedure elev_plot: REM plot elevation profile on screen
5580 DEFine PROCedure move_label(sign): REM position time label on plot
5630 DEFine PROCedure sdump: dump screen as graphics to FX80 printer
5680 DEFine PROCedure f_label: REM Print reminders for F-key actions on screen
5730 DEFine PROCedure plot (choice,base,factor,flag): REM plot elevations/ Erosion
5970 DEFine FuNction form$(val,sig%): REM precise format for printing numbers
6150 DEFine FuNction led$(chanl,row,col,cue$): REM provides cued input with screen
   editing
6490 DEFine PROCedure locate (row,col): REM locates cursor for led$
6530 DEFine PROCedure ed_win: REM sets up text windows
6600 DEFine PROCedure hilite(ff): REM shows cursor in editing strata values
```

LISTING FOR 'Asymm'.

```
100 RESTORE : READ t,v,k,gamma,u,l,g0
110 x=0:z=0:OPEN #3,ser:ed_win: OPEN #4,scr_320x10a96x0: CSIZE #4,3,0: CLOCK #4
120 INPUT \\'Enter dx (m) and print frequency: ';dx,m:DIM
    el(101/dx):test_sed=1E9
125 INPUT 'Enter L/R sediment ratio: ';lamda: PRINT #3,'Run at ';DATE$
130 PRINT #3, 'dx = ';dx\'T, V = ';t,v\'K, gamma = ';k,gamma\'U = ';u\'l, g0 =
    ';l,g0\' '
140 sign=1: REPeat side
150 x=0:z=0:g=0:loop=0
160 AT 5,1:PRINT 'Press ALT/SHIFT to ';:IF sign=1: PRINT 'do other side of
    slope':ELSE PRINT 'stop'
170 FOR ch=1 TO 3 STEP 2:PRINT #ch,\'Dist(m)';TO 20;'Elev drop(m)';TO 40;'Sedi
    Yd (sqcm/y)';TO 60;'Gradient'
180 print_out: g=z/dx: x=dx:z=t/2/k*x*x: el(1)=z
190 FOR loop=1 TO 100/dx
200   tx=x+dx/2: tz=z+dx/2*g: IF loop MOD m=0:print_out
205   IF x*t-v*z*sign>test_sed: EXIT loop
210   a=tx/u: a=1+a*a: REPeat inner
220     ex=EXP(gamma*g*sign)
230     f=k*g*ex*a+l*(g-g0)*(g>g0)-t*tx+v*tz*sign
240     df=k*ex*a*(1+gamma*g*sign)+l*(g>g0)
250     dg=-f/df: g=g+dg: IF ABS(dg)<1E-5: EXIT inner
260     END REPeat inner
270   x=x+dx: z=z+g*dx: el(loop+1)=z
280   IF KEYROW(7)=5: EXIT loop
290   END FOR loop: IF sign=1:test_sed=(t*100-v*el(100/dx))*lamda
300 IF sign=-1: EXIT side: ELSE sign=-1: PRINT #3: CLS
310 END REPeat side
315  x=x-dx:z=el(x/dx):oz=z:os=t*x+v*z:print_out: x=x+dx:z=el(x/dx): print_out
317
    ns=t*x+v*z:prop=(test_sed-os)/(ns-os):x=x-dx+prop:z=oz+prop*(z-oz):print_-
    out
320 PRINT #3,\\\' ':PRINT \\'Run finished';:EDIT 990: STOP
330 DEFine PROCedure print_out
340   FOR ch=1 TO 3 STEP 2: PRINT #ch,  x;TO 20;z;TO 40;10000*(t*x-v*z*sign);
    TO 60;g
350 END DEFine print_out
990 REMark : Data for  t,v(rates of vertical and lateral cutting in
    m/y),k(creep etc rate in sqm/y),gamma(1/gradient value at which  k is
    increased by e*),u(dist beyond which wash>creep etc in m),l(landslide rate
    above threshold in sqm/y),g0(threshold gradient)
1000 DATA 1.1E-4,3.3E-4,1E-3,0,20,.7,10
30210 REMark ----------------------
30220 DEFine PROCedure ed_win
30230 WINDOW #0,512,256,0,0:INK#0,5:PAPER #0,0:BORDER#0,4,2
30240 OPEN#1, con_512x256a0x0_128:INK 7:PAPER 2:BORDER 4
30250 OPEN#2, con_:CLOSE#2:MODE 4:CLS#0
30260 END DEFine ed_win
```