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A FAMILY OF DEMOGRAPHIC ACCOUNTS AND MODELS

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Abstract

A family of nine spatially explicit, demographic accounts is defined and developed in the paper. The most fundamental type of accounts are the closed demographic accounts involving initial state to final state population flows. Migration, birth and death, and combinations of these events are built into the accounts. Simpler and commonly used accounts are input-output accounts which turn out to be rearrangements of the marginal totals of closed demographic accounts. These marginal totals appear in a multi-period context in open demographic accounts. The way in which people spend their time can be accounted for in time and person-time accounts from which can be derived proper multi-regional population at risk accounts. An alternative basis for accounts is to count all state-to state transitions and arrange them in movement accounts. All accounts may be disaggregated in various ways, and most importantly by age. Closed demographic accounts can be broken down by age group but the accounts that should be used as the basis of life tables demand exact age disaggregation. Cohort accounts combine the age group and exact age points of view. One or more of this family of accounts should be employed when constructing a population model for projection purposes.

1. Introduction

Webster's dictionary defines "accounts" to be "records of business transactions". People replace money as the unit of accounting in demographic accounts. These are tabulations of the changes that people experience from one state (age group, regional location, social class, marital status) to another in an interval of time. Demographic accounts are thus records of the transactions of people between social and spatial states. Ideally, demographic accounts should be constructed from the records of the histories of individuals held in a population register. In practice, in most countries less satisfactory data sources have to be used (census tabulations, vital statistics records, immigration departmentl tallies) and the accounts so derived are estimates only of the "true"situation. Even such estimates, however, give a much more consistent and illuminating picture of the ways in which regional populations change than hitherto available.

In this paper a review will be attempted of the concepts that underpin accounting. These were introduced in a demographic context by Stone (1965, 1971a, 1971b, 1972a, 1972b, 1973a, 1973b) and extended by Rees and Wilson (1973, 1975a, 1975b 1075c, and Wilson and Rees 1971a, 1971b) from a national to a multi-regional basis. Connections will be forged between demographic accounts and demographic projection models. Relatively simple and naive examples for a single region, all-age, all-sex population will be presented to illustrate the fundamentals of demographic accounting. It should be stressed, however, that for serious work on regional populations the types of accounts described should all be disaggregated by region, age and sex. Details of spatial and age-sex disaggregation are given for some of the accounts' family in Rees and Wilson (1975b).

In the next section of the paper, section 2, the family of accounts is described. Closed demographic accounts are described in section 3. In section 4 transition rates and birth rates of various kinds are derived from the closed demographic accounts and incorporated in demographic projection models. A simpler accounting format is introduced in section 5, called input-output accounts. The connections between these accounts and the closed accounts are described in section 5. Sections 6 presents an integrated framework for multi-period accounting based on Stone's open demographic accounts. Section 7 described the time-based members of the family of accounts and section 8 briefly reviews the other members of the family. Section 9 summarizes the use of demographic accounts as the essential foundation of modelling with national, regional or local populations.

2. The family of demographic accounts

In Figure 1 a simple genealogy of demographic accounts is sketched out. The source of all the different types of demographic accounts is shown at the top of the diagram to be the set of relevant individual life histories. In the second row of Figure 1 are shown three sets of accounts: closed demographic accounts, input-output accounts and life table accounts all involving persons as the unit of accounting except that moves (of persons) may be counted in some parts of the input-output accounts. These are the most basic and frequently used types of accounts.

The next row contains accounts which rearrange the basic information contained in the accounts above. Open demographic accounts concentrate on the period to period transitions implied in closed demographic accounts and are often multi-period. Movement accounts organize systematically moves of persons between states. Vintage or cohort accounts follow the life history of a group of individuals through many periods.

The final row of accounts organize a set of concepts involving exposure to risk and time spent in various states. These concepts are implicit in or connected with previous accounts. In population at risk accounts the unit of accounting is persons; in person-time accounts it is persons multiplied by units of time; and in time accounts it is units of time.

References are given in Figure 1 below each type of accounts. These citations are a selection of the works in which these accounts are either described initially or outlined definitely. Each type of accounts can be set up in tabular form. This we will generally call the accounts table. The interior of the table "(the table minus its row and column totals) we will call the accounts matrix. All nine types of accounts can be disaggregated in a variety of ways:"by age, by sex, by social class, by educational achievement, by income and by many other characteristics (see Stone, 1973b). All accounts have a spatial reference in that they refer to a single country or region, or to many countries or regions. The simple examples presented in the rest of the paper mostly refer to an aggregate, all-age, all-sex population but could all, in principle be disaggregated in one or more of these ways.

3. Closed demographic accounts

Closed demographic accounts are the most fundamental type. The other types of accounts can either by derived from them or connected to them.

Figure 1. The family of demographic accounts.

Files of the Life histories of individuals in time, space, age and other characteristics [P,M,T]

Closed demographic accounts [P] (One period) Stone, 1971a, 1971b, 1972a. Rees, 1972. Rees and Wilson, 1973. Wilson and Rees, 1974.	Input/Output accounts [P,M] (One period) Stone, 1971b. Rees, 1974.	Life table accounts [P] (Two period) Rogers, 1973a, 1973b. Rees and Wilson, 1975a.
Open demographic accounts [P] (One period or multi-period) Stone, 1965, 1971a. Rees, 1974.	Movement accounts [M] (One period) Rees, 1974. Illingworth, 1975.	Vintage or cohort accounts [P] (Multi-period) Stone, 1971a.
Population-at-risk accounts [P] (One period) Rees and Wilson, 1973, 1975b.	Person-time accounts [P-T] (One period) Rogers, 1975.	Time accounts [T] (One period) Illingworth, 1975. Stone, 1973a, 1973b.

Notes

[P] indicates the unit of accounting involved
P Persons M Moves T Time

Accounts refer to the number of periods indicated in brackets.

Closed demographic accounts can be presented in the following form (Table 1)*. The letter K denotes the variable that refers to population and there is a population variable in each cell of the table. If we were to disaggregate the accounts table each of these letters would represent a submatrix. The superscripts attached to the K variable refer to the initial life states, existence and birth, and the final life states, deaths and survival.** Regional locations of these initial and final life states are given in the brackets that follow the life state label. Region i refers to the region of interest and region R to the rest of the world. If we were to construct a multi-regional set of accounts, region i would be replaced by a set of regions, i = 1, 2, ..., N-1, of interest, and region R would be the Nth region. The terms in the table have the following meanings.

$K^{\varepsilon(i)\sigma(i)}$	denotes the people who begin the period alive in region i and survive to the end of the period in region i.
$K^{\varepsilon(i)\sigma(R)}$	denotes the people who begin the period alive in region i, who migrate out of region i to region R, and survive the period alive in the rest of the world, region R.
$K^{\varepsilon(i)\delta(i)}$	refers to the people who begin the period in existence in region i and who die there.
$K^{\beta(i)\sigma(i)}$	denotes the infants*** born in region i during the period who survive there at the end of the period.
$K^{\beta(i)\sigma(R)}$	are infants born in region i who migrate out to the rest of the world and survive there.
$K^{\beta(i)\delta(i)}$	are infants born in region i who die there within the same period.
$K^{\beta(i)\delta(R)}$	denote infants born in the region of interest, who migrate from region i to region R, and die there.

* The equivalence of this format for the closed demographic accounts and that used by Stone was originally outlined in Rees (1974) and Rees and Wilson (1975b).
** Existence and survival are really two aspects, prospective and retrospective, of the same thing, being alive. It is useful however, for present purposes to distinguish between them.
*** "Infants" will be the term usually used to refer to persons born in the period. "Persons" will normally be used to refer to people already alive at the beginning of the period.

Table 1. Closed demographic accounts: basic concepts

Initial State \ Final State		Survival $\sigma(\ )$ at time t+T		Death $\delta(\ )$ in period t to t+T		Totals
		The region of interest i	The rest of this world R	The region of interest i	The rest of the world R	
Existence $\varepsilon(\ )$ at	The region of interest i	$K^{\varepsilon(i)\sigma(i)}$	$K^{\varepsilon(i)\sigma(R)}$	$K^{\varepsilon(i)\delta(i)}$	$K^{\varepsilon(i)\delta(R)}$	$K^{\varepsilon(i)*(*)}$
Time t	The rest of the world R	$K^{\varepsilon(R)\sigma(i)}$	$K^{\varepsilon(R)\sigma(R)}$	$K^{\varepsilon(R)\delta(i)}$	$K^{\varepsilon(R)\delta(R)}$	$K^{\varepsilon(R)*(*)}$
Birth $\beta(\ )$ in Period	The region of interest i	$K^{\beta(i)\sigma(i)}$	$K^{\beta(i)\sigma(R)}$	$K^{\beta(i)\delta(i)}$	$K^{\beta(i)\delta(R)}$	$K^{\beta(i)*(*)}$
t to t+T	The rest of the world R	$K^{\beta(R)\sigma(i)}$	$K^{\beta(R)\sigma(R)}$	$K^{\beta(R)\delta(i)}$	$K^{\beta(R)\delta(R)}$	$K^{\beta(R)*(*)}$
TOTALS		$K^{*(*)\sigma(i)}$	$K^{*(*)\sigma(R)}$	$K^{*(*)\delta(i)}$	$K^{*(*)\delta(R)}$	$K^{*(*)*(*)}$

The terms $K^{\varepsilon(R)\sigma(R)}$, $K^{\varepsilon(R)\sigma(i)}$, $K^{\varepsilon(R)\delta(R)}$, $K^{\beta(R)\sigma(R)}$, $K^{\beta(R)\sigma(i)}$, $K^{\beta(R)\delta(R)}$ and $K^{\beta(R)\delta(i)}$ are the equivalent terms for region R with R substituted for i and i for R in the above definitions.

The marginal totals of Table 1 refer to the opening and closing stocks of population in the two regions and to the totals of birth and death occurrmces in the regions. The marginal terms are specified as follows

$K^{\varepsilon(i)*(*)}$ refers to the population of region i at the start of the period. The asterisk denotes summation over the superscript which the asterisk replaces. Thus,

$$K^{\varepsilon(i)*(*)} = \sum_{\omega}\sum_{j} K^{\varepsilon(i)\omega(j)}$$

where is a general superscript for final life state in a period and j is a general regional label.

$K^{\beta(i)*(*)}$ denotes the number of (live) births in region i during the period.

$K^{*(*)\delta(i)}$ refers to the number of deaths recorded during the period with usual residence in region i.

$K^{*(*)\sigma(i)}$ denotes the population of region i at the end of the period.

The terms $K^{\varepsilon(R)*(*)}$, $K^{\beta(R)*(*)}$, $K^{*(*)\delta(R)}$ and $K^{*(*)\sigma(R)}$ are the equivalent totals for the rest of the world.

Although we have not included them in Table 1 time labels are implicitly attached to each variable. We define t to be a point in time that starts a period, T to be the length of time occupied by a period, and t+T to be the point in time that ends the period. Periods can be defined by pairs of labels thus: t, t+T refers to the period that begins with time t and ends with time t+T. It is sometimes convenient instead to use Θ as a period label and to use Θ_t to refer to the period Θ that begins with time t. All K variables in the interior of Table 1 (the accounts matrix) and the birth and death totals can be labelled (t, t+T), and the initial and final population totals labelled (t) and (t+T) respectively. In most cases, however, the time context is clear and the labels are omitted.

Often it is unnecessary or impractible to estimate those terms that refer to population transitions that have region R labels attached to both initial and final life states. In that case the closed demographic accounts framework changes from that presented in Table 1 to that shown in Table 2. Table 2 differs from Table 1 in the omission of the $K^{\varepsilon(R)\sigma(R)}$, $K^{\varepsilon(R)\delta(R)}$, $K^{\beta(R)\sigma(R)}$ and $K^{\beta(R)\delta(R)}$ terms, and in the different nature of the totals in the rest of the world rows and columns. These totals have the following meaning.

$K^{\varepsilon(R)*(i)}$ refers to the total of persons starting the period in existence in region R who migrate to region i during the period and either die or survive there.

$K^{\beta(R)*(i)}$ refers to the total of infants born during the period in the rest of the world who migrate to region i and either die or survive there.

$K^{*(i)\sigma(R)}$ refers to the total of people either in existence in region i at time t or born in region i in period t to t+T who migrate to region R and survive there.

$K^{*(i)\delta(R)}$ denotes the total of people either alive in region i at the start of the period or born in region i during the period who migrate to region R and die there.

We will refer to $K^{\varepsilon(R)*(i)}$ as the total of in-migrants to region i, to $K^{\beta(R)*(i)}$ as the total of infant in-migrants to region i, to $K^{*(i)\sigma(R)}$ as the total of surviving out-migrants from region i, and to $K^{*(i)\delta(R)}$ as the total of non-surviving out-migrants from region i.

In order to make the concepts outlined above a little clearer we present in Table 3 an example of a closed demographic accounts table for England and Wales (labelled EW) as the region of interest and for the rest of the world as the world less England and Wales (labelled RTW). The table is an aggregated version of the multi-regional, age and sex disaggregated accounts for the West Riding of Yorkshire, the rest of England and Wales, the rest of the world presented in Smith and Rees (1974). The figures are stated more exactly than likely errors of estimation may warrant. This is done to preserve correct accounting relationships.

Table 2. Closed demographic accounts: rest of the world terms omitted

Initial Stage \ Final State		Survival $\sigma(\ )$ at time t+T: The region of interest i	Survival $\sigma(\ )$ at time t+T: The rest of this world R	Death $\delta(\ )$ in period t to t+T: The region of interest i	Death $\delta(\ )$ in period t to t+T: The rest of this world R	Totals
Existence $\varepsilon(\ )$ at Time t	The region of interest i	$K^{\varepsilon(i)\sigma(i)}$	$K^{\varepsilon(i)\sigma(R)}$	$K^{\varepsilon(i)\delta(i)}$	$K^{\varepsilon(i)\delta(R)}$	$K^{\varepsilon(i)*(*)}$
	The rest of the world R	$K^{\varepsilon(R)\sigma(i)}$	-	$K^{\varepsilon(R)\delta(i)}$	-	$K^{\varepsilon(R)*(i)}$
Birth $\beta(\ )$ in period t to t+T	The region of interest i	$K^{\beta(i)\sigma(i)}$	$K^{\beta(i)\sigma(R)}$	$K^{\beta(i)\delta(i)}$	$K^{\beta(i)\delta(R)}$	$K^{\beta(i)*(*)}$
	The rest of the world R	$K^{\beta(R)\sigma(i)}$	-	$K^{\beta(R)\delta(i)}$	-	$K^{\beta(R)*(i)}$
Totals		$K^{*(*)\sigma(i)}$	$K^{*(i)\sigma(R)}$	$K^{*(*)\delta(i)}$	$K^{*(i)\delta(R)}$	$K^{*(*)*(*)}$

The first row of the accounts shows what happened to the 46,104,543 people living in England and Wales at census date 1961. Some 42,597,211 of them survived the period and stayed in England and Wales, 829,346 of them also survived the period but migrated in the meantime to the rest of the U.K. or abroad. Many more, 2,670,671, died during the five years while still resident in England and Wales. A few, 7,315, died after migrating to the rest of the world. The first column of the table reveals where the 1966 census date population of England and Wales came from. The largest number were the over 42½ million who survived and stayed. Just over a million people were contributed by the rest of the world. This period, 1961-66, was one of heavy in migration from the counties of the New Commonwealth. Some 4,115,430 were infants born in the period in England and Wales, some of whom were the second generation children of immigrants. An additional 47,533 members of the Census date 1966 population were children who migrated into England and Wales, having been born in Scotland, Northern Ireland or abroad. The other terms in the table consist of people who showed up in England and Wales for some of the period but who appeared neither in the opening stock population nor in the final stock population.

There are a set of identities referring to the rows and columns of the accounts table. In the notation of Table 2, these can be stated as follows.

Initial population

$$K^{\varepsilon(i)*(*)} = K^{\varepsilon(i)\sigma(i)} + K^{\varepsilon(i)\sigma(R)} + K^{\varepsilon(i)\delta(i)} + K^{\varepsilon(i)\delta(R)} \qquad (1)$$

Total in-migrants

$$K^{\varepsilon(R)*(i)} = K^{\varepsilon(R)\sigma(i)} + K^{\varepsilon(R)\delta(i)} \qquad (2)$$

Total births

$$K^{\beta(i)*(*)} = K^{\beta(i)\sigma(i)} + K^{\beta(i)\sigma(R)} + K^{\beta(i)\delta(i)} + K^{\beta(i)\delta(R)} \qquad (3)$$

Total infant in-migrants

$$K^{\beta(R)*(i)} = K^{\beta(R)\sigma(i)} + K^{\beta(R)\delta(i)} \qquad (4)$$

Final population

$$K^{*(*)\sigma(i)} = K^{\varepsilon(i)\sigma(i)} + K^{\varepsilon(R)\sigma(i)} + K^{\beta(i)\sigma(i)} + K^{\beta(R)\sigma(i)} \qquad (5)$$

Surviving out-migrants

$$K^{*(i)\sigma(R)} = K^{\varepsilon(i)\sigma(R)} + K^{\beta(i)\sigma(R)} \tag{6}$$

Total deaths

$$K^{*(*)\delta(i)} = K^{\sigma(i)\delta(i)} + K^{\sigma(R)\delta(i)} + K^{\beta(i)\delta(i)} + K^{\beta(R)\delta(i)} \tag{7}$$

Non-surviving out-migrants

$$K^{*(i)\delta(R)} = K^{\varepsilon(i)\delta(R)} + K^{\beta(i)\delta(R)} \tag{8}$$

A check will show that these identities hold for the Table 3 accounts. Each of these identities can be made explicit for many regions and for individual age-sex groups (see Rees and Wilson, 1975b). These identities do not hold for any two region system but only for regional systems that are closed, which in practice means that they must include a proper rest of the world region.

To conclude this section of the paper we should say something about how available demographic information can be used in constructing closed demographic accounts. Figure 2 shows the variety of situations that can occur. If there is a national population register (section II of Figure 2) most cells of the table can be filled in directly. Without a population register a great deal of the table can be filled in from census population and migration tables if these are well specified together with information from birth and death tables produced by the vital registration authorities. This is the section III situation in Figure 2. Rees and Wilson have defined a model that makes less demands for data (section IV of Figure 2) and the empty cells are estimated using the row and column identities and a number of hypotheses about demographic rates. The minimum amount of information that could yield a reasonable estimate of the accounts is shown in section V of Figure 2. Here, infant migration must be estimated on the basis of parental migration. Although the accounts based model does not use all the data available it has the advantage of fairly easy conversion into a projection model (see Rees and Wilson, 1975b for details). If less information is available than that shown in section V of Figure 2 construction of closed demographic accounts is probably infeasible.

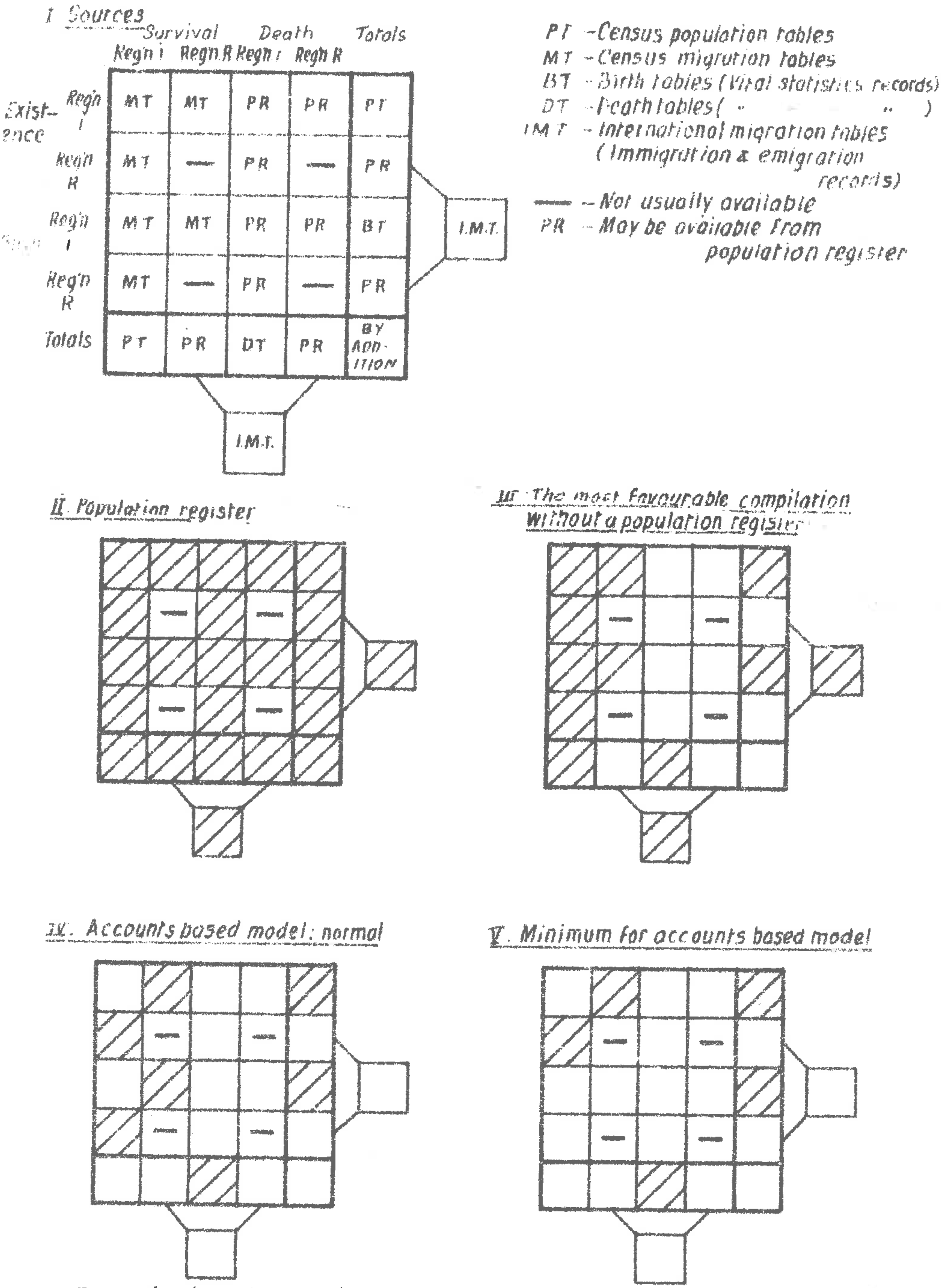

Fig.2. Available information for closed demographic accounts

4. Transition rates, admission rates and demographic projection models

Before describing input-output accounts we digress slightly from the main theme of the paper to show how rates of transition from one state to another over a period may be calculated and how these may be used in demographic projection models.

If we divide each element in the Table 3 accounts by the relevant total for its row, we obtain the transition rates for the period 1961-66 for the England and Wales population. These are shown in Table 4 in the top triangle of each cell of the table. Transition rates are the rates at which persons transfer from initial states to final states in the period. Since they sum row-wise to unity they are true probabilities.

If we divide each element in the Table 3 accounts by the appropriate column total, we calculate admission rates or the rates at which final state populations, death totals or out-migrant totals admit persons from the initial states. The admission rates are shown in the bottom triangle of each cell and sum to unity columnwise.

To forecast the population we use some of the transition rates generated in Table 4 together with other rates that enable us to generate the births total and in-migrant totals to which many of the transition rates refer. We can define birth rates by one of several methods.

(1) We could divide the births total by the initial population.

(2) We could divide the births total by the conventional population at risk, the average population of the period.*

(3) We could divide the births total by the multi-regional population at risk, defined as a weighted sum of the elements in the accounts.

The first method results in a one-step model; the second in a two-step model; the third in a multi-step model involving iteration.

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* In many instances the mid-period population is used rather than the average, and is slightly more "accurate". In practice, however, there is little numerical difference.

Table 4. Transition and admission rates based on the closed demographic accounts table for England and Wales, 1961-66.

Initial State	Final State	Survival at Census Date 1966: Eng. & Wales EW	Survival at Census Date 1966: Rest of the world RTW	Death in 1961-66: Eng. & Wales EW	Death in 1961-66: Rest of the world RTW	Totals
Exist. at Census Date 1961	England & Wales EW	.923927 / .890569	.017988 / .935085	.057926 / .966157	.000159 / .925832	1.000000
	Rest of the world RTW	.992499 / .022397	-	.007501 / .002929	-	1.000000
Birth in 1961-66	England and Wales EW	.966390 / .086040	.013520 / .[illegible]64915	.019953 / .030739	.000138 / .074168	1.000000
	Rest of the world RTW	.989941 / .000994	-	.010059 / .000175	-	1.000000
Totals		1.000000	1.000000	1.000000	1.000000	

Notes

1. Transition rates are above the diagonal in each cell. Admission rates are below the diagonal in each cell.

In-migrants can be treated in rather the same way by dividing the totals by the initial, final, average or multi-regional population at risk but most often it is better to treat migration from outside the system of interest as an exogeneous input. Often such in-migration streams are controlled by numerical quotas defined in legislation rather than by the propensity of persons to leave the rest of the world for a destination region in the system of interest.

The simplest kind of transition rate model can be specified as follows using method (1) to define birth rates and treating in-migrants exogeneously. We continue with the notation previously defined for the accounts and use h as the transition rate variable. Thus, $h^{\varepsilon(i)\sigma(i)}(t,t+T)$ is the rate of surviving and staying in region i in the period t,t+T, $h^{\beta(i)\sigma(i)}$ is the corresponding rate for infants born in the period. Let $b^i(t,t+T)$ be the birth rate for region i in the period t to t+T and $\hat{K}^{Bi}(t,t+T)$ the population at risk of giving birth. Then, survivors in the final population will be

$$K^{\varepsilon(*)\sigma(i)}(t+T) = h^{\varepsilon(i)\sigma(i)}(t,t+T)\, K^{\varepsilon(i)*(*)}(t)$$

$$+\, h^{\varepsilon(R)\sigma(i)}(t,t+T)\, K^{\varepsilon(R)*(i)}(t.t+T) \qquad (9)$$

The term $K^{\varepsilon(i)*(*)}(t)$ is the base population and the term $K^{\varepsilon(R)*(i)}(t,t+T)$ the exogeneously derived in-migrant total.

The population at risk of giving birth is set to the initial population.

$$\hat{K}^{Bi}(t,t+T) = K^{\varepsilon(i)*(*)}(t) \qquad (10)$$

The birth totals are then

$$K^{\beta(i)*(*)}(t,t+T) = b^i(t,t+T)\, \hat{K}^{Bi}(t,t+T) \qquad (11)$$

and the surviving infants are

$$K^{\beta(*)\sigma(i)}(t,t+T) = h^{\beta(i)\sigma(i)}(t,t+T)\, K^{\beta(i)*(*)}(t,t+T)$$

$$+\, h^{\beta(R)\sigma(i)}(t,t+T)\, K^{\beta(R)*(i)}(t,t+T) \qquad (12)$$

The term $K^{\beta(R)*(i)}$ is determined exogeneously.

We can then add survivors and infants

$$K^{*(*)\sigma(i)}(t+T) = K^{\varepsilon(*)\sigma(i)}(t,t+T) + K^{\beta(*)\sigma(i)}(t,t+T) \qquad (13)$$

In our England and Wales example these equations read

survivors

$$K^{\dot{\varepsilon}(*)\sigma(EW)}(c.d.1966)$$

$$= (0.923927)\ 46{,}104{,}543 + (0.992499)\ 1{,}079{,}354 \qquad (14)$$

$$= 43{,}668{,}490$$

population at risk

$$\hat{K}^{B^{EW}} = 46{,}104{,}543 \qquad (15)$$

births

$$K^{\beta(EW)*(*)}(1961\text{-}66)$$

$$= (0.092367)\ 46{,}104{,}543 \qquad (16)$$

$$= 4{,}258{,}560$$

and surviving infants

$$K^{\beta(*)\sigma(EW)}$$

$$= (0.966390)\ 4{,}258{,}560 + (0.989941)\ 48{,}016 \qquad (17)$$

$$= 4{,}162{,}963$$

The transition rate model becomes a projection model when the rates involved are extrapolated into the future, and the equations are run through for future time periods. It should, however, be stressed that the equation (9) to (13) model is only one of many ways in which the closed demographic accounts framework may be used to underpin population forecasting. Other models may be appropriate in other contexts.

5. Input-output accounts

Closed demographic accounts are fairly demanding in terms of population statistics. Simpler sets of accounts have been used for some time that demand less in the way of demographic data but which yield less information about the ways in which the population is changing. These accounts we call input-output accounts, and are built on the fundamental relationship of most dynamic systems that

Initial stocks plus inflows = outflows plus final stocks (18).

This relationship is displayed diagramatically for a population system consisting of one region and one period (with the rest of the world and other periods recognised as the external environment) in Figure 3.

Inputs to the region-period are the initial population, births, and in-migrants. Outputs from the region-period are deaths, out-migrants and the final population. Note that no links are specified between any of these quantities. We organize this information in the form of input-output accounts in Table 5. The columns of the table are the inputs to the "region-period", outputs from the "region-period" and the differences between inputs and outputs, the balances. The rows give the source of the population inputs, outputs and balances. These input-output accounts are purely a single region device and cannot be generalized to many regions unlike the closed demographic accounts.

In Table 6 are listed the inputs to and outputs from England and Wales during the intercensal period 1961-66. Note that, following equation (18) inputs equal outputs at 51,490,473 (all the people who lived at least some time in England and Wales in the intercensal period 1961-66), but that inflows do not equal outflows. There is a net inflow of 1,726,889 persons over the period consisting of 1,494,340 births surplus to deaths and 232,549 in-migrants surplus to out-migrants. A negative change in stocks in the table means an increase in the population.

Tables 5 and 6 display or contain a number of frequently used relationships that follow from equation (19). The first relationship might be termed the "conservation of population" equation:

$$\begin{array}{l} \text{Population of a} \\ \text{region at the end} \\ \text{of the period} \end{array} = \begin{array}{l} \text{Population of a} \\ \text{region at the start} \\ \text{of the period} \end{array} + \text{Births} + \text{In-migrants} - \text{Deaths} - \text{Out-migrants} \qquad (19)$$

In our England and Wales example this reads

$$K^{*(*)\sigma(EW)}(c.d.1966) = 46{,}104{,}543 + 4{,}258{,}560 + 1{,}127{,}370 - 2{,}764{,}220 - 894{,}821 = 47{,}831{,}432 \qquad (20)$$

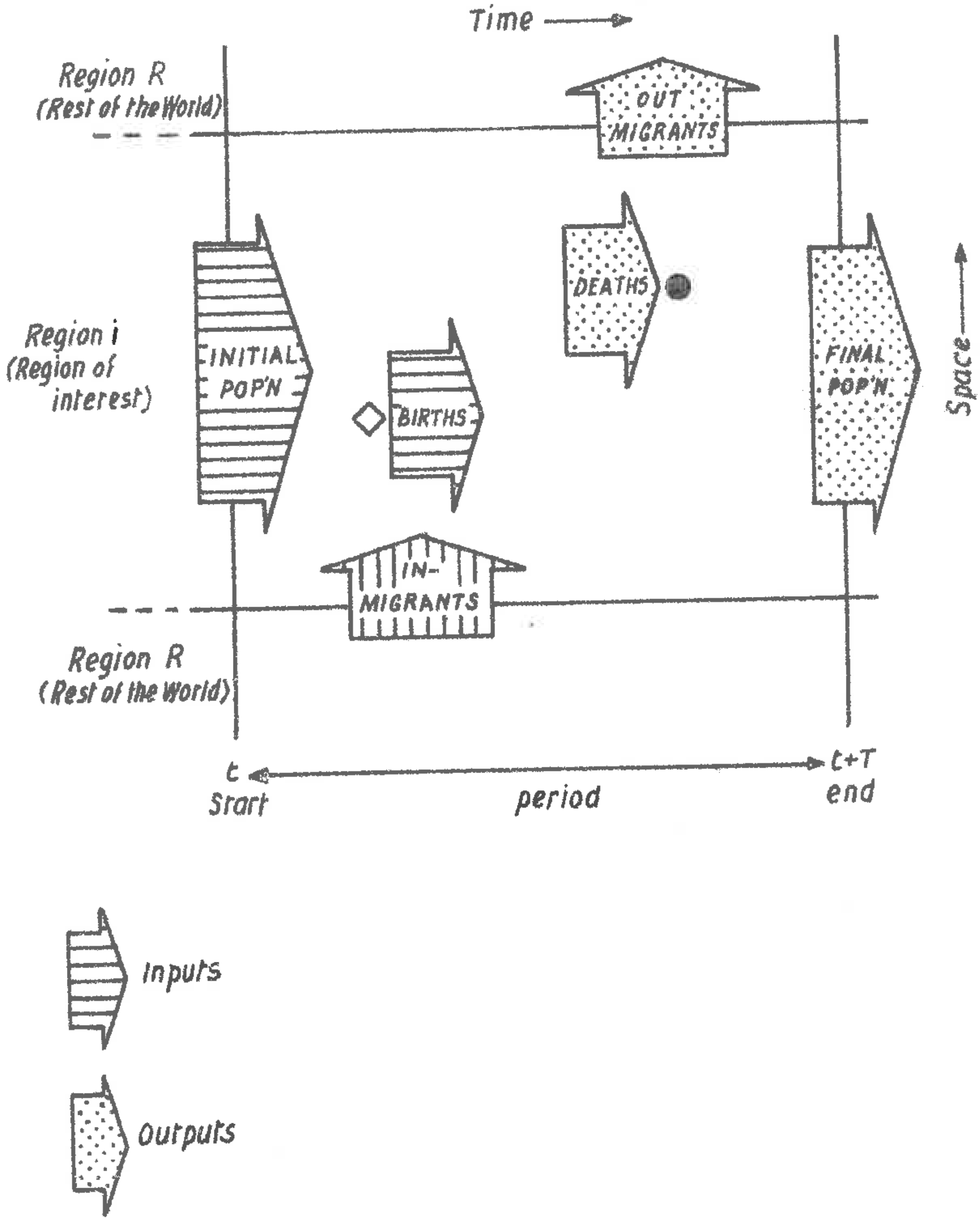

Fig 3. A time-space diagram illustrating input-output accounts

Table 5 Input-output-accounts: verbal definitions.

Source of Change (1)		Inputs (2)	Outputs (3)	Balances (4)
Population Stocks		Initial Population	Final Population	Population Change.
Population Flows	Natural Change	Births	Deaths	Natural Increase
	Migration Change	In Migrants	Out Migrants	Net Migration
	Total Change	In Flow	Out Flow	Net Inflow
Totals		Total Inputs	Total Outputs	Zero

Notes

1. Column (4) items are defined as column (2) items less column (3) items.

Table 6 Input-output-accounts for England and Wales, 1961-66 intercensal period.

Source of Change		Inputs +	Outputs −	Balances ±
Population Stocks		46,104,543	47,831,432	-1,726,889
Population Flows	Natural Change	4,258,560	2,764,220	+1,494,340
	Migration Change	1,127,370	894,821	+ 232,549
	Total Change	5,385,930	3,659,041	+1,726,889
Totals		51,490,473	51,490,473	0

Notes

1. The figures derive from Table 3.

The last column of Tables 5 and 6 show how population change is made up

$$\text{Population change in a period} = \text{Natural increase} + \text{Net migration} \qquad (21)$$

where

$$\text{Population change in a period} = \text{Population at the end of a period} - \text{Population at the beginning of a period} \qquad (22)$$

$$\text{Natural increase} = \text{births} - \text{deaths} \qquad (23)$$

$$\text{Net migration} = \text{in-migrants} - \text{out-migrants} \qquad (24)$$

Where migration information is not available, equation (21) can be rearranged to give an equation that estimates net migration.

$$\text{Net migration} = \text{Population change} - \text{natural increase} \qquad (25)$$

All these relations apply to a single region, and are implicit in Table 5.

The question we might now pose is how are input-output accounts connected with the closed demographic accounts previously described. We can answer that question by using the mathematical notation defined previously to substitute appropriate variables for the words in Table 5. This is done in Table 7. The population, births and deaths correspond exactly with variables used earlier. The in-migrants turn out to be a sum of terms used earlier

$$K^{*(R)*(i)} = K^{\varepsilon(R)*(i)} + K^{\beta(R)*(i)} \qquad (26)$$

and similarly with the out-migrants

$$K^{*(i)*(R)} = K^{*(i)\sigma(R)} + K^{*(i)\delta(R)} \qquad (27)$$

Table 7 <u>Input-output accounts: mathematical definition</u>

Source of Change (1)		Inputs (2)	Outputs (3)	Balances (4)
Population Stocks		$K^{\varepsilon(i)*(*)}$	$K^{*(*)\sigma(i)}$	$K^{\varepsilon(i)*(*)}$ $-K^{x(*)\sigma(i)}$
Population Flows.	Natural Change	$K^{\beta(i)*(*)}$	$K^{*(*)\delta(i)}$	$K^{\beta(i)*(*)}$ $-K^{*(*)\delta(i)}$
	Migrant Change	$K^{*(R)*(i)}$	$K^{*(i)*(R)}$	$K^{*(R)*(i)}$ $-K^{*(i)*(R)}$
	Total Change	$K^{\beta(i)*(*)}$ $+K^{*(R)*(i)}$	$K^{*(*)\delta(i)}$ $+K^{*(i)*(R)}$	$K^{\beta(i)*(x)}$ $+K^{*(R)*(i)}$ $-K^{*(*)\delta(i)}$ $-K^{*(i)*(R)}$
Totals		$K^{*(*)*(*)}$	$K^{*(*)*(*)}$	$K^{*(*)*(*)}$ $-K^{*(*)*(*)}$ $= 0$

<u>Notes</u>

1. $K^{*(*)*(*)}$ means in this case, as in Table 2, the sum of all items in the closed demographic accounts less the rest of the world-rest of the world terms, that is

$$K^{*(*)*(*)} = \sum_{\alpha}\sum_{j}\sum_{\omega}\sum_{k} K^{\alpha(j)\omega(k)}$$

$$- \sum_{\alpha}\ \sum_{j\epsilon R}\ \sum_{\omega} K^{\alpha(j)\omega(j)}$$

where α is a general label for initial life state, ω is a general label for final life states, and j and k are regional labels.

Input -output accounts are thus a slightly aggregated rearrangement of the marginal totals of closed demographic accounts for a single region. The two sets of accounts are therefore very closely related, as is natural in a family!

This familial relationship has implications for the kind of migration statistics that should be used in input-output accounts. They should refer to persons rather than moves. Since one person can make many moves use of move statistics will result in overcounting. Particular care should be taken in using international migration statistics which may include return migrants and multiple migrations. These issues and other related points have been discussed by Courgeau (1973) and by Rees (1974).

6. Open demographic accounts and the integrated accounting system

We postpone for the moment discussion of life table accounts, the last type listed in the second row of Figure 1, and describe briefly open demographic accounts. These are accounts defined and used by Stone (1965, 1971a) in an educational context, and connected with the Rees and Wilson accounts in Rees (1974). They show the connections between successive time periods using the statistics shown in Figure 3 and Tables 5 and 7. Each region-period in the system of interest receives transfers from the previous period and experiences inflows and outflows within the period (births, in-migrants; deaths, out-migrants). The final population of the region in the previous period is transferred into the current period as its initial population. In the educational system changes of state (promotion from one grade into the next) take place at this time of transfer from one year to the next.

A set of open demographic accounts are shown in Figure 4 together with the relevant, within-period closed demographic accounts. Period to period transfers take place from columns to rows and the final/initial populations which are transferred are shown in the first subdiagonal. In the principal diagonal are shown sets of closed demographic accounts labelled according to period (θ-1,θ,θ+1). Births and in-migrants are inflows to each period and deaths and out-migrants are outflows from each period. Total population flows, $K_{\theta}^{*(*)*(*)}$, are the sums of the diagonal elements, and the off-diagonal elements (taken separately).

To \ from	Period $\theta+1$				Period θ				Period $\theta-1$				Births	In-migrants	Total popn flows
Period $\theta-1$									$K^{\varepsilon(i)\sigma(i)}_{\theta-1}$	$K^{\varepsilon(i)\sigma(R)}_{\theta-1}$	$K^{\varepsilon(i)\delta(i)}_{\theta-1}$	$K^{\varepsilon(i)\delta(R)}_{\theta-1}$			$K^{*(*)*(*)}_{\theta-1}$
									$K^{\varepsilon(R)\sigma(i)}_{\theta-1}$	—	$K^{\varepsilon(R)\delta(i)}_{\theta-1}$	—		$K^{\varepsilon(R)*(i)}_{\theta-1}$	
									$K^{\beta(i)\sigma(i)}_{\theta-1}$	$K^{\beta(i)\sigma(R)}_{\theta-1}$	$K^{\beta(i)\delta(i)}_{\theta-1}$	$K^{\beta(i)\delta(R)}_{\theta-1}$	$K^{\beta(i)*(*)}_{\theta-1}$		
									$K^{\beta(R)\sigma(i)}_{\theta-1}$	—	$K^{\beta(R)\delta(i)}_{\theta-1}$	—		$K^{\beta(R)*(i)}_{\theta-1}$	
Period θ					$K^{\varepsilon(i)\sigma(i)}_{\theta}$	$K^{\varepsilon(i)\sigma(R)}_{\theta}$	$K^{\varepsilon(i)\delta(i)}_{\theta}$	$K^{\varepsilon(i)\delta(R)}_{\theta}$	$K^{*(*)\sigma(i)}_{\theta-1}$ $K^{\varepsilon(i)*(*)}_{\theta}$						$K^{*(*)*(*)}_{\theta}$
					$K^{\varepsilon(R)\sigma(i)}_{\theta}$	—	$K^{\varepsilon(R)\delta(i)}_{\theta}$	—						$K^{\varepsilon(R)*(i)}_{\theta}$	
					$K^{\beta(i)\sigma(i)}_{\theta}$	$K^{\beta(i)\sigma(R)}_{\theta}$	$K^{\beta(i)\delta(i)}_{\theta}$	$K^{\beta(i)\delta(R)}_{\theta}$					$K^{\beta(i)*(*)}_{\theta}$		
					$K^{\beta(R)\sigma(i)}_{\theta}$	—	$K^{\beta(R)\delta(i)}_{\theta}$	—						$K^{\beta(R)*(i)}_{\theta}$	
Period $\theta+1$	$K^{\varepsilon(i)\sigma(i)}_{\theta+1}$	$K^{\varepsilon(i)\sigma(R)}_{\theta+1}$	$K^{\varepsilon(i)\delta(i)}_{\theta+1}$	$K^{\varepsilon(i)\delta(R)}_{\theta+1}$	$K^{*(*)\sigma(i)}_{\theta}$ $K^{\varepsilon(i)*(*)}_{\theta+1}$										$K^{*(*)*(*)}_{\theta+1}$
	$K^{\varepsilon(R)\sigma(i)}_{\theta+1}$	—	$K^{\varepsilon(R)\delta(i)}_{\theta+1}$	—										$K^{\varepsilon(R)*(i)}_{\theta+1}$	
	$K^{\beta(i)\sigma(i)}_{\theta+1}$	$K^{\beta(i)\sigma(R)}_{\theta+1}$	$K^{\beta(i)\delta(i)}_{\theta+1}$	$K^{\beta(i)\delta(R)}_{\theta+1}$									$K^{\beta(i)*(*)}_{\theta+1}$		
	$K^{\beta(R)\sigma(i)}_{\theta+1}$	—	$K^{\beta(R)\delta(i)}_{\theta+1}$	—										$K^{\beta(R)*(i)}_{\theta+1}$	
Deaths			$K^{*(*)\delta(i)}_{\theta+1}$				$K^{*(*)\delta(i)}_{\theta}$				$K^{*(*)\delta(i)}_{\theta-1}$				$K^{*(*)\delta(i)}_{*}$
Out-migrants		$K^{*(i)\sigma(R)}_{\theta+1}$		$K^{*(i)\delta(R)}_{\theta+1}$		$K^{*(i)\sigma(R)}_{\theta}$		$K^{*(i)\delta(R)}_{\theta}$		$K^{*(i)\sigma(R)}_{\theta-1}$		$K^{*(i)\delta(R)}_{\theta-1}$			$K^{*(i)*(R)}_{*}$
Total popn flows	$K^{*(*)*(*)}_{\theta+1}$				$K^{*(*)*(*)}_{\theta}$				$K^{*(*)*(*)}_{\theta-1}$				$K^{\beta(i)*(*)}_{*}$	$K^{*(R)*(i)}_{*}$	$K^{*(*)*(*)}_{*}$

Fig. 4. An integrated demographic accounting sysrem

Stone (1971a) has defined a set of demographic models based on the open demographic accounts framework. These have exactly the same formal structure as those based on the closed framework. They differ in the definition of the source accounts. Models based on the closed accounts are generally more appropriate (Stone 1975b).

7. Population-at-risk, person-time and time accounts

Continuing vertically downward in Figure 1 we arriveat the fourth row accounts, all in the same way involving time, and all linked to closed demograpl accounts.

Demographic rates may be divided into two kinds: those in which a population flow (initial state to final state transition) is divided by an initial state population and those in which a count of state to state transitions (the states are not necessarily initial or final) is divided by a population at risk of undergoing the state to state transition. The transitio rates calculated in section 4 of the paper are of the first kind; birth rates and death rates are of the second kind.

The death rate for a region i, d^{*i}, may be defined as

$$d^{*i} = K^{*(*)\delta(i)}/\hat{K}^{D*i} \qquad (28)$$

where the observed total deaths in the region, $K^{*(*)\ (i)}$, are divided by a population K, at risk, , of death, D, in region i, irrespective of origin state, *. The problem is to define the proper population at risk.*

Figure 5 illustrates the problem involved. Lifelines of individuals corresponding to each cell of Table 2 of the closed demographic accounts are represented in the diagram and the dark circles represent deaths. Of the definitional choices for the population at risk mentioned in Section 4 the initial population is a poor choice because in-migrants from region R also contribute to the deaths total as do persons born in the period. More satisfactory is the average or mid-period population of the period which makes allowance for births and net in-migrants. Better, however, would be to construct a weighted average of all the population flows involved, the weights being the exposure to the risk of the event involved in the rate denominator, in this case, death in region i. We count the lengths of lifelines in region i to obtain a person-time index and divide by the length of the period to get a person index that is a population at risk.

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* This death rate is used in the accounts based model that estimates multi-regional examples of closed demographic accounts.

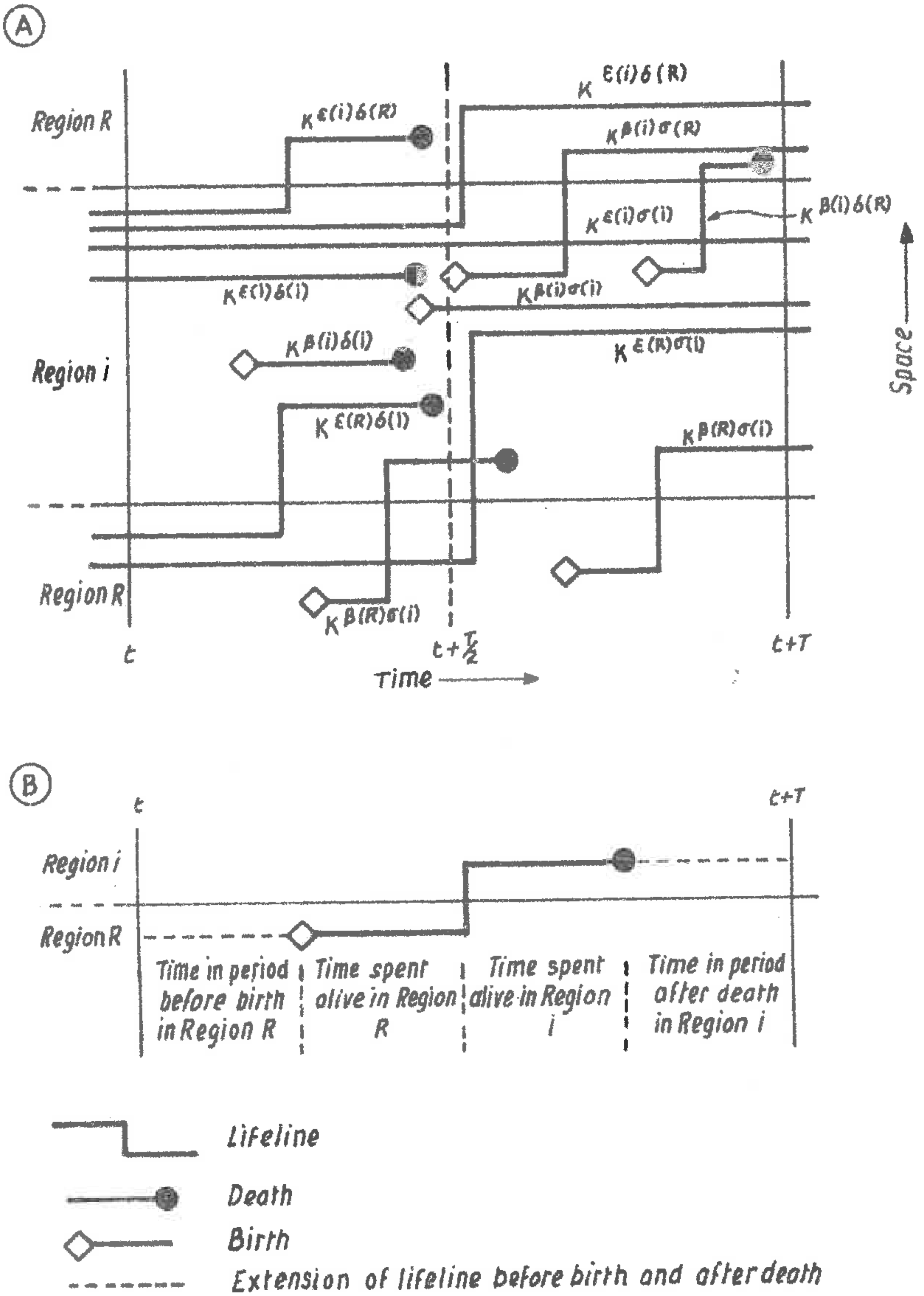

Fig. 5 A time-space diagram illustrating the concept of multi-regional population at risk.

We could also divide the person-time index by the number of persons involved to obtain an average time index. Three sets of numbers are thus generated which can be placed within the framework of the closed demographic accounts table to give the three types of accounts listed in the bottom row of Figure 1.

An example using the Table 3 closed demographic accounts for England and Wales is shown in Figure 6. We have no individual life history information and so cannot actually calculate average time spent in the state of existence in England and Wales. Instead we make assumptions about the average proportion of the period spent in this state by persons in each cell of the accounts, and these assumptions are displayed in Figure 6B. These assumptions correspond with the typical lifelines shown in Figure 5. The average time of death is assumed to occur half-way in a period; births are assumed to occur half-way between the beginning of the period and subsequent death or survival; and migrations are assumed to take place half-way on average between initial and final states in the period. The results of this logic are the proportions of Figure 6B.* These proportions were multiplied by 5 to yield assumed averages of time spent in years with state of existence in England and Wales. If empirical averages had been available the reverse procedure would have been used (division of averages times by 5 to yield average proportions).

Then the number in each cell of Figure 6B and 6C can be multiplied by the number in the corresponding cell of Figure 6A to yield population at risk components and person-time components in Figures 6D and 6E respectively. If we sum these components we obtain the total population at risk of dying in England and Wales, 42,992,094. 125 and the total person-time index value of 234,960,370.625. The death rate for England and Wales, using this multi-regional population at risk definition is

$$\begin{aligned} d^{*EW} &= 2,764,220/46,992,094.125 \\ &= 0.058823 \end{aligned} \tag{29}$$

It is interesting to note that the average population definition of the population at risk implies the following average proportions:

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* This logic may not be fully sound, but the proportions so generated are consistent and useful and in the right relative order of magnitude. We might note that in practice the proportion for $K^{\varepsilon(i)\sigma(i)}$ persons will be less than 1 because of time spent in region R by those persons, and other proportions will similarly differ. There will also be a contribution from persons in the R-R cells who were neglected in the Table 2 version of the closed demographic accounts.

A. Closed demographic accounts (Table 3)

From \ To		Survival E.W.	Survival R.T.W.	Death E.W	Death R.T.W
Existence	E.W.	42,597,211	829,346	2,670,671	7,315
	R.T.W	1,071,258	0	8,096	0
Birth	E.W.	4,115,430	57,574	84,970	586
	R.T.W.	47,533	0	483	0

B. Average proportion of period in state ε(EW) (Assumed)

1	0·5	0·5	0·25
0·5	0	0·25	0
0·5	0·25	0·25	0·125
0·25	0	0·125	0

C. Average time (Yrs) in state ε(EW) (Assumed)

5	2.5	2.5	1·25
2·5	0	1·25	0
2·5	1·25	1·25	0·625
1·25	0	0·625	0

Assumed time × T

Real time × T

D. Population at risk : Components

42,597,211	414,673	1,335,335·5	1,828·75
535,629	0	2,024	0
2,057,715	14,393·5	21,242·5	73·25
11,883·25	0	60·375	0

Grand Total 46,992,069·125

E. Person-time index : Components.

212,986,055	2,073,365	6,676,677·5	9,143·75
2,678,145	0	10,120	0
10,288,075	71,967·5	106,212·5	366·25
59,416·25	0	301·875	0

Grand Total 234,960,345·625

Fig. 6. Population at risk, person-time and time accounts.

1	0.5	0.5	0.5
0.5	0	0	0
0.5	0	0	0
0.5	0	0	0

In effect the assumptions are made (relative to the multi-regional population at risk) that

$$_K\varepsilon(R)\delta(i) = K^{\varepsilon(i)\delta(R)} \tag{30}$$

$$_K\beta(i)\sigma(R) = K^{\beta(R)\sigma(i)} \tag{31}$$

and the lower right quadrant terms are neglected. Usually this will mean that the conventional population at risk (46,867,987.5) is an underestimate of the multi-regional population at risk (46,992,074.125).

The time-related accounts in Figure 6 refer only to the state of existence in England and Wales. The lifeline at the bottom of Figure 4 shows that persons can spend time alive in the rest of the world. It is also useful to define "states of" pre-existence " before birth in a region, and states of "post-existence" in a region. The result is six sets of accounts like those of Figure 6, the sums of which yield 1's in each cell of aver proportions, 5's in each cell of average time, reproduce Figure 5A in the Figure 6D equivalent and show the numbers in Figure 6A multiplied by 5 in the equivalent of Figure 5E. The reader may wish to work out this set of accounts in full.

8. <u>Other members of the family of accounts</u>

The other members of the family of accounts movement accounts, life table accounts, and vintage or cohort accounts, which we have so far neglected will be touched upon here very briefly. Each has a basis different from that of the closed demographic accounts which had underpinned our discussion to date.

Movement accounts involve the counting of all the state-to-state transitions that take place rather than the initial state-to-final state transition Figure 7 illustrates the differences between these two counting systems. In Figure 7 are depicted the lifelines of eight persons who fall in the existence-survival quadrant of the closed demographic accounts. By examining their initial and final locations we can set up the accounts shown in Table 8. We count two <u>migrants</u> shifting their location from the region of interest to the rest of the world, and two <u>migrants</u> locating from region R to region i. There are four non-migrants, three in region i and one in the rest of the world.

However, more than 4 migrations are executed in the period. If we count the numbers of migrations rather than the number of migrants, we obtain the accounts of Table 9. The difference between Table 8 and Table 9 is due to multiple migration. Consider, for example, person B. This person first migrates from region i to region R, then returns to region i and then migrates again from region i to R, staying there. The person contributes 2 i to R migrations, 1 R to i migration and is entered in the $K^{\varepsilon(i)\sigma(R)}$ cell of the closed demographic accounts. Person B makes 3 migrations, 2 of which are surplus to placing him or her in the $K^{\varepsilon(i)\sigma(R)}$ cell. In all there are 8 such surplus migrations, 4 in one direction and 4 in the other.

Note that the number of surplus migrations from region i to region R is equal to the number of surplus migration from region R to region i, of necessity. In formal terms, if we define M^{iR} and M^{Ri} to be the number of migrations between region i and region R, and region R and region i respectivel; then

$$M^{iR} - (K^{\varepsilon(i)\sigma(R)} + K^{\varepsilon(i)\delta(R)} + K^{\beta(i)\sigma(R)} + K^{\beta(i)\delta(R)}$$

$$= M^{Ri} - (K^{\varepsilon(R)\sigma(i)} + K^{\varepsilon(R)\delta(i)} + K^{\beta(R)\sigma(i)} + K^{\beta(K)\delta(i)}) \tag{32}$$

This relationship does not hold between any pair of regions, only between a system of interest and the rest of the world.

The other state to state movements involve birth in a location and death in a location. In these cases the problem of multiple events does not occur since a person can be born only once and can die only once. Births totals and deaths totals can be added to the migrations to tables to give movement accounts (Illingworth, 1975). No direct information was available in our 1961-66 England and Wales example on the number of migrations and surplus migrations in particular. But, if we assume that some 200,000 surplus migrations occurred over the five years, we can construct the set of movement accounts shown in Table 10. These have close connections with the input out-put accounts discussed earlier.

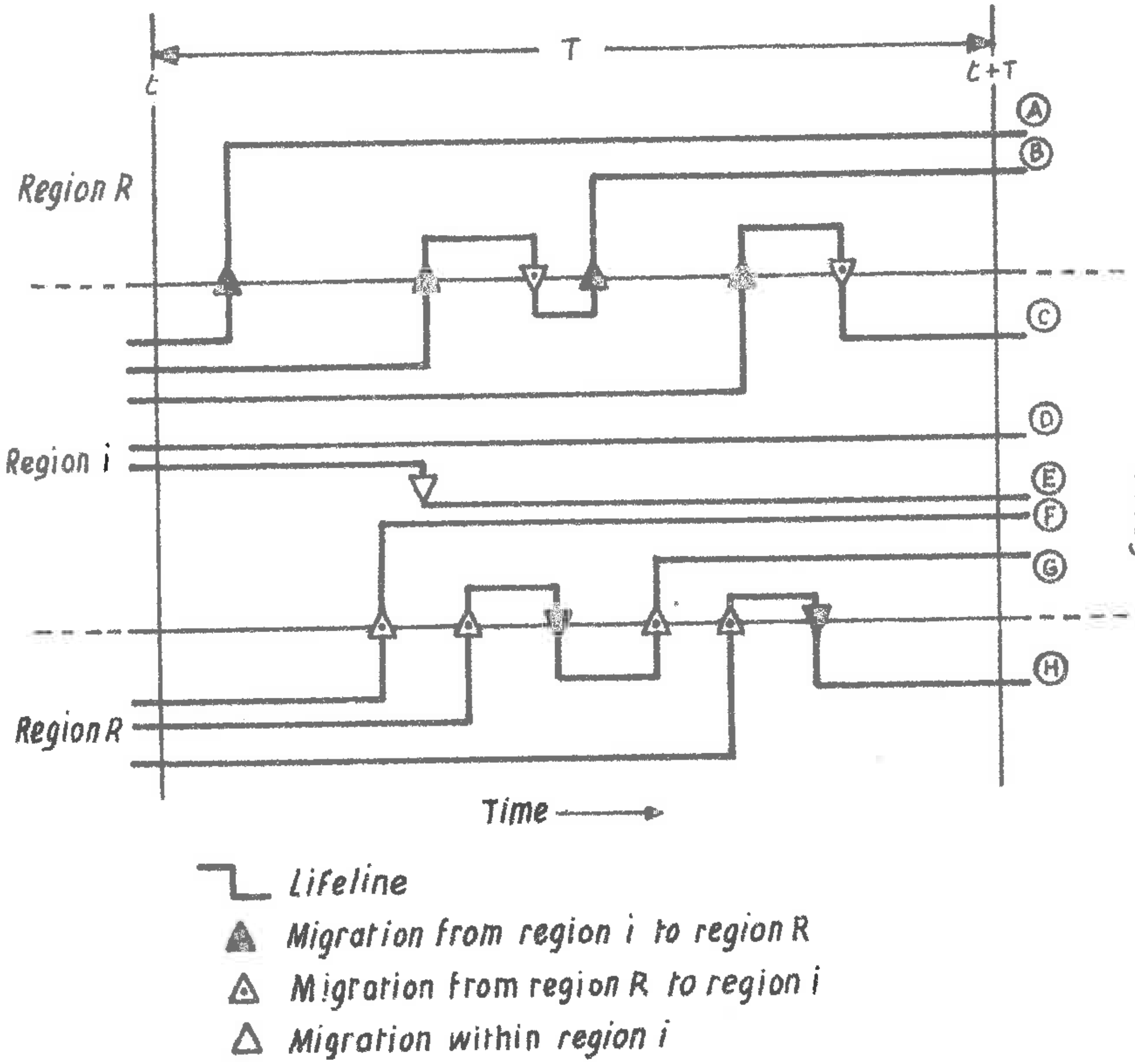

Fig. 7. The concept of multiple migration

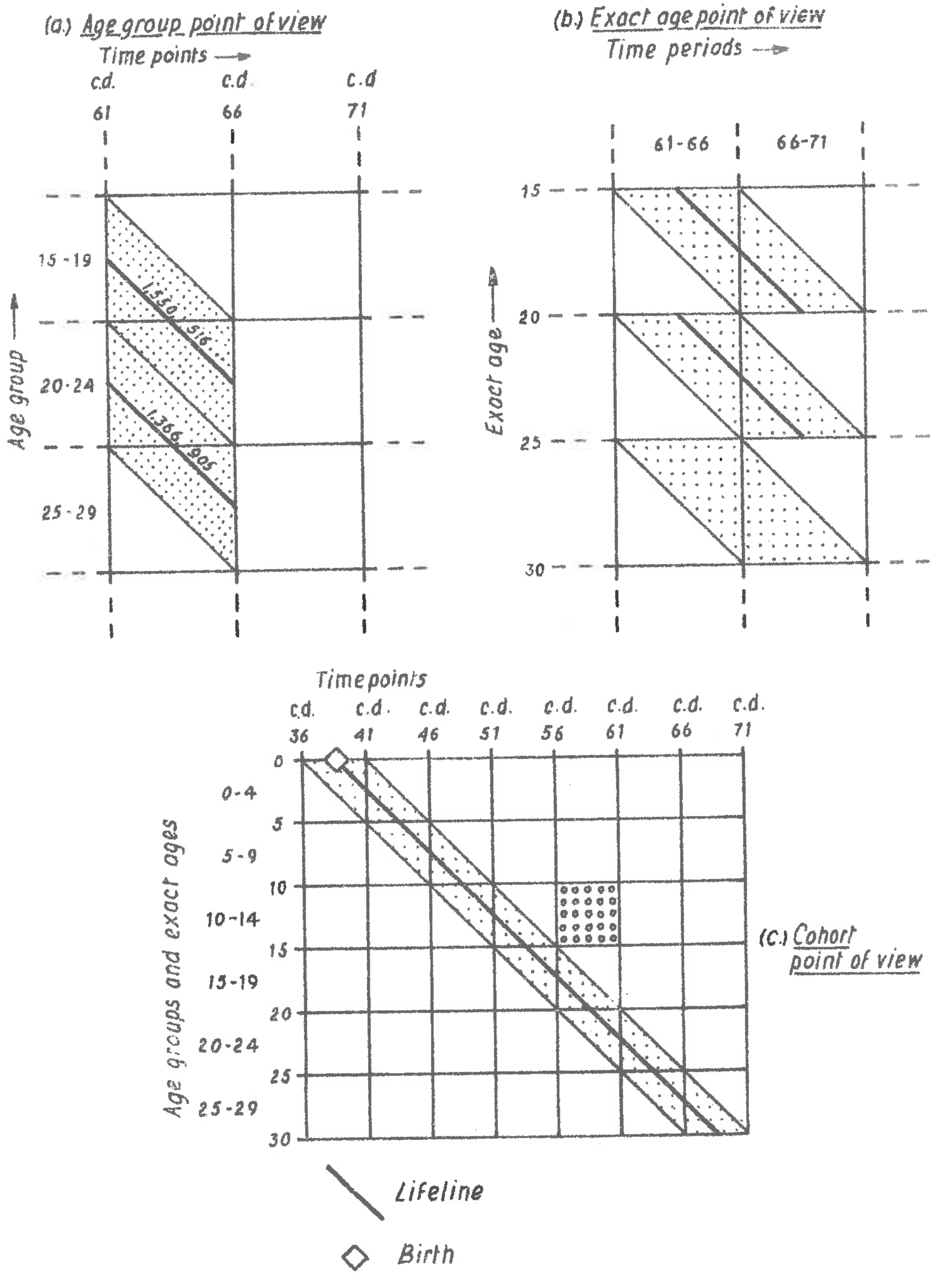

Fig.8. Lexis diagram showing age group, exact age and cohort points of view

The final pair of accounts, life table and cohort accounts can be considered together and in conjunction with the problem of disaggregating closed demographic accounts by age. Closed demographic accounts like those of Table 3 can be produced for each age group. Table 11 sets out part of such a table for the age group 20-24 years old (at last birthday). The rows are classified by age group at census date 1961, and the columns by age group at census date 1966 or by age group at time of death in the inter-censal period 1961-66. People in five year age groups survive into the next age group: the 20-24 year olds survive into the 25-29 age group. However, they can die either in the next age group to the one they start in or in that age group itself. The 20-24 year olds can die in age group 20-24 or in age group 25-29. From this kind of table the chances of survival can be calculated for persons in an age group. Some 1,426,877 women who were located in England and Wales at census date 1961 aged 20-24 survive the 1961-66 inter-censal period, some 1,366,905 of them in the same country and 59,972 elsewhere. The survival anywhere rate for these women was o.994766 and the survival-in-England-and-Wales rate was o.952955.

Table 8. A closed demographic accounts table for a sample population

Survival / Existence		Final location: Region i of interest	Final location: Region R, the rest of the world	Totals
Initial location	Region i of interest	3	2	5
	Region R, the rest of the world	2	1	3
Totals		5	3	8

Table 9 A Migration table for a sample population

From \ To		Region i of interest	Region R the rest of the world	Total
Location before migration	Region i of interest	-	6	6
	Region R, the rest of the world	6	-	6
Totals		6	6	12

(Location after migration: Region i of interest; Region R the rest of the world)

Table 10 A hypothetical set of movement accounts for England and Wales, 1961-66

Location after movement

From \ To		England and Wales EW	Rest of the world RTW	Deaths	Outflow totals
Location before movement	England and Wales EW		1,094,821	2,764,220	3,859,041
	Rest of the world RTW	1,327,370	-	-	-
	Births	4,258,560	-	-	-
Inflow totals		5,585,930	-	-	-

Table 11. A closed demographic accounts table for England and Wales, 20-24 year old women, intercensal period 1961-66

Initial State \ Final State			Survival at Census Date 1966				Death in 1961-1966				Totals
			England and Wales EW		Rest of the World RTW		England and Wales EW		Rest of the world RTW		
			20-24	25-29	20-24	25-29	20-24	25-29	20-24	25-29	
Existence at Census Date 1961	EW	15-19	1,550,516		62,637		4,303		85		(1,621,960)
		20-24		1,366,905		59,972	3,714	3.636	79	78	1,434,385
	RTW	15-19	77,620		-		105		-		77,831
		20-24		91,888		-	121	120	-	-	92,129
Totals			1,628,137	1,458,793	62,637	59,972	8,245	(7,361)	164	(152)	

Notes

1. The totals in parentheses includes figures from tables for younger or older age groups.

These survival rates refer to women moving from one age group to another over one period of time. The life table, on the other hand, is concerned with the chances of survival from one exact age to another over the interval of time required to move from one to the other. This interval of time always falls in part of two periods. These survival rates for life table construction can be calculated from sets of accounts like those in Table 11 with exact ages, 15,20,25 substituted for age groups. The differences between age group accounts and exact age accounts can best be illustrated using the time-age figure called the Lexis diagram. Figure 8 shows the relevant situation. Age is plotted on the vertical axis against time on the horizontal axis. Lifelines are represented as bold lines at 45° in each of the three diagrams because persons age at the same rate as time passes. There is a particular Lexis diagram for each cell of the accounts and in the age group case the survivors and stayers in England and Wales are illustrated.

When the age group point of view is adopted interest is focussed on events in the shaded parallelograms that start at census date 1961 and end at census date 1966. When the exact age point of view is adopted the shaded parallelograms of interest start at one exact age and finish at the next, five years hence, spanning two periods. The cohort point of view is one in which the subsequent histories of a set of persons experiencing the same event in the same period of time are followed over time. The shaded portion of Figure 8c follows a cohort born (originating at exact age of zero) in the period 1936-41 and having their 30th birthdays in the period 1966-71.

Accounts for life tables or for cohorts are difficult to estimate because the migration statistics rarely come in the correct form, although approximate forms are commonly generated on a single region, non-accounting basis. A fuller discussion of these and other issues relating to life table accounts is given in Rees and Wilson, 1975a, 1975b, and in Rogers 1973a, 1973b and 1975.

9. The case for accounting

Most work in macro-economics is based on data derived from sets of national economic accounts. No model of the national economy is today constructed without recourse to the national income accounts. It is hoped that this paper

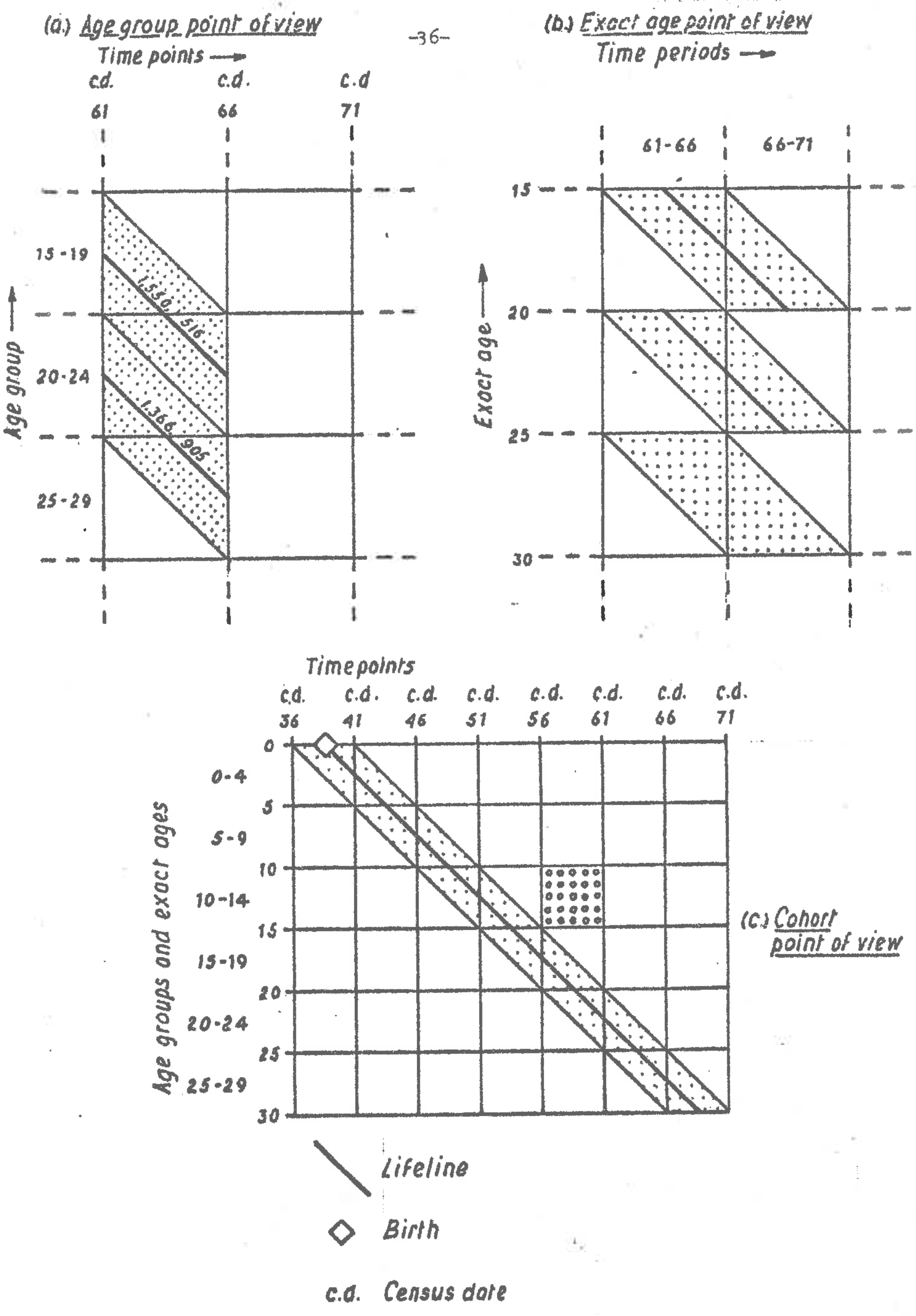

Fig.8. Lexis diagram showing age group, exact age and cohort points of view

has served to demonstrate that no model of the national or regional population should be constructed without recourse to the family of demographic accounts defined in a spatially explicit fashion. However, good models require good statistics, and proper sets of national and regional population accounts based on files of individual life histories have yet to be produced. Perhaps this paper will stimulate the preparation of a full-blooded family of accounts.

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