WORKING PAPER 279

CHANGING ENERGY PARAMETERS AND THE EVOLUTION OF URBAN SPATIAL STRUCTURE

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June, 1980.

Contents

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1. Introduction

It is generally recognised that the 'structure' of cities and city regions is in part determined by a variety of parameters. Measures of structure would include the population density pattern and the location and size structure of commercial centres. An obvious and topical set of parameters to be concerned with are those arising from energy prices. In this paper, we focus on the spatial structure of shopping centres, examining both the form and stability of equilibrium patterns and the dynamics of the evolution of these patterns.

The argument is somewhat more general in two ways. First, the pattern of shopping centres serves as an index of 'centre structure' in a broader sense. Commercial centres, for example, and a wide range of service activities have patterns which are correlated with shopping centre patterns. Secondly, and more importantly, the methods we present are applicable in principle to a wider range of 'sectors' of cities.

In section 2, we outline the model to be used and the methods of dynamical analysis which are employed in conjunction with it. In section 3, we outline a spatial system which we use as the basis for 'numerical experiments' to illustrate the theory. The results are presented in sections 4 and 5 - for equilibrium patterns and growth patterns, respectively. A number of concluding comments are made in section 6.

2. Modelling urban spatial structure: equilibria and dynamics

2.1 The underlying model of consumer and supplier behaviour

We focus on consumers use of shopping centres and of agents who 'supply' such centres. For consumers behaviour, we assume that it takes place according to standard spatial interaction models. These can be assumed to be derived from any preferred basis: entropy maximising, random utility theory, and so on. The main results we are interested in depend only on particular properties of the model which are more or less independent of the derivation and indeed are probably shared by many other models. We assume suppliers operate in such a way as to 'balance' revenue and the costs of supply. They can be interpreted as entrepreneurs in a private market making 'normal' profits under competition, or as government or other agencies balancing the books, and, as it turns out in this simple model, maximising consumers surplus. This result, and the rest of the theoretical argument to be presented in outline below, are to be found in Harris and Wilson (1978) and Wilson (1980). These references also contain a much more extensive bibliography.

Assume that the city of interest is divided into discrete zones labelled i,j, ..., that S_{ij} is the flow of cash from residents of zone i to shops in zone j, that e_i is the per capita expenditure on shopping goods by residents of zone i, P_i is the population of zone i, W_j is the 'size' of the centre in j (and is taken as a measure of 'attractiveness' or to be related to utility of using a centre of that size and c_{ij} is the cost of travel from i to zone j. Since we are taking $\{P_i\}$ as given, the set of variables $\{W_j\}$ are the main 'measures' of urban spatial structure we are using in this simple example.

A lot of information about spatial structure, which we are going to take as given exogenously for the models below, is contained in the matrix of travel costs $\{c_{ij}\}$. It is common in exploratory models to use something like straight line distance between zones, but since, here we are particularly interested in transport supply, we have also introduced a modified scheme which can be taken to reflect congestion or spatially differentiated supply of transport capacity. The way we do this is described in detail in the appendix. The parameters of the model which determine the 'shape' of the $\{c_{ij}\}$ matrix can then be varied so that their impact on urban spatial structure can be investigated. Then the standard aggregate model, not distinguishing, for the moment, types of good is

$$S_{ij} = \frac{e_i P_i W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} \qquad (2.1)$$

where α and β are parameters. α is interpreted as a measure of consumers' scale economies: the higher it is, the more beneficial it is to go to large centres rather than small ones. β obviously related to 'ease of travel'. In particular, at a time of low energy prices, β will be low relative to a time of high prices (other things being equal).

The model equation (2.1) is designed to ensure that all consumers' money is spent:

$$\sum_j S_{ij} = e_i P_i \tag{2.2}$$

It is then possible to use the model, by summing the flows from each residential zone, to calculate the total revenue, D_j, say, attracted to each centre for the given distribution of centres $\{W_k\}$. Thus, for zone j

$$D_j = \sum_i S_{ij} \tag{2.3}$$

Suppose now that suppliers expand if revenue is exceeding costs and contract otherwise. Let k be the cost per unit of supplying centre space, so that kW_j is the total cost at j. A way of recording the assumption stated above is through a differential equation which takes a form like

$$\dot{W}_j = \varepsilon(D_j - kW_j) \tag{2.4}$$

where ε is a new parameter which measures the speed of the system's response to disequilibrium. It is now clear that the equilibrium condition, when

$$\dot{W}_j = 0 \tag{2.5}$$

is given by

$$D_j = kW_j \tag{2.6}$$

and this is exactly what we had in mind above as a 'balancing' condition.

In effect, we have added a mechanism to the model to determine the location and size of centres, $\{W_j\}$. Equations (2.1), (2.3) and (2.6) can be solved simultaneously for $\{S_{ij}\}$ and $\{W_j\}$. This is not a straightforward matter because the system is strongly non-linear in the W_k's, mainly through the denominator in the right hand side of equation (2.1). It is these non-linearities which create the main features of interest in our analysis of the stability of equilibrium in subsection 2.3 below. If the system is not in equilibrium, then an equation like (2.4), or an equivalent difference equation, must be used, and this is explored in subsection 2.4. First, however, in the next subsection, we explore the kinds of patterns we would expect for different parameter values.

2.2 Parameters and the patterns of spatial structure

Here, and in the analysis which follows, we focus on the parameters α, β and k and on the parameters which characterise $\{c_{ij}\}$. Other exogenous variables, such as the sets $\{e_i\}$ and $\{P_i\}$ could also be treated in the same way. We also mention in a later section how the whole approach can be broadened so that $\{P_i\}$ in particular can be generated from another interdependent submodel.

We can begin with the following reasonably obvious observations. High α values will be associated with larger centres (and therefore possibly fewer centres, since the total stock of cash, $\sum_i e_i P_i$ is being taken as fixed at present). So other things being equal, this is likely to generate a concentrated pattern. The same thing can be said for low β, though the argument is more easily understood if stated in reverse: high β forces the consumer to shop near to their residence (and indeed, in the $\beta \to \infty$ limit, to shop at the nearest centre). So low β is likely to go with a concentrated pattern, high β with a dispersed one - a large number of small centres. The effect of k is in some ways more difficult to predict, as we shall see, but generally, high k means that centres are expensive and are likely to be concentrated. Spatial variation (over and above 'distance') in $\{c_{ij}\}$ will affect structure. High central congestion will produce a more suburban-oriented pattern for example.

What is more difficult to predict, *a priori*, is the patterns which result when the parameters are acting in different ways in consort. This is why the results of the numerical experiments to be presented below are useful.

Historically, for a long time α has probably been increasing as consumers with increasing incomes demand higher standards: β has been decreasing; and probably (with improved technology), k has been decreasing. The first two factors combine to generate more concentrated patterns, and this has been widely reflected in western cities in the decline of the small shop and the growth of different kinds of large centre. Increasing energy prices can be represented in this model by increasing β, but it is possible that this increase will take place without the (historically opposite) decline in α - which could stay put or even continue to increase. We explore the consequences of this kind of change later.

2.3 The analysis ot equilibrium point stability

The stability of equilibrium in a zone can be tested using the differential equation (2.4) and a geometrical trick. Equations (2.3) and (2.4) in effect give two ways of calculating D_j - call them $D_j^{(1)}$ and $D_j^{(2)}$ respectively, and these have to be equal at equilibrium. $D_j^{(2)}$ is obviously a straight line, and it can be shown that $D_j^{(1)}$ takes a variety of forms according to the value of α. In effect $\alpha=1$ turns out to be a critical value. These forms are presented in figure 2.1. For the $\alpha=1$ and $\alpha>1$ cases, there are two sub-cases: does the $D_j^{(2)}$ line intersect the curve or not? Both are illustrated on the figure. In all parts of the figure, stable points of intersection are shown by a circle, unstable points by a cross (and the identification can be checked by using equation (2.4) on either side of a point). The interesting feature to emerge, therefore, is that there are values of k which generate the non-intersecting case for which no non-zero value of W_j for that zone is possible. This also implies there is a critical k-value - at which the $D_j^{(2)}$ line touches the $D_j^{(1)}$ curve - and that as k slowly and smoothly increases through that value, a stable equilibrium value for W_j in that zone disappears. This information can be presented in another way by plotting stable W_j values against a changing k. This is done for the three-cases in figure 2.2. This produces curves reminiscent of the fold catastrophe for the $\alpha>1$ case.

It is convenient to single out k for the above analysis, but the result obtained can be broadened. If a zone is in a 'critical' state, then a change in any parameters or exogenous variable will take it to one 'side' or the other. If α or β change, for example, then the position of the $D_j^{(1)}$ curve changes relative to the $D_j^{(2)}$ line and this has analogous effects to k changing through a critical value: a stable equilibrium point can disappear; or, if the change is in the opposite direction, the possibility of it can re-appear (but it remains a possibility only, because the zero equilibrium solution is still stable for $\alpha>1$).

What implication does this analysis have for the evolution of urban spatial structure? Consider any zone: if the parameters are such that a non zero W_j-equilibrium is possible, then we can call this the DP or development-possible state. If not, we can call it the NDP state. We have shown how to discover, at any particular time, whether a zone is in the DP or NDP state. However, we have also shown that there is a complicated surface of critical parameter values, the detailed structure of which we do not yet understand. As the whole system evolves parameters change, exogenous variables change (and W_k, $k \neq j$ must be treated as exogenous variables for any j) and we can now provide a picture if not yet a totally detailed one, of zones 'flipping' from DP to NDP state, or vice versa. We will see examples of this in the results of our numerical experiments presented below. In particular, then, this analysis can be seen as a contribution to a more detailed understanding of the patterns of the measures of urban structure, particularly where the 'zeros' must be in particular circumstances.

This analysis implies continuous transitions from one equilibrium position to another for the whole system, even though some of the transitions involve discrete jumps of elements of the system state. The next step in the argument is to explore what happens when the system is not in equilibrium, and we do this in the next subsection.

2.4 Dynamics: differential equations and difference equations

We presented an argument for a set of simultaneous differential equations governing the growth of the structural variables W_j as equations (2.4). We note first that we can generalise the argument by adding a factor W_j^n for a parameter n without changing the equilibrium points. The equations then become

$$\dot{W}_j = \varepsilon(D_j - kW_j)W_j^n \qquad (2.7)$$

Different values of n then represent different pick-up rates for growth from low W_j to the upper bound determined by the equilibrium condition (which will be achieved if the conditions for stability are satisfied). Example for n=0 and n=1, the logistic case, are shown in figure 2.3.

An alternative formulation involves the use of a corresponding difference equation. This would arise either as an approximation for the differential equation, or because it was thought there were lags in the way the suppliers of structure responded to changes in profit or loss. Because the example turns out to be the most interesting one, let us convert equation (2.7) with n=1, to difference equation form. Write

$$W_{jt+1} - W_{jt} = \varepsilon(D_{jt} - kW_{jt})W_{jt} \qquad (2.8)$$

using an obvious notation. This can be written

$$W_{jt+1} = (1 + \varepsilon D_j)W_{jt} - kW_{jt}^2 \qquad (2.9)$$

We can now explore the possibility of bifurcation at critical parameter values in a broader sense than hitherto (where we have concentrated on the appearance or disappearance of stable equilibrium points). It turns out that the equations (2.9) take the same form as those explored by May (1976) and his results can be applied directly. He shows that the conditions for a stable equilibrium point to exist is

$$0 < \varepsilon D_j < 2 \qquad (2.10)$$

and that for

$$2 < \varepsilon D_j < 3 \qquad (2.11)$$

different kinds of oscillatory solution take over - beginning with two cycles, then four-cycles and so on through all 2^n-cycles, then mixes of three-cycles, and finally chaotic behaviour. For

$$\varepsilon D_j > 3 \qquad (2.12)$$

the system is divergent, which in the case of our model is to be interpreted as a collapse of W_j to zero.

May's results were presented for a single equation

$$W_{t+1} = aW_t - bW_t^2 \qquad (2.13)$$

in which a and b are constants. In our case, of course, we have one such equation for each zone, and since D_j grows with W_j, the 'constant' varies! However, it turns out that May's results still provide the basis of an interpretation of system evolution. When εD_j reaches 2 for example, then periodic behaviour can set in shortly afterwards. An example is shown in figure 2.4. More detailed results are presented in section 5 below.

We recall from the previous subsection, that a zone will only grow to a stable equilibrium W_j-value if the parameters satisfy certain conditions which put it into the DP-state. Here, we are arguing that if the system is growing towards this from a non-equilibrium position, then further difficulties can set in if the adjustment parameter, ε, is too large so that εD_j is not in the (2.10) category. But, both the condition on whether a zone is in the DP state, and whether εD_j is too large or not can both change as the system evolves, partly in relation to what is happening in other nearby zones. Thus the possible range of structures which can evolve is very broad. It is easier, however, to discuss the full implications of this in the context of our results, and so we take this further below.

2.5 Hierarchical structure

So far we have concentrated on an aggregate model. It is obviously important to distinguish types of good and functional relationships between centres of different orders. This can be done by extending the basic shopping model given by equation (2.1).

A simple extension would be to disaggregate by goods and services, using a superscript, g,

$$S^g_{ij} = \frac{e^g_i P_i W_j^{g\alpha g} e^{-\beta^g c_{ij}}}{\sum_k W_k^{g\alpha g} e^{-\beta^g c_{ik}}} \tag{2.14}$$

However, in this formulation, each order is functionally independent of each other and therefore fails to incorporate important interdependencies. For instance, consumers usually purchase a number of items on one shopping trip and do not make a different trip for every single item. Indeed, as the cost of travel increases, it can be suggested that multipurpose trips will be of greater importance. The basic idea we wish to incorporate is that a centre which provides a wide range of goods and services is more attractive than one which offers a more restricted selection (assuming, for the present, no distance effect). An appropriate model, therefore, should incorporate a joint attractiveness function, which explicitly describes the combination of goods and services supplied from a specific centre.

Following Christaller (1966), it is assumed that a successively inclusive hierarchy exists; that is, a centre of order n, offers all the goods and services of order n and below. The joint attractiveness function is thus taken to be

$$(W^g_j)^{\alpha g} \left(\prod_{g'>g}^{n_j} (W^{g'}_j)^{\alpha g'*} \right) \tag{2.15}$$

where n_j is the 'order' of j. Thus we assume an explicit interdependency between the different orders. In our numerical experiments described later we begin the iterative routine with all nj set to 3 for all zones, but successively recalculate n_j each iteration.

An alternative method, which would not have to satisfy the Christaller set-inclusive principle would be to have an additive attractiveness function, for example

$$(W_j^g)^{\alpha g}(W_j^*)^{\alpha_*^g} \tag{2.16}$$

where W_j^* is the sum of all the different orders present in j.

Relative values for some of the order specific parameters can be suggested. For instance, the β^g parameter for higher order goods and services is lower than for lower-order goods and services, because, for the former, transportation costs are relatively less significant in the travel decision. Similarly, α^g is assumed to increase with order, and the combined attractiveness parameter, $\alpha^{g'*}$ would be much smaller than α^g, because the value of $\left(\sum_{g'>g}^{n_j} (W_j^{g'})\right)$ would be typically very large.

3. The spatial system for experiments

To provide the basis for numerical experiments, an idealised simple, geometric zoning system is employed in which the 149 zones are arranged in a lattice of equilateral triangles (see figure 3.1). The spatial distribution of population (P_i) and demand (e_iP_i) is, for convenience, assumed to be uniform, although, of course, it can easily be modified to enable the effects of spatial heterogeneity to be analysed.

In the particular results, which are described in the following sections, 35 outer zones, as indicated in figure 3.1, are treated as exogenous to the system of interest. Whilst this introduces some bias to the urban spatial structure, specifically an element of elongation, various assumptions about the system boundary could be used, for example the exogenous zones could be treated has having a level of demand of some proportion of the demand in the 114 zones.

In the shopping model, the term c_{ij}, representing transport costs between zone i and j, is usually determined, given the locations of i and j (x_i,y_i), (x_j,y_j), respectively, by applying the Euclidean distance metric

$$c_{ij} = \{(x_i - x_j)^2 + (y_i - y_j)^2\}^{\frac{1}{2}} \qquad (3.1)$$

Transport costs (and energy consumption) are, therefore, directly related to physical distance. This cost matrix is used, initially here, together with a set of alternatives, which reflect the problems of different levels of congestion in the city centre. (In the appendix, particular details of the construction of this cost matrix are outlined.) The cost matrix could be further modified by changing the underlying Euclidean metric, and imposing congestion penalties in such a way as to enable us to examine the effects on urban structure of proposed road-building schemes and public transport policy in general. However, we do not go as far as this in this paper.

Four different accessibility maps are given in figures 3.2 to 3.5 (and it is noted that the lack of symmetry is related to the definition of the quadrants - see appendix); each one is generated by a different penalty matrix (representing varying levels of city-centre congestion).

Accessibility, A_i is defined in the following way

$$A_i = 1/\sum_j W_j e^{-\beta c_{ij}} \qquad (3.2)$$

where c_{ij} represents the congested cost matrix and W_j is constant for all zones. The maps are plots of lines of equal accessibility, scaled appropriately (the lower the value the higher the accessibility).
The first map, figure 3.2 has a very low level of congestion and the central area is very accessible, with decline away from the centre.
As congestion increases (figures 3.3, 3.4, 3.5) the central area becomes relatively less accesssible, until eventually (figure 3.5) the central area looses out to two areas away from the centre. The bounded effect is caused by firstly the bias in the zoning system and also by the use of the rectilinear route system (see appendix).

4. Equilibrium spatial patterns

In this section, equilibrium spatial patterns of centres, the $\{W_j\}$ or $\{W_j^g\}$ distributions, are presented for different values of the model parameters. Particular attention is focussed on the effect of systematically increasing the value of the β parameter, which relates to 'ease of travel', and effectively represents the elasticity of travel demand. Specifically, when β is zero, transport costs are no longer important in the trip decision-making procedure, and, as β increases, transportation costs become more important and non-transportation factors, such as tastes, become less important. When β is infinite, consumers travel to their nearest centre, because of the overriding significance attributed to the cost of travel. By varying the β parameter, it is, therefore, possible to consider potential implications for urban spatial structure of future increases in the real price or physical shortages of petrol.

The β parameter reflects consumers' response to travel costs, whereas the α parameter represents their response to centre size. If increases in the price of petrol are acting as a deterrent to travelling long distances, producers may attempt to maintain their market share by increasing their attractiveness, for example by introducing new marketing technologies to bring about cost savings. This could then be reflected in an increased α parameter.

A series of results in which the β parameter is varied is described, using both the simple Euclidean distance metric and the results of the congestion algorithm.

The results presented in figures 4.1 to 4.4 were derived using the Euclidean cost matrix. Each figure shows the resultant $\{W_j^g\}$ pattern for three orders, with spatial demand held constant (ie. all $e_i^g P_i$'s are equal for each order). Figure 4.1 is the result for high β^g values. The spatial pattern of centres is dispersed, there being a large number of order one (the lowest) centres. This reflects the fact that for low order goods under conditions of high perceived travel costs consumers will not travel far. For higher order goods we also note there are many centres, but of relatively small size. As the value of β decreases, figure 4.2, the spatial pattern changes, with the emergence of two areas of spatial concentration, to the north and south of the centre. Note also that the average size of the higher order centres is greater than in 4.1. However the β value is still too high to allow the emergence of a central domination of services. This occurs in the example shown in figure 4.3. β is now sufficiently low to allow consumers to travel further, on average, and thus they trade off the disutility of travel in favour of the utility associated with larger centres. Again the average size of centres of higher order increases. As β continues to decreases (figure 4.4) the general pattern becomes slightly more concentrated, but the city centre now dominates in terms of the proportion of the total floorspace in the system. Thus we have progressed from a dispersed pattern of service provision to a highly concentrated one.

Figures 4.5 to 4.8 repeat the experiments for the same parameter values, except now that we employ a congested cost matrix, thus penalising trips to the city centre. The first noticeable effect, demonstrated in figure 4.5 is that the symmetry present in the results using the Euclidean cost matrix, disappears due to the way the cost matrix is derived (see appendix). Secondly, the distribution of centres, for the corresponding β values is more concentrated. This, we assume, is due to the effect of congestion on the system, although further analysis is needed to ensure that this is not a local solution. As β decreases, in figures 4.6-4.8 we note the emergence of initially three areas of spatial concentration of services, then two and finally, in figure 4.8, just one. The main point to be noted here is that the city centre obtains none of the three orders of services, even when β is very low. The effect of congestion appears to be that the system is 'partitioned' into distinct off-centre areas which function as fairly independent units, until β becomes so low that the most accesible of these units dominates.

It is clear that these preliminary results are but a small subset of the large number of possible experiments that can be undertaken. They, do however, illustrate the range of different types of spatial patterns that can be expected.

5. Evolution of urban spatial structure: numerical experiments

In this section, the evolution of urban spatial structure is described through a series of figures for different time periods. Two simulation runs are presented, each with a different value for the ε parameter, and, in both cases, the congestion algorithm is used to determine the cost matrix. Moreover, for simplicity, it is assumed that the size of all the centres (except certain boundary centres) is identical at the outset (that is, W_j equal 10 units). Particular attention is focussed on the differential growth (or decline) of centres.

In the first numerical experiment, when ε is 0.05, the influence of the relatively congested city-centre is apparent by the first-time period; (see figure 5.1); as to be expected, the more relatively accessible centres grow faster, and an element of spatial heterogeneity in supply is already present. This pattern is accentuated by the second time period, but the associated growth in revenue (D_{jt}) has caused the numerical magnitude of the term εD_{jt} to increase above 2, resulting, eventually, in the onset of oscillating behaviour. This oscillating behaviour continues, as does the pattern of differential change associated with the congestion cost matrix, until the centres in the two regions of relative large centres have grown sufficiently to take their behaviour into a divergent regime. This leads to their extinction by the eighth time period, and the subsequent reduction in spatial competition results in the emergence of three new regions of relatively large centres. It is noted that, whilst the numerical results are only presented for the first ten time-periods, a stable spatial structure has not been attained, and further alterations can be expected.

In the second simulation, ε is fixed at a value of 0.02. As figure 5.2 shows, city-centre congestion again has noticeable effects on the different pattern of growth exhibited by the centres. However, in contrast to the previous numerical experiment, the value of εD_{jt} never exceeds a value of two, and, therefore, as to be expected, the behaviour of each follows a monotonic trajectory to a stable steady-state size.

The results presented in this section have illustrated that individual centres possess a variety of patterns of behaviour and that interesting alterations in urban spatial structure are found. Obviously, this numerical work can be extended, and further interesting patterns are envisaged. One interesting test would be to hypothesise how ϵ, the response rate to change, varies over time. It may be that as energy prices, and hence transport costs, rise then producers may be more conservative in relation to expanding their services in response to increasing profit. Hence ϵ may decline over time.

6. Conclusions

We have demonstrated that under different parameter conditions the equilibrium structures and evolution of urban spatial patterns can vary quite markedly. While we have restricted our discussion to retail facility models there is every reason to believe that this is just one example of a wide range of models that demonstrate similar behaviour. One of our immediate research tasks is demonstrating that this is in fact the case. The next logical progression is the linking of sub-models to allow for the interaction between different sectors of the urban system. We would then expect changes in one sector to feed through to others, thus leading to both complex and interesting dynamical behaviour.

As to how the sorts of parameters we have discussed in this paper will change in the future, there remains an element of speculation. Undoubtedly, however changes in both the price and availability of energy will have significant effects in terms of urban spatial structure.

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APPENDIX

A SIMPLE METHOD OF PRODUCING APPROXIMATE CONGESTED COST MATRICES

A1. Introduction

Congestion in an urban transport network may have a number of associated effects. Amongst these are two that interest us here; the way in which travel costs rise with increasing congestion, and whether in response to this a longer, but in effect cheaper route is chosen, that avoids congestion. We assume that people who travel from one zone to another will compare the cost of travelling by the direct route with the cost of travelling on a number of alternative routes and chose the cheapest. In this appendix we present a relatively simple method for producing the resultant cost matrix for travel between zones under varying degrees of congestion. The method used, because of its simplicity is seen to have certain advantages over more elaborate methods, for example Angel and Hyman's (1975) velocity field concept for continuous space, or the more traditional congested assignment routines although it does have some rather strong assumptions built in, as will be discussed later.

A2. The method

Assume we have a zoning system of the same form as described in section 3, and we know the Cartesian coordinates of all points or zone centroids. It is thus a relatively straightforward matter to calculate the Euclidean distance d_{ij} between any two points, i and j. With congestion this Euclidean distance may be transformed into a non-Euclidean distance d^*_{ij}, where the transformation is related to the nature and degree of congestion in the system.

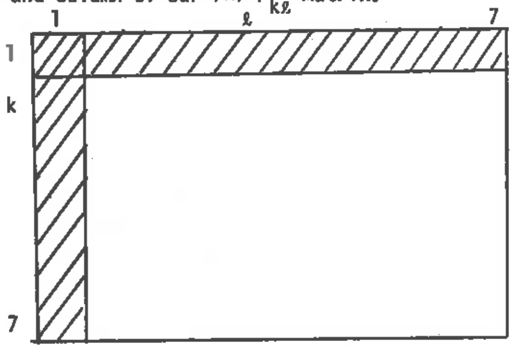
Let us assume we can classify the points in our zoning system in terms of distance from the centre, as shown in figure A1. We then assume that there is a penalty $P_{k\ell}$ associated with travelling between zones which line on different rings, k and ℓ. This penalty will vary with the degree of congestion. If congestion is heavy then the penalty of travelling along routes into the city centre will be high in relation to routes not passing through the city centre. The penalty matrix $P_{k\ell}$ may be defined in the following way.

For any set of i's and j's there will be two associated rings, denoted as $k(i)$ and $\ell(j)$. The penalty for travelling between two rings is defined as $P_{k(i)\ell(j)}$. This is calculated for the 7×7 matrix in our example in the following way.

1. Set $\ell=1$ (city centre) and calculate

$$P_{k1} = a_1 - b_1 e^{d_{k1}}$$

where a_1, b_1 are suitable parameters, relating to the level of congestion in the centre and its decline outwards respectively, and d_{kl} is the average distance of ring k to the centre. We can now fill in the first row and column of our 7×7 $P_{k\ell}$ matrix.

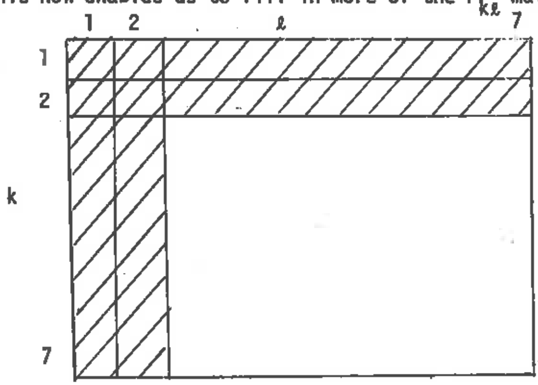


2. Set $\ell=2$ and calculate

$$P_{k2} = a_2 - b_2 e^{d_{k2}} \qquad , k \neq 1$$

where a_2 and b_2 are different parameters than a_1, b_2. (a_1 would be greater than a_2 with city centre congestion.)

This now enables us to fill in more of the $P_{k\ell}$ matrix.



3. Continue for ℓ=3,4,5,6 and 7.

To obtain our transformed d^*_{ij} matrix we multiply the Euclidean distance, d_{ij} by $P_{k\ell}$, where k is the ring associated with zone i and ℓ the ring associated with zone j.

However we noted earlier that when congestion increases a variety of alternative routes will be considered, and the cheapest one, ie. the one with the smallest value of d^*_{ij}, will be chosen. To allow for this we now make the following assumptions.

The zoning system is divided into four quadrants, as shown in figure A2. The segmented division is to allow for all zones to be within only one quadrant. This has the effect of producing some asymmetry into the analysis.

We assume therefore that the following types of trips may occur

(i) Intra-quadrant trips;

(ii) Inter-quadrant trips, potentially passing through the city-centre, ie. I→IV, II→III and vice versa.

(iii) Inter-quadrant trips not even potentially passing through the city centre ie. I→II, III→IV, I→III, II→IV and vice versa.

However in the case of (ii) and (iii) we assume the following:

(a) The trip is made either by the direct route - ie. a straight line,or a straight line to and from the city centre in case (ii), or by the two alternative rectilinear routes. The rectilinear route is taken as the alternative to the congested route. These are shown in figure A3.

(b) The trip chosen, and the d^*_{ij} that is entered in the cost matrix is the alternative with the lowest value of d^*_{ij}. The following procedure is then computed for any set of origins and destinations, i and j.

(i) Determine which quadrants i and j lie in.

(ii) From (i) and the set of assumptions listed above determine which type of trips are to be considered.

(iii) Determine $P_{k\ell}$, the congestion penalty to be applied to each alternative trip or separate parts of the trip. For example on a rectilinear trip, typically a different penalty will be applied to each half of the trip, and trips passing through the city centre will be similarly treated.

(iv) Calculate the Euclidean distance d_{ij} of separate parts of each possible trip. For direct trips the distance is given by

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

where:

$(x_i y_i),(x_j y_j)$ are the Cartesian coordinates of points i and j.

and for rectilinear trips

$$d_{ij} = |(x_i - x_j)| + |(y_i - y_j)|$$

Now we multiply the distance by the penalty:

For direct trips

$$d^{*1}_{ij} = P_{k(i)\ell(j)} d_{ij}$$

(where the superscript 1 denotes a direct trip).

For rectilinear trips we need to consider two different cases:

$$d^{*2}_{ij} = P_{k(i)\ell(i)}|x_i - x_j| + P_{k(i)\ell(j)}|y_i - y_j|$$

$$d^{*3}_{ij} = P_{k(i)\ell(j)}|x_i - x_j| + P_{k(j)\ell(j)}|y_i - y_j|$$

The difference between d^{*2}_{ij} and d^{*3}_{ij} is that for the two rectilinear alternatives different levels of congestion may be experienced, and this is shown in figure A4. In this case d^{*2}_{ij} would invariably be less than d^{*3}_{ij}.

(v) Simply choose

$$d^{*}_{ij} = \text{Minimum } (d^{*1}_{ij}, d^{*2}_{ij}, d^{*3}_{ij})$$

and proceed to the next set of i and j.

(vi) When all d^{*}_{ij}'s have been computed write out the congested cost matrix, $\{d^{*}_{ij}\}$. This now forms the input to the models described in section 2.

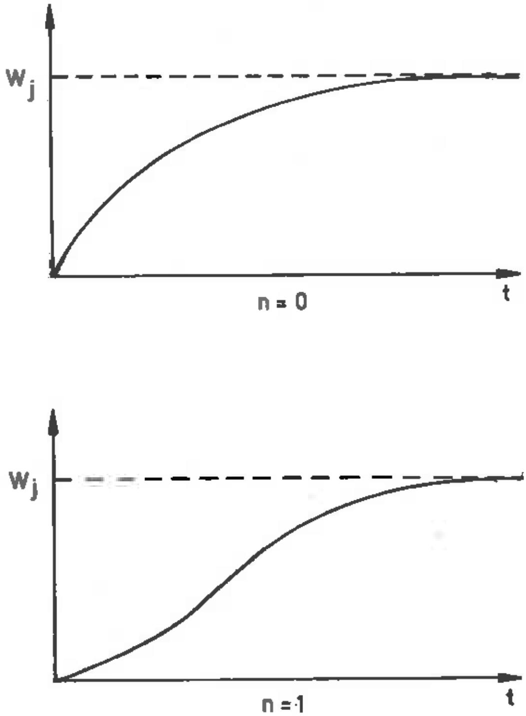


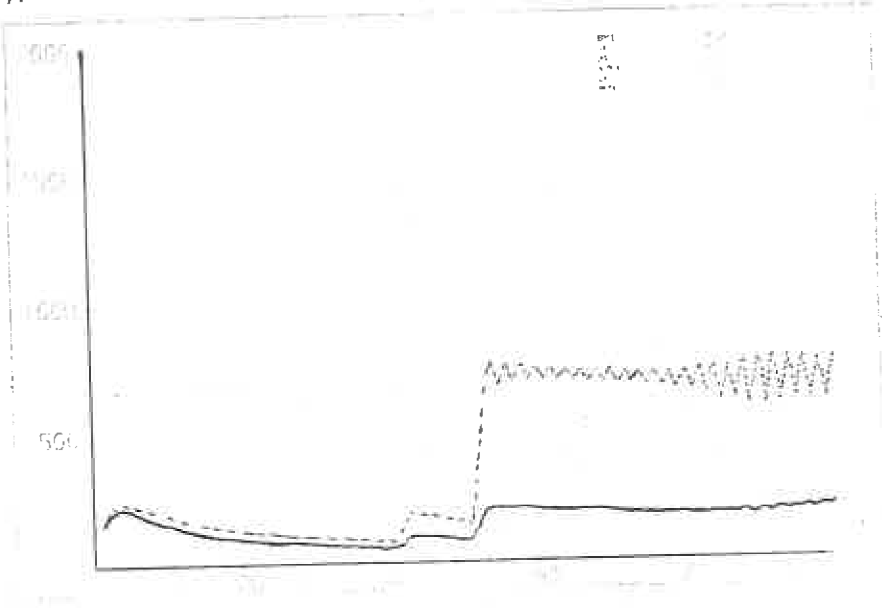
FIGURE 2.3

NUMERICAL EXPERIMENTS FOR 8 ZONES

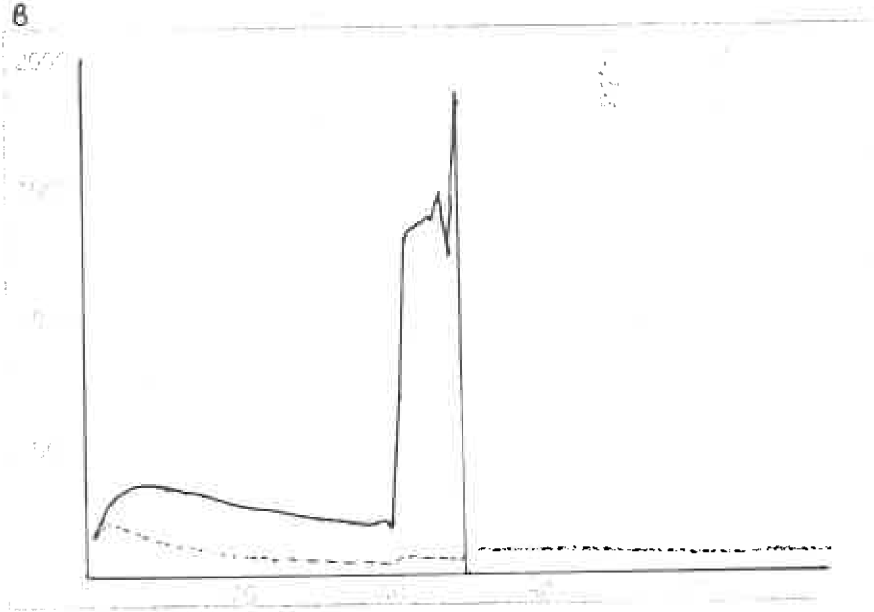
$\epsilon = 0.0035; \beta = 0.1; \alpha = 1.0; k = 1.0$

Figure 2.4

A



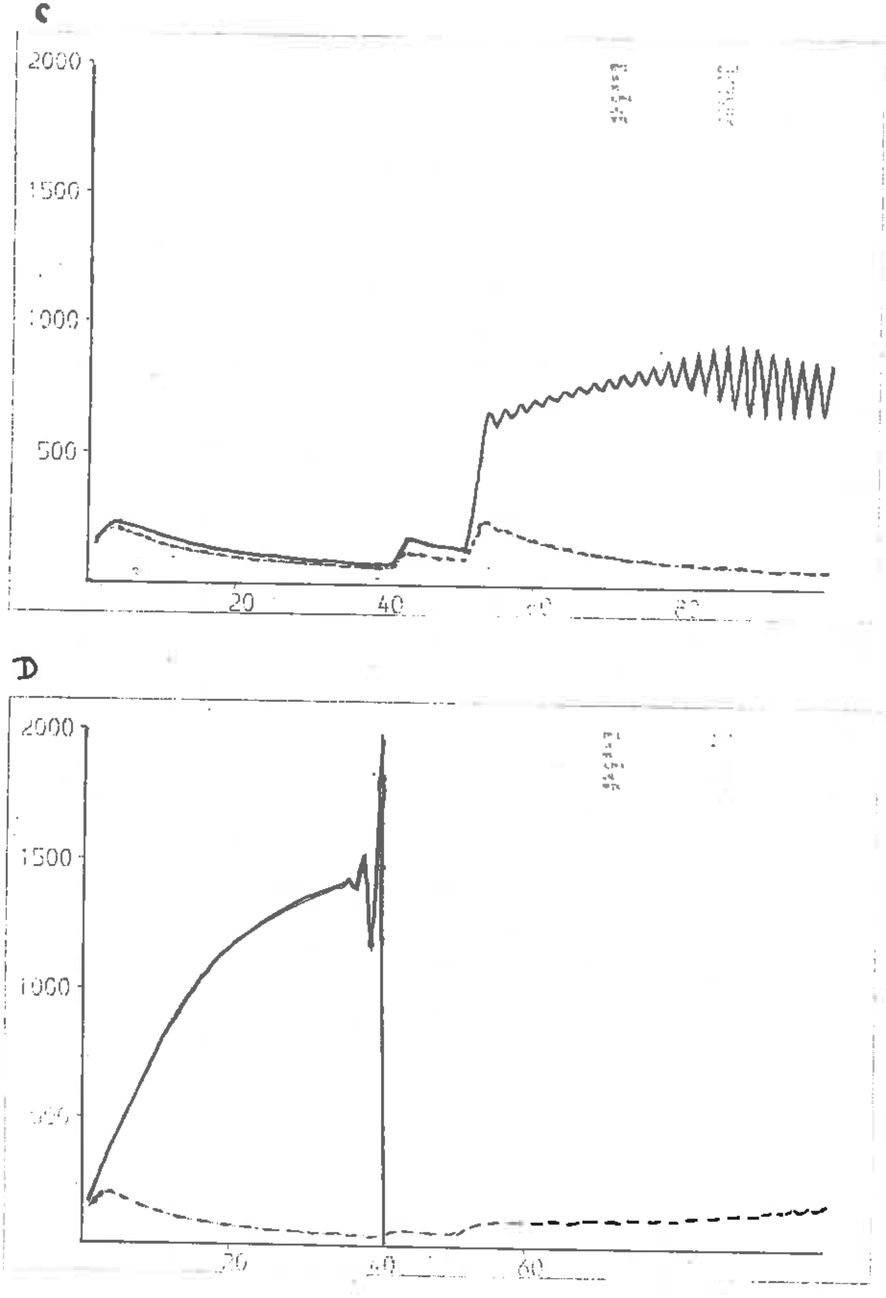
B



NUMERICAL EXPERIMENTS FOR 8 ZONES

$\varepsilon = 0.0035; \beta = 0.1; \alpha = 1.0; k = 1.0$

Figure 2.4 (Continued)



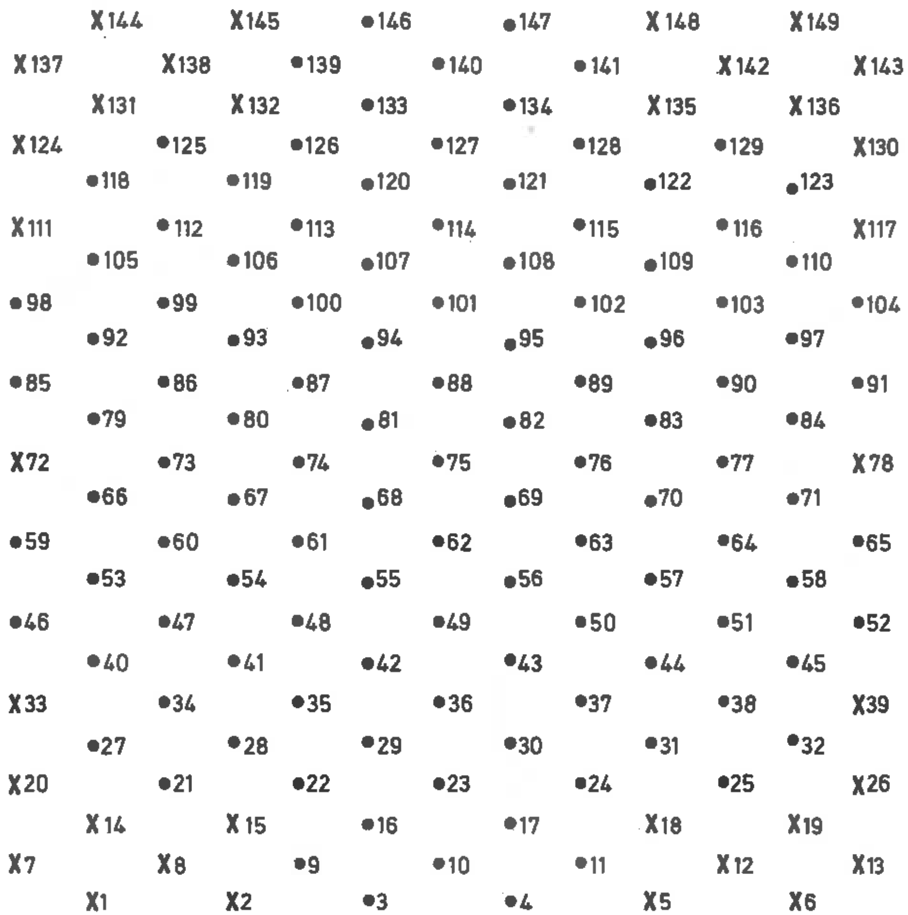


FIGURE 3.1 The zoning system

X indicates exogenous zone

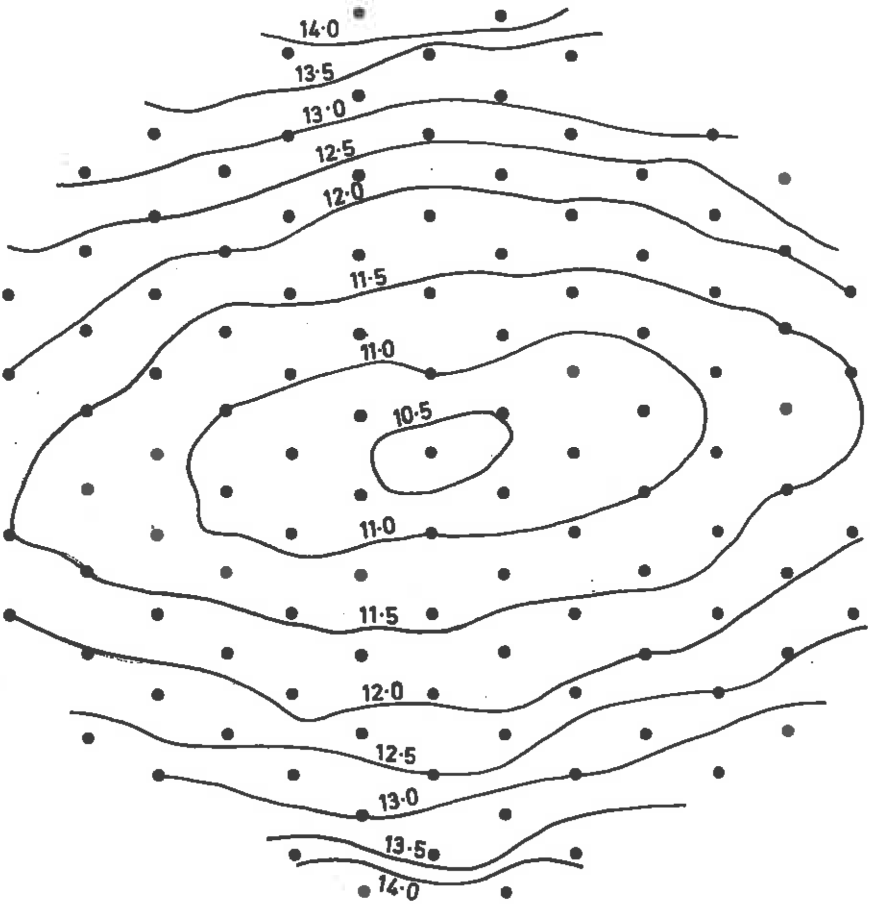


FIGURE 3.2 Accessibility Plot I

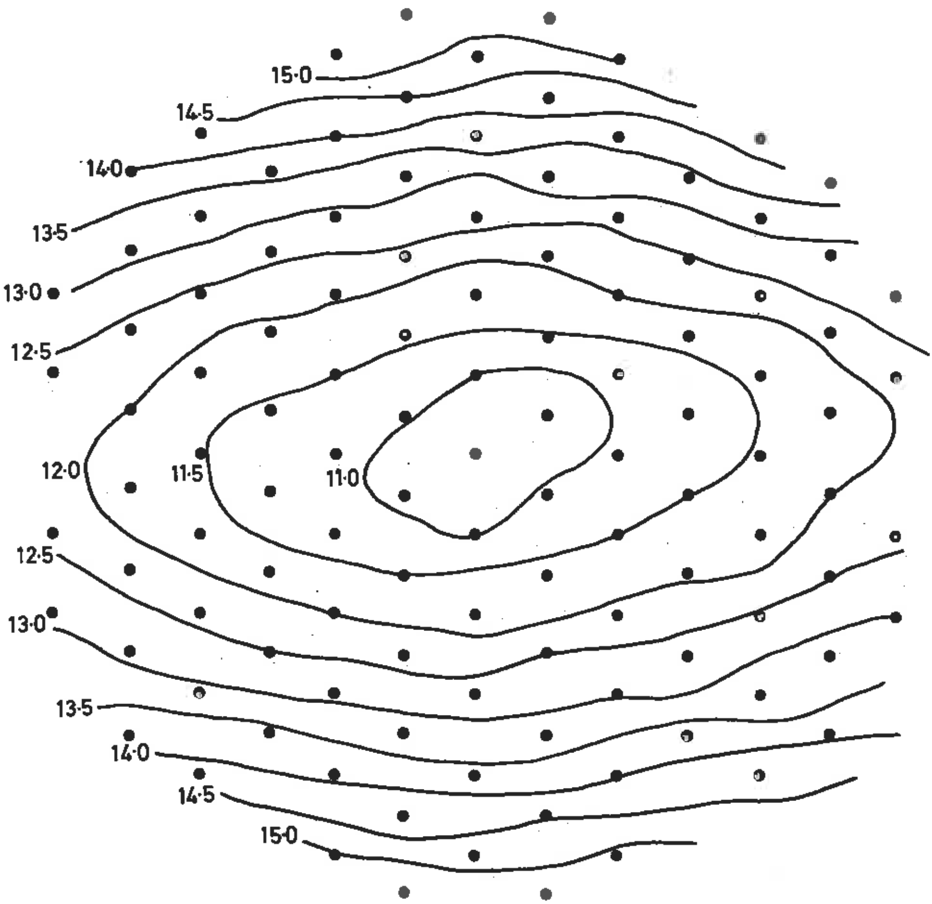


FIGURE 3.3 Accessibility Plot II

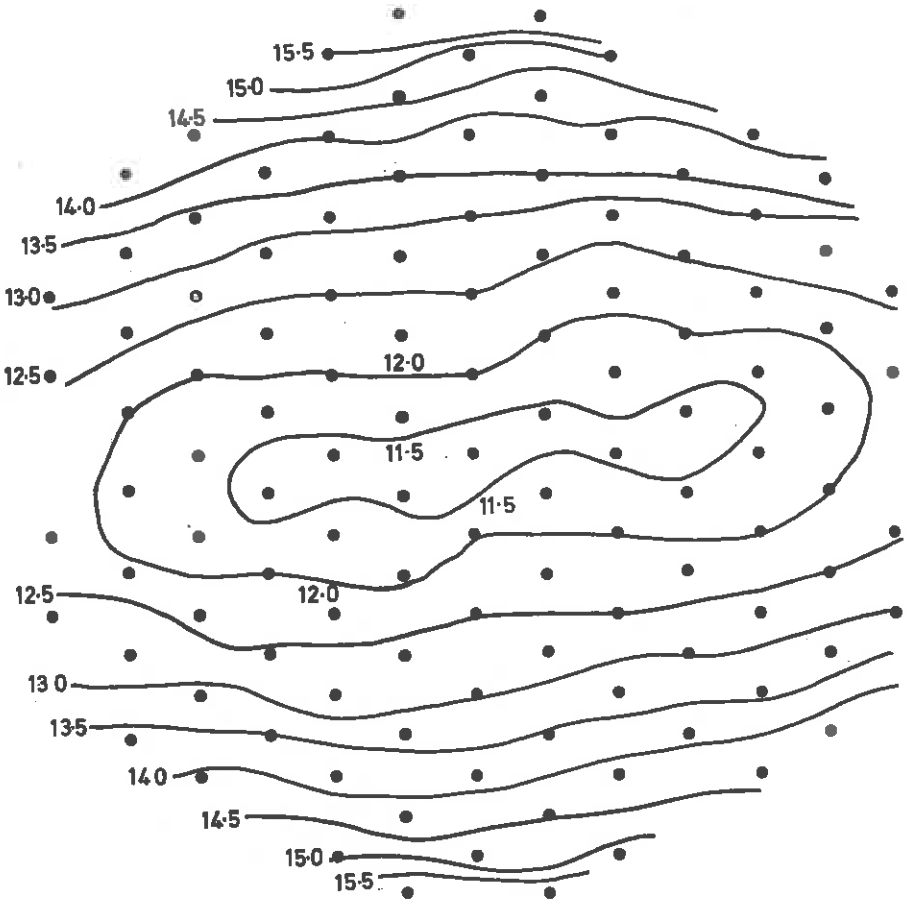


FIGURE 3.4 Accessibility Plot III

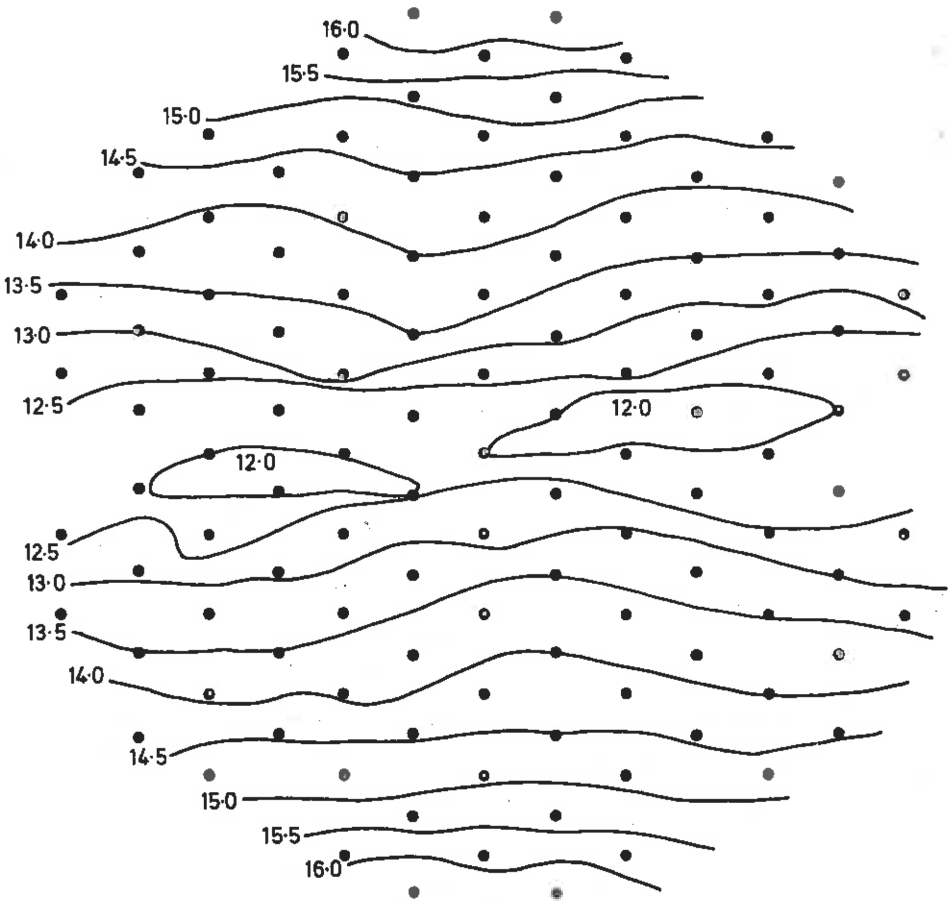


FIGURE 3.5 Accessibility Plot IV

$\beta^1 = 3{\cdot}5$

$\beta^2 = 1{\cdot}75$

$\beta^3 = 1{\cdot}0$

$\alpha = 1{\cdot}0$ (all g)

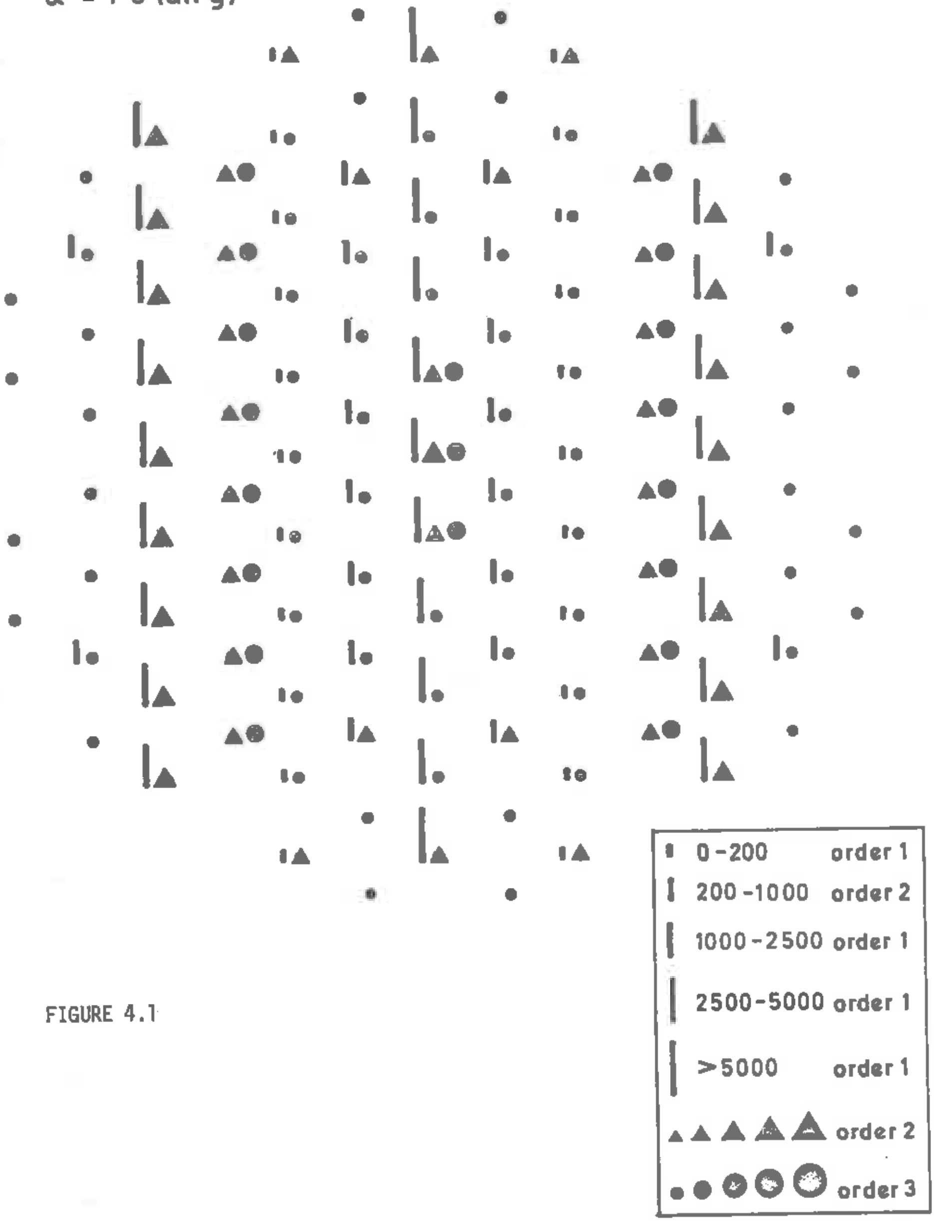
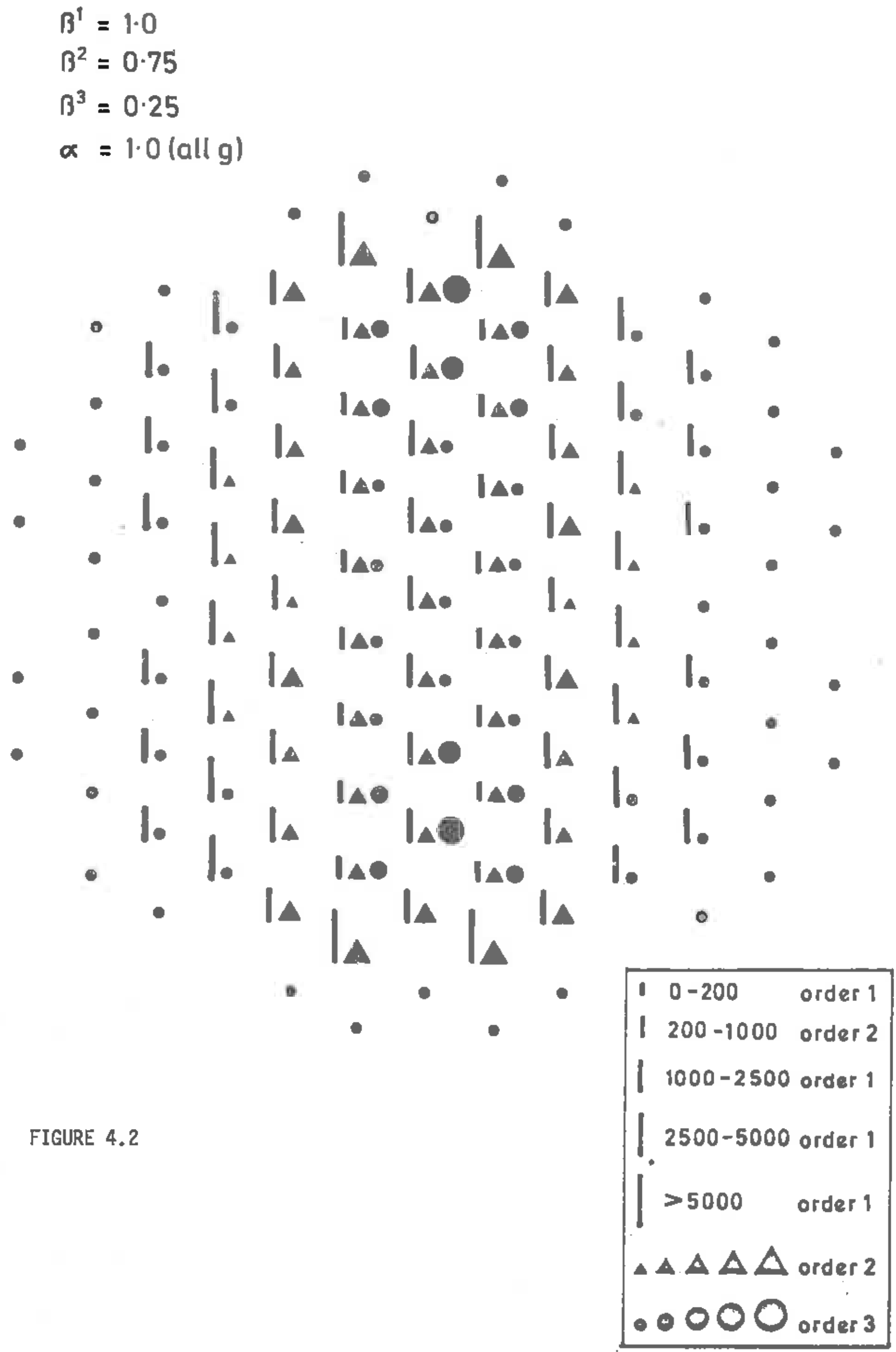


FIGURE 4.1



$\beta^1 = 1{\cdot}0$
$\beta^2 = 0{\cdot}75$
$\beta^3 = 0{\cdot}25$
$\alpha = 1{\cdot}0$ (all g)
0 - 200 order 1
200 - 1000 order 2
1000 - 2500 order 1
2500 - 5000 order 1
>5000 order 1
order 2
order 3

FIGURE 4.2

$\beta^1 = 0{\cdot}25$
$\beta^2 = 0{\cdot}15$
$\beta^3 = 0{\cdot}1$
$\alpha = 1{\cdot}0$ (all g)

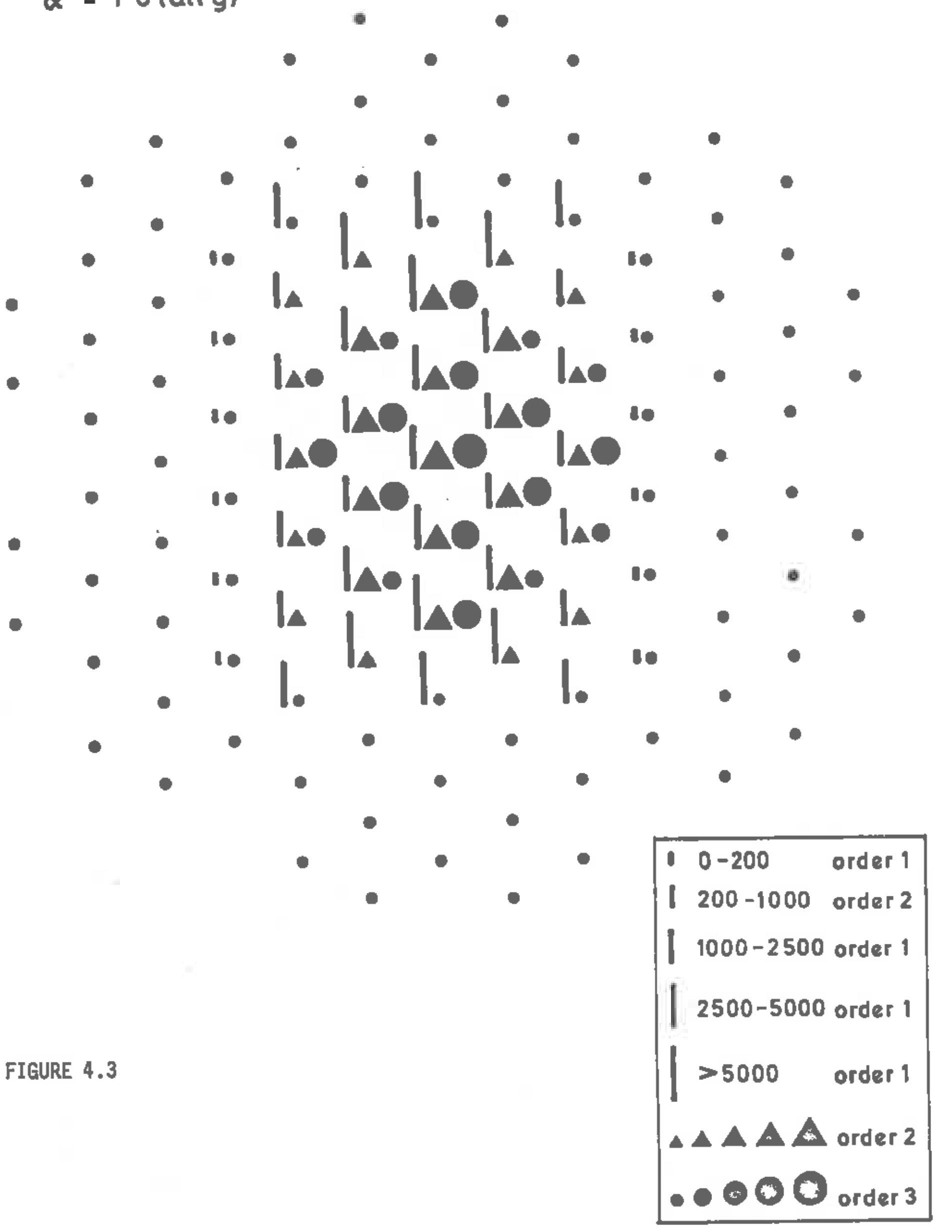


FIGURE 4.3

$\beta^1 = 0{\cdot}05$
$\beta^2 = 0{\cdot}05$
$\beta^3 = 0{\cdot}05$
$\alpha = 1{\cdot}0$ (all g)

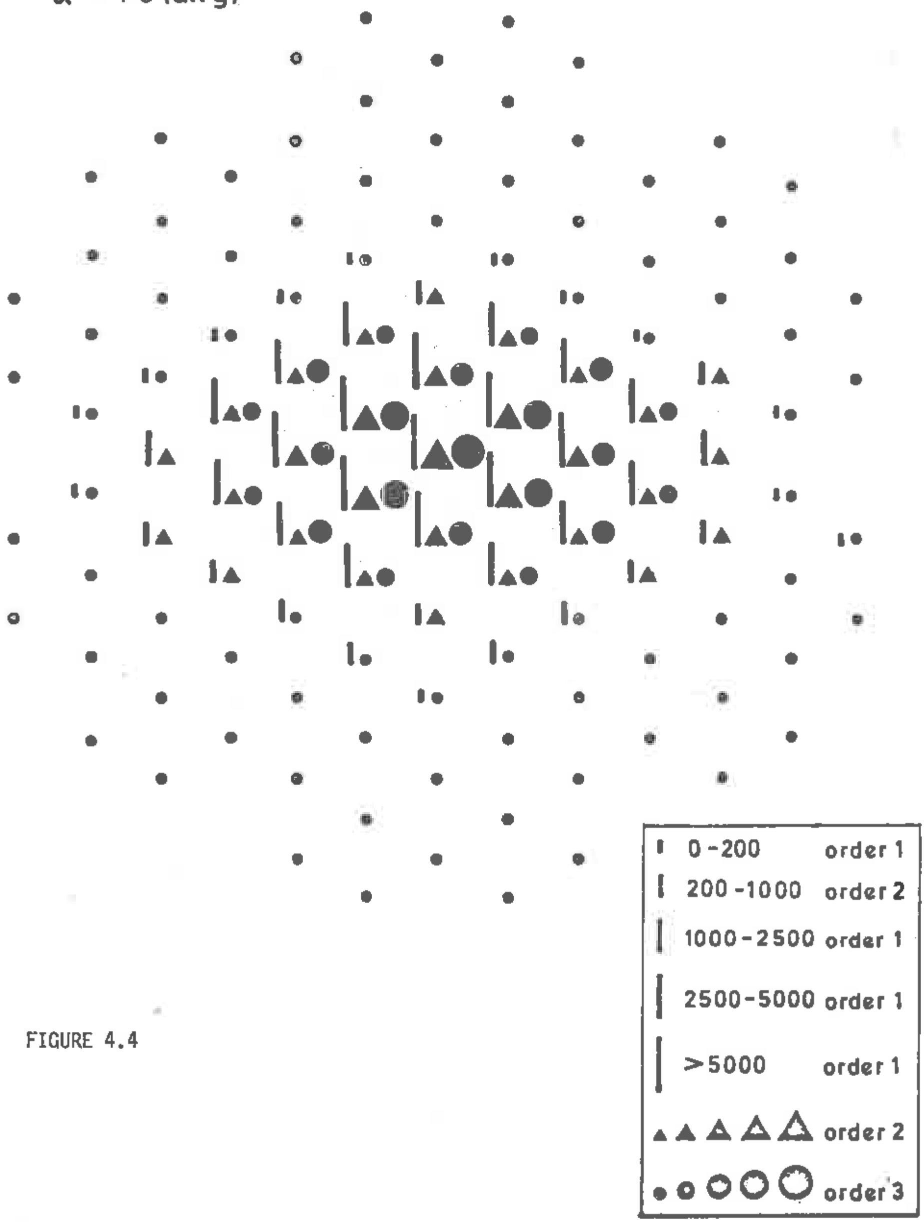


FIGURE 4.4

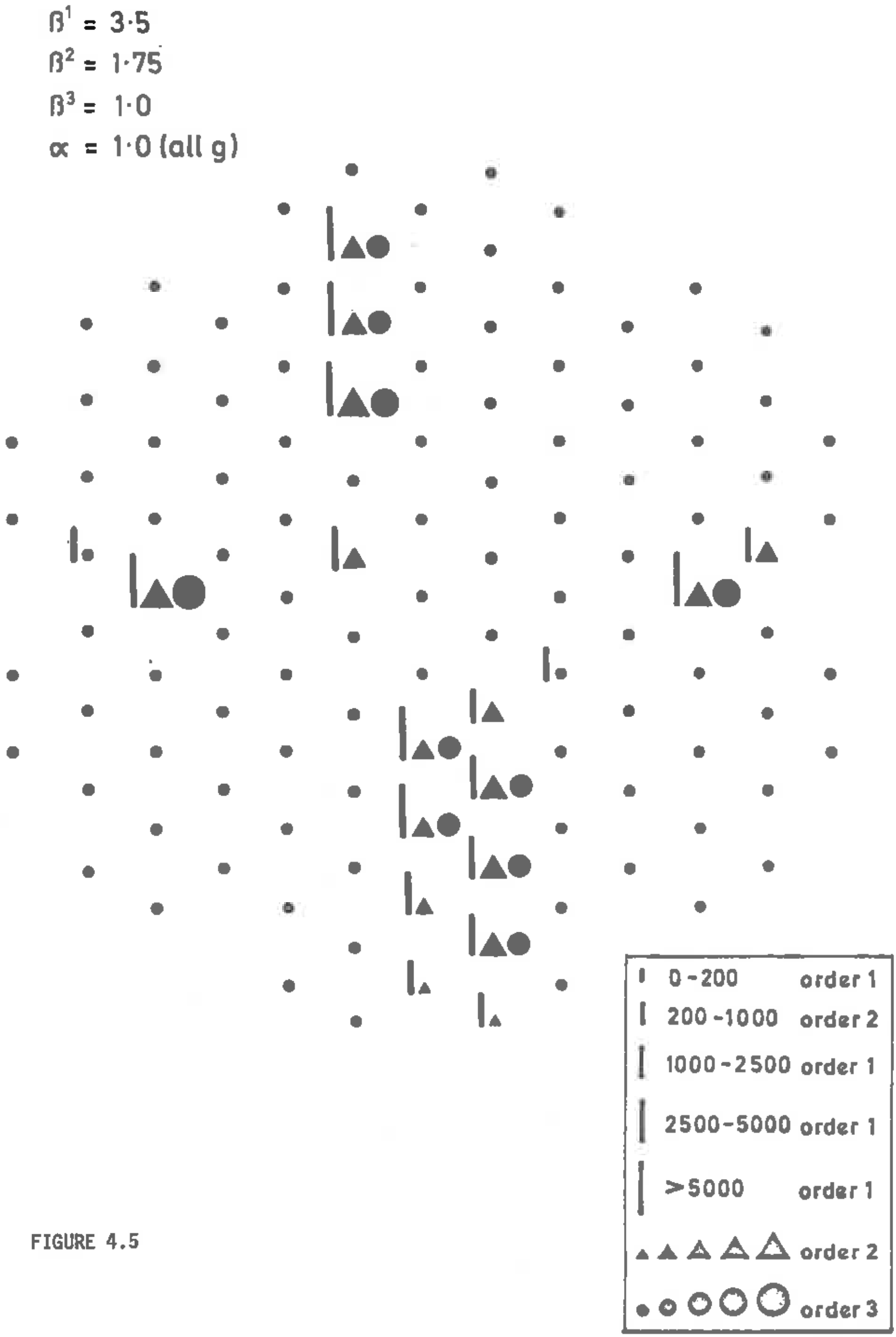

$\beta^1 = 3{\cdot}5$
$\beta^2 = 1{\cdot}75$
$\beta^3 = 1{\cdot}0$
$\alpha = 1{\cdot}0$ (all g)
0 - 200 order 1
200 - 1000 order 2
1000 - 2500 order 1
2500 - 5000 order 1
>5000 order 1
order 2
order 3

FIGURE 4.5

$\beta^1 = 1{\cdot}0$

$\beta^2 = 0{\cdot}75$

$\beta^3 = 0{\cdot}25$

$\alpha = 1{\cdot}0$ (all g)

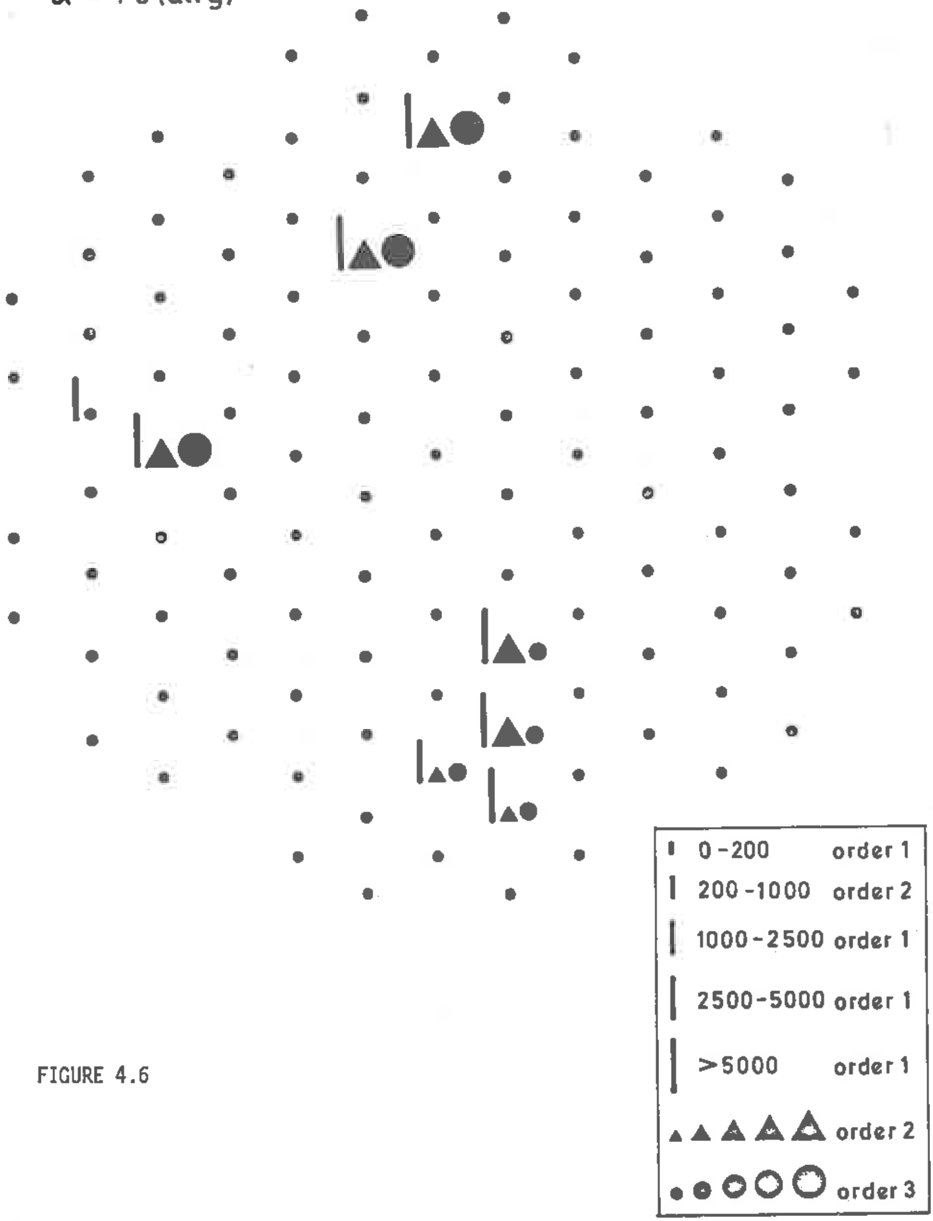


FIGURE 4.6

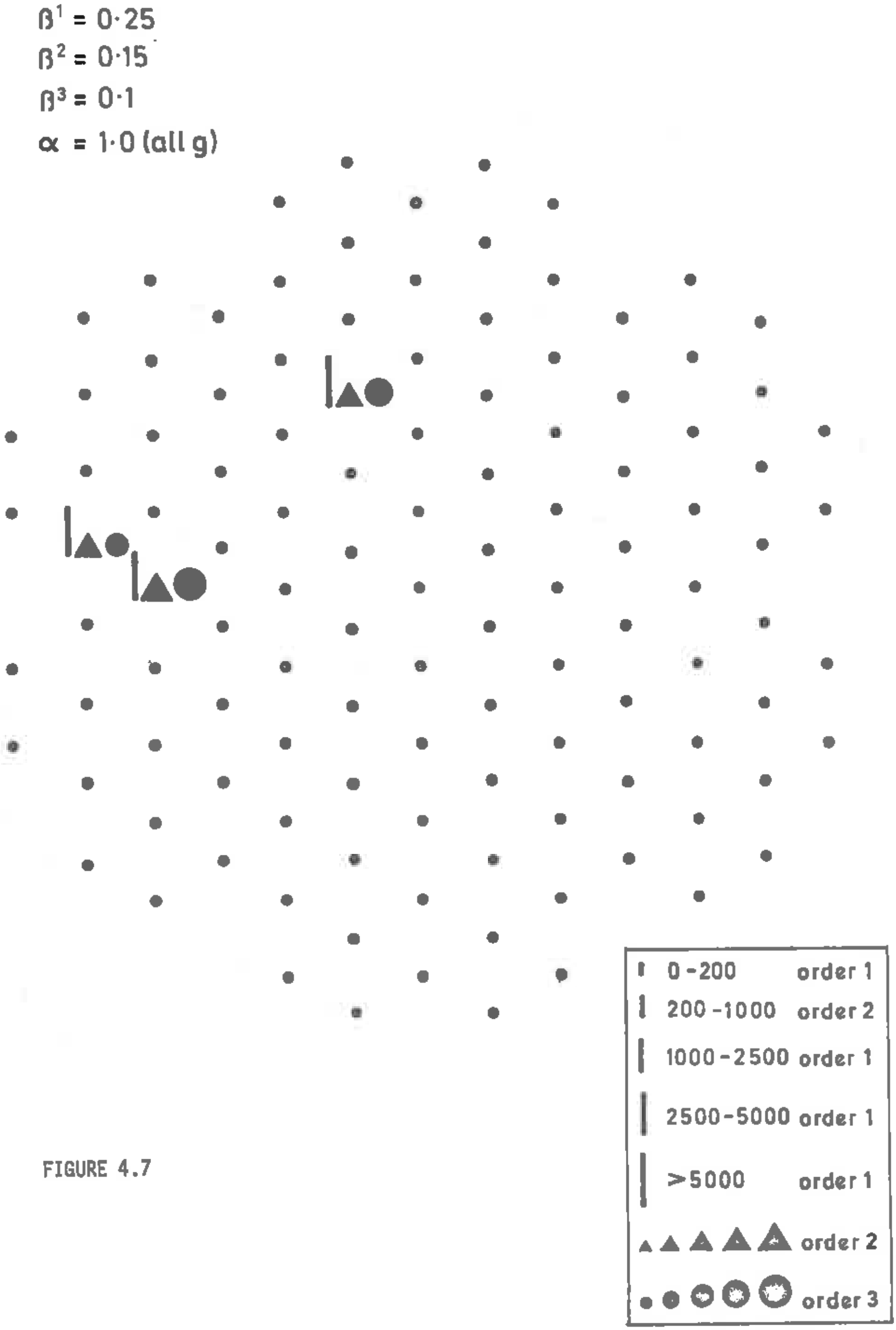

$\beta^1 = 0{\cdot}25$
$\beta^2 = 0{\cdot}15$
$\beta^3 = 0{\cdot}1$
$\alpha = 1{\cdot}0$ (all g)
0 - 200 order 1
200 - 1000 order 2
1000 - 2500 order 1
2500 - 5000 order 1
>5000 order 1
order 2
order 3

FIGURE 4.7

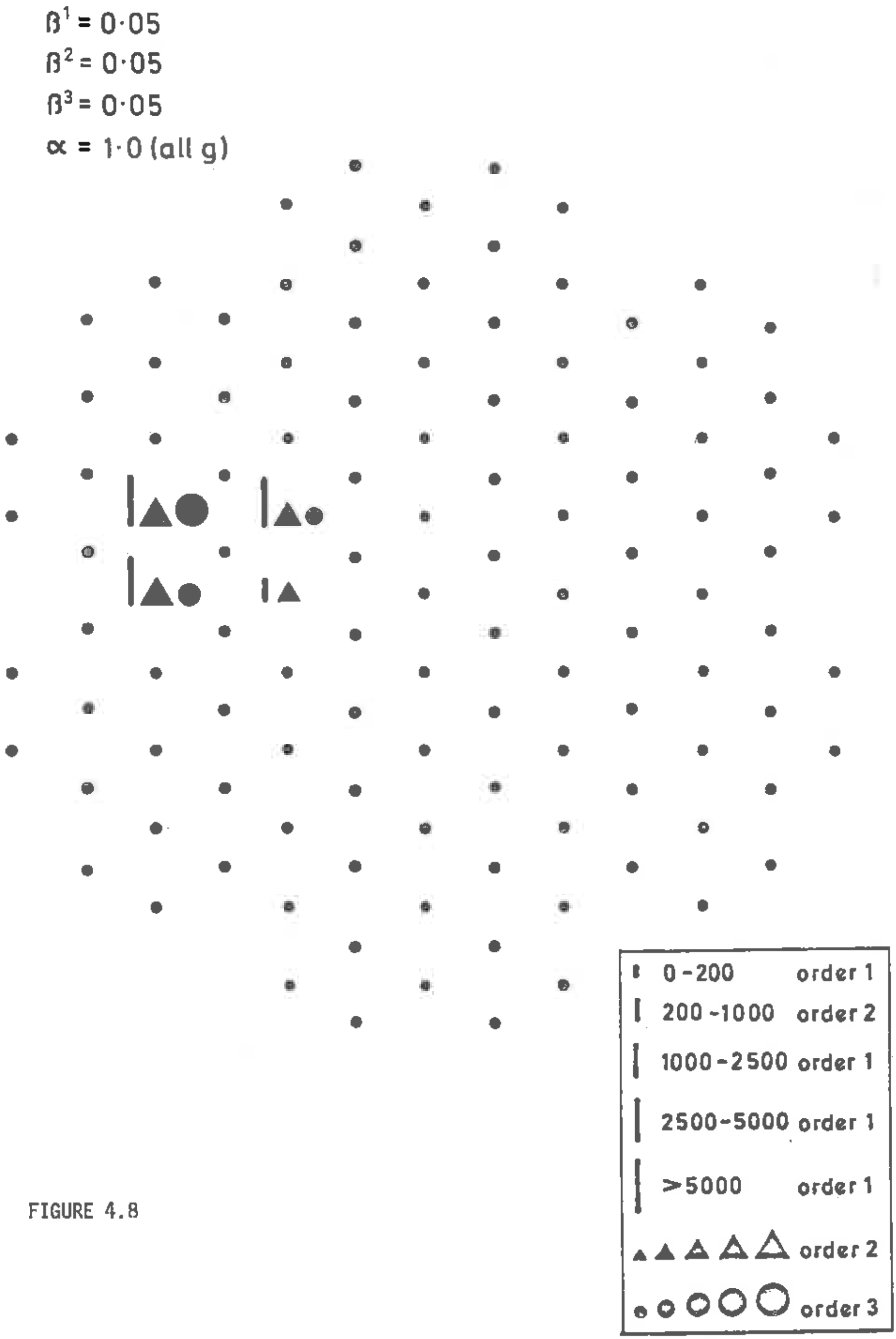

ß¹ = 0·05
ß² = 0·05
ß³ = 0·05
∝ = 1·0 (all g)
0 - 200 order 1
200 - 1000 order 2
1000 - 2500 order 1
2500 - 5000 order 1
>5000 order 1
order 2
order 3

FIGURE 4.8

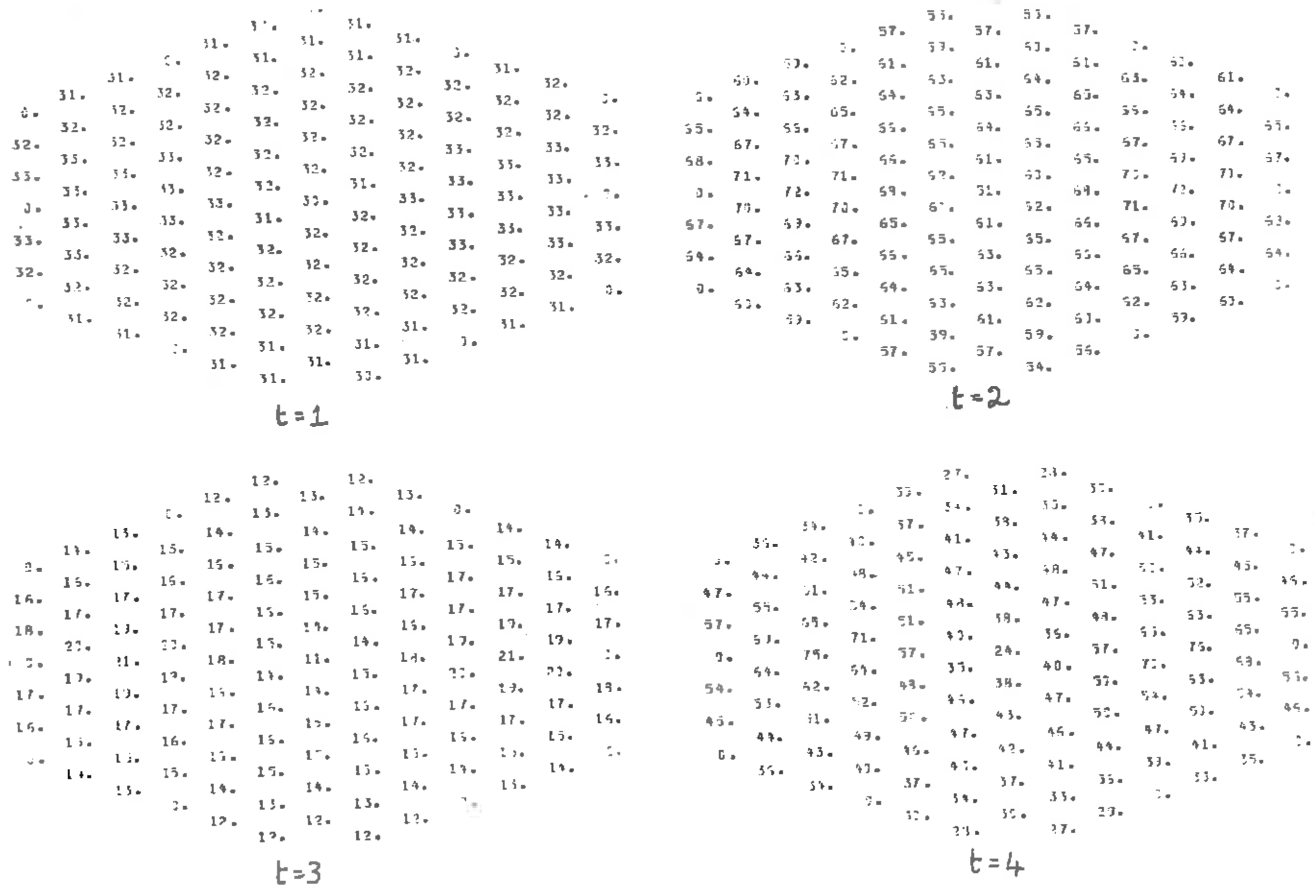


Fig. 5.1a. Difference equation simulation. $\varepsilon = 0.05$.

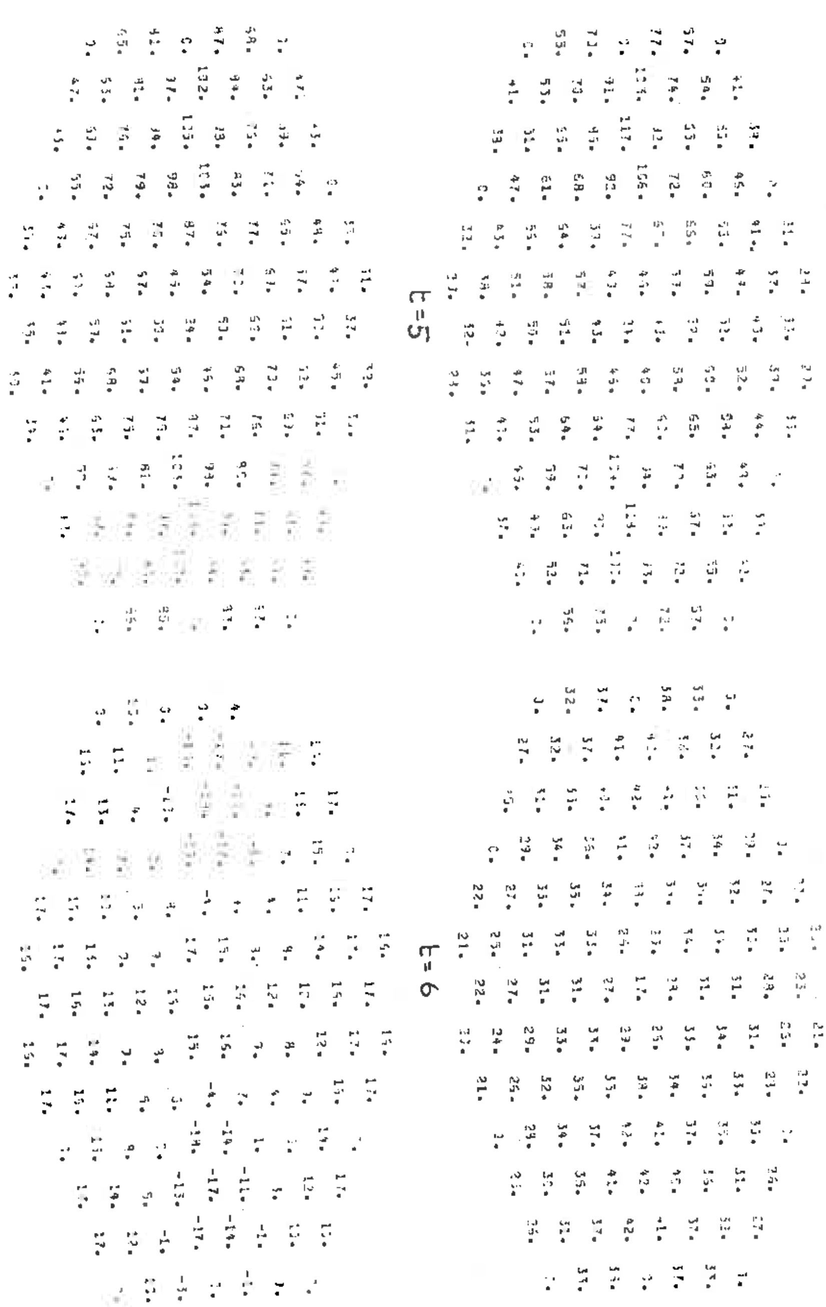
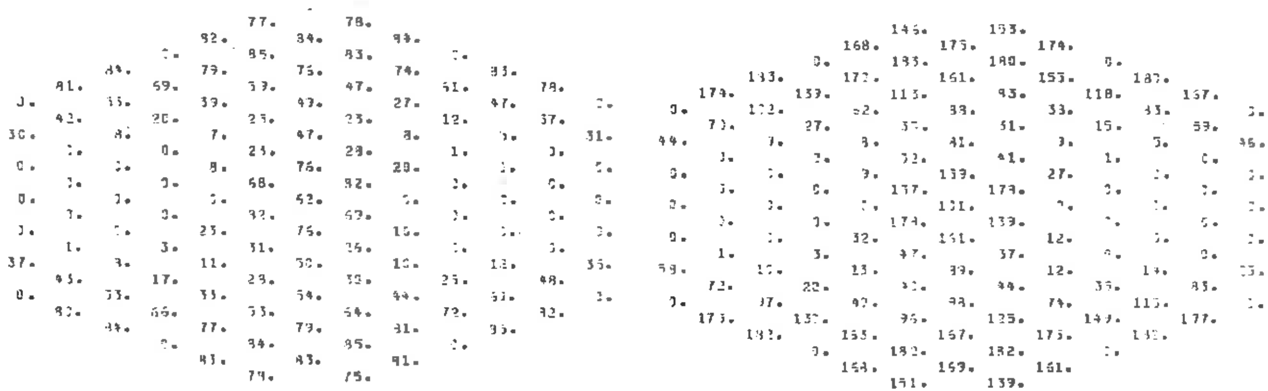


Fig. 5.1b. Difference equation simulation. ε = 0.05



t = 9

t=10

Fig. 5.1c. Difference equation simulation. $\varepsilon = 0.05$

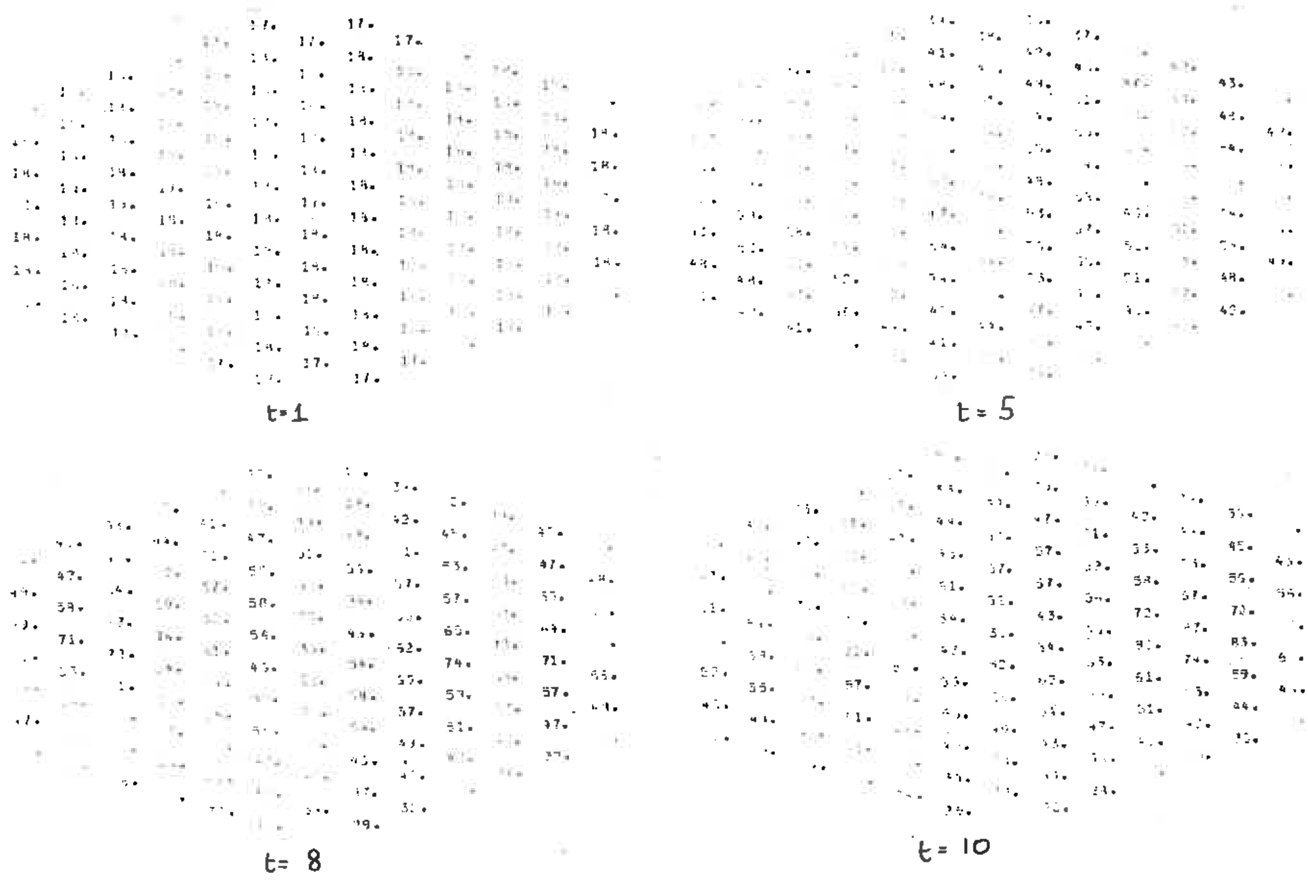


Fig. 5.2. Difference equation simulation. $\varepsilon = 0.02$

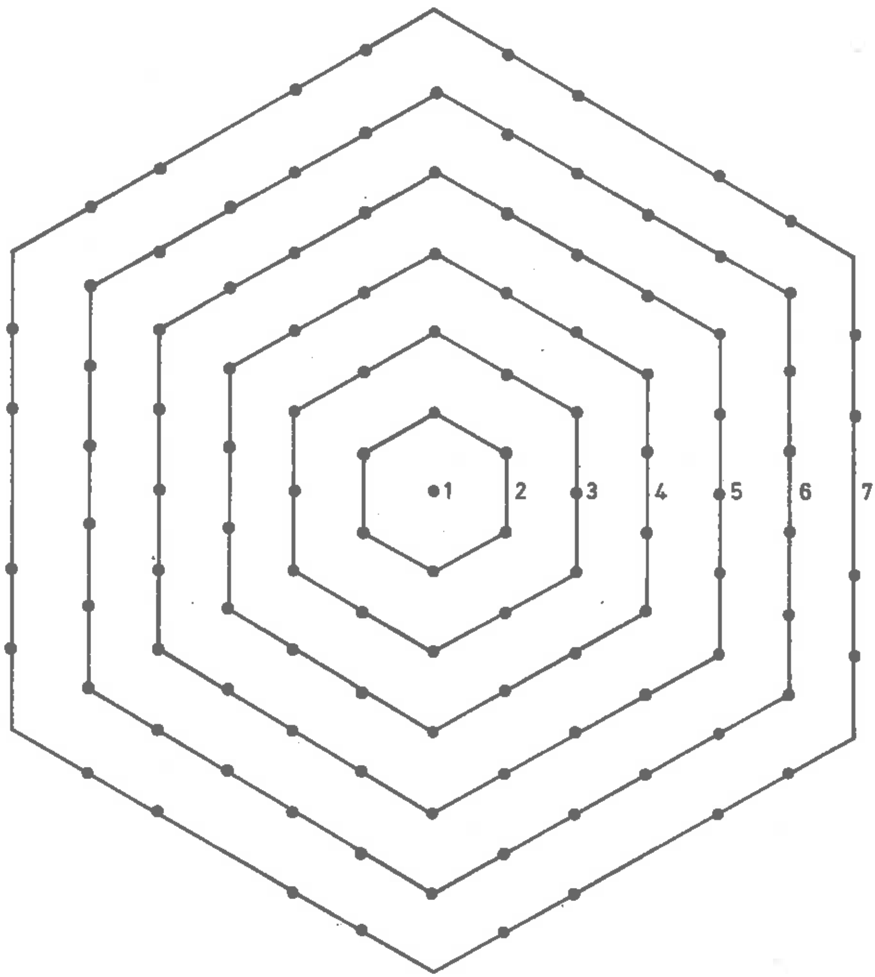


FIGURE A1 Classification of the Zoning System into 7 rings

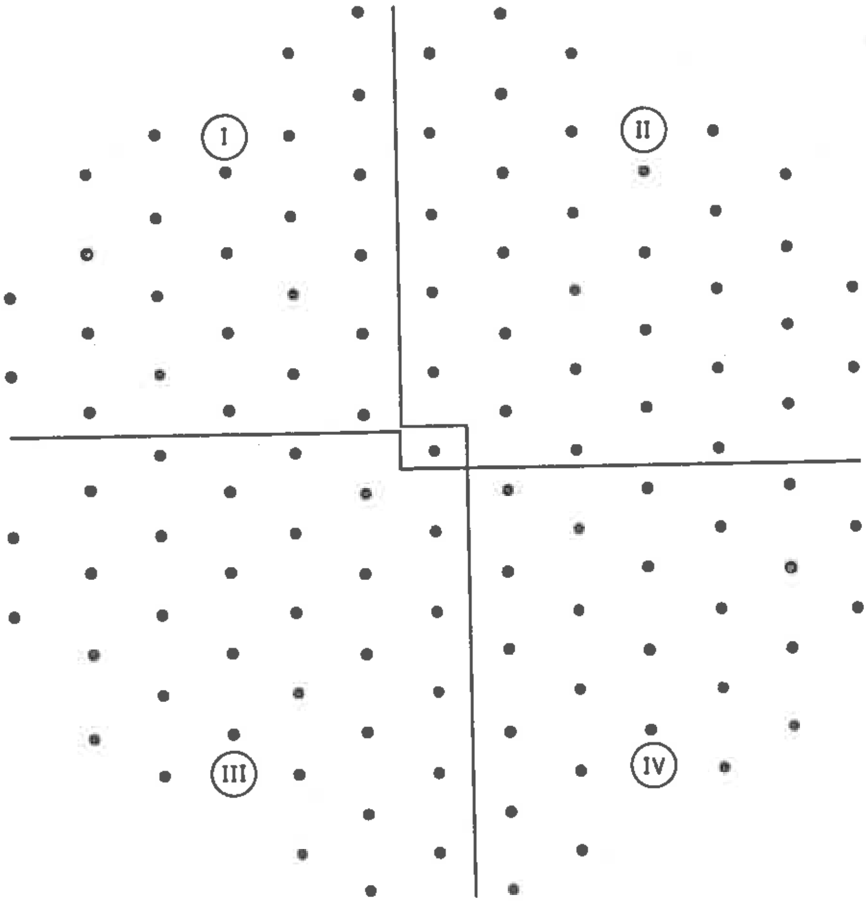


FIGURE A2 The four quadrants

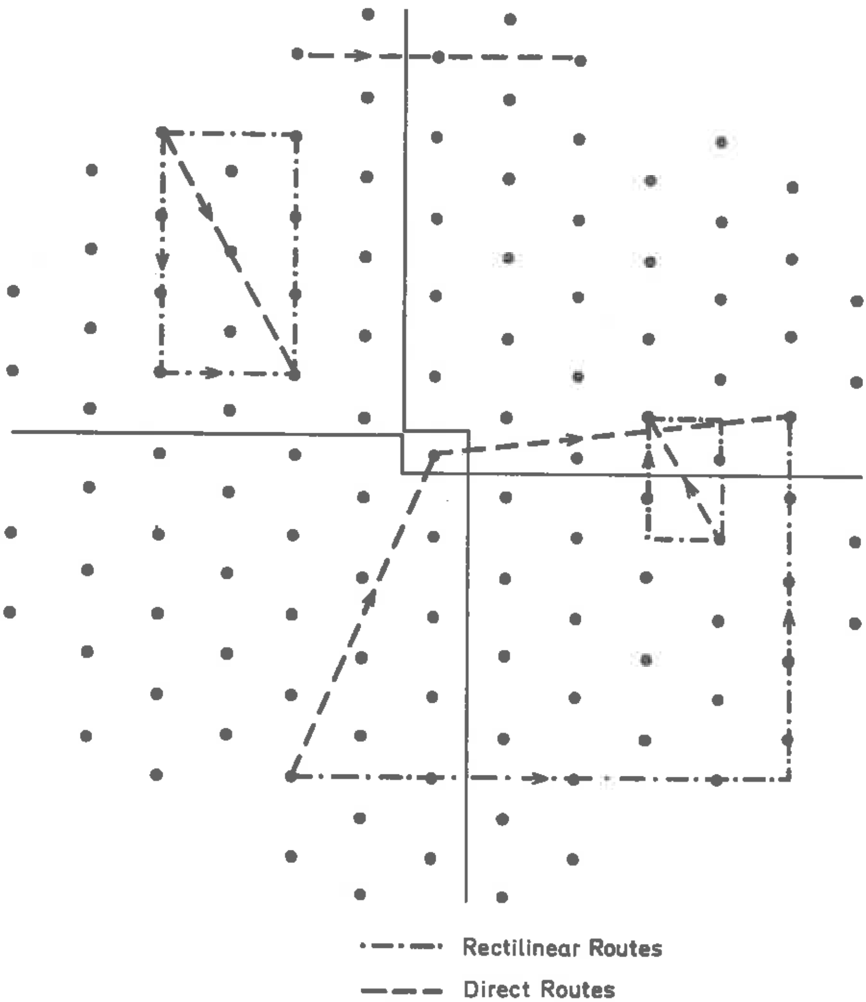


FIGURE A3 Alternative ways of travelling between various sets of points

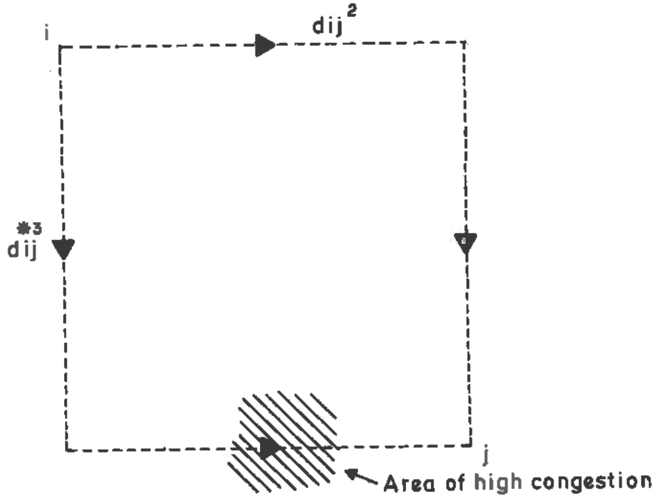


FIGURE A4 Effect of longestion on alternative rectilinear routes