Working Paper 282

TOWARDS A CONCEPTUALISATION OF EVOLUTION IN ENVIRONMENTAL SYSTEMS

J.R. Beaumont

School of Geography,
University of Leeds,
Leeds LS2 9JT.

June 1980.

(From 1st October 1980:

Department of Geography,
University of Keele,
Keele,
Staffordshire.)

CONTENTS:

"There are many difficulties facing those who have interests both in the natural environment and in the man-made environment together with the problems of their interfacing. One of the currently most intractable is that, having been imbued with the ecosystem model, with the emphasis on balance, equilibrium, cycling and stability, scholars are increasingly faced with the methodological necessity of also accommodating active control involving the impelling of systems on time trajectories through sequences of state, each different, probably non-recoverable and presumably ever more adapted to the evolving needs of man in society".

Bennett and Chorley (1978, p.471)

1. INTRODUCTION.

1.1 Aims.

Geographers' investigations of the dynamics of environmental systems involve the spatial-temporal dimension of their social, economic, political and physical features. Over the last decade, however, locational analyses have generally excluded time, whereas theories of economic growth have generally excluded space. Although in particular contexts some justification can be made to examine purely spatial processes (Bennett and Chorley, 1978), there is a generaly necessity to fuse these strands of work; recently, a number of geographers have considered space-time systems (see, for example, Bennett and Chorley, 1978; Bennett and Haining, 1976; Carlstein *et.al.*, 1978; Hordijk and Nijkamp, 1977; and Martin and Oeppen, 1975).

An alternative approach towards this desired goal is outlined in this paper; it is primarily conceptual, introductory and speculative, focusing on explorations of new principles and a framework to enhance out comprehension and representation of basic processes in environmental systems. It is suggested that a new evolutionary conceptualisation in geography is long overdue, and that the work of Prigogine and his associates in the field of physical chemistry deserves the attention of geographers. This has been transferred into the domain of social systems in Jantsch's (1975) book, '*Design for Evolution*'. Whilst his interest was specifically in human design it is important to describe the general implications of the employment of this conceptualisation in an analysis of social system evolution. More specific comments related to planning are examined later.

The basic tenet is that it is plausible to see history as a series of fairly separate eras with individual characteristics rather than as a continuous progression of incremental alterations. Whilst it is, obviously, possible to offer reasons to corroborate either perspective (and, to a certain degree, there is authenticity in both views), in a variety of situations, there is a tendency for system responses to be non-incremental in nature - system organisation is restructured.

For example, in arguing that a long-term view of economic development and cultural change is necessary to adequately analyse the growth of the city system in Wales, Carter (1980) states that

> "...there are distinct phases of urban genesis. They are implicit in all texts on the history of town plan but are never translated into urban system terms. There is not a continuous process of urban creation but a number of distinct periods related to political or economic convulsion, or both".

In a similar way, Davis' model of landscape evolution was in terms of a sequence of three stages: youth, maturity and old age (see Chorley, *et.al.*, 1973).

As a general argument it can be postulated that whenever a human system is confronted by a set of problems that the 'conventional wisdom' cannot accommodate, new and original ways must be developed. Interestingly, such a conceptualisation is found to represent the history of science in Kuhn's (1962, 1970) much discussed treatise, '*The Structure of Scientific Revolutions*'. (Johnston (1979), for instance, has considered the usefulness and shortcomings of this framework in his description of paradigm origination and succession in post-war Anglo-American human geography). 'Normal science', in Kuhn's opinion does not '... aim to produce major novelties....'; rather, it seeks to extract the existing paradigm's inherent potency, by adding '... to the scope and precision with which the paradigm can be applied' (Kuhn, 1962, pp. 35-36; quoted by Johnston, 1979, p.16). This procedure can create results which are inconsistent with the established paradigm, and these form the foundation of a crisis and the ensuing generation of new paradigms which incorporate facets of both the original paradigm and the anomalous facts; Kuhn, in fact, recognised four phases which are recurrent: pre-paradigm research, normal science, crises, and revolution.

In brief, evolution in environmental systems can be thought to involve an interplay between processes of self-organisation and self-maintenance (see also Berry (1973) and Laszlo (1972)), and explanations should explicitly combine functional and morphological aspects of the processes involved.

1.2 Contents.

As a mathematical formalisation is employed, some discussion of this foundation is needed at the beginning. In Section Two, therefore, a detailed discussion of a mathematical representation of various systemic properties is undertaken. This acquaints the reader with the fundamental concepts, and permits a comprehension of the work by Prigogine and his colleagues which is described in detail in Section Three. A simple example from physical chemistry is examined; this facilitates a clear understanding of the integrated conceptualisation of system evolution, and, as the example has been analysed at a variety of different levels, it also permits the interested reader to build on this introductory account. Particular attention focuses on the complementary nature of deterministic and stochastic modelling, and some of the recent developments in bifurcation and catastrophe theory are discussed. Finally, some general implications for applied and theoretical geographical research are raised.

2. SYSTEM FORMALISATION.

2.1 System structure: some issues.

An essential prerequisite of this discussion is an exposition of some of the central concepts associated with systems, including their mathematical representation. A number of difficulties are also noted. Philosophical and methodological features of systems have been widely considered by geographers (see, for example, Ackerman, 1963; Bennett and Chorley, 1978; Chapman, 1977; Chorley and Kennedy, 1971; Harvey, 1969; and Huggett, 1980), and, therefore, in this sub-section, consideration will be restricted to aspects which will be used later.

In our present concern with system dynamics, geographers emphasis on spatial form rather than functional structure is of fundamental importance. Langton (1972) examined this discord in detail, and concluded that it had not been demonstrated that it is especially expedient to employ a systems approach in studying change in human geography. Emphasis is on morphology, recognising the existence of objects (the so-called Landschaft approach), and little attention is given to the explicit analysis of process and the production of the actual artifacts; comprehension of underlying mechanisms is extremely difficult without an understanding of functional linkages.

A basic tenet of the systems approach is that well-defined systems of interest can be perceived. Three distinct elements can be recognised: inputs, throughput and outputs. Flows of mass, energy, ideas, information, and so on are involved, and it is common practice to relate inputs and outputs, particular in forecasting. Often analysis of input-output relationships does not explicitly consider the complex interplay of a system's elements, and cannot uniquely ascertain the nature of a system; this is the so-called 'black box' approach.

A number of system classifications exist, although one based on functional characteristics is often employed. A tripartite division is recognised: open, closed and isolated systems (Chorley and Kennedy, 1971). Open systems exchange both energy and mass with their environment. For example, cities within a particular urban system have direct linkages with cities in other urban systems, and, in fact, the system in general is affected by external events. The significance of multi-locational, often multi-national, organisations is well documented. Closed systems are assumed to have exchanges of only energy and not of mass, whilst in isolated systems there is no exchange of either energy or mass.

The majority of environmental systems are open, although it is common to examine closed and isolated systems because of their relative empirical and theoretical tractability. In our present concern with evolutionary processes, it is interesting to refer to Pred's (1977) examination of urban growth. Two attributes are deemed by Pred to be of fundamental importance: the degree of system openness and the span and structural arrangement of its internal interdependencies.

At a philosophical level, it can be postulated that the complex interdependence necessitates the holistic quality of a systems approach. Moreover, the gestalt principle (that is, the entire system behaves in a manner which is not a straightforward reflection of the sum of the individual parts) has important implications in a mathematical representation of a system (see below). Linear models are used which have involved some suitable mathematical transformation, although often the non-linearities have been disregarded or approximated in some distorted way. Multiple steady-states, however, are only present in non-linear models, and their existence, indicating the possibility of more than one future, is shown to be of fundamental significance in the conceptualisation of system evolution outlined in this paper. The non-linearity, which is likely to characterise any mathematical models representing an environmental system (Bennett and Chorley, 1978), is a fundamental feature of the present analysis. Unlike linear systems which have one unique steady-state, non-linear systems possess multiple steady-states. This results in some analytical problems. Stability for example, is related to properties in the neighbourhood of a specific steady-state, and is, therefore, a local phenomenon; analysis, particularly of system dynamics must, however, take a global perspective, encompassing all the different steady-state regimes. In the next sub-section, a simple logistic population growth model is described to illustrate this idea of multiple steady-states (see also Huggett, 1980).

It is possible to think of a system's phase space as comprising of a number of well-defined zones, which are termed basins of attraction. (The particular arrangement of basins in state space is dependent on the values of the parameters). Each basin possesses a particular attractor, a stable steady-state, which can take a variety of forms (see below). This paper is essentially concerned with a system's change from one basin to another at a critical point, and its resulting qualitative alteration. The extension of a particular basin is an important factor in a system's degree of persistence, because it indicates the largest perturbation that will not result in structural change. (Hirsch and Smale (1974) consider how Lyapunov functions can assist in this direction). The description of system evolution as a series of phases (with transitions between different steady states at specific times) presupposes the existence of more than one attractor, and, whilst this is to be expected in most dynamical systems, (mathematical) theoretical justification only exists for a special type of system, a gradient system. (This relates to the recent developments of catastrophe theory in which there is a growing literature (Postle, 1980; Poston and Stewart, 1978; Thom 1975; Woodcock and Davis, 1978; and Zeeman, 1977)).

At this junction, an illustration of some of these notions is worthwhile. It is noted that Thom's (1975) elementary catastrophe theory classifies transitions between attractors (so-called 'catastrophes') for gradient systems, and the Hopf bifurcation describes a system which alters from a point attractor to a limit cycle as the control parameters are changed through a critical value (see, for example, Marsden and McCracken, 1976). (Elementary catastrophe theory is constrained to a system with up to five state variables and two control parameters, whilst the power of Hopf's theorem is that there is no restriction on the number of state variables). May and Oster (1976) have demonstrated that complex dynamical behaviour can be found in ecological models represented by simple difference equations. For example, with parameter changes, a system may evolve from a stable point through a sequence of bifurcations into stable cycles of varying periodicity and chaotic oscillations and finally to divergent behaviour leading to ultimate extinction; Wilson (1979) has argued that such transitions may appear in spatial dynamics, and Beaumont *et.al.* (1980) found these transitions in their simulations of the effects on the evolution of urban spatial structure of an increase in the price of petrol. At a philosophical level, the existance of a chaotic regime in which the output is indistinguishable from random means that there is no mathematical argument to suggest that it is an outcome of a stochastic, rather than a deterministic, process. With regard to this point, partiuclarly in view of the use of statistics rather than mathematics in geography, it is interesting to note that mathematical modelling is especially significant in theoretical geomorphology (see, for example, Thornes and Brunsden, 1977).

Whatever the specific characteristics of a non-linear system, the existence of discrete basins of attraction has significant corollaries for the modelling of system evolution. For each stable, steady-state, there is an associated region or 'domain' of attraction; the steady-state acts as a type of magnet in this sphere of influence, attracting any initial state within it. The question of a system's persistence in a particular state can be examined, and transitions between different regimes permits an illustration of the threshold phenomenon which is a major feature of many processes taking place in environmental systems. Such a system is termed metastable (Chorley and Kennedy, 1971). It is interesting to note Chorley and Kennedy's (1971, p. 239) phase space description of system behaviour, because, in their discussion on thresholds, they recognised the occurrence of

> "... discontinuities in the phase space separating it into zones of differing system economies or as deformities in the manifold".

A common structural component is the existence of hierarchically arranged sub-systems, and, in fact, Mesarovic and a number of colleagues (Mesarovic *et. al.*, 1970; Mesarovic and Takahara, 1975) have developed a mathematical theory of hierarchical systems. Whilst not every system is of this type, hierarchical ordering is found in geographical systems (for example, the vast literature on central place theory), and it has interesting implications for man's role in co-ordinating systems.

All these systemic properties must be included in an adequate formulation of system evolution. Towards that end, a mathematical representation is employed, and it is described in the next sub-section.

2.2 A Mathematical representation: dynamical systems theory.

A number of alternative means of system representation are available. Geometry is widely viewed as the language of spatial form by geographers (see, in particular, Bunge (1962) and Harvey (1969)); for example, the growing literature on the geometry of central place systems is exemplified by the work of Dacey and his colleagues (see Alao, *et.al.*, 1977). Such representations are static formulations and no comprehension of process is tenable (Gale, 1976; Sack, 1972).

> "Indeed, if there will be anything made of geography, it will undoubtedly come as the result of a shift toward a perspective based on the role of geographic theories as instruments and agents for the design of change". (Gale, 1976, p. 29).

A study of system change is, therefore, difficult. The basic problem is finding an adequate language to express dynamical behaviour. It is suggested that mathematics is an appropriate formal language.

In general, social scientists, with the exception of economists and regional scientists, employ statistical, rather than mathematical techniques; whilst an awareness that the reduction to a mathematical formulation may undermine the very nature of a problem, the usefulness of such expressions has been exemplified by some geographers (see Thomas and Huggett, 1980; Wilson, 1978). Specific details of a mathematical representation pertinent to the present context will be given below, but the problem of balancing mathematical elegance and tractability with realism must be stressed at the outset (King, 1976); mathematical models must be operational. The analytical power and rigour of a mathematical representation often proves extremely useful in investigations of complex systems which comprise of a large number of interacting components. As the framework of non-equilibrium thermodynamics and 'order through fluctuation' demonstrates, mathematics is both a tool of analysis and a language of conceptualisation (c.f. Gould, 1975). Basically, it has been demonstrated that in open systems, which are far from thermodynamic equilibrium and whose (macroscopic) state variables' behaviour can be represented by non-linear differential equations, random fluctuations drive the system to new spatial-temporal structures; this process is termed 'order through fluctuation' and is discussed in more detail later. Fundamentally, it permits a study of system evolution, the topic of this paper, and an attempt is made to place, using a mathematical representation, dynamical analyses of environmental systems into a coherent and unified conceptual structure.

A particular system state can be defined by a set of state variables with given values. These represent individual elements of a system, and their relationships with each other can be portrayed by suitable functions. The magnitude and nature of a function's impact is determined by their associated set of parameters. They are often assumed to be invariant over time, although, as is shown below, interesting results are generated if alterations are introduced, (see also Bennett, 1975, 1975a; Brown, *et.al.*, 1975; and Hepple, 1979). Whilst difficulties are associated with the actual definition of the variables, further problems are present because of the existence of (positive and/or negative) feedbacks within the system. These loops lead to an element of ambivalence in the interpretation of dependent and independent variables, and, in so doing, creating complications in parameter estimation.

The concept of a dynamical system refers to the way a system evolves over time, and, therefore, a description of system dynamics must include relationships between system states at various times, specifically the sequence of states. Different dynamical properties can be found, including unlimited growth (or decline), periodicity, and trajectories to steady-states. This can be formally written as a set of simultaneous differential equations,

$$\frac{d\underline{x}}{dt} = f(\underline{x}, \underline{\alpha}) \qquad (2.1)$$

where $\underline{x}$ is the set of state variables and $\underline{\alpha}$ is the set of parameters. (In conjunction, with the gestalt principle, notice that the no state variables can be considered separately because they are all functions of each other). For example,

$$\frac{dx_1}{dt} = f(x_1, x_2, \ldots, x_n, \alpha). \qquad (2.2)$$

Attention often focuses on a system's steady-state, that is, its time independent state. As Huggett (1980) points out, confusion has been generated by the apparent interchangeability between the terms, 'steady-state' and 'equilibrium'. Basically balance achieved in a (theoretically) isolated system is called equilibrium, but when flows of matter and energy are included change is important and balance is refered to as a steady-state (Chorley and Kennedy (1971) examine the variety of 'equilibrium' concenpts in detail). As the discussion is concerned with system dynamics, it is important to remember that an equilibrium-centred perspective is essentially static and traditionally, it offers little insight into evolutionary processes and structural change (see Marchand, 1978). A basic notion contends that a spatial structure will evolve to an equilibrium pattern, and, therefore, distance from equilibrium is seen as a major driving force behind system change. Central place theory highlights the restrictions of this approach (Beaumont, 1979),

> "The prime assumption ... is an evolution to a final state of order, with the concomitant notion that any departure from that state of order represents a clearly preliminary situation which will soon be 'improved' so as to conform to the theoretical mechanism" (Vance, 1970, p.8).

Following Bellman (1968), Macgill and Wilson (1979) have argued that steady-states can be incorporated into dynamic structures, and, although this results in a representation of additional dynamical properties, this introduction of the time dimension into the static, equilibrium state fails to consider the dynamics of individual elements from the outset. The discussion of non-equilibrium thermodynamics, in Section Three, demonstrates that micro-level variations act as an internal or endogenous driving force, and, therefore, exclusive study of steady-states, which are macroscopic descriptions of a system, is inadequate and must be extended. (It should be noted that the concept of equilibrium applied in geographical work is not usually in the thermodynamic sense. For instance, a high level of system organisation, can still be identified as a steady state, whereas evolution towards thermodynamic equilibrium involves an increase in disorder).

For the system described by equation (2.1), a steady state is when

$$f(\underline{x}, \underline{\alpha}) = \underline{0}. \qquad (2.3)$$

Determination of this state is usually analytically intractable, although numerical integration has been facilitated by computer developments. For instance, numerical analyses of shopping models have demonstrated the possibility of interesting changes in behaviour (see Beaumont, *et.al.*, 1980; Beaumont and Clarke, 1980; Wilson and Clarke, 1979 for more details). However, in this paper concern focuses on the qualitative theory of differential equations and dynamical systems, and the interested reader is referred to the volume by Andronov and his associates (1973) for a more extensive discussion.

The qualitative perspective adopts an essentially geometrical viewpoint, which can be traced back to the work of Poincaré in the last century. This geometrical representation of a solution allows an indirect examination of its analytical characteristics (Lefschetz, 1963). Particular attention centres on the phase space description of a system. The phase space has a dimension equal to the order of the vector of state variables, which enables a unique labelling of a system's state in the space. It, therefore, permits a global analysis of system behaviour for any initial conditions. A phase space representation is exemplified below by an examination of the variety of solution trajectories which two interacting variables may exhibit. The recent books on systems by Bennett and Chorley (1978), Chapman (1977) and by Huggett (1980) include a discussion of phase space analysis, although a more detailed exposition is offered by Sansone and Conti (1964).

Phase Space Representation of Systems Behaviour.

Given two interacting state variables, x and y, the associated phase space is two dimensional. Such a representation has been widely employed in the ecological literature to consider predator-prey systems. A point (x_*, y_*) represents a system state, and a trajectory plotted in the phase space plots the change of a system from any arbitrary initial state.

In the neighbourhood of a steady-state, it is possible to make a linear approximation of the system's set of differential equations (as described by equation (2.1)), which enables the local character of the paths to be ascertained. As Andronov, *et.al.* (1973) and Jordan and Smith (1977) demonstrate, this method permits a classification of the steady-states, and in so doing, enhances the comprehension of a system's behaviour. Moreover, it determines the stability of a steady-state, a topic which is examined in more detail in the next sub-section.

Intuitively, it is possible to imagine a large variety of trajectories in a phase plane. Jordan and Smith (1977), in fact, present a general classification for a linear system which includes: nodes, inflected nodes, spirals, saddle points and centres (see Figure 1); a limit cycle, illustrated in Figure 4, can also be added to this list. (Whilst the linear techniques prove analytically tractable, it is necessary to be aware of a number of problems. Limit cycles cannot result from linear systems with constant coefficients. Fortunately, alternative approaches, such as the Hopf bifurcation theorem and the Poincaré-Bendixson theorem (see, for example, Hirsch and Smale, 1974; Jordan and Smith, 1977; Marsden and McCracken, 1976) are available to demonstrate the existence of limit cycles. Moreover, employment of the technique assumes a congruency between the phase paths of the original, non-linear equations and those of their corresponding linearised equations near the steady-state. This is true, in general, for nodes, spirals and saddle points, but not for centres).

These diagrams can be assembled together in inumerable different ways to present a system's behaviour as a combination of several types of steady state. (Stable) steady-states are 'attractors', and have trajectories leading to them; if they are unstable, trajectories leave them and they are termed 'repellors'. A saddle point is a special type of steady-state. The majority of trajectories are repelled, although there are two trajectories which act as attractors. These are called a separatrix, and they are particularly important in the later discussion on criticality because the separatrix separates the phase space into two zones with trajectories on either side tending towards different stable steady-states. Attention is drawn to the fact that slight parameter changes may alter the form of a trajectory; this feature is of fundamental and central importance to the present discussion.

The existence of multiple steady-states has already been discussed and this is further exemplified now. A simple model which possesses multiple steady-states is the logistic growth, which is commonly used in population modelling. As Figure 2 illustrates, there is a population saturation level (or carrying capacity). For example, for a population x, the rate of growth can be written as a non-linear, differential equation

$$\frac{dx}{dt} = (g - cx)x \tag{2.4}$$

where g is the intrinsic growth rate (excluding carrying capacity) and c relates to the effect of the carrying capacity; specifically, it is the rate at which the addition of population reduces its intrinsic growth rate. For a steady-state to occur, that is when the population stops growing (or decreasing),

$$gx = cx^2 \tag{2.5}$$

because

$$\frac{dx}{dt} = 0. \tag{2.6}$$

This situation arises for two population values, x equal to zero (extinction) and x equal to g/c (the saturation level). Huggett (1980) presents other examples in which multiple steady-states exist.

2.3 Stability of steady-states.

As with the concept of a steady-state, it is imperative to possess a clear and unambiguous definition of the concept of stability. Unfortunately, the employment of the word 'stability' is not unequivocal, and, therefore, it is necessary to briefly consider the concept to avoid confusion later. For the purposes of this paper, it is important to distinguish between 'classical' stability and structural stability. (The latter is considered in more detail in a later sub-section). Classically, stability analysis was concerned with perturbations to a system's initial conditions or its external environment, whereas recent interest in structural stability is concerned with perturbations to a system's structure itself. In both cases, analysis focuses on whether system behaviour is altered by the disturbances, although 'classical' stability analysis involves an examination of trajectories in the neighbourhood of a steady-state for one particular system and structural stability involves an examination of trajectories for a set of systems which are only slightly different.

Precise mathematical definitions of 'classical' stability can be found in numerous text-books, such as Hirsch and Smale (1974) and Jordan and Smith (1977). It has already been noted that stability can be analysed by using the method of linearisation, although Lyapunov's direct method is the most widely applied analytic technique of stability analysis (see, for example, LaSalle and Lefschetz, 1961). Once again, the precise meaning is important - in 'the sense of Lyapunov'. Basically, if one can find a so-called Lyapunov function (a function that is positive definite and whose first-order derivative is negative semi-definite), necessary and sufficient conditions for stability can be given. Unfortunately, the derivation of such a function is not a trivial problem; no general approaches exist for their construction, although it should be remembered that an inability to discover an associated Lyapunov function does not imply that the system is unstable. To examine the stability of a steady-state of equation (2.1), it is unnecessary to explicitly solve the differential equations. A further advantage of this method is its geometrical insight into stability.

An intuitive idea of the notion, however, is given by the diagrams given in Figure 1 - a steady-state

> "... is stable if nearby solutions stay nearby for all future time". (Hirsch and Smale, 1974, p. 185).

Lewontin (1969) undertook a comprehensive review of the alternative meanings given to stability and May (1973) examined a family of ecological models whose dynamics are described by coupled differential equations, and, in fact, stability is an important topic around which a vast literature has developed, but analysis will be restricted to that which has direct relevance here - structural stability.

Structural stability is concerned with the qualitative effects upon a system's steady-state solution produced by continuous alterations of its parameters. Thom's (1975) catastrophe theory demonstrates the important relationship between structural stability and morphogenesis (although bifurcation theory, which is not restricted to gradient systems described by a potential function, is likely to have wider applicability). Introductions to this field which utilise geogrphical applications have been recently undertaken by Beaumont and Clarke (1980) and by Wilson (1980).

More specific details are explored later when the principle of 'order through fluctuation' is examined. For the moment, however, it is important to comprehend the general significance of the concept in modelling system evolution. It can be argued, in fact, that, due to the inherent simplifications and uncertainties which characterise all mathematical models, a system's state should not generally be altered by a small perturbation of its parameters.

2.4 Stability and Complexity.

Debate by ecologists on the relation between stability and complexity has a number of significant implications for the evolution of environmental systems, especially for man's role (see Chadwick, 1977). As we know, stability relates to the tendency to return to a particular steady-state. A system's complexity concerns the number and nature of the individual linkages within it and between its environment. (It should be noted that in a representation of a system, complexity is not wholly a system characteristic, but is also related directly to an analyst's perspective). This important topic is much more complicated than it initially appears, and a number of viewpoints will be considered here. Indeed, no uniquivocal general agreement exists on whether increase complexity enhances stability or not. Do complex systems, for example, remain stable after undergoing structural perturbations (the addition or subtraction of sub-system connection)?

MacArthur (1955), for instance, argued that there was a direct relationship between system stability and the number of linkages. A similar conclusion was formed in Elton's (1958) empirical investigations, because the variety of mechanisms in a complex system facilitates the absorption of stress. In fact, Wilson and Bossert (1971) were able to write that the explanation of observed ecological stability as being ecosystem complexity was part of the disciplines 'conventional wisdom'.

More recently, the development and analysis of mathematical mdoels of ecosystems has led to a contention of this viewpoint (see May, 1973; and Maynard Smith, 1974). For example, May (1973) shows mathematically that randomly compiled, complex systems are usually less stable, and never more stable, than less complex ones; thus, it would appear that the relation between stability and complexity is not 'a mathematical truism'.

> "The emergent moral is that theoretical work should not try to prove any general theorem that 'complexity implies stability', but instead should focus on elucidating the very special sorts of complexity, the singular strategies, which may promote such mathematically atypical stability". (May, 1973, p.4).

Towards this end, Siljak (1978) has derived stability conditions for structural perturbations using the theory of differential equations and inequalities. He examines the complexity-stability issue in terms of a system's so-called 'connective stability'; that is, analysing system stability in terms of the effects of modifying its inter-relationships. For example, does failure in one sub-system result in an enormous change in the behaviour of a complex system?

Siljak (1978) differentiates between two common forms of sub-system interdependence - competition and cooperation - and mathematically presents stability conditions for both cases. These conditions have important implications for system control in general. In the competitive situation, stability is dependent on all the individual sub-systems being stable in isolation and on the magnitude of interactions between them being limited. In contrast, in the cooperative situation, system stability does not require each subsystem to be stable in isolation, because cooperative interactions, which increase system complexity, can achieve this. However, the latter situation is connectively unstable, because the system is dependent on particular sub-system interactions and, therefore, is unlikely to stabilise after structural perturbations which have altered the inter-relationships of its sub-systems. For a system to be connectively stable, the stability of individual sub-systems or groups of sub-systems must be independent of each other. With regard to system control, a systems level of achievement must be traded off against its connective stability. If sub-systems cooperate, it is likely that a system's overall performance will be enhanced, but there remains the possibility of system collapse because of some structural perturbation. Therefore, perhaps system control should explicitly concentrate on the stability of individual subsystems.

In summary, therefore, stability is dependent on the type of sub-systems and their interactions, and no simple answer to the stability-complexity issue exists. Connective stability is significant, because it is often difficult, if not impossible, to define the inter-relationships between sub-systems exactly. Moreover, these findings provide a useful foundation for the conceptualisation of system evolution described in the next section, but some of the ideas on complexity can be directly extended in a discussion of hierarchies which is undertaken in the next sub-section. Simon (1962), for instance, has argued that, intuitively, evolution of complex, systems is stable if it is a hierarchical process involving linking of stable subsystems.

2.5 Hierarchies.

Vertical complexity, involving interactions between different structural levels, such as trophic levels, must be considered in conjunction with horizontal complexity, which involves linkages at each level (Harte and Levy, 1975; May, 1973; Maynard Smith, 1974). Hierarchies are important structural components of various systems, and whilst this organisational principle is of interest per se, in this sub-section, some comments are made regarding evolution from a hierarchical perspective. (It should be noted that Atkin's (1974, 1976, 1981) q-analysis methodology offers an algebraic language for the definition and description of multi-dimensional system structure, although, at most, it is possible to infer dynamics from changes in the structure).

Although no universality is suggested, evolution has been viewed as a progression towards greater integration including additional levels of development (see Berry, 1973; Laszlo, 1972; and Simon, 1962). Indeed, Berry (1973, p.10), following Simon (1962), has recognised an 'architecture of complexity', and also stressed the potentiality of a system experiencing sudden transitions between different regimes. It is, therefore, possible to suggest a scheme which sees system evolution as a succession of stages of re-organisation (morphogenesis) and stabilisation (morphostasis), and Lampard (1968), for instance, applied these concepts in his analysis of 'The evolving system of cities in the United States'.

Whilst such conceptualisations may be attractive, further research is required to specify dynamic mechanisms. An obvious starting place is the interactions between different levels; this can be illustrated by two recent investigations of change in a central place system. Beaumont and Keys (1980) have outlined a general model of hierarchical diffusion which explicitly recognises the nesting between different orders of centres. Attention should also be drawn to Parr's (1980) analysis of structural transformations in central places in Southern Germany, and the possibility of extending it by relaxing the assumption of a successively inclusive hierarchy (c.f. Losch, 1944). Additional insights may be generated by combining features of both these studies to analyse diffusion in Parr's more flexible 'general hierarchical model'.

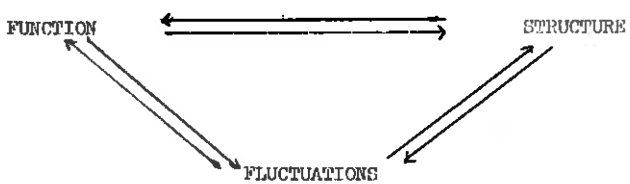
Moreover, it can be suggested that a hierarchical organisation, combining aspects of the regulatory restrictions of the whole and the degree of autonomy of the parts, will enhance a system's adaptability to local fluctuations and therefore, its structural stability. Siljak (1978) has demonstrated mathematically that dynamical, stable systems can be generated by hierarchic feedback control. Specifically, each sub-system is stabilised by feedback controls. Furthermore, processes of differentiation and integration facilitate the development of this ordering principle. Interestingly, Pred (1977) has observed that the hierarchical structure of city systems in advanced economies is actually reinforced (exhibiting long-term stability with respect to both the rank and relative size of the largest cities), because of spatial biases in information flows resulting from the existence of a hierarchy.

3. SELF-ORGANISATION IN NON-EQUILIBRIUM SYSTEMS.

3.1 Introduction.

Many of the concepts examined in the previous section are of significance in the conceptual framework for a dynamic theory introduced in this section. The structure was developed, by Prigogine and his associates in Brussels, in the field of non-equilibrium thermodynamics (Nicolis and Prigogine, 1977; Prigogine, 1978). Although the original theoretical and applied expositions stemmed from investigations in physical chamistry, some of the fundamental notions have been employed in urban and regional analyses. For instance, Allen and Sanglier's (1979) paper on the evolution of a central place system, which was derived from a more extensive research programme (Allen, *et.al.*, 1978), and sections of Isard and Liossatos's (1979) book on spatial dynamics are founded on the theory of non-equilibrium thermodynamics.

Fluctuations only play an important role in the self-organisation process at bifurcation (or critical) points by determining the specific state which the systems evolve to; away from criticality, evolution of a system is described by deterministic equations, whereas, in the neighbourhood of a bifurcation point, stochastic (microscopic) elements are of paramount importance (see Figure 3). It must be remembered that critical points are reached, in the first case, by changes in parameter values over time. Nicolis and Prigogine (1977) schematically portray the complementary role of determinism and fluctuations in system evolution as



For example, a system's function is described by differential equations, its various spatial-temporal structures arise from the instabilities inherent in the system, and the fluctuations actually trigger the transitions between states at critical points.

It is noted that although elementary catastrophe theory classifies the types of transitions between stable regimes in (gradient) systems, this framework presents a mechanism for change through the fluctuations.

3.2 Non-equilibrium thermodynamics.

In physics, it is common practice to describe development in terms of the second law of thermodynamics; that is, there is the tendency for a system to become increasingly disorganised as it progresses inexorably towards a state of maximum entropy. Although pointing the direction of a system's evolution – the so-called 'time's arrow' (Angrist and Hepler, 1973) – no indication of the path to this ultimate state is offered by the law (Ramsay, 1971). This notion of advancing disorganisation is, in general, incongruent to the history of societal development, which has involved greater organisational complexity and integration.

However, the second law is restricted to a closed system (see Bertalanffy (1950) for a discussion of system closure). (Wilson's (1970) employment of entropy in urban and regional modelling, for example, assumes a closed system). In contrast, if a system is believed to be an open system, as it can be argued for many environmental systems, it does not necessarily have to follow a trajectory to a state of disorder in order to be consistent with the second law. For example, by the second law of thermodynamics, entropy production (dS) in a closed system is non-negative, that is,

$$dS = d_i S \geqslant 0 \qquad (3.1)$$

where d_iS is the internal entropy production. At thermodynamic equilibrium,

$$dS = d_iS = 0. \tag{3.2}$$

If the system, however, is open, making exchanges with the environment, it is necessary to extend the equation for entropy production. Now

$$dS = d_iS + d_eS \tag{3.3}$$

where d_eS is the entropy exchanged with the environment. Although equation (3.1) is always satisfied, d_eS does not have a well-defined sign. Thus, the net change of entropy need not necessarily lead to increasing disorder; if

$$-d_eS > d_iS \tag{3.4}$$

entropy will decrease. Imports of 'negentropy' (negative entropy) permit a system to organise itself; and Chapman (1977), for example, examined this feature with respect to human and environmental systems.

Self-organisation to states of lower entropy can, therefore, occur without violating the second law of thermodynamics. An important characteristic of such systems is that they are not at thermodynamic equilibrium, but 'far from equilibrium'. Multiple steady states exist in open, non-linear systems functioning away from thermodynamic equilibrium; in the literature, these systems are frequently referred to as 'dissipative structures' (Lefever and Nicolis, 1974; Nicolis and Prigogine, 1977), because of the presence of a large flux between the system and the environment. Indeed, dissipative structures cannot exist independently; linkages with the environment are necessary for their development and maintenance. In fact, although closed systems are frequently assumed in both practical and theoretical studies in order to isolate phenomena of interest (for example, see one of Marshall's (1969) seven diagnostic criteria for a central place system), in reality, they are a scarcity; communication networks, for instance, extend beyond the boundaries of urban and regional systems in both phenomenal and spatial terms.

The presence of multiple steady states provides a connection to the previous discussion of bifurcation and catastrophe theory. New regimes of qualitatively different dynamic behaviour relate to an enhanced interaction between system and environment, and this behaviour has been termed, 'evolutionary feedback' (Babloyantz, 1972). A number of these features are developed in the following sub-sections, although the next sub-section offers an example of a dissipative structure which has been widely investigated by the Brussels group.

3.3 The 'Brusselator'.

Since Turing's (1952) work on morphogenesis, the potential occurrence of bifurcations in chemical kinetics has become well-known. Extensive experimental and theoretical investigations by the Brussels group have demonstrated that chemical instabilities can lead to spatial-temporal re-organisation. Their model, which is sometimes referred to as the 'Brusselator', will be outlined in this sub-section (and the interested reader is directed to Nicolis and Prigogine's (1977) book for a more detailed discussion), because many of the concepts seem appropriate in the modelling of environmental systems. An extension incorporating diffusion is mentioned and two additional examples provide simple illustrations of the ideas. The development of spatial ordering from a uniform state, for instance, has some parallels with the growth of a central place system.

The reaction is described as a trimolecular scheme incorporating two intermediary products (X and Y)

$$\begin{aligned} A &\rightarrow X \\ 2X + Y &\rightarrow 3X \\ B + X &\rightarrow Y + D \\ X &\rightarrow E. \end{aligned} \qquad (3.5)$$

To maintain the system 'far from equilibrium', the concentration of the products, A and B and D and E, inputs and outputs, respectively, are kept constant. Interest focuses on the chemical kinetics of the intermediary products, X and Y, whose evolution can be represented by the following differential equations,

$$\frac{\partial X}{\partial t} = A + X^2Y - (B + 1)X \qquad (3.6)$$

and,

$$\frac{\partial Y}{\partial t} = BX - X^2Y. \qquad (3.7)$$

The derivation of these equations can be readily recognised from the schematic description of the reaction. The magnitude of interaction is proportional to the product of state sizes; that is, the law of stoichiometry is assumed to hold (which is the same as the law of mass action employed in the Lotka-Volterra ecological systems (see later)).

The steady state concentrations for the intermediary products, X_o and Y_o, are

$$X_o = A \quad ; \quad Y_o = \frac{B}{A} \qquad (3.8)$$

and the investigation of the linearised equations around these states enables an analysis of their stability. The solution, given by equation (3.8) becomes unstable when

$$B > 1 + A^2. \qquad (3.9)$$

A 'limit cycle' is found beyond this critical value of B, and this is an example of the important Hopf bifurcation (see Marsden and McCracken (1976) for an examination of this topic). As Figure 3 shows, the periodic paths followed by X and Y occur independently of the initial conditions.

It is interesting to compare this result with the oscillating behaviour found in ecological systems, (which are founded on the much-discussed Lotka-Voltera equations) and the change in behaviour found in a simple population migration model. A simple prey-predator ecology, comprising of one prey and one predator (whose population sizes are represented by x_1 and x_2, respectively) can be modelled by two, non-linear, coupled, differential equations

$$\frac{dx_1}{dt} = \alpha x_1 - \beta x_1 x_2 \tag{3.10}$$

$$\frac{dx_2}{dt} = \gamma x_1 x_2 - \varepsilon x_2. \tag{3.11}$$

α represents the intrinsic birth rate of the prey population and ε represents the intrinsic death rate of the predator population. The coupled terms, $\beta x_1 x_2$ and $\gamma x_1 x_2$, represent the interactions between the predator and the prey according to the law of mass action and β and γ are parameters. Two steady-states (x_1^*, x_2^*) exist: the trivial case when both the prey and predator are extinct and when

$$x_1 = \frac{\varepsilon}{\gamma} \; ; \; x_2 = \frac{\alpha}{\beta}. \tag{3.12}$$

Figure 1 shows a phase-space representation of prey-predator dynamics. In contrast to the 'Brusselator', the periodicity is dependent on the initial conditions and it is called a centre rather than a limit cycle. This is of fundamental significance with regard to structural stability, which is considered further in the next sub-section. This system is structurally unstable, because small perturbations in the parameters describing system dynamics alter its orbit, and, therefore, an infinite number of different centres exist.

A further simple example of how system behaviour is altered by a change in a parameter value is given by Allen *et.al.*'s (1978) somewhat contrived migration model. In a region, the attractiveness for a population x_1 is represented by $1 + x_1^2$, it is $1 + x_2^2$ for a population x_2. Thus

$$\frac{1 + x_1^2}{(1 + x_1^2) + (1 + x_2^2)} \tag{3.13}$$

is the relative attractiveness of the region for the population x_1. If it is assumed that the level of emigration of both population is dependent on their own size and that α is the parameter of interest which represents the regions' population density, the system can be described by two differential equations

$$\frac{dx_1}{dt} = \frac{\alpha(1 + x_1^2)}{x_1^2 + x_2^2 + 2} - x_1 \tag{3.14}$$

and

$$\frac{dx_2}{dt} = \frac{\alpha(1 + x_2^2)}{x_1^2 + x_2^2 + 2} - x_2. \tag{3.15}$$

Three steady-state solutions (x_1^*, x_2^*) exist: the equality case

$$x_1^* = x_2^* = \alpha/2 \tag{3.16}$$

and the two opposite cases

$$x_1^* = a, \text{ and } x_2^* = b \tag{3.17}$$

and

$$x_1^* = b, \text{ and } x_2^* = a \tag{3.18}$$

where

$$a = \frac{\alpha + \sqrt{\alpha^2 - 4}}{2} \quad \text{and} \quad b = \alpha - a. \tag{3.19}$$

From equation (3.19), it is clear that α equal to two is a critical value, separating qualitatively different stable steady-state solutions. Specifically, when α is less than two, the solution given by equation (3.16) is stable and the other two solutions are unstable and complex. However, when α is greater than two, these two solutions are stable and the solution given by equation (3.16) is unstable. Thus, as a region's population density increases, the situation where x_1 and x_2 coexist together eventually becomes unstable (at the critical value) and segregation occurs. (It is argued later that the particular resulting segregation is dependent on random fluctuations).

Analysis of the 'Brusselator' reaction can be extended to incorporate diffusion, and, in general it is noted that the non-linear coupling between reactivity and diffusional propagation can give rise to different, stable spatial structures and a variety of interesting bifurcations (Diekmann and Temme, 1976). Assuming, for simplicity, a one-dimensional meduim, the presence of diffusion can be included into the previous differential equations representing the concentrations of X and Y to give

$$\frac{\partial X}{\partial t} = A + X^2Y - (B + 1)X + D_X \frac{\partial^2 X}{\partial r^2} \qquad (3.20)$$

and,

$$\frac{\partial Y}{\partial t} = BX - X^2Y + D_Y \frac{\partial^2 Y}{\partial r^2} \qquad (3.21)$$

which should be compared with equations (3.6) and (3.7). D_X and D_Y are the diffusion coefficients, and the terms,

$$D_X \frac{\partial^2 X}{\partial r^2} \quad \text{and} \quad D_Y \frac{\partial^2 Y}{\partial r^2}$$

are based on Fick's law of diffusion (see Bennett and Chorley, 1978; Crank, 1975; Thornes and Brunsden, 1977; Thomas and Huggett, 1980). Furthermore, the boundary conditions are assumed to be the steady state, that is,

$$X(0) = X(1) = A \quad ; \quad Y(0) = Y(1) = B/A \qquad (3.22)$$

where

$$0 \leqslant r \leqslant 1. \qquad (3.23)$$

Given this general problem, investigation concerns linear stability analysis in order to examine whether the system possesses self-organisation processes.

A unique and spatially uniform steady state solution,

$$X_0(r) = A \quad \text{and} \quad Y_0(r) = B/A \quad ; \quad \forall r \qquad (3.24)$$

can be found. Additional steady-state solutions, however, are found for values of B greater than a critical value, B^{crit}, and, therefore, the possibility of a qualitative change in system behaviour is present. The determination of these bifurcation points is shown in detail by Nicolis and Prigogine (1977); in the present discussion, however, it is sufficient to note that it is feasible to model symmetry-breaking transitions by which a system's behaviour alters from a stable, spatially homogeneous state to a stable, heterogenous state. Thus beyond criticality, which defines a bifurcation point of the differential equations, the spatially uniform solution ceases to be stable. Moreover, Marinez (1972), using computer simulations, has demonstrated that similar results (although possessing a greater number of spatial-temporal patterns) can be derived by considering diffusion in two dimensions (see also Tobler, 1970).

The inclusion of diffusion in a dynamic model is a means of representing flux between areas. In fact, this reaction-diffusion framework could have wide applicability, particularly in physico-ecological systems. For instance, in their discussion of atmospheric pollution, Bennett and Chorley (1978, p.377) recognise two important components in addition to emission inputs:

> "(i) Transport and diffusion transfer functions, concerned with atmospheric transport and dispersive processes between spatial cell units,
> (ii) Reaction kinetics, involving meteorological inputs and photochemical transfer functions, describing rates of reactions occuring in the atmosphere as a function of pollution concentration, intensity of radiation, temperature, etc."

Empirical evidence has illustrated the existence of changing spatial concentrations of pollution, and it can be suggested that this framework may present interesting insights. Qualitative changes in system behaviour, for example, have been long recognised in fluid dynamics and in the general circulation of the atmosphere: for example, Ruelle and Rakens (1971) discuss

multiple bifurcations in relation to the Reynold number, which can be thought of as a parameter accounting for changes in the pattern of flow (laminar and turbulent flows are qualitatively different types of flow).
Furthermore, its general characterisation of spatial and temporal autocorrelation suggest the possible use for modelling features of socio-economic systems, for example epidemics (see Cliff, *et.al.*, 1975).

Recently, the concept of structural stability has been employed to discuss the evolution of urban and regional spatial structure. For example, Puu (1979; 1980), using an optimising model and applying the results of elementary catastrophe theory differently, has analysed the qualitative nature of structurally stable flows. In the model, there are two state variables, which are directly related to a two-dimensional, co-ordinate system representation of geographical space, and, using Thom's (1975) classification for two state variables, Puu specifically analysed the hyperbolic and elliptic umbilics in order to gain insights into how the spatial pattern of flows changed as parameters passed through critical points. In addition, using numerical experiments for illustrative purposes, Beaumont *et.al.* (1980) demonstrated that under different parameter conditions the evolution of urban spatial patterns can vary quite markedly.

In conclusion, the conceptualisation of a reaction-diffusion framework suggests, at least, a beginning for the analysis of spatial structure developments by geographer. Complex spatial-temporal regimes can be generated out of simple ones, because spatial interactions and diffusion can produce instabilities. This occurs when particular, slowly changing kinetic or transport coefficients cross critical values, at bifurcation points - the spatially uniform state becomes unstable and a non-uniform state results. The feature of diffusion-driven instability, which is seen to be of paramount importance in the establishment of spatially heterogeneous structures, deserves particular attention from geographers.

3.4 System evolution - structural stability.

The idea of a system's self-organisation, in the previous sub-sections, has only incorporated alterations of initial conditions; the discussion of 'Brusselator' does not examine the possibility of changes in the chemical kinetics, and, in fact, implicitly, assumes them to be constant throughout. The question of stability when perturbations may modify system kinetics is related to the concept of structural stability, which has been given recent impetus by the work of Thom (1975) and Zeeman (1977) on catastrophe theory. (In catastrophe theory, structural stability/instability is related to degenerate singularities, but it can also be considered with respect to positional changes of eigenvalues in the imaginary plane (see, for example, Marsden and McCracken's (1976) discussion of the Hopf bifurcation)). It is interesting to recall that in a paper entitled, 'Problems in the building of mathematical models for geographic systems', Robsman (1973) alluded to the significance of structural stability. In this sub-section, the concept will be illustrated by a simple example, because this is one of the fundamental notions in the conceptualisation.

Structural stability is concerned with whether a system keeps its qualitative structure after perturbations; a system is deemed to be structurally stable if a minor alteration of a control parameter does not produce a change in the form that the function of the state variables takes. The concept of a system's stability of form can be illustrated, following the graphical argument used by Beaumont and Clarke (1980) , by considering the shape of the following simple functions.

Figure 5 portrays a whole family of curves for the function,

$$f(x) = x^2 + a \qquad (3.25)$$

where a is any real constant. We note that the minimum point is always at x equals zero, and the shape of the curves is the same.

In contrast, consider the function,

$$g(x) = x^3 - ax \tag{3.26}$$

where a is any real constant, and we know that

$$\frac{\partial g(x)}{\partial x} = 3x^2 - a. \tag{3.27}$$

It is possible to examine three different cases, depending on the value of the constant, a. First, when a equals zero, the function, $g(x)$, has a point of inflexion at x equals zero. Second, when a is greater than zero, the function, $g(x)$, has two stationary points, a minimum and a maximum. Third, when a is less than zero, the function, $g(x)$, has no stationary points (because there is no real solution to the problem of finding the square root of a negative number). These three cases are graphed in Figure 6, and it is clearly demonstrated that the forms of the curves are different. Thus, if a is progressively increased from a negative value to a positive value, there would be qualitative change in the function's structure.

More generally, structural stability involves an analysis of how domains of attraction, particularly their boundaries, are modified by changes in the value of a system's parameters. Specifically, interest focuses on the transition from being in the domain of one to steady-state to being in the domain of another. Following Casti (1979, p.52), this is illustrated by Figure 7.

> "The point x initially lies in the domain of the attractor P. Because of changes in the system dynamics, the domain of attraction of P shrinks from I to II, while that of Q expands from 1 to 2. The point x is now drawn toward Q rather than P. Of course, the locations of P and Q themselves depend upon the system structure, so the points in the figure are actually regions containing P and Q. What is important is that the regions of P and Q are disconnected".

This phase space representation is not used directly in catastrophe theory; instead, it gives the parameter values which cause a change in domain. Such alterations in the boundaries of particular domains relate to the idea of system persistence which is discussed in more detail in the section on planning.

3.5 Incorporation of stochastic elements into a deterministic framework.

To date, the discussion of dissipative structures has been wholly embodied within a deterministic description. That is, whilst appreciating the problem associated with multiple steady states, given initial conclusions and a set of differential equations, the trajectory of system development can be uniquely determined. Such a method refers only to the mean values of the macrovariables, and, accordingly, is apposite as long as the variance around the mean values remain small. In relation to this point, it is interesting to note Bennett's (1978) exposition on forecasting and control in spatial time-series analyses. Furthermore, in his review of concepts in economic geography, Haggett (1965) suggests that random variables should be incorporated into models because of indeterminancy at the individual level.

Whilst appreciating that, statistically, as the number of each state variable increases, the relative importance of the variance is reduced, the topic of criticality suggests that near an unstable point deviations from the mean value may be crucially significant. For non-linear systems, it is the random fluctuations which determine the particular stable solution branch a system follows when it is at a critical point (or on an unstable branch). The idea that a combination of deterministic and stochastic elements can facilitate a representation of dynamical processes is addressed briefly in this sub-section.

In order to encompass deviations around the mean value, Nicolis and Prigogine (1977) assume that chemical reactions give rise to a Markov process for individual state elements and they derive a so-called stochastic 'master equation' which represents the evolution of the probability $P(x, t)$ of finding the value x at time t. Isard and Liossatos (1979) exemplify this approach in their study of spatial dynamics.

Alternatively, it is possible to allow the parameter values themselves to fluctuate. May (1973), for instance, in his analysis of ecosystems examined this type of fluctuation. Another simple approach is to incorporate the fluctuations directly into the differential equations in the following manner

$$\frac{d\underline{x}}{dt} = f(\underline{x}, \underline{\alpha}) + \underline{r} \qquad (3.28)$$

where r is a small random number which is only a significant feature in the system dynamics near a critical point.

It has already been illustrated that the macroscopic reference state, around which local fluctuations are considered, is linked with the system-environment flux which maintains a system away from thermodynamic equilibrium. In addition, non-linearities present in the transition probabilities have been shown to be significant (Nicolis and Prigogine, 1977); this is, obviously, of paramount importance in attempts to analyse the effects of diffusion within this framework. In fact, diffusion links local fluctuations to the macroscopic description of the system, and can actually dempen the effect of some fluctuations.

3.6 The Brussels group and human geography.

The essence of the work by the Brussel's group has now been outlined, and sufficient details have been given to make geographers aware that fruitful research can be undertaken in this direction. Thermodynamic non-equilibrium is a facet inherent in many of the phenomena examined by human geographers; and, consequently, the concept, 'order through fluctuation', can be considered a fundamental (spatial-temporal) ordering principle influencing the dynamics of evolving human and environmental systems.

The emerging conceptualisation encompasses an integration of two interdependent perspectives: the macroscopic and the microscopic. It is interesting to note that human geographers have considered such viewpoints in a variety of situations; an early example is the well-cited paper by Jones (1956) on 'Cause and effect in human geography'. The basic complementarity is portrayed in the structure which combines a macroscopic view, represented, for example, by (deterministic) differential equations, and a microscopic view, represented, for example, by stochastic formulations which examine the ramifications of particular (usually random) fluctuations. This conceptualisation, therefore, recognises necessary inter-connections between the micro and the macro levels; whilst a number of geographers have advocated an adoption of micro-level studies (see, for example, Hagerstrand (1973), and the review of micro-analytic simulation techniques by Clarke, Keys and Williams (1980)), it is essential to consider more than a number of independent, decision-making units. Such an outlook is particularly important if geographical studies are to be relevant to societal issues. The implications of specific public policies can only be examined if the macroscopic constraints on individuals' behaviour are incorporated into an analysis (see also Clarke, Keys and Williams, 1980). In fact, the consequences for planning raised by the conceptualisation outlined in this section are far-reaching, and are discussed in the next section.

Basically, central facets include criticality, and, consequently, system persistence, that is, whether a specific state can absorb a given fluctuation without being forced to another (temporary) state. With fluctuations less than a critical threshold, there is no evolution to a new structure.

The inclusion of stochastic elements in a representation of the evolution of a human and environmental system is essential to mirror significant random features. Monod (1974), for instance, presents a detailed and general exposition on this topic, arguing that particular structures are a result of stochastic processes. No mechanistic view of development is assumed in the work by the Brussels group, and, in fact, extensive simulations of urban system evolution each beginning with identical conditions, produced a variety of results (Allen, *et.al.*, 1978). Furthermore, this conceptualisation is able to describe society's 'progress' towards enhanced specialisation, integration and differentiation.

Two final comments onthe approach developed by Prigogine and his colleagues relate to the actual conceptualisation of the stochastic elements and to the possibility of the formation of new state variables. Time-series analysis, for instance, incorporates continuously changing system inputs, such as tastes and technology (see Bennett (1978) for an example of such applications). These features are included in this framework; however, it must be stressed that fluctuations are also characterised as endogenous, local characteristics. These are seen as a system's driving force, and, therefore, are related to the concept of structural stability and the idea of sudden transitions between states.

These changes in regime are described by alterations in the value of the state variables and, therefore, involve a movement of position in phase space. Such changes, however, do not necessarily involve description by new state variables. Prigogine and Lefever (1975) demonstrate how the modelling of self-organisation processes can permit the emergence of new variables; at this stage it is sufficient to be aware of this possibility and to note that Isard and Liossatos (1979) has examined such a process,and that,in considering the complexity-stability issue, MacArthur (1970) investigated the effect on stability of adding one species to a system of n-competing species (see also Case, 1980).

In summary, system evolution incorporates the complementarity of micro- and macro-characteristics. System description is at the macro-level, and, therefore, it possesses an average/mean perspective. Transition processes between different system states, which are macroscopic alterations, can result both from exogeneous linkages with its environment and from endogeneous fluctuations. Such a conceptualisation offers a framework for modelling evolution in human and environmental systems, and more detailed research into this topic, based on concepts outlined in a simple way here, deserves and requires further attention. Sufficient details, however, have been introduced to suggest that such a framework has significant implications for both practical and theoretical work. Some planning implications are considered in Section 4, and, in the next sub-section, a number of general remarks are made with regard to the pertinence of this conceptualisation to describe human systems.

4. PLANNING: THE ROLE OF MAN.

In his exposition on human design, which used some of the work of the Brussels group, Jantsch (1975, p.24) offered a general statement that

> "A real crisis of a serious but not yet catastrophic character may have the ultimately desirable effect of facilitating the inevitable mutation to a new regime",

and became more specific by arguing that

> "... the more freedom a system is able to live out and the more freely it can absorb or generate instabilities, the less random its behaviour will ultimately be and the greater the chances for new dynamic order".

A number of significant consequences for planning result from the present conceptualisation of an evolving system. Some relate to the previous discussion of stability and persistence, but further consideration is required because one of geographers' current, major tasks is seen to be an efficient control of environmental systems (see, for example, the books by Bennett and Chorley (1978) and by Chapman (1977) which are reviewed by Kennedy (1979)). Indeed, a major stimulant to the development of systems theory has been a wish to intervene in order to regulate events.

Current preoccupation with efficiency and optimal solutions commonly involves active control to either preserve a given state or restructure a system into a desirable state. In fact, a criterion for control efficiency is often the time to attain a particular steady state (Bennett and Chorley, 1978). However, with regard to some of the ideas relating to persistence, it can be suggested that such actions may produce transient benefits at the cost of a contraction of the particular state's domain of stability. That is, a system's persistence capabilities will actually be undermined. Consequently, there is a possibility of sudden, potentially disasterous, irreversible transitions resulting from apparently insignificant decisions. There is, therefore, an indication of a need to consider alternative courses of action.

At worst, appropriate planning strategies must not reduce a system's ability to counter perturbations. Otherwise, at a future time, some fluctuations, which could formerly be absorbed, will trigger a sudden transition between regimes. It is suggested that any system management should adopt a perspective founded on persistence rather than on stability; that is, there should be a shift away from the current focus on steady states to the broader features of attractor domains (Holling, 1976). The stability of states within a domain is less significant than the probability that a systems behaviour will change as it moves from one domain into another. Moreover, it is important to appreciate that a return to the original state is likely to be difficult, if not impossible - note the 'hysteresis effect' where a return to a previous state does not follow the original path away from it. One example, Blase's (1979) empirical investigation in the field of transport, has demonstrated this effect.

Thus, there is an implicit suggestion that planning cannot indefinitely constrain development processes within a given structure. It has already been postulated that change is inevitable (even if man desires otherwise), and, a further corollary is that such evolutionary processes should be identified and appreciated. Perhaps the role of planning should be thought of as one of assisting evolutionary processes. Obviously, transitions to new regimes will eventually occur, and, although it can be argued that a laissez-faire approach relying on invisible hands is more appropriate than one founded on stability, planners could act as catalysts in the evolution of different behavioural patterns.

These notions have many parallels with the Baconian philosophy that man can master nature only by obeying her (Chappell (1975), for instance, examines this philosophy with respect to underlying aspects of human ecology, and Smith (1973) stresses the need for man in harmony with nature). Accordingly, the common thesis that man's relationship with nature is an ever-increasing one of control and ascendency must be carefully interpreted. No proposal, however, is made to reduce man's intervention in a system's evolution; far from it, the approach may require increased control. Fundamentally, the implications of an emphasis on persistence must be appreciated - ultimately, man's power is restricted, although it is potentially crucial.

A number of present-day features of the planning process will remain of paramount importance. For instance, planning must be continuous and not exhibit discontinuities associated with the system's behaviour. In addition, results attained from monitoring and forecasting procedures will continue to provide essential information to planners (Bennett, 1979). The recognition of sudden transitions between states raises a fundamental problem with respect to current forecasting methods (Bennett, 1978; Haggett, 1973). Applications of (spatial-temporal) forecasts usually involve trend extrapolations (as exemplified by Chisholm, *et.al.* (1971) and by Cliff, *et.al.* (1975)), and, in so doing, they do not necessarily take account of the possibility of discontinuous behaviour (Hay, 1978). Research embodying concepts of bifurcation and catastrophe theory within a forecasting methodology is, therefore, required. The potentiality of alternative future geographies presents a philosophical and methodological challenge for geographers (Garrison, 1973), which must be addressed in order to consider alternative solutions to contemporary societal problems.

In looking to the future with its inherent uncertainites, a system exhibiting structural flexibility is, at least intuitively, highly desirable. (Note the problems associated with agriculture management founded on monoculture). Given this aim, it is more appropriate to consider the whole (stability) boundary of an attractor than confine attention to the specific steady state (unless all trajectories instantaneously, or rapidly, approach it - this is unlikely). This is because if a system is maintained near a boundary, its ability to act on new information is enhanced - a new state, for instance, may become desirable. This suggests that analyses must attempt to determine a system's set of critical points. For example, in a paper concerned with catastrophe theory, Beaumont and Clarke (1979) demonstrate how concepts of elementary matrix algebra are useful in ascertaining and interpreting critical points.

At first sight, such a strategy may appear to be unwise and even detrimental. At criticality, for instance, minor fluctuations may result in a sudden transition to another, qualitatively different, unfitting domain. If, however, the system is maintained out of the critical region, such disturbances will not be a problem. In addition, planners still possess the opportunity to produce purposeful system transitions when necessary. Furthermore, it is noted that as systems evolve through progressively more complex and integrated structures, the capability to persist in a particular state seems to increase and therefore enlarged fluctuations are required to trigger a transition (Holling, 1976). (Laszlo (1972) for instance, suggests that contemporary society has traded stability for adaptability). Further research is requried into this topic, and a possible avenue is through the ecological debate on complexity and stability which has already been briefly examined.

A fundamental conclusion is that the role of planners must be seen within the perspective of the evolving environmental system, assisting, rather than impairing, endogenous organising mechanisms. Moreover, uncertainty is explicitly appreciated, and future options are, therefore, left open. In the end, the success of the planning process is dependent upon the political environment in which it functions and the theory employed. The former is briefly considered in the final section; with regard to the latter, a general weakness of the models used by planners is that they do not adequately represent evolutionary behaviour. This shortcoming was one of the reasons behind the present discussion of the work undertaken by Prigogine and his colleagues; it should not be seen as a panacea, but it does highlight a number of significant paradoxes.

5. CONCLUSION AND EVALUATION.

A number of geographers, such as Langton (1972), have pointed out methodological problems of modelling system change. In fact, there is still a need for a suitable conceptualisation to provide the necessary foundation for the analysis of system evolution. Such a structure is a prerequisite for geographers to comprehend the evolution of environmental systems towards states of increasing (hierarchical) organisation.

Given this requirement, the recent development of a theory of non-equilibrium thermodynamics and dissipative structures, by Prigogine and his colleagues in Brussels, should be of intrest to many geographers. They have demonstrated the principle of 'order through fluctuation', by which open systems, away from thermodynamic equilibrium, develop instabilities permitting minor fluctuations to drive the system to a new spatial-temporal regime.

The successful applications of this conceptualisation in physical chemistry do not defend any advocacy for its employment in the analysis of environmental systems. The quest for analogies from other disciplines is justified, and perhaps even desirable, as long as the dangers of such projections are remembered (see Wilson's (1969) discussion of entropy). New insights into system behaviour may be generated by the transference of concepts, although it should not be a substitute for a detailed investigation of a particular (geographical) problem; the problem must be fitted to the approach, the approach should not be permitted to dictate the geography. As Wilson (1969, p.232) concluded, the geographer

> "... should only remember, in the end, that it is his task to make the concepts thus acquired stand up in their own right, with new interpretations, in geographical theories".

There is, obviously, no suggestion that Prigogine's work has universal application; their work does, however, deserve attention by geographers, particularly at a time when there is

> "... little new theoretical work to replenish the subject's conceptual stock". (Cooke and Robson, 1976, p.89).

Geography has not obtained, if it is ever possible, an exhaustive selection of available approaches of analysis. Additional conceptualisations are always welcome. A comprehension of the power and limitations of the principle of 'order through fluctuation' in processes of self-organisation is essential.

Specific mathematical models and particular environmental systems have not been examined in detail in this introductory outline, although there is sufficient discussion to demonstrate that it is feasible for analyses to go further than at present. In fact, whilst appreciating that system intricacies precludes wholly conclusive modelling, the power of the conceptualisation is its integration of a number of seemingly unconnected system characteristics. For instance, the principle of 'order through fluctuation' provides a mechanism by which particular state transitions can be explained (and, in so doing, overcomes a major problem of catastrophe theory. Alternatively, it would be possible to follow Zeeman's (1977) representation of 'slow' equations).

This framework also has significant ramifications for empirical work. Whilst much work needs to be done in conjunction with the operationalisation of the mathematical models, additional problems relate to the present data difficulties - the scarcity or the poor quality. Detailing a particular system, its elements and their characteristics will remain a serious issue. In fact, as with the system approach in general, initial advance is more likely to be in conceptual, than operational, terms (Langton, 1972).

An area where further research is required relates to the reciprocal symbiosis between spatial and social processes (see Harvey, 1973; and Smith, 1979). Whilst it has been shown that spatial heterogeneity and diffusion are significant aspects in system evolution, ultimately explanation must be linked to the underlying social organisation. Interestingly, in applying the principle of 'order through fluctuation' to describe the dynamics of spatial form, Liossatos (1979) stressed that such analysis should be undertaken in the context of the theroy of capitalism and utilised the loose 'stage-process' theory.

It is, perhaps, too early to make definitive statements concerning the relevance of the work by Prigogine and his colleagues in geography. As more phenomena are described using their conceptualisation, problems will clearly arise, but significant progress may be made in comprehending real world processes. Whatever future awaits it in geography, it is hoped that this paper has introduced the basic concepts to geographers and will allow them to make their own evaluation of its true potential.

ACKNOWLEDGEMENT.

The author wishes to acknowledge the encouragement of Alan G. Wilson who made helpful comments on an earlier draft. The final outcome is the sole responsibility of the author.

REFERENCES.

Ackerman, A.E. (1963) Where is a research frontier? Annals of the American Geographers, 53, pp.429-440.

Alao, N., Dacey, M.F., Davies, O., Denike, K.G., Huff, J., Parr, J.B. and Webber, M.J. (1977) Christaller Central Place Structures: an introductory statement. Department of Geography, Studies in Geography No. 22, Northwestern University, Evanston, Illinois.

Allen, P.M., Deneubourg, J.L., Sanglier, M., Boon, F. and de Palma, A. (1978) The Dynamics of Urban Evolution. Vol.1: Inter-urban evolution; Vol.2: Intra-urban evolution. University of Brussels, Prepared for U.S. Department of Transportation Research and Special Programs Administration.

Allen, P.M. and Sanglier, M. (1979) A dynamic model of growth in a central place system. Geographical Analysis, 11(3), pp.256-272.

Andronov, A.A., Leontovich, E.A., Gordon, I.I. and Maier, A.G. (1973) Qualitative theory of second-order dynamic systems. John Wiley and Sons, New York.

Angrist, S.W. and Hepler, L.G. (1973) Order and chaos: law of energy and entropy. Pelican, Harmondsworth.

Ashby, W.R. (1956) An introduction to cybernetics. Chapman and Hall, London.

Atkin, R.H. (1974) Mathematical structure in human affairs. Heineman, London.

Atkin, R.H. (1976) Combinatorial connectivities in social systems. Birkhauser, Basel.

Atkin, R.H. (1981) Multidimensional man. Penguin, Harmondsworth.(Forthcoming).

Babloyantz, A. Far from equilibrium synthesis of prebiotic polymers. Biopolymers, 11, pp.2349-2356.

Beaumont, J.R. (1979) Some issues in the application of mathematical programming in human geography. Presented at I.B.G. Quantitative Methods Study Group's Conference on "Spatial applications in operational research", University of Salford, September. Working Paper 256, School of Geography, University of Leeds.

Beaumont, J.R. and Clarke, M. (1979) The determination of critical points in urban retailing models. Working Paper (forthcoming), School of Geography, University of Leeds.

Beaumont, J.R. and Clarke, M. (1980) An introduction to elementary catastrophe theory, with applications in urban planning. Working Paper 274, School of Geography, University of Leeds.

Beaumont, J.R. and Keys, P. (1980) A theoretical basis for diffusion in a central place system. Geographical Analysis.

Beaumont, J.R., Clarke, M. and Wilson, A.G. (1980) Changing energy parameters and the evolution of urban spatial structure. Presented at the Symposium on 'Energy Policies and Systems Analysis of Structural Change. University of Umea, Sweden, June.

Bellman, R. (1968) Vistas of modern mathematics. University of Kentucky Press, Lexington.

Bennett, R.J. (1975) Dynamic systems modelling of the North West region: adaptive-parameter policy model. Environment and Planning, A, 7, pp.617-36.

Bennett, R.J. (1975a) Dynamic systems modelling of the North West region: spatio-temporal forecasts. Environment and Planning, A, 7, pp.887-898.

Bennett, R.J. (1978) Spatial time series: analysis, forecasting and control. Pion, London.

Bennett, R.J. (1979) Space-time models and urban geographical research. In: Herbert, D.T. and Johnston, R.J. (Eds.) Geography and the urban environment: Progress in research and application. Vol.2, John Wiley, London. pp.27-58.

Bennett, R.J. and Chorley, R.J. (1978) Environmental systems: Philosophy and control. Methuen, London.

Bennett, R.J. and Haining, R.P. (1976) Space-time models: an introduction. Seminar Paper 28, Department of Geography, University College, London.

Berry, B.J.L. (1973) A paradigm for modern geography. In: Chorley, R.J. (Ed.) Directions in Geography, Methuen, London. pp.3-21.

Blase, J.H. (1979) Hysteresis and Catastrophe theory: empirical identification in transport modelling. Environment and Planning, A, 11, pp.675-688.

Brown, R.L., Durbin, J. and Evans, J.M. (1975) Techniques for testing the constancy of regression relationships over time. Journal of the Royal Statistical Society, 37B, pp.149-192.

Bunge, W. (1962) Theoretical geography. Lund studies in Geography, Series C1, Gleerup, Lund.

Carlstein, R., Parkes, D. and Thrift, N.J. (Eds.) (1978) Timing spacing and spacing time. (3 Volumes) Edward Arnold, London.

Carter, H. (1980) The growth of the city system in Wales: a long term view of economic development and cultural change. Paper presented at the Annual conference of the Institute of British Geographers, University of Lancaster, January.

Case, T.J. (1980) MacArthur's minimisation principle: a footnote. American Naturalist, 115, pp.133-138.

Casti, J.L. (1979) Connectivity, complexity and catastrophe in large-scale systems. Wiley and Sons, Chichester.

Chadwick, G.F. (1977) The limits of the plannable: stability and complexity in planning and planned systems. Environment and Planning A, 9, pp.1189-92.

Chapman, G.P. (1977) Human and environmental systems: a geographer's appraisal. Academic Press, London.

Chappell, J.E. (1975) The ecological dimension: Russian and American views. Annals of the Association of American Geographers, 65, pp.144-162.

Chisholm, M., Frey, A.E. and Haggett, P. (Eds.) (1971) Regional Forecasting. Butterworth, London.

Chorley, R.J., Dunn, A.J. and Beckinsale, R.P. (1973) The history of the study of landforms - The life and work of William Morris Davis, Vol.2., London.

Chorley, R.J. and Kennedy, B. (1971) Physical geography: a systems approach. Prentice-Hall, London.

Christaller, W. (1933) Central Places in Southern Germany. (Translated by Baskin, C.W., 1966), Prentice-Hall, Englewood Cliffs, N.J.

Clarke, M., Keys, P. and Williams, H.C.W.L. (1980) Micro-analysis of socio-economic systems: Progress and prospects. In: Bennett, R.J. and Wrigley, N. (Eds.) Quantitative Geography in Britain: Retrospect and Prospect. Routledge and Kegan Paul, London.

Cliff, A.D., Haggett, P., Ord, J.K., Bassett, K.A. and Davies, R.B. (1975) Elements of spatial structure: a quantitative approach. Cambridge University Press, Cambridge.

Cooke, R.U. and Robson, B.T. (1976) Geography in the United Kingdom, 1972-1976. Geographical Journal, 142, pp.81-100.

Crank, J. (1975) The mathematics of diffusion. Oxford University Press, Oxford.

Diekmann, O. and Temme, N.M. (Eds.) (1976) Non-linear diffusion problems. Mathematical centre, Amsterdam.

Elton, C.S. (1958) The ecology of invasion by animals and plants. Methuen, London.

Galbraith, J.K. (1972) The new industrial state. Pelican, Harmondsworth.

Gale, S. (1976) Geography: from 'form and function' to 'process and design'. Published in International Geography 76: General economic geography (Volume 6). 23rd International Congress, Moscow. Pergamon, Oxford. pp.27-9.

Garrison, W.L. (1973) Future Geographies. In: Chorley, R.J. (Ed.) Directions in Geography. Methuen, London. pp. 237-249.

Gould, P.R. (1975) Mathematics in Geography: Conceptual revolution or new tool? International Social Science Journal, 27. pp. 303-327.

Haggett, P. (1965) Changing concepts in economic geography. In: Chorley, R.J. and Haggett, P. (Eds.) Frontiers in geographical teaching. Methuen, London. pp. 101-117.

Haggett, P. (1973) Forecasting alternative spatial, ecological and regional futures: problems and possibilities. In: Chorley, R.J. (Ed.) Directions in Geography. Methuen, London. pp.217-236.

Harte, J. and Levy, D. (1975) On the vulnerability of ecosystems disturbed by man. In: Van Dobben, W.H. and Lowe-McConnell, R.H. (Eds.) Unifying concepts in ecology. Junk, The Hague, pp.208-223.

Harvey, D. (1969) Explanation in geography. Edward Arnold, London.

Hay, A.M. (1978) Some problems in regional forecasting. In: Clarke, J.I., and Pelletser, J. (Eds.) Regions Geographiques et Regions d'Amenagement. Collection les Rommes et les Lettres, 7, Editions Hermes, Lyon.

Hepple, L.W. (1979) Regional dynamics in British unemployment and the impact of structural change. In: Wrigley, N. (Ed.) Statistical Analysis in the Spatial Sciences. Pion, London.

Hill, A.R. (1975) Ecosystem stability in relation to stress caused by human activity. Canadian Geographer, 19(3), pp.206-220.

Hirsch, M.W. and Smale, S. (1974) Differential equations, dynamical systems and linear algebra. Academic press, London.

Holling, C.S. (1976) Resilience and stability of ecosystems. In: Jantsch, E. and Waddington, C.H. (Eds.) Evolution and consciousness: human systems in transition. Addison-Wesley, London. pp. 73-92.

Hordijk, K.L. and Nijkamp, P. (1977) Dynamic models of spatial autocorrelation. Environment and Planning, A, pp. 505-519.

Huggett, R. (1980) Systems analysis in geography. Oxford University Press, Oxford.

Isard, W. and Liossatos, P. (1979) Spatial dynamics and optimal space-time development. Studies in regional science and urban economics 4, North-Holland, Oxford.

Jantsch, E. (1975) Design for evolution: self-organisation and planning in the life of human systems. Braziller, New York.

Johnston, R.J. (1979) Geography and geographers: Anglo-American human geography since 1945. Edward Arnold, London.

Jones, E. (1956) Cause and effect in human geography. Annals of the association of American geographers, 46, pp.369-377.

Jordan, D.W. and Smith, P. (1977) Non-linear ordinary differential equations. Oxford University Press, Oxford.

Kennedy, B.A. (1979) A Naughtly World. Transactions, I.B.G. (New series), 4(4), pp. 550-558.

King, L.J. (1946) Alternatives to a positive economic geography. Annals of the Association of American Geographers, 66, pp. 293-308.

Kirkby, M.J. (1978) The stream head as a significant geomorphic threshold. Working Paper 216, School of Geography, University of Leeds.

Kuhn, T.S. (1962) The structure of scientific revolutions. University of Chicago Press, Chicago. First Edition.

Kuhn, T.S. (1970) The structure of scientific revolutions. Univeristy of Chicago Press, Chicago. Second Edition.

Lampard, E.E. (1968) The evolving system of cities in the U.S.A.: urbanisation and economic development. In: Issues in Urban Economics (Eds.) Perloff, H.S. and Wingo, L. Resources for the Future, John Hopkins Press, Baltimore.

Langton, J. (1972) Potentialities and problems of adopting a systems approach to the study of change in human geography. Progress in Geography, 4, pp. 125-179.

LaSalle, J.P. and Lefschetz, S. (1961) Stability by Lyapunov's direct method with applications. Academic Press, London.

Laszlo, E. (1972) Introduction to systems philosophy. Gordon and Breach, London.

Lefever, R. and Nicolis, G. (Eds.) (1974) Membranes, Dissipative Structures and Evolution. Wiley, London.

Lefschetz, S. (1963) Differential equations: geometric theory. John Wiley, London.

Lewontin, R.C. (1969) The meaning of stability. In: Diversity and Stability in ecological systems. Brookhaven Symposuim in Biology No.22, National Bureau of Standards, U.S. Department of Commerce, Springfield, Virginia. pp. 13-24.

Liossatos, P. (1979) Spatial dynamics: some conceptual and mathemaitcal issues. Paper presented at 19th European Congress of the Regional Science Association, London, August.

Losch, A. (1944) Die Raumliche Ordnung der Wirtschaft. 2nd edition. Gustav Fischer, Jena. Translated by Woglom, W.H. and Stolper, W.F. (1954) The Economics of Location. Yale University Press, New Haven, Conn.

MacArthur, R.H. (1955) Fluctuations of animal populations, and a measure of community stability. Ecology, 36, pp. 533-536.

MacArthur, R.H. (1970) Species packing and competitive equilibrium for many species. Theoretical Population Biology, 1, pp. 1-11.

Macgill, S.M. and Wilson, A.G. (1979) Equivalences and similarities between some alternative urban and regional models. Sistemi Urbani, 1.

Marchand, B. (1978) A dialectical approach in geography. Geographical Analysis, 10(2), pp. 105-119.

Marsden, J.E. and McCracken, M. (1976) The Hopf bifurcation and its applications. Springer, Heidelberg.

Marshall, J.U. (1969) The Location of Service Towns. Department of Geography Research Publication No.3, University of Toronto, Toronto.

Martin, R.L. and Oeppen, J.E. (1975) The identification of regional forecasting models using space-time correlation functions. Transactions of the Institute of British Geographers, 66. pp. 95-118.

Martinez, H.M. (1972) Morphogenesis and chemical dissipative structures: a computer simulated case study. Journal of Theoretical Biology, 36, pp. 479-501.

May, R.M. (1973) Model Ecosystems. Princeton University Press, Princeton, New Jersey.

May, R.M. and Oster, G.F. (1976) Bifurcation and dynamic complexity in simple ecological models. The American Naturalist, 110, (974), pp. 573-599.

Maynard Smith, J. (1974) Models in Ecology. Cambridge University Press, Cambridge.

Mesarovic, M.D., Macko, D. and Takhara, Y. (1970) Theory of hierarchical, multilevel, systems. Academic Press, London.

Mesarovic, M.D. and Takahara, Y. (1975) General Systems Theory: mathematical foundations. Academic Press, London.

Monod, J. (1974) Chance and Necessity. Knopf, New York.

Nicolis, G. and Prigogine, I. (1977) Self-organisation in non-equilibrium systems. John Wiley, New York.

Parr, J.B. (1980) Frequency distributions of central places in Southern Germany: a further analysis. Paper presented at the Annual Conference of the Institute of British Geographers, University of Lancaster, January.

Postle, D. (1980) Catastrophe Theory. Fontana, Glasgow.

Poston, T. and Stewart, I. (1978) Catastrophe theory and its applications. Pitman, London.

Pred, A. (1977) City-systems in advanced economies. Hutchinson, London.

Prigogine, I. (1978) Time, structure and fluctuations. Science, 201, (4358). pp. 777-785.

Prigogine, I. and Lefever, R. (1975) Stability and self-organisations in open systems. In: Nicolis, G. and Lefever, R. (Eds.) Advances in Chemical Physics, Vol. 29. Wiley, New York.

Puu, T. (1979) Regional modelling and structural stability. Environment and Planning A, 11, pp. 1431-1438.

Puu, T. (1980) Catastrophic structural change in a continuous regional model. Paper presented at Symposuim in 'Energy Policies and Systems Analysis of Structural Change', University of Umea, Sweden.

Ramsay, J.A. (1971) A Guide to Thermodynamics. Chapman and Hall, London.

Robsman, V.A. (1973) Problems in the building of mathematical models for goegraphic systems. Soviet Geography, 14(4). pp. 266-273.

Ruelle, D. and Takens, F. (1971) On the nature of turbulence. Communications in Mathematical Physics, 20, pp. 167-192.

Sack, R.D. (1972) Geography, geometry and explanation. Annals of the Association of American Geographers, 62, pp. 61-78.

Sansone, G. and Conti, R. (1964) Non-linear Differential Equations. Pergamon, Oxford.

Siljak, D.D. (1978) Large-scale dynamic systems: stability and structure. North Holland, New York.

Simon, H.A. (1962) The architecture of complexity. Proceedings of the American Philosophical Society, 106.

Simon, H.A. (1973) The organisation of complex systems. In: Pattee, H.H. (Ed.) Hierarchy theory: the challenge of complex systems. Braziller, New York.

Smith, D.M. (1973) Alternative 'relevant' professional roles. Area, 5, pp.1-4.

Smith, N. (1979) Goegraphy, science and post-positivist modes of explanation. Progress in Human Geography, 3 (3), pp. 356-383.

Thom, R. (Translated by Fowler, D.H.) (1975) Structural stability and morphogenesis. Benjamin, London.

Thomas, R.W. and Huggett, R.J. (1980) Modelling in Goegraphy: a mathemaitcal approach. Harper and Row, London.

Thornes, J.B. and Brunsden, D. (1977) Geomorphology and Time. Methuen, London.

Tobler, W.R. (1970) A computer movie simulating urban growth in the Detroit region. Economic Geography, 26, (2), pp. 234-240.

Turing, A. (1952) A chemical basis for biological morphogenesis. Philosophical transations of the Royal Society (London), Series B, 237. pp.37-72.

Vance, J.E. (1970) The merchant's world: the geography of wholesaling. Prentice-Hall, Englewood Cliffs, N.J.

Von Bertalanffy (1950) The theory of open systems in physics and biology. Science, 111. pp. 23-29.

Wilson, A.G. (1969) The use of analogies in geography. Geographical Analysis, 1. pp. 225-233.

Wilson, A.G. (1970) Entropy in Urban and Regional Modelling. Pion, London.

Wilson, A.G. (1979) Some new sources of instability and oscillation in dynamic models of shopping centres and other urban structures. Working Paper 267, School of Geography, University of Leeds.

Wilson, A.G. (1980) Catastrophe theory and bifurcation with applications in Urban Geography. Croom Helm, London.

Wilson, A.G. and Clarke, M. (1979) Some applications of catastrophe theory to urban retailing structures. In: Breheny, M. (Ed.) London Papers in Regional Science, 10, Pion, London.

Wilson, E.O. and Bossert, W.H. (1971) A primer of population biology. Sinaller Associates, Stanford.

Woodcock, A. and Davis, M. (1978) Catastrophe theory. Dutton, New York (also in Penguin, Harmondsworth, 1980).

Zeeman, E.C. (1977) Catastrophe theory: Selected papers 1972-1977. Addison-Wesley, Reading, Mass.

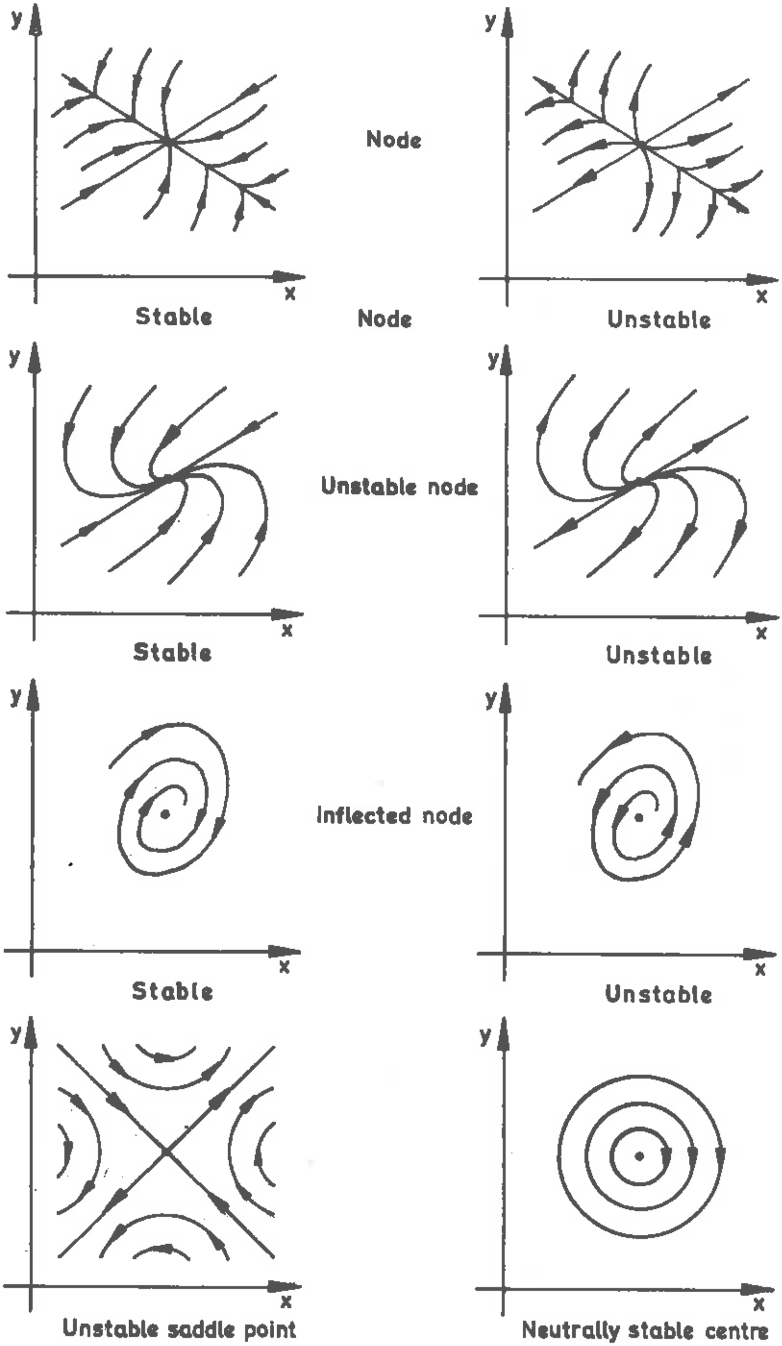


Fig. 1 – Phase space diagrams

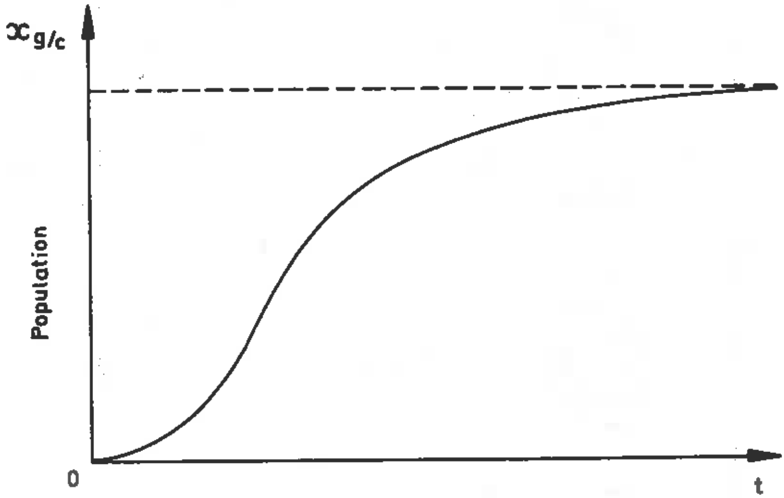


Fig. 2. Logistic growth

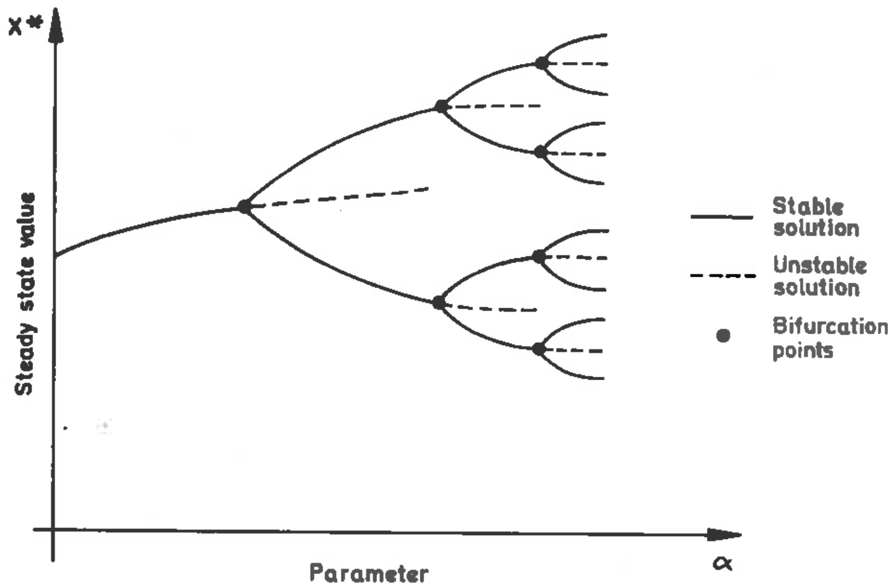
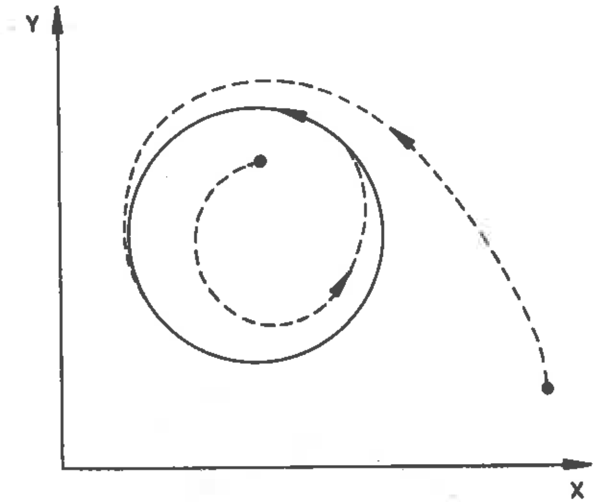
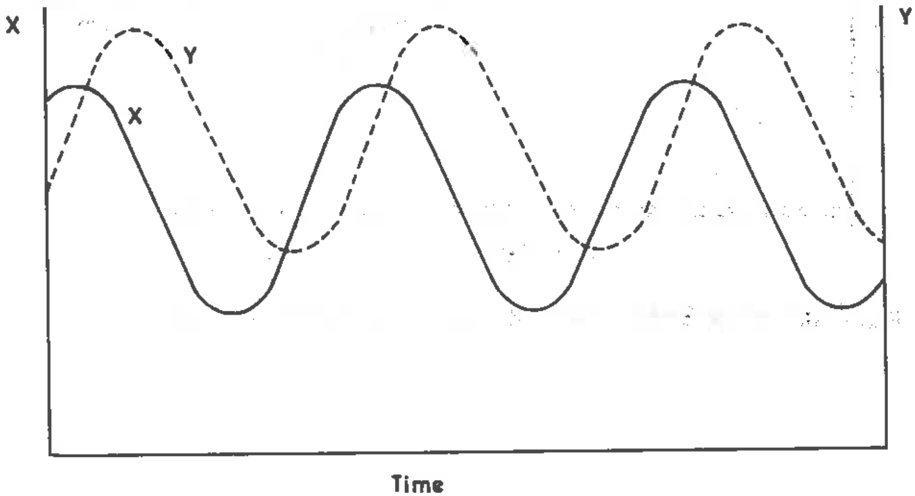


Fig. 3 - Bifurcating steady-state solutions as a parameter changes



(a) Phase space diagram



(b) A representation of the cyclic concentration values

Fig. 4

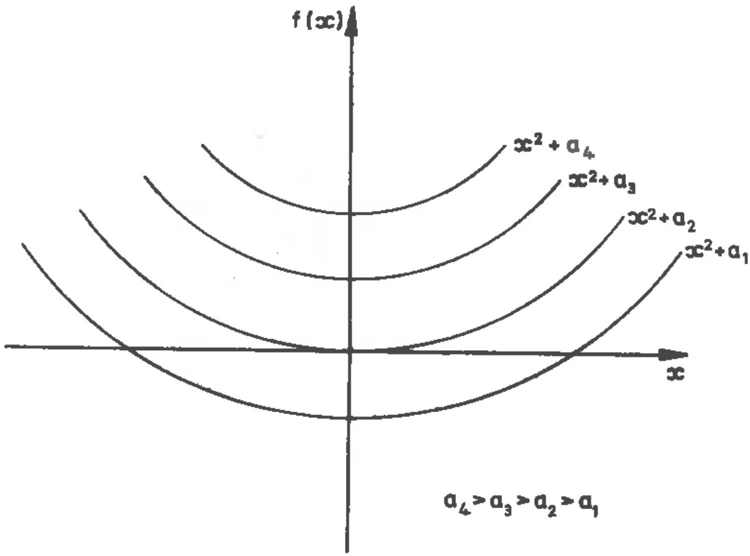


Fig 5 - A family of curves for the function $f(x) = x^2 + a$

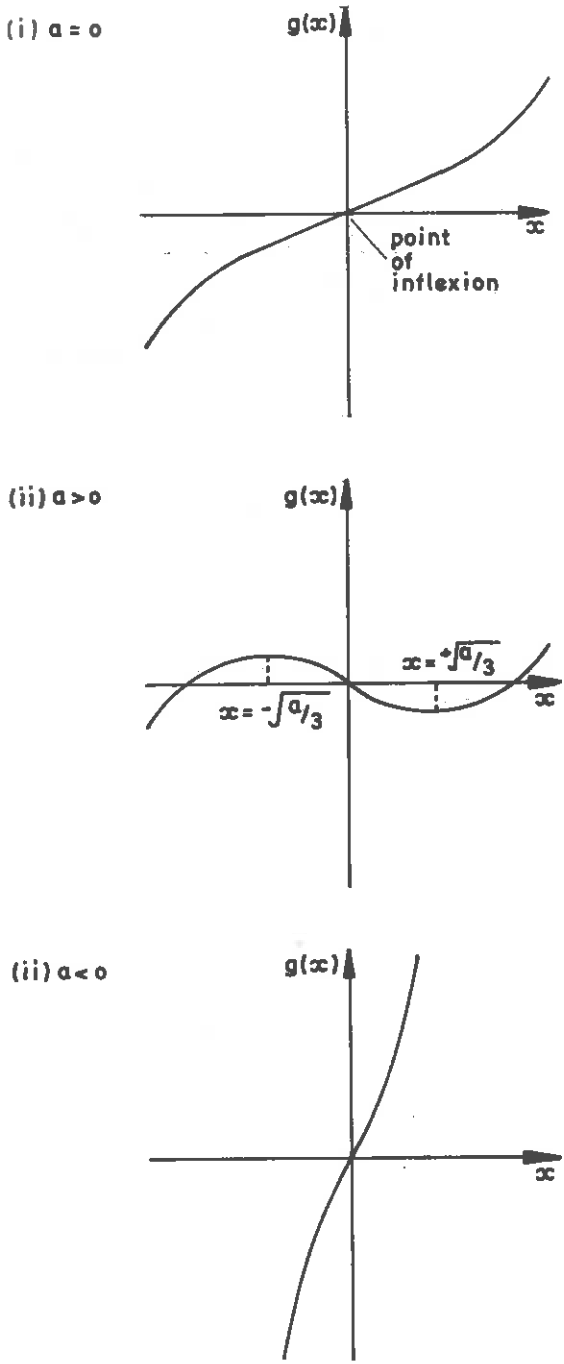


Fig. 6- The function $g(x) = x^3 - ax$

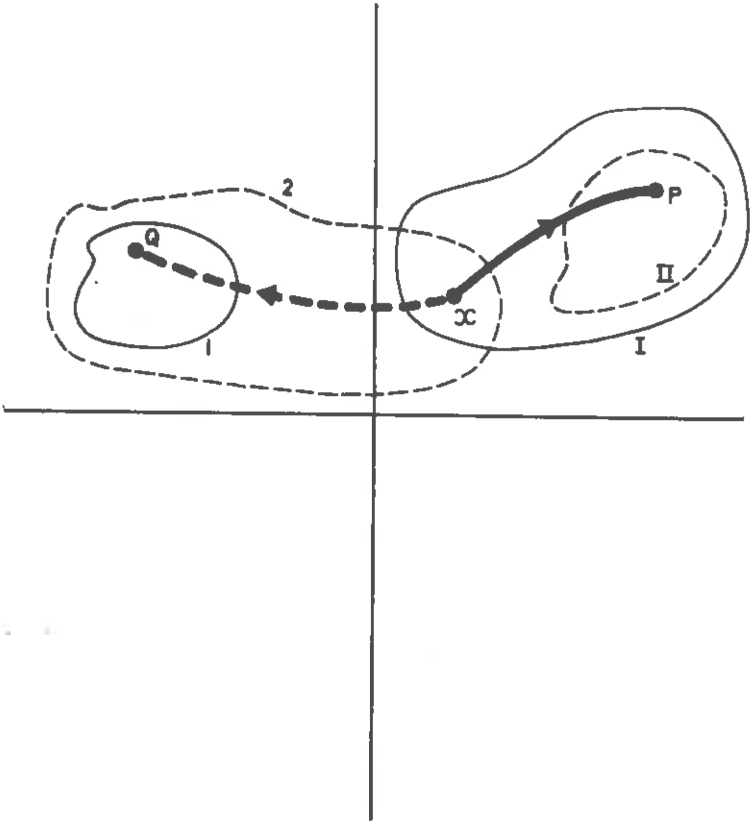


Fig. 7. Shifting domains of attraction (Source : Casti, 1979)