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A RUNOFF SIMULATION MODEL BASED ON HILLSLOPE TOPOGRAPHY

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Abstract

The model presented is a development of an exponential store model (TOPMODEL) previously described (Beven & Kirkby, 1979). The continuity equation for saturated sub-surface flow is expressed in terms of unit runoff which emphasises the relatively low level of spatial variability in rates. For this reason kinematic wave solutions have not been used to solve the partial differential equation.

By assuming that saturated sub-surface flow is carried by a network of macro-pores of constant density and decreasing size with depth, an explicit expression is obtained for the decrease in macroporosity and saturated permeability with depth, linking the main exponential store parameter, m with published permeability profile data. The saturated zone largely determines flood recession curves: above it an unsaturated store helps to control the delay and rate of rise of the flood hydrograph. This unsaturated store receives additions of net rainfall, and is depleted at a rate related to its proportional moisture content, but with an upper limit on percolation determined by the total store value and current rainfall intensity.

The parameter m has been held constant downslope, but the total subsurface flow at saturation, slope gradient and hillslope plan form are used to generate flow differences down the length of the hillslope profile. Simulations show the generation of saturated overland flow at downslope sites where flow converges, superimposed on a hydrograph which is largely controlled by the convex (in profile) divide area. This suggests that for most natural slopes, the runoff delivered to channel banks may be estimated efficiently from two separate linked component models. The first forecasts spatially uniform flow at rates determined by topography and soils in the hilltop divide areas. This model is able to forecast the changing saturated area on which the second

component model forecasts the saturated overland flow. This type of model, when convoluted with appropriate channel network routing, offers some promise as a means of forecasting runoff within arbitrary grid areas as well as for natural catchments.

1. Introduction

In the absence of rainfall and runoff records for a particular catchment, runoff estimation relies on comparison with data from other catchments. Where ,as is usual the comparison is not exact, corrections are needed on the basis of meteorological and drainage basin characteristics. These corrections may rely wholly on empirical correlation between instrumented catchments, or may depend to some extent on physical models for runoff generation. The advantages of a model approach are that it may suggest novel forms of catchment parameter which are not evident from correlation; and that parameters reached through a model are likely to be more reliable under modest extrapolation beyond the data. These advantages will however only be realised in a useful way if the models developed are rather simple, so that the relevant parameters may be both comprehensible and not too many. An additional reason for using a simple model is that hydrographs show only a limited range of forms, and the number of independent model parameters should not greatly exceed the dimensionality of hydrograph form if its parameters are to be sensibly determined.

The model explored here is a development of the exponential store model described by Beven & Kirkby (1979) and tested on UK catchments of up to 10km² (Beven *et.al.* 1984). In the simplest form of this model both sub-surface and overland flow is generated from a single saturated store which is considered to extend downwards indefinitely into the ground and reaches up to a level which changes over time and with position. Zero storage is defined as the condition of saturation to the ground surface, so that negative storage values are associated with saturated sub-surface flow and positive values with overland flow (together with a maximal rate of sub-surface flow). If net rainfall is added directly to such a store then hydrograph peaks occur immediately, at the end of simple storms. An unsaturated store is therefore commonly added to the minimal model, serving to delay percolation into the saturated store. It has been assumed that there is no lateral unsaturated flow. The unsaturated store may also be used to forecast infiltration capacity and Hortonian overland flow, and to correct potential to actual rates of evapo-transpiration. The form of unsaturated store used here most closely follows that described by Beven & Wood (1983).

In this paper the model equations are set out in terms of runoff rates, for the general case of a hillside flow-line strip. Topography and soils are specified in terms of gradient, flow-line width (allowing flow con- or di-vergence) and saturated lateral permeability. Other soil/slope parameters have been held constant along the strip. Flow is routed downslope through distributed unsaturated and saturated stores to generate slope-base and hillside hydrographs in response to storm rainfalls. The behaviour of this kind of model is explored below, considering appropriate simple forms for the unsaturated store(s), and examining the behaviour of the model in response to changes in its main parameters. The first purpose of this exploration is to determine what hillside parameters have the greatest influence on hydrograph form. The second is to consider whether the dominant parameters provide a sufficient basis for a still simpler model which may then be incorporated into a whole-basin model which combines the influence of many flow-line strips interacting with a channel network.

2. Model equations

Flow-line strips are defined as following lines of greatest slope, orthogonal to elevation contours. Distance along the strip, x is measured from the divide in a horizontal direction following the local flow-line direction. The width of the strip is defined by its width, w at each point along it. Drainage area is described per unit strip (*i.e.* contour) width, by the relationship:

[1] $a = \int_0 w.dx$

The geometry of the strip may also be described by two identities which are derived here for use below. From [1];

[2] $\int w.dx = aw$ or $w = d(aw)/dx$

Also from [1];

$1/a = w/\int w.dx$ or $\int (dx/x) = \ln(\int w.dx)$

Taking exponentials;

[3] $\exp[\int (dx/a) = \int w.dx = aw.$

Along a flow strip, the continuity equation may be written:

[4] $\partial(qw)/\partial x + \partial(Sw)/\partial t = iw$

where q is the local saturated discharge per unit width,
S is the local saturated water storage (0 at the ground surface and negative for deficits below saturation),
t is elapsed time
and i is the local rate of percolation to the saturated zone.

Replacing discharge by runoff, j using the geometrical relationship $q = ja$;

[5] $\partial(ajw)/\partial x + \partial(Sw)/\partial t = iw.$

The exponential flow law for saturated flow at any site may be written in the form:

[6] $q = aj = q_0 g \exp(S/m)$ or $S = m \ln\{j.[a/(q_0 g)]\}$

where the soil parameter, m is assumed to be spatially uniform along the strip.

Expanding the first term in equation [5] and substituting for S, it may be seen that j is the only time-dependent term in the partial differentiation with respect to time, so that:

$j\partial(aw)/\partial x + aw\partial j/\partial x + (m/j)\partial j/\partial t = iw$

Substituting equation [2], rearranging and dividing through by w;

[7] $a\partial j/\partial x + m\partial j/\partial t = i-j$

Saturated flow will be routed below using the flow equation in this form.

Two special cases may be readily be solved analytically; for a steady state over time and for spatially uniform runoff under various assumptions. For the st

eady state $\partial j/\partial t = 0$ by definition, so that the flow equation reduces to the ordinary differential equation:

[8] $a\,dj/dx + j = i$

Using the integrating factor $\exp(\int dx/a)$, and the identity [3];

[9] $j = [1/aw] \int (iw).dx$

For a spatially uniform input, i then runoff $j = i$. For other steady inputs the result is non trivial, though it may be derived directly from continuity considerations.

The more interesting special case is of spatial uniformity, defined by $\partial j/\partial x = 0$. In this case the ordinary differential equation is:

[10] $dj/dt = (j/m)(i-j)$

For the case of zero input, this gives the recesssion curve;

[11] $j = j_0/(1+j_0 t/m)$

where j_0 is the runoff at time zero,

which is an adequate fit to many observed catchment regression curves. For constant non-zero input at rate i;

[12] $j = i/[1+(i/j_0-1)\exp(-it/m)]$

For a short burst of percolation, totalling R, the final runoff and the average during the burst are respectively:

[13] $j = j_0 \exp(P/m)$ and $\bar{j} = [mj_0/R][\exp(R/m)-1]$

Equation [7] may be solved as a kinematic wave equation, with a wave velocity of aj/m but this approach has not been used here because solutions show rather strong convergence towards spatially uniform runoff as soon as percolation has become spatially uniform. This condition is generally reached during flood recessions when rainfall stops. The time to reach near-zero percolation is controlled by the unsaturated store. Subsequently spatial differences are carried away at a rate given by the kinematic wave velocity above, which normally suggests survival times of only a few hours.

3. The saturated store

Perhaps the most important soil parameter for the hillslope flow strip is the soil constant, m. By making some assumptions about the saturated soil behaviour, values for m may be linked to more readily measured soil parameters, and in particular to the profile of lateral saturated permeability. Velocities of downslope subsurface flow commonly require effective lateral permeabilities of the order of 100 metres per hour, which can only plausibly be supported if most of the flow occurs in relatively large connected systems of macropores. It is here assumed that all of the lateral flow occurs in macropores, which are assumed for simplicity to form a network of linear cracks, at equal frequency and of decreasing diameter with depth. A microporosity is also assumed to be present, supporting most of the saturated storage but none of its flow. Effective microporosity is also assumed to decline with depth, and to be related to the decline in macroporosity: here simple proportionality is assumed as a basis for proceeding. With these assumptions the relationship between saturated storage level, S and depth in the soil z may be established, together with a form for the associated permeability profile with depth.

For laminar flow in a linear crack of diameter d, the mean flow velocity is given by:

[14] $v = \rho g/(6\eta)\ d^2 = d^2/\kappa$

where ρ is water density,

g is the gravitational acceleration,

η is the kinematic viscosity of the water

and $\kappa = 6\eta/(\rho g)$

The mean flow velocity for all the water is obtained by correcting by the ratio of macroporosity ε to total porosity ε'. This mean velocity is also given by the marginal velocity dq/dS obtained from the form of the saturated store in equation [6] above for unit gradient (g=1), so that;

[15] $[dq/dS]_{g=1} = (q_0/m)\exp(S/m) = (\varepsilon'/\varepsilon)d^2/\kappa$

If macropore cracks occur at constant frequency of n per unit length, then macroporosity is given by:

[16] $\varepsilon = nd$

By definition, the marginal change of storage with depth is equal to the total available porosity ε', so that, applying [16];

[17] $\varepsilon' = -dS/dz = \lambda n d$

where λ is the ratio ε'/ε of total to macro- porosity. Substituting for d from [15]:

[18] $\varepsilon' = \lambda n(\lambda\kappa q_0/m)^{1/2} \exp[S/(2m)] = \varepsilon'_0 \exp[S/(2m)]$

where $\varepsilon'_0 = \lambda n(\lambda\kappa q_0/m)^{1/2}$ is the total porosity at the soil surface.

Integrating over depth, the relationship between depth and storage is:

[19] $z = (2m/\varepsilon'_0)\{\exp[-S/(2m)]-1\}$

and the saturated lateral permeability K_s is:

[20] $K_s = (\varepsilon' q_0/m) \exp(S/m) = (\varepsilon'_0 q_0/m) = (\varepsilon'_0 q_0/m)/[(\varepsilon'_0 z/2m)+1]^3$

Equation [20] shows a decline in permeability with depth at an initial proportional rate of $3\varepsilon'_0 z/(2m)$. The form of the decline may be compared with that fitted to a number of soil profile data by Beven (1983), using the form :

[21] $K_s = K_0 \exp(-fz)$

Comparing rates of decline near the surface, it may be seen that $f \doteq 3\varepsilon'_0/(2m)$. Estimated quartile values of $1/f$ (Beven, 1983, Appendix) for 27 soils lie at about 200 and 400 mm. If available porosity for saturated soil at the surface is 5%, then this range of values corresponds to m-values of 15 to 30 mm. These values may be compared to those estimated for UK experimental catchments of 5 to 40mm, although the lowest of these m-values is for soil with a much higher clay content than any of those analysed for permeability. It is also worth noting that measured values for permeability generally underestimate the contribution of macropores, so that only order-of-magnitude comparisons should be attempted from such data.

This analysis may be extended to estimate the maximum rate of percolation into the saturated zone from the soil above. The result of this analysis is not reported here in detail but relies on an extension of the approach already outlined. Rates forescast in this way are at least an order of magnitude higher than observed rates of percolation. It is therefore suggested that percolation downward is effectively controlled entirely through the possible rate of outflow from the unsaturated zone, which is considered next.

4. The unsaturated store

Flow in the unsaturated zone may be modelled in detail using the unsaturated flow equations in conjunction with characteristic curves for soil moisture *versus* hydraulic potential (with or without hysteresis). Treating the unsaturated zone as a single store is inevitably imperfect, but may be adequate given the low dimensionality of hydrograph forms. These imperfections are greatest where a given total moisture content may be associated with very different vertical moisture distributions. An extreme example is the difference between conditions in a capillary fringe in equilibrium with a static water table, and conditions of rapid infiltration into a dry soil with moisture concentrated near the surface. In the former case there is zero downward flow, gravity balancing hydraulic potential gradients; whereas in the latter case the two gradients combine to maximise downward percolation. In a conceptual single store, percolation must be estimated as an average lying between such extremes, normally for the case of zero hydraulic gradient. Flow is then approximately equal to the vertical permeability, which depends primarily on moisture content. On this view the downward percolation should depend on the ratio $h/(-S)$ where h is the unsaturated water storage, so that the ratio $h/(-S)$ is equal to the proportion of moisture saturation in the zone above that of saturated flow. For simplicity and because it appears to give the most reasonable results, a linear dependence has been used in modelling, even though the relationship between permeability and moisture content is generally found to be more than linear. It has also been assumed throughout that the unsaturated zone is one of downward percolation only, with no lateral component.

An alternative view of the unsaturated store is as a means of delaying the hydrograph peak. From this point of view percolation at rate directly proportional to $h/(-S)$ is equivalent to treating the unsaturated store as linear, with a delay time proportional to the saturated deficit, $-S$. Both points of view appear to give a reasonable direction of dependence on the variables involved, and closely follow the model of Beven & Wood (1983). Thus greater deficits increase the delay to hydrograph peak following rainfall, and greater unsaturated storage levels give greater percolation for a given saturated deficit. The relationship is however likely to break down for low positive deficits, and certainly does so for negative deficits. It is proposed here that at low deficits, the maximum rate of percolation is controlled either by the absolute unsaturated storage level, h or by the rainfall intensity if that is higher.

The argument for using total storage as a determinant of percolation may be related to the condition of maximum percolation described above, namely of rapid infiltration above an advancing wetting front. In that case downward percolation from the front responds to the depth of the front directly, moisture content above it lying close to saturation. Again a linear proportionality has been found to be suitable. On the alternative view of the unsaturated zone as a linear store, an upper limit proportional to h is equivalent to a minimum time for drainage. Plainly if the drainage time is able to decrease indefinitely then percolation outflow may in some circumstances greatly exceed storm rainfall intensity, a situation which is rarely if ever encountered in practice. When rain is falling on a saturated or almost saturated soil, then its intensity is also thought to be an appropriate upper limit for percolation if it is higher than the percolation estimate just described. In this case the unsaturated zone is acting simply as a shunt to transmit rainfall pulses directly.

To maintain continuity between percolation at rate proportional to $h/(-S)$ and percolation at a rate proportional to h, the critical transition must occur at a fixed deficit, which may sensibly be related to the scale depth m. This version of the unsaturated store has been adopted even though it appears to violate continuity which requires that for positive deficit, $-S$, the unsaturated storage h should not exceed the deficit if space is to be found in the soil for the unsaturated moisture.

The unsaturated store may also serve in a model to limit transpiration rates and to determine infiltration capacity.

Transpiration may for example be allowed to exhaust all or part of the unsaturated store, and then draw on the saturated store down to some maximum deficit. The adoption of an abrupt cut-off of this sort is thought adequate at a single site, and will produce a progressive decline in transpiration rates when applied to a drying catchment area, following the areal distribution of deficits. Infiltration capacities may similarly rely on the unsaturated storage level, using the Green & Ampt (1911) equation which is based on storage, with a 'steady' leakage rate based on the downward percolation discussed above. Excess rainfall may then be routed overland at the same marginal rate as for saturated overland flow, q_0/m. These facets of the model have been adopted but not critically tested.

The conceptual model adopted for the unsatuated store is formally described by the following equations.

[22] $i = \rho h/(-S)$

valid for large deficits,

where i is the rate of percolation into the saturated store,

and ρ is a percolation rate parameter (e.g. in $mm.hr^{-1}$)

For small deficits;

[23] $i = \rho h/(\gamma m)$

where γ is a constant parameter.

This expression is valid for deficits less than γm, where γ is thought to be of the order of unity. At high rainfall intensities, equation [23] is replaced by:

[24] $i = r$

where r is the rainfall intensity, valid when $r > \rho h/(\gamma m)$.

The infiltration capacity f may be estimated as:

[25] $f = i + A/h$

for constant A, following Green and Ampt (1911).

Analysis of this equation for infiltration into a soil with large saturated deficit and $h = 0$ initially gives, with no saturated outflow, an initial rapid decrease in $f \propto t^{-1/2}$, followed by an eventual very gradual rise $\propto t^{1/8}$. If deficit is held constant during infiltration the initial fall is followed by a sharper eventual rise $\propto t^{1/2}$. These relationships depart somewhat from standard infiltration theory which requires an eventual constant infiltration rate, but the departures appear to be acceptably small in most cases.

5. General model behaviour

Figure 1 is a schematic flow diagram for the water balance of each segment along the flow strip modelled. Equation [7] has been solved computationally, using a simple explicit scheme on a small micro-computer. Stability has been maintained by calculating rates of change of saturated runoff, and reducing the time step so that this change nowhere exceeds a threshold proportion (usually 0.01). Initial conditions are set by calculating equilibrium with an assumed steady rate of rainfall and transpiration. In the simulations described below, the potential transpiration rate has been set to zero, and the rate of infiltrationto exclude Horton overland flow (by setting the constant A in [25] high). Unsaturated and saturated storages are calculated for up to 20 downslope segments, each one specified by its width and gradient x permeability. Initial conditions have been obtained by assuming equilibrium with a steady rate of antecedent net precipitation, and an equal rate of percolation and runoff at every point.

Simulations have been made with a standard set of parameter values and topographic form, with a simple storm at the start of the simulation period. The topographic form is shown in figure 2(a), which illustrates a convexo-concave profile on a flow strip which shows strong convergence of flow near the base of the hillslope. The form is therefore representative of a valley-head hollow in a temperate environment. The area drained per unit contour length, a is shown in figure 2(b). It may be seen to be initially equal to distance from the divide and then to increase rapidly, reaching a value of 2,355m. at the base of the slope, due to the effect of flow convergence. Figure 2(b) also shows the runoff at saturation, $j_* = q_m g/a$, for the adopted value of q_m. The effect of flow convergence and profile concavity reinforce one another at the base of the slope, so that saturation occurs at a runoff of 0.089mm.hr^{-1} at the slope base and at 10mm.hr^{-1} near the divide. The relevant 'standard' parameter values, antecedent runoff and storm size are listed in Table 1. For these standard conditions the main hydrograph peak occurs about 8.5 hours after the start of the 60mm. storm. The form of this standard response is illustrated in figure 3(b). The upper part of the slope (0-30m. from divide) generates the solid curve labelled '0.5', while the slope base generates the broken-line curve labelled '0.5'. It may be seen that the saturated hollow area generates a quick response, which is superimposed on a larger sub-surface response from the slope as a whole. The results are thought to represent flood response from a hollow with moderately permeable soils.

It has not been practicable to explore the complete parameter space, but simulations have followed changes in one parameter at a time about the standard values described above. Figure 3(a) shows the response of the upper part of the hillslope to changing storm volume (60mm as standard). Storms have all been evenly distributed over a three-hour period, although changes in the distribution have only a limited influence in the overall hydrograph form. As may be expected the responses show moderate non-linearity, with lag times decreasing, and peak flows increasing more than in proportion to storm volume. At storm volumes above about 200mm the peak coincides with the end of the storm. For storms of less than about 20mm the trend in lag times is reversed, until for storms less intense than the antecedent runoff the lag is zero.

Figure 3(b) shows the response of the divide area (solid curves) and the slope base (broken lines) to changes in antecedent runoff (0.5mm.hr^{-1} as standard). Again lag times decrease and peak flows increase for wetter conditions. At the slope base the quick response peak also becomes increasingly important, although it inevitably peaks at the end of the storm. It each case it may be noted that the slow response peak from the slope base is in time with the upslope peak, and this ocurs for all intermediate points.

Changes in parameter values generally influence both the magnitude of the hydrograph peak and its timing, as well as having some influence on other aspects of hydrograph form. Figure 4(a) shows the proportional change in peak flow for proportional changes in each of the major model parameters. Storm volume and antecedent runoff are also included for comparison. It may be seen that the saturated permeability $k = q_0/m$ has least influence on peak flow, and that k, the soil parameter m and the percolation threshold parameter γ all reduce the peak as they increase; whereas peak flow increases with antecedent runoff j_0, storm volume R and the percolation rate parameter β. The figure is plotted for the upslope response where this is delayed beyond the end of the storm. In some cases the change to a quick response at extreme parameter values leads to a discontinuity in the relationships shown, as the peak flow adopts a value related to peak rainfall intensity.

As may be expected from considerations of total runoff volume, increases in peak flow for a given storm volume generally lead to decreased lag times. Figure 4(b) shows the relationships between proportional changes in simulated peak flow and time to peak (from the storm centroid) as parameters are changed, one at a time. It is evident that the form of the hydrograph can be changed most readily through the parameters k, which changes lag more than peak flow, and γ which changes peak flow more than timing. The parameters β and m both have a similar effect on lag as they change the peak flow. However changes in m tend to reduce the quick and slow peaks roughly in proportion whereas changes in β reduce the slow peak faster than the quick peak, which therefore gains in relative importance. Thus opposing changes in m and β change the ratio of quick to slow peak flow for a given time lag.

The effect of topography is somewhat more complex than the response to single parameters because it is represented by the two values of gradient x permeability and width for each slope segment, providing 40 parameters in all. They have the advantage in a forecasting sense that they may, ignoring permeability differences be rather unambiguously measured for a particular site, but that does little to explain their influence on hydrograph form. It has however already been observed above that for the topography used in the simulations, the hydrographs have a slow peak which is almost perfectly synchronous all down the slope. It is worth noting here that for slopes where $a \propto s$ along the profile, the storage levels, both saturated (S) and unsaturated (h), are spatially constant at all points, although changing through time. This condition is met for a number of topographic forms, of which the most immediately relevant is that of a simple convex profile (gradient $\propto$ distance from divide) combined with a ridge, spur or col plan-form on which the strip width, $w \propto x^{\lambda}$ at distance x from the divide for constant λ. The unit area a is then given by:

[26] $a = x/(1+\lambda)$ for $\lambda > -1$.

This class of slopes includes the convex ridge at the top of the standard model slope. It should also be noted that the parameter $k = q_o/m$ scales saturated flow velocities in exactly the same way as gradient, so that changes in k produce identical effects to overall proportional changes in gradient.

Figure 5 shows the influence of the ratio $a/(kg)$ on peak flow and lag, for the cases where there is a distinct slow-response peak, and for slopes or parts of slopes where the ratio is constant from the divide. This ratio which depends on topography, and to some extent on soils, has the dimensions of time, and will be referred to below as the 'topographic scale time', denoted by t_a. For other topographic forms it is very clear that local values of t_a no longer give the same functional dependence of time lag on topographic factor. For slopes with an appreciable convex element near the divide (*i.e.* with constant t_a), then the slow peak is seen to be very strongly influenced by the near-divide timing even where the slope base departs very markedly from constancy of t_a, as is the case for the standard slope used. Retaining the same convexo-concave profile and changing the plan form of the flow strip, dominance by the divide convexity remains strong for a strip of uniform width (a ridge), but the slope base shows the influence of more local factors where the plan form is of linearly increasing width (a spur). Examination of these and other topographic forms suggests that the integrated value of t_a for comparison with the convex slope cases is best calculated as a weighted mean of $a/(kg)$ values from the divide to the point of interest, and that the weighting factor is roughly equal to the strip width squared. Thus local slope factors tend to be much more important on spurs than in hollows for determining the timing of the slope base hydrograph.

6. Towards a 2-component basin-scale model

The analysis of topography above for a distributed flow strip allows approximations to be made about the performance of the whole strip as a lumped unit. A two-component model may be obtained by separately modelling the 'slow' and 'quick' responses. The 'slow' response is used here to indicate the hydrograph peak which occurs after rainfall has ceased for a simple storm, which is largely produced by subsurace flow. The 'quick' response indicates the (usually) secondary peak reached at the end of a simple storm, which generally has a substantial saturated overland flow component. The topographic scale time may be used to re-write the storage-flow relationship of equation [6] in the form:

[27] $\quad j = (m/t_a) \exp(S/m)$.

Together with the other parameters m, β and γ or their average values for the flow strip,it may be used to generate the subsurface flow peak (or the main flow peak if there is no slow peak) for the slope base. The procedure is identical to that for the distributed model except that only a single point is required. The slope is treated as if it were simply convex with the average value of t_a describing the convexity.

The amount of runoff in the quick response peak may be calculated from the area contributing runoff at the current rainfall intensity. Figure 6 illustrates the necessary analysis of the topographic data. The structure of the unsaturated store requires that rainfall is transmitted direct to the saturated store if the saturated deficit is less than γm, so that the contributing area at any time is that for which the current runoff $j >$ $j_* \exp(-\gamma)$, where $j_* = m/t_a$ is the local runoff at saturation. Figure 6 illustrates the form of the dependence for a number of the topographic forms used in model simulations. The current contributing area may be read directly in terms of the current runoff level, calculated using the slow runoff component described above. At each time interval, the volume of rainfall supplying quick runoff is equal to the current rainfall intensity multiplied by the contributing area. This estimate, summed over a storm, gives an estimate of total quick runoff contribution which compares very favourably with that obtained for the distributed flow strip above, although not all the rainfall contributed in this way flows out of the slope immediately. Where the quick response is large, it may be necessary to iterate this process, using a reduced rainfall amount to generate the slow response etc.

This description of a two component model largely repeats the model described by Beven & Kirkby (1979) and Beven & Wood (1983), but makes useful progress in two directions. The first of these is in showing that the simpler model does in fact simulate the behaviour of the distributed flow strip model with reassuring fidelity. The second, and more important value of this exploration has been in making the form of topographic dependence much more explicit in the simple model, particularly for the slow response peaks.

Topography is in fact rather less variable than the range of forms explored above. In most landscapes true slope profiles are dominated by convexities. This is perhaps self-evident for humid-temperate landscapes, but is also true for the typically much shorter slopes of semi-arid lands. Impressions of long concave slopes are commonly illusory, a visual impression being obtained by looking across interfluves which mirror the concavity of the densely spaced intervening channels, as has been noted by Carson (Carson & Kirkby, 1972). True slopes are typically short and more than 50% convex in profile. The predominance of such forms is a necessary conclusion for landscapes of net fluvial erosion, since long concavities are inherently unstable in the sense of developing into permanent eroded channels, following the argument of Smith & Bretherton (1972). Thus for most natural slopes there is in fact a large proportion of their length for which the topographic scale time is genuinely constant, and this convex divide area dominates the (slow) hydrograph form.

At the basin scale, the combination of flow strip hydrographs may be achieved over an area of constant storm intensity through the distribution of data on divide convexity, which also tends to show some consistent pattern within a catchment. These data are rather easily obtained from maps or air photographs etc, and provide one important source of variability on the basin scale. Variability can also be considered in flow strip parameters, which have for simplicity (and in the case of ■ from necessity) been held constant within a strip but which may vary between strips. These data are however much less readily available than topographic data and it is worth exploring the degree of explanation which can be achieved through topography alone. For example headwater hollows within first order humid catchments commonly show lower gradients and more gentle divide convexities than side-slopes within the same catchment. This difference in topography is enough to explain both a more rapid slow response and a greater proportion of quick response from the hollow area than from the side slopes. Field evidence can decide whether these topographic effects are great enough to explain the observed differences of these types, or whether a part of the difference must be attributed to changes in other parameters.

7. Conclusion

The model presented here is seen to offer some advance on existing versions of TOPMODEL, though developed from them. It allows either simple distributed modelling based on hillslope flow strips or explicit forms for integrating topography into a lumped flow strip model, particularly when slopes have substantial upper convexities, as is normal. Aggregation to the basin scale is then readily achieved through the distribution of topographic and soil parameters, and there is some hope that the more accessible topographic parameters will be the more important.

There is also seen to be a real possibility of estimating runoff generated within arbitrary grid squares, for linking with atmospheric circulation or other large-area models. A suitable procedure for a single grid square is thought to consist of first selecting a number of points at random (subject to a suitably stratified design) from the square. It is assumed here that non-topographic parameters (*e.g.* β, γ and m) are constant within the grid square, and a more complex procedure is needed if this is not the case. For each point the flow line strip through it is identified and its upper convexity used to give the topographic scale time, t_0. The area drained per unit strip width, a and the local gradient, g are also obtained for the point to give the saturated runoff, $j_* \exp(-\gamma)$ at which the point belongs to the contributing area. A curve similar to one of those in figure 6 can then be drawn from the sample points. In order to represent the hollow areas adequately, it is anticipated that a higher density of sampling points would be required in potential hollow areas. The two component model outlined above can then give estimates of the slow and quick runoff responses to each storm at slope bases. What the procedure above fails to allow for is differences in timing due to channel flow velocities. The grid basis therefore appears adequate to account for changes in soil moisture due to slope runoff (except along major flood plains), but unsatisfactory for estimating the timing of large stream hydrographs.

A further speculation is that the geomorphology may give additional information to help estimate some other hydrological parameters for each area from remotely sensed data. The channel network density revealed either on a grid or catchment basis reflects the geomorphological evolution of an area in response to its hydrology, and is thought to respond to it over periods of the order of 10^1 to 10^3 years, and so broadly reflect current rather than fossil conditions. Following the stability argument of Smith & Bretherton (1972), Kirkby (1980) has argued that the average slope length [1/(2xDrainage density)] is of the same order as the distance at which overland flow erosion becomes dominant over creep and rainsplash. This distance strongly reflects the rainfall regime, but is also influenced by soil and vegetation. If the rainfall regime is independently known, and vegetation cover can be measured remotely, then hydrological soil factors remain as the unknown to be deduced. Clearly this approach is unlikely to provide all the information required, but may be another worthwhile step towards the assessment of runoff response remotely, and on a grid square as well as a catchment basis.

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Table 1

Standard parameter values adopted in model runs

Soil storage parameter, m = 10 mm [Equation 6]
Mean saturated flow velocity on unit gradient, q_0/m = 100m.hr^{-1} [Equation 6]
Percolation rate constant, β = 2 mm.hr^{-1} [Equation 22]
Threshold deficit for percolation = γm for γ = 1 [Equation 23]
Rain storm at 20mm.hr^{-1} for 3 hours
Initial runoff at 0.5mm.hr^{-1}

Figure captions

1. Schematic flow diagram for hillslope runoff model.

2. Standard flow strip topography adopted in model runs.

(a) Relative flow-line widths and gradients assumed

(b) Resulting areas drained per unit strip width (m.) and saturated runoff rates ($mm.hr^{-1}$).

3. The influence of storm rainfall and antecedent conditions on simulated hillsope floods.

(a) Effect of storm rainfall (60mm. standard) on convex upper slope: Storms in all cases spread over 3hr. period. Arrows and dotted line indicate peaks.

(b) Effect of antecedent equilibrium runoff ($0.5mm.hr^{-1}$ standard) on response to 60mm. storm. Solid line for upper convexity: broken line for slope base.

4. The influence of parameter values on modelled hydrograph form as one parameter is changed at a time.

(a) The proportional response of the slow hydrograph peak to proportional changes in parameter values.

(b) The proportional changes in peak flow and lag to peak from storm centroid as parameter values are changed.

5. The influence of topography on peak flow and lag to peak. Proportional changes are shown in response to proportional changes in the topographic scale time, $t_a = a/(kg)$.

6. The relationship between local runoff rates and contributing area for exampls of differing topography:

(i) Standard slope

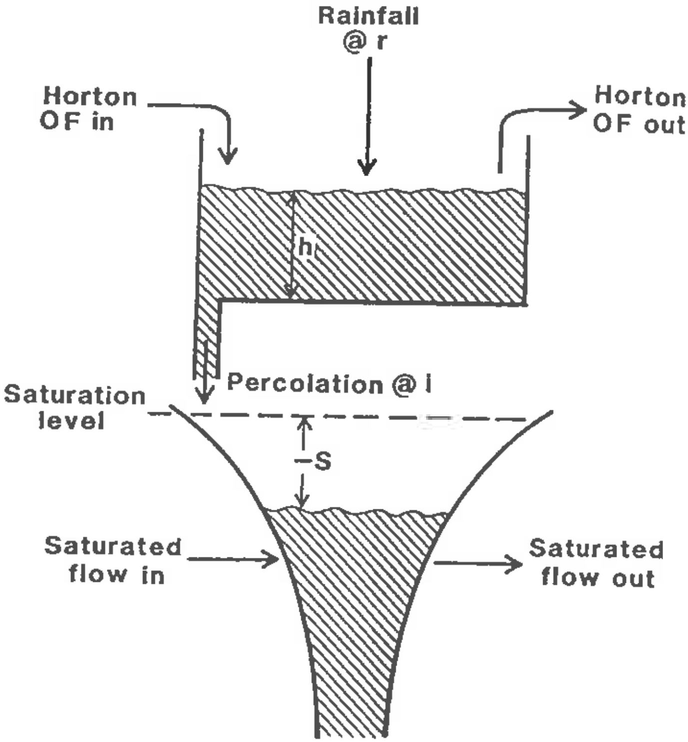
(ii) Straight (constant gradient) profile on ridge (Constant width)

(iii) Convex (uniformly increasing gradient) profile on ridge

(iv) Standard profile on ridge

(v) Straight profile in standard hollow

(vi) Standard profile on spur (linearly increasing width)



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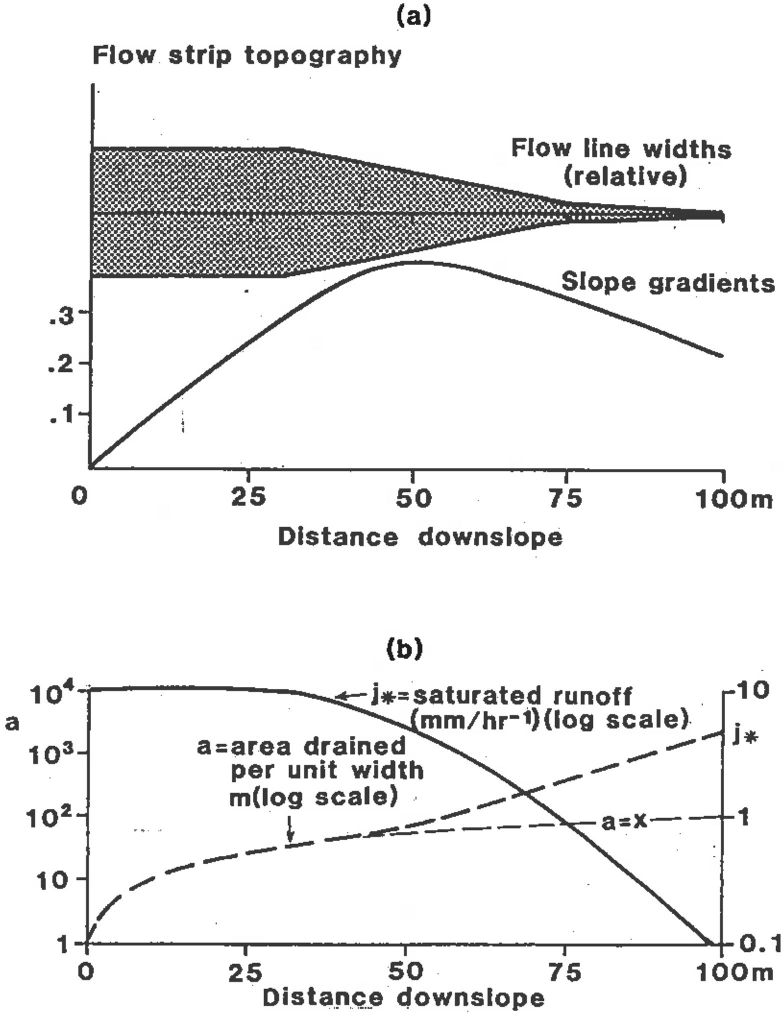

Parameters

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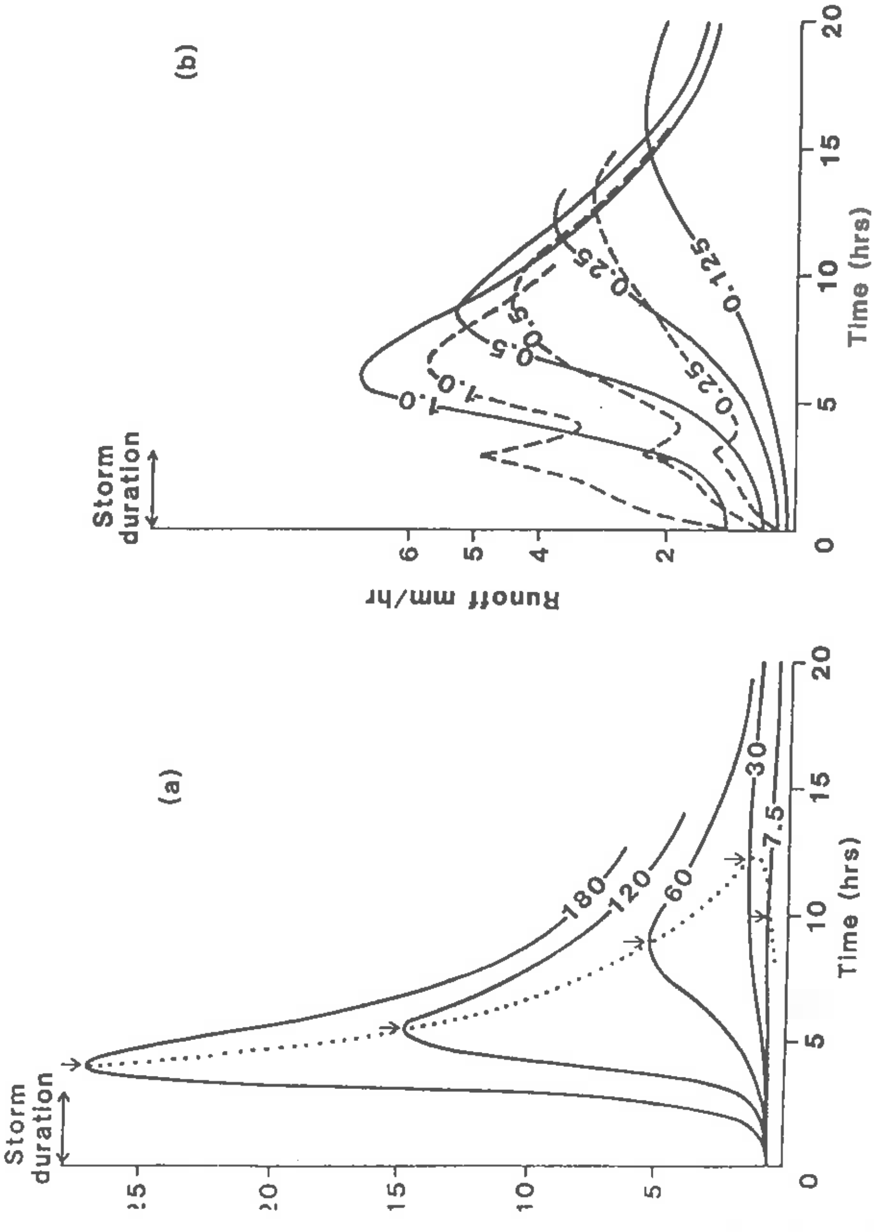

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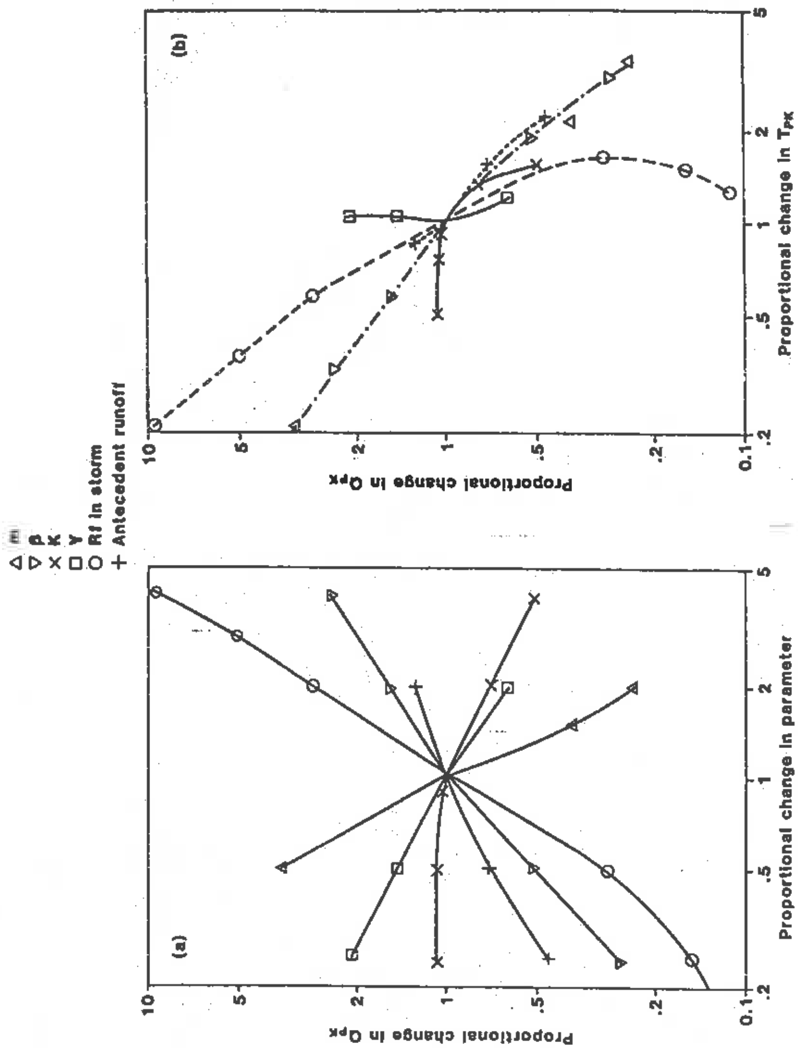
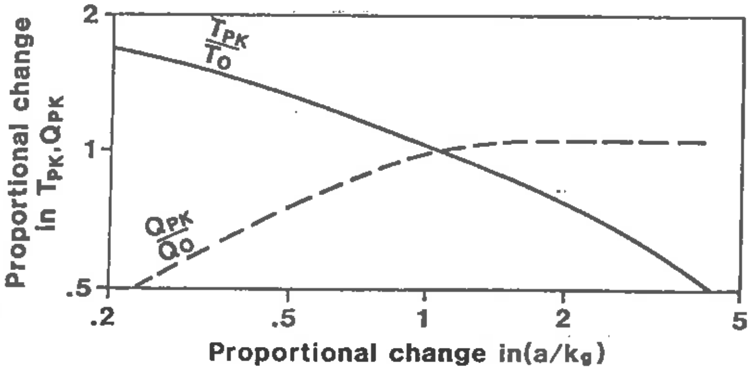


Sensitivity of Q_{PK} to changes in non-topographic parameter, in cases where a peak occurs after the end of rainfall

Changes in Q_{PK} and T_{PK} (where peak occurs after rainfall) as one parameter is changed at a time



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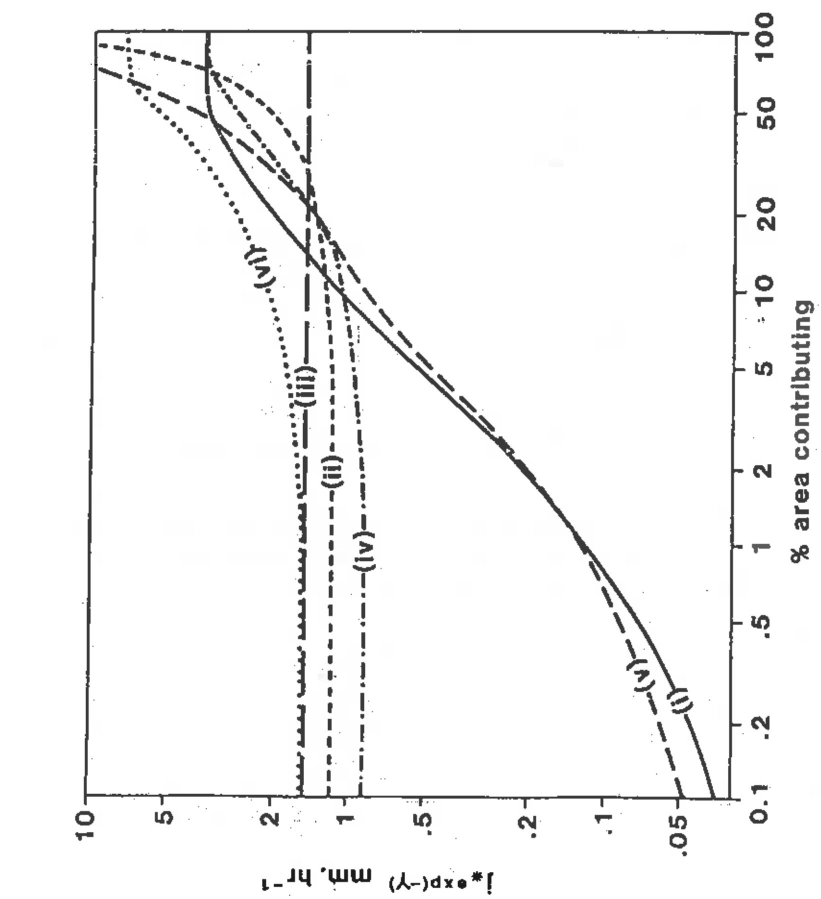

Curve for estimating effective contributing areas at a given runoff level

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