AN EROSION–LIMITED HILLSLOPE EVOLUTION MODEL

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WORKING PAPER 92/5

SCHOOL OF GEOGRAPHY • UNIVERSITY OF LEEDS

Working Paper 92/5

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April 1992

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Summary

An erosion limited slope evolution model is proposed as a generalisation for both flux (transport) limited and supply (weathering) limited conditions, which appear as extreme special cases. Although there is a slight increase in model complexity, and there are no very simple steady state or steady decline solutions, there are several advantages. Firstly, there is scope to include a wide range of processes within a single model; secondly there is improved representation of processes, particularly solution and wash. Distinctions can be made between rainsplash, rainflow and rillwash, and there is scope for forecasting the evolution of an armour layer with grain size selectivity. The model has been implemented in finite difference form. Five model runs are shown as illustrations of the scope of the erosion limited approach.

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Process Response hillslope models

Simulation models for hillslope evolution contain four essential elements. First, a mass balance framework which transforms spatial differences in sediment transport to rates of erosion or deposition over time; second, a representation of sediment transport process rates which respond dynamically to changes in topography; thirdly, an initial form for the landscape and lastly a set of boundary conditions at the margins of the profile or area simulated. It is also assumed that simulations will generally be implemented through numerical solution of the one or more partial differential equation, using a computer algorithm. This framework is taken as given, as a basis for discussing alternative styles of process representation and model implementation, largely in the context of one-dimensional slope profiles although most of the discussion is equally relevant for two-dimensional models representing an area.

The mass balance equation takes the form:

Input - Output = Net increase in storage.

For a section of a slope profile, the inputs and outputs are of sediment transport along the slope (generally, though not necessarily, neglecting aerial inputs), and the resulting change in storage represents erosion or deposition within the section. In mathematical notation, for a one dimensional profile:

$$\frac{\partial z}{\partial t}+\frac{\partial S}{\partial x}=i \tag{1}$$

where z is elevation,
x is distance from the divide,
S is the sediment transport rate,
i is the net rate of external (aerial) addition
and t is elapsed time.

It should be noted that the sediment transport rate, S, in this equation is the actual transport rate, and not the transporting capacity, C. Two extreme cases can conveniently be distinguished. Where the sediment supply is unconstrained, then the actual sediment transport is equal to the transporting capacity, and removal is said to be 'transport limited' or 'flux limited'. For example, soil creep within a deep soil is generally considered to be transport limited. At the other extreme, where sediment transport is far below its capacity ($S << C$), the rate of removal is determined by the supply of material, through weathering for example, and removal is said to be 'weathering limited' or 'supply limited'. For example, rock fall from a steep cliff is at a rate far below the potential transporting capacity, and removal is limited by the rate of release of joint blocks by weathering.

Process rates have generally been specified in terms of sediment transporting capacity, and for flux limited removal, this provides a complete description which allows solution of equation (1). For the supply limited case, however, the critical measure is of the detachment rate, and transporting capacity is largely irrelevant. In this case, an additional equation is required, taking the form:

$$-\frac{\partial z}{\partial t}=D \tag{2}$$

where D is the rate of detachment.

It can be seen that in general, there are two components of process rates which need to be specified to give a complete description.

What we formally propose here is the acceptance of an equation which generalises equation (2), and allows both flux limited and supply limited removal as special cases. This is a novel suggestion only in the sense that it is proposed for use with all the relevant sediment transport processes (BENNETT, 1974; KIRKBY, 1991). It is here offered as a general theoretical framework with wide application to hillslope evolution models.

For a one dimensional hillslope evolution model, representing a slope profile, an erosion or deposition balance can be written as follows.

$$\frac{dS}{dx}=\frac{\partial S}{\partial x}+\frac{1}{u}\frac{\partial S}{\partial t}=D-\frac{S}{h} \tag{3}$$

where D is the detachment rate,
h is the sediment travel distance,
and u is the mean sediment velocity.

This equation provides the total increase in sediment transport along a short reach. In the central expression, it is made up of changes in sediment transport due to position ($\partial S/\partial x$) and changes due to the sweeping through of fluctuations in transport rate over time ($1/u\ \partial S/\partial t$). This second term can generally be ignored in studies of hillslope evolution, but is important in short term sediment studies. In the right hand expression, the rate of change is provided by a rate of detachment of material from the underlying surface, D, which is offset by a rate of sedimentation, S/h. The transport distance is thus defined as the mean distance over which the sediment in transit would be deposited (under uniform conditions). More usefully, 1/h may be interpreted as the proportion of the sediment load deposited per unit distance. This sediment balance should generally be estimated for each process separately, and the total value of $\partial S/\partial x$ obtained by summing the contribution from each individual process.

Transporting capacity is defined as a state of equilibrium, in which detachment exactly balances re-deposition, so that the transporting capacity is the product of detachment rate and travel distance. Equation (3) may also be seen to incorporate the concept of erosion rate as dependent on surplus capacity. The right hand expression may be re-written in the form (C-S)/h, showing that erosion rate is proportional to the excess of transporting capacity, C over actual transport rate, S, with constant of proportionality 1/h. This concept has also been proposed in several contexts (eg FOSTER & MEYER 1972). It plainly has the correct sign, in that transport in excess of capacity must lead to deposition, and erosion can only occur when transport falls below capacity. At this level, equation (3) may also be considered simply as the first order linear model.

Combining equations (1) and (3), for the case where the transient $\partial S/\partial t$ terms may be neglected, we obtain:

$$-\frac{\partial z}{\partial t}=\frac{\partial S}{\partial x}=D-\frac{S}{h} \tag{4}$$

where z is elevation,
x is distance from the divide
t is elapsed time
S is the sediment transport rate
D is the detachment rate
and h is the travel distance

In this formulation, sediment transport processes must be described by two independent parameters, detachment rate and travel distance, instead of by the single parameter of transporting capacity in a simple transport limited model. Equation (4) is referred to here as formally describing 'erosion limited' removal.

As travel distances become small, it may be seen that the transport rate converges on the transport capacity, so that the model converges on the flux limited case. At very large transport distances, on the other hand, $S << C$, and the model converges on the supply limited form of equation (2) which in simple cases behaves as a kinematic wave of parallel slope retreat, corresponding to detachment limited removal. The erosion limited model allows us to include both of these cases as extreme end members within the single model. Where the travel distance is short in comparison with the relevant length of slope, then behaviour converges on the transport limited case, and where is long compared to the slope length of interest, behaviour converges on the detachment limited case. For intermediate cases, for example for landslides where travel distances are of the same order as the slope length, the forecast behaviour will be intermediate in type.

Equation (4) may be formally solved for S in the form:

$$S=I^{-1}\int_0^x D\,I\,dx'+S_0 I^{-1} \tag{5}$$

where $I = \exp(\int dx/h)$
and S_0 is the sediment transport at $x=0$ (commonly zero)

If D and h are both constant, this gives a simple exponential saturation, over scale distance h:

$$S=C\{1-\exp(-x/h)\}.$$

Alternatively, the (generally unknown) sediment transport rate S may be eliminated from equation (4) to give, for $S=0$ at $x=0$:

$$-\frac{\partial z}{\partial t}=D-\frac{\int_0^x DIdx'}{Ih} \tag{6}$$

with the same notation as before

or

$$T+\frac{\partial(Th)}{\partial x}=\frac{\partial C}{\partial x} \tag{7}$$

where $T = -\partial z/\partial t$

<u>Constant Downcutting profiles</u>

In general, the rate of detachment and travel distance are needed as functions of topography for each process. Dependence on lithology and climate may be explicit, but are commonly implicit in the choice of coefficients. It is normally sufficient to express both as functions of distance, x from the divide, and gradient Λ, although there may also be dependence on elevation,z and differentials of gradient. Distance from the divide acts as the topographic surrogate for discharge, and gradient is a direct driving potential for sediment movement. Where the dependence is on gradient alone, then equation (4) and its consequences may be seen as an important special case of the more general relationship due to LUKE (1972):

$$-\frac{\partial z}{\partial t}=f(q,\Lambda,S) \tag{8}$$

where q is the discharge per unit width
and L is the slope gradient.

In general, there are no simple solutions to equation (7) such as characteristic forms (KIRKBY, 1971) or simple parallel retreat (LUKE, 1972), because of the presence of both T and dT/dx terms. These possibilities still exist as special cases associated with the extremes of flux or supply limitation, and as useful approximations over a wider range. Constant downcutting solutions (fixed T) can, however, be generated readily, by setting dT/dx=0 in equation (7), and rearranging to give:

$$T=D\left(1+\frac{x}{h}\right) \tag{9}$$

where D and h generally vary downslope.

This expression can be very clearly seen to bridge between the supply limited and flux limited cases, for which the first and second terms are respectively dominant. For the supply limited case ($h \rightarrow \infty$), it becomes simply $D = T$; and for the flux limited case ($h \rightarrow 0$) it can be written as $C = Dh = Tx$. It is clear that travel distance is only an important parameter when it is of the same order of magnitude as the slope length of interest. The influence of travel distance on slope profile form can be seen fairly readily through comparing forms under conditions of constant downcutting. First, however, we should consider likely forms for the rate of detachment and travel distance, for different categories of sediment transport processes.

Detachment rates have been discussed most widely with respect to wash and splash processes, and the general consensus appears to be that raindrop detachment rates are more or less independent of position, and only weakly dependent on gradient (TORRI & POESEN, in press). For solution rates, it is also commonly assumed that, other things being equal, detachment rates are uniform over areas of similar parent material.

For wash processes and mass movements, detachment is usually associated with a threshold of movement or a safety factor of 1.0, and detachment rates may be taken as a power of shear in excess of the threshold value. For transport by running water, the threshold is commonly expressed in terms of critical shear stress or critical flow power in relation to bed grain sizes. If detachment is close to equal mobility (ANDREWS ,1983), then the mean grain size is relevant, and all sizes are detached equally. There may also be an additional gradient influence for slopes close to their stable angle. For mass movements, the critical stress conditions are related mainly to gradient and bulk material properties which may be assumed constant or may vary systematically with topography with, for example, thickening or finer grained soils downslope.

The definition of travel distance is to some extent linked to the choice of an appropriate time scale. For the long-term view implicit in slope evolution models, the relevant distance is best associated with movement in an entire flood or slide event, rather than the individual hops taken by sediment in the course of a flow, for example. Effective travel distances can also be inferred *post hoc* from the distribution of grains after one or more events. This issue is discussed more fully in KIRKBY (1991).

For transport by running water, travel distance is mainly related to flow and probably to grain size of each particle, giving the potential for size-selective deposition. For transport by rainsplash, creep, solifluction and mass movements, travel distance is driven mainly by gradient, with varying degrees of grain size selectivity. For solution, travel distances are generally long, although it will be seen below that another interpretation can be made where soils are highly permeable.

For simple processes like soil creep or splash then, our understanding of process mechanism suggests that the rate of detachment is roughly constant on a hillside, and that travel distance increases more or less linearly with gradient. If the transporting capacity $C = Dh$ is set at $\beta\Lambda$, with travel distance $(h_0\Lambda)$ allowed to vary with h_0 as total transporting capacity (given by $\beta\Lambda$) is held constant, then the constant downcutting slope form is given by:

$$\Lambda = Tx/(\beta - Th_0).$$

This is a smoothly convex parabolic ridge, but it can be seen that as the travel distance parameter h_0 increases, the convexity required to maintain a given rate of downcutting, T, also increases. Alternatively the same convexity (defined by the constant ratio x/Λ) generates lower and lower rates of downcutting as h_0 increases, following the expression:

$$T = \beta/(h_0 + x/\Lambda).$$

If rainsplash is combined with a simple form of rillwash without a threshold for motion, we may hypothesize the processes as follows:

Rainsplash:	$C_s = \beta\ \Lambda$:	$h = h_s\ \Lambda$:	$D_s = \beta/h_s$
Rillwash:	$C_r = \beta(x/u)^2$:	$h = \alpha\ x$:	$D_r = x\Lambda/x_r$

where $x_r = \alpha\ u^2/\beta$,

u is the distance at which the transporting capacity for rillwash exceeds that for rainsplash, and distance x is included as a surrogate for overland flow discharge. As for rainsplash, these expressions allow transporting capacities to be held constant (through β and u), while travel distances are varied (through α and h_s).

The corresponding constant downcutting form is given by solving for the rates of downcutting, which are generally no longer constant downslope for each individual process, but only for their sum. Although the equations can be solved numerically, there is no convenient analytical form where both splash and rillwash have non-zero travel distances. Figure 1 illustrates the changes in profile and gradient as the travel distance parameters α (for rillwash) and h_s (for rainsplash) are varied. The range of values shown is rather extreme, to emphasize the nature and direction of the changes. As may be expected, increases in h_s, which reduce the splash transport, produce a narrower convexity than with zero travel distances [(ii) and (iv) in comparison with (i) in Figure 1]. Similarly increases in α, which reduce rillwash transport, increase the width of the convexity [(iii) and (iv) in comparison with (i) and (ii) respectively].

Profile convexity or concavity

A qualitative idea of the form of the hillslope profiles predicted by the erosion limited model can be obtained by examining the conditions for profile convexity or concavity. These can be obtained from equation (7). Let us assume that the transporting capacity, C can be locally fitted by an expression of the form:

$$C = f(x)\,\Lambda^n$$

for an exponent n, which can be defined in general as $1/C\ \partial C/\partial \Lambda$. Then integration of equation (7), and rearrangement gives:

$$\Lambda = -\frac{\partial z}{\partial x} = \left[\frac{\int_0^x T\,dx + Th}{f(x)}\right]^{\frac{1}{n}}$$

(10)

Differentiating, the convexity (rate of increase in gradient) is given by:

$$\frac{\partial \Lambda}{\partial x} = \left[\frac{\int_0^x T\,dx + Th}{\mathrm{f}(x)}\right]^{\frac{1}{n}} \left\{\frac{\mathrm{f}(x)\left[T + \frac{\partial (Th)}{\partial x}\right] - \left[\int_0^x T\,dx + Th\right]\mathrm{f}'(x)}{[\mathrm{f}(x)]^2}\right\}$$

(11)

where the prime (') indicates differentiation with respect to x.

In this expression, it is clear that:

$f(x) > 0$

$n >= 0$

$h > 0$

$\int_0^x T\,dx + T\,h > 0$

The truth of the last statement follows because the left hand side is equal to the transporting capacity, which is necessarily positive on a single slope profile (i.e. provided that the slope gradient remains positive). It follows that the concavity is given by the sign of the simpler expression:

$$\mathfrak{L} = \frac{x\,\mathrm{f}'(x)}{\mathrm{f}(x)} - \frac{Tx + x\frac{\partial (Th)}{\partial x}}{\int_0^x T dx + Th}$$

(12)

where $\mathfrak{L}$ is positive for concavities, negative for convexities.

We may distinguish three informative cases, corresponding to the divides, the slope base, and mature slopes as a whole, with constant downcutting forms as a special case. Close to the divide, variations in the local rate of downcutting, T, are negligible. If the distance function f(x) behaves locally as x^m [i.e. $m = x\, f'(x)/f(x)$, evaluated near $x=0$], Equation (12) simplifies to:

$$\mathfrak{L} = m - (1+dh/dx)/(1+h/x)$$

For typical processes, like those discussed above, h/x is approximately constant near divides, so that the critical value of the exponent m is very close to 1.0. It can be seen, therefore, that differences in travel distance have only a very slight influence on the generally strong argument for convexity around divides (GILBERT, 1909; CARSON and KIRKBY, 1972, p.438), since creep, rainsplash and solifluction processes are thought to have exponents, m, close to zero. Thus divides are generally convex throughout their history, and significant travel distances make little difference to this conclusion.

Near the slope base, travel distance is generally changing downslope much less rapidly than the rate of denudation. Neglecting the term in dT/dx, we have:

$$\mathfrak{L} = m - \frac{T + h\, dT/dx}{\mathfrak{S} + T\, h/x}$$

where $\mathfrak{S}$ is the mean slope denudation,

and h,T refer to local slope base values.

Under conditions of rapid incision of a gorge, the ratio $T/\mathfrak{S}$ may be very high, so that convexity is generally associated with the early stages of incision. Over a period, slope base incision generally falls below the average, and incision generally declines downslope as the influence of base level is felt. Thus $T < \mathfrak{S}$ and $dT/dx < 0$. The critical value for the exponent m therefore generally falls below 1.0, and if the slope base is close to base level, may fall below zero. Since effective slope base exponents, m are generally at least 1.0, and usually more in association with wash processes and/or mass movements, the base of the slope will generally become concave as incision slows down.

For mature slopes in general, the local rate of denudation declines sharply downslope, and the mean rate declines with it, but more gradually. The term Th generally increases near the divide and then levels off or falls towards the slope base. the minimum value of the exponent, m, for slopes to become concave, therefore falls from about 1.0 near the divide to a value which may be zero or less at the slope base. Since the locally relevant value of m generally increases downslope, from an effective value of zero at divides to 1, 2 or more downslope, it is plain that hillslope profiles generally become convexo-concave. This conclusion for erosion limited removal is the same in general terms as for transport limited removal. For the special case of constant downcutting, equation (12) takes the particularly simple form:

$$\mathfrak{L} = m - (1+dh/dx)/(1+h/x).$$

As with transport limited removal, the condition for the constant downcutting from to be concave is identical to the condition for unstable enlargement of small hollows in the SMITH & BRETHERTON (1972) sense.

Before presenting example runs from a more general model which illustrates these principles, we will discuss the process formulations in greater detail. Wash processes have been very briefly outlined above, but require fuller development. There is also scope to discuss some detail of landslide and solution processes.

Wash processes

We have noted above that raindrop detachment rates have only a weak dependence on gradient. For a more fully realistic model, it should also be noted that detachment is also constrained by the depth of flow. Using Manning's equation as a first approximation, then flow depth r at distance x from the divide is given by:

$$r \propto \left(\frac{x}{\Lambda^{0.5}} \right)^{0.6} \tag{13}$$

Forecast detachment is then attenuated in the ratio:

$(1+y) \exp(-y)$

where y is the ratio of depth to a critical depth, which is usually taken (PALMER, 1963) to be 5-6mm. This effect is important for limiting rainflow on long, very gentle slopes.

Rainsplash travel distance is strongly, and to a first approximation linearly, dependent on gradient, though largely independent of distance from the divide (representing overland flow collecting area). Empirical exponents of gradient vary, with values as low as 0.5 and occasionally greater than 1.0, but the value of 1.0 is adopted here for simplicity.

The use of a travel distance model allows very effective forecasting of rainflow and rillwash. For this purpose, rainflow is defined as detachment by raindrops and transport in a water flow. Rillwash is defined as detachment by fluid traction, and transport in the flow. Thus we need to define two modes of detachment, by raindrops and by fluid traction; and two modes of travel, splashing through the air and moving as bed or suspended load in the water flow. The theoretically possible motion consisting of fluid detachment and aerial transport is generally discounted!

On steep slopes and for relatively large grains, the normal Shields/Andrews (ANDREWS, 1983) analysis requires considerable revision. Firstly it is not fair to assume that the depth of flow is large relative to the grain diameter, and secondly the gradients are large enough that the gravity term in grain traction cannot be ignored.

The forces acting are then:

(i) A downslope component of the submerged grain weight,

(ii) An upslope frictional resistance, related to submerged grain weight and angle of friction and

(iii) A fluid traction stress integrated over the range of depths of grain submergence.

These can be combined to give:

$r/d_c = \psi + 0.5$ for values greater than 1.0

and $= (2\psi)^{½}$ for values less than 1.0,

where $\psi = 0.06\ \Delta\ (\cot\alpha - \cot\phi)$,

Δ is the ratio of submerged grain density to water density,

r is the flow depth,

d_e is the effective grain diameter,

α is the slope gradient angle

and ϕ is the angle of friction.

This relationship gives a roughly linear increase in critical grain diameter with gradient up to angles of 20-25° and then a very rapid rise as the angle of friction is approached. It is assumed here that the relevant grain size is the mean, and that there is equal mobility during grain detachment. Thus, for a surface layer containing a mixture of grain sizes, the detachment rate for each size class is proportional to its concentration in the surface layer.

For travel distance, there is the potential for strong size selectivity, which appears to be demonstrated in many downstream and downslope fining sequences. If grains are given an equal initial velocity at detachment, then travel distance may be calculated from the net deceleration of the grains. If the angle of friction is kept constant, then deceleration is greatest for large grains, and grains below a critical size (for the flow) will be transported indefinitely. However, the effective angle of friction has been shown to increase as the size of a moving grain is reduced relative to the mean bed size, producing an opposite effect. Another factor is that initial grain velocities are not equal, and that grains are initially displaced upwards into the flow. An alternative view of travel distance is that it is the distance taken to fall to the bed from a position initially high in the flow. On this view, the travel distance is given by:

$h = r\ u/w$

where h is the travel distance

u is the flow velocity

and w is the grain settling velocity.

This estimate also forecasts much greater travel distances for larger grains, though with some convergence of values at high flows. For present purposes, it is proposed to use a travel distance which broadly follows the latter proposal, with distance inversely proportional to grain size, and proportional to discharge (= r u).

Mass movement processes

Mass movements can only be dealt with to a limited extent in a continuous (or discretised continuous) time framework. Macroscopic movements modify the topography, producing distinctive head scars and toe forms. Subsequent movements are changed by the detailed topography of individual previous slides. If there are positive feedbacks and/or non-linearities, as seems probable, then the continuous approximation must be in error. These errors will be least where individual slides are small. In practical computation, the size of the landslide is set by the choice of time step. Where relatively large time steps are used, the computed 'events' can be seen as visible waves of material, which mimic the feedbacks related to distribution of mass, but not those related to dynamic behaviour of piezometric pressures etc. There is, therefore, some merit in choosing a time step which is related to slide recurrence intervals, provided that it does not compromise computational stability.

Accepting this limitation on the representation of large slides, the proposed formulation for landslides follows KIRKBY (1984) and KIRKBY (1987) in assigning a detachment rate which increases above a threshold gradient:

$$D = D_0 \Lambda (\Lambda - \Lambda_0)^m \tag{14}$$

where Λ_0 is a threshold gradient, below which no slides occur,
D_0 is a rate parameter
and m is an empirical exponent, which takes values from 1-3.

In this formulation, the parameters depend, in general, on rock type and climate. At a site, over time, they also depend on the degree of weathering, but constant values have been used in the model presented here.

The stable gradient Λ_0 is generally less steep than the stable talus gradient, Λ_T, on which moving material will come to rest. The travel distance formulation for mass movements is similar to the simple sled model of SCHEIDEGGER (1973). The mass is assumed to have an initial velocity, so that its travel distance, measured horizontally, becomes:

$$h = \frac{h_0}{\Lambda_T - \Lambda} \tag{15}$$

where Λ_T is the gradient on which moving material will just come to rest.

At gradients steeper then Λ_T, it is clear that material will never come to rest, so that the slope undergoes free degradation (HUTCHINSON 1967), and is completely supply limited, but at lower gradients, removal is impeded and the travel distance has an important role in allowing some material to remain on the slope in a realistic manner. Below the stable gradient, material is never detached by mass movements, but material already detached (from steeper slopes) can be re-deposited to form a landslide 'toe' in a realistic way.

Between the talus and threshold gradients (Λ_T and Λ_0), where both D and h are defined, the concept of transporting capacity is also meaningful, though not outside this range. it can be seen that the model is applicable through the whole range of gradients, and not only for the limited range for which transporting capacity is defined. Where model mass movements are allowed to operate in addition to diffusive processes such as solifluction or soil creep, which tend to smooth and round sharp breaks in slope, there is a significant interaction with the landslides. At the head of the slide, the sharp break in slope becomes rounded, on average, although with episodic variations related to slide events. Where rounding rates are high, they may be seen to influence the rate at which the head of the landslide begins to decline in angle. Where diffusive processes are slight, or absent, the landslide head retreats parallel to itself, as a shock wave. The presence of diffusive processes introduces significant decline in the slide-head gradient, which works downslope to reduce the maximum gradients within the slide, so that slide areas decline in gradient over time, unless maintained by active basal downcutting. The rate of this decline is directly influenced by the rate of the diffusive processes. Figure 2 illustrates the form of cliff top convexities under conditions of constant cliff retreat. It may be seen that the convexity narrows, and the cliff gradient steepens, as rates of retreat increase.

Solution processes.

As stated above, the simplest possible assumption about solution processes is that their detachment rate is spatially uniform, with infinite travel distances. In this case they provide the simplest possible example of a supply limited process. A number of factors may, however, complicate this simple case.

The assumption of areally uniform detachment is based on the view that rainfall is able to come to chemical equilibrium with the soil on which it falls, and that residence times in the soil are sufficiently great (say more than 100 hours) for equilibration to occur in situ before significant lateral movement takes place. The assumption about large travel distances assumes that all sites along the subsequent flow path have similar chemistry. The equilibrated water is therefore mixed with other equilibrated water at the same solute concentration, and no further reaction takes place. Both of these assumptions need qualification.

Two factors which may influence the local rate of denudation are the pattern of overland flow, and consistent soil differences. For the important case of temperate stream head hollows, it is generally true that overland flow is greater than on neighbouring straight slopes and spurs. The overland flow is unable to reach chemical equilibrium with the soil, so that there some tendency for a reduction in solutional denudation from the hollow. The second factor is the consistent difference in soil acidity due to wetter average conditions in the hollows. This tends to increase the rate of local solutional loss, and allows water from less acid side slopes to pick up additional solutes. Evidence from 200m elevation in Somerset, England (CRABTREE and BURT, 1982), shows increased (25-30% greater) losses from rock pills in hollows compared to spurs and side slopes, indicating that soil differences were the dominant factor. On downslope transects, there was a similar reduction in loss downslope, indicating dominance of the overland flow factor. If these summaries are taken together, it may be inferred that the acidity factor is the more important in hollows for this rather wet site. At somewhat dryer sites, with less frequent soil saturation, it seems likely that the overland flow factor might be dominant.

Whichever factor is more important, there is likely to be some amplification over time. If hollows lose more than side slopes by solution, then preferential leaching will enhance the soil difference which is initiated by differences in organic decomposition. If hollows lose less than side slopes, then material dissolved on side slopes may be re-deposited in the less leached, and therefore less acid hollows. The interaction with overland flow is incorporated into the current model to constrain solution loss on very low gradients. Where the ratio of distance to gradient exceeds a critical value, corresponding to a saturation level for subsurface flow, then the overland flow is excluded from taking part in solution, and detachment is reduced pro rata. This effect becomes important where very gentle gradients develop through solution near the slope base, since reversed gradients might otherwise occur.

The assumption about long effective travel distances is based on the assumption of long residence times, close to the point where rainfall initially hits the surface. Flow residence times are typically long in the unsaturated phase of infiltration, when water is in diffusive contact with textural water already in the soil. Once water reaches a level of saturated lateral flow within the soil, lateral flow is generally more rapid. Thus the residence time in situ may be significantly longer than the subsequent time taken to reach the channel. In general, this may be the norm for soils without exceptional macro-pore bypassing. Where macropores are sufficiently large however, water passing through them may not have time to mix with the soil textural water which has had previously equilibrated. In this case, the significance residence time is during lateral flow downslope, and kinetic effects become crucial. These effects are probably most significant for karst areas, where much rainfall passes through the soil via enlarged joints etc.

For the kinetic case, we have:

$$\frac{\partial V}{\partial x} = \frac{1}{u} \cdot \frac{\partial V}{\partial t}$$

where V is the rate of solute transport,

and u is the downslope flow velocity = KΛ by Darcy's Law.

V may be also written as

$$V = ixc$$

where i is the net rainfall

and c is the solute concentration.

Substituting on the right hand side above,

$$\frac{\partial V}{\partial x} = \frac{ix}{K\Lambda} \cdot \frac{\partial c}{\partial t}$$

Making the simplest possible kinetic assumption,

$$\partial c/\partial t = \alpha(c_* - c)$$

where c_* is the equilibrium concentration

and α is the kinetic rate constant.

Substituting and rearranging, writing h_s for K/α and D_s for ic_*:

$$\frac{\partial V}{\partial x} = \frac{D_s x}{h_s \Lambda} - \frac{V}{h_s \Lambda} \tag{16}$$

with the same notation as before.

Equation (16) is exactly equivalent in form to Equation (4), so that we may interpret this kinetic solution process as an erosion limited process, with transporting capacity $C = D_s x = ic_* x$, and travel distance $h = h_s \Lambda$. The travel distance in this case is the distance travelled while equilibration is taking place.

Notice that, in this view, the soil covered case does not corresponds with very large travel distances, but appears to go with small travel distances. On the kinetic viewpoint, the soil covered case appears to relate to supply limited removal at transport rate $D_s x$. To resolve this apparent contradiction, where both residence times and downslope flow times may both be significant, rain water removes a denudation of γD_s during the infiltration period. If infiltration lasts for time T_0, and the same kinetic pickup applies in the unsaturated zone, then $\gamma=[1\text{-}\exp(-\alpha T_0)]$. An additional term is then added to equation (16), so that the travel distance is unchanged, but transporting capacity is modified to:

$$C = D_s(\gamma K\Lambda/\alpha + x).$$

The first term in this expression is the downslope distance which would be travelled during the period of unsaturated infiltration. Where infiltration is slow (large T_0), relative to lateral flow time, then this first term is dominant, but the same condition requires that travel distances are much longer than the slope. In other words, the original interpretation, as a supply limited process with detachment rate $D_s\gamma$, is appropriate.

For normal soils, an order of magnitude estimate for K is 250 m/day, and, for silicates, α is about 0.01 /hour, so that the travel distance $h = K/\alpha \approx 10^3$ m, which is generally long compared to the slope length. If residence time in the soil is also T = 100 hours, say, then the proportion of saturation within the soil, $\gamma = 0.63$, and the first term above is at least two orders of magnitude greater than the second, so that the simple supply limited model is appropriate. For a limestone area with large open joints at the surface, relevant values might change to:

$$\alpha = 0.1\text{/hour (for carbonates)},\ K = 1000\text{ m/day},\ T = 1\text{ hr}.$$

The travel distance then falls somewhat to $\approx$ 400m, and γ to 0.1. With these values, the second term above becomes dominant beyond 40m from the divide, and must be taken into account.

Solution rates also imply soil development, which is also highly relevant for slope form development. This aspect of the model is not explored here, but it is worth noting that it has important cross-connections both with mass movements and wash. For mass movements, the degree of weathering of the residual soil has a direct influence on geotechnical properties. For wash, the degree of weathering influences the grain size distribution of material available for transport. Where the model is used to give grain size selective transport, then slopes will produce greater wash transport for a given gradient as they age. Again, these interactions are not developed here, and a fixed grain size distribution is supplied from the model soil.

Model implementation and behaviour

It has been shown that it is both reasonable and practicable to adopt an erosion limiting framework for modelling slope evolution. For some processes, there is little advantage over flux limited models, and for others little advantage over supply limited models, but it has been shown that the erosion limited scheme is able to represent a wider range of processes than either alone. The main disadvantage of the erosion limited approach may lie in the lack of empirical values at present, but it has been shown that there is a rational basis for deriving values from what we already know, although there is clearly scope for much more experimental work.

Because the erosion limited approach contains diffusive flux limited models and kinematic supply limited models as special cases, there are plainly different possible strategies for implementing a model. The natural choice is between a finite difference scheme which works well for flux limited processes, and the method of characteristics which works well for many supply limited processes. Neither, however works well for the opposite extreme. Figure 3 shows the effect of using a finite difference solution, with a dynamic time step to limit instabilities, for a case in which the exact solution is a pure lateral translation. At long times, the exponential growth of a high frequency oscillation becomes highly unacceptable! Even a small amount of diffusion damps these oscillations. In practice, an adequate amount of damping is introduced if the diffusive sediment flux rate D is constrained by:

$$D \geq 0.2\ c\ \Delta x$$

where D is the diffusive rate (Sediment transport = D times gradient),

c is the translational kinematic wave velocity,

and Δx is the horizontal distance increment in the computation.

For creep or splash, the diffusion rate is generally taken as about 10 cm^2a^{-1}, so that retreat rates of up to 1 mm a^{-1} are acceptable with a 5m increment, 0.5 mm a^{-1} with a 10m increment, and so on. For the ten times greater diffusive rates shown for solifluction in Figure 2 and the maximum rates of 5mm a^{-1}, the minimum acceptable distance increment is $\Delta x =$ 10m.

The method of characteristics generates straight lines or simple curves along which slope gradient remains constant for supply limited processes where the rate of detachment is a function of gradient. Although the method may still be applied for purely diffusive processes, notions of parallel retreat are no longer applicable, and the curves of equal gradient become increasingly complex. Figure 4 shows the paths along which gradient is constant for a wholly diffusive process, beginning with a steep cliff at time zero. The paths show some initial retreat, but ultimately converge on the slope base in a pattern which does not lend itself to convenient computation by the method of characteristics.

It is plain that either method of solution is a compromise. In the implementation illustrated here, a finite difference scheme has been adopted, and has not produced any pathological forms like Figure 3 in practice. All examples include a significant amount of diffusion, and kinematic wave retreat is generally limited to much less extreme amounts than shown in Figure 3, because gradients decline below stability thresholds. Without continuous regeneration of steep gradients, the condition above, for minimum rates of diffusion, may therefore usually be relaxed somewhat. At long times, the slope evolution converges increasingly on a diffusive form, for which the finite difference solution is the more appropriate. Stability has been enhanced by using a dynamic time step, which constrains local changes in gradient to 20% or less in any time step.

At present, wash processes are the only group of processes which have been represented as grain size selective. The management of grain size is achieved through supplying debris from the soil in a fixed distribution of sizes. At the surface, an armour layer is generated from the balance of erosion and deposition. Initially the armour layer has zero thickness, and a composition equal to that of the soil. For processes which are not size selective, the sediment balance of equation (4) is used to calculate sediment transport and net erosion at each point down the length of the slope, from the detachment rate and travel distance as they respond to the topography. Net degradation ($-\partial z/\partial t$) or aggradation ($\partial z/\partial t$) is distributed across grain size classes in proportion to their frequency in the armour size distribution.

For wash, and other size selective processes if appropriate, detachment rates are calculated from the median grain size of the armour layer, and is distributed across size classes in proportion to class frequencies. Equation (4) is evaluated for each size class separately, with size selective travel distances. We obtain a net degradation or deposition by size class for each point down the slope profile, which is added to the contribution from non size selective processes. The dynamic time step is calculated on the initial assumption that these changes can all be accommodated within the armour layer. Where there is net deposition, or degradation which can be accommodated within the current depth of the armour layer for all size classes over the chosen time step, the net changes are simply added to the armour layer, by size class. Implicitly the layer is re-mixed to modify both its total depth and grain size composition. Where, on the other hand, degradation exceeds the amount of material in the armour layer for at least one size class, then additional material is taken from the soil beneath. The total depth of soil transferred to the armour layer is calculated as the maximum of the depths required for individual size classes. This transferred layer of soil is mixed into the armour layer as before. At present no mechanism has been incorporated to transfer the base of a thick armour layer back into the 'soil'.

It should be noted again that there are no weathering effects incorporated in the current version of the model. Solution is included as a denudational process, but is not allowed to modify soil composition or geotechnical properties in response to the degree of soil weathering, although that potential exists. The movement of surface debris is also based solely on selective grain transport, and there is no allowance for grain breakdown and/or weathering on the surface or in the armour layer. There is scope for adding these effects at a later date.

The model has been implemented in MicroSoft QuickBASIC (v 4.5) for MSDOS desktop computers. It consists of a 28KB program file, with access to a library of common graphical and Input/Output routines. Output is largely graphical, with detail of slope profiles over time, rates of denudation etc, and properties of the surface armour layer if the grain size selective option is taken. Copies and listings may be obtained from the author, and may be freely used with acknowledgement.

Example output

It is impracticable to represent more than a small number of the possible cases which can be generated by the erosion limited model. Here we focus on a small number of cases for which the model appears to offer substantial advantages over previous formulations. We will examine five cases. Firstly a straightforward profile undergoing creep, mass movements and wash. Secondly an example of solutional lowering, where there are significant kinetic effects; thirdly a mass movement example, with slope decline and rock core development; fourthly an example of wash without grain size effects, and finally of wash with grain size sorting.

Figure 5 shows an example profile, developed under the action of creep, mass movements and wash. The relatively rapid rates of landsliding (D_0 = 100 mm a^{-1}) give rapid degradation and slope replacement towards the stable gradient of 22° (40%) at 5,000 to 50,000 years. Subsequent development approximates to Davisian decline at much lower rates.

Figure 5 also shows two curves which summarise the slope evolution. The curve (i) plots mean relief (vertical axis) against the ratio of mean to summit relief, all defined relative to the slope base. Curve (ii) shows the ratio of mean rate of denudation to mean relief, on a logarithmic scale, against the same horizontal scale. The first curve follows a straight line directed towards the origin as long as the original summit elevation is preserved, as is clearly shown in this example. If the slope profile achieves a characteristic form, in which the profile becomes a progressively gentler version of it current form (KIRKBY, 1971), then the horizontal coordinate will tend to a constant value of mean:summit relief. A characteristic form also implies that the mean rate of denudation is proportional to mean relief (AHNERT, 1970), so that curve (ii) should converge on a single point. The global average value for this ratio is approximately 10^{-7} a^{-1}. It may be seen that this is approximately true for the latter phase of the development, after landslide stability. The increase in the ratio of mean: summit elevation is associated with the formation of a strong summit convexity as creep and wash take over the dominant role in slope development.

The predicted effects of solution are shown for the second set of parameters suggested under the discussion of solution above. (α = 0.1/hour (for carbonates), K = 1000 m/day, T = 1 hr). These values provide strong kinetic effects, which dominate at more than 50m from the divide, with a travel distance of 400m on unit gradient. Figure 6 shows the forecast slope evolution with a creep rate of $10cm^2a^{-1}$, and for a fixed base level. It may be seen that the effect of solution is to produce a sharp basal concavity, the shape of which is controlled by the critical value of x/Λ needed to generate overland flow. In figure 6, the critical value is set at 250m, corresponding to the high value for hydraulic conductivity, and requiring a flow depth of 25% of the mean daily net rainfall to give saturated conditions. A higher value gives an even more dramatic concavity. This pattern of slope evolution seems to correspond with some tendency for tower karst development, with the steepening of side slopes associated with the development of a basal plain with saturated overland flow. Figure 6 illustrates a quite different pattern for the summary curves (i and ii) from Figure 5, in which the ratio of denudation to mean relief remains close to $10^{-5}\ a^{-1}$, but the form is far from characteristic.

Where kinetic effects are not present, then slope evolution by solution and creep still produces a strong basal concavity associated with saturated overland flow, but there is no tendency for oversteepening of the side slopes, giving a topography perhaps more closely related to tropical dambos.

Figure 7 shows the interaction between diffusive processes and mass movements in a time development frame, expanding the discussion of equilibrium forms above. (a), (b) and (c) show increasing rates of diffusion. This results in a reduction in slope which is introduced at the top of the cliff, so that with higher rates of diffusion, the cliff is reduced in gradient more rapidly. In (a), where there is zero diffusion, slope development ceases once landslide stability has been attained. Here the relative rates of denudation fall rapidly as slopes decline towards a stable value, and then increase again towards a stable value which depends on the diffusive process rate. The algorithm used introduces a small amount of spurious numerical diffusion from the slope base.

The effect of wash without grain size effects is shown in figure 8. In this example, rainsplash is dominant up to 10m from the divide, giving rise to a narrow summit convexity. From 10-20m, rainflow is dominant, tending to provide a rather straight slope section. Beyond 20m, rillwash is predominant, and gives a gentle concavity. In the initial stages of evolution, there is slope replacement at a gradient of 35° (70%), corresponding to the talus gradient, above which transported material cannot come to rest. Although not observable in one-dimensional profile, this stage is associated with considerable rill incision. Although the addition of grain sizes to this example makes very little difference to the slope forms, the angle of repose basal slope is covered with a thin armour layer with a grain size approximately 1ϕ class coarser than the average for the soil. Ultimately this coarse armour spreads all over the surface, but without evident sorting patterns. With processes like these, for which travel distances are generally small relative to the total slope length, there is a fair approximation to a characteristic form, shown by the near-vertical segment of curve (i), and the inactivity of curve (ii) past 10Ka.

The final example is also for wash processes, but with appreciable deposition on a basal slope, as may occur for example beneath a fault scarp. In the depositional area the model shows a clear area of downslope fining, and a strong relationship between gradient and grain size in this zone. Elsewhere, as in the previous example, the relationship between armour size, soil grain size and gradient is less clear. Figure 9 shows two similar runs of the model in which splash, rainflow and rillwash are the only processes operating. The only difference in inputs is in the soil grain sizes. In (a), the distribution is equally divided among 5 grain size classes, while in (b) there is 50% in each of the same two extreme classes, and none in intermediate classes. Although the course of profile evolution is very similar in general, there is an interesting difference in the depositional area. For the bimodal case (b), the armour grain size flips from one extreme to the other, as might be expected, and the gradient curve shows a sharp break in slope at the same point. For the evenly graded soil in (a), there is a much broader transitional zone in which downslope fining occurs, and the gradient curve echoes it in a broader concave break in slope.

This behaviour appears to mimic the behaviour at many semi-arid breaks in slope. Bedrocks yielding a continuous range of grain sizes (eg basalts) are associated with gentle breaks in slope; and bedrocks which break down discontinuously (eg granites) tend to be associated with sharp breaks in slope. In the model, it is currently possible to reproduce this behaviour in areas of net deposition. Areas modelled with net erosion show a generally coarse armour layer but little grading. Perhaps there is scope to extend the scope of surface grading by inclusion of in situ weathering and breakdown of surface material.

Conclusions

By disaggregating sediment transporting capacity into two components, a rate of detachment and a travel distance, it is possible to simulate the behaviour of one or more processes under conditions which depart significantly from the flux limited case where transport is always at its transporting capacity. This erosion limited approach to modelling slope development has been shown to have a number of advantages over simpler supply or flux limited approximations. Although it has not yet been possible to include all relevant processes in the present model, nevertheless, it has been shown that erosion limited removal can be used as a basis for combining a wide range of processes within a single model framework.

The simulations behave in a similar way to conventional supply limited models where travel distances are short relative to the position of interest on the slope, but show significant divergences where travel distances are long. These differences may be relevant on small experimental plots, even for processes like rainsplash with travel distances of the order of 1 metre; and are certainly relevant for wash, solution and landslides on the scale of complete slope profiles.

By considering the mechanics of each process, it has been possible to make reasonable inferences about how travel distance and detachment rates behave for each process, although the conclusions require empirical confirmation. The erosion limited view adds considerably to the treatment of wash processes in particular, by allowing rainsplash, rainflow and rillwash to be properly integrated as two detachment and two travel processes. It also allows a rational analysis of grain size selection in transport, with a gradation from supply limited transport of fines to flux limited transport of coarse debris.

Erosion limited removal is also highly relevant to mass movements, although the episodic nature of large mass movements must limit the applicability of any continuum approach. For solution, there is scope to separate explicitly the roles of unsaturated infiltration and saturated lateral flow to produce effective travel distances which modify the pattern of solution where there is strong macropore bypassing.

Overall, the erosion limited approach is seen as a way of unifying the treatment of sediment transport in a proper way, with the present implementation as no more than a first illustrative step.

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List of Figures

8. Example of slope profile evolution by wash processes, without incorporating grain size effects. Rainsplash is dominant to 10m from the divide; then Rainflow to 20m and subsequently Rillwash. Initial slope replacement is associated with steepest slopes on which transported material will come to rest. Curves (i) and (ii) as above.

9. Evolution of a stepped profile (eg a fault scarp) by wash processes, with grain size selectivity. (a) and (b) differ only in the grain size distribution of the soil. The upper graph shows initial and final slope profiles, together with final gradients and armour thicknesses. The lower curve shows final grain size distributions and mean grain size, in comparison with original soil grain size (shown at far right).
(a) is for a soil with 20% in each of five ϕ classes from -4ϕ to +1ϕ.
(b) is for a soil with 50% in -4ϕ to -3ϕ class, and 50% in 0ϕ to +1ϕ.
Although general evolution is similar, note the different curves for gradient and grain sizes in the depositional area.

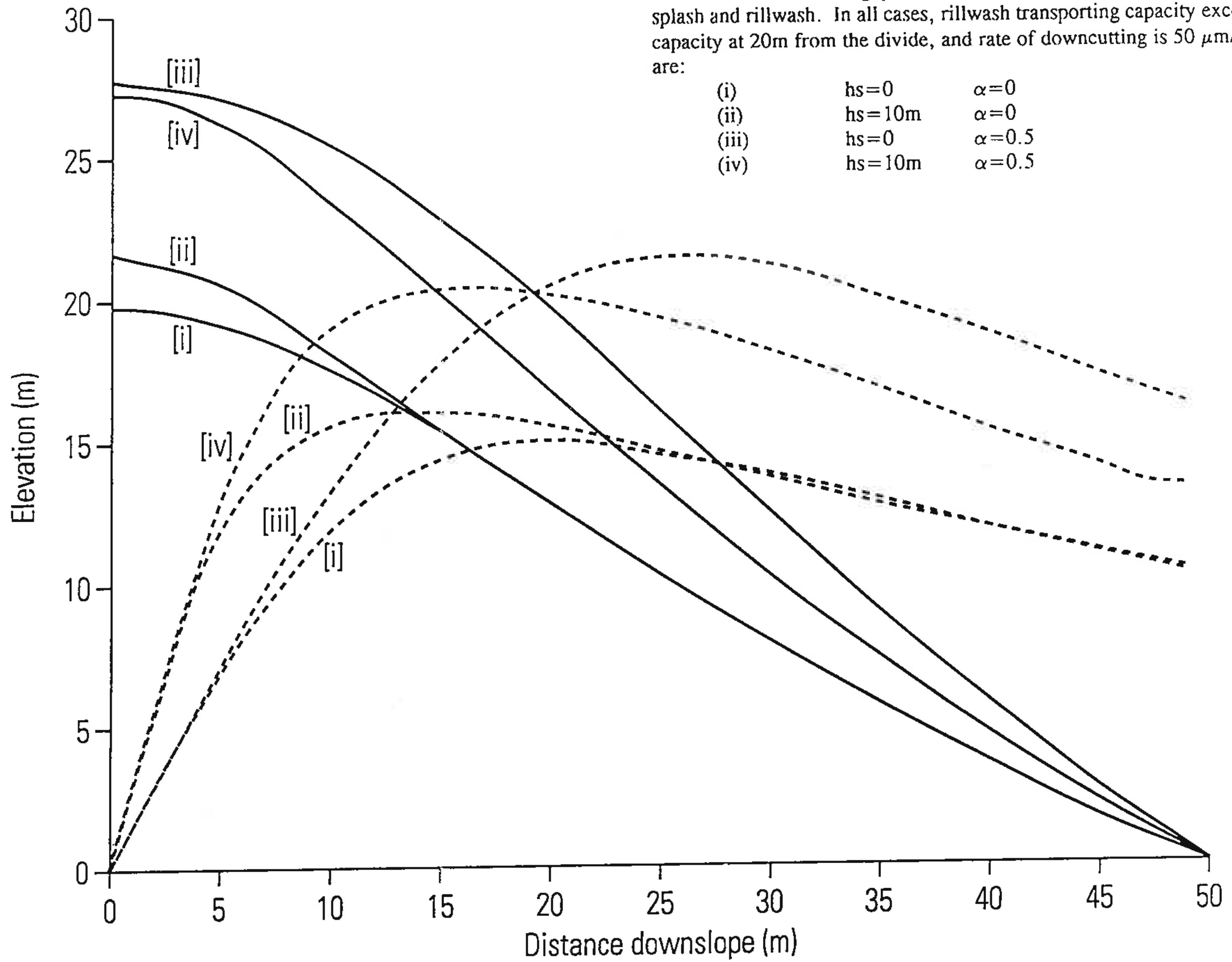


Figure 1 Constant downcutting profile forms and gradients for a combination of splash and rillwash. In all cases, rillwash transporting capacity exceeds splash capacity at 20m from the divide, and rate of downcutting is 50 $\mu m/y$. Curves are:

(i)	hs=0	$\alpha=0$
(ii)	hs=10m	$\alpha=0$
(iii)	hs=0	$\alpha=0.5$
(iv)	hs=10m	$\alpha=0.5$

acting are supply limited solifluction or creep, in combination with erosion limited mass movements. The curves therefore show the average profile around a cliff top. Solid curves are for $D_0 = 2$ mm a^{-1}: Broken curves for 1 mm a^{-1}. The four pairs of curves are for equilibrium rates of retreat of, from left to right, 5, 2, 1 and 0.5 mm a^{-1}. Other values are: $\Lambda_0 = 0.4$ (22°), $\Lambda_T = 0.7$ (35°) and rate of solifluction, $\beta = 100$ cm^2 a^{-1}.

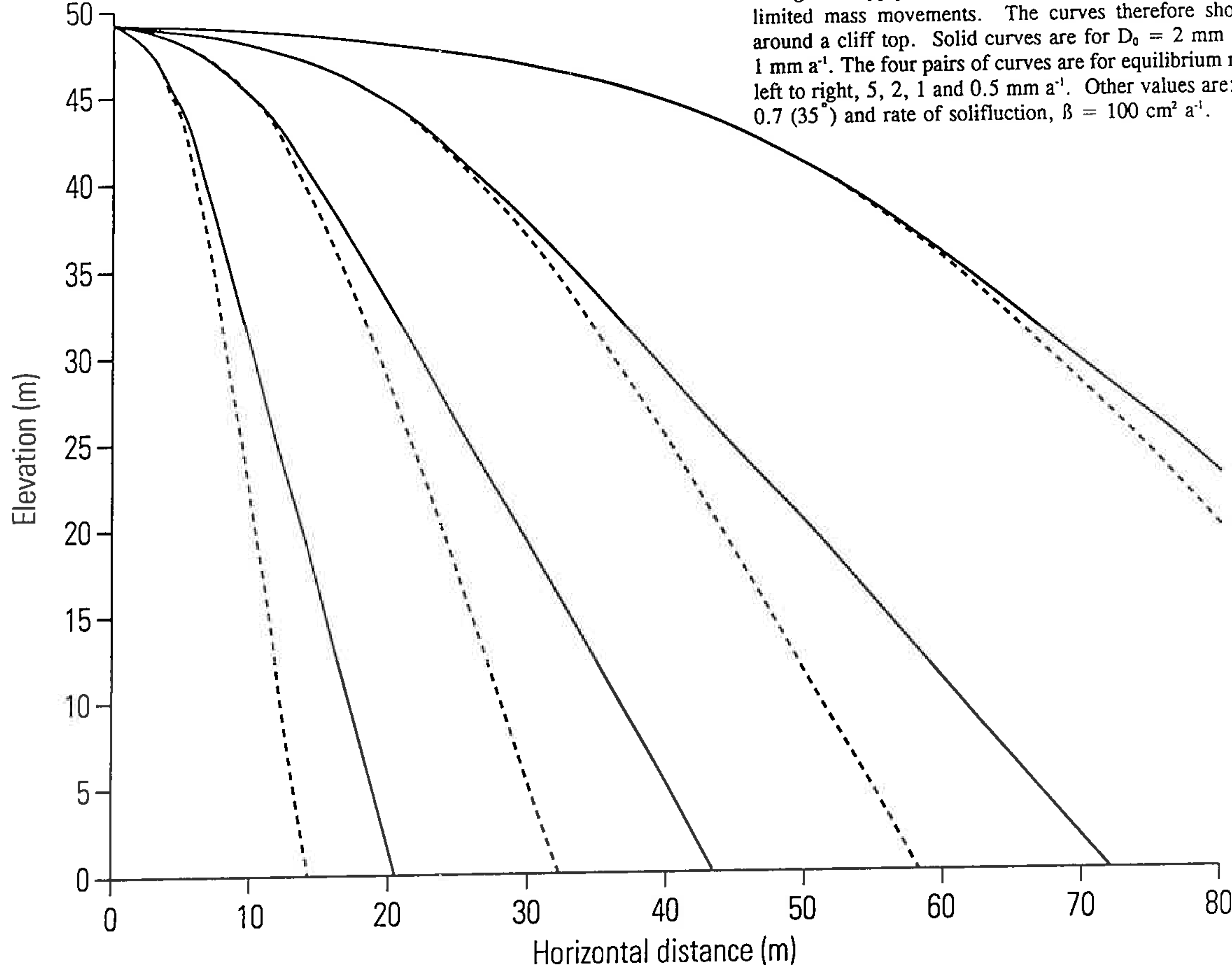
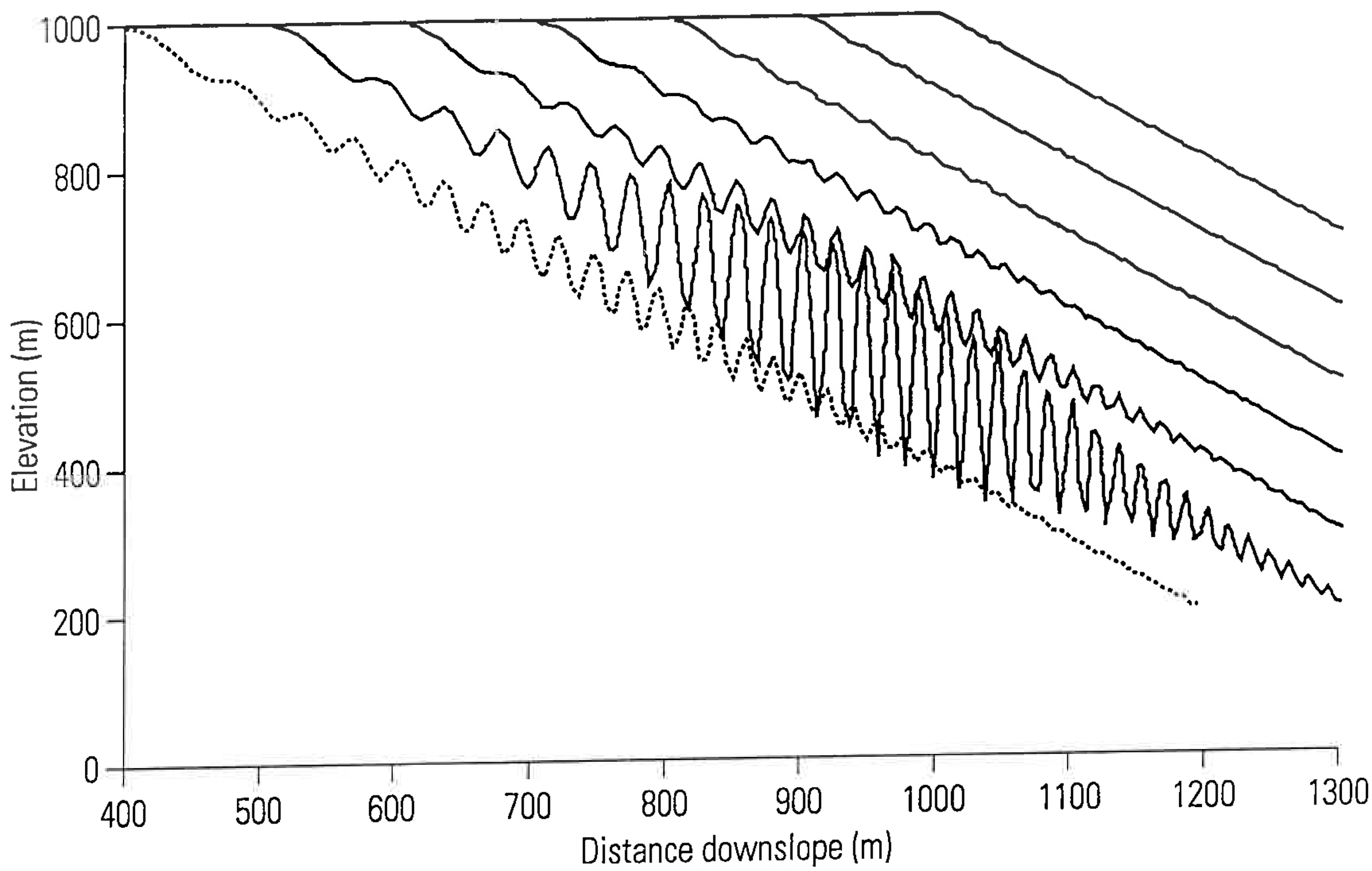
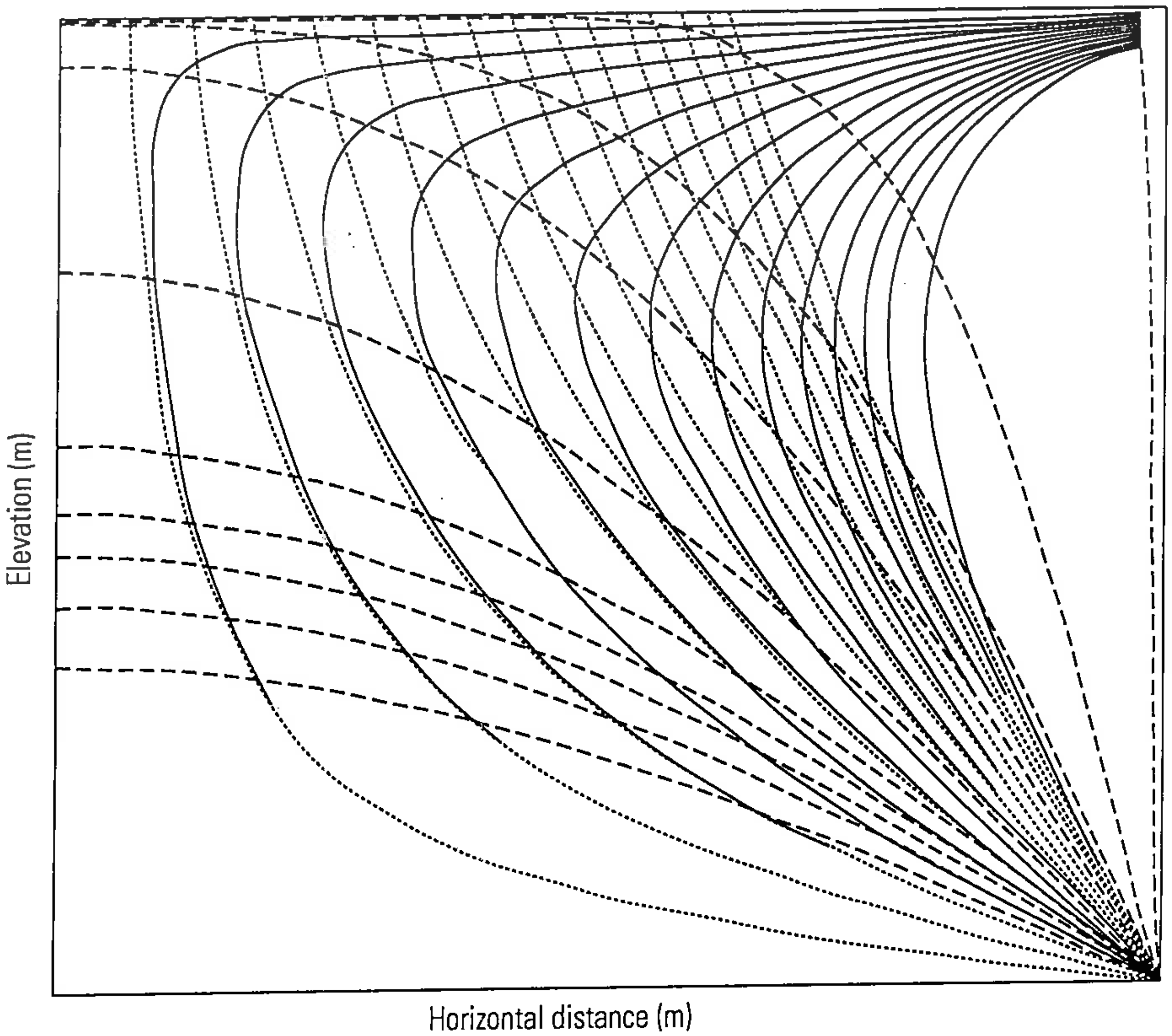


Figure 3 Computational instability arising from use of finite difference solution for a case of pure retreat. The high frequency oscillation grows exponentially, and is propagated downwards from the original break in slope at A, as can be seen from successive curves from right to left. The dotted curve shows the effect of a small amount of diffusion in damping these spurious oscillations. It corresponds to the same time (and total slope retreat) as the left hand solid curve. The amount of damping is only 5% of that recommended in the text to provide adquate stability.



processes over time. Dotted lines show successive slope profiles. Broken lines show the characteristic form solutions on which the solution converges.



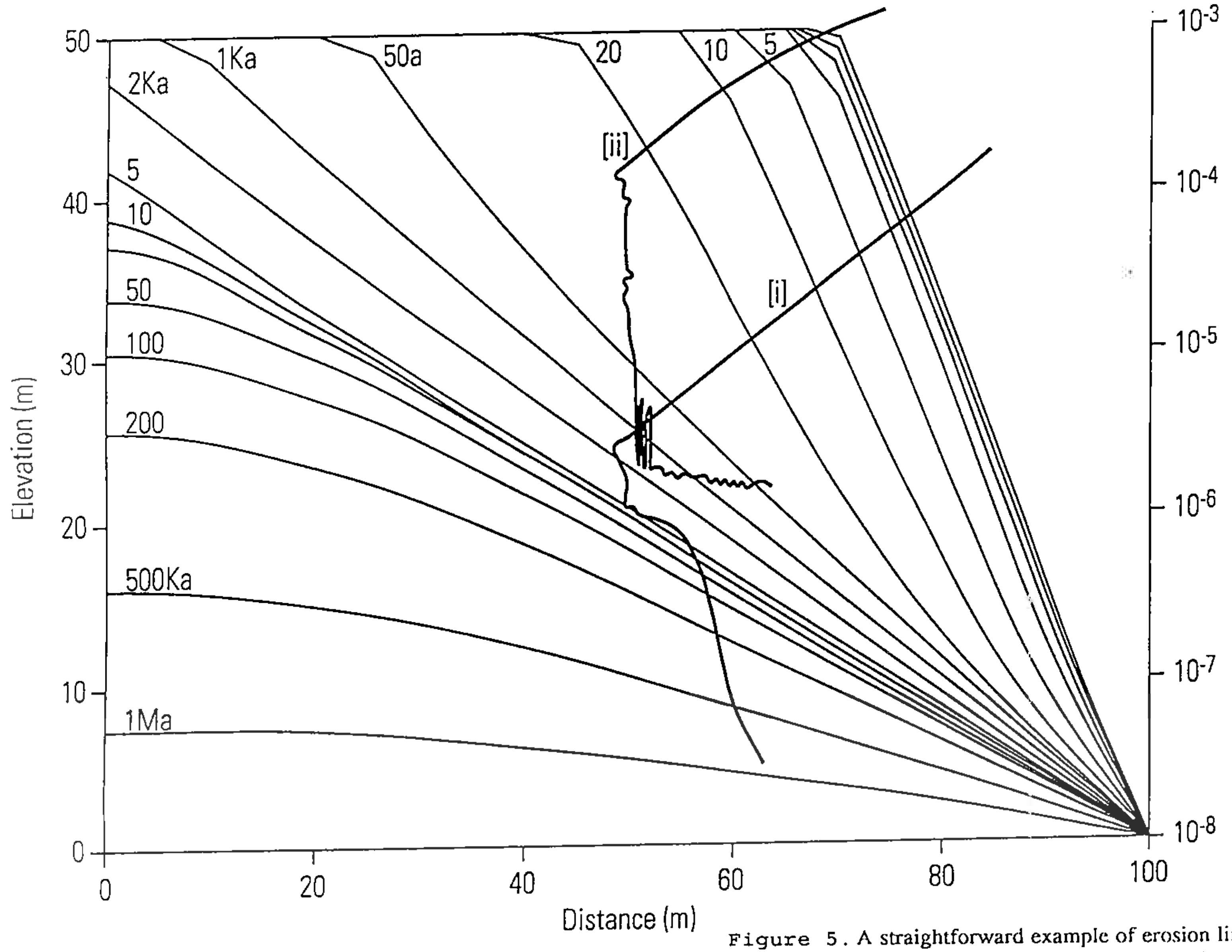


Figure 5. A straightforward example of erosion limited slope development by creep, mass movements and wash. The slope base is held fixed, and the initial form is as shown. The main curves show slope profiles as successive times. The additional summary curve (i) shows the mean relief plotted against the ratio of mean to summit relief. Curve (ii) shows ratio of mean denudation to mean elevation against the ratio of mean to summit relief.

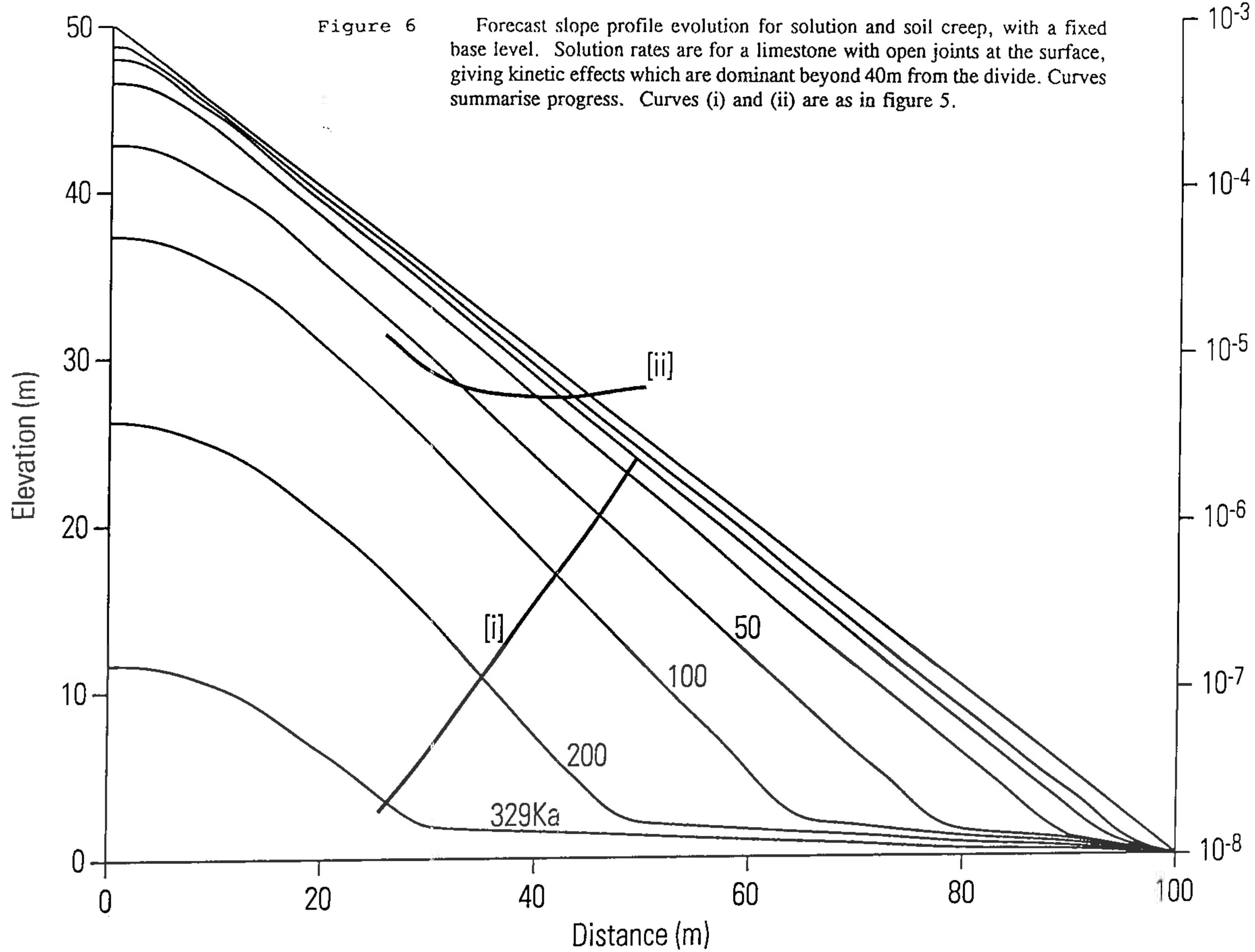


Figure 6 Forecast slope profile evolution for solution and soil creep, with a fixed base level. Solution rates are for a limestone with open joints at the surface, giving kinetic effects which are dominant beyond 40m from the divide. Curves summarise progress. Curves (i) and (ii) are as in figure 5.

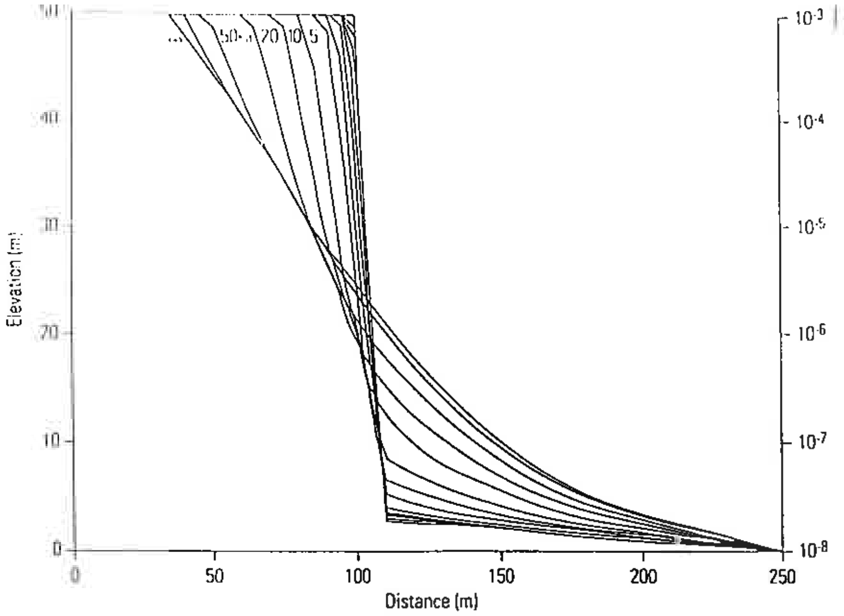


Figure 7a

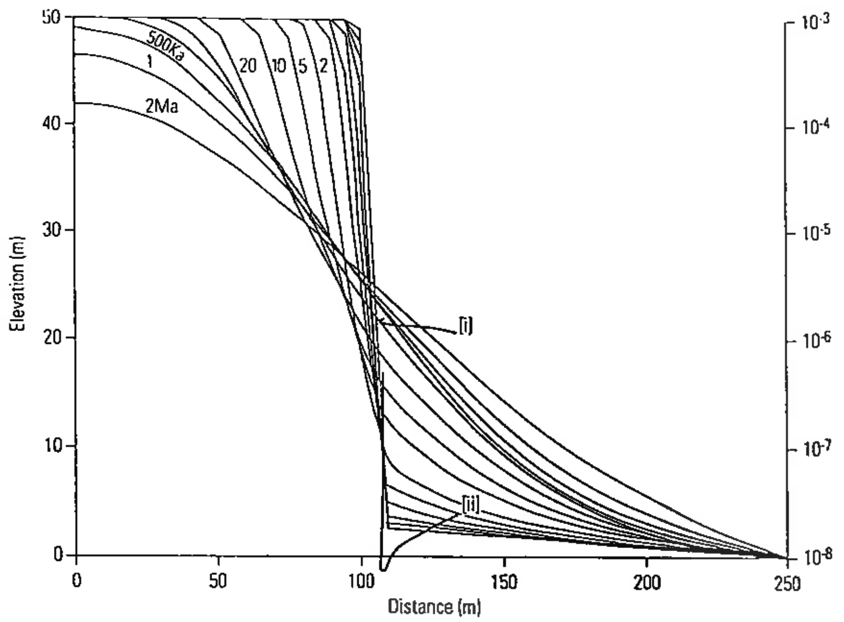


Figure 7b

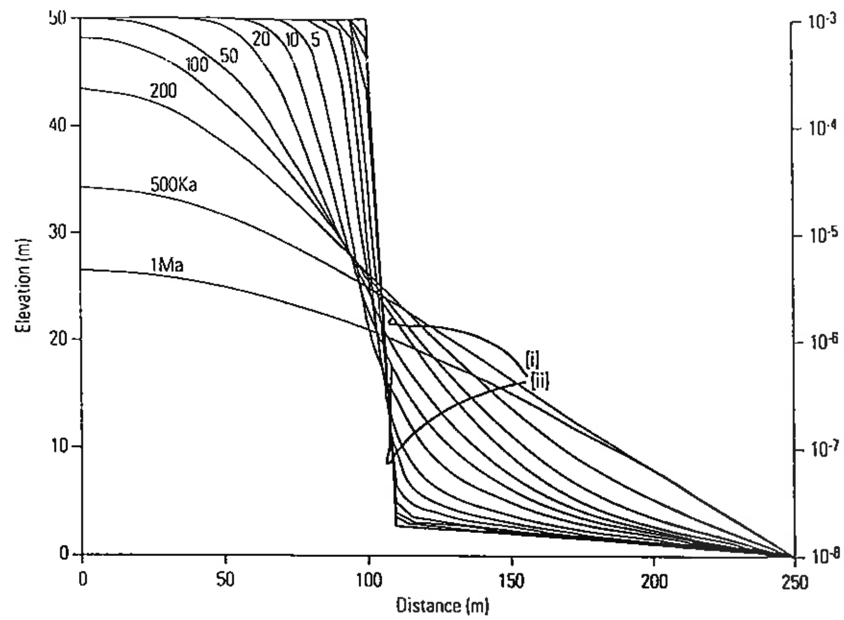


Figure 7c

Figure 7 Slope evolution due to landsliding, with debris accumulating at the slope foot. (a) ,(b) and (c) differ in the amount of rounding introduced from the cliff top by diffusive processes. In (a) there is zero diffusion, in (b) $ß = 10$ cm^2a^{-1} (Creep) and in (c) $ß = 100$ cm^2a^{-1} (Solifluction). It can be seen that the rounding speeds the recline of the cliff from the top.

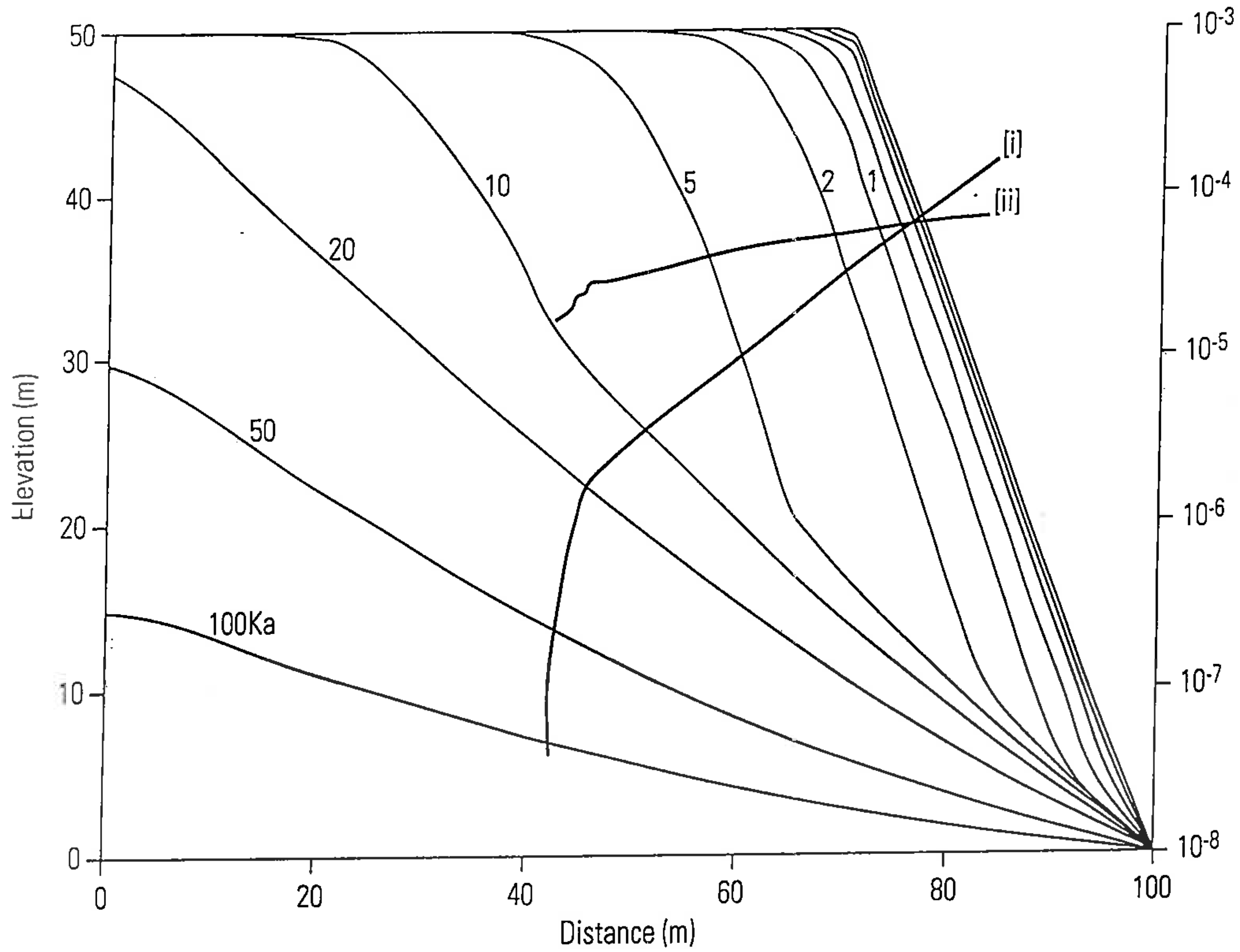


Figure 8 Example of slope profile evolution by wash processes, without incorporating grain size effects. Rainsplash is dominant to 10m from the divide; then Rainflow to 20m and subsequently Rillwash. Initial slope replacement is associated with steepest slopes on which transported material

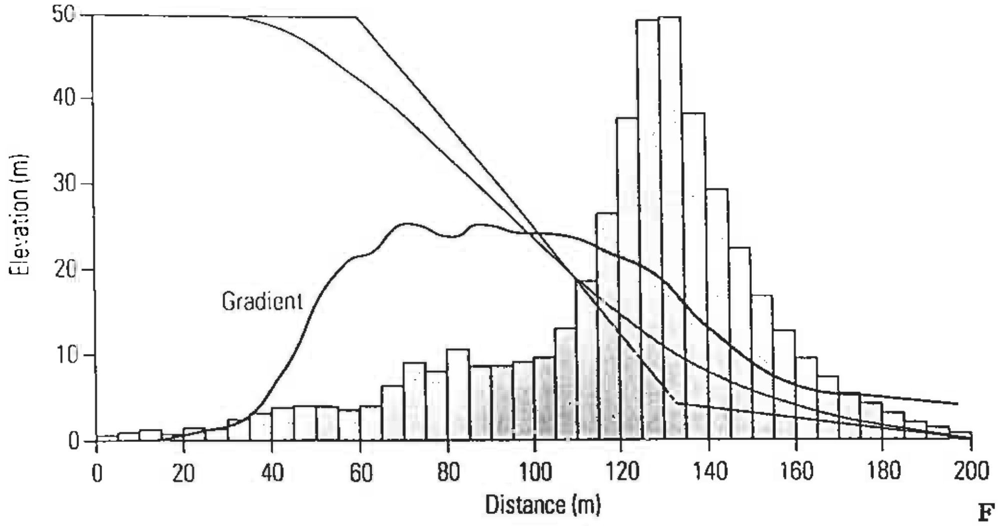
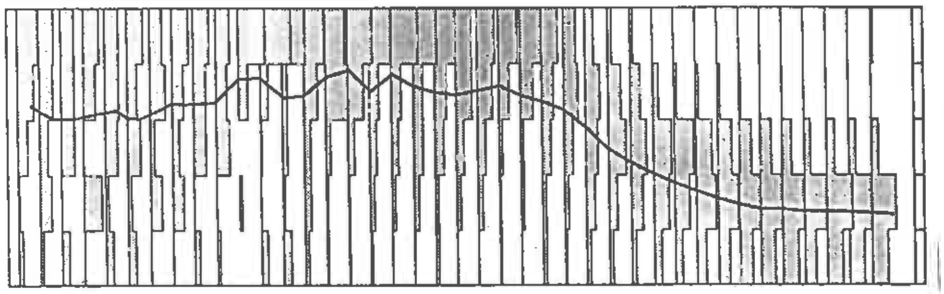


Figure 9a



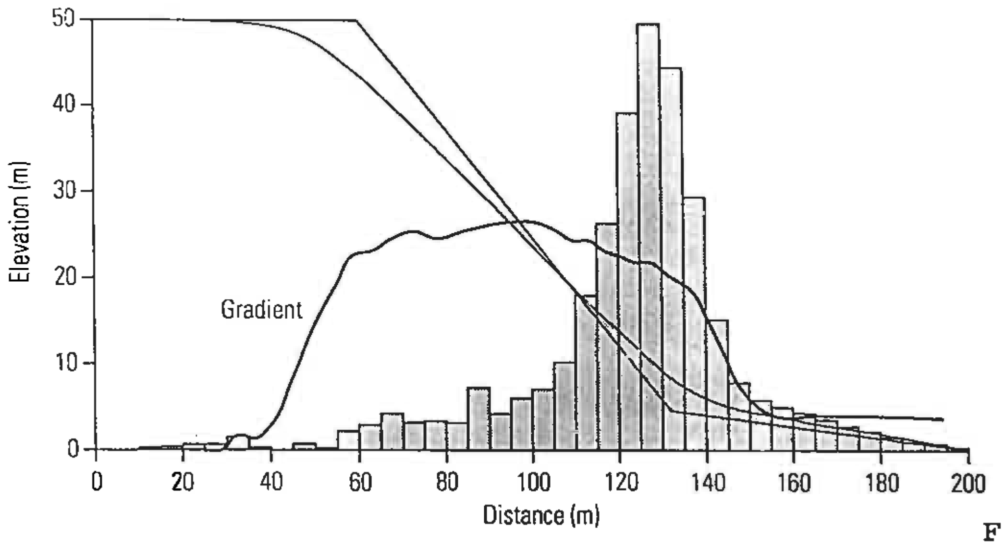


Figure 9b

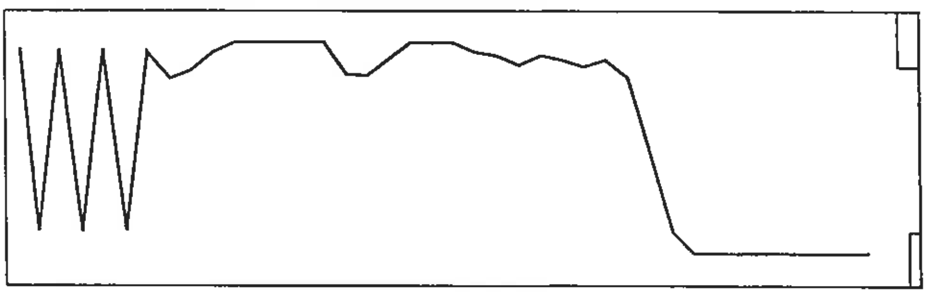


Figure 9. Evolution of a stepped profile (eg a fault scarp) by wash processes, with grain size selectivity. (a) and (b) differ only in the grain size distribution of the soil. The upper graph shows initial and final slope profiles, together with final gradients and armour thicknesses. The lower curve shows final grain size distributions and mean grain size, in comparison with original soil grain size (shown at far right).
(a) is for a soil with 20% in each of five ϕ classes from -4ϕ to +1ϕ.
(b) is for a soil with 50% in -4ϕ to -3ϕ class, and 50% in 0ϕ to +1ϕ.
Although general evolution is similar, note the different curves for gradient and grain sizes in the depositional area.

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licrosoft QuickBASIC 4.5 (for MSDOS systems)

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LARE SUB Slide (ix)
LARE SUB Wash (ix)
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LARE SUB InitData (iflag)
LARE SUB plot (ip)
LARE SUB plothist ()
LARE SUB baselevel ()
LARE SUB UpDate ()
LARE SUB gsizes ()
LARE SUB ArmourLayer ()
LARE SUB vdu (nb, nu)
LARE FUNCTION Sout (den, RECH, SSin, iy, jg)
LARE SUB intro ()
LARE SUB ExitOptions ()
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 SHARED CreepRate, CreepDen
 SHARED SolRate, soltrav, ascrit, solgamma, solK, solRes, solEQ
 SHARED SlideRate, Travel, gth, gtal
 SHARED RainDen, FlowDen, RainTrav, FlowTrav, xrd, xthr
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ata = 0: nsize = 20: conl = 1 / LOG(2): nHist = 25
de = 0: relden = 0: basel = 0: ibase = 1: FpWidth = 10
TART:
IM SHARED xms(5000), xrr(5000), xm0(5000)
udata = 0 THEN
tro
 igsize = 1 THEN
REDIM SoilSize(nsize), xas(nsize)
IM SHARED xArm, igmin, igmax
gsizes
D IF
E vdu 0, 1
 IF
Data nudata
IM elev(nx), Soil(nx), dsoil(nx), dElev(nx)
IM ElevHist(nx, nHist), THist(nHist), dElevHist(nx, nHist)
```

```
REDIM SoilHist(nx, nHist), dSoilHist(nx, nHist)
khist = 0
IF igsize = 1 THEN
  ' Armour characteristics for (i) refer to  mid point in [i,i+1]
  REDIM Armour(nx - 1), ArmourSize(nx - 1, nsize), ArMean(nx - 1), gDelev(nx, nsize)
  REDIM ArmourHist(nx - 1, nHist)
  FOR j = 0 TO nx - 1
    FOR i = igmin TO igmax
      ArmourSize(j, i) = SoilSize(i): NEXT
    Armour(j) = 0: ArMean(j) = dphi: NEXT
END IF

InitForm
vdu 1, 2
Tnext = 1
ON KEY(1) GOSUB F1
ON KEY(2) GOSUB F2
ON KEY(3) GOSUB F3
ON KEY(4) GOSUB F4
ON KEY(5) GOSUB F5
ON KEY(6) GOSUB F6
KEY(1) ON
KEY(2) ON
KEY(3) ON
KEY(4) ON
KEY(5) ON
KEY(6) ON
DO
  ' non size-selective processes first
  oz = elev(0): REDIM dElev(nx)
  FOR i = 1 TO nx
    SedTpt = 0
    cz = elev(i)
    grad = (oz - cz) / dx: xx = (i - .5) * dx
    Creep i
    Solution i
    Slide i
    oz = cz
    NEXT

  'Now redistribute lowering pro rata amongst armour grain sizes
  IF igsize = 1 THEN
    FOR i = 0 TO nx - 1
      dd = dElev(i)
      FOR j = igmin TO igmax
        IF i = 0 THEN jk = 0 ELSE jk = i - 1
        gDelev(i, j) = dd * (ArmourSize(i, j) + ArmourSize(jk, j)) * .5
        NEXT
      NEXT
  END IF

  'Now size-selective process(es)
  oz = elev(0)
```

```
igsize = 1 THEN
rmourLayer
D IF
R i = 1 TO nx
z = elev(i)
F igsize = 1 THEN
 xam = ArMean(i - 1)
 FOR j = igmin TO igmax: xas(j) = ArmourSize(i - 1, j): NEXT
ND IF
rad = (oz - cz) / dx: xx = (i - .5) * dx
ash i
z = cz
XT

selevel
Date

ey1 = 1 THEN
F ign = 2 THEN vdu 1, 2
ey1 = 0: plot 1
D IF

key2 = 1 THEN
F ign = 1 THEN vdu 2, 2
ey2 = 0: plot 2
D IF

key6 = 1 THEN
nitData 2
du 1, 2
 = pMenu(menu, "")
ey6 = 0
D IF

key4 = 1 THEN key4 = 0:  EXIT DO
key3 = 1 THEN key3 = 0: vdu 3, 1
P UNTIL time >= 1E+08
Options
O RESTART



y1 = 1: RETURN

y2 = 1: RETURN

y3 = 1: RETURN

y4 = 1: RETURN

D

y6 = 1: RETURN
```

```
F7:
 key7 = 1: RETURN

REM $STATIC
SUB ArmourLayer
'sets mean phi grain size for armour at all slope positions
IF igsize = 0 THEN EXIT SUB
FOR i = 0 TO nx - 1
  tot = 0: sum = 0: FOR j = igmin TO igmax
    z = ArmourSize(i, j): sum = sum + z
    tot = tot + (j - 9.5) * z: NEXT
  IF sum > 0 THEN
    z = tot / sum
    ArMean(i) = EXP(-z / conl)
  ELSE ArMean(i) = dphi
  END IF
  NEXT

END SUB

SUB baselevel
dElev(nx) = 0

den = (absden + relden * (elev(nx) - basel)) * .000001
SELECT CASE ibase
CASE 1
    'Absolute control on elevation
    dElev(nx) = -den
CASE 2
    'Control on sediment increment in stream
    zsed = SlowTpt + SlideTpt
    dElev(nx) = (SedTpt * 2 - FpWidth * den) / (FpWidth + dx)
END SELECT
'Distribute among grain sizes in proportion to Armour composition
IF igsize = 1 THEN
  FOR j = igmin TO igmax: gDelev(nx, j) = dElev(nx) * ArmourSize(nx - 1, j): NEXT
END IF

'Neutral assumption for soil depth
dsoil(nx) = dsoil(nx - 1)




 END SUB

 SUB Creep (ix) STATIC
 ' non size-selective
 IF grad = 0 THEN Rh = 1E+29 ELSE Rh = 1 / (CreepRate * grad)
  CreepTpt = Sout(CreepDen, Rh, CreepTpt, ix, -1)
  SedTpt = SedTpt + CreepTpt

 END SUB
```

```
 ExitOptions
RED key1, key2, key3, key4, key5, key6, key7, nudata
(4) OFF
size = 0 THEN KEY(3) OFF
EY(7) GOSUB F7
(7) ON
t(khist) = time
 i = 0 TO nx
vHist(i, khist) = elev(i)
evHist(i, khist) = dElev(i)
lHist(i, khist) = Soil(i)
ilHist(i, khist) = dsoil(i)
T
n <> 3 THEN plot ign ELSE vdu 3, 2
size = 0 THEN
nu="<F1>Graph<F2>Numbers<F5>Quit<F6>Edit Parameters/Restart<F7>File/Restart

E
nu="<F1Graph<F2>Numbers<F3>GrSizes<F5>Quit<F6>Edit/ReStart<F7>File/Restart

 IF
pMenu(menu, "")

key1 = 1 THEN
 ign <> 1 THEN vdu 1, 2
ey1 = 0
D IF

key2 = 1 THEN
 ign <> 2 THEN vdu 2, 2
ey2 = 0
D IF

key3 = 1 THEN
 ign <> 3 THEN vdu 3, 2
ey3 = 0
D IF

key6 = 1 THEN nudata = 1: key6 = 0: EXIT DO
key7 = 1 THEN nudata = 0: key7 = 0: EXIT DO
OP
(7) OFF

 SUB

 gsizes
LOR 14, 7
INT "GRAIN SIZE ANALYSIS FOR SOIL"
INT "from Data File with extension .MM"
INT "Format of data should be:"
INT "    mm (b-axis) or phi classes for grain size: smallest first"
INT "    weights in each class or numbers in Pebble count"
INT "Largest (open-ended) class must contain no stones (0 or blank in file)."
```

```
 PRINT : readfile "mm"
 tn = dat(1, 1): tx = tn: FOR i = 2 TO nrow
  a = dat(1, i): IF a > tx THEN tx = a ELSE IF a < tn THEN tn = a
  NEXT
 ig = 0: IF tn < 0 THEN ig = 2
 IF tx > 10 THEN ig = 1
 IF ig = 0 THEN ig = pMenu("Indicate type of size data in file: <M> for mm: <P> for Phi",
"MP")
 DIM diam(nrow), phi(nrow), wt(nrow), cum(nrow), dbar(nrow), phibar(nrow)
 'If smallest grain sizes are NOT at top of file, then invert data!
 IF (ig = 1 AND dat(1, 1) > dat(1, nrow)) OR (ig = 2 AND dat(1, 1) < dat(1, nrow))
THEN
   FOR i = 1 TO INT(nrow / 2): SWAP dat(1, i), dat(1, nrow + 1 - i): NEXT
   FOR i = 1 TO INT((nrow - 1) / 2): SWAP dat(2, i), dat(2, nrow - i): NEXT
   END IF
 SELECT CASE ig
   CASE 1: 'Diameters
     FOR i = 1 TO nrow: diam(i) = dat(1, i): phi(i) = -LOG(dat(1, i)) * conl: NEXT
   CASE 2: 'Phi classes
     FOR i = 1 TO nrow: phi(i) = dat(1, i): diam(i) = EXP(-dat(1, i) / conl): NEXT
   END SELECT
 od = diam(1): op = phi(1): FOR i = 1 TO nrow - 1
   cd = diam(i + 1): dbar(i) = SQR(od * cd): od = cd
   cp = phi(i + 1): phibar(i) = (op + cp) * .5: op = cp: NEXT
 FOR i = 1 TO nrow - 1: wt(i) = dat(2, i): NEXT
 cum(1) = 0: FOR i = 1 TO nrow - 1: cum(i + 1) = cum(i) + wt(i): NEXT
 tot = cum(nrow)
 FOR i = 1 TO nrow: cum(i) = cum(i) / tot: NEXT
 s0 = 0: s1 = 0: s2 = 0: FOR i = 1 TO nrow - 1
   t = wt(i): s0 = s0 + t
   u = phibar(i) * t: s1 = s1 + u
   u = phibar(i) * u: s2 = s2 + u
   NEXT
 phimean = s1 / s0: sdphi = SQR((s2 / s0 - phimean * phimean))
 dphi = EXP(-phimean / conl)
 PRINT : PRINT USING "Phi mean = ###.#. Grain size equivalent = ###.####mm";
phimean; dphi
 iphi = 10: j = 1: xcum = 0: xphi = 10: DO
   IF j > nrow THEN cphi = -10: ccum = 1 ELSE cphi = phi(j): ccum = cum(j)
   IF iphi >= cphi THEN
     SoilSize(iphi + 10) = xcum + (ccum - xcum) * (iphi - xphi) / (cphi - xphi)
     iphi = iphi - 1
   ELSE
     xcum = ccum: xphi = cphi: j = j + 1
   END IF
   LOOP UNTIL iphi < -10
 'Now convert to weights in each whole phi size class
 'soilsize(i) contains material from phi=(i-10) to (i-9)
 igmax = 20: igmin = 0: igg = 0
 FOR i = 0 TO nsize - 1
   u = SoilSize(i)
   u = u - SoilSize(i + 1): SoilSize(i) = u
   IF ((igg = 0) AND (u = 0)) THEN igmin = i + 1
```

```
F u > 0 THEN igg = 1
F ((igg = 1) AND (u = 0)) THEN igmax = i - 1: igg = 2
IEXT
'RINT USING "Material in classes from ###ϕ to###ϕ"; igmin - 10; igmax - 9
ilSize(0) = 0
 SUB

 InitData (iflag) STATIC
ag=0 new data file: =1 amend existing data file: =2 to change during run
lag = 2 THEN vdu 0, 1: COLOR 14, 4: CLS
lag = 0 THEN
INT : PRINT "Select a file for process rate parameters"
adflag$ = readpar("tra")
 IF
lag > 0 THEN
n = 2
E
n = pMenu("<A> to Accept or <M> to Modify process rate parameters", "AM")
 IF
LE iam = 2
LOR 0, 7: CLS
 = pMenu("Revise Process Rate Parameters or accept by pressing <ENTER>", "")
LOR 4, 7: modpar -(iflag = 2): COLOR 0, 7
 = par!(20): IF xs > 0 THEN xs = par!(17) / xs ELSE xs = 1E+09
 = par!(18): IF xr > 0 THEN xr = par!(15) / xr ELSE xr = 1E+09
 = SQR(xs * xr)
 xs < xr THEN
F xs > 0 THEN
 PRINT USING "Wash Dominance: 0 -- Splash --####m -- RainFlow --####m -- Rills";
 xr
LSE
 PRINT USING "Wash dominance: 0 -- Rainflow ---####m --- Rills"; xr
ND IF
SE
F xf > 0 THEN
 PRINT USING "Wash Dominance: 0 -- Splash --####m -- Rills: Rainflow never"; xf
LSE PRINT "Rills always dominant"
ND IF
D IF
itepar
n = pMenu("<A> to Accept or <M> to Modify process rate parameters", "AM")
ND

 = par!(1): Summit = par!(2): nx = par!(3): dx = xlen / nx: thr = .2
epRate = par!(5): CreepDen = par!(4) * .0001 / CreepRate
ate = par!(6) * .000001: solK = par!(7): solEQ = par!(8)
es = par!(9): ascrit = par!(10): soltrav = solK * solEQ / 24
amma = 1 - EXP(-solRes / solEQ)
eRate = par!(11) * .001: con! = ATN(1) / 45: gth = TAN(par!(12) * con!)
vel = par!(14): gtal = TAN(par!(13) * con!)
tal = 0 THEN gtal = .001
Den = par!(15) * .001: FlowDen = par!(18) * .001
 = par!(16): xthr = par!(19)
```

```
RainTrav = par!(17): FlowTrav = par!(20)
DO
 PRINT : PRINT "BASE LEVEL CONTROL and RATES"
 ibase = pMenu("Control on <E> for Elevation or <S> for Sediment increment", "ES")
 suggest "Rate of absolute downcutting (µm/y)", absden
 suggest "Rate of downcutting relative to BaseLevel (µm/m/y)", relden
 IF relden <> 0 THEN
   suggest "Base Level elevation (m: relative to initial slope base)", basel
 END IF
 IF ibase = 2 THEN
   suggest "Flood plain width to spread sediment across (m)", FpWidth
 END IF
 IF ((ibase = 2) OR ((absden <> 0) OR (relden <> 0))) THEN
   izmaxmin = pMenu("<H> to plot range of historic elevations: <C> for current range
only", "HC")
 ELSE izmaxmin = 1
 END IF
 LOOP UNTIL pMenu("<A> to accept values: <M> to modify them", "AM") = 1


END SUB

SUB InitForm
IF igsize = 0 THEN
  menu = "<F1> Graphs: <F2> Numbers:            <F4> Restart: <F5> Quit: <F6> Rates"
ELSE
  menu = "<F1> Graphs: <F2> Numbers: <F3> Gr Sizes: <F4> Restart: <F5> Quit: <F6>
Rates"
END IF
zmax = Summit + 1: zmin = 0
IF ign <> 1 THEN vdu 1, 3
VIEW PRINT tc% TO tc% + 4: CLS 2
iq = pMenu("Enter Initial slope form as a series of straight 'legs'", "")

 DO
  j = 0: oz = Summit: ox = 0: elev(0) = Summit
  DO
    LOCATE tc%, 3: PRINT USING "Enter % of remaining horiz dist (####.#m) for next leg:
"; (nx - ox) * dx;
    DO: LOCATE tc%, 59: pu = fetch(100) * .01: LOOP UNTIL pu <= 100
    LOCATE tc% + 1, 5: PRINT USING "and % of remaining vertical fall (###.##m): "; oz;
    DO: LOCATE tc% + 1, 49: pv = fetch(100) * .01: LOOP UNTIL pv <= 100
    cz = oz * (1 - pv): cx = ox + pu * (nx - ox)
    DO
      ex = (j + 1)
      IF ((ex >= ox) AND (ex <= cx)) THEN
        j = j + 1: elev(j) = oz - (ex - ox) / (cx - ox) * (oz - cz)
        LINE (dx * (j - 1), elev(j - 1))-(dx * j, elev(j)), 9
      END IF
    LOOP UNTIL ex > cx
    oz = cz: ox = cx
   LOOP UNTIL j >= nx
 LOOP UNTIL pMenu("Hit <A> to Accept: <R> to Revise Initial Form", "AR") = 1
```

```
ı = 0: Iter = 0
) SUB

3 intro
 0, 1
= "Erosion limited Slope Evolution Model"
= "C 1992: Mike Kirkby, School of Geography, Univ of Leeds, UK"
= "Fax (44)-532-336758/3308: Phone -333310": PRINT
.OR 15
:ATE 3, 3: PRINT "╔"; STRING$(73, "═"); "╗"
R i = 4 TO 10: LOCATE i, 3: PRINT "║"; STRING$(73, " "); "║": NEXT
:ATE 11, 3: PRINT "╚"; STRING$(73, "═"); "╝"
.OR 1: l = LEN(a1$): LOCATE 5, 40 - l \ 2: PRINT a1$;
.OR 0: l = LEN(a2$): LOCATE 7, 40 - l \ 2: PRINT a2$;
.EN(a3$): LOCATE 9, 40 - l \ 2: PRINT a3$;

:ATE 13, 1
ze = pMenu("<G> for grain size selectivity: <X> if not", "XG") - 1
gsize = 1 THEN
RINT : PRINT "GRAIN SIZE SELECTIVITY applies for wash processes only, with
ımptions:"
RINT "1. Given rill threshold and splash/flow travel distances for 1mm grainsize."
RINT "2. Equal detachment mobility, ie rates proportional to frequency in armour
ır."
RINT "3. Raindrop and flow Detachments rates proportional to 1/(mean diameter)"
RINT "4. Rill threshold is directly proportional to average of"
RINT "    mean surface diameter in mm and its reciprocal (ie minimum at 1mm)."
RINT "5. Splash & rill travel Distances selective & proportional to 1/diameter."
RINT
) IF

) SUB

3 plot (ip)
p <> ign THEN
lu ip, 2
:IT SUB
) IF
fl = 1 THEN ipc = 15 ELSE ipc = 11
.OR 15

.ECT CASE ip
SE 1
)CATE tc%, 1
RINT USING "Last Plot @ ###########, yrs"; time;
)R i = 0 TO nx
/ = elev(i)
SELECT CASE i
 CASE 0: oy = y
 CASE 1: LINE (0, oy)-(dx, y), ipc
 CASE ELSE: LINE -(i * dx, y), ipc
END SELECT
:XT
```

```
CASE 2:
 LOCATE 3, 51
 PRINT USING "Last Plot @ ###########, yrs"; time;
 LOCATE 4, 51: PRINT "Plots @ 1,2,5,10,20,50.. yrs";
 FOR i = 0 TO nx
  y = elev(i)
  SELECT CASE i
   CASE 0: oy = y
   CASE 1: LINE (0, oy)-(dx, y), ipc
   CASE ELSE: LINE -(i * dx, y), ipc
  END SELECT
 NEXT
 LOCATE 1, 1
 PRINT "Pt #  Dist   Elev  Grad  Eros'n";
 IF igsize = 1 THEN PRINT "  Surface Armour" ELSE PRINT
 PRINT "      (m)   (m)   (°) (um/y) ";
 IF igsize = 1 THEN PRINT "Depth(mm) Diam(φ)" ELSE PRINT
 ix = 1: IF nx > 20 THEN ix = nx \ 20 + 1
 FOR i = 0 TO nx STEP ix
  PRINT USING "### ####.# ###.## "; i; i * dx; elev(i);
  IF i > 0 THEN
   g = (elev(i - 1) - elev(i)) / dx
   PRINT USING "##.# "; ATN(g) / con!;
  ELSE PRINT "     ";
  END IF
  PRINT USING "#######"; -dElev(i) * 1000000!;
  IF ((igsize = 1) AND (i < nx)) THEN PRINT USING " #####.## ####.##"; Armour(i) *
1000; -LOG(ArMean(i)) * conl ELSE PRINT
  NEXT

CASE 3

 FOR z = .2 TO 1 STEP .2
  NuStr xlen - xASC * 3.1, pmin + (pmax - pmin) * z - 1.1 * yASC, NuNum$(z)
  NEXT
 sc = (pmax - pmin) / xArm
 FOR i = 0 TO nx - 1
  y = Armour(i) * sc + pmin: iuc = i MOD 13 + 1
  LINE (i * dx, 0)-((i + 1) * dx, y), 7, B
  IF y > 0 THEN PAINT ((i + .5) * dx, y * .9), iuc, 7
 NEXT

 FOR i = 0 TO nx
  y = elev(i)
  SELECT CASE i
   CASE 0: oy = y
   CASE ELSE
    LINE ((i - 1) * dx, oy)-(i * dx, y), ipc
    gp = (oy - y) / dx
    IF i > 1 THEN LINE ((i - 1.5) * dx, pmin + ogp * (pmax - pmin))-((i - .5) * dx,
zmin + gp * (zmax - zmin)), 12
  END SELECT
  oy = y: ogp = gp
```

```
XT
 SELECT

 SUB

 plothist
OR 15
ECT CASE ign
E 1: LOCATE tc%, 1
E 2: LOCATE 3, 51
E 3: LOCATE 1, 1
 SELECT
NT USING "Last Plot @ ###########, yrs"; time;

 k = 0 TO khist
R i = 0 TO nx
 = ElevHist(i, k)
ELECT CASE i
 CASE 0: oy = y
 CASE 1: LINE (0, oy)-(dx, y), 15
 CASE ELSE: LINE -(i * dx, y), 15
ND SELECT
XT
T
gn = 1 THEN
 Iter <= 5000 THEN
OR k = 0 TO Iter - 1
 SELECT CASE k
 CASE 0: x = xms(0): y = xm0(0)
 CASE 1: LINE (x, y)-(xms(1), xm0(1)), 9
 CASE ELSE: LINE -(xms(k), xm0(k)), 9
 END SELECT
EXT
OR k = 0 TO Iter - 1
 SELECT CASE k
 CASE 0: x = xms(0): y = xrr(0) * (pmax - pmin) + pmin
 CASE 1: LINE (x, y)-(xms(1), xrr(1) * (pmax - pmin) + pmin), 10
 CASE ELSE: LINE -(xms(k), xrr(k) * (pmax - pmin) + pmin), 10
 END SELECT
EXT
SE
ter = Iter MOD 5000
OR k = jter + 1 TO 5000
 SELECT CASE k
 CASE jter + 1: x = xms(k): y = xm0(k)
 CASE jter + 2: LINE (x, y)-(xms(k), xm0(k)), 9
 CASE ELSE: LINE -(xms(k), xm0(k)), 9
 END SELECT
EXT
OR k = 0 TO jter: LINE -(xms(k), xm0(k)), 9: NEXT
OR k = jter + 1 TO 5000
 SELECT CASE k
 CASE jter + 1: x = xms(k): y = xrr(k) * (pmax - pmin) + pmin
```

```
    CASE jter + 2: LINE (x, y)-(xms(k), xrr(k) * (pmax - pmin) + pmin), 9
    CASE ELSE: LINE -(xms(k), xrr(k) * (pmax - pmin) + pmin), 9
    END SELECT
   NEXT
   FOR k = 0 TO jter: LINE -(xms(k), xrr(k) * (pmax - pmin) + pmin), 9: NEXT
 END IF

END IF

END SUB

SUB Slide (ix) STATIC
' non size-selective
IF grad > gtal THEN RhSlide = 0 ELSE RhSlide = (gtal - grad) / Travel
Dslide = SlideRate * (ABS(grad) - gth) * grad
IF SedTpt = 0 AND Dslide < 0 THEN Dslide = 0
SlideTpt = Sout(Dslide, RhSlide, SlideTpt, ix, -1)
SedTpt = SedTpt + SlideTpt
END SUB

SUB Solution (ix) STATIC
' non size-selective
IF ix = 1 THEN xsol = 0
u = grad / (xsol + dx) * ascrit: u = -u * (u > 0)
xsol = xsol + dx * (1 + (1 - u) * (u < 1)) / (1 - (ix = 1))
IF ((soltrav = 0) OR (grad = 0) OR (xsol = 0)) THEN
 DSol = SolRate: RecSol = 0
ELSE
 RecSol = 1 / (soltrav * grad)
 DSol = SolRate * (xsol * RecSol + solgamma)
END IF
IF u < 1 THEN DSol = DSol * u
IF (elev(ix) + elev(ix - 1)) <= 0 THEN DSol = 0
ChemTpt = Sout(DSol, RecSol, ChemTpt, ix, -1)
SedTpt = SedTpt + ChemTpt
END SUB

FUNCTION Sout (den, RECH, SSin, iy, jg)
IF RECH < 0 THEN RECH = RECH - EXP(RECH * dx) / dx
 'for -ve travel distances on reverse slopes, arbitrarily prevents values < -dx
IF ABS(RECH) > 1E+25 THEN
 s = 0
ELSE
 IF iy > 1 THEN
  Extra = (den - SSin * RECH) / (1 / dx + RECH)
  s = SSin + Extra
 ELSE
  s = den / (2 / dx + RECH)
 END IF
 IF s <= 0 THEN s = 0
END IF
dElev(iy) = dElev(iy) + s / dx
'Symmetrical divide at x=0
```

```
ev(iy - 1) = dElev(iy - 1) - s / dx * (1 - (iy = 1))
g >= 0 THEN
elev(iy, jg) = gDelev(iy, jg) + s / dx
elev(iy - 1, jg) = gDelev(iy - 1, jg) - s / dx * (1 - (iy = 1))
 IF
t = s

 FUNCTION

 UpDate
x = thr * .00000001#: oz = elev(0): odz = dElev(0)
 i = 1 TO nx
 = elev(i): cdz = dElev(i): grad = ABS(oz - cz)
 igsize = 1 THEN ca = Armour(i - 1) ELSE ca = 0
 grad > .001 THEN grad = ABS(cdz - odz) / grad ELSE grad = ABS(cdz / Summit)
 grad > gmax THEN gmax = grad
 ca > .01 THEN grad = .2 * ABS((cdz + odz) * .5 / ca): IF grad > gmax THEN gmax
rad
 = cz: odz = cdz: NEXT
 thr / gmax
 0: IF time + dt >= Tnext THEN dt = Tnext - time: ifl = 1: khist = khist + 1
 = time + dt: sum = 0: ds = 0
zmaxmin = 2 THEN zmin = 1E+09: zmax = -zmin
 i = 0 TO nx
lo change in form if line below rem-ed out !
 = elev(i) + dElev(i) * dt: elev(i) = zz
 zz > zmax THEN zmax = zz
 zz < zmin THEN zmin = zz
 = ds - dElev(i) * (2 + (i = 0) + (i = nx))
m = sum + elev(i) * (2 + (i = 0) + (i = nx)): NEXT
 = sum - nx * elev(nx) * 2
ean = ds / sum: zmean = sum * .5 / nx
(Iter MOD 5000) = zmean / (elev(0) - elev(nx)) * xlen
(Iter MOD 5000) = zmean
lzmean > 1E-08 THEN
r(Iter MOD 5000) = (8 + LOG(dzmean) / LOG(10)) * .2
E
r(Iter MOD 5000) = 0
) IF

gsize = 1 THEN
pdate thickness and composition of armour layer
 too thin to provide any component of denudation, then topped up from soil
Arm = 0
)R i = 0 TO nx - 1
dd = 0: ga = Armour(i): dz = (dElev(i) + dElev(i + 1)) * .5
FOR j = igmin TO igmax
  xgD = (gDelev(i, j) + gDelev(i + 1, j)) * .5
  dg = SoilSize(j)
  IF dg > 0 THEN
    de = -(xgD * dt + ArmourSize(i, j) * ga) / dg
    IF de > dd THEN dd = de
  END IF
```

```
    NEXT
   sum = 0: FOR j = igmin TO igmax
    xgD = (gDelev(i, j) + gDelev(i + 1, j)) * .5
    xa = ArmourSize(i, j) * ga + xgD * dt + dd * SoilSize(j)
    IF xa < 0 THEN xa = 0
    sum = sum + xa: ArmourSize(i, j) = xa: NEXT
   Armour(i) = sum: IF sum > xArm THEN xArm = sum
   IF sum > 0 THEN
    FOR j = igmin TO igmax: ArmourSize(i, j) = ArmourSize(i, j) / sum:
    NEXT
    '
   ELSE
    FOR j = igmin TO igmax: ArmourSize(i, j) = SoilSize(j):
    NEXT
   END IF
  NEXT
END IF


IF ((Iter > 0) AND (ign = 1)) THEN
 jter = Iter MOD 5000
 IF jter <> 0 THEN
  LINE (xms(jter), xm0(jter))-(xms(jter - 1), xm0(jter - 1)), 9
  LINE (xms(jter), xrr(jter) * (pmax - pmin) + pmin)-(xms(jter - 1), xrr(jter - 1) *
(pmax - pmin) + pmin), 10
 END IF
END IF
Iter = Iter + 1
IF ifl = 1 THEN
 IF ign < 3 THEN plot ign ELSE vdu 3, 1
 THist(khist) = time
 FOR i = 0 TO nx
  ElevHist(i, khist) = elev(i)
  dElevHist(i, khist) = dElev(i)
  SoilHist(i, khist) = Soil(i)
  dSoilHist(i, khist) = dsoil(i)
 NEXT
 SELECT CASE Tnext
 CASE .2, 2, 20, 200, 2000, 20000!, 200000!, 2000000!, 2E+07, 2E+08, 2E+09
  Tnext = Tnext / 2 * 5
 CASE ELSE: Tnext = Tnext + Tnext
 END SELECT
END IF
SELECT CASE ign
CASE 1
 LOCATE tc% + 1, 1: COLOR 15: PRINT USING " Time =############, yrs: Step =
##########,.### yrs after #######, Iterations"; time; dt; Iter;
CASE 2
 LOCATE 6, 51: COLOR 7: PRINT USING "Time = ############, yrs"; time;
 LOCATE 7, 51: PRINT USING "Step = #########,.### yrs"; dt;
 LOCATE 8, 52: PRINT USING "after #######, Iterations"; Iter;
CASE 3
 LOCATE 1, 32: COLOR 7
```

```
PRINT USING ": Currently @ ##############, yrs: ######, Iters"; time; Iter;
ND SELECT

ND SUB

JB vdu (nb, nu)
ELECT CASE nb
ASE 0
SCREEN 0: COLOR 15, 7: sc% = 0: tc% = 21
IF nu > 0 THEN CLS
ign = 0
ASE 1
IF nu < 3 THEN sc% = 12
IF nu > 0 THEN
 scr 12: CLS 1: axes 0, zmin, xlen, zmax: pmax = zmax: pmin = zmin
 FOR x = 1 TO 5
   y = (x - 8):
   NuStr xlen - (4.3) * xASC, zmin + x / 5 * (zmax - zmin) - 1.1 * yASC, "10"
   NuStr xlen - (2.3) * xASC, zmin + x / 5 * (zmax - zmin) - .7 * yASC, NuNum$(y)
   NEXT
ELSE
 VIEW PRINT tc% TO tc% + 4: VIEW (2, 2)-(634, 394)
 WINDOW (0, pmin)-(xlen, pmax * 1.02)
END IF
gn = 1
IF Iter > 0 THEN
 SELECT CASE nu
 CASE 1: plot 1
 CASE 2: plothist
 END SELECT
END IF
LOCATE tc% + 2, 1: COLOR 15: PRINT "Coloured Graphs: x=Mean Elev/Summit: ";
COLOR 9: PRINT "y=Mean Elev"
COLOR 10: PRINT "y=Rate of Denudation/Mean Elev per Year: Log scale 10^-8 to
^-3";
VIEW PRINT tc% TO tc% + 4
q = pMenu(menu, ""): COLOR 10

ASE 2
F nu > 0 THEN
 SCREEN 12: COLOR 15: VIEW (0, 0)-(639, 479), 0
 VIEW (394, 288)-(638, 453), 7, 7
END IF
VIEW (395, 289)-(637, 452), 8, 7
VIEW PRINT 1 TO 30
WINDOW (0, zmin)-(xlen, zmax * 1.02): pmax = zmax: pmin = zmin
gn = 2
F Iter > 0 THEN
 plot 2
 IF nu = 2 THEN plothist
END IF
q = pMenu(menu, ""): COLOR 15
```

```
CASE 3
 IF igsize = 0 THEN EXIT SUB
 IF ign <> 3 THEN iq = 0: ign = 3 ELSE iq = 1
 IF iq = 0 THEN SCREEN 12: VIEW (0, 0)-(639, 479), 0
 VIEW PRINT 1 TO 30: COLOR 15
 LOCATE 1, 1: PRINT USING "Last Plot @ ###########, yrs"; time;
 IF iq = 0 THEN VIEW (0, 32)-(637 * nx / (nx + 2) + 2, 287), 7, 7
 VIEW (2, 34)-(637 * nx / (nx + 2), 285), 0, 7
 axes 0, zmin, xlen, zmax
 VIEW PRINT 1 TO 30: pmax = zmax: pmin = zmin

 plot 3: plothist
 LOCATE 6, 4: COLOR 15: PRINT "Elevation";
 LOCATE 2, 1: COLOR 12: PRINT "Gradient scaled to 1.0";
 LOCATE 2, 38: COLOR 10
 PRINT USING "Armour depth scaled to Max of ###.### m"; xArm;
 LOCATE 19, 1: COLOR 12
 PRINT USING "Armour Size Dist: ###φ to###φ: Largest at top: "; igmin - 10; igmax -
9;
 COLOR 15: PRINT "--Soil —Armour Mean";
 COLOR 7: PRINT " Soil Dist'n";
 IF iq = 0 THEN VIEW (0, 304)-(639, 463), 7, 7
 VIEW (2, 306)-(637, 462), 0, 7
 WINDOW SCREEN (0, igmin - 10)-(nx + 2, igmax - 9)
 COLOR 15
 FOR i = nx - 1 TO 0 STEP -1
  iuc = (i MOD 13) + 1
  IF Armour(i) <> 0 THEN
   FOR j = igmin TO igmax
    cs = ArmourSize(i, j) * 2
    LINE (i + .5, j - 10)-(i + .5 + cs, j - 9), iuc, BF
   NEXT
  END IF
 NEXT
 FOR j = igmin TO igmax
  cs = SoilSize(j) * 2
  LINE (2 + nx, j - 10)-(2 + nx - cs, j - 9), 7, B
  NEXT
 FOR i = 0 TO nx - 1
  y = -LOG(ArMean(i)) * conl
  SELECT CASE i
  CASE 0: oy = y
  CASE 1: LINE (.5, oy)-(1.5, y), 15
  CASE ELSE: LINE -(i + .5, y), 15
  END SELECT
  NEXT
 FOR i = 0 TO nx - 1: iuc = (i MOD 13) + 1
  LINE (i + .5, igmin - 10)-(i + .5, igmax - 9), iuc, , &H3333
 NEXT
 y = -LOG(dphi) * conl
 LINE (0, y)-(nx + 2, y), 15, , &HF0F
 LOCATE 28, 2: COLOR 15:
 PRINT USING "Mean Soil grain size = ###.#φ"; y
```

```
ID SELECT

q = pMenu(menu, ""): COLOR 15



ID SUB

JB Wash (ix) STATIC
rain-size specific if igsize=1
 igsize = 1 THEN
IF ix = 1 THEN REDIM gSplash(nsize), gRillFlow(nsize), gRainFlow(nsize)
.SE xam = 1
ID IF
qual mobility, with rates of rill detachment proportional to power in excess
f Threshold, which depends on grain size
plash detachment independent of grain size
ravel dist taken as inversely proportional to grain size for splash & rills
 grad <= .00001 THEN
dRa = 0
.SE
y = (xx / (xrd * SQR(grad))) ^ .6
'Reduction of splash detachment beneath a water film
dRa = (1 + y) * EXP(-y) * RainDen
ID IF
 igsize = 1 THEN
rtheta = xam * xthr / xx
IF grad > .00001 THEN psi = .1 * (1 / grad - 1 / gtal) ELSE psi = 10000!
IF rtheta < 1 THEN rthr = (psi + .5) * xthr * xam ELSE rthr = SQR(2 * psi) * xthr * xam
.SE rthr = xthr
ID IF
 rthr < 0 THEN rthr = 0
Fl = FlowDen * (xx * ABS(grad) - rthr)
gt = gtal - ABS(grad): IF ggt < 0 THEN ggt = 0

ELECT CASE igsize
ASE 0:
 grad = 0 THEN RRa = 1E+29 ELSE RRa = ggt / (RainTrav * grad)
 = FlowTrav * xx: IF u = 0 THEN RFl = 1E+29 ELSE RFl = -ggt / u * (grad > 0)
plash = Sout(dRa, RRa, Splash, ix, -1)
ainFlow = Sout(dRa, RFl, RainFlow, ix, -1)
illFlow = Sout(dFl, RFl, RillFlow, ix, -1)

ASE 1:
plash = 0: RainFlow = 0: RillFlow = 0
OR j = igmin TO igmax
gdRa = dRa * xas(j): gdFl = dFl * xas(j): gd = 2 ^ (9.5 - j)
IF grad = 0 THEN gRRa = 1E+29 ELSE gRRa = ggt * gd / (RainTrav * grad)
u = FlowTrav * xx
IF u = 0 THEN
  gRFl = 1E+29
ELSE gRFl = -ggt * gd / u * (grad > 0)
END IF
```

```
  gSplash(j) = Sout(gdRa, gRRa, gSplash(j), ix, j)
  gRainFlow(j) = Sout(gdRa, gRFl, gRainFlow(j), ix, j)
  gRillFlow(j) = Sout(gdFl, gRFl, gRillFlow(j), ix, j)
  Splash = Splash + gSplash(j)
  RainFlow = RainFlow + gRainFlow(j)
  RillFlow = RillFlow + gRillFlow(j)
  NEXT
  SedTpt = SedTpt + Splash + RainFlow + RillFlow
END SELECT


END SUB
```

Produced By
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