WORKING PAPER 442

THE DEVELOPMENT AND APPLICATION OF URBAN MODELS

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October, 1985

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1. Introduction: some key problems of geographical analysis and the potential role of modelling

Geography can be seen as concerned with the synthesis of knowledge about regions at a variety of spatial scales; or with particular sub-systems, such as cities, or urban housing. These concerns are shared with a number of other disciplines and inevitably discipline-based theories overlap. What further distinguishes geography is a focus on spatial analysis and in particular on location and spatial interaction. Contemporary modelling methods provide a basis for building theories based on these topics, and these have a variety of fields of application.

The purpose of this paper is to outline, first, the methods and, secondly, the range of application. As a preliminary, the history of urban modelling is reviewed briefly to provide a context and to explain the particular style of modelling which forms the main subject matter of the paper.

2. A sketch of the history of urban modelling

There are 'classical' contributions to geographical theory which serve both to illustrate the range of tasks of geographical analysis and to provide the initial impulse for modelling endeavours. These include the works of von Thunen (1826), Weber (1909), Ravenstein (1885), Burgess (1927), Hoyt (1939) and the Chicago ecologists, and Christaller (1933) and Losch (1940). Much subsequent geographical theory can be described as 'neo-classical': its style and roots remain clearly connected to the classical works. This involves a disciplinary basis which is mostly close to conventional economics and a geographical

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* prepared for the Anglo-Chinese Seminar to be held at Wuxi, November, 1985

style which is based on the treatment of space as continuous. Thus, for geographical problems concerned with dispersed activities (agriculture and residential location), the focus is on identifying the boundaries which demarcate different land uses (e.g. von Thunen's rings). For concentrated activities, the task is the optimal location of points or the explanation of settlement patterns (e.g. Weber on industrial location, Christaller and central place theory). The exception is the spatial interaction model, which has its roots in the nineteenth century, where space is divided into discrete zones and the emphasis is on the study of flows between each pair of zones, or the locational pattern of structures and activities distributed across zones. It is argued below that this spatial representation, though in a sense approximate, allows much more powerful tools to be used.

Modelling, however, has developed in relation to different disciplinary styles. One, the so-called new urban economics (cf. Richardson, 1977), is also based on continuous space and ultimately experiences difficulties with its mathematical tools which inhibit operational formulations and applied work. Another long-running thread can be identified as ecological (cf. Dendrinos and Mullally, 1984). However, this is relatively narrow and can be considered to be a special case of the third 'school'. This is based on spatial interaction concepts and was originally characterised as 'social physics' because of the borrowing of concepts such as the 'gravity' model. However, the approach has been restated as 'first-principles' geography in recent years and can also be considered to have a multi-disciplinary base. It is this approach, it is argued, that has the potential to offer a broad core for geographical theory and it provides, therefore, the main focus of this paper.

3. Key components of modelling methodology

3.1 Introduction

In this section, we take a rather broad and abstract perspective to demonstrate the main concepts which than have a broad range of application. There are six steps in the argument:

(i) spatial interaction models;

(ii) activity-location models, based on spatial interaction;

(iii) optimization models;

(iv) comprehensive models, representing high levels of inter-dependence;

(v) static structural models;

(vi) dynamic models.

In this introduction, we review some key papers in the modern history of these elements and then we proceed to discuss each in turn.

Spatial interaction models were thought of in geography very much as gravity models until the 1960's. But from the late 1950's onwards, developments were taking place in civil engineering and transport planning which transformed the usual 'constant of proportionality' into more complex factors which formed the basis of more realistic models. By the late 1960's, improved derivations were available which, in spite of the terminology, avoided physics analogies (for example, entropy maximising - cf. Wilson, 1967, 1970) and it was understood that there was not a single model but a rich family of models from which examples could be drawn for specific applications (cf. Wilson, 1971). Yet more derivations were available, a particularly fruitful one being random utility theory (cf. Williams, 1977 and Macgill and Wilson, 1979, for a review of a wide range of alternative derivations).

It was recognised at a relatively early stage that singly-constrained interaction mdoels would function as <u>location</u> models. The first applications were in the fields of retailing and services (Harris, 1965, Huff, 1964, Lakshmanan and Hansen, 1965).

A somewhat confusing element of the history of urban modelling was provided by the invention in operational research of flow and locational <u>optimization</u> models, one of the best early examples being the transportation problem of linear programming (cf. Koopmans and Beckmann, 1957). This predicted the trip matrix which minimised overall transport costs while still ensuring correct origin and destination totals. There were many applications in commercial and public sectors involving totally-controlled single-commodity flows. There was an element of 'competition' between this model and the doubly-constrained spatial interaction model (e.g. Chisholm and O'Sullivan, 1971) until this was removed (along with the mystery) when it was realised that the programming model was a special case of the interaction model (Evans, 1973, Wilson and Senior, 1974).

The need to build comprehensive models stems from the interdependence of major urban subsystems. The essence of this was captured in Lowry's (1964) model. Although the component spatial interaction models were very simplistic, Lowry's work had a profound effect on the future of urban modelling because of his identification of the principal structures of interdependence.

The developments mentioned so far are all centred on the 1960's. Subsequently, although there was a wide range of applications - some of them obviously useful - there was also much criticism (for example, by Lee, 1973). Much of this was related to the inadequacy of existing models in addressing problems of dynamical analysis. There were inadequate attempts to respond, as with Forrester's (1969) Urban dynamics model, but it was not until the late 1970's that the beginnings of a new understanding started to emerge. This turned on the application of ideas of dynamical systems theory - and particularly catastrophe theory and bifurcation theory (cf. Harris and Wilson, 1978, Wilson and Clarke, 1979, Wilson, 1981-A). These developments can be subdivided into explorations of static structures on the one hand, and disequilibrium dynamics on the other. It was in this area that there turned out to be more assistance from ecological ideas than, in the first instance, economic ones.

We now proceed to discuss each of these elements in turn.

3.2 Spatial interaction models

To fix ideas, consider a spatial system with typical zones labelled i and j and the variables defined on Figure 1. We will work with the so-called production-constrained model as an example and then indicate later how to extend to other members of the family; and then how to extend further by disaggregation.

Rather than give a formal derivation, we will simply appeal to the intuitively sensible hypotheses that

$$S_{ij} \propto E_i \qquad (1)$$

$$S_{ij} \propto W_j \qquad (2)$$

[illegible] (3)

In the last of these, we could substitute any decreasing function of travel cost, and we will return to this issue later. Then we can take

$$S_{ij} = A_i E_i W_j e^{-\beta c_{ij}} \tag{4}$$

where we have a <u>set</u> of constants of proportionality, A_i, given by

$$A_i = 1/\sum_k W_k e^{-\beta c_{ik}} \tag{5}$$

which ensures that the production constraint

$$\sum_j S_{ij} = E_i \tag{6}$$

holds. This is, for example, a model of retail sales, where E_i is expenditure from residential zone i and W_j a measure of the attractiveness of shops in j.

W_j can be assumed to be made up from a set of factors

$$W_j = X_{1j}^{\alpha_1} \; X_{2j}^{\alpha_2} \; X_{3j}^{\alpha_3} \; \dots \tag{7}$$

where $X_{\ell j}$ are the factors and the α_ℓ are parameters which measure the 'strength' of each one. If there is only one factor, say shopping centre size in the retail case, then W_j might be used for that and W_j^α used in (4) and (5), which then become

$$S_{ij} = A_i E_i W_j^\alpha e^{-\beta c_{ij}} \tag{8}$$

$$A_i = 1/\sum_k W_k^\alpha e^{-\beta c_{ik}} \tag{9}$$

We should also note that the travel cost term might be made up of a set of elements, this time combined additively:

$$c_{ij} = m_{ij} + at_{ij} + be_{ij} + p_i^{(1)} + p_j^{(2)} \tag{10}$$

where m_{ij} is the money cost of travel between i and j, t_{ij}, travel time, e_{ij}, the so-called 'excess' time involved in the journey from i to j, and $p_i^{(1)}$ and $p_j^{(2)}$ are 'terminal costs' at i and j respectively. a and b are coefficients which can be estimated empirically and which value time (e.g., Bruzelius, 1979).

Thus, by specifying the attractiveness function, (7), and the generalised cost function, (10), for specific applications, a great variety of situations can be modelled in this way.

The other members of the basic family are derived by making alternative assumptions about which interaction totals are exogenously given and constrained. And this family can be further extended by disaggregation, for example by modelling flows (in the retail case) by types of goods, g and person, w, and using a set of arrays $\{S_{ij}^{gw}\}$ for each relevant (g,w) combination.

3.3 Interaction models as location models

When one set of interaction totals are unconstrained, then they can be estimated as a by-product of the interaction model. In the model used in the preceding subsection, for example, it is possible to calculate

$$D_j = \sum_i S_{ij} \tag{11}$$

$$= \sum_i \frac{E_i W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} \tag{12}$$

(if we substitute first for A_i and then for S_{ij}). This, of course, is a locational variable and plays a crucial role in a number of location models. In the retail example, it is an estimate of the amount of revenue attracted to each j zone. It also plays an important role in the dynamical-structural analyses, as we will see later.

3.4 Optimization models

We can use this subsection to illustrate the doubly-constrained spatial interaction model as well as the concept of optimization. To emphasise the difference, let us use T_{ij} as the interaction variable and now assume that both origin and destination totals are given, say O_i and D_j. Then the transportation problem of linear programming is to find the T_{ij}'s such that

$$Z = \sum_{ij} T_{ij} c_{ij} \tag{13}$$

is a minimum subject to

$$\sum_{j} T_{ij} = O_i \tag{14}$$

and

$$\sum_{i} T_{ij} = D_j \tag{15}$$

There is no analytical solution to this problem, but there are computer algorithms. We can also get some insight from a theorem which states that if there are N zones, at most N-1 of the N^2 T_{ij} variables are non-zero at the optimum.

The corresponding doubly-constrained spatial interaction model is

$$T_{ij} = A_i B_j O_i D_j e^{-\beta c_{ij}} \tag{16}$$

with

$$A_i = 1/\sum_{k} B_k D_k e^{-\beta c_{ik}} \tag{17}$$

and

$$B_j = 1/\sum_{k} A_k O_k e^{-\beta c_{kj}} \tag{18}$$

which ensure that the constraints (14) and (15) are satisfied. In this case, none of the N^2 T_{ij} variables would actually be zero, though some may be very small.

Evans (1973) showed that as $\beta \to \infty$ in the model (16)-(18), then the T_{ij}'s tend to the solution of (13)-(14). Thus the transportation problem can be thought of as the '$\beta = \infty$' case of the doubly-constrained spatial interaction model. This result has important implications for model building. First, apparently 'competitive' models formulated in the two paradigms can be reinterpreted and seen to be closely related. Secondly, one can be transformed into the other. This is particularly important for optimising economic models which suffer from the 'not more than N-1 variables non-zero' theorem when compared against reality. They can be transformed into the second kind of model, and still be given an economic interpretation; but they will be much more realistic (cf. Wilson and Senior, 1974, and for an extended treatment of this problem, see Wilson, Coelho, Macgill and Williams, 1981).

3.5 Comprehensive models

The key interdependencies built into the Lowry (1964) model were as follows. Workers were assigned to residences on the basis of a spatial interaction model. And residents 'demanded' services on the basis of a second spatial interaction model. Of course, these services generated employment, and these workers then had to be assigned to residences. This defines a (convergent) iterative process and an urban structure is created from the feedback relationships between the two spatial interaction models. Most of the other assumptions in the Lowry model were very crude and have since been replaced, but the feedback remains an essential feature of most attempts to build comprehensive models. Indeed, the argument can be extended further. In Lowry's case, industrial employment was given exogenously and was the starting point. It is also possible in principle to model industry endogenously (and indeed agriculture) and this leads to more complicated feedbacks. More broadly, what has to be modelled is the mutual interdependence of the labour market, the housing market and the supply of services. For a detailed review, see Wilson (1974), and for a recent example, see Birkin, Clarke and Wilson (1984).

3.6 Geographical structures 1 : statics

Until the late 1970's, relatively few attempts were made to model endogenously basic geographical structures. The problem can be illustrated with the spatial interaction model presented in 3.2 above. A typical use of this model would be to take a trial set of W_j's as given, and to calculate $\{S_{ij}\}$ and $\{D_j\}$ and then to assess the effectiveness of this trial plan in some appropriate way. The model is then being used to assess the impact of such a plan. It is interesting, however, to go a step further and to predict the W_j variables within the model. This can then be interpreted as a model of the outcome of the behaviour of suppliers of W_j's - service facilities or shopping centres for instance - in some 'market' as they compete for clients or customers. Even in a situation where there is no market, and the W_j's are planned, such a calculation gives an idea of the structure which tends to arise from the interplay of economic forces. The best way to proceed is to show how the model can be extended in this way, and then to interpret the outcome. We will use the model given by equations (8) and (9) as a starting point. The argument is a general one, but it may help, to fix ideas, to think of the system as a retail or service system. The argument follows Harris and Wilson (1978).

Suppose we use the model in a conventional way for impact analysis. W_j is a measure of facility size and D_j of 'revenue' or 'demand attracted'. Then it is interesting to look at the values of D_j/W_j, the demand attracted to j per unit size of provision. If D_j/W_j is relatively large, then there is an argument for saying that j is underprovided, i.e. W_j is too small; and vice versa. We could therefore define a process whereby the W_j's are adjusted, step by step, until all the D_j/W_j terms take the same value. It is this kind of process which is at the basis of dynamical analysis and, in particular, the equilibrium structures which we explore in this section.

To state this more formally, we need a preliminary step which usefully broadens the picture painted so far. Let C_j be the cost (in some appropriate, say annualised, units) of supplying a facility of size W_j at j. This will obviously be a function of W_j, $C_j(W_j)$; but it may be a nonlinear function. Then an appropriate ratio to examine in impact analysis would be D_j/C_j and we might hypothesise a dynamical process in which the W_j's are adjusted to equalise these ratios. Indeed, in the case where the D_j's are revenues and measured in the same units as C_j, then these ratios should be unity. Note that if we make a simple linear assumption that

$$C_j = kW_j \tag{19}$$

for some constant, k, then this description reduces to the earlier one.

This dynamical adjustment process can be described formally by differential or difference equations:

$$\frac{\partial W_j}{\partial t} = \varepsilon (D_j - C_j) \tag{20}$$

or

$$\Delta W_j = \varepsilon (D_j - C_j)\Delta t \tag{21}$$

for some suitable constant ε and, in the second case, a time increment, Δt. These equations simply formalise the hypotheses suggested earlier. The system is in equilibrium if $\frac{\partial W_j}{\partial t} = 0$, or $\Delta W_j = 0$, and this clearly occurs if

$$D_j = C_j \tag{22}$$

It is interesting, therefore, to investigate the geographical patterns, the set of W_j's, for which the system is in equilibrium. Formally, if there are N zones, we have added N equations (22) to the model equations (8) and (9) so that the additional N variables, $\{W_j\}$, can be determined endogenously. In the rest of this section, we explore these equilibrium solutions, and in the following sections, we return to (20) and (21) and a full dynamical analysis.

Suppose we combine (8) and (9) by substituting for A_i:

$$S_{ij} = \frac{E_i W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} \tag{23}$$

so that

$$D_j = \sum_i \frac{E_i W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} \tag{24}$$

as noted earlier. For simplicity and illustration, take

$$C_j = k_j W_j \tag{25}$$

which repeats equation (19) but with a constant for each j. This allows us to deal with, for example, the spatial variation in land prices. Then the equilibrium conditions (22), written out in full and to be solved for $\{W_j\}$, are

$$\sum_i \frac{E_i W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} = k_j W_j \tag{26}$$

There are nonlinear equations in the W_j's which also exhibit a high degree of interdependence. The nonlinearities arise partly from the parameter α, but mainly from the denominator in each term of the sum on the left hand side; the interdependence from the presence of all the W_k's in each j-equation. The process which is being represented is one of spatial competition among the suppliers of W_j's for the 'customers', spatially distributed through the given variables $\{E_i\}$.

At first sight, it is not clear how to set about solving these equations except through some iterative process. This is what is in fact done: roughly speaking, insert trial values of the W_j's on the left hand side of (26); if the result is less than k_jW_j, then increase the estimate, and vice versa. There are some remaining research questions about uniqueness of solution, but it appears that the global equilibrium solution is reached if the starting values of the W_j's are all equal.

None of this offers much analytical insight. This can be achieved, however, by means of a trick and the outcome is exciting and of fundamental importance for geographical theory. Denote the left hand side of (26) by $D_j(W_j)$ and the right hand side (for the more general cost function) by $C_j(W_j)$. It can be shown that $D_j(W_j)$ takes a distinctive functional form for $\alpha < 1$, $\alpha = 1$ and $\alpha > 1$. These cases are shown, with $C_j(W_j)$ as a straight line, in Figure 2. When $\alpha < 1$, the C_j line always intersects the D_j curve because the curve has an infinite gradient at the origin. This means that W_j will be non-zero for each zone, since the equilibrium values are at the intersections of D_j and C_j curves. In the $\alpha = 1$ case, the D_j curve has a finite gradient at the origin. When $\alpha > 1$, the gradient at the origin is zero and the D_j curve has a logistic shape. The $\alpha = 1$ and $\alpha > 1$ cases have the same properties in crucial respects and we consider them together henceforth. In this case, the C_j-line <u>may or may not</u> intersect the D_j curve, as shown in Figure 2. There will be an overall pattern with some non-zero W_j's and some zero. This provides a key analytical insight into geographical structure. It can be shown, for example, that the lower α and the higher β, the larger number of smaller W_j's there will be; and vice versa. Further, the particular W_j-structures in particular situations will also be functions of the E_i's and the c_{ij}'s. And finally but most importantly, when any of these parameters and exogenous variables change, the intersection/non-intersection situation changes in particular zones, often accompanied by 'jump behaviour' - discrete or sudden structural change which would manifest itself in reality as relatively rapid change. The situation is even more complicated in two respects: first, the analysis above assumes that the system is in a state of global equilibrium, and there are many alternative 'local' solutions; and secondly, it assumes the system is in an equilibrium state. (We tackle the second of these issues in the next subsection.)

One task of geographical theory can now be seen as the interpretation of geographical structures in terms of the kind of analysis sketched above. In section 4, we say a little more about the nature of this task across the full range of application.

3.7 Geographical structures 2 : dynamics

We now return to equations (20) and (21), and for convenience we work with the difference equations (21) which are repeated here for convenience and in a slightly different form:

$$\frac{\Delta W_j}{\Delta t} = \varepsilon(D_j - C_j)W_j^n \tag{27}$$

A factor W_j^n has been added. Commonly, n is taken as 0 or 1, and its effect is to change the way in which a particular W_j increases from a small value - that is, the behaviour near to $W_j = 0$ rather than the equilibrium value.

If the equations (27) are solved numerically, and if there are no exogenous impulses, then the W_j's will tend to their equilibrium values. But the possibilities are more complicated. They may tend to local rather than global values. And May (1978) (and cf. Wilson, 1981-B, for the urban analysis) has shown that there is a new kind of bifurcation effect: if ε exceeds a critical value, the W_j's may oscillate either periodically or chaotically. Finally, we should note that typically, there will be a series of exogenous shocks and the system will never actually be in equilibrium.

Thus, while considerable insight can be achieved from the static analysis, which will remain relevant to explain tendencies even when the system is not in equilibrium, in the long run we should be prepared to seek full dynamical analyses. For this purpose, the argument needs to be carried one step further, however (Wilson, 1985-A, Birkin and Wilson, 1978). So far, the focus has been entirely on the adjustment of the W_j's. In a market economy, there would also be adjustment of, say, the prices p_j - taken here as an index - of the goods or services considered and of the land rents, r_j. Thus (27) could be replaced by the following set of equations:

$$\frac{\Delta W_j}{\Delta t} = \varepsilon_1(D_j - C_j)W_j^{n_1} \tag{28}$$

$$\frac{\Delta p_j}{\Delta t} = \varepsilon_2(D_j - C_j)p_j^{n_2} \tag{29}$$

$$\frac{\Delta r_j}{\Delta t} = \varepsilon_3(D_j - C_j)r_j^{n_3} \tag{30}$$

The relative magnitude of the parameters ε_1, ε_2 and ε_3 in different situations can then be interpreted as representing the relative strengths of the different kinds of agents involved in the process, developers, retailers and land owners, for example, in a private sector retail case. n_1 - n_3 are additional parameters.

There is one further complication which can be demonstrated first from a mathematical perspective and then interpreted. A set of model structures can be generated in the following way. In each time interval t to t + 1, each of the equations (28)-(30) could be computed for one step or iterated for a number of steps up to a maximum. If this maximum is large, then this can be interpreted as iteration to equilibrium for that variable (provided the ε-parameter is small enough to permit this). There are at least three useful cases for different situations:

(i) Iteration to equilibrium for all variables at each t. This is comparative statics.

(ii) Single steps for the quantity variables, W_j, iteration for the prices, p_j and r_j. This is the usual assumption of neo-classical economics.

(iii) Single steps for each variable. This is continuous disequilibrium.

Thus, understanding and decoding the details of the dynamic mechanism in any particular application can be quite a difficult task. Sometimes, it is worthwhile being entirely explicit about the details of mechanisms, and in this case a master equations format is appropriate. An example for retailing, in the style of this paper, is presented in Haag and Wilson (1985).

4. The range of application, and some examples

4.1 Principles for building particular models

The argument of section 3 has been carried through on a relatively abstract basis, though the examples of retailing and service subsystems have been used to fix ideas throughout. In this section, we discuss the broad range of potential application and hence the contribution of this style of modelling to geographical theory. We begin in this subsection by stating in general terms the principles which can be used for building a model in any particular application. There is, typically, a sequence of steps as follows.

(i) Decide on the appropriate level of disaggregation. For example, instead of using S_{ij} as the flow of expenditure from residents of i to j we also identify types of good, g, and person types, w and work with the set of arrays S_{ij}^{gw}. In other words, the system of interest must be properly defined to reflect the appropriate categorisation of its components.

(ii) Offer an hypothesis which determines the 'need' or demand terms, E_i.

(iii) Collect together the factors which are relevant to the attractiveness function, as in equation (7).

(iv) Specify the elements of generalised cost, as in equation (10).

(v) Articulate the components of the cost function, C_j, at the appropriate level of disaggregation. This may itself involve other spatial interaction models to describe the flows of inputs into a production process, say.

(vi) Identify the main agents who are determining the processes of change; and articulate any associated planning problems. This step will show whether optimization models are relevant and how to assemble the various components of an overall model into an appropriate comprehensive and dynamic framework.

4.2 The range of application

In the rest of this section, we summarise an argument which has been presented in more detail in Clarke and Wilson (1985-A). In broad terms, the range of application can be summarised as follows:

* agriculture
* industry
* private services, such as retailing
* public services, such as health and education
* residential location and housing
* transport flows and transport infrastructure
* settlement structure

In many cases, an appropriate population or economic model has to be incorporated as a 'backcloth'; and an important task is the linking of subsystem models in a comprehensive framework which facilitates the modelling of settlement structure.

Many of the applications involve more finely-categorised sectors within one of the above headings.

It is useful to note that this range of application covers the 'classical problems' of geographical theory briefly referred to in section 2 above. But the power of the modelling technique and the use of discrete zone systems on the basis for analysis means that the restrictive features of the classical approaches are removed. To give a flavour of the kinds of generalisations which can be achieved, Figures 3-5 show the kinds of results which are obtained from models of agriculture, industry and residential location and housing respectively. In each case, it is possible to reproduce the 'classical' results, such as von Thunen's concentric rings, but then to deal with any more general case in which symmetries are broken and restrictive assumptions removed.

In the rest of the section, we consider the main ideas resulting from three applications, each chosen because they use the general form of the models presented in section 3 more or less directly.

4.3 Retailing

We noted in section 3 that the model presented as an illustration could easily be used to represent a retailing or similar service system. In Figure 6, taken from Clarke and Wilson (1983-A), we show a range of patterns which can be obtained by varying the key parameters α and β. It can be shown that, as α and β vary, there are 'jumps' in the form of the pattern and in some sectors, there is empirical evidence that such jumps have taken place. In the UK in the 1960's, for example, there was a rapid transition in food retailing from a system with a large number of widely-scattered small shops to one with a much smaller number of very large shops. This 'jump' was brought about by a combination of increasing consumer incomes, increasing car ownership and the possibilities of scale economies in the retailing production function. In modelling terms, these could be reflected in changes in E_i's, the α and β parameters, the c_{ij}'s and the C_j's. It is then possible to give a model-based account of such transitions and, in suitable circumstances, to predict other kinds of structural change in the future.

Examples of what can be achieved by varying parameters other than α and β are presented in Figure 7. This shows the effects of reducing supply-side costs away from the centre, of representing the shorter journey times which can be achieved in trips to the centre, and these in combination.

A more recent survey of retail modelling and applications is available in Birkin, Clarke and Wilson (1985).

4.4 Health services

This example can be used to show how a suitable model can not only be useful for prediction, but can also provide the basis for some useful conceptual thinking on system performance (cf. Clarke and Wilson, 1984, 1985-B). Assume that a spatial interaction model can be built of flows of patients to hospitals:

$$N_{ij} = A_i E_i W_j e^{-\beta c_{ij}} \tag{31}$$

with

$$A_i = 1/\sum_j W_j e^{-\beta c_{ij}} \tag{32}$$

E_i is now the number of cases generated in residential zone i and W_j a measure of the capacity - say beds - in hospitals in j. The problem is: how to plan the distribution of W_j's so that the whole system runs equitably and efficiently.

Let P_i be the population of i. Then we can calculate the <u>catchment population</u> of hospitals in j, π_j, as follows

$$\pi_j = \sum_i P_i \frac{N_{ij}}{N_{i*}} \tag{33}$$

(where an asterisk replacing an index denotes summation). In effect this is a partition of the population of each zone i into the catchments of different hospital areas in proportion to the fraction treated there. (The exact form of hypothesis for catchment populations is controversial, but this will suffice for illustrative purposes.) Similarly, we can calculate $\hat{W}_i$ as the <u>notional number of beds</u> available to residents of i:

$$\hat{W}_i = \sum_j W_j \frac{N_{ij}}{N_{*j}} \tag{34}$$

Then an indication of <u>efficiency</u> is

$$W_j/\pi_j \tag{35}$$

the number of beds per head of catchment population, while an indication of equity is

$$\hat{W}_i/P_i \tag{36}$$

the number of beds available to residents per head of population. The planning task is then to find a distribution of W_j's which is <u>both</u> efficient and equitable and the model can be used to help achieve this. It should be noted that without this simple but effective conceptual analysis, there is a temptation to use W_j/P_j, beds per head of resident population in the same zone, as the indicator, but this is hopelessly inadequate as it takes no account of cross-boundary flows.

Figure 8 shows a set of performance indicators calculated along the lines sketched here for a region of Italy.

4.5 Settlement structures

We end with a rather unusual example, showing how the static structural model can be used to throw some light on ancient settlement structures, in this case in Greece in the eighth century BC. The argument follows Rihll and Wilson (1985).

Obviously, in this kind of situation, there is very little data. One of the interesting characteristics of the entropy-maximising foundation of spatial-interaction based modelling is that it makes maximum use of available information and it is interesting in this case to see what can be extracted from minimal data. In fact, all that is available is archaeological and historical evidence on the existence of sites. The issue then is: given the topology of these sites, can anything be said about which ones are likely to be the most important? One way to answer this question is to take the structural model given in section 3.6, equations (23)-(26), to assume that all the E_i's are equal and the c_{ij}'s given as Euclidean distances (with some modifications for sea crossings and mountain ranges), and then to predict $\{W_j\}$. In effect, this is saying: taking site information only, and assuming flows to the most important sites will be largest, compute the W_j's as indices of importance.

In this case, the results were plotted using the Nystuen and Dacey (1961) method, as this gives a portrayal of hierarchical structure. They are obtained for a variety of α and β values, and some of the plots are quite striking - see Figure 9 (a)-(d) and the associated key. It does seem that most of the major cities of the time are identified by this method and in some instances, elements of pattern have persisted for thousands of years.

5. The future of urban modelling

In this section, a brief summary is presented of an argument which is given in greater detail elsewhere (Wilson, 1984). At the outset, it is possible to assert that with the relatively-recent development of various styles of dynamic modelling, it is now possible for modelling to provide the means for constructing a wide range of geographical theory. There are, of course, a number of technical and mathematical research problems which will need considerable effort and skill. But the new insights offered are important, and it becomes possible not only to operationalise particular styles of theory with modelling techniques, but also to use these insights to integrate apparently different approaches to theory - for example, by focussing on the great variety of geographical configurations which can arise and then dynamics (Wilson, 1985-B).

Given the position which has now been reached, there is one clear broad priority for further research: that is to carry out more related empirical research. This has been difficult because the testing of dynamic models involves time series data. However, it will become increasingly possible, both for particular subsystems and for cities as a whole. In the longer run, this should provide a new feedback to systematic and regional geography. The development of empirical work will involve a research focus on geographical information systems, coupled with a useful by-product of the style of modelling described in this paper - statistical methods for 'making the best' of incomplete data (cf. Clarke and Wilson, 1985-C).

Finally, it should be emphasised that the state-of-the-art is such that a wide range of applications are now possible. An increasing degree of connection of geographical modelling to a variety of applications in different kinds of planning will be useful in itself, will increase the volume of empirical work carried out, and will strengthen the links between geography and associated planning disciplines.

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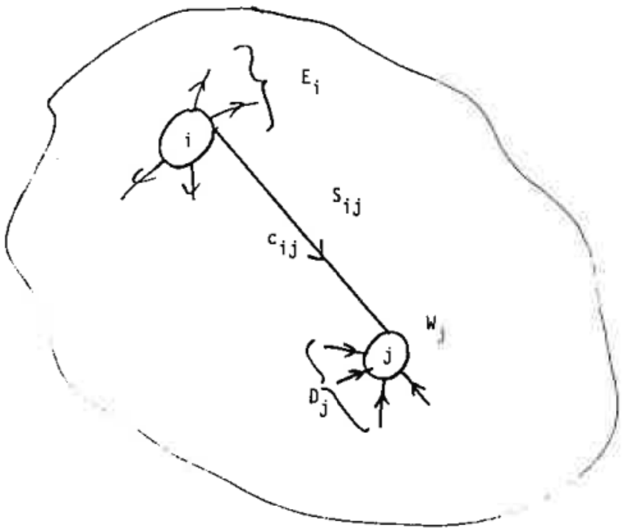
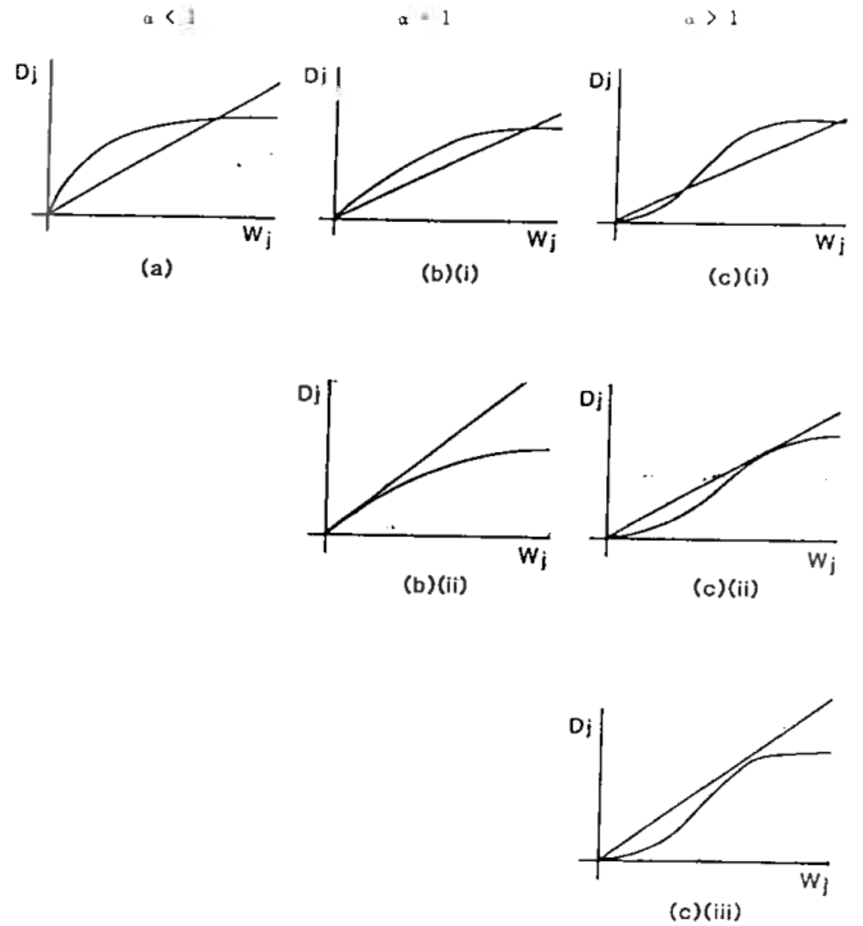
Figure 1 The main variables for a spatial system



S_{ij} = flow from i to j
E_i = total outflow from i (given)
W_j = attractiveness of j
D_j = to inflow to j (predicted)
c_{ij} = travel cost from i to j

<u>Figure 2</u> Revenue-cost relationships for different α values



Sources: Harris and Wilson (1978), Wilson (1981)

Figure 3 von Thunen's rings calculated from a discrete zone structural model

Source: Wilson and Birkin (1984)

Figure 3(a) Concentric ring structure

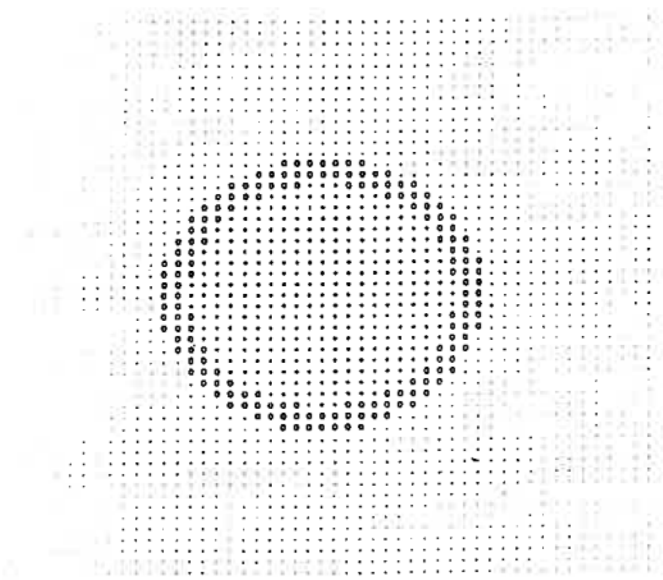


Figure 3(b)   Ring structure with low cost corridor

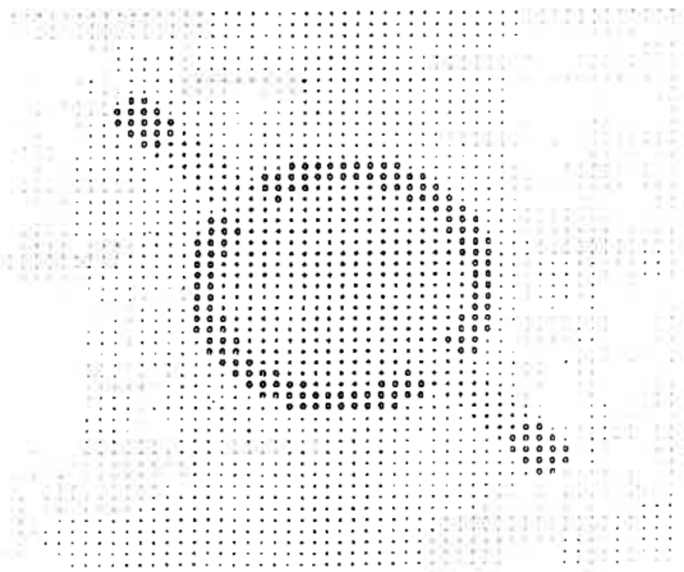


Figure 3(c)   Variable fertility

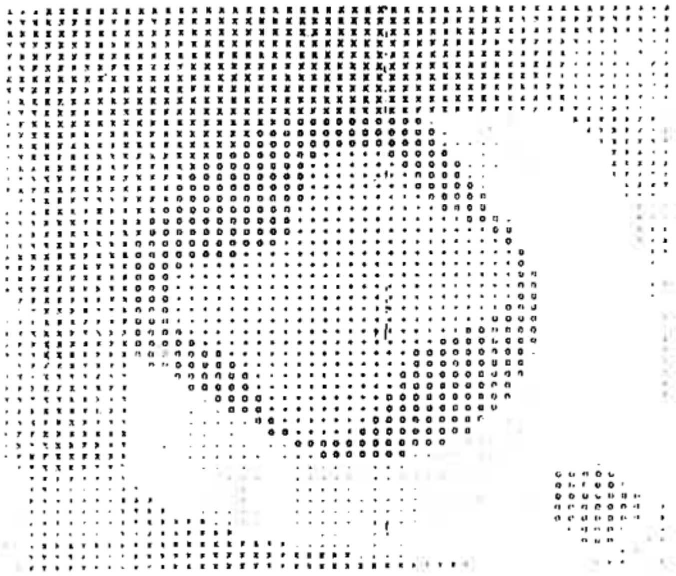


Figure 4 Industrial location with a discrete zone system

Sources: Birkin and Wilson (1984-A, 1984-B), Wilson and Birkin (1983)

Figure 4(a) The main 'Weber triangle' results

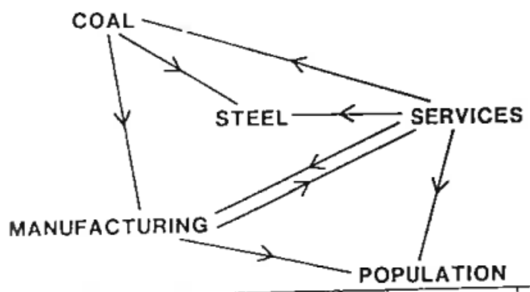


Figure 4(b) Illustrative linkages in a five sector economy

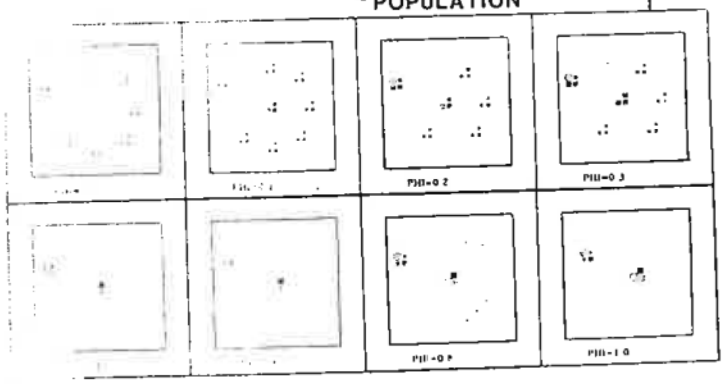


Figure 4(c) Industrial location patterns for varying population densities

Figure 5 Housing and residential location

Source: Clarke and Wilson (1983-B)

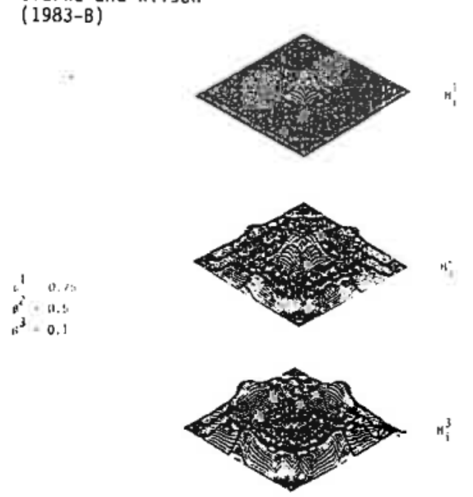


Figure 5(a) Housing distribution with low β^W

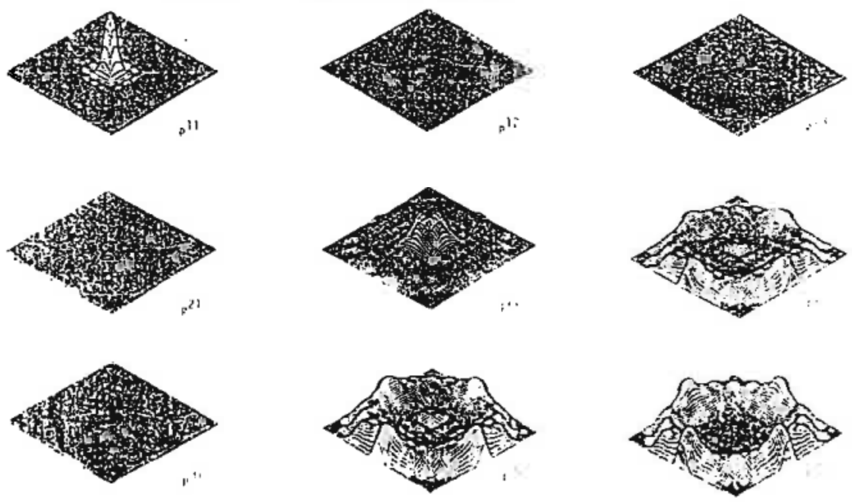


Figure 5(b) Population groups by housing type, low β^W

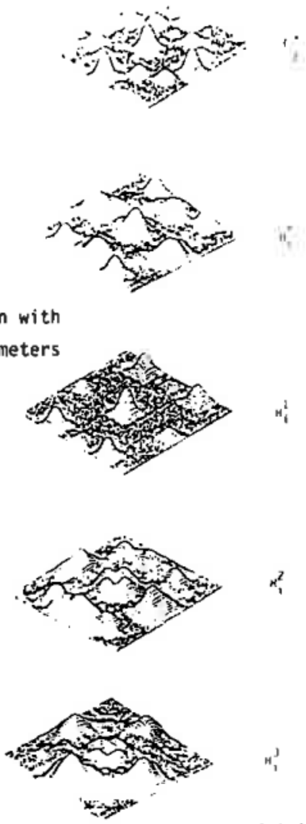


Figure 5(c) Housing distribution with high α parameters

β^1 1.7

β^2 1.5

β^3 1.0

$\alpha_3^{*1} = 0.7$

$\alpha_3^{*2} = 0.7$

$\alpha_3^{*3} = 0.7$

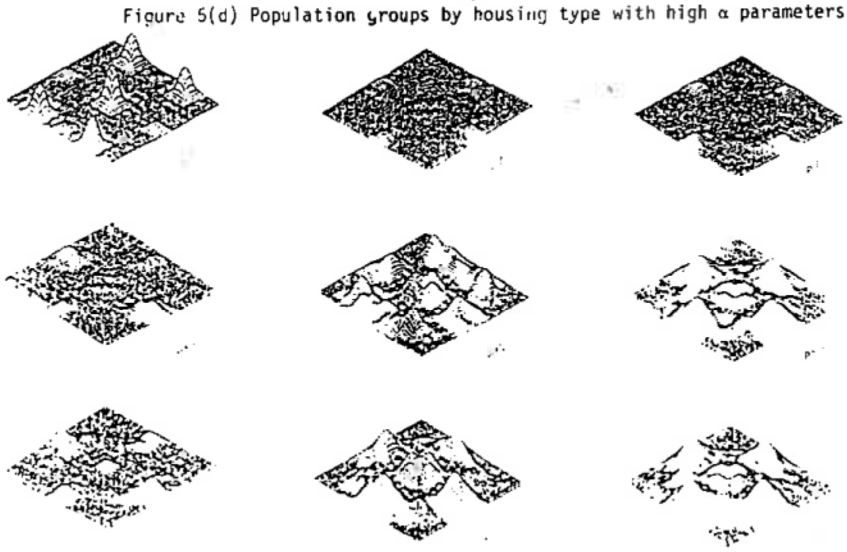
$\alpha_4^{1} = 0.6$

$\alpha_4^{2} = 0.5$

$\alpha_4^{'3} = 0.34$

$\alpha_4^{''3} = 0.5$

Figure 5(d) Population groups by housing type with high α parameters



<u>Figure 6</u> Retail patterns for various alpha and beta values

Source: Clarke and Wilson (1983-A)

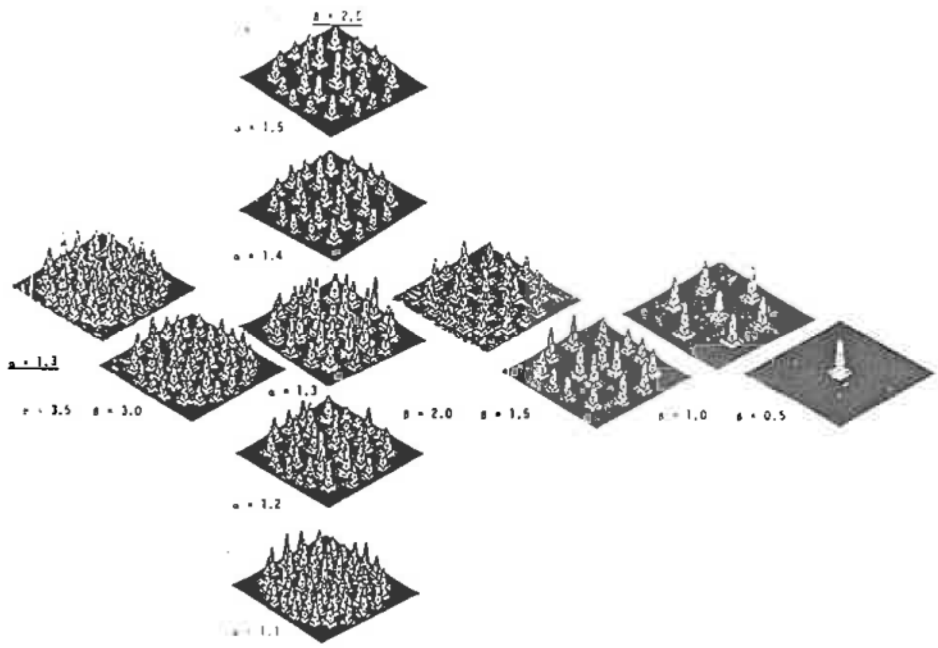


Figure 7 Retail patterns with additional parameter variation

Source: Clarke and Wilson (1985-A)

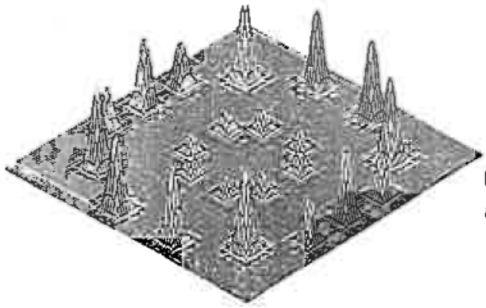


Figure 7(a)
Declining supply costs away from city centre ($1/c_{ij}$)

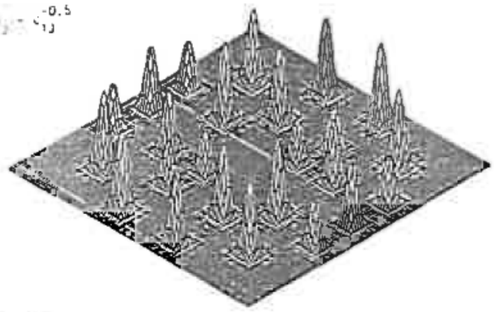


Figure 7(b)
Declining supply costs ($1/c_{ij}^{0.5}$)

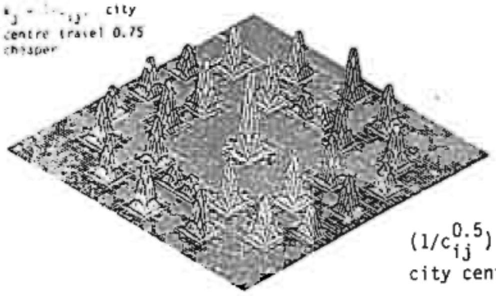


Figure 7(c)
Combination of declining supply costs ($1/c_{ij}^{0.5}$) with cheaper travel to city centre (factor of 0.75)

<u>Figure 8</u> Performance indicators for health service provision for Piedmont, Italy

Source: Clarke and Wilson (1985-B)

Figure 8(a)
The 51 districts (USLs) in the region

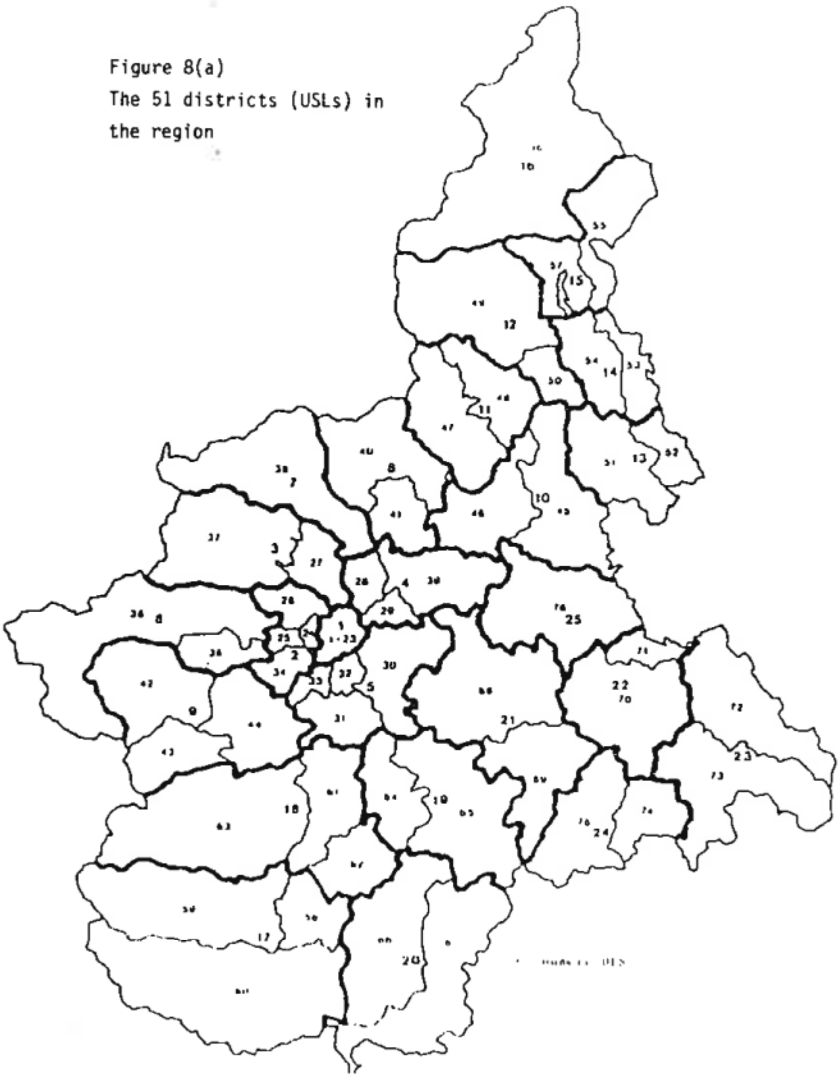


Figure 8(b) Definitions of performance indicators

RESDENCE-BASED INDICATORS

The indicators calculated at present are (for (1) - (8), for both aggregate and specialty cases):

(1) For each district I, proportion of cases treated within the district.

(2) Proportion of cases treated in the next 'ring' of contiguous districts.

(3) Proportion treated in the next 'ring'.

(4) Proportion treated in the rest of the system.

(5) Hospitalisation rate: number of cases generated in I divided by total population of I.

(6) Notional number of beds available to I-residents from all districts, per head of I-population.

(7) Notional available expenditure on I-residents by all districts per head of I-population.

(8) Average distance travelled to facilities by I-residents.

Figure 8(b) continued

FACILITY-BASED INDICATORS

In all cases, calculated for aggregate and specialties:

(1) Proportion of J-facility patients who live with J.

(2) Proportion who live in contiguous districts.

(3) Proportion who live in the next 'ring'.

(4) Proportion who live elsewhere in the region.

(5) Beds/head of catchment population.

(6) Budget (expenditure)/head of catchment population.

(7) Case capacity/head of catchment population.

(8) Cases attracted/beds.

(9) Cases attracted/budget (unit of expenditure).

(10) Treatment intensity rate: cases attracted/head of catchment population.

(11) Average distance travelled to that facility.

(12) % of capacity used: (cases attracted/case capacity) X 100.

Figure 8(c) Residential-zone and facility zone performance indicators for base year.

8(c)(i) Residential

	(1)	(2)	(3)	(4)
1	0.949726	0.024909	0.025376	0.025376
2	0.000000	0.786746	0.213263	0.213263
3	0.491014	0.267162	0.241836	0.241836
4	0.590204	0.238919	0.170787	0.170787
5	0.798030	0.100720	0.101260	0.101260
6	0.000000	0.498870	0.501137	0.501137
7	0.000000	0.539494	0.460507	0.460507
8	0.671504	0.218946	0.109558	0.109558
9	0.811284	0.050146	0.138582	0.138582
10	0.342702	0.602898	0.054407	0.054407
11	0.000000	0.944835	0.055175	0.055175
12	0.758954	0.190092	0.050962	0.050962
13	0.425030	0.262088	0.312882	0.312882
14	0.773813	0.022688	0.203507	0.203507
15	0.636680	0.074725	0.288594	0.288594
16	0.646799	0.157885	0.195322	0.195322
17	0.494427	0.107075	0.398501	0.398501
18	0.734807	0.090456	0.174742	0.174742
19	0.618777	0.168038	0.213194	0.213194
20	0.555029	0.036515	0.408457	0.408457
21	0.132425	0.128500	0.739075	0.739075
22	0.438154	0.137360	0.424487	0.424487
23	0.762143	0.094177	0.143685	0.143685
24	0.348098	0.443863	0.208046	0.208046
25	0.812404	0.039210	0.148388	0.148388
26	0.347282	0.380709	0.272015	0.272015
27	0.828686	0.029671	0.141648	0.141648
28	0.675901	0.226577	0.097529	0.097529
29	0.824176	0.087690	0.088136	0.088136
30	0.643326	0.161455	0.195225	0.195225
31	0.567823	0.263661	0.168521	0.168521
32	0.652319	0.219346	0.128339	0.128339
33	0.724726	0.123351	0.151926	0.151926
34	0.866367	0.041758	0.091878	0.091878
35	0.585737	0.176931	0.237336	0.237336
36	0.909147	0.032528	0.058328	0.058328
37	0.000000	0.547197	0.452809	0.452809
38	0.579275	0.226146	0.194584	0.194584
39	0.763673	0.127384	0.108946	0.108946
40	0.557497	0.282266	0.160241	0.160241
41	0.517764	0.138736	0.343503	0.343503
42	0.490340	0.262439	0.247223	0.247223
43	0.508185	0.127936	0.363880	0.363880
44	0.598509	0.198966	0.202526	0.202526
45	0.650767	0.071180	0.278053	0.278053
46	0.631378	0.137915	0.230708	0.230708
47	0.579183	0.199583	0.221236	0.221236
48	0.812153	0.115512	0.072336	0.072336
49	0.513157	0.319906	0.166939	0.166939
50	0.712535	0.178879	0.108587	0.108587
51	0.719725	0.164478	0.115797	0.115797
52	0.503021	0.279521	0.217458	0.217458
53	0.643433	0.139478	0.217090	0.217090
54	0.747124	0.123369	0.129507	0.129507

8(c) (ii) Residential - aggregate - continued

	(5)	(6)	(7)	(8)
1	0.118980	0.005720	0.183137	11.171148
2	0.119001	0.005621	0.162252	36.014023
3	0.126376	0.005677	0.155852	16.467209
4	0.129263	0.004411	0.165757	24.314163
5	0.136433	0.003223	0.165831	14.789124
6	0.118899	0.005241	0.146472	44.041382
7	0.101148	0.004399	0.122085	39.593872
8	0.142151	0.004808	0.149165	17.559052
9	0.159944	0.007000	0.138634	10.902503
10	0.144205	0.006358	0.203337	11.605224
11	0.140957	0.006819	0.209177	16.189606
12	0.129500	0.007315	0.152068	12.290760
13	0.118274	0.007977	0.188194	27.691467
14	0.128832	0.004742	0.137991	23.795242
15	0.126249	0.007258	0.172458	28.109390
16	0.160920	0.005962	0.165582	25.388321
17	0.155260	0.006055	0.160148	25.657227
18	0.139869	0.006748	0.165630	18.096481
19	0.124235	0.005078	0.109446	19.024063
20	0.156080	0.012041	0.176255	21.744675
21	0.150881	0.007883	0.237148	31.491089
22	0.150968	0.007662	0.213413	19.445694
23	0.198949	0.010367	0.237441	15.716727
24	0.210614	0.008280	0.237132	22.581268
25	0.155126	0.007245	0.150293	15.465292
26	0.159006	0.006292	0.171013	25.796600
27	0.173880	0.009956	0.172597	16.033249
28	0.176695	0.007056	0.157292	12.421432
29	0.155962	0.008111	0.200423	13.972667
30	0.138631	0.004646	0.125305	15.679955
31	0.137502	0.005139	0.159795	19.471405
32	0.145447	0.005995	0.187958	14.981613
33	0.154992	0.006312	0.231317	21.146027
34	0.183994	0.006774	0.150141	18.074860
35	0.182526	0.007669	0.189010	23.991196
36	0.152841	0.009781	0.208352	11.063938
37	0.142014	0.007530	0.182357	51.376694
38	0.151554	0.006554	0.190800	24.298645
39	0.182592	0.008016	0.212310	13.461343
40	0.180706	0.008233	0.186124	17.458206
41	0.156375	0.006435	0.177540	22.452591
42	0.180008	0.006858	0.201699	20.582397
43	0.133711	0.005614	0.192363	27.217926
44	0.158663	0.006566	0.174299	23.981873
45	0.131948	0.006475	0.163424	26.775528
46	0.152384	0.006662	0.195597	19.989349
47	0.142909	0.005856	0.182997	22.900375
48	0.170314	0.009112	0.203956	13.902085
49	0.162727	0.007813	0.193853	18.834015
50	0.160153	0.009075	0.167740	15.026179
51	0.150484	0.007335	0.165305	15.625429
52	0.145353	0.007013	0.173970	24.261780
53	0.166455	0.008013	[illegible]	[illegible]
54	[illegible]	0.007919	0.176091	[illegible]

c)(ii) Facility - aggregate

	(1)	(2)	(3)	(4)
1	0.705980	0.148177	0.145852	0.145852
2	1.000000	-1.000000	-1.000000	-1.000000
3	0.565772	0.282396	0.151845	0.151845
4	0.789278	0.127636	0.083104	0.083104
5	0.882952	0.067559	0.049499	0.049499
6	-1.000000	-1.000000	-1.000000	-1.000000
7	-1.000000	-1.000000	-1.000000	-1.000000
8	0.814358	0.084254	0.101398	0.101398
9	0.666341	0.121129	0.212540	0.212540
10	0.458783	0.352464	0.188764	0.188764
11	-1.000000	-1.000000	-1.000000	-1.000000
12	0.442155	0.340678	0.217171	0.217171
13	0.613150	0.132113	0.254737	0.254737
14	0.943793	0.009256	0.046960	0.046960
15	0.675164	0.142649	0.182186	0.182186
16	0.883780	0.065894	0.050336	0.050336
17	0.582186	0.276030	0.141791	0.141791
18	0.655923	0.169841	0.174240	0.174240
19	0.641180	0.226423	0.132400	0.132400
20	0.502062	0.065679	0.432259	0.432259
21	0.560645	0.119238	0.320121	0.320121
22	0.533728	0.185095	0.281184	0.281184
23	0.535746	0.180903	0.283352	0.283352
24	0.677164	0.205558	0.117277	0.117277
25	0.665615	0.148655	0.185731	0.185731
26	0.800507	0.096331	0.103175	0.103175
27	0.806340	0.052962	0.140704	0.140704
28	0.575874	0.236177	0.187957	0.187957
29	0.616820	0.153920	0.229260	0.229260
30	0.817760	0.061629	0.120618	0.120618
31	0.774663	0.129492	0.095855	0.095855
32	0.603561	0.239755	0.156688	0.156688
33	0.800624	0.142529	0.056850	0.056850
34	0.927090	0.040193	0.032720	0.032720
35	0.766503	0.138469	0.095033	0.095033
36	0.538230	0.280170	0.181602	0.181602
37	1.000000	-1.000000	-1.000000	-1.000000
38	0.869677	0.053219	0.077110	0.077110
39	0.545558	0.277261	0.177183	0.177183
40	0.56[illegible]752	0.225237	0.206016	0.206016
41	0.725332	0.100797	0.173874	0.173874
42	0.691274	0.183723	0.125006	0.125006
43	0.763288	0.122613	0.114101	0.114101
44	0.811019	0.107855	0.081128	0.081128
45	0.807415	0.090720	0.101865	0.101865
46	0.705740	0.137717	0.156544	0.156544
47	0.795110	0.102989	0.101903	0.101903
48	0.608441	0.255377	0.136182	0.136182
49	0.687939	0.152048	0.160013	0.160013
50	0.652049	0.160929	0.187022	0.187022
51	0.738833	0.119715	0.141453	0.141453
52	0.766281	0.131648	0.101972	0.101972
53	0.771564	0.093882	0.134554	0.134554
54	0.709458	0.110243	0.180299	0.180299

d(c)(iv) Facility - aggregate - continued

	(5)	(6)	(7)	(8)
1	0.006004	0.194259	0.125089	20.771102
2	-1.000000	-1.000000	-1.000000	-1.000000
3	0.004451	0.153258	0.158206	28.602859
4	0.003139	0.159967	0.089509	41.119080
5	0.002342	0.162658	0.112613	58.051773
6	-1.000000	-1.000000	-1.000000	-1.000000
7	-1.000000	-1.000000	-1.000000	-1.000000
8	0.003875	0.128798	0.136019	36.581161
9	0.006702	0.119032	0.141900	23.340225
10	0.004866	0.162743	0.184188	27.987122
11	-1.000000	-1.000000	-1.000000	-1.000000
12	0.007785	0.144157	0.081003	16.918747
13	0.011042	0.250537	0.273763	11.296632
14	0.004234	0.128473	0.100478	30.534912
15	0.008536	0.183041	0.164560	15.128097
16	0.005044	0.150848	0.137230	31.323273
17	0.004553	0.124549	0.145032	31.865265
18	0.007164	0.176180	0.177221	20.268158
19	0.005099	0.095343	0.000669	26.122726
20	0.014762	0.134137	0.059168	9.997418
21	0.009918	0.406437	0.186149	14.753507
22	0.007491	0.208295	0.245821	19.350189
23	0.009478	0.212072	0.177178	18.477249
24	0.004525	0.207322	0.174505	40.859085
25	0.007261	0.142134	0.163610	21.376465
26	0.004103	0.163111	0.034188	38.697189
27	0.010120	0.163206	0.193680	16.796967
28	0.006024	0.122948	0.155240	27.807373
29	0.008278	0.202898	0.174500	18.621902
30	0.003607	0.101274	0.070384	39.036575
31	0.004447	0.155136	0.103017	31.670532
32	0.005757	0.203883	0.183337	25.849594
33	0.006211	0.257884	0.178778	25.222672
34	0.006454	0.134493	0.165384	28.112030
35	0.007027	0.151019	0.150617	24.899323
36	0.010025	0.209926	0.221332	15.154010
37	-1.000000	-1.000000	-1.000000	-1.000000
38	0.005244	0.186870	0.079957	28.863373
39	0.007363	0.195710	0.206748	22.970963
40	0.007215	0.145586	0.169814	23.460848
41	0.005068	0.159709	0.142859	30.549316
42	0.005054	0.173104	0.195367	33.480270
43	0.005255	0.224891	0.171749	26.352386
44	0.005637	0.158473	0.116504	27.859116
45	0.006788	0.168807	0.113730	19.961639
46	0.006166	0.192128	0.156987	24.382552
47	0.005183	0.185399	0.092037	27.869659
48	0.008899	0.194911	0.170731	18.287109
49	0.007281	0.197305	0.127754	22.145401
50	0.009406	0.156370	0.143222	16.853607
51	0.007205	0.161594	0.119425	21.034744
52	0.006922	0.176994	0.137993	21.307648
53	0.007769	0.235733	0.186820	20.880173
54	0.007438	0.160193	0.136606	22.184157

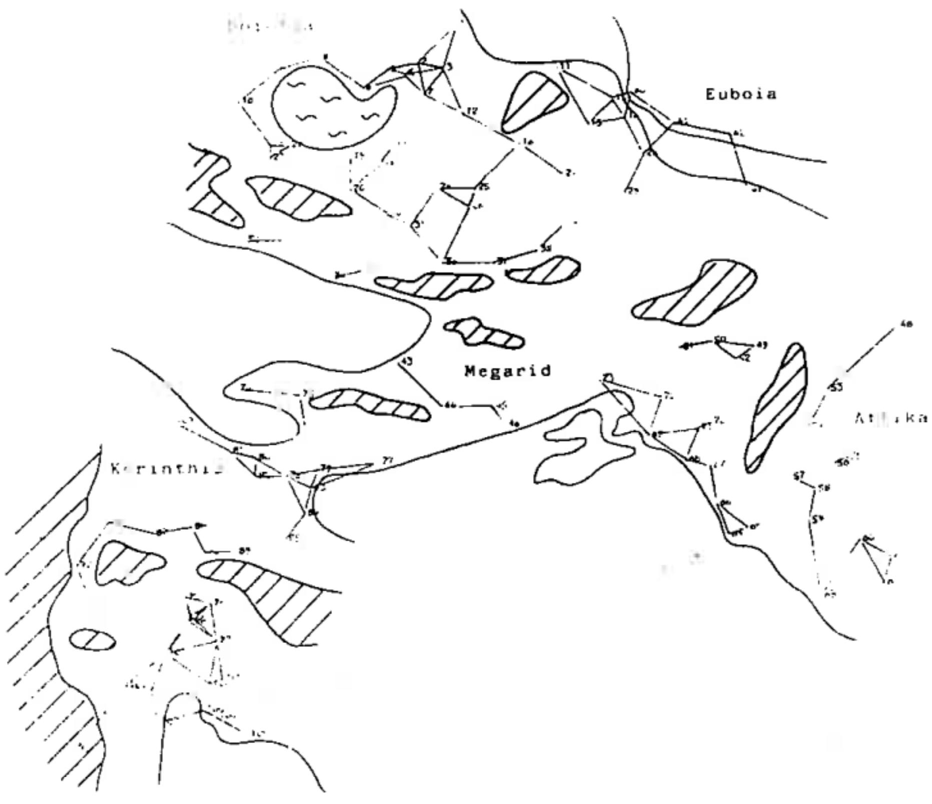
:(c)(v) Facility - aggregate con:'nue.

	(9)	(10)	(11)	(12)
1	0.641962	0.124707	19.674866	99.694550
2	-1.000000	-1.000000	-1.000000	-1.000000
3	0.830693	0.127310	15.018292	80.471283
4	0.806907	0.129079	11.584565	144.208145
5	0.835688	0.135931	8.310228	120.706924
6	-1.000000	-1.000000	-1.000000	-1.000000
7	-1.000000	-1.000000	-1.000000	-1.000000
8	1.100436	0.141734	12.350161	104.201843
9	1.314155	0.156426	16.233932	110.236542
10	0.836760	0.136177	15.841341	73.933777
11	-1.000000	-1.000000	-1.000000	-1.000000
12	0.913721	0.131719	21.889771	162.609146
13	0.497876	0.124736	24.444427	45.563599
14	1.006315	0.129284	11.366137	128.669159
15	0.705490	0.129133	20.086807	78.472122
16	1.047297	0.157983	12.509160	115.122314
17	1.164958	0.145095	16.507050	100.043304
18	0.824210	0.145209	18.923172	81.936569
19	1.396937	0.133188	15.43366919903.023400	
20	1.100223	0.147581	28.395386	249.428436
21	0.360010	0.146322	22.438339	78.604721
22	0.695889	0.144950	20.759460	58.965836
23	0.825780	0.175125	22.345688	98.841187
24	0.891766	0.184883	12.945644	105.946793
25	1.092080	0.155222	18.630447	94.872681
26	0.973310	0.158757	12.405187	464.366211
27	1.041519	0.169982	17.249786	87.764023
28	1.362512	0.167518	15.430510	107.909103
29	0.759716	0.154145	20.652557	88.334824
30	1.390262	0.140798	10.471803	200.041061
31	0.907829	0.140837	12.678705	136.712494
32	0.729853	0.148805	15.860987	81.164474
33	0.607504	0.156666	14.570755	87.631317
34	1.349113	0.181446	11.990792	109.711426
35	1.158658	0.174979	14.945108	116.174683
36	0.723674	0.151918	21.331253	68.638229
37	-1.000000	-1.000000	-1.000000	-1.000000
38	0.810011	0.151366	11.908195	189.309448
39	0.864209	0.169134	17.578979	81.807129
40	1.162756	0.169281	16.826828	99.686539
41	0.969380	0.154819	14.932310	108.372147
42	0.977561	0.169219	13.850463	86.616058
43	0.615737	0.138473	16.199738	80.625290
44	0.990962	0.157041	14.137783	134.794006
45	0.802747	0.135509	14.977987	119.150314
46	0.782534	0.150347	18.511520	95.770096
47	0.779115	0.144447	14.229289	156.943985
48	0.834937	0.162738	20.988037	95.318481
49	0.817174	0.161233	16.013519	126.205841
50	1.013777	0.158523	19.061172	110.683197
51	0.937821	0.151546	15.815596	126.896698
52	0.811336	0.147495	14.327041	106.885834
53	0.688117	0.162217	16.028412	86.827866
54	[illegible]	0.145027	18.566696	120.804611

<u>Figure 9</u>
Settlement
structures in
Ancient Greece

Source: Rihll and Wilson (1985)

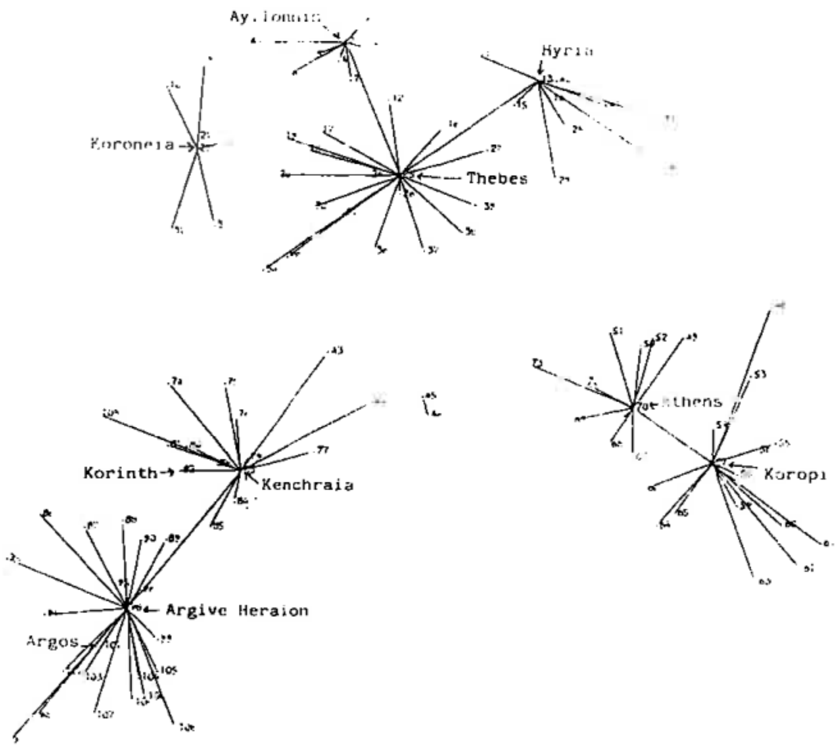
Figure 9(b) The topography



Model I

[illegible]	1.15
[illegible]	[illegible]
[illegible]	[illegible]

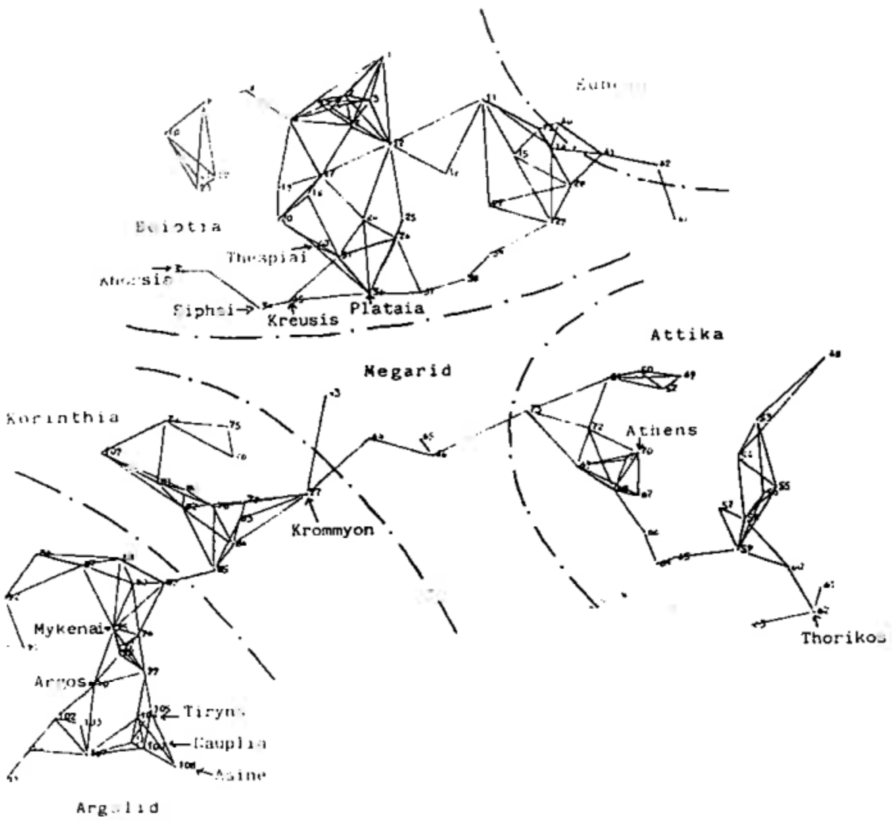
Figure 9(c) The development of centres



Model 3

α 1.5
β .2
TND 1

Figure 9(d) Intra- and inter-regional interaction



<u>Model 1</u>

α	1.15
β	.1
TND	.5