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Dynamic models of agricultural location in a spatial interaction framework

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1. INTRODUCTION

Agricultural location theory rests firmly on the work of von Thunen (1826) and particularly the concept of bid rent. Some of the restrictions - for example that there is a single market centre - have been removed by authors like Stevens (1968), but remain unrealistic in other ways (cf Found, 1971, Chapter 5). Two problems in particular are deserving of further attention. Firstly, the implication of linear programming representations that all flows go to the nearest market needs to be relaxed. The second requirement is the development of dynamic models concerned with the evolution of spatial pattern. Harvey (1966) identified this as an essential precursor to the development of a general theory of agricultural location "which is both operational and intuitively appealing" (p374).

In this paper, we show how to apply spatial interaction models linked to hypotheses about change to generate a new dynamic theory of agricultural location which can also be used to generate equilibrium patterns. Many existing models can then be seen as special cases of this theory.

We begin, in Section 2, by reviewing some important contributions to the literature on agricultural location theory. In Section 3 our own model framework is discussed in more detail and we outline a set of plausible assumptions which may be used to derive specific models. Sections 4, 5, and 6 focus on the crucial interaction component and the utility of different kinds of interaction model is discussed. A secondary outcome here concerns the applicability of alternative theoretical assumptions in different contexts, and we begin to get a feel for the range of alternative models available, thus reinforcing the general argument of Section 3. Some more complex examples including an explicit dynamic component are introduced and evaluated in Sections 7 and 8. There remain, however, many unexplored avenues for the future, the most important of which are discussed in Section 9.

2. AGRICULTURAL LOCATION: THEORY AND MODELS

The classic work here is that of von Thunen (1826), and it is so well-known that only a brief review is necessary. Initially a simplified and abstract situation is proposed in which a single point of consumption is located at the centre of an homogeneous plain with the following characteristics (cf Stevens, 1968):

1) Uniform fertility;
2) Uniform transport costs;
3) Uniform production costs;
4) Infinitely elastic demand at a given price;
5) A single market centre.

For each of a number of different crops, the prices, production and transport costs and productivity of land vary. It is assumed that all profits will be imputed as rent to land-owners, and the land rent which could be paid for a particular crop at a particular location may then be interpreted as a bid rent. The objective of the land-owners is to maximise bid rents at each location with respect to the land-use activity represented there. It is possible to define the bid-rent (B) rigorously in terms of production cost (C), market price (P), transport rate (T), distance to market (D) and crop yield (Y):

$$B = Y(P-C) - YDT \tag{1}$$

The essence of the von Thunen argument is that the distance to market is the only non-constant term in this expression, hence the ratio of production to transport costs will vary as the distance to the market centre changes. Accordingly the type of land-use which pays most rent may vary with location. Since the plain is homogeneous, this variation with distance produces a series of concentric rings of land-use in which activities with high transport to input cost ratios locate centrally, while activities with lower relative transport costs are forced to the periphery. Land ceases to be used at a distance from the centre when no activity is profitable.

The development of von Thunen's rings in a situation with three crops is illustrated in Figure 1. In Figure 1A we assume that 'Activity 1', say dairy farming, has a high unit profitability (P-C), but also a high tariff for transportation (T). The bid rent function for activity 1 is consequently rather steep. The reverse applies to the 'Activity 3' (e.g. grazing sheep) with low profitability and transport charges, and thus a flatter bid rent curve. 'Activity 2' (wheat cultivation) is intermediate between the two. Up to distance D_1 from the market activity 1 may offer the highest bid rents, but at this point the influence of transport costs allows activity 2 to take over. Similarly at distance D_2 the low yield-low transport charge activity becomes dominant. Finally, at distance D_3, no activity is profitable, so beyond here land is uncultivated.

Since von Thunen's backcloth is an isotropic plain with uniform characteristics in any direction, the one-dimensional distances of Figure 1A may be seen as representing radii about the market in two dimensional space. Thus a series of 3 concentric rings of land-use develops about the market, with radii of length D_1, D_2 and D_3. This is shown in Figure 1B.

While the original von Thunen model operates in continuous space, it is possible to define discrete space models of a similar structure which are amenable to the powerful solution procedures of linear programming. Such a discrete space problem may be written (following Lefeber, 1966; cf Stevens, 1968) as:

$$\underset{\{Z,L\}}{\text{Max}} \quad \sum_{jk} (p_i^k - c_{ij}^k - v_j^k) * Z_j^k \qquad (2)$$

where p_i^k is the price of good k at the market i; c_{ij}^k is the cost of transporting good k to i from j; v_j^k is the variable cost of producing k at j; and Z_j^k is production of k at j (all of which must be sold at i).

The constraints are:

$$q^k Z_j^k = L_j^k \qquad (3)$$

where L_j^k is land used in zone j to produce k; and q^k is an input coefficient for land to k.

$$\sum_k L_j^k \quad \bar{L}_j \leq 0 \qquad (4)$$

where $\bar{L}_j$ is land available for agriculture in zone j.

Finally there are the non-negativity conditions on the decision variables:

$$Z_j^k \geq 0 ; \quad L_j^k \geq 0. \qquad (5)$$

The specification of the model as a linear program facilitates numerical simulation, and may be used to attempt an answer to practical policy questions (e.g. Henderson, 1959); it is also an aid to description or explanation of real-world patterns (cf Chisholm, 1961).

However, as noted by Stevens (1968) the use of programming methods also allows us to extend rigorously the scope of the initial problem. In particular assumptions 1, 2 and 3 of the original von Thunen setup are easily relaxed here. Stevens notes that this is also the case for assumption 5, although this is not spelled out explicitly, as MacMillan (1979) observes. One of the characteristics of this paper will be an explicit focus on interaction patterns and the possibility of modelling situations with numerous markets.

A more problematic assumption is number 4, and it is in this direction that Stevens' paper is most directly aimed. He attempts to show that an assumption of demand varying linearly with price may be used to generate a class of soluble (quadratic) programming problems. This idea has been extended and refined by MacMillan (1979) who derives the agricultural location problem as a special case of the 'Walras-Wald model' of general economic equilibrium.

In this paper, it is argued that prices are so important that we need to treat consumers explicitly as one set of actors within the system. This allows a more flexible treatment of this element of the problem, although prices may, ofcourse, be treated as fixed by making a simplifying assumption at the appropriate point. Thus in Section 3 below we introduce a system with only four backcloth characteristics, and three sets of actors, labelled 'land owners', 'producers' and 'consumers'.

3. FRAMEWORKS INCORPORATING SPATIAL INTERACTION MODELS.

As noted in the introduction, one of the problems with the linear programming representations discussed above is that they imply a pattern in which all flows go to the nearest market. This is unrealistic, and spatial interaction models with their associated overlapping market areas provide a more flexible basis. According to the form of the constraints in particular circumstances, we may apply production-constrained, attraction-constrained, or doubly-constrained models.

It has been shown in earlier work that such models can then be 'embedded' within modified mathematical programming models - ones which are then non-linear rather than linear because of the entropy term which is needed to generate the interaction pattern (cf. Coelho and Wilson, 1977; Coelho, Williams and Wilson, 1978). In all these models, flows can then 'overlap' in a realistic way. Not all go to the nearest centre.

It is then possible to extend the framework further by developing an alternative mechanism for predicting land use in each zone. This can be done by analogy with recent research on the prediction of the distribution of retail structure (cf. Harris and Wilson, 1978). If Z^k_j is a measure of land used in zone j for crop k, D^k_j is the revenue obtained from the sale of this crop; and C^k_j the cost of producing it (including land rent), then the analogue of the retail hypothesis is that Z^k_j should grow if $D^k_j > C^k_j$ and decline otherwise. The system will be in equilibrium when $D^k_j = C^k_j$. Formally, this argument can be expressed in terms of producers' surplus, E^k_j, as follows

$$E_j^k = D_j^k - C_j^k \tag{6}$$

The model may be operated on a comparative static basis using the equilibrium condition which is:

$$E_j^k = 0 \quad \forall \; j,k \tag{7}$$

or in dynamic mode, say by making the rate of change a function of profitability:

$$\dot{Z}_j^k = f\,(E_j^k) \tag{8}$$

Typically, for example in the retail model equivalent, $f\,(E_j^k)$ is taken as $\varepsilon^k E_j^k Z_j^k$ (where ε^k is a suitable response parameter). Indeed this is the formulation proposed by Wilson (1983). It is clear, however, that land rents will also be a function of profitability in this model. Hence a slightly more subtle framework is required to handle the interdependence between rent and production levels. The solution adopted here is to fix total land availability in a zone, and then to use profitability as the driving force behind competition between activities for that land. This is discussed further in relation to the first example below (Section 4).

Before proceeding with more detailed model development, it is useful to review in a broader way the decisions to be made in model design. This will provide a basis for generating a great variety of models, not all of which can be treated explicitly here. The decisions can be seen as an extension of the basic principles outlined earlier as underpinning the von Thunen model.

Six different kinds of decision to be considered are outlined in Table 1. The spatial backcloth is defined by the series of assumptions

A1-A4. Model flows between activities may then be generated by one of a family of spatial interaction models (choice B), which may be treated in a static fashion; or embedded in a comparative static or dynamic framework (C). The second group of decisions (D-F) relate to the behaviour of the various actors in the system, whom we refer to as 'land-owners', 'producers', and 'consumers'. Choices D-F are concerned with the assumptions we make in relation to the model variables associated with these actors, which are rents, land under cultivation and prices (quantity demanded) respectively. The first method is to assume a fixed distribution, with the variable treated as external to the model. This is assumption D1 (E1,F1). In a commodity flow model to determine interactions between producers and consumers, rents, land use and prices might all be specified in this way. A second possibility is to deduce an activity level as a model OUTPUT, but without allowing the appropriate variable any role in model OPERATION. For example, in the von Thunen model rents are imputed to land-owners "after the fact", i.e. once land use has been determined through an appropriate mechanism. This procedure would be represented as D2 ('internal and passive'). In dynamic models, however, we may wish to model these variables as active determinants of system structure. Thus the pattern of land use at any one time will usually affect the pattern in the next time period. This would be represented as E3 ('internal and active').

One of the most interesting features of this framework is the existence of three sets of actors, whereas mainstream economic theory is concerned with the interaction between only two activities, 'supply' and 'demand'. Here we have producers operating as both a demand sector in relation to the supply of land, and as a supply sector in relation to market demand.

It is well-known that dynamic supply-demand relations may be either stable or unstable, as shown in Figure 2. In Figure 2A, where the supply curve is 'flatter' than the demand curve, we see the stable case in which equilibrium results at point (q,p) after starting from some non-equilibrium state (q_o, p_o). In Figure 2B the supply curve is the steeper and no stable solution emerges. This relationship is described in an appropriate COBWEB mechanism.

An important third situation is illustrated in Figure 2C, where the basic state is unstable, but dampening the adjustment procedure allows stability to be achieved. This mechanism is significant in the current context, because the level of spatial and temporal resolution adopted implies a high degree of instability through fluctuation in supply-demand relations between time periods. The whole system is much more complex, therefore, than the schematic illustration of Figure 2, with equilibrium resulting from shifts in the curves aswell as from shifts along them. In such a situation it is possible that the strength of adjustment or the 'reaction speed' of the actors to the state of the system can actually have an important effect on the equilibrium achieved. Further complexity is introduced by the fact that we have three sets of actors here, and therefore in principle a three- dimensional cobweb adjustment is possible. However by varying assumptions D-F, one, two, or even all three, may be removed.

It is clear that variation in the set of choices B-F allows us to create a wide variety of different models for the agricultural location problem. Within such models the set of choices A1-A4 allows us to vary the scenario away from isotropic plane assumptions. It is hardly possible to demonstrate the model possibilities exhaustively here. However we do wish to present a variety of cases which will hopefully demonstrate the richness of the framework. Three aspects are focussed on. Initially, we wish to demonstrate that the behaviour of classical models (specifically the von Thunen model) can be reproduced. Secondly, that the scope of these problems can be extended by varying the backcloth. Finally, it is possible to demonstrate that new kinds of insight are offered, particularly in relation to dynamics.

With these objectives in mind, the remainder of the material is structured around the construction of some alternative models. In Sections 4, 5 and 6 we investigate issues related to the model interactions, under a variety of pricing assumptions. It is shown that the choice of an interaction model also affects the other choices to be made. However an appropriate set of decisions will in each case allow simple concentric ring patterns to be generated. Section 7 presents a more complex version of the attraction- constrained model incorporating the internal and active treatment of both prices and land use. The backcloth assumptions A1-A4 are varied to demonstrate the generality and power of the approach. Finally, Section 8 treats dynamics explicitly and looks in more detail at the effects of variation within the cobweb mechanism on stability, solution paths and model equilibria.

4. A PRODUCTION-CONSTRAINED MODEL

Let us begin by specifying the form of the interaction model:

$$Y_{ij}^{k} = A_i^k X_i^k Z_j^k \exp(-\beta^k c_{ij}^k) \tag{10}$$

with

$$A_i^k = \sum_j Z_j^k \exp(-\beta^k c_{ij}^k) \tag{11}$$

where X_i^k is the demand for good k at market i; Z_j^k is production of k at location j; Y_{ij}^k is the flow of goods type k from j to i; c_{ij}^k is the cost of transporting a unit of k from j to i; and β^k is a distance deterrence parameter.

Assume that the total stock of land in each zone j is fixed to $\bar{L}$:

$$\sum_k q^k Z_j^k \leq \bar{L} \tag{12}$$

where q^k is land needed to produce a unit of k.

Rent is given as a simple surplus term, and treated as a residual (assumption D2) i.e.

$$r_j = \sum_k E_j^k \tag{13}$$

Since rent is fixed after the fact, it is not an element of the cost function. Profit is given by:

$$E_j^k = D_j^k - C_j^k$$

$$= \sum_i (p_i^k \quad c_{ij}^k \quad v_j^k) Y_{ij}^k \tag{14}$$

It is at this point that we need a specification of the function f in equation (8). We assume that land-use change between time periods is determined by the equation:

$$Z_j^k(t+1) = Z_j^k(t) + \sum_{s \neq k} d_j^{sk}(t)\, \varepsilon_1\, q^s\, Z_j^s(t) / q^k \qquad (15)$$

where ε_1 is a parameter governing the rate at which land may be converted between uses; $d_j^{sk}(t)$ is given by:

$$d_j^{sk}(t) = \begin{cases} 1 & \text{if } E_j^k(t) > E_j^v(t) \quad \forall\, v \neq k \\ -1 & \text{if } E_j^{st} > E_j^{vt} \quad \forall\, v \neq s \\ 0 & \text{otherwise} \end{cases} \qquad (16)$$

It can be seen that in the case when k is the most profitable land-use in zone j at time t, then the first of the relationships in (16) will be satisfied for all s, and therefore an amount of land proportional to ε_2^s will be transferred from less to more profitable uses (i.e. k). In the case when k in (15) is not the most profitable, then $d_j^{sk}(t)$ in (16) will be set to -1 only for the s which is the most profitable. This is therefore the complement of the first relation: some land is transferred from less profitable uses to the most profitable one.

The model is applied to a simple situation with three goods in which production costs are uniform, transport costs are proportional to euclidean distance, and there is a single market at the centre of a 51 X 51 zone spatial grid. A set of parameter values for this problem is given in Table 2, with goods differentiated only by their transport costs. In Figure 3 we see the effect of setting $\beta^k = 0 \;\forall\, k$. Intuitively one would expect activities of type 1 to locate at the centre of the plain, since they have the lowest transport costs and therefore the highest bid rents, but this is clearly not the case. The problem arises because of an imbalance between supply and demand in this model. Although the demand levels are fixed, the total area of land under cultivation is determined in the usual way (see above, page 4) according to the distance from the centre at which non-negative profits can be made. Activities with low transport costs have a high area of potential production - hence high profits in the centre can be competed away by cultivation at the fringe. On the other hand, although profits in the centre are initially lower for high transport cost activities, the area of potential production is here much lower, and so the profits may not be competed away to the same extent. This competitive mechanism therefore allows that high transport cost activities may generate the greatest bid rents near to the market.

By altering the nature of the assumptions, however, the structure of the model outputs may be altered dramatically. For example, suppose we respecify the rent function (13) so that rents are derived actively (assumption D3). The approach adopted here is to set rents as a function of the intensity of land use. This may be expressed as:

$$r_j = f_j \left(\sum_k Z_j^k , L \right) \qquad (13)'$$

where L is a land use constraint (assumed equal in each zone), but one which may be relaxed by the substitution of capital (or labour) for land. In fact a particular functional form of r_j was taken as:

$$r_j = a * \exp \left[b \left(\sum_k Z_j^k - L \right) \right] \qquad (17)$$

This relationship between rent and the utilisation of land is illustrated in Figure 4. It implies that the substitution process is subject to rapidly diminishing returns beyond L. The form of the function below L (i.e. it is an increasing function) may be justified in simple supply- demand terms the greater the price, the more land that will be released onto the market. The rent element must now be included explicitly, hence:

$$E^k_j = \sum_i (p^k_i - c^k_{ij} - v^k_j r_j) * Y^k_{ij} \qquad (14)'$$

and it is assumed that the intensity of land use increases in accordance with its profitability (assumption E3) viz:

$$Z^k_j(t+1) = Z^k_j(t) + \varepsilon^k_1 E^k_j(t) \qquad (15)'$$

An operational model of agricultural land use based on production- constrained interactions may be described, in terms of the framework of Section 3, as {A11,A21,A31,A41: B2: C2: D3: E3: F1}. Since it is the first of a series of such models to be presented in the paper, we refer to this as "MODEL 1". Some simple results are given in Figure 5. Figure 5A demonstrates that the introduction of land-owners as active participants in the allocation process draws low transport cost (i.e. high bid rent) activities back to the centre of the region, because these are the only ones that can afford the rents being charged there. The associated rent cross-section is shown in Figure 5B. The pattern of land-use may now be modified by variation in assumptions A1-A4. For example, if we assume a low-cost avenue of transportation (e.g. a river) running diagonally from "top left" to "bottom right" in our figures, then there is a consequent distortion in the rings as shown in Figure 5C.

5. ATTRACTION-CONSTRAINED MODELS

In Section 4, we saw that problems arise in the production-constrained model because of the lack of restriction on supply. Here we reverse the direction of the constraints and explore the properties of the resulting model. We now have a situation in which the interaction terms govern the way in which suppliers distribute a fixed output between markets. Thus:

$$Y_{ij}^{k} = A_{i}^{k} W_{i}^{k} Z_{j}^{k} \exp(-\beta^{k} c_{ij}^{k}) \tag{18}$$

with

$$A_{j}^{k} = \left(\sum_{i} W_{i}^{k} \exp(-\beta^{k} c_{ij}^{k}) \right)^{-1} \tag{19}$$

so that the constraint:

$$\sum_{i} Y_{ij}^{k} = Z_{j}^{k} \tag{20}$$

is always met. W_{j}^{k} is now a generalised measure of the attractiveness of a market to a producer - it could be a price, a population measure or whatever.

Although the nature of the interactions is now fundamentally different, we can still apply the equilibrium mechanism first outlined in the last section i.e. equations (13)-(16). This is 'MODEL 2', with characteristics {A11,A21,A31,A41: B3: C2: D2: E3: F1}. It operates in a similar way to the original von Thunen model in that producers can dispose of all their output, and land-use is determined by the transport to production cost ratios. Figure 6A shows the result of applying an attraction-constrained model for the set of parameter values described in Table 2. We now see a 'desired' von Thunen result, with land-use 1 dominating the plain, giving way to unused land when cultivation is no longer viable.

By altering the structure of input to transportation costs, a typical concentric ring structure can be generated. The set of parameter values in Table 3 generates the distribution of activities shown in Figure 6B. Note that the value of beta is immaterial with a single centre, since output can go to that centre only. However the parameter beta becomes crucially important when there is more than one demand centre. For $\beta=0$, the pattern of distribution is the same, regardless of the producer's location. As the value of β is increased, producers are more inclined to direct output to the nearest market - hence activity patterns will tend to become nucleated. This effect is demonstrated in Figure 7.

To develop more interesting models here, we need to find a way of limiting demand levels. If they are to be constrained explicitly, then we arrive at a set of doubly- constrained models which are considered in the next section. An alternative is to introduce some price dependence into the model, so that the revenue for an individual producer now varies with the total quantities supplied at each i. Thus:

$$p_i^k = f_i^k \left(\sum_j Y_{ij}^k \right) \tag{21}$$

The overall structure would then be:

A. Define an arbitrary pattern of land-use

B. Calculate the interactions in equation (19) and sum these to provide cost and revenue for each activity at each location

C. Calculate a pattern of land-use change from (16)

D. Sum the flows at the demand locations to determine quantity demanded, and hence generate a price needed to sustain the level of supply. These prices will be inputs to the revenue equations in the next round

E. If both the pattern of land-use and commodity prices are stable, an equilibrium pattern has been achieved. Otherwise set new patterns of land-use and return to Step B

The iteration adjusts both land use and prices at demand locations as in a cobweb model, the relative importance of each being determined by ε^k_2 and ε^k_3 respectively. We return to a model of this type in Section 7.

6. DOUBLY-CONSTRAINED MODELS

As we have seen in the previous two sections, the use of singly-constrained models can give rise to problems at the unconstrained end. A third alternative is to apply a doubly- constrained model in which both output and demand totals are explicitly fixed. In this section we fix rents internally (assumption D3) as in Model 1, but also remove land use change as an explicit dynamic and indeed treat it as a passive response to changing rents (assumption E2). The interaction model may be written as:

$$Y^k_{ij} = A^k_i B^k_j X^k_i Z^k_j \exp(-\beta^k c^k_{ij}) \qquad (22)$$

with

$$A^k_i = (\Sigma_j B^k_j Z^k_j \exp(-\beta^k c^k_{ij}))^{-1} \qquad (23)$$

$$B^k_j = (\Sigma_i A^k_i X^k_i \exp(-\beta^k c^k_{ij}))^{-1} \qquad (24)$$

to ensure that

$$\sum_j Y^k_{ij} = X^k_i \tag{25}$$

$$\sum_i Y^k_{ij} = Z^k_j \tag{26}$$

Equations (25) and (26) together imply that:

$$\sum_i X^k_i = \sum_j Z^k_j \tag{27}$$

To ensure that this constraint is maintained, it is necessary to introduce a dummy demand location with characteristics:

$$X^k_D = \sum_j Z^k_j - \sum_i X^k_i \quad \forall \ k \tag{28}$$

$$c^k_{Dj} = 0 \quad \forall \ j,k \tag{29}$$

$$p^k_D = 0 \quad \forall \ k \tag{30}$$

These dummy sectors are, in effect, 'mountains' of useless surplus production.

Revenues are defined as:

$$D^k_j = \sum_i p^k_i Y^k_{ij} \tag{31}$$

and costs as

$$C^k_j = (r^k_j + v^k_j) * Z^k_j + \sum_i c^k_{ij} Y^k_{ij} \tag{32}$$

This doubly- constrained example becomes our 'MODEL 3', with structure (A11,A21,A31,A41: B4: C2: D3: E2: F1). A suitable solution algorithm comprises the following steps:

A. Fix a pattern of rents and land-use arbitrarily

B. Calculate $\{Y_{ij}^{k}\}$ in (22) - (24), and sum to costs and revenues in (31), (32)

C. Calculate profits by zone and by activity, and transfer land between uses accordingly by (16). Set a new pattern of rents based on profitability across uses by the method detailed below

D. Iterate through B and C until zonal rents and patterns of land-use are stable

As in the previous sections, land is diverted to more profitable uses according to equation (16). In this model the rent adjustment procedure of Step C above is represented as:

$$r_j^{t+1} = r_j^{t} + \varepsilon_2 \Sigma_k (D_j^{k} - C_j^{k}) / \Sigma_k Z_j^{k} \qquad (33)$$

ε_2 is a parameter determining the rate of adjustment of land prices in relation to the profitability of uses. This is more realistic than the assumption of previous sections that rents are only determined once a stable pattern of land-use has been achieved. The rate of adjustment may also have an important influence on the trajectory to equilibrium. While this effect is examined further in the next section, we assume for the present that ε_2 takes a sufficiently small value to generate a smooth trajectory. In this case, the relative magnitudes of ε_1 and ε_3 determine the relative importance of land use and rent in the cobweb mechanism.

A major drawback of the doubly constrained approach is that the solution time required is greatly increased because of the need to iterate through the balancing factor equations (23) and (24) each time the interactions need to be generated. For this reason, the model is only applied to a 13 X 13 zone spatial grid. Assuming the data of Table 4, the land-use pattern of Figure 8 is generated.

7. A MORE COMPLEX EXAMPLE

In this section, we present a version of the attraction-constrained model of Section 5 which has been extended through the introduction of price- dependent consumer demand functions. Thus the initial assumptions of "MODEL 4" are:

A11 - Uniform fertility by good
A21 - Transport rates proportional to euclidean distance
A31 - Uniform production costs by good
A41 - A single, centrally located market

B3 - Attraction- constrained flows

C3 - The model is dynamic

D2 - Rents are set as residual profits

E3 - A fixed proportion of land under cultivation is diverted to the most profitable use in any one time period.

F3 - There is a linear relationship between price and quantity demanded

We assume that the dispersion parameters (β^k) and the cost terms (c^k_{ij}) are both invariant over time, but the interaction term must be modified to take account of the time dependence of the other variables and thus becomes:

$$Y^k_{ij}(t) = A^k_j(t)\, Z^k_j(t)\, W^k_i(t) \exp(-\beta^k c^k_{ij}) \tag{34}$$

with

$$A^k_j(t) = \{ \sum_i W^k_i(t) \exp(-\beta^k c^k_{ij}) \}^{-1} \tag{35}$$

The attractiveness of location i as a market for k, W^k_i, is assumed to be proportional to the money value of sales in that market:

$$W^k_i(t) = p^k_i(t)\, X^k_i(t) \tag{36}$$

The demand for k at i is a function of the delivered price:

$$X_i^k(t) = [\,\Theta_i^k - p_i^k(t)\,] * \bar{X}_i^k \tag{37}$$

where $\bar{X}_i^k$ is a measure of the market size, being the quantity demanded when the delivered price is equal to the 'demand decay' parameter Θ_i^k. Price changes are determined by a supply- demand mechanism:

$$p_i^k(t+1) = p_i^k(t) + \varepsilon_3^k * (\,X_i^k(t) - \sum_j Y_{ij}^k(t)\,) \,/\, X_i^k(t) \tag{38}$$

where ε_3^k is a parameter governing the rate of adjustment of the market.

There is an obvious equilibrium condition that quantity demanded be equal to the quantity supplied:

$$X_i^k(t) = \sum_j Y_{ij}^k(t) \tag{39}$$

Suppliers costs and revenues are calculated in the usual way:

$$D_j^k(t) = \sum_i p_i^k(t)\, Y_{ij}^k(t) \tag{40}$$

and

$$C_j^k(t) = v_j^k\, Z_j^k(t) + \sum_i c_{ij}^k\, Y_{ij}^k(t) \tag{41}$$

Land-use changes between time-periods according to profitability i.e. through equations (14) - (16) of Section 4 (which are repeated here for convenience) :

$$E_j^k(t) = D_j^k(t) - C_j^k(t) \tag{14'}$$

$$Z_j^k(t+1) = Z_j^k(t) + \sum_{s \neq k} d_j^{sk}(t)\, \varepsilon_1^s\, q^s\, Z_j^s(t) \,/\, q^k \tag{15'}$$

where

$$d_j^{sk}(t) = \begin{cases} 1 & \text{if } E_j^k(t) > E_j^v(t) \ \forall \ v \neq k \\ -1 & \text{if } E_j^s(t) > E_j^v(t) \ \forall \ v \neq s \\ 0 & \text{otherwise} \end{cases} \qquad (16)$$

Here the equilibrium conditions require that there be no changes in land-use:

$$d_j^{sk}(t) = 0 \qquad \forall \ s, k, j \qquad (42)$$

We begin by setting the demand levels, $\{\bar{X}_i^k\}$, and the various parameter values, and set arbitrary starting values for the land-use mix $\{Z_j^k\}$, the delivered prices $\{p_i^k\}$ and hence the quantities demanded $\{X_i^k\}$. Each iteration then comprises the following steps:

A. Calculate market attractiveness in (35) and hence the interactions in (33) and (34)

B. Sum the interactions at the demand end and test for equilibrium in (38). If equilibrium has not been achieved, reset prices from (37)

C. Sum the interactions at the supply end and test for equilibrium in (41). If equilibrium has not been achieved, reset activity levels in (15).

The process may be repeated until convergence is achieved (i.e. all the equilibrium conditions are satisfied) or for a specified number of iterations/ time periods.

The operation of the model is now investigated for a variety of different backcloths. We introduce the assumption of nine market centres of equal size ($\bar{X}_i^k$ = 500. $\forall$ i,k) as shown in Figure 9A; and fertility increasing from the 'north-west' to the 'south-east' of the region, as in Figure 9B. The marriage of these two new scenarios with the original cases yields four different possibilities. In Figure 10A, we have constant fertility and a single, central market location as previously. Figure 10B retains the single market in conjunction with varying fertility, and we see clearly a corresponding distortion in the concentric rings.

Figure 10C illustrates the situation when nine market centres are to be supplied from land with constant fertility. Here we see the distortion of the concentric rings in accordance with the new configuration of demand, along with the introduction of outliers of production for good 3 to supply the peripheral markets. Notice also the absence of activity 2 in the outlying areas. This situation is the result of a complex dynamic in the evolution of land-use, essentially arising because on the one side demand in the central area for good 1 allows this activity to outbid activity 2, while towards the peripheral markets activity 3 generates the highest bid rents because of its lower variable costs.

The situation in Figure 10D, where variable fertility and multiple market areas are combined, is rather different once more. Here the main focus of interest is again in the activity patterns around the peripheral markets. In the north-west we see that low fertility denies the potential for profitable cultivation despite the presence of a local market. In the north-east and south-west rather higher fertility appears to have extended the comparative advantage of good 3 over the whole area of profitable land-use. Finally, in the south-east the highest fertility implies that activity 2 is now able to outbid activity 3 through its lower transport costs.

8. MODEL DYNAMICS

The point has already been made (in Section 3) that dynamics may be incorporated through the behaviour of each of the three sets of actors - land-owners, producers and consumers. In Sections 4-7 a series of models were considered with either one or two dynamic actors, but the parameters of growth were sufficiently small to generate a smooth path to equilibrium.

Let us return now to the model of Section 7. Suppose we assume a fixed rate of land use adjustment, such that in a given period only one tenth of the land in a zone may be transferred between uses ($\varepsilon_2^* =$ 0.1). What will now be the effect of price adjustment in relation to such a pattern of land use change? If we initially take $\varepsilon_3^* = 0.01$, a stable equilibrium pattern emerges, which is illustrated for a radius of the system in Figure 11.

Taking a value of $\varepsilon_3^* = 0.1$, however, the stability of the system collapses and oscillations set in as demonstrated in Figure 12. It appears, therefore, that somewhere between $\varepsilon_3^* = 0.01$ and $\varepsilon_3^* = 0.1$ there exists a critical value (for $\varepsilon_2^* = 0.1$) below which the transition to equilibrium is smooth and above which some kind of periodic behaviour sets in.

A third element may be introduced to the cobweb if we allow rents to be specified actively. Using the technique employed in Model 3 above, this implies the application of an equation of the form

$$r_j(t+1) = r_j(t) + \varepsilon_1^* \sum_k (D_j^k(t) - C_j^k(t)) / \sum_k Z_j^k(t) \qquad (33)'$$

at each round of the iterative procedure. The cost equations must also be modified to include a land rent term

$$C^k_j(t) = (r_j + v^k_j)\, Z^k_j(t) + \sum_i c^k_{ij}\, Y^k_{ij}(t) \qquad (41)'$$

Interestingly, the value of the parameter ε^*_1 has no influence over a wide range of values. With $\varepsilon^*_2 = 0.1$ and $\varepsilon^*_3 = 0.01$, the model is convergent for values of $\varepsilon^*_1 = 0.1$ and $\varepsilon^*_1 = 1.0$, the latter implying that all profits are instantaneously absorbed as rents.

9. CONCLUDING COMMENTS

We have shown that a new method for dynamic model building, originally intoduced in relation to services, can be applied to the traditional problem of agricultural location. This forces a close scrutiny of the roles of prices in such models (both of goods and land) and this, together with other choices which have to be made in relation to spatial interaction hypotheses, enables a wide variety of models to be generated. The results of the paper should therefore be seen as a deeper-than-hitherto theoretical analysis and a model-building 'kit'. The numerical experiments show that a suitable variety of results can be generated. What is now needed is a set of empirical tests.

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TABLE 1: A FRAMEWORK FOR MODEL CONSTRUCTION

A. Definition of the Spatial Backcloth

A1. Patterns of fertility or productivity
- A11 - spatially uniform
- A12 - location-specific

A2. Structure of transportation charges
- A21 - independent of load and distance
- A22 - varying with quantities or distance

A3. Variation in production costs
- A31 - independent of location and output
- A32 - dependent on location only
- A33 - dependent on level of output only
- A34 - dependent on both location and output levels

A4. The number and location of markets
- A41 - single (usually central) market
- A42 - two markets
- A43 - a large number of markets

B. Model Flow Patterns

B1. Unconstrained
B2. Production-constrained
B3. Attraction-constrained
B4. Doubly-constrained

C. Temporal Context

C1. Static
C2. Comparative static
C3. Dynamic

D. Determination of Land Rents

D1. Exogenously specified
D2. Internal and passive
D3. Internal and active

E. Determination of Land Use

E1. Exogenously specified
E2. Internal and passive
E3. Internal and active

F. The Price Mechanism

F1. Exogenously specified
F2. Internal and passive
F3. Internal and active

TABLE 2: PARAMETER VALUES FOR PRODUCTION-CONSTRAINED MODEL

	Activity 1	Activity 2	Activity 3
X^k	5000	5000	5000
v^k	0.5	0.5	0.5
p^k	1.0	1.0	1.0
t^k	0.10	0.12	0.14
$\bar{L}$	5.0	5.0	5.0

$$c^k_{ij} = t^k d_{ij}$$

TABLE 3: PARAMETER VALUES FOR ATTRACTION-CONSTRAINED MODEL

	Activity 1	Activity 2	Activity 3
X^k	5000	5000	5000
v^k	0.5	0.4	0.3
p^k	1.0	1.0	1.0
t^k	0.15	0.20	0.30

$\bar{L} = 5.0$

$$c^k_{ij} = t^k d_{ij}$$

TABLE 4: DATA FOR A DOUBLY-CONSTRAINED SPATIAL INTERACTION MODEL

	Activity 1	Activity 2	Activity 3
X^k	500	500	500
v^k	0.60	0.62	0.66
p^k	1.0	1.0	1.0
t^k	0.04	0.03	0.02

k

$$c^k_{ij} = t^k d_{ij}$$

$\bar{L} = 30.0$; $\varepsilon^*_2 = 0.1$

FIGURE 1A: CHANGING BID RENTS WITH DISTANCE

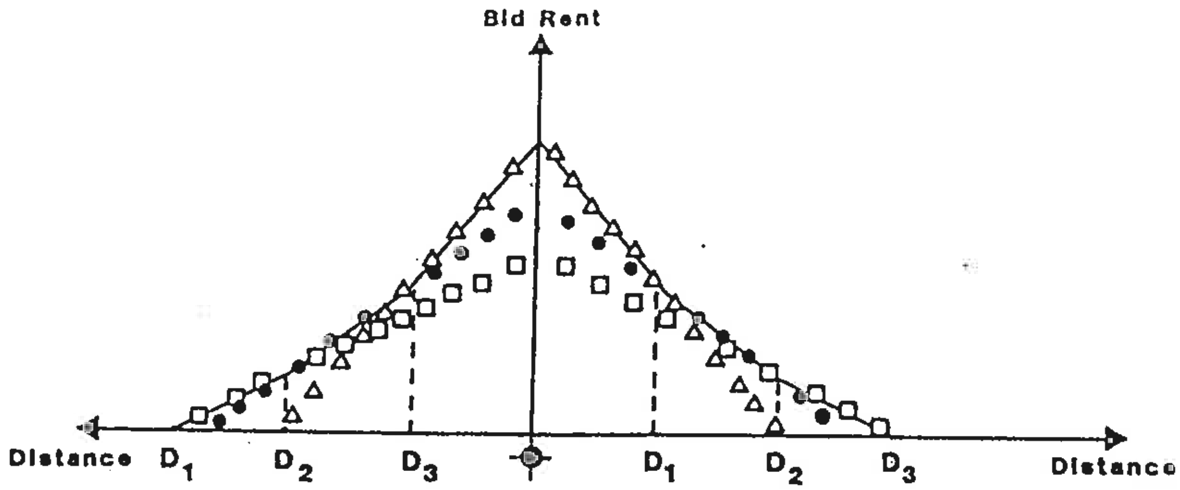


FIGURE 1B: CONCENTRIC RINGS OF LAND USE

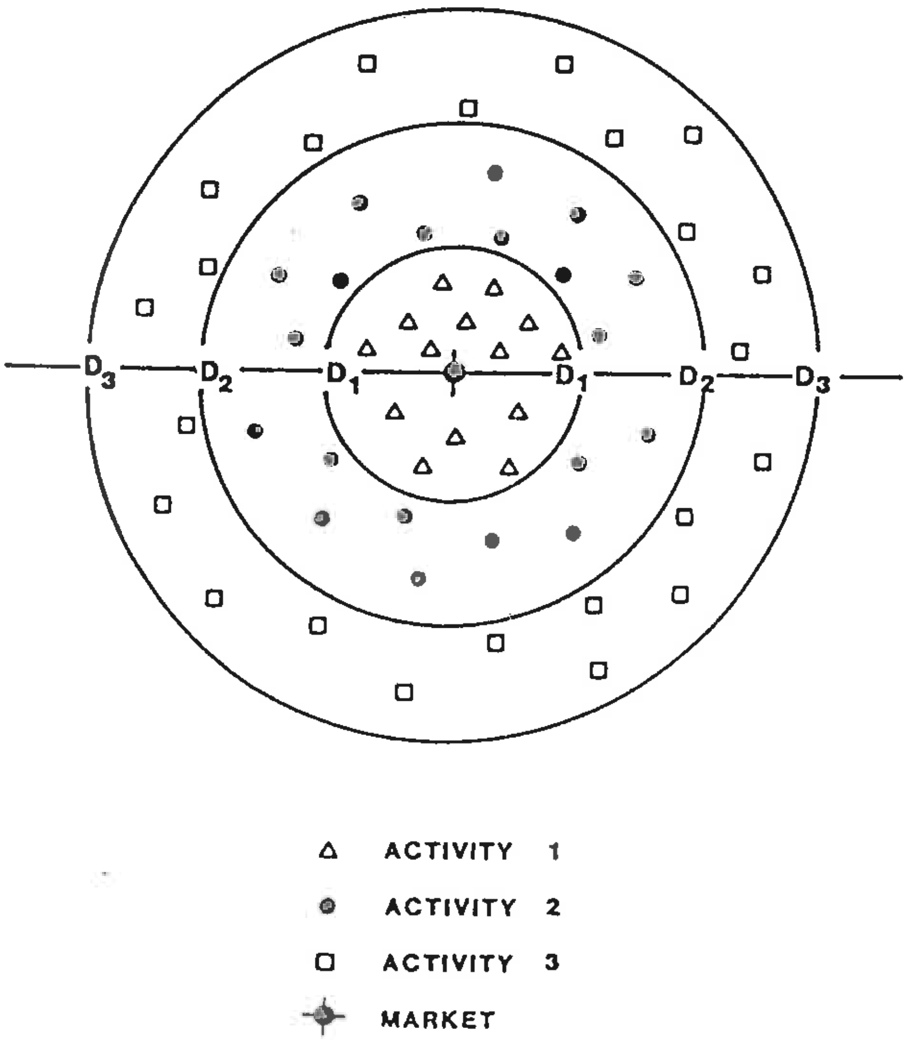


FIGURE 2: THE COBWEB THEOREM

Figure 2A: Stable and convergent

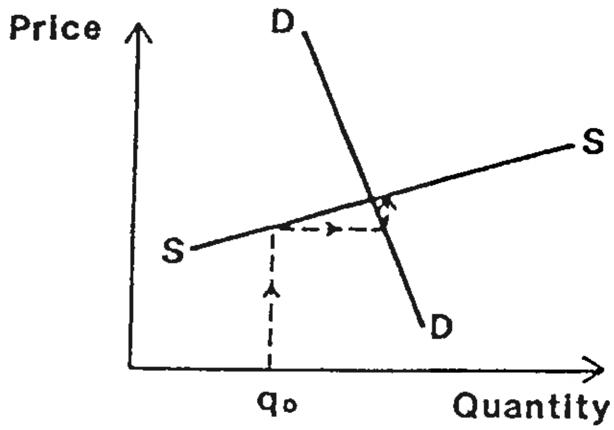


Figure 2B: Unstable and divergent

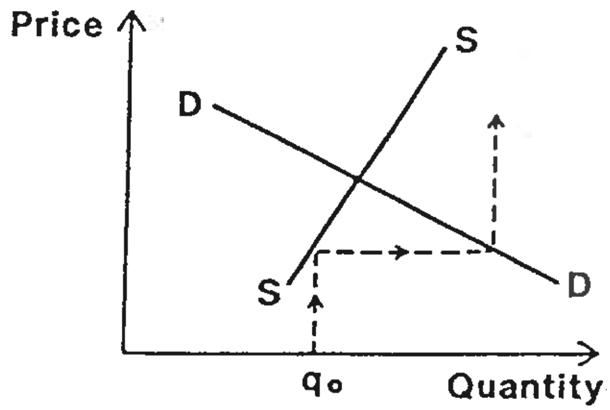


Figure 2C: Modified divergent case

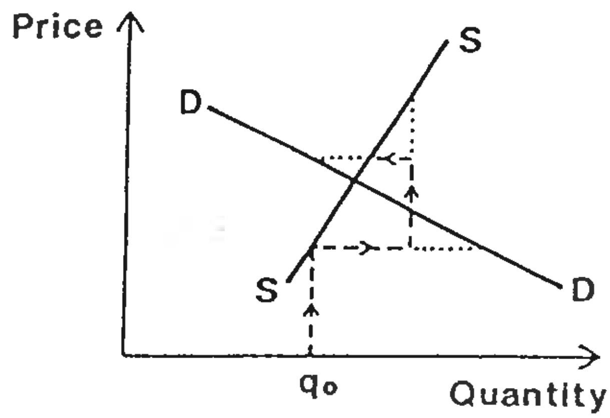
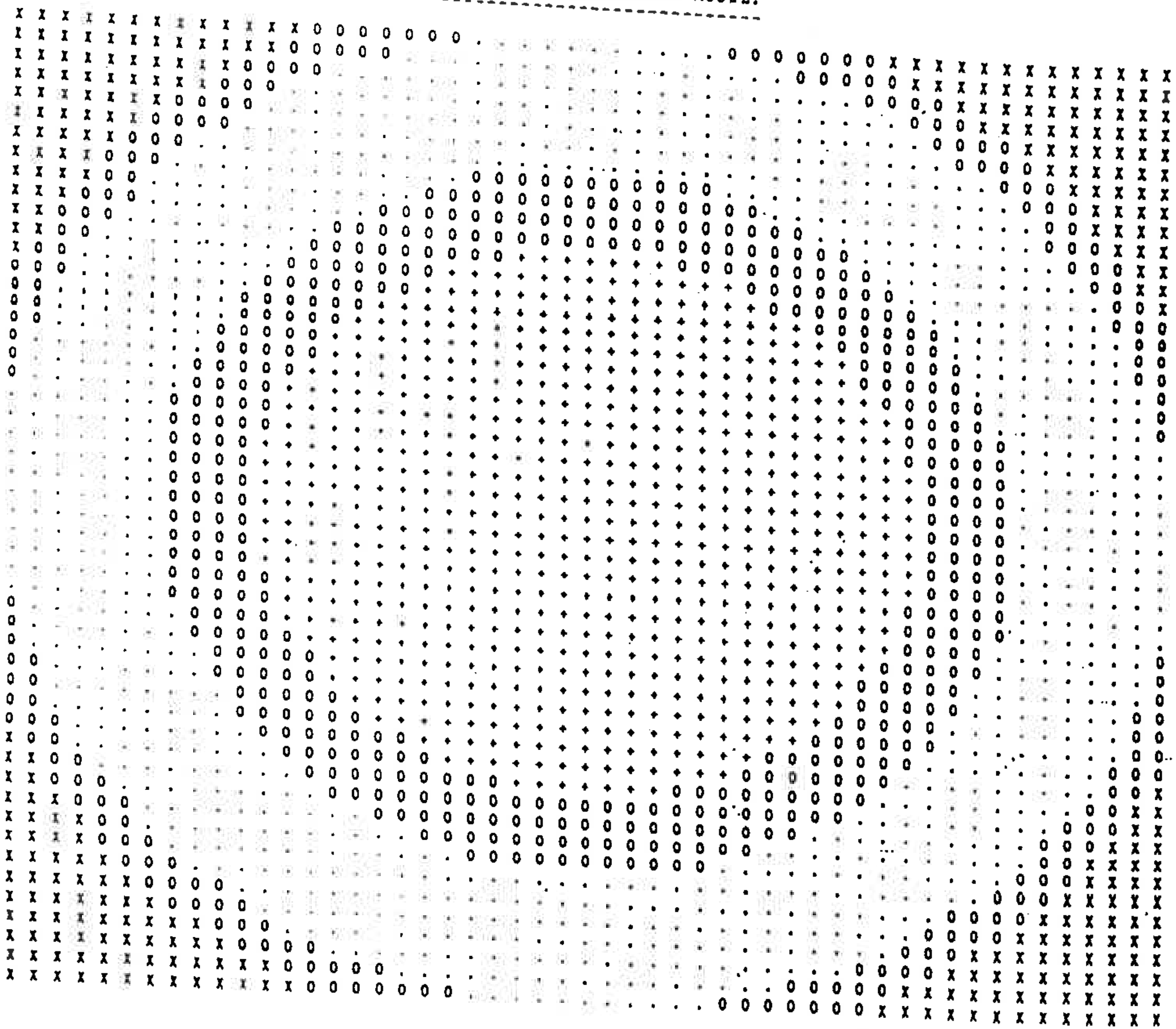


FIGURE 3: LAND-USE PATTERN UNDER PRODUCTION-CONSTRAINED MODEL.



KEY TO ACTIVITIES:

- . ONE (HIGHEST BID RENT)
- 0 TWO
- + THREE (LOWEST BID RENT)
- X FOUR (NO CULTIVATION)

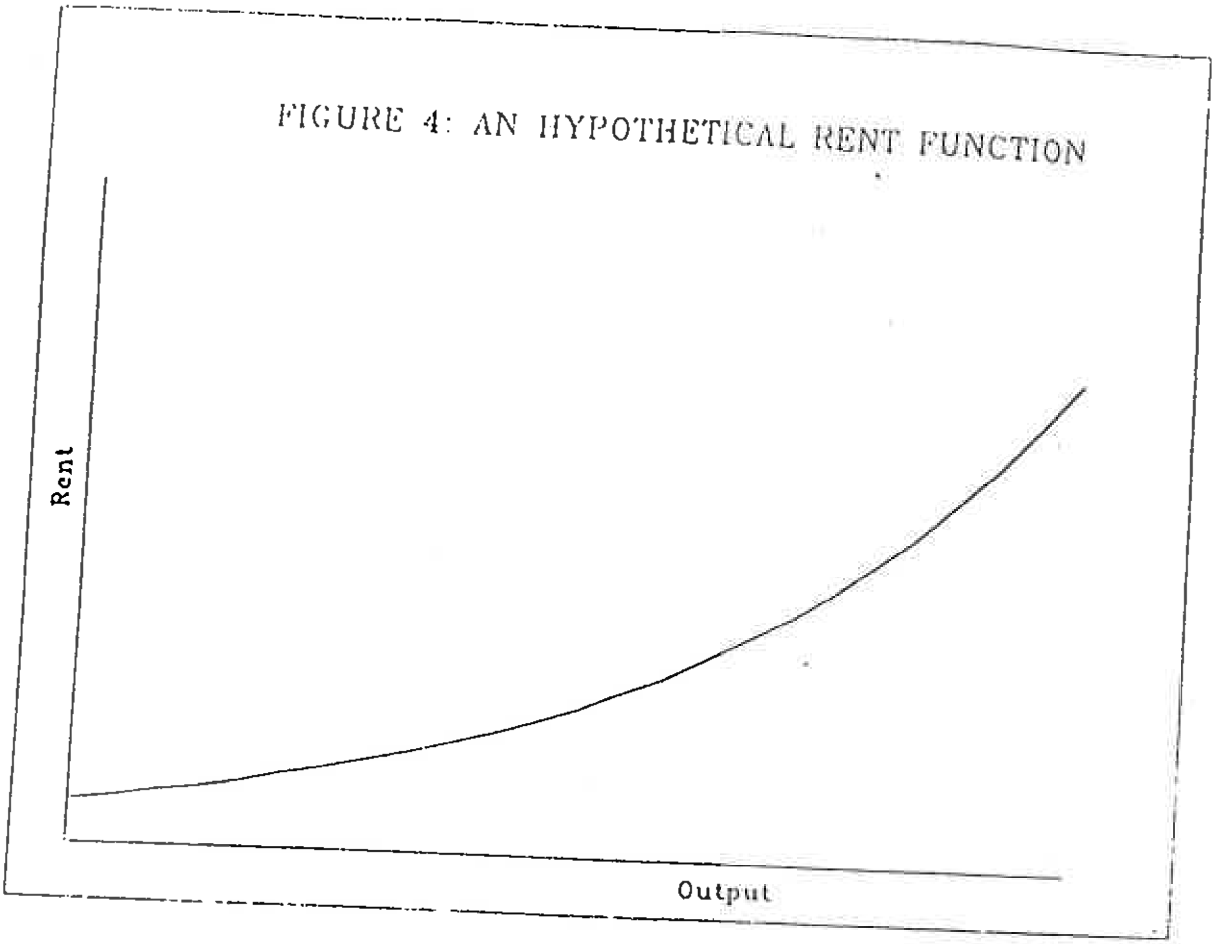


FIGURE 4: AN HYPOTHETICAL RENT FUNCTION

FIGURE 5A: CONCENTRIC RING STRUCTURE.

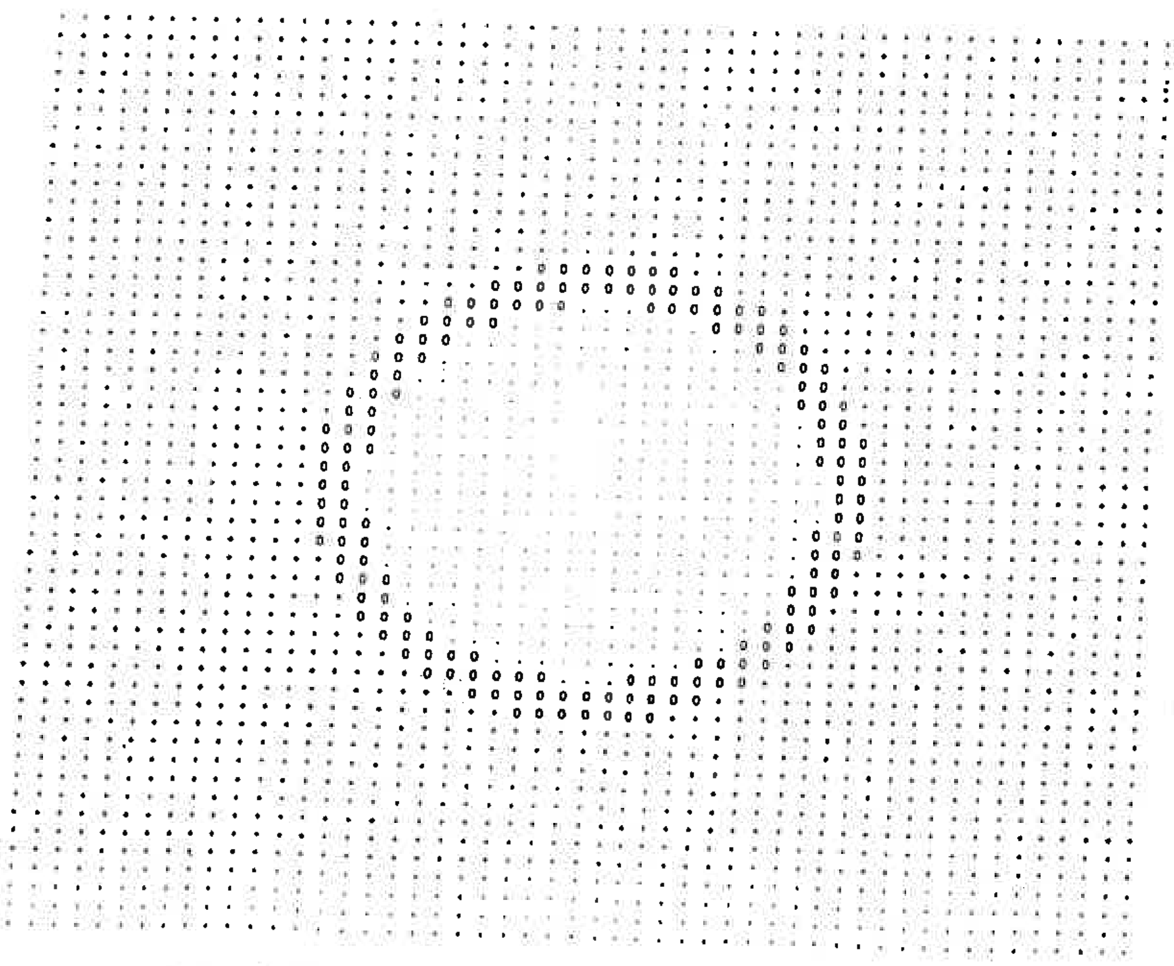


FIGURE 5B: RENT CROSS-SECTION FOR FIGURE 5A

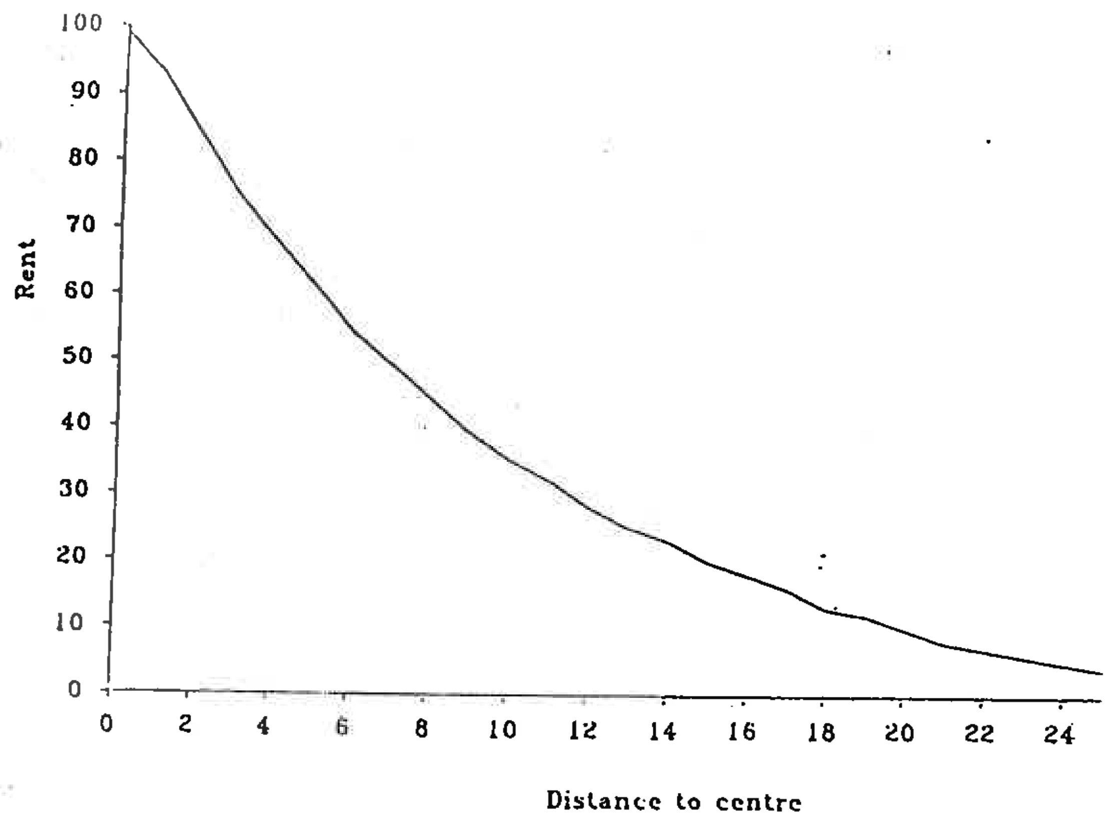


FIGURE 5C: RING STRUCTURE WITH LOW COST CORRIDOR

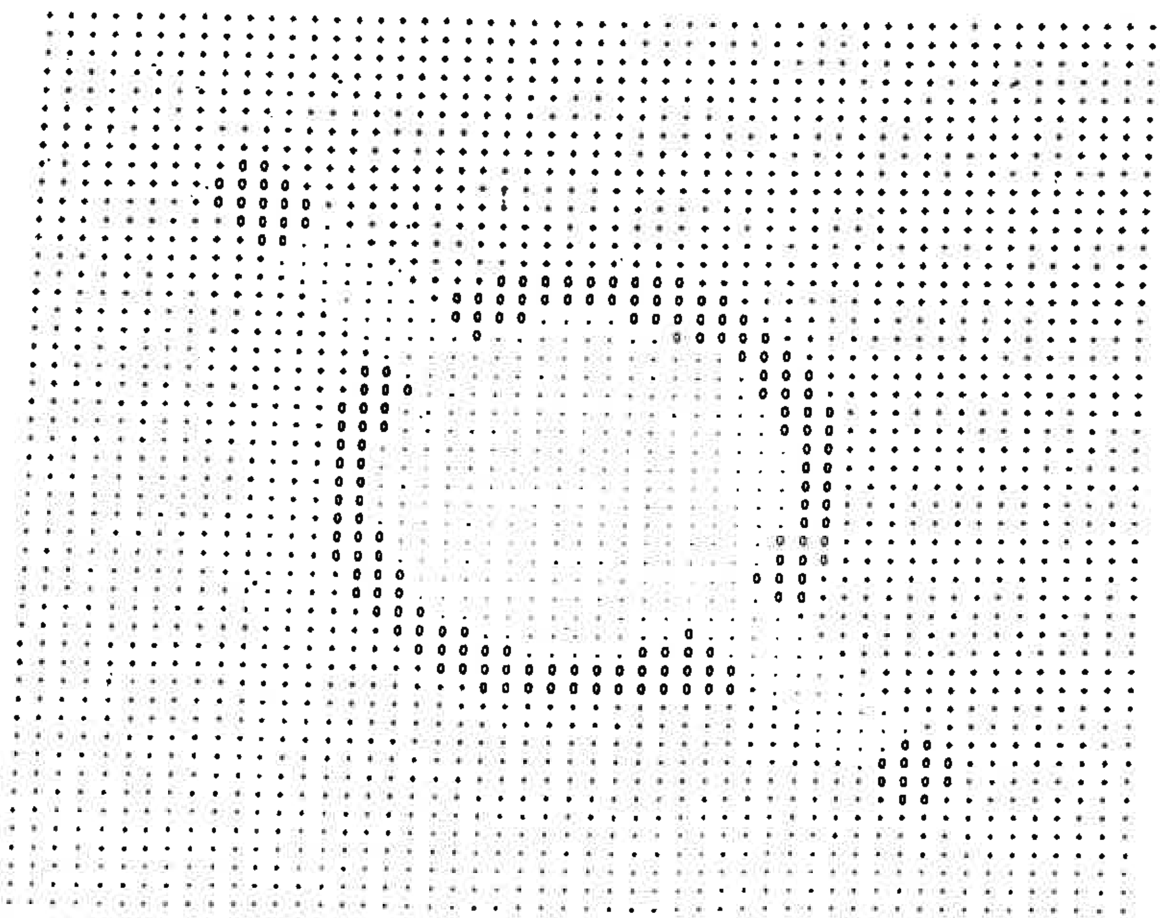


FIGURE 6A: ATTRACTION CONSTRAINED MODEL WITH TABLE 2 VALUES

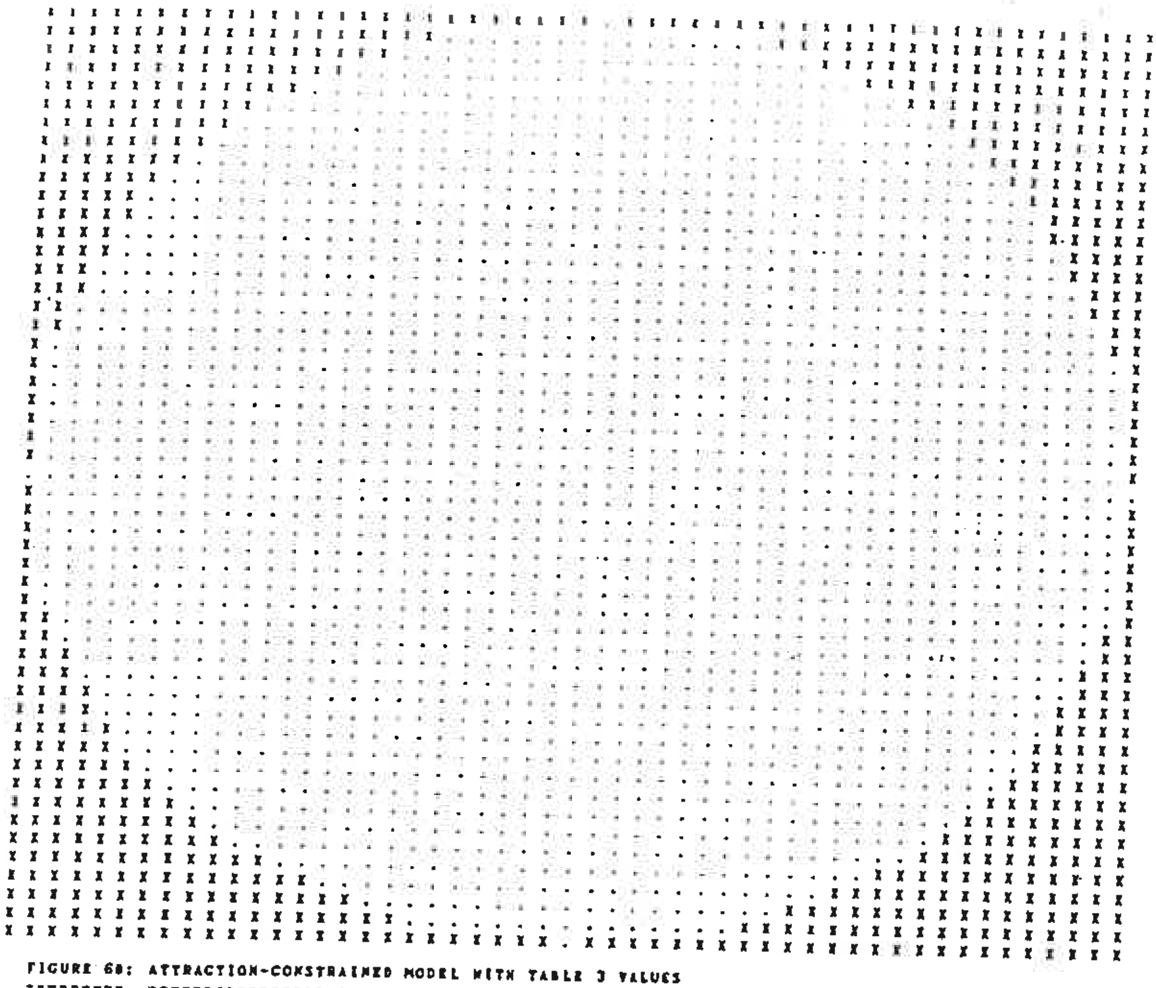


FIGURE 6B: ATTRACTION-CONSTRAINED MODEL WITH TABLE 3 VALUES

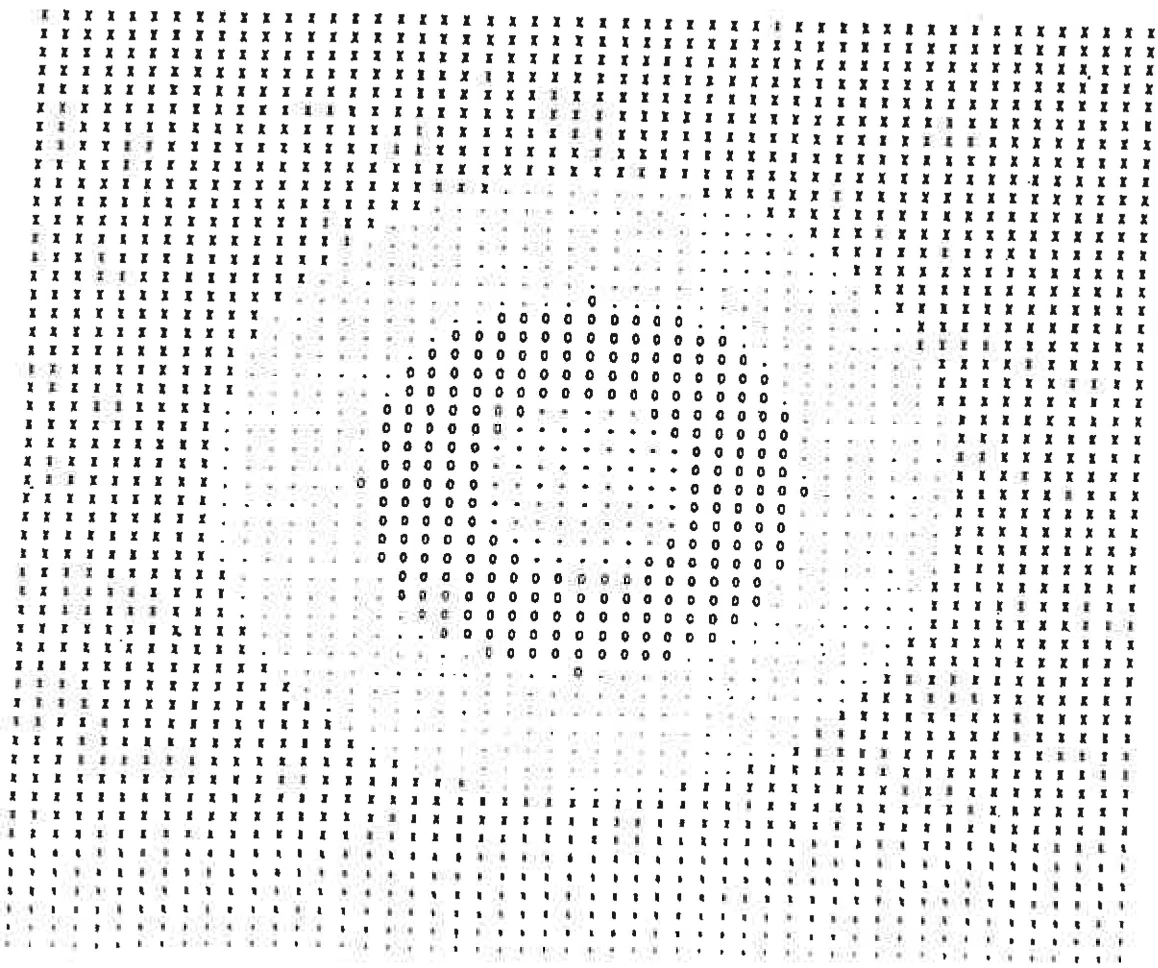


FIGURE [illegible]: LAND-USE WITH TWO MARKET CENTRES (B=[illegible])

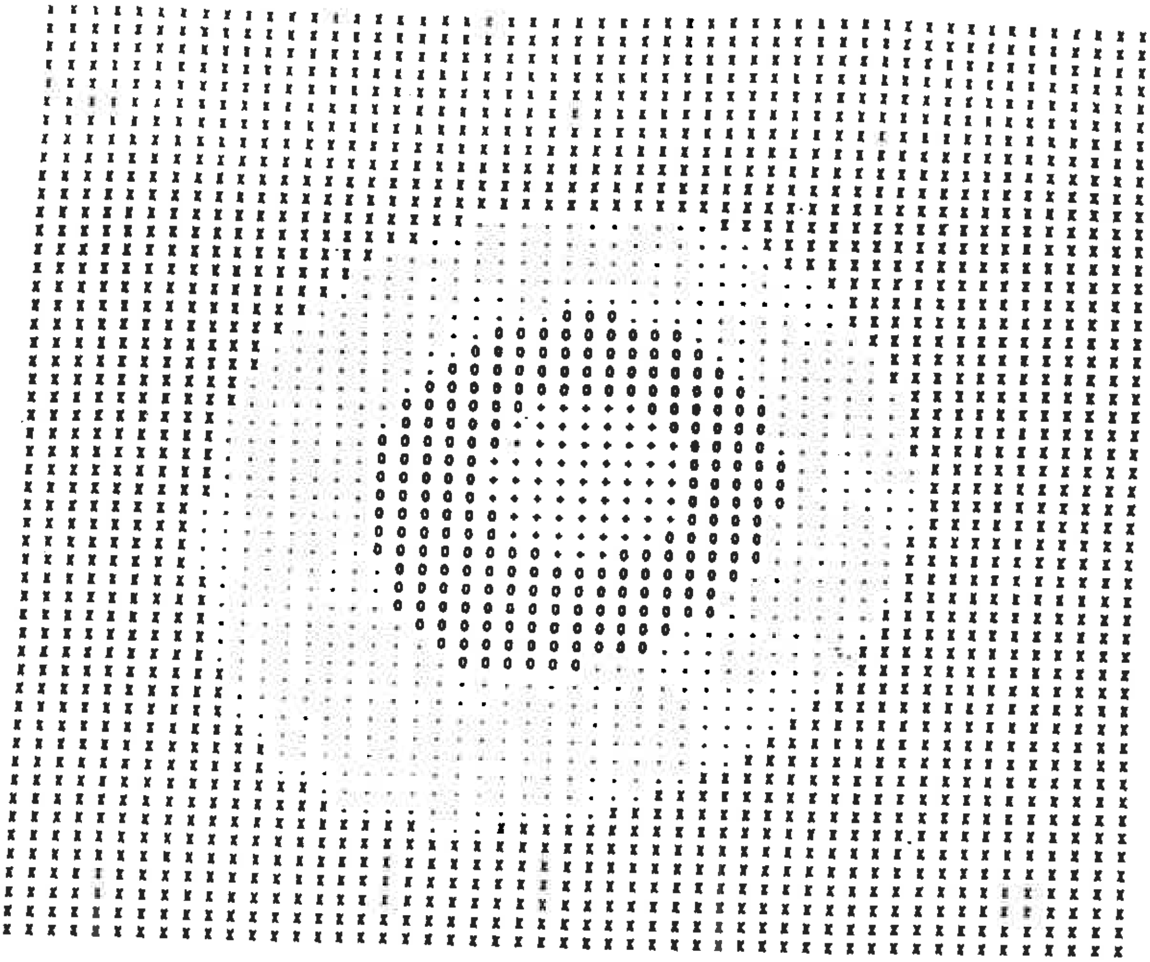


FIGURE 7B: LAND-USE WITH TWO MARKET CENTRES (B=5)

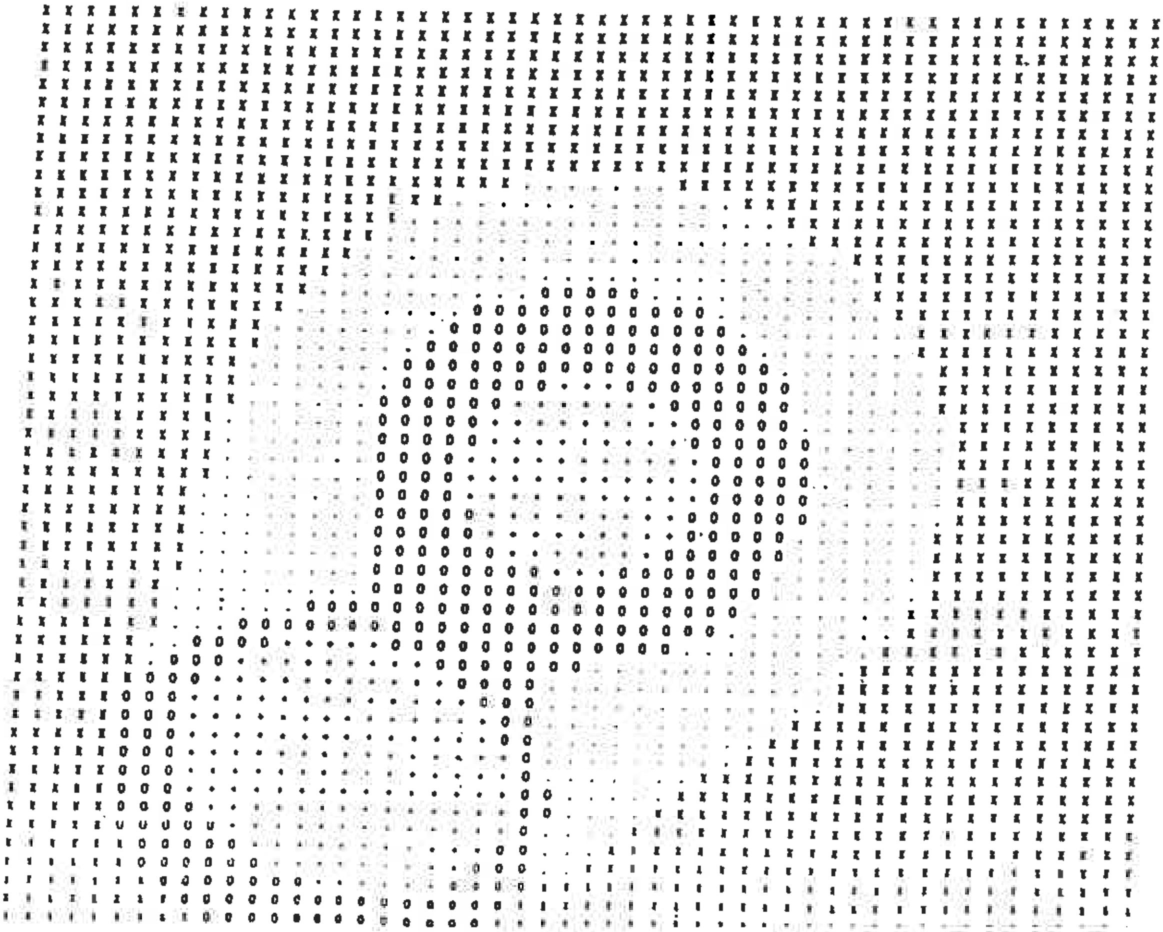


FIGURE 8: CONCENTRIC RINGS IN A DOUBLY-CONSTRAINED MODEL

```
X X X X X X X X X X X X X
X X X X X X X X X X X X X
X X X X X . . . X X X X X
X X X X . 0 0 0 . X X X X
X X X . 0 + + + 0 . X X X
X X . 0 + + + + + 0 . X X
X X . 0 + + + + + 0 . X X
X X . 0 + + + + + 0 . X X
X X X . 0 + + + 0 . X X X
X X X X . 0 0 0 . X X X X
X X X X X . . . X X X X X
X X X X X X X X X X X X X
X X X X X X X X X X X X X
```

KEY TO ACTIVITIES:

```
.    ONE
0    TWO
+    THREE
X    FOUR
```

FIGURE 9A LOCATION OF NINE MARKET CENTRES

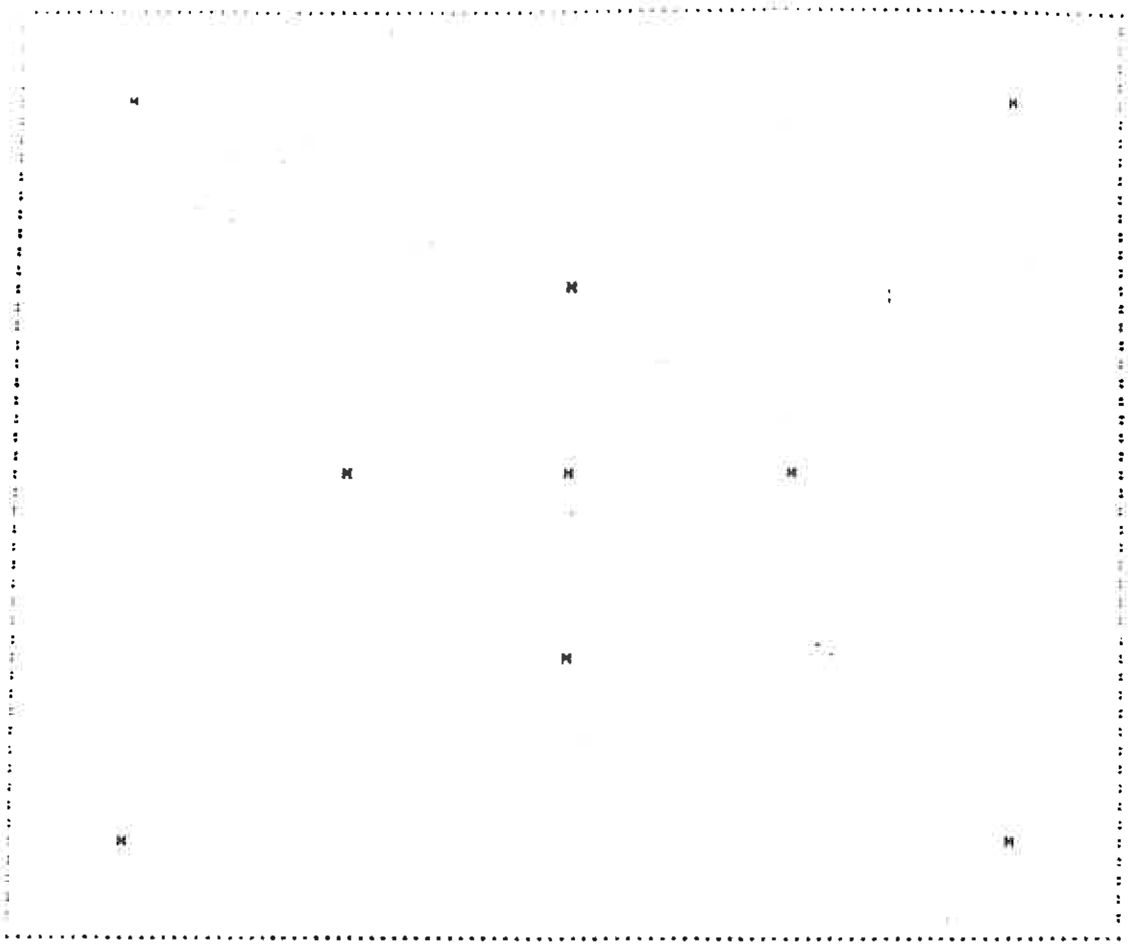


FIGURE 9B: RELATIVE PRODUCTIVITY (OUTPUT PER UNIT COST) OF LAND WITH VARIABLE FERTILITY

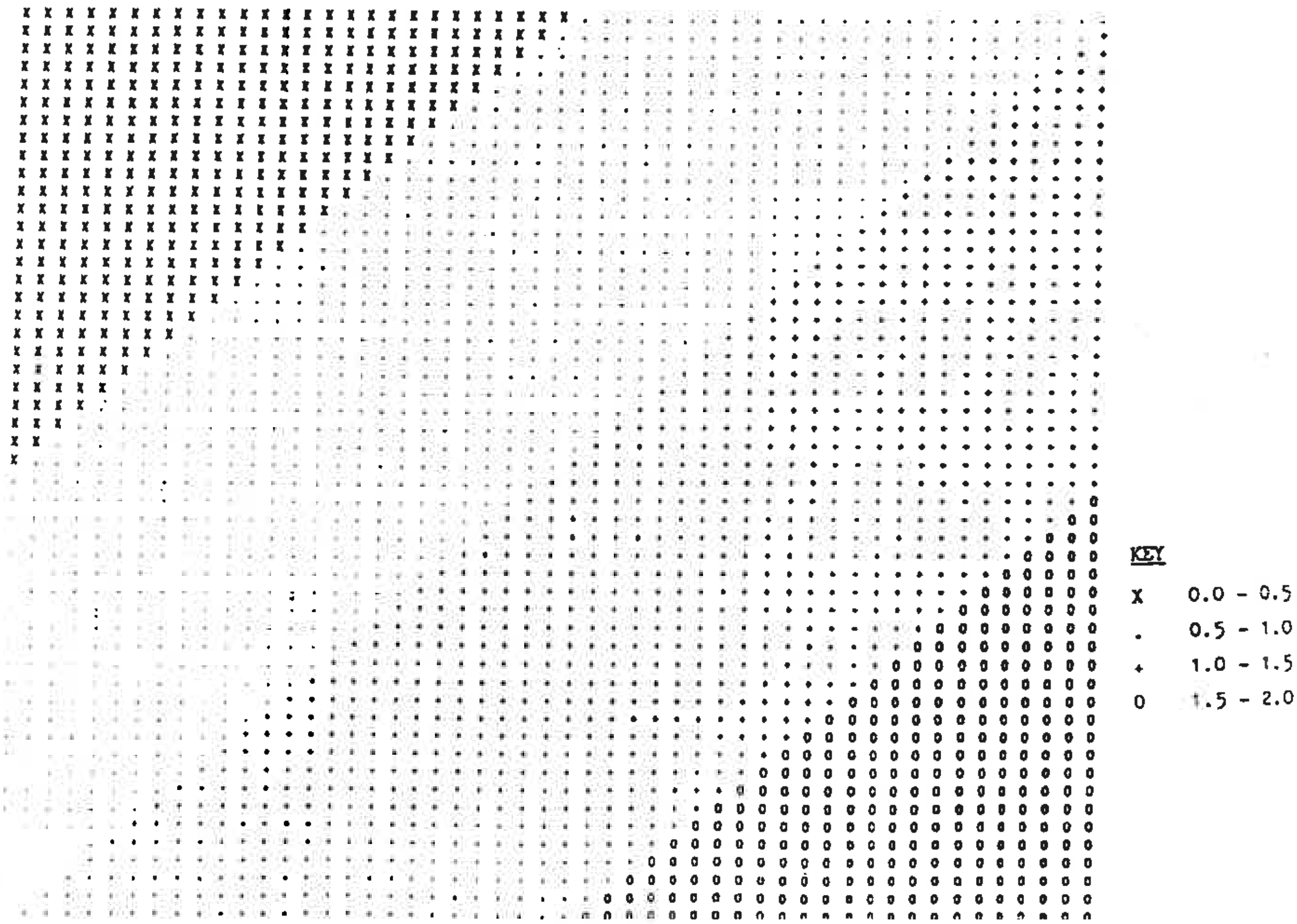


FIGURE 10A: LAND-USE WITH CONSTANT FERTILITY AND A SINGLE MARKET CENTRE

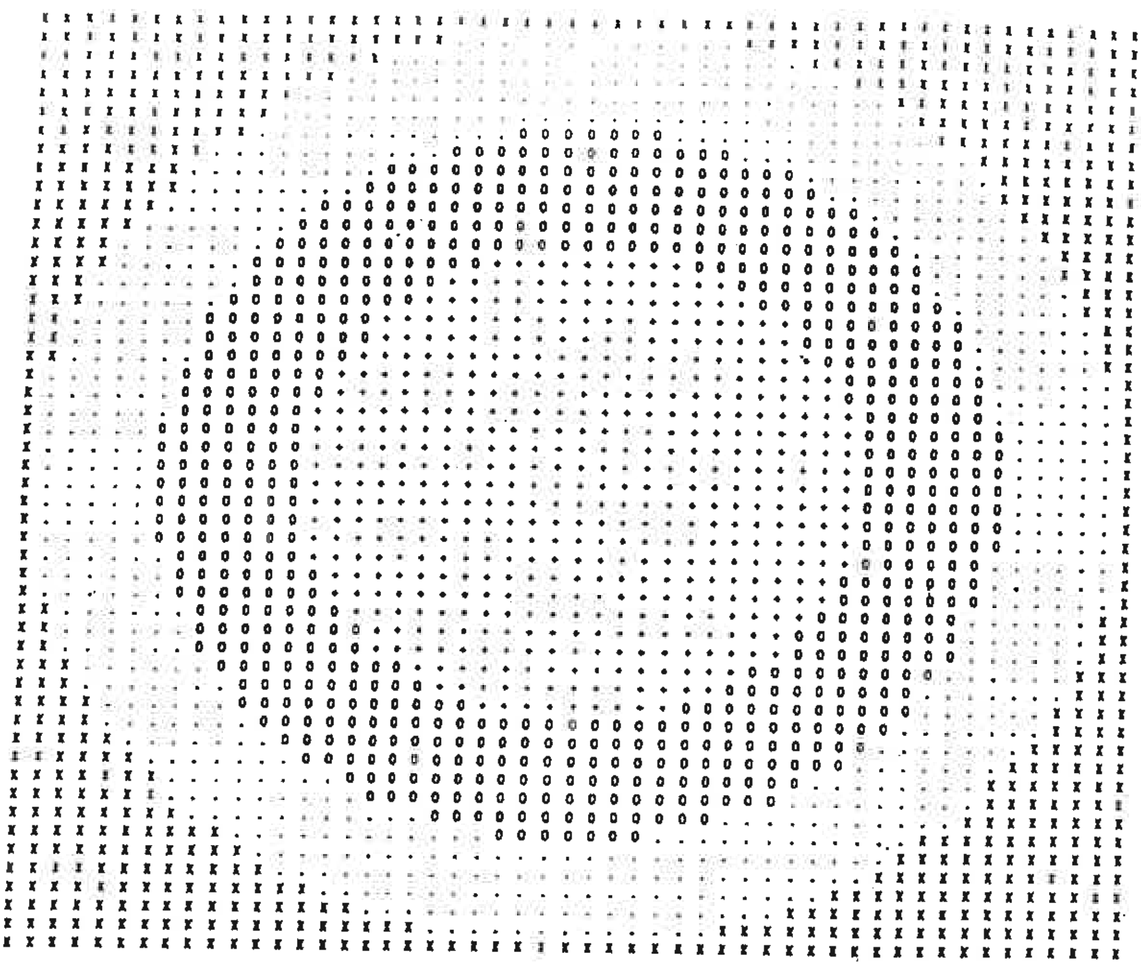


FIGURE 10B: LAND-USE WITH VARIABLE FERTILITY AND A SINGLE MARKET CENTRE

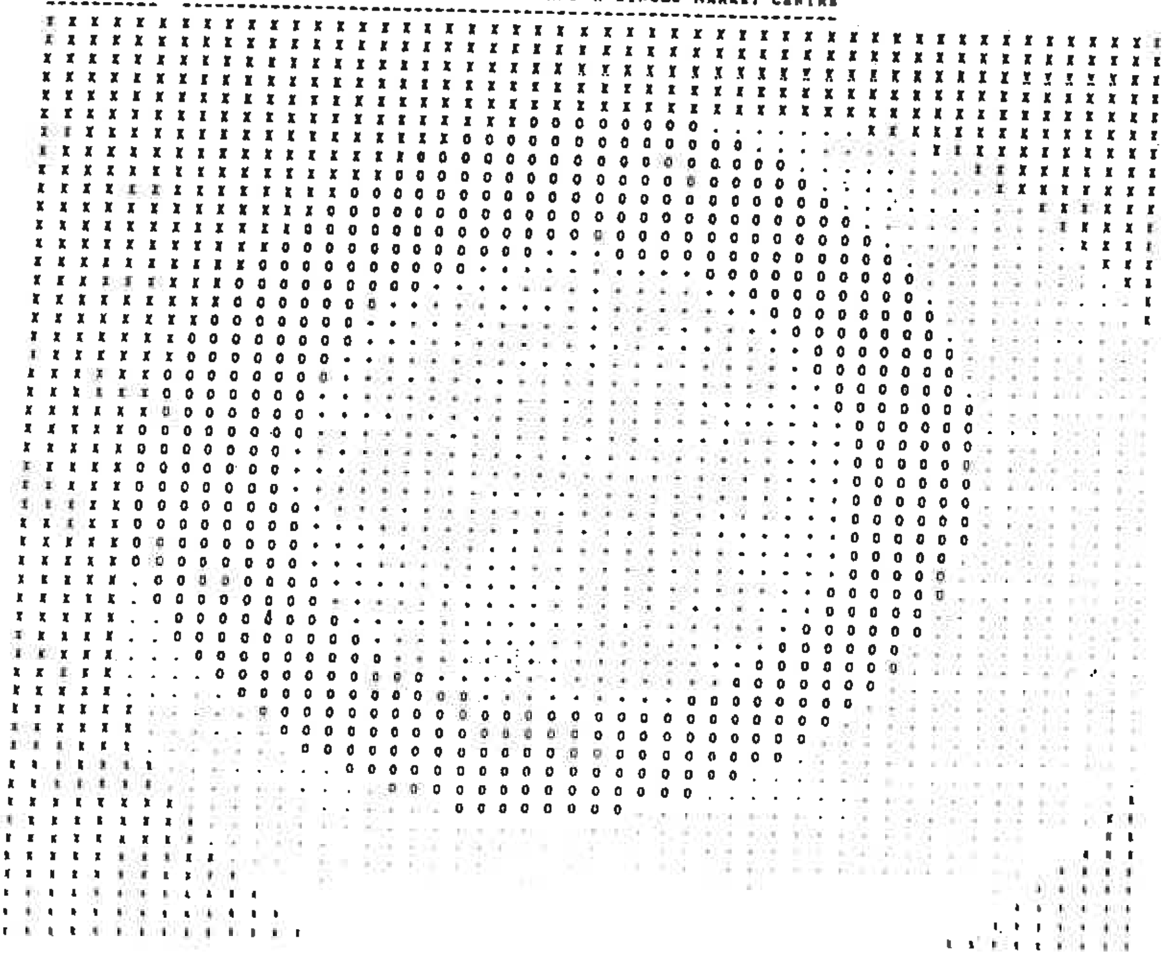


FIGURE 10C. LAND-USE WITH CONSTANT FERTILITY AND NINE MARKET CENTRES

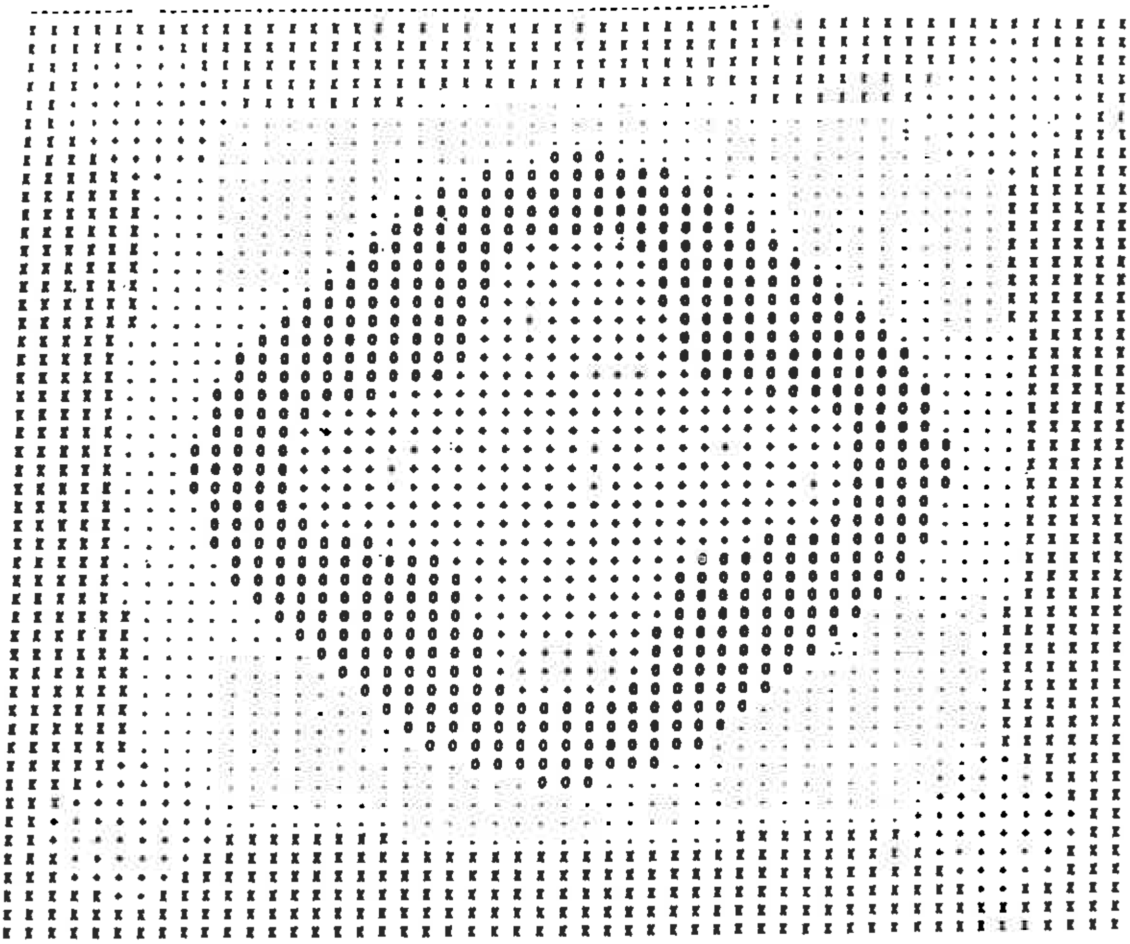


FIGURE 10D: LAND-USE WITH VARIABLE FERTILITY AND NINE MARKET CENTRES

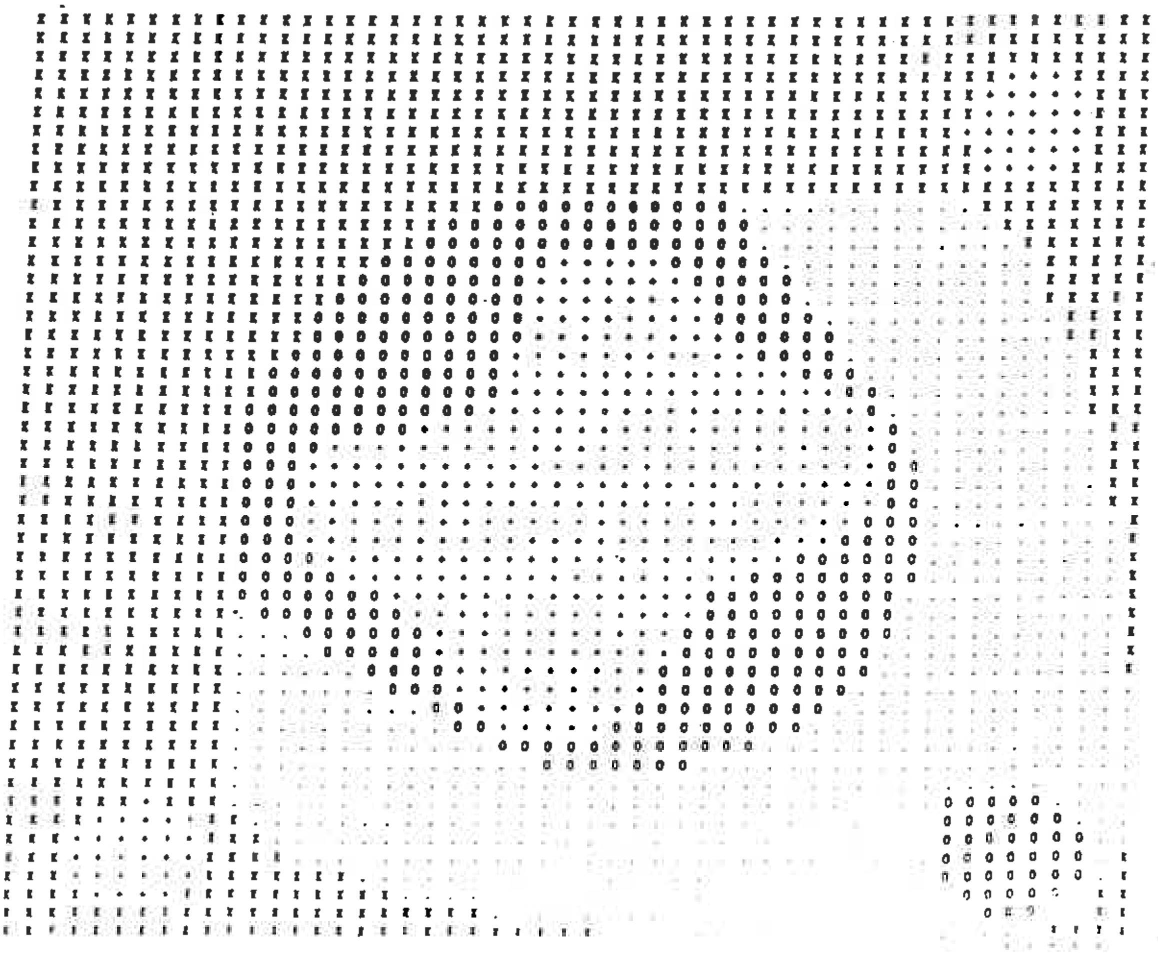


FIGURE 11: Radial development in convergent case

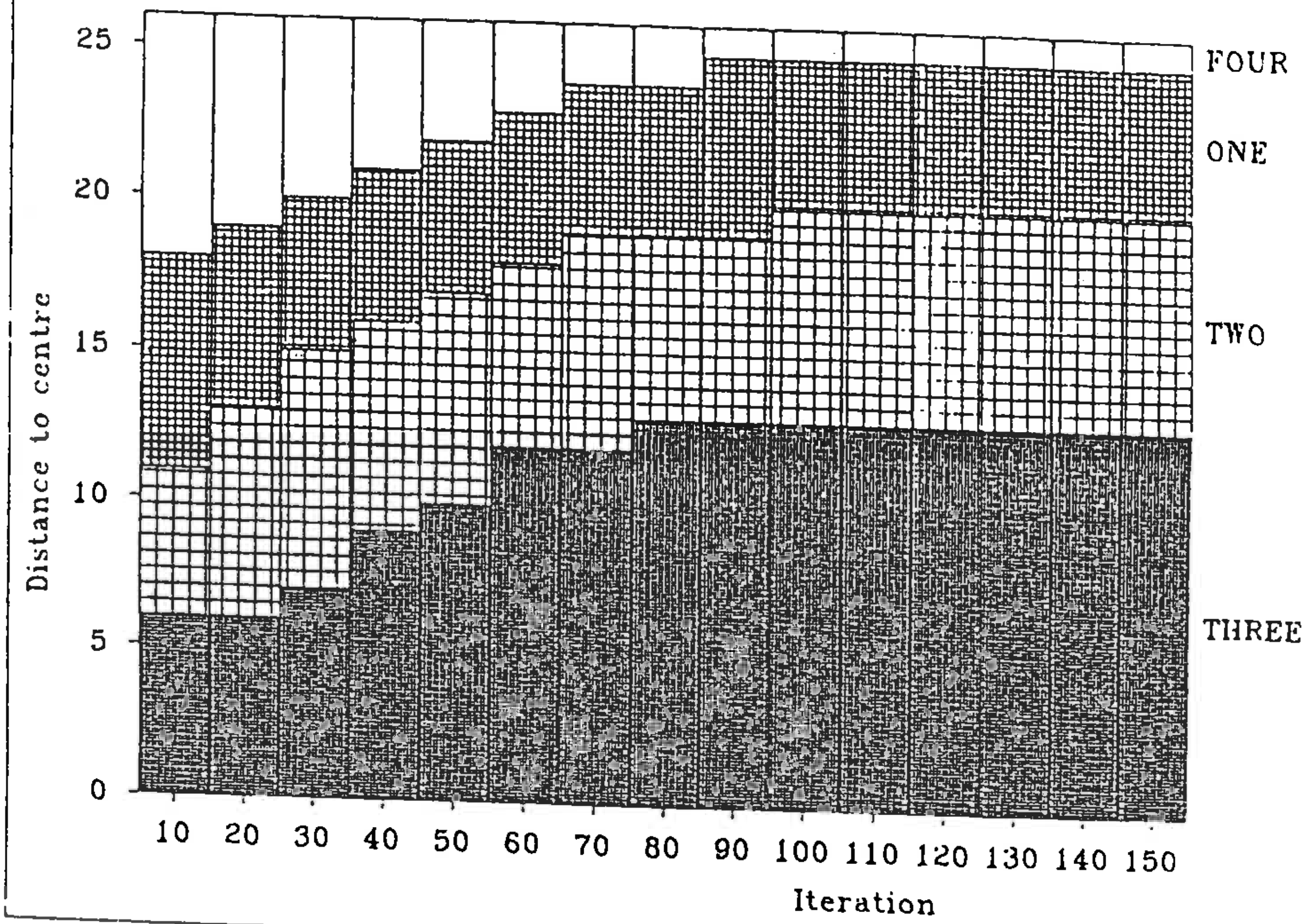


FIGURE 12: Radial development in non-convergent case