Working Paper 148

MODELLING THE REGIONAL SYSTEM: THE POPULATION COMPONENT

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Paper to be presented at the Second Franco-British Geographical Conference, Lyons, France, July, 1976

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May, 1976

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Abstract

The paper presents a linked set of analyses that model the population component of the regional system. Aggregate population accounts are presented for British standard regions for 1965-66 and 1970-71. A transition rates matrix is derived from the 1970-71 accounts, transformed into a growth rates matrix and used in a model that forecasts the population of the regions to 2001. The drawbacks and the strengths of the forecast are discussed.

MODELLING THE REGIONAL SYSTEM: THE POPULATION COMPONENT

P.H. REES

1. Definition of terms

The first part of the title of this paper, 'Modelling the regional system', immediately conjures up in the mind of the reader a vast array of possible topics and models ranging from land use-transportation models through to ecosystem models. The second part of the title is meant to deflate these expectations and to narrow the discussion to a concern with population, that is, population change, population accounting and population forecasting. The regional system which will be used to illustrate the discussion will be that of the standard regions of Britain.

The principal aim of the paper is to sketch out a linked set of analyses and models that describe and attempt to forecast population change in a set of regions. In order to accomplish this aim in the short space available the focus is narrowed to that of the aggregate population only and only a small subset of the many possible models are outlined. The set of analyses described in the paper thus form a prototype for a more extensive investigation of the changing British population (Rees, 1974, 1975).

The structure of the paper is as follows. In the next section, the framework of spatial population accounting is briefly reviewed. Population accounts for British regions for 1965-66 and 1970-71 are then presented in section 3, and the information they provide about the direction and pace of change in regional populations analyzed. Absolute measures of change are converted into relative measures through the computation of rates of birth, death, migration, survival and so on in section 4. Also generated in that section is the matrix of transition rates associated with the accounts.

Change in this matrix from 1965-66 to 1970-71 is examined. Then the matrix of growth rates of the regional population system is calculated from the transition rates matrix and from other information in the accounts.

The $\underline{G}$ matrix for 1970-71 is then used (section 5) in a model, first developed by Rogers (1966) for forecasting regional population and inter-regional flows, and results for a preliminary run of this model to 2001 are presented. These results are compared with official forecasts, and modifications to the initial model suggested. Criticisms of this model leads to proposals for the use of the alternative accounts based model in which easy to incorporate recent trends in birth rates, death rates and migration rates if available. A number of conclusions are reached about the appropriate strategy for modelling the demographic component of the regional system in this final section (section 8).

2. The theory of spatial demographic accounting

Accounting methods were first applied to demographic problems by Stone (1965, 1971a, 1971b) in a spatially aggregate form that recognised just two 'regions': 'our country' and 'the outside world'. Demographic accounts were first expressed in a spatially explicit form in Rees (1972) and the underlying concepts explored in Rees and Wilson (1973), Wilson and Rees (1974), and Rees and Wilson (1975) with a full statement appearing in Rees and Wilson (1976). Alternative perspectives on demographic accounting are provided in Rees (1975b, 1975c). Here a very brief review is given and the reader is referred to the works cited for fuller details.

We can define a matrix $\underline{K}$ with elements $K^{\alpha(i)\omega(j)}$ to be a matrix of population flows over a period of time (measured by numbers of people involved) between a set of 'origin' states represented by the rows and 'destination' states represented by the columns. The superscript $\alpha(i)$

attached to the K variable refer to 'origin' states, α being the initial 'life state' and i the initial region of a person. The superscripts w(j) refer to final 'life state' and final region. There are two initial life states, existence at the start of a period (represented by superscript ε) and birth during the period (represented by superscript β), and two final life states, death during the period (represented by superscript δ) and survival at the end of the period (represented by superscript σ). The K matrix can be partitioned into four parts therefore recognizing the four life state to life state transitions that can take place

$$K = \left| \begin{array}{c|c} \{K^{\varepsilon(i)\sigma(j)}\} & \{K^{\varepsilon(i)\delta(j)}\} \\ \hline \{K^{\beta(i)\sigma(j)}\} & \{K^{\beta(i)\delta(j)}\} \end{array} \right| \qquad (1)$$

where $K^{\varepsilon(i)\sigma(j)}$ refers to persons who exist in region i at the start of the period and survive in region j at the end; where $K^{\varepsilon(i)\delta(j)}$ are persons likewise starting in existence in a region i who end the period dying in region j; where $K^{\beta(i)\sigma(j)}$ and $K^{\beta(i)\delta(j)}$ are the corresponding flows for persons born in the period in region i.

The accounts matrix can be specified for any number of regions but must always include a residual, 'rest of the world', region to close the system of accounts. In the case of the British regional system we describe later in the paper, we specify accounts with 11 regions, 10 regions within Britain and the 11th referring to the rest of the world. The resulting accounts matrix looks like this

$$
\underline{K} = \left[\begin{array}{cccc|cccc}
K^{\varepsilon(1)\sigma(1)} & K^{\varepsilon(1)\sigma(2)} & \ldots & K^{\varepsilon(1)\sigma(11)} & K^{\varepsilon(1)\delta(1)} & K^{\varepsilon(1)\delta(2)} & \ldots & K^{\varepsilon(1)\delta(11)} \\
K^{\varepsilon(2)\sigma(1)} & K^{\varepsilon(2)\sigma(2)} & \ldots & K^{\varepsilon(2)\sigma(11)} & K^{\varepsilon(2)\delta(1)} & K^{\varepsilon(2)\delta(2)} & \ldots & K^{\varepsilon(2)\delta(11)} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
K^{\varepsilon(11)\sigma(1)} & K^{\varepsilon(11)\sigma(2)} & \ldots & K^{\varepsilon(11)\sigma(11)} & K^{\varepsilon(11)\delta(1)} & K^{\varepsilon(11)\delta(2)} & \ldots & K^{\varepsilon(11)\delta(11)} \\
\hline
K^{\beta(1)\sigma(1)} & K^{\beta(1)\sigma(2)} & \ldots & K^{\beta(1)\sigma(11)} & K^{\beta(1)\delta(1)} & K^{\beta(1)\delta(2)} & \ldots & K^{\beta(1)\delta(11)} \\
K^{\beta(2)\sigma(1)} & K^{\beta(2)\sigma(2)} & \ldots & K^{\beta(2)\sigma(11)} & K^{\beta(2)\delta(1)} & K^{\beta(2)\delta(2)} & \ldots & K^{\beta(2)\delta(11)} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
K^{\beta(11)\sigma(1)} & K^{\beta(11)\sigma(2)} & \ldots & K^{\beta(11)\sigma(11)} & K^{\beta(11)\delta(1)} & K^{\beta(11)\delta(2)} & \ldots & K^{\beta(11)\delta(11)}
\end{array}\right] \tag{2}
$$

Since interest is focused on regions 1 to 10, the terms $K^{\varepsilon(11)\sigma(11)}$, $K^{\varepsilon(11)\delta(11)}$, $K^{\beta(11)\sigma(11)}$ and $K^{\beta(11)\delta(11)}$ involving rest-of-the-world to rest-of-the-world population flows can be omitted from the analysis.

The population numbers that replace the $K^{\alpha(i)\omega(j)}$ in an empirical study are normally unavailable in direct form, and have to be estimated using a model (here referred to as 'accounts based model') involving what information is known, involving the row and column constraints, and a set of hypotheses about the rates at which migrants ($K^{\varepsilon(i)\sigma(j)}$ flows where $i \neq j$) and infant migrants ($K^{\beta(i)\sigma(j)}$ flows) die. In the matrix below (equation (3)) the 'known' terms are picked out and row and column constraints have been added.

$$
\underline{K} = \begin{array}{cccc|cccc|c}
- & K^{\varepsilon(1)\sigma(2)} & \ldots & K^{\varepsilon(1)\sigma(11)} & - & - & \ldots & - & K^{\varepsilon(1)*(*)} \\
K^{\varepsilon(2)\sigma(1)} & - & \ldots & K^{\varepsilon(2)\sigma(11)} & - & - & \ldots & - & K^{\varepsilon(2)*(*)} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\
K^{\varepsilon(11)\sigma(1)} & K^{\varepsilon(11)\sigma(2)} & \ldots & 0 & - & - & \ldots & 0 & 0 \\
\hline
- & - & \ldots & - & - & - & \ldots & - & K^{\beta(1)*(*)} \\
- & - & \ldots & - & - & - & \ldots & - & K^{\beta(2)*(*)} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\
- & - & \ldots & 0 & - & - & \ldots & 0 & 0 \\
- & - & \ldots & 0 & K^{*(*)\delta(1)} & K^{*(*)\delta(2)} & \ldots & 0 &
\end{array} \tag{3}
$$

The terms of equation (2) which in equation (3) are represented by a dash (-) are those which must be estimated by an accounts based model (see Rees and Wilson (1973) and (1976) for a full description of these). This model uses as input the initial populations of the regions, $K^{\varepsilon(i)*(*)}$, with the asterisk representing summation over the superscript in the same position, to which the elements in the top half of the accounts matrix sum row-wise; the birth totals in the regions, $K^{\beta(i)*(*)}$, to which the elements in the bottom half of the matrix sum row-wise; the death totals, $K^{*(*)\delta(i)}$, to which elements in the right-hand half of the matrix sum column-wise; and the 'exist-survive' migrants, $K^{\varepsilon(i)\sigma(j)}$, $i \neq j$, who occupy the off diagonal positions in the top-left quadrant of the $\underline{K}$ matrix. The totals of columns in the left-hand half of $\underline{K}$ are the final populations in the period, the $K^{*(*)\sigma(i)}$, which are a product of the model, although they may be used in some variants of the accounts based model.

In this brief survey of the theory underlying spatial demographic accounting, disaggregation of the population by age and sex (or by any other classification) is neglected. Readers are referred to Wilson and Rees (1974) and Rees and Wilson (1976) for full treatment. The neglect is continued in the applications that follow in order to highlight the regional or spatial issues involved, as opposed to those involving age and sex which are well treated elsewhere (Pressat, 1972; Keyfitz, 1968).

3. Demographic accounts for British regions

Figure 1 shows the boundaries of the regions for which accounts are constructed. These boundaries have changed twice since the 1961 census and the map shows those existing at the 1966 and 1971 census dates, together with the revised boundaries now current as a result of local government reorganization. Together the 10 regions make up 'Great Britain'.

Figure 1 The standard regions of Great Britain

Addition of Northern Ireland to the list would have converted the regional system into one for the United Kingdom (of Great Britain and Northern Ireland), but figures for emigration from Northern Ireland to the other British regions are unavailable. Northern Ireland was therefore placed in the rest-of-the-world region.

Figure 2 shows the set of accounts prepared for the one year period 1965-66 (April 23/24 to April 23/24) prior to the date of the Sample Census 1966. This period was chosen as it was the one for which migrant information was available. This is displayed in the top left-hand quadrant (off-diagonal elements). The numbers given in the census migration tables have been corrected for underenumeration (see Smith and Rees, 1974; and Illingworth, 1975a, 1975b, for details of method). Emigrants to the rest-of-the-world are estimated by multiplying the total of immigrant figures for all regions (given in the census migration tables) by the ratio of emigrants to immigrants for 1965-66 (1.3516) given in the International Passenger Survey statistics (General Register Office, 1966) and then disaggregating this total by the regional shares revealed in a later survey for 1971. One might note, in passing, that the migration figures given by the retrospective census tables and by this method are about 50 per cent greater in magnitude than those recorded by the 'current' measuring devices (International Passenger Survey, Commonwealth Immigrant Act Statistics).

The initial populations of the regions (3307961...5206304) are estimated by backwards extrapolation from the mid-year estimate for 1965. The births and deaths totals are made up of appropriate shares of the births and deaths of 1965 and 1966 calendar years (see Rees and Wilson, 1976, Chapter 3 for methods). Infant migrants (children under 1 year of age migrating

Existence at G.B. 1965

Final State / Initial State	1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RW	Sub Totals
1. North	3195707	10774	6113	3397	[illegible]	1675	13412	[illegible]	1288	4170	24764	[illegible]
2. Yorkshire & Humberside	[illegible]	4556799	10774	13057	[illegible]	[illegible]	[illegible]	4545	8171	3410	27604	4651393
3. North West	6057	10815	[illegible]	6330	7211	[illegible]	21737	6513	8411	[illegible]	44797	6614986
4. East Midlands	2739	12439	5479	3151540	[illegible]	4201	15650	3774	1400	2334	23247	3231270
5. West Midlands	3247	4961	5610	9910	[illegible]	2074	21300	11176	5750	2003	31307	4315409
6. East Anglia	1521	[illegible]	2180	[illegible]	2322	[illegible]	17055	3397	[illegible]	1262	26319	1536614
7. South East	12018	15435	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	53747	[illegible]	14930	105199	[illegible]
8. South West	2769	4036	4554	3874	7303	2094	40801	[illegible]	4900	2030	34449	[illegible]
9. Wales	1167	1947	5192	1077	4000	710	12038	5493	[illegible]	1795	9515	2656115
10. Scotland	5407	4750	6754	4707	5550	1835	19786	3470	1137	5037917	48755	5140774
11. Rest of the world	13052	17636	25263	13742	25500	17170	175013	[illegible]	7932	21602	0	340401
Sub totals	[illegible]	4641750	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3545210	[illegible]	[illegible]	470026	[illegible]

Final State / Initial State	Deaths 1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RW	Sub Totals	TOTALS
1. North	[illegible]	61	39	20	29	9	79	17	8	27	149	39042	3307551
2. Yorkshire & Humberside	65	55992	[illegible]	72	21	16	97	30	14	22	147	[illegible]	[illegible]
3. North West	36	65	[illegible]	35	41	13	123	42	55	25	232	83476	[illegible]
4. East Midlands	16	76	34	35102	43	24	92	25	9	15	127	[illegible]	[illegible]
5. West Midlands	19	30	62	54	[illegible]	14	126	73	38	[illegible]	164	[illegible]	4567116
6. East Anglia	9	17	14	25	12	17341	101	20	5	8	151	17709	1554373
7. South East	72	93	129	105	103	173	190043	340	75	96	1118	[illegible]	[illegible]
8. South West	17	24	29	21	39	17	238	45345	32	19	224	[illegible]	[illegible]
9. Wales	7	12	33	11	33	4	71	33	34815	12	61	35093	2691209
10. Scotland	33	29	42	26	29	9	117	22	8	65499	313	66127	[illegible]
11. Rest of the world	78	107	159	75	123	99	1634	[illegible]	48	139	0	2031	342431
Sub totals	[illegible]	56416	[illegible]	35552	[illegible]	17720	[illegible]	[illegible]	35109	65878	2757	[illegible]	53254171

Births in 1965-66

Final State / Initial State	1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RW	Sub Totals
1. North	57413	[illegible]	[illegible]	35	48	14	119	[illegible]	11	37	221	58114
2. Yorkshire & Humberside	93	83538	97	110	37	25	138	41	20	31	249	84412
3. North West	56	99	120374	58	72	27	128	55	77	35	409	121458
4. East Midlands	25	114	50	58751	76	39	143	35	13	22	213	59481
5. West Midlands	31	47	94	95	93012	23	204	107	55	27	299	94274
6. East Anglia	13	24	19	40	20	25711	150	28	7	11	223	26246
7. South East	105	135	184	176	177	270	[illegible]	401	102	134	1695	301647
8. South West	24	35	33	33	[illegible]	26	347	60400	42	25	297	61331
9. Wales	10	10	44	17	53	6	102	46	[illegible]	15	79	45477
10. Scotland	52	45	64	46	53	15	169	33	11	97603	466	98578
11. Rest of the world	227	307	449	239	410	[illegible]	3647	451	123	377	0	[illegible]
Sub totals	53107	84172	[illegible]	[illegible]	[illegible]	26430	[illegible]	61704	[illegible]	98317	4151	956943
TOTALS	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1565131	[illegible]	[illegible]	[illegible]	5195404	[illegible]	[illegible]

Final State / Initial State	Deaths 1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RW	Sub Totals	TOTALS
1. North	344	0	0	0	0	0	0	0	0	0	1	345	58459
2. Yorkshire & Humberside	0	503	0	0	0	0	0	0	0	0	1	509	84921
3. North West	0	0	761	0	0	0	1	0	0	0	1	763	122221
4. East Midlands	0	0	0	324	0	0	0	0	0	0	1	325	59806
5. West Midlands	0	0	0	0	491	0	1	0	0	0	1	493	94767
6. East Anglia	0	0	0	0	0	149	0	0	0	0	1	150	[illegible]
7. South East	0	0	1	0	0	1	1772	2	0	0	5	1781	303428
8. South West	0	0	0	0	0	0	1	395	0	0	1	397	[illegible]
9. Wales	0	0	0	0	0	0	0	0	298	0	0	298	45775
10. Scotland	0	0	0	0	0	0	1	0	0	629	1	631	99209
11. Rest of the world	1	1	1	1	1	1	9	1	0	1	0	17	5942
Sub totals	345	509	763	325	[illegible]	151	[illegible]	[illegible]	[illegible]	[illegible]	13	5709	952652
TOTALS	[illegible]	[illegible]	84175	[illegible]	[illegible]	17870	[illegible]	[illegible]	[illegible]	66509	[illegible]	[illegible]	54216825

Figure 2 Aggregate population accounts for British regions, 1965-66 (one year)

between regions - the $K^{\beta(i)\sigma(j)}$ terms) are estimated by applying the migration rates of the 'exist-survive' migrants to the regional birth totals with a division by 2 to reflect the shorter life span in the period of these infants. From these inputs the accounts matrix of Figure 2 was generated with sets of sub-totals and totals added using the SDAT computer programme (Rees and Wilson, 1974).

Exactly parallel accounts for British regions for the one year period 1970-71 prior to the 1971 census date (April 23/24, 1971) are shown in Figure 3. The known data input to the accounts based model is the same as that for 1965-66 except that the initial populations were interpolated between the June 30, 1969 and June 30, 1970 estimates by the Registrar General.

These two sets of demographic accounts present comprehensive pictures of population change among British regions 5 years ago and 10 years ago respectively. The system appears to have shifted relatively little in the five years between 1965-66 and 1970-71 in structure with the latter containing larger population flows than the former. The gross totals of all population flows involved in the regional system are 54.217 millions in 1965-66 and 55.448 millions in 1970-71. We will look in the next section at the detailed differences in growth regime by calculating the demographic rates involved.

To get a picture of the gross changes taking place in the regional system we can summarize the two sets of accounts in more conventional form in terms of input flows and output flows. However, these tables differ from most conventional tables in having explicitly dealt with all possible flows into and out of the system - because they are derived from sets of accounts. Figure 4 shows these inflow-outflow-accounts for two regions - the first, Scotland, showing slight population decline, and the second, East Anglia, showing substantial population increase.

Initial State / Final State	Survival at C.D. 1971: 1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 R.W	Sub Totals
Existence at C.D. 1970												
1. North	3241941	9290	7374	3630	7790	2190	13040	3870	1230	3910	35275	3315526
2. Yorkshire & Humberside	12730	4644500	13290	13190	5970	4320	21580	5610	2570	3470	27486	4755176
3. North West	6790	10650	6567400	6320	7750	2980	23210	9950	10370	5200	43032	6701572
4. East Midlands	4200	12170	5300	3123569	9610	6030	19110	6680	2350	3100	25545	3323644
5. West Midlands	3550	5400	3750	12600	5009250	5040	25050	12550	6350	3340	27877	5139377
6. East Anglia	1150	2510	1970	4450	2510	1584277	20460	3820	1170	1650	28194	1652171
7. South East	12070	16570	22920	23390	21410	35410	16697512	66370	12050	17610	193299	17121811
8. South West	2970	4600	4860	4230	8600	3390	40050	3549220	4730	4810	35450	3711970
9. Wales	1450	2060	6350	2600	5650	1280	13160	7300	2647069	1650	10185	2696294
10. Scotland	5530	4610	7220	4910	4630	2070	23720	5330	1700	5033830	57612	5151442
11. Rest of the world	14970	19630	27230	16650	23140	20280	157220	23390	6840	29100	0	385910
Sub totals	5307021	4732170	6673[illegible]	3324339	5104470	[illegible]	17111742	37[illegible]	2698399	5107730	475475	[illegible]
Birth in 1970-71												
1. North	52151	73	58	29	30	17	119	31	10	39	200	52757
2. Yorkshire & Humberside	109	60569	113	112	51	37	184	49	22	30	230	81513
3. North West	57	90	112263	55	82	25	246	75	37	44	370	113392
4. East Midlands	35	103	45	55667	81	51	161	56	20	25	215	56461
5. West Midlands	31	47	85	110	88686	25	223	110	55	39	263	89645
6. East Anglia	9	30	15	36	20	25073	163	30	9	13	224	25413
7. South East	94	129	179	206	167	277	165676	540	94	138	1510	258990
8. South West	23	35	37	32	66	26	175	56273	56	37	272	57219
9. Wales	11	15	51	19	45	10	104	55	42533	15	81	42938
10. Scotland	47	41	61	42	39	18	201	42	14	55708	405	57704
11. Rest of the world	256	344	479	292	446	356	1459	515	155	494	0	6756
Sub totals	52805	81467	113307	56598	89673	26710	270312	57764	43035	57571	3812	883788
TOTALS	3359844	4813637	6787167	3381297	5194093	1591015	17381654	3796794	2741434	5132301	479117	54824[illegible]

Initial State / Final State	Death in 1970-71: 1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 R.W	Sub Totals	TOTALS
Existence at C.D. 1970													
1. North	38271	55	[illegible]	20	19	12	83	24	8	29	147	38714	3357240
2. Yorkshire & Humberside	74	55971	59	72	21	24	119	34	16	21	165	55470	4011645
3. North West	39	63	82920	39	50	17	151	55	65	31	774	83710	6785282
4. East Midlands	24	72	33	35111	40	35	115	41	75	18	140	36542	3560286
5. West Midlands	21	32	61	69	50000	17	142	77	40	20	183	52622	5171999
6. East Anglia	7	15	12	25	13	17952	113	25	7	10	156	18563	1670534
7. South East	70	58	143	145	110	156	187500	409	76	105	1065	189925	17311740
8. South West	17	27	30	23	44	19	271	45000	30	29	218	45708	3757678
9. Wales	8	12	30	13	29	7	75	43	33607	10	64	33906	2752200
10. Scotland	52	28	65	27	24	11	132	33	11	56533	349	61512	5212760
11. Rest of the world	85	116	170	91	118	112	1090	191	55	167	0	2185	387795
Sub totals	38643	56349	83583	36631	52487	18430	189798	45[illegible]	33939	60936	2718	619557	54556760
Birth in 1970-71													
1. North	305	0	0	0	0	0	0	0	0	0	1	306	53063
2. Yorkshire & Humberside	0	450	0	0	0	0	1	0	0	0	1	402	81955
3. North West	0	0	705	0	0	0	1	0	0	0	1	705	114097
4. East Midlands	0	0	0	308	0	0	0	0	0	0	1	309	56770
5. West Midlands	0	0	0	0	457	0	1	0	0	0	1	459	90104
6. East Anglia	0	0	0	0	0	145	0	0	0	0	1	146	25559
7. South East	0	0	1	1	0	1	1478	2	0	0	4	1487	270477
8. South West	0	0	0	0	0	0	1	349	0	0	1	351	57570
9. Wales	0	0	0	0	0	0	0	0	268	0	0	268	43206
10. Scotland	0	0	0	0	0	0	1	0	0	518	1	520	58224
11. Rest of the world	1	1	1	1	1	1	10	2	0	7	0	19	6775
Sub totals	306	461	705	310	459	147	1483	353	268	519	12	5032	888840
TOTALS	37954	56630	84288	36941	53445	18577	191281	46273	34198	61492	2730	624619	55447600

Figure 3 Aggregate population accounts for British regions, 1970-71 (one year)

Figure 4 **Inflow-Outflow Accounts for Scotland and East Anglia**

SCOTLAND

Item	1965-66 Inflow	1965-66 Outflow	1965-66 Net flow	1970-71 Inflow	1970-71 Outflow	1970-71 Net flow
Initial population	5206305			5212760		
Migrants E-S	59170	102261	-43091	73900	117612	-43712
B-S	714	975	-261	863	996	-133
E-D	380	629	-249	440	685	-412
B-D	1	2	-1	1	2	-1
Total	60265	103867	-43602	75204	119265	-44258
Births	99209			88224		
Deaths		66508			61592	
Natural increase			+32701			+26632
Final population		5195404			5195301	
Totals	5365779	5365779		5376188	5376188	
Total change			-10901			-17626

EAST ANGLIA

Item	1965-66 Inflow	1965-66 Outflow	1965-66 Net flow	1970-71 Inflow	1970-71 Outflow	1970-71 Net flow
Initial population	1554323			1670534		
Migrants E-S	65986	62919	+3067	80990	67954	+13036
B-S	739	535	+204	843	540	+303
E-D	379	368	+11	448	381	+67
B-D	2	1	+1	2	1	+1
Total	67106	63823	+3283	82283	68876	+13407
Births	26396			26559		
Deaths		17871			18577	
Natural increase			+8525			+7982
Final population		1566131			1691923	
Totals	1647825	1647825		1779376	1779376	
Total change			+11808			+21389

Scotland's regime of population change is roughly the same in both years with a falling natural increase failing to offset fairly constant net-out-migration resulting in a rise in the size of population decline. In East Anglia, natural increase has also fallen somewhat but has been counterbalanced by an increase in net immigration. By looking back at the accounts table (Figure 3) one can see that a substantial portion of the 44,000 net migrant loss in 1970-71 in Scotland was to the rest-of-the-world (29,500). Hence the importance of including in the analysis a rest-of-the-world region. This is essential if an accounting framework is adopted.

Much more could be said about the pattern of population flows revealed in Figures 2 and 3 but since the main aim of the paper is to outline the prototype modelling system this analysis is omitted here.

4. Rates from the accounts

Using the demographic accounts we can define a variety of rates which form the input to historical analyses of trends and forecasting analyses of future numbers. The different birth and death rates which can be defined utilize the same numerators (total births, total deaths) but employ populations at risk appropriate to the forecasting model adopted.

The conventional population at risk of giving birth or of dying is usually taken to be the mid-period population or the average of the initial and final populations of the region in question. The second version of the conventional at risk population can be calculated from the accounts. A more precise alternative which only population accounts make possible is the multi-regional population at risk (see Rees and Wilson, 1973 and 1976), although there is relatively little difference between the two. A third alternative, the initial population of the region, can be rather different.

The variation in vital rates amongst the regions is relatively small. Versions of these rates, or rather their age-sex disaggregated equivalents, are used in cohort survival models.

An alternative rate is the transition rate formed by dividing the row element in the accounts by the appropriate row total. This kind of rate has to be used when migration is considered, and forms part of the raw material for the growth rates matrix developed by Rogers (1966, 1971, 1975) which will be utilized in section 5 of the paper. The transition rates or $\underline{H}$ matrix for 1970-71 is displayed in Figure 5.

The elements in any row of this matrix show how persons originating in that row are distributed in the ensuing year. In the case of Scotland and East Anglia illustrated earlier the chances of surviving within the region are 0.96567 and 0.94833, of migrating to and surviving in the South East are 0.00456 and 0.01225 respectively; of migrating to and surviving in the rest-of-the-world are 0.01105 and 0.01688 respectively and so on.

Now, the $\underline{H}$ matrix of transition rates can be used in a population change model directly as long as a birth sub-model is added that utilizes one of the sets of the vital rates referred to earlier (see Rees and King, 1970). However, Rogers (1966, 1971, 1975) has used them in a different form in a simpler matrix multiplication model. The $\underline{G}$ matrix of growth rates involved in that model is calculated as follows from the $\underline{H}$ and $\underline{K}$ matrices. The transition rates in the existence-survival quadrant (top-left) of the $\underline{H}$ matrix are retained and to each element is added birth-and-transition rates formed by dividing each element in the birth-survival quadrant (bottom-left) of the $\underline{K}$ matrix by the corresponding initial population total. The resulting matrix of growth rates for 1970-71 is shown in Figure 6. Normally this matrix is transposed before use and this is what is termed the $\underline{G}$ matrix. These rates are rates of transition and

Initial State \ Final State	1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RTW	Sub Totals
	Survival at C.B. 1971											
Existence at C.B. 1970												
1. North	.96546	.00277	.00220	.00101	.00113	.00045	.00140	.00115	.00034	.00145	.00759	
2. Yorkshire & Humberside	.00266	.96526	.00275	.00274	.00128	.00050	.00449	.00117	.00053	.00072	.00510	
3. North West	.00100	.00157	.97188	.00033	.00144	.00044	.00470	.00152	.00153	.00077	.00545	
4. East Midlands	.00129	.00112	.00158	.96110	.00155	.00179	.00563	.00113	.00069	.00057	.00760	
5. West Midlands	.00069	.00104	.00189	.00244	.96853	.00355	.00495	.00245	.00123	.00075	.00539	
6. East Anglia	.00069	.00150	.00116	.00069	.00152	.94637	.01225	.00229	.00070	.00029	.01503	
7. South East	.00036	.00098	.00132	.00152	.00124	.00209	.95452	.00335	.00070	.00150	.01117	
8. South West	.00079	.00122	.00120	.00113	.00231	.00090	.01305	.95517	.00125	.00128	.00343	
9. Wales	.00053	.00015	.00234	.00237	.00227	.00047	.00462	.00255	.96834	.00070	.00373	
10. Scotland	.00106	.00032	.00133	.00094	.00030	.00040	.00456	.00132	.00033	.96567	.01165	
11. Rest of the world	$.0^{5}41$	$.0^{5}55$	$.0^{5}77$	$.0^{5}47$	$.0^{5}65$	$.0^{5}57$	$.0^{4}553$	$.0^{5}32$	$.0^{5}25$	$.0^{5}72$	.0	
Sub totals												
Birth in 1970-71												
1. North	.96279	.00130	.001[illegible]	.00055	.00057	.00032	.00204	.00058	.00019	.00073	.00377	
2. Yorkshire & Humberside	.00133	.96[illegible]	.00138	.00137	.00162	.00045	.00224	.00059	.00027	.00037	.00270	
3. North West	.00050	.00079	.98373	.00046	.00072	.00022	.00216	.00066	.00076	.00039	.00324	
4. East Midlands	.00062	.00181	.00079	.96057	.00143	.00090	.00284	.00093	.00035	.00015	.00300	
5. West Midlands	.00034	.00052	.00094	.00122	.98426	.00069	.00247	.00122	.00061	.00032	.00270	
6. East Anglia	.00034	.00075	.00050	.00135	.00075	.97417	.00114	.00113	.00034	.00049	.00343	
7. South East	.00031	.00045	.00066	.00076	.00062	.00102	.98225	.00192	.00035	.00051	.00556	
8. South West	.00043	.00061	.00064	.00036	.00115	.00045	.00693	.97750	.00063	.00064	.00472	
9. Wales	.00025	.00037	.00118	.00044	.00104	.00023	.00241	.00127	.98442	.00030	.00187	
10. Scotland	.00035	.00046	.00049	.00044	.00044	.00020	.00228	.00051	.00016	.95202	.00553	
11. Rest of the world	$.0^{5}20$	$.0^{5}28$	$.0^{5}$[illegible]	$.0^{5}23$	$.0^{5}32$	$.0^{5}28$	$.0^{4}277$	$.0^{5}41$	$.0^{5}12$	$.0^{5}40$	.0	
Sub totals												
TOTALS												

Initial State \ Final State	1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RTW	Sub Totals	Totals
	Death in 1970-71												
Existence at C.B. 1970													
1. North	.1140	$.0^{4}143$	$.0^{4}137$	$.0^{5}56$	$.0^{5}56$	$.0^{5}33$	$.0^{4}247$	$.0^{5}31$	$.0^{5}23$	$.0^{5}$[illegible]	$.0^{4}437$		1.00000
2. Yorkshire & Humberside	$.0^{4}153$	.01160	$.0^{4}172$	$.0^{4}143$	$.0^{5}64$	$.0^{5}43$	$.0^{4}247$	$.0^{5}70$	$.0^{5}32$	$.0^{5}42$	$.0^{4}347$		1.00000
3. North West	$.0^{5}57$	$.0^{5}93$	.01222	$.0^{5}52$	$.0^{5}23$	$.0^{5}23$	$.0^{4}237$	$.0^{5}31$	$.0^{5}56$	$.0^{5}31$	$.0^{4}233$		1.00000
4. East Midlands	$.0^{5}71$	$.0^{4}214$	$.0^{5}96$	.01075	$.0^{4}145$	$.0^{5}36$	$.0^{4}315$	$.0^{4}172$	$.0^{5}45$	$.0^{5}24$	$.0^{4}317$		1.00000
5. West Midlands	$.0^{5}41$	$.0^{5}62$	$.0^{4}119$	$.0^{4}133$	.01005	$.0^{5}33$	$.0^{4}275$	$.0^{4}149$	$.0^{5}77$	$.0^{5}29$	$.0^{4}176$		1.00000
6. East Anglia	$.0^{5}42$	$.0^{5}30$	$.0^{5}72$	$.0^{4}150$	$.0^{5}79$	.01076	$.0^{4}635$	$.0^{4}139$	$.0^{5}42$	$.0^{5}60$	$.0^{4}731$		1.00000
7. South East	$.0^{5}40$	$.0^{5}57$	$.0^{5}83$	$.0^{5}84$	$.0^{5}64$	$.0^{4}113$	.01063	$.0^{4}234$	$.0^{5}44$	$.0^{5}61$	$.0^{4}613$		1.00000
8. South West	$.0^{5}45$	$.0^{5}72$	$.0^{5}80$	$.0^{5}61$	$.0^{4}117$	$.0^{5}51$	$.0^{4}721$	.01193	$.0^{4}130$	$.0^{4}772$	$.0^{4}590$		1.00000
9. Wales	$.0^{5}59$	$.0^{5}44$	$.0^{4}146$	$.0^{5}48$	$.0^{4}100$	$.0^{5}26$	$.0^{4}267$	$.0^{4}157$	.01230	$.0^{5}37$	$.0^{4}251$		1.00000
10. Scotland	$.0^{5}61$	$.0^{5}54$	$.0^{5}36$	$.0^{5}52$	$.0^{5}41$	$.0^{5}21$	$.0^{4}253$	$.0^{5}63$	$.0^{5}21$	.01163	$.0^{4}655$		1.00000
11. Rest of the world	$.0^{7}24$	$.0^{7}33$	$.0^{7}48$	$.0^{7}26$	$.0^{7}33$	$.0^{7}31$	$.0^{6}31$	$.0^{7}51$	$.0^{7}15$	$.0^{7}47$	.0		1.00000
Sub totals													
Birth in 1970-71													
1. North	.00375	.0	.0	.0	.0	.0	.0	.0	.0	.0	$.0^{4}108$		1.00000
2. Yorkshire & Humberside	.0	.00385	.0	.0	.0	.0	$.0^{4}17$[illegible]	.0	.0	.0	$.0^{4}122$		1.00000
3. North West	.0	.0	.00516	.0	.0	.0	$.0^{5}68$	.0	.0	.0	$.0^{5}83$		1.00000
4. East Midlands	.0	.0	.0	.00543	.0	.0	.0	.0	.0	.0	$.0^{4}176$		1.00000
5. West Midlands	.0	.0	.0	.0	.00507	.0	$.0^{4}111$	.0	.0	.0	$.0^{4}111$		1.00000
6. East Anglia	.0	.0	.0	.0	.0	.00516	.0	.0	.0	.0	$.0^{4}277$		1.00000
7. South East	.0	.0	$.0^{5}37$	$.0^{5}37$	.0	$.0^{5}37$	.00545	$.0^{5}74$	.0	.0	$.0^{4}140$		1.00000
8. South West	.0	.0	.0	.0	.0	.0	$.0^{4}173$	.00606	.0	.0	$.0^{4}175$		1.00000
9. Wales	.0	.0	.0	.0	.0	.0	.0	.0	.00520	.0	.0		1.00000
10. Scotland	.0	.0	.0	.0	.0	.0	$.0^{4}113$	.0	.0	.00587	$.0^{4}113$		1.00000
11. Rest of the world	$.0^{8}80$	$.0^{8}80$	$.0^{8}80$	$.0^{8}80$	$.0^{8}80$	$.0^{8}80$	$.0^{7}80$	$.0^{7}16$	.0	$.0^{8}80$	.0		
Sub totals													
TOTALS													

N.B. $.0^{5}20$ means .0000020

Figure 5 The H or transition rates matrix for British regions, 1970-71

Figure [illegible] The [illegible] rates [illegible] for 1970-71 British [illegible]

Initial State (Existence at C.D. 1970) \ Final State (Survival at C.D. 1971)	1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RTW	
1. North	.98119	.00279	.00221	.00109	.00114	.00066	.00452	.00116	.00037	.00147	.00759	
2. Yorkshire & Humberside	.00268	.98201	.00279	.00276	.00125	.00091	.00452	.00118	.00054	.00073	.00584	
3. North West	.00101	.00158	.98443	.00094	.00145	.00044	.00434	.00133	.00154	.00077	.00653	
4. East Midlands	.00126	.00365	.00159	.97767	.00288	.00182	.00573	.00200	.00070	.00093	.00767	
5. West Midlands	.00069	.00105	.00190	.00246	.98568	.00059	.00500	.00246	.00124	.00065	.00544	
6. East Anglia	.00069	.00151	.00120	.00271	.00153	.96382	.01235	.00230	.00071	.00100	.01701	
7. South East	.00070	.00096	.00133	.00154	.00125	.00206	.97987	.00388	.00070	.00103	.01125	
8. South West	.00080	.00123	.00130	.00113	.00233	.00091	.01315	.97015	.00127	.00129	.00951	
9. Wales	.00053	.00075	.00236	.00207	.00208	.00047	.00485	.00258	.98441	.00061	.00516	
10. Scotland	.00107	.00093	.00140	.00095	.00090	.00040	.00460	.00103	.00033	.98231	.01115	
11. Rest of the world	$.0^{5}42$	$.0^{5}56$	$.0^{5}78$	$.0^{5}55$	$.0^{5}66$	$.0^{5}58$	$.0^{4}563$	$.0^{5}83$	$.0^{5}25$	$.0^{5}80$	.02072	
Sub totals												

survival plus birth, transition and survival between the regions. Thus, the g^{55} rate for the West Midlands, with a value of 0.98568 is made up of a transition rate 0.96853 from Figure 6 and a birth and survival rate of 0.01715. Whereas in the $\underline{H}$ matrix all rates must be equal or less than one, in the $\underline{G}$ matrix rates may exceed one although in Figure 6 this is true only for the rest-of-the-world*.

The $\underline{G}$ matrix represents the operator that transforms an initial distribution of population amongst a set of regions into a final one. The off-diagonal elements spread out migrants and infant migrants from origin regions to destination regions and gather. in migrants and infant migrants to destination regions from origin regions. The diagonal elements deal with the propensity of persons to stay in initial regions, survive there and to reproduce.

5. A forecast for British regions using the Rogers' model

The model of population change developed by Rogers (1966, 1968, 1971, 1975) can be stated as follows

$$\underline{w}(t+1) = \underline{G}\,\underline{w}(t) \qquad (4)$$

where $\underline{w}$ is a column vector of populations, $\underline{G}$ is a matrix of growth rates, t is the start of the period and t+1 is end of the period of one unit in length. The $\underline{G}$ matrix can be applied successively thus

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*It is convenient to close the system when using the $\underline{G}$ matrix. To do this estimates were made of the population of the rest-of-the-world, births and deaths in the rest-of-the-world and adjusted estimation equations applied to yield the following estimates: $K^{\varepsilon(11)\sigma(11)} = 3{,}513{,}090{,}583$; $K^{\varepsilon(11)\delta(11)} = 49{,}499{,}685$; $K^{\varepsilon(11)*(*)} = 3{,}563{,}362{,}873$; $K^{\beta(11)\sigma(11)} = 124{,}104{,}583$; $K^{\beta(11)\delta(11)} = 874{,}904$; $K^{\beta(11)*(*)} = 124{,}986{,}244$.

$$\underline{w}(t+2) = \underline{G}\ \underline{w}(t+1) \tag{5}$$

$$\underline{w}(t+3) = \underline{G}\ \underline{w}(t+2) \tag{6}$$

$$\vdots$$

$$\underline{w}(t+n) = \underline{G}\ \underline{w}(t+n-1) \tag{7}$$

or in other words the vector of populations after θ periods will be

$$\underline{w}(t+\theta) = \underline{G}^{\theta}\ \underline{w}(t) \tag{8}$$

where $\underline{G}$ is raised to the power θ. This model makes the assumption that the system continues to be characterized by the same $\underline{G}$ growth matrix, and this assumption can be relaxed by adopting a time series of $\underline{G}$ matrices (if available) and using instead the equation

$$\underline{w}(t+\theta) = \prod_{\lambda=1}^{\theta} \underline{G}_{\lambda}\ \underline{w}(t) \tag{9}$$

where λ is an index running from 1 to θ indicating which time period the $\underline{G}$ refers.

We can illustrate the operation of equation (4) of the model to the British population system. We adopt census date 1971 as the initial time t and assume that the $\underline{G}$ matrix for 1970-71 holds for 1971-72:

$$\begin{bmatrix} 3301845 \\ 4809037 \\ 6761499 \\ 3414521 \\ 5138552 \\ 1683310 \\ 17308532 \\ 3799723 \\ 2734606 \\ 5205852 \\ 3637672207 \end{bmatrix} = \begin{bmatrix} .98119 & .00268 & .00101 & \dots & 0^{5}42 \\ .00279 & .98201 & .00158 & \dots & 0^{5}56 \\ .00221 & .00279 & .98443 & \dots & 0^{5}78 \\ \vdots & \vdots & \vdots & & \vdots \\ .00759 & .00584 & .00653 & \dots & 1.02072 \end{bmatrix} \times \begin{bmatrix} 3298185 \\ 4807240 \\ 6759380 \\ 3388910 \\ 5115760 \\ 1661730 \\ 17236840 \\ 3761400 \\ 2725365 \\ 5223725 \\ 3563362873 \end{bmatrix}$$

$$\underline{w}(\text{c.d.}1972) = \underline{G}\ \underline{w}(\text{c.d.}1971) \tag{10}$$

This particular forecasting model was run forward from census date 1971 to census date in 2001. The resulting population vectors for every fifth year (unfortunately not all of them will have a census taken in them) are displayed in Figure 7, along with the population breakdown within Great Britain in percentage form.

The redistribution of population seen in the 1960's towards the South East, East Anglia, the South West and the East Midlands (Eversley, 1971; Department of the Environment, 1971) continues under this scenario. The West Midlands gains marginally and the other regions continue to lose their share of the national population cake, particularly Scotland.

Gains are made in absolute numbers in all regions by 2001 although Scotland declines to 1988 before recovering. The South East makes particularly large population gains - nearly 4 millions in the 30 years to the turn of the century. The reason within the model for this sequence of events is the operation of the growth rates associated with the rest-of-the-world. This 'region' grows massively in population through natural increase and this is transmitted to the other regions in the system through the set of constant rates associated with the 11th region. In other words, the population scenario depicted by the model is one of immigration continuing at present rates and increasing in absolute numbers substantially as population builds up in the rest-of-the-world. This flow and the question of future immigrant numbers are the subject of much debate today.

6. Comments on the forecasts and suggestions for improvement

What comments can we make about this prototype model derived that will enable us to improve our forecasts?

Firstly, we may note that there is probably a case for revising the $\underline{K}$ accounts matrix for 1970-71. The reader may have noticed the discrepancy between the end of period accounts populations (Figure 3) and the census 1971

POPULATION	*1971	1976	1981	1986	1991	1996	2001
1. North	3298.2	3320.3	3352.6	3395.8	3450.9	3519.0	3601.5
2. Yorkshire and Humberside	4807.2	4822.8	4855.3	4905.4	4974.3	5063.1	5173.3
3. North West	6759.4	6778.2	6818.5	6881.6	6969.4	7083.6	7226.5
4. East Midlands	3388.9	3518.3	3652.4	3793.2	3943.1	4104.1	4278.7
5. West Midlands	5115.8	5234.5	5366.0	5512.2	5674.8	5856.0	6058.2
6. East Anglia	1661.7	1768.8	1875.4	1984.1	2097.2	2216.9	2345.3
7. South East	17236.8	17633.2	18126.8	18722.5	19426.6	20247.0	21192.7
8. South East	3761.4	3952.6	4145.4	4343.6	4551.3	4772.1	5009.8
9. Wales	2725.4	2773.7	2827.7	2888.4	2956.5	3033.1	3119.3
0. Scotland	5223.7	5145.5	5094.9	5072.2	5077.7	5112.2	5176.6
1. Rest-of-the-world	3363362.9	3950647.1	4379803.6	4855364.0	5382349.9	5966326.7	6613461.5
PERCENTAGES							
1. North	6.11	6.04	5.97	5.91	5.84	5.77	5.70
2. Yorkshire and Humberside	8.91	8.78	8.65	8.53	8.41	8.30	8.19
3. North West	12.52	12.34	12.15	11.97	11.79	11.61	11.44
4. East Midlands	6.28	6.40	6.51	6.60	6.67	6.73	6.77
5. West Midlands	9.48	9.53	9.56	9.59	9.60	9.60	9.59
6. East Anglia	3.08	3.22	3.34	3.45	3.55	3.63	3.71
7. South East	31.93	32.09	32.30	32.56	32.86	33.19	33.54
8. South West	6.97	7.19	7.39	7.55	7.70	7.82	7.93
9. Wales	5.05	5.05	5.04	5.02	5.00	4.97	4.94
0. Scotland	9.68	9.36	9.08	8.82	8.59	8.38	8.19
TOTAL	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Figure 7 The forecast populations of British regions ('000s)

*Base population

figures given in Figure 8. The mid-year estimates just prior to the census proved to be overestimates. The accounts matrix could be adjusted by inserting the census 1971 population figures and by recalculating the $K^{\varepsilon(i)\sigma(i)}$ and $K^{\varepsilon(i)*(*)}$ terms holding the others constant. New $\underline{H}$ and $\underline{G}$ matrices would then be generated with slightly higher migration and slightly lower 'staying' rates, and a new forecast made.

Secondly, we should undoubtedly move from the constant $\underline{G}$ assumption of equations (4) to (7) to the variable $\underline{G}$ framework of equation (9). This is necessary because we already know that birth rates, for example, have continued to fall considerably in the 1970-76 period, and migration rates may have also shifted a little. Figure 8 shows estimates of the regional birth rates in 1970 and 1976, and compares this forecast with two official forecasts prepared on a 1969 and a 1973 base respectively. The lower birth rate levels of 1972-73 are assumed to continue in part in the rest of the century in the 1973 forecast and this is reflected in the lower total projected for the total population of Great Britain compared with either the 1969 forecast or the one outlined in this paper. It is probably possible to adjust the $\underline{G}$ matrix to reflect the trend in falling birth rates and the possible alteration in the pattern of migration picked up in the 1973 based forecast, though there is the problem of comparing differently defined regions (see Figure 1).

The third comment that should be made concerns the status of populations flows from and to the rest-of-the-world. In the section 5 model these were modelled endogeneously. It might be more realistic to assume instead that the flows, being subject to legal restrictions, quotas and so on, behaved as exogeneous variables to be specified at more constant levels than those generated in the section 5 model. The growth model of equation (4) would then become, for the British regional system, either

Figure 8 Birth Rates 1970 and 1976, and Forecast Population 1991 for British Regions

Region		Birth Rate 1970 (per 1000)	Birth[1] Rate 1976 (per 1000)	1991 Populations (share & total) 1969[2] Based Forecast %	1970-71[3] Based Forecast %	1973[4] Based Forecast %
1	North	15.9	11.1	5.9	5.8	5.5
2	Yorkshire & Humberside	17.0	11.4	8.5	8.4	8.5
3	North West	16.7	11.6	12.3	11.8	11.5
4	East Midlands	16.6	11.8	6.8	6.7	7.4
5	West Midlands	17.5	11.8	9.7	9.6	9.3
6	East Anglia	15.6	11.8	3.5	3.6	4.0
7	South East	15.5	11.2	32.0	32.9	30.8
8	South West	15.1	10.9	7.3	7.7	8.4
9	Wales	15.5	11.5	4.9	5.0	5.1
10	Scotland	16.8	11.6	9.2	8.6	9.3
	Total, Great Britain %			100.0	100.0	100.0
	'000s			60463	59122	56348

Notes

1 Estimated from Weekly Returns to 19th March 1976, O.P.C.S. Monitor VS 76/11.

2 Source: Table 3.2 in Department of the Environment (1971).

3 Source: Figure 8.

4 Source: O.P.C.S. (1974) and O.P.C.S. (1975). These forecasts are for the Standard Regions after Local Government Reorganization.

$$\underset{11\times1}{\underline{w}(t+1)} = \underset{11\times10}{\underline{G}}\ \underset{10\times1}{\underline{w}(t)} + \underset{11\times1}{\underline{I}(t,t+1)} \tag{11}$$

if just immigrants were treated exogeneously, or

$$\underset{10\times1}{\underline{w}(t+1)} = \underset{10\times10}{\underline{G}}\ \underset{10\times1}{\underline{w}(t)} + \underset{10\times1}{\underline{I}(t,t+1)} - \underset{10\times1}{\underline{E}(t,t+1)} \tag{12}$$

where $\underline{I}(t,t+1)$ is a vector of immigrants from the rest-of-the-world to the regions over the period t to t+1 and $\underline{E}$ is a vector of emigrants from the regions over the period t to t+1.

Fourthly, one might note that each of these problems can probably be dealt with more effectively by using the forecasting version of the accounts based model employed earlier to generate the accounts for 1963-66 and 1970-71. Such a forecasting version is being developed.

A fifth problem posed is that of adjusting the forecast to the new regional basis shown in Figure 1 and for which new regional forecasts have already been prepared. Revised population vectors are already available but adjustment of rates requires care. Rogers (1969) has outlined how adjustment to the $\underline{G}$ matrix can be made by pre-multiplication by a consolidation matrix $\underline{C}$ and post-multiplication by a deconsolidation matrix $\underline{D}$:

$$\hat{\underline{G}} = \underline{C}\ \underline{G}\ \underline{D} \tag{13}$$

where $\hat{\underline{G}}$ is the newly adjusted matrix. The $\underline{C}$ and $\underline{D}$ matrices are prepared from a mapping of population stocks from one set of regions to the other. Alternatively, the accounts matrix itself may be aggregated in a fashion described by Stillwell (1976) using row, $\underline{R}$, and column, $\underline{C}$, aggregation/disaggregation operators:

$$\hat{\underline{K}} = \underline{R}\ \underline{K}\ \underline{C} \tag{14}$$

Care, however, has to be taken in this process to allow for the situation where persons who have moved only within the region previously and are classified as surviving stayers in the accounts become inter-regional migrants as a result of boundary change (Illingworth, 1975b). Figure 1 shows that there could be a serious problem in the North of England.

Finally, there are a set of problems which have been ignored in this paper and which require solution in revising the forecasts of the population of British regions. The models need to be disaggregated by age and sex, and by other variables such as ethnicity or social class. The fertility rate forecasts need to be tied to leading indicators of family size norms and intentions and possibly economic indicators. The migration rate forecasts need to be tied to leading or forecast economic indicators.

Having commented at length on some of the drawbacks of the models and forecasts developed to date, one should perhaps conclude by emphasizing their strengths. An accounting framework forces the researcher to pay attention to all population flows, and in particular those to and from the rest-of-the-world. The accounts based model connects the framework with data normally available. The growth model of Rogers represents one use of the information represented in the accounts and lays bare the multi-regional interactions at work in the population system. Thus, we can say on the basis of our forecast in section 5 that the year 2000-2001 will see the emigration of the following numbers of persons from each region to the other regions (Figure 9). Although our confidence in those forecast may be low, other projections do not to date ever produce such a picture.

Figure 9 Part of the Accounts Matrix for 2000-2001

Birth in 2000-2001 OR Existence at C.D. 2000: Initial State \ Final State	Survival at C.D. 2001: 1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RW
1. North	3516408	9995	7930	3906	4078	2356	16182	4164	1313	5283	27194
2. Yorkshire & Humberside	13804	5056838	14343	14236	6443	4662	23292	6055	2774	3746	30098
3. North West	7261	11389	7083526	6758	10433	3186	31237	9570	11089	5561	47001
4. East Midlands	5347	15495	6748	4147824	12236	7720	24331	8504	2967	3946	32524
5. West Midlands	4165	6335	11440	14784	5929841	3566	30072	14772	7450	3919	32708
6. East Anglia	1609	3512	2780	6282	3553	2234888	28626	5344	1636	2308	39445
7. South East	14750	20250	28011	32249	26164	43275	20570324	81356	14727	21520	236234
8. South West	3951	6119	6465	5626	11546	4510	65223	4812619	6292	6398	47158
9. Wales	1658	2334	7311	6434	6464	1464	15055	8008	3052902	1887	11653
10. Scotland	5532	4803	7209	4903	4622	2067	23754	5322	1697	5069999	57526
11. Rest of the world	27023	36255	50754	35697	42843	37570	364554	54058	16475	52076	661289998
Sub totals											

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