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SPATIAL INTRACTION AND STRUCTURAL MODELS IN HISTORICAL ANALYSIS: SOME POSSIBILITIES AND AN EXAMPLE

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Abstract

Heightened awareness of the importance of the spatial dimension in historical societies, which is evident in recent publications, can be complemented by the employment of more sophisticated methods of spatial analysis. After a brief retrospect of methods and models, a general spatial interaction and location model is presented, with particular attention paid to presenting the mathematics as simply as possible by using an illustration which can easily be related to everyday experience - the workings of a retail system.

This general model can be adapted to suit the requirements of a particular historical society: Ancient Greece, in the example here. The results of such analysis of Ancient Greek poleis are presented and discussed, and the value of undertaking the analysis is assessed.

1. Introduction

The purpose of this paper is to show how modelling techniques which are being developed as part of contemporary geographical theory have interesting applications in ancient history (and, in a related way, in archaeology). Interest in the study of settlement and spatial organization in the past has recently been invigorated owing to a heightened awareness that knowledge of historical societies can be extended and deepened through investigation and incorporation of information present in the spatial distribution and organization of social phenomena. Most studies are currently discursive and pictoral however, with little or no attempt to analyse the spatial information easily available in the innumerable maps which accompany such studies (eg. many of the papers in Faull (ed.), 1984 ; Roberts and Glasscock (eds.), 1985). A growing minority do involve analysis, but such analyses are usually confined to drawing Thiessen polygons around known or inferred 'centres' and/or conducting statistical tests in order to try to identify patterns or correlations between sites/ structures/finds (eg. Bintliff, 1984 ; Knight, 1984). Spatial flow models have been used occasionally (eg. Crumley, 1979 ; Doorn, 1985 ; Hodder, 1974 ; Jochim, 1976) but in most cases the focus has been on interaction alone and the models used have typically been of a rather primitive kind - the old 'gravity' model, in one of various guises. Rarely, more interesting models and methods have been employed (eg. Zubrow and Harbargh, 1978), and those developed by Wilson and colleagues in the 1970's have been considered (by eg. Clarke, 1977 ; Crumley, op cit) and employed (Chadwick, 1978). We aim to show here that there is a wider family of models from which to choose for particular applications and that the modelling concepts can now be extended to cover aspects of locational analysis as well as interaction. It is this particular feature which we use in our illustration later.

We proceed in the following stages. First, in section 2, we outline spatial interaction models and show that they can be used in locational analysis in two ways. In section 3, secondly, we discuss in broad terms the potential application of these models in history and archaeology. Thirdly, in section 4, we present a particular example which appears to demonstrate quite strikingly the potential of the approach. Some concluding comments are presented in section 5.

2. Spatial interaction and locational analysis

Spatial interaction modelling is concerned with reproducing, predicting and explaining flows between one set of located entities and another. In a contemporary context, the most frequent applications are at the urban scale: the journey-to-work pattern linking residential and workplace areas, the retail flow pattern linking residences and shopping centres, patient flows linking residences and hospitals, and so on. But there are also inter-urban or inter-regional applications - for example, in relation to migration or long-distance freight. There are even inter-national applications - to migration again, or to trade flows.

The origins may be spatially-extensive zones, or points; the destinations similarly can be either. Sometimes the same spatial system is used for both origins and destinations, but they can be different. In the example to be used below, origins and destinations are settlements, considered as an approximation to be located at points and the same system is used for both origins and destinations. This is illustrated in Figure 1. There are 109 settlements, numbered consecutively from 1 to 109. For convenience and generality in the specification of model equations, a typical origin zone is labelled i, and a typical destination zone, j. A flow can then be defined as T_{ij}, the flow from i to j. In this case, i = 1, 2, ... 109; j = 1, 2, ... 109; and so there are 109 x 109 = 11,881 possible flows. The model is used to estimate these flows in terms of a much smaller number of variables. Usually, this involves an i-variable (or set of variables), a j-variable (or set) and some function of a measure of the distance (or cost) of travel between i and j. Thus the traditional gravity model might take the form

$$T_{ij} = K \frac{P_i P_j}{d_{ij}^2} \tag{1}$$

with P_i as the population of zone i; P_j, the population of zone j; d_{ij} the distance from i to j; and K, a constant. Most of the models previously used in history and archaeology are not far removed from this level of sophistication. The first step in improvement is to recognise that attenuation of trips with distance may not be an inverse square law, and so a parameter β could be used:

$$T_{ij} = K \frac{P_i P_j}{d_{ij}^{\beta}} = KP_i P_j d_{ij}^{-\beta} \tag{2}$$

The parameters K and β can be estimated to give a 'best fit' of model estimates against data. Alternatively again, another function could be used instead of $d_{ij}^{-\beta}$. There are good theoretical reasons in many circumstances for believing that a negative exponential function is better than a power function (cf. Wilson, 1967, 1970). This is also a convenient point to introduce a more generalized measure of 'distance', usually represented as c_{ij} rather than d_{ij} - which may be cost, travel time or some combination of these and other variables when data is available. Henceforth, we will use c_{ij} rather than d_{ij} but leave it to be specified in particular contexts. Thus

$$T_{ij} = KP_i P_j e^{-\beta c_{ij}} \tag{3}$$

where e is the base of natural logarithms.

The next step towards refinement is to replace P_i and P_j by measures of total trips originating at i, say O_i, and total trips with destination at j, say D_j. The O_i's and D_j's may be functions of population, but also other variables:

$$O_i = O_i(P_i, \ldots) \tag{4}$$

$$D_j = D_j(P_j, \ldots) \tag{5}$$

$$T_{ij} = KO_i D_j e^{-\beta c_{ij}} \tag{6}$$

This model still has basic 'gravity' characteristics and was common (though probably with a power-distance function) in various guises in the 1930s, 40s and 50s, though it has its origins in migration studies in the nineteenth century (eg. Ravenstein, 1885).

The main breakthroughs came in the late 1950s and the 1960s, first in the context of flow modelling for transport planning - mainly by civil engineers; and secondly, in the broader context of land-use planning (with both retail modelling specifically and more broadly-based urban models). These advances were systematised in the late 60s and absorbed into the corpus of geographical theory and associated planning disciplines. There were a variety of possible theoretical justifications - each more or less appropriate for a variety of circumstances. For a recent review, see Wilson and Bennett (1985), especially Chapter 12.

The main thrust of these breakthroughs related to the constant, K. It was quickly realised that in many applications, the model predictions of the flows, $\{T_{ij}\}$, were inconsistent with information on known sub-totals. In particular, either or both of O_i and D_j in (4) and (5) were often known independently of T_{ij}, so that one or both of the following sets of constraints had to be satisfied:

$$\sum_j T_{ij} = O_i \tag{7}$$

$$\sum_i T_{ij} = D_j \tag{8}$$

The Σ-signs represent summation, so, for example,

$$\sum_j T_{ij} = \sum_{j=1}^{N} T_{ij} = T_{i1} + T_{i2} + T_{i3} + \ldots + T_{iN} \tag{9}$$

For (7) to be satisfied, the constant K had to be replaced by a set of constants, say A_i - each calculated 'internally' in the model; and (8) demanded a set, say B_j. Thus the models become, instead of (6),

$$T_{ij} = A_i O_i W_j e^{-\beta c_{ij}} \tag{10}$$

$$T_{ij} = B_j W_i D_j e^{-\beta c_{ij}} \tag{11}$$

$$T_{ij} = A_i B_j O_i D_j e^{-\beta c_{ij}} \tag{12}$$

according to whether O_i, D_j or both are known exogenously. Note that where O_i or D_j are not known, they are replaced in the model by a factor W_i (or W_j) which is taken as an 'attractiveness' term, representing push or pull factors, but not directly a trip total. Thus (6), (10), (11) and (12) represent different members of a family of models, one being selected in a particular case which is appropriate to the circumstances (Wilson, 1971).

To take the argument a stage further, we focus on (10), both because it is particularly interesting and because it is the one we use in our example below.

As a preliminary, we extend the model slightly. When a term like W_j is used in a model like that of equation (10), it is being used to represent the pulling-power or 'attractiveness' of j. In many applications, a size measure is used as a proxy for attractiveness; but, then there is no reason to assume that it appears in a linear way. So, as we introduced the β parameter, we can now, in such cases, replace W_j by W_j^{α}, where W_j is some measure of size. The model now becomes

$$T_{ij} = A_i O_i W_j^{\alpha} e^{-\beta c_{ij}} \tag{13}$$

Some algebraic manipulation shows that if (7) is to be satisfied, then A_i in (10) has to be:

$$A_i = 1/\sum_k W_k^{\alpha} e^{-\beta c_{ik}} \tag{14}$$

where the summation on the right hand side is over all sites, k. We begin to get a clearer idea of how to interpret this if we substitute from (14) into (11) to give

$$T_{ij} = O_i \frac{W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} \tag{15}$$

It may even help to spell out the summation sign explicitly in the denominator of the right hand side:

$$T_{ij} = O_i \frac{W_j e^{-\beta c_{ij}}}{W_1 e^{-\beta c_{i1}} + W_2 e^{-\beta c_{i2}} + .. + W_j e^{-\beta c_{ij}} + .. W_N e^{-\beta c_{iN}}} \tag{16}$$

Thus the known total of flow, O_i, which originates at i is being shared out to each j in proportion to $W_j^\alpha e^{-\beta c_{ij}}$ (an attractiveness term multiplied by a distance function) but in a way which takes account of the spatial distribution of the 'competition' of other k's, $k \neq j$. This spatial competition is represented by the sum of terms in the denominator. A little thought and experimentation with a variety of cases shows that this mechanism can in principle work very well; and in practice, it does - for example, in modelling retail flows. Indeed, to fix ideas, it is convenient to think in terms of retailing as an example - and those unfamiliar with this modelling style will be able to see how the model concepts illuminate everyday experience. At this stage in the argument, a leap in imagination may be needed before the relevance of such an analogy to historical analysis can be seen - but the connections will be established in section 3.

It is at this point that the first element of locational analysis can be introduced. Since the flows into destination zones are unconstrained, D_j can be calculated from the flows, T_{ij}, for each j in turn:

$$D_j = \sum_i T_{ij} \tag{17}$$

In retail analysis, this is invaluable: it is an estimate of the revenue attracted into each shopping centre (or, at a finer level of detail, store).

We can also use the retailing analogy to explain the next step in the deployment of spatial interaction concepts in locational analysis. This takes us from the mid 1960s to Harris and Wilson (1978) and to a spate of work in the current decade (cf. for recent examples, Wilson, 1981, Birkin and Wilson, 1985-A, 1985-B, Wilson and Birkin, 1986, M Clarke and Wilson, 1983, 1985; G P Clarke and

Wilson, 1986-A, 1986-B). The focus now shifts to the attractiveness term, W_j, for each j. For simplicity, in the retailing case, suppose it can be measured by size - eg. square footage. (This is, of course, a proxy for a number of variables, many of which can be included explicitly - but this simple assumption serves the illustration adequately.) Thus the set of W_j's, $\{W_j\}$ say, represents a locational pattern. For instance,

$$\{W_j\} = \{W_1, W_2, W_3 \ldots W_N\} = \{100,0,0,0, \ldots 200,0, \ldots 0, \ldots 300,0,0\} \tag{18}$$

represents a small number of large centres, while

$$\{W_j\} = \{5,6,4,2,3,1,5, \ldots 7,5\} \tag{19}$$

represents a large number of small centres - very different geographical structures.

In the case of the retail model described above, this pattern is given, and the sets of flows and revenues are found as functions of it. The question now arises: can we develop the model further to predict this pattern? The answer is affirmative and the procedure is best explained in relation to the retail example. For a hypothesised structure, $\{W_j\}$, a set of revenues $\{D_j\}$ can be calculated using (11), (14) and (17). In each centre, j, the retailer or developer will have a cost of running the centre, say C_j. Then, if $D_j > C_j$, j is profitable and W_j might grow; and vice versa. Clearly, there is the possibility of equilibrium when

$$D_j = C_j \tag{20}$$

for each j.

To take matters further, let us make a further simplifying assumption - that

$$C_j = k_j W_j \tag{21}$$

That is, cost is proportional to square footage in each j. Then the equilibrium condition is

$$D_j = k_j W_j \tag{22}$$

for each j. This set of simultaneous equations can be made explicit if we substitute for D_j from (17) and T_{ij} from (15):

$$\sum_i O_i \frac{W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} = k_j W_j \tag{23}$$

These equations, one for each j = 1,2, ... N, are highly non-linear in the W_j's, not only because α may not be equal to 1, but also because of the denominators - the competition terms - in each term of the summation on the left hand side. Nonetheless, it is possible to achieve some analytical insights and also to solve the equation numerically on a sufficiently large and powerful computer.

Broadly speaking, the higher α and the lower β, the smaller number of larger centres there is in the pattern, and vice versa. This makes intuitive sense because α measures the importance of 'size' to consumers and β (inversely) the 'ease' of travel. The patterns are also sensitive to the underlying transport networks as reflected in the c_{ij}'s and the pattern of demands, $\{O_i\}$.

Before concluding this section, it is worth drawing attention to some important general features of this type of model which arise from their nonlinear nature. The models we have used to illustrate the argument are <u>equilibrium</u> models (though dynamic extensions are also possible - see Wilson, 1981). The nonlinearities imply that, when the equations are solved, the solutions are not unique. There is usually a 'global' optimum, which we use in the examples below; but there are other possibilities also. It is in this way that the mathematical analysis coincides with historical common sense: there may be 'forces' (say economic) driving a system but particular acts of human agents also determine the state reached. This interaction of structure and agency can be seen as determining which specific equilibrium is achieved (or, at least approached) out of the many available. A second important feature is that discontinuities are possible, which should be interpreted in the historical case as structurally-based rapid change. This connects this kind of analysis to catastrophe theory (cf. Thom, 1975) and bifurcation theory, and although this has been used before in historical contexts (eg. Amson, 1974, Mees, 1975, and Wagstaff , 1978) this approach is richer in that it involves a much larger number of interdependent variables.

The method described above has been used to give a model-based account of the evolution of shopping centre structures in Leeds (G P Clarke, 1986; G P Clarke and Wilson, 1986-A, 1986-B) - and in this particular case, there are discontinuities, as in the shift from small-shop food retailing to supermarket structures (Wilson and Oulton, 1983). The next step in the argument is to show that this apparently very specific model has a very general structure and has a potentially useful application in historical analysis. This we do in the next section in the context of our example.

3. <u>Spatial interaction and structural analysis as a contribution to method in history: a general argument and an example</u>

History is enacted on a spatial backcloth. Explanation in history must recognise this and spatial relationships are often of considerable, even crucial, importance. Models can potentially play two roles: first, to help in the articulation of spatial relationships; and secondly, to fill in gaps in data or knowledge through a capacity for inference based on hypothesised spatial relationships which are thought to have existed. The first kind of application is probably of most relevance in fields like economic history or historical demography. It is the second, and more unusual on which we concentrate here.

The general principle involved is this: if enough evidence exists to 'calibrate' a model from partial information, then model outputs can be used as estimates for the 'data' which are missing. To fix ideas, it is best to proceed in terms of a specific example, though this does not exhaust the possibilities.

The example we take is Greece in the Ninth and Eighth Centuries BC. A map is shown as Figure 1 showing the location of settlements which were thought, on the basis of a variety of types of evidence, to have been occupied at that time. We make the assumption that these locations represent the only known 'data'. This neglects the fact that we know that some settlements were much more important than others: we use this information partly to calibrate the model and partly to validate it.

Consider the model given by equations (15), for the flows, and (23) for the geographical structures. How can we begin to apply it to the settlement patterns of Ancient Greece? There are, in this case, 109 settlement sites and all we assume at the outset is the location. Suppose they each have a notional population, P_i, $i = 1,2, \ldots 109$ and that W_j is some other measure of size or 'importance'. Let c_{ij} be a measure of the 'distance' between i and j. Then the model flow equations, (15), are

$$T_{ij} = P_i \frac{W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} \tag{24}$$

and the structural equations to be solved for $\{W_j\}$ are, from (23),

$$\sum_i \frac{P_i W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}} = k_j W_j \tag{25}$$

It is ludicrous, of course, to interpret these equations in retail or service terms, but for present purposes, T_{ij} can be thought of as a measure of 'influence' of j over i - and this may indeed correlate with different but for ever unknown flows. So we cannot use the flow equations directly. However, we can set all the P_i's to some common value, P, so as not to make any prior judgements about 'size' or 'importance' and then solve the non-linear simultaneous equations (25) for $\{W_j\}$. It is then possible to set $P_i = W_i$ and to recalculate new W_j's, and to proceed iteratively. This reinforces the results obtained without this feedback and therefore we use them for illustrative purposes here.

In order to solve the equations, we need specific values for α and β; and then the equations can be solved numerically on a suitably large and fast computer: typically a mainframe rather than a micro is necessary at the time of writing. As with the flows, we cannot 'know' α and β because the usual data for model calibration does not exist. However, we do have additional information which can be brought into play: as a first stage, we could argue that we know, roughly, the number of major settlements out of the 109; as a second, we could add information about which particular ones of the 109 are known to be historically important. In either case, we try to find values of α and β which reproduce the appropriate types of structure. However, it is nice to be able to make predictions at stage 1 and to compare the model's predictions of 'importance' with historical knowledge. It turns out that this is indeed possible and that the results are quite striking. We present these in section 4.

What, then, in principle, can these models achieve in a historical context? They can illuminate structures by showing how they can arise from hypothesised spatial relationships. In some cases, where data is very poor, provided minimalist calibration is possible, model estimates of all the 'missing quantities' can be treated as estimates of what might have been - though caution in interpretation needs to be uppermost. One possibility illustrates this: if a site is consistently shown to be 'important' in model runs, and has not previously been thought important by historians, in the absence of evidence, then in suitable circumstances it may be worth encouraging archaeological exploration at that site before others!

4. Some results for an example: settlement structure in Ancient Greece

The specific model presented here was developed in order to assist investigation of the emergence of the polis, or city-state, in Ancient Greece around the eighth century BC. The spatial dimension of the historical problem requiring explanation is basically two-fold. Firstly, there is the question of urbanisation; in particular, the emergence of one settlement within a community territory as the unequivocal capital (or primate) settlement of the community, in many cases the name of which was taken as an identifier by the populace of the whole territory (eg. Athens-Athenians) even though the majority of the populace did not reside in the city itself. Secondly, there is the question of state formation; in particular, the union of people living in dispersed settlements to form one sovereign community. Greek tradition remembered this episode in their history as a wilful human act of creation involving a synoikism, a 'coming together' to form a united association, and there is no good reason to reject the essence of this tradition: a union of communities involving the transfer of sovereignty from village (or group of villages) to a larger association as a conscious human act. Physical migration might have been involved in some cases,[1] but the important union was psychological: the recognition by all members of sharing a common but exclusive identity. It is no linguistic accident that 'community' derives from in communis, and in modelling this aspect of Ancient Greek settlement structures we are assuming that the relative level of communication between discrete settlements was the chief factor determining membership of one multi-settlement community rather than another.

From a modelling point of view, and in relation to the available data, these two questions can be put slightly differently, but in an obviously related way. Given the location of known settlements (the only 'data'), first, what can the model say about relative sizes? Secondly, there is a further, regionalisation, task: can the model suggest how settlements can be grouped together in some functional way? The historical question of the interpretation of these regions as possible poleis can then be tackled. We can proceed, therefore, with the model presented as an example at the end of the previous section and we now present results based on this.

Consider Figures 2 and 3 . They are plots of the same analysis (α = 1.01, β = 0.15), but in Figure 2 only the maximum flows from i to j are depicted, in Figure 3 all flows at or above 50% of the maximum are depicted. There is also a prior selection of 'more important' j's as 'terminals' in these plots - building on Nystuen and Dacey (1961) as developed in Wilson and Kirkby (1976). In effect, terminals are those settlements which dominate the associated (hypothesised) flow matrix. Figure 2 better indicates local centres, Figure 3 interaction between local sites.

As the parameter values are varied, so the results of the analysis vary. For example, compare Figures 4 and 5, which as before, depict maximum flows (4) and $\geq$ 50% maximum flows (5), but with slightly different parameter values (α = 1.025, β = 0.15). Notice how small a change in parameter values (ie. change in α of 0.015) can produce significantly different (but not inexplicably different) results. This illustrates the model's sensitivity and robustness.

An indication of how results change across a range of parameter values is shown in Figure 6. The x-axis is beta values, the y-axis is alpha values. The number of terminals, which are assumed to be higher order centres, at any point in the parameter space is indicated and 'contours' have been drawn around each area of equal terminal count. The terminal count serves as an index of degree of centralization: one terminal would mean that there is one highest order (or primate) site within the survey area and that all other sites are nested into hierarchies within the same system; three terminals means that here are three highest order sites and thus three settlement systems into which all sites in the survey area are grouped; ten terminals means that there are ten highest order sites and thus ten settlement systems, and so on.

Finding the appropriate parameter range for the survey area and society under examination is initially a matter of trial and error, where 'appropriate' is dependent also upon what kind of interaction one is attempting to model. If, for example, the system of interest is political units and the survey area envelopes two or more sovereignties, as in this case, then the α,β values which define an area of parameter space in which only one terminal is predicted are inappropriate. On the other hand, the sovereignties will probably vary in territorial size, as (markedly) in this case,[2] and the parameter range should therefore have as maxima those α,β values which define the least number of terminals in the system as a whole necessary to define the largest territorial unit as one unit (ie. with one terminal), in this case three. To establish minima for the range of parameter values the procedure might be inverted; the α,β values which define the most terminals in the whole system necessary to define the smallest unit as a unit. In this case there is insufficient evidence to establish minima; the partial state of the evidence was the stimulus to develop this model and is what gives it a predictive potential.

As stated above (section 3), the model must reproduce observed data reasonably accurately, but because the only data input is location co-ordinates, sites for which observed (ie. known) data is, minimally, location alone, can be included in an analysis. In such a case, the model potentially goes beyond reproduction of the known to prediction of the (currently) unknown by inference. Therefore a survey area must always include some sites[3] which are well known in order to test results against observed data; else there is no justification for prediction from those results. We now test the results against this criterion.

Consider Figures 2 and 4. Athens, Korinth, Argos and Thebes are within the top ten ranked sites, as we should expect: though recall that the only information input to the model was site location. The relatively significant change in rank in this group (Korinth, 1:10) is easily explained by the shift of regional centre to Kromna in Figure 2, which models a situation in which the importance of size is less significant than it is simulated to be in Figure 4 (by means of a smaller α value), where Korinth dominates. Such shifts are noticeable in attenuated fashion elsewhere in the same analyses, for example, Argos and the Heraion in the Argolid, Merenda and Kalyvia in the Mesogeia plain (Attika). Brief comment on some of the sites which are also in the 'top ten' but which are much less well known should suffice to illustrate the potential value of the model.

Akraiphnion has become a familiar name in Greek studies only recently. The cemeteries began to receive serious archaeological attention in 1974, and quickly became recognized as one of the most exciting discoveries of the decade in Greek archaeology. After one season some 400 richly furnished graves had been excavated, producing, for example, over 2000 vases, from Attika, Euboia and Korinthia as well as other areas of Boiotia. (See Archaeological Reports 1974/5: 18, 1975/6: 16; 1980/1: 22 for summaries pending proper publication.) The total number of graves revealed was in excess of 1100 (Hodkinson, 1986). Kromna, on the other hand, has received no such attention, and may serve to suggest the predictive potential of the model. This potential cannot, of course, be tested until and unless someone excavates Kromna (but had it been excavated, it could not serve to illustrate this potential utility of modelling settlement structures in historical contexts). Currently, knowledge of Kromna is meagre; but what exists is consistent with model predictions.[4] There has been sporadic excavation of odd graves; one found in 1960, for example, contained 26 vases (closely dated to 560 BC; see Wiseman, 1978: 66). John Salmon walked the area a few years ago, and the sherd-scatter suggested to him a large and substantial settlement, occupied for over a thousand years (from at least the seventh century BC to the fourth century AD; (1984: 25, 35, 156)).

The prediction of Hyria(?) would appear to be erroneous, arising from an assumption that the three sites on the island of Euboia, which were included in the survey area specifically to experiment with modelling the topographic problem of an island, should be unfavourably weighted vis-a-vis the sites on the mainland to reflect the difficulty of crossing the Euripos. In the analyses above (and indeed, in most experimental running) Khalkis, Lefkandi and Eretria were rendered half as distant again from all other sites - but not from each other - as their location coordinates and an isotropic plain (which is assumed unless so modified by the user) would suggest. The same is true of the three sites on the Perakhora peninsula (numbers 74, 75, 76) to reflect the mountaneous terrain of the peninsula. A plot of an unmodified simulation with precisely the same parameter values as (2/3) is given in Figures 7 and 8 which, like (2) and (3), depict maximum and $\geq$ 50% maximum flow. [For completeness, plots of the same parameter values as 4/5 are given in Figures 9 and 10 which again depict maximum and >50% flows.]

The overall similarity between the two pairs of plots again illustrates the model's robustness, while the differences illustrate its sensitivity. Note the rank changes generally: Argos and the Heraion swap order (as they do between $\alpha = 1.01$ (Figures 2 and 3) and $\alpha = 1.025$ (Figures 4 and 5)); Korinth and Kromna are more closely (in fact, sequentially) ranked; the rank difference between Athens and Merenda has increased; Thebes has risen in rank by two; Akraiphnion and Koroneia have dropped by one rank place each. Khalkis, now unprejudiced by user prejudice, has been predicted instead of Hyria (?), and at a higher rank than Hyria(?) hitherto. Khalkis is another site within the survey area which we should definitely expect to rank highly, in the light of the archaeological reports which consistently affirm and enlarge upon her importance in the early period as attested by literary evidence (Sampson, 1980, 1981). Contrary to our assumptions on this matter, an isotropic plain would seem to be a more reasonable landscape on which to model settlement interaction - at least for this case.

The plots and rank orderings mask more subtle differences and similarities revealed in the numeric output (this is true generally). These should not be overlooked - the former can be misleading. For example, the results of the analysis in Figures 7 and 8 show that the Heraion takes precedence in rank over Argos because its numeric estimate of importance is 103.78, whereas that of Argos is 102.13 (in a range extending from 0 (many sites) to 128.37 (Athens, rank 1), and where eg. site number 104 'scores' 1.12). That is, there is about 1% difference in calculated importance: a negligible difference. Likewise, Kromna is ranked higher than Korinth because the former scores 69.69, the latter 67.92. Khalkis' Figure is 95.41, about 4 more (or < 5%) than Akraiphnion. In general, we could say that less than 10% difference in final numeric results should be ignored, given the state of the evidence by which 'accuracy' must be judged.

The analysis so far has been concerned with the relative size or 'importance' of centres, and the relationships between them based on estimates of interaction. We conclude by drawing out the regionalisation features of the analysis and draw some conclusions on the development of poleis.

Figures 11-15 depict increasing levels of centralisation within the survey area, effected by either increasing α or decreasing β values.[5] Alpha reflects importance over and above size; beta reflects ease of communication, and we can achieve the same system level through setting relatively high α, high β values <u>or</u> relatively low α, low β. However, whilst the <u>level</u> thus simulated may be identical, the systems themselves may be quite different, as we shall see below, because the correspondence between α and β changes is not 1:1.

Figure 11 reveals thirteen settlement systems: a relatively devolved structure in which the enormous Athenian territory (Attika) is composed of four discrete regions, but in which a more normal sized <u>polis</u>, such as Kleonai (number 88), has been incorporated into a larger system, in this case that centering on the Argive Heraion (number 98). Note, however, that this site is identified as a sub-regional centre in the N. Argolid, and that for much of its known history, Kleonai "belonged to Argos, or at least was in some way attached to Argos" (Tomlinson, 1972: 29). It is significant too that Krommyon (number 77), once thought to be a border town between Korinth and Megara, but recently shown in the style of its artistic products and the burial practices of its residents to be wholly Korinthian (Salmon, 1972: 202 ff; <u>ibid</u>, 1984: 48), is even at this level firmly part of the Korinthian system.

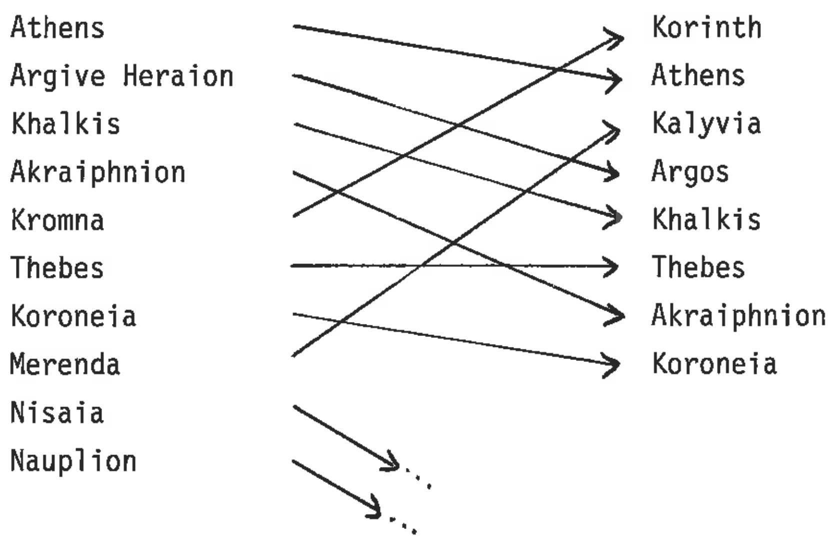
At a slightly higher level of centralisation, Figures 12 and 13 each depict ten settlement systems. Allowing for regional centre shifts (see below) they map as follows:

Koropi	→	Athens
Argive Heraion	→	Argive Heraion
Khalkis	→	Khalkis
Thebes		Akraiphnion
Ay. Ionnis		Kromna
Athens		Thebes
Ay. Gerasimos		Koroneia
Koroneia		Merenda
Nisaia	→	Nisaia
Kaki Thalassa	...	Nauplion

In Figure 12 there is an independent settlement system in S. Attika centred on Kaki Thalassa; in Figure 13 there is instead one in the S. Argolid centred on Nauplion. This is the most obvious difference between the two analyses; the Argive Heraion, Khalkis and Nisaia demonstrate the most obvious similarity. More subtle changes are revealed in the mapping: the image is not always the same site as the domain element. Ay. Ionnis, for example, is mapped to Akraiphnion. This settlement system in N. Boiotia has, between the two sets of parameter values, gained Aspledon (number 8), but lost Askra and Thespiai (numbers 20, 30) to Thebes (which has nevertheless lost two rank positions), and the centre of the system has shifted from Ay. Ionnis to Akraiphnion, which now ranks one higher than the 'former' centre. That is, the mapping is of system centre (terminal) to system centre, not site to site. This better indicates how the ten settlement systems as units change over different parameter values. So, in the Korinthia, Ay. Gerasimos is mapped to Kromna. Notice that in Figure 12 the second centre,Isthmia (number 79), ranks 9, two lower than the system primate; in Figure 13 the second centre, now Korinth (number 82), also ranks 9, but this is now four rank positions lower than Kromna. In raising the centralization of the system, the first and second centre have shifted inwards, towards each other, and the primate has gained several ranks vis-a-vis the other systems in the survey area. Slightly more centralization leads to the Korinthian system centre shifting and coalescing in the neighbour Korinth, leaving Kromna, for example, as second centre with rank 16, but with a total 'score' of 1.24 compared with Korinth's 148.83 (Figure 14) (in an analysis where the site ranked 10, Medeon (number 17) scores 45.30, and the site ranked 14, Koropi (number 57, scored 11.32). This renders Korinth, the system centre, rank 1 in the whole survey area.

For the purpose of the mapping Attika is treated as a single system (since we know that it was to become one system); complete consistency would demand that Athens be mapped to Athens, a rank gain of six, and Koropi to Merenda, a rank loss of eight. Over this change in parameter values which, we recall, produces the same number of settlement systems (or the same system level), the N. Attika system gains Draphi (number 53) and Aliki (number 66), has strong influence over (attracts $\geq$ 75% of the maximum flow from) Spata (number 54) and Vouliagmeni (number 64), and receives the maximum flow from Marathon (number 48, which hitherto flowed to Koukouvaones, number 52). Despite gaining the four most southerly sites (three indirectly via Kalyvia, number 59, rank 10) the S. Attika system nevertheless has lost ground vis-a-vis Athens and the survey area as a whole.

If we compare Figure 13 with Figure 14, which depicts an analysis in which the only change is an increase in the α parameter, we see that there are now only eight systems, the smaller having been incorporated into larger neighbouring systems (that centring on Nauplion has been incorporated into that centring on Argos/the Heraion; that of the Megarid has been incorporated into that centring on Athens). We can map them as before:



We have already made reference to the Korinthian system. Notice how at these more centralized levels importance is less diffused, particularly obvious in S. Attika where the centre, now shifted further south to Kalyvia (number 59) gains six rank positions, comparing favourably with Korinth's gain of five, despite losing Spata (number 54) to Athens.

Apart from Thebes, which retains its rank although losing Askra to Akraiphnion (via Medeon, number 17) whilst gaining Eleon (?)

(number 27) from Khalkis (number 40), all other rank changes are losses, occasioned by the considerable gains of Korinth and Kalyvia. The centre of the Argolid shifts south to Argos and the system accommodates that which had been centred on Nauplion, whilst losing Tenea (number 89), Zygouries (number 90) and Kleonai (number 88) to Korinth, which centre also exerts considerable influence ($\geq$ 75% maximum flows) on Phlious (number 86) and Nemea (number 87) at Argos' expense. The Argolid now experiences what the Korinthia was simulated to have experienced at the lower α value: a two-centre or shared foci system. Although Argos is the primate (terminal), the Heraion occupies the next rank position (rank 5), scoring 103 compared to Argos' 109 (in an analysis in which many sites score 0) - a negligible difference.

Likewise, at least some of Akraiphnion's rank demise can be explained by the rise of Medeon (number 17), which now ranks 10 and scores 45, about half the primate's 94. If the level of centralization is increased to the sort of level required to produce a united Attika (see Figure 15), Medeon or its neighbour Onkhestos (number 18) is often predicted to be the centre of Boiotia (and the N. survey area in general).(6)

5. Conclusion

It has been shown that a new generation of spatial interaction and location models developed in contemporary geography have great potential as aids in historical studies. Necessary data can be pared down to the minimum of a site's existence and location - nothing more - and the result that all other knowledge of the sites in the survey area can be used to test and to validate the model results. Having established the appropriate parameter space for the time and place in question (the calibration process), settlement structures in Ancient Greece have been reproduced, with a reasonable degree of accuracy (in view of the state of the evidence by which 'accuracy' must be judged), from a database consisting entirely of location co-ordinates. The high-level abstraction of the model components, necessitated by the relatively poor state of the evidence for many sites in this instance, forces an explanation framed in very general terms, such as 'size' and 'importance'. But even at this level of resolution the results can illuminate the growth of cities, and in particular 'capital' settlements of poleis, 'city-states', in Ancient Greece; the development (and perhaps also the (sudden) decline - the Argive Heraion?) of major intra-regional sanctuaries; and of settlement structures and communication networks from which emerged the multi-settlement poleis and the multi-poleis federations and leagues. By minimizing necessary input data to the location of known sites it is also possible to use the model to predict (rather than merely guess) which of the (currently) unknown sites might have been the more important (eg. Kromna; Merenda and/or Kalyvia in the Mesogeia plain of Attika).

It is the aim of future research to disaggregate some of these high-level abstractions by experimentation with better known historical societies.

Notes

1. As we know to have happened in later times, eg. Megalopolis, founded c.369 after the liberation of Messenia from Spartan occupation.

2. The average polis had a territory of approximately 70 m^2; Korinth was large with approximately 350 m^2. Athens is of a different order of magnitude with about 1000 m^2 territory.

3. Say at least 5%, or, in absolute terms, not less than three.

4. Kromna features regularly in the top ten in the same approximate area of parameter space as known major sites.

5. There are no weightings in these analyses, which therefore do not correspond exactly with Figure 6, produced from analyses in which the three sites on Euboia and the three on the Perakhora peninsula were 'distanced' with a 1.5 weight factor.

6. This is, of course, substantially beyond the level of political centralisation achieved in Ancient Boiotia, but it is perhaps significant that the sanctuary of Poseidon, which was one of the most venerated sanctuaries of Boiotia and the locale of the Pamboiotia ('all-Boiotia') Games, was located at Onkhestos.

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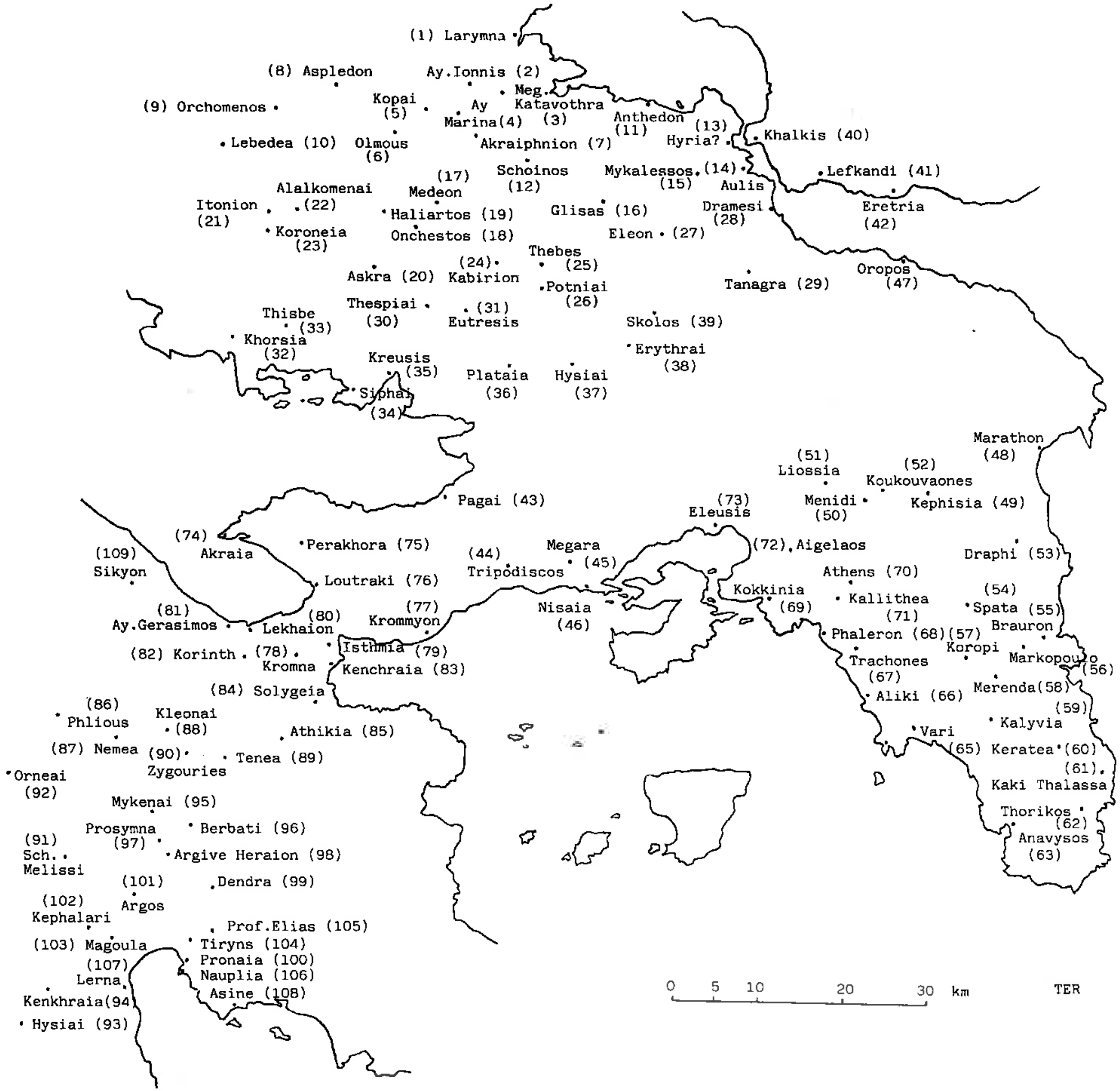
Figure 1



(1) Larymna
(8) Aspledon
Ay.Ionnis (2)
(9) Orchomenos
Kopai (5)
Ay Marina(4)
Meg. Katavothra (3)
Anthedon (11)
(13) Hyria?
Khalkis (40)
Lebedea (10)
Olmous (6)
Akraiphnion (7)
Lefkandi (41)
(17) Medeon
Schoinos (12)
Mykalessos (15)
(14) Aulis
Alalkomenai (22)
Itonion (21)
Haliartos (19)
Glisas (16)
Dramesi (28)
Eretria (42)
Koroneia (23)
Onchestos (18)
Eleon (27)
Thebes (25)
(24) Kabirion
Askra (20)
Potniai (26)
Tanagra (29)
Oropos (47)
Thespiai (30)
(31) Eutresis
Thisbe (33)
Skolos (39)
Khorsia (32)
Erythrai (38)
Kreusis (35)
Plataia (36)
Hysiai (37)
Siphai (34)
Marathon (48)
(51) Liossia
(52) Koukouvaones
Pagai (43)
Menidi (50)
Kephisia (49)
(73) Eleusis
(74) Akraia
Perakhora (75)
(72) Aigelaos
(109) Sikyon
(44) Tripodiscos
Megara (45)
Draphi (53)
Athens (70)
Loutraki (76)
Kokkinia (69)
(54) Spata (55)
Kallithea (71)
(81) Ay.Gerasimos
(80) Lekhaion
(77) Krommyon
Nisaia (46)
Brauron
Phaleron (68)
(57) Koropi
(82) Korinth
(78) Kromna
Isthmia (79)
Kenchraia (83)
Trachones (67)
Markopoulo (56)
Merenda(58)
(84) Solygeia
Aliki (66)
(86) Phlious
Kleonai (88)
(59) Kalyvia
Athikia (85)
Vari
(65) Keratea (60)
(87) Nemea
(90) Zygouries
Tenea (89)
Orneai (92)
(61) Kaki Thalassa
Mykenai (95)
Thorikos (62)
Prosymna (97)
Berbati (96)
Anavysos (63)
(91) Sch. Melissi
Argive Heraion (98)
(101) Argos
Dendra (99)
(102) Kephalari
Prof.Elias (105)
(103) Magoula
Tiryns (104)
(107) Lerna
Pronaia (100)
Nauplia (106)
Kenkhraia(94)
Asine (108)
Hysiai (93)
0 5 10 20 30 km
TER

*Figure 2: Eight systems (with topographical weightings).

$\alpha = 1.01$
$\beta = 0.15$
Maximum flows only depicted.

* In all figures the number is the predicted rank of a named site. High ranking but non-terminal sites are given in brackets.

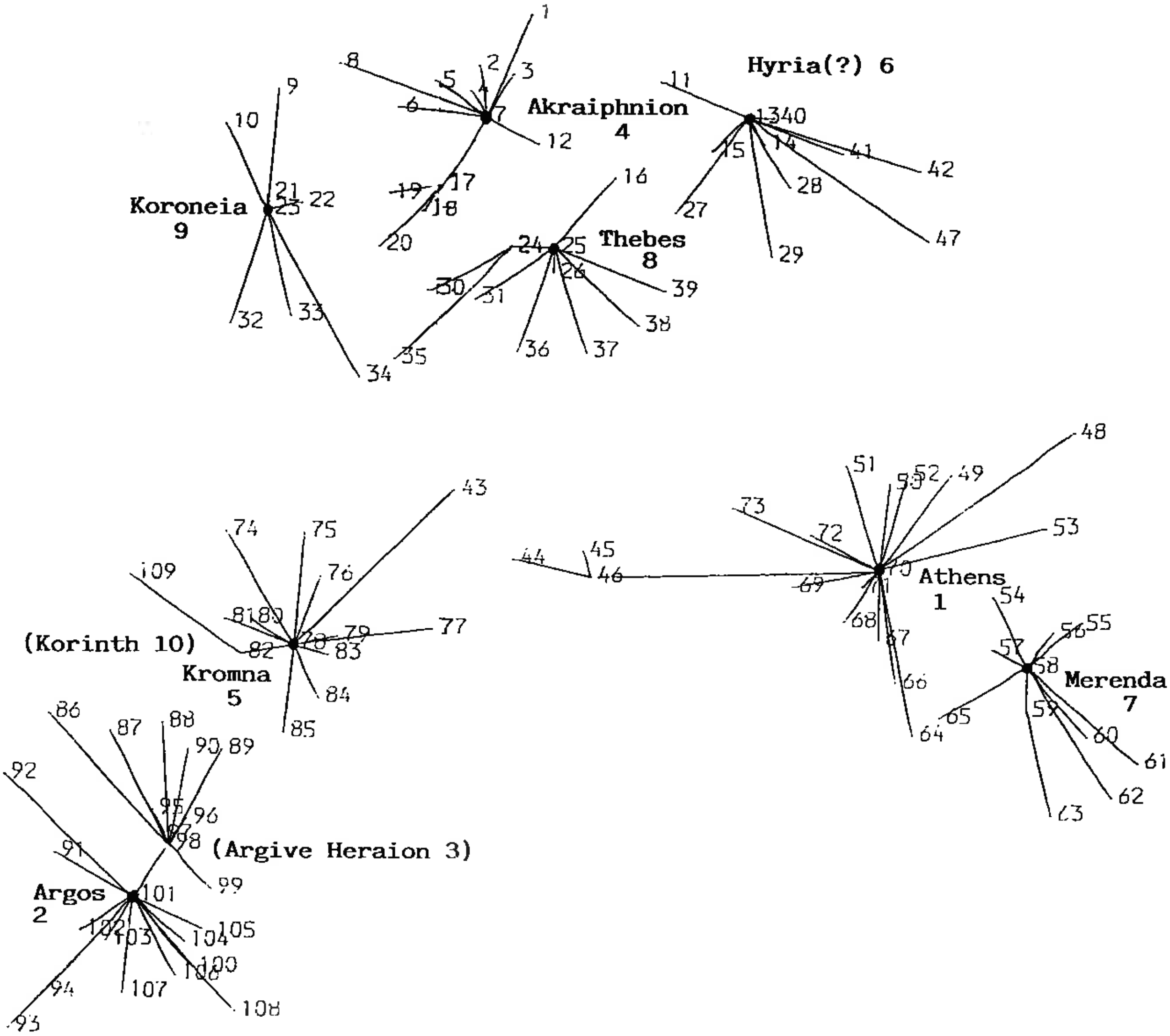


Figure 3: As figure 2, but ≥ 50%
maximum flows depicted.

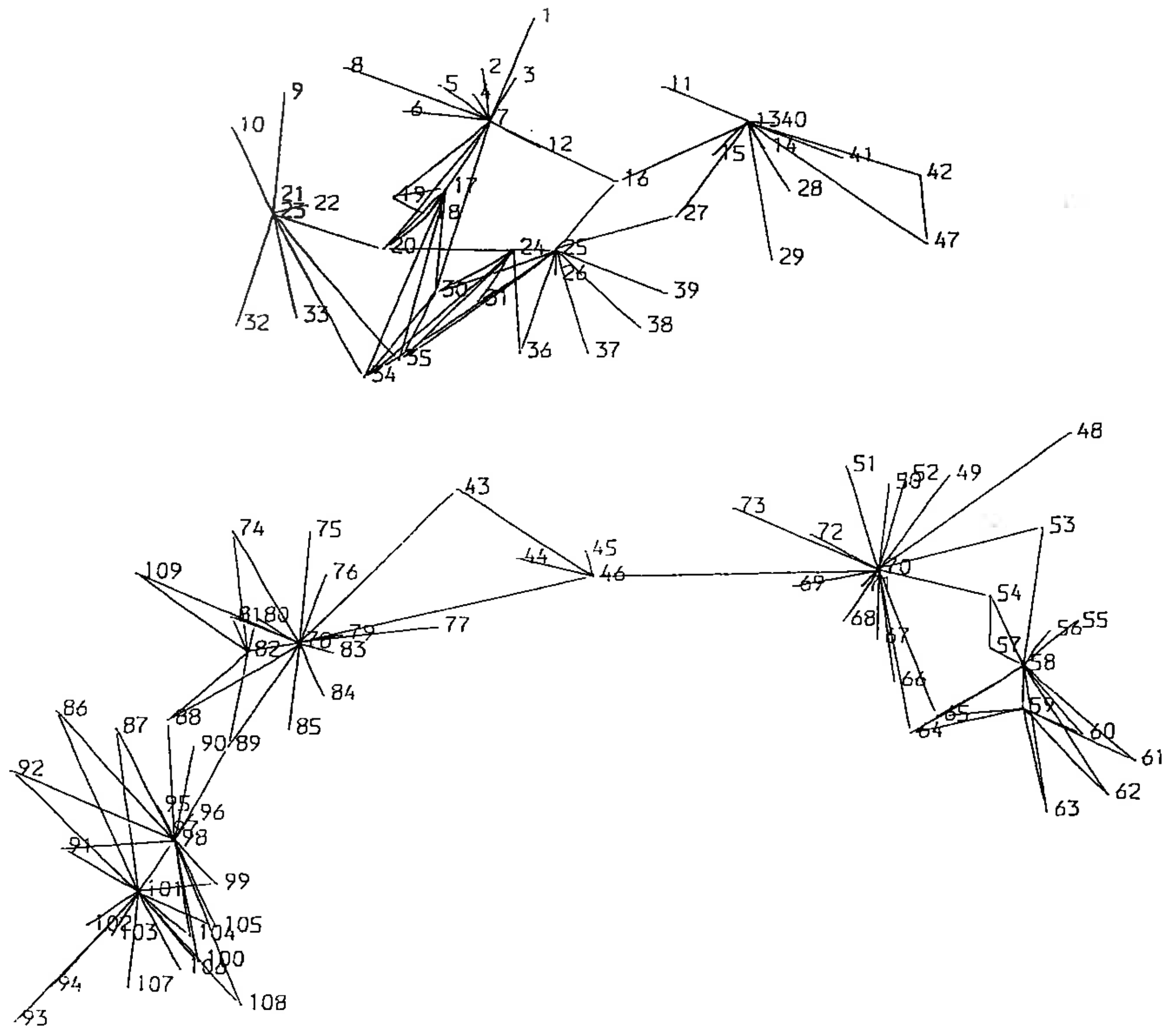


Figure 4: Seven systems (with topographical weightings).

$\alpha = 1.025$
$\beta = 0.15$
Maximum flows only depicted.

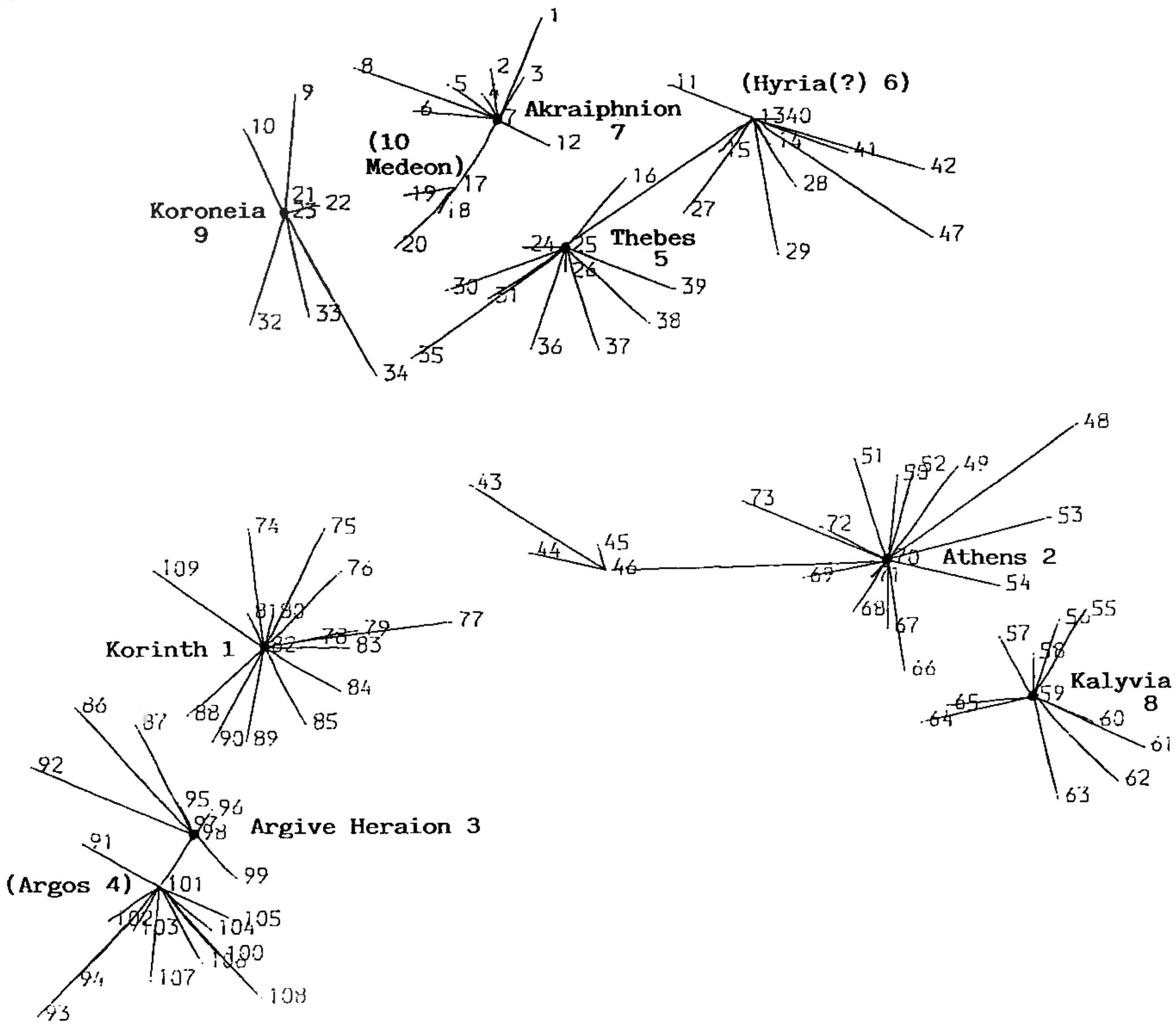


Figure 5: As figure 4, but ⩾ 50%
maximum flows depicted.

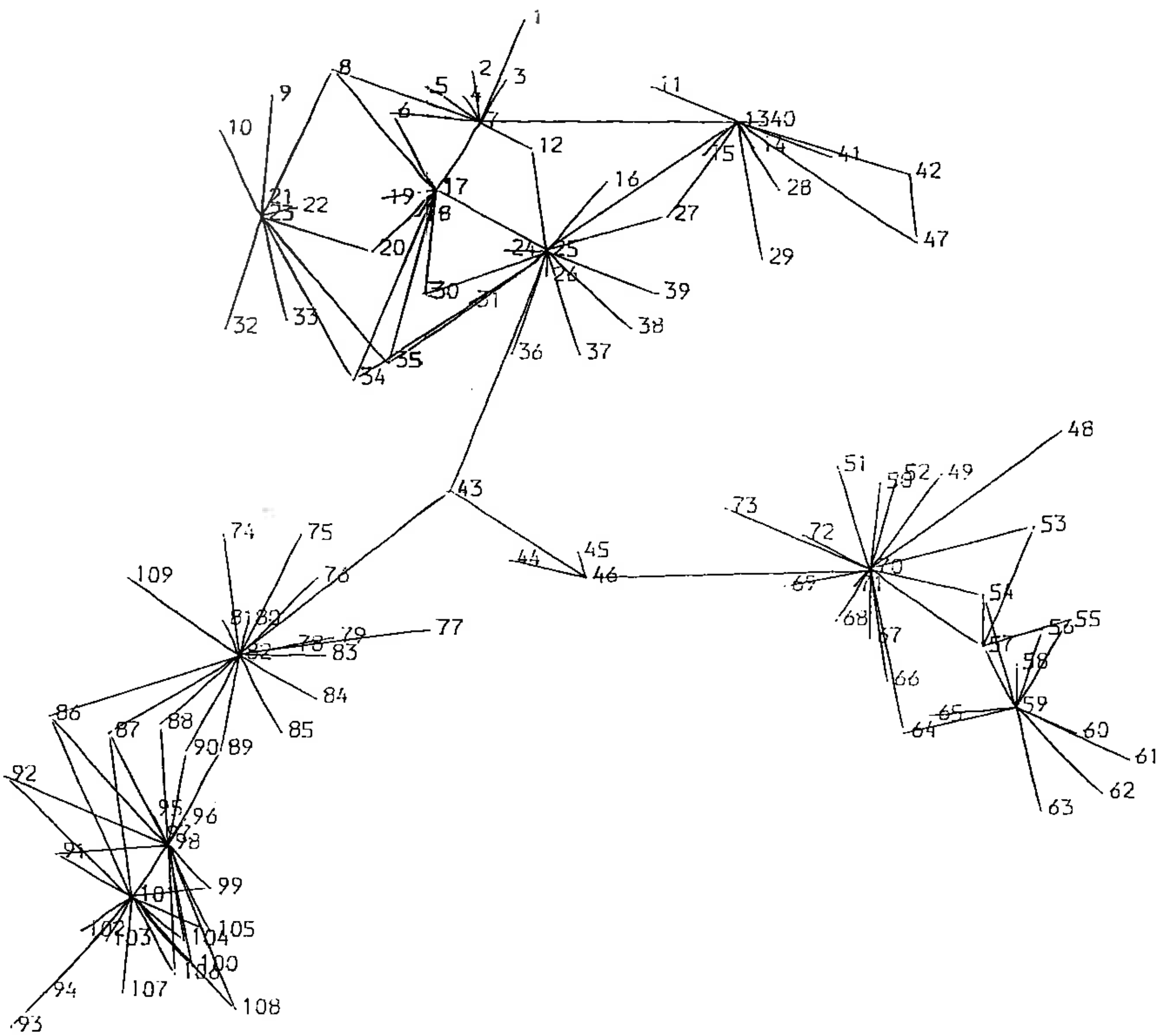


Figure 6: Terminal count over changing parameter values.

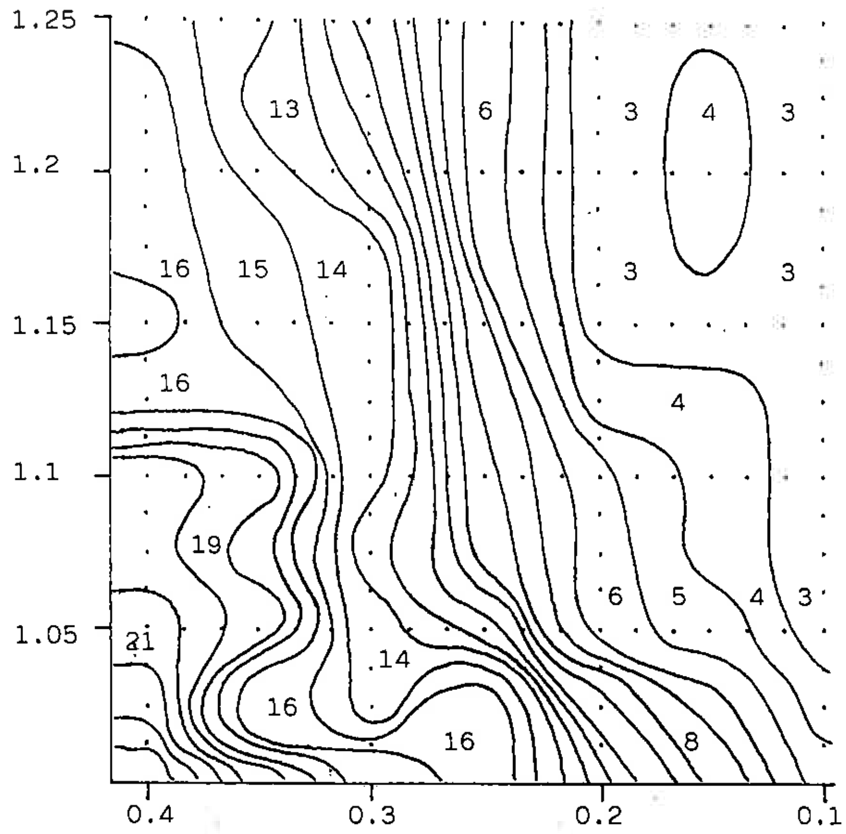


Figure 7: Eight systems (no weightings).

α = 1.01
β = 0.15
Maximum flows only depicted.

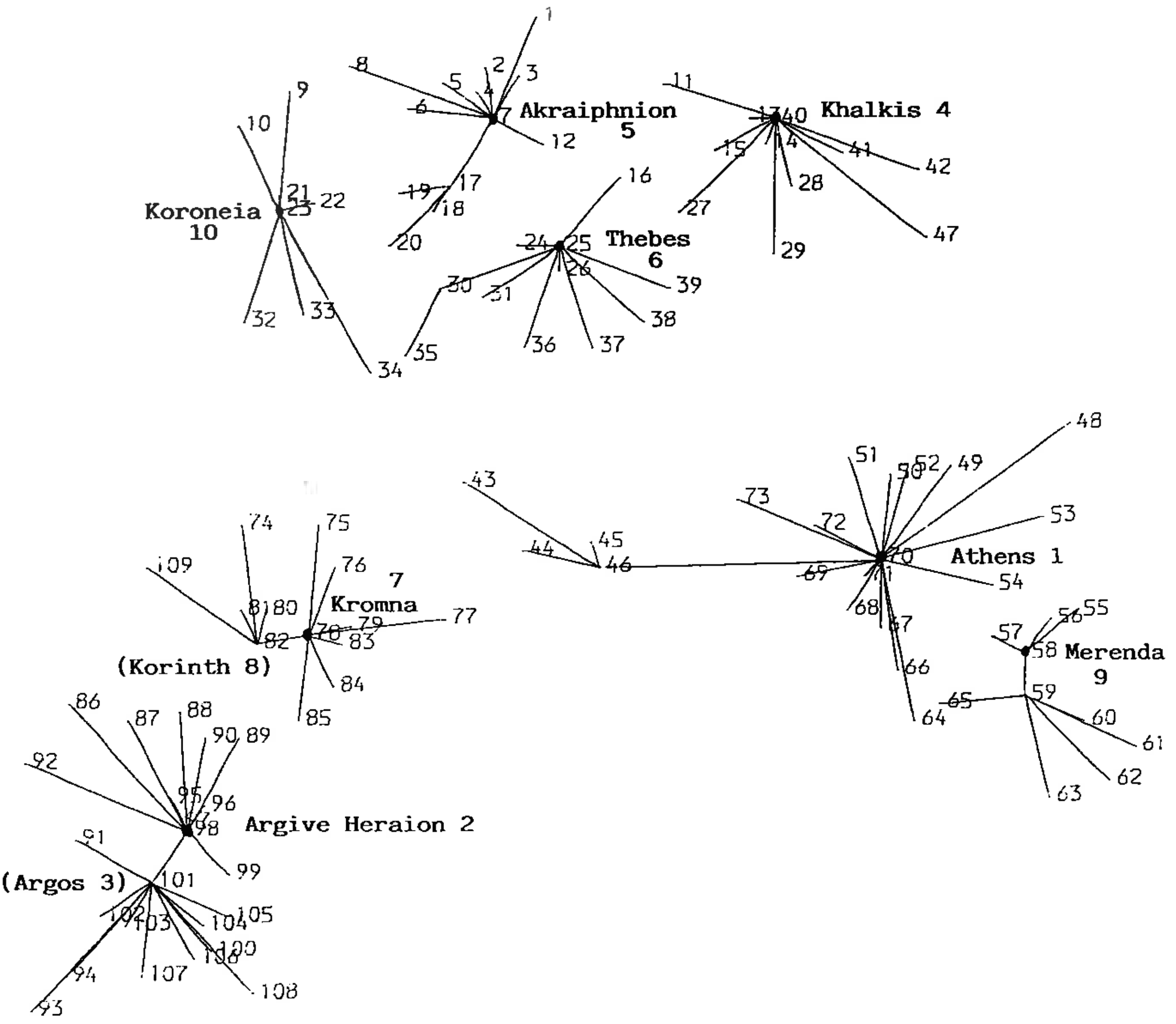


Figure 8: As figure 7, but ⩾ 50%
maximum flows depicted.

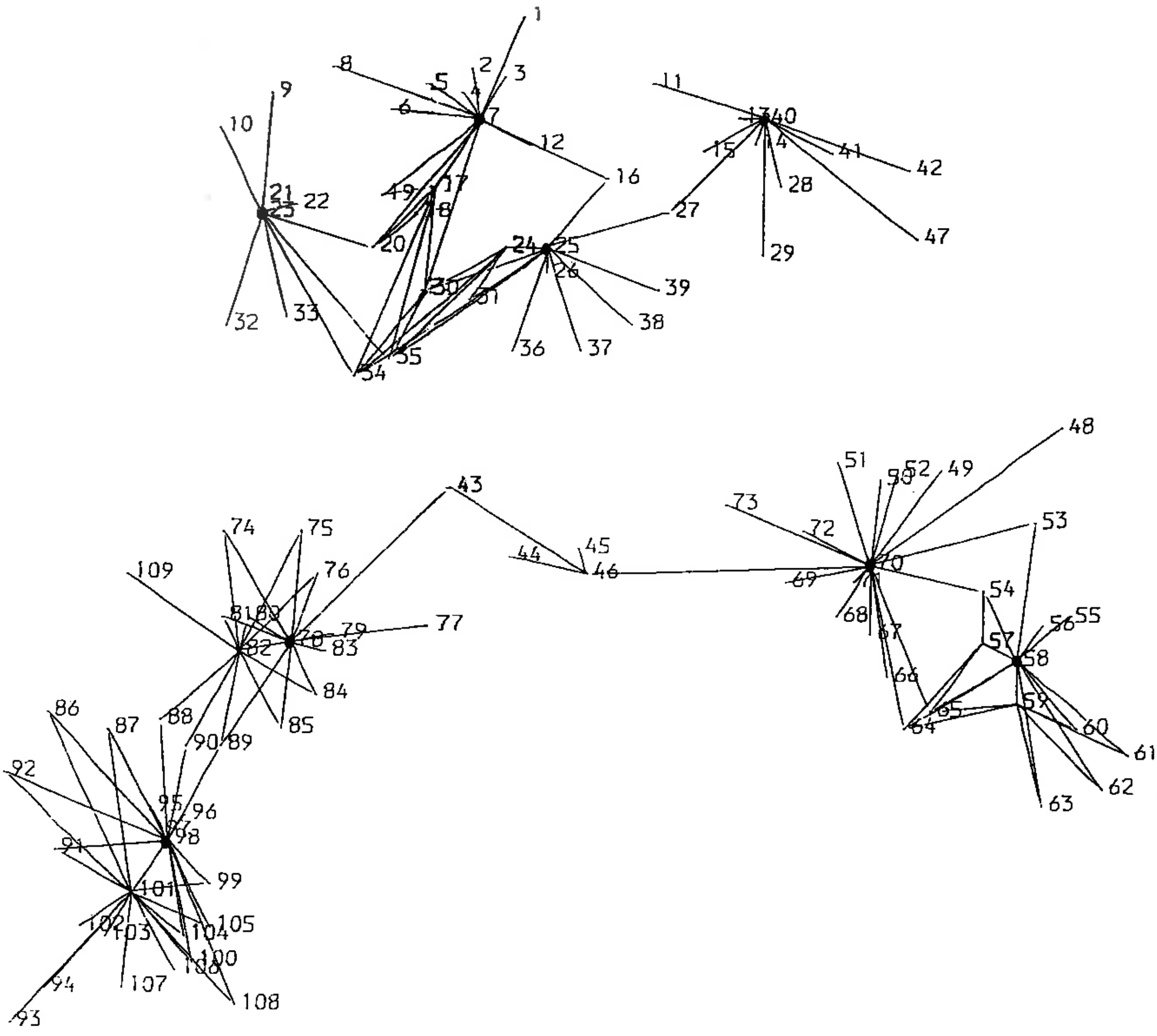


Figure 9: Eight systems (no weightings)

$\alpha = 1.025$
$\beta = 0.15$
Maximum flows only depicted.

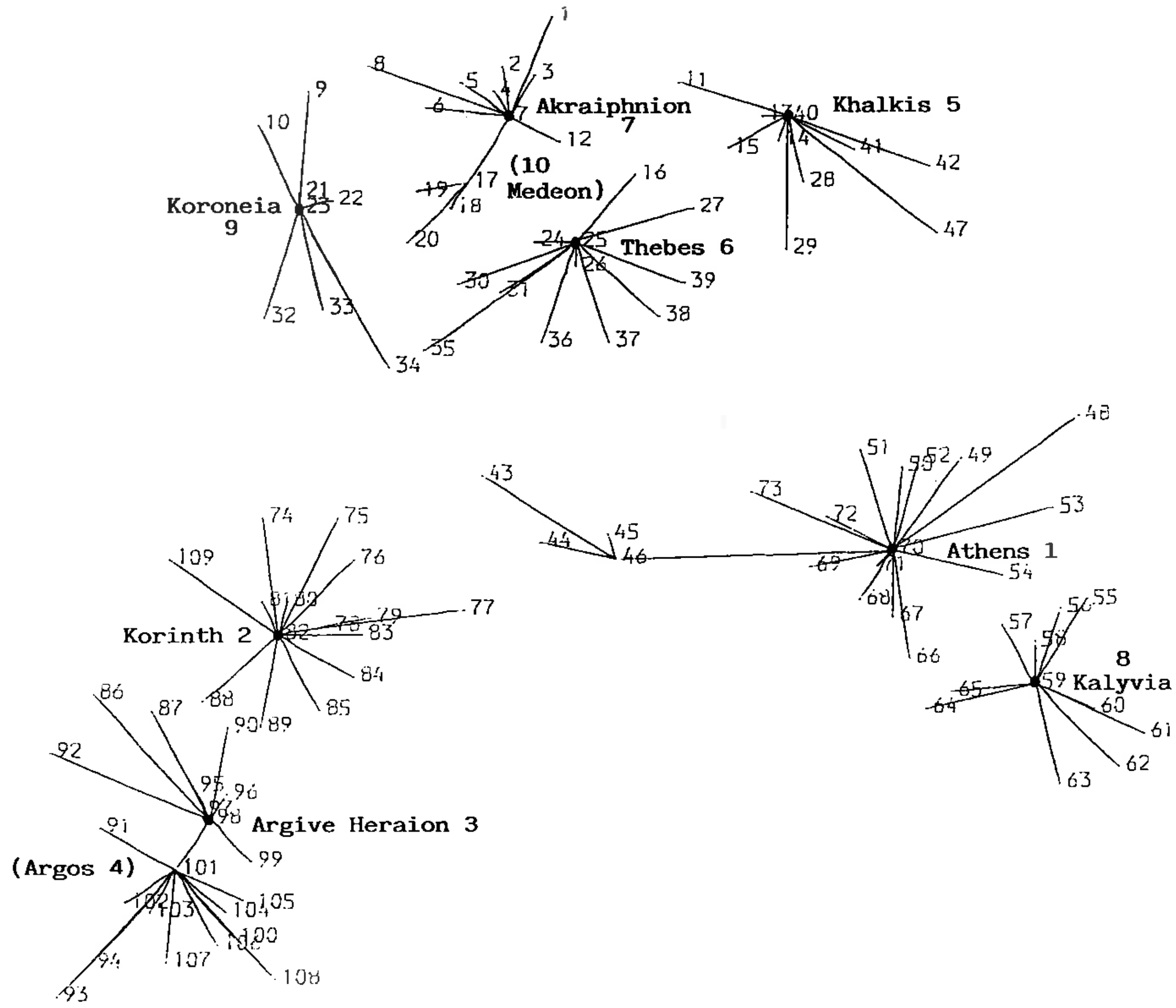


Figure 10: As figure 9, but $\geqslant$ 50% maximum flows depicted.

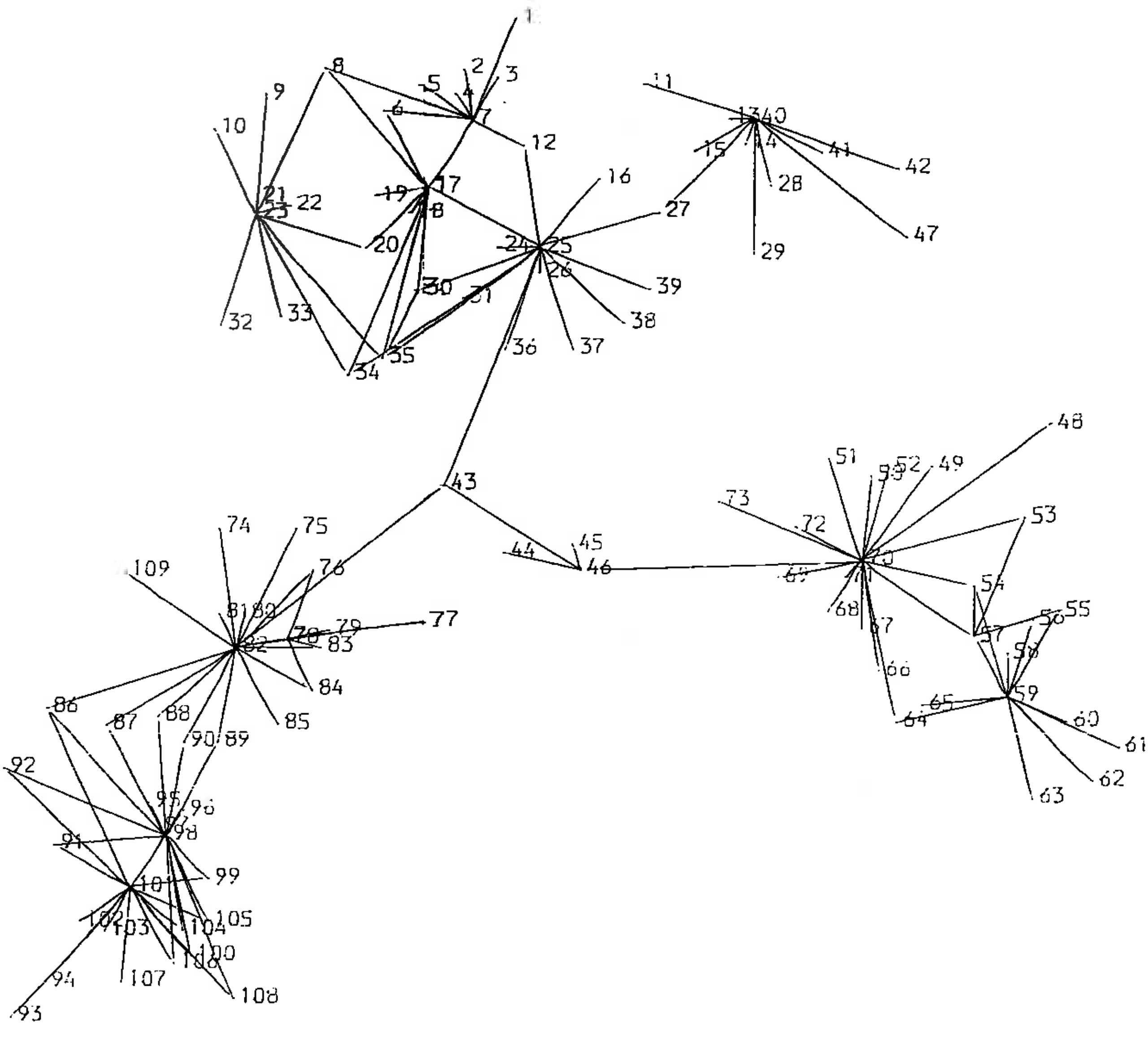


Figure 11: Devolved structure with thirteen systems.

$\alpha = 1.025$
$\beta = 0.25$

$\geqslant$75% maximum flows depicted; arrows indicate direction of greatest flow, and system boundaries are drawn accordingly: they are schematic.

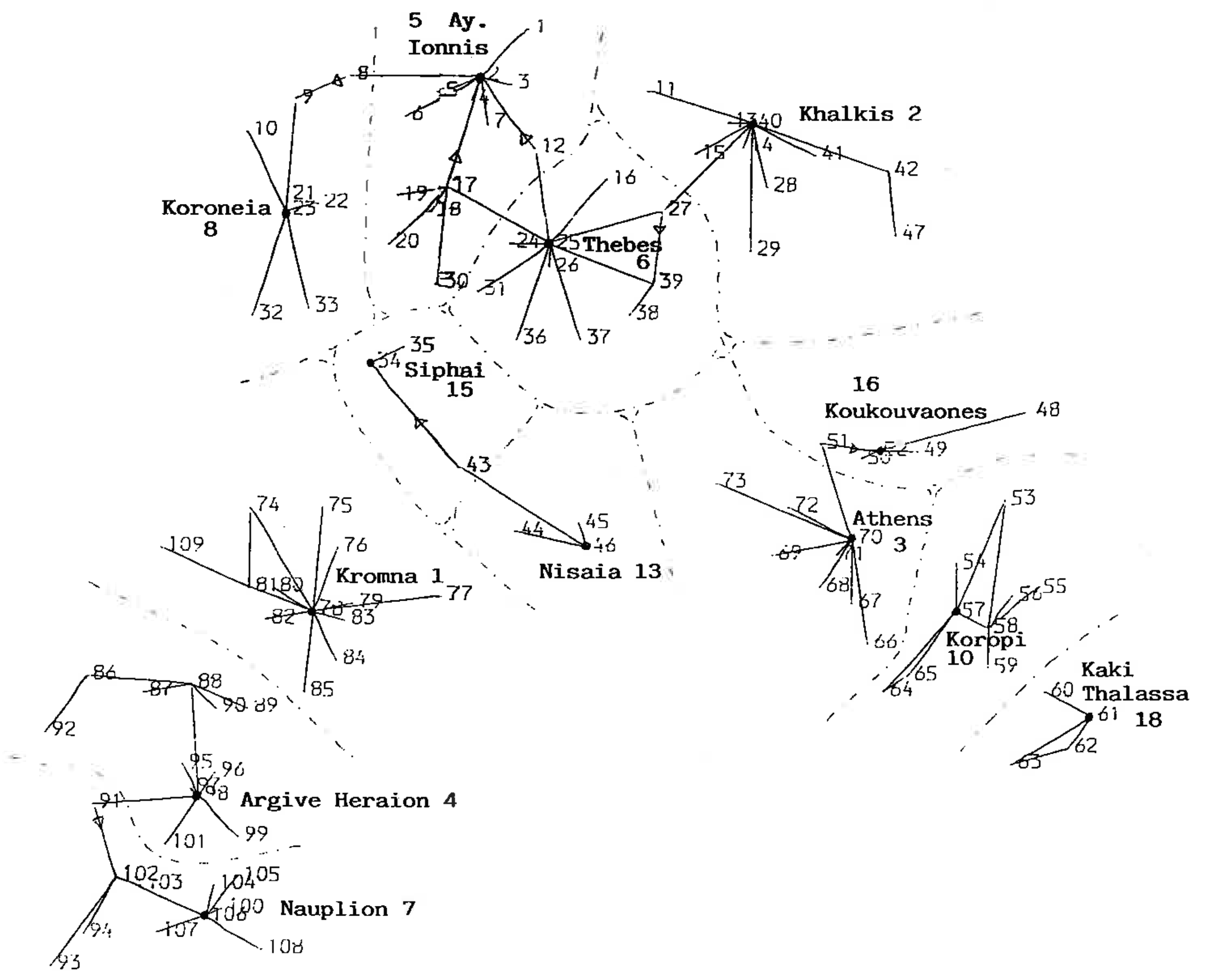


Figure 12: Ten systems.

$\alpha = 1.225$
$\beta = 0.3$

$\geqslant$ 75% maximum flows depicted, boundaries drawn schematically.

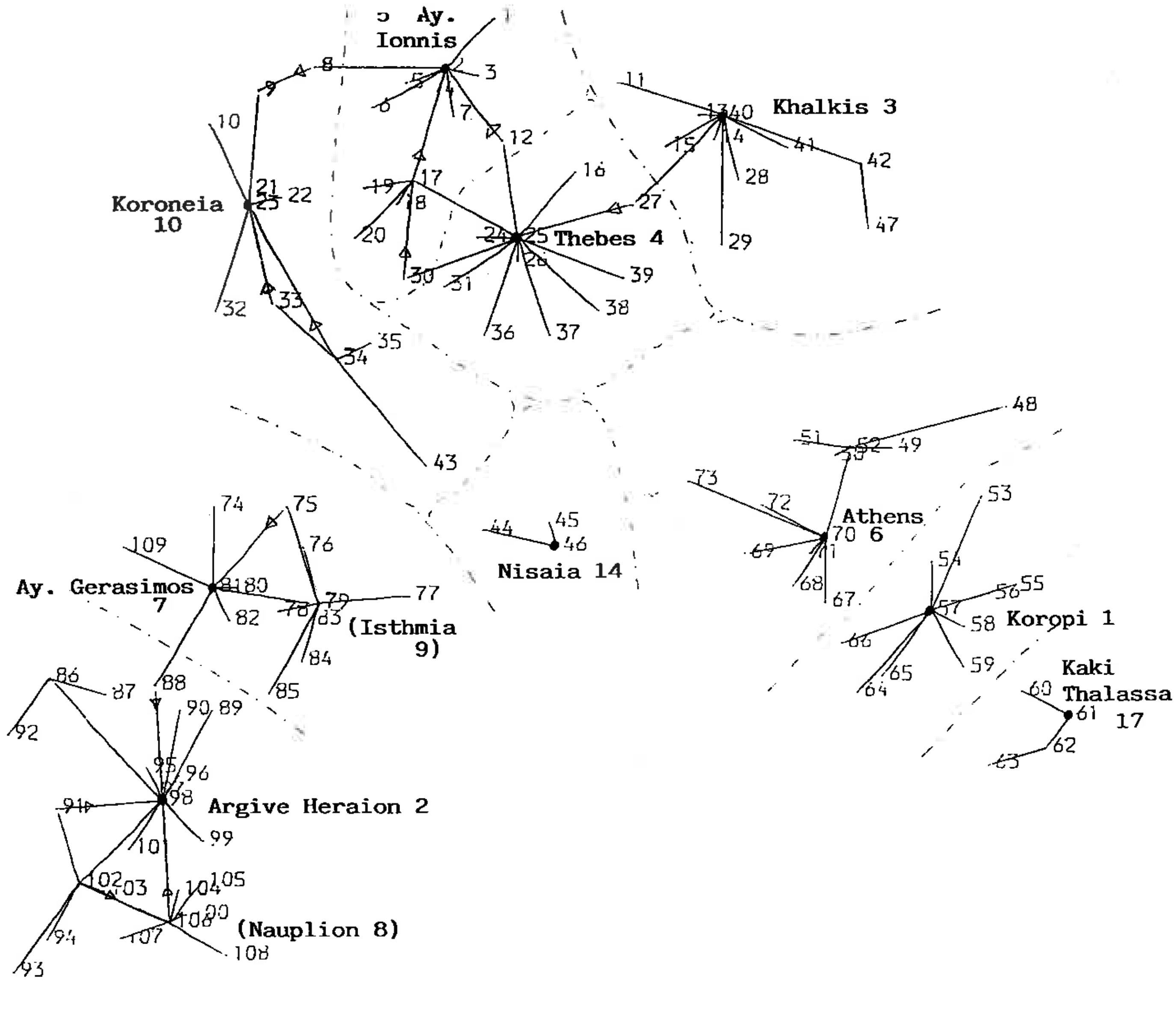


Figure 13: Ten (different) systems.

$\alpha = 1.005$
$\beta = 0.175$

$\geqslant$ 75% maximum flows depicted, system boundaries drawn schematically.

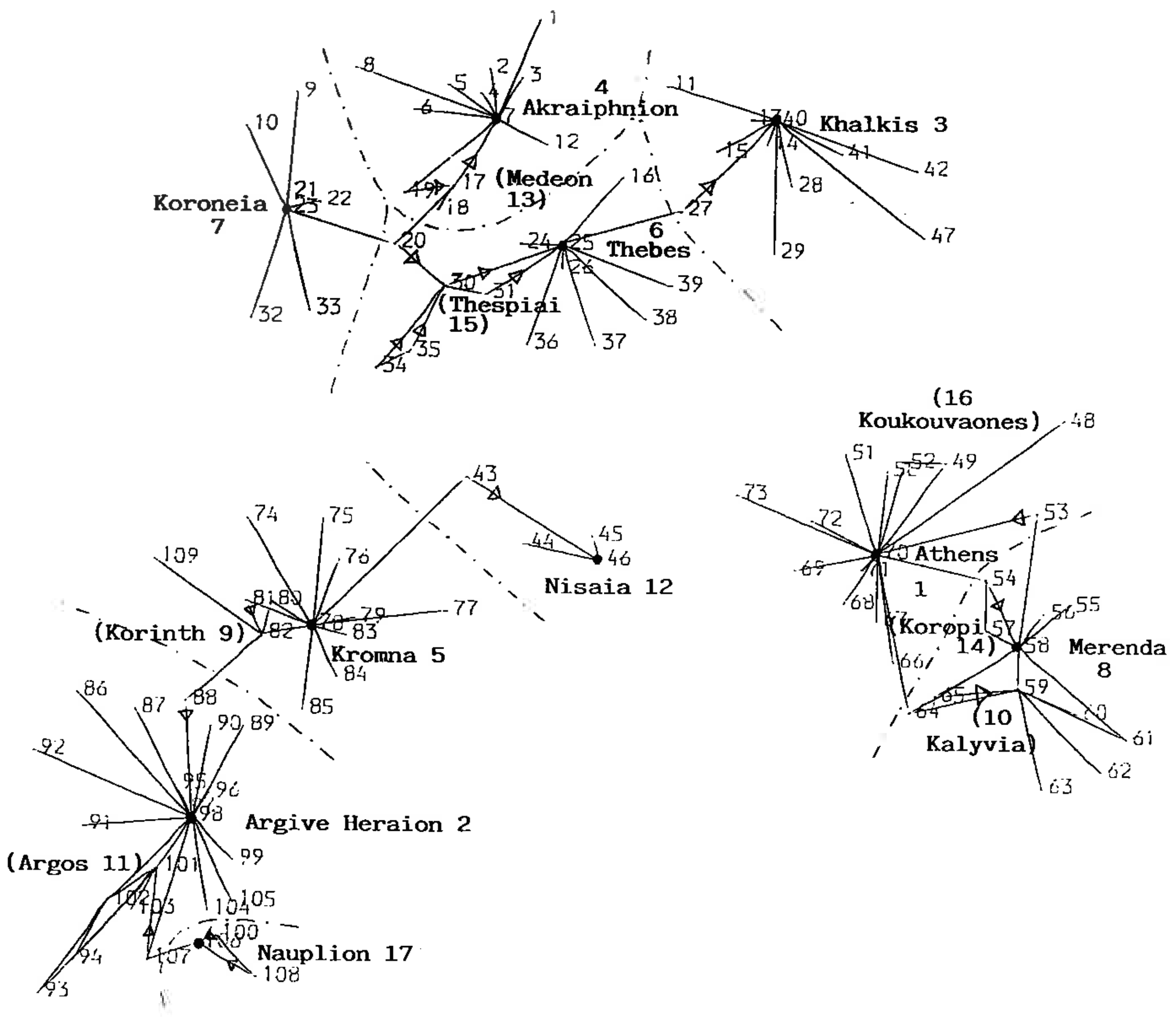


Figure 14: Eight systems.

$\alpha = 1.05$
$\beta = 0.175$

$\geqslant$75% maximum flows depicted, boundaries drawn schematically.

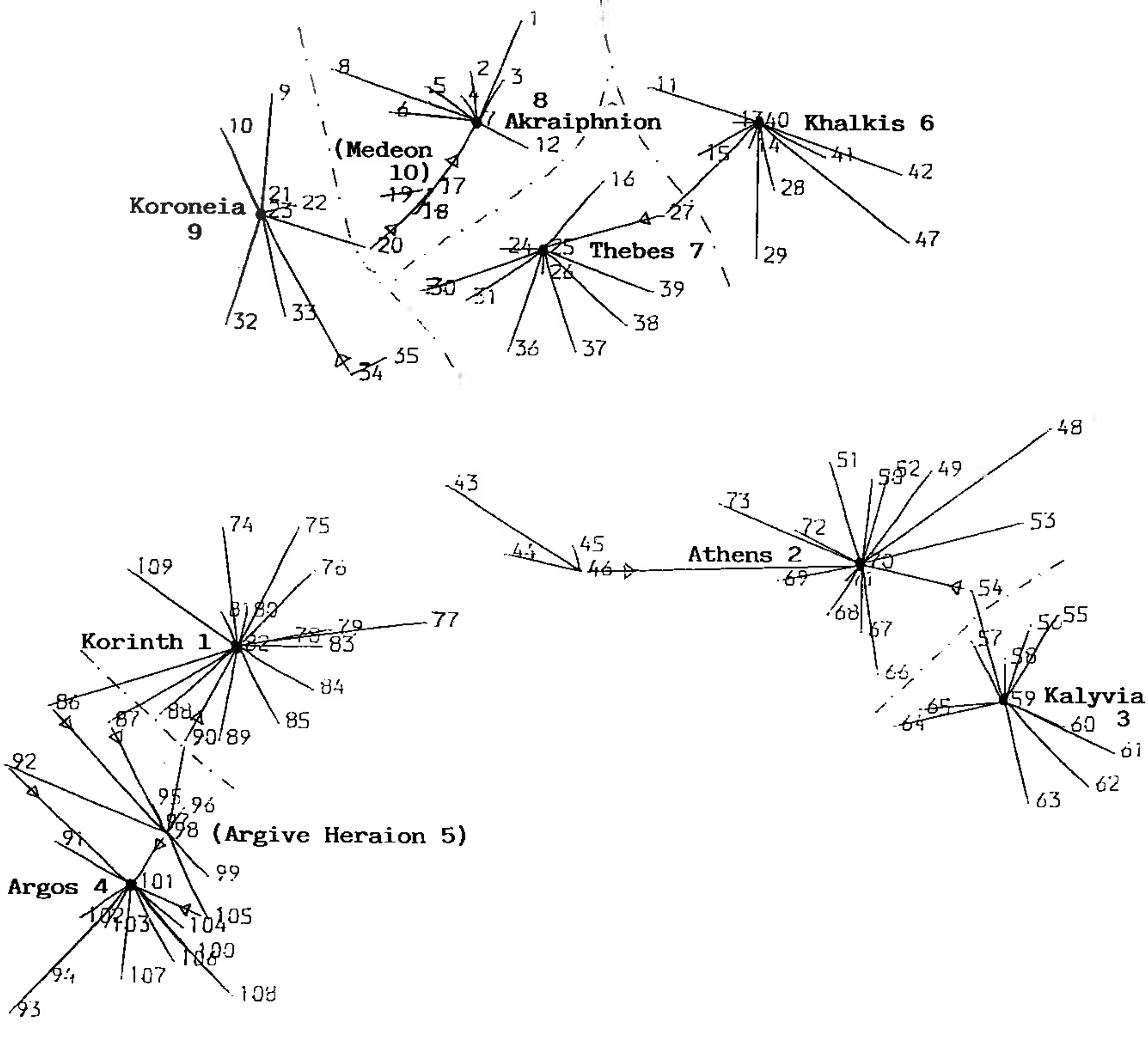


Figure 15: Highly centralised structure with three systems.

$\alpha = 1.15$
$\beta = 0.2$

$\geqslant$75% maximum flows depicted.