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A TWO-DIMENSIONAL SIMULATION MODEL FOR SLOPE AND STREAM EVOLUTION

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Abstract

A computer simulation models landform evolution by wash and creep/splash over a two dimensional 64x64 grid. The same process law is used for all points to allow the development of a valley network minimally constrained by model rules, to represent erosion on a surface of constant composition. The initial surface consists of a uniform slope, perturbed by subtracting a low-relief fractal surface from it. Fractal perturbation was chosen to minimise scale constraints; and some form of perturbation in either the process law or in the initial surface is necessary, as symmetry otherwise prevents the formation of channels at all.

Three model runs (#1 to 3) are presented, differing mainly in the citical unit area (*i.e.* area per unit contour width) at which wash transport becomes equal to creep/splash. Critical unit areas of 5000,1280 and 320 metres gave respectively no channels and average channel spacings of 2200 and 560m. Evident contour crenulations indicating an eroded channel occured at unit areas of about twice the critical values. Simulation results are compared with the theoretical criterion for unstable growth of surface hollows, and good agreement is found, supporting the theory and suggesting ways of making it operational.

1: The interaction between streams and slopes

The single most important parameter to describe a fluvially produced landscape is almost certainly some measure of its drainage texture. The most commonly used measure is drainage density, expressed as total channel length (however defined) per unit area; its reciprocal is then a measure of average stream spacing. Drainage density varies from less than 1.0 km^{-1} for some humid areas to about 1000 km^{-1} in some areas of badland dissection. Although drainage density has been shown to be well correlated with vegetation cover (Melton, 1957), there have been few attempts to explain this or other dependence in terms of the processes operating on hillslopes and/or channels.

The most fundamental approach to the problem has been by Smith and Bretherton (1972), who established the conditions under which small random depressions in the surface topography would grow into valleys in an initally unstable fashion, rather than fill in. This condition is met when converging flows of sediment have a combined transporting capacity in excess of their previous summed capacities: that is when transporting capacity increases more than linearly with drainage area for a given gradient. In mathematical notation, there is instability if and only if:

[1] $$\partial C/\partial a > C/a$$

where C is the sediment transporting capacity of the flow per unit width

and a is drainage area per unit width (Smith & Bretherton, *ibid*, p. 1517-19: Carson & Kirkby, 1972, p. 394).

Thus for example soil creep and rainsplash processes, which depend entirely on gradient, do not produce instabilities but smooth landforms without growth of depresssions. Wash processes, on the other hand, with a more than linear dependence on overland flow discharge and hence on drainage area, tend to enlarge small depressions into macroscopic valleys. Equation [1] is also the condition for hillslopes to become convex in profile under conditions of downcutting at a constant rate for all points.

This argument has been applied to the stable length for individual slope profiles (Kirkby, 1980), where it was argued that it would be efficient for valleys to adjust towards a state in which slope lengths are close to their maximum stable value. Suppose that, for example, transport-limited erosion occurs with a combination of splash and wash, at rate:

[2] $$C = k(1+a^2/u^2)s$$

where k is the constant of proportionality for splash,
u is the distance beyond which wash transport exceeds splash
and s is the tangent slope gradient.

This expression is reasonable adequate for semi-arid areas where overland flow discharge increases more or less linearly with area: it is less satisfactory for humid areas where overland flow is generated by saturation excess mechanisms. With this transport law, it may be seen that instability occurs for all drainage areas, $a > u$, irrespective of gradient. If the efficiency argument is valid, it follows that the slopes have lengths close to u so that drainage density is approximately given by 1/(2u). There is then a simple relationship betweenthe process rates from which u is estimated, and the drainage texture.

To pursue this relationship further, it is necessary to resort to representations of the landscape with two horizontal dimensions, since the growth or infilling of hollows cannot be represented with a single dimension, even though the stability criterion may be evaluated. The development equations, although easily stated, do not offer an analytical solution in any but the simplest and most symmetrical cases. It has therefore been necessary to proceed through a computer simulation.

There have been a number of 2-dimensional slope simulations, but none to date has satisfactorily addressed the problem of drainage texture. The most relevant are those of Ahnert (1976), Armstrong (1976) and Cordova, Rodriguez-Iturbe & Vaca (1983). Ahnert's model is limited by a rather small grid-size (10x10) and by introducing geological and other patterns into his model which force the location and spacing of the resulting drainage pattern. Although such areal differences are of interest and relevance, they side-step the theoretical question of where drainage is located in the absence of evidient control. Armstrong's work is for a larger grid (40x40), but begs the question of stream spacing by assuming a fixed network of channels and different transport laws for slopes and channels.

Cordova *et al* have come much closer to a solution of the problem, in that they have used a uniform area, or one with random perturbations only and a uniform transpot law. They have however used a rather small grid size (16x16) and, more seriously, have used a transport law which is based only on hydraulic transport. In effect their flow law is equivalent to a power law of both area and gradient, with exponents greater than unity. The process modelled is therefore inherently unstable in the Smith & Bretherton sense, so that hollows tend to grow everywhere, without a divide area of smooth slopes. Their simulation, as might be expected, generates a valley network which obeys Horton's Law of stream numbers. They also demonstrate a result which may also be argued from considerations of symmetry, although perhaps less convincingly! Namely that a uniform slope with no initial irregularities at all will develop no channels, but simply into a uniform ridge; and that a slope which is irregular only in having a notch in the centre of its bottom edge, develops a single central channel. They recognise the vital importance of some pattern of irregularities, however small, to initiate perturbations which can grow into valleys, and their final simulation shown is based on a random assignment of the process rate parameter, which then remains constant at each point over time.

2: Model formulation

A rather simple model has been used here. It is based on transport limited removal at a rate given be equation [2] above, constrained by a mass balance calculated for each point of a two-dimensional grid. Initial irregularities have been introduced through small perturbations of the initial topography, whereas the process has been held spatially uniform. The initial perturbations have been generated from a pseudo-fractal surface. These choices are discussed below.

There are good a *priori* reasons, even though with only slight empirical backing, for assuming that soil creep and rainsplash transport should be proportional to slope gradient. For both processes, a proportionality constant of 10^{-3} m^2 a^{-1} is of the correct order of magnitude. These processes are thought to be dominant near divides. It is therefore important to simulate these processes, if divides as well as channels are to be modelled.

For wash processes, a good fit to many data is obtained from empirical relationships of the form:

[3] $$C \propto q^m s^n$$

for constant exponents m and n which usually lie between 1 and 3. A good fit may generally be obtained with m = 2 and n = 1. These exponents are used for annual overland flow totals: a slightly higher exponent of discharge (2.5 - 3) is appropriate for individual storm discharges. Runoff in individual storms may be adequately modelled, in the absence of datailed antecedent moisture and intensity data, from daily rainfall data, which is commonly distributed, at least for each season, as:

[4] $$N(r) = N\exp(-r/r_m)$$

where $N(r)$ is the number of days with rainfall $> r$,

$N = N(0)$ is the total number of rain days,

and r_m is the mean rain per rain day, = Total annual Rainfall, $R \div N$.

The runoff on any day is estimated as a proportion λ of any rainfall exceeding a threshold r_c. Integrating over the frequency distribution, the total annual runoff production per unit area is then given by:

[5] $$F = \Sigma f = \lambda R\exp(-r_c/r_m).$$

The effect of raising Q to the m'th power may be compared with a storm based sum;

$$\Sigma(f^m) = m!(\lambda r_m)^m N\exp(-r_c/r_m)$$

It may be seen that the first two terms on the right hand side are raised to the m'th power, while the remainder are not: the effect is, in practise similar to using a somewhat lower exponent for the annual total than for the sum of storm totals. Appropriate values for λ and r_c have been derived from Meginnis (1935) for some Mississippi loess soils as:

	λ	r_c (mm)
Bare soil	0.60	4.7
Grass	0.12	36.6
Forest	0.025	37.6

Conversion of runoff production to area has been done on the assumption of uniform production, so that total discharge per unit width is directly proportional to area per unit width, and unit discharge $q = aF$, where F is obtained from [5] above. Wash erosion is then proportional to q^2s, with a constant empirically evaluated at about 0.017 a m^{-2}. This gives a power law for the wash processes which is similar in its effect to the law used by Cordova *et al*, but contains an explicit summation over the frequency distribution of storm rainfalls.

The transport law used in this simulation takes the form of equation [2] above, which may be seen to be a sum of a creep or splash term and a wash term, based on the values quoted above. It may be seen that the distance u, at which wash overtakes creep and splash processes, is given by:

[6] $$u = 240\exp(r_c/r_w)/(\lambda R),$$

where the annual rainfall, R is expressed in mm and the critical distance, u is in metres.

This relationship is as given in Kirkby (1976a) for the case of $\lambda = 1$. For an annual rainfall of 500m falling on 50 days per year, the values of λ and r_c tabulated above give u-values of 1.3,155 and 825 metres respectively, representing a substantial part of the range of values implicit in the variety of drainage textures. During individual major storms, the tendency for hollow enlargement is expected to occur at unit areas much smaller then u, but periods of low intensity or no rainfall allow creep and splash to refill any rills thus formed. If the creep/splash and wash terms are disaggregated by storms, the short-term history or frequency distribution of cut and fill may be examined. The headward end of valleys is thus thought to consist of smooth slopes with a detailed cut and fill stratigraphy representing the relevant local historic sequence of large storms and intervening periods of slope infilling.

More complex models for hillslope runoff may be used, to allow for the development of partial saturated areas in humid soils. Some suggestions have been made (Kirkby 1976a, 1978), in which the unit areas of equation [2] are replaced by terms of the form:

[7] $$\left[\int_0^w \phi(x/s)\,dx\right]/w$$

for a suitable slope hydrology function, ϕ. The case considered here corresponds the the simplest possible case, of $\phi \equiv 1$. More general cases may be investigated within the framework adopted here.

Initial perturbations may be introduced either in the initial surface or in the process parameters. Each produces conditions which are inherited by the subsequent landforms. In the case of a perturbed topography, small perturbations generate an initial distribution of unit drainage areas which determine where erosion is initally most rapid. Erosion then smooths or enhances the topographic irregularities, and inheritance is therefore conditional on this evolutionary process. This topographic perturbation is preferred here because the effect of initial disturbances is not re-imposed at subsequent times, so that the area modelled is treated as if it were fully uniform. If process parameters are varied over the model area, then systematically or randomly (as in Cordova *et al*) distributed differences are re-imposed indefinitely. This is thought to be appropriate for describing geological differences, as in Ahnert's (1976) model, but appears to violate the assumption of a uniform area for landscape evolution.

A second problem about introducing initial perturbations lies in avoiding an initial pattern which deliberately or accidentally imposes a scale on the resulting drainage pattern. The solution adopted here is to use a deliberately scale-free perturbation, derived from a fractal surface, which is then superimposed, at very low relief, on a smooth initial surface.

3: Computational procedure

This model has been implemented, with some approximations, on a 32 K-byte micro-computer, over a 64x64 grid. Figure 1 gives a skeleton flow diagram for the computational procedure. Elevation and other values are stored as 2-byte values, giving a resolution of about 4mm, over a total elevation range of 250m. The initial surface generated is in fact a pseudo-fractal obtained by successive mid-point interpolation with a suitably reducing variance about linearly interpolated values. Strictly this surface is only scale-free at the 2:1 scales used to generate it, but is is thought to be an adequate approximation for the present purpose. This surface is generated within a square, or distorted square (rhomboidal) frame at elevation zero, in terms of a central height, a standard deviation k, and an excess dimension h. The central point (32,32) is either specified, or drawn from a normal distribution (generated as a sum of rectangularly distributed random numbers) with zero mean and standard deviation k, which is then reduced to:

[8] $$k' = k(1-2^{-2h})^{1/2}$$

The next set of 4 points is obtained by linear interpolation from the four corners of an upright square: for (16,16) from (0,0),(0,32),(32,0) & (32,32) and similarly for (48,16),(16,48) and (48,48). The interpolation is weighted inversely by distance to each corner if the square is distorted. To the interpolated value is added a normally distributed random variate of zero mean and standard deviation k'. The standard deviation is then reduced, at this and successive interpolation scans, in the ratio $2^{-(1-h)/2}$. The next four points are obtained by interpolating from the corners of a diagonal square; (32,16) from (16,16), (48,16), (32,0) & (32,32) and so on. Alternate scans thus interpolate alternately from the corners of the nearest upright and diagonal squares around them, with steadily reducing variance corresponding to the reducing interpolation span.

This initial surface is then linearly combined with a specified regular surface. In the examples shown this surface is a uniform slope, falling to zero at the base. The range of elevations on the slope is scaled over the range 0 to 65535 to give maximum resolution in the subsequent simulation. Each point in the grid is considered to be the mid point of a square or rhombus. the following boundary conditions have been adopted. The upper edge of the grid (y=0) is taken as a divide, with symmetrical removal on the opposing slope. The slope base (y=64) is considered to be a base-level of fixed position and zero elevation, at which all material is removed. The two sides of the slope (x=0 and x=64) are allowed to connect to one another in a circular fashion, so that drainage lines may go off at x=0 and continue at x=64 on the other edge. This condition is thought to provide less forcing of stream spacing than a condition of reflecting boundaries at the sides.

At each model time step, the flow direction from each point to its neighbours is assigned on the basis of the lowest of these points: all flow is then assumed to follow this path. This is an approximate algorithm which greatly speeds computation, but which forces the drainage to adopt a dendritic network without distributaries, as flow is never divided between alternative routes. Using this procedure, surfaces on which flow is generally divergent tend to have a large number of internal sources, so that drainage areas on them are nowhere large, and wash processes tend to be insignificant using equation [2] above, in accordance with expectation.

In early runs of the model the simulation was carried out in a square grid, and flow was assigned from a point to any of its eight immediate neighbours, or to the point itself. Although this provides a maximum number of possible flow direction, the simple algorithm used takes no account of the greater distance to the diagonal corners, which may therefore be assigned flow in cases where the gradient to some other point is greater. In later runs this difficulty was avoided by distorting the grid into a 60° rhombus. Flow was then assigned to any of the six equally near neighbours in the surrounding hexagon.

A second approximation, which is difficult to avoid in this type of model, is the assumption that flow occupies the entire width of an individual grid square. It may be seen, from the form of equation [2], that if flow becomes concentrated over a width much less than the dimension of a grid element, than the total sediment transported within that element may become arbtrarily large. It is therefore essential to maintain a grid element which is of the correct order of magnitude for observed channel widths. A grid element widths of 10 m. has been adopted here as a compromise between meeting this criterion and simulating over an adequate total area 640x640 m^2).

Drainage area at each point in the grid is evaluated by starting at each element, adding unity to the area of this element and to all elements to which it successively drains down the length of the slope. This procedure was continued from each starting point either to the base of the slope (y=64) or to any enclosed depression (which was thus treated as a sinkhole for water). Sediment transport may now be evaluated from each grid element, using equation [2] or an alternative. Each element provides a rate of erosion for the element itself, and deposition in the element to which it immediately drains. No sediment was considered to leave the lowest point of an enclosed depression, which therefore innevitably fills up.

This explicit solution procedure is stabilised by choosing a time step which allows gradient to change by a maximum of 50%(or a lower threshold) at the worst point. To allow slope reversals, stream capture *etc*, this condition is not applied where points differ by less than 4mm (*ie* gradient is calculated as zero for the computed resolution). Elevations are then updated, and the next time step begun. At chosen intervals, the data matrices are stored for analysis of erosion rates, drainage areas and network patterns.

4. Example simulations.

The results of three runs are presented here to illustrate the way in which the model behaves. In the first run (#1), the grid consists of square cells, with flow possible to eight directions. In the other two runs (#2 & 3)the grid is rhomboidal, with flow possible to six directions in a hexagonal pattern. In each case the creep/splash rate constant (*k* in equation [2]) has been taken as 0.001 $m^2\ a^{-1}$, the unit cell dimension is 10m, and the initial surface consists of a perturbed uniform slope with a relief of 255m, giving an initial gradient of 21.7° (40%) in #1 and of 24.6° (46%) in #2 & 3. The main difference in the runs was in the critical threshold distance (*u* in Equation [2]) beyond which transport due to wash is greater than that due to creep/splash; and beyond which hollows should grow in size. In the three runs, *u* has the respective values 5000, 1280 and 320m, so that drainage from 500,128 and 32 cells are required to reach the threshold value.

Figure 2 shows the initial surface for #1. Figure 2(a) shows the fractal surface with elevations from -20 to +35. In (b) a uniform gradient with 255m of total relief has been perturbed by subtracting 10% of the fractal values to give a surface with cross-relief of about 4 metres. Drainage lines are also shown for cells with catchments larger than the critical value, u. It may be seen that the topography has little obvious relationship to their position, but that the random combination of lines leads to areas of potential instability, even though this critical unit area (5000m) is about 8x the average (640m) at the slope base. In (c) it may be seen that, after 1.6 million years the irregularities of the initial surface have been very much reduced, although the remaining small differences still define substantial drainage areas which frequently shift in position through processes similar to the micro-piracy described by Horton (1945, p.---) for rilled surfaces of negligible cross-relief. The general trend towards smoothing is also shown by the halving of the largest drainage areas present. A total of 0.86m of denudation has taken place, largely through rounding of the upper convexity, wih 15m lowering of the upper divide. The evidence from this run is in accordance with the stability argument for the upper slopes, but appears to show that the growth of hollows with drainage areas only slightly above the critical threshold is too slow to be effective in forming valleys. The overall course of the slope evolution is closely similar to that obtained from the analogous *slope profile* model, although with a very slightly raised average near-base denudation rate due to the existence of some larger drainage areas.

Figure 3 shows a similar set of maps for Run #2, but with flow in six directios towards the corners of a hexagon, and with the grid consequently deformed into a 60° rhombus as described above. Figure 2(a) shows the original fractal surface, in this instance with a marked cantral peak of 100m elevation. In (b) the initial uniform gradient has been perturbed by subtracting 1/20 of the fractal surface, producing an initial surface with a 5m central depression. The most significant difference in conditions for #2 is the reduction of the critical unit area, u to 1280m, about a quarter of its value in #1. In consequence the initial network defined by drainage areas above the critical value is denser, and clearly follows the central depression. In (b) and (c), after respectively 40,000 and 240,000 years, and 0.08m and 0.43m denudation, the process of upper slope smoothing has proceeded as before, but at the same time tha central valley has gradually developed into a subtantial erosional feature with strong channel incision. There has also been some evolution of side-walls with valley widening and the establishment of convexities along the valley rim. Some smaller streams have been eliminated, and at the same time there has been a slight enlargement of the largest catchment area. Examining detailed cross-profiles, clear evidence of crenulation may be seen only for cells with drainage areas greater than about *twice* the critical unit area. This criterion is equally applicable for all times at which crenulations are clearly present (from about 20,000 years in this case).

In figure 4 (run #3) the critical unit area has once more been quartered, tp 320m. Flow directions are again hexagonal, as in #2. In (a) an irregular fractal surface with 30m total relief is used to perturb the initial surface shown in (b), giving it a maximum cross-relief of 2m. The lower critical area produces a dense network of channels, as defined by initial drainage areas. Over time (c) after 20,000 years and 0.34m denudation, and (d) after 39,000 years and 0.63m show smoothing of inter-stream areas, incision of the largest streams at or close to their original positions, and a reduction in the number and density of smaller streams though capture of their drainage areas by the largest. As in #2, the threshold for observable crenulation is about twice the theoretical crtical unit area. Figure 4 also shows one clear example of a channel and valley bifurcation. Given the 40% initial gradient it is perhaps not surprising that stream courses show a stong downslope trend which reduces tendencies towards bifurcation.

5. Analysis of simulated landscapes

The example runs above are far from exhaustive, but show some clear patterns which may be compared with, and in minor ways extend the theoretical concept of valley stability. The only simple operational definition of channels which can be applied to these simulations is through contour crenulations. In all cases this definition appears to correspond to unit areas of about twice the theoretical value (u). For the sediment transport law used (equation [2] above), this corresponds to a wash transport of 4x the splash/creep transport rate. The existence of unit areas greater than $2u$ on the initial surface does not appear to guarantee effective incision because of subsequent piracy of catchment areas. As a rough rule of thumb a unit area of $4u$ usually guarantees continuity of drainage lines to develop into valleys, but this criterion is a poor substitute for examining the detailed pattern of capture on a surface of low cross-relief.

In #1 the initial cells draining unit areas of more than $2u$ are eliminated, and no areas drain as much as $4u$: little or no coherent pattern may be seen in maps of erosion. In #2 the main valley has a basal unit area of more than 10,240m throughout. Initally a second catchment, flowing out near the centre of the base, initially exceeds 5,120m ($4u$): after 40,000 years it still survives with a crenulation, but with a halved catchment area through headwater capture by the main stream. At 240,000 years further capture has reduced its unit catchment area to 640m and the crenulation has filled. This example may be seen as a model for stream capture in general: diversion of unchannelled divide areas leading to a loss of collecting area for the disadvantaged stream-head, which therefore falls below the threshold for hollow enlargement and so fills with slope sediment with a consequent loss of stream length. Other examples of this process may be seen in #3, although less clearly because of the greater density of channels: a probable candidate for subsequent capture may be seen near the lower left corner of figure 4 (b,c & d).

The continuity equation following a flow-line in the downslope direction, x may be written in the form:

[9] $$-\partial z/\partial t = \partial S/\partial x - S/\rho$$

where z is elevation,

t is elapsed time,

S is sediment transport along the flow path

and ρ is the radius of contour curvature, taken positive in hollows.

The left hand side of the expression is thus the rate of lowering at any point. Substituting a slope transport law which is linear in gradient, g:

[10] $$s = f(a)\ g$$

for a suitable function f of unit area a and substituting the geometrical identity $\partial a/\partial x = 1+a/\rho$ leads to:

[11] $$-\partial z/\partial t = g\partial f/\partial a + f\partial g/\partial x + (g/\rho)(a\partial f/\partial a - f)$$

On the right hand side of this expression, the first term is always zero or positive; the second is positive on profile convexities and negative in concavities. The sign of the final term depends on both the direction of plan curvature (ρ) and the stability criterion, $a\partial f/\partial a - f$: overall it is positive for unstable hollows and stable spurs. Because small perturbations in the surface influence curvature very much more strongly than area or gradient, the final term in equation (11) may be seen as one way of obtaining the stability criterion. The magnitude of this term determines the rate at which hollows erode within the unstable area, or in other words the sensitivity of the landscape to the degree of instability. For the transport law used here in equation [2], $f(a) = k(1 + a^2/u^2)$, giving for the stability term:

[12] $$a\partial f/\partial a - f = k(a^2/u^2 - 1)$$

which takes the value of $-k$ on divides ($a = 0$), zero for unit area u and $+3k$ for unit area $2u$ which has been seen above to be appropriate for a channel-head defined by contour crenulation. For the initial near-uniform slopes used in the simulations, the rate of smoothing near divides appears low, but may be seen to be strongly dominant over other terms, particularly until marked convexities develop. Downslope the hollow enlargement term needs to be subtantial to dominate the first term on the right hand side (RHS) of equation [11]. As initially small hollows grow, enlargment is ultimately stopped by the increase in profile concavity which acts through the second term of the RHS. Any subsequent loss of drainage area through slope capture then leads to partial or total infilling of the valley.

Figure 5 shows some aspects of the course of evolution of the model landscapes. In all three runs there is a reduction in drainage density over time, as measured by the length of channels draining unit areas of at least $2u$. For #1, the channel length finally drops to zero, after some fluctuations, by 600,000 years. For #2, the length appears to have stabilised quickly, perhaps because of the relatively definite valley in the initial surface. The final channel length is estimated as 350m. This length has been converted to a drainage density by allocating it the area (2280 out of the 4096 cells) of the two catchments containing the streams, as headwater divide areas typically underestimate density, giving a density of 1.77 $km.km^{-2}$, or an average stream spacing of 560m. In #3, drainage density also declines over time, but more slowly, and tending towards an estimated total length of 2600m. This represents a drainage density (over the full area) of 7.33 $km.km^{-2}$, or a stream spacing of 136m. These are in ratio of 4.1x, which is close to the ratio of 4 for the critical unit areas (u). This result gives support to the concept that stream spacing should be directly proportional to unit critical area. The empirical constant of proportionality of 0.43 rather then unity is thought to reflect first the problems of using an operational definition of stream heads and second catchment shape, which for these rather early stages of development may remain strongly influenced by the overall dowmslope gradient tending to elongate drainage basins. If the constant of proportionality is used to extrapolate to #1, then the expected stream spacing of 2200m is consistent with an absence of channels in the area simulated (640mx640m).

Average denudation rates are also shown in figure 5. Initial differences between the runs reflect the different cell shape for #1, but also the difference in the average basal sediment transport rates forecast by equation [2] and summed over the distribution of unit areas. Comparing values for #2 and #3, the initial denudation rates are compatible with a coefficient of variation for slope-base unit area of about 2.5 around its mean of 640m. In detail the distribution of unit areas is similar to that predicted by the random network model (Kirkby, 1976b, p.201, Table 1), applied to all cells, with truncation of the distribution for large magnitudes to satisfy the constraint of finite mean magnitude. As the model landscape evolves, figure 5 shows a very slight increase in denudation rate for #1, and a marked decrease for #2 and #3. In #1,it is thought that the overall smoothing of the landscape has steepened the gradients of random initial depressions, so that, with the wash term everywhere negligible, this infilling of depressions is the dominant influence. In #2 and #3 the development of marked valleys produces lower gradients along their axes, so that the initially high sediment outputs from the highest drainage areas which dominate total output, progressively decrease over time.

It was surmised in the introduction that side-slopes leading to permanent channels would tend to adjust so that all tended towards a unit area not far short of the critical value, and general arguments about efficiency of drainage might tend to this conclusion. The model may most readily be compared with this surmise for the final stage of #2, where the central drainage basin is beginning to show signs of overall adjustment. Figure 6 shows the distribution of numbers of cells drainage to each channel (defined as above by unit area >=2u) cell from each bank separately, for the 240,000 year stage shown in figure 3(d) above. The average number of cells draining to the 72 channel banks (36 cells x 2 banks) is 31.7, which is compatible with the drainage spacing (560m) quoted above, allowing for cell shape. The distribution is strongly bimodal, with a peak for very small catchments and a second broader peak centred between 2^5 and 2^6 cells drained. The distribution is necessarily truncated for areas of more than 2^7 cells. Initially (at time zero) this second peak is negligible and grows over time: at the same time the initial peak becomes smaller. A rather similar pattern of distributions is found for areas draining to the slope base, although with a broader peak of small areas. It is hard to avoid the conclusion that the right-hand peak reflects progressive organization of the drainage in response to valley incision and catchment piracy. It is tempting to conjecture that the low area peak will ultimately be eliminated completely, leaving a truncated, approximately log-normal distribution of tributary areas for a maturely developed valley. Because of the difficulties of measuring the distribution of tributary areas in the field on slopes with necesarily have only very slight cross-relief, a derivation through modelling may have considerable relevance to problems of rill location and overland flow hydrology (for the transport equation adopted here).

6. Conclusions

The simulations carried out support Smith & Bretherton's (1972) theory of slope and valley stability as an effective mechanism for the formation of valleys. It appears to lead to a stable drainage pattern with a texture which may be related to sediment transport laws through the theory. For a given unit critical area, u, the unit area needed to support an eroded channel is about $2u$, at which point wash is 4x more effective than splash/creep for the process law used. The mean distance of overland flow streams stabilizes at about $0.22u$. It is conjectured that this may be distributed approximately log-normally when the catchment is fully integrated.

There remain a number of uncertainties about the interpretation of the model used here, even though it apears to perform realistically overall. The most serious problems are concerned with the significance of the unit cell size, and with the assumption of a a dendritic network throughout by carrying flow from each cell to only one other. The choice of unit cell size inevitably conceals some distribution of unit areas within each cell, which may, as has been seen have substantial effects if flow threads are very much narrower than the individual cell dimension. Some confidence is given by the constant of proportionality of drainage spacing to unit critical area, but the problem must remain as a possible source of error. The use of small cells plainly increases accuracy, but at the expense of the total area which may realistically be modelled.

The assumption of dendritic drainage patterns throughout has proved advantageous in simplifying drainage area algorithms, and is thought to be effective in practise but may be a source of error in establishing the size and development of hillslope areas. In practise the dendritic assumption is usually seen to be correct for drainage areas large enough to carry stream flows and substantial sediment loads, so that its adoption will not introduce errors in macroscopic channels. For an alluvial fan context too, the dendritic assumption will suppress ditributaries but will allow channel shifting, so that a fan may be built up in a reasonable way. The biggest possible error is for unchanneled slopes where the dendritic assumption may severely over-estimate tributary areas and so distort the kind of distribution shown in figure 6. This problem must remain unresolved in the present context, as a topic for further research.

The transport law used here (equation [2]) is thought to be appropriate for semi-arid rather than humid areas. It may be seen that under these assumptions the drainage density is strongly influenced by climate, together with hydrological properties probably dominated by vegetation influences, while gradient has little or no influence. If the same arguments are carried through to well vegetated humid areas where sub-surface flow is important, drainage density should respond most sensitively to soil properties and should decrease as slope gradient increases, other things being equal (Kirkby, 1980, p.70). The present paper is thought to make a small contribution to the problem of how slope and channel processes interact to scale the landscape. Much remains to be done, both at a theoretical level and in the field. The topic is a central one in process geomorphology, and merits a greater research effort.

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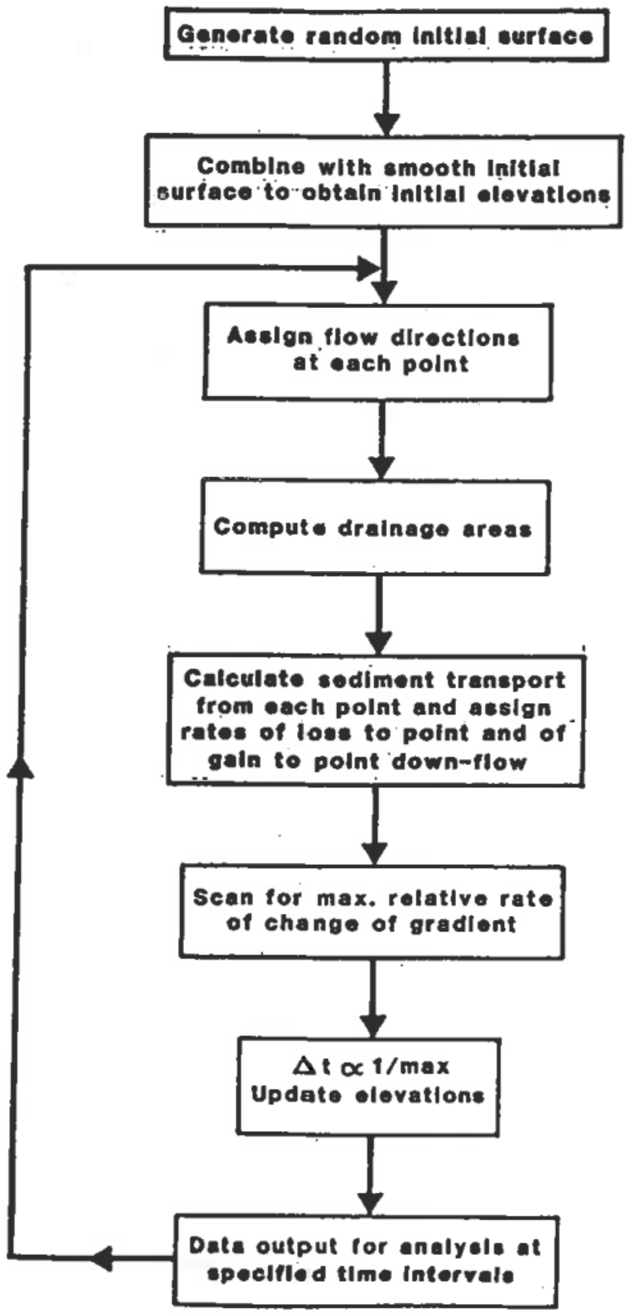
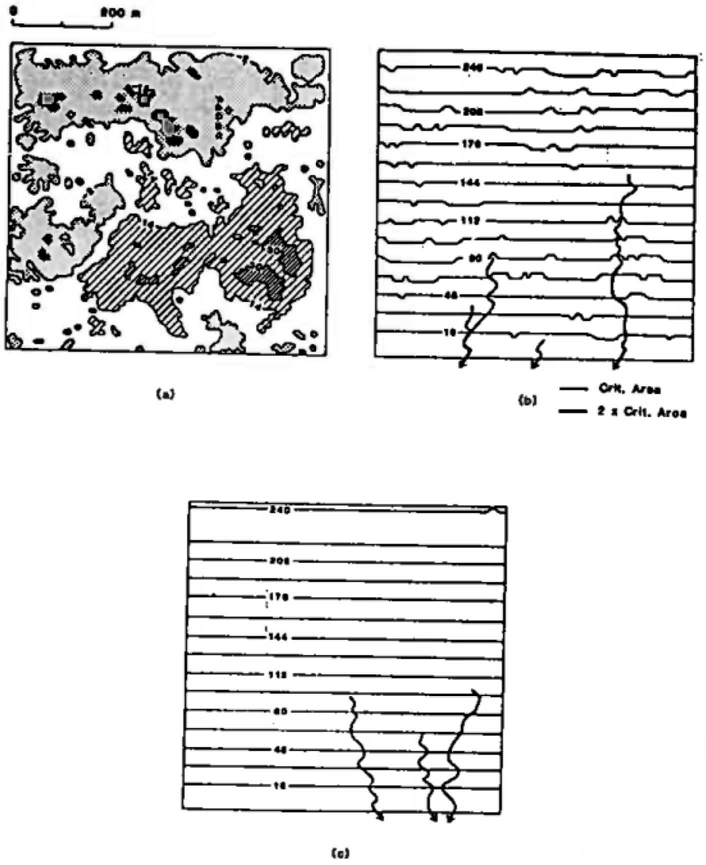
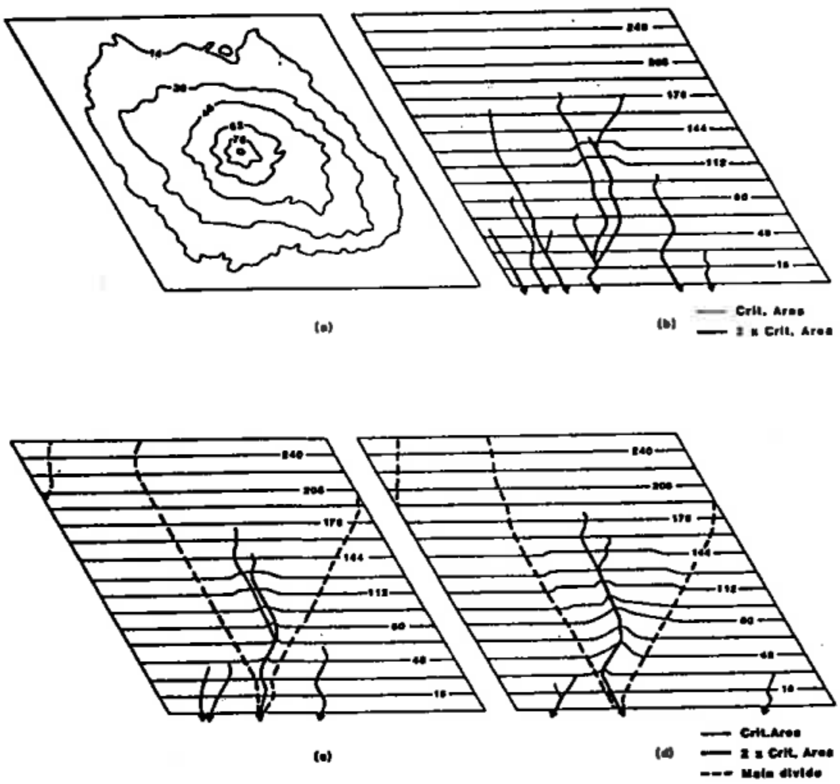
Figure list

1. Outline flow diagram for 2-dimensional simulation model.





2. Run #1 simulation.
(a) Fractal surface used to perturb initial surface.
(b) Initial surface perturbed by subtracting 1/10x fractal in a.
(c) Surface after 1.6 million years (176 iterations) and 0.86m average denudation.



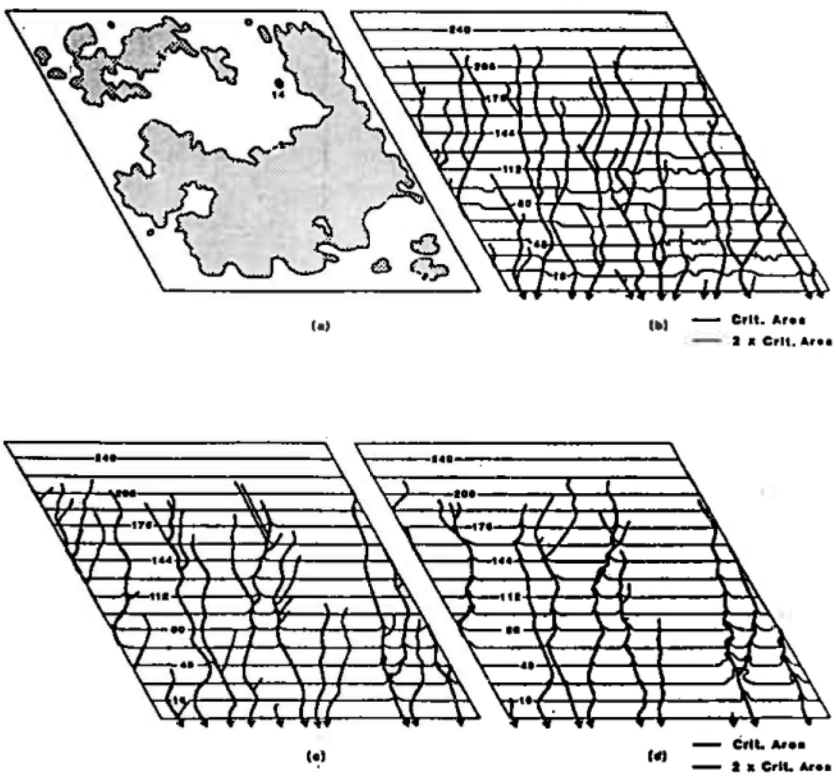
3. Run #2 simulation.

(a) Fractal surface with marked cantral peak

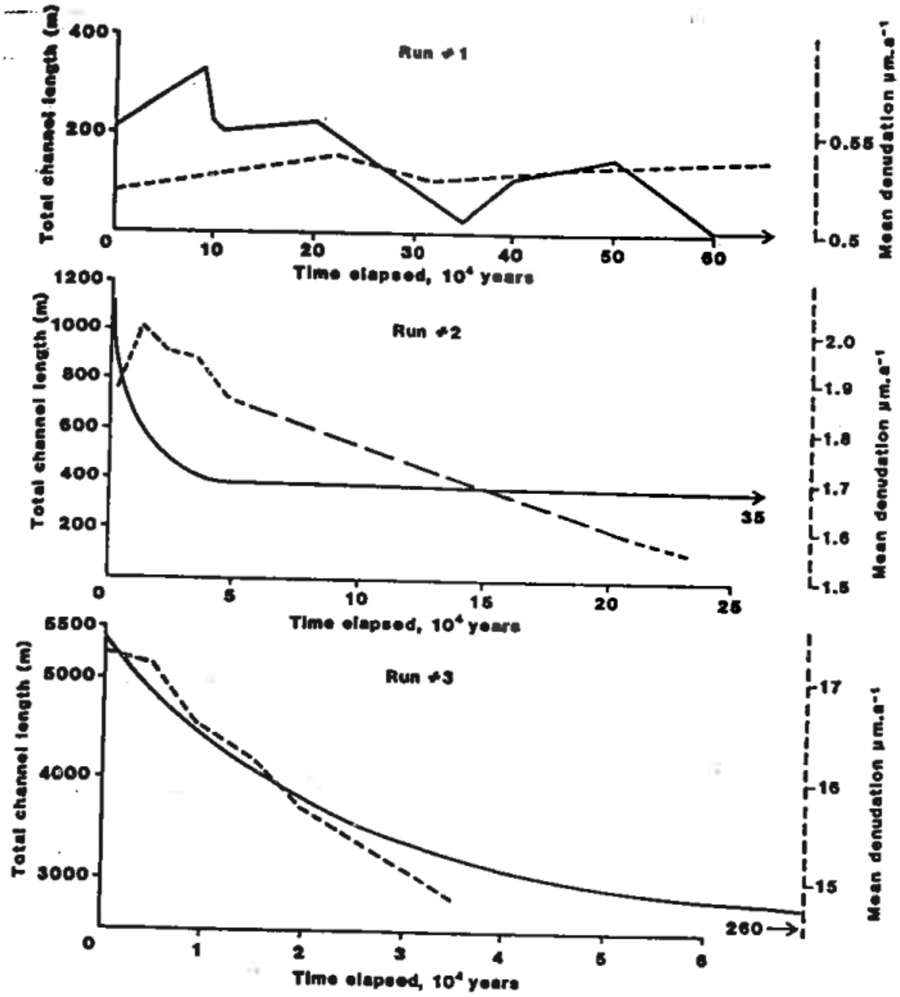
(b) Initial surface perturbed by subtracting 1/20x fractal in a.

(c) Surface after 40,000 years (52 iterations) and 0.08m average denudation.

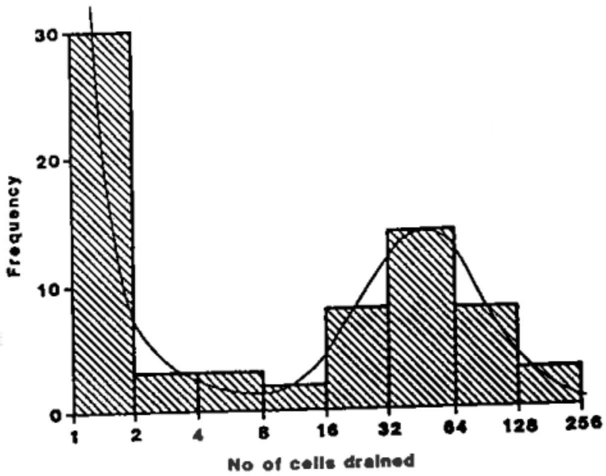
(d) Surface after 240,000 years (287 iterations) and 0.43m denudation.



4. Run #3 simulation.
(a) Fractal surface of low relief.
(b) Initial surface perturbed by subtracting 1/20x fractal in a.
(c) Surface after 20,000 years (118 iterations) and 0.34m denudation.
(d) Surface after 39,000 years (238 iterations) and 0.63m denudation.



5. Changes in effective channel length (unit area >=2u) and mean denudation rate over time in simulations #1 (a), #2 (b) and #3 (c).



6. The distribution of catchment areas tributary to effective channels for the 240,000 year stage of #2.