Working Paper 148

MODELLING THE REGIONAL SYSTEM: THE POPULATION COMPONENT

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Paper to be presented at the Second Franco-British Geographical Conference, Lyons, France, July, 1976

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May, 1976

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Abstract

The paper presents a linked set of analyses that model the population component of the regional system. Aggregate population accounts are presented for British standard regions for 1965-66 and 1970-71. A transition rates matrix is derived from the 1970-71 accounts, transformed into a growth rates matrix and used in a model that forecasts the population of the regions to 2001. The drawbacks and the strengths of the forecast are discussed.

MODELLING THE REGIONAL SYSTEM: THE POPULATION COMPONENT

P.H. REES

1. Definition of terms

The first part of the title of this paper, 'Modelling the regional system', immediately conjures up in the mind of the reader a vast array of possible topics and models ranging from land use-transportation models through to ecosystem models. The second part of the title is meant to deflate these expectations and to narrow the discussion to a concern with population, that is, population change, population accounting and population forecasting. The regional system which will be used to illustrate the discussion will be that of the standard regions of Britain.

The principal aim of the paper is to sketch out a linked set of analyses and models that describe and attempt to forecast population change in a set of regions. In order to accomplish this aim in the short space available the focus is narrowed to that of the aggregate population only and only a small subset of the many possible models are outlined. The set of analyses described in the paper thus form a prototype for a more extensive investigation of the changing British population (Rees, 1974, 1975).

The structure of the paper is as follows. In the next section, the framework of spatial population accounting is briefly reviewed. Population accounts for British regions for 1965-66 and 1970-71 are then presented in section 3, and the information they provide about the direction and pace of change in regional populations analyzed. Absolute measures of change are converted into relative measures through the computation of rates of birth, death, migration, survival and so on in section 4. Also generated in that section is the matrix of transition rates associated with the accounts.

Change in this matrix from 1965-66 to 1970-71 is examined. Then the matrix of growth rates of the regional population system is calculated from the transition rates matrix and from other information in the accounts.

The G matrix for 1970-71 is then used (section 5) in a model, first developed by Rogers (1966) for forecasting regional population and inter-regional flows, and results for a preliminary run of this model to 2001 are presented. These results are compared with official forecasts, and modifications to the initial model suggested. Criticisms of this model leads to proposals for the use of the alternative accounts based model in which easy to incorporate recent trends in birth rates, death rates and migration rates if available. A number of conclusions are reached about the appropriate strategy for modelling the demographic component of the regional system in this final section (section 8).

2. The theory of spatial demographic accounting

Accounting methods were first applied to demographic problems by Stone (1965, 1971a, 1971b) in a spatially aggregate form that recognised just two 'regions':'our country' and 'the outside world'. Demographic accounts were first expressed in a spatially explicit form in Rees (1972) and the underlying concepts explored in Rees and Wilson (1973), Wilson and Rees (1974), and Rees and Wilson (1975) with a full statement appearing in Rees and Wilson (1976). Alternative perspectives on demographic accounting are provided in Rees (1975b, 1975c). Here a very brief review is given and the reader is referred to the works cited for fuller details.

We can define a matrix K with elements $K^{\alpha(i)\omega(j)}$ to be a matrix of population flows over a period of time (measured by numbers of people involved) between a set of 'origin' states represented by the rows and 'destination' states represented by the columns. The superscript $\alpha(i)$

attached to the K variable refer to 'origin' states, α being the initial 'life state' and i the initial region of a person. The superscripts $w(j)$ refer to final 'life state' and final region. There are two initial life states, existence at the start of a period (represented by superscript ε) and birth during the period (represented by superscript β), and two final life states, death during the period (represented by superscript δ) and survival at the end of the period (represented by superscript σ). The $\underline{K}$ matrix can be partitioned into four parts therefore recognizing the four life state to life state transitions that can take place

$$K = \left|\begin{array}{c|c} \{K^{\varepsilon(i)\sigma(j)}\} & \{K^{\varepsilon(i)\delta(j)}\} \\ \hline \{K^{\beta(i)\sigma(j)}\} & \{K^{\beta(i)\delta(j)}\} \end{array}\right| \tag{1}$$

where $K^{\varepsilon(i)\sigma(j)}$ refers to persons who exist in region i at the start of the period and survive in region j at the end; where $K^{\varepsilon(i)\delta(j)}$ are persons likewise starting in existence in a region i who end the period dying in region j; where $K^{\beta(i)\sigma(j)}$ and $K^{\beta(i)\delta(j)}$ are the corresponding flows for persons born in the period in region i.

The accounts matrix can be specified for any number of regions but must always include a residual, 'rest of the world', region to close the system of accounts. In the case of the British regional system we describe later in the paper, we specify accounts with 11 regions, 10 regions within Britain and the 11th referring to the rest of the world. The resulting accounts matrix looks like this

$$\underline{K} = \left[\begin{array}{cccc|cccc}
K^{\varepsilon(1)\sigma(1)} & K^{\varepsilon(1)\sigma(2)} & \ldots & K^{\varepsilon(1)\sigma(11)} & K^{\varepsilon(1)\delta(1)} & K^{\varepsilon(1)\delta(2)} & \ldots & K^{\varepsilon(1)\delta(11)} \\
K^{\varepsilon(2)\sigma(1)} & K^{\varepsilon(2)\sigma(2)} & \ldots & K^{\varepsilon(2)\sigma(11)} & K^{\varepsilon(2)\delta(1)} & K^{\varepsilon(2)\delta(2)} & \ldots & K^{\varepsilon(2)\delta(11)} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
K^{\varepsilon(11)\sigma(1)} & K^{\varepsilon(11)\sigma(2)} & \ldots & K^{\varepsilon(11)\sigma(11)} & K^{\varepsilon(11)\delta(1)} & K^{\varepsilon(11)\delta(2)} & \ldots & K^{\varepsilon(11)\delta(11)} \\
\hline
K^{\beta(1)\sigma(1)} & K^{\beta(1)\sigma(2)} & \ldots & K^{\beta(1)\sigma(11)} & K^{\beta(1)\delta(1)} & K^{\beta(1)\delta(2)} & \ldots & K^{\beta(1)\delta(11)} \\
K^{\beta(2)\sigma(1)} & K^{\beta(2)\sigma(2)} & \ldots & K^{\beta(2)\sigma(11)} & K^{\beta(2)\delta(1)} & K^{\beta(2)\delta(2)} & \ldots & K^{\beta(2)\delta(11)} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
K^{\beta(11)\sigma(1)} & K^{\beta(11)\sigma(2)} & \ldots & K^{\beta(11)\sigma(11)} & K^{\beta(11)\delta(1)} & K^{\beta(11)\delta(2)} & \ldots & K^{\beta(11)\delta(11)}
\end{array}\right] \tag{2}$$

Since interest is focused on regions 1 to 10, the terms $K^{\varepsilon(11)\sigma(11)}$, $K^{\varepsilon(11)\delta(11)}$, $K^{\beta(11)\sigma(11)}$ and $K^{\beta(11)\delta(11)}$ involving rest-of-the-world to rest-of-the-world population flows can be omitted from the analysis.

The population numbers that replace the $K^{\alpha(i)\omega(j)}$ in an empirical study are normally unavailable in direct form, and have to be estimated using a model (here referred to as 'accounts based model') involving what information is known, involving the row and column constraints, and a set of hypotheses about the rates at which migrants ($K^{\varepsilon(i)\sigma(j)}$ flows where $i \neq j$) and infant migrants ($K^{\beta(i)\sigma(j)}$ flows) die. In the matrix below (equation (3)) the 'known' terms are picked out and row and column constraints have been added.

$$\underline{K} = \begin{array}{c}
\left[\begin{array}{cccc|cccc}
- & K^{\varepsilon(1)\sigma(2)} & \ldots & K^{\varepsilon(1)\sigma(11)} & - & - & \ldots & - \\
K^{\varepsilon(2)\sigma(1)} & - & \ldots & K^{\varepsilon(2)\sigma(11)} & - & - & \ldots & - \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
K^{\varepsilon(11)\sigma(1)} & K^{\varepsilon(11)\sigma(2)} & \ldots & 0 & - & - & \ldots & 0 \\
\hline
- & - & \ldots & - & - & - & \ldots & - \\
- & - & \ldots & - & - & - & \ldots & - \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
- & - & \ldots & 0 & - & - & \ldots & 0
\end{array}\right]
\begin{array}{c}
K^{\varepsilon(1)*(*)} \\ K^{\varepsilon(2)*(*)} \\ \vdots \\ 0 \\ K^{\beta(1)*(*)} \\ K^{\beta(2)*(*)} \\ \vdots \\ 0
\end{array} \\
\begin{array}{cccc|cccc}
- & - & \ldots & 0 & K^{*(*)\delta(1)} & K^{*(*)\delta(2)} & \ldots & 0
\end{array}
\end{array} \tag{3}$$

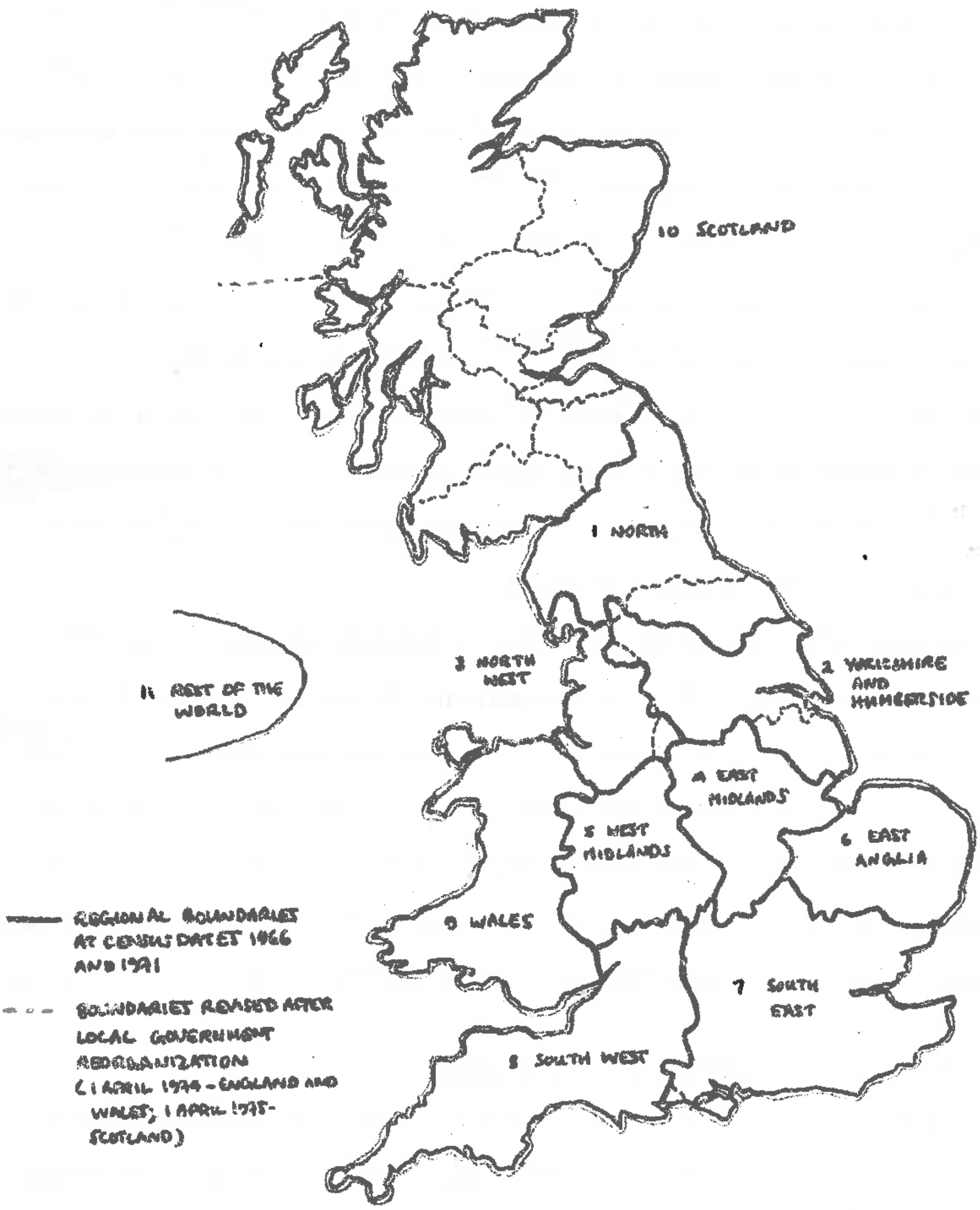
The terms of equation (2) which in equation (3) are represented by a dash (-) are those which must be estimated by an accounts based model (see Rees and Wilson (1973) and (1976) for a full description of these). This model uses as input the initial populations of the regions, $K^{\varepsilon(i)*(*)}$, with the asterisk representing summation over the superscript in the same position, to which the elements in the top half of the accounts matrix sum row-wise; the birth totals in the regions, $K^{\beta(i)*(*)}$, to which the elements in the bottom half of the matrix sum row-wise; the death totals, $K^{*(*)\delta(i)}$, to which elements in the right-hand half of the matrix sum column-wise; and the 'exist-survive' migrants, $K^{\varepsilon(i)\sigma(j)}$, $i \neq j$, who occupy the off diagonal positions in the top-left quadrant of the $\underline{K}$ matrix. The totals of columns in the left-hand half of $\underline{K}$ are the final populations in the period, the $K^{*(*)\sigma(i)}$, which are a product of the model, although they may be used in some variants of the accounts based model.

In this brief survey of the theory underlying spatial demographic accounting, disaggregation of the population by age and sex (or by any other classification) is neglected. Readers are referred to Wilson and Rees (1974) and Rees and Wilson (1976) for full treatment. The neglect is continued in the applications that follow in order to highlight the regional or spatial issues involved, as opposed to those involving age and sex which are well treated elsewhere (Pressat, 1972; Keyfitz, 1968).

3. Demographic accounts for British regions

Figure 1 shows the boundaries of the regions for which accounts are constructed. These boundaries have changed twice since the 1961 census and the map shows those existing at the 1966 and 1971 census dates, together with the revised boundaries now current as a result of local government reorganization. Together the 10 regions make up 'Great Britain'.

Figure 1 The standard regions of Great Britain



Addition of Northern Ireland to the list would have converted the regional system into one for the United Kingdom (of Great Britain and Northern Ireland), but figures for emigration from Northern Ireland to the other British regions are unavailable. Northern Ireland was therefore placed in the rest-of-the-world region.

Figure 2 shows the set of accounts prepared for the one year period 1965-66 (April 23/24 to April 23/24) prior to the date of the Sample Census 1966. This period was chosen as it was the one for which migrant information was available. This is displayed in the top left-hand quadrant (off-diagonal elements). The numbers given in the census migration tables have been corrected for underenumeration (see Smith and Rees, 1974; and Illingworth, 1975a, 1975b, for details of method). Emigrants to the rest-of-the-world are estimated by multiplying the total of immigrant figures for all regions (given in the census migration tables) by the ratio of emigrants to immigrants for 1965-66 (1.3516) given in the International Passenger Survey statistics (General Register Office, 1966) and then disaggregating this total by the regional shares revealed in a later survey for 1971. One might note, in passing, that the migration figures given by the retrospective census tables and by this method are about 50 per cent greater in magnitude than those recorded by the 'current' measuring devices (International Passenger Survey, Commonwealth Immigrant Act Statistics).

The initial populations of the regions (3307961...5206304) are estimated by backwards extrapolation from the mid-year estimate for 1965. The births and deaths totals are made up of appropriate shares of the births and deaths of 1965 and 1966 calendar years (see Rees and Wilson, 1976, Chapter 3 for methods). Infant migrants (children under 1 year of age migrating

Initial State \ Final State	Survival at C.D. 1966: 1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RW	Sub Totals	Death in 1965-66: 1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RW	Sub Totals	TOTALS
Existence at C.D. 1965																									
1. North	3195207	10024	6118	3397	5468	1635	13412	2630	1288	4170	24964	3268919	38603	61	38	22	29	9	79	17	6	27	149	39042	3307961
2. Yorkshire & Humberside	10856	4556799	10774	13057	4089	2300	15259	4545	2171	3419	27624	4551393	65	55902	68	72	21	16	90	30	14	22	167	56167	4707860
3. North West	6097	10815	6450139	6392	7913	2252	21732	6513	8411	3825	44797	6614886	36	65	82732	35	41	13	128	42	55	25	282	83476	6698360
4. East Midlands	2739	12499	5479	3151530	8279	4231	15654	3774	1400	2334	23247	3231276	16	76	34	35102	-43	24	92	25	9	15	127	35565	3266839
5. West Midlands	3247	4961	9878	9918	4812591	2424	21368	11176	5750	2809	31307	4915429	19	30	62	54	51089	14	126	73	38	18	164	51680	4967116
6. East Anglia	1521	2870	2160	4665	2322	1473695	17656	3357	781	1268	26319	1536614	9	17	14	26	12	17341	104	22	5	6	151	17709	1554323
7. South East	12018	15435	20547	19664	19705	30141	16347869	53740	11330	14938	189199	16734594	72	93	129	108	103	173	196043	349	75	96	1118	198359	16932953
8. South West	2789	4036	4554	3874	7363	2592	40201	3424762	4909	2890	34449	3532809	17	24	29	21	39	17	238	45345	32	19	224	46005	3578815
9. Wales	1187	1927	5192	1978	6288	710	12038	5405	2610250	1795	9315	2556115	7	12	33	11	33	4	71	35	34815	12	61	35093	2691209
10. Scotland	5497	4756	6754	4767	5558	1633	19786	3438	1187	5037917	48865	5140178	33	29	42	26	29	9	117	22	8	65498	313	66127	5206304
11. Rest of the world	13052	17636	25263	13742	23569	17170	175073	25902	7332	21662	0	340401	78	107	159	75	123	99	1034	168	48	139	0	2031	342431
Sub totals	3254210	4641758	6592668	3233654	4903145	1539681	16700048	3545230	2654847	5097087	460086	52622614	38954	56416	83361	35552	51563	17720	198124	46129	35109	65678	2757	631562	53254171
Birth in 1965-66																									
1. North	57463	89	54	35	48	14	119	23	11	37	221	58114	344	0	0	0	0	0	0	0	0	0	1	345	58459
2. Yorkshire & Humberside	98	83558	97	118	37	25	138	41	20	31	249	84412	0	508	0	0	0	0	0	0	0	0	1	509	84921
3. North West	56	99	120374	58	72	21	158	59	77	35	409	121458	0	0	761	0	0	0	1	0	0	0	1	763	122221
4. East Midlands	25	114	50	58751	76	39	143	35	13	22	213	59481	0	0	0	324	0	0	0	0	0	0	1	325	59806
5. West Midlands	31	47	94	95	93292	23	204	107	55	27	299	94274	0	0	0	0	491	0	1	0	0	0	1	493	94767
6. East Anglia	13	24	19	40	20	25711	150	28	7	11	223	26246	0	0	0	0	0	149	0	0	0	0	1	150	26396
7. South East	106	138	184	176	177	270	298182	481	102	134	1695	301547	0	0	1	0	0	1	1772	2	0	0	5	1781	303428
8. South West	24	35	39	33	63	26	347	60408	42	25	297	61331	0	0	0	0	0	0	1	395	0	0	1	397	61728
9. Wales	10	16	44	17	53	6	102	46	45089	15	79	45477	0	0	0	0	0	0	0	0	298	0	0	298	45775
10. Scotland	52	45	64	46	53	16	189	33	11	97603	466	98578	0	0	0	0	0	0	1	0	0	629	1	631	99209
11. Rest of the world	227	307	440	239	410	299	3047	451	128	377	0	5925	1	1	1	1	1	1	9	1	0	1	0	17	5942
Sub totals	58107	84472	121459	59608	94301	26450	302819	61704	45555	98317	4151	956943	345	509	763	325	492	151	1785	393	298	630	13	5709	962652
TOTALS	3312317	4726230	6714327	3293262	4997446	1566131	17002867	3606934	2700402	5195404	464237	53585482	39300	56925	84123	35877	52054	17870	199907	46527	35405	66509	2769	637271	54216823

Figure 2 Aggregate population accounts for British regions, 1965-66 (one year)

between regions - the $K^{\beta(i)\sigma(j)}$ terms) are estimated by applying the migration rates of the 'exist-survive' migrants to the regional birth totals with a division by 2 to reflect the shorter life span in the period of these infants. From these inputs the accounts matrix of Figure 2 was generated with sets of sub-totals and totals added using the SDAT computer programme (Rees and Wilson, 1974).

Exactly parallel accounts for British regions for the one year period 1970-71 prior to the 1971 census date (April 23/24, 1971) are shown in Figure 3. The known data input to the accounts based model is the same as that for 1965-66 except that the initial populations were interpolated between the June 30, 1969 and June 30, 1970 estimates by the Registrar General.

These two sets of demographic accounts present comprehensive pictures of population change among British regions 5 years ago and 10 years ago respectively. The system appears to have shifted relatively little in the five years between 1965-66 and 1970-71 in structure with the latter containing larger population flows than the former. The gross totals of all population flows involved in the regional system are 54.217 millions in 1965-66 and 55.448 millions in 1970-71. We will look in the next section at the detailed differences in growth regime by calculating the demographic rates involved.

To get a picture of the gross changes taking place in the regional system we can summarize the two sets of accounts in more conventional form in terms of input flows and output flows. However, these tables differ from most conventional tables in having explicitly dealt with all possible flows into and out of the system - because they are derived from sets of accounts. Figure 4 shows these inflow-outflow-accounts For two regions - the first, Scotland, showing slight population decline, and the second, East Anglia, showing substantial population increase.

Initial state \ Final state	Survival at C.D. 1971: 1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RTW	Sub totals
Existence at C.D. 1970												
1. North	3241941	9290	7370	3630	3790	2150	15040	5870	1220	4910	25275	3318526
2. Yorkshire & Humberside	12790	4644500	13290	13190	5970	4320	21580	5610	2570	3470	27886	4755176
3. North West	6790	10650	6567400	6320	5750	2580	23210	8950	10370	5200	43952	6701572
4. East Midlands	4200	12170	5300	3229569	9610	6030	19110	6680	2330	3100	25545	3323644
5. West Midlands	3550	5400	9750	12600	5009250	3040	25630	12590	6350	3340	27877	5119377
6. East Anglia	1150	2510	1970	4490	2510	1584217	20460	3820	1170	1650	29194	1652171
7. South East	12070	16570	22920	26390	21410	35410	16597512	66570	12050	17610	193259	17121011
8. South West	2970	4600	4860	4230	8680	3390	49030	3589220	4730	4810	35450	3711970
9. Wales	1450	2040	6390	2420	5650	1280	13160	7000	2647069	1650	10185	2698294
10. Scotland	3540	4610	7220	4910	4630	2070	23790	5330	1700	5033830	57512	5151442
11. Rest of the world	14570	19630	27330	16650	23140	20280	197220	29390	8840	28160	0	385210
Sub totals	3307021	4732170	6673800	3324399	5104420	1665207	17111742	3739030	2698399	5107730	475275	53339193
Birth in 1970-71												
1. North	52151	73	56	29	30	17	119	31	10	39	200	52757
2. Yorkshire & Humberside	109	80569	113	112	51	37	184	48	22	30	238	81513
3. North West	57	90	112263	53	82	25	246	75	87	44	370	113392
4. East Midlands	35	103	45	55667	81	51	161	56	20	26	216	56461
5. West Midlands	31	47	85	110	88686	26	223	110	55	29	243	89645
6. East Anglia	9	20	16	36	20	25873	163	30	9	13	224	26413
7. South East	94	129	179	206	167	277	265676	520	94	138	1510	268990
8. South West	25	35	37	32	66	26	376	56279	36	37	272	57219
9. Wales	11	16	51	19	45	10	104	55	42533	13	81	42938
10. Scotland	47	41	61	42	39	18	201	45	14	86708	488	87704
11. Rest of the world	256	344	479	292	406	356	3459	515	155	494	0	6756
Sub totals	52823	81417	113387	56598	89673	26716	270912	57764	43035	87571	3842	883788
TOTALS	3359844	4813637	6787187	3380997	5194093	1691923	17382654	3796794	2741434	5195301	479117	54222981

Initial state \ Final state	Death in 1970-71: 1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RTW	Sub totals	TOTALS
Existence at C.D. 1970													
1. North	36271	55	46	20	19	12	83	24	8	29	147	38714	3357240
2. Yorkshire & Humberside	74	55851	63	72	31	24	119	34	16	21	165	56470	4811646
3. North West	39	63	82920	35	50	17	161	55	65	31	274	83710	6785282
4. East Midlands	24	72	33	36111	49	33	106	41	15	18	140	36642	3360286
5. West Midlands	21	32	61	69	52000	17	142	77	40	20	143	52622	5171999
6. East Anglia	7	15	12	25	13	17382	113	83	7	10	156	18363	1670534
7. South East	70	93	143	145	110	196	187508	409	76	105	1069	189929	17311740
8. South West	17	27	30	25	44	19	271	45000	30	29	218	45708	3757678
9. Wales	8	12	40	13	29	7	73	43	33607	10	64	33906	2732200
10. Scotland	32	28	45	27	24	11	132	33	11	60633	342	61318	5212760
11. Rest of the world	85	116	170	91	116	112	1090	181	55	167	0	2185	387395
Sub totals	38648	56349	83583	36631	52487	18430	189798	45920	33930	60936	2718	619567	54558760
Birth in 1970-71													
1. North	305	0	0	0	0	0	0	0	0	0	1	306	53063
2. Yorkshire & Humberside	0	480	0	0	0	0	1	0	0	0	1	482	81995
3. North West	0	0	703	0	0	0	1	0	0	0	1	705	114097
4. East Midlands	0	0	0	308	0	0	0	0	0	0	1	309	56770
5. West Midlands	0	0	0	0	457	0	1	0	0	0	1	459	90104
6. East Anglia	0	0	0	0	0	145	0	0	0	0	1	145	26558
7. South East	0	0	1	1	0	1	1478	2	0	0	4	1487	270477
8. South West	0	0	0	0	0	0	1	349	0	0	1	351	57570
9. Wales	0	0	0	0	0	0	0	0	268	0	0	268	43206
10. Scotland	0	0	0	0	0	0	1	0	0	518	1	520	88224
11. Rest of the world	1	1	1	1	1	1	10	2	0	1	0	19	6775
Sub totals	305	481	705	310	458	147	1495	353	268	519	12	5052	888840
TOTALS	38954	56830	84288	36941	52945	18577	191291	46273	34198	61432	2730	624619	55447600

Figure 3 Aggregate population accounts for British regions, 1970-71 (one year)

Figure 4 Inflow-Outflow Accounts for Scotland and East Anglia

SCOTLAND

Item		1965-66 Inflow	1965-66 Outflow	1965-66 Net flow	1970-71 Inflow	1970-71 Outflow	1970-71 Net flow
Initial population		5206305			5212760		
Migrants	E-S	59170	102261	-43091	73900	117612	-43712
	B-S	714	975	-261	863	996	-133
	E-D	380	629	-249	440	685	-412
	B-D	1	2	-1	1	2	-1
	Total	60265	103867	-43602	75204	119265	-44258
Births		99209			88224		
Deaths			66508			61592	
Natural increase				+32701			+26632
Final population			5195404			5195301	
Totals		5365779	5365779		5376188	5376188	
Total change				-10901			-17626

EAST ANGLIA

Item		1965-66 Inflow	1965-66 Outflow	1965-66 Net flow	1970-71 Inflow	1970-71 Outflow	1970-71 Net flow
Initial population		1554323			1670534		
Migrants	E-S	65986	62919	+3067	80990	67954	+13036
	B-S	739	535	+204	843	540	+303
	E-D	379	368	+11	448	381	+67
	B-D	2	1	+1	2	1	+1
	Total	67106	63823	+3283	82283	68876	+13407
Births		26396			26559		
Deaths			17871			18577	
Natural increase				+8525			+7982
Final population			1566131			1691923	
Totals		1647825	1647825		1779376	1779376	
Total change				+11808			+21389

Scotland's regime of population change is roughly the same in both years with a falling natural increase failing to offset fairly constant net-out-migration resulting in a rise in the size of population decline. In East Anglia, natural increase has also fallen somewhat but has been counterbalanced by an increase in net immigration. By looking back at the accounts table (Figure 3) one can see that a substantial portion of the 44,000 net migrant loss in 1970-71 in Scotland was to the rest-of-the-world (29,500). Hence the importance of including in the analysis a rest-of-the-world region. This is essential if an accounting framework is adopted.

Much more could be said about the pattern of population flows revealed in Figures 2 and 3 but since the main aim of the paper is to outline the prototype modelling system this analysis is omitted here.

4. Rates from the accounts

Using the demographic accounts we can define a variety of rates which form the input to historical analyses of trends and forecasting analyses of future numbers. The different birth and death rates which can be defined utilize the same numerators (total births, total deaths) but employ populations at risk appropriate to the forecasting model adopted.

The conventional population at risk of giving birth or of dying is usually taken to be the mid-period population or the average of the initial and final populations of the region in question. The second version of the conventional at risk population can be calculated from the accounts. A more precise alternative which only population accounts make possible is the multi-regional population at risk (see Rees and Wilson, 1973 and 1976), although there is relatively little difference between the two. A third alternative, the initial population of the region, can be rather different.

The variation in vital rates amongst the regions is relatively small. Versions of these rates, or rather their age-sex disaggregated equivalents, are used in cohort survival models.

An alternative rate is the transition rate formed by dividing the row element in the accounts by the appropriate row total. This kind of rate has to be used when migration is considered, and forms part of the raw material for the growth rates matrix developed by Rogers (1966, 1971, 1975) which will be utilized in section 5 of the paper. The transition rates or $\underline{H}$ matrix for 1970-71 is displayed in Figure 5.

The elements in any row of this matrix show how persons originating in that row are distributed in the ensuing year. In the case of Scotland and East Anglia illustrated earlier the chances of surviving within the region are 0.96567 and 0.94833, of migrating to and surviving in the South East are 0.00456 and 0.01225 respectively; of migrating to and surviving in the rest-of-the-world are 0.01105 and 0.01688 respectively and so on.

Now, the $\underline{H}$ matrix of transition rates can be used in a population change model directly as long as a birth sub-model is added that utilizes one of the sets of the vital rates referred to earlier (see Rees and King, 1970). However, Rogers (1966, 1971, 1975) has used them in a different form in a simpler matrix multiplication model. The $\underline{G}$ matrix of growth rates involved in that model is calculated as follows from the $\underline{H}$ and $\underline{K}$ matrices. The transition rates in the existence-survival quadrant (top-left) of the $\underline{H}$ matrix are retained and to each element is added birth-and-transition rates formed by dividing each element in the birth-survival quadrant (bottom-left) of the $\underline{K}$ matrix by the corresponding initial population total. The resulting matrix of growth rates for 1970-71 is shown in Figure 6. Normally this matrix is transposed before use and this is what is termed the $\underline{G}$ matrix. These rates are rates of transition and

Survival at C.D. 1971

Initial State \ Final State	1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RW	Sub totals
Existence at C.D. 1970												
1. North	.96566	.00277	.00220	.00103	.00113	.00065	.00445	.00115	.00036	.00146	.00753	
2. Yorkshire & Humberside	.00266	.95526	.00276	.00274	.00124	.00090	.00449	.00117	.00053	.00072	.00508	
3. North West	.00100	.00157	.96159	.00093	.00144	.00044	.00430	.00132	.00153	.00077	.00645	
4. East Midlands	.00125	.00362	.00158	.96110	.00256	.00179	.00569	.00199	.00069	.00092	.00760	
5. West Midlands	.00069	.00104	.00169	.00244	.96853	.00059	.00436	.00243	.00123	.00065	.00539	
6. East Anglia	.00069	.00150	.00118	.00259	.00152	.94833	.01225	.00229	.00070	.00099	.01603	
7. South East	.00070	.00096	.00132	.00152	.00124	.00205	.96452	.00385	.00070	.00102	.01117	
8. South West	.00079	.00122	.00129	.00113	.00231	.00090	.01305	.95517	.00126	.00128	.00343	
9. Wales	.00053	.00075	.00234	.00207	.00307	.00047	.00462	.00256	.95884	.00060	.00373	
10. Scotland	.00106	.00092	.00159	.00094	.00039	.00040	.00456	.00102	.00033	.96557	.01105	
11. Rest of the world	$.0^{5}41$	$.0^{5}55$	$.0^{5}77$	$.0^{5}47$	$.0^{5}65$	$.0^{5}57$	$.0^{4}553$	$.0^{5}82$	$.0^{5}25$	$.0^{5}79$	.0	
Sub totals												
Birth in 1970-71												
1. North	.98279	.00138	.00109	.00055	.00057	.00032	.00224	.00058	.00019	.00073	.00377	
2. Yorkshire & Humberside	.00153	.98261	.00136	.00137	.00062	.00045	.00224	.00059	.00027	.00037	.00290	
3. North West	.00050	.00079	.98335	.00046	.00072	.00032	.00216	.00066	.00076	.00039	.00324	
4. East Midlands	.00062	.00181	.00079	.98357	.00143	.00030	.00284	.00099	.00035	.00046	.00380	
5. West Midlands	.00034	.00052	.00074	.00122	.98426	.00029	.00247	.00122	.00061	.00032	.00270	
6. East Anglia	.00034	.00075	.00060	.00136	.00075	.97417	.00614	.00113	.00034	.00049	.00843	
7. South East	.00035	.00043	.00066	.00076	.00062	.00102	.98225	.00192	.00035	.00051	.00558	
8. South West	.00040	.00061	.00064	.00056	.00115	.00045	.00653	.97758	.00063	.00064	.00472	
9. Wales	.00025	.00037	.00118	.00044	.00104	.00023	.00241	.00127	.98442	.00030	.00187	
10. Scotland	.00033	.00046	.00069	.00048	.00044	.00020	.00228	.00051	.00016	.98282	.00553	
11. Rest of the world	$.0^{5}20$	$.0^{5}28$	$.0^{5}38$	$.0^{5}25$	$.0^{5}32$	$.0^{5}23$	$.0^{4}277$	$.0^{5}41$	$.0^{5}12$	$.0^{5}40$	.0	
Sub totals												
TOTALS												

Death in 1970-71

Initial State \ Final State	1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RW	Sub totals	TOTALS
Existence at C.D. 1970													
1. North	.1140	$.0^{4}163$	$.0^{4}137$	$.0^{5}56$	$.0^{5}56$	$.0^{5}35$	$.0^{4}247$	$.0^{5}71$	$.0^{5}23$	$.0^{5}65$	$.0^{4}437$		1.00000
2. Yorkshire & Humberside	$.0^{4}153$	.01160	$.0^{4}172$	$.0^{4}149$	$.0^{5}64$	$.0^{5}49$	$.0^{4}247$	$.0^{5}70$	$.0^{5}37$	$.0^{5}43$	$.0^{4}342$		1.00000
3. North West	$.0^{5}57$	$.0^{5}93$	.01222	$.0^{5}52$	$.0^{5}25$	$.0^{5}25$	$.0^{4}237$	$.0^{5}81$	$.0^{5}36$	$.0^{5}46$	$.0^{4}403$		1.00000
4. East Midlands	$.0^{5}71$	$.0^{4}214$	$.0^{5}96$	.01075	$.0^{4}146$	$.0^{5}98$	$.0^{4}315$	$.0^{4}122$	$.0^{5}45$	$.0^{5}54$	$.0^{4}417$		1.00000
5. West Midlands	$.0^{5}41$	$.0^{5}62$	$.0^{4}118$	$.0^{4}135$	.01005	$.0^{5}33$	$.0^{4}275$	$.0^{4}149$	$.0^{5}77$	$.0^{5}39$	$.0^{4}276$		1.00000
6. East Anglia	$.0^{5}42$	$.0^{5}90$	$.0^{5}72$	$.0^{4}150$	$.0^{5}78$	.01076	$.0^{4}676$	$.0^{4}138$	$.0^{5}42$	$.0^{5}60$	$.0^{4}934$		1.00000
7. South East	$.0^{5}40$	$.0^{5}57$	$.0^{5}83$	$.0^{5}84$	$.0^{5}64$	$.0^{4}113$	.01083	$.0^{4}236$	$.0^{5}44$	$.0^{5}61$	$.0^{4}618$		1.00000
8. South West	$.0^{5}45$	$.0^{5}72$	$.0^{5}60$	$.0^{5}61$	$.0^{4}117$	$.0^{5}51$	$.0^{4}721$	.01198	$.0^{4}738$	$.0^{4}772$	$.0^{4}580$		1.00000
9. Wales	$.0^{5}29$	$.0^{5}44$	$.0^{4}146$	$.0^{5}48$	$.0^{4}106$	$.0^{5}26$	$.0^{4}267$	$.0^{4}157$	.01230	$.0^{5}37$	$.0^{4}234$		1.00000
10. Scotland	$.0^{5}61$	$.0^{5}54$	$.0^{5}86$	$.0^{5}52$	$.0^{5}46$	$.0^{5}21$	$.0^{4}253$	$.0^{5}63$	$.0^{5}21$	.01163	$.0^{4}656$		1.00000
11. Rest of the world	$.0^{7}24$	$.0^{7}33$	$.0^{7}45$	$.0^{7}26$	$.0^{7}33$	$.0^{7}31$	$.0^{6}31$	$.0^{7}51$	$.0^{7}15$	$.0^{7}47$	.0		1.00000
Sub totals													
Birth in 1970-71													
1. North	.00575	.0	.0	.0	.0	.0	.0	.0	.0	.0	$.0^{4}108$		1.00000
2. Yorkshire & Humberside	.0	.00585	.0	.0	.0	.0	$.0^{4}122$	.0	.0	.0	$.0^{4}122$		1.00000
3. North West	.0	.0	.00616	.0	.0	.0	$.0^{5}88$	.0	.0	.0	$.0^{5}88$		1.00000
4. East Midlands	.0	.0	.0	.00543	.0	.0	.0	.0	.0	.0	$.0^{4}176$		1.00000
5. West Midlands	.0	.0	.0	.0	.00507	.0	$.0^{4}111$	.0	.0	.0	$.0^{4}111$		1.00000
6. East Anglia	.0	.0	.0	.0	.0	.00545	.0	.0	.0	.0	$.0^{4}377$		1.00000
7. South East	.0	.0	$.0^{5}37$	$.0^{5}37$	.0	$.0^{5}37$	.00546	$.0^{5}74$	.0	.0	$.0^{4}149$		1.00000
8. South West	.0	.0	.0	.0	.0	.0	$.0^{4}173$	.00606	.0	.0	$.0^{4}173$		1.00000
9. Wales	.0	.0	.0	.0	.0	.0	.0	.0	.00620	.0	.0		1.00000
10. Scotland	.0	.0	.0	.0	.0	.0	$.0^{4}115$	.0	.0	.00587	$.0^{4}113$		1.00000
11. Rest of the world	$.0^{8}80$	$.0^{8}80$	$.0^{8}80$	$.0^{8}80$	$.0^{8}80$	$.0^{8}80$	$.0^{7}80$	$.0^{7}15$	.0	$.0^{8}80$	.0		
Sub totals													
TOTALS													

NOTE $.0^{5}20$ means .0000020

Figure 5 The H or transition rates matrix for British regions, 1970-71

Figure [illegible] [illegible] rates Matrix for 1970–71 British Regions

Initial State (Existence at C.D. 1970) \ Final State (Survival at C.D. 1971)	1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RTW	
1. North	.98119	.00279	.00221	.00109	.00114	.00066	.00452	.00116	.00037	.00147	.00759	
2. Yorkshire & Humberside	.00268	.98201	.00279	.00276	.00125	.00091	.00452	.00118	.00054	.00073	.00584	
3. North West	.00101	.00158	.98443	.00094	.00145	.00044	.00434	.00133	.00154	.00077	.00653	
4. East Midlands	.00126	.00365	.00159	.97767	.00288	.00182	.00573	.00200	.00070	.00093	.00767	
5. West Midlands	.00069	.00105	.00190	.00246	.98568	.00059	.00500	.00246	.00124	.00065	.00544	
6. East Anglia	.00069	.00151	.00120	.00271	.00153	.96382	.01235	.00230	.00071	.00100	.01701	
7. South East	.00070	.00096	.00133	.00154	.00125	.00206	.97987	.00388	.00070	.00103	.01125	
8. South West	.00080	.00123	.00130	.00113	.00233	.00091	.01315	.97015	.00127	.00129	.00951	
9. Wales	.00053	.00075	.00236	.00207	.00208	.00047	.00485	.00258	.98441	.00061	.00376	
10. Scotland	.00107	.00093	.00140	.00095	.00090	.00040	.00460	.00103	.00033	.98231	.01115	
11. Rest of the world	$.0^{5}42$	$.0^{5}56$	$.0^{5}78$	$.0^{5}55$	$.0^{5}66$	$.0^{5}58$	$.0^{4}563$	$.0^{5}83$	$.0^{5}25$	$.0^{5}80$	1.02072	
Sub totals												

survival plus birth, transition and survival between the regions. Thus, the g^{55} rate for the West Midlands, with a value of 0.98568 is made up of a transition rate 0.96853 from Figure 6 and a birth and survival rate of 0.01715. Whereas in the $\underline{H}$ matrix all rates must be equal or less than one, in the $\underline{G}$ matrix rates may exceed one although in Figure 6 this is true only for the rest-of-the-world*.

The $\underline{G}$ matrix represents the operator that transforms an initial distribution of population amongst a set of regions into a final one. The off-diagonal elements spread out migrants and infant migrants from origin regions to destination regions and gather in migrants and infant migrants to destination regions from origin regions. The diagonal elements deal with the propensity of persons to stay in initial regions, survive there and to reproduce.

5. A forecast for British regions using the Rogers' model

The model of population change developed by Rogers (1966, 1968, 1971, 1975) can be stated as follows

$$\underline{w}(t+1) = \underline{G}\ \underline{w}(t) \tag{4}$$

where $\underline{w}$ is a column vector of populations, $\underline{G}$ is a matrix of growth rates, t is the start of the period and t+1 is end of the period of one unit in length. The $\underline{G}$ matrix can be applied successively thus

---

*It is convenient to close the system when using the $\underline{G}$ matrix. To do this estimates were made of the population of the rest-of-the-world, births and deaths in the rest-of-the-world and adjusted estimation equations applied to yield the following estimates: $K^{\varepsilon(11)\sigma(11)} = 3{,}513{,}090{,}583$; $K^{\varepsilon(11)\delta(11)} = 49{,}499{,}685$; $K^{\varepsilon(11)*(*)} = 3{,}563{,}362{,}873$; $K^{\beta(11)\sigma(11)} = 124{,}104{,}583$; $K^{\beta(11)\delta(11)} = 874{,}904$; $K^{\beta(11)*(*)} = 124{,}986{,}244$.

$$\underline{w}(t+2) = \underline{G}\ \underline{w}(t+1) \tag{5}$$

$$\underline{w}(t+3) = \underline{G}\ \underline{w}(t+2) \tag{6}$$

$$\vdots$$

$$\underline{w}(t+n) = \underline{G}\ \underline{w}(t+n-1) \tag{7}$$

or in other words the vector of populations after θ periods will be

$$\underline{w}(t+\theta) = \underline{G}^{\theta}\ \underline{w}(t) \tag{8}$$

where $\underline{G}$ is raised to the power θ. This model makes the assumption that the system continues to be characterized by the same $\underline{G}$ growth matrix, and this assumption can be relaxed by adopting a time series of $\underline{G}$ matrices (if available) and using instead the equation

$$\underline{w}(t+\theta) = \prod_{\lambda=1}^{\theta} \underline{G}_{\lambda}\ \underline{w}(t) \tag{9}$$

where λ is an index running from 1 to θ indicating which time period the $\underline{G}$ refers.

We can illustrate the operation of equation (4) of the model to the British population system. We adopt census date 1971 as the initial time t and assume that the $\underline{G}$ matrix for 1970-71 holds for 1971-72:

$$\begin{bmatrix} 3301845 \\ 4809037 \\ 6761499 \\ 3414521 \\ 5138552 \\ 1683310 \\ 17308532 \\ 3799723 \\ 2734606 \\ 5205852 \\ 3637672207 \end{bmatrix} = \begin{bmatrix} .98119 & .00268 & .00101 & \ldots & 0^{5}42 \\ .00279 & .98201 & .00158 & \ldots & 0^{5}56 \\ .00221 & .00279 & .98443 & \ldots & 0^{5}78 \\ \vdots & \vdots & \vdots & & \vdots \\ .00759 & .00584 & .00653 & \ldots & 1.02072 \end{bmatrix} \times \begin{bmatrix} 3298185 \\ 4807240 \\ 6759380 \\ 3388910 \\ 5115760 \\ 1661730 \\ 17236840 \\ 3761400 \\ 2725365 \\ 5223725 \\ 3563362873 \end{bmatrix}$$

$$\underline{w}(c.d.1972) = \underline{G}\ \underline{w}(c.d.1971)$$

(10)

This particular forecasting model was run forward from census date 1971 to census date in 2001. The resulting population vectors for every fifth year (unfortunately not all of them will have a census taken in them) are displayed in Figure 7, along with the population breakdown within Great Britain in percentage form.

The redistribution of population seen in the 1960's towards the South East, East Anglia, the South West and the East Midlands (Eversley, 1971; Department of the Environment, 1971) continues under this scenario. The West Midlands gains marginally and the other regions continue to lose their share of the national population cake, particularly Scotland.

Gains are made in absolute numbers in all regions by 2001 although Scotland declines to 1988 before recovering. The South East makes particularly large population gains - nearly 4 millions in the 30 years to the turn of the century. The reason within the model for this sequence of events is the operation of the growth rates associated with the rest-of-the-world. This 'region' grows massively in population through natural increase and this is transmitted to the other regions in the system through the set of constant rates associated with the 11th region. In other words, the population scenario depicted by the model is one of immigration continuing at present rates and increasing in absolute numbers substantially as population builds up in the rest-of-the-world. This flow and the question of future immigrant numbers are the subject of much debate today.

6. Comments on the forecasts and suggestions for improvement

What comments can we make about this prototype model derived that will enable us to improve our forecasts?

Firstly, we may note that there is probably a case for revising the $\underline{K}$ accounts matrix for 1970-71. The reader may have noticed the discrepancy between the end of period accounts populations (Figure 3) and the census 1971

POPULATION	*1971	1976	1981	1986	1991	1996	2001
1. North	3298.2	3320.3	3352.6	3395.8	3450.9	3519.1	3601.5
2. Yorkshire and Humberside	4807.2	4822.8	4855.3	4905.4	4974.3	5063.1	5173.3
3. North West	6759.4	6778.2	6818.5	6881.6	6969.4	7083.6	7226.5
4. East Midlands	3388.9	3518.3	3652.4	3793.2	3943.1	4104.1	4278.7
5. West Midlands	5115.8	5234.5	5366.0	5512.2	5674.8	5856.0	6058.2
6. East Anglia	1661.7	1768.8	1875.4	1984.1	2097.2	2216.9	2345.3
7. South East	17236.8	17633.2	18126.8	18722.5	19426.6	20247.0	21192.7
8. South East	3761.4	3952.6	4145.4	4343.6	4551.3	4772.1	5009.8
9. Wales	2725.4	2773.7	2827.7	2888.4	2956.5	3033.1	3119.3
10. Scotland	5223.7	5145.5	5094.9	5072.2	5077.7	5112.2	5176.6
11. Rest-of-the-world	3363362.9	3950647.1	4379803.6	4855364.0	5382349.9	5966326.7	6613461.5
PERCENTAGES							
1. North	6.11	6.04	5.97	5.91	5.84	5.77	5.70
2. Yorkshire and Humberside	8.91	8.78	8.65	8.53	8.41	8.30	8.19
3. North West	12.52	12.34	12.15	11.97	11.79	11.61	11.44
4. East Midlands	6.28	6.40	6.51	6.60	6.67	6.73	6.77
5. West Midlands	9.48	9.53	9.56	9.59	9.60	9.60	9.59
6. East Anglia	3.08	3.22	3.34	3.45	3.55	3.63	3.71
7. South East	31.93	32.09	32.30	32.56	32.86	33.19	33.54
8. South West	6.97	7.19	7.39	7.55	7.70	7.82	7.93
9. Wales	5.05	5.05	5.04	5.02	5.00	4.97	4.94
10. Scotland	9.68	9.36	9.08	8.82	8.59	8.38	8.19
TOTAL	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Figure 7 The forecast populations of British regions ('000s)

*Base population

figures given in Figure 8. The mid-year estimates just prior to the census proved to be overestimates. The accounts matrix could be adjusted by inserting the census 1971 population figures and by recalculating the $K^{\varepsilon(i)\sigma(i)}$ and $K^{\varepsilon(i)*(*)}$ terms holding the others constant. New $\underline{H}$ and $\underline{G}$ matrices would then be generated with slightly higher migration and slightly lower 'staying' rates, and a new forecast made.

Secondly, we should undoubtedly move from the constant $\underline{G}$ assumption of equations (4) to (7) to the variable $\underline{G}$ framework of equation (9). This is necessary because we already know that birth rates, for example, have continued to fall considerably in the 1970-76 period, and migration rates may have also shifted a little. Figure 8 shows estimates of the regional birth rates in 1970 and 1976, and compares this forecast with two official forecasts prepared on a 1969 and a 1973 base respectively. The lower birth rate levels of 1972-73 are assumed to continue in part in the rest of the century in the 1973 forecast and this is reflected in the lower total projected for the total population of Great Britain compared with either the 1969 forecast or the one outlined in this paper. It is probably possible to adjust the $\underline{G}$ matrix to reflect the trend in falling birth rates and the possible alteration in the pattern of migration picked up in the 1973 based forecast, though there is the problem of comparing differently defined regions (see Figure 1).

The third comment that should be made concerns the status of populations flows from and to the rest-of-the-world. In the section 5 model these were modelled endogeneously. It might be more realistic to assume instead that the flows, being subject to legal restrictions, quotas and so on, behaved as exogeneous variables to be specified at more constant levels than those generated in the section 5 model. The growth model of equation (4) would then become, for the British regional system, either

Figure 8 **Birth Rates 1970 and 1976, and Forecast Population 1991 for British Regions**

Region	Birth Rate 1970 (per 1000)	Birth[1] Rate 1976 (per 1000)	1991 Populations (share & total) 1969[2] Based Forecast %	1970-71[3] Based Forecast %	1973[4] Based Forecast %
1 North	15.9	11.1	5.9	5.8	5.5
2 Yorkshire & Humberside	17.0	11.4	8.5	8.4	8.5
3 North West	16.7	11.6	12.3	11.8	11.5
4 East Midlands	16.6	11.8	6.8	6.7	7.4
5 West Midlands	17.5	11.8	9.7	9.6	9.3
6 East Anglia	15.6	11.8	3.5	3.6	4.0
7 South East	15.5	11.2	32.0	32.9	30.8
8 South West	15.1	10.9	7.3	7.7	8.4
9 Wales	15.5	11.5	4.9	5.0	5.1
10 Scotland	16.8	11.6	9.2	8.6	9.3
Total, Great Britain %			100.0	100.0	100.0
Total, Great Britain '000s			60463	59122	56348

Notes

1 Estimated from Weekly Returns to 19th March 1976, O.P.C.S. Monitor VS 76/11.

2 Source: Table 3.2 in Department of the Environment (1971).

3 Source: Figure 8.

4 Source: O.P.C.S. (1974) and O.P.C.S. (1975). These forecasts are for the Standard Regions after Local Government Reorganization.

$$\underset{11\times1}{\underline{w}(t+1)} = \underset{11\times10}{\underline{G}}\ \underset{10\times1}{\underline{w}(t)} + \underset{11\times1}{\underline{I}(t,t+1)} \qquad (11)$$

if just immigrants were treated exogeneously, or

$$\underset{10\times1}{\underline{w}(t+1)} = \underset{10\times10}{\underline{G}}\ \underset{10\times1}{\underline{w}(t)} + \underset{10\times1}{\underline{I}(t,t+1)} - \underset{10\times1}{\underline{E}(t,t+1)} \qquad (12)$$

where $\underline{I}(t,t+1)$ is a vector of immigrants from the rest-of-the-world to the regions over the period t to t+1 and $\underline{E}$ is a vector of emigrants from the regions over the period t to t+1.

Fourthly, one might note that each of these problems can probably be dealt with more effectively by using the forecasting version of the accounts based model employed earlier to generate the accounts for 1963-66 and 1970-71. Such a forecasting version is being developed.

A fifth problem posed is that of adjusting the forecast to the new regional basis shown in Figure 1 and for which new regional forecasts have already been prepared. Revised population vectors are already available but adjustment of rates requires care. Rogers (1969) has outlined how adjustment to the $\underline{G}$ matrix can be made by pre-multiplication by a consolidation matrix $\underline{C}$ and post-multiplication by a deconsolidation matrix $\underline{D}$:

$$\underline{\hat{G}} = \underline{C}\ \underline{G}\ \underline{D} \qquad (13)$$

where $\underline{\hat{G}}$ is the newly adjusted matrix. The $\underline{C}$ and $\underline{D}$ matrices are prepared from a mapping of population stocks from one set of regions to the other. Alternatively, the accounts matrix itself may be aggregated in a fashion described by Stillwell (1976) using row, $\underline{R}$, and column, $\underline{C}$, aggregation/disaggregation operators:

$$\underline{\hat{K}} = \underline{R}\ \underline{K}\ \underline{C} \qquad (14)$$

Care, however, has to be taken in this process to allow for the situation where persons who have moved only within the region previously and are classified as surviving stayers in the accounts become inter-regional migrants as a result of boundary change (Illingworth, 1975b). Figure 1 shows that there could be a serious problem in the North of England.

Finally, there are a set of problems which have been ignored in this paper and which require solution in revising the forecasts of the population of British regions. The models need to be disaggregated by age and sex, and by other variables such as ethnicity or social class. The fertility rate forecasts need to be tied to leading indicators of family size norms and intentions and possibly economic indicators. The migration rate forecasts need to be tied to leading or forecast economic indicators.

Having commented at length on some of the drawbacks of the models and forecasts developed to date, one should perhaps conclude by emphasizing their strengths. An accounting framework forces the researcher to pay attention to all population flows, and in particular those to and from the rest-of-the-world. The accounts based model connects the framework with data normally available. The growth model of Rogers represents one use of the information represented in the accounts and lays bare the multi-regional interactions at work in the population system. Thus, we can say on the basis of our forecast in section 5 that the year 2000-2001 will see the emigration of the following numbers of persons from each region to the other regions (Figure 9). Although our confidence in those forecast may be low, other projections do not to date ever produce such a picture.

Figure 9 Part of the Accounts Matrix for 2000-2001

Initial State \ Final State	Survival at C.D. 2001										
	1 N	2 YH	3 NW	4 EM	5 WM	6 EA	7 SE	8 SW	9 W	10 S	11 RTW
Birth in 2000-2001 OR Existence at C.D. 2000											
1. North	3516408	9995	7930	3906	4078	2356	16182	4164	1313	5283	27194
2. Yorkshire & Humberside	13804	5056838	14343	14236	6443	4662	23292	6055	2774	3746	30098
3. North West	7261	11389	7083526	6758	10433	3186	31237	9570	11089	5561	47001
4. East Midlands	5347	15495	6748	4147824	12236	7720	24331	8504	2967	3946	32524
5. West Midlands	4165	6335	11440	14784	5929841	3566	30072	14772	7450	3919	32708
6. East Anglia	1609	3512	2780	6282	3553	2234888	26626	5344	1636	2308	39445
7. South East	14750	20250	28011	32249	26164	43275	20570324	81356	14727	21520	236234
8. South West	3951	6119	6465	5626	11546	4510	65223	4812619	6292	6398	47158
9. Wales	1658	2334	7311	6434	6464	1464	15055	8008	3052902	1887	11653
10. Scotland	5532	4803	7209	4903	4622	2067	23754	5322	1697	5069999	57526
11. Rest of the world	27023	36255	50754	35697	42843	37570	364554	54058	16475	52076	6612899998
Sub totals											

References

Department of the Environment (1971) Long term population distribution in Great Britain - a study, H.M.S.O., London.

Eversley, D.E.C. (1971) 'Population changes and regional policies since the war', Regional Studies, 5, pp. 211-228.

General Register Office (1966) The Registrar General's Statistical Review of England and Wales, 1964. Part II. Tables, Population, H.M.S.O., London.

Illingworth, D.R. (1975a) 'Aggregate population accounts for the districts of the West Yorkshire Metropolitan County: Part I - 1961-66', Working Paper 125, School of Geography, University of Leeds.

Illingworth, D.R. (1975b) 'Aggregate population accounts for the districts of the West Yorkshire Metropolitan County: Part II - 1966-71', Working Paper 126, School of Geography, University of Leeds.

Keyfitz, N. (1968) Introduction to the mathematics of population, Addison-Wesley, Reading, Massachusetts.

Office of Population Censuses and Surveys (1974) Population projections No. 4 1973-2013, H.M.S.O., London.

Office of Population Censuses and Surveys (1975) Regional population projections - mid-1973 based. New Standard regions of England and Wales, H.M.S.O., London.

Office of Population Censuses and Surveys (1976) Monitor. Week ended 19 March 1976. Reference VS 76/11 and WR 76/11, H.M.S.O., London.

Pressat, R, (1972) Demographic analysis, Edward Arnold, London.

Rees, P.H. (1972) 'Accounts and models for social groups within cities, in Wilson, A.G. (Editor) Patterns and processes in Urban and regional systems, Pion, London.

Rees, P.H. (1974) 'The Spatial analysis of demographic growth in British regions.' Research proposal submitted to the Social Science Research Council, London.

Rees, P.H. (1975a) Progress report, first year, on grant Hr 2914/2 submitted to the Social Science Research Council, London.

Rees, P.H. (1975b) 'Population analysis of the city region', Working Paper 112, School of Geography, University of Leeds. To be published in T. Hancock (Editor) Growth and change in the future city region, International Text Book Co., London.

Rees, P.H. (1975c) 'A family of demographic accounts and models', Working Paper 119, School of Geography, University of Leeds.

Rees, P.H. and King, J.R. (1974) 'A simple model for population projection applied to ethnic groups and small area populations', Working Paper 76, School of Geography, University of Leeds.

Rees, P.H. and Wilson, A.G. (1973) 'Accounts and models for spatial demographic analysis 1: aggregate population', Environment and Planning, 5, pp. 61-90.

Rees, P.H. and Wilson, A.G. (1974) 'A computer program for constructing spatial demographic accounts for aggregate population: user's manual', Working Paper 60, School of Geography, University of Leeds.

Rees, P.H. and Wilson, A.G. (1975) 'Accounts and models for spatial demographic analysis 3: rates and life tables', Environment and Planning A, 7, pp. 199-231.

Rees, P.H. and Wilson, A.G. (1976) Spatial population analysis, Edward Arnold, London, forthcoming.

Rogers, A. (1966) 'Matrix methods of population analysis', Journal of the American Institute of Planners, 32, pp. 40-44.

Rogers, A. (1968) Matrix analysis of inter-regional population growth and distribution, University of California Press, Berkeley and Los Angeles..

Rogers, A. (1969) 'On perfect aggregation in the matrix cohort survival model of inter-regional population growth', Journal of Regional Science, 9, pp. 417-424.

Rogers, A. (1971) Matrix methods in urban and regional analysis, Holden Day, San Francisco.

Rogers, A. (1975) Introduction to multiregional mathematical demography, Wiley, London.

Smith, A.P. and Rees, P.H. (1974) 'Methods of constructing spatial demographic accounts for a British region', Working Paper 59, School of Geography, University of Leeds.

Stillwell, J.C.H. (1976) 'A user's guide to a simple matrix aggregation program', Working Paper 141, School of Geography, University of Leeds.

Stone, R. (1965) 'A model of the educational system', Minerva, 3, pp. 172-186.

Stone, R. (1971a) Demographic accounting and model building, O.E.C.D., Paris.

Stone, R. (1971b) 'An integrated system of demographic, manpower and social statistics and its links with the system of national economic accounts', Sankhya B, 33, pp. 1-184.

Wilson, A.G. and Rees, P.H. (1974) 'Accounts and models for spatial demographic analysis 2: age-sex disaggregated populations', Environment and Planning A, 6, pp. 101-116.