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# A GENERALISED AND UNIFIED APPROACH TO THE MODELLING OF SERVICE-SUPPLY STRUCTURES

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## 1. Introduction

Substantial progress has been made in recent years with the development of models of shopping supply and similar service systems. (See, for example, Harris and Wilson, 1978, Wilson and Clarke, 1979, Wilson, 1981-A). This involved taking an aggregate retail model in the form

$$S_{ij} = A_i e_i P_i W_j^{\alpha} e^{-\beta c_{ij}} \quad (1.1)$$

where

$$A_i = 1/\sum_k W_k^{\alpha} e^{-\beta c_{ik}} \quad (1.2)$$

where $S_{ij}$ is the flow of cash from residents in zone i to shops in zone j; $e_i$ is the per capita expenditure by residents of zone i; $P_i$ is the population of i; $W_j$ is the attractiveness of shops in j; and $c_{ij}$ is the generalised cost of travel from i to j. $\alpha$ and $\beta$ are parameters. Attractiveness is usually measured by the 'size' of supply. If $D_j$ is the total revenue attracted to j, then

$$D_j = \sum_i S_{ij} \quad (1.3)$$

and a suitable hypothesis about the dynamics of the system is

$$\dot{W}_j = \varepsilon(D_j - k_j W_j) W_j \quad (1.4)$$

where $k_j$ is the cost per unit size of supply in j and $\varepsilon$ is a parameter. The equilibrium condition is

$$D_j = k_j W_j \quad (1.5)$$

By substituting for $D_j$ from (1.3) and $S_{ij}$ from (1.1) and (1.2), the equations (1.5) form a set of simultaneous equations for $\{W_j\}$ and hence constitute an equilibrium model of the supply side. Alternatively, the differential equations (1.4) (or equivalent difference equations) can be solved if the system does not reach equilibrium. The kinds of equilibrium patterns which emerge, and the alternative modes of system evolution, have been extensively explored (see, for example, Clarke and Wilson, 1983).

These ideas have also been applied to disaggregated versions of the model. A superscript g is added to most of the variables and parameters in the above equations for example. The attractiveness factor can then be taken as a product of the size of facilities for good g in j, say $W_j^g$, and perhaps also of the total scale of supply for higher order goods. Thus, if $\hat{W}_j^g$ is the attractiveness factor, we would typically have

$$\hat{W}_j^g = (\sum_{g'>g} W_j^g)^{\alpha_1^g} (W_j^g)^{\alpha_2^g} \tag{1.6}$$

where g'>g indicates a sum over g' of higher order (or, the same order) as g. At this level of disaggregation, the model becomes a replacement for more traditional central place theory, and this case has been argued explicitly elsewhere (Wilson, 1978, 1981-B).

The purpose of this paper is to extend this argument so that the model systems can be made more realistic. We continue with retail systems as the main example, but we show that the new developments have a much more general application. This can be borne in mind throughout the paper and is spelled out explicitly in section 7 below.

The main proposed extensions are threefold: (i) to expand the usual disaggregated representation to have a finer categorisation of shop (or service) types; (ii) to incorporate a greater variety of effects in the attractiveness term; and (iii) to investigate and specify supply-side costs in more detail. These extensions of the framework are outlined in section 2. In section 3, we show how to build an appropriate spatial interaction model of consumer utilisation of facilities. In section 4, we explore the dynamics of the supply side by formulating appropriate differential and difference equations. This is a non-trivial matter because of the number of interrelated elements which have been incorporated into the attractiveness function, but the problem is solved using some of the concepts of activity analysis. In section 5, we present a specific example to fix ideas. In section 6, we then speculate about the range of equilibrium solutions which can be generated with the extended model and in section 7, we show how the ideas are applicable to a wide range of systems. Some concluding comments are offered in section 8.

The motivation for these developments lies in contemporary phenomena which demonstrate the importance of moving beyond traditional central place theory as well as the straightforward disaggregation of spatial-interaction systems. It is usually implicit in these approaches that centres are nested in a neat hierarchy, and that, at a given order of centre, all lower order goods are sold. In fact, this is no longer visibly the case. The situation has changed with the development for example, of superstores of different kinds, at locations away from traditional centres, and sometimes specialising in the types of goods they sell - for example, furniture or D.I.Y. equipment. To explain this kind of phenomenon, we not only need the further disaggregation of shop types, but also the previously-mentioned developments of attractiveness functions and supply-side costs which then enable us to represent scale economies for both consumers and suppliers as a basis for understanding different kinds of agglomeration tendencies.

## 2. Disaggregation, attractiveness functions and costs

We will disaggregate by person type, w, and by type of good, g; but, crucially, we add shop type, h. Some h's are defined as sellers of single g's, but often, we need to list a number of g's associated with a particular h. To fix ideas, consider the simple example presented in table 1.

Table 1.

| g | | h | | | |
|---|---|---|---|---|---|
| 1 | food | 1 | 5 | | |
| 2 | hardware/DIY | 2 | | 6 | |
| 3 | clothes | 3 | | | 7 |
| 4 | jewellery | 4 | | | |

h = 5: supermarket
h = 6: superstore
h = 7: department store

A basic issue to be dealt with at the outset is how to relate broad concepts of 'shops in a zone' or 'shopping centres' with the characteristics of individual shops in that zone. For the time being we continue to assume that zonal descriptions are adequate, but in section 7 we briefly explore what is involved in relating this level of description to a finer one - for example one which specified number of shops by size groups, say $N_{j\ell}^{h}$ being the number of type-h shops in zone j in size category $\ell$.

At the level of disaggregation now defined, the main spatial interaction variable will be $S_{ij}^{ghw}$, the flow of cash from w-type residents of zone i to h-type shops in zone j to buy goods of type g. This obviously provides the basis of a rich representation. Similarly, we take the attractiveness function as $W_{ij}^{ghw}$. This is largely as would be expected, but note that we also at this stage add the subscript i to make it specific to the residents of a particular zone. This will enable us to include travel costs, $c_{ij}$, as a (negative) component of attractiveness and thus demonstrate how it is on the same footing as other terms.

It is interesting that, while great attention has been given in the literature to the components of generalised travel cost, the same concern has not been devoted to attractiveness. The next step in the argument, therefore, is to look at the possible components of the attractiveness function $W_{ij}^{ghw}$.

Attractiveness can be made up of components which vary with j, (j,g), (i,j), (j,h) and (j,g,h) - as well as other combinations which we neglect for the time being. Let $z_{jk}^{(1)}$, $z_{jk}^{(2)g}$, $z_{ijk}^{(3)}$, $z_{jk}^{(4)}$ and $z_{jk}^{(5)gh}$ be the kth component of each of these elements respectively. Then $W_{ij}^{ghw}$ could take the form

$$W_{ij}^{ghw} = \sum_k \alpha^{(1)w} z_{jk}^{(1)} + \sum_k \alpha^{(2)w} z_{jk}^{(2)g} + \sum_k \alpha^{(3)gw} z_{ijk}^{(3)} + \sum_k \alpha^{(4)hw} z_{jk}^{(4)h} + \sum_k \alpha^{(5)ghw} z_{jk}^{(5)ghw} \qquad (2.1)$$

for a set of coefficients, $\underline{\alpha}$, to which appropriate superscripts have been added. Note that some (or all) of the z's could be measured logarithmically - and then such terms would appear in the corresponding spatial interaction model multiplicatively and raised to the power of the corresponding $\alpha$. (The overall attractiveness term is going to be $e^{\beta W_{ij}^{ghw}}$, with $W_{ij}^{ghw}$ defined as in (2.1).) Examples of the elements of each type of component are as follows:

(i) $z_{jk}^{(1)}$. Collective measure of supply of facilities at j; parking costs and facilities at j; measures of environmental quality at j.

(ii) $z_{jk}^{(2)g}$. Price and quality indices of g at j. Supply of g at j (though this might be more appropriately taken as given by one of the $z_{jk}^{(2)gh}$ terms below - supply of g in h's at j.

(iii) $z_{ijk}^{(3)}$. Elements of generalised cost of travel from i to j.

(iv) $z_{jk}^{(4)h}$. Availability of h in j. (Perhaps best expressed through $z_{jk}^{(5)gh}$ below?)

(v) $z_{jk}^{(5)gh}$. Availability of g in h-type shops in j.

The next step is to look at the components of supply and their costs. We designate these as $x_{jk}^{gh}$, the kth component of (g,h) supply in j. These can be taken as the inputs of the supply process with unit costs $\gamma_{jk}^{gh}$. These might include: (i) $x_{j1}^{gh}$, land; (ii) $x_{j2}^{gh}$, floorspace; (iii) $x_{j3}^{gh}$, labour; (iv) $x_{j4}^{gh}$, capital; (v) $x_{j5}^{gh}$, costs of other inputs (whose origins and transport arrangements would be endogenous in a more detailed model); (vi) $x_{j6}^{gh}$, the cost of goods for sale. It will also be necessary in some circumstances to specify some technical information like $q_j^{gh}$, the proportion of h-resources used in selling good g, though in the formulation above, by retaining both g and h disaggregation on the x's, $q_j^{gh}$ will be determined implicitly in the model.

The corresponding unit costs of the various inputs can be taken as $\gamma_{j\ell}^{gh}$. These can be taken as functions of $x_{j\ell}^{gh}$ so that various non-linearities, scale economies or diseconomies, for example, can be taken into account. Formally

$$\gamma_{jk}^{gh} = \gamma_{jk}^{gh}(x_{jk}^{gh}, \dots) \tag{2.2}$$

(where the dots indicate that such unit costs may be functions of other resource inputs too.) The cost of (g,h) supply at j is then $C_j^{gh}$ where

$$C_j^{gh} = \sum_k \gamma_{jk}^{gh}(x_{jk}^{gh})x_{jk}^{gh} \tag{2.3}$$

We complete our specification of the system by introducing the idea of a *level of activity* of (g,h) in j, and we define this, using some suitable scale, as $y_j^{gh}$. Then each input can be a function of these activity levels. A number of the supply side characteristics used in the attractiveness function will be functions of the x's, and hence of the y's. The unit costs $\gamma_{jk}^{gh}$ and the costs $c_j^{gh}$ will become functions of the y's through equations (2.2) and (2.3), respectively. Thus, formally, we now have

$$x_{jk}^{gh} = x_{jk}^{gh}(y_j^{gh}) \tag{2.4}$$

$$z_{j1}^{(1)} = z_{j1}^{1}(\sum_{gh} x_{j2}^{gh}) \tag{2.5}$$

$$z_{j1}^{(5)gh} = z_{j1}^{(5)gh}(x_{j2}^{gh}) \tag{2.6}$$

where $z_{j1}^{(1)}$ plays the role of $\sum_g W_j^g$ in the traditional model and $z_{j1}^{(5)gh}$ the role of $W_j^g$ (now disaggregated by h).

The relationship (2.4) can be substituted in (2.2), (2.3), (2.5) and (2.6), thus making all variables functions of activity levels. The detailed specification of all the formal functions constitutes a specification of the *production function* of the supply side. Though nonlinearities will be present in these functions, possibly in quite complex ways, this production function does represent all the relevant interdependencies. This is the advantage of the activity analysis formulation. We will see in the next section that the differential equations can be formulated in terms of the activity levels $y_j^{gh}$. Otherwise, they would have to be formulated in terms of the resource inputs $x_{jk}^{gh}$. This is a much longer list of variables and the representation of the interdependencies would be much more complicated. We proceed to this shortly, but first we write down the appropriate spatial interaction model for consumer behaviour.

## 3. A disaggregated spatial interaction model for consumer behaviour

We have noted how travel costs can be taken as an element of the attractiveness function. Let $e_i^{gw}$ be the expenditure per capita by type-w people resident in zone i on type-g goods. (Note that this varies with g but not with h; we let the model determine the h-split.) Let $P_i^w$ be the number of type-w people resident in zone i. Then an obvious disaggregation of the model given by equations (1.1) and (1.2) is:

$$S_{ij}^{ghw} = A_i^{gw} e_i^{gw} P_i^w e^{\beta W_{ij}^{ghw}} \tag{3.1}$$

where

$$A_i^{gw} = 1/\sum_{jh} e^{\beta W_{ij}^{ghw}} \tag{3.2}$$

The attractiveness function is given as equation (2.1) above and some of the z's are specified formally as functions of the x's and the y's through (2.5) and (2.6), and (2.4) respectively.

This is straightforward as a formal model. In practice, there are likely to be some difficulties. For example, there is likely to be a problem of correlation - at least for certain definitions of h's - which makes it difficult to obtain unbiassed estimates of the α-coefficients. This will arise because, in some case, different h's are selling the same g's. This is analogous to the red bus-blue bus problem in modal split studies (cf. Williams, 1977). This will demand careful definition of the h's to minimise this problem. If necessary, it may be appropriate to build a degree of hierarchy into the definitions of g's and h's, so that sequential choices can be made - first between levels and then within levels.

To fix ideas, we offer a specific example in section 6 below, but first we specify the appropriate differential equations for the study of dynamics and then sketch some forms of structure and evolution which can be predicted by the overall model.

## 4. Differential equations and equilibrium analysis

The revenue attracted to a (g,h) combination in j is $D_j^{gh}$ given by

$$D_j^{gh} = \sum_{iw} S_{ij}^{ghw} \tag{4.1}$$

We have already specified the (g,h,j) costs in equation (2.3). It is repeated here for convenience with the costs shown as functions of activity levels using (2.4):

$$C_j^{gh} = \sum_k \gamma_{jk}^{gh}[x_{jk}^{gh}(y_j^{gh}), \ldots] \; x_{jk}^{gh}(y_j^{gh}) \tag{4.2}$$

The $D_j^{gh}$ terms are also functions of activity levels: $S_{ij}^{ghw}$ is taken from (3.1) and (3.2); it depends on $W_{ij}^{ghw}$ from (2.1). This depends on the z's and some of the z's on the x's and hence the y's as shown. Formally, then, we can write

$$\dot{y}_j^{gh} = \varepsilon^{gh}[D_j^{gh} - C_j^{gh}]y_j^{gh} \qquad (4.3)$$

as an appropriate set of differential equations with $\varepsilon^{gh}$ as a new set of parameters; and equilibrium condition

$$D_j^{gh} = C_j^{gh} \qquad (4.4)$$

This can be made explicit as a function of the y's by making all the appropriate substitutions and deriving the main result as follows:

$$\sum_{iw} \frac{e_i^{gw} P_i^w e^{\beta W_{ij}^{ghw}[\underline{z}(\underline{x}(\underline{y}))]}}{\sum_{jgh} e^{\beta W_{ij}^{ghw}[\underline{z}(\underline{x}(\underline{y}))]}}$$

$$= \sum_k \gamma_{jk}^{gh}[x_{jk}^{gh}(y_j^{gh}, \ldots)]\ x_{jk}^{gh}(y_j^{gh}) \qquad (4.5)$$

It is necessary to work towards the type of analysis carried out in Harris and Wilson (1978) for this more complicated model. We can obtain a clearer idea of what this entails in relation to the specific example in section 6 below and we pursue the issue further in section 6 following the presentation of an example.

## 5. An example

Let g and h be defined as in table 1 and let there be two w-classes (well-off, not-well-off; ie. car owning, non car owning, etc). Then, if there are I origin zones and J destination zones, the array $\{S_{ij}^{ghw}\}$ has $2\times4\times7\times IJ = 56IJ$ elements. In practice, the number will be reduced because not all (g,h) combinations are defined to be possible in table 1. For example, if $n_g^h$ is the number of g's sold by an h category, the number of (g,h) categories is $\sum_{gh} n_g^h$ and an inspection of table 1 shows this to be 13 in this case rather than $4\times7 = 28$. So there are $2\times13IJ = 26IJ$ possibly non-zero $\{S_{ij}^{ghw}\}$ elements.

Suppose the attractiveness function has three elements: $z_j^{(1)}$, $z_{ij}^{(3)}$ and $z_j^{(5)gh}$. We will consider there to be only two x's: $x_{j1}^{*h}$, land for h in j and $x_{j2}^{gh}$, floorspace for g in h in j. These can be designated $L_j^h$ and $W_j^{gh}$ respectively at unit costs $\gamma_j^{(1)}$ and $\gamma_j^{(2)gh}$. Let $g_j^{gh}$ be the activity levels in the usual way. Then equation (2.4) becomes

$$L_j^h = L_j^h(y_j^{1h}, y_j^{2h}, y_j^{3h}, \ldots) \tag{5.1}$$

$$W_j^{gh} = W_j^{gh}(y_j^{gh}) \tag{5.2}$$

We define, for these illustrative purposes, $z_j^{(1)}$ to be the log of the accessibility to all retail facilities, so equation (2.5) will be

$$z_j^{(1)} = \log \sum_{g'h'j'w} W_{j'}^{g'h'} e^{-\hat{\beta}^{g'w} c_{jj'}} \tag{5.3}$$

where we use as a parameter $\hat{\beta}^{gw}$. This differs from that used in the spatial interaction part of the model below. This latter effect is introduced into the main model by taking

$$z_{ij}^{(2)} = c_{ij} \tag{5.4}$$

and the corresponding parameter, $\alpha^{(2)gw}$, as $-\beta^{gw}$. $\hat{\beta}^{gw}$ measures the relative importance of access to g from a particular shopping location; $\beta^{gw}$ is measuring access to g from a residential location. For later use, we define

$$\hat{X}_j = \sum_{g'h'j'w} W_{j'}^{g'h'} e^{-\hat{\beta}^{gw} c_{jj'}} \tag{5.5}$$

and

$$X_i = \sum_{ghjw} W_j^{gh} e^{-\beta^{gw} c_{ij}} \tag{5.6}$$

as the two accessibilities. The second is only used as an output indicator, not internally within the model. It is also useful to define population potential as

$$X_i^P = \sum_{wgj} P_i^w e^{-\beta^{gw} c_{ij}} \tag{5.7}$$

In order to get the usual kind of attractiveness term for $W_j^{gh}$, we represent equation (2.6) as

$$z_j^{(5)gh} = \log W_j^{gh} \tag{5.8}$$

To complete the picture, we note that the unit costs could also be functions of scale:

$$\gamma_j^{(1)} = \gamma_j^{(1)}(X_j^P, \hat{X}_j) \tag{5.9}$$

where we make land costs a function of access to population and access to other facilities. Then, also

$$\gamma_j^{(2)gh} = \gamma_j^{(2)gh}(W_j^{gh}) \qquad (5.10)$$

In each case, the $\gamma$'s will be functions of the activity levels, y, through equations (5.1) and (5.2).

Equations (5.1), (5.2), (5.9) and (5.10) constitute a representation of the supply-side production function, and the z's in equations (5.3), (5.4) and (5.8) a representation of how this is perceived by consumers. Then the attractiveness function (cf. equation (2.1)) can be written

$$W_{ij}^{ghw} = \mu \log \hat{X}_j - \beta^{gw} c_{ij} + \alpha^{gw} \log W_j^{gh} \qquad (5.11)$$

with suitable definitions of parameters replacing the $\alpha$'s in (2.1). The spatial interaction model (equation (3.1) and (3.2)) then becomes

$$S_{ij}^{ghw} = A_i^{gw} e_i^{gw} P_i^w X_j^\mu (W_j^{gh})^{\alpha^{gw}} e^{-\beta^{gw} c_{ij}} \qquad (5.12)$$

$$A_i^{gw} = 1/\sum_{jh} X_j^\mu (W_j^{gh})^{\alpha^{gw}} e^{-\beta^{gw} c_{ij}} \qquad (5.13)$$

The cost of supplying (g,h) at j is (cf. equation (2.3)):

$$C_j^{gh} = \gamma_j^{(1)} (X_j^P) L_j^h + \gamma_j^{(2)gh}(W_j^{gh}) W_j^{gh} \qquad (5.14)$$

which can be written

$$C_j^{gh} = \gamma_j^{(1)} [X_j^P(\{y_j^{gh}\})] L_j^h(y_j^{1h}, y_j^{2h}, \ldots)$$

$$+ \gamma_j^{(2)gh}[W_j^{gh}(y_j^{gh})] W_j^{gh}(y_j^{gh}) \qquad (5.15)$$

if we show all the variables as functions of activity levels.

Total revenue is obviously

$$D_j^{gh} = \sum_{iw} S_{ij}^{ghw} \qquad (5.16)$$

and the differential equations in activity levels are (say)

$$\dot{y}_j^{gh} = \varepsilon^{gh}(D_j^{gh} - C_j^{gh})y_j^{gh} \qquad (5.17)$$

The equilibrium condition is

$$D_j^{gh} = C_j^{gh} \qquad (5.18)$$

which, written out in full after making the appropriate substitutions, is

$$\sum_{iw} \frac{e_i^{gw} p_i^w X_j^{\mu} (W_j^{gh})^{\alpha^{gw}} \; e^{-\beta^{gw} c_{ij}}}{\sum\limits_{jgh} X_j^{\mu} (W_j^{gh})^{\alpha^{gw}} \; e^{-\beta^{gw} c_{ij}}}$$

$$= \gamma_j^{(1)h}(W_j)L_j^h + \gamma_j^{(2)gh}(W_j^{gh})W_j^{gh} \qquad (5.19)$$

which is a set of simultaneous equations in $\{y_j^{gh}\}$ when the various variables in (5.19) are shown as functions of activity levels.

We can recall the earlier discussion about possibly non-zero $S_{ij}^{ghw}$ elements and note that in all this analysis, a $W_j^{gh}$ (and $y_j^{gh}$) is permanently zero if such an h-type does not sell good type-g by definition. So in the case of the table 1 definitions, there are 13 possibly non-zero $y_j^{gh}$'s for each j.

## 6. The variety of patterns and modes of evolution

We first consider the equilibrium patterns which can be generated by such a system. It is clearly a rich system and a great variety of patterns would be possible. We discuss a number of examples in turn to illustrate this.

(i) A part of the Harris-Wilson (1978) analysis will carry over: high $\alpha^{gw}$ and/or low $\beta^{gw}$ will mean relatively fewer larger (g,h) facilities. There will also be fewer facilities the larger the scale of the unit costs, but the most interesting effects of costs will arise from nonlinearities and these are considered separately below. The $\alpha^{gw}$ and $\beta^{gw}$ effects are determined by consumer tastes and behaviour.

(ii) The larger $\mu$, and the larger the $\hat{\beta}^{gw}$'s, the greater will be the tendency to consumer-forced agglomeration. At some stage in future explorations of this kind of model, it may be appropriate to break down the $\hat{X}_j$ term into components to reflect the tendency for particular (g,h), (g',h') combinations to group together because of consumer demand. One way to do this would be to replace $Z_j^{(1)}$ by a $Z_j^{(2)g}$ term which measured access from g in j to other g''s, say through $\sum_{j'g'h'} W_{j'}^{g'h'} e^{-\hat{\beta}^{gg'w} c_{ij}}$, with a set of coefficients $\hat{\beta}^{g'gw}$ which, in effect, measured the strength of (g,g') spatial connections. In practice, the number of parameters could be reduced by having values specified for (g,g') *groups*.

(iii) The next step is to consider the variation in unit costs. They will clearly vary with location - for example, because land prices will vary with accessibility, say. They will also vary with scale. It will be possible, therefore, even if there are relatively strong agglomeration forces, for certain h-types to develop in isolation at suburban or ex-urban locations. The cheapness of the site together with scale economies would allow a supplier to offer a sufficiently large $W_j^{gh}$ to overcome the $X_j^{\mu}$-agglomeration attractiveness relative to a smaller $W_j^{gh}$ at a central location.

(iv) It will be particularly important to explore the different and changing unit costs as between different h-types for the same g. This may offer some clues about what seems to be, for example, the relative decline in importance of the city-centre department store relative to suburban large, but more specialised stores.

The first two items in the list above could in principle be derived from the traditional disaggregated model; but the last two need the h-disaggregation and the more detailed submodel of supply side costs.

The second stage of this part of the argument is to consider the evolution of the system over time. To a large extent, this can be carried forward on the basis of the preceding analysis by using comparative static assumptions. The main task then is to specify changes in parameters and variables which are at present exogenous to the model and to trace the evolution of patterns. There is one serious complication in this however. For any particular set of parameters, there will be multiple equilibrium solutions and, even if most of these by definition are suboptimal, they are accessible. One task therefore is to chart the multiplicity of possible paths of evolution and to give historical analyses of the paths adopted in particular cases. Finally, there is the possibility of new kinds of bifurcation arising from the $\varepsilon^{gh}$ parameters - see Wilson (1981) for an analysis of the traditional model in this respect.

## 7. Applications to other sectors

Although we have presented the argument in terms of retail systems, it will already be clear that the model is a general one. This general structure is made clear in figure 1. One the left hand side, we show a demand for goods or services by groups at locations; on the right hand side, the supply of these in different kinds of organisations, also at locations. The supply is represented in terms of 'attractiveness' and is matched against demand to generate the assignment. The revenue and costs can be estimated and we require these to be in balance.

By an appropriate interpretation of w, g and h, this framework can be seen to be widely applicable. Here, we present a brief description of four very different kinds of examples.

(i) Hospital serivces. Let $e_i^{gw}$ be the per capita demand for speciality service g by persons of type w by residential location. The supply is represented by $A_j^{gh}$, which represents number of bed-days, say, in facility h at j for service g. In this case, a different mechanism will be needed for 'matching' to produce the equivalent of $\{S_{ij}^{ghw}\}$. 'Revenue' and 'cost' balancing will be in terms of bed days rather than money (in a public sector system), and the supply offered will probably have to be designed to satisfy budget constraints. In other words, an alternative money-cost mechanism is required, but the similarities could still offer some useful insights. For a full account of the framework for a health service model developed along these lines, see Wilson and Clarke (1982).

(ii) Education. In this case, g would have to be taken as 'subjects at a given level' say and h could range over types of school and other relevant educational institutions (Sixth Form Colleges, Colleges of Further Education, and so on). The interest would almost certainly lie in the form of the unit cost functions which could be used to explore efficient spatial forms for supply systems. (For a full discussion of this kind of model, see Crouchley and Wilson, 1983).

(iii) Residential location and housing. This framework suggests an extension of some current residential location and housing models - cf. Clarke and Wilson (1983-B) for an example. The extension arises from a decision as to what to do with h? We can take g as specifying a dwelling of a given size, say (possibly a list of characteristics), and then we might take h as representing, say, tenure. The usual residential location-housing array of $T_{ij}^{gw}$ (using g for house type) would then become $T_{ij}^{ghw}$. The mechanism would then work in a similar way to retailing, though with appropriate changes in the definition of the elements of the attractiveness function. We should also add in this case that, in the formulation as presented, demand at i would arise at the workplace of (say) the head of household, and housing would be supplied at j.

(iv) Industrial location. To build a model of industrial location, we also need to consider at least certain serivces simultaneously if only because these will form part of the list of inputs to manufacturing industries. We can thus define g to cover a list of goods and related services, both w and h to be lists of organisation types (eg. 'industries'), but with w ranging over a larger list to include final demand sectors. The framework can then be expected to apply. The $S_{ij}^{ghw}$ arrays would represent flows of goods, commodities or services (g) from sector h in j to sector w in i. It is analogous to the $\{x_{ij}^{mnq}\}$ array used by Cripps, Macgill and Wilson (1974) and subsequently developed by Macgill (1977). The mechanism outlined for retailing for the specification of production functions using ideas about unit costs and activity levels would be directly applicable here. The main advantage of this approach over standard approaches is that it offers explicit modelling of spatial flows, a topic largely neglected by economists.

In all of these four examples, it could be argued that too many prices are assumed exogenously. This is an important point which will be taken up explicitly as one of the concluding comments of the next section.

## 8. Concluding comments: the next steps in a research programme

It seems intuitively correct to seek to expand the form of service sector models as suggested here. As indicated at the outset, there are observable phenomena which cannot be represented in traditional models. The next crucial step, however, is the testing of the model empirically; and this will lead to a theoretical filling-out (and, undoubtedly, further modification). A range of attractiveness functions will have to be explored. The most difficult part of the exercise, however, is likely to be the assembling of information on the unit cost functions. A useful first step may be numerical experiments with alternative forms of such functions to see if recognisable patterns can be obtained.

There then remain a number of theoretical steps to pursue which will be the subject of future papers. We mention six problems below which demand further work.

(i) An interesting theoretical exercise will be to attempt to repeat the Harris-Wilson (1978) analysis for this more disaggregated problem - particularly the examination of the intersections of revenue and cost curves for equilibrium points. In the simplest case, the cost 'curve' was a straight line; now it will be much more complicated and the results of the analysis correspondingly more interesting.

(ii) We mentioned in the introduction that there is a fundamental difficulty of the representation of service units - shops say. At present, we take as our main variables 'provision in a zone' and we relate, for example, our cost variables to these. It would be more fundamental to use actual shops, or at least distributions of shop sizes, by zone, so that zonal cost functions were explicit aggregates of individual shop cost functions. This would probably imply a more realistic treatment of scale economies.

(iii) In this paper, we have treated different service sectors as though they are independent (except insofar as they play a role in the example of the industrial location model). There are obviously important linkages to be added to a model at some stage - for instance in relation to competition for land and the effect this has on land prices; or whether there are tendencies for different services to agglomerate. An important research question beyond the formulation of such comprehensive models is the investigation of the significance of such linkages: the extent to which they can be neglected to a reasonable degree of approximation; or equivalently, whether the problem can be largely decomposed with some coupling through exogenously-supplied land prices (perhaps from a linking submodel).

(iv) One feature of the models presented is that they contain a large number of general *functions*, particularly with respect to costs. An important task is to seek to specify them more precisely in terms of theory, and then to explore the consequences of alternative assumptions in a series of numerical experiments.

(v) Many, especially those with a training in economics, will argue that too many prices are specified exogenously. We first make a comment on this and then some remarks on how to extend the models.

While economists might argue that the determination of prices is the most interesting part of the problem, the consequence of this strategy is that a number of approximations have to be made. The alternative strategy, of assuming some prices but making fewer further approximations may well be fruitful in some circumstances. Perhaps most importantly of all, it provides a basis from which the problem of price determination can be tackled. There are a number of aspects to this. Public services for which consumers are not charged market prices are basically different from, say, retailing. Secondly, there are, from the point of view of a regional system, 'macro' prices - essentially those determined in the national economy. Thirdly, there is the task of modelling spatially varying prices such as land prices. Fourthly, there is the need to model consumer-perceived prices - in particular, those which incorporate transport costs between point of demand and point of supply. Since it is these prices which should appear in the consumers' demand function, this introduces elasticity into the demand in an interesting way. There is a substantial literature which bears on these questions, and the integration of the various approaches, and the formal posing of new research questions, will be the subject of another paper.

(vi) The final remark is intended to situate this kind of model building within the broader radical critique of this kind of geography. It is often argued that modelling is essentially tied to the neo-classical economic paradigm and indeed there is an element of this in the models presented here. It should be emphasised, however, that the underlying structures will remain the same whatever specific theories are employed. There will be a demand for services (or 'need'); there will be a supply, which should be efficient in some way; and demand (need) will have to be allocated to supply in some way. Many of the concepts presented here will remain useful and necessary, therefore, whatever the outcome of future research on social processes.

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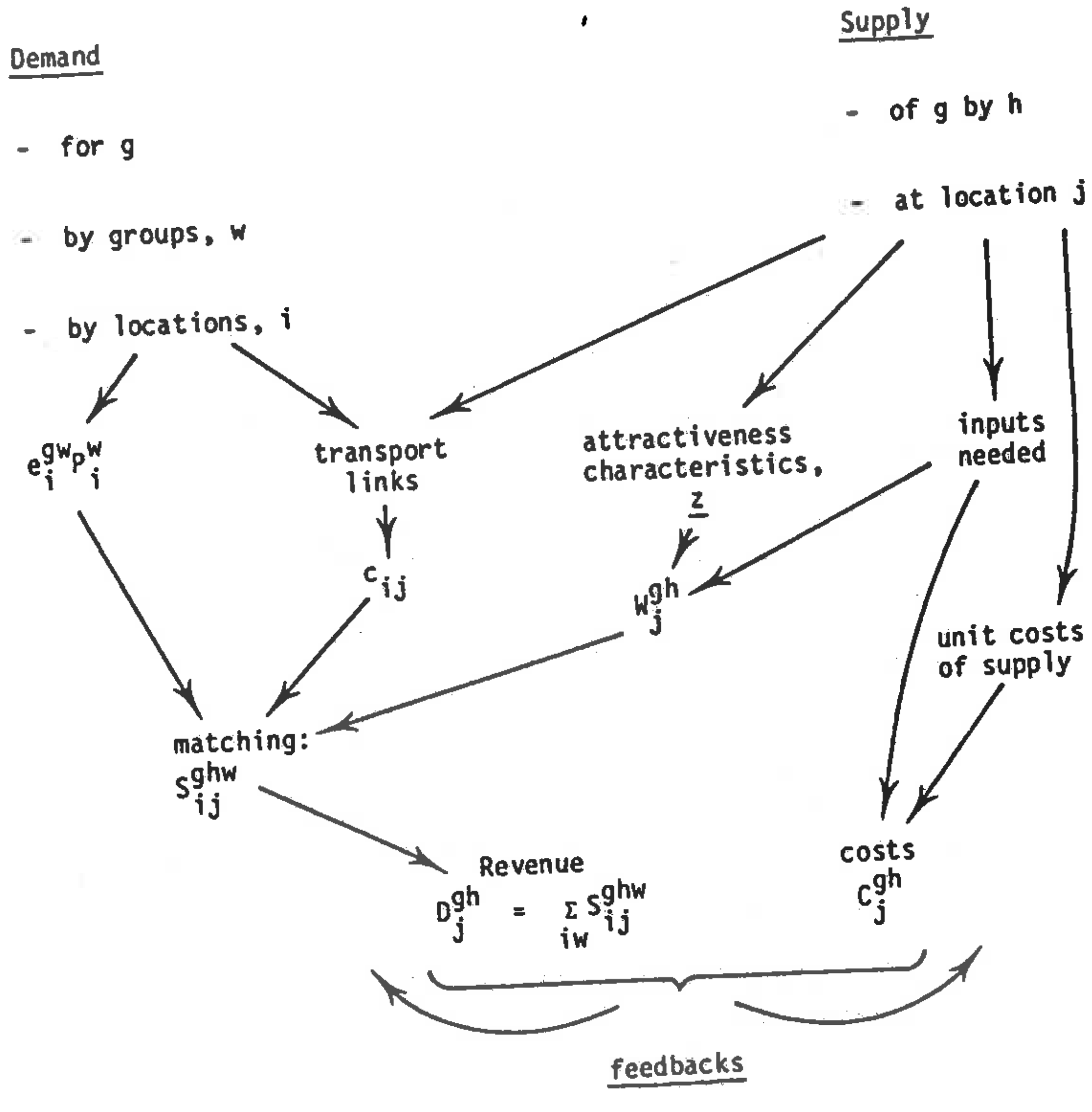
Figure 1


Demand
- for g
- by groups, w
- by locations, i
Supply
- of g by h
- at location j
$e_i^{gw} p_i^w$
transport links
attractiveness characteristics, $\underline{z}$
inputs needed
$c_{ij}$
$W_j^{gh}$
unit costs of supply
matching: $S_{ij}^{ghw}$
Revenue
$D_j^{gh} = \sum_{iw} S_{ij}^{ghw}$
costs
$C_j^{gh}$
feedbacks