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Industrial Location Models II: Weber, Palander, Hotelling and extensions within a new framework

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2. ELEMENTS OF THE GENERAL FRAMEWORK

It has been argued elsewhere (Wilson, 1983) that the first step in model design should be 'entitation'. Thus we begin with an attempt to conceptualise our system of interest in Figure 1. The focus of our interest is on production activities which we define as Z_j^{ng} - the amount of a good g produced by activity n at j. We need to distinguish production sectors from final demand sectors in which no output is generated. Formally, we may let N_1 be the set of production sectors, N_2 the set of final demand sectors and N_* is $N_1 \cup N_2$. Two types of interaction variable may then be specified (see Figure 1B). Y_{ij}^{mnh} represents the flow of good h from sector/ activity m in i as inputs to activity n at j ($m,n \in N_1$). Y_{jk}^{nrg} represents the flow of good g from sector n at j as outputs to sector r at k ($n \in N_1$, $r \in N_*$).

The next step is an attempt to spell out the nature of the interaction variables in a formal manner. Initially, we define W_{ij}^{mng} as the attractiveness of activity m in i as a supplier of input g to n at j; and c_{ij}^{g} the unit cost of transport of g from i to j. The attractiveness is assumed to be a function of production levels and other appropriate variables, such as prices and transport costs. The formal expression of this assumption is:

$$W_{ij}^{mng} = W_{ij}^{mng}\left(Z_i^{mg}, p_i^{mg}, c_{ij}^{g}, \ldots \right) \tag{1}$$

As with the interaction variables, there are two types of demand situation. We define:

$$\overline{X}_k^{rg} = \sum_h a^{rgh} Z_k^{rh} \qquad r \in N_1 \tag{2}$$

as intermediate demands, assuming a Leontief technology with input-output relations defined by the fixed coefficients $\{a^{rgh}\}$.

Alternatively:

$$\overline{X}_{k}^{rg} = X_{k}^{rg} \qquad r \in N_2 \qquad (3)$$

where X_{k}^{rg} is final demand for g in sector r at k.

The spatial interaction flows governing the input requirements of activity n at j may be specified formally as

$$Y_{ij}^{mnh} = Y_{ij}^{mnh}\left(W_{ij}^{mnh}, \Sigma_{h}\, a^{hg}\, Z_{j}^{ng}, c_{ij}^{h} \right) \qquad m,n \in N_1 \qquad (4)$$

If we wish to further disaggregate these flows according to the good produced in process n at the destination, j, then this may be achieved by removing the summation carried over into (4) from (2). Thus:

$$\overline{Y}_{ij}^{mnhg} = \overline{Y}_{ij}^{mnhg}\left(W_{ij}^{mnh}, a^{nhg}\, Z_{j}^{ng}, c_{ij}^{h} \right) \qquad m,n \in N_1 \qquad (5)$$

Changing the focus to outputs of good g for activity n at j, the interaction flows may be taken as:

$$Y_{jk}^{nrg} = Y_{jk}^{nrg}\left(W_{jk}^{nrg}, \overline{X}_{k}^{rg}, c_{jk}^{g} \right) \qquad n \in N_1,\ r \in N_* \qquad (6)$$

with $\overline{X}_{k}^{rg}$ given by (2), (3). Notice that in the case where $r \in N_1$, when $\overline{X}_{k}^{rg}$ is given by (2), then the flows would take the same form as (4). This emphasises the circularity of the input-output relationships within the production sector, as implied by Figure 1A. However by specifying the input and output flows distinctly, we are also able to stress their relationship to the individual production sectors, thus facilitating the specification of costs and revenue for a particular activity, which now follows.

Let us assume for the present that appropriate price vectors may be specified. The revenue from production of good g by sector n at j is now given as:

$$D_j^{ng} = \sum_{kr} p_k^{rg} Y_{jk}^{nrg} \qquad n \in N_1 \tag{7}$$

Similarly, a fairly general form for the cost of the activity may be specified as:

$$C_j^{ng} = f_j^{ng} + k_j^{ng} Z_j^{ng} + \sum_{ihm} \bar{Y}_{ij}^{mnhg} p_j^{ng} \qquad n \in N_1 \tag{8}$$

where f_j^{ng} represents fixed costs, k_j^{ng} a variable cost and the right-hand term is a measure of the cost of materials inputs.

A key idea now (cf Harris and Wilson, 1978) is that an equilibrium will exist when:

$$D_j^{ng} = C_j^{ng} \tag{9}$$

that is, cost and revenue are equal for each activity and the associated commodity at each location. Typically this equilibrium may be seen as the steady state in a dynamic mechanism related to the activity levels $\{Z_j^{ng}\}$. For example:

$$\dot{Z}_j^{ng} = \varepsilon^{ng} (D_j^{ng} - C_j^{ng}) Z_j^{ng} \tag{10}$$

For ease of presentation an input-output basis to the production system has been assumed throughout here, and this is by far the most common case in the literature (see Birkin and Wilson, 1984). However, two further points are worth making. The first is that our presentation is at a finer level of detail than is usual for input-output analysis, since it concentrates neither on good-by-good relations, nor on sector-by-sector relations, but a combination of both. Secondly, the whole framework could in principle be easily raised to a higher level of generality by specifying production functions throughout. The point is that although the cost functions would then need to be respecified, the dynamic mechanism offered above is still an appropriate basis for modelling.

In more general cases, the prices also need to be determined through the model (see Takayama and Judge, 1972; Birkin and Wilson, 1984), as functions of the demand levels, output levels, the friction of distance, and other appropriate paramaters. Formally:

$$p_k^{rg} = p_k^{rg}\left(\bar{X}_k^{rg}, Y_{jk}^{nrg}, c_{jk}^{g}, \ldots\right) \tag{11}$$

This might again be expressed in terms of a set of differential equations, for example:

$$\dot{p}_k^{rg} = \theta^{rg}\left(\bar{X}_k^{rg} - \sum_{jm} Y_{jk}^{mrg}\right) p_k^{rg} \tag{12}$$

if the price change were related to the difference between (latent) demand and the quantity supplied. The practical solution to this set of equations then becomes a complex iteration involving adjustments in the quantity produced, Z_j^{ng}, through (10), and the market price of goods, p_k^{rg}, through (12). It is possible that such solutions are then highly dependent on the relative values of the ε^{ng} and θ^{rg} parameters which between them represent a cobweb mechanism for return to equilibrium. This matter is explored further, in the context of agricultural location, by Wilson and Birkin (1984).

The detailed model structure depends on the assumptions we make in relation to:

a) the treatment of interdependence;

b) cost functions;

c) revenue functions;

d) pricing strategies;

e) the structure of demand;

and f) whole system objectives.

In short, these are the 'elements of theory' outlined by Wilson (1983) and presented in the context of industrial location models in Birkin and Wilson (1984). We now attempt to demonstrate that, by making an appropriate choice of assumptions in relation to these dimensions, the model can be used to imitate the behaviour of a variety of well-known location models.

3. CREATION AND TESTING OF STANDARD MODELS

3.1 A multi-sector model

We begin by spelling out a particular model using the general structure presented in Section 2. A similar model has been constructed elsewhere (Wilson and Birkin, 1983) and subjected to numerical experimentation. Let us assume fixed prices, and that each activity produces a single, unique good so that the label g can be dropped. Spatial interaction flows are generated from an attraction-constrained model:

$$Y_{jk}^{nr} = A_k^n \overline{X}_k^{nr} (Z_j^n)^{\gamma^n} \exp\{-\beta^n (p_j^n + t_j^n d_{jk})\} \quad n \in N_1 , \; r \in N_* \qquad (13)$$

where:

$$A_k^n = 1 / \sum_j (Z_j^n)^{\gamma^n} \exp\{-\beta^n (p_j^n + t_j^n d_{jk})\} \qquad (14)$$

to ensure that:

$$\sum_j Y_{jk}^{nr} = \overline{X}_k^{nr} \qquad (15)$$

These equations represent the flow of a good, now effectively labelled n, as one of the inputs to sector r ($r \in N_1$) or to final demand in sector r ($r \in N_2$). Their purpose is essentially to distribute a fixed demand between a number of production sources according to the relative cost advantage of a particular source to a particular market (the negative exponential term) and assuming attractiveness to be related only to the characteristics of the origin (production) zones:

$$W_j^n = (Z_j^n)^{\gamma^n} \tag{16}$$

for some parameter γ^n.

There are, once again, two demand situations:

$$\overline{X}_k^{nr} = a^{nr} Z_k^r \qquad n,r \in N_1 \tag{17}$$

$$\overline{X}_k^{nr} = X_k^{nr} \qquad n \in N_1, \; r \in N_2 \tag{18}$$

In our earlier paper (Wilson and Birkin, 1983) we defined a total of five sectors, in which $N_1 = \{1,2,3,4\}$ and $N_2 = \{5\}$.

Revenue may be taken as:

$$D_j^n = p_j^n \sum_{kr} Y_{jk}^{nr} \tag{19}$$

and costs by:

$$C_j^n = f_j^n + k_j^n Z_j^n + \sum_{im} p_j^m Y_{ij}^{mn} \tag{20}$$

Beginning with a trial allocation of $\{Z_j^n\}$, the costs and revenues may be calculated and the solution approached iteratively assuming:

$$Z_j^n (t+1) = Z_j^n (t) \; D_j^n (t) \; / \; C_j^n (t) \tag{21}$$

– the 'quasi-balancing factor method', so-called because $Z_j^n (t+1)$ is calculated as a value of Z which yields a quasi-solution to the equations $D_j^n (t) = C_j^n (t)$.

The main features of this model are:

- spatial interaction flows
- linear production functions (no economies of scale)

- complex revenue functions dependent on the scale of activity, its location and other elements of its 'attractiveness'

- fixed prices

- a given regime of consumer demand. Demand for intermediate products (as inputs) is dictated by the spatial coordination of input-output relations between activities

- the whole system objective is something like the maximisation of consumers' surplus, given a fixed aggregate expenditure on transport costs.

We attempted to demonstrate the power of this kind of model in an earlier paper (Wilson and Birkin, 1983), to which the reader is referred for further results.

3.2 The Weber model

Let us suppose that we have an industrial location model with three production sectors, with input-output linkages, and a single final demand sector ($N_1 = \{1,2,3\}$; $N_2 = \{4\}$). The first stage in attempting to generate results compatible with the Weber model is to fix the locations of two of the production activities and the market, and to identify from that the location of the other production sector. The situation is illustrated graphically in Figure 2. Activity 1 locates at I_1 in the top left-hand corner, activity 2 at I_2 to the top right and activity 4 at I_4 at the bottom. Let us assume that an intermediate good (type 3) is to be produced at a location to be determined. Good 3 uses inputs from sectors 1 and 2, and sells its products to sector 4. For each unit of product sold to sector 4, the input requirements from sectors 1 and 2 are determined by the input-output multipliers a_{13} and a_{23} respectively.

The next step is to transfer the problem to a discrete zone system. In this case we assume a regular 13 X 13 grid (see Figure 3). Let us assume it is possible to number these zones such that i = 1 is the zone containing I_1, and similarly i = 2 has I_2 and i = 4 the market. To focus on transport costs both fixed and variable costs are set to zero, as are the f.o.b prices of goods 1 and 2:

$$p_1^1 = p_2^2 = 0$$

$$k_j^n = f_j^n = 0 \quad \forall j,n \tag{22}$$

Thus costs are now given simply as the sum of weighted transport flows (cf (19)):

$$C_j^3 = c_{1j}^1 Y_{1j}^{13} + c_{2j}^2 Y_{2j}^{23} + c_{j4}^3 Y_{j4}^{34} \tag{23}$$

The Weber model assumes that the ability to generate revenue is space-independent. This will clearly apply by default to the flows from sectors 1 and 2 to sector 3, since a single origin zone is involved. To eliminate the element of spatial competition for activity 3 we need to assume that $\beta^3 = 0$, so the interactions between sectors 3 and 4 are given as:

$$Y_{j4}^{34} = X_4^{34} \frac{(Z_j^3)^{\gamma^3}}{\sum_j (Z_j^3)^{\gamma^3}} \tag{24}$$

(setting $\beta^n = 0$ in (13) and substituting for A_k^n and $\overline{X}_k^{nr}$). In the particular case where $\gamma^3 = 1$, firms are clearly able to sell goods in proportion to their total market share. If $\gamma^3 > 1$ then we have an effect which may be thought of as economies of scale in distribution i.e. large scale activities are able to control a greater than proportional share of the market. Similarly $\gamma^3 < 1$ implies diseconomies. Hence $\gamma^3 = 1$ is a further necessary assumption for location-independent revenue accumulation.

The total revenue generated by zones is:

$$D_j^3 = p_4^4 Y_{j4}^{34} \qquad (25)$$

although typically we would assume $p_4^4 = 1$ as a numeraire. The quasi-balancing factor method may now be applied to find a distribution of $\{Z_j^3\}$ for which revenue and costs are in equilibrium in all zones.

We are now in a position to set up some simple numerical experiments with this model. The fixed locations are specified as I_1 = (5,2); I_2 = (4,11) and I_4 = (11,7). An euclidean measure of distance is assumed and transport cost multipliers are 0.03 on each good, that is:

$$c_{ij}^n = 0.03\, d_{ij} \qquad \forall\ n \qquad (26)$$

The distances along the edges of the locational figure are d_{12} = 9.06, d_{14} = 8.60 and d_{24} = 8.95.

In Figure 4 we see the effect of varying the materials pulls through the values of the coefficients a^{13} and a^{23}. Here we can see examples of some standard Weberian situations. In extreme cases (3 and 6) where the weight requirements of one material exceed the combined requirements of the other two, production occurs at the material source. In intermediate cases where no single material dominates, the production site may still tend towards the most important input (case 5), but the market has a natural advantage, albeit a slight one, as the most accessible location (case 4). Hence the market does not need to have a dominant materials pull to obtain the production site (case 2), although clearly this will be retained if the market pull is dominant (case 1).

3.3 Market Area Models

In our review paper (Birkin and Wilson, 1984) it was argued that SIA models effectively function as overlapping market area models. In this sub-section, we wish to demonstrate this point explicitly with reference to the well-known work of Palander (1935). Once again a summary is provided by Birkin and Wilson (1984), and also Paelinck and Nijkamp (1975, Chapter 3).

Palander assumes that firms adopt an f.o.b. pricing strategy, and that consumers choose their suppliers to minimise the sum of mill price and transport costs (the delivered price). To mimic Palander, we will need to do likewise. We assume (cf. (25)) that:

$$c^n_{jk} = t^n_j d_{jk} \tag{27}$$

where c^n_{jk} is the cost of transporting one unit of product n from production location j to market location k; t^n_j the transport charge on goods type n originating at j; and d_{jk} is the euclidean distance from j to k.

The interactions are derived as an appropriate modification of (13):

$$Y^{nr}_{jk} = A^n_k \overline{X}^{nr}_k (Z^n_j)^{\gamma^n} \exp\{-\beta^n (p^n_j + t^n_j d_{jk})\} \tag{28}$$

with

$$A_k^n = 1 / \Sigma_j (Z_j^n)^{\gamma^n} \exp(-\beta^n (p_j^n + t_j^n d_{jk})) \tag{29}$$

Since the model is concerned with interactions only, we need pursue the structural issues no further. Let us assume then that we have a sector for which there are two suppliers, both in equilibrium and located at points (4,7) and (11,7) on our 169-zone grid. Demand for their products is spread uniformly across the plain, and the suppliers are identical in terms of aggregate attractiveness.

A key issue here concerns the role in the model of the parameter β, which controls the degree of overlap between market areas - this is demonstrated in Figure 5. Figure 5a shows the distribution of demand between the two suppliers at locations across the region assuming b = 1.0. We observe that in this case (with $p_1^n = p_2^n = 1.0$; $t_1^n = t_2^n = 0.1$) spatial variation in costs is relatively small and the allocation of market shares is fairly uniform. Thus suppliers are able to control only 70% of the market at their own locations, this share falling steadily towards the other supplier. This situation changes radically if we increase the value of β from 1.0 to 10.0 (Figure 5b). In this case, we can see that the two suppliers exert complete control over their local market areas, and there is only a narrow band of overlap between them. Furthermore, it is clear from Figure 5b that this division is a linear one, as one would expect when the prices and transportation rates are equal. Careful inspection of Figure 5a, however, reveals that in this case the distributions are symmetrical about the same market area boundary, and if one measures the market in terms of the dominant supplier, then the market areas are uniform with respect to β.

It is possible to demonstrate quite easily that if we define market areas in this way, the boundaries will never be affected by the value of β as long as the attractiveness of the activities is equal. Equations (28) and (29) may be expanded for the case of two suppliers to yield:

$$Y^{nr}_{jk} = \bar{X}^{nr}_{k} \frac{(W^{n}_{j})\exp\{-\beta^{n}(p^{n}_{j}+t^{n}d_{jk})\}}{(W^{n}_{1})\exp\{-\beta^{n}(p^{n}_{1}+t^{n}d_{1k})\} + (W^{n}_{2})\exp\{-\beta^{n}(p^{n}_{1}+t^{n}d_{2jk})\}} \qquad (30)$$

Given that $W^{n}_{1} = W^{n}_{2}$, the market share of activity 1 in zone k (ϕ^{nr}_{1k}) is given by:

$$\phi^{nr}_{1k} = \frac{\exp\{-\beta^{n}(p^{n}_{1}+t^{n}d_{1k})\}}{\exp\{-\beta^{n}(p^{n}_{1}+t^{n}d_{1k})\} + \exp\{-\beta^{n}(p^{n}_{2}+t^{n}d_{2k})\}} \qquad (31)$$

Hence the condition that ϕ^{nr}_{1k} be greater than ϕ^{nr}_{2k} (or alternatively, that ϕ^{nr}_{1k} be greater than 0.5) is clearly:

$$\exp\{-\beta^{n}(p^{n}_{1}+t^{n}d_{1k})\} > \exp\{-\beta^{n}(p^{n}_{2}+t^{n}d_{2k})\}$$

i.e.

$$p^{n}_{1} + t^{n}_{1}d_{1k} < p^{n}_{2} + t^{n}_{2}d_{2k} \qquad (32)$$

which correspond to the original Palander conditions that a demand point locates in the market area for which the delivered price is lower. The difference is that the degree of market 'control' exerted is mediated through (31). For higher values of β, the difference in the left- and right-hand terms of the denominator will be greater and hence market area shares will be more sensitive to delivered price.

In Figure 6 we have reproduced the three fundamental Palander cases with our discrete zone representation. We have assumed here a value of $\beta = 1.0$ and mapped the dominant market influence for each demand zone. In Figure 6a, in which $p_1^n = p_2^n$; $t_1^n = t_2^n$ we see the straight-line market area boundary again. In Figure 6b, we force the mill price of the first supplier to rise, and consequently its market area boundary is pushed back into the characteristic hyperbolic configuration. Thirdly, if prices are equal but the second supplier enjoys more favourable tariffs on transportation we arrive at Figure 6c, when the market area for the first supplier becomes circular.

3.4 Models of Competition

In this sub-section we move on to consider models in the style of Hotelling (1929). One of the characteristic features here is the assumption of a linear distribution of demand, and so we begin by specifying a linear spatial system of fifteen zones, each separated from its nearest neighbours by a distance of one unit. Let us now specify a standard single-sector production-constrained SIAS model (cf. Harris and Wilson, 1978). This model may be considered as a special case of the multi-sector situation outlined above, in which $N_1 = \{1\}$; $N_2 = \{2\}$. Assuming there to be no spatial flows within a sector, we may drop the superscripts, since there is now only one set of flows (from sector 1 - production - to sector 2 - final demand), and one set of activity levels to be determined. The interactions are given by:

$$Y_{jk} = A_k X_k (Z_j)^{\gamma} \exp(-\beta c_{jk}) \tag{33}$$

where:

$$A_k = \{ \Sigma_j (Z_j) \exp(-\beta c_{jk}) \}^{-1} \tag{34}$$

Revenue for zone j is:

$$D_j = \Sigma_k Y_{jk} \tag{35}$$

and costs:

$$C_j = f_j + k_j Z_j \tag{36}$$

allowing no material inputs in this instance.

We assume a uniform distribution of demand, thus $X_k = 10.$ ($\forall$ k) and, initially, a uniform distribution of production too - $Z_j = 10.$ ($\forall$ j). Once the interactions have been calculated in (33) given this distribution of production, costs and revenue are calculated in (36) and (35) respectively. Now outputs for the next time period (t+T) are set such that:

$$Z_j^{t+T} = Z_j^t D_j^t / C_j^t \tag{37}$$

Given (arbitrarily) that $f_j = 1.0$ ($\forall$ j) and $k_j = 1.0$ ($\forall$ j), it is interesting to investigate the outcomes of this simple model over a range of gamma and β values. Some examples are presented in Figure 7, with gamma equal to 1.0, 1.1 or 1.2, and β taken as 0.5, 1.0, 2.0, or 5.0. We can see that in certain cases { (γ=1.1,β=0.5); (γ=1.2,β=0.5) } the Hotelling situation of transport cost minimisation, with two locations at the quartiles (zones 4 and 12) is duplicated. However since we are no longer restricted to two firms, a greater number of non-zero output locations are found in other situations.

These model outcomes are comparable with, but not identical to, the predictions of Serck-Hanssen (1970), who deduced that in a continuous one-dimensional market with an even distribution of consumers, all activities would be of equal size and located at the centre of their markets. Our solutions appear to obey the second condition but not the first. However Serck-Hanssen assumed consumers to be utility-maximisers with respect to quantity consumed only, whereas our implied welfare function has a more involved consumers' surplus term (Birkin and Wilson, 1984).

Recall now that the Hotelling argument is essentially that these equilibrium solutions related to consumers' surplus are not stable in relation to competition between suppliers. To incorporate such notions, we need to add an element of spatial search to our heuristic solution procedure. Suppose we begin as before with each supplier calculating his revenue and costs, and resetting output levels by equation (37). The supplier then tests, given the existing pattern of output, whether this level of activity would be more profitable at another location, which is:

$$\max_{\{R_j\}} R_j = D_j(Z_j^{t+T}) / C_j(Z_j^{t+T}) \tag{38}$$

This framework may now be used to scrutinise the stability of the results presented in Figure 7 when spatial competition is introduced. Let us begin with the classic Hotelling case with production located at the quartiles [(γ=1.1, β=0.5)]. This is input as the initial solution to the revised model. We observe in Figure 8a that after the first iteration or time period, the activities have 'leapfrogged' over one another and relocated nearer to the centre of the line. By the second time period both suppliers have adopted a stable central location. This is all very much as we would expect.

Consider now what happens if we take a situation with more than two firms e.g. the (γ=1.1,β=1.0) case. Here we find there is no stable solution (Figure 8b) because in the pseudo-Hotelling situation in which all activities located at the centre of the region, it would always be beneficial for one or more of the activities to break the agglomeration. This effect is also discussed by Gunnarsson (1977, pp58-59) in relation to Serck-Hanssen's 'Weber model' framework. In fact, from iteration 8 onwards, the model settles into a simple three-stage oscillating solution - an intuitively appealing outcome.

4. CONCLUSIONS

In this paper we have attempted to demonstrate the ability of models of spatial interaction and structure to reproduce the results of classical industrial location theory. Numerical solution has been facilitated by the adoption of a discrete zone representation, but the nature of the results is not affected.

While the approaches considered in this paper may be among the more elementary ones, they are also crucial in that they explore unambiguously three of the fundamental determinants of spatial industrial pattern viz. location (Weber), interaction (Palander) and competition (Hotelling). The extension to multi-firm models may be technically more demanding in view of their increased (sectoral) complexity, but is facilitated by the approach to the rationalisation of theory ourlined in out earlier paper (Birkin and Wilson, 1984).

One of the oddities of industrial location theory on which we have not yet commented has been a tendency for it to develop autonomously with respect to the modelling of other socio-economic subsystems, notably residential and service location. In particular, the supposedly 'integrated' Lowry model frameworks typically assume 'basic' industry as an exogenous or driving force. In reality, the notion of a basic industry as independent of the local area is not sustainable, if only in view of their dependence on local labour markets (this interdependence has been commented on in depth by Webber, 1984). An important long-term aim in urban and regional modelling is, therefore, the integration of industrial location as an endogenous subsystem. We would hope that the work presented here provides a useful foundation towards such an objective.

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FIGURE 1A: Production and distribution relationships in an industrial location model

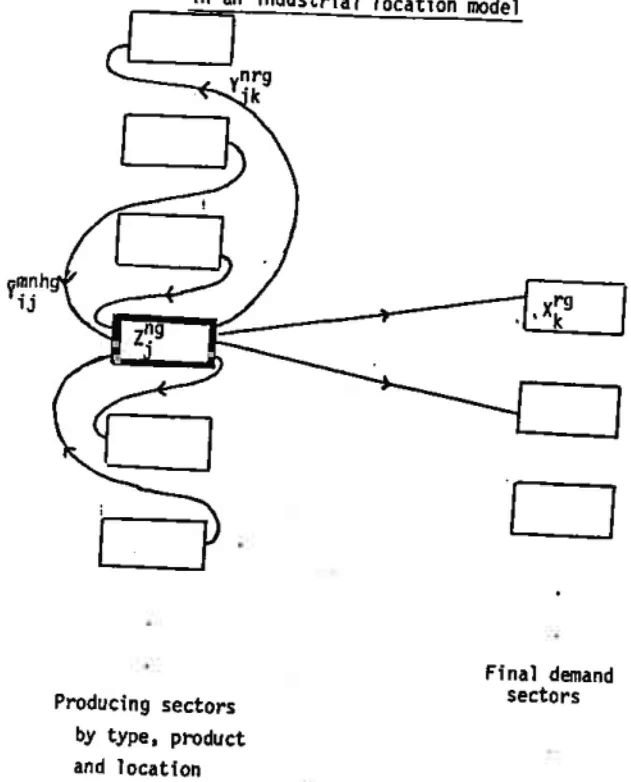


FIGURE 1B: Types of interaction flow

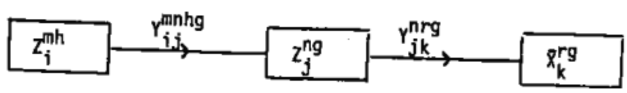


FIGURE 2: A simple Weberian polygon

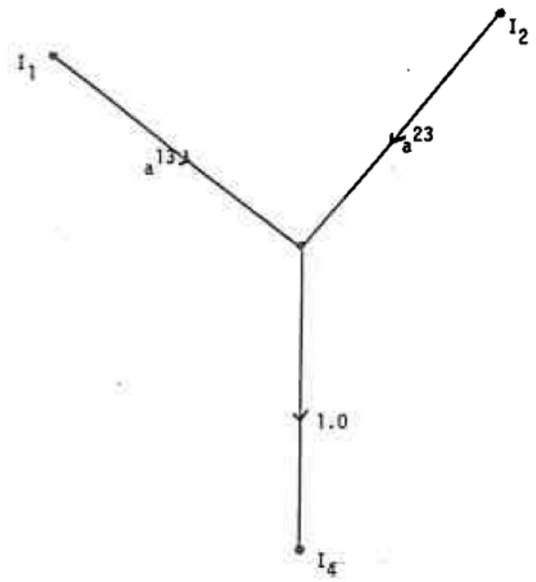
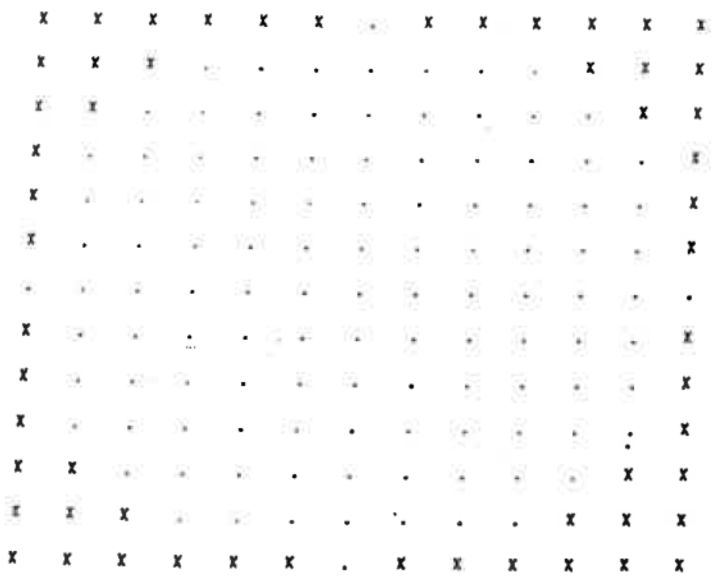


FIGURE 3. A 169-ZONE SPATIAL SYSTEM



X = EXOGENOUS ZONE

FIGURE 4: Weber model simulations with varying input ratios.

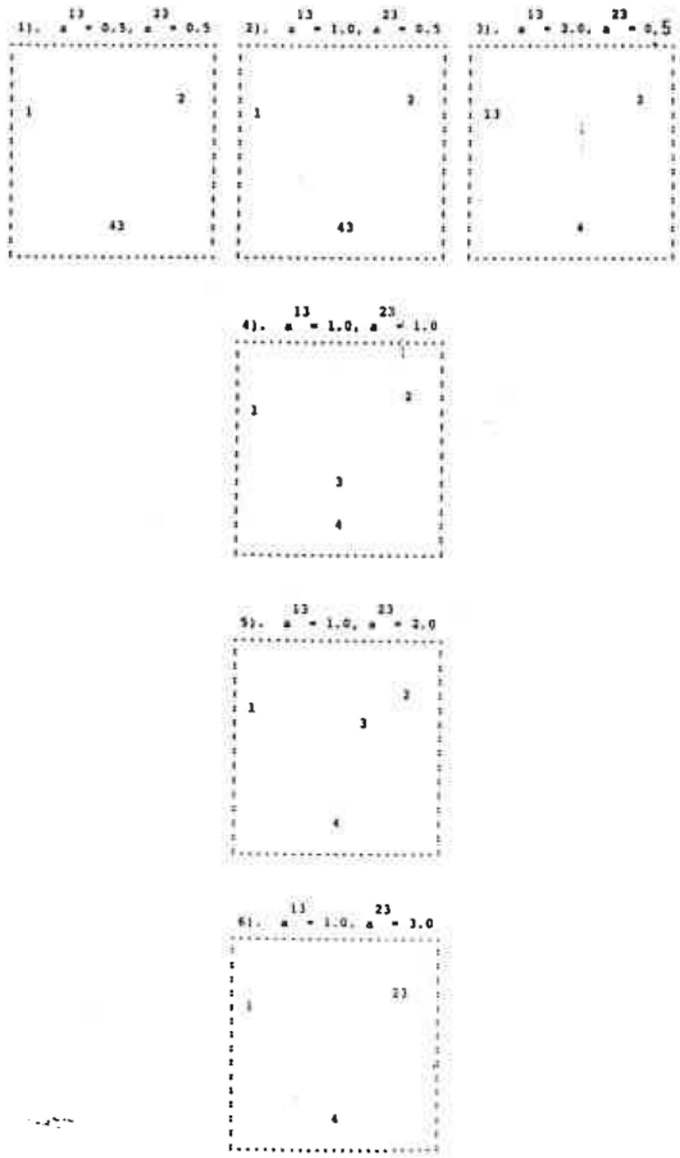


FIGURE 5: Effect of varying beta on market shares

Market shares to zone 1

5A Beta = 1.0

6	6	6	6	6	5	5	5	5	4	4	4	4
6	6	6	6	6	5	5	5	5	4	4	4	4
6	6	6	6	6	5	5	5	5	4	4	4	4
7	6	6	6	6	6	5	5	4	4	4	4	4
7	7	6	6	6	6	5	5	4	4	4	4	3
7	7	7	6	6	6	5	5	4	4	4	3	3
7	7	7	6	6	6	5	5	4	4	3	3	3
7	7	7	6	6	6	5	5	4	4	4	3	3
7	7	6	6	6	6	5	5	4	4	4	4	3
7	6	6	6	6	6	5	5	4	4	4	4	4
6	6	6	6	6	5	5	5	5	4	4	4	4
6	6	6	6	6	5	5	5	5	4	4	4	4
6	6	6	6	6	5	5	5	5	4	4	4	4

5B Beta = 10.0

*	*	*	*	*	*	7	3	0	0	0	0	(
*	*	*	*	*	*	8	2	0	0	0	0	(
*	*	*	*	*	*	8	2	0	0	0	0	(
*	*	*	*	*	*	8	2	0	0	0	0	(
*	*	*	*	*	*	8	2	0	0	0	0	(
*	*	*	*	*	*	9	1	0	0	0	0	(
*	*	*	*	*	*	9	1	0	0	0	0	(
*	*	*	*	*	*	9	1	0	0	0	0	(
*	*	*	*	*	*	8	2	0	0	0	0	(
*	*	*	*	*	*	8	2	0	0	0	0	(
*	*	*	*	*	*	8	2	0	0	0	0	(
*	*	*	*	*	*	8	2	0	0	0	0	(
*	*	*	*	*	*	7	3	0	0	0	0	(

Market shares to zone 2

5A Beta = 1.0

4	4	4	4	4	5	5	5	5	6	6	6	6
4	4	4	4	4	5	5	5	5	6	6	6	6
4	4	4	4	4	5	5	5	5	6	6	6	6
3	4	4	4	4	4	5	5	6	6	6	6	6
3	3	4	4	4	4	5	5	6	6	6	6	7
3	3	3	4	4	4	5	5	6	6	6	7	7
3	3	3	4	4	4	5	5	6	6	7	7	7
3	3	3	4	4	4	5	5	6	6	6	7	7
3	3	4	4	4	4	5	5	6	6	6	6	7
3	4	4	4	4	4	5	5	6	6	6	6	6
4	4	4	4	4	5	5	5	5	6	6	6	6
4	4	4	4	4	5	5	5	5	6	6	6	6
4	4	4	4	4	5	5	5	5	6	6	6	6

5B Beta = 10.0

0	0	0	0	0	0	3	7	*	*	*	*	*
0	0	0	0	0	0	2	8	*	*	*	*	*
0	0	0	0	0	0	2	8	*	*	*	*	*
0	0	0	0	0	0	2	8	*	*	*	*	*
0	0	0	0	0	0	2	8	*	*	*	*	*
0	0	0	0	0	0	1	9	*	*	*	*	*
0	0	0	0	0	0	1	9	*	*	*	*	*
0	0	0	0	0	0	1	9	*	*	*	*	*
0	0	0	0	0	0	2	8	*	*	*	*	*
0	0	0	0	0	0	2	8	*	*	*	*	*
0	0	0	0	0	0	2	8	*	*	*	*	*
0	0	0	0	0	0	2	8	*	*	*	*	*
0	0	0	0	0	0	3	7	*	*	*	*	*

* indicates saturatior

FIGURE 6: Palander market area simulations

MARKET AREAS

Figure 6A.

$p_1 = p_2$; $t_1 = t_2$

1	1	1	1	1	1	1	2	2	2	2	2	2
1	1	1	1	1	1	1	2	2	2	2	2	2
1	1	1	1	1	1	1	2	2	2	2	2	2
1	1	1	1	1	1	1	2	2	2	2	2	2
1	1	1	1	1	1	1	2	2	2	2	2	2
1	1	1	1	1	1	1	2	2	2	2	2	2
1	1	1	1	1	1	1	2	2	2	2	2	2
1	1	1	1	1	1	1	2	2	2	2	2	2
1	1	1	1	1	1	1	2	2	2	2	2	2
1	1	1	1	1	1	1	2	2	2	2	2	2
1	1	1	1	1	1	1	2	2	2	2	2	2
1	1	1	1	1	1	1	2	2	2	2	2	2

Figure 6A.

$p_1 > p_2$; $t_1 = t_2$

1	1	2	2	2	2	2	2	2	2	2	2	2
1	1	1	2	2	2	2	2	2	2	2	2	2
1	1	1	1	2	2	2	2	2	2	2	2	2
1	1	1	1	2	2	2	2	2	2	2	2	2
1	1	1	1	1	2	2	2	2	2	2	2	2
1	1	1	1	1	2	2	2	2	2	2	2	2
1	1	1	1	1	2	2	2	2	2	2	2	2
1	1	1	1	1	2	2	2	2	2	2	2	2
1	1	1	1	1	2	2	2	2	2	2	2	2
1	1	1	1	2	2	2	2	2	2	2	2	2
1	1	1	1	2	2	2	2	2	2	2	2	2
1	1	1	2	2	2	2	2	2	2	2	2	2
1	1	2	2	2	2	2	2	2	2	2	2	2

Figure 6A.

$p_1 = p_2$; $t_1 > t_2$

2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2
2	1	1	1	2	2	2	2	2	2	2	2	2
1	1	1	1	1	2	2	2	2	2	2	2	2
1	1	1	1	1	2	2	2	2	2	2	2	2
1	1	1	1	1	2	2	2	2	2	2	2	2
2	1	1	1	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2

FIGURE 7: Results for static linear model

```
KOUNT = 15 AT ITERATION  93
GAMMA =  1.0 BETA =  0.5 VCOST =  1.0 FCOST =  1.0
SOLUTION    0.  0. 47.  0.  0.  0.  0. 53.  0.  0.  0.  0. 47.  0.  0.

KOUNT = 15 AT ITERATION  75
GAMMA =  1.0 BETA =  1.0 VCOST =  1.0 FCOST =  1.0
SOLUTION    0. 28.  0.  0. 30.  0.  0. 29.  0.  0. 30.  0.  0. 28.  0.

KOUNT = 15 AT ITERATION  87
GAMMA =  1.0 BETA =  2.0 VCOST =  1.0 FCOST =  1.0
SOLUTION    0. 25.  0. 18.  0. 19.  0. 19.  0. 19.  0. 18.  0. 25.  0.

KOUNT = 15 AT ITERATION   3
GAMMA =  1.0 BETA =  5.0 VCOST =  1.0 FCOST =  1.0
SOLUTION    9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.

KOUNT = 15 AT ITERATION  53
GAMMA =  1.1 BETA =  0.5 VCOST =  1.0 FCOST =  1.0
SOLUTION    0.  0.  0. 74.  0.  0.  0.  0.  0.  0.  0. 74.  0.  0.  0.

KOUNT = 15 AT ITERATION  52
GAMMA =  1.1 BETA =  1.0 VCOST =  1.0 FCOST =  1.0
SOLUTION    0. 33.  0.  0.  0. 40.  0.  0.  0. 40.  0.  0.  0. 33.  0.

KOUNT = 15 AT ITERATION  50
GAMMA =  1.1 BETA =  2.0 VCOST =  1.0 FCOST =  1.0
SOLUTION    0. 25.  0. 20.  0. 11. 16.  0. 16. 11.  0. 20.  0. 25.  0.

KOUNT = 15 AT ITERATION   3
GAMMA =  1.1 BETA =  5.0 VCOST =  1.0 FCOST =  1.0
SOLUTION    9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.

KOUNT = 15 AT ITERATION  40
GAMMA =  1.2 BETA =  0.5 VCOST =  1.0 FCOST =  1.0
SOLUTION    0.  0.  0. 74.  0.  0.  0.  0.  0.  0.  0. 74.  0.  0.  0.

KOUNT = 15 AT ITERATION  31
GAMMA =  1.2 BETA =  1.0 VCOST =  1.0 FCOST =  1.0
SOLUTION    0. 33.  0.  0.  0. 40.  0.  0.  0. 40.  0.  0.  0. 33.  0.

KOUNT = 15 AT ITERATION  44
GAMMA =  1.2 BETA =  2.0 VCOST =  1.0 FCOST =  1.0
SOLUTION    0. 29.  0.  0. 25.  0. 18.  0. 18.  0. 25.  0.  0. 29.  0.

KOUNT = 15 AT ITERATION   3
GAMMA =  1.2 BETA =  5.0 VCOST =  1.0 FCOST =  1.0
SOLUTION    9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.  9.
```

FIGURE 8A: Stability under competition for two-firm case

GAMMA = 1.1 BETA = 0.5 VCOST = 1.0 FCOST = 1.0

ITERATION															
1	0	0	0	74	0	0	0	0	0	0	0	74	0	0	0
2	0	0	0	0	0	74	0	0	0	74	0	0	0	0	0
3	0	0	0	0	0	0	0	148	0	0	0	0	0	0	0

FIGURE 8B: Instability under competition with four firms

ITERATION															
1	0	0	0	33	0	40	0	0	0	40	0	33	0	0	0
2	0	0	0	73	0	0	0	0	0	0	0	73	0	0	0
3	0	0	0	0	40	33	0	0	0	33	40	0	0	0	0
4	0	0	0	28	45	0	0	0	0	0	45	28	0	0	0
5	0	0	0	73	0	0	0	0	0	0	0	73	0	0	0
6	0	0	0	44	0	0	29	0	29	0	0	44	0	0	0
7	0	0	23	50	0	0	0	0	0	0	0	50	23	0	0
8	0	0	0	52	0	0	0	42	0	0	0	52	0	0	0
9	0	18	0	0	0	0	55	0	55	0	0	0	0	18	0
10	0	31	0	42	0	0	0	0	0	0	0	42	0	31	0
11	0	0	0	49	0	0	0	48	0	0	0	49	0	0	0
12	0	19	0	0	0	54	0	0	0	54	0	0	0	19	0
13	0	27	0	46	0	0	0	0	0	0	0	46	0	27	0
14	0	0	0	51	0	0	0	44	0	0	0	51	0	0	0
15	0	18	0	0	0	55	0	0	0	55	0	0	0	18	0
16	0	27	0	46	0	0	0	0	0	0	0	46	0	27	0
17	0	0	0	52	0	0	0	43	0	0	0	52	0	0	0
18	0	18	0	0	0	55	0	0	0	55	0	0	0	18	0
19	0	27	0	46	0	0	0	0	0	0	0	46	0	27	0
20	0	0	0	52	0	0	0	43	0	0	0	52	0	0	0