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MODELLING THE URBAN LABOUR MARKET: THEORETICAL AND PRACTICAL ISSUES.

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1. INTRODUCTION

In general discussion of model design, various topics continually appear as factors to be considered: the level of aggregation to be adopted, data availability for example. In the following, four of these topics shall be discussed as they affect the design of a model of an urban labour market. These mutually interdependent factors are representation, aggregation, data requirements and the theory underpinning the model.

Before undertaking this exercise it is worthwhile to pose the question: why is this model to be created? The usual answers to this question depend upon the intent of the modeller but range from, to "validate or refute theory" to "to forecast future events on the basis of past behaviour". It is hypothesised here that a model useful for the latter should be able to provide theoretical insights also but this may not necessarily hold in reverse. It will be seen during the course of the following discussion that the factors involved in model design will affect the usefulness of a model in any particular direction. It is likely that the assumptions required to create a model able to answer particular questions make it unsuitable to answer competently other questions. The model to be developed here is designed to retain as much information as possible about the individual actors in the labour market, and to examine the impact of labour market policies upon these individuals. It has two advantages. Firstly the model can then be built upon theories of individual behaviour and via the information retained on each person can encompass the large amount of heterogeneity found in the population. Secondly, the wealth of data available in the model at the individual level allows a more complete investigation of the impact of different policies to be made than in a more aggregate model which deals with group totals.

A description of the representation of such a model and the result of aggregation upon this representation is developed in the third section, preceeding this a description of the labour market seen as a set of state transitions is given in Section Two. Having obtained the representation, the interface between the model and theoretical statements will become obvious. In Section Four a discussion of presently available theory and the usefulness of it to the present model is undertaken. This will lead to a theoretically adequate model but in order to be implementable the data required for such a model is needed, if data is not available, aggregation must be undertaken until a level at which the data is available is reached. This may lead to a model which is theoretically on less solid ground at the micro level. The impact of such an eventuality is addressed in Section Five. Finally some comments on the task of model design are offered and in particular the impact on the model described herein of the issues discussed.

2. THE LABOUR MARKET AS A SET OF STATE TRANSITIONS

One major effect of the development and application of systems theory (see von Bertalanffy (1968) for an early statement of this approach) is the realisation that entities are not independent of each other and that strong causal chains can exist between them either as individuals or as groups. Prime examples of such chains can be identified in the labour market although it did not need systems theory to realise their observation. Systems theory does however provide a useful and efficient means of studying such chains and a major tool of systems theory is the flow chart. This identifies the direct effects observable in the system of interest and helps to then identify the indirect effects which are often of great importance. They are less easy to predict intuitively, and thus are less expected when they occur.

Here the usefulness of the flow chart is not that of predicting qualitative changes but that it identifies a series of state transitions which are present in the labour market. The relationship between the idea of state transitions and general system theory is close as the former provides a logical and formal basis for the latter as developed by Mesarovic and Takehara (1975). This relationship is observed in the reverse direction here as the model to be developed from the flow chart is of the state transition type and could be formulated as a formal model in the way described by Mesaroric and Takehara.

Holt and David (1966) present a scheme which can be taken as a useful starting point for the definition of a flow chart of the labour market. In describing the labour market in this way they saw it as

> "a complex stochastic process that involves interaction among many participants. Their actions are governed by fairly complex

> relationships arising from the objectives and constraints of firms and families. Thus, the labour market is not considered in terms of such relatively simple constructs as supply and demand schedules" (p.74).

By taking such a view Holt and David are implicitly appealing to systems theory as an aid to understanding and then to a state transition representaiton in order to formalise their model.

It is preferable here to define the flow chart corresponding to the model to be developed later in the paper rather than give Holt and David's presentation. The close comparisons will however be obvious. The labour market is considered as a usual economic market and is seen to comprise three fundamental sectors: the supply, the demand, and the market (or allocation) mechanism. These three central sectors are each influenced by external events, and in particular by one type of event. The supply pool is under the direct and important influence of demographic change. These changes act through the medium of the household and affect the individuals in that household. This is in immediate agreement with the Holt and David viewpoint where the objectives and constraints of the family determine the actions of the constituent individuals in the labour market. In a likewise fashion the demand pool is directly affected by the overall economic condition which, via the firms in the economy, will determine the level of labour demand. Thirdly, the allocation mechanism which allows supply and demand to interact is affected by the information network in the system and the ability of supply and/or demand pools to be mobile in order to achieve a mutually suitable allocation. In reality of course the relatively immobile nature of employment demand requires that the suppliers of labour should be mobile and not the demanders.

Having given a broad overview of the conceptual framework it is now appropriate to put some flesh on these bones and give a detailed description of the flowchart in figure 1. In this diagram the supply pool is comprised of a set of individuals {I}, and the demand pool of a set of jobs {J}. As noted the individuals are all members of a family which forms a household {H,I}. Similarly each job belongs to a family which together form a firm or sub-unit of a firm, {F,J}. The allocation mechanism, represented by the shaded area A, interfaces the supply and demand pools and yeilds the allocation of individuals to jobs {H,I,J,F}. The family is subject to demographic changes which are seen as demographic processes imposing themselves upon the household and again the firm is subject to similar processes acting from the macro-economic environment. The allocation mechanism is dependent upon the ways in which information can flow between supply and demand and this is considered as exogenous to the allocation mechanism.

To fully specify such a scheme it is useful to formalise the flow diagrams of figures 1 and 2 in a system of accounts. These will be of a similar nature to the system of spatial demographic accounts of Rees and Wilson (1977). The accounts describe the liklihood of transitions within a system between time t and t + 1. They are formed as a matrix $X = \{x_{k\ell}\}$ where $x_{k\ell}$ is the probability of an individual in state k at time t being in state l at time t + 1. Then if $S_k(t)$ is the number of individuals in state k at time t, $S_k(t)x_{k\ell}$ is the number of individuals moving between states k and ℓ between time t and t + 1.

It is convenient to segment the states into three classes to generate the simplest possible set of accounts. These classes are comprised of those people in education, those in the labour force and those in neither. This classification enables every individual to be placed in a unique class. A matrix is formed with this classification on each set of rows and columns, see figure 3. The cells of the matrix contain the probabilities of transition between the different classes. A simple example of this accounting system is given in figure 3. The definition of the matrix demands that the rows sum to 1.0 as any individual in the system must remain in the system. By making certain assumptions certain cells of the matrix may be set to zero. For example in figure 3 it is assumed that no one can enter education once they have left it.

As noted above this 3 x 3 matrix is the simplest set of accounts which can be used to model the labour market. Any more complicated accounts are obtained by disaggregating the classes defined above. For example the level of education may be defined to be school, college or university, the type of employment defined by industry, occupation and wage level can be given, retirement and death may be different ways in which the individual is not in the labour force or education. A further way of disaggregating the accounts is to include different types of person in each category. Thus age, sex, marital status may all be included. An example of such a framework, not fully specified, is given in figure 4. It will be seen that this simply is an extension of the simpler scheme in figure 3.

A concern of the type of models to be discussed in the later sections is with the different types of individual and firm found in the labour system. To include individual and firm attributes in such a system of accounts will lead to many different states and it is convenient to form a general representation for the different states within the system. This will also allow the correspondence between the system of accounts and the earlier description of the labour market transitions to become apparent. Each state is dependent upon the state of its elements, the individuals, the households, the firms and the jobs, that is the system state is the quadruple {H,I,J,F}. Each of these four components is itself a set of smaller units and so can be written alternatively as a full set of single items. For example H can be decomposed into the separate households within the system. A result of considering the components of H, I, J and F is that the system is now defined at the level of decision-making units. A household is the level at which housing and budgeting decisions are made, an individual is concerned with personal issues, a job (in what may be seen as a slightly contrived interpretation) can decide the level of output from a machine, and a firm is concerned with more aggregate issues of production and marketing. This level of resolution is of value as will be shown in the next section.

A further outcome of this individual level representation is that it is possible to differentiate between different units of the same type. For example households will differ in many ways of which a few will affect the labour market processes. Thus each element of H, I, J and F can be seen as being represented by a sequence of identifying attributes. A household can be represented by size and tenure of dwelling, firms by location and ownership, an individual by age and sex and a job by occupation and industry for example. In the model of Clarke, Keys and Williams (1979) such a vector of attributes contained, amongst other variables, the following string of labour market attributes,

{size of household, tenure of house, age, sex, education, marital status, labour market status, wage}.

In that model the "labour market status" variable was defined to include "in education", "unemployed", "retired" and "not in the labour force" as well as various employment types.

Each of the arrows in figure 1 and the non-zero entries in figures 3 and 4, denote a causal link between two classes of the system. What is not specified here is the exact cause and effect of the link, this is developed in the next section. In this section only the types of link will be specified as the more formal statement is best made after the formal state transition representation has been made. Firstly then, consider the entries to the supply pool. These are of two types: from the household and from the set of workers in work. The former are comprised of school-leavers, secondary workers re-entering the labour market, immigrants to the area of interest who have no job. The latter are workers who wish to or are forced to seek a job different from their present one. Movements of this type may be due to voluntary change, or change forced by the employer through firing or redundancy. In this case the movements are a result of the firms action to remove the job with which that individual is associated. The entries to the demand pool are also of two types, originating either from the firm creating new jobs (as the household created new workers) or by the removal of the individual from the worker-job association. This latter type is caused by changes originating in the household such as retirement, death or the desire to change job. Finally the allocation A adjoins individuals and jobs in some way to yield a new set of workers in work {H,I,J,F}. The probabilities corresponding to such an allocation are those which are given in the accounts.

Although all these effects are taking place simultaneously they do not affect any one person or job simultaneously but sequentially. In order to model the effects of any one action, in the household say, several steps need to be considered and this requires a dynamic representation and leads ultimately to a dynamic model.

In this section the qualitative framework of a model of the labour market has been presented and a typology of the various transitions present in that system given. In the next section a formal representation of such transitions is developed and this allows two important areas to be discussed, aggregation and the interface between models and theory. These are partially developed in the next section, but treated more fully later.

3. REPRESENTATION AND SOLUTION METHODS.

In the set of accounts derived above the classification of states in the system was seen as being described by the combination of individual, household, job and firm units {H,I,J,F}. Further, an individual may be identified by age and sex, a household by the size and tenure of their dwelling, a job by an occupation and industry label and a firm by location and type of ownership. It is seen that very rapidly the number of possible states reaches a large number. The greater the number of identifying attributes the more heterogeneity of the actual system can be modelled. An individual, job, household or firm will have an infinite set of attributes which will identify that entity uniquely. The problem facing the modeller of these entities is to identify those attributes which contribute to the effects that are being modelled. This is related to the problem of data availability and if a desired attribute does not appear in the data a proxy attribute may be required.

The set of accounts can be used to generate a dynamic model based upon matrix multiplication. Thus if $\underline{s}(t)$ is a vector of the number of occurances of each state at time t, m is a transition matrix giving the probability of state transitions, then

$$\underline{s}(t+1) = m\underline{s}(t). \qquad (3.1)$$

For a problem with many classes, and hence many attributes, the matrix m contains many zero or small values. This means that the matrix multiplication may prove to be expensive computationally. There are also theoretical problems associated with the use of a matrix such as m for labour market problems. A fundamental assumption made is that the liklihood of any transition is assumed to be constant over time. This does not allow such a model to take into account the effect upon transitions of changes in policy which will effect these probabilities. A second point is that no explicit modelling of the allocation mechanism is made. This may be affected by changing policy and hence requires a more explicit representation. Also the changing levels of supply and demand will affect the allocation at any one time. The basic requriements to overcome these theoretical objections is to allow the elements of the accounts matrix to become time-dependent and to explicitly model the supply/demand interactions.

One way of allowing such requirements to be met is to alter the representation of the model. As was noted above the use of many attributes for each of the four system elements, individuals, households, firms and jobs, generates a large transition matrix m. An alternative representation is to represent each individual, household, firm and job explicitly together with their particular attributes. For example $I_t^{i_1 \ldots i_m}$ could be an individual in the system at time t. The attributes of this individual, are given by the values of the indices $i_1 \ldots i_m$. In addition to those previously indicated, it will be necessary to number the individual and indicate which household, job and firm they belong to. Similarly $H_t^{h_1 \ldots h_n}$ will be a household and its attributes, together with a list of the individuals which comprise it. A completely analogous representation can be given for jobs $J_t^{j_1 \ldots j_k}$ and firms $F_t^{f_1 \ldots f_\ell}$.

In this new representation individual actors are identified, this means that individual actions can be incorporated directly. This has the result that any theory introduced must be specified at the individual level. This is seen as being of importance if the model is to have a sound theoretical basis. It also means that the simple matrix multiplication used in (3.1) to generate forecasts of future system states cannot be used. Instead it becomes necessary to model each individual unit's transition separately.

As noted in Section two the constituent actors in the system are identified at the level at which decisions are made. They are therefore correctly specified to allow such transitions to be modelled. The outcome of each transition, which can be related to an underlying problem in decision-theory, is that the actor is associated with a different set of attributes. The decision made by an actor is determined by that actor's preferences and the range of options open. Such problems can be formalised, as in Williams and Ortuzar (1979) and Clarke et.al. (1979) as a consideration of the set, here given for individuals,

$$\{I_t^{i_1-i_m}, \underline{O}, \underline{\psi}, \underline{Q}\} \tag{3.2}$$

where

$I_t^{i_1-i_m}$ is the individual,

$\underline{O}$ is a set of options open to that individual,

$\underline{\psi}$ is a set of constraints which act upon the individual,

$\underline{Q}$ is the set of objectives by which the individual decides which option to take.

If a probabilistic choice theory is taken to be valid in this case and if the options can be described by the attributes which are already describing the individual then an outcome of the above decision problem can be given as a probability of moving between different attribute sets for the individual. Let $P(I_t^{i_1 \ldots i_m} \rightarrow I_t^{i'_1 \ldots i'_m})$ is the probability of an individual at time t with attributes i_1-i_m being described by attributes

$i_1'-i_m'$ at time t'. Then the probability is a function of $\underline{O}$, $\underline{\psi}$ and $\underline{Q}$ as in (3.2) and can be written,

$$P(I_t^{i_1 \ldots i_m} \rightarrow I_t^{i_1' \ldots i_m'}) = f(I_t^{i_1 \ldots i_m}, \underline{O}, \underline{\psi}, \underline{Q}). \qquad (3.3)$$

The function f depends upon the nature of the choice theory used to generate the model. It may for example be a multinomial logit model if a choice theory based upon random utility theory is invoked (McFadden, 1974). Similar equations to (3.2) and (3.3) can be written for decisions made at the household, firm or job level.

In some circumstances the constraint set $\underline{\psi}$ and the set of options $\underline{O}$ act so as to make the set of feasible options small. This may make the choice problem trivial. This is the case in the labour market, for example, where the educational qualifications of an individual makes them unsuitable for many jobs. Then the feasible set of possible jobs is reduced to a small subset of all jobs and the individual may be forced to consider only one type of job if the local economy does not provide all of them.

The constraint set $\underline{\psi}$ and the set of options $\underline{O}$ are functions of the factors external to the decision-making unit. This is seen from figure 2 where the variables $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\gamma}$ which are external to the labour market system affect the transitions. Here this is reflected in the effect they have upon the solutions of the choice models. The variables in $\underline{\alpha}$, $\underline{\beta}$ and $\underline{\gamma}$ may also be affected by the policy adopted by various authorities. Thus they are also the mechanism by which policy interfaces with the model and in this representation this interface is explicitly at the decision-making level.

An advantage allowed by solving the system equation (3.1) in this disaggregate fashion is that each decision problem can be solved separately and hence the assumption of constant transition probabilities need not be made. In particular the interaction between various parameters, such as supply and demand for jobs, can be introduced into the decisions made by individuals. A second advantage is that it now becomes possible to model the allocation between supply and demand explicitly. The adoption of the new representation has the effect that the major theoretical disadvantages of the transition matrix method has been overcome.

It is interesting to consider how the above representation of individual actors can be aggregated to give different model representations. The system at its most disaggregate is described by the set,

$$\{(H_t^{h_1 \ldots h_n}),\ (I_t^{i_1 \ldots i_m}),\ (J_t^{\gamma_1 \ldots \gamma_k}),\ (F_t^{f_1 \ldots f_\ell})\}. \tag{3.4}$$

A more aggregate representation involves the removal of any dependence upon a particular set of attributes. Thus if a model in which the type of firm was irrelevant is to be generated the above representation will be aggregated over all firm attributes $f_1 \ldots f_\ell$. This will result in the following set,

$$\{(H_t^{h_1 \ldots h_n}),\ (I_t^{i_1 \ldots i_m}),\ (J_t^{\dot{\gamma}_1 \ldots \dot{\gamma}_k})\} \tag{3.5}$$

where all dependency upon any $f_1 \ldots f_\ell$ is removed. A simple relationship between (3.4) and (3.5) is obtainable,

$$\{(H_t^{h_1 \ldots h_n}),\ (I_t^{i_1 \ldots i_m}),\ (J_t^{\dot{\gamma}_1 \ldots \dot{\gamma}_k}) = \bigcup_{f_1 \ldots f_\ell} \{(H_t^{h_1 \ldots h_n}),\ (I_t^{i_1 \ldots i_m}) \\ (J_t^{\dot{\gamma}_1 \ldots \dot{\gamma}_k}),\ (F_t^{f_1 \ldots f_\ell})\} \tag{3.6}$$

where $\bigcup_{f_1 \ldots f_\ell}$ is the operation of set addition (or union) and is equivalent to removing all dependency upon the attributes which it takes place over.

Now consider a method by which state transitions can be modelled in the above framework. Such a method is Monte-Carlo sampling in which a random number between 0 and 1 is compared with a probability of making a transition. If the random number exceeds the probability the transition is not made, that is the new state is the same as the old, otherwise the transition occurs and a new state representation is obtained for the individual actor concerned. Such a solution method is referred to as list-processing (see Clarke, Keys and Williams, 1979).

Let the level of aggregation be such that there are e possible system states and let c and d be particular states. Let the choice used (3.3) give a probability of moving from state c to state d be P_{cd}. Let $T_c(t)$ be the number of actors in state c at time t. Then, the expected number of moves from state c to state d is $T_c(t)$, P_{cd} and,

$$T_c(t+1) = T_c(t) + \sum_{d=1}^{k\ell} T_d(t)P_{dc} - \sum_{d=1}^{e} T_c(t)P_{cd}. \qquad (3.7)$$

Thus, if a household decision was being considered here, the number of households in state c at time t+1 is equal to the net change from the number at time t. The overall system behaviour is made up of a variety of such decisions by different decision-making units. The transition matrix m in (3.1) is a summary of all the separate decisions and the vector s(t) is a summary of the system states $T_c(t)$ for all types of decision-making unit.

In this section a discussion of the formal nature of model representations has been made. In particular a representation at the smallest unit level for the labour market has been presented. The introduction of such a representation has several advantages for a modelling exercise. Firstly, it allows a decision model to be used for each decision-making unit which is more flexible than a transition matrix approach. In particular it enables the effects of endogenously determined variables to be explicitly considered within the model. Secondly it enables theory to be directly interfaced with the model and that theory needs to be specified at the level of the individual decision making unit. Thirdly, because of the way the model is solved, by considering individual actors, it is possible to generate supply and demand pools for the allocation and explicitly solve the allocation problem. This is of considerable theoretical interest as it allows the effects of the allocation mechanism to feedback to the household budget and decision making contexts. Finally because the transition probabilities are variable and not pre-specified it is possible to introduce parameters into the model which represent the policy of various government agencies. This will increase the ability of the model to address policy impact questions and widen its range of applications.

4. THEORETICAL UNDERPINNINGS FOR THE LABOUR MARKET MODEL.

In order to be able to understand the model response to parameter changes and the natural dynamics of the model it is necessary to understand the theoretical statements upon which the model is built. In this section each transition, or class of transitions, which can occur in the model will be discussed in relation to this desire. It will be seen that suitable micro-level theories can be found for each transition, in the next section the implementation of these theories will be discussed and it will there be seen that for reasons of data availability some aggregation must be made. Also in this section similar models to that proposed here will be discussed in order to examine the theoretical underpinnings that have been used in them. To enable a systematic discussion to take place the transitions are classified as supply-side, based on household decisions, demand-side, based on the firm, and the market allocation.

Firstly consider how these transitions relate to the accounts described in Section two. The supply-side transitions relate to those decisions made in the household concerning the participation of individuals in the labour force. Similarly the demand-side transitions relate to firm-based decisions to increase or decrease the number of jobs they make available on the market. These transitions will generate entry to and withdrawal from the labour market. In particular they will lead to a pool of workers looking for jobs and a pool of jobs requiring workers. The market allocation transitions will allocate individual workers to individual jobs. In the attribute string representation this sequence of decision making contexts can be explicitly represented. Each unit first enters the supply or demand part and then is allocated. In the accounts framework this pair of decisions is represented purely as a single probability, the probability of a person or job changing their state. As the state includes the characteristics of a job and an individual the dual nature of the decision is subsumed within a single probability and this does not allow the separate consideration of the two decision contexts or the effects of different market conditions to be taken into account. Therefore the discussion to follow is directly related to the attribute string representation method. It is split into three sections, supply-side transitions, demand-side transitions (both as defined above) and the market allocation.

Supply-side transitions.

Here three transitions are relevant, school-leaving, retirement, and the re-entry to the labour force of secondary workers (typically married women). An implicit assumption made here is that the primary worker in any household is always in the labour force and hence the discussion here is mainly to do with the entry into the labour force of workers previously out of the labour force. The exception, retirement, is, in Britain, for the majority automatic at a certain age, 65 for men and 60 for women. The increase in voluntary redundancy with the provision of a "golden handshake" and no loss in any pension rights may lead to an increase in the number of early retirements and this may then necessitate a formal theory to determine which workers are more susceptible to this type of transition. As it is presently only an option for the minority and since some of this minority still remain in the labour force until the legal retirement age, the topic will not be discussed further than to note the work of Rones (1978) in the U.S.A.

It remains to discuss firstly the decision to leave school and enter the labour force. This decision has been studied since the early 1960s within the framework of human capital theory (Becker, 1964). In the theory the individual and his family are considered to invest in an individuals formal education in order to achieve higher income once the individual is in the labour market. It is an investment because the individual and family will lose earnings that may have been received during the extra time at school and college and may also need to support the person financially when they may be out at work and supporting themselves. This immediately implies that the ability of a household to be independent of the extra earnings is of great importance in the decision to stay in education or not. Apart from this the ability of an individual to remain in education is vital and is independent of the financial state of the household.

In the choice model framework set out in the previous section the set of constraints depends upon the individuals innate ability to remain in education and the households ability to do without the income that may be earned during the spell in education. The model (3.3) should therefore include directly or indirectly some measure of these variables. The individuals educational ability is an unobservable and unmeasurable quantity and as such is accounted for in the model by random behaviour. In the micro-simulation approach here adopted this random behaviour is modelled by the Monte-Carlo sampling technique and it is argued that this is a suitable method of generating behaviour which can be seen as originating from these unobservable variables. The choice models need only concern themselves with the observable variables and in this case the model is of the form

$$P(\text{leaving school and entering labour force}) = P(\text{households ability to forego potential earnings}). \tag{4.1}$$

The model of Orcutt et.al. (1976) took the right hand side of (4.1) to be a function of parent's education alone basing this on empirical work in the U.S. which showed that parental education and occupation are of greater importance than income.

Now consider the decision facing secondary workers as to whether to work or not. The early work in the area was done in the U.S. by Mincer (1964) but since then work has been performed in the U.K. recently by McNabb (1977) and Greenhalgh (1977). The decision of to work or not is usually considered to be made as a function of household needs and not of individual needs. To encompass this Mincer modified the usual neo-classical dichotomous trade-off between work and leisure to become a trichotomous trade-off between work for money, unpaid work in the house and leisure. Following this theoretical statement Mincer then considers the trade-off between the foregone earnings of the secondary worker against the household budgetry needs expressed as a function of household size and structure. This purely economic analysis failed to take into account the level of supply and demand which may exert an effect upon the supply decision. Two such effects have been hypothesised, the so-called added worker effect and the discouraged worker effect which are in conflict and to some extent cancel each other out. The former suggests that in times of high unemployment more secondary workers will seek work because the chances of the primary worker being unemployed are high. The latter suggests that in similar circumstances the secondary workers will be deterred from seeking work because they perceive that the chances of finding work are low. Studies have consistently shown that the "discouraged worker" effect is stronger the the "added worker" effect both in the U.S. (Tella, 1964, 1965 and Dernburg and Strand, 1966) and in the U.K. (McNabb, 1977, and Greenhalgh, 1977). The decision model to be considered here is of the typical form,

$$P(\text{secondary worker working}) = P(\text{supply and demand in the labour market, household structure, earning potential of secondary worker}). \quad (4.2)$$

Orcutt et.al. (1976) consider the above effects but do not have any measure of the husbands employment state directly but include this indirectly by the level of benefit payments and income the previous year.

Demand-side Transitions.

Due to the symmetry of the labour market transition the two flows relevant here correspond closely to those in the supply side. The transitions are the creation of new jobs, and the removal of old jobs causing redundancies corresponding to entry and withdrawal from the labour force of jobs. The two transitions can be seen as results of the same decision process, that of determining the level of labour required by a firm or production unit and the type of labour required. A decrease in requirements leading to redundancy and an increase to the creation of new jobs.

Studies of such decisions date back to the late 1950s and the early theories have been subject to a continual process of revision since that time. In all the studies the problem is considered as that of determining the optimal labour input to produce a given amount of goods subject to a fully specified production process. As such it falls neatly into the paradigm expressed by (3.1) and objective under which the decision is made is that of minimising costs, (Brechling, 1965 and Ball and St.Cyr, 1965 in U.K., Dhrymes, 1969, Solow, 1968, in the U.S.). The analytic solution to this problem is then framed as a regression equation and estimated.

The analysis concentrated initially upon labour demand as being the only input to a production process which is able to respond in the short-run to changes in output level. It has now been extended to allow changes in capital input to also be made in the short-run (Nadiri and Rosen, 1969, 1973). Recent surveys of the many studies have been provided by Killingsworth (1970), Hazeldine (1976) and Briscoe and Peel (1975) and it is not the purpose to fully describe the differences between models here. It will suffice to note that, following Hazeldine, two classes of models can be identified. In one the labour input determined purely as the number of men and/or women employed. This is the earlier approach to the problem and Fair (1969) was the first to modify this type of model to allow for labour hoarding. In his model the number of workers and the amount of hours worked is estimated separately in a two-equation model. This development led the way for Nadiri and Rosen to extend the models to a more complete specification. Having determined the level of labour input at any time the change in demand from the previous time will then indicate the probability of a new job being formed or a redundancy of a certain type of worker occuring. This can be described by,

P(redundancy, of a given type of worker or new job being formed of a given type)

= P(output of the firm, wage rate, the production process used). (4.3)

Previous models of this type of decision are due to Kain et.al. (1976), Orcutt (1976) and Wishart (1976). The NBER model (Kain et.al., 1976) uses a trend forecast based on the changes between 1961 and 1971 in Chicago and Pittsburgh, the areas to which the model is applied. Orcutt et.al. have built into their model a macro-economic model from which the forecasts of labour, income, wage rates and unemployment level are obtained, but they do not produce a level of labour demand. The MANSIM model (Wishart, 1976) is different in its intention to the large scale urban and national models of Kain and Orcutt, as it models the labour market in a specific organisation, the Civil Service. As such the demand is provided by the users, and hence there is little need to model demand in the same manner as the larger scale models do.

The Market Allocation.

In this sub-section apart from the market itself, or the allocation mechanism, two other transitions need to be considered. These relate closely to the state of the market and hence are discussed here. They are the firing of workers by employers and the voluntary quitting of a job by a worker. The former of these depends upon the performance of the worker and the expected performance of that worker by the management. The ease with which a replacement worker can be found will affect the liklihood of firing and hence this transition can be defined,

P(a worker being fired) = P(level of unemployment and vacancies, workers expected and actual performance). (4.4)

Similarly the liklihood of a worker quitting is dependent upon the liklihood of their finding a new job. A worker will quit if it is felt that a greater utility (either financial, psychological or both) will be found by obtaining another job. Thus,

P(a worker quitting) = P(level of unemployment and vacancies, workers perception of potential and actual utility). (4.5)

These two models can be coalesced as they have a common structure revolving around the utility which a worker can provide and that which he receives or is expected to receive from the job. If he does not provide sufficient the management will be inclined to fire him and if he provides too much for the utility he receives he may look elsewhere for a job providing a greater level of remuneration. This can be described by,

$$P(\text{a job and worker separating}) = P(\text{level of unemployment and vacancies, difference between utility provided and received}). \tag{4.6}$$

Models of turnover are included in the NBER model and the MANSIM model. MANSIM estimate the rate of leaving the Civil Service and of movement by either specifying absolute rates or a function of length of service or by specifying the expected numbers of leavers and the liklihood of a particular individual leaving, the attributes affecting these weights are not given. The NBER model uses survey data to estimate job turnover rates by race, industry, occupation and workplace zone.

The remainder of this sub-section is taken up with a discussion of the allocation mechanism, the central underlying theory of which is provided by search theories. The first statements of this theory are contained in Phelps et.al. (1971) and in this a micro-economic foundation for the empirically observed Philips curve and the U-V relation are given. The search models are one class of the wider area of optimal stopping models (Chow, et.al., 1971) as discussed by Weibull (1977). Each individual searching for a job is assumed to investigate offers which appear at random intervals. With each offer is associated a utility (usually measured as a wage) and if the searcher is prepared to accept this utility the job is taken. The level of utility at which a job is refused or not is referred to the "acceptance utility" and this is taken to decrease with time, at the start of the search it is higher than at the end. The individual's attributes will tend to determine the rate at which the acceptance utility declines. The early models have no notion of spatial effects upon the search process and the spatial distribution of supply and demand can be held to have a significant effect upon the resulting success of finding employment. Miron (1978) and David (1974) have extended the basic model to a search over space and linked up immigration to the levels of demand and supply but these models are simple and consider inter- as opposed to intra-urban imigration and search. Liddle and Roper (1979) have attempted to outline a model of employers search behaviour, an area which has been ignored up to the present, and this is important because the way in which employers notify their vacancies determines which type of workers will receive information upon them. This will then have ramifications in the distribution of unemployment in an urban area (see Keys, 1978). The general model of the allocation mechanism can then be given as,

$$\begin{aligned} &P(\text{job } j \text{ and searcher } i \text{ being allocated to each other}) \\ &\quad = P(\text{information network, expected utility of searcher,} \\ &\qquad \text{utility offered by the employer}). \qquad (4.7) \end{aligned}$$

The MANSIM model allocates individuals to jobs by estimating a liklihood of promotion as a function of seniority, age and education level. A probability of each individual receiving the promotion is then found and using Monte-Carlo sampling an individual identified for each vacancy. The NBER model on the other hand implements a heuristic model of job search on behalf of the worker over space. Firstly the searcher is allocated to a new job in the same zone, industry and occupation, failing this a search over all zones for a job in the same industry and occupation is made. If this is unsuccessful a search ensues over all industries for a job of the same occupation type. If then no job is found the worker is unemployed.

A connection between the market allocation and the quit and firing transitions is provided by search for jobs whilst remaining in employment. Models of turnover in the labour market built within a job search framework are also influenced by human capital considerations and this type of transition is therefore determined by household or firm considerations, as are the supply and demand side transitions, and by the utility provided and utility received arguments used in the allocation transition.

The above section has contained a brief review of theories which can be invoked to provide a behavioural basis for the model outlined in Section three. In order to implement these models the amount of data required and the availability of that data must be considered. If micro-data is not available, assuming that a survey is not to be made, then some aggregation must be made.

5. COMPUTATIONAL AND THEORETICAL EFFICIENCY.

In the previous two sections a complete model of the labour market has been specified and a sufficient theoretical basis established to allow it to be formulated in accord with the desire that a behavioural base exists. In order to implement such a model in a non-ideal world, computational requirements and data availability need to be considered. Although the problem of data is a large one it is not one that cannot be overcome given sufficient resources and time to enable a suitable survey to be completed. The computational requirements are limited by technology available and it is this problem which shall now be of concern.

Firstly, it is necessary to establish how the model is constrained by the available technology and hence how it may need to be refined to comply with these constraints. By technology it is meant the type of computer and associated data storage available to a model builder. Typically the constraints imposed are the time taken by the machine to solve the model equations and the amount of space available to store data. As the cost of using computers increase proportionaly with the size of the model it is advantageous to make the size of the model as small as possible whilst still retaining the desired level of information in the model. This usually results in a certain amount of aggregation over variables and functional relationships. In this section the aggregation most suited to modify the model outlined and to answer the type of questions described in Section one will be discussed. The first step in the process is to establish for which areas of the model it is least necessary to specify at the full micro-level.

It will be remembered that the intention was to create a model capable of answering questions posed regarding the supply side and allocation mechanism effects of different macro-environments. It is therefore taken that a fully specified model of the demand side is not absolutely necessary and hence it is suitable for aggregation. It is this process which is to be now discussed further. The problems of aggregation in general have been examined for many years and the problem is still not yet fully resolved. The problem here is to aggregate equations of the form (4.3) in such a way that good empirical results are found and that can be explained by a behavioural theory. The satisfactory solution of this problem revolves upon two issues, the existence of production function at the aggregate level and the assumption of homogeneous behaviour of different firms. If these two requirements are met the behavioural theory of Section three can be carried over to the aggregate level and the macro-theory then becomes a special case of the micro-theory.

Firstly consider the possible existence of aggregate production functions. Since the first derivation of the Cobb-Douglas function in the 1920s this analytic representation of the production process has been used for all levels of aggregation and has consistently given good empirical results. This is however in conflict with the simplistic conditions which must hold for aggregation to be possible (Fisher, 1969, 1971) and the theory is not therefore built upon a sound theoretical base when applied to aggregates. In order to create such a theoretical base whilst still providing an empirically testable analysis Johansen (1959, 1972) has generated a schema in which different theories are seen as relavent to different levels of aggregation. Within this schema it is shown that in certain cases a short-run macro-production function can be obtained by aggregating over the individual units. Bosworth (1976) has criticised this analysis on the grounds that perfect competition in an environment in which the intention is to maximise output is assumed. He has attempted to generalise the Johansen analysis to include different objectives than output maximisation and different types of market than that in which perfect competition exists.

The second requirement, that all firms in a particular class can be characterised by the same parameters, is now considered. The need to meet this requirement is derived from the need to estimate regression equations which model (4.3). If the parameters in this equation are defined by a vector let $\underline{\alpha}_i$ be the vector of parameters for firm i. The aggregation over all these firms will lead to an aggregate equation with parameters $\underline{\alpha}$. Theil (1954) and others have shown that if all the $\underline{\alpha}_i$'s are equal then the aggregation does not introduce any bias into the regression equation. Thus if all the firms are assumed to behave in a similar way then the aggregation can be performed.

Under certain conditions a theoretical and statistical basis for the aggregation of the demand side variables can be found. The amount of aggregation which can be performed is controlled by the assumptions made and the critical one here is the homogeneous behaviour of firms with respect to (4.3). In this function wage rates and production processes over the different firms in any one class play a central role. It is obvious that aggregation can only be realistically performed up to a level equivalent to industry classes. Within each industry class it is reasonable, but not certain, that different firms will pay workers of the same type, the same rate. As geographical location can explain a large part of the variation in wage rates over the country as a whole this assumption is increasingly valid as spatial area decreases. The concern here with urban labour markets enables the assumption of constant wage rates to be made with less apprehension than for a national or regional model. The assumption of similar production processes is less valid over an urban area than that of similar wage rates. It needs to be made however and it is this which is the assumption most open to doubt. The success of the aggregate production functions in empirical work would suggest that such an assumption does not hinder their empirical performance.

This section has described theoretical work which can be used to support aggregation over firms in the demand side. This aggregation is forced upon the modeller by computational constraints and results in a model which is both computationally efficient and theoretically sound. It is also directed at being able to answer the specific type of questions for which it was designed. It would not be as useful at answering questions for which a more detailed demand side is required.

6. CONCLUSION.

In the above sections the conflict of two requirements of an implementable model of the labour market have been discussed. The requirements, computational and theoretical, are seen as necessary rather than desirable and should be considered in all modeling exercises. In order to consider them here four areas of discussion relating to model design were identified, representation, aggregation, data availability and theoretical underpinnings of the model. Traces of each of these can be identified in each area of the model. It is also necessary to bear in mind the desired use of the model as this affects the form of the model most suited to be implemented, this is seen in Section five clearly.

The model design process, as given here, consists of defining first a framework in which the model is to be built. This then identifies the representation and the solution methods available to solve the problem in mathematical form. It is then possible to identify the relationships and causal chains in the model at the lowest level of aggregation. These should be embedded in a consistant behavioural theory. This model should satisfy the theoretical requirements and in order to satisfy the computational requirements aggregation over the micro-units is performed. This involves making assumptions about the characteristics of the micro-units involved and by aggregating from the fundamental units these are explicitly identified.

REFERENCES.

Ball, R.J. and St. Cyr, E.B.A. (1966) Short term employment functions in British manufacturing industry. Review of Economic Studies 33, pp.199-207.

Becker, G.S. (1964) Human Capital. N.B.E.R., New York.

Bertalanffy, L. Von (1973) General Systems Theory. Penguin Books, Harmondsworth.

Bosworth, D.L. (1976) Production Functions. Saxon House, Farnborough, Hants.

Brechling, F.P.R. (1965) The relationship between output and employment in British manufacturing industries. Review of Economic Studies 32. pp.187-216.

Briscoe, G. and Peel, D.A. (1975) The specification of the short-run employment function. Oxford Bulletin of Economics and Statistics 37. pp.115-142.

Chow, Y.S., Robbins, N.S. and Siegmund, D. (1971) Great Expectations: The Theory of Optimal Stopping. Houghton-Mifflin.

Clarke, M.C., Keys, P. and Williams, H.C.W.L. (1979) Household dynamics and socio-economic forecasting: a micro-simulation approach. Working Paper 257, School of Geography, University of Leeds.

David, P.A. (1974) Fortune, risk and the microeconomics of migration. In: Nations and Households in Economic Growth. Eds. P.A. David and M.W. Reder. Academic Press, New York. pp.21-38.

Dernburg, R. and Strand, K. (1964) Cyclical variation in civilian labour force participation. Review of Ecnomics and Statistics 46, pp.378-391.

Dhrymes, P.J. (1969) A model of short-run labour adjustment. In: The Brookings Model: Some Further Results, J.S. Dvesenberry et.al., North Holland, Amsterdam.

Fair, R.C. (1969) The Short-Run Demand for Workers and Hours. North Holland, Amsterdam.

Fisher, F.M. (1969) The existence of aggregate production functions. Econometrica 37. pp.553-77.

Fisher, F.M. (1971) Aggregate production functions and the explanation of wages: a simulation experiment. Review of Economics and Statistics 53. pp.305-325.

Greenhalgh, C. (1977) A labour supply function for married women in Great Britain. Economica 44. pp.249-265.

Holt, C.C. and David, M.H. (1966) The concept of vacancies in a dynamic theory of the labour market. In: Measurement and Interpretation of Job Vacancies. N.B.E.R., New York. pp.73-141.

Johansen, L. (1959) Substitution versus fixed production coefficients in the theory of economic growth: a synthesis. Econometrica 27, pp.157-76.

Johansen, L. (1972) Production Functions. Contributions to Economic Analysis 75, North Holland, Amsterdam.

Kain, J.F., Apgar, N.C. and Ginn, J.R. (1978) Simulation of the market effects of housing allowances: Vol.1: Description of the N.B.E.R. urban simulation model. Research Report, Department of City and Regional Planning, Harvard University.

Keys, P. (1978) An information theoretic approach to the labour market allocation process. Working Paper 226, School of Geography, University of Leeds.

Killingsworth, M.R. (1970) A critical survey of 'Neoclassical' models of labour. Bulletin of the Oxford University Institute of Economics and Statistics 32. pp.133-65.

Liddle, B. and Roper, B.A. (1979) Towards an economic model of recruitment technology in the urban labour market. Paper presented to the 19th European congress of the Regional Science Association, London.

McFadden, D. (1974) Conditional logit analysis of quantative choice behaviour. In: P. Zarembka Frontiers in Econometrics. Academic Press.

McNabb, R. (1977) The labour force participation of married women. The Manchester School 45. pp.221-235.

Mesarovic, M.D. and Takahara, Y. (1975) General Systems Theory: Mathematical Foundations. Academic Press, New York.

Miron, J.R. (1978) Job-search perspectives on migration behaviour. Environment and Planning A, 10. pp.519-35.

Nadiri, M.I. and Rosen, S. (1969) Inter-related factor demand functions. American Economic Review 59. pp.457-71.

Nadiri, M.I. and Rosen, S. (1973) A Disequilibrium Model of Demand for Factors of Production. N.B.E.R., New York.

Orcutt, G., Caldwell, S. and Wertheimer, R. (1976) Policy Exploration through Micro-Analytic Simulation. The Urban Institute, Washington D.C.

Phelps, E.S. et.al. (1971) Microeconomic Foundations of Employment and Inflation Theory. Macmillan, London.

Rees, P.H. and Wilson, A.G. (1977) Spatial Population Analysis. Edward Arnold, London.

Rones, P.L. (1978) Older men - the choice between work and retirement. Monthly Labour Review 101. pp.3-10.

Solow, R.M. (1968) Short-run adjustment of employment to output. In: J.N. Wolfe Value, Capital and Growth. University Press, Edinburgh.

Tella, A.J. (1964) The relation of labor force to employment. Industrial and Labour Relations Review 17. pp.454-69.

Tella, A.J. (1965) Labour force sensitivity to employment by age, sex. Industrial Relations 4. pp.69-83.

Theil, H. (1954) Linear Aggregation of Economic Relations. Contributions to Economic Analysis 7, North Holland, Amsterdam.

Weibull, J.W. (1977) A search model for micro-economic analysis - with spatial applications. Paper presented to an International Research Conference 'Spatial Interaction Theory and Planning Models', Bastad, Sweden.

Wishart, D. (1976) Manpower supply models III and IV. In Manpower Planning in the Civil Service, Ed. A.R. Smith.

Williams, H.C.W.L. and Ortuzar, J.D. (1979) Behavioural travel theories, model specification and the response error problem. Working Paper 263, School of Geography, University of Leeds.

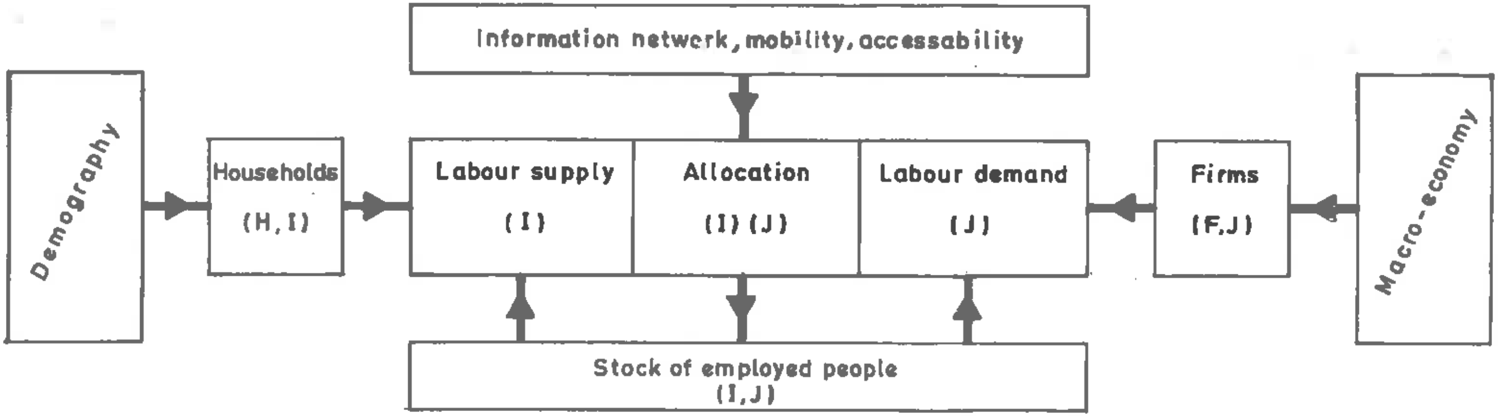

FIGURE 1.

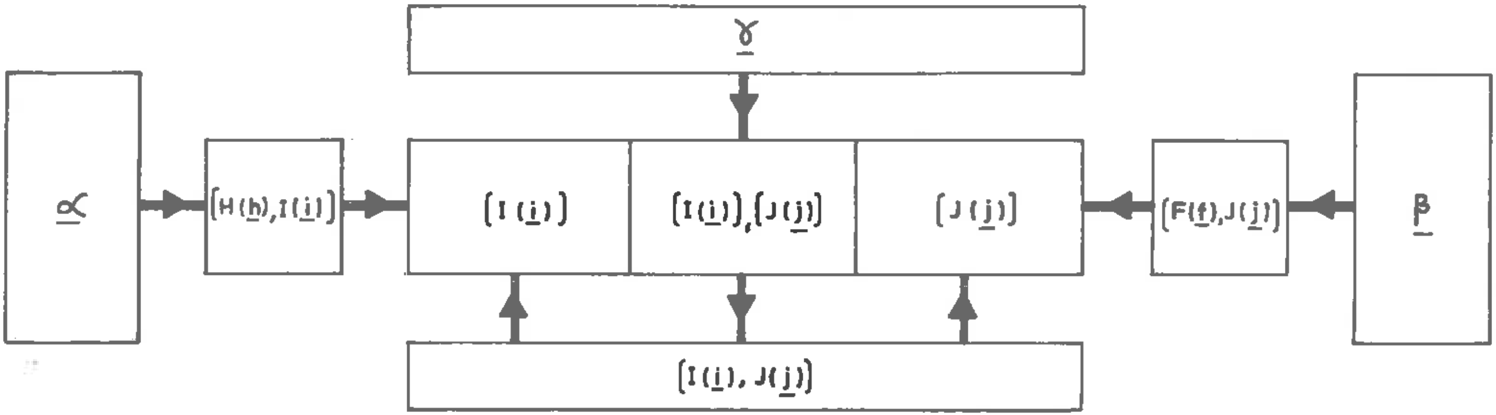

FIGURE 2.

Time t \ Time t+1	In education	In the labour force	In neither
In education	0·7	0·2	0·1
In the labour force	0·0	0·7	0·3
In neither	0·0	0·2	0·8

For example:-
Of those people in the labour force at time t 70% will be in the labour force at time t1 and 30% will not be in either the labour force or in education

FIGURE 3.

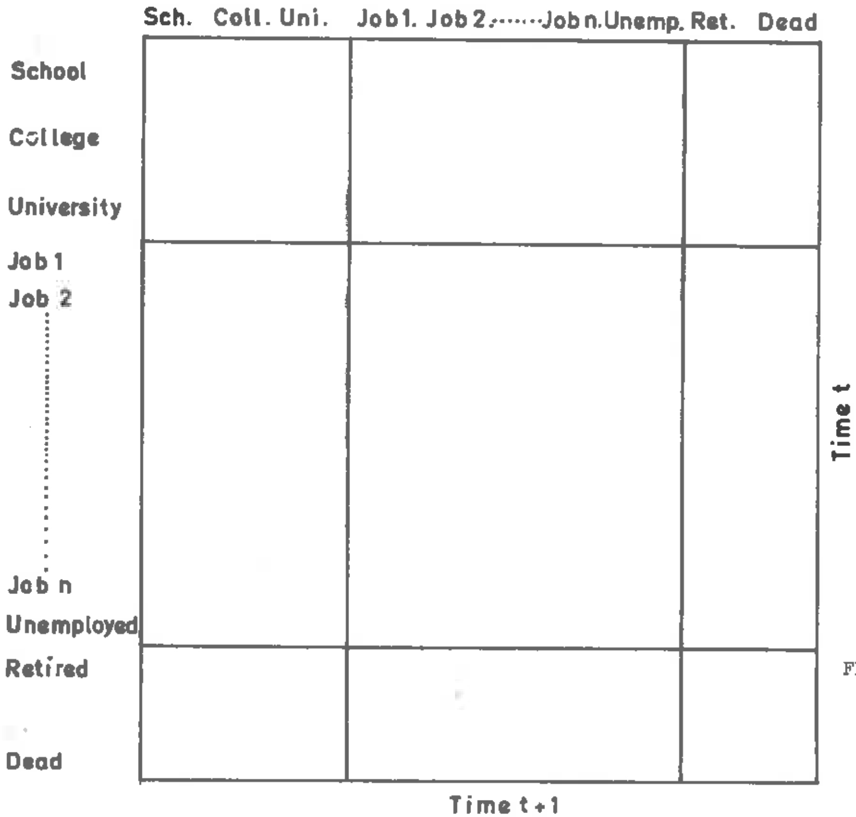

FIGURE 4.