WORKING PAPER 334

FRAMEWORKS FOR MODELLING IN RELATION TO STRATEGIC PLANNING IN A HEALTH SERVICE

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WORKING PAPER
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1. Introduction

This paper is offered as the first step in an attempt to 'transfer' some of the methods of geographical modelling and planning to corresponding problems in relation to health services*. In particular, we aim to use micro-simulation models and mathematical programming models (including location-allocation models). As a preliminary, we attempt to characterise algebraically the main features of a public health service. This enables us to examine alternative ways of modelling such a service and to formulate different planning problems which can be tackled using the corresponding models.

In this paper, we operate more or less from first principles, using our experience with geographical modelling in relation to a different kind of system. We do not, at this stage, systematically compare the approach which evolves with others in the literature. We reserve that for a future paper.

At the outset, we can argue that the system is made up of three main subsystems relating to demand, supply and allocation, as shown in Figure 1. The modelling tasks to be accomplished include the following. First, to represent the demand side in sufficient detail that the medical 'conditions' to be treated are 'visible' in sufficient detail, and in such a way that they can be related to social, economic or other characteristics of the population concerned. Similarly, the supply side has to be represented in sufficient detail that it can be matched to the demand conditions in the allocation programme. However, in each case, there will be alternative ways of representing this detail.

There are going to be obvious complications: that particular conditions can be treated in a number of different ways - in-patient or out-patient, home help or hospital care, cottage hospital or general hospital. In the next section, we begin to tackle these complexities by looking at that set of labels we need to define an appropriate set of algebraic variables. Then we look at the three major subsystems in more detail before formulating some examples of particular modelling and planning problems in subsequent sections.

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*This is part of an SSRC-sponsored project entitled: "Urban systems: models for planning and policy".

2. Algebraic notation and system representation

It is convenient for the argument to begin with the supply side. A health service is divided into a number of subservices (or services for short) for administrative and planning purposes and it is obviously important that we should build these directly into the model. A typical list is shown in Table 1. We can number these sequentially and refer to the k^{th} (say) service.

An important feature to be recognised is that a particular building, or group of buildings, will typically contain a mix of these services - that is, more than one. Since we are ultimately interested in the geography of health services, and since these buildings and mixes define the location of services, we need to build them directly into our notation and models. We therefore use the index j to label a facility, and a facility is, typically, taken to be a set of buildings at a location - and so, for the present, we also use this as a location label. (The complication to deal with at some time is that in the health service, there are facilities known as 'units' defined in this way in which the buildings are scattered across a number of locations).

We now need to define a set of (k,j)-labelled variables to represent the output and level of service j provided at k. There are obviously going to be difficulties in defining an output or a level, but we suppose for now this can be done and we call it A_{kj}. (For a particular k, it might be in-patient bed-days.) We return to the question of what this means in section 4 below. This service at a unit will need various resource inputs. We let $r_{kj\ell}$ be the ℓ^{th} such input and $c_{kj\ell}$ be the costs associated with these. One modelling task to be taken up below concerns the way resource inputs, $\{r_{kj\ell}\}$ relate to outputs $\{A_{kj}\}$ and at what costs $\{c_{kj\ell}\}$ - essentially the specification of the production function of the (k,j) service at a facility. In aggregate, we then have a representation of the production function of the health service as a whole.

We can now turn to the demand side. At the very least, we will need the number of people at a location (i, say) with a set of personal characteristics, $\underline{n}$ (say) with condition for treatment, m, which can be allocated to facilities (k,j). The total demand is therefore an array like $\{D^{i\underline{n}m}\}$, and the allocation array will be $\{T^{i\underline{n}m}_{kj}\}$ - which has five *plus* dimensions, the actual number depending on the number of elements in n. (If $\underline{n} = (n_1,n_2)$, for example, {T} is a six-dimensional array.) This argument suggests that it may be valuable at the outset to set up a micro-simulation representation. The reasons for doing this are set out elsewhere (for example, in terms of Leeds work, in Wilson and Pownall, 1976, Clarke, Keys and Williams, 1979, 1980, 1981). This will not only help with the accounting, but will also enable us to represent higher degrees of system-interdependence than would be possible otherwise.

We can then take h to label an individual and let $\underline{x}^h$ be the set of individual characteristics. We would thus have, developing the earlier notation, $\underline{x}^h = (i^h, n_1^h, n_2^h, n_3^h, \ldots)$. To this we can add the characteristics which describe the condition m. If we now treat this as a vector, $\underline{s}^{hm}$, for individual h and condition m and we can add not only the label of the condition, but (via a probability distribution) the likely demand for treatment and so on (as elements of the vector). Thus, in a micro-simulation model, we seek to develop a list $(\underline{x}^h, \underline{s}^{hm})$.

In arguing for a micro-simulation approach, we introduced the variable $T_{k\underline{j}}^{i\underline{n}m}$ and this, of course, is an allocation variable. We specified this array earlier so that the a great number of dimensions required would be visible. This array arose from a demand $D^{i\underline{n}m}$, say, being matched to a supply, A_{kj}. If a micro-simulation approach is used, then the relevant demand information will be contained in the set of vectors $\underline{s}^{hm}$, h = 1, ..., N, where N is the total population. The matching process would then add the (k,j) characteristics for each h to the $\underline{s}^{hm}$ list to make the whole vector for individual h $(\underline{x}^h, \underline{s}^{hm}, k^h, j^h)$. We discuss in section 5 below some of the different ways of making allocations to obtain either aggregate variables like $T_{k\underline{j}}^{i\underline{n}m}$ or population lists like $(\underline{x}^n, \underline{s}^{hm}, k^h, j^h)$.

We can now reverse the order of discussion again and take in turn the demand side and the supply side before progressing to allocation.

3. The demand side

We have considerable experience in generating lists of people and their characteristics which are in accord with particular models or probability distributions. These distributions are typically obtained from sources such as the census, the FES, GHS, The New Earnings Survey and so on. A typical list is shown in Table 2. The task now is to find ways of adding the $\underline{s}^{hm}$ vector. At its simplest, this could be a single variable indicating presence or absence of condition m, obtained by the usual random number procedure, based on the probability that, at a given time, a person with various characteristics would have this condition. The probability could be conditional on variables from the $\underline{x}^{h}$ list, such as age, location, occupation, and so on. (In a more sophisticated model, the probability for a condition m might also be dependent on a concurrent - or earlier - condition m'). To obtain this information, we need to convert known morbidity information such as published in the Hospital In-Patients Inquiry or the Survey of General Practitioners into probabilities of a suitable form.

We leave open at present the extent to which we can add other elements to the vector $\underline{s}^{hm}$. For example, we may wish to add, again on a probabilistic basis, a measure of the seriousness of the condition - possibly measured in terms of the 'quantity of treatment' which should ideally be supplied. (The 'supply' of this may not, of course, be available in practice). Such information would obviously form the basis of a more sophisticated allocation model. This notion, however, presupposes a known relationship between a 'condition' and the service used to treat it. Often, there will be a one-one correspondence, as between 'broken limb' and 'orthopaedic'. But if the condition is 'being elderly' for example then a range of services are available from domicilliary support via sheltered accommodation to hospitalisation (in either a medical ward or a geriatric ward). We will assume that we know, in order to make progress, the probability that condition m is treated in service k - P_{mk}, say. This is published at a national level, triennally in HIPE (The Hospital In-Patients Survey). More precisely this can be written $P(k|m)$, the probability of being treated by k given the existence of m. This also opens up the possibility of making this conditional on other factors such as age, place of residence or seriousness of condition, and we could then write it more formally as $P(k|\underline{x}^h,\underline{s}^{hm})$. A further extension would be to introduce the notion of elastic demand by introducing a factor that related the overall demand for a service with the amount of that service provided.

By summing over the population lists (and using P_{mk}) - or by constructing an alternative aggregate demand model - we can obtain aggregate measures of demand. These are various alternative aggregates of the array $D^{i\underline{n}m}$ defined earlier. We could compute D^m, total number with condition m, or $\sum_m P_{mk} D^m$, some crude measure of total demand for k. If $\underline{s}^{hm}$ contained more detailed estimates of demand, then these could be incorporated. Alternatively, we could aggregate, retaining some of the socio-economic information contained in $\underline{x}^h$. For example, we could sum to obtain D^{iam}, the number of people resident in i in age group a with condition m. This might be a useful basis for certain approximate spatial models.

4. The supply system

As noted in section 2, the supply system can be considered to be made up of a set of resource inputs $\{r_{jk\ell}\}$ by service and unit. A typical list of resources for a number of services are shown in Table 3.

It is obviously important to identify the costs of these resource inputs, and hence the array $\{c_{kj\ell}\}$ defined earlier. The crucial step, however, is to represent the way the resource inputs are combined to offer a service. There are obviously a variety of ways in which resources can be combined to offer a particular level of output and the way this is done is a matter of management decision and the technical possibilities. It is likely that in many cases there will be a hierarchical structure on the technical side of this specification of the production function and that it is the details of this which will be filled in by management decision. Consider Figure 2, for example.

For a particular service k within unit j, it is desired to offer A_{kj} bed-days of service. This needs r_{kj1} consultants, r_{kj2} junior doctors, r_{kj3} theatre days, r_{kj4} pathological services, r_{kj5} drugs, r_{kj6} nurses and r_{kj7} maintenance and miscellaneous services. The hierarchical structure shown on the figure suggests a set of functional relationships like

$$\begin{aligned} r_{kj1} &= f_1(A_{kj}, y_{kj1}) \\ r_{kj2} &= f_2(r_{kj1}) \\ r_{kj3} &= f_3(r_{kj1}) \\ r_{kj4} &= f_4(r_{kj1} + r_{kj2}) \\ r_{kj5} &= f_5(r_{jk1} + r_{kj2}) \\ r_{kj6} &= f_6(A_{kj}) \\ r_{kj7} &= f_7(A_{kj}) \end{aligned} \tag{1}$$

The total cost of the service at j is

$$C_{kj} = \sum_{\ell} r_{kj\ell} c_{kj\ell} \tag{2}$$

y_{kj1} is a management decision which describes the number of days, an average say, allocated by the (k,j) consultants to a patient in his care. Or, this can be treated as an adjustment factor above and below the mean. Either way, it will facilitate the allocation process. The remaining management decisions will be incorporated in the parameters and coefficients associated with the functions $f_1, \ldots f_7$. Let these be denoted by a vector $\underline{y}_{kj}$. Then the whole service is characterised by the following vector:

$$R_{kj} = \{A_{kj}, \underline{y}_{kj}, (r_{kj\ell}), (c_{kj\ell}), C_{kj}\} \tag{3}$$

And we have the following fomal functional relationship (as a kind of inverse of the system (1)):

$$A_{kj} = A_{kj}(\underline{y}_{kj}, \underline{r}_{kj}, \underline{c}_{kj}) \qquad (4)$$

Formally, we might now consider the task of health service planning to be the manipulation of the variables $\{r_{jk\ell}\}$ - allocation of resource inputs of different kinds to (k,j)'s and the management variables $\underline{y}_{kj}$ to produce a set of outputs $\{A_{kj}\}$ to meet demands $\Sigma p_{mk} D^m$. It is important to establish whether feasible solutions exist and to seek to adjust the resource allocations and management variables to improve the overall service. It is important even at this early stage to recognise that this can be done at several levels. Each (k,j) unit of service must be maximally efficient (via the $\underline{y}_{kj}$ variables) for example. We would then, for instance, have to consider the nature of scale economies or diseconomies related to size of facilities, j, and this generates a problem of efficient allocation of resources between facilities. Finally, we seek efficiency within services. In all cases, 'efficiency' is measured from twin perspectives: minimisation of costs for a given level of output; and secondly, satisfaction of 'demand' at least cost to the consumer.

This hierarchical pattern of supply-side planning is sketched in Figure 3, and we return to it in a later section.

As a more specific illustration of these kinds of ideas, we consider what can be done to characterise supply using standard unit-cost information which is available by region (Health Service Costing Returns) We consider two kinds of cost measures. Let A_{kj} be the output measures for the k^{th} service at j and assume that unit costs C_{kj} can be related to them. Then the total cost of a facility is

$$C_j = \sum_k C_{kj} A_{kj} \qquad (5)$$

The alternative is to break down the unit costs further. Let $c_{kj\ell}$ be the unit cost of the ℓ^{th} resource input to (k,j) and let $r_{jk\ell}(A_{kj})$ be the level of ℓ needed to produce A_{kj}. Then

$$C_j = \sum_{k\ell} c_{kj\ell} r_{kj\ell}(A_{kj}) \qquad (6)$$

This kind of approach would enable us to cost alternative output levels. It is also important to 'invert' the procedure, as noted in relation to the more general framework above. In relation to (5), this procedure would work as follows. Given a total budget C_*, allocate it to facilities, j, as C_j's such that

$$\sum_j C_j = C_* \tag{7}$$

Let y_{kj} (a management variable) be the proportion of j's resources to be devoted to k. Then

$$A_{kj} = \frac{y_{kj} C_j}{c_{kj}} \tag{8}$$

is the level of output achievable.

With the more detailed model (6), it is slightly more complicated to obtain an inverse. We can allocate C_* to obtain C_j's. Again let y_{kj} be the proportion allocated to service k at j. Then we require

$$\sum_\ell c_{kj\ell} r_{kj\ell}(A_{kj}) = y_{kj} C_j \tag{9}$$

Clearly the left hand side will vary monotonically, though not necessarily linearly, with A_{kj}, and the equation can in principle be solved for A_{kj}.

These simple notions can form the basis of a cost-accounting system which is related to outputs which will form a basis for elementary planning and budget allocation. Note also that through an array like T_{kj}^{inm}, we can allocate costs to users (and so use this to assess, if necessary, the 'equity' of current policies). In the simple model, with C_{kj} as a unit cost, if the average amount of A_{kj} consumed by one person is a_{kj}, say, then $a_{kj} C_{kj} T_{kj}^{inm}$ is the average 'cost' taken by an $(i,\underline{n},m)$ person using service k in j. $\sum_{kj} a_{kj} c_{kj} T_{kj}^{inm}$, then, is the amount of money allocated to $(i,\underline{n},m)$ people.

5. The allocation process

We consider two kinds of allocation (or 'matching') procedure - the first at the micro-simulation scale, the second at more aggregative scales.

At the micro-scale, we need a probability distribution such as $P(j|i,k)$, the probability that someone resident in i and demanding service k, will be allocated to j. This could itself be the subject of a sub-model or it could be known and decided as a matter of administrative decision. Then we can use this, and the usual random number procedure, to process each individual on the list and generate additional variables like k^h and j^h for each individual. This would allow us to compute aggregate variables of the form T_{kj}^{inm}, for example, if required.

The alternative is to allocate at the aggregate scale. Indeed, this can be considered to be a way of modelling distributions like $P(j|i,k)$. Suppose, for definiteness, we wish to estimate an allocation variable like T_{kj}^{im} (in which case $P(j|i,k) = T_{kj}^{i*}/T_{**}^{**}$, where the asterisk denotes summation). A suitable entropy-maximising model would be:

$$\underset{\{T_{kj}^{im}, W_{kj}^{im}, F_{kj}\}}{\text{Max}} \quad Z = \sum_{imkj} T_{kj}^{im} \log \frac{T_{kj}^{im}}{P_{mk} A_{kj}} \tag{10}$$

such that

$$\sum_{im} (T_{kj}^{im} + W_{kj}^{im}) = A_{kj} \tag{11}$$

and

$$\sum_{kj} (T_{kj}^{im} + F_{kj}) = D_{**}^{im} \tag{12}$$

$$W_{kj}^{im}, F_{kj} > 0 \tag{13}$$

$$\Sigma T_{kj}^{im} c_{ij} = C \tag{14}$$

Note that the W_{kj}^{im}'s are patient waiting lists and F_{kj}'s are underutilisations, and where the c_{ij}'s are patient travel costs.

At this stage, in presenting frameworks, we simply need to know that allocation models can in principle be built. In practise, the task is a very hard one and we discuss some possible methods in more detail in another paper (including the possibility of building mathematical programming models, which include allocation variables, at the inter-district scale).

6. The whole model system

The whole system is now sketched in figure 4, showing the points at which the main variables are determined. This, of course, is a very general framework. The outputs of the allocation programme and supply system model are shown on figure 4 as the inputs to various evaluation programmes. The main output of the allocation programme is an array of the form $\{T_{kj}^{im}\}$. This, together with the 'waiting lists' array, $\{W_{kj}^{im}\}$, form the main inputs to the patient-consumer evaluation programme. The other subsidiary output of the allocation programme is the set of under-utilisation rates, $\{F_{kj}\}$. These are inputs to the supply system evaluation programme together with the $\{A_{kj}\}$ vector and the other supply list arrays.

The patient evaluation will be in terms of demands being met (or when) at particular places relative to the consumers' residence. The supply system evaluation is concerned with efficiency, cost minimisation and avoidance of underutilisation. There are competing objectives here, in a sense, and these need to be combined, as shown in a joint evaluation programme.

7. Discussion: flows and degrees of control

What we have defined so far is essentially an accounting framework for an overall model system. In particular, we follow patient flows from the demand side and resource flows from the supply side (keeping track of costs and measuring service outputs).

There are many intermediate variables which can either be considered as measurable system parameters (reflecting present practise) or as management or control system parameters. Various strategies can be adopted for the use of the model and these are defined by the extent to which variables are put in the former (measurable) category or the latter (control). It seems sensible first to build a model which reflects current practice, minimising the number of controllable variables. This at least allows us to test the effectiveness of the model system as a representation. The model can then be used in two ways: by taking some variables to be controllable, but adjusting them and assessing the impacts 'manually' (which could mean interactively with the computer). For example, a hospital could be closed and its resources re-directed to other hospitals. The impact of such a change could be assessed by comparing 'before and after'. This is equivalent to using the retail sales model 'manually' to assess the impact of new allocations of shopping centres. Alternatively, we could seek optimum values of the controllable variables by using the accounting and other structural equations as constraints within an overall mathematical programming framework. In some cases, this would be an 'outer' framework within which other programmes - such as the entropy-maximising allocation programme of section 5 - could be embedded. This is equivalent to seeking optimum $\{W_j\}$ patterns of shopping centre distribution subject to consumer behaviour which is described by another (embedded) mathematical programme. One difference in this case is that there is an immense range and type of mathematical programmes which can be defined because of the different ways of choosing the control variables. And it is by no means clear in many cases how to define objective functions (or to find the weights to combine different elements of objective functions - for example, minimising supply system costs while maximising patient accessibility to facilities). The variety of mathematical programmes (and potential depth of embedding) is further complicated by the hierarchical structure of the allocations of resources: between services, between facilities, among facilities of a given type, and within facilities. Optimum hospital location problems may well be of the location-allocation type, while the higher level ones will not be. And, to make matters more difficult, the programming problems at different hierarchical levels will be interdependent.

The potential complexities are obviously interesting. However, our first aim will be to build illustrative model systems at fairly coarse levels of aggregation (for example in relation to published cost information). We also intend to develop such model systems within planning frameworks which can be used interactively from a computer terminal.

Table 1A. Typical services (coarse listing)

Acute hospital

Long-stay hospital

Geriatric

Convalescent

Maternity

Mental illness

Mental handicap

Orthopaedic

Children's

Tuberculosis and Chest

Community

Ambulance

Family Practitioner

Table 1D. Speciality Group

1.	General Medicine Dermatology Neurology Cardiology Physical Med/Rehab Rheumatology
2.	Diseases of Chest
3.	General Surgery Plastic Surgery Thoracic Surgery Dentistry
4.	Urology
5.	Ear, Nose & Throat
6.	Opthalmology
7.	Traumatic & Orthopaedic
8.	Gynaecology
9.	Paediatrics
10.	Obstetrics C.P. Maternity
11.	Special Care
12.	Geriatric
13.	GP Other Medical
*14.	Infectious Diseases
*15.	Radiotherapy
*16.	Neurosurgery
*17.	Other Special Units

*Outside area

Table 2

Individual Label	Age	Sex	Marital Status	Race	Education Status	Occupation	Wage	No of Weeks Worked	SEG
1.1	36	1	2	1	3	17	2984	52	2
1.2	34	2	2	1	2	0	0	0	4
1.3	12	1	1	1	1	0	0	0	-
2.1	64	1	3	1	4	1	8726	52	1

Table 3. Typical resource inputs

Medical staff

Nursing staff

Medical supplies

Medical services (Diagnostic departments)

Catering

Laundry

Other general services

Figure 1.

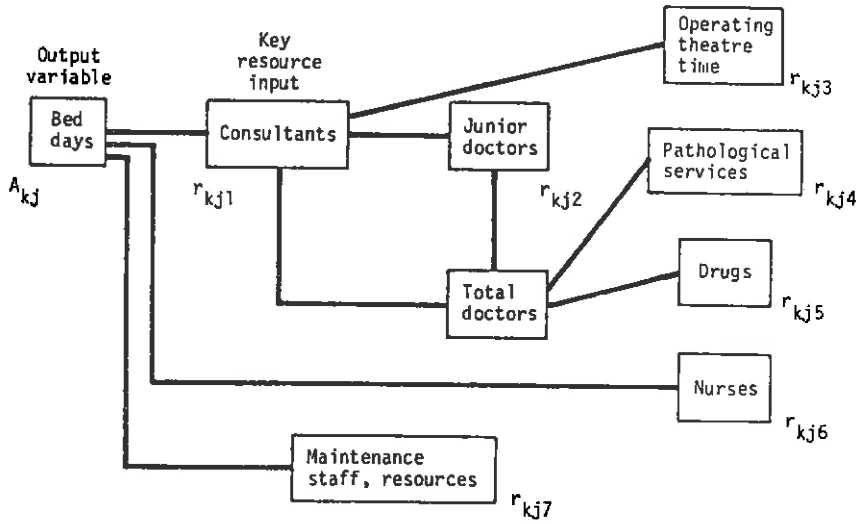

Figure 2.

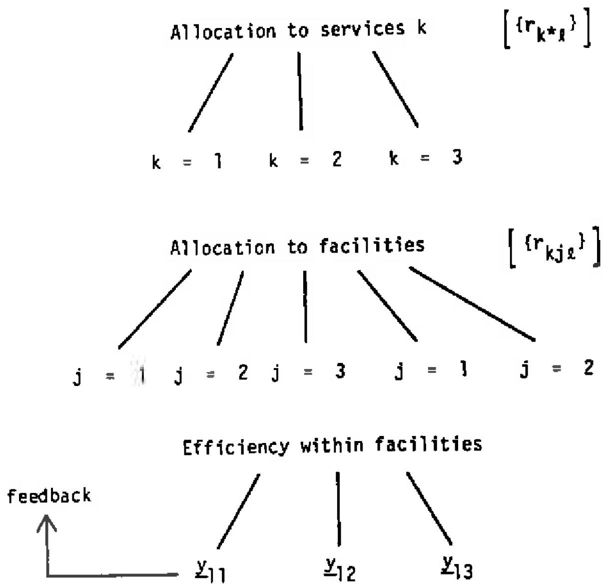
Allocation to services k
[{r k*ℓ}]
k = 1
k = 2
k = 3
Allocation to facilities
[{r kjℓ}]
j = 1
j = 2
j = 3
j = 1
j = 2
Efficiency within facilities
feedback
y11
y12
y13

Figure 3.

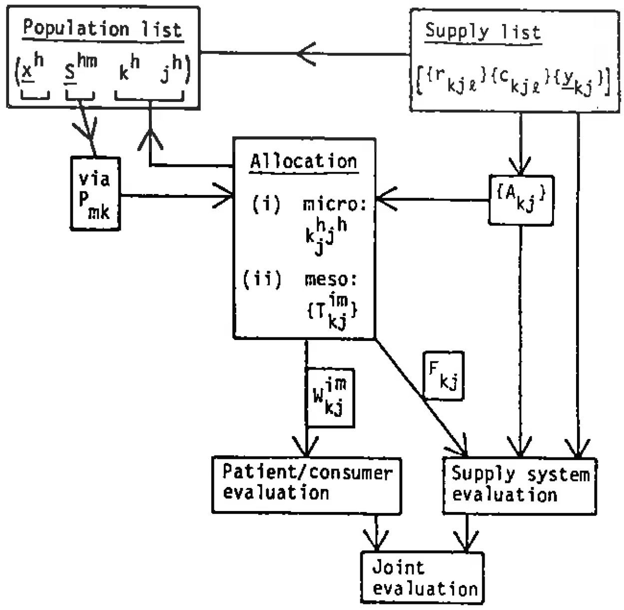
Population list
$(\underline{x}^h \quad \underline{s}^{hm} \quad k^h \quad j^h)$
Supply list
$[\{r_{kj\ell}\}\{c_{kj\ell}\}\{\underline{y}_{kj}\}]$
via P_{mk}
Allocation
(i) micro: $k_j^h j^h$
(ii) meso: $\{T_{kj}^{im}\}$
$\{A_{kj}\}$
W_{kj}^{im}
F_{kj}
Patient/consumer evaluation
Supply system evaluation
Joint evaluation

Figure 4.

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