WORKING PAPER 361

INDUSTRIAL LOCATION THEORY: EXPLORATIONS OF A NEW APPROACH

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1. Introduction: the basis of the approach

Many subjects for geographical theory have two main phenomena which form the main elements of models: the location of activities and the associated pattern of spatial interaction. The problem of building models of industrial location is no exception. The simple Weber problem, of locating a single firm in relation to its inputs and a market illustrates this. The optimum location is determined by minimising the costs of the various spatial interactions involved.

The spatial interaction components of the models have proved easier to build than the locational ones. It has been possible to build models of person trips from work to home, or from home to shop for example, and because these models are singly-constrained spatial interaction models, it was possible to sum in relation to the unconstrained trip ends and obtain an index of housing demand or an estimate of shopping centre revenue (cf. Wilson, 1971). It was difficult, however, to model the supply side. This position began to change in the late 1970s when it became possible to model the retail centre supply side (Harris and Wilson, 1978; Wilson and Clarke, 1979; Wilson, 1981-A, 1981-B). It has since been argued that this represents the basis of a unified approach to location theory (Wilson, 1983) and in the same paper, the bones of an industrial

location model were outlined. The purpose of this paper is to present an industrial location model in more detail along with some numerical experiments for an idealised system which provide insights into the nature of the model.

In the case of industrial location, much of the interest arises from interdependence between sectors and so this feature must be built in from the outset. We also decide to use a discrete zone system, which is a significant shift from the Weber 'continuous space' picture - but it turns out to make the mathematical analysis much more tractable with very little price to be paid in terms of loss of resolution. We can now begin to define our main variables as follows. Let Y_{ij}^{rs} be the flow of goods from sector r in i to sector j in s. Let Z_j^r be total production of r in j; let Z_j^{rT} be the final demand for r in j. Let C_j^r be the total cost of producing r in j, and let D_j^r be the total amount of revenue received. Let c_{ij}^r be the cost of shipping a unit of r from i to j.

Then it is possible to build a spatial interaction model for the array $\{Y_{ij}^{rs}\}$ together with a corresponding array $\{Y_{ij}^{rT}\}$ of flows of r from i to final demand in j, and to solve for the equilibrium distribution of $\{Z_j^r\}$ by imposing the balancing condition

$$C_j^r = D_j^r \tag{1}$$

for each sector r in each zone j.

These turn out to be a set of interdependent nonlinear simultaneous equations in $\{Z_j^r\}$ when all the appropriate substitutions have been made in an explicitly formulated model. They therefore potentially have multiple solutions and equilibrium solutions appear and disappear at critical parameter values - and so we have connections to catastrophe theory and bifurcation theory.

In the next section, we spell out the experimental model explicitly, and then report on a range of numerical experiments in Section 3. Some concluding comments, including a discussion of directions for further research, are offered in Section 4.

2. A new approach to industrial location modelling

We begin by stating the model and then compare it with possible alternative assumptions. Let α^{rs} be the quantity of r needed to produce a unit of s, so if Z_j^s is produced by s in j, then inputs of r totalling $\alpha^{rs}Z_j^s$ are needed from all locations. Suppose these are attracted in proportion to $(Z_i^r)^{\gamma^{rs}}$ from i and to a travel cost term $e^{-\beta^{rs}c_{ij}^r}$. γ^{rs} and β^{rs} are parameters. Then a suitable spatial interaction model is

$$Y_{ij}^{rs} = \alpha^{rs}Z_j^s \frac{(Z_i^r)^{\gamma^{rs}} e^{-\beta^{rs}c_{ij}^r}}{\sum_k (Z_k^r)^{\gamma^{rs}} e^{-\beta^{rs}c_{kj}^r}}. \tag{2}$$

The flows from sector r in i to final demand can similarly be modelled as

$$Y_{ij}^{rT} = Z_j^{rT} \frac{(Z_i^r)^{\gamma^{rT}} e^{-\beta^{rT}c_{ij}^r}}{\sum_k (Z_k^r)^{\gamma^{rT}} e^{-\beta^{rT}c_{kj}^r}}. \tag{3}$$

Then, for a trial allocation of $\{Z_i^r\}$, if producers receive price p_i^r for r at i, then the total revenue received is

$$D_i^r = p_i^r(\sum_{js} Y_{ij}^{rs} + \sum_j Y_{ij}^{rT}). \tag{4}$$

The cost is assumed proportional to Z_i^r:

$$C_i^r = K_i^r Z_i^r \tag{5}$$

for suitable constants, K_i^r. The equilibrium conditions to be satisfied are:

$$D_i^r = C_i^r \tag{6}$$

which is

$$p_i^r\left(\sum_{js} Y_{ij}^{rs} + \sum_j Y_{ij}^{rT}\right) = K_i^r Z_i^r. \tag{7}$$

Making appropriate substitutions from (2) and (3), equation (7) becomes

$$\sum_{js} \alpha^{rs} Z_j^s \frac{(Z_i^r)^{\gamma^{rs}} e^{-\beta^{rs} c_{ij}^r}}{\sum_k (Z_k^r)^{\gamma^{rs}} e^{-\beta^{rs} c_{ij}^r}} + \sum_j Z_j^{rT} \frac{(Z_i^r)^{\gamma^{rT}} e^{-\beta^{rT} c_{ij}^r}}{\sum_k (Z_k^r)^{\gamma^{rT}} e^{-\beta^{rT} c_{ij}^T}}$$

$$= \frac{K_i^r}{p_i^r} Z_i^r. \tag{8}$$

This is obviously a complicated set of nonlinear simultaneous equations in $\{Z_i^r\}$.

Fortunately, it is possible to identify an iterative procedure for solving these equations by analogy with the retail case. This proceeds in the following steps:

(i) take a trial $\{Z_i^r\}$, either beginning with a uniform distribution or taken from the end of the previous step in the iteration.

(ii) Calculate the flows from (2) and (3) and the revenues from (4). Calculate the costs from (5).

(iii) Check equation (6). If it is not satisfied, replace the trial Z_i^r by $\hat{Z}_i^r$ given by

$$Z_i^r = Z_i^r . \frac{D_i^r}{C_i^r} . \tag{9}$$

There are some possible problems, at least for some areas of parameter space, of finding global optima by this procedure (cf. Crouchley, 1983) but this is the procedure used in the numerical experiments below. Before proceeding to review these experiments, we examine the nature of the model and the kinds of solutions we would expect from it.

The key exogenous variables of the model are the prices $\{p_i^r\}$, the costs $\{K_i^r\}$, the transport costs $\{c_{ij}^r\}$, the final demands $\{Z_i^{rT}\}$ and the various arrays of parameters $\{\alpha^{rs}\}$, $\{\beta^{rs}\}$ and $\{\gamma^{rs}\}$. It may also be appropriate to fix some of the $\{Z_i^r\}$ for particular r at particular locations - for example to represent sources of natural resources such as coal. An appropriate starting point will be to assume that the unit prices and costs are independent of location. This is in effect to assume that purchasers of inputs on final goods pay transport costs - but that this assumption is represented only approximately in the model through the nature of the spatial interaction models. If one particular set of transport costs is high, for example, then the corresponding β^{rs}'s will be high and locations adopted, other things being equal, which shorten these particular trip lengths. At a later point, we need to explore how to make these various prices endogenous.

We can get some clue as to what to expect in relation to the parameters $\underline{\beta}$ and $\underline{\gamma}$ from retail and related work. This would suggest (cf. Clarke and Wilson, 1983-A) that higher γ (which corresponds to α in most of these other models) and lower β would generate fewer larger facilities; that is, fewer larger Z_i^r. The detailed analysis in the manner of Harris and Wilson (1978) has not been carried out, but it is a reasonable conjecture (cf. Wilson, 1981-B, p. 124) that D_i^r as a function of Z_i^r is a sum of logistic-shaped functions and that this sum is likely to be itself logistic though possibly with more than one inflexion. The effect of the $\underline{\alpha}$ parameters remains to be explored. Again, we can look to the retail model - in this case the disaggregated one for some rough and obvious guidance.

This suggests that sectors which are heavily linked through the $\underline{\alpha}$ array may to some extent follow each other.

Of more interest here is the detailed effect of the $\underline{\beta}$ parameters. The interest here is again in the interdependence. D_i^r will be greater for (i,r) combinations which are (a) 'near to' corresponding final demand, (b) 'near to' significantly large inputs. The 'nearness' referred to here will be determined in part by the $\underline{\beta}$-parameters. The new interest arises because there will be a mix of these for each $\{Z_i^r\}$. One of the points of interest in the numerical experiments will be to see how this feature works out.

3. <u>Numerical experiments</u>

In this section we present a set of numerical results from the model outlined above (equations (2)-(9)). Let us assume that the economy can be broken down into four sectors: to fix ideas, let us call these 'coal', 'steel', 'manufacturing' and 'services'. A reasonable set of intersectoral linkages might be that presented in Figure 1.

The nature of the forward and backward linkages between the sectors are clearly quite varied. For example, the service sector has rather high forward linkages to all the other sectors, but depends on inputs only from manufacturing. On the other hand, the steel sector appears to be rather more strongly linked backwards to the sources of its inputs (coal and services) than forwards to its markets in the manufacturing sector. Of course, the exact pattern depends on the relative weights of the individual links. This is determined by the precise structure of the input-output matrix $\{\alpha^{rs}\}$ and the vector of final demands $\{Z_*^{rT}\}$. In the long-term of course, the 'technology' of the economy will be changing and this will generate its own bifurcations, but for the time being we assume the following production relations and that they remain constant for our explorations.

Table 1

$$\begin{array}{lc}
 & \{\alpha^{rs}\} \\
\begin{array}{l}\text{coal}\\ \text{steel}\\ \text{manufacturing}\\ \text{services}\end{array} & \begin{bmatrix} 0.0 & 0.28 & 0.2 & 0.0 \\ 0.0 & 0.0 & 0.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.8 \\ 0.3 & 0.3 & 0.3 & 0.0 \end{bmatrix}
\end{array}
\qquad
\begin{array}{c}
\{Z_*^{rT}\} \\
\begin{bmatrix} 0 \\ 0 \\ 22{,}600 \\ 11{,}300 \end{bmatrix}
\end{array}$$

Total production in the economy is now fixed as the solution to the spatially-aggregated simultaneous equations:-

$$\sum_{s=1}^{4} \alpha^{rs}.Z_*^s + Z_*^{rT} = Z_*^r \qquad r = 1, 4 \tag{10}$$

which can be solved for the four unknowns Z_*^1, Z_*^2, Z_*^3, Z_*^4. Alternatively in matrix notation:-

$$\underline{\alpha}.\underline{Z} + \underline{Z}^T = \underline{Z} \tag{11}$$

$$(\underline{I}-\underline{\alpha})^{-1}\underline{Z}^T = \underline{Z}. \tag{12}$$

This is a well-known result of input-output analysis. The actual solution with the data of Table 1 is:-

$$\underline{Z} = \begin{bmatrix} 16{,}640 \\ 21{,}335 \\ 53{,}340 \\ 38{,}695 \end{bmatrix}.$$

The full-set of aspatial product flows (with final demans repeated in the final column) is therefore:-

$$\begin{bmatrix} 0 & 5{,}970 & 10{,}670 & 0 & 0 \\ 0 & 0 & 21{,}340 & 0 & 0 \\ 0 & 0 & 0 & 30{,}960 & 22{,}600 \\ 4{,}990 & 6{,}400 & 16{,}000 & 0 & 4{,}300 \end{bmatrix}.$$

A spatial grid of 113 zones is used in the subsequent experiments - a square 13 x 13 network which has been made approximately circular by excluding all zones at a distance of greater than six units from the centre (Fig. 2).

As we noted in Section 2, the following are all potential parameters of the model: the technical structure of the economy, $\{\alpha^{rs}\}$; the final demands $\{Z^{rT}\}$; unit costs of production $\{K_j^r\}$; and product prices $\{p_i^r\}$. However we wish to concentrate for the moment on the effect of varying α- and β-parameters. The variables {K} and {p} are therefore all assigned a value of unity, and the relative values of the individual sectors' products is assumed to be built in through the α^{rs} array, i.e. all flows are measured in terms of a unit value of product 4 (services) - say one pound. Thus £11,300 worth of product 4 is required by the final demand sector. Similarly each pound's worth of production by Sector 2 requires 28 pence worth of product 1, and 30 pence worth of product 3, and so on. The technology of the economy, which we assume fixed, was given in Table 1 above, along with the final demand sector $\{Z_*^{rT}\}$. For the time being, it is assumed that final demand is evenly distributed over the region.

The final element to the backcloth is to fix the location(s) of one of the production sectors, which simplifies the numerical solution of this model. We have chosen to locate a single coal mine on the north-western periphery of the region (marked with a C in Figure 2).

Within this framework we now wish to examine the effects of changing the $\{\gamma\}$ and $\{\beta\}$ parameters. This still represents a large parameter space with 4 origin sectors and 5 destination sectors (including final demand). Thus we choose to simplify still further and assume the parameters to be destination independent. In addition, the parameters are redundant in relation to sector 1 since there is only one supply source anyway. This leaves just the six parameters $\{\beta^2, \beta^3, \beta^4, \gamma^2, \gamma^3, \gamma^4\}$.

Previous experience (eg. Clarke, 1983; Clarke and Wilson, 1983-A, 1983-B) suggests that a reasonable range of parameter values would see beta varying between 1 and 2.5, and gamma going from 1 to 1.5. Although rather arbitrary, these provide a convenient set of limits to reduce further the parameter space we wish to investigate.

For the first set of model runs, we fix $\gamma^2 = 1.1$, $\gamma^3 = 1.1$, $\gamma^4 = 1.5$ and $\beta^4 = 1.0$. The other two parameters - β^2 and β^3 - are allowed to vary between 1 and 2.5. The results are shown in Figure 3. The patterns here are very much as we would expect with a substantial increase in the dispersion of facilities with increasing beta. Furthermore the patterns are rather more dependent on the variation of β^3 than β^2. Again this is unsurprising, reflecting as it does the dominance of sector 3 as a market for the goods of other sectors. Excluding coal, sector 2 is the least important receiving sector. The patterns of the steel and manufacturing sectors are closely tied together since manufacturing is the only market for steel. On the other hand, sector 4 activity levels show a surprising degree of independence with respect to the other sectors.

These patterns are largely maintained in Figure 4 where the previous set of experiments has been repeated with the value of γ^4 reduced to 1.2. As we would expect, the number of type 4 activities increases (from 3 to 4), except at very low values of β^3, but in general there is rather less dispersion of activities in sectors 2 and 3. The intuitive explanation for this is that the creation of a new type 4 centre facilitates an expansion of sector 2 and 3 activities around it. These new larger centres replace a number of smaller centres from the $\gamma^4 = 1.5$ case. Thus a kind of agglomeration effect appears to be at work in the model. There is also a clear hierarchy of centres. The most diverse grouping is in a sense a trivial case since this occurs around the coal mine and appears to be always present, as we shall see in further runs below. Where one type 4 activity is present, types 2 and 3 appear to come too, and this generates a second kind of centre. Three others can also be readily identified - those having sector 2 only, those having sector 3 only and those with both sectors 2 and 3. The identification of this somewhat crude hierarchy provides a possible basis for classification which is perhaps clearer in some cases than the graphical presentation, particularly at the scale necessary here. Consider the case for $\beta^2 = \beta^3 = 1.5$ in Figure 4C. We describe this case as 13012, by which we mean that it has one centre with all activities, 3 centres with activities type 2, 3 and 4, no centres with types 2 and 3, two centres with steel only and a single centre with manufacturing. Any increase in this simple 'hierarchical index' represents a more dispersed pattern of activities and vice versa. It is a somewhat crude measure in that it takes no account of centre size, only its composition, but it serves our present purposes well enough. Table 2 gives the hierarchical indices for the plots of Figure 4 and may be taken as an alternative demonstration of our earlier assertions, that increasing beta creates more dispersion, and that β^3 is a more important parameter than β^2.

Table 2: Hierarchical indices for Figure 3.2

$\beta^2 \rightarrow$			12000	
$\beta^3 \downarrow$	13003	13012	13111	13111
			13139	
			13446	

Restoring γ^4 to its original value of 1.5, Fig. 5 shows the effect on the overall pattern of increasing β^4 from 1.0 to 1.5. This time we see a big increase in the number of triple activity centres. Thus between $\beta^4 = 1.0$ and $\beta^4 = 1.5$ we have identified a major 'pattern bifurcation' where the basic structure of the system changes fundamentally, and furthermore a similar change occurs over a wide range of β^2 and β^3 centres. However looking again at Figure 5 we see that the same thing applies in relation to the β^3 parameter between the values of 1.0 and 1.5. In other words, the pivotal case of Figure 5 with $\beta^2 = 2.0$, $\beta^3 = 1.5$, $\beta^4 = 1.5$ is highly sensitive to fairly small reductions in either the β^3 or β^4 parameters, although robust with respect to changes in β^2.

The next step we take is to examine the effect of a wider range of variations in the γ^4 and β^4 parameters for fixed β^2 and β^3. For the purposes of this illustration we choose the pivotal plots of Figures 3, 4 and 5 with $\beta^2 = 2.0$ and $\beta^3 = 1.5$. The results are shown in Figure 6. The thing to note is that the patterns are fairly constant across a fairly wide range of intermediate parameter values, but breaking up at both ends of the spectrum. For low values of beta we find that the pattern bifurcations discovered above are prevalent across the whole range of β^4 values, but while the structures are similar across all gamma values for $\beta^4 = 1.5$, they are significantly different when $\beta^4 = 1.0$. This is illustrated in terms of the hierarchical index in Table 3.

<u>Table 3</u>

		β^4	
		1.5	1.0
	1.6	17000	14001
γ^4	1.7	17000	13111
	1.5	16100	12213

This suggests the existence of a classic phenomenon of bifurcation theory - *divergence*. In particular for some critical value of γ^4 between 1.0 and 1.2 a gradual reduction in the β^4 parameter from 1.5 towards 1.0 may generate a transition from the 17000 state to either a 14001 state or a 13111 state (possibly via other states) through a relatively small change in this value of the γ^4 parameter.

On the other hand, for high beta values we begin to see the break-up of sector 4 activities in the $\gamma^4 = 1.0$ case. In particular the sequence $\beta^4 = 3.0 \rightarrow 2.5 \rightarrow 2.0 \rightarrow 1.5$ shows a strong sensitivity to the cornershop to supermarket transition discussed, amongst others, by Poston and Wilson (1977) albeit from a situation ($\beta^4 = 3.0$) in which the larger centres are already dominant. The large number of smaller centres which exist in the $\beta^4 = 3.0$ case are rapidly eliminated with reduction in beta. Notice here how higher values of gamma accelerate the transition, i.e. it now occurs at higher values of beta. Our sector 4 thus acts in a similar way to the equilibrium retail model in respect of gamma-alpha variation (cf. Clarke, 1983; Clarke and Wilson, 1983-A).

3.2 Experiments with centralised populations

We have seen in the last section how it is possible to generate a variety of patterns for an idealised model through relatively restricted slices of the whole parameter space (for example, we have not considered possible variations in γ^2 or γ^3 at all). In this section we would like to explore the effect of varying a third kind of parameter which we call phi - a coefficient of population distribution. This affects the system through the zonal demands Z_i^{rT}. Let us define Z_i^{rT} in terms of a constant total demand, Z_*^{rT}:-

$$Z_i^{rT} = \frac{e^{-\phi d_{i*}}}{\sum_i e^{-\phi d_{i*}}} Z_*^{rT} \tag{13}$$

where d_{i*} is the distance from zone i to the centre. For small positive values of phi, final demand declines exponentially from the centre therefore. The experiments of Section 3.1 are now in a sense a special case with ϕ =0. In Figure 7 the population distributions associated with some sensible values of phi as illustrated across a radius of the 169 zone system. In Figure 8 the effects of these types of population distribution are illustrated with respect to the pivotal plot of Figure 5. Here we start with a fairly orderly 'annular' structure, but as phi is slowly increased the ring of activity gradually loses out in influence to the centre, and the second level activities begin to break up. By the time we reach phi = 0.4 no second level centres are in the ring, only third level centres, and the central zone has taken over from the coal mine as the principal source of activity. Now we enter a period of relative stability in which the influence of the centre gradually builds up, and the outside zones break-up still further, tending toward singular activities within a zone. Finally the population distribution becomes so concentrated that satellite activities can no longer be supported, but where the centre is clearly pre-eminent, the raw materials source is still important enough to maintain a large subsidiary focus of activity.

Once again we are showing here the ability to pick up quite an important effect - by working 'backwards' from high to low phi we have a suburbanisation of activity driven by the decentralisation of population. Moreover, slow changes in pattern may be accompanied by intermittent bursts of activity - for example there is a very marked change between phi = 1.0 and 0.8, but relatively little from 0.8 to 0.4.

Of course, one must not claim too much from this analysis, for example decentralisation here is also associated with an increase in the importance of the raw materials source, and increasing agglomeration in the satellite centres. These trends are the reverse of what one might expect in an evolutionary sense. However one still has the effects of beta and gamma to build on.

As a final set of experiments, the variation of γ^4 and β^4 explored in Figure 6 is revisited with a rather high value of phi (0.6) in Figure 9. The much higher degree of centralisation in the $\phi = 0.6$ case does little to disguise the basic similarity between the two sets of results for high values of the beta parameters, but at lower beta values more significant differences are apparent. In particular the patterns now appear to be more sensitive to low level changes in the beta, notably between 1.5 and 2. While group 2 centres (with three activities) persist at higher beta values in the $\phi = 0.6$ case, they break up with low beta, which was not previously the case. The centralisation of sector 4 activity is associated with increased dispersion in sectors 2 and 3 - a phenomenon which has already been observed above in relation to Figure 4.

A further interesting feature is the way in which changes in alpha and beta can replicate very closely the effect of varying phi in Figure 8. In particular note the similarity between the $\beta = 2.0$, $\gamma = 1.5$ case of Figure 9 and $\phi = 0.1$ in Figure 8 : and between the $\beta = 1.5$ $\gamma = 1.2$ case to $\phi = 0.2$.

4. Concluding comments: directions for further research

The numerical experiments presented in Section 3 do offer generally plausible results. A good example is the interpretation of Figure 8 as an account of the impact of the suburbanisation of population on industrial location. It is also clear from these results (and comparing them with the references noted on the retail sector) that there are critical parameter values at which pattern bifurcations take place. These demonstrations are all achieved with a very simple model. In a review paper (Birkin and Wilson, 1983), we show in more detail how the model here relates to alternatives in the literature, and how other features of some of these models can be built into this one. In the rest of this section, we simply note briefly the obvious extensions. They will each change the behaviour of the model in a (generally) predictable way but typically will not affect the character of the results. In terms of model modification, or of further analyses to be carried out, we note five directions for further research and the results of our own explorations will be reported in further papers.

(i) There is a need to build more detailed representations of industrial production functions to allow for scale economies and so on. This can be done to some extent by making constants like K_i^r in equation (5) functions of scale - as $K_i^r(Z_i^r)$.

(ii) We need to explore the consequences of making final demand elastic with respect to accessibility to suppliers.

(iii) By a shift of scale, we can take zones as large scale regions and the spatial interactions as inter-regional trade flows. The model presented, therefore, offers an alternative way of solving the problem posed by Leontief and Strout (1963) and developed in an entropy-maximising framework by Wilson (1970).

(iv) It will be possible to integrate the industrial and input-output elements of this model with a more comprehensive urban or regional model. It would be possible, for instance, to take a modified Lowry model with dynamic structural submodels (cf. Wilson, 1981-C, pp. 268-273) and to add the model of this paper (suitably modified in respect of the service model) as the industrial submodel. Industrial location usually has to be taken as exogenous in such models.

(v) We need to analyse the nature of the bifurcations which take place more explicitly. We could use the method of Harris and Wilson (1978) by examining C_i^r and D_i^r as functions of Z_i^r and look at the stability of the intersection of these curves (which represent equilibrium points). However, the industrial case will be complex relative to the retail case because of the role of the input-output coefficients connecting sectors. These analyses will be reported in a later paper.

We conclude, then, by arguing that the model presented here, while relatively simple, represents a new approach which is rich in potential. Preliminary numerical experiments show the right kinds of results. Bifurcation phenomena are present - reflecting the interdependence and nonlinearities both in the model and the real-life situation. It is a straightforward if demanding task to develop the model further in the direction of greater realism.

References

Birkin, M. and Wilson, A.G. (1983) Models of industrial location: a review. Forthcoming Working Paper, School of Geography, University of Leeds.

Clarke, M. (1983) Integrating dynamical models of urban structure and activity: an application to urban retail systems. Forthcoming Ph.D. thesis, School of Geography, University of Leeds.

Clarke, M. and Wilson, A.G. (1983-A) The dynamics of urban spatial structure: progress and problems. *Journal of Regional Science*, 23(1), pp. 1-18.

Clarke, M. and Wilson, A.G. (1983-B) The dynamics of urban spatial structure. Forthcoming.

Crouchley, R. (1983) Equilibrium conditions and solution procedures for the production-constrained spatial-interaction model with a general attractiveness function. Working Paper 353, School of Geography, University of Leeds.

Harris, B. and Wilson, A.G. (1978) Equilibrium values and dynamics of attractiveness terms in production-constrained spatial-interaction models. *Environment and Planning A*, 10, pp. 371-88.

Leontief, W. and Strout, A. (1963) Multi-regional input-output analysis. In T. Barna (Ed.) *Structural interdependence and economic development*, Macmillan, London, pp. 119-150.

Poston, T. and Wilson, A.G. (1977) Facility size vs distance travelled: urban services and the fold catastrophe. *Environment and Planning A*, 9, pp. 681-68.

Wilson, A.G. (1970) Inter-regional commodity flows: entropy-maximising approaches. *Geographical Analysis*, 2, pp. 255-282.

Wilson, A.G. (1971) A family of spatial interaction models and associated developments, *Environment and Planning A*, 3, pp. 1-32.

Wilson, A.G. (1981-A) The evolution of urban spatial structure: a review of progress and research problems using SIA models. *Bulletin of the Institute of Mathematics and its Applications*, 18, pp. 90-100.

Wilson, A.G. (1981-B) *Catastrophe theory and bifurcation: applications to urban and regional systems.* Croom Helm, London and University of California Press, Berkeley.

Wilson, A.G. (1981-C) *Geography and the environment: systems analytical methods*, John Wiley, Chichester.

Wilson, A.G. (1983) Location theory: a unified approach. Working Paper 355, School of Geography, University of Leeds.

Wilson, A.G. and Clarke, M. (1979) Some illustration of catastrophe thoery applied to urban retailing structures. In M.J. Breheny (Ed.) *Developments in urban and regional analysis*, Pion, London, pp. 5-27.

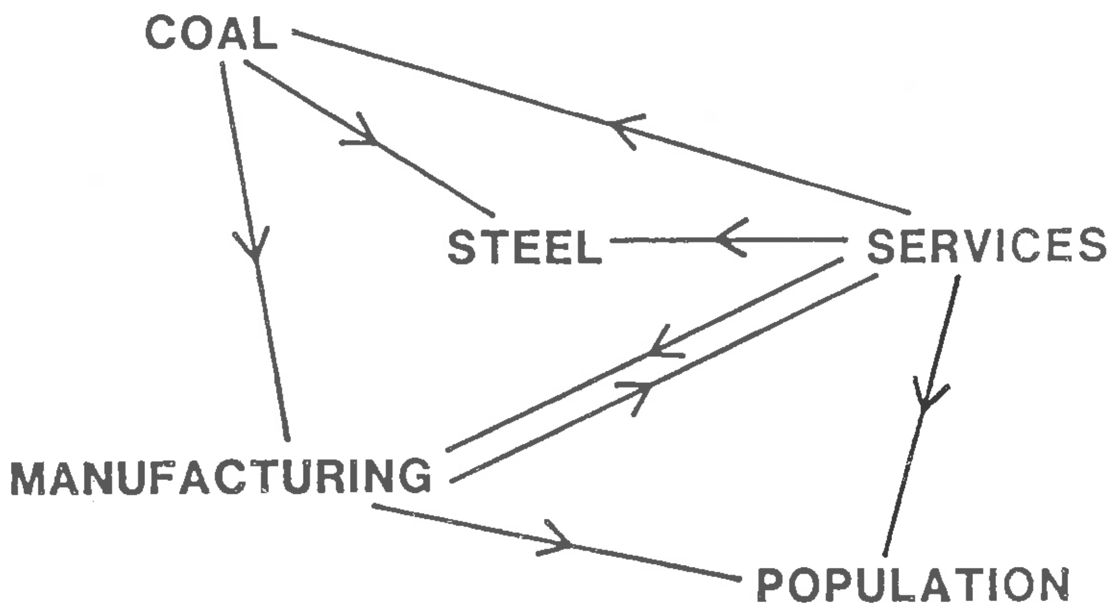


FIGURE 1 : Linkages in a four-sector economy

FIGURE 2: A 169-ZONE SPATIAL SYSTEM.

X	X	X	X	X	X	.	X	X	X	X	X	X
X	X	X	.	.	.	.	.	.	.	X	X	X
X	X	.	.	.	.	.	.	.	.	.	X	X
X	C	.	.	.	.	.	.	.	.	.	.	X
X	.	.	.	.	.	.	.	.	.	.	.	X
X	.	.	.	.	.	.	.	.	.	.	.	X
.	.	.	.	.	.	.	.	.	.	.	.	.
X	.	.	.	.	.	.	.	.	.	.	.	X
X	.	.	.	.	.	.	.	.	.	.	.	X
X	.	.	.	.	.	.	.	.	.	.	.	X
X	X	.	.	.	.	.	.	.	.	.	X	X
X	X	X	.	.	.	.	.	.	.	X	X	X
X	X	X	X	X	X	.	X	X	X	X	X	X

X = EXOGENOUS ZONE

C = COAL MINE

FIGURE 3 : Varying β^2 against β^3 with $\gamma^4 = 1.5$, $\beta^4 = 1.0$

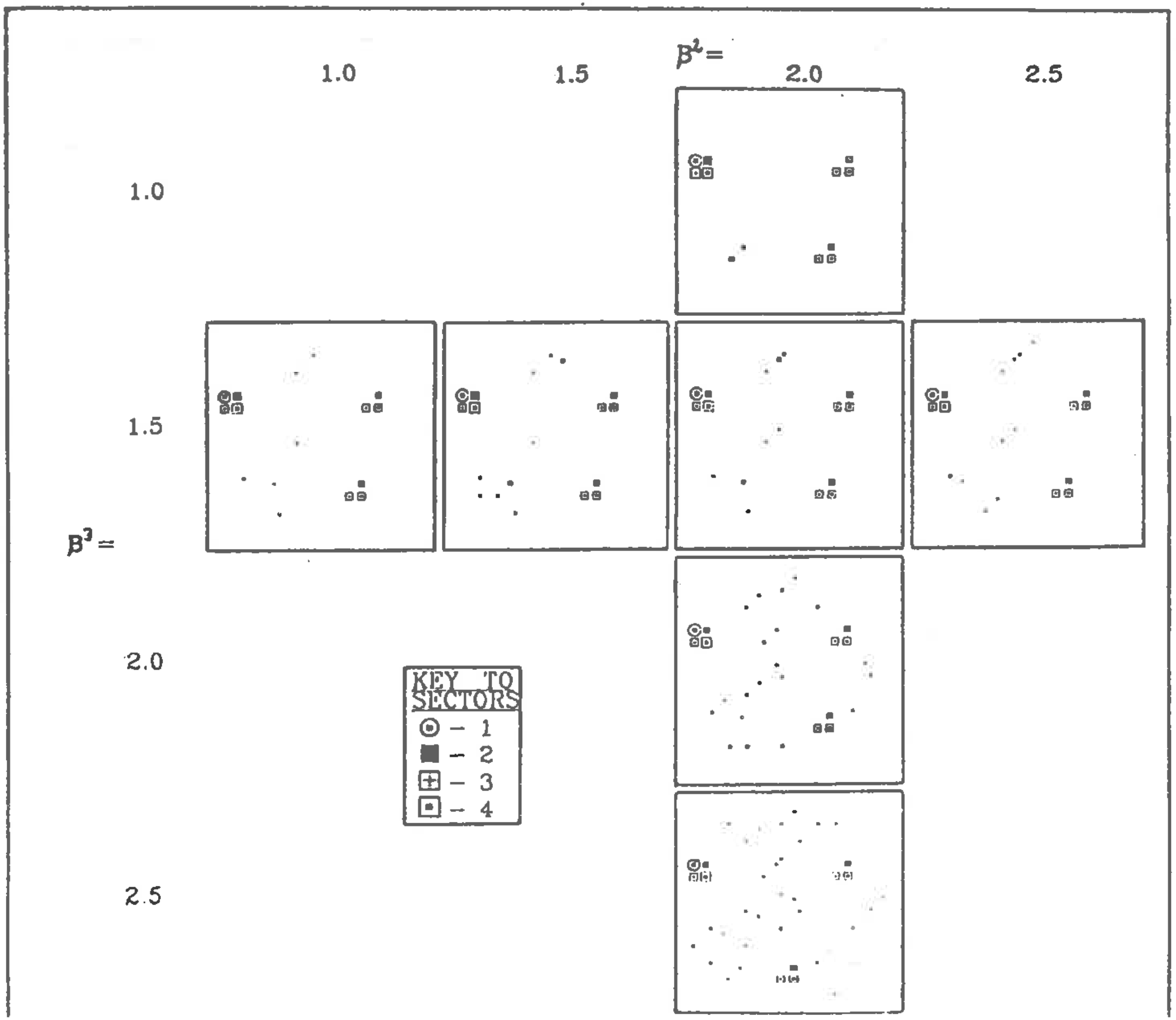


FIGURE 4 : Varying β^2 against β^3 with $\gamma^4 = 1.2$, $\beta^4 = 1.0$

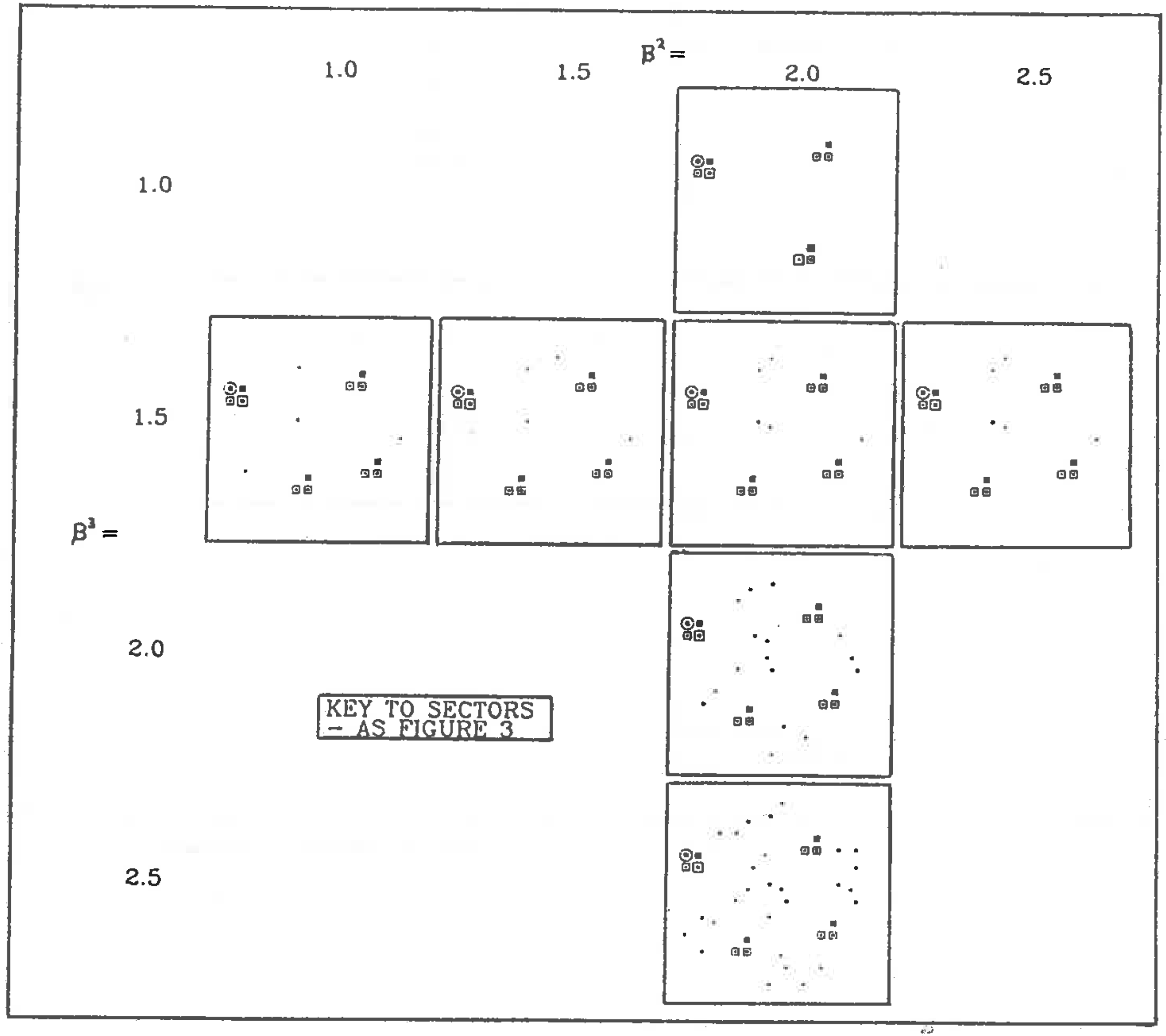


FIGURE 5 : Varying β^2 against β^3 with $\gamma^4 = 1.5$, $\beta^4 = 1.5$

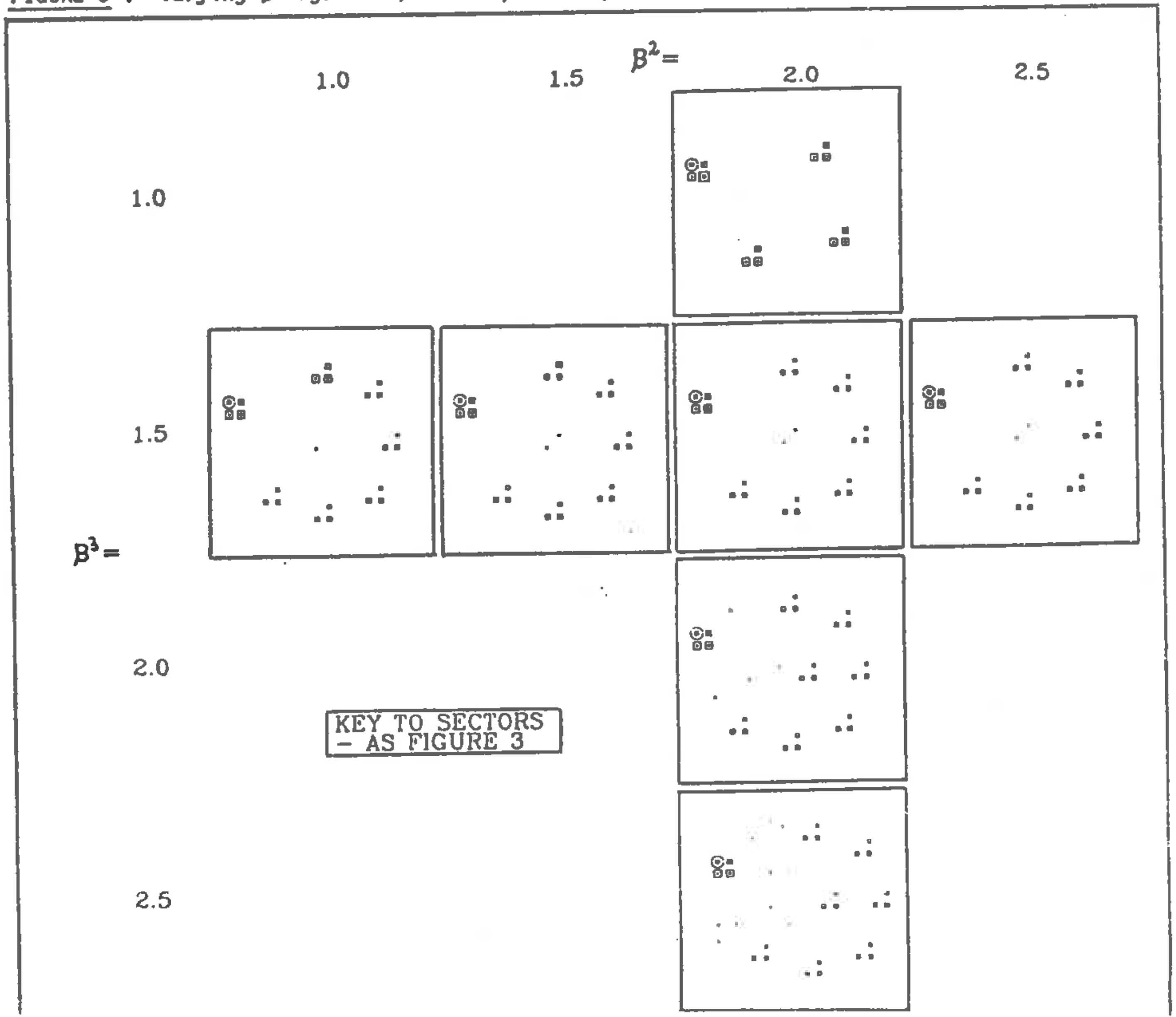
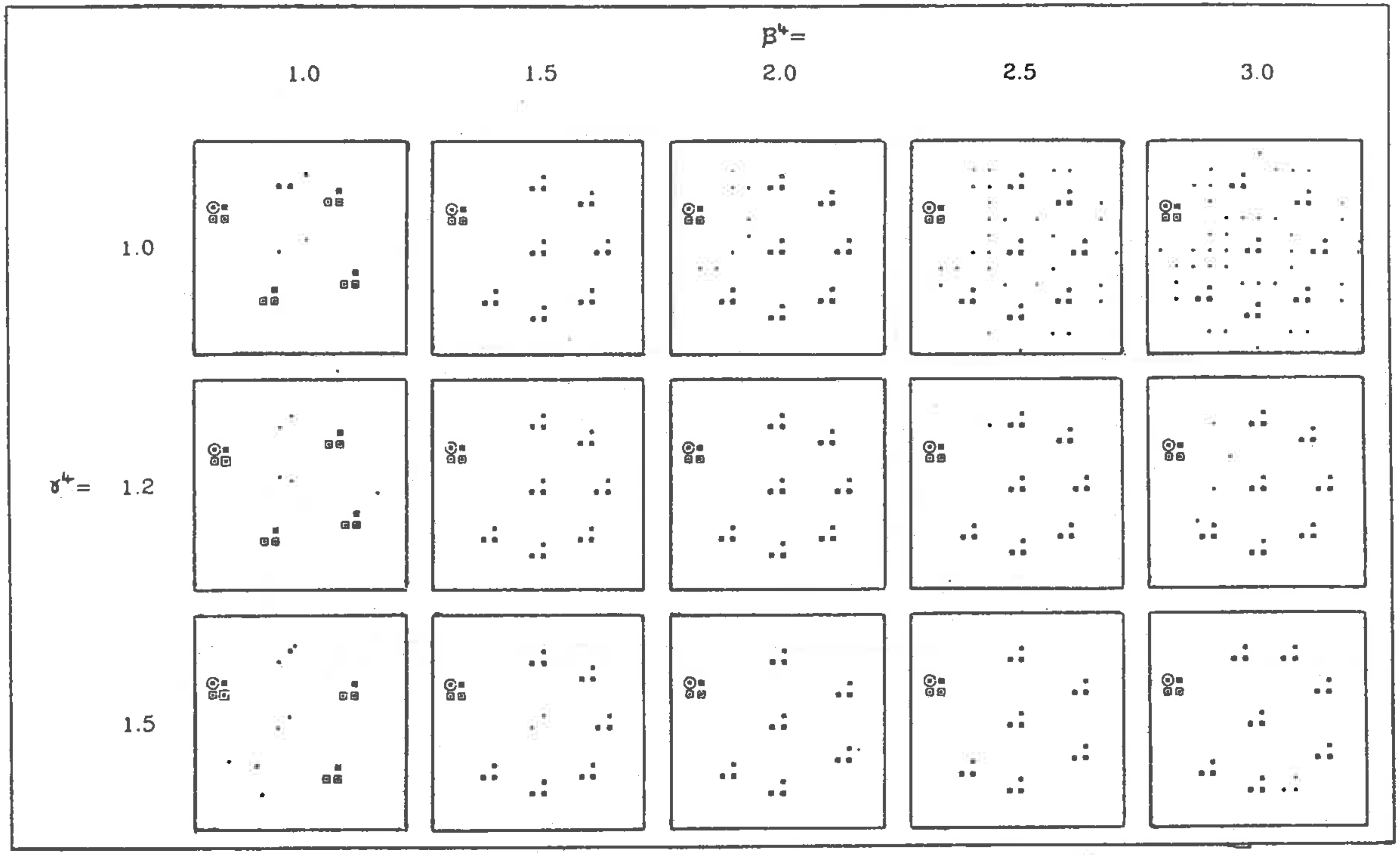


FIGURE 6 : Varying γ^4 against β^4



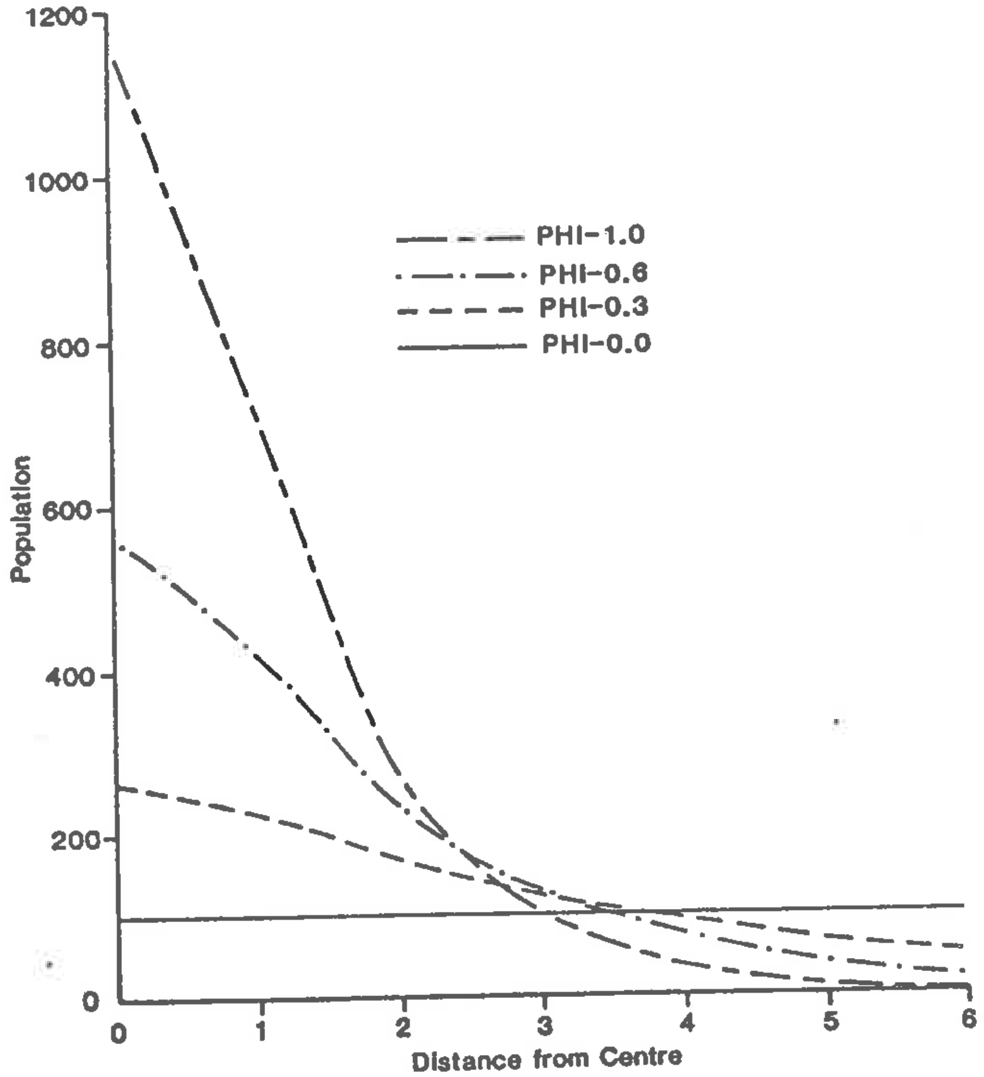
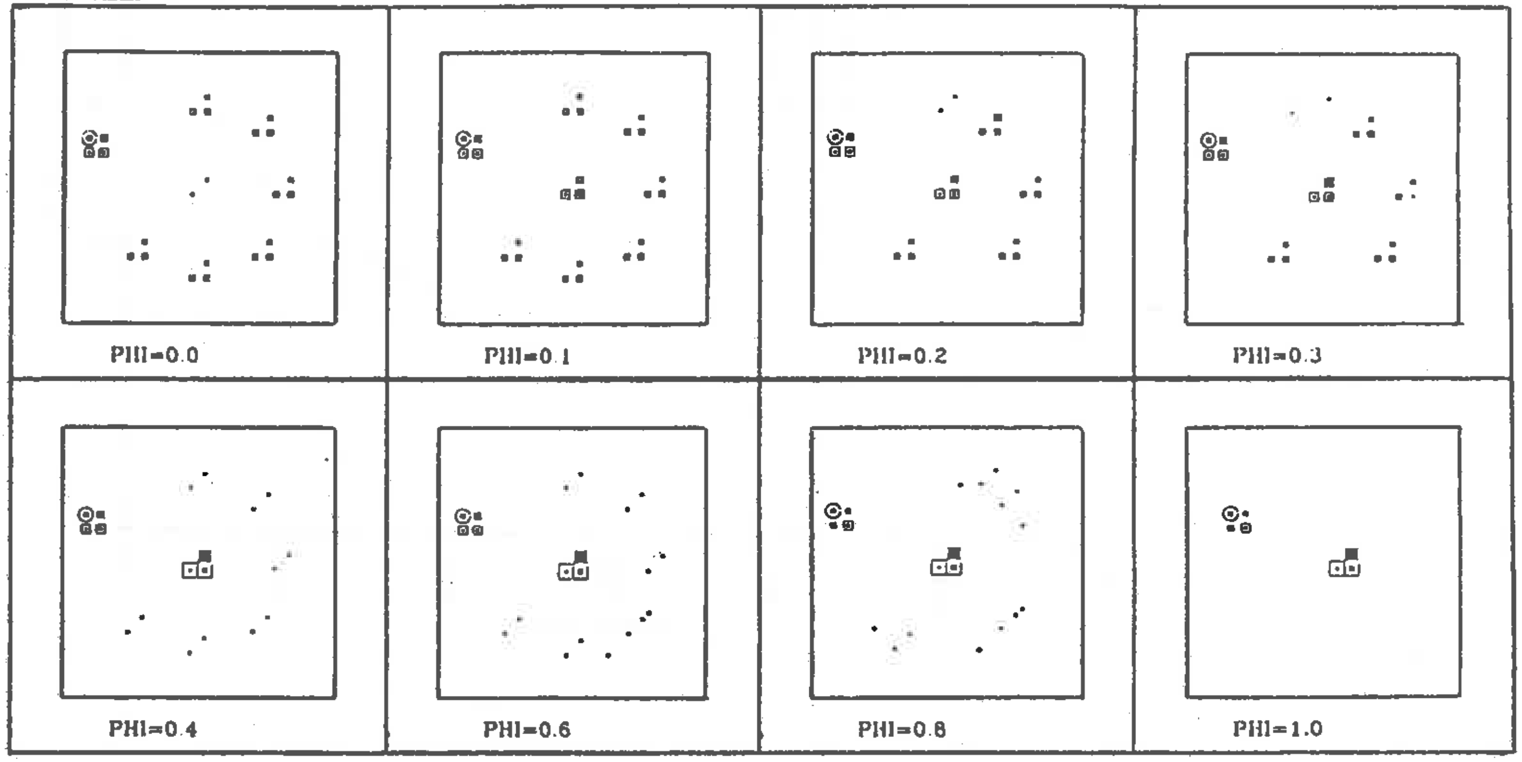


FIGURE 7 : Population distribution for selected values of phi.

FIGURE 8 : The effects of changing population distributions.



<u>FIGURE 9</u> : Varying γ^4 against β^4 with $\phi = 0.6$.