EKN-812: Problem Set 2

There are ??? points available on this problems set.

Due Date: ???*

1. This problem introduces "home production", which we can see an application of weak separability. The basic observation is that many important goods are not sourced only from the market, but also are partly or entirely produced in the household. Meals, children's education, cleaning services, and some aspects of health would be natural examples.

For simplicity, consider a consumer who consumes two goods, z and y. y is purchased directly in the market for a given price of p_y. z is produced at home for individual consumption using a fixed input F and a variable input x. The output of z is given by a production function

$$z = A(F)x$$

where $A(F)$ increasing in F.

For concreteness, you can assume $A(F) = A_0 F^\theta$, where $A_0 > 0$ and $\theta > 0$ are given constants. And, you may assume that preferences over y and z are CES with elasticity of substitution σ:

$$u(y, z) = (\alpha y^\rho + (1 - \alpha) z^\rho)^{1/\rho}.$$

Recall that with these preferences, $\sigma = (1 - \rho)^{-1}$ is the elasticity of substitution between y and z; $\alpha \in (0, 1)$ is a share parameter.

(a) If the level of F is fixed, will the consumption of z and y increase proportionally with income?
(b) Now suppose the level of F can be freely chosen. For a given level of z, what are the optimal choices of x and F?
(c) Could an increase in the price of x lead to an increase in the consumption of z? That is, will x and z tend to be substitutes? Why or why not?
(d) Now, let's interpret z to be the internal temperature of people's homes, and suppose F represents goods like the efficiency of heating equipment and insulation. Suppose, in particular ???

[10 points]

2. Suppose students have to take an exam with N questions. The time available for the exam is T, and the score on each question is a function of the student's level of knowledge, K, and the amount of time he or she spends on each question:

$$S_i = f_i(t_i, K).$$

Assume that f_i is increasing in t_i and K for each question i. Also, each question i has a maximum of $\overline{S}_i$ points available.

(a) If students try to maximize their total score on the exam, how much time will they spend on each question? Work out the solution for the particular case

$$f_i(t_i, K) = \max\{K t_i^\theta, \overline{S}_i\}.$$

Would any student ever not attempt to answer a question at all? When will this happen?

(b) Would any student ever get the maximum possible score for a question? Which ones? [**Hint**: consider the case $\theta < 1$ and $\theta \geq 1$ separately.]

(c) How would your answers to (a) and (b) change if the production function was

$$f_i(t_i, K) = \overline{S}_i \left[1 - \exp(-Kt_i)\right]$$

Do students spend more time on the questions with more points available? Is your answer different for the well-prepared students as compared to the badly prepared ones (as defined by their K)?

(d) Now suppose that all students value their time outside of the classroom at rate v. Set up the student's time allocation problem and write down the first-order conditions.

(e) Do any students walk out of the exam early? Which ones?

(f) Do students have any incentive to collude on how much time they spend on the exam?

(g) Now suppose that the exam is graded on a "curve", e.g. the top 10% get an A, the next 20% get a B, etc. What rationale can you think of for such a system?

(h) Would students want to collude under the system of relative grading? Would they have an incentive to cheat on such an agreement?

3. Consider a monopolist with costs $c(y)$ who sells his output in two separate markets, with demand curves $D_1(p_1)$ and $D_2(p_2)$.

(a) Express the relationship between the equilibrium prices in the two markets in terms of the relative elasticities of demand. Let y_1 and y_2 be the quantities sold in each market, respectively.

(b) Suppose the two markets represent different countries, and the government in country 1 puts a per-unit tax of t on the monopolist's product. What happens to output in market 2?

(c) Could *total* output actually increase as a consequence of the new tax in market 1? When will this be the case?