EKT-816 Lecture 4

Mechanical Properties of OLS

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Linear Projections

- let (X_1, X_2) have some joint distribution
- define

$$b = \frac{\text{cov}(X_1, X_2)}{V[X_2]}$$

$$a = E[X_1] - bE[X_2]$$

- and let $X_1^* = X_1 - (a + bX_2)$
- what is
  - $\text{cov}(X_1^*, X_2)$?
  - $E[X_1^*]$
- is X_1^* *independent* of X_2?
- we say $a + bX_2$ is the *linear projection* of X_1 onto X_2
  - X_1^* is the component of X_1 that is orthogonal to X_2

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Linear Projections: Example

- suppose the joint distribution of (D, X) is as follows:
  - $P(D = 0, X = 0) = 0.1$
  - $P(D = 0, X = 1) = 0.2$
  - $P(D = 1, X = 0) = 0.5$
  - $P(D = 1, X = 1) = 0.2$
- you can confirm that:
  - $\text{cov}(D, X) = -0.08$ and $V[X] = 0.24$
  - $D^* = D + X/3 - 5/6$, $E[D^*] = 0$, and $\text{cov}(D^*, X) = 0$
  - nevertheless, X and D^* are *not* independent

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Derivation of OLS Formula

- Before we talk about the statistical properties of regression estimates, we need to understand exactly what OLS does mechanically
- given N data points $(Y_i, X_{i1}, \dots X_{iK})$, consider

$$\min_{\beta_0, \dots \beta_K} \sum_{i=1}^{N} (Y_i - \beta_0 - \beta_1 X_{i1} \dots - \beta_K X_{iK})^2$$

  - what are the first-order conditions for this problem?
- we can write the data in matrix form
  - X is the $N \times (K+1)$ matrix of regressors
  - Y is a $N \times 1$ vector of "outcomes"

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- when we do this, the FOC become

$$(X'X)\hat{\beta} = X'Y \Longrightarrow \hat{\beta} = (X'X)^{-1}X'Y$$

- now let
  - $\hat{e} = Y - X\hat{\beta}$ be the residuals
  - $M = I - X(X'X)^{-1}X'$ be the "residual maker" matrix ($N \times N$)
    - notice that M is symmetric ($M' = M$) and idempotent ($M \times M = M$)
    - also notice that $MY = \hat{e}$ and $MX = 0$
    - and, $X'\hat{e} = 0$, by construction
- we can write $Y = X\hat{\beta} + \hat{e}$
  - i.e. decomposition into predicted values + residuals
  - these follow from facts about linear algebra, *not* anything about causality!
  - in fact, you could do this with purely deterministic data: no statistics necessary

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  - $M = I - X(X'X)^{-1}X'$ be the "residual maker" matrix ($N \times N$)
    - notice that M is symmetric ($M' = M$) and idempotent ($M \times M = M$)
    - also notice that $MY = \hat{e}$ and $MX = 0$
    - and, $X'\hat{e} = 0$, by construction
- we can write $Y = X\hat{\beta} + \hat{e}$
  - i.e. decomposition into predicted values + residuals
  - these follow from facts about linear algebra, *not* anything about causality!
  - in fact, you could do this with purely deterministic data: no statistics necessary

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- now, suppose we have $X = [X_1 \; X_2]$ where X_1 contains a constant and X_2 is a single column
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  - why we want to do this will become clear later
- let $M_1 = I - X_1(X_1'X_1)^{-1}X_1'$
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  - premultiply again by X_2' (check that the dimensions are appropriate!)
  - then, we get

$$X_2'M_1Y = X_2'M_1X_2\hat{\beta}_2$$

- use the fact that M_1 is symmetric and idempotent to write

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- the OLS coefficient on X_2 is numerically identical to the one we would obtain from:
  - regressing X_2 on X_1 and obtaining the residuals x_2^*
  - regressing Y on x_2^*
- implications:
  - if $X_2 \perp X_1$, we get the same coefficient on X_2 whether we include "controls" for X_1 or not
  - as we will see, richer conditioning sets are *not* always weakly better
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Wages, Experience and IQ

- suppose we have a sample of workers observed at the same date
  - we observe their wages (w), and the results of an IQ test (x)
  - all workers in the sample were tested at the same date, say 20 years ago
  - we also have their age in years which we encode in a vector of dummies (D)
- suppose we want to estimate

$$\log w = \alpha x + D\gamma + e$$

- suppose age has the following effects:
  - as workers gain experience their productivity rises and employers may pay them more
  - some cohorts differ in ability because of changes in e.g. school quality or environmental factors
- also assume that age at testing affects measured IQ, with older kids doing better on average

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- does it make sense to adjust the IQ scores for age?
  - it depends: do we want
    - the effect of ability on wages, holding experience constant?
    - or, the effects of experience, holding ability constant?
- notice that all of the following regressions will give the same $\hat{\alpha}$
  - regress $\log w$ on $\alpha x + D\gamma$ (1)
  - regress $\log w$ on $\alpha x^* + D\gamma$ (2)
  - regress $\log w$ on αx^* (3)
- however, the estimated *age* effects will change
  - consider regressing $\log w$ on D alone (4)
  - because $x^* \perp D$ by construction, (4) gives the same $\hat{\gamma}$ as (2)

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    - is it possible to determine whether a given cohort earns more at a given point in time because of
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        - greater experience
        - some combination of the two?
- clearly, without including the IQ measure, the age coefficients pick up both effects
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  - the component of ability that predicts cohort gets residualized out by OLS
  - and, our D^* will be variation in experience that is orthogonal to cohort ability
- HOWEVER, if you age-adjust z to form z^*, you undo the benefit of having kids tested at the same age
  - you would then be regressing log wages on $\alpha z^* + D\gamma$
  - because z^* is orthogonal to D by construction, we know the estimated γ will be identical to the one from regressing log wages on D alone
    - and, as we discussed, that reflects both cohort and experience effects

Wages, Experience and IQ

- now, what if you had instead z, an IQ test administered at the same *age*?
  - would controlling for z help us isolate the experience effects?
- answer: depends whether you adjust z for age.
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  - the component of ability that predicts cohort gets residualized out by OLS
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- the bottom line here is that your choice of specification determines the residual variation used to estimate your coefficient of interest
- many specification choices, e.g.
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    - measurement error
    - normalizations
    - omitted variables
- the key to thinking clearly about the costs and benefits of these choices is to think about how they change the residual variation "in the denominator"

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Conditional Independence and Causal Effects

- When does OLS give us an estimate of the *causal* effect of D?
    - we have already seen one simple case: where D is assigned at random (as in an experiment)
    - in this case, you can compute the difference between treatment and controls by regressing Y on D
    - but, why would we "control" for X then?
        - and, what needs to be true about the joint distribution of (Y_1, Y_0, D, X) for us to recover a causal effect?
- the basic answer is that we need D to be independent of the potential outcomes, conditional on X
    - sometimes people call this the "conditional independence assumption" or CIA
    - you can interpret this to mean: once we have accounted for X, people choose D at random
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- next, we will work through this model of the "data generating process" and ask whether its assumptions make sense in context
- one interpretation is:
    - distribution of education differs across locations for reasons to do with
        - the characteristics of the location itself (δX, e.g. local education policies)
        - and person-specific factors (u, e.g. ability) which are independent of location
        - why might $X \perp\!\!\!\perp u$ fail?
    - further, earnings differ across people because of
        - direct effects of location-specific factors γX, e.g. differences in labor demand
        - education itself (βD)
        - individual-specific factors ε (e.g. individual labor supply preferences, family obligations)
- does it make sense to assume $u \perp\!\!\!\perp \varepsilon$? What does this assumption mean?
    - mechanically, it gives us that $D|X$ is independent of potential outcomes
    - in context, it means location is the *only* thing that affects both earnings and education
        - once we have accounted for it, education is not affected by (e.g.) earnings potential or ability

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