Stochastic:

Diagram, schematic

Description automatically generated with medium confidence4) I chose the Normal Inverse Gaussian Process (nig process) to be the parametric Levy model of your choice with jumps of infinite variation as it had well defined moments which would be generated from the cumulant function. At each point in time the process has a nig(alpha,delta,beta,mu) distribution. The parameters of the nig alpha,beta,delta,mu control the tail heaviness, skew, dispersion and location which makes the model very flexible to input data. The nig2 function integrates over a very small time period to simulate the realised return which represents the drift component mu in a nig process as shown in figure 1 due to alpha being 0.

**The nig distribution is suitable for modelling stock prices as introduced by Barndorff and Nielsen(1998) and I therefore calibrated the distributed on stock price data of a highly volatile stock ‘TSLA” using a function in R while use Maximum Likelihood Estimation to fit the model and obtain the best estimates of the parameter. We used daily return data over the last 5 years to train the data on a sufficiently large sample size and reduce biases. I then estimated the levy triplet (0, l\_x, gamma\_x) where gamma\_x = sqrt(alpha^2-beta^2). We use the returns data to create a more stationary series with jumps which better fits a levy process of infinite variation which can be thought of as a drift and large jump function. However, mle is non linear which can lead to instability in the parameter estimation and a distribution test.**

The cumulant generating function is given by the equation on the left and can be used to define the whole levy process. Diagram, text, letter

Description automatically generatedDiagram

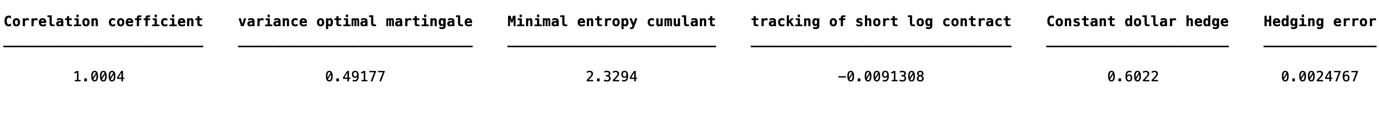
Description automatically generatedThe moments could be estimates using the following formula Matsuda, K. (1970.) I was therefore unable to generate the tables in item (1) as the process is alpha stable and the moments are simply a function of the parameters and time. I deliberately fit the data to a volatile stock in order to general trend(drift) but with the possibility for large jumps as we will be modelling riskier derivatives such as options which have large jumps and volatile pricing.

If we calibrate the model we should be able to forecast a risky asset as we capture the heavy tails and assymtery with the parameters as the nig model is very flexible. As the distribution is suitable for modelling heavy tailed data, we used empirical realised stock returns to form the distribution of the iid random variables (Hanssen, Alfred, Øigård 2001.) As the moments of the process can be easily obtained from the moments two calibration strategies present themselves. Train the model on data for the instrument you are trying to model then compute the mean, variance statistics for the process as a time dependant process for forecasting as these parameters can define the process. Or use the method of moments with a system of 4 equations to solves for the parameters. Once you have the parameters the drift at each iid Random

(fig1)

Variable X\_i wll simply be a function of the mean. The specification on the left holds true for all levy processes with alpha representing the small jump process. However, as the nig has infinite variation alpha would be equal to 0. In this way we can simulate the levy measure simply from the parameters of the distribution. Additionally, the gaussian variation of the process is 0 simply leaving the mean as the drift at each realisation x\_i. This makes it relatively easy to calibrate a model from a data set. For the most accurate forecast of your deritive recalibrate the model on past data as past empiracle data is the best way to model the future X\_1 r.v. However, this is dependand on context with the risk of poor data, at risk of errors such as hindsight bias and overweighting of old data.

The results of my calculations of question 1 with the nig model are shown below : Due to not being properly calibrated to the exact data type some of the results are slighty wrong.



Diagram

Description automatically generated5) The size of the hedging error generated by the trust and the nig process are extremely small suggesting that the risk in the derivative being traded is sufficiently covered from risk which implies it would be a strategy more suitable for a very risk averse investor. Over time as seen in the cash gamma squared equation the hedging risk will decrease over time as in levy models kurtosis is inversely proportional to time. The hedging error exists because the investor used the Black-Scholes model to delta gamma hedge the portfolio however, due to markets being unpredictable the BS approximation cannot hold all the risk information and therefore the portfolio makeup will be slightly skewed from a fully hedged portfolio, however, the error is small which says the approximation by the BS model is very accurate. As the investor is delta gamma hedging using options and stocks the difference represents potential profit for the investor as the volatility of the option increases its value. However, while the investor is long more long volatile derivatives as they have greater payoff potential, they are are likely to have higher kurtsosis. The BS model has zero kurtosis while levy processes model kurtosis. The cash-gamma squared formula (from week 5) shows the difference between levy and BS hedge error is simply the kurtosis with BS underestimating the hedge risk error. This means the investor is systematically taking on more risk than they believe they are. As options are typically traded quickly the kurtosis effect will be worse as kurtosis is inversely proportional to the time period in levy models. An investor will also frequently rebalance under the BS assumption to maintain delta gamma neutrality however the more frequently rebalancing your portfolio will also increase kurtosis risk under the cash gamma squared equation.

Bibliography;

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