EXACT METHODS EXAMPLE QUESTIONS

Question 1

Consider the following statements. Indicate whether they are true or false and justify your answer.

a) In a mixed integer linear programming problem, some of the variables may be binary.

b) Problems with an unbounded objective function value and infeasible problems both have an open feasible region.

c) Rounding the optimal solution of the LP relaxation of a BIP or ILP problem is a good strategy for obtaining a solution for that BIP or ILP problem.

d) When maximizing, the optimal objective function value of a relaxation provides a LB for the optimal objective function value of the original problem.

Question 2

Consider a travelling salesman problem with 6 cities (including the starting location). Consider the subset containing cities 1, 2 and 4. Write the constraint that specifies that there must be at least one connection between this subset and its complement. Write the full expression, that is, do not use the summation operator (that is, do not use $\sum$).

Question 3

Consider a traveling salesman problem with 5 cities (including the starting location). Also consider the following two constraints for this problem.

1) $X_{31} + X_{32} + X_{34} + X_{35} = 1.$
2) $X_{21} + X_{24} + X_{25} + X_{31} + X_{34} + X_{35} \geq 1.$

Explain the meaning of these two constraints.

Question 4

Consider a set covering problem in which we wish to decide where to create warehouses, so that several stores are within a reasonable distance of at least one warehouse. The cost of opening a warehouse at each of the possible locations, as well as the covering information, are given next.

		warehouse locations				
		1	**2**	**3**	**4**	**5**
stores	**1**	Y	Y			
	2		Y	Y		
	3	Y			Y	
	4	Y		Y		
	5		Y	Y		
	6			Y		Y
	7			Y	Y	Y
	8				Y	Y
	cost	1500	1250	1900	1600	1500

Y: the store is covered by the warehouse

a) Write this problem's objective function. Write the full expression, that is, do not use the summation operator (that is, do not use $\sum$).

b) Write the covering constraint for store 7. Write the full expression, that is, do not use the summation operator (that is, do not use $\sum$).

c) Consider the following statement: "in this problem's optimal solution, a warehouse will be opened at location 2 and / or at location 3". Indicate whether this statement is true or false. Justify your answer.

Question 5

Consider a set covering problem with 6 subsets and 7 items. In the optimal solution for this problem, we have: $X_1 = 0, X_2 = 0, X_3 = 1, X_4 = 1, X_5 = 1, X_6 =?$. The coefficients of the variables in the functional constraints are given in the following table.

	X_1	X_2	X_3	X_4	X_5	X_6
item 1	1	1	0	1	0	0
item 2	1	1	0	1	1	1
item 3	1	0	1	0	0	1
item 4	0	1	1	1	1	0
item 5	0	1	1	0	0	1
item 6	0	0	0	0	1	1
item 7	0	0	0	0	1	0

a) Indicate the items which are included in subset 5 (that is, the items covered by subset 5).

b) What is the value of variable X_6 in the optimal solution? Justify your answer.

Question 6

Consider a cutting stock problem, in which a standard-sized piece of size 45 must be cut to satisfy the demand of smaller pieces of sizes 22, 15 and 10. The demand for each of the smaller pieces, and information about the possible cutting patterns, are given in the following tables. Let X_j denote the number of standard-sized pieces cut according to pattern $j, j = 1,2, \ldots ,7$.

size	22	15	10
demand	150	120	140

		cutting patterns						
		1	**2**	**3**	**4**	**5**	**6**	**7**
sizes	**22**	2	1	1	0	0	0	0
	15	0	1	0	3	2	1	0
	10	0	0	2	0	1	3	4
	waste	1	8	3	0	5	0	5

a) Write the demand constraint for the piece of size 15. Write the full expression, that is, do not use the summation operator (that is, do not use $\sum$).

b) Assume the goal is to minimize the total waste. Write the objective function. Write the full expression, that is, do not use the summation operator (that is, do not use $\sum$).

Question 7

A certain company has ten jobs to do in the next week. This company employs technicians (TECH), mechanical engineers (ME) and electrical engineers (EE). The company currently has 7 mechanical engineers and 7 electrical engineers (the number of technicians is high enough not to be a concern). Each job can be done by one of the following six combinations of personnel.

Combination	Personnel Used
1	TECHs
2	1 ME
3	2 ME
4	1 EE
5	2 EE
6	1 ME + 1 EE

Let t_{ij} denote the time it takes to do job $j, j = 1,2, \dots ,10$ with personnel combination $i, i = 1,2, \dots ,6$. Also let X_{ij} be binary variables that are equal to 1 if job j is performed by personnel combination i, with $i = 1,2, \dots ,6, j = 1,2, \dots ,10$. Write an ILP model that assigns personnel to jobs, in order to minimize the total time required to do all the jobs.

Question 8

Consider a problem with only binary variables X_j representing yes / no decisions (variable X_j corresponds to decision j). Write the following constraints. Write the full expression, that is, do not use the summation operator (that is, do not use $\sum$).

a) We have to choose at least three among decisions 1, 5, 7, 10, 12 and 13.

b) If we choose decision 2, then decision 4 must be chosen; also, if we do not choose decision 2, decision 4 must also not be chosen.

c) If we choose at least three of decisions 3, 6, 9, 11, 14 and 15, then decision 20 must also be chosen.

Question 9

Consider the following three constraints:

$g_1(X_1, X_2, \dots , X_n) \geq b_1;$
$g_2(X_1, X_2, \dots , X_n) \leq b_2;$
$g_3(X_1, X_2, \dots , X_n) \geq b_3.$

Assume we wish that at least two of these constraints are met. Write the modified set of constraints that assures this.

Question 10

Consider the following ILP model:

Max $\mathrm{F} = 24x_1 + 2x_2 + 20x_3 + 4x_4$

subject to:

$$8x_1 + x_2 + 5x_3 + 4x_4 \leq 9$$
$$x_1, x_2, x_3, x_4 \geq 0 \text{ and integer}$$

Write the LP relaxation that will be solved at node 1.

Question 11

Consider the following BIP model:

Max $\mathrm{F} = 24x_1 + 2x_2 + 20x_3 + 4x_4$

subject to:

$$8x_1 + x_2 + 5x_3 + 4x_4 \leq 9$$
$$x_1, x_2, x_3, x_4 \in \{0,1\}$$

A B&B method starting with an empty incumbent is being used to solve this problem. The LP relaxation solved at node 1 is:

Max $\mathrm{F} = 24x_1 + 2x_2 + 20x_3 + 4x_4$

subject to:

$$\begin{array}{lllll} 8x_1 + & x_2 + & 5x_3 + & 4x_4 & \leq 9 \\ x_1 & & & & \leq 1 \\ & x_2 & & & \leq 1 \\ & & x_3 & & \leq 1 \\ & & & x_4 & \leq 1 \\ \multicolumn{5}{l}{x_1, x_2, x_3, x_4 \geq 0} \end{array}$$

The optimal solution of this LP relaxation is:

$$x_1 = 0.5;\ x_2 = 0;\ x_3 = 1;\ x_4 = 0; F_1 = 32.$$

a) What is the objective function value of the initial incumbent solution? Justify your answer.

b) What kind of bound (upper or lower) does the incumbent provide? Justify your answer.

c) Can node 1 be fathomed? Justify your answer.

d) What kind of bound (upper or lower), if any, does the objective function of the optimal solution of the LP relaxation solved in node 1 provide? Justify your answer

e) What are the branching conditions (variable and values) for node 1? Justify your answer?

Question 12

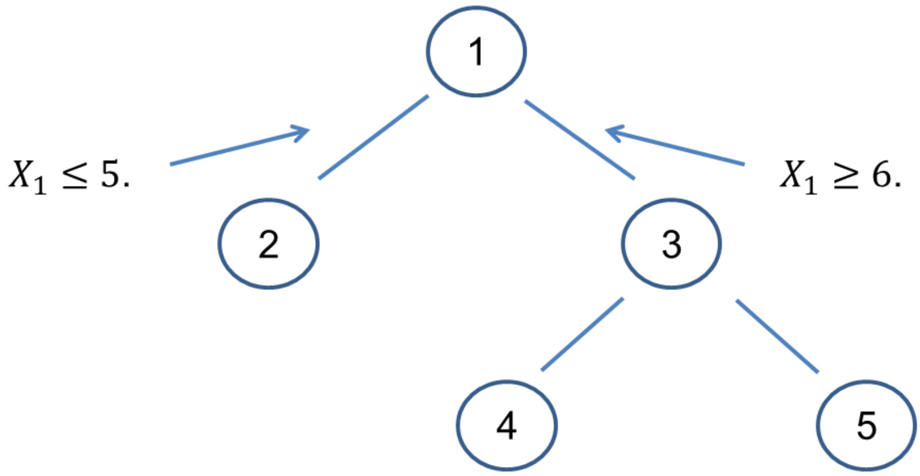
Consider a Branch-and-Bound algorithm for a BIP problem with three variables (X_1, X_2, X_3) and a maximization objective function (epsilon-optimality is not used, so the algorithm will only fathom a node when it cannot possibly lead to a solution that is better than the incumbent). The optimal solution of the linear programming relaxation of a node has just been calculated (this node does not correspond to a complete solution). The objective function value of this optimal solution is equal to 750. Also, in this optimal solution we have $X_1 = 1$ and $X_3 = 0$. When can this node be fathomed by the Branch-and-Bound algorithm? Also, if the node is not fathomed, how will branching be performed on this node? Justify your answer.

Question 13

Consider a Branch-and-Bound algorithm for a BIP problem with a minimization objective function. Assume the objective function value of the incumbent is equal to 1000. Also assume the optimal objective function value of the LP relaxation for a certain node is equal to 925. In the context of epsilon-optimality, consider a tolerance, defined in percentage. Assume the node mentioned above was fathomed by bounding. What can you say about the tolerance value (%)? Justify your answer.

Question 14

Consider a minimization ILP problem with two non-negative variables, that is, $X_1, X_2 \in Z_{\geq 0}$. Consider a B&B procedure that started with an empty incumbent. Also consider the following tree and table with all the nodes that were generated by B&B until it stopped with the optimal solution. Nodes are numbered in the order in which they were generated. Also, the information regarding the optimal solution of the LP relaxation is given in the following format: (optimal value of X_1, optimal value of X_2; optimal objective function value).



node	optimal solution of LP relaxation
1	(?, 4; 3800)
2	(5, 6; 4200)
3	(7.5, 8; 4000)
4	(?, ?; 4100)
5	(?, ?; 4500)

a) What are the values that X_1 may take in the optimal solution of the LP relaxation of node 1?

b) What is the first (non-empty) incumbent found by the B&B procedure?

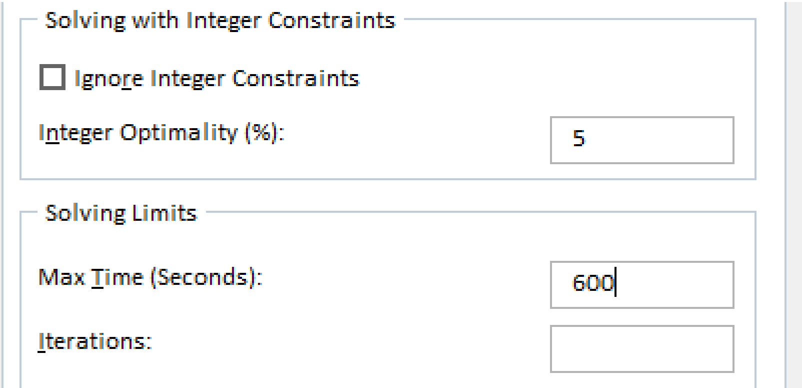
c) When node 3 was branched, what constraints were used to create its subproblems? (Note: just mention the constraints – you may not be able to associate them with a specific subproblem!)

d) What is the optimal objective function value of the original ILP problem?

e) What are the possible reason or reasons for the fathoming of nodes 4 and 5?

Question 15

Consider the following screenshot from the Solver Options dialog box.



If we apply the B&B method with these settings, will we obtain a solution that is not worse than the optimum by more than 5%? Justify your answer.