ECE 595 HW6

nakulupadhya1

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1 Problem 1

$\odot$ is negative and $\oplus$ is positive

1.1 Part A

The VC dimension of this hypothesis set is 2. Possible Combinations with their respective hypothesis:

- $\odot \odot | \rightarrow$
- $\odot | \rightarrow \oplus$
- $\oplus \leftarrow | \odot$
- $| \rightarrow \oplus \oplus$

If we extend this to $N = 3$, we find that we cannot generate a hypothesis set for $\odot \oplus \odot$, therefore we cannot shatter this set.

1.2 Part B

The VC dimension of this hypothesis set is 2.

- $\odot \odot ||$
- $\odot | \oplus |$
- $| \oplus | \odot$
- $| \oplus \oplus |$

If we extend this to $N = 3$, we find that we cannot generate a hypothesis set for $\oplus \odot \oplus$, therefore we cannot shatter this set.

1.3 Part C

The VC dimension of this is 1, since we are stuck with a zero center. If we extend to $N = 2$, then we can't have an orientation where a hypothesis can be generated for both points being positive, without it being unable to generate a hypothesis where only one is positive.

1.4 Part D

The VC dimension of this hypothesis set is 3.

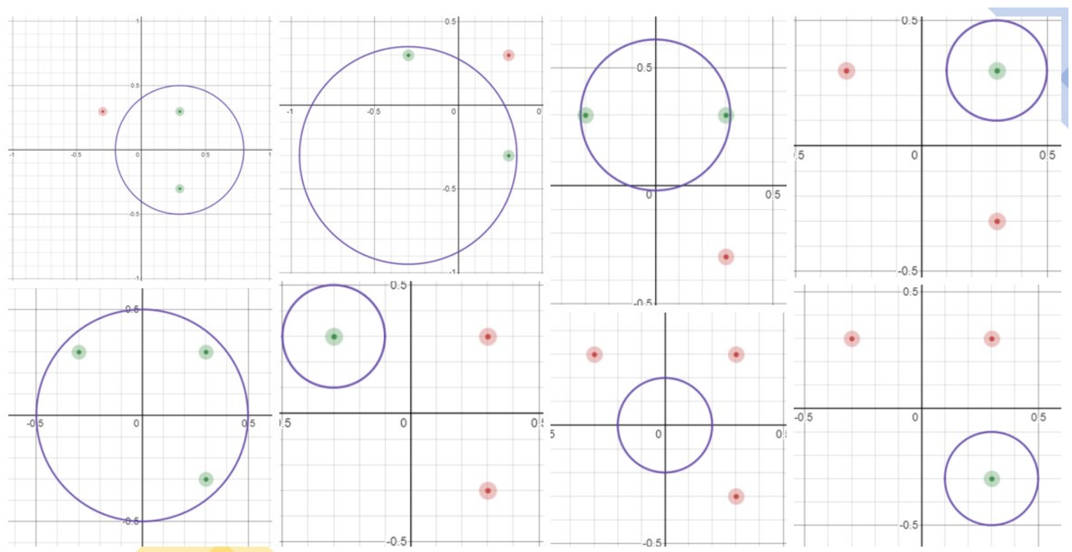

Figure 1: N=3 orientations

If we extend this to $N = 4$, we cannot shatter since a circle wouldn't work in any orientation of the points if the 2 points farthest to each other are positive while the other 2 points are negative.

2 Problem 2

Any x can be represented as $x_N = 10^N; N \in \mathbb{R}$. α can then be represented as $0.d_1 d_2 ... d_N$, where $d_n = 1 \iff y_n = -1; d_n = 2 \iff y_n = 1$. Therefore, our hypothesis set is now $\{-1, +1\}^N$ or $m_{\mathcal{H}}(N) = 2^N$ for all n, which means or VC dimension is infinity.

3 Problem 3

3.1 Part A

$$
\begin{aligned}
g^{(\mathcal{D})}(x') &= \hat{\theta}^T(x') = (x')^T\hat{\theta} && (1)\\
&= (x')^T[(X^TX)^{-1}X^Ty] && (2)\\
&= (x')^T[(X^TX)^{-1}X^T(X\theta+e)] && (3)\\
&= (x')^T[\theta+(X^TX)^{-1}X^Te] && (4)\\
&= (x')^T\theta+(x')^T(X^TX)^{-1}X^Te && (5)
\end{aligned}
$$

3.2 Part B

$$
\begin{aligned}
\bar{g}^{(\mathcal{D})}(x') &= \mathbb{E}_{\mathcal{D}}[g^{(\mathcal{D})}(x')] && (6)\\
&= \mathbb{E}_e[(x')^T\theta+(x')^T(X^TX)^{-1}X^Te] && (7)\\
&= (x')^T\theta+(x')^T(X^TX)^{-1}X^T\mathbb{E}_e[e]; \mathbb{E}_e[e]=0 && (8)\\
&= (x')^T\theta && (9)
\end{aligned}
$$

This is an unbiased estimator of the original model since its the same form of the original model, and θ is the same as the model's θ

3.3 Part C

$$
\begin{aligned}
\mathrm{var}(x') &= \mathbb{E}_{\mathcal{D}}[(g^{(\mathcal{D})}(x')-\bar{g}^{(\mathcal{D})}(x'))^2] && (10)\\
&= ((x')^T(X^TX)^{-1}X^T)\mathbb{E}_e[ee^T](X(X^TX)^{-1}x') && (11)\\
&= ((x')^T(X^TX)^{-1}X^T)\sigma^2I(X(X^TX)^{-1}x') && (12)\\
&= \sigma^2(x')^T(X^TX)^{-1}x' && (13)
\end{aligned}
$$