Page No.
Date

Ashish Upadhyay
CST SPL 1
Roll no. 16
DAA Tutorial 1

Ans. 1 Asymptotic notations are the mathematical notations used to describe the running time of an algorithm when the input tends towards a particular limiting value.

Asymptotic notations are of 3 type:

1. Big O notation : It is the upper bound of the running time of an algorithm and it gives the worst ~~time~~ case complexity.

2. Omega Notation : It is the lower bound of the running time of an algorithm and gives the best case complexity of an algorithm.

3. Θ - Notation (Theta) : This encloses the function from above and below. It is used to determine average case complexity of an algorithm.

~~e.g. The~~

Ans. 2 ~~i = 1, 2, 3,~~ i = 1, 2, 4, 8, 16, ... , n

This is a G.P.

$a_k = a r^{k-1}$

$a = 1$, $r = 2$

$n = 2^{k-1}$

$\Rightarrow \quad k = \log_2 n + 1$

$\therefore \quad T(n) = O(\log_2 n)$

Ans

Ans. 3 $\quad T(n) = 3T(n-1)$ ——— ①

$T(0) = 1$ ——— ②

putting $n = n-1$ in ①

$T(n-1) = 3T(n-2)$ ——— ③

putting ③ in ①

$T(n) = 3(3T(n-2)) = 3^2 T(n-2)$ ——— ④

putting $n = n-2$ in equ. ①

$T(n-2) = 3T(n-2-1) = 3T(n-3)$ ——— ⑤

putting value of $T(n-2)$ from ⑤ to ④

$T(n) = 3^2[3T(n-3)] = 3^3 T(n-3)$

$\therefore$ we can see the below trend:

$T(n) = \cancel{3^k(}\; 3^k T(n-k)$

Let $n-k = 0 \quad \therefore \; n = k$

$T(n) = 3^n T(0)$

as $T(0) = 1$

$\therefore \quad T(n) = 3^n$

$T(n) = O(3^n)$

Ans.

Page No.
Date

Ans (4.) $T(n) = \left[2T(n-1) - 1 \; ; \because n > 0 \; ; \text{ otherwise } 1 \right]$

$T(n) = 2T(n-1) - 1$ ——— (1)
$T(0) = 1$ ——— (2)

putting $n = n-1$ in (1):
$T(n-1) = 2T(n-2) - 1$ ——— (3)

putting values in (3) to (1)
$T(n) = 2(2(T(n-2) - 1) - 1$
$T(n) = 2^2T(n-2) - 2 - 1$ ——— (4)

putting value $n = n-2$ in (1):
$T(n-2) = 2T(n-2-1) - 1$
$= 2(T(n-3)) - 1$ ——— (5)

putting value of (5) in (4):
$T(n) = 2^2[2T(n-3) - 1] - 2 - 1$
$T(n) = 2^3T(n-1) - 2^2 - 2^1 - 2^0$ ——— (6)

$\therefore$ the trend is as below:
$T(n) = 2^kT(n-k) - 2^{k-1} - 2^{k-2} - 2^{k-3} - \dots 2^0$
let $n-k=0$
$\therefore n = k$
$T(n) = \cancel{2^n T(0)}\, 2^n\, T(n-n) - 2^{n-1} - 2^{n-2} \dots 2^0$
~~as~~
$T(n) = 2^n T(0) - 2^{n-1} - 2^{n-2} \dots 2^0$
$\Rightarrow T(n) = 2^n - 2^{n-1} - 2^{n-2} \dots 2^0$ $\{T(0) = 1\}$
$\cancel{T(n) = 2^n - 2^{n-1}}$ $T(n) = 2^n - (1 + 2^1 + 2^2 \dots 2^{n-1})$
$T(n) = 2^n - \left(\frac{1 \times (2^n - 1)}{2-1}\right)$

$T(n) = \cancel{2^n} - \cancel{2^n} + 1$

$\therefore T(n) = O(1)$ Ans.

Ans. 5

for $i = 1$, $s = 1$
for $i = 2$, $s = 1+2$ $\quad s = 3$
for $i = 3$, $s = 1+2+3$ $\quad s = 6$

$\therefore$ this is the sum of 'n' natural nos.

$s = \frac{k(k+1)}{2}$

To stop the iteration, s should be greater than n.

$\therefore$

$\frac{k(k+1)}{2} > n$

$\Rightarrow k^2 + k > 2n$

$\Rightarrow k^2 > n$

$\Rightarrow k > \sqrt{n}$

$\therefore O(k) = \sqrt{n}$ Ans.

Ans. 6

$i = 1, 2, 3, \ldots n$
$i^2 = 1, 4, 9, \ldots n^2$

$\therefore i^2 <= n$ or $i <= \sqrt{n}$

now, $a_k = a + (k-1)d$
$\Rightarrow a = 1$, $d = 1$
$a_k <= \sqrt{n}$
$\sqrt{n} = 1 + (k-1) \cdot 1$
$\sqrt{n} = k$
$\therefore T(n) = O(\sqrt{n})$ Ans.

Page No.
Date

Ans. 7

(i)	(j)	(k)
$\frac{n}{2}$	$(\log_2 n)$	$(\log_2 n)$
⋮	⋮	⋮
n		
$(\frac{n}{2}+1)$ times	$\log_2 n$	$(\log_2 n)$

$$O(i * j * k) = O\left(\left(\frac{n}{2}+1\right) \times \log_2 n \times \log_2 n\right)$$

$$= O\left(\left(\frac{n}{2}+1\right) \times (\log_2 n)^2\right)$$

$$\therefore T(n) = O(n(\log_2 n)^2)$$ Ans.

Ans. 8

$T(n) = T(n-3) + n^2$ —— ①

$T(1) = 1$ —— ②

Putting $n = n-3$ in ①:

$T(n-3) = T(n-3-3) + (n-3)^2$ —— ③

Putting ③ in ①:

$T(n) = T(n-6) + (n-3)^2 + n^2$ —— ④

putting $n = n-6$ in ①:

$T(n-6) = T(n-9) + (n-6)^2$ —— ⑤

putting ⑤ in ④:

$T(n) = T(n-9) + (n-6)^2 + (n-3)^2 + n^2$

after generalization we get:

$$T(n) = T(n-3k) + (n-3(k-1))^2 + (n-3(k-2))^2 + \cdots + n^2$$

let $n-3k=1$,

$$\frac{n-1}{3} = k$$

$$T(n) = T(1) + \left(n-3\left(\frac{n-1}{3}-1\right)\right)^2 + \left(n-3\left(\frac{n-1}{3}-2\right)\right)^2 + \cdots + n^2$$

$$T(n) = T(1) + [n - \cancel{3}(n-1\cancel{3}-3]^2 + [n-(n-1\cancel{3}-6]^2 + [n-(n-1-1)]^2 + \cdots + n^2$$

$$T(n) = 1 + (3+1)^2 + (6+1)^2 + \cdots + n^2$$

$$T(n) = 1 + 4^2 + 6^2 + \cdots + n^2$$

$$\cancel{= n^2 + \ldots}$$

$$\therefore T(n) = O(n^2)$$

Ans.