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MATH 170 Sec. 03

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MATH 170 Final Project Report

1) **Description of the Disease Model and All Assumptions Made:**

Introduction:

Our disease model is based on the new COVID-19 Variant called Omicron and how it will coexist with the previous Delta Variant in the US. We are looking to find what happens when the Omicron Variant is introduced into the population while the Delta Variant is still widespread across the nation. Additionally, based on our findings, we will be trying to predict what will become of these two strains in the future.

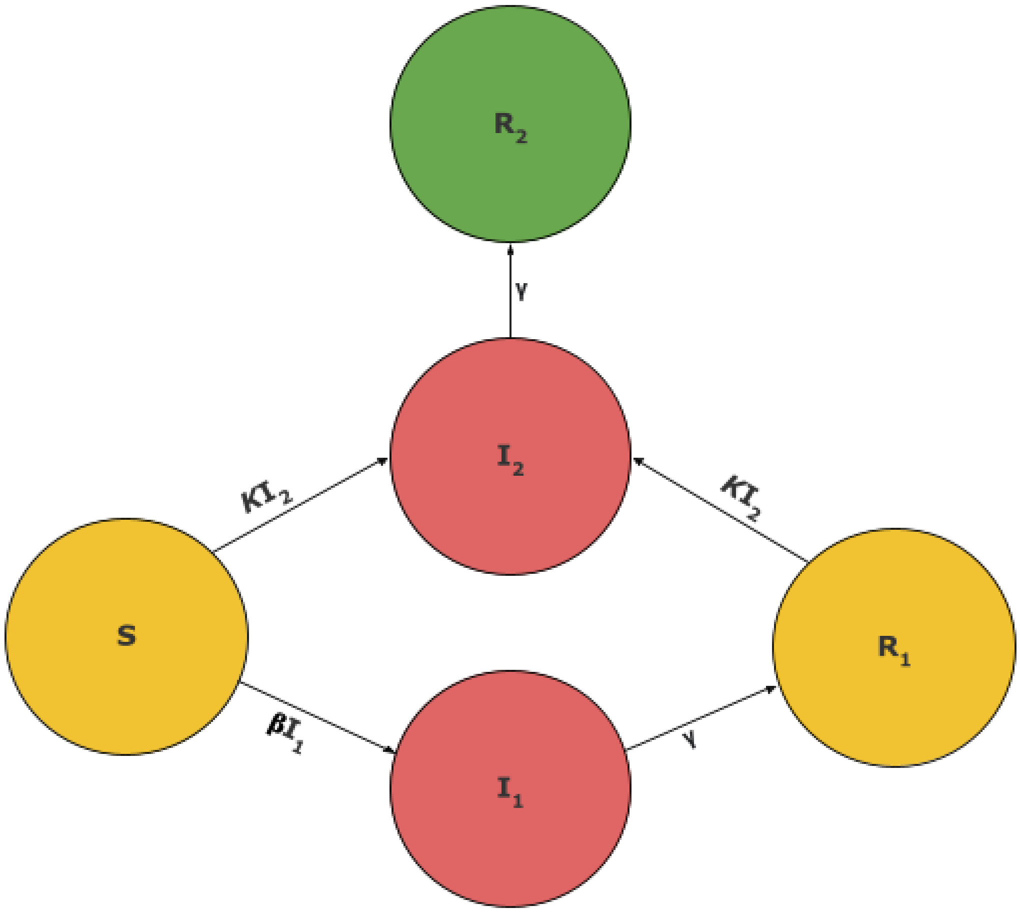
Assumptions:

To find the answers to the model, this would only be possible if we made some assumptions to set it up and these are what we came up with.

- There are no vaccines to help prevent both variants.
- A person can only be infected by one variant at a time.
- If a person were infected by the Delta variant, they could still be susceptible to the Omicron variant after recovery. But, if they were infected by the Omicron variant, they would be immune to both Omicron and Delta once recovered.
- The recovery rate for both variants are the same because with the lack of data and knowledge about Omicron, we thought it would be in the best interest to make both recovery rates the same value.

- The Omicron variant is far more transmissible than the Delta variant meaning that the infection rate would be higher for Omicron.

2) Block Diagram of Our Model With Various Compartments and Transition Rates:



Compartments:

S = Susceptible Population (to both Delta and Omicron)

I_1 = Infectious Population with COVID-19 Delta Variant

I_2 = Infectious Population with COVID-19 Omicron Variant

R_1 = Recovered Population from COVID-19 Delta Variant

R_2 = Removed Population from COVID-19 Omicron Variant

Rates:

β = COVID-19 Delta Variant's average # of contacts per host per time

K = COVID-19 Omicron Variant's average # of contacts per host per time

γ = COVID-19 Delta and Omicron Variants' recovery rate

Sidenote: $K > \beta$ due to the transmissibility of Omicron being higher than Delta

3) The System of ODEs and Its Relevant Parameters:

ODE System:

$$\frac{dS}{dt} = -\beta I_1 S - K I_2 S$$

$$\frac{dI_1}{dt} = \beta I_1 S - \gamma I_1$$

$$\frac{dI_2}{dt} = K I_2 S + K I_2 R - \gamma I_2$$

$$\frac{dR_1}{dt} = \gamma I_1 - K I_2 R_1$$

$$\frac{dR_2}{dt} = \gamma I_2$$

Relevant Parameters:

COVID-19 Delta Variant's R_o: $R_B = \frac{NB}{\gamma}$

COVID-19 Omicron Variant's R_o: $R_K = \frac{NK}{\gamma}$

Sidenote: $R_K > R_B$ due to the transmissibility of Omicron being higher than Delta

4) Long Term Behaviors and the Their Dependance on the Parameters:

Non-Dimensionalization Work:

Let $S \rightarrow Ns$, $I_1 \rightarrow Ni_1$, $I_2 \rightarrow Ni_2$, $R_1 \rightarrow Nr_1$, $R_2 \rightarrow Nr_2$, $t \rightarrow \frac{t}{\gamma}$

$N\gamma \frac{dS}{dt} = -\beta N^2 I_1 S - K N^2 I_1 S \rightarrow -\frac{NB}{\gamma} I_1 S - \frac{NK}{\gamma} I_1 S \rightarrow$ Plug in Corresponding R_o

N$\gamma \frac{dI_1}{dt} = \beta N^2 I_1 S - \gamma N I_1 \rightarrow \frac{NB}{\gamma} I_1 S - I_1$ → Plug in Corresponding R_o

N$\gamma \frac{dI_2}{dt} = KN^2 I_1 S + KN^2 I_1 R_1 - \gamma N I_2 \rightarrow \frac{NK}{\gamma} I_1 S + \frac{NK}{\gamma} I_1 R_1 + I_2$ → Plug in Corresponding R_o

N$\gamma \frac{dR_1}{dt} = \gamma N I_1 - KN^2 I_2 S \rightarrow I_1 - \frac{NK}{\gamma} I_2 S$ → Plug in Corresponding R_o

N$\gamma \frac{dR_2}{dt} = \gamma N I_2 \rightarrow I_2$

Finalized Non-Dimensionalized ODE System:

$\frac{dS}{dt} = -R_K I_1 S - R_B I_2 S$

$\frac{dI_1}{dt} = R_B I_1 S - I_1$

$\frac{dI_2}{dt} = R_K I_2 S + R_K I_2 R_1 - I_2$

$\frac{dR_1}{dt} = I_1 - R_K I_2 R_1$

$\frac{dR_2}{dt} = I_2$

More Assumptions:

Before we found data on the Omicron and Delta Variants, we set up a couple more assumptions. We assumed that people who caught previous COVID-19 variants such as the Alpha and Beta variants were still susceptible to the Delta and Omicron variants. Furthermore, the time periods between finding the first Delta variant case and the first Omicron variant case will all be allocated as Delta variant cases.

COVID-19 Data:

- United States Population: ≈ 330 Million
- First Case of COVID-19 Delta Variant Detected in the US: May 20th, 2021
- First Case of COVID-19 Omicron Variant Detected in the US: Dec. 1st, 2021

- Number of Cases in the US on May 20th, 2021: ≈ 33 Million Cases
- Number of Cases in the US on Dec. 1st, 2021: ≈ 48.5 Million Cases
- Number of Delta Cases From May 20th to Dec. 1st: ≈ 15.5 Million Cases
- Average Covid Recovery Time: 2-6 Weeks → ≈ 1 Month
- Number of Cases in the US on Nov. 1st, 2021: ≈ 46 Million
- Number of Delta Cases From Nov. 1st to Dec. 1st (Active Cases): ≈ 2.5 Million
- Number of Delta Cases From May 1st to Nov. 1st (Recovered Cases): ≈ 13 Million
- COVID-19 Delta Variant's R_o or $R_B = \frac{NB}{\gamma}$: ≈ 5.08
- COVID-19 Omicron Variant's R_o or $R_K = \frac{NK}{\gamma}$: N/A but it is known to be much higher than the Delta Variant's R_o because recent studies show that it is 4x more transmissible. With this information we will assume that $R_K = R_B \times 4 \approx 20.32$

Sources: Google Covid Tracker and CDC Covid Database

Known Information for Analyzing COVID-19 Data:

According to the notes we took in class, "An epidemic occurs when $\frac{dI}{dt} > 0$" and "We expect $\frac{dI}{dt} < 0$ when $R_o < \frac{1}{S(t)}$". With these things in mind, we calculated what $\frac{dI_1}{dt}$ and $\frac{dI_2}{dt}$ are to find whether the two variants are Endemic or not.

Solving for Parameters at Beginning of COVID-19 Omicron Variant Endemic:

- Delta $S(0) = \frac{Total\ Population - Total\ Number\ of\ Delta\ Cases\ Till\ Dec\ 1st}{Total\ Population} \approx \frac{330 - 15.5}{330} \approx 0.95$
- Omicron $S(0) \approx 1$ or (0.999)
- $I_1(0) = \frac{Active\ Delta\ Cases}{Total\ Population} \approx \frac{2.5}{330} \approx 0.01$
- $I_2(0) \approx$ very small amount (0.001)

- $R_1(0) = \frac{Recovered\ Delta\ Cases}{Total\ Population} \approx \frac{13}{330} \approx 0.04$
- $R_2(0) = 0$

Sidenote: Since people aren't immune to the Omicron Variant even if they were infected by the Delta Variant, the S(0) for Omicron will be very close to 1 because whether they are in the Delta Susceptible, Infectious, or Recovered compartments, they will all be susceptible to the Omicron variant sooner or later.

Implementation of Analysis Information, COVID-19 Data, and the Parameters Found:

<u>*Method 1:*</u>

For the variants to be Endemic we will need to see if their R_o's are greater than $\frac{1}{S(t)}$ or not.

$R_B \approx 5.08$ and $\frac{1}{S(t)} = \frac{1}{0.95} \approx 1.05 \rightarrow 5.08 > 1.05 \rightarrow$ Delta is Endemic

For the Omicron Variant to be Endemic as well, it's the same process.

$R_K \approx 20.32$ and $\frac{1}{S(t)} = \frac{1}{\approx 1} \approx 1 \rightarrow 20.32 > 1 \rightarrow$ Omicron is Endemic

<u>*Method 2:*</u>

Another method to to check if the variants are endemic or not is to find what their $\frac{dI}{dt}$ is and whether or not they are greater than 0.

$\frac{dI_1}{dt} = R_B I_1 S - I_1 \rightarrow 5.08 \text{ x } 0.01 \text{ x } 0.95 - 0.01 \approx 0.04 \rightarrow 0.04 > 0 \rightarrow$ Delta is Endemic

For the Omicron Variant to be Endemic as well, it's the same process.

$\frac{d2}{dt} = R_B I_1 S - I_1 \rightarrow 20.32 \text{ x } 0.001 \text{ x } 1 - 0.001 \approx 0.02 \rightarrow 0.02 > 0 \rightarrow$ Omicron is Endemic

Sidenote: Used the parameter values found above and plugged them into our DEs.

Analysis of the Current State and Predicting the Future of Both Variants:

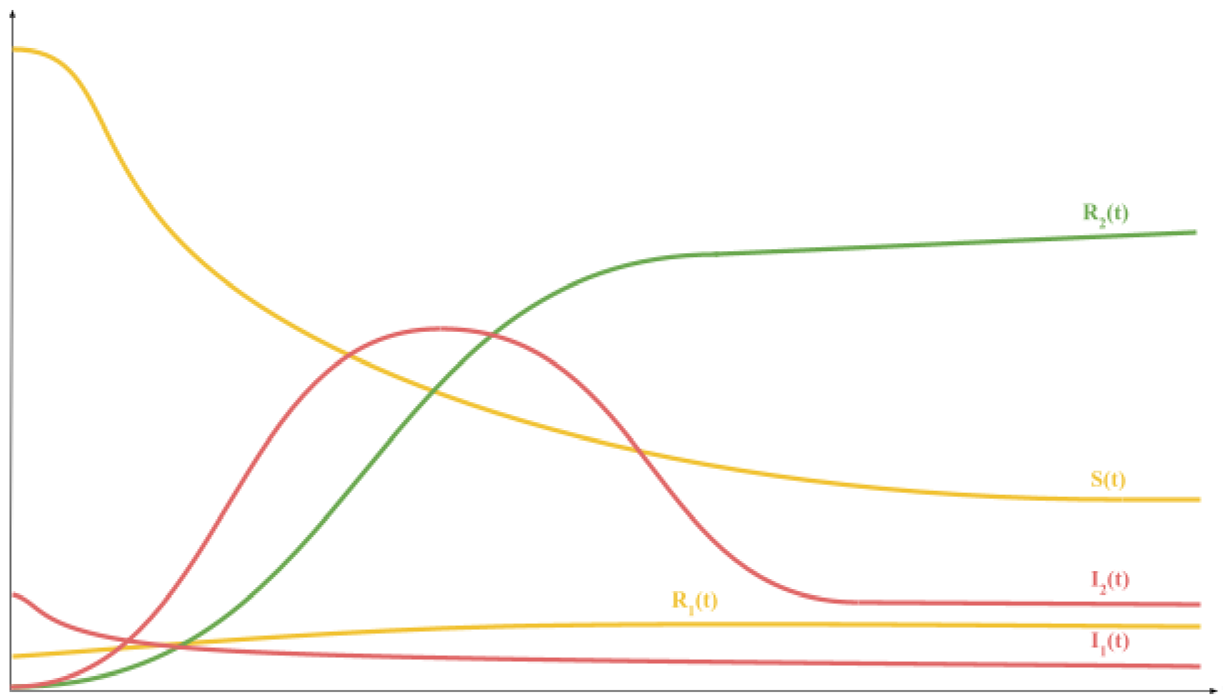
The application we did above was when the Omicron variant was at its beginning portion of its rise to becoming an endemic. It also showed that the Delta variant is endemic when the Omicron variant was introduced into the system. This is because according to the notes from class, the R_o values of the variants are just too high that no matter how much $\frac{1}{S(t)}$ is, it is still not significant enough to become a Disease Free Equilibrium. It could possibly happen if most of the population is infected and crosses the herd immunity threshold but as we know of the COVID-19, antibodies and vaccines only work for so long that you can get reinfected or get a breakthrough case.

Adding onto the model, there was also one thing that we failed to focus on due to it being very complex and it is how both variants will interact with each other as they trend into the future. To answer this question, we made an educated guess by taking all of the stuff we found into account and most importantly focused on the R_o values of each variant. With the R_o value of the Omicron variant being so much higher than the R_o value of the Delta variant, it will most probably descend into a competition between the two variants. This will depend on which variant will spread faster and ultimately, we think it will end with the Omicron variant showing dominance due to its high R_o value.

We predict this scenario based on past experiences such as the Delta variant taking dominance over the Alpha strain and as well as what our ODE model suggests. When the Delta variant was detected in the US we saw how it rose to become the dominant strain in the country while the Alpha strain sort of died out. It wasn't wiped out completely but due to its low R_o value and low transmissibility it couldn't compete with the Delta variant. This is what we think will happen to the Delta variant eventually as the Omicron variant's R_o value is bound to be much higher than Delta's.

When looking at it from our ODE Systems perspective we see the same thing happening. With K being higher in value than β and there being two K for the infectious compartment of the Omicron variant, we can clearly see that it has a step up in transmission. Due to this, the Delta variant will probably not be able to keep up with the Omicron variant in the future and end up in the same scenario as the Alpha variant when the Delta variant was introduced into the population. This, however, does not mean that the Delta variant will be eradicated unless it runs out of hosts to infect. It will probably have some spikes in infecting certain groups of hosts but then die back down.

Graph of Our Compartmental Model:



Conclusion:

Based on what we found through our Compartmental Model and its ODE system, we can see that at the beginning of the Omicron Variant induced endemic, both the Delta variant and Omicron variant will be at an Endemic Equilibrium. As time goes along, we as a group predict that the Omicron Variant will effectively stay in that Endemic Equilibrium state while the Delta variant will slowly decline until it is no longer the dominant strain and quite possibly even die out if it no longer has hosts to infect.