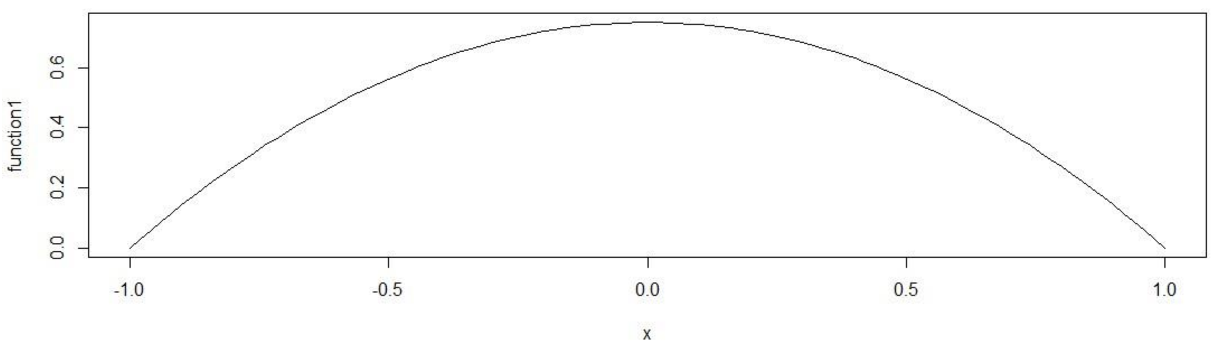
Probability Density Function

Imagine that a continuous random variable X defined on the range (−s, s) follows the probability density function:

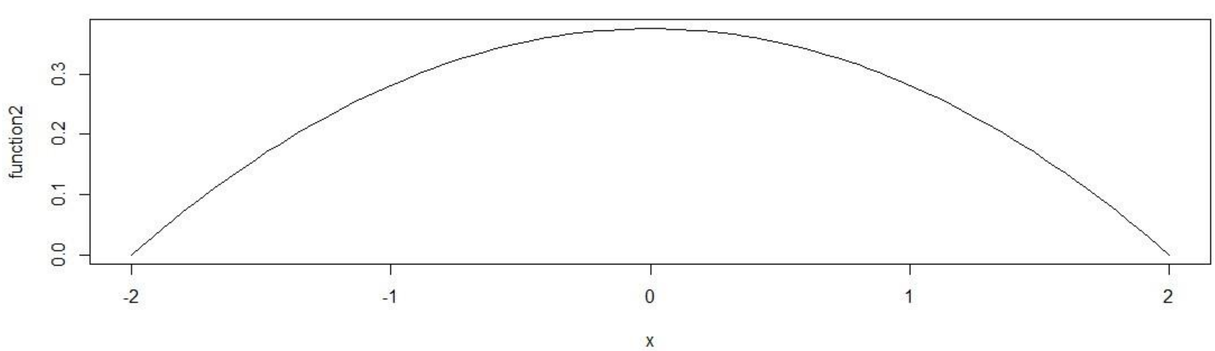
Plot the probability density function of X when s = 1 and s = 2:

For function having boundery: (-2,2)

```
function1=function(x) ((3/4)*(1-x^2))  >
plot.function(function1,from=-1, to=1)
```




```
function2=function(x) ((3/(4*2)*(1-(x/2)^2))) >
plot.function(function2,from=-2, to=2)
```



b) Determine the expected value of X, i.e., E [X]:

$$= 6(E(X_1))^2 + 3(V(X_1) + (E(X_1))^2)$$

$$= 9(E(X_1))^2 + 3V(X_1))$$

$$= 9 \times 12.25 + 3 \times 2.91$$

$$= 110.25 + 8.73$$

$$= \boxed{118.98}$$

Q→4(b)

$$\rightarrow E(X) = \int_{-s}^{s} x\, p(x)\, dx$$

$$= \int_{-s}^{s} x \times \frac{3}{4s}\left(1-\left(\frac{x}{s}\right)^2\right) dx$$

$$= \frac{3}{4s}\int_{-s}^{s}\left(x - \left(\frac{x^3}{s}\right)\right) dx$$

$$= \frac{3}{4s}\left[\left[\frac{x^2}{2}\right]_{-s}^{s} - \left[\frac{x^4}{4s}\right]_{-s}^{s}\right]$$

$$= \frac{3}{4s}\left[0-0\right]$$

$$\rightarrow E(X) = \boxed{0}$$

c) Determine the variance of X, i.e., V [X] (it will be a function of s):

4(c)

→ Variance of X,

$$V(X) = \int_{-s}^{s} (x - E(x))^2 \, p(x)\, dx,$$

where $E(x) = 0$ as above,

$$= \int_{-s}^{s} x^2 p(x)\, dx$$

$$= \int_{-s}^{s} x^2 \times \frac{3}{4s}\left(1 - \left(\frac{x}{s}\right)^2\right) dx$$

$$= \frac{3}{4s} \int_{-s}^{s} \left(x^2 - \frac{x^4}{s^2}\right) dx$$

$$= \frac{3}{4s}\left[\left(\frac{x^3}{3}\right)_{-s}^{s} - \left(\frac{x^5}{5s^2}\right)_{-s}^{s}\right]$$

$$= \frac{3}{4s}\left(\frac{2s^3}{3} - \frac{2s^5}{5s^2}\right)$$

$$= \frac{3}{\cancel{4}_2 \cancel{s}} \times \cancel{2} s^{\cancel{3}\,2}\left(\frac{1}{3} - \frac{1}{5}\right)$$

$$\rightarrow V(X) = \frac{3}{\cancel{2}} \times s^2 \left(\frac{\cancel{2}}{15}\right) = \boxed{\frac{s^2}{5}}$$

d) Determine the cumulative distribution function for this distribution, i.e., P(X ≤ x).

e) Determine the expected value of |X|, i.e., E [abs(X)].

4(d): $P(X \le x) = \int_{-\infty}^{x} \frac{3}{45} \times \left(1-\left(\frac{x}{5}\right)^2\right) dx$

$= \frac{3}{45}\left((x)_{-\infty}^{x} - \left(\frac{x^3}{3 5^2}\right)_{-\infty}^{x} \right)$

$= \frac{3}{45}\left(x+\infty - \frac{x^3}{3 5^2} + \infty \right)$

$=$ Indefined for $(-\infty, x)$

4(e) → The expected value of $|x|$,

→ E(abs(X)) = (?)

→ E(X) = 0 so according to definition of absolute values,

→ $\boxed{E(abs(x)) = 0}$