# Dice Probability

Imagine that we roll two fair six-sided dice (i.e., all six sides have equal probability). Let X1 and X2 be the independent random variables representing these outcomes. Let S = X1 + X2 be the sum of the two rolls.

3a) What is the variance of S, i.e., what is V [S]?

→ P(losing two games)

$$= \frac{P(W_t = 0, W_{t-1} = 0)}{P(W_{t-1} = 0)} \times \frac{P(W_t = 0, W_{t-1} = 1)}{P(W_{t-1} = 1)}$$

$$= \frac{0.2985}{0.2985 + 0.1791} \times \frac{0.1940}{0.1940 + 0.3283}$$

$$= 0.6250 \times 0.3715$$

$$= \boxed{0.2321}$$

3(a)

→ $X_1$ and $X_2$ : the independent random variable

→ $S = X_1 + X_2$ (the sum of the two rolls)

* Expected value of S,

$$\rightarrow E(S) = \sum_{s \in S} s\,P(s)$$

* where, $S = \{2, ----, 12\}$, $X_1 = \{1, 2, 3, 4, 5, 6\}$, $X_2 = \{1, 2, 3, 4, 5, 6\}$, (which are the dice rolls)

$$* \quad E(S) = 2P(2) + 3P(3) + 4P(4) + 5P(5) + 6P(6) + 7P(7) + 8P(8) + 9P(9) + 10P(10) + 11P(11) + 12P(12)$$

$$= 2 \times \frac{1}{36} + 3 \times \frac{2}{36} + 4 \times \frac{3}{36} + 5 \times \frac{4}{36} + 6 \times \frac{5}{36} + 7 \times \frac{6}{36} + 8 \times \frac{5}{36} + 9 \times \frac{4}{36} + 10 \times \frac{3}{36} + 11 \times \frac{2}{36} + 12 \times \frac{1}{36}$$

$$= \frac{1}{36}\left(2+3(2)+12+20+30+42+40+36+30+22+12\right)$$

$$= \frac{252}{36}$$

$$\rightarrow E(s) = \boxed{7}$$

$$\Rightarrow Var(s) = E\left(s - E(s)^2\right)$$

$$= \sum_{s \in S} (s - E(s))^2 \, p(s)$$

$$= \frac{1}{36}\times(2-7)^2 + \frac{2}{36}\times(3-7)^2 + \frac{3}{36}\times(4-7)^2$$

$$+ \frac{4}{36}\times(5-7)^2 + \frac{5}{36}\times(6-7)^2 + \frac{6}{36}\times(7-7)^2$$

$$+ \frac{5}{36}\times(8-7)^2 + \frac{4}{36}\times(9-7)^2 + \frac{3}{36}\times(10-7)^2$$

$$+ \frac{2}{36}\times(11-7)^2 + \frac{1}{36}\times(12-7)^2$$

$$= \frac{1}{36}\left(25+32+27+16+5+0+5+16+27+32+25\right)$$

$$= \frac{1}{36}\times 2\times 105$$

$$= \frac{210}{36}$$

$$\rightarrow Var(s) = \boxed{5.83}$$

3b) Determine the probability distribution of S, i.e., the probability that S = {2, . . . , 12}.

(b.) The probability distribution is as below,

| $X$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $P(X)$ | $\frac{1}{36}$ | $2/36$ | $\frac{3}{36}$ | $\frac{4}{36}$ | $\frac{5}{36}$ | $\frac{6}{36}$ | $\frac{5}{36}$ | $\frac{4}{36}$ | $\frac{3}{36}$ | $\frac{2}{36}$ | $\frac{1}{36}$ |

→ It seems to be discrete uniform distribution where probability is similar with each other.

| $X_2 =$ \ $X_1 =$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

→ As above can be the number of occurrence of S values, which are summed up from $X_1$ and $X_2$.

3c) Write a simple one-line formula describing this probability distribution.

3d) What is the expected value of √ S, i.e., what is E h√ S i ?

(c.) probability dist. function,

$$\rightarrow P(X) = \begin{cases} \dfrac{n-1}{36} & \text{for } x \leq 7 \\ \dfrac{n-8}{36} & \text{for } x > 7 \end{cases}$$

(d.) Expected value of $\sqrt{S}$,

$$\Rightarrow E(\sqrt{S}) = \sum_{s \in S} \sqrt{s}\, P(s)$$

$$= \sqrt{2}\times\frac{1}{36} + \sqrt{3}\times\frac{2}{36} + \sqrt{4}\times\frac{3}{36} + \sqrt{5}\times\frac{4}{36} + \sqrt{6}\times\frac{5}{36} + \sqrt{7}\times\frac{6}{36} + \sqrt{8}\times\frac{5}{36} + \sqrt{9}\times\frac{4}{36} + \sqrt{10}\times\frac{3}{36} + \sqrt{11}\times\frac{2}{36} + \sqrt{12}\times\frac{1}{36}$$

$$= \frac{1}{36}\left(\sqrt{2} + 2\sqrt{3} + 6 + 4\sqrt{5} + 5\sqrt{6} + 6\sqrt{7} + 5\sqrt{8} + 12 + 3\sqrt{10} + 2\sqrt{11} + \sqrt{12}\right)$$

$$\Rightarrow E(\sqrt{S}) = \boxed{2.60}$$

3e) Imagine we roll a third dice, X3. What is the expected value of (X1 + X2 + X3) 2 , i.e., what is E (X1 + X2 + X3) 2?

(e.) for a third dice,

* Expect value $[(x_1+x_2+x_3)^2]$

$$= E((x_1+x_2+x_3)^2)$$

$$= E(x_1^2+x_2^2+x_3^2+2x_1x_2+2x_3x_1+2x_2x_3)$$

$$= E(x_1^2)+E(x_2^2)+E(x_3^2)+2E(x_1)E(x_2)$$
$$+2E(x_2)E(x_3)$$
$$+2E(x_3)E(x_1)$$

→ However, $E(x_1) = E(x_2) = E(x_3)$ as there are same dice rolls,

$[E(x_1) = 2.91$ and $V(x_1) = 3.5]$

$$\Rightarrow E((x_1+x_2+x_3)^2) = 3E(x_1^2) + 2(E(x_1))^2 + 2(E(x_1))^2$$
$$+2(E(x_1))^2$$

$$= 3E(x_1^2) + 6(E(x_1))^2$$