

# PRICING VULNERABLE OPTIONS WITH CORRELATED CREDIT RISK UNDER JUMP-DIFFUSION PROCESSES

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This study extends the framework of Klein [Journal of Banking & Finance, 20, 1211–1229] to price vulnerable options. We provide a pricing model for vulnerable options which face not only default risk but also rare shocks encountered by the underlying asset and the assets of the counterparty. The dynamics of asset prices are governed by jump-diffusions with two sorts of assets correlated with each other. Jumps are divided into idiosyncratic component for each asset price and systematic component affecting the prices of all assets. A closed-form valuation formula is derived for vulnerable European options. Numerical analysis compares the results of this model with those of other pricing formulae, and illustrates jump effects on the vulnerable option prices. © 2013 Wiley Periodicals, Inc. Jrl Fut Mark 34:957–979, 2014

## 1. INTRODUCTION

In recent years, the OTC derivatives market has experienced tremendous development. In mid-2012, the notional amounts outstanding of the OTC derivatives market reached 638 trillion dollars.<sup>1</sup> Since there is no organized exchange, holders of OTC contracts are vulnerable to counterparty credit risk. To protect counterparties from each other's default, the clearing counterparty (CCP) was introduced, which is a process by which financial transactions in equities are cleared by a single central counterparty. Although a large amount of the OTC market has moved towards the CCP, the OTC derivatives remain a significant part of the world of global finance. Counterparty credit risk has gained particular emphasis due to the financial crisis, as evidenced by the sudden bankruptcy of Lehman Brothers and the U.S. government bailout of AIG.

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To model the effect of credit risk when pricing contingent claims, structural approach has been adopted by a lot of previous works, in which credit events are triggered when the firm's value is less than some boundary.<sup>2</sup> Based on this approach, Johnson and Stulz (1987) first incorporate credit risk into option pricing model. They assume that the option is the sole liability of the counterparty and default will happen when the value of the option is greater than the value of the assets of the counterparty. Klein (1996) addresses this issue by allowing the option writer to hold other liabilities which rank equally with payments under the option. In addition, the correlation between the underlying asset and the assets of the counterparty is also considered. In contrast to constant default barrier assumption in Klein (1996), Rich (1996), and Hui, Lo, and Lee (2003) consider the stochastic default barrier. Taking the stochastic interest rate into account, Klein and Inglis (1999) derive a closed-form pricing formula of the vulnerable option via the partial differential equation approach. Klein and Inglis (2001) extend the model of Klein (1996) by incorporating the potential liability of the written option into the default boundary. Arguing that vulnerable options face not only the default risk but also the risk of illiquidity, Hung and Liu (2005) price vulnerable options when the market is incomplete.

Most of the literature on vulnerable options assume that the dynamics of the assets follow the log-normal diffusion process. However, this assumption ignores sudden shocks in price due to the arrival of important new information.<sup>3</sup> The purpose of this study is to provide a new pricing model for vulnerable options, where the dynamics of the underlying asset and the assets of the counterparty follow jump-diffusion processes. Besides retaining the attractive features of Klein (1996), this study has three main characteristics. First, we incorporate jumps in both the underlying asset and the assets of the option writer, which is an improvement over the model of Klein (1996). Second, to make the pricing model more flexible, the correlation between the jumps of the two sorts of assets is considered. Specifically, we divide the jumps into individual jumps for each asset price and common jumps that affect the prices of all assets.<sup>4</sup> Individual jump component reflects the shocks on asset prices experienced by some specific firms, such as mergers and restructuring. Analogously, common jump component reflects the shocks linked to market factors such as financial crisis. Third, we derive a closed-form pricing formula with credit risk and jump risk jointly considered in our framework. The pricing formulae of Black and Scholes (1973), Merton (1976), and Klein (1996) can be regarded as special cases of our result.

In our framework, individual shocks to the underlying asset and the assets of the counterparty are governed by two independent Poisson processes, respectively. To model common shocks, another compound Poisson process is added into the dynamics of the two sorts of assets. After deriving the pricing formula, we compare the results of the proposed model with those of other pricing formulae. Numerical analysis illustrates that jump risk of the assets of the counterparty reduces the value of the vulnerable option prices, and the impact of the jumps of the underlying asset on the prices is more sensitive than the assets of the counterparty.

The remainder of our study is organized as follows. In Section 2, a jump-diffusion model with correlated credit risk is proposed. Moreover, we derive an explicit pricing formula of

<sup>2</sup>Examples of structural approach include Black and Scholes (1973), Merton (1976), Black and Cox (1976), Leland (1994), Longstaff and Schwartz (1995), Leland and Toft (1996), and Anderson and Sundaresan (2000).

<sup>3</sup>There is evidence for jump risk, for example, Merton (1976), Bakshi, Cao, and Chen (1997), Aït-Sahalia (2002), Andersen, Benzoni, and Lund (2002), Carr, Geman, Madan, and Yor (2002), Bakshi, Kapadia, and Madan (2003), Kou and Wang (2004), Bates (2008), and Cai and Kou (2011).

<sup>4</sup>Following the referee's constructive suggestion, we consider the impact of correlation in a more general framework. Similar approach is employed when pricing basket options under jump-diffusions (e.g., Bae, Kang, & Kim, 2011; Xu & Zheng, 2009).

vulnerable European options. Section 3 presents numerical simulations to illustrate our results. Finally, concluding remarks are contained in Section 4.

## 2. PRICING VULNERABLE OPTIONS

### 2.1. Model Description

In this subsection, we adopt jump-diffusion processes to describe our framework for valuing vulnerable European options. Here we mainly focus on the discontinuous changes of prices and correlated credit risk. Our formulation incorporates a jump process to describe discontinuous changes, and it allows for correlation between the underlying asset and the assets of the counterparty for both the continuous and the marked point process components. Correlations are linked to market factors in the marked point process component.

The dynamics of the underlying asset and the assets of the counterparty are assumed under the risk neutral measure  $\mathbb{Q}$ . Suppose that the default-free term structure is flat with an instantaneous riskless rate  $r$ . On a complete probability space  $(\Omega, \mathcal{F}, \mathbb{Q})$ , the dynamics of the underlying asset is governed by the following jump-diffusion process,

$$\frac{dS_t}{S_{t-}} = (r - k_S \lambda_S^*) dt + \sigma_S dW_t^{(1)} + (e^{Z_t^{(1)}} - 1) dM_t^{(1)}, \quad (1)$$

where  $\sigma_S$  is the volatility of the underlying asset and  $W_t^{(1)}$  is a standard Brownian motion. Jumps in the underlying asset price are modeled by the last term with  $dM_t^{(1)}$  which is a Poisson process with intensity  $\lambda_S^*$ . In our formulation, both the jump term  $M_t^{(1)}$  and the intensity  $\lambda_S^*$  consist of two parts,

$$\begin{aligned} M_t^{(1)} &= N_t^{(1)} + N_t, \\ \lambda_S^* &= \lambda_S + \lambda. \end{aligned}$$

Specifically, shocks to the underlying asset price are also composed of two parts: individual shocks, corresponding to  $N_t^{(1)}$ , and common shocks, corresponding to  $N_t$ , where  $N_t^{(1)}$  and  $N_t$  are independent Poisson processes with intensities  $\lambda_S$  and  $\lambda$ , respectively. Here we take common shocks as market factors, which also influence the assets of the counterparty. We make a simple yet reasonable assumption that the jumps which simultaneously happen on the underlying asset and the assets of the counterparty, even if not market risk, are also recognized as common jumps. For example, industry shocks occur and the two parties belong to the same industry.

If the jump happens at time  $t$ , the jump amplitude of the underlying asset is controlled by  $Z_t^{(1)}$ . For any time  $t \neq s$ , we assume that  $Z_t^{(1)}$  and  $Z_s^{(1)}$  are independently and identically distributed. Moreover, the mean percentage jump of the price,  $k_S = \mathbb{E}[e^{Z_t^{(1)}}] - 1$ , is assumed to be finite. In particular, as in Merton (1976), the jump size is assumed to be drawn from a log-normal distribution, and in this situation,  $Z_t^{(1)}$  is normally distributed with mean  $\mu_1$  and standard deviation  $\sigma_1 > 0$ . Then,  $k_S = e^{\mu_1 + \frac{1}{2}\sigma_1^2} - 1$ . Furthermore, the situation where the underlying asset price evolves continuously is modeled by setting  $\lambda_S = \lambda = 0$ .

In the following, we consider counterparty risk with structural approaches as in Klein (1996). A credit loss occurs if the market value of the assets of the counterparty,  $V_T$ , is less than some amount  $D^*$ . Also, this amount is not set to the value of the option but corresponds to the amount of claims  $D$  outstanding at exercise time  $T$ . Once a credit loss occurs at exercise time  $T$ , the recovery is  $\frac{(1-\alpha)V_T}{D}$ , where  $\alpha$  represents the deadweight costs due to the bankruptcy

or reorganization. Taking jump risk into consideration, we also assume that  $V$  is driven by the following jump-diffusion process,

$$\frac{dV_t}{V_{t-}} = (r - k_V \lambda_V^*) dt + \sigma_V dW_t^{(2)} + (e^{Z_{t-}^{(2)}} - 1) dM_t^{(2)}, \quad (2)$$

where  $\sigma_V$  is the volatility of the assets of the counterparty and  $W_t^{(2)}$  is a standard Brownian motion on  $(\Omega, \mathcal{F}, Q)$ . Suppose that  $W_t^{(1)}$  and  $W_t^{(2)}$  have a correlation coefficient  $\rho$ . Shocks to  $V_t$  also consist of the individual component and the common component. Similarly, both the jump term  $M_t^{(2)}$  and intensity  $\lambda_V^*$  are expressed as,

$$\begin{aligned} M_t^{(2)} &= N_t^{(2)} + N_t, \\ \lambda_V^* &= \lambda_V + \lambda, \end{aligned}$$

where  $N_t^{(2)}$  is a Poisson process with intensity  $\lambda_V$ , and independent of  $N_t$  and  $N_t^{(1)}$ . If the jump occurs at time  $t$ , the jump amplitude is controlled by  $Z_t^{(2)}$  with distribution  $N(\mu_2, \sigma_2^2)$ . We assume that for  $t \neq s$ ,  $Z_t^{(2)}$  and  $Z_s^{(2)}$  are independently and identically distributed. When the jump arrives, the mean percentage jump is  $k_V = \mathbb{E}[e^{Z_t^{(2)}}] - 1 = e^{\mu_2 + \frac{1}{2}\sigma_2^2} - 1$ . Moreover, we assume that  $(W_t^{(1)}, W_t^{(2)}), N_t, Z_t^{(1)}, N_t^{(1)}, Z_t^{(2)}$ , and  $N_t^{(2)}$  are mutually independent.

In our framework, the correlated credit risk is considered in both the continuous part and the marked point process part. For the continuous part,  $W_t^{(1)}$  and  $W_t^{(2)}$  have the correlation coefficient  $\rho$ . As for the marked point process part, correlation is linked to market factors  $N_t$ . Conditional on the path of  $N_t$ , the jump components between the underlying asset and the assets of the counterparty are independent. Based on the methods used in Merton (1976), we get an explicit formula for the valuation for vulnerable European options. Due to the similarity between call options and put options, we only show specific calculations and numerical analysis for vulnerable European call options in this study.

## 2.2. Valuation of Vulnerable Options

As in Merton (1976), we assume that  $Z_t^{(i)}, i = 1, 2$  are normally distributed with mean  $\mu_i$ ,  $i = 1, 2$  and standard deviation  $\sigma_i > 0$ ,  $i = 1, 2$ . The value of a vulnerable option is the expectation of the value of the cash flow from a non-vulnerable call option times the value of a claim on the risky counterparty. Denote the value of a vulnerable option by  $C^*$ , which is represented by

$$C^* = e^{-rT} \mathbb{E} \left[ (S_T - K)^+ \left( \mathbf{1}(V_T \geq D^*) + \frac{1 - \alpha}{D} V_T \mathbf{1}(V_T < D^*) \right) \right].$$

According to Itô formula, the following equalities hold:

$$\begin{aligned} \ln S_T &= \ln S_0 + \left( r - \frac{1}{2} \sigma_S^2 - k_S \lambda_S^* \right) T + \sigma_S W_T^{(1)} + \sum_{k=1}^{M_T^{(1)}} Z_{\tau_k^{(1)}}^{(1)}, \\ \ln V_T &= \ln V_0 + \left( r - \frac{1}{2} \sigma_V^2 - k_V \lambda_V^* \right) T + \sigma_V W_T^{(2)} + \sum_{k=1}^{M_T^{(2)}} Z_{\tau_k^{(2)}}^{(2)}, \end{aligned}$$

where  $\tau_k^{(i)}, i = 1, 2$  denote the  $k$ th jump time of  $M_t^{(i)}, i = 1, 2$  respectively. Conditional on  $\mathcal{G}_T^{(n, n_1^{(1)}, n_2^{(2)})} := \{N_T = n, N_T^{(1)} = n_1, N_T^{(2)} = n_2\}$ , the total jump times of  $S_t$  and  $V_t$  are

denoted by

$$\begin{aligned} m_1 &\triangleq n + n_1, \\ m_2 &\triangleq n + n_2, \end{aligned}$$

for simplicity. It is clear that  $\left(\ln \frac{S_T}{S_0}, \ln \frac{V_T}{V_0}\right)$  are bivariate normally distributed with the following properties:

$$\begin{aligned} M_1(m_1) &= \mathbb{E} \left[ \ln \frac{S_T}{S_0} \right] = \left( r - \frac{1}{2} \sigma_S^2 - k_S \lambda_S^* \right) T + m_1 \mu_1, \\ M_2(m_2) &= \mathbb{E} \left[ \ln \frac{V_T}{V_0} \right] = \left( r - \frac{1}{2} \sigma_V^2 - k_V \lambda_V^* \right) T + m_2 \mu_2, \\ \text{Cov} \left( \ln \frac{S_T}{S_0}, \ln \frac{V_T}{V_0} \right) &= \rho \sigma_S \sigma_V T. \end{aligned} \quad (3)$$

Denote

$$\begin{aligned} \ln S_{T,m_1} &= \ln S_0 + \left( r - \frac{1}{2} \sigma_S^2 - k_S \lambda_S^* \right) T + \sigma_S W_T^{(1)} + \sum_{k=1}^{m_1} \xi_k^{(1)}, \\ \ln V_{T,m_2} &= \ln V_0 + \left( r - \frac{1}{2} \sigma_V^2 - k_V \lambda_V^* \right) T + \sigma_V W_T^{(2)} + \sum_{k=1}^{m_2} \xi_k^{(2)}, \end{aligned}$$

where  $\xi_k^{(i)}$ ,  $i = 1, 2$  are independent normally distributed with mean  $\mu_i$ ,  $i = 1, 2$  and standard deviation  $\sigma_i > 0$ ,  $i = 1, 2$ . Note that the whole probability space  $\Omega = \bigcup_{n=0}^{\infty} \bigcup_{n_1=0}^{\infty} \bigcup_{n_2=0}^{\infty} \mathcal{G}_T^{(n,n_1,n_2)}$  and  $\mathcal{G}_T^{(i,i_1,i_2)} \cap \mathcal{G}_T^{(j,j_1,j_2)} = \emptyset$  for any  $i \neq j$ ,  $i_1 \neq j_1$  or  $i_2 \neq j_2$ , we can rewrite  $C^*$  as follows:

$$\begin{aligned} C^* &= e^{-rT} \mathbb{E} \left[ (S_T - K)^+ \left( \mathbf{1}(V_T \geq D^*) + \frac{1-\alpha}{D} V_T \mathbf{1}(V_T < D^*) \right) \mathbf{1}(\omega \in \Omega) \right] \\ &= e^{-rT} \sum_{n=0}^{\infty} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \mathbb{E} \left[ (S_T - K)^+ \left( \mathbf{1}(V_T \geq D^*) + \frac{1-\alpha}{D} V_T \mathbf{1}(V_T < D^*) \right) \mathbf{1}(\omega \in \mathcal{G}_T^{(n,n_1,n_2)}) \right] \\ &= \sum_{n=0}^{\infty} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} Q(N_T = n, N_T^{(1)} = n_1, N_T^{(2)} = n_2) \\ &\quad \cdot e^{-rT} \mathbb{E} \left[ (S_{T,m_1} - K)^+ \left( \mathbf{1}(V_{T,m_2} \geq D^*) + \frac{1-\alpha}{D} V_{T,m_2} \mathbf{1}(V_{T,m_2} < D^*) \right) \right] \\ &= \sum_{n=0}^{\infty} \sum_{m_1=n}^{\infty} \sum_{m_2=n}^{\infty} \frac{(\lambda_T T)^n}{n!} \frac{(\lambda_S T)^{m_1-n}}{(m_1-n)!} \frac{(\lambda_V T)^{m_2-n}}{(m_2-n)!} e^{-\lambda_T T - \lambda_S T - \lambda_V T} C_{m_1,m_2}, \end{aligned} \quad (4)$$

where

$$C_{m_1,m_2} = e^{-rT} \mathbb{E} \left[ (S_{T,m_1} - K)^+ \left( \mathbf{1}(V_{T,m_2} \geq D^*) + \frac{1-\alpha}{D} V_{T,m_2} \mathbf{1}(V_{T,m_2} < D^*) \right) \right].$$

Clearly,  $C_{m_1,m_2}$  represents the value of the vulnerable option, conditional on  $m_1$  and  $m_2$  jumps on the underlying asset and the assets of the counterparty respectively.  $C^*$  is just the weighted sum of each price where the weight equals the probability that the three Poisson random variables take on the values  $n$ ,  $n_1$ , and  $n_2$ .

Next we show the explicit formula of  $C_{m_1, m_2}$  so that we can get the explicit expression of  $C^*$ . For further calculation, we divide  $C_{m_1, m_2}$  into four parts,

$$\begin{aligned} C_{m_1, m_2} &= e^{-rT} \left( \mathbb{E}[(S_{T, m_1} - K)^+ \mathbf{1}(V_{T, m_2} \geq D^*)] + \mathbb{E}\left[(S_{T, m_1} - K)^+ \frac{1 - \alpha}{D} V_{T, m_2} \mathbf{1}(V_{T, m_2} < D^*)\right] \right) \\ &= e^{-rT} [A_1(m_1, m_2) + A_2(m_1, m_2) + A_3(m_1, m_2) + A_4(m_1, m_2)], \end{aligned} \quad (5)$$

where  $A_1(m_1, m_2)$ ,  $A_2(m_1, m_2)$ ,  $A_3(m_1, m_2)$ , and  $A_4(m_1, m_2)$  are given by

$$\begin{aligned} A_1(m_1, m_2) &= \mathbb{E}[S_{T, m_1} \mathbf{1}(S_{T, m_1} \geq K, V_{T, m_2} \geq D^*)], \\ A_2(m_1, m_2) &= -K \mathbb{E}[\mathbf{1}(S_{T, m_1} \geq K, V_{T, m_2} \geq D^*)], \\ A_3(m_1, m_2) &= \frac{1 - \alpha}{D} \mathbb{E}[S_{T, m_1} V_{T, m_2} \mathbf{1}(S_{T, m_1} \geq K, V_{T, m_2} < D^*)], \\ A_4(m_1, m_2) &= -\frac{1 - \alpha}{D} K \mathbb{E}[V_{T, m_2} \mathbf{1}(S_{T, m_1} \geq K, V_{T, m_2} < D^*)]. \end{aligned}$$

Equation (5) indicates that the value of the vulnerable option depends on whether the underlying asset is greater than the strike price  $K$  and whether the value of the assets of the counterparty is greater than the default barrier  $D^*$ . Then we can get the closed form of  $A_1(m_1, m_2)$ ,  $A_2(m_1, m_2)$ ,  $A_3(m_1, m_2)$ , and  $A_4(m_1, m_2)$  respectively,

$$\begin{aligned} A_1(m_1, m_2) &= S_0 e^{(r - k_S \lambda_S^*)T + m_1 \mu_1 + \frac{1}{2} m_1 \sigma_1^2} N_2(a_1(m_1), a_2(m_2), \bar{\rho}(m_1, m_2)), \\ A_2(m_1, m_2) &= -K N_2(b_1(m_1), b_2(m_2), \bar{\rho}(m_1, m_2)), \\ A_3(m_1, m_2) &= \frac{1 - \alpha}{D} S_0 V_0 e^{(r - k_S \lambda_S^*)T + m_1 \mu_1 + (r - k_V \lambda_V^*)T + m_2 \mu_2 + \rho \sigma_S \sigma_V T + \frac{1}{2} m_1 \sigma_1^2 + \frac{1}{2} m_2 \sigma_2^2} \\ &\quad \cdot N_2(c_1(m_1), c_2(m_2), -\bar{\rho}(m_1, m_2)), \\ A_4(m_1, m_2) &= -\frac{1 - \alpha}{D} K V_0 e^{(r - k_V \lambda_V^*)T + m_2 \mu_2 + \frac{1}{2} m_2 \sigma_2^2} N_2(d_1(m_1), d_2(m_2), -\bar{\rho}(m_1, m_2)), \end{aligned}$$

where specific calculations can be found in the Appendix and the parameters are expressed as follows:

$$\begin{aligned} a_1(m_1) &= \frac{\ln \frac{S_0}{K} + \left(r + \frac{1}{2} \sigma_S^2 - k_S \lambda_S^*\right)T + m_1 \mu_1 + m_1 \sigma_1^2}{\sqrt{\sigma_S^2 T + m_1 \sigma_1^2}}, \\ a_2(m_2) &= \frac{\ln \frac{V_0}{D^*} + \left(r - \frac{1}{2} \sigma_V^2 - k_V \lambda_V^*\right)T + m_2 \mu_2 + \rho \sigma_S \sigma_V T}{\sqrt{\sigma_V^2 T + m_2 \sigma_2^2}}, \\ b_1(m_1) &= \frac{\ln \frac{S_0}{K} + \left(r - \frac{1}{2} \sigma_S^2 - k_S \lambda_S^*\right)T + m_1 \mu_1}{\sqrt{\sigma_S^2 T + m_1 \sigma_1^2}}, \\ b_2(m_2) &= \frac{\ln \frac{V_0}{D^*} + \left(r - \frac{1}{2} \sigma_V^2 - k_V \lambda_V^*\right)T + m_2 \mu_2}{\sqrt{\sigma_V^2 T + m_2 \sigma_2^2}}, \end{aligned}$$

$$\begin{aligned}
 c_1(m_1) &= \frac{\ln \frac{S_0}{K} + \left(r + \frac{1}{2}\sigma_S^2 - k_S \lambda_S^*\right)T + m_1 \mu_1 + m_1 \sigma_1^2 + \rho \sigma_S \sigma_V T}{\sqrt{\sigma_S^2 T + m_1 \sigma_1^2}}, \\
 c_2(m_2) &= -\frac{\ln \frac{V_0}{D^*} + \left(r + \frac{1}{2}\sigma_V^2 - k_V \lambda_V^*\right)T + m_2 \mu_2 + m_2 \sigma_2^2 + \rho \sigma_S \sigma_V T}{\sqrt{\sigma_V^2 T + m_2 \sigma_2^2}}, \\
 d_1(m_1) &= \frac{\ln \frac{S_0}{K} + \left(r - \frac{1}{2}\sigma_S^2 - k_S \lambda_S^*\right)T + m_1 \mu_1 + \rho \sigma_S \sigma_V T}{\sqrt{\sigma_S^2 T + m_1 \sigma_1^2}}, \\
 d_2(m_2) &= -\frac{\ln \frac{V_0}{D^*} + \left(r + \frac{1}{2}\sigma_V^2 - k_V \lambda_V^*\right)T + m_2 \mu_2 + m_2 \sigma_2^2}{\sqrt{\sigma_V^2 T + m_2 \sigma_2^2}}, \\
 \bar{\rho}(m_1, m_2) &= \frac{\sigma_S \sigma_V T}{\sqrt{\sigma_S^2 T + m_1 \sigma_1^2} \sqrt{\sigma_V^2 T + m_2 \sigma_2^2}} \rho.
 \end{aligned} \tag{6}$$

From Equation (5) we can derive that,

$$\begin{aligned}
 &C_{m_1, m_2} \\
 &= S_0 e^{-k_S \lambda_S T + m_1 \mu_1 + \frac{1}{2} m_1 \sigma_1^2} N_2(a_1(m_1), a_2(m_2), \bar{\rho}(m_1, m_2)) \\
 &\quad - K e^{-rT} N_2(b_1(m_1), b_2(m_2), \bar{\rho}(m_1, m_2)) \\
 &\quad + \frac{1-\alpha}{D} S_0 V_0 e^{rT - k_S \lambda_S^* T - k_V \lambda_V^* T + m_1 \mu_1 + m_2 \mu_2 + \rho \sigma_S \sigma_V T + \frac{1}{2} m_1 \sigma_1^2 + \frac{1}{2} m_2 \sigma_2^2} \\
 &\quad \cdot N_2(c_1(m_1), c_2(m_2), -\bar{\rho}(m_1, m_2)) \\
 &\quad - \frac{1-\alpha}{D} K V_0 e^{-k_V \lambda_V^* T + m_2 \mu_2 + \frac{1}{2} m_2 \sigma_2^2} N_2(d_1(m_1), d_2(m_2), -\bar{\rho}(m_1, m_2)).
 \end{aligned} \tag{7}$$

Therefore, we have got the explicit formula for the valuation of the vulnerable European call options,

$$\begin{aligned}
 &C^* \\
 &= \sum_{n=0}^{\infty} \sum_{m_1=n}^{\infty} \sum_{m_2=n}^{\infty} \frac{(\lambda T)^n (\lambda_S T)^{m_1-n} (\lambda_V T)^{m_2-n}}{n! (m_1-n)! (m_2-n)!} e^{-\lambda T - \lambda_S T - \lambda_V T} \\
 &\quad \cdot (S_0 e^{-k_S \lambda_S^* T + m_1 \mu_1 + \frac{1}{2} m_1 \sigma_1^2} N_2(a_1(m_1), a_2(m_2), \bar{\rho}(m_1, m_2)) - K e^{-rT} N_2(b_1(m_1), b_2(m_2), \bar{\rho}(m_1, m_2)) \\
 &\quad + \frac{1-\alpha}{D} S_0 V_0 e^{rT - k_S \lambda_S^* T - k_V \lambda_V^* T + m_1 \mu_1 + m_2 \mu_2 + \rho \sigma_S \sigma_V T + \frac{1}{2} m_1 \sigma_1^2 + \frac{1}{2} m_2 \sigma_2^2} N_2(c_1(m_1), c_2(m_2), -\bar{\rho}(m_1, m_2)) \\
 &\quad - \frac{1-\alpha}{D} K V_0 e^{-k_V \lambda_V^* T + m_2 \mu_2 + \frac{1}{2} m_2 \sigma_2^2} N_2(d_1(m_1), d_2(m_2), -\bar{\rho}(m_1, m_2))).
 \end{aligned} \tag{8}$$

Similarly, the explicit form of vulnerable European put option is expressed as

$$\begin{aligned}
 P^* &= \sum_{n=0}^{\infty} \sum_{m_1=n}^{\infty} \sum_{m_2=n}^{\infty} \frac{(\lambda T)^n (\lambda_S T)^{m_1-n} (\lambda_V T)^{m_2-n}}{n! (m_1-n)! (m_2-n)!} e^{-\lambda T - \lambda_S T - \lambda_V T} \\
 &\quad \cdot (Ke^{-rT} N_2(-b_1(m_1), b_2(m_2), -\bar{\rho}(m_1, m_2)) \\
 &\quad - S_0 e^{-k_S \lambda_S^* T + m_1 \mu_1 + \frac{1}{2} m_1 \sigma_1^2} N_2(-a_1(m_1), a_2(m_2), -\bar{\rho}(m_1, m_2)) \\
 &\quad + \frac{1-\alpha}{D} K V_0 e^{-k_V \lambda_V^* T + m_2 \mu_2 + \frac{1}{2} m_2 \sigma_2^2} N_2(-d_1(m_1), d_2(m_2), \bar{\rho}(m_1, m_2)) \\
 &\quad - \frac{1-\alpha}{D} S_0 V_0 e^{rT - k_S \lambda_S^* T - k_V \lambda_V^* T + m_1 \mu_1 + m_2 \mu_2 + \rho \sigma_S \sigma_V T + \frac{1}{2} m_1 \sigma_1^2 + \frac{1}{2} m_2 \sigma_2^2} N_2(-c_1(m_1), c_2(m_2), \bar{\rho}(m_1, m_2))).
 \end{aligned}$$

Moreover, we can also get the explicit expression for vulnerable European options at  $t < T$  when the underlying stock pays a continuous dividend  $q$ , where  $S_0$ ,  $r$ , and  $T$  are replaced with  $S_t$ ,  $r - q$ , and  $T - t$  respectively and  $S_t$  is discounted by  $e^{-q(T-t)}$ .

### 2.3. Specific Cases of the Pricing Formula

Compared with the classical Black–Scholes model, our proposed model adds two pieces-credit risk and jump risk. Option price with default risk has been investigated by Johnson and Stulz (1987) and Klein (1996). We show the contribution of the jumps to the formula by illustrating four specific examples: Black–Scholes formula (non-jump and non-vulnerable), Merton’s jump-diffusion formula (non-vulnerable) and the vulnerable Black–Scholes formula in Klein (1996) (non-jump), as well as independent case. The relationship among the models is expressed in Figure 1.

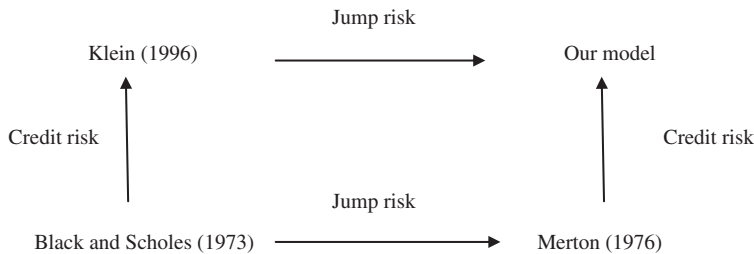
#### Case 1: Black–Scholes model

It can be seen from Equation (8) that if there is no counterparty risk and jump risk, that is,  $D^* = 0$ ,  $\lambda = \lambda_S = \lambda_V = 0$  and  $n = n_1 = n_2 = 0$ , Equation (8) becomes the Black–Scholes formula. As the default threshold  $D^*$  converges to zero,

$$\begin{aligned}
 \lim_{D^* \rightarrow 0} a_2(0) &= \lim_{D^* \rightarrow 0} b_2(0) = \infty, \\
 \lim_{D^* \rightarrow 0} c_2(0) &= \lim_{D^* \rightarrow 0} d_2(0) = -\infty.
 \end{aligned}$$

Moreover,  $a_1(0)$  and  $b_1(0)$  reduce to  $A_1 = \frac{\ln \frac{S_0}{K} + (r + \frac{1}{2} \sigma_S^2)T}{\sigma_S \sqrt{T}}$  and  $B_1 = A_1 - \sigma_S \sqrt{T}$  respectively. As  $N_2(A_1, \infty, \rho)$  is the marginal distribution of the first random variable  $\xi_1$ , which is distributed as  $N_1(A_1)$  and  $N_2(B_1, -\infty, \rho)$  equals zero, Equation (8) can be rewritten as

$$C^* = S_0 N_1(A_1) - Ke^{-rT} N_1(B_1).$$



**FIGURE 1**  
Model comparison.



**Case 2: Merton's jump-diffusion model**

When there is no default risk, that is,  $D^* = 0$ ,

$$C_{m_1, m_2} = S_0 e^{-k_S \lambda_S^* T + m_1 \mu_1 + \frac{1}{2} m_1 \sigma_1^2} N_1(a_1(m_1)) - K e^{-rT} N_1(b_1(m_1)).$$

As  $C_{m_1, m_2}$  is irrelevant to  $m_2$ ,

$$\begin{aligned} C^* &= \sum_{n=0}^{\infty} \sum_{m_1=n}^{\infty} \sum_{m_2=n}^{\infty} \frac{(\lambda T)^n}{n!} \frac{(\lambda_S T)^{m_1-n}}{(m_1-n)!} \frac{(\lambda_V T)^{m_2-n}}{(m_2-n)!} e^{-\lambda T - \lambda_S T - \lambda_V T} C_{m_1, m_2} \\ &= \sum_{n=0}^{\infty} \sum_{m_1=n}^{\infty} \frac{(\lambda T)^n}{n!} \frac{(\lambda_S T)^{m_1-n}}{(m_1-n)!} e^{-\lambda T - \lambda_S T} C_{m_1, m_2} \cdot \left( \sum_{n_2=0}^{\infty} \frac{(\lambda_V T)^{n_2}}{n_2!} e^{-\lambda_V T} \right) \\ &= \sum_{n=0}^{\infty} \sum_{m_1=n}^{\infty} e^{-\lambda T - \lambda_S T} \frac{(\lambda T)^n}{n!} \frac{(\lambda_S T)^{m_1-n}}{(m_1-n)!} \\ &\quad \cdot \left( S_0 e^{-k_S \lambda_S^* T + m_1 \mu_1 + \frac{1}{2} m_1 \sigma_1^2} N_1(a_1(m_1)) - K e^{-rT} N_1(b_1(m_1)) \right), \end{aligned}$$

where  $a_1(m_1)$  and  $b_1(m_1)$  can be found in (6). Noting that  $(a+b)^m = \sum_{i=0}^m C_m^i a^i b^{m-i}$ ,  $C^*$  can be represented as Merton's jump-diffusion form,

$$C^* = \sum_{i=0}^{\infty} e^{-\lambda_S^* T} \frac{(\lambda_S^* T)^i}{i!} \left( S_0 e^{-k_S \lambda_S^* T + i \mu_1 + \frac{1}{2} i \sigma_1^2} N_1(a_1(i)) - K e^{-rT} N_1(b_1(i)) \right).$$

**Case 3: Vulnerable Black-Scholes model in Klein (1996)**

If there is no jump, that is,  $\lambda = \lambda_S = \lambda_V = 0$ ,  $C^*$  can be restated as follows,

$$\begin{aligned} C^* &= S_0 N_2(A_1, A_2, \rho) - K e^{-rT} N_2(B_1, B_2, \rho) + \frac{1-\alpha}{D} S_0 V_0 e^{rT + \rho \sigma_S \sigma_V T} N_2(C_1, C_2, -\rho) \\ &\quad - \frac{1-\alpha}{D} K V_0 N_2(D_1, D_2, -\rho), \end{aligned}$$

where the parameters are given by

$$\begin{aligned} A_1 &= B_1 + \sigma_S \sqrt{T}, \\ A_2 &= B_2 + \rho \sigma_S \sqrt{T}, \\ B_1 &= \frac{\ln \frac{S_0}{K} + \left( r - \frac{1}{2} \sigma_S^2 \right) T}{\sigma_S \sqrt{T}}, \\ B_2 &= \frac{\ln \frac{V_0}{D^*} + \left( r - \frac{1}{2} \sigma_V^2 \right) T}{\sigma_V \sqrt{T}}, \\ C_1 &= B_1 + (\sigma_S + \rho \sigma_V) \sqrt{T}, \\ C_2 &= -B_2 - (\sigma_V + \rho \sigma_S) \sqrt{T}, \\ D_1 &= B_1 + \rho \sigma_V \sqrt{T}, \\ D_2 &= -B_2 - \sigma_V \sqrt{T}. \end{aligned}$$

**Case 4: Independence**

If the underlying asset is not correlated with the assets of the counterparty,  $\rho = 0$  and common shocks will disappear, that is,  $\lambda = 0$ . Note that when  $\rho = 0$ ,  $a_1(n_1) = c_1(n_1)$ ,  $b_1(n_1) = d_1(n_1)$ ,  $a_2(n_2) = b_2(n_2)$ , and  $c_2(n_2) = d_2(n_2)$ . Then Equation (8) simplifies as

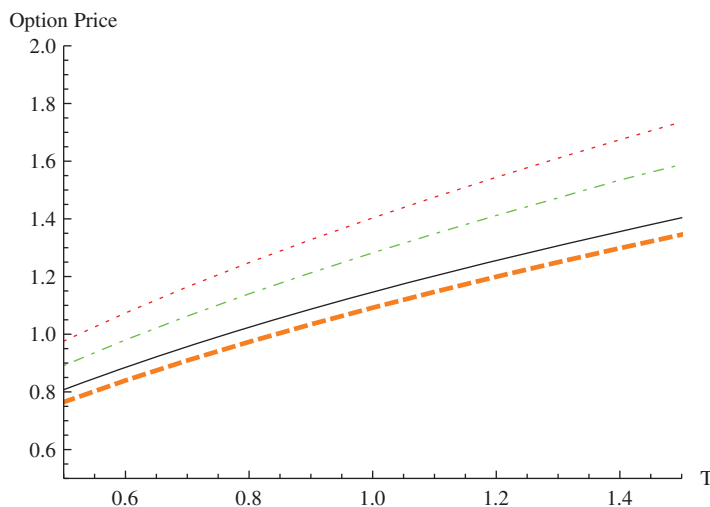
$$C^* = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \frac{(\lambda_S T)^{n_1}}{n_1!} \frac{(\lambda_V T)^{n_2}}{n_2!} e^{-\lambda_S T - \lambda_V T} \cdot \left( S_0 e^{-k_S \lambda_S T + n_1 \mu_1 + \frac{1}{2} n_1 \sigma_1^2} N(a_1(n_1)) - K e^{-rT} N(b_1(n_1)) \right) \cdot \left( N(b_2(n_2)) + \frac{1-\alpha}{D} V_0 e^{rT - k_V \lambda_V T + n_2 \mu_2 + \frac{1}{2} n_2 \sigma_2^2} N(d_2(n_2)) \right).$$

Provided that  $n_1$  and  $n_2$  are the jump times of the underlying asset and the assets of the counterparty,  $S_0 e^{-k_S \lambda_S T + n_1 \mu_1 + \frac{1}{2} n_1 \sigma_1^2} N(a_1(n_1)) - K e^{-rT} N(b_1(n_1))$  is the option price without default and  $N(b_2(n_2)) + \frac{1-\alpha}{D} V_0 e^{rT - k_V \lambda_V T + n_2 \mu_2 + \frac{1}{2} n_2 \sigma_2^2} N(d_2(n_2))$  just corresponds to the undiscounted expected value of a nominal claim of one dollar on the counterparty.

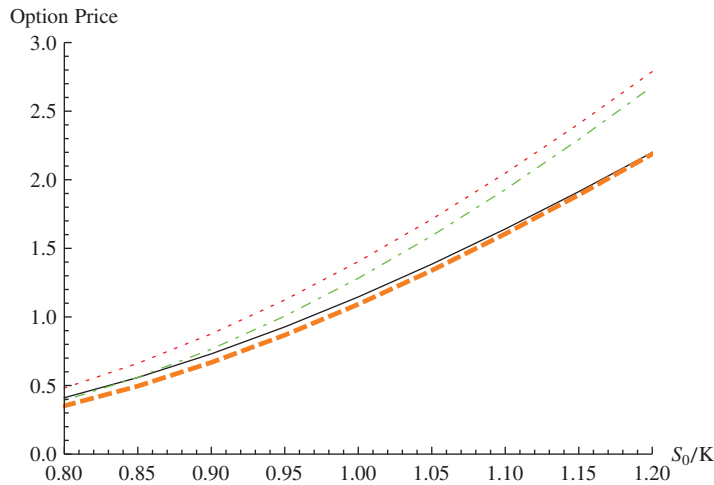
**3. NUMERICAL SIMULATIONS**

In this section, derived formulae are used to perform a numerical experiment. Some examples are illustrated to understand the impact of the parameters on the vulnerable option. The Black–Scholes model, the vulnerable Black–Scholes model in Klein (1996) and Merton's jump-diffusion model are chosen as reference models. Our choice of the three reference models is in order to observe the impact of jump risk on vulnerable option prices and to observe the impact of credit risk on option prices under jump-diffusions.

The effects of some basic variables on vulnerable option price are illustrated in Figures 2–6, including time to maturity, spot-to-strike ratio, correlation coefficient,

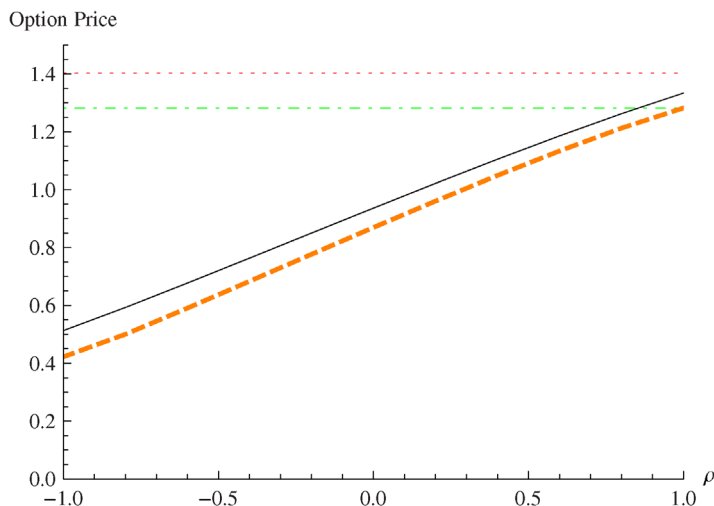
**FIGURE 2**

Option price against time to maturity. The solid, dot-dashed, thick dashed, and dotted lines correspond to the proposed model, BS model, BS model with correlated risk, and to Merton's jump-diffusion model, respectively. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

**FIGURE 3**

Option price against spot-to-strike ratio. The solid, dot-dashed, thick dashed, and dotted lines correspond to the proposed model, BS model, BS model with correlated risk, and to Merton's jump-diffusion model, respectively. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

outstanding claims, default barrier, and deadweight cost associated with bankruptcy. For comparison, the three reference models are included in Figures 2–6. Numerical analysis for the impact of jump is shown in Figures 7–13. The effects of jump intensity, mean jump size and standard deviation of the jump size on the option prices are considered in our model. Preference parameters listed in Table I represent a typical business situation. In the base case, the vulnerable option is at the money, and is written by a highly leveraged firm. Time to

**FIGURE 4**

Option price against correlation coefficient. The solid, dot-dashed, thick dashed, and dotted lines correspond to the proposed model, BS model, BS model with correlated risk, and to Merton's jump-diffusion model, respectively. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

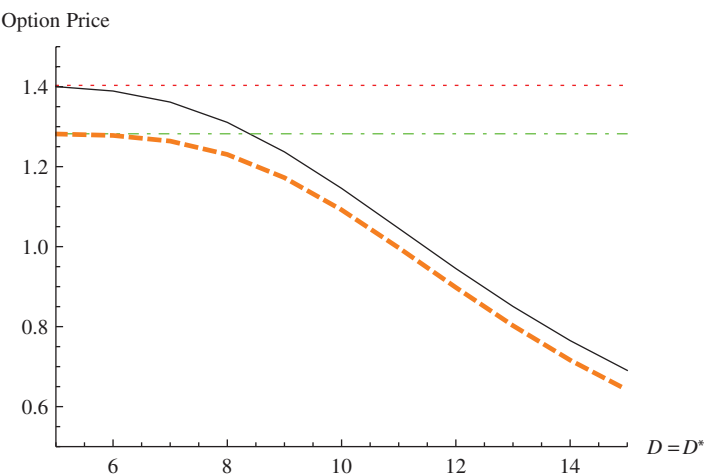


FIGURE 5

Option price against outstanding claims and default barrier. The solid, dot-dashed, thick dashed, and dotted lines correspond to the proposed model, BS model, BS model with correlated risk, and to Merton’s jump-diffusion model, respectively. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

maturity is assumed to be one year. The market value of the option writer’s assets and the market value of the asset underlying the option are correlated with instantaneous correlation  $\rho = 0.5$ . Default barrier is assumed to be the total value of the debt. Shocks to the stock price are assumed to happen once a year. In the following tables and figures, we change one of the parameter values to investigate the impact on the vulnerable option price with other variables taking on the values listed in Table I.

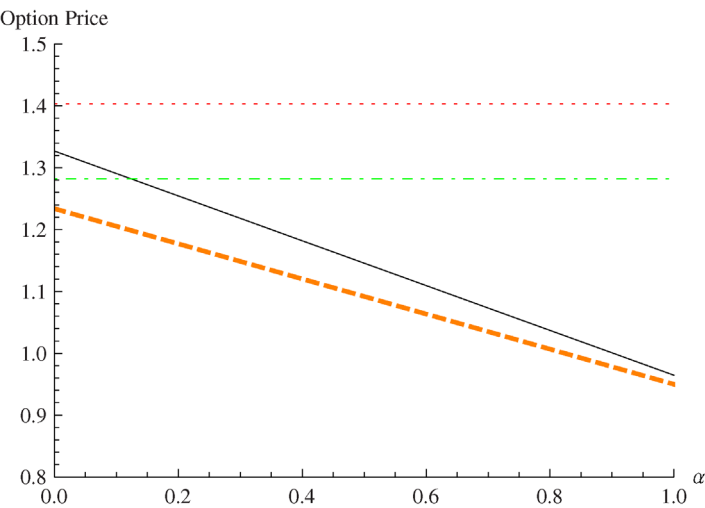
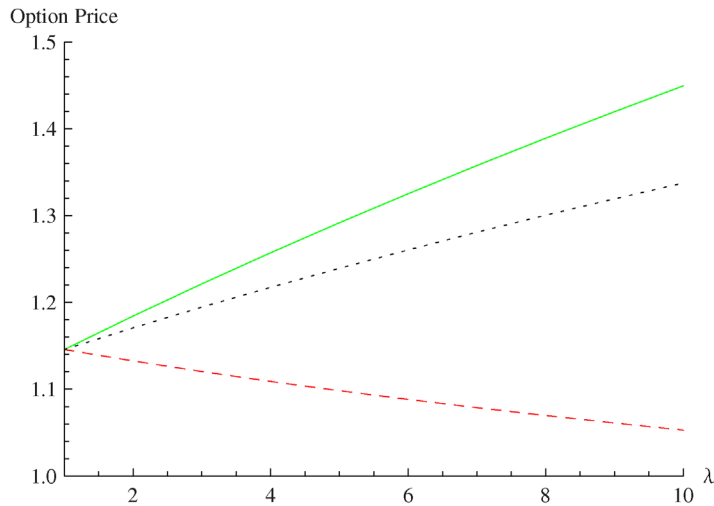


FIGURE 6

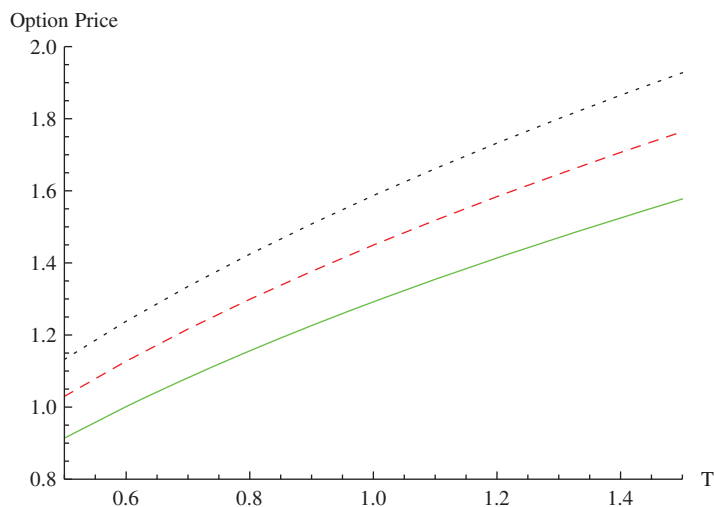
Option price against deadweight cost associated with bankruptcy. The solid, dot-dashed, thick dashed, and dotted lines correspond to the proposed model, BS model, BS model with correlated risk, and to Merton’s jump-diffusion model, respectively. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]


**FIGURE 7**

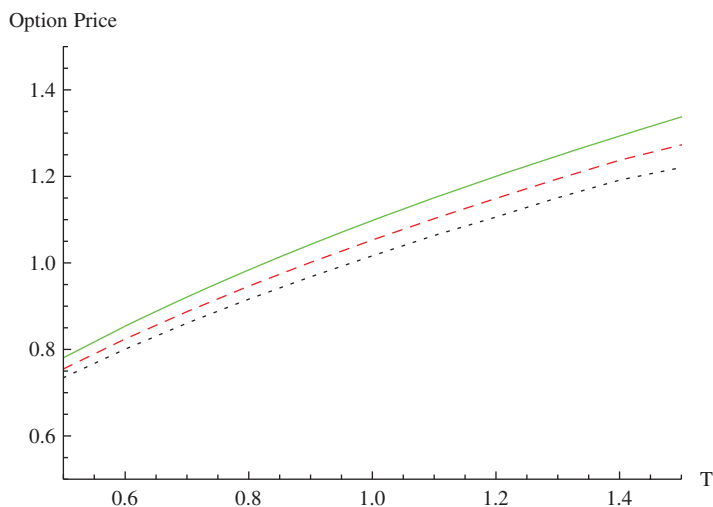
Option price against the jump intensities of the underlying asset and the assets of the counterparty. The dotted, solid, and the dashed lines correspond to  $\lambda$ ,  $\lambda_S$  and to  $\lambda_V$ , respectively. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

Table II depicts the convergence analysis for the formulae of the proposed model and Merton's jump-diffusion model. We take the sum of  $n$ ,  $n_1$ ,  $n_2 = 0-5, 10, 30, 50, 100$  from the series of the formulae, respectively. It can be seen from Table II that the various series converge quickly. In the following numerical analysis, we take

$$\sum_{n=0}^{50} \sum_{m_1=n}^{n+50} \sum_{m_2=n}^{n+50} \frac{(\lambda T)^n}{n!} \frac{(\lambda_S^* T)^{m_1-n}}{(m_1-n)!} \frac{(\lambda_V^* T)^{m_2-n}}{(m_2-n)!} e^{-\lambda T - \lambda_S T - \lambda_V T} C_{m_1, m_2}$$


**FIGURE 8**

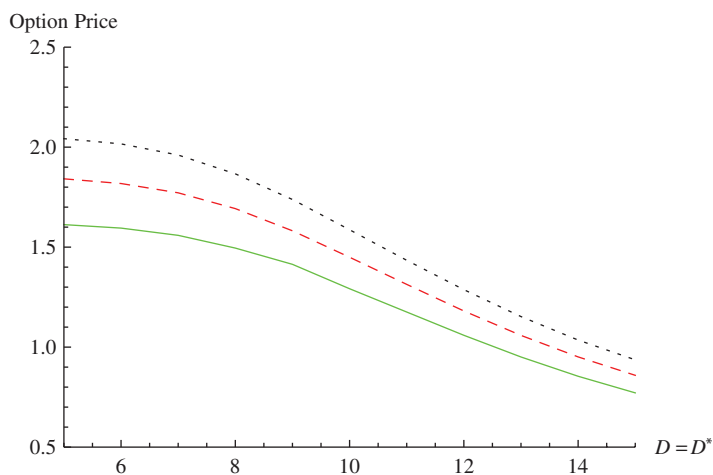
Option price against time to maturity for the proposed model. The solid, dashed, and dotted lines correspond to  $\lambda_S = 5$ ,  $\lambda_S = 10$ , and  $\lambda_S = 15$ , respectively.  $\lambda_S$  is the intensity of the individual jump from the underlying asset. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

**FIGURE 9**

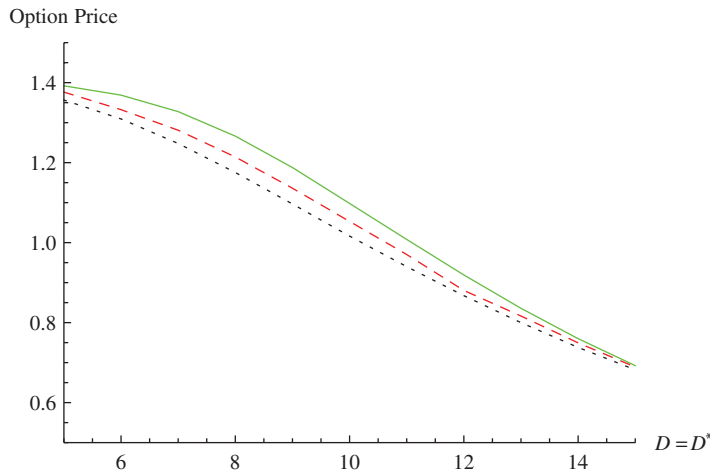
Option price against time to maturity for the proposed model. The solid, dashed, and dotted lines correspond to  $\lambda_V = 5$ ,  $\lambda_V = 10$ , and  $\lambda_V = 15$ , respectively.  $\lambda_S$  is the intensity of the individual jump from the assets of the counterparty. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

as approximated option price. For consistency, the jump intensity in Merton's model is taken as  $\lambda_S^* = \lambda_S + \lambda$  where specific steps are listed in Section 2.3 (Case 2).

Table III shows the numerical results obtained by the proposed model. The values of Black and Scholes (1973), Merton (1976), and Klein (1996) are reported in the last three columns for comparison. Values calculated by Merton's jump-diffusion model are the largest among the four models, which overestimate the option price due to overlooking the

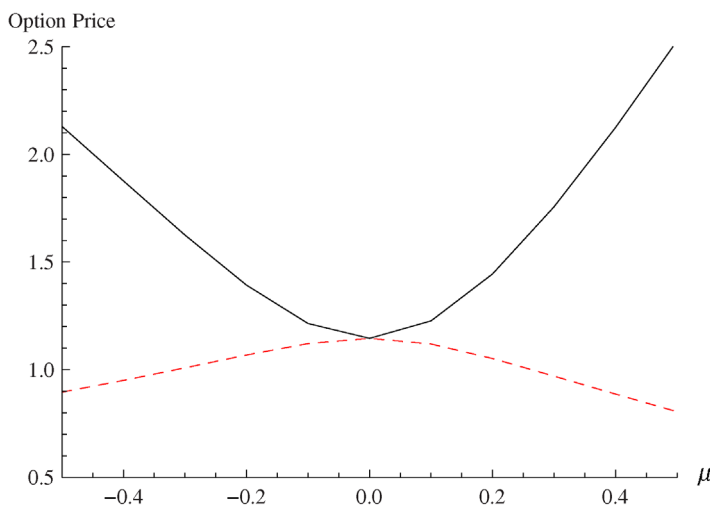
**FIGURE 10**

Option price against outstanding claims and default barrier for the proposed model. The solid, dashed, and dotted lines correspond to  $\lambda_S = 5$ ,  $\lambda_S = 10$ , and  $\lambda_S = 15$ , respectively.  $\lambda_S$  is the intensity of the individual jump from the underlying asset. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]


**FIGURE 11**

Option price against outstanding claims and default barrier for the proposed model. The solid, dashed, and dotted lines correspond to  $\lambda_V = 5$ ,  $\lambda_V = 10$ , and  $\lambda_V = 15$ , respectively.  $\lambda_V$  is the intensity of the individual jump from the assets of the counterparty. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

counterparty risk. Increasing the correlation coefficient  $\rho$  from  $-0.3$  to  $0.3$ , values of the proposed model and Klein (1996) change from  $0.807$  and  $0.730$  to  $1.064$  and  $1.005$ , respectively. Higher correlation between the two assets makes the vulnerable option more valuable. In contrast, option prices decrease with the default barrier and the outstanding claims. Given  $\lambda_S = \lambda_V$ ,  $\mu_1 = \mu_2$ , and  $\sigma_1 = \sigma_2$ , values of our model are higher than those of Klein (1996), from which we can find that the jump risk of the underlying asset has a more significant impact on the price than that of the assets of the counterparty. Values with


**FIGURE 12**

Option price against mean jump size for the proposed model. The solid and the dashed lines correspond to  $\mu_1$  and  $\mu_2$ , respectively. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

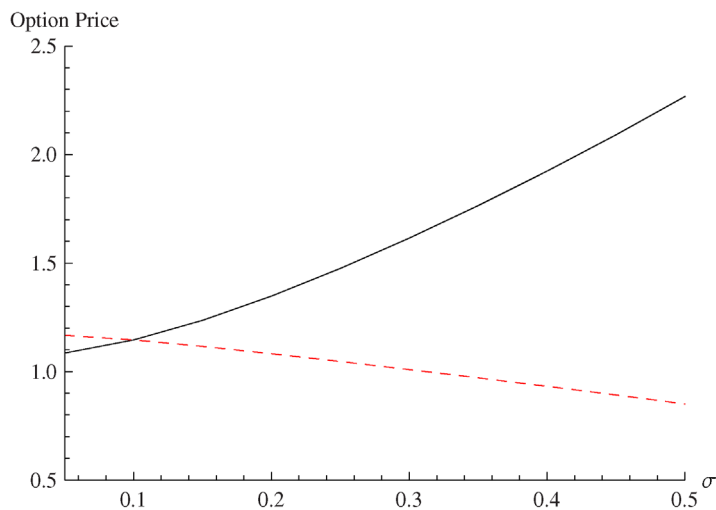


FIGURE 13

Option price against standard deviation of the jump size for the proposed model. The solid and the dashed lines correspond to  $\sigma_1$  and  $\sigma_2$ , respectively. [Color figure can be viewed in the online issue, which is available at [wileyonlinelibrary.com](http://wileyonlinelibrary.com).]

TABLE I  
Parameter Values in the Base Case

Parameter	Value	Parameter	Value
Volatility	$\sigma_S = 0.3$	Volatility	$\sigma_V = 0.3$
Initial price	$S_0 = 10$	Initial price	$V_0 = 10$
Mean jump size of $S$	$\mu_1 = 0$	Mean jump size of $V$	$\mu_2 = 0$
Annual jump intensity	$\lambda_S = 1$	Annual jump intensity	$\lambda_V = 1$
Standard deviation of the jump size	$\sigma_1 = 0.1$	Spot rate	$r = 0.02$
Standard deviation of the jump size	$\sigma_2 = 0.1$	Correlation coefficient	$\rho = 0.5$
Time to maturity	$T = 1.0$	Strike price	$K = 10$
Default barrier	$D^* = 10$	Outstanding claims	$D = 10$
Deadweight cost associated with bankruptcy	$\alpha = 0.5$	Common jump intensity	$\lambda = 1$

TABLE II  
Convergence Analysis

	$n, n_1, n_2$				
	5	10	30	50	100
Proposed model					
Base	1.14334	1.14570	1.14570	1.14570	1.14570
$\lambda_S = 10$	0.07684	0.77973	1.44949	1.44949	1.44949
$\lambda_V = 10$	0.07221	0.61862	1.05286	1.05286	1.05286
$\lambda = 10$	0.07231	0.72387	1.33748	1.33748	1.33748
Merton (1976)					
Base	1.37388	1.40322	1.40324	1.40324	1.40324
$\lambda_S = 10$	0.05076	0.75353	1.84851	1.84851	1.84851



**TABLE III**  
Values of Vulnerable European Call Options

<i>Cases</i>	<i>Proposed model</i>	<i>B-S (1973)</i>	<i>Merton (1976)</i>	<i>Klein (1996)</i>
Base	1.146	1.282	1.403	1.092
$T=0.5$	0.808	0.891	0.976	0.765
$T=1.5$	1.404	1.589	1.736	1.345
$S_0/K=0.8$	0.411	0.392	0.482	0.352
$S_0/K=1.2$	2.199	2.680	2.789	2.187
$\rho=-0.3$	0.807	1.282	1.403	0.730
$\rho=0.3$	1.064	1.282	1.403	1.005
$\sigma_S=0.2$	0.847	0.892	1.061	0.752
$\sigma_S=0.4$	1.460	1.670	1.763	1.433
$\sigma_V=0.2$	1.158	1.282	1.403	1.120
$\sigma_V=0.4$	1.125	1.282	1.403	1.066
$D=D^*=8$	1.311	1.282	1.403	1.230
$D=D^*=12$	0.945	1.282	1.403	0.898
$D^*=6$	1.385	1.282	1.403	1.277
$D^*=8$	1.297	1.282	1.403	1.222
$\alpha=0.3$	1.218	1.282	1.403	1.149
$\alpha=0.7$	1.073	1.282	1.403	1.035
$\lambda_S=5$	1.292	1.282	1.617	1.092
$\lambda_S=10$	1.449	1.282	1.849	1.092
$\lambda_V=5$	1.098	1.282	1.403	1.092
$\lambda_V=10$	1.053	1.282	1.403	1.092
$\lambda=5$	1.239	1.282	1.617	1.092
$\lambda=10$	1.337	1.282	1.849	1.092
$\mu_1=-0.5$	2.130	1.282	2.809	1.092
$\mu_1=0.5$	2.527	1.282	3.459	1.092
$\mu_2=-0.5$	0.896	1.282	1.403	1.092
$\mu_2=0.5$	0.805	1.282	1.403	1.092
$\sigma_1=0.05$	1.085	1.282	1.314	1.092
$\sigma_1=0.45$	2.091	1.282	2.806	1.092
$\sigma_2=0.05$	1.167	1.282	1.403	1.092
$\sigma_2=0.45$	0.892	1.282	1.403	1.092

different jump intensities are also considered. When  $\lambda_S$  changes from 5 to 10, the value of the proposed model increases from 1.292 to 1.449 although the same increasing amplitude on  $\lambda_V$  causes the value to decrease from 1.098 to 1.053. The vulnerable option price is more sensitive to  $\lambda_S$  than to  $\lambda_V$ , which is also indicated by the fact that the value increases with the common jump intensity  $\lambda$ .

In Figure 2, the gaps among the values of the four models become large as time to maturity increases. It is evident that the probability of a jump occurrence in the underlying asset or in the firm's value increases with time, which indicates that the jump risk and the default risk become more and more pronounced as the life of the option increases. Figure 3 plots the option prices against the spot to strike ratio. In the case of  $S_0/K = 0.8$ , the difference between the values in Merton's model and in BS model is 0.090. The difference increases slowly to 0.108 when  $S_0/K = 1.2$ . In contrast, the difference between the values in Merton's model and in the proposed model is 0.071 when  $S_0/K = 0.8$ , and it dramatically rises to 0.590 when  $S_0/K = 1.2$ . For deep-in-the-money options, the probability that the stock price will exceed the strike price at expiration date is relatively high. It is more likely for holders to exercise their options. Thus, deep-in-the-money option holders will suffer more potential credit losses than deep-out-of-the-money option holders, once credit events occur. Hence,

credit risk has a more significant impact on the vulnerable options with high spot-to-strike ratio than those with low spot-to-strike ratio. Figure 4 illustrates the effect of the correlation coefficient  $\rho$ . Since there is no counterparty, the prices in the BS model and Merton's model do not vary with  $\rho$ . The prices of vulnerable options increase with the correlation coefficient. A stronger correlation between the underlying asset and the assets of the counterparty corresponds to a smaller effect of credit risk on the vulnerable option. In the special case of  $\rho = 1$ , we find that the value of BS model is 1.282, which equals that of Klein's model. For the B-S model and Klein's model,  $S_t = V_t$  at any time when  $\rho = 1$ . Default happens only when the investor decides not to exercise the option.

Figures 5 and 6 show the effects of the variables relating to counterparty risk on the vulnerable option price. Figure 5 presents the prices varying with outstanding claims and default barrier, respectively. We assume that  $D$  and  $D^*$  vary simultaneously, that is, the counterparty cannot continue in operation if the assets at expiration date are less than the outstanding claims. The counterparty shall encounter greater credit risk as the default threshold turns higher. When  $D = D^* = 5$ , which is half of the assets of the counterparty, the difference between the values in the Merton's model and in the proposed model is 0.003. Similarly, the difference between the values in the BS model and in Klein's model is merely 0.001. If  $D^* \ll V_0$ , default seldom happens. As the outstanding claims increase, the price in the proposed model decreases more dramatically than that in Klein (1996) does. Figure 6 presents the prices varying with deadweight cost associated with bankruptcy. Compared with outstanding claims and default barrier,  $\alpha$  affects only the recovery when default happens, not default probability. Therefore, the prices in two models experience linear decline instead of accelerated decline as in Figure 5. The change of  $\alpha$  has a more significant impact on the values of the proposed model than those of Klein's model.

Figure 7 shows the effects of the jump intensities  $\lambda$ ,  $\lambda_S$ , and  $\lambda_V$ . When  $\lambda_S$  changes from 1 to 10, the range of the vulnerable option prices is (1.146, 1.450). In contrast, the vulnerable option price decreases from 1.146 to 1.053 as  $\lambda_V$  increases from 1 to 10. A stronger jump intensity of the underlying asset corresponds to a higher price whereas the price decreases with the jump intensity of the assets of the counterparty. Moreover, the common jump intensity produces a positive effect on the vulnerable option price. Intuitively, the jump intensity of the underlying asset has an impact on the expected payoff of the option. On the other hand, default probability partly depends on the jump intensity of the assets of the counterparty. When the option is not exercised, the payoff directly decreases to zero. However, even if the credit event occurs, holders of option contracts could still receive a proportion of the payoff. In a word, jumps on the assets of the counterparty only affect the discounted rate of the expected payoff due to credit risk, which indicates that the impact of  $\lambda_S$  on the vulnerable option prices is more significant than that of  $\lambda_V$ . Figures 8–11 present the option prices varying with time to maturity and bankruptcy threshold under different annual jump intensities values, from which we can also find that the effect of the annual jump intensity of the underlying asset is stronger than that of the assets of the counterparty.

Figure 12 plots option values against the mean jump size of  $S_t$  and  $V_t$ . When the mean jump size of the underlying asset (the assets of the counterparty) changes from  $-0.5$  to  $0.5$ , the vulnerable option prices trace out a U-shaped curve (inverted U-shaped curve). Different from other variables, monotonicity does not exist. Under the risk neutral measure  $Q$ , the drift coefficient of the dynamics of the assets decreases when the mean jump size of the assets rises. The option price arrives at minimum point when the mean jump size of the underlying stock is zero. The reason is that risk-neutral drift also depends on the mean jump sizes of the underlying asset and the assets of the counterparty. Similar cases were also pointed out in Merton (1976) and Kou and Wang (2004). Figure 13 presents option values against the standard deviation of the jump size of  $S_t$  and  $V_t$ . When  $\sigma_1$  and  $\sigma_2$  change from 0.05 to 0.5

**TABLE IV**  
Sensitivity Analysis

<i>Parameter</i>	<i>Effect on option price</i>	<i>Parameter</i>	<i>Effect on option price</i>
$T$	+	$S_0/K$	+
$D$	−	$D^*$	−
$\alpha$	−	$\rho$	+
$\lambda_S$	+	$\lambda_V$	−
$\mu_1$	U-shaped	$\mu_2$	Inverted U-shaped
$\sigma_1$	+	$\sigma_2$	−
$\lambda$	+		

respectively, the range of the option values is (1.085, 2.268) and (0.850, 1.167). The curve of  $\mu_2$  ( $\sigma_2$ ) is smoother than that of  $\mu_1$  ( $\sigma_1$ ), which indicates that the impact of the mean (standard deviation of) jump size of the underlying asset on the price is more sensitive than the assets of the counterparty.

In conclusion, the effects of the parameters on the vulnerable option prices are summarized in Table IV.

#### 4. CONCLUSION

We investigate vulnerable option pricing where the dynamics are governed by a jump-diffusion process with correlated credit risk. Compared with the existing models for vulnerable option pricing, the main advantage of our proposed model is that we take jump risk into consideration. Both the underlying asset and the assets of the counterparty are driven by jump diffusions. Correlation between the assets of the counterparty and the underlying stock of the options is also considered. Under the risk neutral measure, the closed-form pricing formula for vulnerable European options is derived. We also present and discuss several numerical simulations of this pricing formula.

In the numerical illustrations, our proposed model is compared with Black and Scholes model, Merton's jump-diffusion model and the vulnerable B-S model in Klein (1996). We further examine the performance of the proposed model under different parameters assumptions. We find that jump risk of the underlying asset has a positive effect on the price, whereas the impact of shocks from the assets of the counterparty on the price is negative. The option price increases with common jump intensity in our analysis. Furthermore, jump risk of the underlying asset has a more significant impact on the vulnerable option price than that of the assets of the option issuer.

#### APPENDIX

Recall that  $(\ln \frac{S_{T,m_1}}{S_0}, \ln \frac{V_{T,m_2}}{V_0})$  is bivariate normally distributed. Let  $\xi_1$  and  $\xi_2$  be standard normal random variables with correlation  $\bar{\rho}(m_1, m_2) = \frac{\sigma_S \sigma_V T}{\sqrt{\sigma_S^2 T + m_1 \sigma_1^2} \sqrt{\sigma_V^2 T + m_2 \sigma_2^2}} \rho$ , the following equalities hold,

$$\ln \frac{S_{T,m_1}}{S_0} = \left( r - \frac{1}{2} \sigma_S^2 - k_S \lambda_S^* \right) T + m_1 \mu_1 + \sqrt{\sigma_S^2 T + m_1 \sigma_1^2} \xi_1,$$

$$\ln \frac{V_{T,m_2}}{V_0} = \left( r - \frac{1}{2} \sigma_V^2 - k_V \lambda_V^* \right) T + m_2 \mu_2 + \sqrt{\sigma_V^2 T + m_2 \sigma_2^2} \xi_2.$$

For simplifying the expression form, denote  $a_1(m_1)$ ,  $a_2(m_2)$ ,  $b_1(m_1)$ , and  $b_2(m_2)$  as follows,

$$\begin{aligned} a_1(m_1) &= b_1(m_1) + \sqrt{\sigma_S^2 T + m_1 \sigma_1^2}, \\ a_2(m_2) &= b_2(m_2) + \bar{\rho} \sqrt{\sigma_S^2 T + m_1 \sigma_1^2}, \\ b_1(m_1) &= \frac{\ln \frac{S_0}{K} + \left(r - \frac{1}{2} \sigma_S^2 - k_S \lambda_S^*\right) T + m_1 \mu_1}{\sqrt{\sigma_S^2 T + m_1 \sigma_1^2}}, \\ b_2(m_2) &= \frac{\ln \frac{V_0}{D^*} + \left(r - \frac{1}{2} \sigma_V^2 - k_V \lambda_V^*\right) T + m_2 \mu_2}{\sqrt{\sigma_V^2 T + m_2 \sigma_2^2}}. \end{aligned}$$

Recall that in (3),

$$\begin{aligned} M_1(m_1) &= \mathbb{E} \left[ \ln \frac{S_{T,m_1}}{S_0} \right] = \left( r - \frac{1}{2} \sigma_S^2 - k_S \lambda_S^* \right) T + m_1 \mu_1, \\ M_2(m_2) &= \mathbb{E} \left[ \ln \frac{V_{T,m_2}}{V_0} \right] = \left( r - \frac{1}{2} \sigma_V^2 - k_V \lambda_V^* \right) T + m_2 \mu_2. \end{aligned}$$

Then  $A_1(m_1, m_2)$  in (5) is given by

$$\begin{aligned} A_1(m_1, m_2) &= \mathbb{E}[S_{T,m_1} \mathbf{1}(S_{T,m_1} \geq K, V_{T,m_2} \geq D^*)] \\ &= S_0 \mathbb{E} \left[ \frac{S_{T,m_1}}{S_0} \mathbf{1} \left( \ln \frac{S_{T,m_1}}{S_0} \geq \ln \frac{K}{S_0}, \ln \frac{V_{T,m_2}}{V_0} \geq \ln \frac{D^*}{V_0} \right) \right] \\ &= S_0 \mathbb{E} \left[ \frac{S_{T,m_1}}{S_0} \mathbf{1} \left( \frac{\ln \frac{S_{T,m_1}}{S_0} - M_1(m_1)}{\sqrt{\sigma_S^2 T + m_1 \sigma_1^2}} \geq \frac{\ln \frac{K}{S_0} - M_1(m_1)}{\sqrt{\sigma_S^2 T + m_1 \sigma_1^2}}, \frac{\ln \frac{V_{T,m_2}}{V_0} - M_2(m_2)}{\sqrt{\sigma_V^2 T + m_2 \sigma_2^2}} \geq \frac{\ln \frac{D^*}{V_0} - M_2(m_2)}{\sqrt{\sigma_V^2 T + m_2 \sigma_2^2}} \right) \right] \\ &= S_0 \mathbb{E} \left[ e^{M_1(m_1) + \sqrt{\sigma_S^2 T + m_1 \sigma_1^2} \xi_1} \mathbf{1} \left( \xi_1 \geq \frac{\ln \frac{K}{S_0} - M_1(m_1)}{\sqrt{\sigma_S^2 T + m_1 \sigma_1^2}}, \xi_2 \geq \frac{\ln \frac{D^*}{V_0} - M_2(m_2)}{\sqrt{\sigma_V^2 T + m_2 \sigma_2^2}} \right) \right] \\ &= S_0 \mathbb{E} \left[ e^{M_1(m_1) - \sqrt{\sigma_S^2 T + m_1 \sigma_1^2} (-\xi_1)} \mathbf{1} \left( -\xi_1 \leq -\frac{\ln \frac{K}{S_0} - M_1(m_1)}{\sqrt{\sigma_S^2 T + m_1 \sigma_1^2}}, -\xi_2 \leq -\frac{\ln \frac{D^*}{V_0} - M_2(m_2)}{\sqrt{\sigma_V^2 T + m_2 \sigma_2^2}} \right) \right] \\ &= S_0 \int_{-\infty}^{b_1(m_1)} \int_{-\infty}^{b_2(m_2)} e^{M_1(m_1) - \sqrt{\sigma_S^2 T + m_1 \sigma_1^2} x} \frac{1}{2\pi \sqrt{1 - \bar{\rho}(m_1, m_2)^2}} \end{aligned}$$

$$\begin{aligned}
& \cdot \exp \left\{ -\frac{1}{2(1-\bar{\rho}(m_1, m_2)^2)} (x^2 - 2\bar{\rho}(m_1, m_2)xy + y^2) \right\} dx dy \\
&= S_0 \int_{-\infty}^{b_1(m_1)} \int_{-\infty}^{b_2(m_2)} e^{M_1(m_1) + \frac{1}{2}(\sigma_S^2 T + m_1 \sigma_1^2)} \frac{1}{2\pi \sqrt{1-\bar{\rho}(m_1, m_2)^2}} \\
& \cdot \exp \left\{ -\frac{1}{2(1-\bar{\rho}(m_1, m_2)^2)} ((x + \sqrt{\sigma_S^2 T + m_1 \sigma_1^2})^2 \right. \\
& \quad - 2\bar{\rho}(m_1, m_2)(x + \sqrt{\sigma_S^2 T + m_1 \sigma_1^2})(y + \bar{\rho}(m_1, m_2)\sqrt{\sigma_S^2 T + m_1 \sigma_1^2}) \\
& \quad \left. + (y + \bar{\rho}(m_1, m_2)\sqrt{\sigma_S^2 T + m_1 \sigma_1^2})^2) \right\} dx dy \\
&= S_0 e^{M_1(m_1) + \frac{1}{2}(\sigma_S^2 T + m_1 \sigma_1^2)} \int_{-\infty}^{a_1(m_1)} \int_{-\infty}^{a_2(m_2)} \frac{1}{2\pi \sqrt{1-\bar{\rho}(m_1, m_2)^2}} \\
& \cdot \exp \left\{ -\frac{1}{2(1-\bar{\rho}(m_1, m_2)^2)} (x^2 - 2\bar{\rho}(m_1, m_2)xy + y^2) \right\} dx dy \\
&= S_0 e^{M_1(m_1) + \frac{1}{2}(\sigma_S^2 T + m_1 \sigma_1^2)} N_2(a_1(m_1), a_2(m_2), \bar{\rho}(m_1, m_2)) \\
&= S_0 e^{(r - k_S \lambda_S^*)T + m_1 \mu_1 + \frac{1}{2}m_1 \sigma_1^2} N_2(a_1(m_1), a_2(m_2), \bar{\rho}(m_1, m_2)),
\end{aligned}$$

where  $N_2(\cdot, \cdot, \cdot)$  is the bivariate normal cumulative distribution function.

For  $A_2(m_1, m_2)$ , the following expressions hold:

$$\begin{aligned}
A_2(m_1, m_2) &= -KE[\mathbf{1}(S_{T, m_1} \geq K, V_{T, m_2} \geq D^*)] \\
&= -KE \left[ \mathbf{1} \left( -\xi_1 \leq -\frac{\ln \frac{K}{S_0} - M_1(m_1)}{\sqrt{\sigma_S^2 T + m_1 \sigma_1^2}}, -\xi_2 \leq -\frac{\ln \frac{D^*}{V_0} - M_2(m_2)}{\sqrt{\sigma_V^2 T + m_2 \sigma_2^2}} \right) \right] \\
&= -K N_2(b_1(m_1), b_2(m_2), \bar{\rho}(m_1, m_2)).
\end{aligned}$$

Denote

$$\begin{aligned}
c_1(m_1) &= b_1(m_1) + \left( \sqrt{\sigma_S^2 T + m_1 \sigma_1^2} + \bar{\rho} \sqrt{\sigma_V^2 T + m_2 \sigma_2^2} \right), \\
c_2(m_2) &= -b_2(m_2) - \left( \sqrt{\sigma_V^2 T + m_2 \sigma_2^2} + \bar{\rho} \sqrt{\sigma_S^2 T + m_1 \sigma_1^2} \right).
\end{aligned}$$

Recall that  $\xi_1$  and  $\xi_2$  are standard normal random variables with correlation  $\bar{\rho}(m_1, m_2)$ . Then we derive  $A_3(m_1, m_2)$  as follows:

$$\begin{aligned}
A_3(m_1, m_2) &= \frac{1-\alpha}{D} \mathbb{E}[S_{T, m_1} V_{T, m_2} \mathbf{1}(S_{T, m_1} \geq K, V_{T, m_2} < D^*)]
\end{aligned}$$

$$\begin{aligned}
&= \frac{1-\alpha}{D} S_0 V_0 \mathbb{E} \left[ e^{M_1(m_1) - \sqrt{\sigma_S^2 T + m_1 \sigma_1^2}(-\xi_1)} e^{M_2(m_2) + \sqrt{\sigma_V^2 T + m_2 \sigma_2^2} \xi_2} \mathbf{1}(-\xi_1 \leq b_1(m_1), \xi_2 \leq -b_2(m_2)) \right] \\
&= \frac{1-\alpha}{D} S_0 V_0 e^{M_1(m_1) + M_2(m_2)} \int_{-\infty}^{b_1(m_1)} \int_{-\infty}^{-b_2(m_2)} e^{-\sqrt{\sigma_S^2 T + m_1 \sigma_1^2} x + \sqrt{\sigma_V^2 T + m_2 \sigma_2^2} y} \frac{1}{2\pi \sqrt{1 - \bar{\rho}(m_1, m_2)^2}} \\
&\quad \cdot \exp \left\{ -\frac{1}{2(1 - \bar{\rho}(m_1, m_2)^2)} (x^2 - 2\bar{\rho}(m_1, m_2)xy + y^2) \right\} dx dy \\
&= \frac{1-\alpha}{D} S_0 V_0 e^{M_1(m_1) + \frac{1}{2}(\sigma_S^2 T + m_1 \sigma_1^2) + M_2(m_2) + \frac{1}{2}(\sigma_V^2 T + m_2 \sigma_2^2) + \rho \sigma_S \sigma_V T} \\
&\quad \cdot \int_{-\infty}^{b_1(m_1)} \int_{-\infty}^{-b_2(m_2)} \frac{1}{2\pi \sqrt{1 - \bar{\rho}(m_1, m_2)^2}} \cdot \exp \left\{ -\frac{1}{2(1 - \bar{\rho}(m_1, m_2)^2)} ((x + c_1(m_1) - b_1(m_1))^2 \right. \\
&\quad \left. - 2\bar{\rho}(m_1, m_2)(x + c_1(m_1) - b_1(m_1))(y + b_2(m_2) + c_2(m_2)) + (y + b_2(m_2) + c_2(m_2))^2) \right\} dx dy \\
&= \frac{1-\alpha}{D} S_0 V_0 e^{M_1(m_1) + \frac{\sigma_S^2}{2} T + M_2(m_2) + \frac{\sigma_V^2}{2} T + \rho \sigma_S \sigma_V T + \frac{1}{2} m_1 \sigma_1^2 + \frac{1}{2} m_2 \sigma_2^2} \int_{-\infty}^{c_1(m_1)} \int_{-\infty}^{c_2(m_2)} \frac{1}{2\pi \sqrt{1 - \bar{\rho}(m_1, m_2)^2}} \\
&\quad \cdot \exp \left\{ -\frac{1}{2(1 - \bar{\rho}(m_1, m_2)^2)} (x^2 - 2\bar{\rho}(m_1, m_2)xy + y^2) \right\} dx dy \\
&= \frac{1-\alpha}{D} S_0 V_0 e^{M_1(m_1) + \frac{\sigma_S^2}{2} T + M_2(m_2) + \frac{\sigma_V^2}{2} T + \rho \sigma_S \sigma_V T + \frac{1}{2} m_1 \sigma_1^2 + \frac{1}{2} m_2 \sigma_2^2} N_2(c_1(m_1), c_2(m_2), -\bar{\rho}(m_1, m_2)) \\
&= \frac{1-\alpha}{D} S_0 V_0 e^{(r - k_S \lambda_S^*)T + m_1 \mu_1 + (r - k_V \lambda_V^*)T + m_2 \mu_2 + \rho \sigma_S \sigma_V T + \frac{1}{2} m_1 \sigma_1^2 + \frac{1}{2} m_2 \sigma_2^2} N_2(c_1(m_1), c_2(m_2), -\bar{\rho}(m_1, m_2)).
\end{aligned}$$

Similarly,  $A_4(m_1, m_2)$  can be written as

$$\begin{aligned}
&A_4(m_1, m_2) \\
&= -\frac{1-\alpha}{D} K \mathbb{E} [V_{T, m_2} \mathbf{1}(S_{T, m_1} \geq K, V_{T, m_2} < D^*)] \\
&= -\frac{1-\alpha}{D} K V_0 \mathbb{E} [V_{T, m_2} \mathbf{1}(-\xi_1 \leq b_1(m_1), \xi_2 < -b_2(m_2))] \\
&= -\frac{1-\alpha}{D} K V_0 e^{M_2(m_2)} \int_{-\infty}^{b_1(m_1)} \int_{-\infty}^{-b_2(m_2)} e^{\sqrt{\sigma_V^2 T + m_2 \sigma_2^2} y} \frac{1}{2\pi \sqrt{1 - \bar{\rho}(m_1, m_2)^2}} \\
&\quad \cdot \exp \left\{ -\frac{1}{2(1 - \bar{\rho}(m_1, m_2)^2)} (x^2 - 2\bar{\rho}(m_1, m_2)xy + y^2) \right\} dx dy \\
&= -\frac{1-\alpha}{D} K V_0 e^{M_2(m_2) + \frac{1}{2}(\sigma_V^2 T + m_2 \sigma_2^2)} \int_{-\infty}^{d_1(m_1, m_2)} \int_{-\infty}^{d_2(m_2)} \frac{1}{2\pi \sqrt{1 - \bar{\rho}(m_1, m_2)^2}} \\
&\quad \cdot \exp \left\{ -\frac{1}{2(1 - \bar{\rho}(m_1, m_2)^2)} (x^2 - 2\bar{\rho}(m_1, m_2)xy + y^2) \right\} dx dy \\
&= -\frac{1-\alpha}{D} K V_0 e^{(r - k_V \lambda_V^*)T + m_2 \mu_2 + \frac{1}{2} m_2 \sigma_2^2} N_2(d_1(m_1), d_2(m_2), -\bar{\rho}(m_1, m_2)),
\end{aligned}$$

where

$$\begin{aligned}
d_1(m_1) &= b_1(m_1) + \bar{\rho}(m_1, m_2) \sqrt{\sigma_V^2 T + m_2 \sigma_2^2}, \\
d_2(m_2) &= -b_2(m_2) - \sqrt{\sigma_V^2 T + m_2 \sigma_2^2}.
\end{aligned}$$

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