Advanced Term Structure Models

Applied Finance Project

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Advanced Term Structure Models

Jump diffusion extensions of classical interest rate models

Abstract

There is growing evidence of the presence of jump risks within the fixed income markets. The crises of 2008, 2013, and 2020 all shocked the bond markets in an instantaneous manner that is not fully captured by the classical interest rate models which assume a more continuous evolution. Our paper extends the classical models such as Vasicek and CIR to incorporate the effects of jump-risks in the market. We explore modern methods to price and calibrate such models and evaluate their pricing performance with respect to classical models and the observed market prices.

Contents

1 Introduction

1.1 Background/Motivation

Many continuous asset-pricing models that follow continuous paths have been derived under the stochastic diffusion process. However recent, empirical studies suggest that the underlying process might follow discontinuous sample paths. Although, the research is still going on the effectiveness of the discontinuous sample paths, the difference between the outcomes of the theoretical models with continuous path and jumps motivates this paper. Recently, the models developed by Ahn and Thompson studied the effect examined the effect of regulatory risk on the valuation of public utilities where they identified that jump risks are one of the contributing factors in the price even though they are uncorrelated with market factors which implies that Jump risks cannot be ignored in the asset pricing.

1.2 Literature Review

Many studies in the past have tried to model the regime change or shocks in financial variables. There is compelling evidence for jumps both in emerging and developed economies. Specific sources of jumps in interest rates, including economic news and moves by central banks, are put forward in [BW97], [ELP07], and [Joh03]. [Joh03] provides evidence of jumps in US treasury bill rates and highlights the importance of incorporating jumps in pricing interest rate options. Many empirical studies note the importance of jump-diffusion in explaining observed interest rates than pure diffusion models.

[BNP07] models traded uncertainty by a Poisson process and extends the HJM framework to derive the forward rate dynamics for jump diffusion under the real world probability measure. [GK03] also characterizes both jumps and diffusion using discretely compounded forward rates evolving continuously or forward rates. They allow for dependence between jump time, jump sizes and interest rates. Using this framework, they price derivative securities like interest rate caps, floors and options on swaps. They argue that the rationale for using simple forwards lies in using observable interest rate data and study the effect of jumps on skew, smile and implied volatilities of interest rate derivatives.

[Zho01] argues that the CIR model is only able to explain mean persistence, long run mean reversion and volatility in short rate. It fails to capture the volatility persistence and therefore advocates introducing a stochastic volatility factor into diffusion function. Conceptually, they argue, a stochastic volatility term is similar to the jump-diffusion approach. They implement a Multivariate Weighted Non-Linear Least Squares estimator (MWNLS-JD) and the instantaneous interest rate is modeled as a mixture of a square root diffusion process and a Poisson jump process.

1.3 Problem Definition/Statement of Hypotheses

We hypothesize that jump-diffusion processes more accurately model the term structure of interest rates than those without. To test this hypothesis, we price debt instruments, including floors and caps, using data obtained from Bloomberg, comparing the accuracy of models with and without jumps.

We shall proceed in 4 steps:

1. Extend classical interest models to include jump diffusion processes.
2. Develop Monte-Carlo with variance reduction techniques to evaluate the price curve for an arbitrary set of parameters.
3. Fit the model parameters using various regression and root-finding techniques to obtain a price curve that agrees with market rates for Treasury bonds.
4. Compare the prices obtained in the Jump diffusion setup versus prices for the classical models. Use standard measures such as MSE, RMSE, and/or MAE to evaluate the pricing error.

Based on the size and significance of the pricing errors of jump models compared to classical models will enable us to make a conclusion regarding the validity of jump models and the existence of jump risks in fixed income product pricing.

2 Experiment Setup

2.1 Model Definition

We begin with two classical interest rate models, Vasicek and Cox-Ingersoll-Ross (CIR):

$$dr(t) = [\theta(t) - a(t)r(t)]dt + \sigma(t)dW(t) \qquad \text{(Vasicek)}$$
$$dr(t) = [\theta(t) - a(t)r(t)]dt + \sigma(t)\sqrt{r(t)}dW(t) \qquad \text{(CIR)}$$

Both have a mean-reversion tendency with speed of mean reversion controlled by the parameters a and θ plus a random Wiener process. The key difference between the two models is the presence of a $\sqrt{r(t)}$ term in CIR that imposes non-negative interest rates on the model (as $r(t)$ approaches 0, the contribution of the random component shrinks to 0).

For simplicity, we begin by assuming $\theta(t)$, a(t), and $\sigma(t)$ to be constants. The exact values of the constants will be calibrated to fit the observed yield curve of Treasury bonds.

Next, we extend the classical model by adding a Jump process. The arrival of the jumps is driven by a Poisson process with a parameterized arrival probability. Furthermore, the jump itself is random with a normally distributed size. This is all we need to do to extend the Vasicek model to include jumps and we obtain the following SDE:

$$dr(t) = [\theta(t) - a(t)r(t)]dt + \sigma(t)dW(t) + JdP \qquad \text{(Jump-Vasicek)}$$
$$J = N(\mu, \gamma), P = Poisson(\lambda)$$

In the case of CIR, we have to be more careful with the model definition. Since the Jumps are normally distributed we cannot allow an unconstrained discretely arriving Jump process for a CIR as such a jump could potentially drive the interest rates negative. Zhou (2001) [Zho01] demonstrated that this is equivalent to imposing the condition $-r(t_-) < J < \infty$. In our model, we go even further and impose a symmetry condition on the normal distribution: $-r(t_-) < J < r(t_-)$. This allows us to preserve symmetry (useful when we proceed to MC with antithetic variates) and use the truncated normal distribution. The CIR model is thus specified as follows:

$$dr(t) = [\theta(t) - a(t)r(t)]dt + \sigma(t)\sqrt{r(t)}dW(t) + JdP \qquad \text{(Jump-CIR)}$$
$$J = N_{\text{trunc}}(\mu, \gamma, \frac{-r(t_-)}{dP}, \frac{r(t_-)}{dP}), P = Poisson(\lambda)$$

The Jump diffusion terms simulate the "jump risk" that may exist in the market. Jumps arrive randomly and can move the interest rate instantaneously. This attempts to simulate real-world shocks to the yield curve such as a surprise Fed rate decisions (2013 and 2018), commercial paper liquidity crisis during COVID-19, and the credit crunch triggered by bank failures in 2008-09.

It is important to note that these SDEs are in the real-world probability measure (P).

2.2 Model PDE

2.2.1 Vasicek

Similar to the work done in MFE230I for the classical models, we use the *market price of risk*, represented by the parameter λ and assume that the jump risk is diversifiable to derive the following bond pricing PDE for Vasicek: [PKK06]:

$$[\theta(t) - a(t)r(t) - \lambda(t)\sigma(t)]V_r + V_t + \frac{1}{2}\sigma^2(t)V_{rr} + hV[-\mu A(t,T) + \frac{1}{2}(\gamma^2 + \mu^2)A(t,T)^2] = 0 \text{ (Vasicek PDE)}$$

With can guess a solution of the form $V(r,t,T) = exp[-A(t,T)r + B(t,T)]$ where $A(t,T)$ and $B(t,T)$ are given by the following PDEs, where $\phi(t) = \theta(t) - \lambda(t)\sigma(t)$ and $A(T,T) = B(T,T) = 0$:

$$-\frac{\delta A}{\delta t} + a(t)A - 1 = 0$$

$$-\frac{\delta B}{\delta t} - \phi(t)A + \frac{1}{2}\sigma^2 A^2 + h[-\mu A + \frac{1}{2}(\gamma^2 + \mu^2)A^2] = 0$$

2.2.2 CIR

Similarly, we have an analogous PDE for the CIR model:

$$[\theta(t) - a(t)r(t) - \lambda(t)\sigma(t)r(t)]V_r + V_t + \frac{1}{2}\sigma^2(t)r(t)V_{rr} + hV[-\mu A(t,T) + \frac{1}{2}(\gamma^2 + \mu^2)A(t,T)^2] = 0 \text{ (CIR PDE)}$$

With the guessed solution of the same form as before and the following equations for $A(t,T)$ and $B(t,T)$:

$$0 = -\frac{\delta A}{\delta t} + \psi(t)A + \frac{1}{2}\sigma^2 A^2 - 1$$

$$0 = -\frac{\delta B}{\delta t} + (\theta(t) + h\mu)A + \frac{1}{2}h(\gamma^2 + \mu^2)A^2$$

2.3 Methodology

2.3.1 Classical Monte-Carlo

Using the Model PDEs derived in section 2.2.2, we proceed to discretize the PDEs in preparation for a Monte-Carlo simulation as follows [PKK06]:

$$r_j = r_{j-1} + [\theta - ar_{j-1} - \lambda\sigma]\Delta t + \sigma\epsilon_j\sqrt{\Delta t} + JP_{\Delta t} \text{ (Vasicek)}$$

$$r_j = r_{j-1} + [\theta - ar_{j-1} - \lambda\sigma\sqrt{r_{j-1}}]\Delta t + \sigma\epsilon_j\sqrt{r_{j-1}\Delta t} + J^{trunc}P_{\Delta t} \text{ (CIR)}$$

ϵ_j is N(0,1), $P_{\Delta t}$ is Poisson with arrival likelihood $h\Delta t$. For Vasicek, J is N(μ, γ) whereas for CIR J^{trunc} is truncated Normal where the truncation is symmetric and such that r_j is guaranteed to be non-negative. This corresponds to a truncated Normal distribution given by:

$$J^{trunc} \sim N^{trunc}(\mu, \gamma, \pm\frac{(r_{j-1} + [\theta - ar_{j-1} - \lambda\sigma\sqrt{r_{j-1}}]\Delta t + \sigma\epsilon_j\sqrt{r_{j-1}\Delta t})}{P_{\Delta t}})$$

The Monte-Carlo simulation will simulate the interest rate over a 20-year period (t = 0, T = 20) with 365 time steps per year (i.e. $\Delta t = \frac{1}{365}$ years). We will use 1000 simulations (n = 1000) for each time step.

A complete interest rate path (consisting of $365(T-t)$ time steps) will be used to discount the bond price to obtain a single sample price curve for the time period from t to T. The different price curves will be averaged out using a simple average to obtain a Classical MC price curve:

$$\frac{1}{n}\sum_{i=1}^{n} exp(-\sum_{j=1}^{m} -r_{ij}\Delta t)$$

2.3.2 Variance Reduction in Monte-Carlo

2.3.2.1 Antithetic Variates

The classical Monte-Carlo simulation provides us with bond price estimates as the average of the bond prices obtained for each spot rate path.

In order to reduce the variance of these estimates, we use antithetical variates. We obtain the antithetic FDE in relation to the FDE(s) derived for the classical Monte-Carlo simulation in 2.3.1

The anithetical FDE(s) are

$$r_j = r_{j-1} + [\theta - ar_{j-1} - \lambda\sigma]\Delta t - \sigma\epsilon_j\sqrt{\Delta t} - JP_{\Delta t} \text{ (Vasicek)}$$

$$r_j = r_{j-1} + [\theta - ar_{j-1} - \lambda\sigma\sqrt{r_{j-1}}]\Delta t - \sigma\epsilon_j\sqrt{r_{j-1}\Delta t} + J^{trunc\_anti}P_{\Delta t} \text{ (CIR)}$$

ϵ_j is N(0,1) and the same sample drawn for the original spot rate. $P_{\Delta t}$ is Poisson with arrival likelihood $h\Delta t$ and the same sample drawn for the original spot rate. For Vasicek, J is N(μ, γ) and the same sample drawn for the original spot rate whereas for CIR J^{trunc_anti} is truncated Normal where the truncation is symmetric and such that r_j is guaranteed to be non-negative. This corresponds to a truncated Normal distribution given by:

$$J^{trunc\_anti} \sim N^{trunc}(\mu, \gamma, \pm\frac{(r_{j-1} + [\theta - ar_{j-1} - \lambda\sigma\sqrt{r_{j-1}}]\Delta t - \sigma\epsilon_j\sqrt{r_{j-1}\Delta t})}{P_{\Delta t}})$$

The sign of the truncated normal is such that $J^{trunc} * J^{trunc\_anti} < 0$ where J_{trunc} corresponds to the original spot rate.

Similar to the classical Monte-Carlo simulation, we simulate over a 20-year period (t = 0, T = 20) with 365 time steps per year (i.e. $\Delta t = \frac{1}{365}$ years). We use 1000 simulations (n = 1000) for each time step. Effectively, we get 2000 paths because for each simulated path, we also obtain the corresponding antithetic path.

A complete interest rate path (consisting of $365(T-t)$ time steps) is used to discount the bond price to obtain a single sample price curve for the time period from t to T. The price for each path and its antithetic path is averaged out followed by the average of all the averaged out prices to obtain the antithetical MC price curve:

$$\frac{1}{n}\sum_{i=1}^{n}\frac{exp(-\sum_{j=1}^{m} -r_{ij}\Delta t) + exp(-\sum_{j=1}^{m} -r_{ij\_anti}\Delta t)}{2}$$

Let the variance of the bond price be V (We use sample variance as an estimator for the true variance). The variance of the above estimate is hence given by:

$$\frac{1}{n^2}\sum_{i=1}^{n}\frac{V+V+2*\rho*\sqrt{V}\sqrt{V}}{4}$$

where ρ is the correlation between the bond prices obtained using spot rates and their antithetic spot rates. With antithetic variates, we try to obtain a negative correlation so the variance of the estimate is reduced.

2.3.2.2 Control Variates

Our interest rate models are an extension to existing models without jumps. The classical models mentioned in 2.1 have closed form solutions for zero-coupon bond prices. Vasicek uses a negative λ as the market price of risk. The PDE for zero coupon bond prices is :

$$0=\frac{1}{2}\sigma^2V_{rr}+[\kappa(\mu-r)+\lambda\sigma]V_r+V_t-rV$$

$$0=\frac{1}{2}\sigma^2V_{rr}+\kappa(\hat{\mu}-r)V_r+V_t-rV$$

where $\kappa=a$, $\mu=\frac{\theta}{a}$, $\hat{\mu}=\frac{\theta}{a}+\frac{\lambda\sigma}{a}$

The closed form solution for bond prices is :

$$Z(t,t+T)=e^{-\hat{\mu}T-\frac{1-e^{-\kappa T}}{\kappa}(r-\hat{\mu})+\frac{\sigma^2}{2\kappa^2}(T+\frac{1-e^{-2\kappa T}}{2\kappa}-2\frac{1-e^{-\kappa T}}{\kappa})}$$

We use the above closed form solution as a control variate for obtaining the bond price using jump model. We define the following estimator for bond price using jump model:

$$\theta=V_{jump}+c(V_{classical}-Z))$$

where V_{jump} is the bond price using jump model, $V_{classical}$ is the bond price using classical model and Z is the true bond price ($E^Q(V_{classical})=Z$). It can be shown that $E(\theta)=E(V_{jump})$, hence θ is an unbiased estimator for bond price using jump model.

It can be shown that the variance is minimum when:

$$c=-\frac{Cov(V_{classical},V_{jump})}{Var(V_{classical})}$$

It can also be shown that the minimum variance of θ is:

$$min(Var(\theta))=Var(V_{jump})-\frac{Cov(V_{classical},V_{jump})^2}{Var(V_{classical})}$$

Hence with a higher covariance between $V_{classical}$ and V_{jump}, we obtain a lower variance for the estimator of bond price using jump model (i.e. θ)

2.4 Data Sources & Description

The data is obtained using the Custom Swap Curve Builder "ICVS" command in Bloomberg.

Term	Unit	Ticker	Bid	Ask	Spread	Bid Spr Val	Ask Spr Val	Final Bid Rate	Final Ask Rate	Rate Type	Daycount	Freq
3	MO	US0003M	0.21575	0.21575		0	0	0.21575	0.21575	Cash Rates	ACT/360	0
20210317	ACTDATE	EDZ20	0.239883731	0.239883731		0	0	0.239883731	0.239883731	Contiguous Futures	ACT/360	0
20210616	ACTDATE	EDH1	0.204494567	0.204494567		0	0	0.204494567	0.204494567	Contiguous Futures	ACT/360	0
20210915	ACTDATE	EDM1	0.198914642	0.198914642		0	0	0.198914642	0.198914642	Contiguous Futures	ACT/360	0
20211215	ACTDATE	EDU1	0.198148308	0.198148308		0	0	0.198148308	0.198148308	Contiguous Futures	ACT/360	0
20220316	ACTDATE	EDZ1	0.232199603	0.232199603		0	0	0.232199603	0.232199603	Contiguous Futures	ACT/360	0
20220615	ACTDATE	EDH2	0.271072732	0.271072732		0	0	0.271072732	0.271072732	Contiguous Futures	ACT/360	0
2	YR	USSWAP2	0.235973001	0.239026994		0	0	0.235973001	0.239026994	Swap Rates	30I/360	2
3	YR	USSWAP3	0.280021012	0.282979012		0	0	0.280021012	0.282979012	Swap Rates	30I/360	2
4	YR	USSWAP4	0.35274601	0.356454015		0	0	0.35274601	0.356454015	Swap Rates	30I/360	2
5	YR	USSWAP5	0.445024997	0.451974988		0	0	0.445024997	0.451974988	Swap Rates	30I/360	2
6	YR	USSW6	0.546862006	0.554138005		0	0	0.546862006	0.554138005	Swap Rates	30I/360	2
7	YR	USSWAP7	0.647571027	0.651428998		0	0	0.647571027	0.651428998	Swap Rates	30I/360	2
8	YR	USSW8	0.737478971	0.741420984		0	0	0.737478971	0.741420984	Swap Rates	30I/360	2
9	YR	USSW9	0.82002598	0.82377398		0	0	0.82002598	0.82377398	Swap Rates	30I/360	2
10	YR	USSWAP10	0.893034995	0.896866024		0	0	0.893034995	0.896866024	Swap Rates	30I/360	2
11	YR	USSWAP11	0.95529002	0.956690013		0	0	0.95529002	0.956690013	Swap Rates	30I/360	2
12	YR	USSWAP12	1.009559035	1.013440967		0	0	1.009559035	1.013440967	Swap Rates	30I/360	2
15	YR	USSWAP15	1.12656796	1.130532026		0	0	1.12656796	1.130532026	Swap Rates	30I/360	2
20	YR	USSWAP20	1.241929054	1.247671008		0	0	1.241929054	1.247671008	Swap Rates	30I/360	2
25	YR	USSWAP25	1.289873958	1.294324994		0	0	1.289873958	1.294324994	Swap Rates	30I/360	2
30	YR	USSWAP30	1.310613036	1.31518805		0	0	1.310613036	1.31518805	Swap Rates	30I/360	2
40	YR	USSWAP40	1.272482991	1.278246999		0	0	1.272482991	1.278246999	Swap Rates	30I/360	2
50	YR	USSWAP50	1.194857001	1.215643048		0	0	1.194857001	1.215643048	Swap Rates	30I/360	2

Figure 1: Bloomberg Custom Swap Curve ("ICVS" command)

The Bloomberg LIBOR curve is estimated using three different types of instruments based on maturity length: LIBOR deposit rates for short maturities (3 months), rates calculated from Eurodollar futures for intermediate maturities (3 months to 2 years), and swap rates for longer maturities (2 years to 50 years).

[PLACEHOLDER FOR ADDITIONAL DATA DESCRIPTIONS]

3 Preliminary Results

To simplify the initial simulation we assume constant values for the model parameters such as θ, a, b, γ, and μ. The following values are assumed:

[PLACEHOLDER FOR PARAMETER VALUES]

3.1 Classical Approach

3.1.1 Vasicek

We run the MC simulation for the Vasicek model to obtain the following yield and price curves for the bonds.

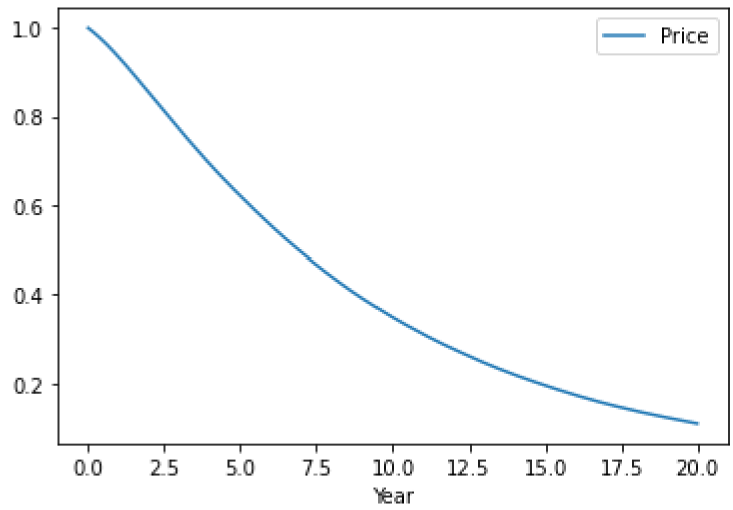

Figure 2: Vasicek Price Curve

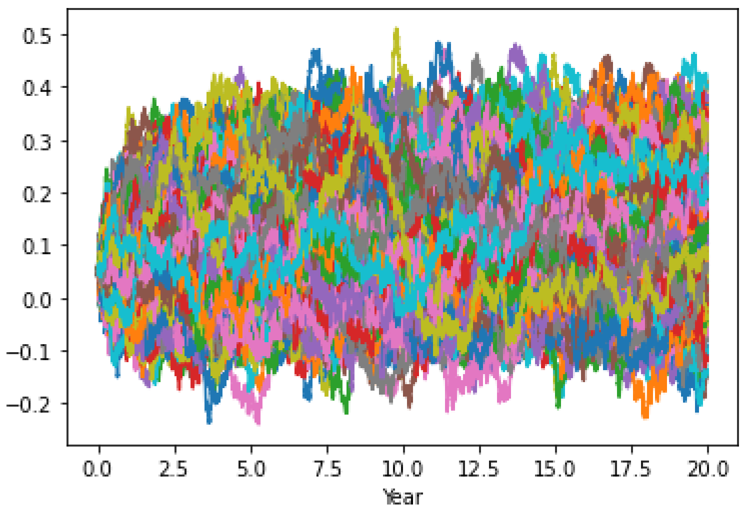

Figure 3: Vasicek Simulated Rates

Figure 2 represents the average price obtained from the 1000 interest rate simulations. Figure 3 plots the simulations. Note that both positive and negative interest rates are observed in the evolution.

3.1.2 CIR

We run the MC simulation for the CIR model to obtain the following yield and price curves for the bonds.

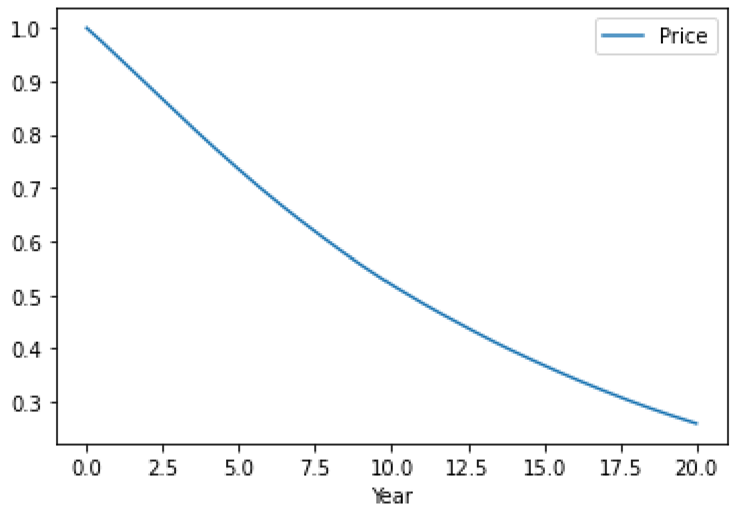

Figure 4: CIR Price Curve

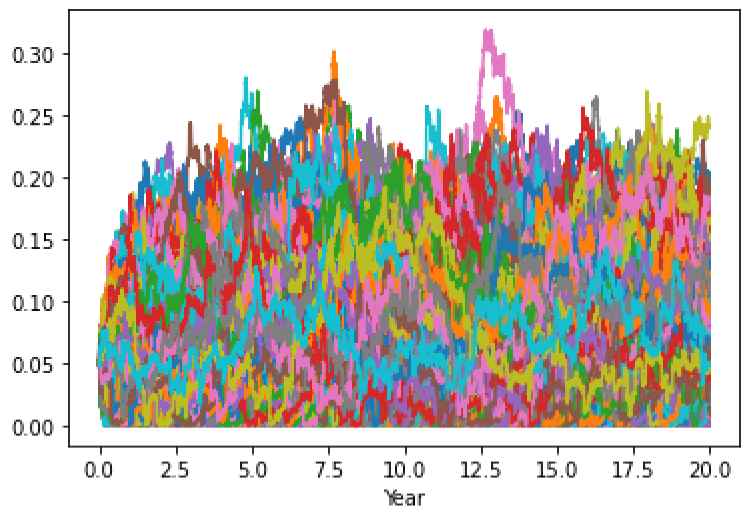

Figure 5: CIR Simulated Rates

Figure 9 represents the average price obtained from the 1000 interest rate simulations. Figure 5 plots the simulations. Note that unlike in the Vasicek, rates are lower bounded by zero. This is because of the truncated normal sampling performed for the CIR simulation.

3.2 Variance Reduction

3.2.1 Antithetic Variates

3.2.1.1 Vasicek

We run antithetic variates based MC simulation for Vasicek. A sample spot rate path along with its antithetic counterpart looks like below

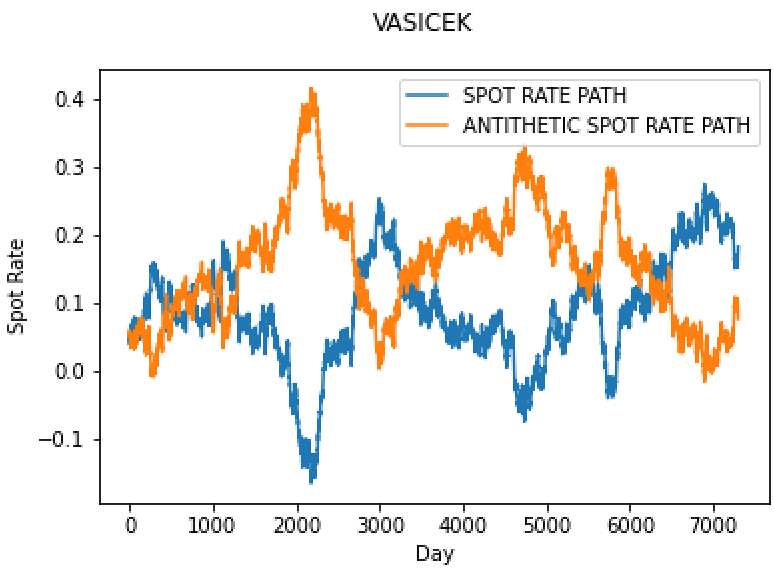

Figure 6: Sample Spot Rate And Antithetic Spot Rate paths for Vasicek

It is evident that the paths are quite negatively correlated.

We also compare the variances of bond price estimates using classical MC and antithetical MC.

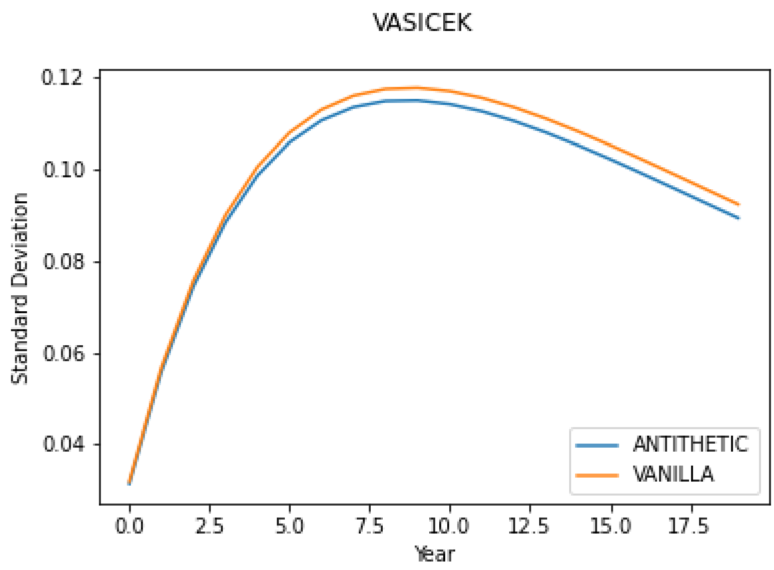

Figure 7: Classical MC and Antithetic MC Variance for Vasicek

As expected, we observe that the variance for antithetic is lower than classical MC.

3.2.1.2 CIR

Likewise, we run MC using antithetic variates for CIR. Here is how a sample spot rate path and it's antithetical counterpart looks like

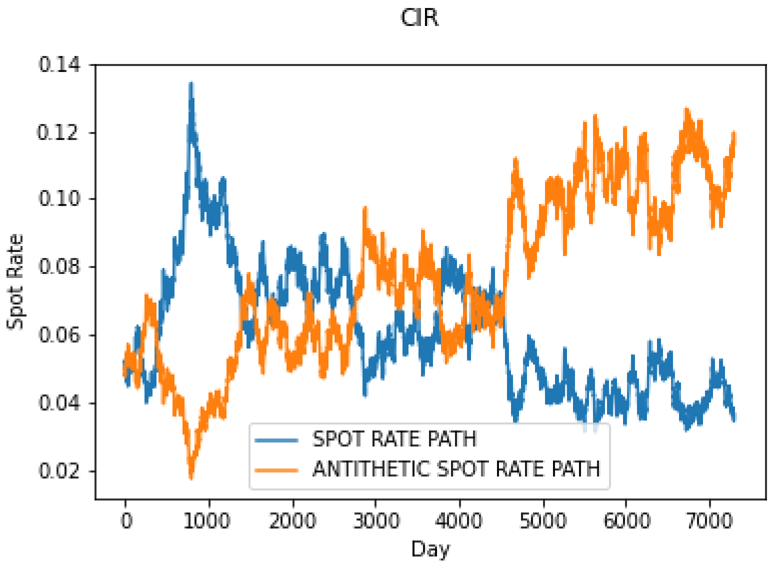

Figure 8: Sample Spot Rate And Antithetic Spot Rate paths for CIR

We observe negative correlation in this case as well. However, the magnitude of jumps are not the same for both the paths.

Theoretically, the variance reduction in CIR should be lesser than that in Vasicek. This is because in CIR, the jump normal is re-sampled for antithetical path and hence can be different from the original spot rate path. We make sure that the two samples have opposite signs in order to maximize the effect of negative correlation on variance reduction.

The variances of bond price estimates using classical MC and antithetical MC looks like the below

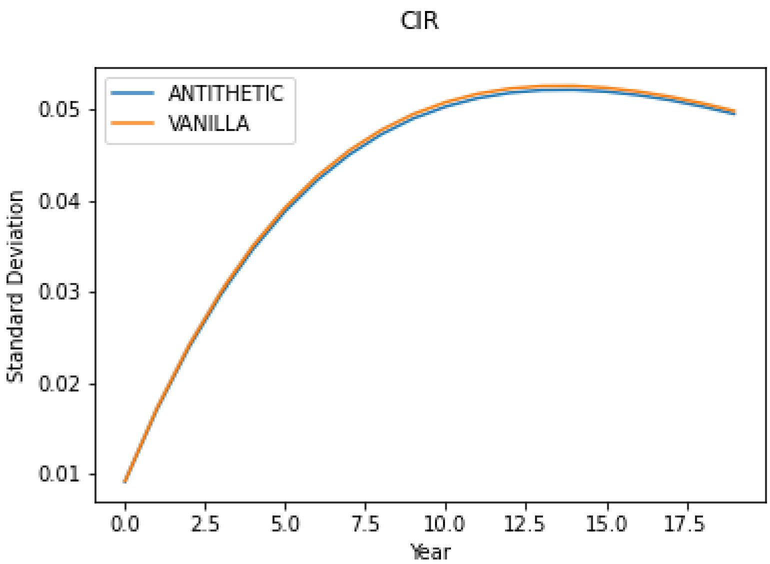

Figure 9: Classical MC and Antithetic MC Variance for CIR

As expected, we note that the variance reduction in CIR is lesser than Vasicek.

3.2.2 Control Variates

[COMPARISON OF VARIANCE FOR VASICEK]

[COMPARISON OF VARIANCE FOR CIR]

[DISCUSSION OF BOTH]

4 Calibration

The Monte-Carlo set up provides us with a framework for pricing yield curves that incorporate jump risk. So far we have used a assumed set of parameters for the model. We now proceed to tune this parameters to obtain yield and price curves that agree with the prevailing market values.

[DETAILS ABOUT NATURE OF MARKET DATA USED. PROBABLY TREASURY YIELD RATES. ALSO REFER TO DATA DESCRIPTION SECTION].

To fit the parameters of the model we propose three broad approaches:

1. Newton-Raphson and other solver based approaches
   - Advantages: Simple to use, can use standard routines available in Python, Excel, and other common financial tools.
   - Disadvantages: Algorithm may take too long or may not converge at all. Existence of multiple solutions could force the user to choose between several compelling solutions with little theoretical basis for doing so.
2. Maximum Likelihood Estimation (MLE) approach
   - Advantages: Will provide consistent, efficient estimator of the model parameters.
   - Disadvantages: Contingent upon assumptions about model parameter distributions.
3. NLS regression estimator (error minimizing estimator)
   - This is the weighted NLS estimator based on the work of Zhou (2001) [Zho01]. Zhou constructs a regression estimator by using Ito's formula to compute the first 4 moments of the interest rate at time t. The estimator minimizes the error between the fitted and actual first moments.
   - Advantages: Provides a regression based, consistent, asymptotically normal estimator of the model parameters.
   - Disadvantages: More technically complex. Contingent on being able to write down expressions for first 4 moments. Moments may have integrals which will have to be estimated using discrete observations of the rate. This approach may be prone to market micro-structure effects.

4.1 Newton-Raphson and Solver Approach

[PLACEHOLDER FOR RESULTS]

[PLACEHOLDER FOR DISCUSSION]

4.2 Maximum Likelihood Estimation (MLE) Approach

[PLACEHOLDER FOR RESULTS]

[PLACEHOLDER FOR DISCUSSION]

4.3 Regression (NLS) Approach

[PLACEHOLDER FOR RESULTS]

[PLACEHOLDER FOR DISCUSSION]

5 Pricing

With our model properly setup and calibrated, we proceed to price common fixed-income derivatives on the financial markets.

In particular we will focus on [DISCUSSION OF TYPES OF DERIVATIVES, WHAT INTEREST RATES THEY USE, HOW WE WILL PRICE THEM].

We will use mean squared error and mean absolute error as metrics of pricing accuracy with respect to the price observed on the market. The parameter estimates from each of the 3 different methods of calibration will be used to evaluate the pricing errors.

5.1 Newton-Raphson

[TABLE FOR PRICING ERRORS]

5.2 Maximum Likelihood

[TABLE FOR PRICING ERRORS]

5.3 Regression (NLS) Estimation

[TABLE FOR PRICING ERRORS]

5.4 Conclusions

[DISCUSSION ABOUT PRICING RESULTS]

6 Final Conclusions

[FINAL DISCUSSIONS ABOUT RESULTS AND WORK DONE FOR PAPER. DISCUSSION OF WHETHER JUMP RISKS ARE/ARE NOT SUPERIOR TO CLASSICAL MODELS BASED ON EXPERIMENTS. DISCUSSION OF WHETHER MARKET IS PRICING IN JUMP RISKS]

7 Future Work

[SUGGESTIONS FOR IMPROVEMENT TO PAPER'S WORK. POSSIBLE DISCUSSION POINTS:

1. TREE BASED MODELS
2. BETTER ESTIMATIONS FOR PARAMETERS
3. WORK TOWARDS PARTIAL/COMPLETE CLOSED FORM SOLUTIONS]

8 Bibliography/References

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9 Appendices

[ADD FINALIZED CODES]