Matrix theory - Assignment1

Banoth Upender
EE20MTECH11011

***Abstract*—This document illustrates the distance of the point from the point of intersection of the line and the plain.**

Download all python codes from

https://github.com/upender20/EE5600/tree/master

1 Problem

Find the distance of the point $\begin{pmatrix} -1 \\ -5 \\ -10 \end{pmatrix}$ from the point of intersection of the line

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} \tag{1.0.1}$$

and the plane

$$\begin{pmatrix} 1 - 11 \end{pmatrix} \mathbf{x} = 5 \tag{1.0.2}$$

2 Construction

We know that equation of the line passing through given a point and a plane

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{m} \tag{2.0.1}$$

Also we can find direction vector from the Cartesian form of equation

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \tag{2.0.2}$$

This can be expressed as

$$\mathbf{x} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \lambda \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{2.0.3}$$

where $\mathbf{a} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ is a point on given line and $\mathbf{m} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is the direction vector.
Distance between the point and point of intersection.

$$D = \sqrt{(x2-x1)^2 + (y2-y1)^2 + (z2-z1)^2} \tag{2.0.4}$$

3 Solution

Writing given equation (1.0.1) in vector form as

$$\mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \tag{3.0.1}$$

substitute (3.0.1) in (1.0.2) to find the value of λ

$$\begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \left\{ \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \right\} = 5 \tag{3.0.2}$$

by multiplying the row vector with the first column vector

$$1(2) - 1(-1) + 1(2) = 5 \tag{3.0.3}$$

by multiplying the row vector with the coefficient column vector of lambda

$$1(3\lambda) - 1(4\lambda) + 1(2\lambda) = \lambda \tag{3.0.4}$$

we get as

$$\lambda = 0 \tag{3.0.5}$$

The line intersects the plane at

$$\mathbf{x}_0 = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \tag{3.0.6}$$

Finally the distance between the point $\mathbf{P} = \begin{pmatrix} -1 \\ -5 \\ -10 \end{pmatrix}$ and intersection point $\mathbf{x}_0 = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ is

$$||x_0 - P|| = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} \tag{3.0.7}$$

$$||x_0 - P|| = 13 \tag{3.0.8}$$

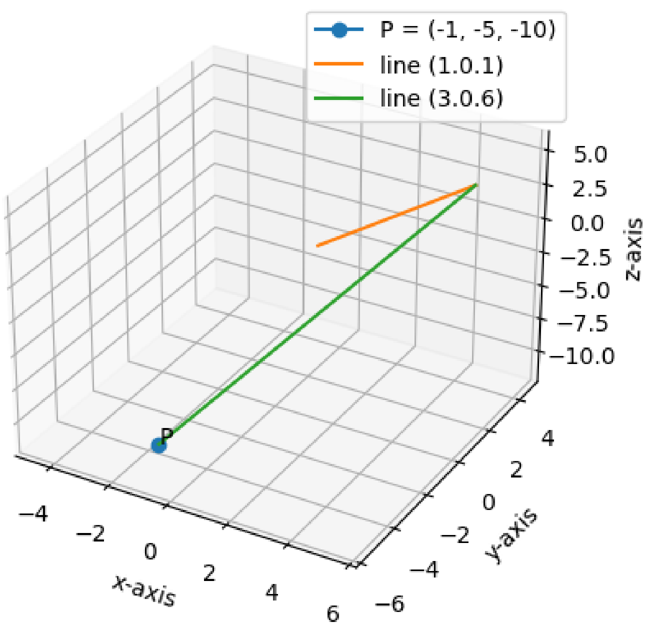

Fig. 0: Equation of line passing through point x_0 and intersection to line (1.0.1)