

Matrix theory - Assignment1

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Abstract—This document illustrates the distance of the point from the point of intersection of the line and the plane.

Download all python codes from

<https://github.com/upender20/EE5600/tree/master>

1 PROBLEM

Find the distance of the point $\begin{pmatrix} -1 \\ -5 \\ -10 \end{pmatrix}$ from the point of intersection of the line

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} \quad (1.0.1)$$

and the plane

$$(1-11)\mathbf{x} = 5 \quad (1.0.2)$$

2 CONSTRUCTION

We know that equation of the line passing through given a point and a plane

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{m} \quad (2.0.1)$$

Also we can find direction vector from the Cartesian form of equation

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \quad (2.0.2)$$

This can be expressed as

$$\mathbf{x} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \lambda \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad (2.0.3)$$

where $\mathbf{a} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ is a point on given line and $\mathbf{m} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is the direction vector.

Distance between the point and point of intersection.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad (2.0.4)$$

3 SOLUTION

Writing given equation (1.0.1) in vector form as

$$\mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \quad (3.0.1)$$

substitute (3.0.1) in (1.0.2) to find the value of λ

$$(1 \quad -1 \quad 1) \left\{ \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \right\} = 5 \quad (3.0.2)$$

by multiplying the row vector with the first column vector

$$1(2) - 1(-1) + 1(2) = 5 \quad (3.0.3)$$

by multiplying the row vector with the coefficient column vector of lambda

$$1(3\lambda) - 1(4\lambda) + 1(2\lambda) = \lambda \quad (3.0.4)$$

we get as

$$\lambda = 0 \quad (3.0.5)$$

The line intersects the plane at

$$\mathbf{x}_0 = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \quad (3.0.6)$$

Finally the distance between the point $\mathbf{P} = \begin{pmatrix} -1 \\ -5 \\ -10 \end{pmatrix}$

and intersection point $\mathbf{x}_0 = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ is

$$\|\mathbf{x}_0 - \mathbf{P}\| = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} \quad (3.0.7)$$

$$\|\mathbf{x}_0 - \mathbf{P}\| = 13 \quad (3.0.8)$$

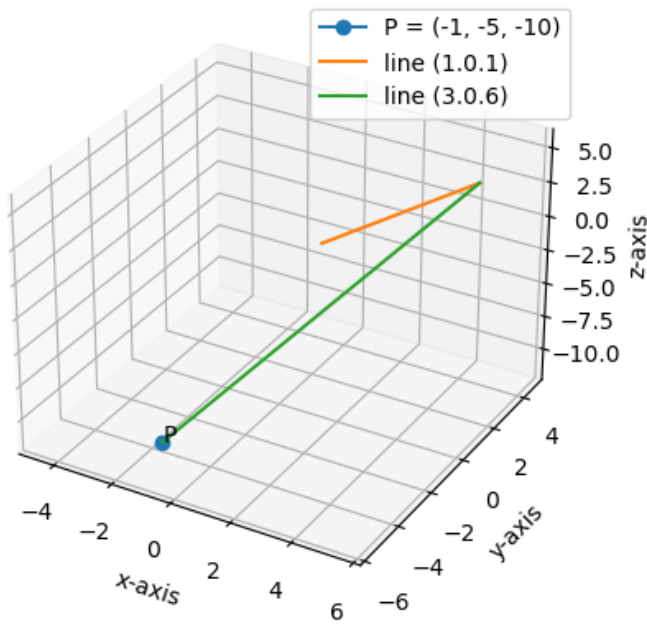


Fig. 0: Equation of line passing through point x_0 and intersection to line (1.0.1)