

# Assignment 1

## Geometry

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### 1 Problem

Find the distance of the points  $\begin{pmatrix} -1 \\ -5 \\ -10 \end{pmatrix}$  from the point of intersection of the line

$$x = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \text{ and the plane } (1, -1, 1) \cdot X = 5.$$

### 2 Solution

Given, the equation of line is

$$x = \begin{pmatrix} 2i \\ -j \\ 2k \end{pmatrix} + \lambda \begin{pmatrix} 3i \\ 4j \\ 2k \end{pmatrix}$$

and the equation of the plane is

$$(i, -j, k) \cdot X = 5$$

To find the point of intersection of line and plane,

Putting value of  $x$  from equation of line into equation of plane.

$$[(2i - j + 2k) + \lambda(3i + 4j + 2k)] \cdot (i - j + k) = 5$$

$$(2i - j + 2k + 3\lambda i + 4\lambda j + 2\lambda k) \cdot (i - j + k) = 5$$

$$[(2 + 3\lambda)i + (-1 + 4\lambda)j + (2 + 2\lambda)k] \cdot (i - j + k) = 5$$

$$(2 + 3\lambda) \cdot 1 + (-1 + 4\lambda) \cdot (-1) + (2 + 2\lambda) \cdot 1 = 5$$

$$2 + 3\lambda + 1 - 4\lambda + 2 + 2\lambda = 5$$

$$\lambda + 5 = 5$$

$$\lambda = 5 - 5$$

$$\lambda = 0$$

So the equation of the line is

$$X = (2i - j + 2k) + \lambda(3i + 4k + 2k)$$

$$X = 2i - j + 2k$$

Let the point of intersection be  $(x, y, z)$

$$\text{so, } X = xi + yj + zk$$

$$xi + yj + zk = 2i - j + 2k$$

$$\text{Hence, } x=2, y=-1, z=2$$

Therefore, the point of intersection is  $(2, -1, 2)$

Now the distance between two points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  is  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Distance between  $(2, -1, 2)$  and  $(-1, -5, -10)$

$$= \sqrt{(-1 - 2)^2 + (-5 + 1)^2 + (-10 - 2)^2}$$

$$= \sqrt{(-3)^2 + (-4)^2 + (-12)^2}$$

$$= \sqrt{9 + 16 + 144}$$

$$= \sqrt{169}$$

$$= 13.$$