Matrix theory - Assignment1

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Abstract—This document illustrates the distance of the point from the point of intersection of the line and the plain.

Download all python codes from

https://github.com/upender20/EE5600/tree/master

1 Problem

Find the distance of the point $\begin{pmatrix} -1 \\ -5 \\ -10 \end{pmatrix}$ from the point

of intersection of the line

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} \tag{1.0.1}$$

and the plane

$$(1-11)\mathbf{x} = 5 \tag{1.0.2}$$

2 Construction

We know that equation of the line passing through given a point and a plane

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{m} \tag{2.0.1}$$

Also we can find direction vector from the Cartesian form of equation

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \tag{2.0.2}$$

This can be expressed as

$$\mathbf{x} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \lambda \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{2.0.3}$$

where $\mathbf{a} = \begin{pmatrix} x_1 \\ y_1 \\ z \end{pmatrix}$ is a point on given line and $\mathbf{m} = \begin{pmatrix} a \\ b \end{pmatrix}$

is the direction vector.

Distance between the point and point of intersection.

$$D = \sqrt{(x^2 - x^1)^2 + (y^2 - y^1)^2 + (z^2 - z^1)^2}$$
 (2.0.4)

3 Solution

Writing given equation (1.0.1) in vector form as

$$\mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \tag{3.0.1}$$

substitute (3.0.1) in (1.0.2) to find the value of λ

$$\begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \left\{ \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \right\} = 5 \tag{3.0.2}$$

by multiplying the row vector with the first column vector

$$1(2) - 1(-1) + 1(2) = 5$$
 (3.0.3)

by multiplying the row vector with the coefficient column vector of lambda

$$1(3\lambda) - 1(4\lambda) + 1(2\lambda) = \lambda \tag{3.0.4}$$

we get as

$$\lambda = 0 \tag{3.0.5}$$

The line intersects the plane at

$$\mathbf{x}_0 = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \tag{3.0.6}$$

Finally the distance between the point $P = \begin{pmatrix} -1 \\ -5 \\ -10 \end{pmatrix}$

(2.0.3) and intersection point $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ is $= \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2} = 13$

$$= \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2} = 13$$
(3.0.7)

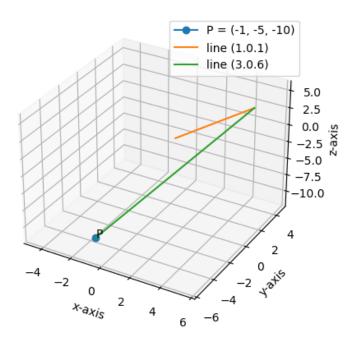


Fig. 0: Equation of line passing through point x_0 and intersection to line (1.0.1)