Matrix theory - Assignment1

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Abstract—This document illustrates the distance of the point from the point of intersection of the line and the plain.

1 Problem

Find the distance of the point $\begin{pmatrix} -1 \\ -5 \\ -10 \end{pmatrix}$ from the point of intersection of the line

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} \qquad (1.0.1)$$

and the plane

$$\begin{pmatrix} 1 & -11 \end{pmatrix} \mathbf{x} = 5 \qquad (1.0.2)$$

2 Construction

We know that equation of the line passing through given a point and a plane

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{m} \qquad (2.0.1)$$

Also we can find direction vector from the Cartesian form of equation

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad (2.0.2)$$

This can be expressed as

$$\mathbf{x} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \lambda \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad (2.0.3)$$

where $\mathbf{a} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ is a point on given line and $\mathbf{m} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is the direction vector.

Distance between the point and point of intersection.

$$D = \sqrt{(x2 - x1)^2 + (y2 - y1)^2 + (z2 - z1)^2} \qquad (2.0.4)$$

3 Solution

Writing given equation (1.0.1) in vector form as

$$\mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \qquad (3.0.1)$$

substitute (3.0.1) in (1.0.2) to find the value of λ

$$\begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \left\{ \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \right\} = 0 \qquad (3.0.2)$$

$$(2 + 3\lambda)1 + (-1 + 4\lambda)(-1) + (2 + 2\lambda)1 = 5 \qquad (3.0.3)$$

$$\lambda + 5 = 5 \qquad (3.0.4)$$

$$\lambda = 0 \qquad (3.0.5)$$

substitute (3.0.5) in (3.0.1)we get the equation of line as

$$\mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \qquad (3.0.6)$$

Finally the distance between the point P = $\begin{pmatrix} -1 \\ -5 \\ -10 \end{pmatrix}$ and intersection point $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ is

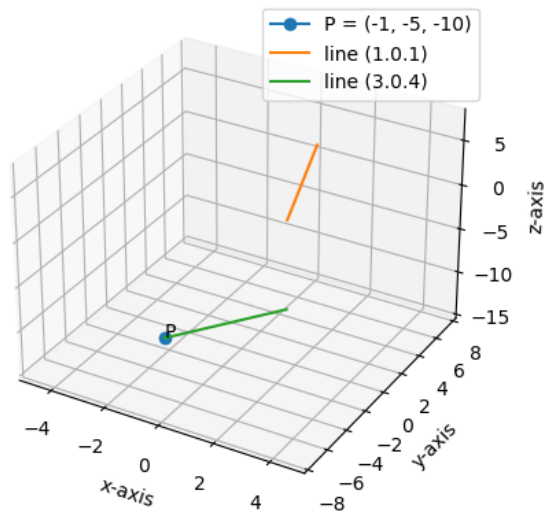$$= \sqrt{(-1 - 2)^2 + (-5 + 1)^2 + (-10 - 2)^2} = 13 \qquad (3.0.7)$$

Fig. 0: Equation of line passing through point P and parallel to line (1.0.1)