8.4: <u>Fuzzy rule based system</u> 8/10/24 ①

Fuzzy linguistic descriptions are formal representation of systems made through fuzzy if then rule.

They encode knowledge about a system in states of the form:

ie

IF (a set of condition) are satisfied THEN (a set of consequents) can be inferred

IF-THEN rule are coded in forms.

IF (x_1 is $\tilde{A}_1$, x_2 is $\tilde{A}_2$, ... x_n is $\tilde{A}_n$) THEN (y_1 is $\tilde{B}_1$, y_2 is $\tilde{B}_2$, ... y_n is $\tilde{B}_n$)

where linguistic variables x_i, y_i takes the values of fuzzy sets A_i & B_i resp.

Example:

If there's heavy rain & strong winds,
then there must be severe flood warning.

here heavy, ~~~~ strong & severe are fuzzy sets quantifying the variable rain, wind & flood warning resp..

* A collection of rules referring to a particular system is known as <u>fuzzy rule base</u>

If conclusion <u>C</u> to be drawn from a rule base <u>R</u> is the conjunction of all the individual consequents <u>C_i</u> of each rule, then.

$$C = C_1 \cap C_2 \cap C_3 \dots C_n \text{ where}$$

$$\mu_c(y) = \min(\mu_{c_1}(y), \mu_{c_2}(y), \mu_{c_3}(y) \dots \mu_{c_n}(y))$$

$\forall y \in Y$ where Y is the universe of discourse

On the oth hand if conclusion C to be drawn from rule base R is the disjunct of individu consequents of each rule, then

$$C = C_1 \cup C_2 \cup C_3 \dots C_n$$

$$\mu_C(y) = \max\left(\mu_{C_1}(y), \mu_{C_2}(y), \mu_{C_3}(y) \dots \mu_{C_n}(y)\right)$$

$$\forall y \in Y$$

DEFUZZIFICATION

It is easier to take a crisp decision if the fuzzy output of a system is represented as a single scalar quantity.
The convers of fuzzy set to a single crisp value is called defuzzification & is reve of fuzzification

Centroid method : or centre of gravity or centre of area method.

If obtain the centre of area (x^*) occupied by the fuzzy set.

$$x^* = \int \frac{\mu(x)\, x\, dx}{\mu(x)\, dx} \quad \Bigg| \quad x^* = \frac{\sum_{i=1}^{n} x_i\, \mu(x_i)}{\sum_{i=1}^{n} \mu(x_i)}$$

here n repre no. of the elements in the sample,
x_i's are the elements & $\mu(x_i)$ is its membership funct.

Centre of sums (COS) method:

in centroid meth. the overlapp. area is count once but in COS the overlappin area's counted twice.

it consid each contributin fuzzy set $\tilde{A}_1, \tilde{A}_2 \ldots$ etc & resulting membr funct is esta as ~~Alg sum~~ algebric sum.

$$\cancel{\phi}\, x^* = \frac{\sum_{i=1}^{N} x_i \cdot \sum_{K=1}^{n} \mu_{\tilde{A}_K}(x_i)}{\sum_{i=1}^{N} \sum_{K=1}^{n} \mu_{\tilde{A}_K}(x_i)}$$

$n \equiv$ no of fuzzy sets

$N \equiv$ no of elements. in sample.

* Commonly used / fast / easy to implement

Mean of maxima (MOM)

it take the crisp value with the high degree of membership.

If more than one elem have the same max value the mean value of the maxim is take

$$x^* = \frac{\sum_{x_i \in M} x_i}{|M|}$$

where $M = \{x_i \mid \mu(x_i)$ is equal to the height of fuzzy set $\}$

$|M|$ is the cardn of set M

$M = \{x \in [-C, C] \mid \mu(x)$ is equal to the heigh of the fuzzy set$\}$

ASSIGNMENT

8.3.1 Fuzzy Quantifier — Absolute & Relative

8.3.2 . Fuzzy Inference.