Fuzzy set Theo. ①

→ real world → uncertainty → Simulate will the problem

Parameter define problem

we use probability theo to handle uncertain but it can be ar to random processes.

Large class has uncertainty charact by non random process.

→ partial inform. ~~of~~ about prob.

→ in reliab information

→ lengua interpretations

FUZZINESS ≡ Vagueness.

in 1965 Lotfi A. Zadeh ~~prop~~ introd. fuzzy set theo..

→ thousa of patents in consumer products

<u>FUZZY VS CRISP</u>

"Is water colourless" — answer can be defined Yes/True/1 | No/False/0

$\boxed{\text{CRISP}}$ — 1, 0

CRISP DOMAIN

ie "Temper is 22°C" "Train time is 20 S"

YES(1) ⟷ No(0)

"Is Ram honest" ≡ Degree of Truth

$\boxed{\text{Fuzzy}}$

Extreme honest (1)

Very hone (0.80)

hone at times (0.4)

Extrem dishonest (0)

CRISP Set $\Longrightarrow$ Classical/Boolean logic

Fuzzy Set $\Longrightarrow$ Fuzzy logic

CRISP SET & FUZZY SET

CRISP SET

Universal set:

of all stude in a universe

Set is a well defin collection of objects:

$A = \{$ Gandhi, Bose, Nehru $\}$

$B = \{$ Swan, Peacock, Dove $\}$

A set may also be defin based on the proper the mem hav to satisfy:

$$A = \{x \mid P(x)\}$$

here $P(x)$ stands of Property P to be satisfied by the members x

Read as "A is the set of all X such that P(x) is satisfied"

$A = \{x \mid x$ is an odd number$\}$

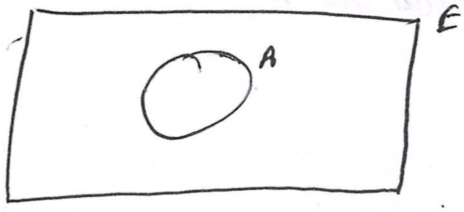
$B = \{y \mid y > 0$ and mod $5 = 0\}$

Venn diagra

Pictorial represen to deno a set

$E \equiv$ Univere set $\equiv$ Set of all st in MCA

$A =$ defn ove univers set E $\equiv$ Set of ~~dtb~~ st of MCA sem 3



Venn dig of set A

Membership 'ϵ'

elem x is said to be a memb of a set A if x belongs to the set.

$x \in A$ — x belon to A

$x \notin A$ — x does n belo t A

$A = \{4, 5, \dots 10\}$ $x = 3$ & $y = 4$

$x \notin A$, $y \in A$

Cardenala '#' conu to | |

No of elemen in a set

Ex $A = \{4, 5, 6, 7\}$, $\#A = 4$ or $|A| = 4$

Famil of set

of each mem is also a set / set of sets

$A = \{\{1, 3, 5\}, \{2, 5, 6\}\{5, 10\}\}$

Null set ϕ

set of PM of age less than 15 y.

Singelton Set : Set with single Element

$A = \{9\}$ this $\# A = 1$ or $|A| = 1$

Subset $A \subset B$

A & B defi on E

A is sub of B if A is fully contai in B

ever elem of A is in B

Superset $\supset$ $\supseteq$ Equivalen

$\wedge$

$A = \{3, 4\}$, $B = \{3, 4, 5\}$, $C = \{4, 5, 3\}$

$B \supset A$ $A \subset B$

$C \supseteq B$ & $B \supseteq C$

Power set : set of all possible subsets that are derivable fr A inclu null

$A = \{1, 2, 3, 4\}$

$P(A) = \{\{1\}, \{2\}, \{3\} \ldots\ldots$

$\# A = 4$

$\# P(A) = ~~B~~ ~~\# A~~ = 2^{\# A} = 16$

Operu..

Union $\cup$ / Inters $\cap$ / Compu A^c / Differ $-$

$A \cup B$ $A \cap B$

$A - B$: set for all elements which ar in A but not in B

$A = \{9, b, c, d, e\}$ $B = \{b, d\}$ $A - B = \{9, c, e\}$

• Fuzzy logic can work with linguistic variables

•

FUZZY SETS

- Supports flexible sense of membership of elements to a set. whil in crisp set member either belong ore ~~not~~ does not belong to it.

- in fuzzy set there many degree of membership (betwee 0 & 1) are allowed.

Membership function $\mu_A(x)$ is associal with a fuzzy set $\tilde{A}$ such that the function maps every element of the univers of discourse X (or the refernce set) to the interval [0, 1].

Formally written as $\mu_{\tilde{A}}(x) : X \rightarrow [0,1]$

A fuzzy set is defined as

If X is a univers of discourse & x is a particular element of X, then a fuzzy set A define on X may be written as a collection of ordered pairs

$$A = \{ (x, \mu_{\tilde{A}}(x)), \ x \in X \}$$

Where each pair $(x, \mu_{\tilde{A}}(x))$ is called a singelton.

in a crisp set $\mu_{\tilde{A}}(x)$ is dropped.

Fuzzy set as union of all $\mu_{\tilde{A}}(x)/x$ singletons is

$$\boxed{A = \sum_{x_i \in X} \mu_{\tilde{A}}(x_i)/x_i}$$ in the discrete case

$$\boxed{A = \int_X \mu_{\tilde{A}}(x)/x}$$ in the continuous case

here Σ & $\int$ ~~represent~~ indicate the union of all $\mu_{\tilde{A}}(x)/x$ singletons

Example

Let $X = \{g_1, g_2, g_3, g_4, g_5\}$ be the reference set of students.

Let $\tilde{A}$ be the fuzzy set of "smart" students, where "smart" is a fuzzy linguistic term

$$\tilde{A} = \{(g_1, 0.4), (g_2, 0.5), (g_3, 1), (g_4, 0.9) (g_5, 0.8)\}$$

here $\tilde{A}$ indicates smartness of g_1 is 0.4 & g_2 is 0.5 ....

when graded over a scale of 0-1

It can also be written as

Union

$$\tilde{A} = \frac{0.4}{g_1} + \frac{0.5}{g_2} + \frac{1}{g_3} + \frac{0.9}{g_4} + \frac{0.8}{g_5}$$

here + indicates union of all $\frac{\mu_{\tilde{A}}(x)}{x}$

MEMBERSHIP Func

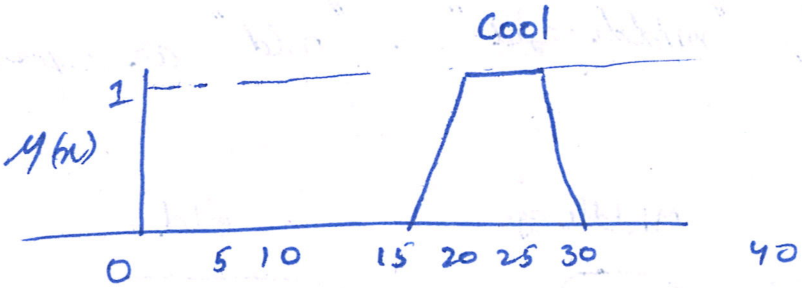
- need not always be described by discrete values,

Quite often this are described by a continuous function.

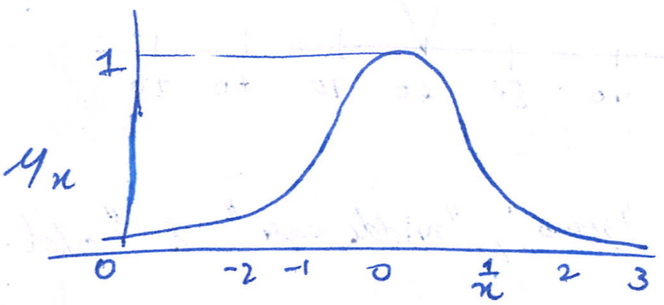
mathematic: as $\mu_{\tilde{A}}(x) = \frac{1}{(1+x)^2}$

ie fuzzy linguistic term "cool" relating to temperature

can be ~~shown~~ described as



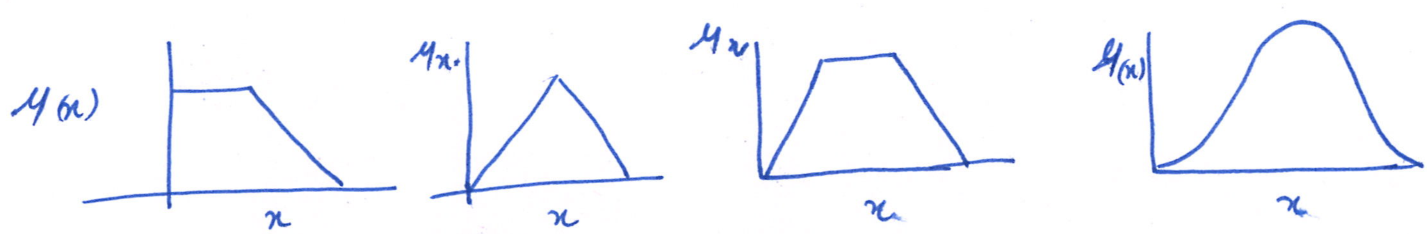
Continuous membership func for cool.



$\mu_x = \frac{1}{(1+x)^2}$

Continuous member func described by a mathematical func:

Different shapes of MF exist (tri, trap, curve or other.



Example.

Consid the set of peopl in follo age grou.

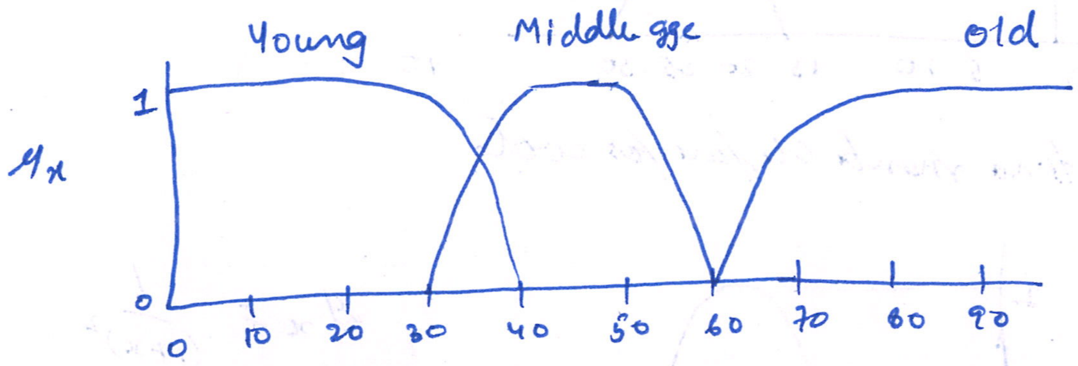
0 - 10 40 - 50

10 - 20 50 - 60

20 - 30 60 - 70

30 - 40 70 & above

fuzzy sets "young", "middle-aged" & "old" are repre by the MF graph as.



fuzz sets expresn "young", "middle-age" & "old.

24/9/24

Fuzzy sets & Crisp sets

α- cut of a fuzzy set

The α- cut of a fuzzy set $\tilde{A}$ denoted by A_α is a set consisting of those elements of the universe X, whose member value exceed the threshold α,

$$A_\alpha = \{x \mid \mu_{\tilde{A}}(x) \geq \alpha\}$$

A_α is a crisp set that contains all elements of the universe X whose membership grade are great than or equal to the specified value of α

Example : Let $\tilde{A}$ refer to a fuzzy set of smart students defined over refer set $X = \{a_1, a_2, a_3, a_4, a_5\}$ as follow

$$\tilde{A} = \{(a_1, 0.4), (a_2, 0.5), (a_3, 1), (a_4, 0.9), (a_5, 0.8)\}$$

The α-cuts A_α for specified values of α are given below :

$$A_{\alpha=0.4} = \{a_1, a_2, a_3, a_4, a_5\}$$

$$A_{\alpha=0.5} = \{a_2, a_3, a_4, a_5\}$$

$$A_{\alpha=0.9} = \{a_3, a_4\}.$$

Representation theorem:

any fuzzy set $\tilde{A}$ can be decompa into a series of its α cuts

$$\tilde{A} = \bigcup_{\alpha \in [0,1]} (\alpha A_\alpha$$

or

$$\tilde{A}(x) = \sup_{\alpha \in [0,1]} [\alpha A_\alpha(x)]$$

GIVEN

$$A_{0.1} = \{a, b, c, d\}$$

$$A_{0.4} = \{b, c, d\}$$

$$A_{0.8} = \{b, c\}$$

$$A_1 = \{c\}$$

a, $\boxed{\mu_A(a) = 1}$ crisp

here not cri

$$\therefore \tilde{A} = \bigcup_{\alpha \in [0,1]} (\alpha . A_\alpha)$$

$$= 0.1 A_{01} + 0.4 A_{0.4} + 0.8 A_{0.8} + 1 A_1$$

union

$$= 0.1\left\{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right\} + 0.4\left\{\frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right\} + 0.8\left\{\frac{1}{b} + \frac{1}{c}\right\} + 1\left\{\frac{1}{c}\right\}$$

$$= \frac{0.1}{a} + \frac{\max(0.1, 0.4, 0.8)}{b} + \frac{\max(0.1, 0.4, 0.8, 1)}{c} + \frac{0.4}{d}$$

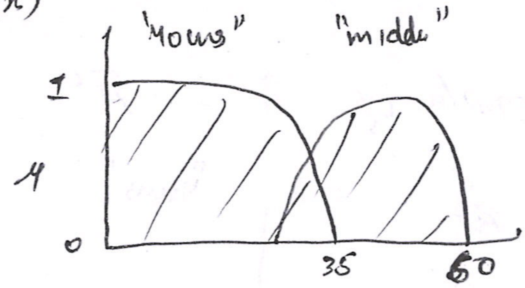
$$= \frac{0.1}{a} + \frac{0.8}{b} + \frac{1}{c} + \frac{0.4}{d}$$

$$\tilde{A} = \{(a, 0.1), (b, 0.8), (c, 1) (d, 0.4)\}$$

Basic fuzzy set operations

X = Universe of Discourse.

$\tilde{A}$ & $\tilde{B}$ are fuzzy set $\mu_A(x)$ & $\mu_B(x)$



$\hat{A} \cup \tilde{B}$

UNION

in its discrete form for x_1, x_2 & x_3

if $A = \{(x_1, 0.5), (x_2, 0.7), (x_3, 0)\}$

$B = \{(x_1, 0.8), (x_2, 0.2), (x_3, 1)\}$

$\therefore A \cup B = \{(x_1, 0.8), (x_2, 0.7)\ (x_3, 1)\}$

Since $\mu_{\tilde{A} \cup \tilde{B}}(x_1) = \max(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$

$= \max(0.5, 0.8)$

$= 0.8$

Similarly $\mu_{\tilde{A} \cup \tilde{B}}(x_2) = 0.7$

$\mu_{\tilde{A} \cup \tilde{B}}(x_3) = 1$

INTERSECTION

$A \cap B$

in its discrete form

$A \cap B = \{(x_1, 0.5), (x_2, 0.2), (x_3, 0)\}$

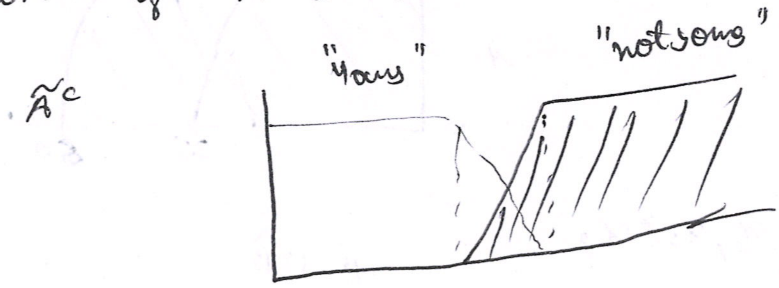
Sim $\mu_{\tilde{A} \cap \tilde{B}}(x_1) = \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{B}}(x_1)) = \min(0.5, 0.8) = 0.8$

Similarly $\mu_{\tilde{A} \cap \tilde{B}}(x_2) = 0.2$ & $\mu_{\tilde{A} \cap \tilde{B}}(x_3) = 0$

Complement

$$\mu_{\tilde{A}^c}(x) = 1 - \mu_{\tilde{A}}(x)$$

Complement of $\tilde{A} = \tilde{A}^c$



consider of $\tilde{A} = \{ (x_1, 0.5), (x_2, 0.7) (x_3, 0)\}$

$\therefore \tilde{A}^c = \{ (x_1, 0.5), (x_2, 0.3), (x_3, 1)\}$

Since $\mu_{\tilde{A}^c}(x_1) = 1 - \mu_{\tilde{A}}(x_1) = 1 - 0.5 = 0.5$

Similarly $\mu_{\tilde{A}^c}(x_2) = 1 - \mu_{\tilde{A}}(x_2) = 1 - 0.7 = 0.3$

$\mu_{\tilde{A}^c}(x_3) = 1 - \mu_{\tilde{A}}(x_3) = 1 - 0 = 1$

Product of two fuzzy set $\qquad \mu_{A \cdot B}(x_1) = \mu_{\tilde{A}}(x_1) \cdot \mu_{\tilde{B}}(x_1)$

$\tilde{A} \cdot \tilde{B}$

$$\mu_{\tilde{A} \cdot \tilde{B}}(x) = \mu_{\tilde{A}}(x) \cdot \mu_{\tilde{B}}(x)$$

Equal $\quad \tilde{A} = \tilde{B}$ if $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$ for all x.

$\tilde{A} = \{ (x_1, 0.2), (x_2, 0.8)\}$

$\tilde{B} = \{ (x_1, 0.6), (x_2, 0.8)\}$

$\tilde{C} = \{ (x_1, 0.2) (x_2, 0.8)\}$.

$\therefore \tilde{A} \neq \tilde{B}$

$\tilde{A} = \tilde{C}$

Product with crisp number

$$\mu_{a\tilde{A}}(x) = a\,\mu_{\tilde{A}}(x)$$

$a = 0.3$

$a.\tilde{A} = \{(x_1, 0.12), (x_2\ 0.18), (x_3, 0.24)\}$

Power of fuzz:

$$\mu_{A^\alpha}(x) = (\mu_{\tilde{A}}(x))^\alpha$$

$\alpha = 2$

$$\mu_{A^2}(x) = (\mu_{\tilde{A}}(x))^2$$

Differen

$\tilde{A} - \tilde{B}$

$$\tilde{A} - \tilde{B} = (\tilde{A} \cap \tilde{B}^c)$$

Disjunct Sum

$\tilde{A}\ \&\ \tilde{B} = \quad A \oplus B$

$$\tilde{A} \oplus \tilde{B} = (\tilde{A}^c \cap B) \cup (\tilde{A} \cap \tilde{B}^c)$$

1 500126791
500126529 24/09/24

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123581
125757
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Gaurav
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Nikhil A

~~EAP~~ CRISP RELATIONS

The concept of relations between sets is build on the cartisian product operator of sets.

CARTESIAN PRODUCT

of two sets A & B denoted by $A \times B$ is a set of all ordered pairs such that first ~~to~~ element in the pair belongs to A & the second element belongs to B.

ie $A \times B = \{(a,b) \mid a \in A, b \in B\}$

if $A \neq B$ & A & B are non-empty then $A \times B \neq B \times A$

generalize to n number of sets

$$\bigtimes_{i=1}^{n} A_i = \{(a_1, a_2, a_3 \ldots a_n) \mid a_i \in A_i \text{ for every } i = 1, 2 \ldots n\}.$$

observe Cardinality $\left| \bigtimes_{i=i}^{n} A_i \right| = \prod_{i=1}^{n} |A_i|$

Example $A_1 = \{a, b\}$, $A_2 = \{1, 2\}$, $A_3 = \{\alpha\}$

$A_1 \times A_2 = \{(a,1), (a,2), (b,1), (b,2)\}$

$|A_1 \times A_2| = 4$ & $|A_1| = 2$, $|A_2| = 2$.

$\therefore$ $|A_1 \times A_2| = |A_1| \cdot |A_2|$

Fm $A_1 \times A_2 \times A_3$

RELATIONS

An *n-ary relation* denoted as $R(X_1, X_2, X_3, \dots X_n)$ among crisp sets $X_1, X_2, \dots X_n$ is a subset of the Cartesian product and is indicative of an association or relation among the tuple elements

For $n = 2$ the Rel $R(X_1, X_2)$ is termed binary

3 ternary

4 quaternary

quinary

∵ if the universe of discourse or sets are finite, the 'n-ary' rel can be expr as an n-dim relation matrix. for binary Rel $R(X, Y)$

$X = \{x_1, x_2, \dots x_n\}$ & $Y = \{y_1, y_2 \dots y_m\}$ the Relation matrix R is a two dimensional matrix where X repr rows & Y rep column.

$X = \{1, 2, 3, 4\}$

$\because X \times X = \{(1,1), (1,2) (1,3), (1,4) \dots \}$ $|X \times X| = 16$

Let the relation R be defin as

$$R = \{(x, y) \mid y = x + 1, \; x, y \in X\}$$
$$= \{(1,2), (2,3) (3,4)\}$$

the relat matr R is given by

$$R = \begin{array}{c|c} & Y \\ & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ X \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Operations on Relations

Given two relation R & S defin on $X \times Y$ represented by relation matrix the follow oper are suppor by R & S

UNION $R \cup S$

$$R \cup S(x,y) = \max\left(R(x,y), S(x,y)\right)$$

Intersection $R \cap S$

$$R \cap S(x,y) = \min\left(R(x,y), S(x,y)\right)$$

Complim $\bar{R}$

$$\bar{R}(x,y) = 1 - R(x,y)$$

COMPOSITION OF RELATIONS $R \circ S$

Given R to be a relation on X, Y & S to be relation on Y, Z

then $R \circ S$ is a compos of relation on X, Z defined as.

$$R \circ S = \{(x,z) \mid \underset{\text{belong to}}{(x,z) \in X \times Z}, \ \underset{\text{there exist}}{\exists}\ y \in Y \text{ such that } (x,y) \in R \ \& \ (y,z) \in S\}$$

- MIN MAX composition

given the rel matrix of Relations R & S the min-max composite T is defined as $T = R \circ S$

$$T(x,z) = \max_{y \in Y}\left(\min\left(R(x,y), S(y,z)\right)\right)$$

Example

Let R, S be defined on the set $\overset{X}{\{1,3,5\}} \times \overset{Y}{\{1,3,5\}}$

Let $R: \{(x,y) \mid y = x+2\}$ $S: \{(x,y) \mid x < 4\}$

$R = \{(1,3),(3,5)\}$, $S = \{(1,3)(1,5)(3,5)\}$

the

$$R: \begin{array}{c} \\ 1 \\ 3 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 3 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \qquad S: \begin{array}{c} \\ 1 \\ 3 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 3 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Using min-max compon

$$R \circ S = \begin{array}{c} \\ 1 \\ 3 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 3 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$R \circ S = \{(1,5)\}$

Also $S \circ R = \{(1,5)\}$

UNIT 1

UNIT 2 & 3 ANN

~~UNIT 4 FUZZY~~

Fuzzy Relations is a fuzzy set defined on the Cartesian product of a crisp set $X_1, X_2 \dots X_n$ where the n-tuples $(x_1, x_2, \dots x_n)$ may have a varying degree of membership within the relation. The membership value indicates the strength of the relation between the tuples.

Let R be the fuzzy relation between two sets. X_1 & X_2

where X_1 is the set of diseases &

X_2 is the set of symptoms.

$X_1 = \{$typhoid, viral fever, common cold$\}$ $\equiv X$

$X_2 = \{$runny nose, high temperature, shivering$\}$ $\equiv Y$

The fuzzy relation R may be defined as

$$
\begin{array}{c|ccc}
 & \text{Runny Nose} & \text{high temperature} & \text{Shivering} \\
\hline
\text{Typhoid} & 0.1 & 0.9 & 0.8 \\
\text{Viral fever} & 0.2 & 0.9 & 0.7 \\
\text{Common cold} & 0.9 & 0.4 & 0.6
\end{array}
$$

Fuzzy Cartesian Product

Let $\tilde{A}$ be a fuzzy set defined on universe X &

$\tilde{B}$ " " " " Y

The cartesian product of $\tilde{A}$ & $\tilde{B}$ indicated by $\tilde{A} \times \tilde{B}$ & resulting a fuzzy relation R is given by

$$\tilde{R} = \tilde{A} \times \tilde{B} \subset X \times Y$$

where $\tilde{R}$ has its membership function given by

$$\mu_{\tilde{R}}(x, y) = \mu_{\tilde{A} \times \tilde{B}}$$