~~FAP~~ CRISP RELATIONS

The concept of relations between sets is build on the cartesian products operator of sets.

CARTESIAN PRODUCT

of two sets A & B denoted by $A \times B$ is a set of all ordered pairs such that first ~~to~~ element in the pair belongs to A & the second element belongs to B.

ie $A \times B = \{(a,b) \mid a \in A, b \in B\}$

if $A \neq B$ & A & B are non-empty then $A \times B \neq B \times A$

generalize to n numb of sets

$$\overset{n}{\underset{i=1}{X}} A_i = \{(a_1, a_2, a_3 \ldots a_n) \mid a_i \in A_i \text{ for every } i = 1, 2 \ldots n\}$$

observe Cardinality $\left| \overset{n}{\underset{i=i}{X}} A_i \right| = \prod_{i=1}^{n} |A_i|$

Example $A_1 = \{a, b\}, A_2 = \{1, 2\}, A_3 = \{\alpha\}$

$A_1 \times A_2 = \{(a,1), (a,2), (b,1), (b,2)\}$

$|A_1 \times A_2| = 4$ & $|A_1| = 2, |A_2| = 2$

$\therefore |A_1 \times A_2| = |A_1| \cdot |A_2|$

Fm $A_1 \times A_2 \times A_3$

RELATIONS

An *n-ary relation* denoted as $R(X_1, X_2, X_3, \ldots X_n)$ among crisp sets $X_1, X_2, .. X_n$ is a subset of the Cartesian product and is indicative of an association or relation among the tuple elements

For $n=2$ the Rel $R(X_1, X_2)$ is termed binary

3 ternary

4 quaternary

quinary

∵ if the universe of discourse or sets are finite, the 'n-ary' rel can be expr as an n- dim relation matrix. i.e. for binary Rel $R(X, Y)$

$X = \{x_1, x_2, \ldots x_n\}$ & $Y = \{y_1, y_2 \ldots y_m\}$ the Relation matrix R is a two dimensional matrix where X repr rows & Y rep column.

$X = \{1, 2, 3, 4\}$

$\because X \times X = \{(1,1), (1,2) (1,3) (1,4) \ldots \ldots \}$ $\quad |X \times X| = 16$

Let the relation R be defn as

$$R = \{(x, y) \mid y = x+1, \; x, y \in X\}$$
$$= \{(1,2), (2,3) (3,4)\}$$

The relat matx R is given by

$$R = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(rows: X, columns: Y)

Operations on Relations

Given two relation R & S defin on $X \times Y$ represen by relation matr the follow opert are suppor by R & S

UNION $R \cup S$

$$R \cup S(x,y) = \max\left(R(x,y),\, S(x,y)\right)$$

Intersection $R \cap S$

$$R \cap S(x,y) = \min\left(R(x,y),\, S(x,y)\right)$$

Complim $\bar{R}$

$$\bar{R}(x,y) = 1 - R(x,y)$$

COMPOSITION OF RELATIONS $R \circ S$

Given R to be a relation on X, Y & S to be relation on Y, Z

then $R \circ S$ is a compos of relation on X, Z defined as.

$$R \circ S = \{ (x,z) \mid \underset{\text{belong to}}{(x,z) \in X \times Z}, \underset{\text{there exist}}{\exists}\; y \in Y \text{ such that } (x,y) \in R \;\&\; (y,z) \in S \}$$

• MIN MAX composition

given the relation matrix of Relation R & S the min-max composite T is defined as $T = R \circ S$

$$T(x_i, z_k) = \max_{\substack{y \in Y \\ \forall j}} \left(\min \left(R(x_i, y_j),\, S(y_j, z_k) \right) \right)$$

Example

Let R, S be defined on the set $\overset{X}{\{1,3,5\}} \times \overset{Y}{\{1,3,5\}}$

Let $R: \{(x,y) \mid y = x+2\}$ $S: \{(x,y) \mid x < 4\}$

$R = \{(1,3),(3,5)\}$, $S = \{(1,3)(1,5)(3,5)\}$

the

$$R: \begin{array}{c|c} & \begin{matrix} 1 & 3 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \qquad S: \begin{array}{c|c} & \begin{matrix} 1 & 3 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Using min-max compon

$$R \circ S = \begin{array}{c|c} & \begin{matrix} 1 & 3 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$R \circ S = \{(1,5)\}$

Also $S \circ R = \{(1,5)\}$

UNIT 1

UNIT 2 & 3 ANN

~~UNIT 4 FUZZY~~

Fuzzy Relations is a fuzzy set defined on the Cartesian product of a crisp set $X_1, X_2 \ldots X_n$ where the n-tuples $(x_1, x_2, \ldots x_n)$ may have a varying degree of membership within the relation. The membership value indicates the strength of the relation between the tuples.

Let R be the fuzzy relation between two sets X_1 & X_2
where X_1 is the set of diseases &
X_2 is the set of symptoms.

$X_1 = \{$typhoid, viral fever, common cold$\}$ $\equiv X$

$X_2 = \{$runny nose, high temperature, shivering$\}$ $\equiv Y$

The fuzzy relation R may be defined as

$$\begin{array}{l} \\ \text{Typhoid} \\ \text{Viral fever} \\ \text{Common cold} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Runny Nose} & \text{high temperature} & \text{Shivering} \end{array} \\ \begin{bmatrix} 0.1 & 0.9 & 0.8 \\ 0.2 & 0.9 & 0.7 \\ 0.9 & 0.4 & 0.6 \end{bmatrix} \end{array}$$

Fuzzy Cartesian Product

Let $\tilde{A}$ be a fuzzy set defined on universe X &

$\tilde{B}$ " " " " Y

The cartesian product of $\tilde{A}$ & $\tilde{B}$ denoted by $\tilde{A} \times \tilde{B}$ & resulting a fuzzy relation R is given by

$$\tilde{R} = \tilde{A} \times \tilde{B} \subset X \times Y$$

where $\tilde{R}$ has its membership function given by

$$\mu_{\tilde{R}}(x,y) = \mu_{\tilde{A} \times B}$$

__Example__ page 206

Let $\tilde{A} = \{(x_1, 0.2), (x_2, 0.7) (x_3, 0.4)\}$ & $\tilde{B} = \{(y_1, 0.5), (y_2, 0.6)\}$ be two fu set defi on the univers of discour $X = \{x_1, x_2, x_3\}$ & $Y = \{y_1, y_2\}$, respectively, then the fuz relat $\tilde{R}$ as an out of the fuz Carten prod $\tilde{A} \times \tilde{B}$ is s.t.

$$\tilde{R} = \tilde{A} \times \tilde{B} = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.2 & 0.2 \\ x_2 & 0.5 & 0.6 \\ x_3 & 0.4 & 0.4 \end{array}$$

Since

$$\tilde{R}(x_1, y_1) = \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{B}}(y_1)) = \min(0.2, 0.5)$$
$$= 0.2$$

$$\tilde{R}(x_1, y_2) = 0.2$$

Oper on Fuz Relat

Let $\tilde{R}$ & $\tilde{S}$ be the fuz relat on $X \times Y$

Union

$$\mu_{\tilde{R} \cup \tilde{S}}(x, y) = \max(\mu_{\tilde{R}}(x,y), \mu_{\tilde{S}}(x,y))$$

Intersec

$$\mu_{\tilde{R} \cap \tilde{S}}(x, y) = \min(\mu_{\tilde{R}}(x,y), \mu_S(x,y))$$

Complem

$$\mu_{\tilde{R}^c}(x, y) = 1 - \mu_{\tilde{R}}(x,y)$$

Compo of relat $\equiv$ Same as crisp set

Min max compo

$$\mu_{\tilde{R} \circ \tilde{S}}(x, z) = \max_{y \in Y} (\min(\mu_{\tilde{R}}(x,y), \mu_{\tilde{S}}(y,z)))$$

Example

for $X = \{x_1, x_2, x_3)$ $Y = \{y_1, y_2\}$, $Z = \{z_1, z_2, z_3\}$.

Let
$$\tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.9 \\ 0.8 & 0.6 \end{bmatrix} \end{array} \qquad \tilde{S} = \begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.6 & 0.4 & 0.7 \\ 0.5 & 0.8 & 0.9 \end{bmatrix} \end{array}$$

then $R \circ S$ by max-min composit yields

$$R \circ S = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.5 & 0.4 & 0.5 \\ 0.5 & 0.8 & 0.9 \\ 0.6 & 0.6 & 0.7 \end{bmatrix} \end{array}$$

$$\begin{aligned} \mu_{\tilde{R} \circ \tilde{S}}(x_1, z_1) &= \max\left(\min\left(R(x_1, y_1), S(y_1, z_1)\right), \min\left(R(x_1, y_2), S(y_2, z_1)\right)\right) \\ &= \max\left(\min(0.5, 0.6), \min(0.1, 0.5)\right) \\ &= \max(0.5, 0.1) \\ &= 0.5 \end{aligned}$$

• Fuzzy Logic & Inference 3/10/24 (1)

CRISP LOGIC (Proposition)

Consid. Statement "Water boils at 90°C" True / "Sky is blue" false — an agreement or disagreement can be indicated by "True" or False"

a statement which is either "true" or false" but not both is called proposition (It is indicated by upper case letter as P, Q, R, S ...)

Examp P: water boils at 90°C / Q: Sky is blue. — both are proposition (or atoms)

To represent real world complex inform sequence of propositions linked using ~~connectives~~ operatorss connectives or operators

Proposition logic connective

Symbol	Connective	Usage	Description
(binary) $\wedge$	and	$P \wedge Q$	P and Q are true
$\vee$	or	$P \vee Q$	Either P or Q ~~are~~ is true.
(unary) $\neg$ or $\sim$	not	$\sim P \equiv \neg P$	P is not true
$\Rightarrow$	implicat	$P \Rightarrow Q$	P implies Q is true
$=$	equal	$P = Q$	P and Q are equal (in truth value) is true.

P	Q	$P \wedge Q$	$Q \vee Q$	$\sim P$	$P \Rightarrow Q$	$P = Q$
T	T	T	T	F	T	T
T	F	F	T	F	F	F
F	T	F	T	T	T	F
F	F	F	F	T	T	T

Interpretatory Tautology contradiction

A logical formula comprising 'n' proposition will have 2^n interpretations in its truth table (TT).
A formula which holds all its inter records true is known as tautology & ones which records false for all its inters is known as contradiction

Q. Obtain a TT for the form $(P \vee Q) \Rightarrow (\sim P)$. Is it a tautology

P	Q	$P \vee Q$	$\sim P$	$P \vee Q \Rightarrow \sim P$
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	T

not all are True

it is not a tautology

Q It $\underbrace{((P \Rightarrow Q) \wedge (Q \Rightarrow P)}_{A:} = \underbrace{(P = Q)}_{B:})$ a tautology

Q TT

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$\overbrace{(P \Rightarrow Q) \wedge (Q \Rightarrow P)}^{A:}$	$\overbrace{P = Q}^{B:}$	$A = B$
						T
						T
						T
						T

TRY

Yes its a tautology

Q) ASSIGNMT

Q $(P \Rightarrow Q) = (\sim P \vee Q)$?

ASSIGNMENT EXAMPLE 8.3 PG 214

Inference in Propositional Logic

Inference is a technique by which, given a set of facts, or postulates, or axioms, or premises $F_1, F_2, \dots F_n$ a goal G is to be derived.

i.e. the fact "where there is smoke there is fire" &

"There is smoke in the hill",

The statement "then the hill is on fire" can be easily deducted.

There are three rules for inferring facts (widely used in proposition logic)

1) Modus Ponens
2) Modus Tollens, and
3) Chain rule

1) MODUS PONENS (mod pons)

Given $P \Rightarrow Q$ & P to be true, Q is true

$$\begin{array}{c} P \Rightarrow Q \\ P \\ \hline Q \end{array}$$

formula above line are the premises

formula below line Q is the goal which can be inferred from the premises

2) Modus tollens

Given $P \Rightarrow Q$ & $\sim Q$ to be true $\sim P$ is true

$$\begin{array}{c} P \Rightarrow Q \\ \sim Q \\ \hline \sim P \end{array}$$

3) Chain rule

Given $P \Rightarrow Q$ and $Q \Rightarrow R$ to be true, $P \Rightarrow R$ is true

$$\begin{array}{c} P \Rightarrow Q \\ Q \Rightarrow R \\ \hline P \Rightarrow R \end{array}$$

it is a representation of the transitivity relation w.r.t "$\Rightarrow$"

Q2 Given

Handwritten assignm

to be submitted on or before

4/10/24

i) $C \vee D$

ii $\sim H \Rightarrow A \wedge \sim B$

iii $(C \vee D) \Rightarrow \sim H$

(iv) $(A \wedge \sim B) \Rightarrow (R \vee S)$

Can $(R \vee S)$ be inferred from the above

Mod Pon

$$\frac{\begin{array}{c} C \vee D \\ (C \vee D) \Rightarrow \sim H \end{array}}{\sim H} \quad (v)$$

Chain r.

$$\frac{\begin{array}{c} \sim H \Rightarrow (A \wedge \sim B) \\ (A \wedge \sim B) \Rightarrow (R \vee S) \end{array}}{\sim H \Rightarrow (R \vee S)} \quad (vi)$$

Mod pon.

$$\frac{\begin{array}{c} \sim H \Rightarrow (R \vee S) \\ \sim H \end{array}}{R \vee S}$$

Read Section 8.1.2 pg 216

Example 8.6 pg 217

Vinay for Placem