TDT4171 Artificial Intelligence Methods
Assignment 1

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Problem 1

The probability that a person has 0, 1, 2, 3, 4, or 5 or more siblings is 0.15, 0.49, 0.27, 0.06, 0.02, 0.01, respectively.

a) What is the probability that a child has at most 2 siblings?

First we have to define whether a *child* is a *person* or not. For all future problems including this one, this fact will be deemed true.

Now that that's out of the way, the child having at most 2 siblings can be seen as a proposition:

$$P(Sibling \leq 2) = \sum_{\omega=0}^{2} P(\omega) = \underline{\underline{0.91}}$$

b) What is the probability that a child has more than 2 siblings given that he has at least 1 sibling?

The formal way to calculate this is:

$$P(Sibling > 2 \mid Sibling \geq 1) = \frac{P(Sibling > 2 \land Sibling \geq 1)}{P(Sibling \geq 1)}$$

$$P(Sibling \geq 1) = \sum_{\omega=1}^{5} P(\omega) = 0.85$$

$$P(Sibling > 2) = \sum_{\omega=3}^{5} P(\omega) = 0.09$$

The probabilities of $P(Sibling > 2)$ and $P(Sibling \geq 1)$ are dependent, meaning that their intersection is calculated as such:

$$P(Sibling > 2 \land Sibling \geq 1) = P(Sibling > 2) \times P(Sibling > 2 \mid Sibling \geq 1)$$

Since $P(Sibling > 2)$ is a subset of $P(Sibling \geq 1)$, meaning that the latter encompasses the former, $P(Sibling > 2 \,|\, Sibling \geq 1)$ equates to 1, which means that

$$P(Sibling > 2 \wedge P(Sibling \geq 1)) = P(Sibling > 2) = 0.09$$

Plugging the numbers in:

$$P(Sibling > 2 \mid Sibling \geq 1) = \frac{P(Sibling > 2)}{P(Sibling \geq 1)} = \frac{0.09}{0.85} = \underline{\underline{0.105}}$$

c) Three friends who are not siblings are gathered. What is the probability that they combined have three siblings?

To calculate this I need to add together all possible permutations of the probabilities:

$$\begin{aligned} P(TotalSib = 3) = {} & P(3,0,0) + P(0,3,0) + P(0,0,3) \\ & + P(2,1,0) + P(2,0,1) + P(1,2,0) \\ & + P(1,0,2) + P(0,1,2) + P(0,2,1) \\ & + P(1,1,1) \end{aligned}$$

This can be expressed like so:

$$\begin{aligned} P(TotalSib = 3) = {} & P(3 \wedge 0 \wedge 0) + P(0 \wedge 3 \wedge 0) + P(0 \wedge 0 \wedge 3) \\ & + P(2 \wedge 1 \wedge 0) + P(2 \wedge 0 \wedge 1) + P(1 \wedge 2 \wedge 0) \\ & + P(1 \wedge 0 \wedge 2) + P(0 \wedge 1 \wedge 2) + P(0 \wedge 2 \wedge 1) \\ & + P(1 \wedge 1 \wedge 1) \end{aligned}$$

Since the $\wedge$ operator is commutative, the expression can be shortened to:

$$P(TotalSib = 3) = P(3 \wedge 0 \wedge 0) \times 3 + P(0 \wedge 1 \wedge 2) \times 6 + P(1 \wedge 1 \wedge 1)$$

Which is the same as:

$$\begin{aligned} P(TotalSib = 3) &= P(3) \times P(0) \times P(0) \times 3 + P(0) \times P(1) \times P(2) \times 6 + P(1)^3 \\ &= 0.06 \times 0.15 \times 0.15 \times 3 + 0.15 \times 0.49 \times 0.27 \times 6 + 0.49^3 \\ &= \underline{\underline{0.24}} \end{aligned}$$

d) Emma and Jacob are not siblings, but combined they have a total of 3 siblings. What is the probability that Emma has no siblings?

$$P(E = 0 \wedge J = 3) = \alpha P(E = 0 \mid J = 3) * \alpha P(J = 3)$$

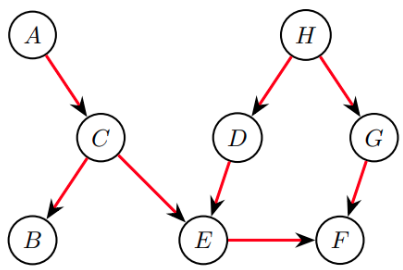
Since Emma and Jacob are not related, $P(E = 0 \,|\, J = 3) = P(E = 0)$, meaning that: $\alpha P(E = 0 \wedge J = 3) = \alpha P(E = 0) \times \alpha P(J = 3) = 0.15\alpha \times 0.06\alpha = \underline{0.009\alpha^2}$

This value has to be normalized, as I don't take into account the probabilities of 4 and 5 siblings.

$$\alpha P(E = 0 \land J = 3) = (\frac{1}{0.15 + 0.49 + 0.27 + 0.06})^2 \times 0.009 = \underline{\underline{0.0095}}$$

Problem 2

Given the Bayesian network structure below, decided whether the statements are true or false. Justify each answer with an explanation.



a) If every variable in the network has a Boolean state, then the Bayesian network can be represented with 18 numbers.

The Bayesian network consisting only of variables with Boolean states is represented by $n2^k$ numbers, where n is the number of nodes and k is a constant representing the amount of parents each node has. This network has nodes with varying amounts of parents however, making the equation a bit more complicated (not by much though). Looking at the two topmost nodes, A and H, these don't have any parents, meaning that they can be represented by one number each (as the second one is implied by 1-number). The rest of the nodes all have 1 parent each, except for E and F, which both have 2. The equation will therefore be $2 + 4 \times 2^1 + 2 \times 2^2 = 2 + 8 + 8 = \underline{\underline{18}}$.

The statement is therefore true.

b) $G \perp\!\!\!\perp A$

This statement is true, because A is not related to G in any way.

c) $E \perp\!\!\!\perp H \mid \{D, G\}$

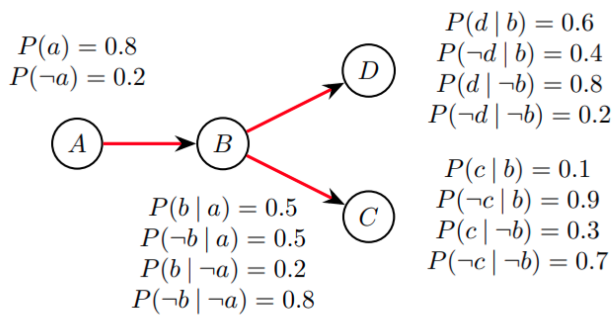
This statement is true, because the nodes D and G are given, meaning that the network won't go to the ancestors of D and G, which is H.

d) $E \perp\!\!\!\perp H \mid \{C, D, F\}$

This statement is false, because the node F is given, which has the ancestor G, which is the child of H.

Problem 3

The Bayesian network below contains only binary states. The conditional probability for each state is listed. From the Bayesian network, calculate the following probabilities:



a) $P(b)$

Using a formula found in the Jiří Kléma paper [1], I can infer that:

$$P(b) = P(b \wedge a) + P(b \wedge \neg a)$$

With this knowledge, I can use the information I have to find out what $P(b \wedge a)$ and $P(b \wedge \neg a)$ are:

$$P(b \mid a) = \frac{P(b \wedge a)}{P(a)}$$

$$P(b \wedge a) = P(b \mid a) \times P(a) = 0.5 \times 0.8 = \underline{0.4}$$

$$P(b \mid \neg a) = \frac{P(b \wedge \neg a)}{P(\neg a)}$$

$$P(b \wedge \neg a) = P(b \mid \neg a) \times P(\neg a) = 0.2 \times 0.2 = \underline{0.04}$$

$$P(b) = 0.4 + 0.04 = \underline{\underline{0.44}}$$

b) $P(d)$

There's no reason not to use the same method as before, but to do that, I need to find out what $P(\neg b)$ is first:

$$P(\neg b \mid a) = \frac{P(\neg b \wedge a)}{P(a)}$$

$$P(\neg b \wedge a) = P(\neg b \mid a) \times P(a) = 0.5 \times 0.8 = \underline{0.4}$$

$$P(\neg b \mid \neg a) = \frac{P(\neg b \wedge \neg a)}{P(\neg a)}$$

$$P(\neg b \wedge \neg a) = P(\neg b \mid \neg a) \times P(\neg a) = 0.8 \times 0.2 = \underline{0.16}$$

$$P(\neg b) = 0.4 + 0.16 = \underline{\underline{0.56}}$$

The answer I get is the same as $1-0.44$ which affirms that this answer is correct. Now I can do the same method as in the a) tsak to find $P(d)$:

$$P(d \mid b) = \frac{P(d \wedge b)}{P(b)}$$
$$P(d \wedge b) = P(d \mid b) \times P(b) = 0.6 \times 0.44 = \underline{0.264}$$
$$P(d \mid \neg b) = \frac{P(d \wedge \neg b)}{P(\neg b)}$$
$$P(d \wedge \neg b) = P(d \mid \neg b) \times P(\neg b) = 0.8 \times 0.56 = \underline{0.448}$$
$$P(d) = 0.264 + 0.448 = \underline{\underline{0.712}}$$

c) $P(c \mid \neg d)$

$$\begin{aligned}
P(c \mid \neg d) &= \alpha P(c \mid \neg d) = \alpha \sum_B \sum_A P(c, \neg d, B, A) \\
&= \alpha \sum_B P(c \mid B) P(\neg d \mid B) \sum_A P(B \mid A) P(A) \\
&= \alpha [P(c \mid b) P(\neg d \mid b) + P(c \mid \neg b) P(\neg d \mid \neg b)] \sum_B \sum_A P(B \mid A) P(A) \\
&= \alpha [< P(c \mid b) P(\neg d \mid b), P(\neg c \mid b) P(\neg d \mid b) > + < P(c \mid \neg b) P(\neg d \mid \neg b) + \\
&+ P(\neg c \mid \neg b) P(\neg d \mid \neg b) >] \sum_B \sum_A P(B \mid A) P(A) \\
&= \alpha [< 0.1 \times 0.4, 0.5 \times 0.4 >, < 0.3 \times 0.2, 0.7 \times 0.2 >] \sum_B \sum_A P(B \mid A) P(A) \\
&= \alpha < 0.1, 0.5 > \sum_B \sum_A P(B \mid A) P(A) \\
&= \alpha < 0.1, 0.5 > [P(b \mid a) P(a) + P(b \mid \neg a) P(\neg a) + P(\neg b \mid a) P(a) + P(\neg b \mid \neg a) P(\neg a)] \\
&= \alpha < 0.1, 0.5 > [0.5 \times 0.8 + 0.2 \times 0.2 + 0.5 \times 0.8 + 0.8 \times 0.2] \\
&= \alpha < 0.1, 0.5 > [0.4 + 0.04 + 0.4 + 0.16] = \alpha < 0.1, 0.5 > \times 1 \\
&= \alpha < 0.1, 0.5 > \\
&= \frac{1}{0.1 + 0.6} \times < 0.1, 0.5 > = < 0.17, 0.83 > \\
P(c \mid \neg d) &= \underline{\underline{0.17}}
\end{aligned}$$

d) $P(a \mid \neg c, d)$

$$
\begin{aligned}
P(a \mid \neg c, d) &= \alpha P(a, \neg c, d) = \alpha \sum_B P(a, B, \neg c, d) \\
&= \alpha \sum_B P(a)P(B \mid a)P(\neg c \mid B)P(d \mid B) = \alpha P(a) \sum_B P(B \mid a)P(\neg c \mid B)P(d \mid B) \\
&= \alpha P(a)[P(b \mid a)P(\neg c \mid b)P(d \mid b) + P(\neg b \mid a)P(\neg c \mid \neg b)P(d \mid \neg b)] \\
&= \alpha P(a)[0.5 \times 0.9 \times 0.6 + 0.5 \times 0.7 \times 0.8] = \alpha P(a) \times 0.55 \\
&= \alpha < 0.8, 0.2 > \times 0.55 = \alpha < 0.44, 0.11 > \\
&= \frac{1}{0.44 + 0.11} < 0.44, 0.11 >=< 0.8, 0.2 > \\
P(a \mid \neg c, d) &= \underline{\underline{0.8}}
\end{aligned}
$$

Problem 4

The code is written in python 3, meaning that to run my code, python 3 must be installed. The code is commented. I didn't get 4b) to work, which is why it is not commented/why it doesn't work.

References

[1] Jiří Kléma. *Bayesian networks – exercises*. Fall 2015-2016. URL: `https://cw.fel.cvut.cz/old/_media/courses/ae4m33rzn/bn_solved.pdf`