```
(%i59) kill(all);
(%o0) done
```

1 Hyperbolic spiral orbit

```
(%i1) D: L^2/m^2*(1/r^2+1/r[0]^2)^(3/2);
```

$$(\%o1)\ \frac{\left(\frac{1}{r^2}+\frac{1}{r_0^2}\right)^{3/2} L^2}{m^2}$$

```
(%i2) X: M*G*L/m*(1/r*cos(r[0]/r) - 1/r[0]*sin(r[0]/r))/D;
```

$$(\%o2)\ \frac{m\left(\frac{\cos\left(\frac{r_0}{r}\right)}{r}-\frac{\sin\left(\frac{r_0}{r}\right)}{r_0}\right) G\,M}{\left(\frac{1}{r^2}+\frac{1}{r_0^2}\right)^{3/2} L}$$

```
(%i3) Y: M*G*L/m*(1/r*sin(r[0]/r) - 1/r[0]*cos(r[0]/r))/D;
```

$$(\%o3)\ \frac{m\left(\frac{\sin\left(\frac{r_0}{r}\right)}{r}-\frac{\cos\left(\frac{r_0}{r}\right)}{r_0}\right) G\,M}{\left(\frac{1}{r^2}+\frac{1}{r_0^2}\right)^{3/2} L}$$

```
(%i4) str: [M=1, G=1, L=1, m=1, r[0]=1];
```

$$(\%o4)\ [M=1, G=1, L=1, m=1, r_0=1]$$

```
(%i5) X1: ev(X, str);
```

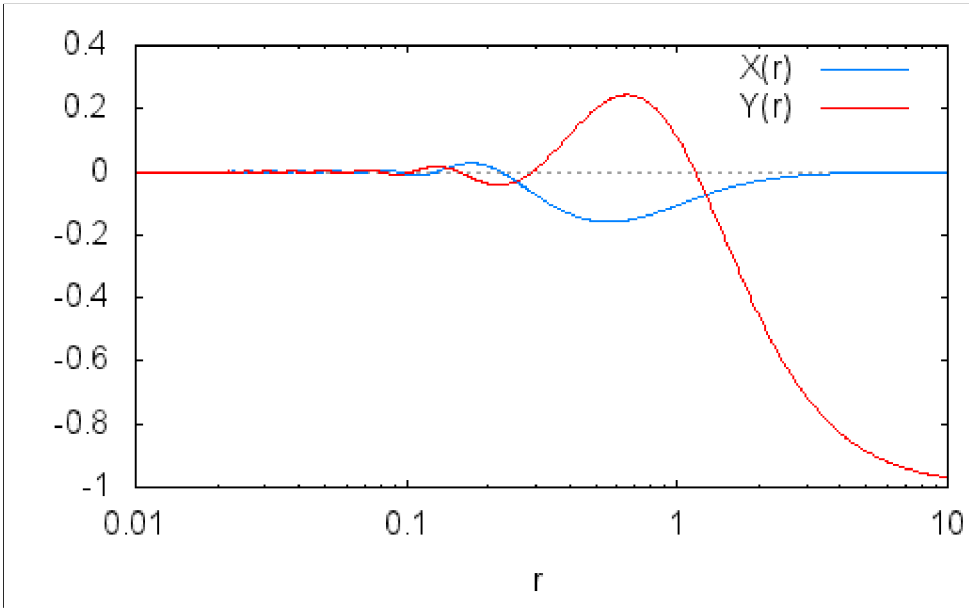
$$(\%o5)\ \frac{\frac{\cos\left(\frac{1}{r}\right)}{r}-\sin\left(\frac{1}{r}\right)}{\left(\frac{1}{r^2}+1\right)^{3/2}}$$

```
(%i6) Y1: ev(Y, str);
```

$$(\%o6)\ \frac{\frac{\sin\left(\frac{1}{r}\right)}{r}-\cos\left(\frac{1}{r}\right)}{\left(\frac{1}{r^2}+1\right)^{3/2}}$$

```
(%i7) wxplot2d ([X1,Y1], [r,0.01,10], [logx],
        [legend, "X(r)", "Y(r)"])$
```

(%t7)




```
(%i54) /*plot2d ([X1,Y1], [theta,0,2*%pi],
          [legend, "X(theta)", "Y(theta)"],
          [gnuplot_term, "png linewidth 3 font 'Arial' 16 size 800,600"],
        [gnuplot_out_file, "D:/Doc/Artikel-Eck/ECE-Theorie/Paper360/Fig11.
incorrect syntax: Premature termination of input at ;.
```

2 Precessing ellipse

```
(%i9) D: (rdot^2+r^2*thdot^2)^(3/2);
```

$$(\%o9)\quad (r^2\,thdot^2+rdot^2)^{3/2}$$

```
(%i10) X: M*G*(rdot*sin(theta) + r*thdot*cos(theta))/D;
```

$$(\%o10)\quad \frac{(rdot\,\sin(theta)+r\,thdot\,\cos(theta))\,G\,M}{(r^2\,thdot^2+rdot^2)^{3/2}}$$

```
(%i11) Y: M*G*(-rdot*cos(theta) + r*thdot*sin(theta))/D;
```

$$(\%o11)\quad \frac{(r\,thdot\,\sin(theta)-rdot\,\cos(theta))\,G\,M}{(r^2\,thdot^2+rdot^2)^{3/2}}$$

```
(%i12) rdot: x*epsilon*L/(m*alpha)*sqrt(1+1/epsilon^2*(alpha/r-1)^2);
```

$$(\%o12)\quad \frac{\varepsilon\,\sqrt{\frac{\left(\frac{\alpha}{r}-1\right)^2}{\varepsilon^2}+1}\;x\,L}{\alpha\,m}$$

```
(%i13) thdot: L/(m*r^2);
```

$$(\%o13)\quad \frac{L}{m\,r^2}$$

```
(%i14) kill(r);
(%o14) done
```

```
(%i15) r = alpha/(1+epsilon*cos(x*theta));
```

$$(\%o15)\quad r=\frac{\alpha}{\varepsilon\cos(\mathit{theta}\,x)+1}$$

```
(%i16) solve(%,theta);
solve: using arc-trig functions to get a solution.
Some solutions will be lost.
```

$$(\%o16)\quad \left[\mathit{theta}=\frac{\operatorname{acos}\left(\frac{\alpha}{\varepsilon\,r}-\frac{1}{\varepsilon}\right)}{x}\right]$$

```
(%i17) theta: rhs(first(%));
```

$$(\%o17)\quad \frac{\operatorname{acos}\left(\frac{\alpha}{\varepsilon\,r}-\frac{1}{\varepsilon}\right)}{x}$$

```
(%i18) str: [alpha=1, L=1, M=1, G=1, m=1, epsilon=0.5, x=1];
(%o18) [α=1,L=1,M=1,G=1,m=1,ε=0.5,x=1]
```

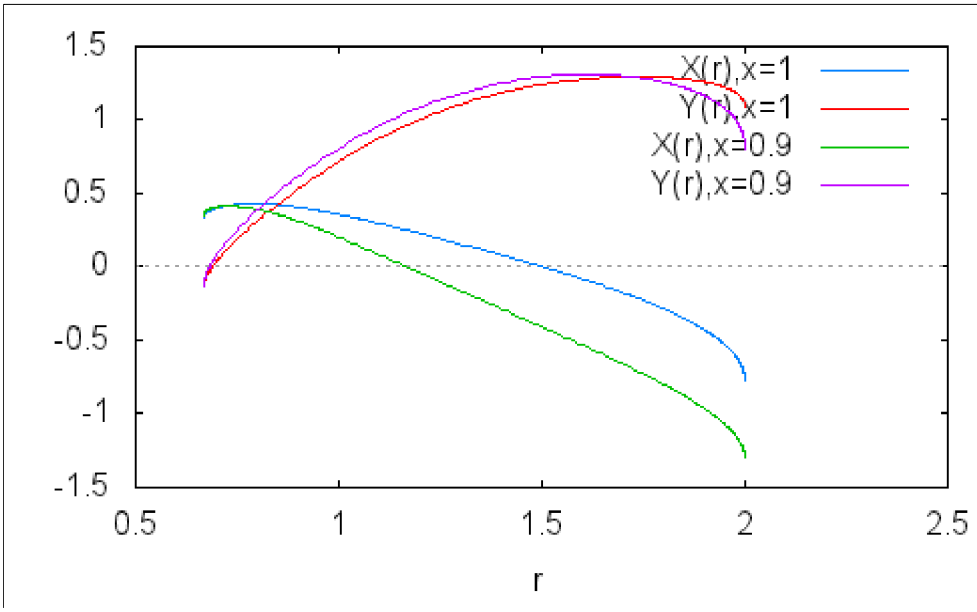
```
(%i19) X1: ev(X, str, eval)$
       Y1: ev(Y, str, eval)$
```

```
(%i21) str: [alpha=1, L=1, M=1, G=1, m=1, epsilon=0.5, x=0.9];
(%o21) [α=1,L=1,M=1,G=1,m=1,ε=0.5,x=0.9]
```

```
(%i22) X2: ev(X, str, eval)$
       Y2: ev(Y, str, eval)$
```

```
(%i24) wxplot2d ([X1,Y1,X2,Y2], [r,0.5,2.5],
         [legend, "X(r),x=1", "Y(r),x=1", "X(r),x=0.9", "Y(r),x=0.9"])$
plot2d: expression evaluates to non-numeric value somewhere in plotting ra
plot2d: expression evaluates to non-numeric value somewhere in plotting ra
plot2d: expression evaluates to non-numeric value somewhere in plotting ra
plot2d: expression evaluates to non-numeric value somewhere in plotting ra
```

(%t24)

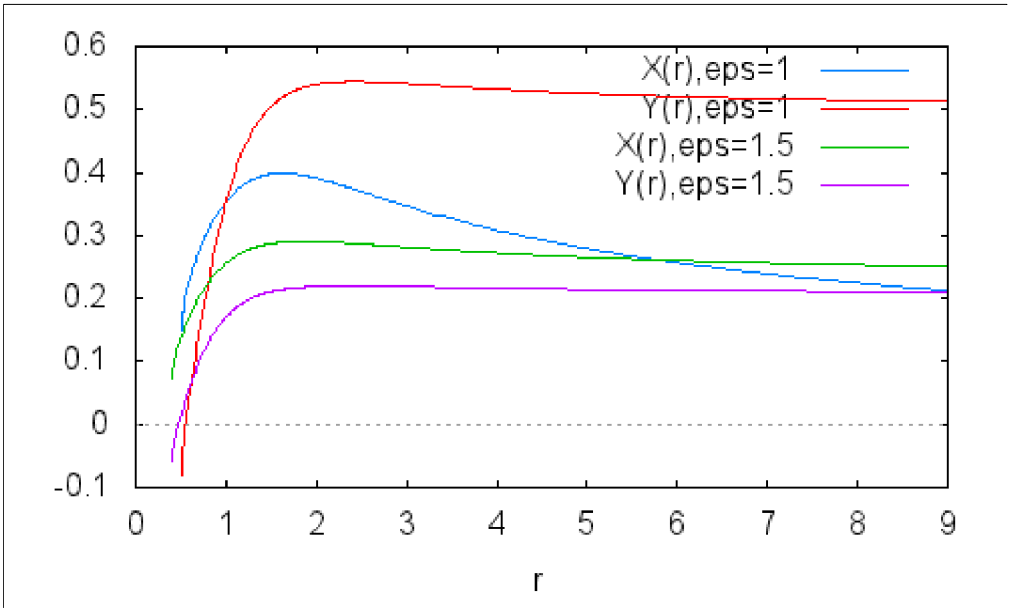


3 Conic sections

```
(%i25) str: [alpha=1, L=1, M=1, G=1, m=1, epsilon=1., x=1.];
(%o25) [α=1,L=1,M=1,G=1,m=1,ε=1,x=1]

(%i26) X1: ev(X, str, eval)$
       Y1: ev(Y, str, eval)$

(%i28) str: [alpha=1, L=1, M=1, G=1, m=1, epsilon=1.5, x=1.];
(%o28) [α=1,L=1,M=1,G=1,m=1,ε=1.5,x=1]

(%i29) X2: ev(X, str, eval)$
       Y2: ev(Y, str, eval)$
```

```
(%i31) wxplot2d ([X1,Y1,X2,Y2], [r,0.0,9],
         [legend, "X(r),eps=1", "Y(r),eps=1", "X(r),eps=1.5", "Y(r),eps=1
plot2d: expression evaluates to non-numeric value somewhere in plotting ra
plot2d: expression evaluates to non-numeric value somewhere in plotting ra
plot2d: expression evaluates to non-numeric value somewhere in plotting ra
plot2d: expression evaluates to non-numeric value somewhere in plotting ra
```

(%t31)



4 Acceleration of hyperbolic spiral orbit

```
(%i32) g: -1/(M*G)*(L^2/m^2*(1/r^2+1/r[0]^2))^2;
```

$$(\%o32)\quad -\frac{\left(\frac{1}{r^2}+\frac{1}{r_0^2}\right)^2 L^4}{m^4\, G\, M}$$

```
(%i33) str: [L=1, M=1, G=1, m=1, r[0]=1];
```

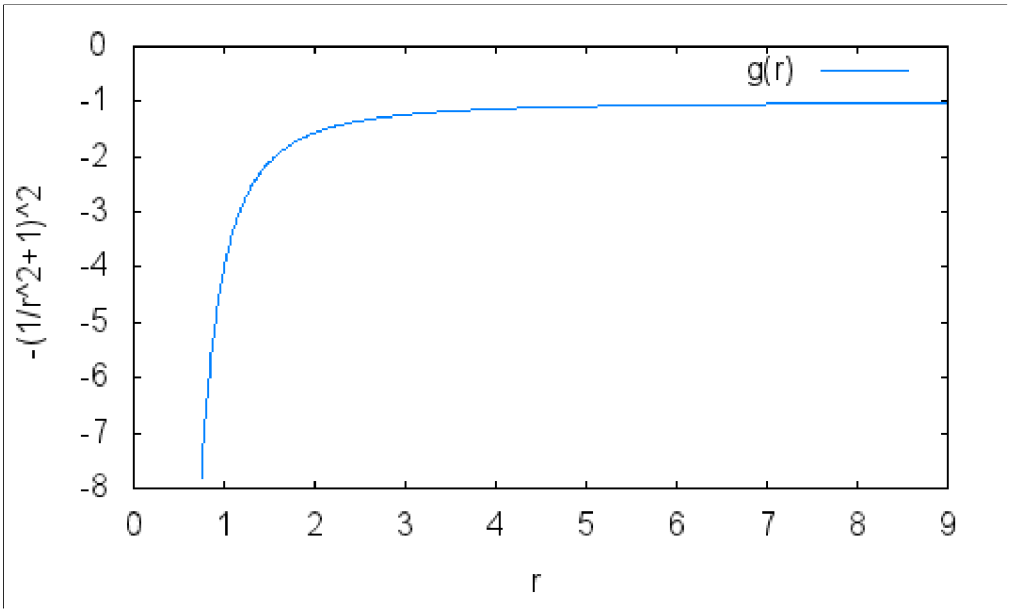
$$(\%o33)\quad [L=1, M=1, G=1, m=1, r_0=1]$$

```
(%i34) g1: ev(g, str, eval);
```

$$(\%o34)\quad -\left(\frac{1}{r^2}+1\right)^2$$

```
(%i35) wxplot2d ([g1], [r,0,9], [y,-8,0],
         [legend, "g(r)"])$
plot2d: expression evaluates to non-numeric value somewhere in plotting ra
plot2d: some values were clipped.
```

(%t35)



5 Acceleration of precessing ellipse

```
(%i36) g: -L^4/(m^4*M*G)*(x^2*epsilon^2/alpha^2*(1-1/epsilon^2*(alpha/r-1
```

(%o36)
$$-\frac{\left(\frac{\varepsilon^2\left(1-\frac{\left(\frac{\alpha}{r}-1\right)^2}{\varepsilon^2}\right)x^2}{\alpha^2}+\frac{1}{r^2}\right)^2 L^4}{m^4\,G\,M}$$

```
(%i37) str: [alpha=1, L=1, M=1, G=1, m=1, epsilon=.5, x=1.];
```

(%o37) $[\alpha=1, L=1, M=1, G=1, m=1, \varepsilon=0.5, x=1]$

```
(%i38) g1: ev(g, str, eval);
```

(%o38)
$$-\left(\frac{1}{r^2}+0.25\left(1-4.0\left(\frac{1}{r}-1\right)^2\right)\right)^2$$

```
(%i39) str: [alpha=1, L=1, M=1, G=1, m=1, epsilon=.5, x=.9];
```

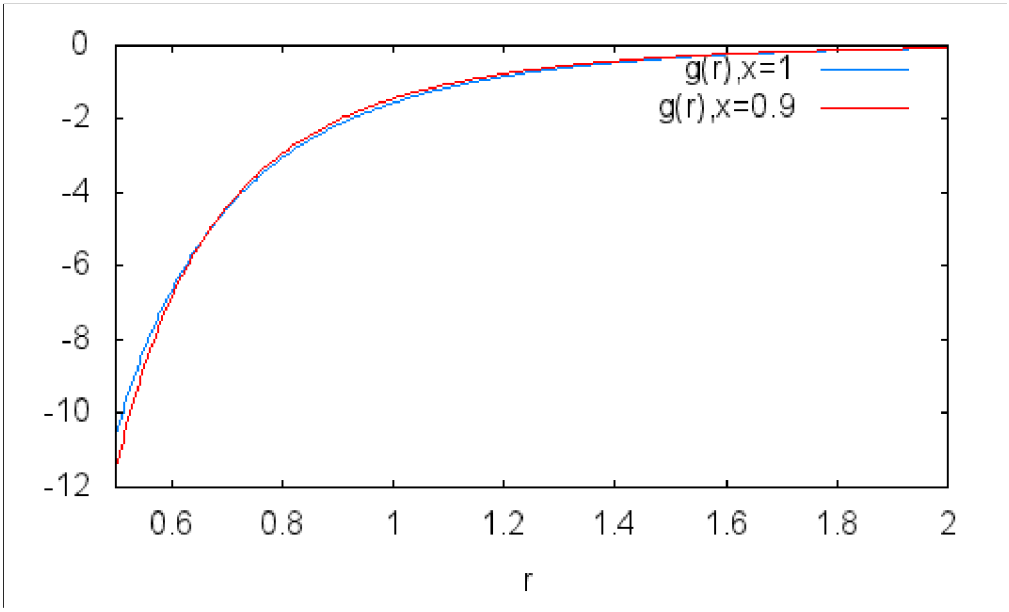
(%o39) $[\alpha=1, L=1, M=1, G=1, m=1, \varepsilon=0.5, x=0.9]$

```
(%i40) g2: ev(g, str, eval);
```

(%o40)
$$-\left(\frac{1}{r^2}+0.2025\left(1-4.0\left(\frac{1}{r}-1\right)^2\right)\right)^2$$

```
(%i41) wxplot2d ([g1,g2], [r,.5,2],
         [legend, "g(r),x=1", "g(r),x=0.9"])$
```

(%t41)



6 Acceleration of conic sections

```
(%i42) g: -L^4/(m^4*M*G)*(x^2*epsilon^2/alpha^2*(1-1/epsilon^2*(alpha/r-1
```

$$(\%o42)\quad -\frac{\left(\frac{\varepsilon^2\left(1-\frac{\left(\frac{\alpha}{r}-1\right)^2}{\varepsilon^2}\right)x^2}{\alpha^2}+\frac{1}{r^2}\right)^2 L^4}{m^4\,G\,M}$$

```
(%i43) str: [alpha=1, L=1, M=1, G=1, m=1, epsilon=1., x=1.];
```

$$(\%o43)\quad [\alpha=1,L=1,M=1,G=1,m=1,\varepsilon=1,x=1]$$

```
(%i44) g1: ev(g, str, eval);
```

$$(\%o44)\quad -\left(\frac{1}{r^2}-\left(\frac{1}{r}-1\right)^2+1\right)^2$$

```
(%i45) str: [alpha=1, L=1, M=1, G=1, m=1, epsilon=1.5, x=1.];
```

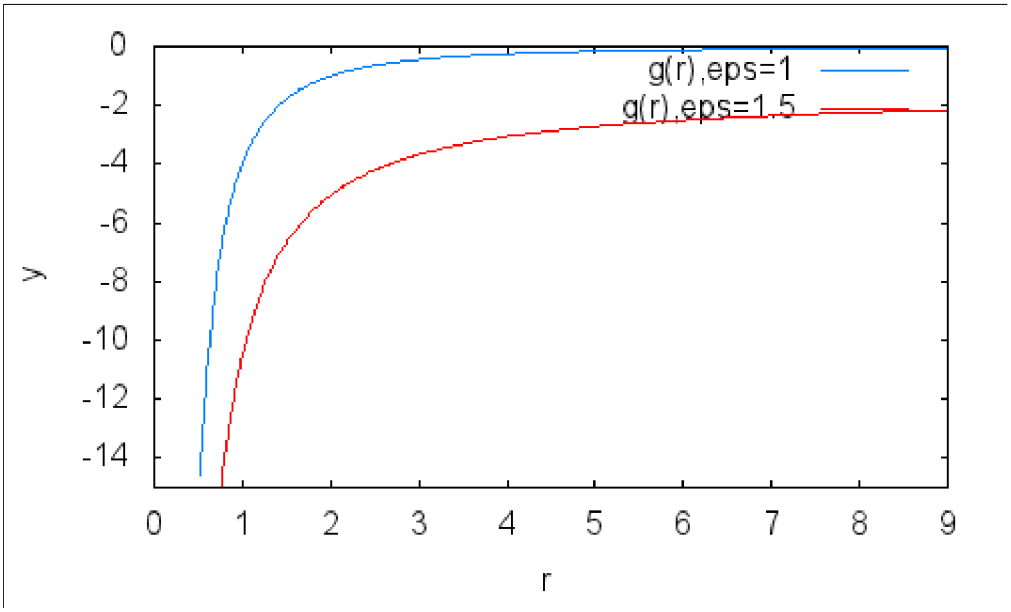
$$(\%o45)\quad [\alpha=1,L=1,M=1,G=1,m=1,\varepsilon=1.5,x=1]$$

```
(%i46) g2: ev(g, str, eval);
```

$$(\%o46)\quad -\left(\frac{1}{r^2}+2.25\left(1-0.44444444444444\left(\frac{1}{r}-1\right)^2\right)\right)^2$$

```
(%i47) wxplot2d ([g1,g2], [r,0,9], [y,-15,0],
         [legend, "g(r),eps=1", "g(r),eps=1.5"])$
plot2d: expression evaluates to non-numeric value somewhere in plotting ra
plot2d: some values were clipped.
plot2d: expression evaluates to non-numeric value somewhere in plotting ra
plot2d: some values were clipped.
```

(%t47)



7 Check

```
(%i48) 1/r^2+0.2025*(1-4.0*(1/r-1)^2)=0;
```

(%o48) $$\frac{1}{r^2}+0.2025\left(1-4.0\left(\frac{1}{r}-1\right)^2\right)=0$$

```
(%i49) %*r^2;
```

(%o49) $$\left(\frac{1}{r^2}+0.2025\left(1-4.0\left(\frac{1}{r}-1\right)^2\right)\right)r^2=0$$

```
(%i50) ratsimp(%);
rat: replaced 0.2025 by 81/400 = 0.2025
rat: replaced -4.0 by -4/1 = -4.0
```

(%o50) $$-\frac{243\,r^2-648\,r-76}{400}=0$$

```
(%i51) solve(%, r);
```

(%o51) $$\left[r=-\frac{2\sqrt{381}-36}{27},\ r=\frac{2\sqrt{381}+36}{27}\right]$$

```
(%i52) %,numer;
```

(%o52) $$[r=-0.1125349108106,\ r=2.77920157747727]$$