Name:

4.7 L'Hopital's Rule

Theorem: L'Hopital's Rule

Suppose $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ is given where $f(x)$ and $g(x)$ are differentiable near a and

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$$

$$OR$$

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = \pm\infty.$$

Then

Indeterminate Forms

When evaluating a limit results in one of the following forms,
there is not yet enough information to evaluate the limit.

$$\frac{0}{0} \qquad \frac{\infty}{\infty} \qquad \frac{-\infty}{-\infty} \qquad \infty - \infty \qquad \infty^0 \qquad 1^\infty \qquad 0^0$$

Evaluate the following limits. Be sure to first determine whether L'Hopital's rule applies.

1. $\lim_{x \to 0} \dfrac{\sin(2x)}{x}$

2. $\lim_{x \to \infty} x^{-2} e^{x}$

3. $\lim_{x \to \infty} \dfrac{e^{-x}}{\sin x}$

4. $\lim_{x \to 1} \dfrac{\ln x}{x^2 - 1}$

5. $\lim_{x \to 0^+} x \ln x$

6. $\lim_{x \to 0} \dfrac{x + \cos x}{x}$

7. $\lim_{x \to 1} \left(1 + \sin\left(\dfrac{3}{x}\right)\right)^{x}$