3.4 Chain Rule

Wednesday, October 4, 2023

Objectives

1. Introduce the chain rule of derivatives
2. Review composite functions

Recall: Rule of Finding Derivatives

1. Power Rule: $f(x) = x^n \rightarrow \frac{df}{dx} = nx^{n-1}$
2. Product Rule: $h(x) = f(x)g(x) \rightarrow \frac{dh}{dx} = f'(x)g(x) + f(x)g'(x)$
3. Quotient Rule: $h(x) = \frac{f(x)}{g(x)} \rightarrow \frac{dh}{dx} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

<u>Composite Functions</u>

A composite function is a function with a function.

$\rightarrow$ Notation: $h(x) = f(g(x))$ or $h = f \circ g$

Examples:

1. $h(x) = 2(x+1) - 1$

   $\rightarrow f(\text{something}) = 2(\text{something}) - 1$

   $\rightarrow \text{something} = x+1$

2. $h(x) = e^{2x+3}$

   $\rightarrow g(x) = 2x+3$

   $\rightarrow f(\text{“}x\text{”}) = e^{\text{“}x\text{”}}$

$$f(g(x)) = e^{g(x)} = e^{2x+3}$$

Chain Rule of Derivatives

Given a composite function $f(g(x))$, then

$$\frac{d}{dx} f(g(x)) = \left(\frac{df}{dg}\right)\left(\frac{dg}{dx}\right)$$

or

$$(f(g(x)))' = f'(g(x))\,g'(x) .$$

Example:

- $h(x) = e^{2x+3}$.

$\longrightarrow$ $g(x) = 2x+3$

$\longrightarrow$ $f(\text{"}x\text{"}) = e^{\text{"}x\text{"}}$

$$\frac{dh}{dx} = \left(\frac{df}{dg}\right)\left(\frac{dg}{dx}\right)$$

$$= e^{2x+3}\,2$$

$$\frac{dh}{dx} = 2e^{2x+3}$$