

Review Exponent Properties

Wednesday, September 27, 2023

Objectives:

1. Polynomials, Exponentials, and Logarithms
2. Law of Exponents
3. Law of Logs

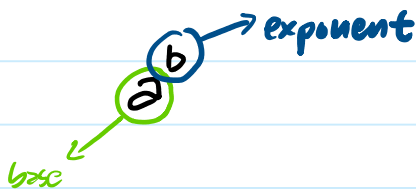
Recall: Inverse functions

If $f^{-1}(x)$ is the inverse of $f(x)$, then

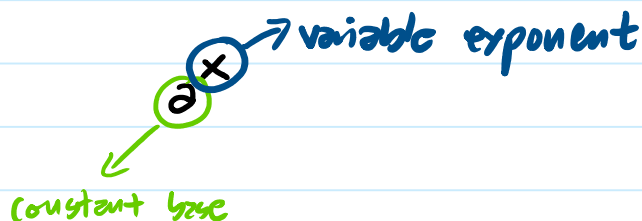
$$\rightarrow f(f^{-1}(x)) = x \text{ or}$$

$$\rightarrow f^{-1}(f(x)) = x.$$

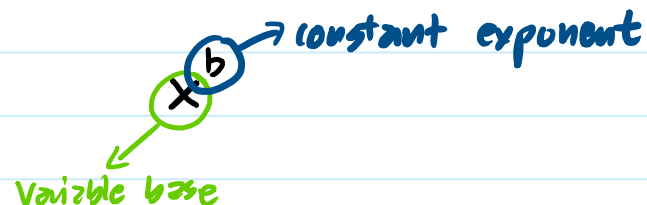
structure of exponents



Exponentials

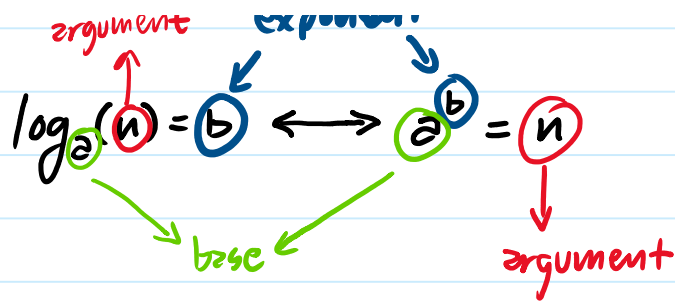


Polynomials



Logarithms & Exponentials





→ logs and exponentials

$$\left. \begin{array}{l} \log_a(a^x) = x \\ a^{\log_a(x)} = x \end{array} \right\} \begin{array}{l} \text{logs and exponentials} \\ \text{are inverses} \end{array}$$

→ If $\log_a x = y$, then $a^{\log_a x} = a^y$

$$x = a^y$$

→ If the base $a = e$, where e is a transcendental number (a constant)
 $e \approx 2.7182818$ (also called Euler's number),
 then

$$\log_e x \rightarrow \ln(x) \rightarrow \text{"natural log"}$$

Law of Exponents

$$1. a^n a^m = a^{n+m}$$

$$2. \frac{a^n}{a^m} = a^{n-m}$$

$$3. (a^n)^m = a^{nm}$$

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$$4. (ab)^n = a^n b^n$$

$$5. \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$6. a^0 = 1$$

$$7. a^{-1} = \frac{1}{a}$$

$$8. a^{1/n} = \sqrt[n]{a}$$

$$9. a^{m/n} = (\sqrt[n]{a})^m$$

Law of logs

$$1. \log_a(u) + \log_a(m) = \log_a(u \cdot m)$$

$$2. \log_a(a^x) = x$$

$$3. a^{\log_a x} = x$$

$$4. \log_a\left(\frac{u}{m}\right) = \log_a(u) - \log_a(m)$$

$$5. \log_a(u^t) = t \log_a(u)$$

$$6. \log_a(1) = 0$$

$$7. \log_a a = 1$$

Examples:

• Simplify $x\sqrt{x}$.

$$x\sqrt{x} = x^1 x^{1/2}$$

$$\begin{aligned}
 x\sqrt{x} &= x^1 x^{1/2} \\
 &= x^{1+1/2} \\
 &= x^{3/2} \text{ or } \sqrt{x^3}
 \end{aligned}$$

- Simplify $\ln(e^2)$.

$$\begin{aligned}
 \ln(e^2) &= 2\cancel{\ln(e)} \\
 &= 2
 \end{aligned}$$

- Simplify $\log_3(3/2)$.

$$\begin{aligned}
 \log_3(3/2) &= \log_3(3) - \log_3(2) \\
 &= 1 - \log_3(2)
 \end{aligned}$$

- Solve for x given $\log(2x^2) = 3$.

$$\begin{aligned}
 {}^{10}\log(2x^2) &= {}^{10}3 \\
 2x^2 &= 10^3 \\
 x^2 &= \frac{10^3}{2} \\
 x &= \sqrt{\frac{10^3}{2}}
 \end{aligned}$$

- Solve for x given $2 \cdot 7^x = 3 \cdot 2^x$.

$$\begin{aligned}
 2 \cdot 7^x &= 3 \cdot 2^x \\
 \ln(2 \cdot 7^x) &= \ln(3 \cdot 2^x)
 \end{aligned}$$

$$2 \cdot 7^x = 3 \cdot 2^x$$

$$\ln(2 \cdot 7^x) = \ln(3 \cdot 2^x)$$

$$\ln(2) + \ln(7^x) = \ln(3) + \ln(2^x)$$

$$\ln(2) + x \ln(7) = \ln(3) + x \ln(2)$$

$$x \ln(7) - x \ln(2) = \ln(3) - \ln(2)$$

$$x (\ln(7) - \ln(2)) = \ln(3) - \ln(2)$$

$$x = \frac{\ln(3) - \ln(2)}{\ln(7) - \ln(2)}$$