

4.6 Related Rates Cont.

Monday, November 13, 2023

Objectives:

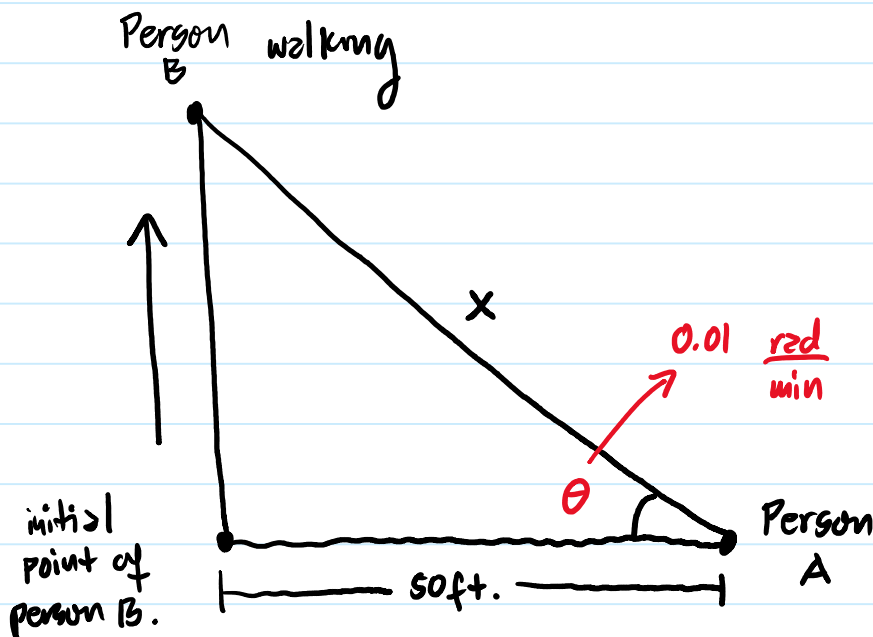
1. Cont. on related rates problems.

Recall: Related Rates

→ A combination of word problems and implicit differentiation.

Example Problem 2:

Two people are 50 ft apart. One of them starts walking north at a rate so that the angle shown in the diagram below is changing at a constant rate of 0.01 rad/min . At what distance between two people changing when $\theta = 0.5$ radians?



What is changing and not changing?

- θ is changing in time
- x is changing in time
- 50 ft. initial distance is not changing.

Goal: Find $\frac{dx}{dt}$, given $\theta = 0.5$ & $\frac{d\theta}{dt} = 0.01 \frac{\text{rad}}{\text{min}}$

Distance between A and B at any point in time:

$$\cos(\theta) = \frac{50}{x} \rightarrow \cos(\theta) = \frac{\text{adj}}{\text{hyp}}$$

or

$$\sec(\theta) = \frac{x}{50} \rightarrow \sec(\theta) = \frac{1}{\cos(\theta)}$$

Let $x = x(t)$ and $\theta = \theta(t)$.

$$D\{\sec(\theta)\} = D\left\{\frac{x}{50}\right\} \rightarrow \text{compute implicit derivatives}$$

$$\sec(\theta) \tan(\theta) \frac{d\theta}{dt} = \frac{1}{50} \frac{dx}{dt}$$

meaning! the rate of change of θ
is related to the rate of change of x .



As θ changes, x also changes.

$$\frac{dx}{dt} = 50 \sec(\theta) + \tan(\theta) \frac{d\theta}{dt}$$

↓ Plug in $\theta = 0.5$ and $\frac{d\theta}{dt} = 0.01$

$$\frac{dx}{dt} = 50 \sec(0.5) + \tan(0.5) (0.01)$$

$$\approx 0.311254 \text{ ft/min.}$$