1.9 Computing Limits

Warm-up

Sketch one graph with all of the following features:

1.
$$\lim_{x \to -1^{-}} f(x) = 2$$

$$2. \lim_{x \to -1^+} f(x) = 0$$

3.
$$f(-1) = 4$$

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4. $\lim_{x \to \infty} f(x) = 4$

1. Evaluate the following limits, or state that the limit does not exist.

(a)
$$\lim_{x \to 3} \frac{x+1}{x+3}$$

(b)
$$\lim_{b \to 2} e^{4-b^2} - 1$$

(c)
$$\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$$

(d)
$$\lim_{r \to -\infty} \frac{r^4 + 5r^2}{5r + r^3}$$

(e)
$$\lim_{t \to 4} \frac{t-4}{\sqrt{t}-2}$$

(f)
$$\lim_{h \to 0} \frac{(3+h)^2 - 9}{h}$$

(g)
$$\lim_{z \to 1} \frac{z - 1}{z^2 - 2z + 1}$$

Squeeze Theorem

If
$$b(x) \le f(x) \le a(x)$$
 for all x close to $x = c$ and $\lim_{x \to c} b(x) = L = \lim_{x \to c} a(x)$, then

2. Evaluate the following limit:

$$\lim_{x \to 0} x^3 \cos\left(\frac{1}{x}\right)$$

3. Bonus limits.

(a)
$$\lim_{t \to 1} \frac{|t| - 1}{t - 1}$$

(b)
$$\lim_{x \to 4} \frac{x - \sqrt{3x + 4}}{4 - x}$$

(c)
$$\lim_{n \to \infty} \sqrt{n^2 + 5n} - n$$

(d)
$$\lim_{x \to 0} \frac{5\cos x \sin x}{x}$$