2.3 Derivative Function Preview

Objectives:
1. Introducing derivative functions.

Previously...

- Limit definition of derivative at a point:
  Suppose we have $f(x)$, then

$$\frac{df}{dx}(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \text{at} \quad x = a.$$

Common Notations of Derivative:

$\frac{df}{dx} \longrightarrow$ derivative of f with respect to x — a ratio / not a fraction

$f' \longrightarrow f$ "prime", assume x is the free variable

$\dot{f} \longrightarrow f$ "dot", the same as f'.
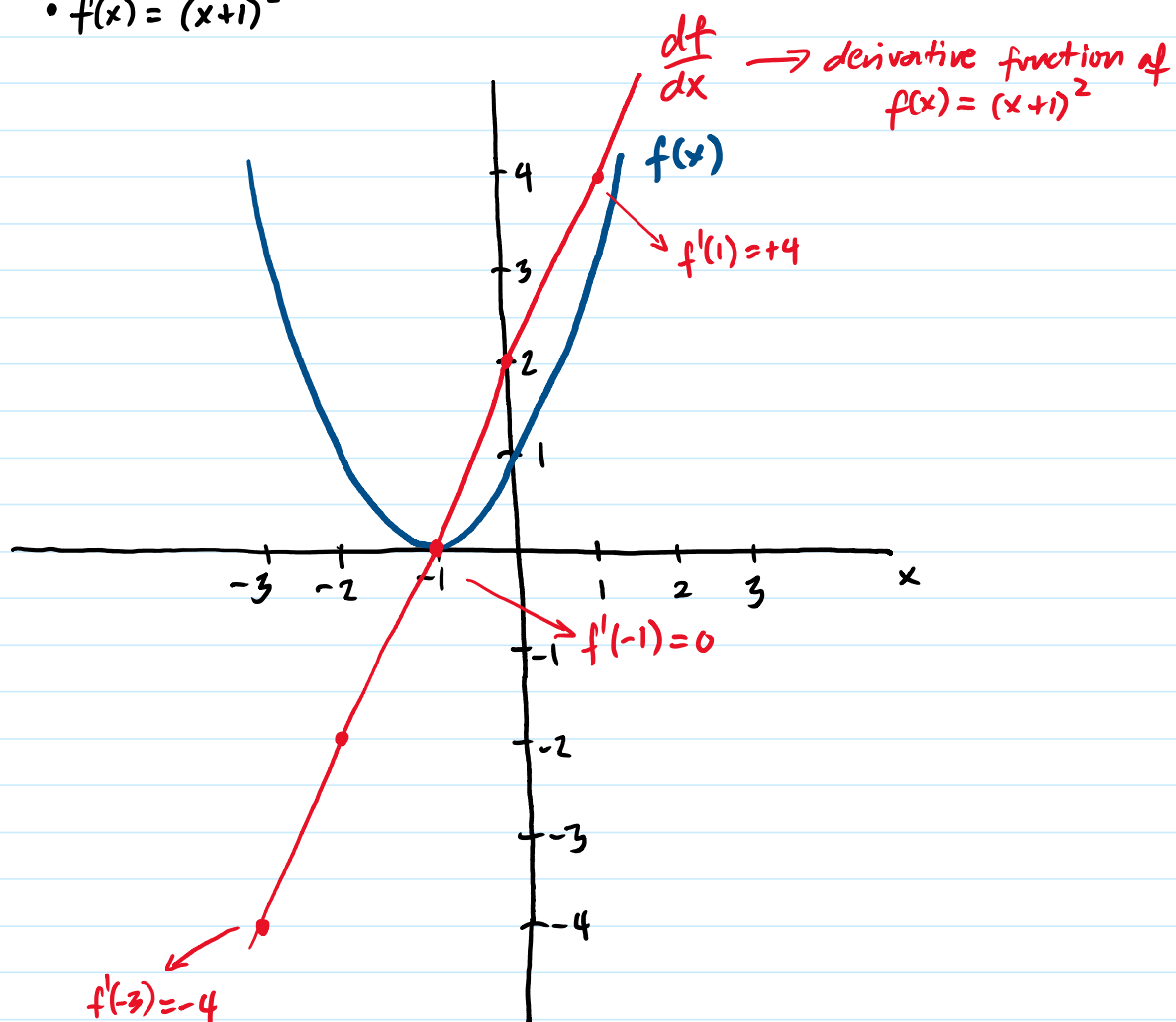
Example:
- $f(x) = (x+1)^2$ at $x = 1$

$$\frac{df}{dx}(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{((1+h)+1)^2 - [(1+1)^2]}{h}$$

$$= \lim_{h \to 0} \frac{(h+2)^2 - 2^2}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 4h + 2^2 - 2^2}{h}$$

$$= \lim_{h \to 0} \frac{h(h+4)}{h}$$

$$= \lim_{h \to 0} h + 4$$

$$\frac{df}{dx}(1) = 4$$

Suppose we find $\frac{df}{dx}$ for all x:

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- $f(x) = (x+1)^2$

$\frac{df}{dx}$ $\rightarrow$ derivative function of $f(x) = (x+1)^2$

$f(x)$

$f'(1) = +4$

$f'(-1) = 0$

$f'(-3) = -4$

- Go to Desmos for more examples: