Review Exponent Properties

Wednesday, September 27, 2023

Objectives:

1. Polynomials, Exponentials, and Logarithms
2. Law of Exponents
3. Law of logs

Recall: <u>Inverse functions</u>

If $f^{-1}(x)$ is the inverse of $f(x)$, then

$\rightarrow f(f^{-1}(x)) = x$ or

$\rightarrow f^{-1}(f(x)) = x$.

<u>structure of exponents</u>

exponent

a^b

base

<u>Exponentials</u>

variable exponent

a^x

constant base

<u>Polynomials</u>

constant exponent

x^b

variable base

<u>Logarithms & Exponentials</u>

argument exponent

argument exponent

$$\log_a(n) = b \longleftrightarrow a^b = n$$

base argument

→ logs and exponentials

$$\log_a(a^x) = x$$

$$a^{\log_a(x)} = x$$

} logs and expontials are inverses

→ If $\log_a x = y$, then $a^{\log_a x} = a^y$

$$x = a^y.$$

→ If the base $a = e$, where e is a transcendental number (a constant) $e \approx 2.7182818$ (also called Euler's number), then

$$\log_e x \longrightarrow \ln(x) \longrightarrow \text{"natural log"}.$$

Law of Exponents

1. $a^n a^m = a^{n+m}$
2. $\dfrac{a^n}{a^m} = a^{n-m}$
3. $(a^n)^m = a^{nm}$

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4. $(ab)^n = a^n b^n$
5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
6. $a^0 = 1$
7. $a^{-1} = \frac{1}{a}$
8. $a^{1/n} = \sqrt[n]{a}$
9. $a^{m/n} = (\sqrt[n]{a})^m$

Law of logs

1. $\log_a(n) + \log_a(m) = \log_a(nm)$
2. $\log_a(a^x) = x$
3. $a^{\log_a x} = x$
4. $\log_a\left(\frac{n}{m}\right) = \log_a(n) - \log_a(m)$
5. $\log_a(n^t) = t\log_a(n)$
6. $\log_a(1) = 0$
7. $\log_a a = 1$

Examples:

- Simplify $x\sqrt{x}$.

$x\sqrt{x} = x^1 x^{1/2}$

$$x\sqrt{x} = x^1 x^{1/2}$$
$$= x^{1+1/2}$$
$$= x^{3/2} \text{ or } \sqrt{x^3}$$

- Simplify $\ln(e^2)$.

$$\ln(e^2) = 2\cancel{\ln(e)}^{\,1}$$
$$= 2$$

- Simplify $\log_3(3/2)$.

$$\log_3(3/2) = \log_3(3) - \log_3(2)$$
$$= 1 - \log_3(2)$$

- Solve for x given $\log(2x^2) = 3$.

$$10^{\log(2x^2)} = 10^3$$
$$2x^2 = 10^3$$
$$x^2 = \frac{10^3}{2}$$
$$x = \sqrt{\frac{10^3}{2}}$$

- Solve for x given $2 \cdot 7^x = 3 \cdot 2^x$.

$$2 \cdot 7^x = 3 \cdot 2^x$$
$$\ln(2 \cdot 7^x) = \ln(3 \cdot 2^x)$$

$$2 \cdot 7^{x} = 3 \cdot 2^{x}$$

$$\ln(2 \cdot 7^x) = \ln(3 \cdot 2^x)$$

$$\ln(2) + \ln(7^x) = \ln(3) + \ln(2^x)$$

$$\ln(2) + x\ln(7) = \ln(3) + x\ln(2)$$

$$x\ln(7) - x\ln(2) = \ln(3) - \ln(2)$$

$$x(\ln(7) - \ln(2)) = \ln(3) - \ln(2)$$

$$x = \frac{\ln(3) - \ln(2)}{\ln(7) - \ln(2)}$$