Name:

4.1 Using First and Second Derivatives

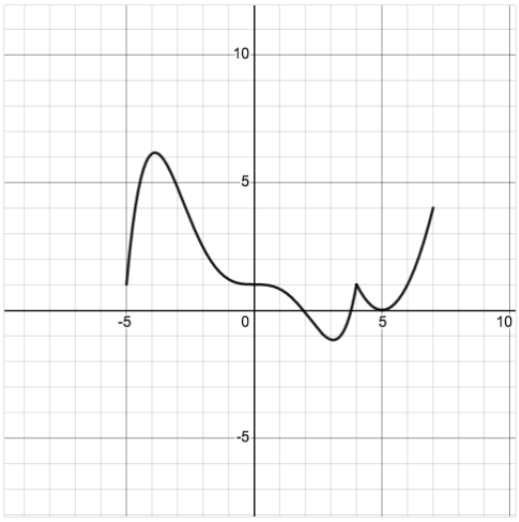
1. The graph of f is shown to the right. Estimate the following. Your solutions should be given as an ordered pair (coordinates of a point).

   (a) Critical point(s) of f.

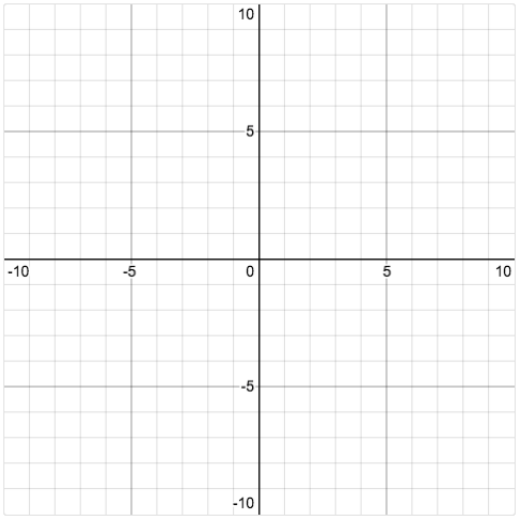
   (b) Local maxima.

   (c) Local minima.

   (d) Inflection point(s) of f.



2. Sketch a graph of a function that has exactly one critical point at $x = 2$ and exactly one inflection point at $x = 4$, or explain why no such function exists.

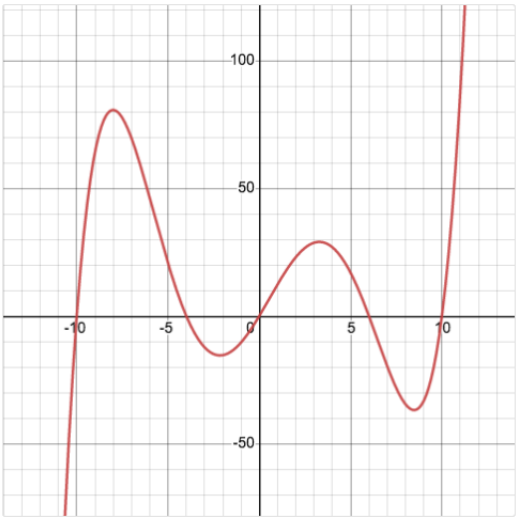


3. The graph of the DERIVATIVE, f' is shown.

   (a) What are the critical points of f? (NOT f').

   (b) Where does f have a local max? A local min?

   (c) What are the inflection points of f? (NOT f').



4. Let $f(x) = \dfrac{a}{x^2} + x$.

   (a) If a is a nonzero constant, find all critical points of f.

   (b) Use the second-derivative test to show that if a is positive then the graph has a local minimum, and if a is negative then the graph has a local maximum.

   (c) Check your work by graphing f in Desmos.

5. The rabbit population on a small Pacific island is approximated by

$$P = \frac{2000}{1 + e^{(5.3-0.4t)}}$$

   with t measured in years since 1774, when Captain James Cook left 10 rabbits on the island.

   (a) Graph P on Desmos. Does the population level off?

   (b) Estimate when the rabbit population grew most rapidly. How large was the population at that time?

   (c) What natural causes could lead to the shape of the graph of P?