Name: ______________________________

# 1.8 Limits and Continuity

1. **Holes, Asymptotes, and Indeterminate forms:** Compare the following functions. For each, Find the limit as $x$ approaches 2, and determine whether the function is continuous at $x = 2$.

   (a) $f(x) = 3x + 5$

   (b) $g(x) = \dfrac{(3x+5)(x-2)}{x-2}$

   (c) $h(x) = \dfrac{3x+5}{x-2}$

   (d) $g(x) = \dfrac{(3x+5)(x-2)}{(x-2)^2}$

**Summary:**

If a function is of the form $f(a) = \dfrac{K}{0}$ where $K$ is a constant, then $\lim_{x\to a} f(x)$....

If a function is of the form $f(a) = \dfrac{0}{0}$, then $\lim_{x\to a} f(x)$....

2. **Limits at Infinity:** Use graphs to find the limits at infinity for the following functions:

   (a) $\lim_{x\to -\infty} e^{-x}$

   (b) $\lim_{x\to \infty} \dfrac{1}{x^2} + 2$

   (c) $\lim_{x\to \infty} \dfrac{4x^2 - 8}{-7x^2}$

   (d) $\lim_{x\to -\infty} \dfrac{3x^3 - 27}{x - 3}$

   (e) $\lim_{x\to -\infty} \dfrac{3\sqrt{x} - 2}{x^5 + 4}$

   (f) $\lim_{x\to \infty} \sin(x)$

**Summary:** For rational function of the form $r(x) = \frac{p(x)}{q(x)}$ with $p(x)$ degree $n$ and $q(x)$ degree $m$, we have:

- If $n < m$...
- If $n > m$...
- If $n = m$...

3. Find the value of $k$ that would make the function continuous.

$$g(x) = \begin{cases} \frac{e^x - 1}{x} & x \neq 0 \\ k & x = 0 \end{cases}$$

4. Find a value of $m$ that would make the limit exist. Find the limit.

$$\lim_{x \to \infty} \frac{2x^3 - 6}{x^m + 3}$$

5. For each description, sketch a graph with the given characteristics:

   (a) $f(4)$ is undefined and $\lim_{x \to 4} f(x) = 2$

   (b) $g(3) = 2$ and $\lim_{x \to 3} g(x)$ does not exist

   (c) $h(4) = 2$, $\lim_{x \to 4^-} h(x) = 0$, and $\lim_{x \to 4^+} h(x) = -\infty$

6. BONUS: The Dirichlet Function is defined as

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{otherwise} \end{cases}$$

Is this function continuous at $x = 0$?