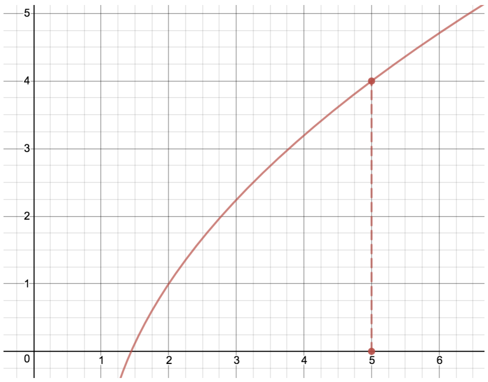
Name: ______________________________

3.9 Linear Approximation

1. Write the equation for the line tangent to $f(x) = 3\sqrt{x-1} - 2$ at $x = 5$.



> **Definition:** The Equation for a tangent line to the function $f(x)$ at the point $(a, f(a))$ with slope $f'(a)$ is
>
> $$L_a(x) = f'(a)(x-a) + f(a)$$
>
> Note: $f(a)$ and $f'(a)$ are NUMBERS
> All functions which are *locally linear* at $x = a$ are differentiable at $x = a$. Also, all functions which are differentiable at $x = a$ are locally linear at $x = a$. A function is *locally linear* if ....

2. Let $f(x) = \frac{1}{2}(x-2)^3 + 3$.

   (a) Find the equation for the linear approximation of f at $x = 3$.

   $$L(x) =$$

   (b) Add a few more x-values to the table below and compare the outputs of the original function and the tangent line approximation.

x_1	$f(x_1)$	$L(x_1)$	$L(x_1) - f(x_1)$
3	3.5		

   (c) How good is the linear approximation at estimating the actual value of the function?

Definition: The error is the difference between the actual value (given by f), and the approximate value (given by L), with equation

$$E(x) = f(x) - L(x) =$$

For a nicely behaved function,

$$E(x) \approx \frac{f''(a)}{2}(x-a)^2$$

3. Use the function $f(x) = \sqrt[5]{x}$ and its tangent line at $x = 1$ to estimate the fifth root of 2 without using your calculator. Is this an over estimate or an under estimate?

4. If $h(0) = 4$ and $h'(0) = -5$, use a linear approximation to estimate $h(0.25)$.

5. Given that g is continuous, differentiable, and contains the value given in the table, estimate $g(3.2)$.

x	$-\frac{3}{2}$	$\frac{5}{2}$	3	7
$g(x)$	6	$-\frac{7}{4}$	$-\frac{1}{2}$	14.3