Name:

3.6 Derivatives of Inverse Functions

1. The equation $C = \frac{5}{9}(F - 32)$ relates a temperature given in F degrees Fahrenheit to the corresponding temperature C measured in degrees Celsius.

   (a) Solve the equation for F to write F (Fahrenheit temperature) in terms of C (Celsius temperature).

   (b) Using the first equation, compute $\frac{dC}{dF}$ and interpret the units.

   (c) Using the inverse equation you found compute $\frac{dF}{dC}$ and interpret the units.

   (d) What do you notice?

Derivatives of Inverse Functions

$$\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{1}{f'\left(f^{-1}(x)\right)}$$

This means....

2. Suppose a function $f(x)$ has an inverse given by $f^{-1}(x) = g(x)$. We also know that $f(3) = -1$ and $f'(3) = 10$. Which of the following are true? (You may select more than one option.)

   - ☐ $g(-1) = 3$
   - ☐ $g'(3) = \frac{1}{10}$
   - ☐ $g'(-1) = -10$
   - ☐ $g'(-1) = \frac{1}{10}$
   - ☐ $g'(10) = 3$

3. Match each function to its derivative function.

(a) $f(x) = \ln(x)$	1. $f'(x) = \frac{1}{\sqrt{1-x^2}}$
(b) $f(x) = \arcsin(x)$	2. $f'(x) = \frac{1}{1+x^2}$
(c) $f(x) = \arctan(x)$	3. $f'(x) = \frac{1}{x}$

4. Compute the derivatives of the following functions. Note the importance of the domain for each.

   (a) $f(x) = \ln(1 - e^{-x})$

   (b) $g(x) = \cos(\arctan 3x)$

5. Average leaf width, w (in mm), in tropical Australia is a function of the average annual rainfall, x (in mm). We have

$$w = f(x) = 32.7 \ln\left(\frac{x}{24.5}\right)$$

   (a) Find $f'(x)$.

   (b) Find $f'(2000)$. Include units.

   (c) Explain how you can use your answer to part (b) to estimate the difference in average leaf widths in a forest whose average annual rainfall is 2000 mm and one whose annual rainfall is 150 mm more.