3.9 Linear Approximation

Thursday, October 26, 2023

Objectives:

1. Introduce linear approximations

Recall: Finding tangent line equation at a point given $f(x)$.

Given $y=f(x)$ and point (x_1, y_1).
the tangent line at point (x_1, y_1) of $f(x)$ is

$$y - y_1 = m(x - x_1)$$

where $m = f'(x_1)$.

Another way of writing tangent line equation

Given a function $y=f(x)$, find tangent line at $x=a$.

We have a point $(a, f(a)) = (x_1, y_1)$ on the function $y=f(x)$.

$$y - y_1 = m(x - x_1)$$
$$\downarrow$$
$$y - f(a) = f'(a)(x - a)$$
$$\downarrow$$
$$y = f(a) + f'(a)(x-a)$$
$$\downarrow$$
$$L(x) = f(a) + f'(a)(x-a)$$

Example:

1. Determine the linear approximation for $f(x) = \sqrt[3]{x}$ at $x = 8$.
   Use the linear approximation to approximate the value of $\sqrt[3]{8.05}$ and $\sqrt[3]{25}$.

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}, \quad f(8) = 2, \quad f'(8) = \frac{1}{12}$$

5 $3\sqrt[3]{x^2}$ 12

$$L(x) = 2 + \frac{1}{12}(x-8) = \frac{1}{12}x + \frac{4}{3}$$

$\longrightarrow L(8.05) \approx 2.00417$, $\sqrt[3]{8.05} = 2.00416$

$\longrightarrow L(25) \approx 3.4167$, $\sqrt[3]{25} = 2.9240$