Name: ______________________________

1.7 What's up with these functions?

1. All of these functions do something weird at $x = 0$. Investigate the functions using your intuition, plugging in values, or using graphs. In particular, be prepared to share with the class:

   - What is $f(0)$? Does it even exist?
   - What happens to the function near $x = 0$? Plug in nearby points or use a graph to support your reasoning.

   (a) $f(x) = \dfrac{|x|}{x}$

   (b) $g(x) = \dfrac{4x^2 - 5x}{x}$

   (c) $h(x) = \dfrac{x^2 + 5}{x^2}$

   (d) $k(x) = \begin{cases} e^x & \text{if } x < 0 \\ 1 - x & \text{if } x \geq 0 \end{cases}$

   (e) $l(x) = \sin\left(\dfrac{1}{x}\right)$

Definitions:

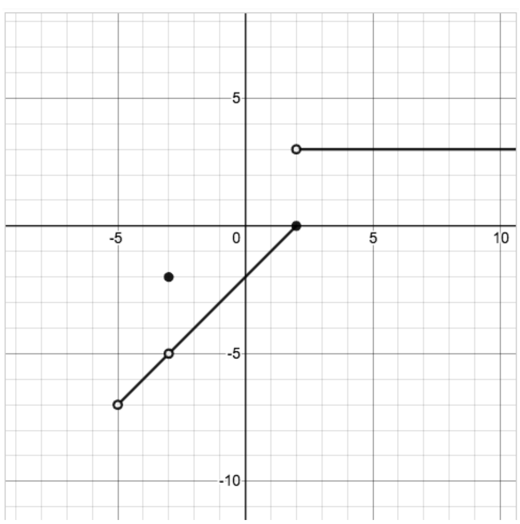
- A function f is continuous at c if $\lim_{x \to c} f(x) = f(c)$, which means ...

- The limit L of a function f at c exists if $\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)$, which means...

2. The piecewise function $f(x)$ is graphed below.

   (a) For what value(s) of x is this function not continuous?

   (b) For what value(s) of x does the limit not exist?



(c) Find the following:

(a) $\lim_{x \to -3^+} f(x) =$

(b) $\lim_{x \to -3^-} f(x) =$

(c) $\lim_{x \to -3} f(x) =$

(d) $f(-3) =$

(e) $\lim_{x \to 2^+} f(x) =$

(f) $\lim_{x \to 2^-} f(x) =$

(g) $\lim_{x \to 2} f(x) =$

(h) $f(2) =$

Intermediate Value Theorem: Suppose f is continuous on a closed interval $[a, b]$. If k is any number between $f(a)$ and $f(b)$, then there exists at least one number c in $[a, b]$ such that $f(c) = k$.