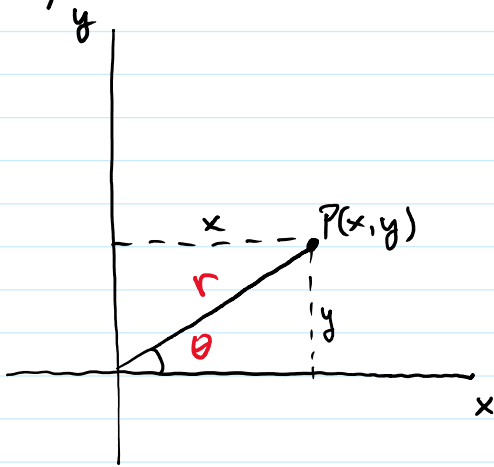# 7.3 Polar Coordinates

Monday, December 5, 2022
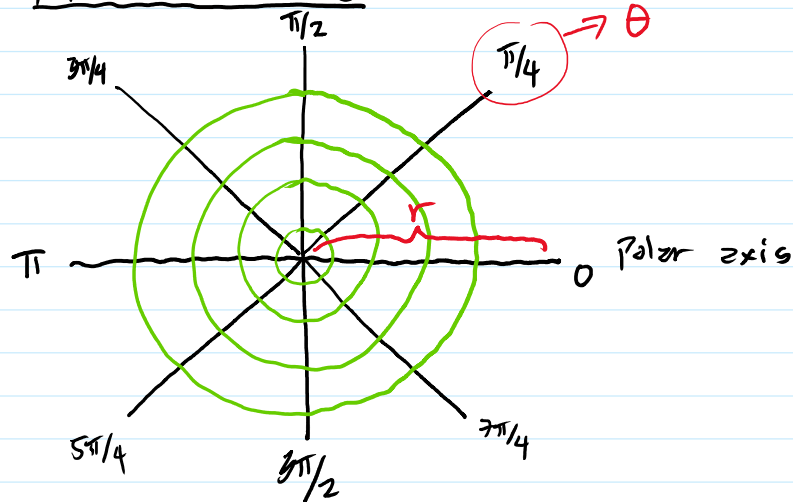
Objectives:
1. Locate points in the plane using polar coordinates.
2. Convert points between rectangular and polar.

## Rectangular Coordinates



## Polar Coordinates



$$\cos(\theta) = \frac{x}{r} \implies x = r\cos(\theta)$$
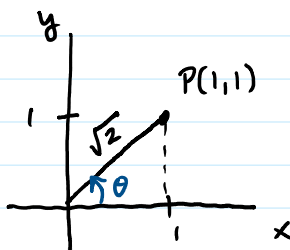
$$\sin(\theta) = \frac{y}{r} \implies y = r\sin(\theta)$$

$$r^2 = x^2 + y^2 \quad \text{and} \quad \tan(\theta) = \frac{y}{x}$$
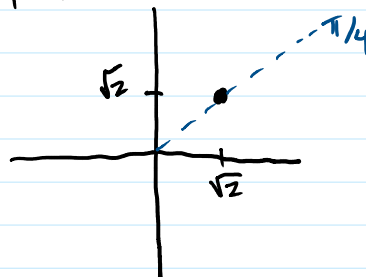
convert $(x,y)$ to $(r, \theta)$

## Converting Points Between Coordinate Systems

Examples: from $(x,y) \rightarrow (r, \theta)$

- point $(1,1)$

Polar



Find r:
$r^2 = x^2 + y^2$

Find $\theta$:
$\tan(\theta) = y/x$

Find r:                     Find θ:
$r^2 = x^2 + y^2$          $\tan(\theta) = y/x$
$\quad = 1^2 + 1^2$   and   $\quad = 1/1$
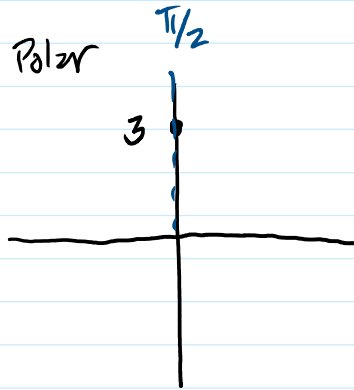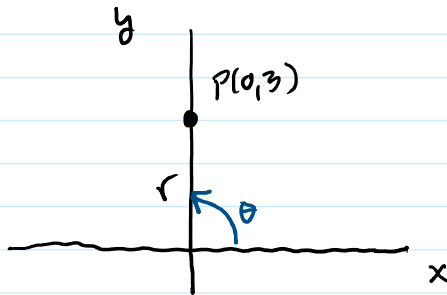$r^2 = 2$                   $\quad = 1$
$r = \sqrt{2}$             $\tan(\theta) = \pi/4$

So,   $(x, y) \rightarrow (r, \theta)$
      $(1, 1) \rightarrow (\sqrt{2}, \pi/4)$

- Point $(0, 3)$



Find r:                          Find θ:
$r^2 = x^2 + y^2$               $\tan(\theta) = \dfrac{y}{x}$
$\quad = (3)^2 + 0^2$
$\quad = 9$                      $\quad = \dfrac{3}{0} \rightarrow$ undefined
$r^2 = 9$
$r = 3$                          $\underbrace{\qquad\qquad}$

                               this means $\theta = \pi/2$

So,   $(x, y) \rightarrow (r, \theta)$
      $(0, 3) \rightarrow (3, \pi/2)$

- point $(5\sqrt{3}, -5)$



Find r:                          Find θ:
$r^2 = x^2 + y^2$               $\tan(\theta) = \dfrac{y}{x}$
$\quad = (5\sqrt{3})^2 + (-5)^2$
$\quad = 75 + 25$        and     $\quad = \dfrac{-5}{}$
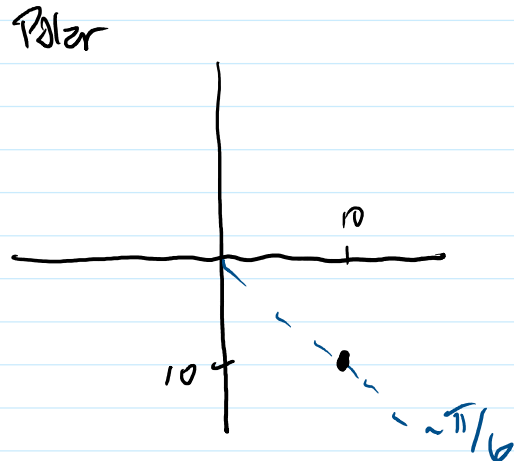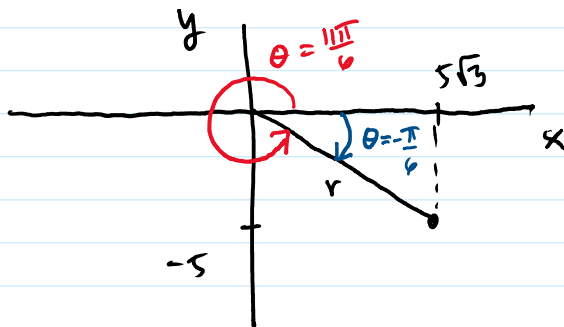
$$= (5\sqrt{3})^2 + (-5)^2$$
$$= 75 + 25 \qquad \text{and}$$
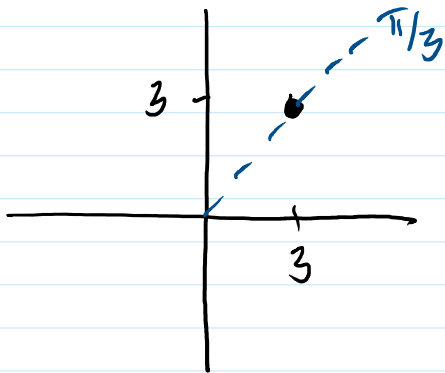$$r^2 = 100$$
$$r = 10$$

$$\frac{-5}{x}$$
$$= \frac{-5}{5\sqrt{3}}$$
$$= -\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$
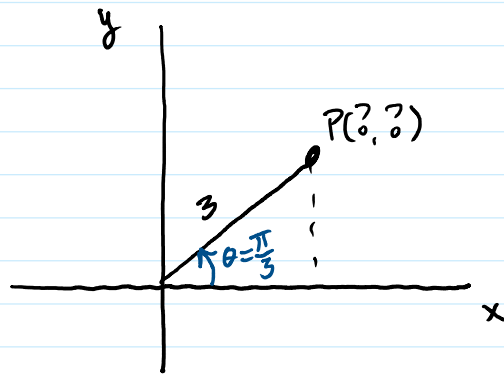$$\theta = -\frac{\pi}{6}$$

So, $(x, y) \to (r, \theta)$

$(5\sqrt{3}, -5) \to (10, -\pi/6)$ clockwise

$\qquad$ or $(10, {}^{11\pi}/_6)$ counterclockwise

## Example: $(r, \theta) \to (x, y)$

- polar point $(3, \pi/3)$



$\qquad\qquad (x, y)$ point



$P(?, ?)$

Find x:
$$x = r\cos(\theta)$$
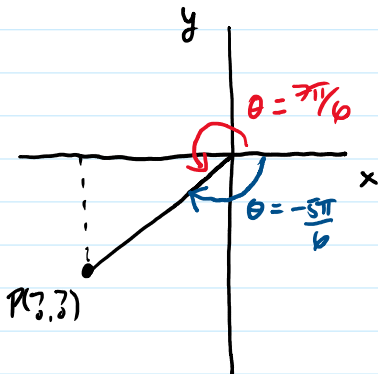$$= 3\cos(\pi/3)$$
$$= 3(1/2)$$
$$x = 3/2$$

Find y:
$$y = r\sin(\theta)$$
$$= 3\sin(\pi/3)$$
$$= 3(\sqrt{3}/2)$$
$$y = \frac{3\sqrt{3}}{2}$$

So, $(r, \theta) \to (x, y)$

$\qquad (3, \pi/3) \to (3/2, 3\sqrt{3}/2)$

- polar point $(6, -{}^{5\pi}/_6)$



$\theta = {}^{7\pi}/_6$

$\theta = \frac{-5\pi}{6}$

$P(?, ?)$

Find x:
$$x = r\cos(\theta)$$
$$= 6\cos\left(-5\pi/6\right)$$
$$= 6\left(-\sqrt{3}/2\right)$$
$$x = -3\sqrt{3}$$

Find y:
$$y = r\sin(\theta)$$
$$= 6\sin\left(-5\pi/6\right)$$
$$= 6\left(-1/2\right)$$
$$y = -3$$

So, $(r, \theta) \rightarrow (x, y)$
$$\left(3, -5\pi/6\right) \rightarrow \left(-3\sqrt{3}, -3\right)$$

## Mini-Activity Part 1

1. Convert $(-8, -8)$ into polar coordinates.
2. Convert $(4, 2\pi/3)$ into rectangular coordinates $(x, y)$