5.2 Problem 4

Saturday, September 3, 2022 2:26 PM

$$S = \sum_{n=6}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+2} \right)$$

* the series above may be different from yours on WebWork but the solution still holds.

Step 1: Compute the kth partial sum.

Note that the series starts at $n=6$!

kth partial sum starting at $n=6$

$n=6$

$$S_1 = \left(\frac{1}{7} - \frac{1}{8} \right)$$

start $n=6$

$n=6$ $\quad$ $n=7$

$$S_2 = \left(\frac{1}{7} - \frac{1}{8} \right) + \left(\frac{1}{8} - \frac{1}{9} \right) = \frac{1}{7} - \frac{1}{9}$$

$n=6$ $\quad$ $n=7$ $\quad$ $n=8$

$$S_3 = \left(\frac{1}{7} - \frac{1}{8} \right) + \left(\frac{1}{8} - \frac{1}{9} \right) + \left(\frac{1}{9} - \frac{1}{10} \right) = \frac{1}{7} - \frac{1}{10}$$

$\vdots$ the pattern looks like $k+7$

$$S_k = \frac{1}{7} - \frac{1}{k+7}$$

$$S_k = \frac{1}{7} - \frac{1}{k+7}$$

So,

$k=3$: $S_k = \frac{1}{7} - \frac{1}{10} = \frac{3}{70}$

$k=4$: $S_4 = \frac{1}{7} - \frac{1}{11} = \frac{4}{77}$

$k=5$: $S_5 = \frac{1}{7} - \frac{1}{12} = \frac{5}{84}$

the infinite sum is

$$S = \lim_{k\to\infty} S_k = \lim_{k\to\infty} \left(\frac{1}{7} - \frac{1}{k+7}\right)$$

$$= \frac{1}{7} - \lim_{k\to\infty} \frac{1}{k+7} \quad (0)$$

$$S = \frac{1}{7}$$

* Lessons to remind ourselves.
  1. notice on where the series starts.
  2. the k in the kth partial sum refers to the 1st term to the kth term in the series.