Integral Techniques

Linearity

$$\int [af(x) \pm bg(x)] \ dx = a\int f(x) \ dx \pm b\int g(x) \ dx, \quad \text{for constants } a \text{ and } b$$

Power Rule

$$\int x^a \ dx = \frac{x^{a+1}}{a+1} + C, \quad \text{for constant } a \neq -1$$

Integration by Substitution

$$\int f(u(x))\frac{du}{dx} \ dx = \int f(u)du$$

Integration by Parts

$$\int udv = uv - \int vdu$$

Trigonometric Substitution

$\sqrt{a^2-x^2}$	$x = a\sin(\theta)$ $dx = a\cos(\theta)d\theta$ $\sqrt{a^2-x^2} = a\cos(\theta)$
$\sqrt{a^2+x^2}$	$x = a\tan(\theta)$ $dx = a\sec^2(\theta)d\theta$ $\sqrt{a^2+x^2} = a\sec(\theta)$
$\sqrt{x^2-a^2}$	$x = a\sec(\theta)$ $dx = \sec(\theta)\tan(\theta)d\theta$ $\sqrt{x^2-a^2} = a\tan(\theta)$

Partial Fractions

$\frac{P(x)}{Q(x)}$, degree$(P(x)) <$ degree$(Q(x))$	distinct	$Q(x) = (ax+b)(cx+d)$	$\frac{P(x)}{Q(x)} = \frac{A}{ax+b} + \frac{B}{cx+d}$
	repeated	$Q(x) = (ax+b)^2$	$\frac{P(x)}{Q(x)} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2}$
	irreducible	$Q(x) = ax^2+bx+c$	$\frac{P(x)}{Q(x)} = \frac{Ax+B}{ax^2+bx+c}$
	repeated irreducible	$Q(x) = (ax^2+bx+c)^2$	$\frac{P(x)}{Q(x)} = \frac{Ax+B}{ax^2+bx+c} + \frac{Cx+D}{(ax^2+bx+c)^2}$
$\frac{P(x)}{Q(x)}$, degree$(P(x)) \geq$ degree$(Q(x))$	long division		