Objectives:

1. Summarize concepts on solving 1st-order ODEs.
2. Review 1st-order ODEs that model physical systems and how to interpret them.

- Ways to slove an ODE we have presented so far.

ODE
- Graphical
  - Slope Field SF
  - Phase Diagram PD
- Numerical
  - Euler's Method EM
- Analytical
  - Separation of Variables SOV
  - Integrating Factors IF

→ Note that you can use multiple methods simultaneously to analyze an ODE.

- Analytical methods and the class of ODEs.

1st-Order ODE
- Separable
  - Linear
    - SOV
    - IF
    - LT
  - Non-Linear
    - SOV
- Inseparable
  - Linear
    - IF
    - LT
  - Non-Linear

- Forms:
  - → Separable: $\frac{dy}{dt} = f(y)g(t)$
  - → 1st-order linear: $\frac{dy}{dt} + P(t)y = Q(t)$
    - If $P(t)$ is a constant, then LT is "easy" to apply.

- Numerical methods and the form of ODEs.

1st-order ODE
- $\frac{dy}{dt} = f(y,t),\ y(t_0) = y_0$
  - EM

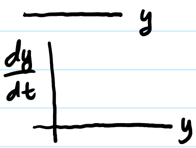
- Graphical methods and the class of ODEs

1st-order ODE
- Autonomous
  - SF
  - PD
- Non-Autonomous
  - SF

- Forms:
  - → Autonomous: $\frac{dy}{dt} = f(y)$
  - → Non-Autonomous: $\frac{dy}{dt} = f(y,t)$

- Axes of SF: y (vertical), t (horizontal)

- Axes of PD: in 1D: / in 2D:



- Modeling Physical Systems

→ Exponential growth: $\frac{dy}{dt} = ky$ with parameter k, $y(0) = y_0$

If $k>0$, then increasing

If $k<0$, then decreasing

$y(0) = y_0$ ← initial condition

→ logistic growth: $\frac{dy}{dt} = ky(N-y) + \beta$ with parameter k and β, $y(0) = y_0$

β ← Bifurcation parameter

$y(0) = y_0$ ← initial condition

If $k>0$, then increasing for $0<y<N$ & decreasing for $y<0$ & $y>N$

If $k<0$, then decreasing for $0<y<N$ increasing for $y<0$ & $y>N$

→ Mixing model:

$$\underbrace{\frac{dS}{dt}}_{\text{rate of change salt amount}} = \underbrace{\mu}_{\text{flow-in rate}} - \underbrace{\left(\frac{\beta S}{V_0 + Mt}\right)}_{\text{flow-out rate with changing or changing volume}}$$

with parameter μ, β, and M, $S(0) = S_0$, V_0 ← initial volume

$S(0) = S_0$ ← initial condition