NAME(S):

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Worksheet: Calculus Review

The purpose of this worksheet is to review the essential concepts of calculus I and calculus II that are necessary for a successful start in differential equations. The problems are designed to help students practice their skills in limits, derivatives, series, and integrals.

Instructions: Worksheets are graded mostly on completion and partially on correctness. Please write your complete solutions on a separate paper.

Part 1: Differentiation

1. Differentiate the following functions with respect to x:
   (a) $f(x) = 3x^2 - 5x + 2$
   (b) $g(x) = \sqrt{x} + \frac{1}{x}$
2. Find the equation of the tangent line to the curve $y = 2x^3 - x^2 + 3$ at the point where $x = 1$.
3. Determine the critical points of the function $h(x) = x^4 - 8x^2$, and classify each point as a local maximum, local minumum, or saddle point.
4. Compute the second derivative of $y = e^{2x} \sin(x)$.

Part 2: Integration

1. Evaluate the definite integral:

$$\int_{1}^{3} (3x^2 - 2x + 1)dx$$

2. Integrate the following using integration by parts:

$$\int x \ln{(x)} dx$$

3. Compute the indefinite integral:

$$\int \left(2e^x + \frac{1}{x}\right) dx$$

4. Evaluate the improper integral:

$$\int_{1}^{\infty} \frac{1}{x^2} dx$$

Part 3: Series

1. Determine whether the sequence a_n defined by $a_n = \frac{2^n}{n!}$ is convergent or divergent.
2. Find the sum of the geometric series:

$$\sum_{n=0}^{\infty} \frac{1}{3^n}$$

3. Compute the sum of the first n terms of the arithmetic series:

$$4 + 8 + 12 + \cdots$$

4. Determine the interval of convergence for the power series:

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{n}$$

Outstanding Question

1. Consider the integral:

$$I = \int_{-\infty}^{\infty} e^{-x^2} dx$$

Use a Taylor series expansion for e^{-x^2} and calculate the value of the integral I using the first few terms of the series. Compare the result to the known value of $\sqrt{\pi}$. Explain the convergence properties of the series for this integral.