**Unit 4: Linear Differential Equations**

Goals/Rationale

This unit begins with a salty tank modeling problem that leads to a linear differential equation. The modeling context is one in which it makes sense for the rate of change to depend explicitly on time (in comparison to the previous population growth contexts) and requires students to set up the differential equation. After setting up the differential equation, student use the tip to tail Geogebra app to find an approximate amount of salt in the tank after 15 minutes. Doing so provides students with an approximate correct answer and revisits previous ideas so that new ideas are not taught in isolation. The technique for solving linear equations is then developed in the same two column table format as separation of variables. We derive the technique from first principles (e.g., the product rule) and hence refer to the technique as the “reverse product rule.” One of the homework problems provides an opportunity for students to do some web searching to figure out how the reverse product rule as presented in here relates to the integrating factor technique.

**Pages 4.1 – 4.2 – A Salty Tank**

Implementation Notes and Student Thinking

Problem 1 (a) asks students to think about whether the rate of change should depend on the amount of salt S in the tank, time t, or both S and t? This will be challenging for students. Some will argue for just S and others for both S and t. Here are some questions/prompts that can help them deepen their reasoning:

* Could the tank be saltier at the top or bottom of the tank?
* What do you think the value of dS/dt will approach in the long run and why? Should dS/dt approach 1 in the long run? 0? Something else?

Part (b) provides students with a rule of thumb for setting up differential equations that gives students some traction on the problem and a hint to consider what the units are for dS/dt. Students usually figure out that the rate in is 1 lb/gal x 2 gal/min = 2lbs/min, but have a much harder time with the rate out. One problem is that they need to think about the amount of salt in the tank as the unknown. Another challenge is how to write an expression for the number of gallons in the tank.

* Earlier you said that the rate of change should depend on S. So where does this fit into the rate out?
* There are 15 gallons of liquid in the tank to start with. How many gallons are there after 1 minute? After 2 minutes? After t minutes?
* What part of your differential equation reflects the assumption that the solution is well-mixed?

Ultimately students should arrive at dS/dt = 2 – S/(15+t) and then use the Geogebra app to estimate the amount of salt in the tank after 15 minutes.

**Pages 4.3 – 4.6**

The problems on these pages develop a method to solve linear differential equations. To make for slightly easier algebra, students solve dy/dt + 2 = 3y and then returns to the DE for the salty tank.

On page 4.5 problem 4(a) you might also ask students to use an earlier Geogebra app where that gives the slope field and allows the user to enter a function that is thought to be a solution. This is another way that they can check to see if their work to get the general and then particular solution is correct.

Page 4.6 is connected to HW problem 9. 4.6 has students create 3 different initial value problems in class. Then they finish the problem in homework.

Notes on Homework

* Definitely assign problem 6, which introduces students to the Runge-Kutta method, which will be used extensively in systems of differential equations.
* Problem 9 makes for a good whole class discussion. You might consider doing part of this in class and assigning the rest for homework.

**Notes for Personal Reflections on Unit 4**