Name:

Collaborators:

Instructions: Worksheets are graded mostly on completion, and partially on correctness. Please write complete solutions showing explanations and key steps to the following problems, unless it says otherwise.

Testing for Linear Independence

This worksheet has a supplementary Jupyter Notebook (Python) in Posit Cloud. Please log-in into this course's Posit Cloud Workspace and navigate to the appropriate environment.

1. Spanning Subsets

A set of vectors $\{\vec{v}_1, \vec{v}_2, \cdots \vec{v}_k\}$ in a vector space V is said to be:

- *Spanning* if for all $\vec{v} \in V$, there exist $c_1, c_2, \cdots c_k \in \mathbb{R}$ so that $\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_k\vec{v}_k$ or $\text{Span}(\{\vec{v}_1, \vec{v}_2, \cdots \vec{v}_k\}) = \vec{v}$.
- *Linearly independent* if and only if the solution to $c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_k\vec{v}_k = \vec{0}$ is $c_1 = c_2, \cdots, c_k = 0$.

For each set of vectors, determine whether they are linearly independent or dependent, and determine if the set spans the given vector space.

a. $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}$ in $\mathbb{R}^2$

b. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \\ 3 \end{bmatrix} \right\}$ in $\mathbb{R}^3$

2. A Questionable Basis

A set that is both spanning and linearly independent forms a *basis* of the vector space V: $\text{Span}\left(\{\vec{v}_1, \vec{v}_2, \cdots \vec{v}_k\}\right) = V$ and $\{\vec{v}_1, \vec{v}_2, \cdots \vec{v}_k\}$ is linearly independent.

Consider the set of vectors

$$S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right\}.$$

a. Show that S is linearly dependent.

b. Explain why S is not a basis for $\mathbb{R}^3$.

c. Change one of the vectors in S to form a basis for $\mathbb{R}^3$. Explain your reasoning.

d. Describe the differences of the set vectors in S and the new set of vectors in Part (c) on how they would look like in $\mathbb{R}^3$.