Name:

Collaborators:

Instructions: Worksheets are graded mostly on completion, and partially on correctness. Please write complete solutions showing explanations and key steps to the following problems, unless it says otherwise.

Row Echelon Form

1. Solution Types and Echelon Forms

Consider the system of m linear equations with n variables x_1, x_2, $\cdots$, x_n in general form:

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m
\end{aligned}
\quad \xrightarrow{\text{augmented matrix}} \quad
\left[
\begin{array}{cccc|c}
a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\
a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn} & b_m
\end{array}
\right]
$$

where a_{ij} and b_i are constants for $i \in \{1, 2, \cdots, m\}$ and $j \in \{1, 2, \cdots, n\}$.

The goal of Gussian elimination is to reach *row echelon form* (REF) of the augmented matrix, a simplified version of the matrix where each nonzero row starts with a leading entry (called a *pivot*) to the right of the one above it, and all rows of zeros are at the bottom.

For each of the following augmented matrices already in REF:

- Identify the number of equations m and variables n.
- Classify if the system is overdetermined, underdetermined, or neither.
- Determine if the system has a unique solution, infinite solutions, or no solution.
- If the the system have infinite solutions, list the free variables.

a) $\left[\begin{array}{cccc|c} 1 & -3 & 2 & -1 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$

b) $\left[\begin{array}{ccc|c} 1 & -3 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

2. More Examples

For each of the following augmented matrices already in *row echelon form*:

- Identify the number of equations m and variables n.
- Classify if the system is overdetermined, underdetermined, or neither.
- Determine if the system has a unique solution, infinite solutions, or no solution.
- If the the system have infinite solutions, list the free variables.

a) $\begin{bmatrix} 1 & 2 & | & 5 \\ 0 & 1 & | & 3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & | & 3 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 2 & | & 4 \\ 0 & 0 & 1 & | & 3 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & -1 & | & 3 \\ 0 & 0 & | & 1 \\ 0 & 0 & | & 0 \end{bmatrix}$

e) $\begin{bmatrix} 1 & 4 & 0 & | & 2 \\ 0 & 1 & -5 & | & -1 \\ 0 & 0 & 1 & | & -1 \end{bmatrix}$

f) $\begin{bmatrix} 1 & 0 & 3 & | & 2 \\ 0 & 1 & -5 & | & 1 \end{bmatrix}$

g) $\begin{bmatrix} 0 & 1 & 3 & | & 1 \\ 0 & 0 & 1 & | & 2 \end{bmatrix}$

h) $\begin{bmatrix} 1 & 3 & 5 & 2 & | & 1 \\ 0 & 0 & 1 & 2 & | & 3 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$