Dimension

Objectives

- 1. Define homogeneous and non-homogeneous systems.
- 2. Define "dimensions" of a vector space.
- 3. Define the "dimensions" of 2 solution set.

Recall: Watrix-Vector Form of Linear Equations

A linear system of m equations and n variable,

$$\partial_{11} \times_1 + \partial_{12} \times_2 + \cdots + \partial_{1N} \times_N = b_1$$

 $\partial_{21} \times_1 + \partial_{22} \times_2 + \cdots + \partial_{2N} \times_N = b_2$
 \vdots
 $\partial_{m_1} \times_1 + \partial_{m_2} \times_2 + \cdots + \partial_{m_N} \times_N = b_m$

can be uritten in <u>matrix-vector</u> form:

$$\begin{bmatrix} \partial_{11} & \partial_{12} & \cdots & \partial_{1N} \\ \partial_{21} & \partial_{22} & \cdots & \partial_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \partial_{m1} & \partial_{m2} & \cdots & \partial_{mN} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_M \end{bmatrix}$$

- · A is size mxn
- · x is size ux1
- . B is size mx1

Homogeneurs vs Non-homogeneurs Equations

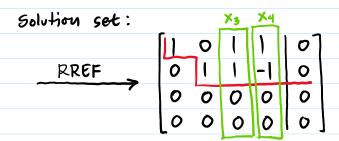
Given a linear equation AX = 5,

- · if b = o, then Ax = o is a homogeneous system.
- · if b ≠ o, then A x = b is a non-homogeneous system.

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} A \\ 4x4 \end{bmatrix} \begin{bmatrix} A \\ 4x1 \end{bmatrix} \begin{bmatrix} A \\ Ax1 \end{bmatrix} \begin{bmatrix} A \\ Ax1 \end{bmatrix} \begin{bmatrix} A \\ Ax1 \end{bmatrix}$$

Here, $\vec{b} = \vec{0}$, so, a homogeneous system.



xy \$ x4 are free variables

Let
$$x_3 = 6$$
 and $x_4 = t$.

the \overrightarrow{o} vector (always for homo equations)

 $x_1 + 6 + t = 0$
 $x_2 + 6 - t = 0$
 $x_3 = 6$
 $x_4 = 6$
 x_4

So, the basis of the solution set is $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$ which is also in the subspace of \mathbb{R}^q of $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$, since it passes through the origin.

the dimension of this solution set is 2 since there are two vectors in the solution set basis.

Vector Space Dimensions

Vector Space Dimensions

The dimension of a vector space is the number of vectors in its basis.

* Examples: 1. TR' has one dimension, only one vectors in its basis
2. TR2 has two dimension, only two vectors in its basis

3. 183 has three dimension, only three vectors in its basis

Solution Set Dimensions

the dimensions of a solution set is the number of vectors in its basis, which is always less tuan the dimension of the linear system.

* Example: Given a linear system AZ= I in 12 and its solution set with 2 basis vectors, then the dimension of the solution set is 2.