Name:

Collaborators:

Instructions: Worksheets are graded mostly on completion, and partially on correctness. Please write complete solutions showing explanations and key steps to the following problems, unless it says otherwise.


Changing the Basis

1. Coordinate Transformation

Changing basis means a transformation of a vector's coordinates relative to a different basis of the same vector space. If $B = \{\vec{b}_1, \vec{b}_2, \cdots, \vec{b}_k\}$ is a basis and $\vec{v}$ is in standard form in $\mathbb{R}^2$, its coordinates relative to B are found by solving $\vec{v} = c_1\vec{b}_1 + c_2\vec{b}_2 + \cdots + c_k\vec{b}_k$.

These coefficients form $\text{Rep}_B(\vec{v}) = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_k \end{bmatrix}_B$, the representation of $\vec{v}$ with respect to B.

a. Draw the standard basis basis vectors $\vec{e}_1$ and $\vec{e}_2$ in $\mathbb{R}^2$ using the blank axis below. Then, draw the vector $\vec{v} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$.
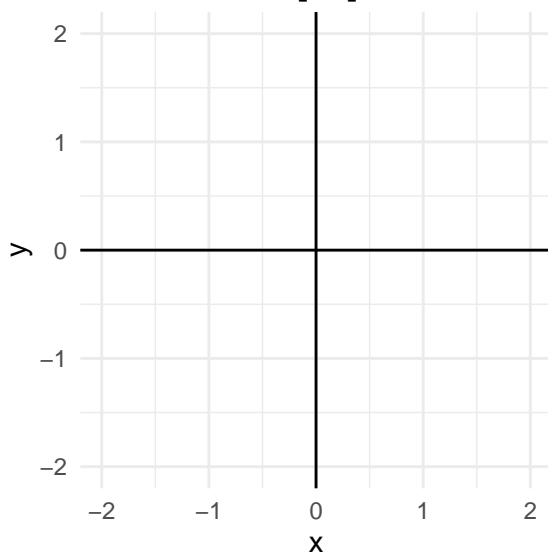
b. Let $B = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$ be a basis in $\mathbb{R}^2$. Determine $\text{Rep}_B(\vec{v})$. Then, draw the vectors in B and $\text{Rep}_B(\vec{v})$ on the blank axis below.
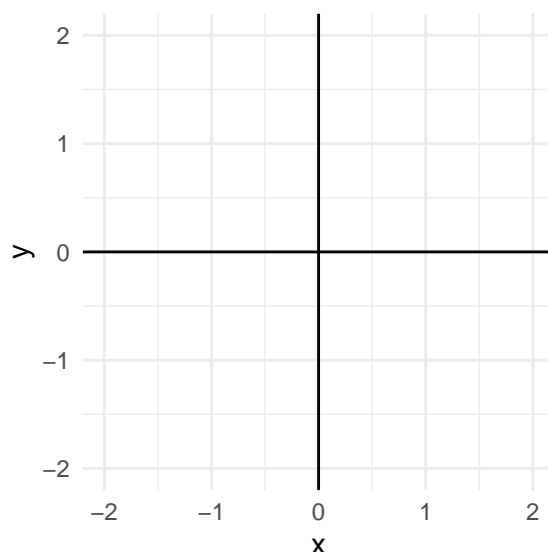
c. Describe your observations between the two basis vectors and the vectors $\vec{v}$ and $\text{Rep}_B(\vec{v})$. Did they stretch, rotate, or both?
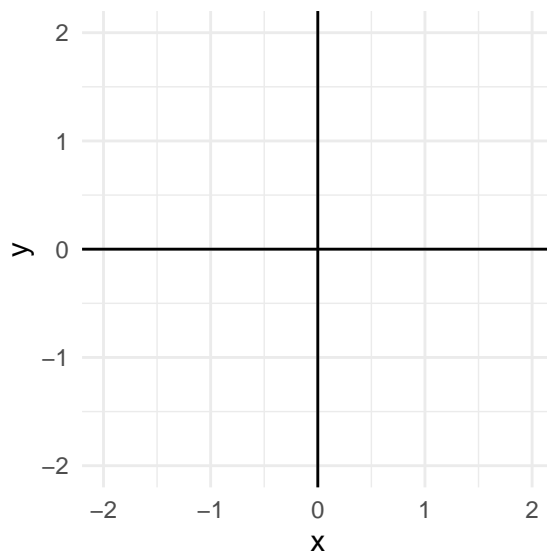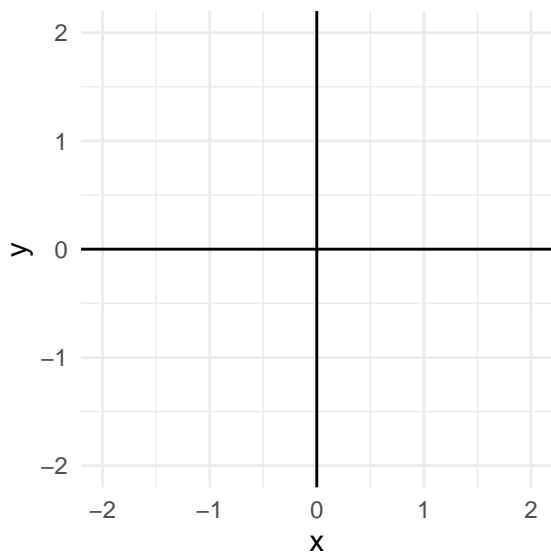
2. Create Your Own Basis

Recall that a set of vectors is a basis when they are both linearly independent and spans the vector space.

Let $B = \left\{ \begin{bmatrix} a \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ b \end{bmatrix} \right\}$ be a set of basis vectors in $\mathbb{R}^2$, where a and b are some real numbers.

  a. Determine values a and b so that given the vector $\vec{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ transforms into $\text{Rep}_B(\vec{v}) = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$.

  b. Confirm that the values a and b determined in Part (a) forms a basis B.

  c. Separately draw the vectors $\vec{v}$ and $\text{Rep}_B(\vec{v})$ and their corresponding basis on the blank axis below.

  e. Describe your observations between the two basis vectors and the vectors $\vec{v}$ and $\text{Rep}_B(\vec{v})$. Did they stretch, rotate, or both?