Name:

Collaborators:

Instructions: Worksheets are graded mostly on completion, and partially on correctness. Please write complete solutions showing explanations and key steps to the following problems, unless it says otherwise.

Exploring Spans and Subspaces

1. Intersecting Planes in Three Dimensions

The system $\begin{aligned} x+y+z &= 1 \\ x+y &= 2 \end{aligned}$ has a solution set S.

a. Show $S = \left\{ \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} : t \in \mathbb{R} \right\}$.

b. Find the subspace parallel to S and give a basis and its dimension.

c. Provide an explanation on how the intersection of the two planes look like in $\mathbb{R}^3$ and the basis.

d. Is S a subspace of $\mathbb{R}^3$?

2. Intersecting Hyperplanes in Four Dimensions

The system $\begin{aligned} x_1 - 2x_2 + x_3 + x_4 &= -1 \\ x_1 - x_2 + 2x_3 - x_4 &= 1 \end{aligned}$ has a solution set S.

a. Show $S = \left\{ \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -3 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \end{bmatrix} : s, t \in \mathbb{R} \right\}$.

b. Give a basis and the dimension of the subspace parallel to S.

c. Provide an explanation on how the intersection of the two planes look like in $\mathbb{R}^4$ and the basis.

d. Decide whether S is a subspace of $\mathbb{R}^4$.