Name: Solutions

Collaborators:

Instructions:

> You must submit your worksheet individually by end-of-class or end-of-day. Your name must exist in your worksheet and the names of your collaborators.

Worksheets are marked mostly on completion, and partially on correctness. It will be marked either pass or fail, there will no detailed feedback on worksheets, and no opportunities for revisions and make-up.

Probabilities vs Sample Proportions

1. Binomial Probability with Dice

Ten fair six-sided die is rolled simultaneously. Let X be the number of times a 4 appears.

a. Determine the appropriate probability distribution for X.

The random variable X follows a Binomial distribution, or $X \sim \text{Binom}\,(n, p)$, as it counts the number of "success" outcomes with success probability p in a finite number of trials n. Here, a "success" is rolling a "4" on a six-sided die, with probability $p = \frac{1}{6}$, over $n = 10$ rolls. Thus, $X \sim \text{Binom}\left(10, \frac{1}{6}\right)$.

b. Find the probability that exactly three 4s appear.

$$\begin{aligned} P(X = 3) &= \binom{10}{3}\left(\frac{1}{6}\right)^3\left(1 - \frac{1}{6}\right)^{10-3} \\ &= 120\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^7 \\ P(X = 3) &\approx 0.1550 \end{aligned}$$

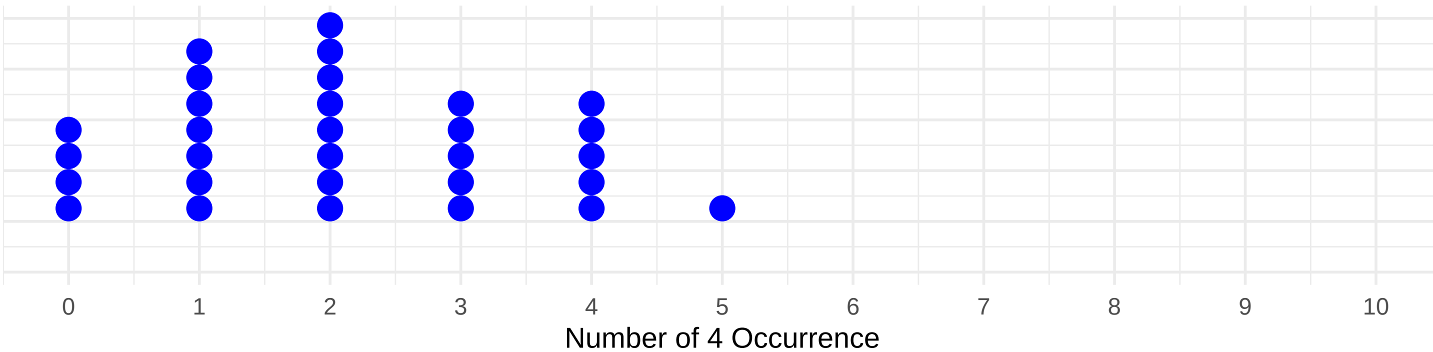
c. Find the probability that at most two 4s appear.

$$\begin{aligned} P(X \leq 2) &= \sum_{k=0}^{2} P(X = k) \\ &= \sum_{k=0}^{2}\binom{10}{k}\left(\frac{1}{6}\right)^k\left(1 - \frac{1}{6}\right)^{10-k} \\ &= \sum_{k=0}^{2}\binom{10}{k}\left(\frac{1}{6}\right)^k\left(\frac{5}{6}\right)^{10-k} \\ &= 1\left(\frac{5}{6}\right)^{10} + 10\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^9 + 45\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^8 \\ P(X \leq 2) &\approx 0.7752 \end{aligned}$$

2. Estimate Probabilities

a. Data Collection

- Work in pairs. Roll 10 six-sided dice 30 times.
- For each roll, count the number of times a 4 appears. This will be your "success" outcomes.
- Record each count as a dot on the blank axis below, stacking dots above each number value to create a dot plot. As you progress, a distribution will appear.



b. Compute the relative frequency (or proportion) of obtaining three 4s. That is the frequency of three 4 outcomes out of the number of rolls. Note that rolling 10 dice simultaneously is considered 1 roll. Then, compare this proportion to the probability computed in Problem (1b).

Answers may vary. Based on the samples, there are 4 three 4s outcomes out of 30 rolls. So, the proportion is $\frac{5}{30} \approx 0.1667$. Compared to the results in Problem (1b), the proportion is slightly more than ≈ 0.1550 but close. This is due to sampling variability.

c. Compute the relative frequency (or proportion) of obtaining at most two 4s. Then, compare this proportion to the probability computed in Problem (1c).

Answers may vary. Based on the samples, there are 19 at most two 4s outcomes out of 30 rolls. So, the proportion is $\frac{19}{30} \approx 0.6333$. Compared to the results in Problem (1c), the proportion is slightly less than ≈ 0.7752 but close. This is due to sampling variability.

d. Would you expect the proportions in Parts (b) and (c) to be closer to the probabilities in Problems (1b) and (1c) if you increased the number of rolls? Explain your reasoning.

As we increase the number of roll, we would expect the proportions to get closer and closer to the probabilities due to the law of large numbers.

Extra Credit: If you rolled where you have all 4s in 10 dice (meaning you have 10 "success" occurrences in all 10 dice, then a +1 will be added to your worksheet totals.

References

1. Speegle, Darrin and Clair, Bryan (2021) Probability, statistics, and data: A fresh approach using r, Chapman; Hall/CRC.
2. Diez DM, Barr CD, Çetinkaya-Rundel M (2012) OpenIntro statistics, OpenIntro.