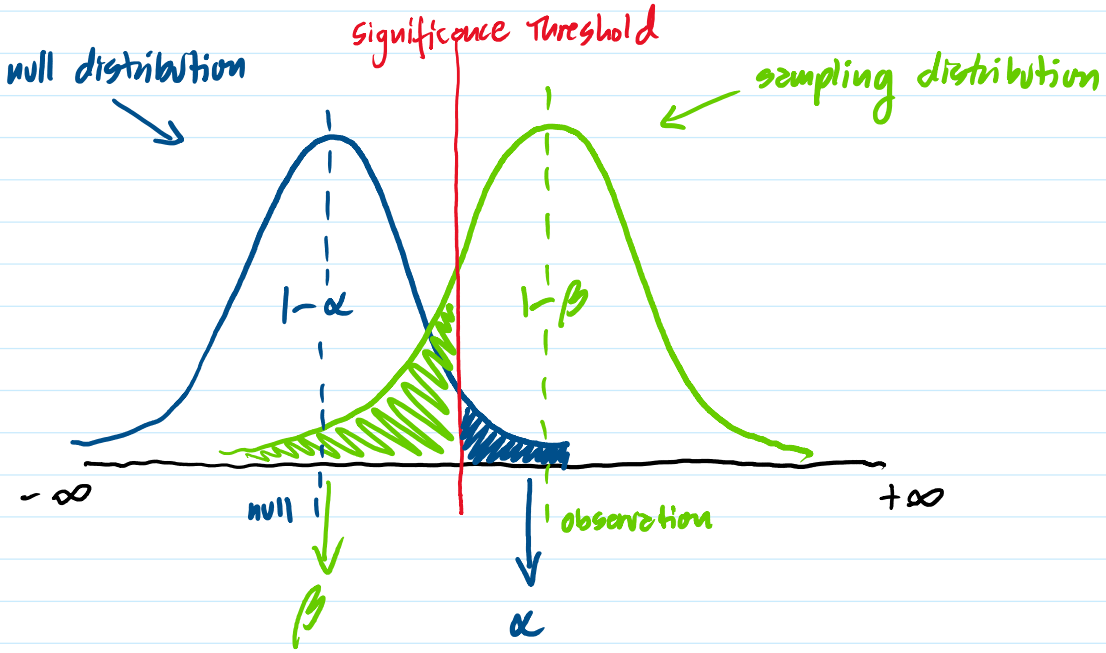
Power Calculations

Wednesday, April 5, 2023

Previously...

Decision Errors

- the null distribution and sampling distribution



- Type I and Type II errors Table:

		reality: H_0 is true	reality: H_0 is false
conclusions given observations	Reject H_0	Type I error (false positive) α	Correct Decision (true positive) $1-\beta$ power → statistical power or power of the test or How good your test is.
	Fail to Reject H_0	Correct Decision (true negative) $1-\alpha$ confidence → confidence level or how confident you are.	Type II error (false negative) β

- 2-sample t-test

standard error: $SE = \sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}$

where s_A & s_B are the sample st. dev. of groups A & B respectively.
and n_A & n_B are the sample sizes of groups A & B respectively.

degrees of freedom: $df = \min(\{n_A - 1, n_B - 1\})$

Compute the power for a 2-sample test

Example:

$n_A = 100$, $n_B = 100$ → sample sizes

$\bar{x}_A - \bar{x}_B = -3$ → Sample difference (point-estimate)

$s_A = 12$, $s_B = 12$ } sample st. devs. → $SE = \sqrt{\frac{12^2}{100} + \frac{12^2}{100}} \approx 1.6971$

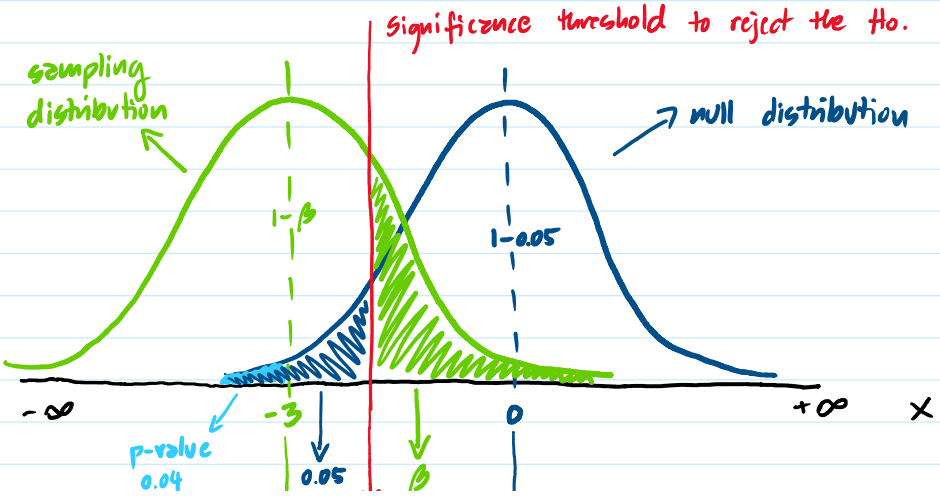
H_0: $\mu_A = \mu_B$ or $\mu_A - \mu_B = 0$

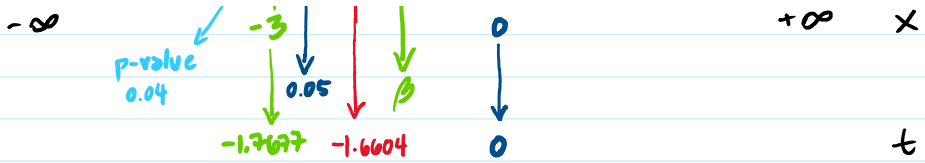
H_A: $\mu_A < \mu_B$ or $\mu_A - \mu_B < 0$

↓

Here we are doing a one-sided test

Set the significance value to be $\alpha = 0.05$.
So, the goal here is to find the power $1-\beta$, which requires us to find β.





Using the null distribution:

$$t_\alpha = qt(0.05, 99) \approx -1.6604$$

$\downarrow$

$df = n-1$

$$t_{\bar{x}_A - \bar{x}_B} = \frac{(\bar{x}_A - \bar{x}_B) - 0}{SE} = \frac{-3}{1.6971} = -1.7677$$

$\rightarrow$ p-value $= pt(-1.7677, 99) \approx 0.04009603 < 0.05$ ✓

Using the sampling distribution:

$$t_{samp} = t_\alpha - t_{\bar{x}_A - \bar{x}_B}$$
$$= -1.6604 - (-1.7677)$$
$$t_{samp} \approx 0.1073$$

$\rightarrow$ distance from the point estimate to the significance threshold

$$\beta = pt(t_{samp}, df)$$
$$= pt(0.1073, 99)$$
$$\approx 0.5426$$

So, power is $1 - \beta = 1 - 0.5426 \approx 0.4574$ or 45.74%

Determining the sample size given a set power level

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