Statistical Analyses of Personal and Economic Freedom from the Human Freedom Index

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Introduction

The number of nations that identify as democracies has fluctuated over the past few centuries. Most new democracies have maintained their pledge to vote by majority rule, but other freedoms have fallen to the wayside. “As freedom declines, so does the quality of democracy.” The Human Freedom Index creates a direct measure of freedom as a composite of economic and personal freedoms.

The Human Freedom Index of 2016 is a global index of personal, economic, and civil freedoms sampled from 2008-2016. The dataset that we picked originally had 123 variables made up of variables from the economic freedom index and the personal freedom index. The index cites that up to their 2015 report, participants included 100,000 citizens and 102 countries. Original data stems from a multitude of sources which cannot be traced back to original samples, and thus the methods are unknown to us.
Our data frame includes the following variables that explore legal, economic, and personal identity based components of the Human Freedom Index:

Variable	Type of Variable	Description
ef_legal_judicial	Continuous numerical	Measurement of the right to life and security, due process of law, and privacy. (0= no adherence to rule of law, 1 = full adherence to rule of law)
ef_legal_courts	Continuous numerical	Impartial Courts
ef_legal_protection	Continuous numerical	Protection of property rights
ef_legal_military	Continuous numerical	Military interference in rule of law and politics
ef_legal_integrity	Continuous numerical	Integrity of the legal system
ef_legal_enforcement	Continuous numerical	Legal enforcement of contracts
ef_legal_restrictions	Continuous numerical	Regulatory restrictions on the sale of real property
ef_legal_police	Continuous numerical	Reliability of police
ef_legal_crime	Continuous numerical	Measurement of whether criminal justice is redressing grievances and bringing action against individuals for offenses committed. (0= no adherence to rule of law, 1 = full adherence to rule of law)

ef_legal_gender	Continuous numerical	Gender adjustment
ef_legal	Continuous numerical	Legal system and property rights - average of all legal subcategories
pf_identity_parental_marriage	Continuous numerical	Whether women and men have the same right to be the legal guardian of a child during marriage
pf_identity_parental_divorce	Continuous numerical	Whether women and men have the same right to be the legal guardian of and have cus- tody rights over a child after divorce
pf_identity_sex_male	Ordinal Categorical	The extent to which male-to-male sexual relationships are legal
pf_identity_sex_female	Ordinal Categorical	The extent to which female-to-female sexual relationships are legal
pf_identity_divorce	Ordinal Categorical	To what extent women and men have the same rights to initiate divorce
pf_identity	Nominal Categorical	Identity and Relationships - average of all identity subcategories
pf_score	Discrete Numerical	Personal Freedom Score - average of all subcategories of personal freedom
hf_score	Discrete Numerical	Human freedom score - average of all subcategories of personal and economic freedom combined
ef_score	Discrete Numerical	Economic freedom score - average of all subcategories of economic freedom
ef_trade	Ordinal Categorical	Freedom to trade internationally
year	Discrete numerical	The year during which data was collected (2008-2016)
countries	Nominal categorical	The nation from which data was collected (n=140)
region	Nominal categorical	The region from which data was collected (n=10)
ef_trade_movement	Continuous numerical	Controls of the movement of capital and people

ef_trade_tariffs	Continuous numerical	Tariffs
pf_ss_disappearances_fatalities	Continuous numerical	Terrorism fatalities

**The human freedom index sourced their data from other parties. Due to this information being obscured, sampling methods for the data are not clear for all of the variables. Each variable is scored by the writers of the Human Freedom Index and the Economic Freedom of the World index on a" 0-10 scale, with 10 representing the most freedom." The main variables (ie. ef_score, pf_score, hf_score) are the averages of each subcategory. It is not disclosed in the document how they determined the individual score for each subcategory.

Over the last 4 years on record in the HFI, there has been a decline in human freedom. Even in countries experiencing an increase in personal freedom, economic freedom has decreased. Populist politicians have targeted minorities and call for rolling back trade and other economic freedoms. They support increasing police powers and state intrusions which threaten personal freedom. Thus, we have decided to ask: ***How strongly is legal economic freedom correlated with freedom to trade internationally? How strong is the relationship between human freedom and economic freedom? How strongly is legal economic freedom correlated with legal gender expression/identity? How do economic freedoms differ from personal freedoms on average? Do economic freedom and personal freedom have a linear relationship?*** By asking these questions we will gain information about within group relationships (economic freedom to economic freedom) and between group relationships (economic freedom to personal freedom), and how these relationships contribute to overall human freedom score.

Figure 1: Economic Freedom and Personal Freedom across all regions in 2016

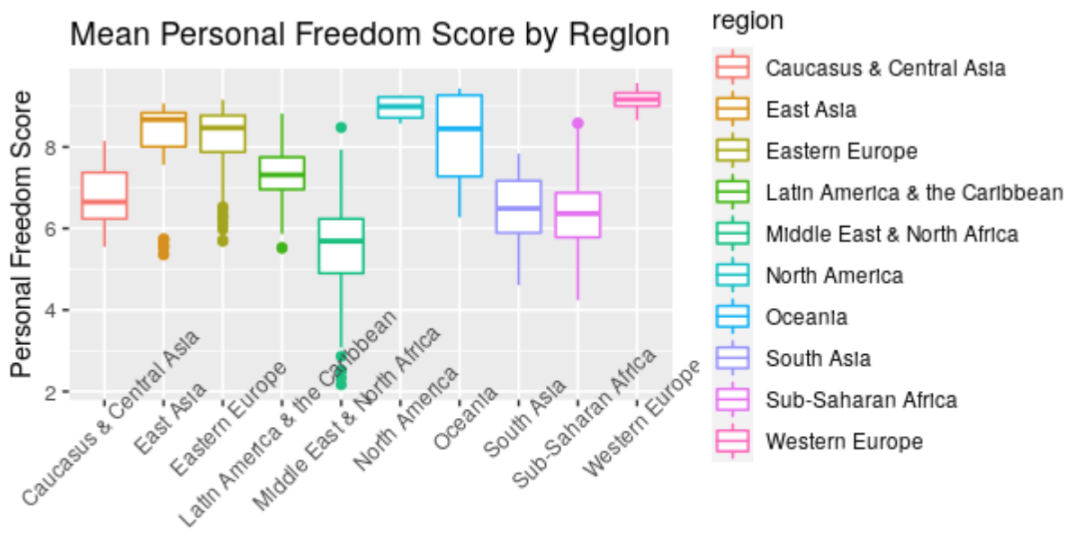


Figure 2: Mean personal freedom score by region

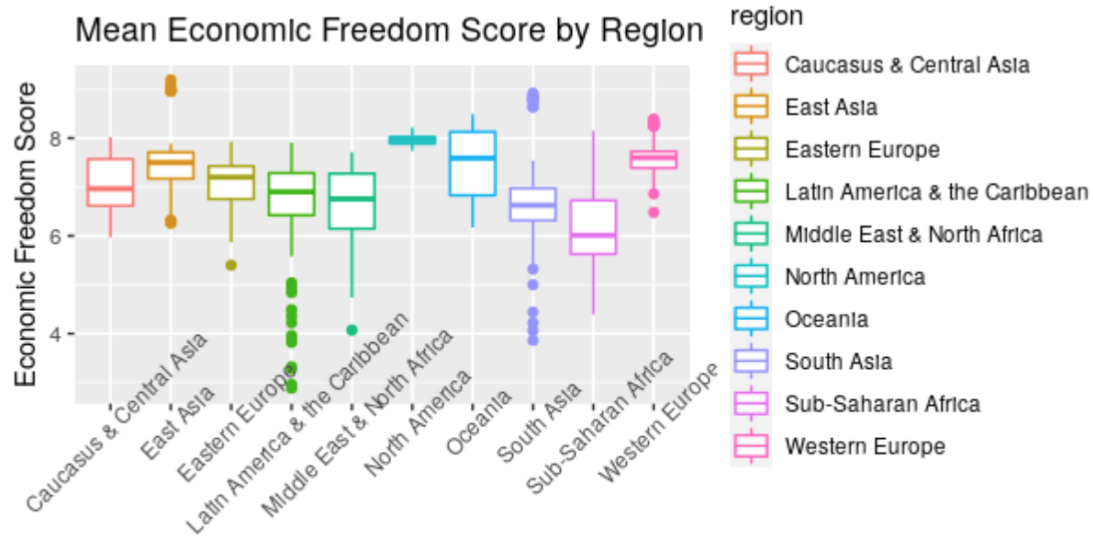


Figure 3: Mean economic freedom score by region

By looking at the relationship and differences between economic freedom scores and personal freedom scores across the ten sampled regions, it appears that some regions have a stronger correlation between the freedom scores than others. This led to the questions: Do economic and personal freedom scores within a region differ significantly on average? Do economic freedom and personal freedom scores within a region have a significant linear relationship?

Another area of exploration was into the variables related to legal systems and rule of law. The law and rights under the law ensure that a society is not just able to secure individual freedom but also expand on that freedom (Human Freedom Index, 11). As the index notes there has been a decrease in economic freedom scores so this was a key area of exploration.

The variable ef_legal_courts scores the country on whether its legal court system is considered Impartial. Essentially, whether citizens of a country can use their civil and criminal justice system effectively without corruption. Additionally, the variable ef_legal_gender scores countries on whether gender adjustment is legal and evaluates the different barriers men and women might face in achieving economic freedom. Both ef_legal_courts and ef_legal_gender are two of the subcategories that were averaged to create the ef_legal score a country gets.

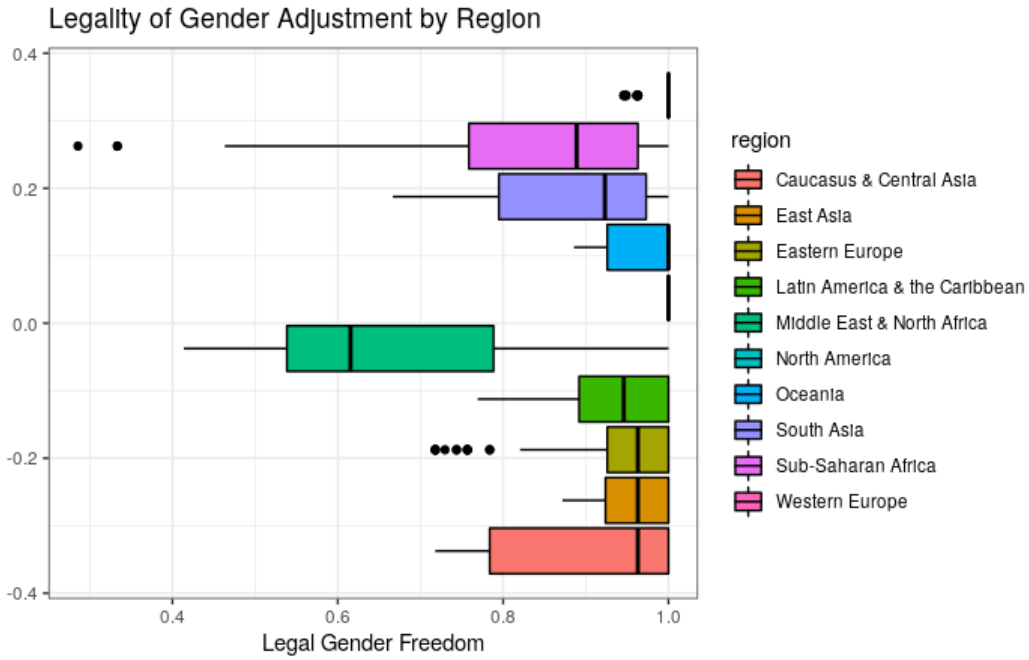


Figure 4: Legality of gender adjustment by region

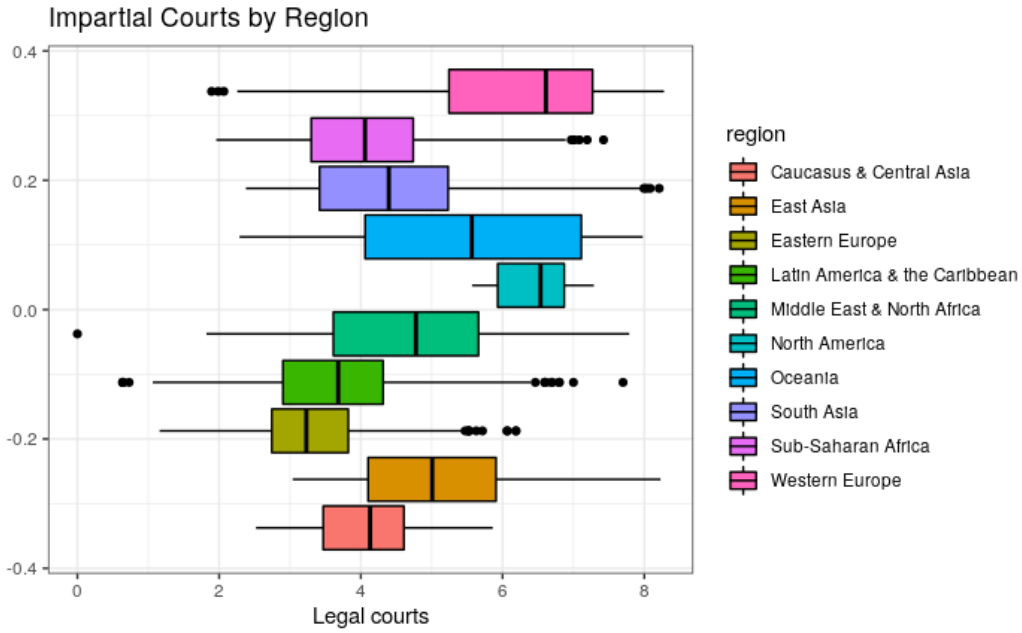


Figure 5: Imperial courts by region

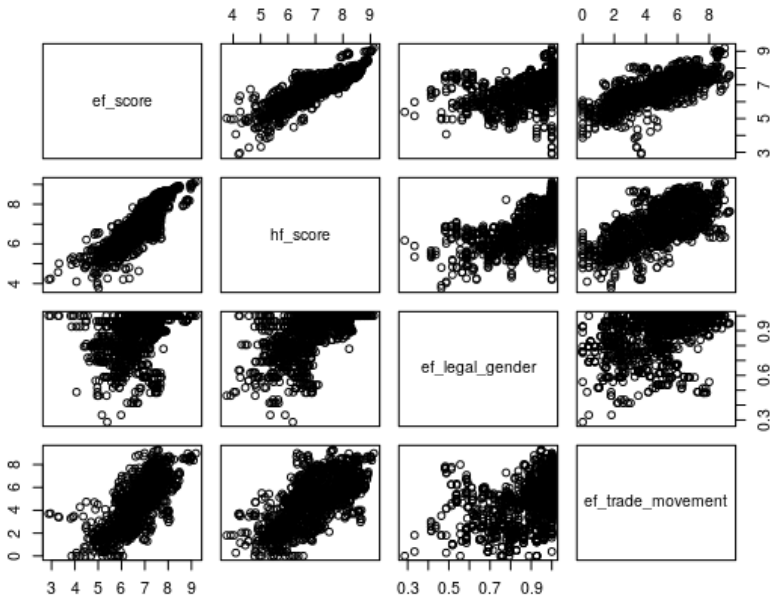


Figure 6: Scatterplot of economic freedom score, human freedom score, legal gender score, and trade movement score.

This plot shows the relationship between the ef score, hf score, legal gender and trade movement variables. Most of these have a positive relationship indicated however the most clear is that of the ef score and hf score.

Our Questions and the Associated Hypothesis:

Do economic freedom and personal freedom differ significantly within region?

Null hypothesis: $\overline{X}_{diff} = 0$

On average, there is no difference between economic and personal freedom in each region.

Alternative hypothesis: $\overline{X}_{diff} \neq 0$

On average, there is a significant difference between economic and personal freedom in each region.

What is the relationship between a country's economic freedom score (ef_score) and a country's regulations or protections with gender adjustment (ef_legal_gender score)?

Null hypothesis: The slope of the Linear regression line is equal to 0

$H_0: \beta = 0$

Alternative hypothesis: The slope of the linear regression is not 0.

$H_A: \beta \neq 0$

What is the relationship between human freedom and economic freedom?

The Null Hypothesis:
There is no relationship between the economic freedom score and the human freedom score.
$H_0: \beta = 0$
The Alternative Hypothesis:
$H_A: \beta \neq 0$

Does freedom to trade internationally (`ef_trade`) vary significantly with the legality of gay relationships between women (`pf_identity_sex_female`)?

Hypotheses:
H_0: Scores on freedom to trade internationally are independent from scores for legality of gay relationships between women.
H_A: Scores on freedom to trade internationally are vary with scores for legality of gay relationships between women.

Methods

How do economic freedoms differ from personal freedoms on average?

We performed a paired T test to determine if the mean economic freedom score and mean personal freedom score differed significantly from one another within each region. Subsets of the data set were created for each region. Each subset contained two variables: ef_score and pf_score. We created a new column, freedom_diff, to show the difference in scores. Since the data did not meet the conditions for normality (n<30), we performed a random sample of n=100 with replacement from each region's subset. We then calculated the T statistic and p-value for each region, and visualized the theoretical null distribution. We performed a bootstrap simulation to find the 95% confidence interval centered on the point estimate, $\overline{X}_{diff}$. Based on the p-value and the presence of the null value inside the 95% confidence intervals, we were able to determine which regions had significantly different mean economic and personal freedom scores.

What is the relationship between a country's economic freedom score (ef_score) and a country's regulations or protections with gender adjustment (ef_legal_gender score)?

The Human freedom index was filtered to only look at pertinent variables for the explorations. The sample size was examined to make sure the conditions were met. Before each region was separated, the dataset was examined for any overarching trends. Following the initial Linear Regression, the data was filtered by region to be able to run region specific linear regressions to see region specific impact on ef_scores and ef_legal_gender score. The R^2 value was produced by running a summary command on the lm_mod. The test statistic was computed using the following equation: T*=(B0-0)/B1. Finally, the p-value was computed using Rstudio 2*(1-pt(T*,df)). The linear regression equation, R^2 values, test

statistic,and p-values were recorded for each region. The Null hypothesis was H_0: $\beta = 0$, where β,the slope would have to be 0. The Alternative hypothesis was that H_A: $\beta \neq 0$. In the alternative case, the slope would indicate a positive or negative relationship present. Based on the data collected and the subsequent statistical tests, we can conclude if the region shows a linear relationship or whether a different model would best describe the relationship.

How strong is the relationship between human freedom and economic freedom?

We used all the human freedom and economic freedom data to compute a linear regression model approximating the relationship between the two variables in the dataset. In our prior analysis, we had not learned the proper techniques for linear regression hypothesis testing, however, in revision we rephrased our hypotheses and achieved a more accurate conclusion. Our new hypotheses were that the slope of the linear regression would be equal to zero for the null and that it would not be equal to zero for the alternative. Our linear model equation was: $hf\ score = 0.25906 + 0.99245(ef\ score)$. We then calculated a t statistic which was 61.11145. Next, we found our degrees of freedom which was 1456. Then, we calculated our p-value which according to R is 0. This is much smaller than a significance level of 95% and so we can conclude our alternative hypothesis was correct. Next, we decided to construct a confidence interval. In our confidence interval, we found that we could be 95% confident that a one unit increase in economic freedom would result in between a .96 and 1.02 unit increase in human freedom.

Does freedom to trade internationally (`ef_trade`) vary significantly with the legality of gay relationships between women (`pf_identity_sex_female`)?

The `ef_trade` variable is continuous numerical, so a new nominal categorical variable was created using the `within` function to assign a range of the ef_trade values to a category. `ef_trade` ranged from 0 to 10, so scores between 0 and 2.5 were assigned to "1," scores between 2.5 and 5 were assigned to "2," scores between 5 and 7.5 were assigned to "3," and scores between 7.5 and 10 were assigned to "1."

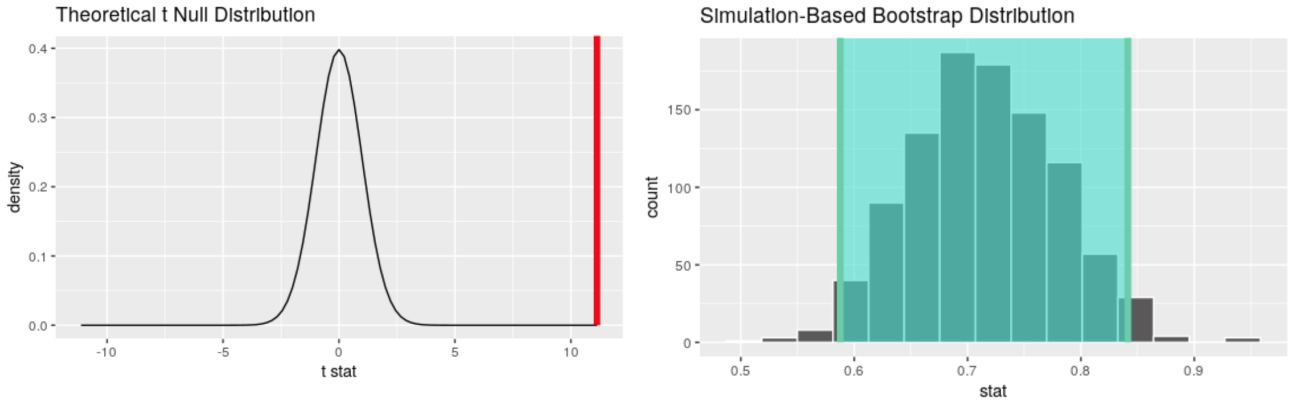
The `pf_identity_sex`_female variable is discrete numerical, with levels 0, 5, and 10. These were converted into strings using the `as.character` function to make a new nominal categorical variable.

With these two categorical variables, Chi Square analyses were done to calculate an χ^2 value and a p-value. The theoretical method was employed because the simulation method yielded p-values of 0. Chi Square tests were performed on both the Human Freedom Index dataset as a whole and also by region.

Results

How do economic freedoms differ from personal freedoms on average? Do economic freedom and personal freedom have a linear relationship?

Eastern Europe

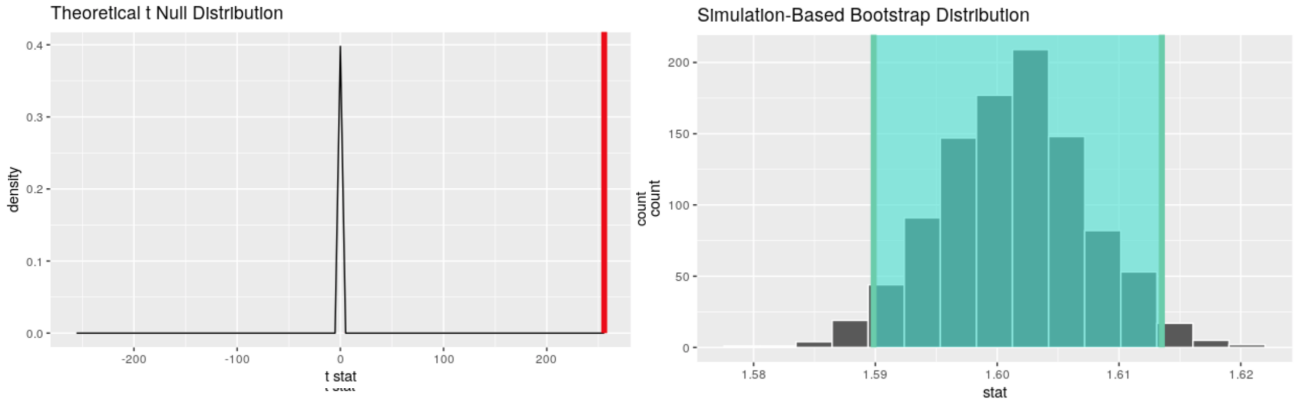


$\overline{X}_{diff} = 0.75, p = 0, 95\% CI = (0.5877062, 0.8414792), T = 11.12344$

Figure 7: Theoretical Null Distribution and Simulation Based Bootstrap Distribution of Mean Differences between Economic and Personal Freedom in Eastern Europe

Based on the p-value less than 0.05, and the fact that the null value, 0, does not lie within the 95% confidence interval, we can reject the null hypothesis and conclude that on average there is a significant difference between economic and personal freedom scores in Eastern European countries.

Western Europe



$\overline{X}_{diff} = 1.601689, p = 0, 95\% CI = (1.589858, 1.61352), T = 255.663$

Figure 8: Theoretical Null Distribution and Simulation Based Bootstrap Distribution of Mean Differences between Economic and Personal Freedom in Western Europe

Based on the p-value less than 0.05, and the fact that the null value, 0, does not lie within the 95% confidence interval, we can reject the null hypothesis and conclude that on average there is a significant difference between economic and personal freedom scores in Western European countries.

Caucasus and Central Asia

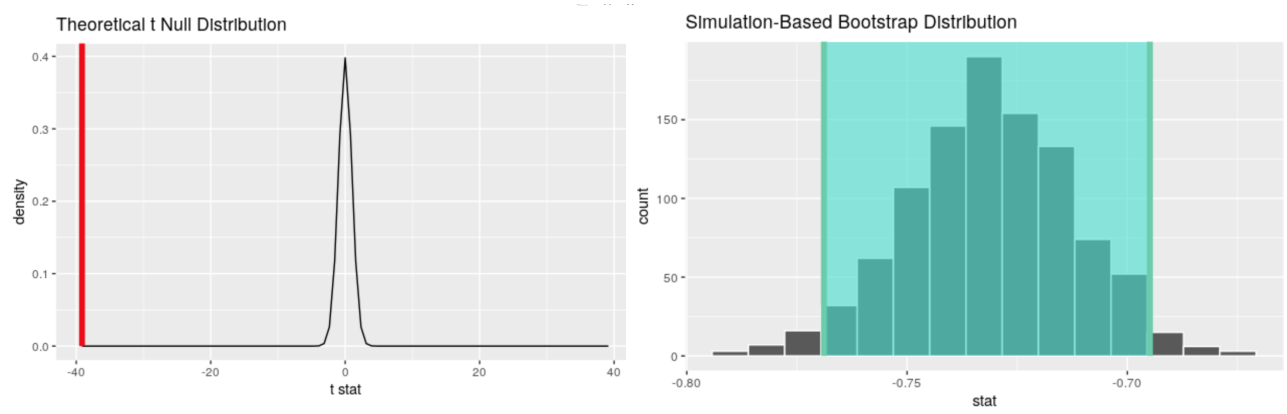


Figure 9: Theoretical Null Distribution and Simulation Based Bootstrap Distribution of Mean Differences between Economic and Personal Freedom in Caucasus and Central Asia $(\overline{X}_{diff} = -0.7318771,$ $p = 4.825798\,x\,10^{-62},\ 95\%\ CI = (-0.7688735, -0.6948807), T = -39.14101).$

Based on the p-value less than 0.05, and the fact that the null value, 0, does not lie within the 95% confidence interval, we can reject the null hypothesis and conclude that on average there is a significant difference between economic and personal freedom scores in Caucasus and Central Asian countries.

East Asia

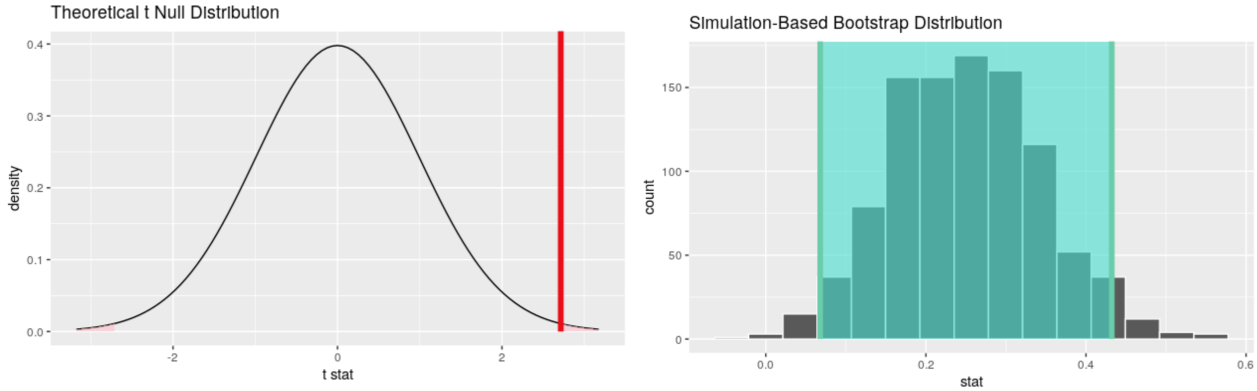


Figure 10: Theoretical Null Distribution and Simulation Based Bootstrap Distribution of Mean Differences between Economic and Personal Freedom in East Asia. $(\overline{X}_{diff} = 0.2501075,$ $p = 0.007830323,\ 95\%\ CI = (0.06795975,\ 0.4322553), T = 2.714556)$

Based on the p-value less than 0.05, and the fact that the null value, 0, does not lie within the 95% confidence interval, we can reject the null hypothesis and conclude that on average there is a significant difference between economic and personal freedom scores in East Asian countries.

South Asia

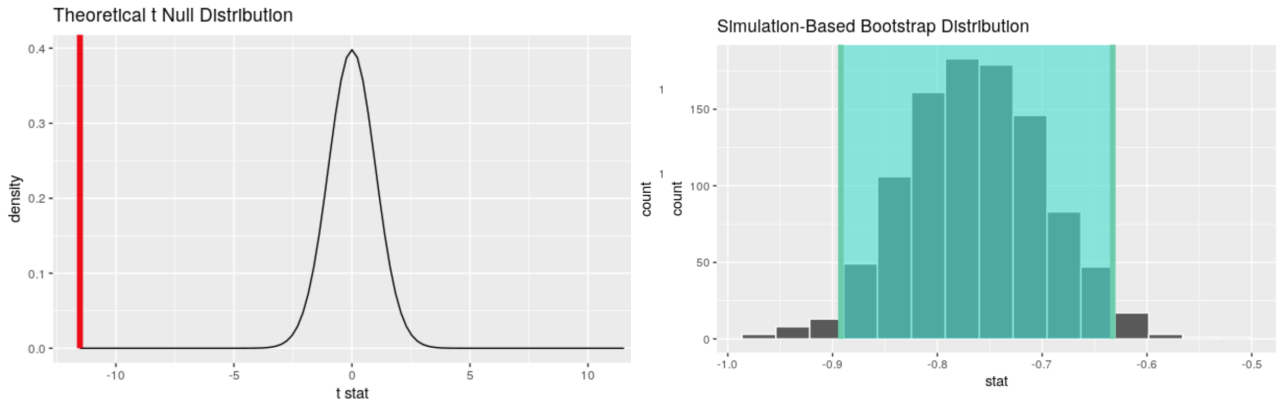


Figure 11: Theoretical Null Distribution and Simulation Based Bootstrap Distribution of Mean Differences between Economic and Personal Freedom in South Asia. ($\overline{X}_{diff} = 0.4992699$, $p = 5.383773e^{-20}$, 95% $CI = (-0.8920749, -0.6326534)$, $T = -11.52415$).

Based on the p-value less than 0.05, and the fact that the null value, 0, does not lie within the 95% confidence interval, we can reject the null hypothesis and conclude that on average there is a significant difference between economic and personal freedom scores in South Asian countries.

Sub Saharan Africa

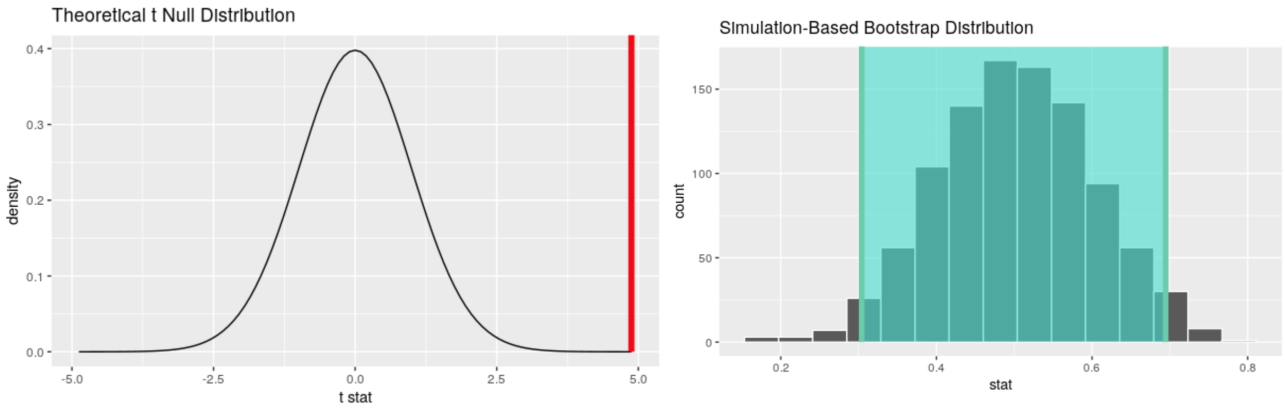


Figure 12: Theoretical Null Distribution and Simulation Based Bootstrap Distribution of Mean Differences between Economic and Personal Freedom in Sub Saharan Africa. ($\overline{X}_{diff} = -0.7623642$, $p = 4.112615e^{-6}$, 95% $CI = (0.3039558, 0.6945841)$, $T = 4.877101$)

Based on the p-value less than 0.05, and the fact that the null value, 0, does not lie within the 95% confidence interval, we can reject the null hypothesis and conclude that on average there is a significant difference between economic and personal freedom scores in Sub Saharan African countries.

North America

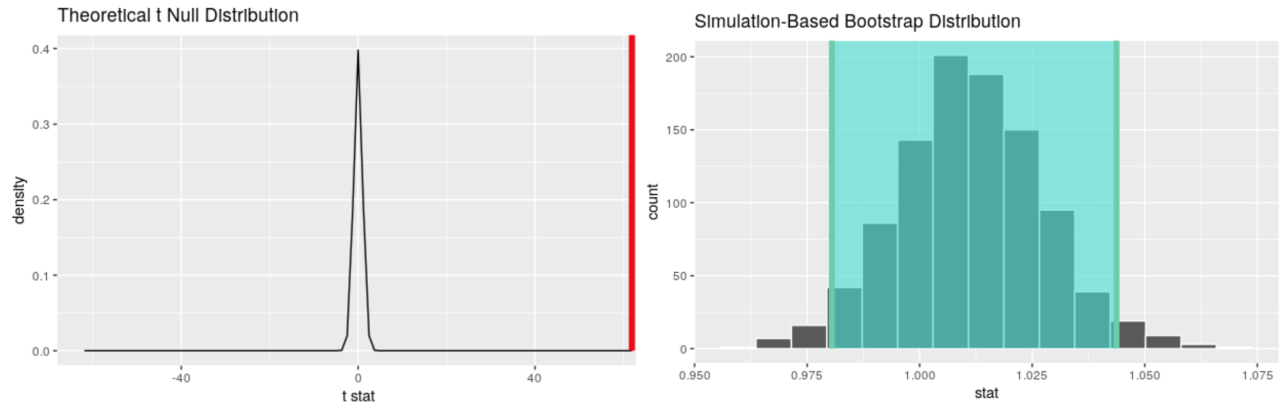


Figure 13: Theoretical Null Distribution and Simulation Based Bootstrap Distribution of Mean Differences between Economic and Personal Freedom in North America. ($\overline{X}_{diff} = 1.012116, p = 0,$ 95% $CI = (0.9805069, 1.043725), T = 61.90213$).

Based on the p-value less than 0.05, and the fact that the null value, 0, does not lie within the 95% confidence interval, we can reject the null hypothesis and conclude that on average there is a significant difference between economic and personal freedom scores in North American countries.

Latin America and the Caribbean

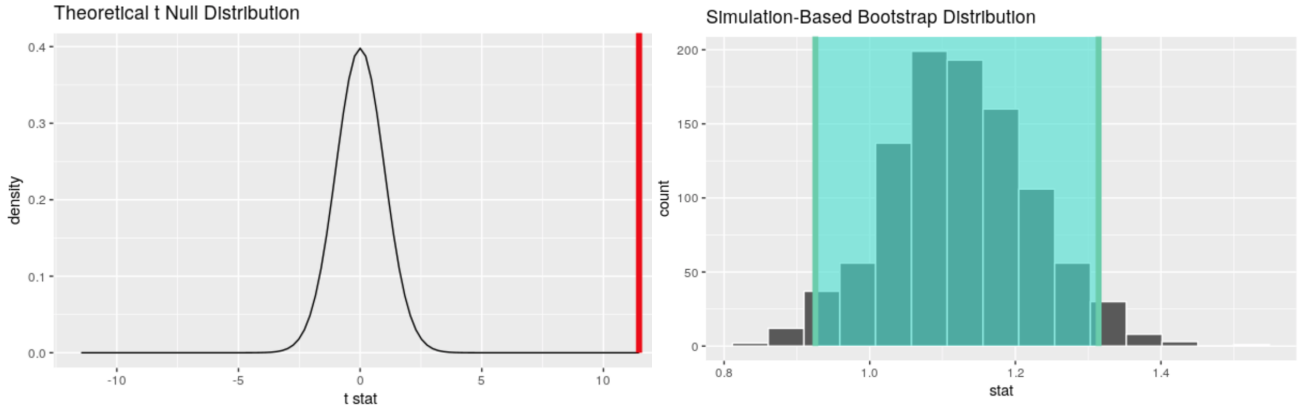


Figure 14: Theoretical Null Distribution and Simulation Based Bootstrap Distribution of Mean Differences between Economic and Personal Freedom in Latin America and the Caribbean. ($\overline{X}_{diff} = 1.119849, p = 0,$ 95% $CI = 0.925346,\ 1.314352), T = 11.46343$).

Based on the p-value less than 0.05, and the fact that the null value, 0, does not lie within the 95% confidence interval, we can reject the null hypothesis and conclude that on average there is a significant difference between economic and personal freedom scores in Latin American and Caribbean countries.

Oceania

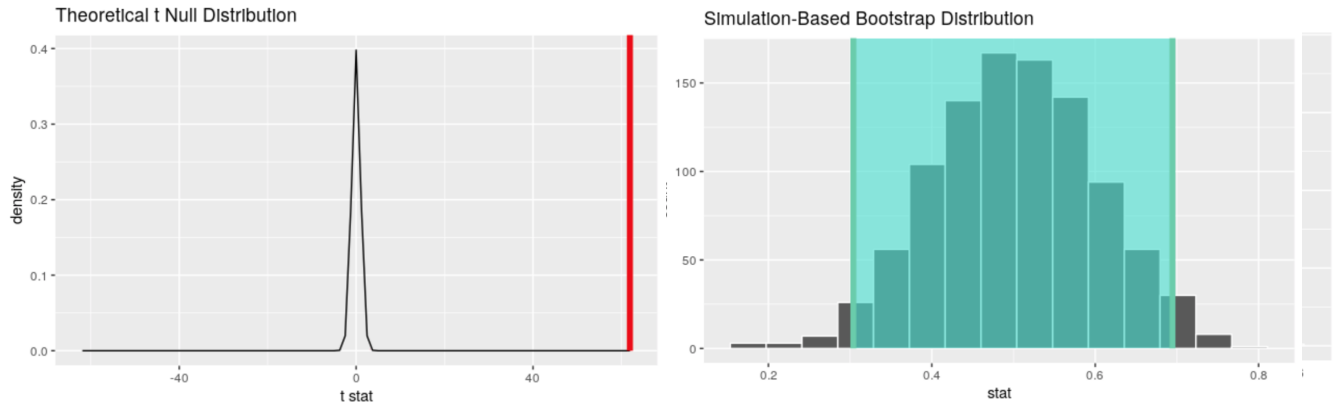


Figure 15: Theoretical Null Distribution and Simulation Based Bootstrap Distribution of Mean Differences between Economic and Personal Freedom in Oceania. ($\overline{X}_{diff} = 1.012284$, $p = 0$, 95% $CI = 0.9841546, 1.040413$), $T = 67.92389$).

Based on the p-value less than 0.05, and the fact that the null value, 0, does not lie within the 95% confidence interval, we can reject the null hypothesis and conclude that on average there is a significant difference between economic and personal freedom scores in Oceania countries.

Middle East and North Africa

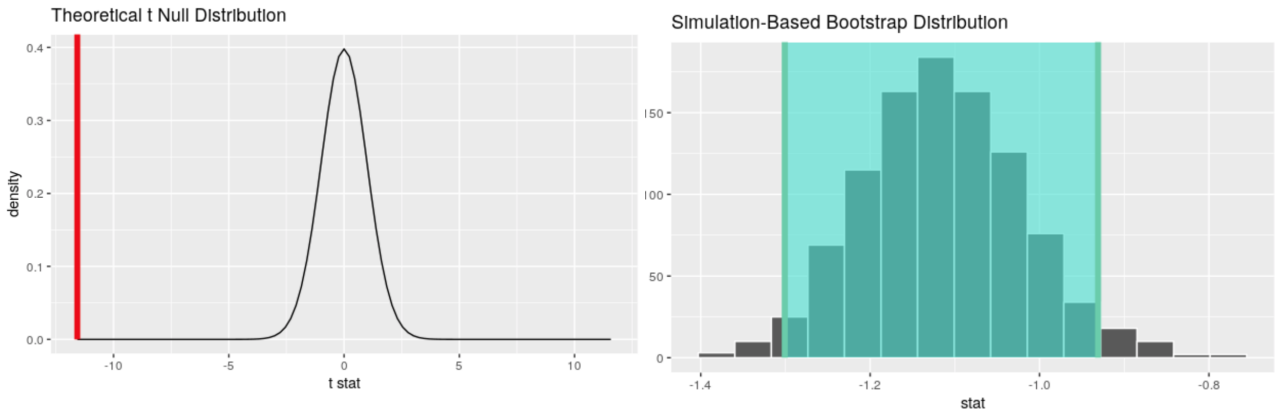


Figure 16: Theoretical Null Distribution and Simulation Based Bootstrap Distribution of Mean Differences between Economic and Personal Freedom in Middle East and North Africa. ($\overline{X}_{diff} = -1.115846$, $p = 4.225833e^{-20}$, 95% $CI = (-1.300777 - 0.9309155)$, $T = -11.57302$).

Based on the p-value less than 0.05, and the fact that the null value, 0, does not lie within the 95% confidence interval, we can reject the null hypothesis and conclude that on average there is a significant

difference between economic and personal freedom scores in Middle Eastern and North African countries.

Table 2:

Region	$\overline{X}_{diff}$	P value	95% CI	T stat
Eastern Europe	0.75	0	$(0.5877062, 0.8414792)$	11.12344
Western Europe	1.601689	0	$(1.589858, 1.61352)$	255.663
Caucasus & Central Asia	-0.7318771	$4.825798\ x\ 10^{-62}$	$(-0.76887, -0.69488)$	-39.14101
East Asia	0.2501075	0.007830323	$(0.06795975,\ 0.43226)$	2.714556
South Asia	0.4992699	$5.383773e^{-20}$	$(-0.89207, -0.63265)$	-11.52415
Sub Saharan Africa	-0.7623642	$4.112615e^{-6}$	$(0.3039558, 0.694584)$	4.877101
North America	1.012116	0	$(0.9805069, 1.043725)$	61.90213
Latin America & Caribbean	1.119849	0	$(0.925346,\ 1.314352)$	11.46343
Oceania	1.119849	0	$(0.9841546, 1.040413)$	67.92389
Middle East & Northern Africa	-1.115846	$4.225833e^{-20}$	$(-1.3008, -0.93092)$	-11.57302

What is the relationship between a country's economic freedom score (ef_score) and a country's regulations or protections with gender adjustment (ef_legal_gender score)

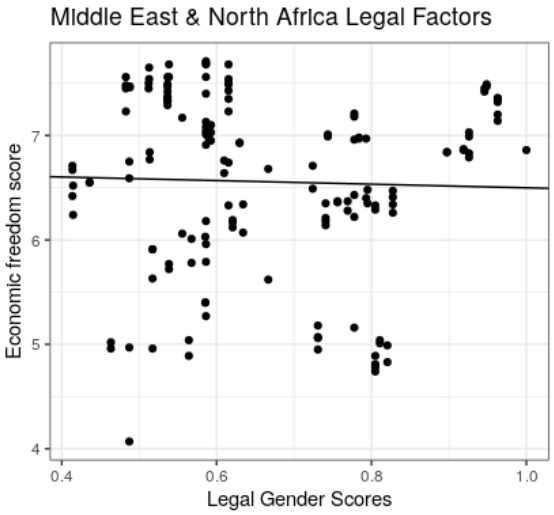


Figure 17: Middle East & North Africa

R-squared:0.0009518
T=-0.3754355
P-value:1.292257

The R^2 is 0.0009518 which is very low and indicates the weak linear relationship between a country's legal gender scores and economic freedom scores. The p-value is above 0.05 meaning that the null can not be rejected.

ef_score=6.6711-0.1724(ef_legal_gender)

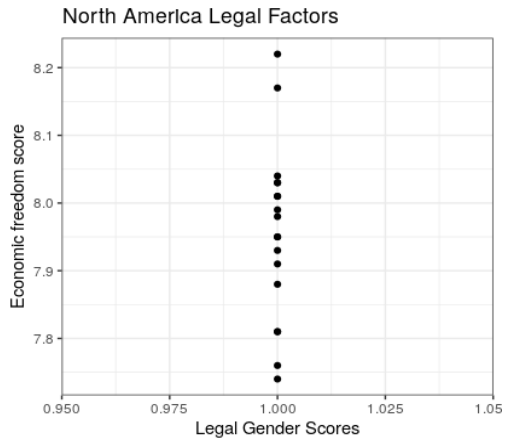


Figure 18: North America

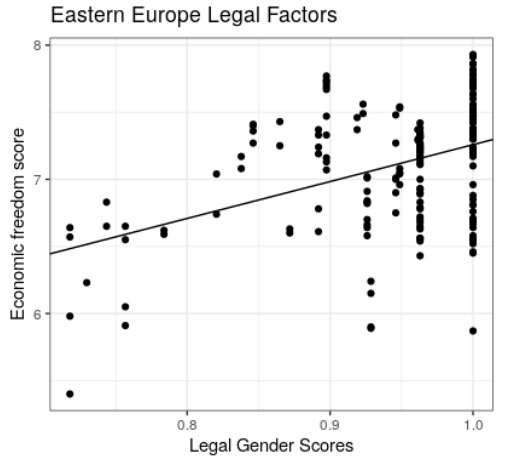
```
Call:
lm(formula = ef_score ~ ef_legal_gender, data = NorthA)

Residuals:
      Min        1Q    Median        3Q       Max
-0.216667 -0.069167  0.008333  0.068333  0.263333

Coefficients: (1 not defined because of singularities)
                Estimate Std. Error t value Pr(>|t|)
(Intercept)      7.95667    0.03011   264.3   <2e-16 ***
ef_legal_gender       NA         NA      NA       NA
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1277 on 17 degrees of freedom
```

Figure 19: Eastern Europe



R-squared: 0.1686
T=0.04218382
P=0.9663949

The R^2 calculation for Eastern European countries was 0.1686. The test statistic was computed at 0.04218382. The p-value is not significant as it is greater than 0.05 and we cannot reject the Null hypothesis.

ef_score=4.5098+ 2.7486(ef_legal_gender)

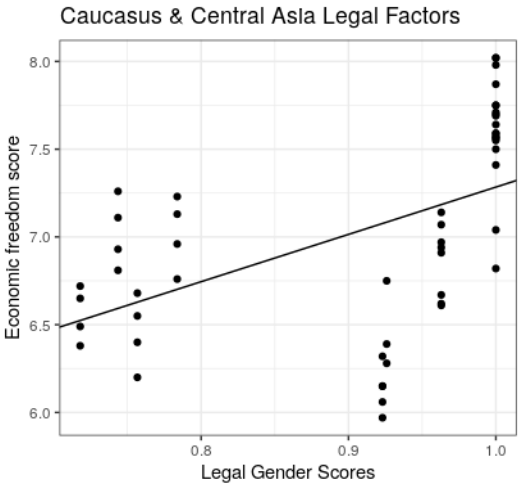


Figure 20: Caucasus & Central Asia

R-squared= 0.2595
T=4.186451
P-value= 0.0001096383

Caucasus & Central Asia had an R^2 of 0.2595 which is normally thought to be an extremely weak linear relationship. The p-value is less than 0.05 meaning we reject the null in favor of the alternative hypothesis.

ef_score=4.5898 + 2.6944(ef_legal_gender)

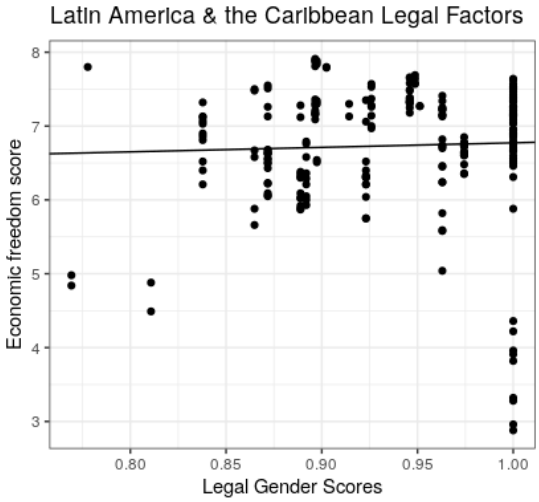


Figure 21: Latin America & the Caribbean

R-squared: 0.001551
T-test:0.5976899
P-value:0.5506349

Latin America & the Caribbean have a R^2 of 0.001551. The T-test statistic was 0.5976899. The P-value is greater than 0.05. We can not reject the null hypothesis.

ef_score=6.1625 + 0.6106(ef_legal_gender)

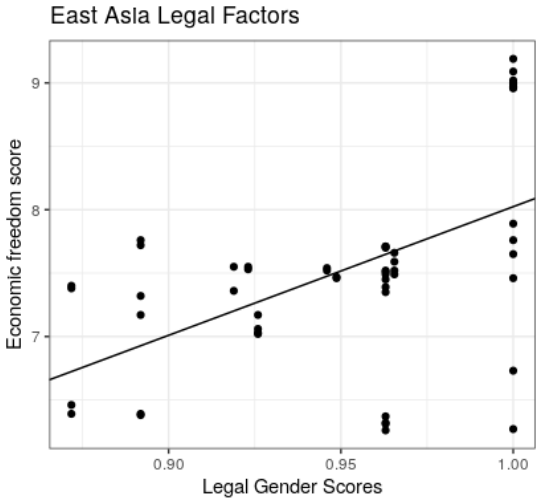


Figure 22: South Asia

R-squared: 0.03755
T=2.286737
P=0.02360105

The R^2 for the region of South Asia is 0.03755. The Test stat that was observed was 2.286737.The p-value is less than 0.05 so we can reject the null. The relationship between ef scores and legal gender score appears to be a weak linear relationship.

ef_score=6.1625 + 0.6106(ef_legal_gender)

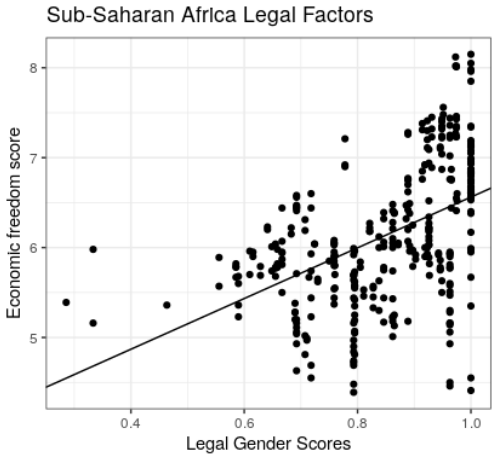


Figure 23: Sub-Saharan Africa

R-squared: 0.2288
T=10.13259
P-value=0Sub-Saharan Africa has a weak linear relationship, R^2=0.2288. The test statistic was 10.13259. The P-value is significant as it is less than 0.05.
ef_score=3.7416 + 2.8199(ef_legal_gender)

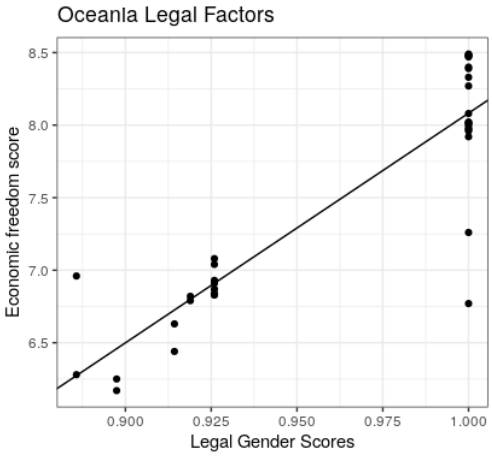


Figure 24: Oceania

R-squared: 0.7966
Test statistic:11.53863
P-value:0
The R^2 for the region of Oceania perceives a strong positive linear correlation. The test statistic was11.53863. The P-value is significant as it is less than 0.05. With a 95% Confidence, results show that the linear regression and R^2 value for Oceania suggests that the data has a strong linear relationship.

ef_score=-7.748 +15.831 (ef_legal_gender)

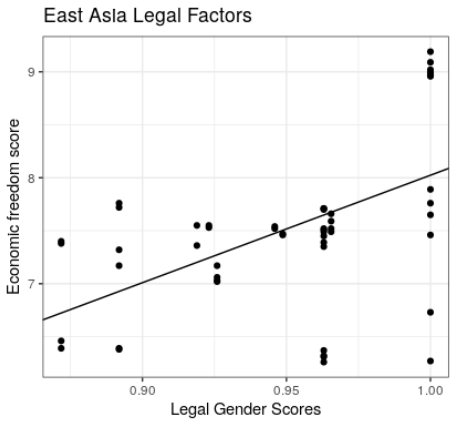


Figure 25: East Asia

R-squared: 0.2696
T=4.380397
p-value=5.760128e-05

East Asa has an R^2 of 0.2696 which corresponds to suggesting a weak linear relationship. The Test statistic is 4.380397l. The p-value is less than 0.5.

ef_score= -2.121 +10.145 (ef_legal_gender)

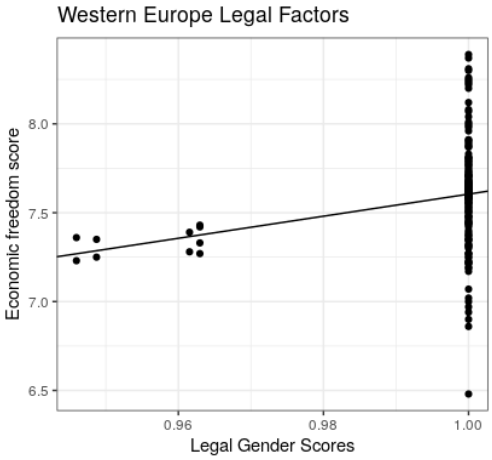


Figure 26: Western Europe

R-squared: 0.04591
Test statistic=2.774799
P-value: 0.006181499

The R^2 is very low and so the data suggests that Western European countries have a weak positive linear relationship. The Test statistic is outside of the CI interval. The p-value allows us to reject the null.

ef_score= 1.395 +6.210 (ef_legal_gender)

How strong is the relationship between human freedom and economic freedom?

Data:

Using a refined dataset omitting all null values and using the LM function in R, we created our linear regression model.

Linear Model:

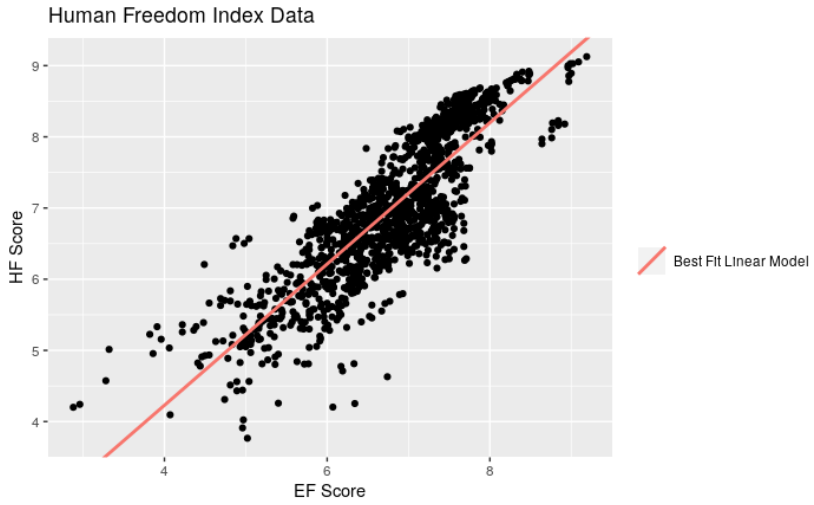


Figure 27: Scatterplot of human freedom scores and economic freedom scores with best fit linear model.

```
Call:
lm(formula = hf_score ~ ef_score, data = freedomindex141)

Residuals:
     Min       1Q   Median       3Q      Max
-2.31864 -0.36668  0.05449  0.41767  1.49198

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.25906    0.11112   2.331   0.0199 *
ef_score     0.99245    0.01624  61.117   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.5324 on 1376 degrees of freedom
  (80 observations deleted due to missingness)
Multiple R-squared:  0.7308,    Adjusted R-squared:  0.7306
F-statistic:  3735 on 1 and 1376 DF,  p-value: < 2.2e-16
```

From this information, we can determine that our linear model is,
$hf\ score\ =\ 0.25906\ +\ 0.99245(ef\ score)$. (Both HF and EF scores should have hats over them but I do not know how to in google docs) This model means that our intercept is .25906 which indicates that if there was an economic freedom score of 0 then there would be a human freedom score of 25906. Further, the slope of the model shows that for every individual one unit increase in EF score, the HF score is expected to increase by .99245.
Here is the visual of our model applied to our dataset:

This plot indicates the strong positive relationship between the EF and HF scores. Further, the model seems to track the data quite well with a fairly consistent variance of data surrounding the best fit line. Using SSE and SST calculations in R, we found an R^2 of .7038. This is very high and further reinforces the notion that the relationship between HF scores and EF scores is positive and fairly strong.

Residuals:

To make sure that we properly evaluated the accuracy and efficacy of our model we created both a histogram and scatter plot of our residuals.

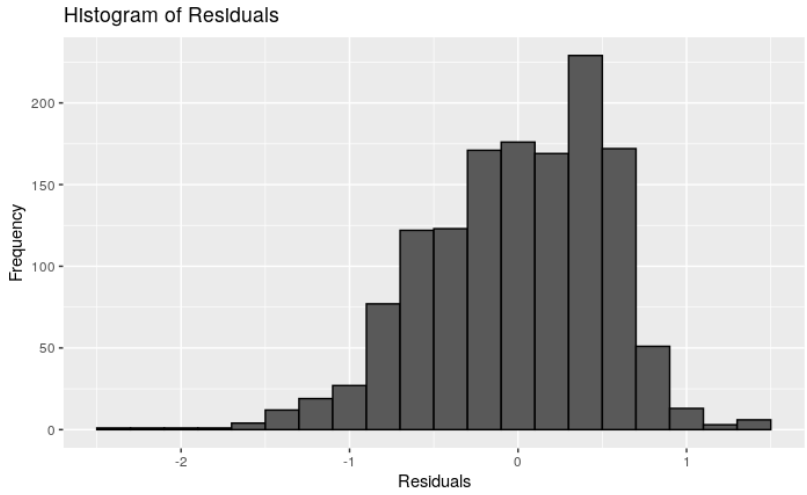


Figure 28: A histogram of residuals plot for the economic freedom score data.

The histogram of residuals plot shows a fairly normal distribution with a slight negative or leftwards skew. This indicates that the residuals are fairly evenly distributed but not perfectly.

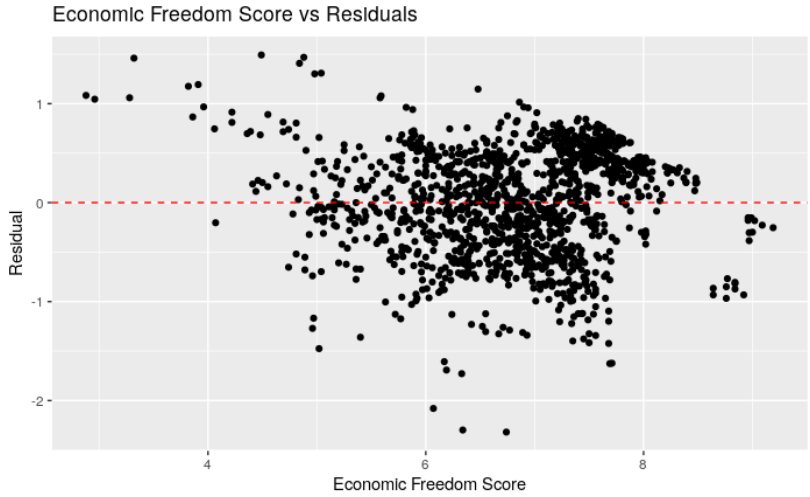


Figure 29: A scatterplot of residuals for the economic freedom score data.

This residual plot shows are mostly unbiased and homoscedastic. However, It looks like at the extremes there is a slightly negative relationship between residual and the economic score. But, I do not think this

is enough to justify a different distribution. The data has a constant enough variance that I believe the linear model is apt.

T-Test:

Next, we calculated the relevant statistics to evaluate the null and alternative hypotheses.
From our initial data analysis we found that that standard error is 0.01624. With this, we can calculate the T statistic using $\beta_1 - null\ value(SE)$ which resulted in a T statistic of 61.11145.

Next, we found our degrees of freedom which was 1456.
Then using R we found our p-value which was 0.
This means it is extremely small, and much smaller than our significance value of 5%. Thus, we can conclude that the alternative hypothesis is most likely correct.

Confidence Interval:

Next, we began to create our confidence interval. First, we used a z* value because we have an extremely large sample size. Thus, our z* will be 1.96.
Our CI then is:
To calculate the confidence interval we used:
$\beta_1 \pm 1.96 \cdot SE = 0.99245 \pm 1.96 \cdot 0.01624 = 0.9606196, 1.02428$
Thus, we are 95% confident that a one unit increase in economic freedom will also increase human freedom by between .96 and 1.02 units.

Does freedom to trade internationally (`ef_trade`) vary significantly with the legality of gay relationships between women (`pf_identity_sex_female`)?

Between the `ef_trade` and `pf_identity_sex_female` variables within the entire Human Freedom Index dataset, the calculated χ^2 value is 74.7561. The p-value, calculated using the null distribution shown in Figure X, is 4.307665 x 10^{-14}. With a significance threshold of 0.05, the p-value is low enough to be significant.

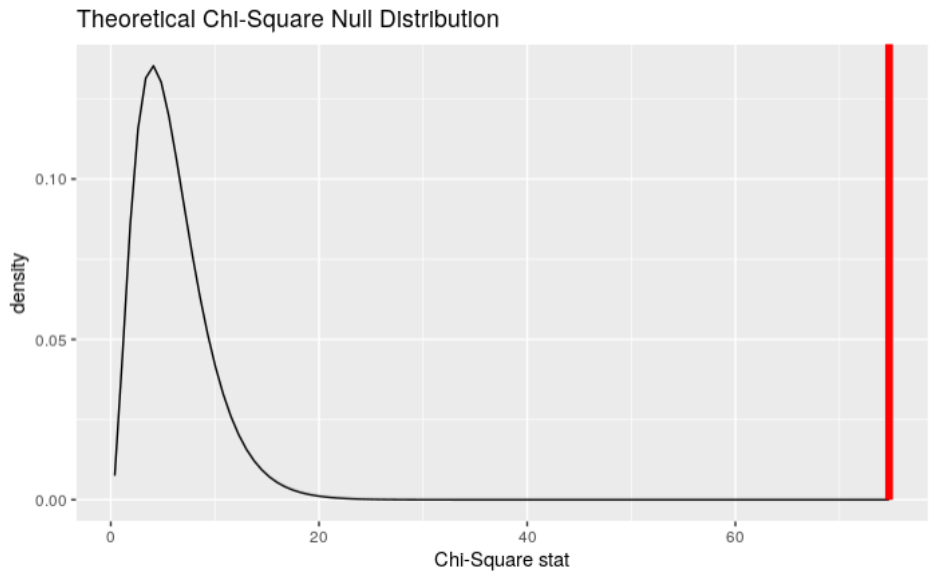


Figure 30: Theoretical Chi Square null distribution of freedom to trade internationally and legality of gay relationships between women

The results for the regional Chi Squares are shown in Table X. The only region where there was a significant p-value was the Middle East & North Africa. East Asia, Oceania, and North America all only had one level for `pf_identity_sex_female`, which wouldn't work in a Chi Square analysis. For these regions, there would be no covariance between `pf_identity_sex_female` and `ef_trade` because every country had the same `pf_identity_sex_female` value of 10. South Asia, Eastern Europe, Latin America & the Caribbean, Sub-Saharan Africa, and Caucasus & Central Asia all had non-significant p-values above 0.05.

Table 3: Chi Square analyses of freedom to trade internationally and legality of gay relationships between women

Region	χ^2	P-Value
East Asia[1]	N/A	N/A
South Asia	3.996262	0.6771823
Eastern Europe	0.1910532	0.9998647
Oceania[1]	N/A	N/A
Middle East & North Africa	19.17156	0.0038835
Latin America & the Caribbean	2.964185	0.8133284
Sub-Saharan Africa	4.165962	0.6542285
North America[1]	N/A	N/A

Caucasus & Central Asia	0.2972411	0.9995104

[1] The variable `pf_identity_sex_female` only had the level 10 and therefore a Chi Square wasn't possible.

Discussion

Do economic freedom and personal freedom differ significantly within region?

The results show that on average, there is a significant difference between economic and personal freedom scores in every region. The largest difference in means lies within the Western European region, which includes Austria, Belgium, Denmark, Finland, France, Germany, Iceland, Ireland, Italy, Luxembourg, Malta, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. The majority of these countries are well developed, and are often thought to have well organized governments, so it is interesting to see such discrepancies between freedom scores. These countries have high overall human freedom scores in comparison to countries in other regions, six of them being listed in the "Top 10 Freest Countries." This leads to further questions regarding the weight of specific variables over others when calculating a final "human freedom score." The regions with the second and third largest difference in means are Latin America & the Caribbean and the Middle East & North Africa, respectively. These regions are stereotypically less developed and organized than Western Europe and other regions in the index, which leads to a further hypothesis that human freedom and the developmental status of countries are not correlated.

Additional research should perform ANOVA tests to explore the differences between mean personal freedom and economic freedom between each region, and between the countries of each region. This would provide insight as to whether or not the region a country is in is any indication of economic and personal freedom. Other future questions could include: How strongly are economic freedom and personal freedom correlated within each country? Is there a significant difference in these correlations across countries?

What is the relationship between a country's economic freedom score (ef_score) and a country's regulations or protections with gender adjustment (ef_legal_gender score)?

During interpretation of the graphs, it becomes important to note the slope of the line as the visual might appear to be deceiving. Additionally, throughout the data manipulation period it was challenging to assess a variable's score because the scores were composed of a combination of index's variables. Sampling methods could not be determined as the Human Freedom index does not provide that information. The results show that there is a relationship between a country's economic freedom score and ef_legal_gender score. The model that would best describe the relationship is however mediated by the region where the country is located. While in some cases the relationship between the economic freedom and the ef_legal_gender score is clearly linear, other models might provide a better explanation of the impact ef_legal_gender scores have on ef_score. Oceania has a high R^2 that is associated with a strong linear relationship. Sub-Saharan Africa and East Asia both also display weak linear relationships. Western Europe is a special case because so many of the countries have the max ef_legal_gender score that they could receive. Countries in North America also have similar data results but to a greater extent where every country has received the score of 1. This suggests that Western regions like Western Euuope and

North America share similar legal views on gender adjustment and the barriers to gaining Economic Freedom. However, this score does not factor in the socio-economic position a country might be in. The laws of a society versus the unspoken rules can be drastically different. The Middle East and North Africa have scores that are significantly less than the scores from other regions. Furthermore, the Middle East and North Africa has one of the largest gender gap in the world (The Economics of Women in the Middle East and North Africa)[1]. Past research has shown that gender equality is associated with regions that have higher economic freedom scores. The Middle East and North Africa have economic platforms that are centered around natural resources. For the Middle East, the economy is focused on oil and the vast oil reserves found throughout the region. As the Middle East has faced a series of drastic changes in social norms it would be especially interesting to see the impact legal gender scores had on the economic freedom scores.

It would be illuminating to examine what variable most impacts the ef score in comparison to which variable most impacts the personal freedom score. Fuerther expiration should be considered before formulating the final answer on the relationship between these two variables. One set of analysis that would be especially interesting would be tests focusing on trends over different years. This enables us to consider the linear models in the context of periods where the scores would be drastically different.

What is the relationship between human freedom and economic freedom?

One key note to address is that the human freedom scores and economic freedom scores are by no means objective or realistic measures of either human or economic freedom. These scores are determined by the creators of the human freedom index and are an amalgam of other also subjective scores. Thus, the linear model does not actually show the relationship between economic freedom and human freedom, rather, it shows the relationship between the economic freedom scores and human freedom scores that the creators of the Human Freedom Index used.

In terms of conditions for the model, all are met. We need to have linearity, independent observations, normally distributed residuals, and constant or equal variability. First, our scatter depicts a linear relationship. Next, our samples are independent. In terms of our residual plot, although we do see some negative skew, there is a mostly bell-curved shape. With so much data I believe the minor skew is not a big issue. Our residuals plot exhibits strong heteroscedasticity except at the absolute extremes where there is a slightly negative relationship between residuals and the economic score. But, in the grand scheme of things, our model exhibits enough equality of data variance that the linear model is apt.

In terms of results, we can see that there is a clear relationship between the economic freedom score and the human freedom score. Further, this relationship is almost one to one. This means that as the economic freedom score increases by one unit, the human freedom score increases by about one unit too. By our T test and confidence interval we can reject our null hypothesis that there is no relationship between the two scores. The T Test value being 0 indicates that at even a 99% confidence level, we would reject the null. Further, the R^2 value we found also indicates a strong positive relationship between the two scores. It means that 70% of the increase in human freedom score can be accounted for by an increase in economic freedom score. And even more, our linear model slope falls within the 95% confidence interval model further reinforcing the efficacy of our conclusion to reject the null hypothesis. The

[1] Shafik, N. (2001), "Closing the gender gap in the middle east and North Africa", *The Economics of Women and Work in the Middle East and North Africa* (*Research in Middle East Economics, Vol. 4*), Emerald Group Publishing Limited, Bingley, pp. 13-31.

confidence interval itself was between 0.9606196 and 1.02428 which means that for every one unit increase in economic freedom score, 95% of the time we would expect an increase in human freedom score of between .96 and 1.02. This all indicates that, as one might expect, there is a relationship between economic freedom score and human freedom score.

In the future it may be important and helpful to more deeply analyze the data behind each of the Human Freedom Index's scores. Each score is an amalgam of information about regions and countries that must be collected from somewhere, however, it is difficult to discern exactly where this information is coming from. If we could do so, we could conduct an audit of sorts that evaluates the validity and reliability of the Human Freedom Index's conclusions. Additionally, as the Human Freedom index is published each year, it would be fascinating to do several linear regressions with focusing on different years enabling investigators to determine trends over time.

Another fascinating research topic would be a deeper look into the weight of the individual scores for the broader human freedom score. For example, how much of an effect does "pf_identity_sex" have in comparison to something like "ef_trade_tariffs". This could give us aspects about two things. One would be an evaluation of whether or not the Human Freedom Index skews their information by over weighting some variables above others. If they do not do this, then it would simply help indicate the importance each variable has upon the overall scores of economic, personal, or human freedom.

Does freedom to trade internationally (`ef_trade`) vary significantly with the legality of gay relationships between women (`pf_identity_sex_female`)?

The p-value generated from a Chi Square analysis between the `ef_trade` and `pf_identity_sex_female` variables for the entire Human Freedom Index dataset (4.307665×10^{-14}) is significant. This means that, based on the dataset, there is a correlation between a country's freedom to trade internationally and the legality of gay relationships between women. Legality of queerness and cultural acceptance of queerness are two different things. Legality only indicates that the government recognizes and protects queerness. The relationship shown here between queer legislation and international trade might be caused by the distinctions between more conservative and more liberal governments. A more liberal government might be more likely to both encourage international trade and legally recognize gay relationships between women.

Only the p-value for the Middle East & North Africa region is significant, while the rest are not. This indicates that the variation between the regions must be a critical element of the covariance between the `ef_trade` and `pf_identity_sex_female` variables; within regions, there isn't enough variation alone.

Future methods could include applying linear regressions to the two variables and using T-Tests to see if they are significant there as well. This would help alleviate the issue of not being able to apply Chi Square tests to the regions where there was only one level for a variable. Visualizations would also be useful for understanding the relationship.

Conclusions

The Human Freedom index incorporates measures of personal and economic freedom to produce a Human Freedom score. The index report has been published every year from 2008 on and further time should be spent investigating longitudinal models of data analysis. The index is tagged as the most comprehensive global analysis, yet does not include data on all countries and does not Additionally, how

scores were determined could shift the outcomes. In terms of linear regression, we can definitively conclude that there is a clear relationship between the economic freedom score and the human freedom score. Further, it is a strong and positive relationship. According to our 95% confidence interval we can conclude that 95% of the time a one unit increase in economic freedom will result in between a .96 and 1.02 unit increase in human freedom score. This relationship we found between the two is validated by the fact that all our conclusions are met and our residuals satisfy the necessary conditions to perform an accurate test. Overall, we have rejected the null hypothesis that there is no relationship between the economic and human freedom scores in favor of the alternative hypothesis.

Based on the four research questions and conclusions, it appears that there are relationships between the economic freedom scores and the personal freedom scores. This indicates that economic and personal freedoms are associated, and future research could investigate a more directional approach to see how the relationships between the variables vary further than simply determining that they are associated.