Binomial Distribution Expected Value

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$$P(x) = \binom{n}{x} p^x (1-p)^{n-x} \rightarrow \text{pmf for the binomial r.v.}$$

Show that $E[X] = np$.

$$E[X] = \sum_{x=0}^{n} x P(x) \rightarrow \text{Expected value definition for discrete r.v.}$$

$$E[x] = \sum_{x=0}^{n} \boxed{x\binom{n}{x}} p^x (1-p)^{n-x}$$

$$x\binom{n}{x} = x \frac{n!}{x!(n-x)!} \rightarrow \binom{n}{x} = \frac{n!}{x!(n-x)!}$$

$$= \cancel{x} \frac{n(n-1)!}{\cancel{x}(x-1)!(n-x)!} \rightarrow n! = n(n-1)!$$

$$= n \frac{(n-1)!}{(x-1)!(n-x)!}$$

$$= n \frac{(n-1)!}{(x-1)!((n-1)-(x-1))!}$$

$$= n \binom{n-1}{x-1}$$

$$E[x] = \sum_{x-1=0}^{n-1} \boxed{n\binom{n-1}{x-1}} \boxed{p p^{x-1}} (1-p)^{\boxed{(n-1)-(x-1)}} \rightarrow n-1-x+1 = n-x$$

Let $j = x-1$.

Let $m = n-1$

$$pp^{x-1} = p^{x-1+1} = p^x$$

$$= \sum_{j=0}^{m} np \binom{m}{j} p^j (1-p)^{m-j}$$

$$= np \boxed{\sum_{j=0}^{m} \binom{m}{j} p^j (1-p)^{m-j}}$$

$$= np \boxed{\sum_{j=0}^{m} \binom{m}{j} p^{j} (1-p)^{m-j}}$$

$$(p+(1-p))^{m} = \sum_{j=0}^{m} \binom{m}{j} p^{j} (1-p)^{m-j} \longrightarrow \text{binomial theorem}$$

$$= np \boxed{(1)^{m}}$$

$$E[X] = np \checkmark$$