Black body radiation

cube with perfectly reflecting walls

standing electromagnetic waves inside cavity

modes

$$\mathbf{E}(\mathbf{x},t) = -2E_0 \sin\frac{2\pi}{\lambda}x\cos 2\pi\nu t$$

$$\mathbf{B}(\mathbf{x},t) = 2B_0 \cos\frac{2\pi}{\lambda}x\sin 2\pi\nu t$$

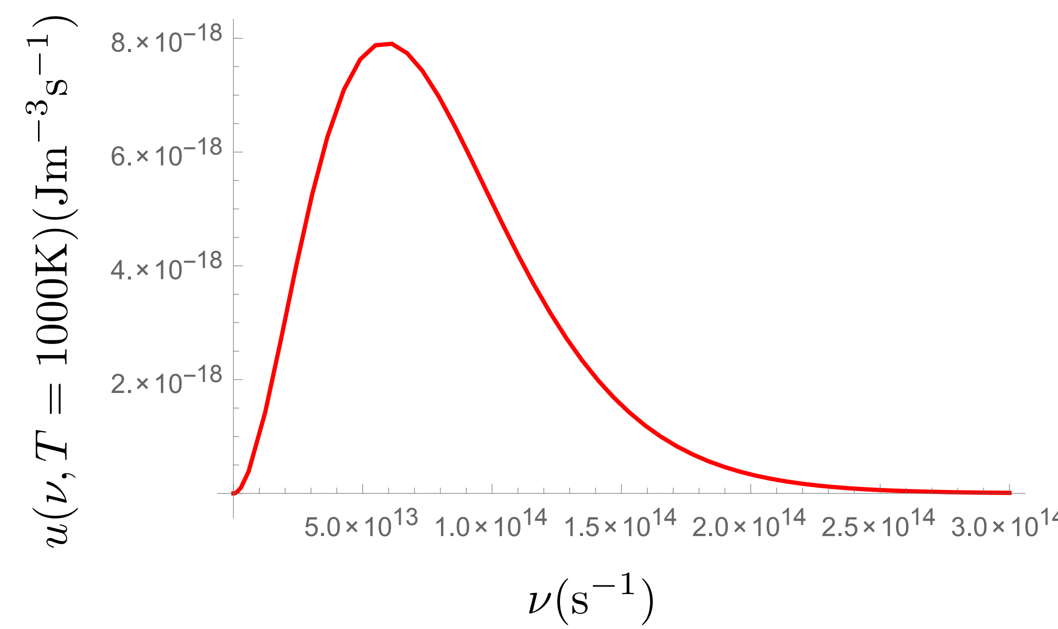
modes are characterized by vector $\mathbf{n}$

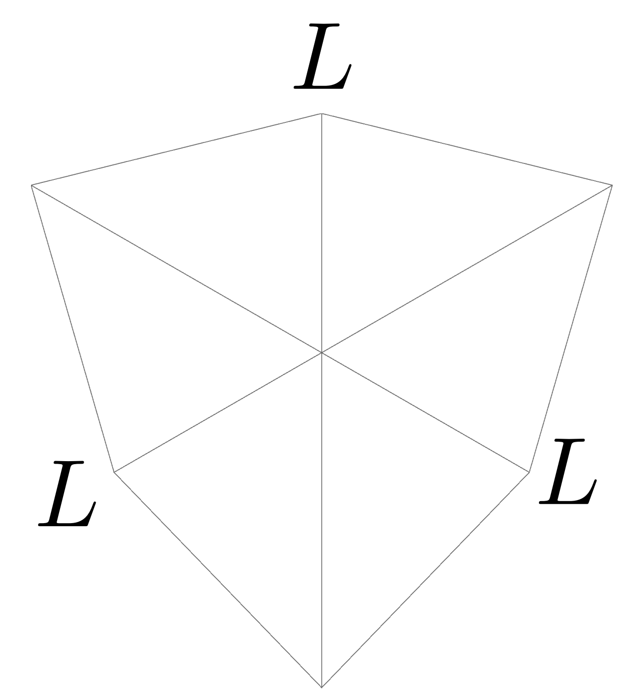
$$\lambda_{n_x} = \frac{2L}{n} \qquad \nu_{n_x} = \frac{n_x c}{2L}$$

what is the energy density in the cube?

number of modes as function of frequency

average energy of each modes





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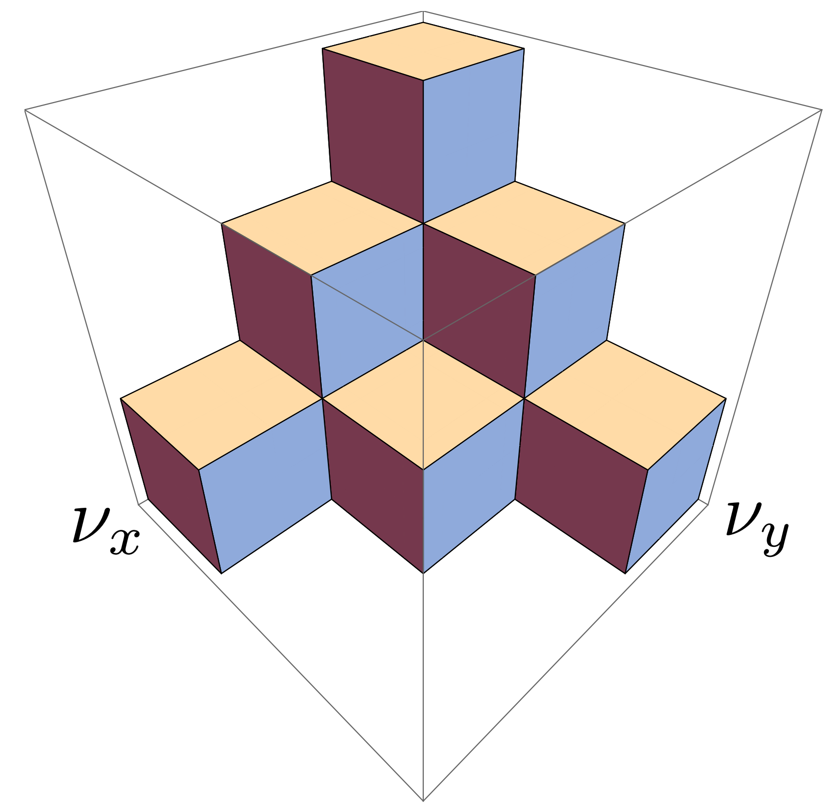
$$\nu_{n_x} = \frac{n_x c}{2L}$$



number of modes as function of frequency?

smallest frequency 'volume'($\mathbf{n} = \mathbf{1}$)

$$V_{111} = \left(\frac{c}{2L}\right)^3$$

'frequency density'

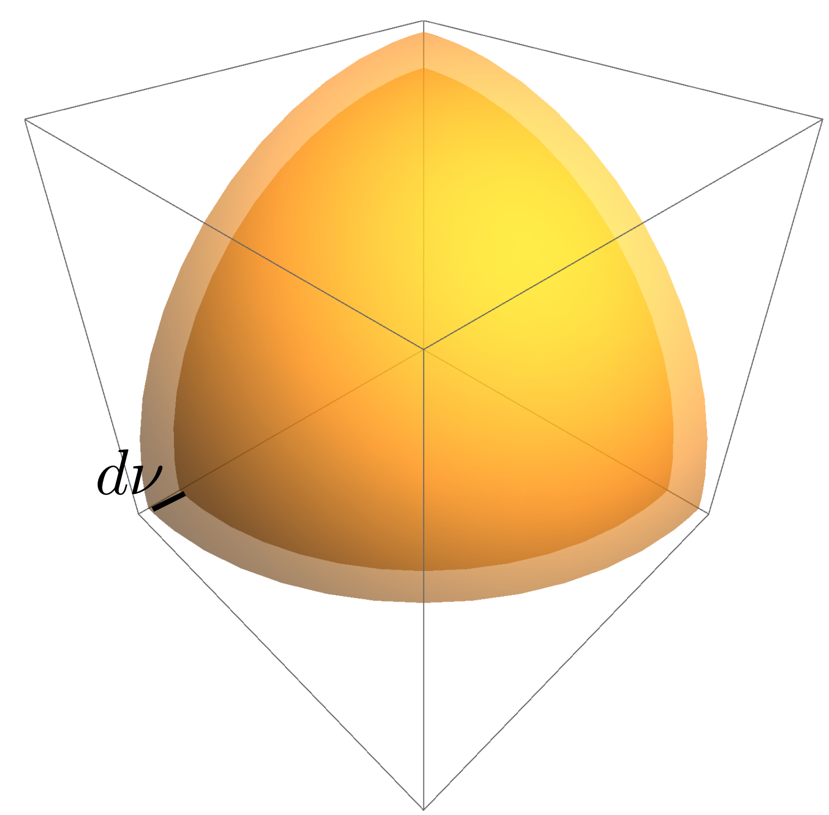
$$\rho(\nu) = \left(\frac{2L}{c}\right)^3$$

total frequency 'volume' in interval $\nu + d\nu$

$$V(\nu)d\nu = \frac{1}{8}4\pi\nu^2 d\nu$$

with polarization up & down

$$V(\nu)d\nu = \pi\nu^2 d\nu$$



number of modes f in interval $\nu + d\nu$

$$f(\nu)d\nu = \pi\nu^2 d\nu \left(\frac{2L}{c}\right)^3 = \frac{8\pi L^3}{c^3}\nu^2 d\nu$$

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average energy per mode?

classical mechanics: equipartition theory

$$\langle E\rangle = k_{\mathrm{B}}T$$

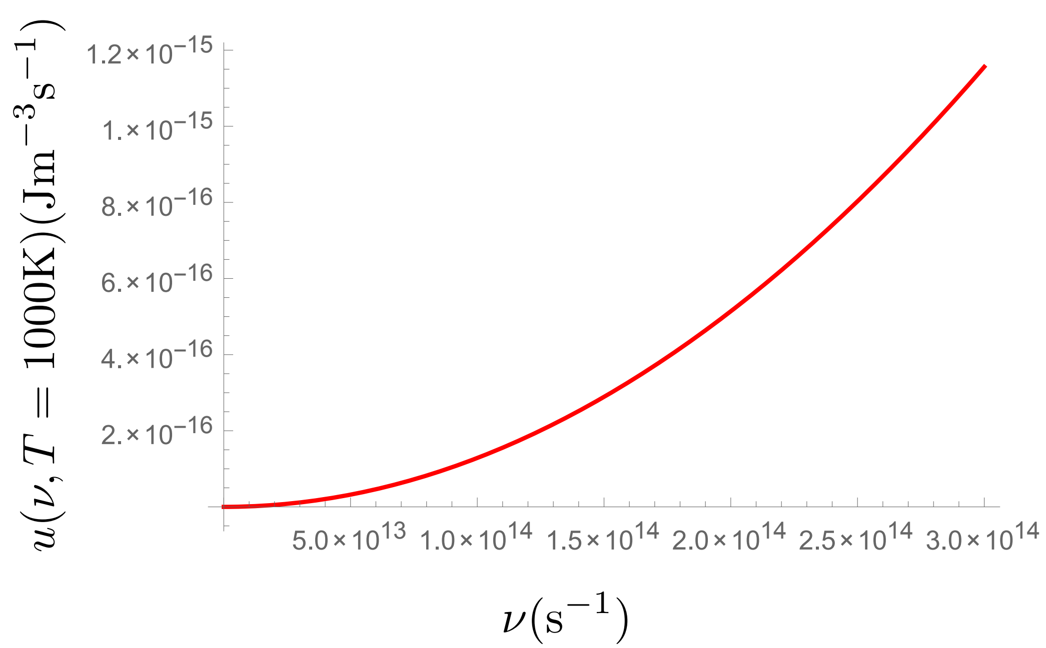
energy density

classical mechanics

$$u(\nu, T) = \frac{1}{L^3}f(\nu)\langle E\rangle = \frac{8\pi\nu^2}{c^3}k_{\mathrm{B}}T$$

ultraviolet catastrophe

$$U(T) = \int_0^\infty \frac{8\pi\nu^2}{c^3}k_{\mathrm{B}}T d\nu \to \infty$$



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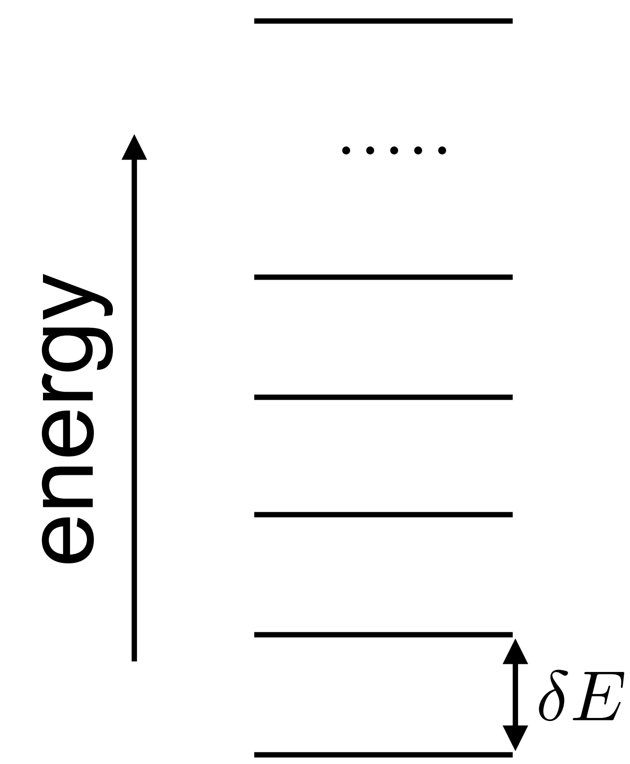
average energy per mode?

Planck

$$\langle E \rangle = \frac{h\nu}{1 - \exp[h\nu/k_{\mathrm{B}}T]}$$

Black body radiation

system with discrete equidistant energy levels



partition function

$$Z = \sum_{n=0}^{\infty} \exp[-\beta n \delta E] = \frac{1}{1-\exp[-\beta\delta E]} \qquad \beta = \frac{1}{k_{\mathrm{B}}T}$$

average energy

$$\langle E \rangle = -\frac{\partial}{\partial\beta}\ln Z = \frac{\delta E \exp[-\beta\delta E]}{1-\exp[-\beta\delta E]} = \frac{\delta E}{\exp[\beta\delta E]-1}$$

using

$$\delta E = h\nu$$

leads to Planck's expression

$$\langle E \rangle = \frac{h\nu}{1-\exp[h\nu/k_{\mathrm{B}}T]}$$

electromagnetic radiation apparently has discrete energies

atoms in the wall (oscillatory sources of EM radiation) emit energy in discrete packages

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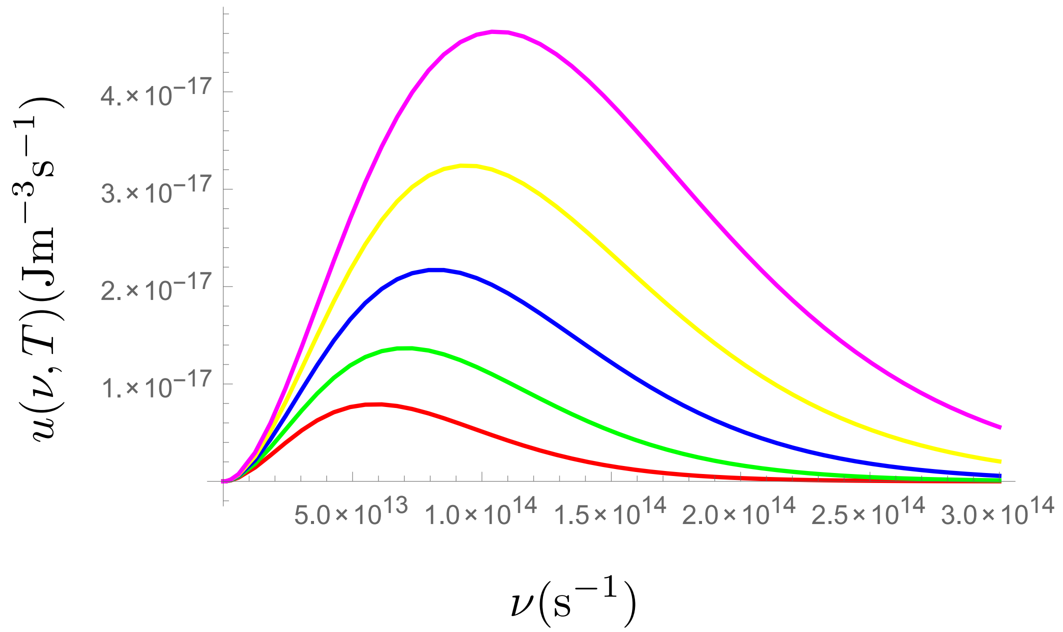
number of modes as function of frequency?

number of modes f in interval $\nu + d\nu$

$$f(\nu)d\nu = \pi\nu^2 d\nu \left(\frac{2L}{c}\right)^3 = \frac{8\pi L^3}{c^3}\nu^2 d\nu$$



average energy per mode?

Planck

$$\langle E\rangle = \frac{h\nu}{1 - \exp[h\nu/k_\mathrm{B}T]}$$

energy density

walls emit radiation in discrete 'quanta' of energy $nh\nu$ with $n = 1, 2, 3, ..$

$$u(\nu,T) = \frac{1}{L^3} f(\nu)\langle E\rangle = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{1 - \exp[h\nu/k_\mathrm{B}T]}$$