Representing Concept Lattices with Euler Diagrams

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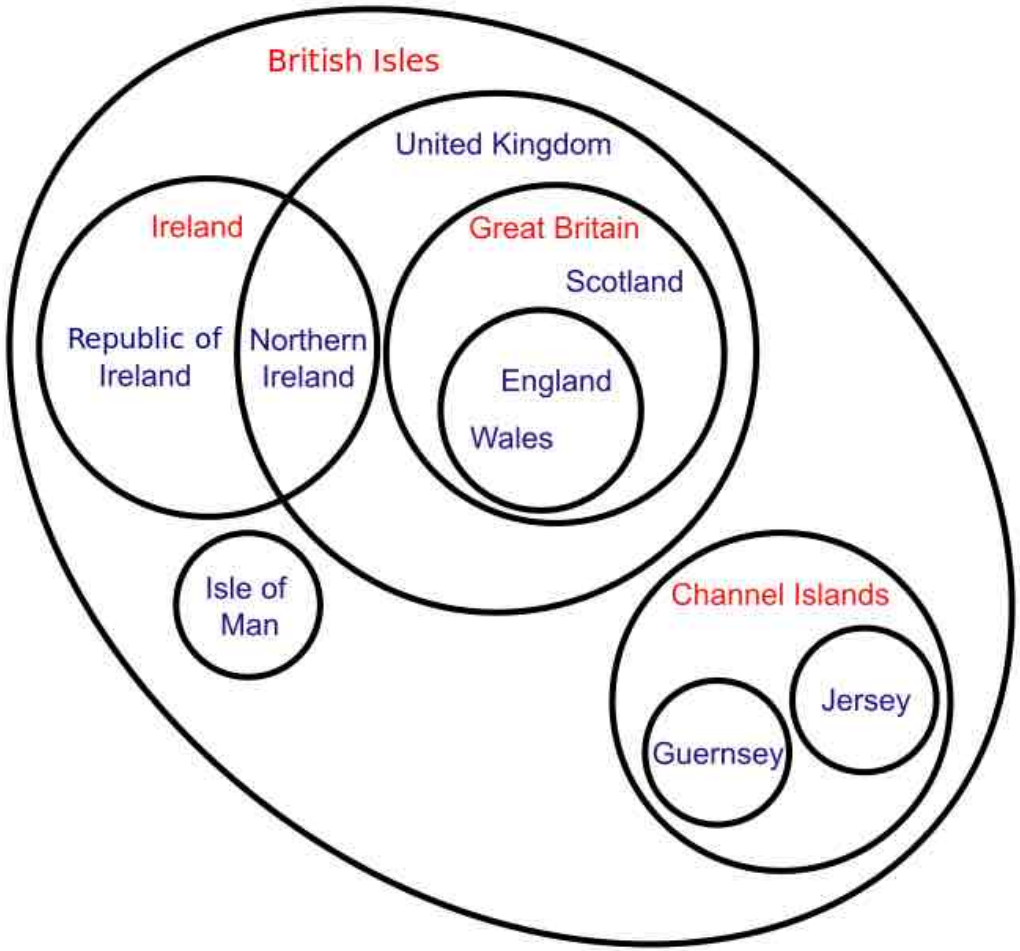
My experience with teaching mathematics to computer science students:

Reading Hasse diagrams of concept lattices can be difficult

Euler diagrams are easier to read



What kinds of diagrams?

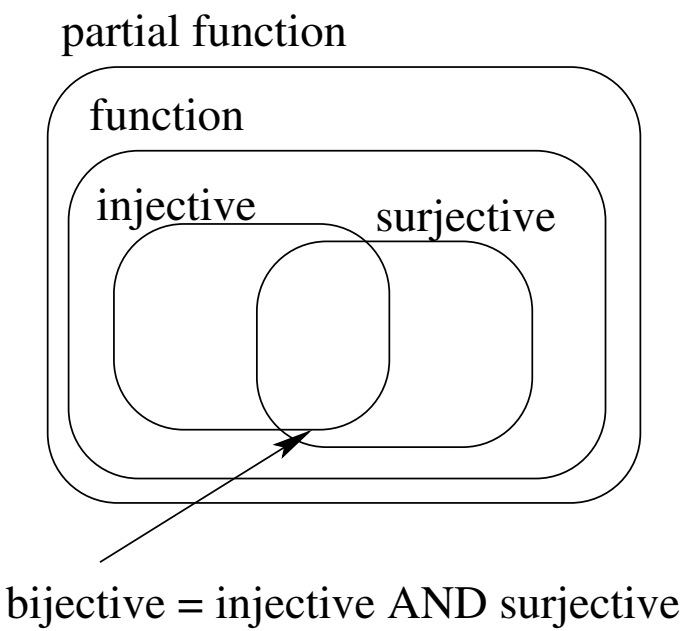
- small diagrams ($<$ 20 concepts)
- crisp, non-fuzzy data
- background knowledge matters
- for example: used in mathematics education

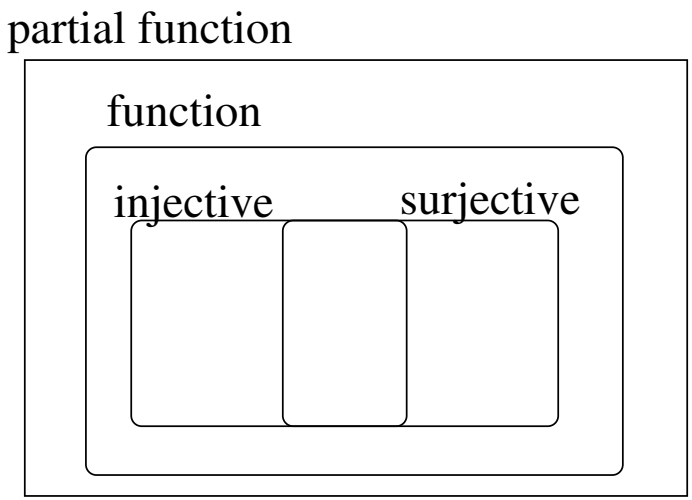
Software:
$\rightarrow$ interactive layout, heuristics
$\rightarrow$ multiple linked representations (text and graphics)

There is a lot to be said about Euler diagrams ...

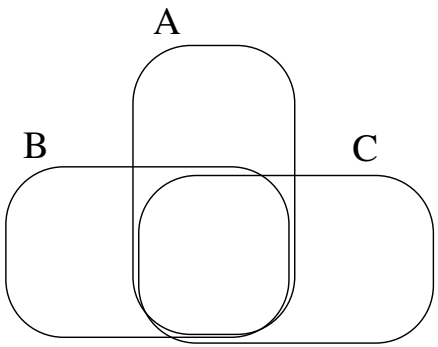
... not everything can be included in this talk.

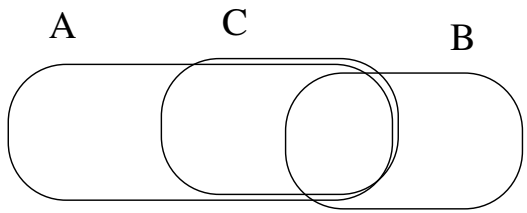
Rectangular Euler diagrams $\rightarrow$ Euler$^+$Diagrams





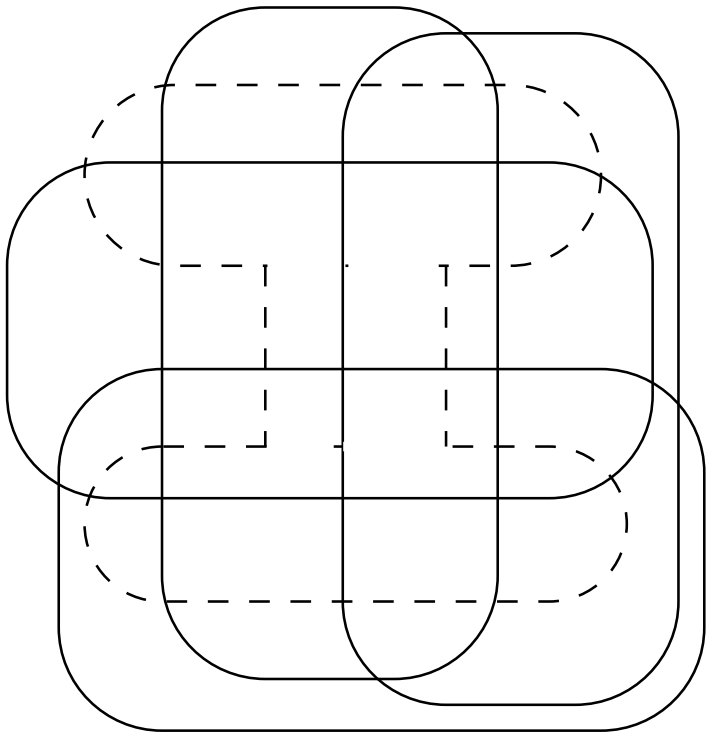
Euler diagrams that cannot be "well-drawn"





(2-dimensional and 1-dimensional)

Boolean lattice with 5 elements



(3-dimensional)

Background knowledge

- 3-valued logic: at least one, none, don't care
- negatable attributes (binary)
- supplemental concepts (contingent extension empty even if more objects are added)

Contrary to conceptual exploration: the data is not changed

Reduction methods

- synonyms ("clarification")
- AND ("reduction")
- OR
- NOT
- negation
- factorisation
- horizontal split
- lower horizontal split
- partitioning attributes
- partitioning objects
- conceptual partitioning

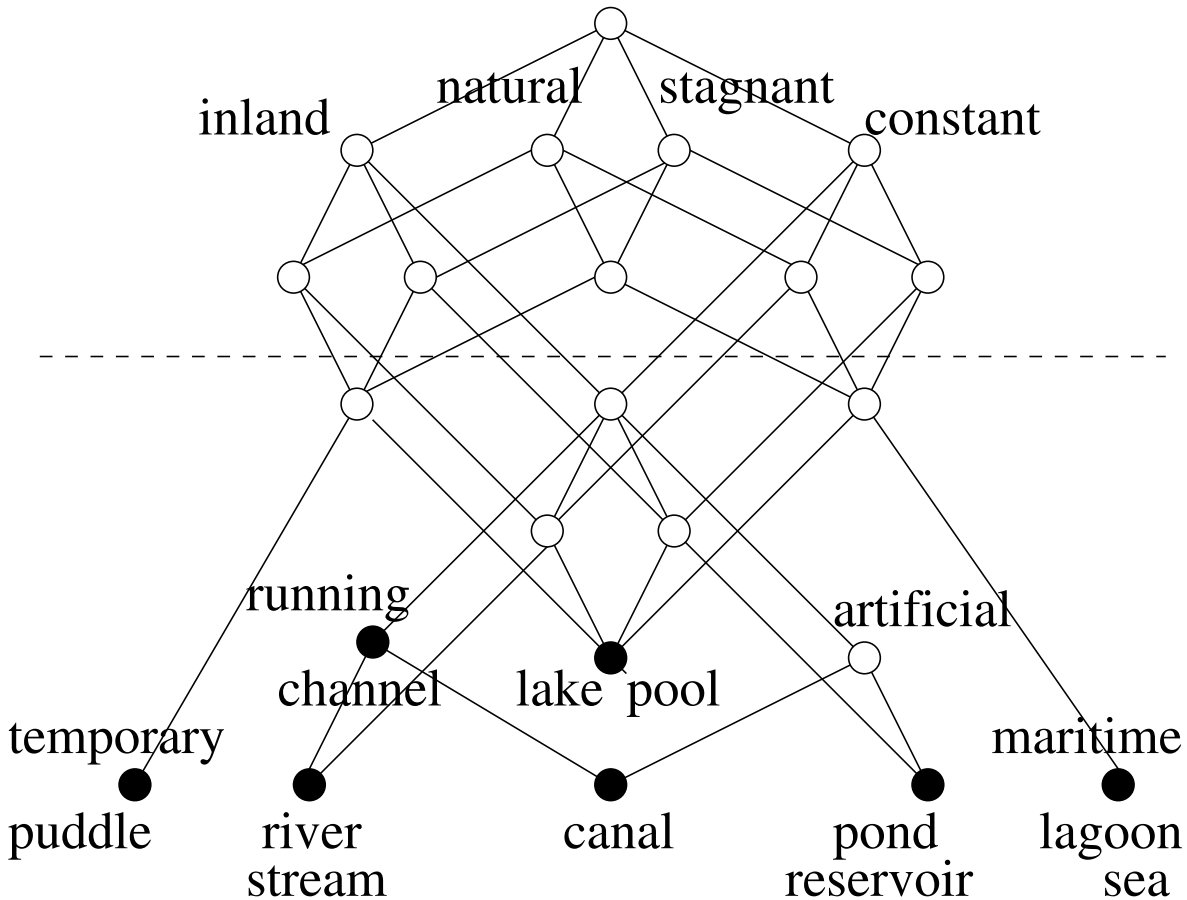
Conceptual partitioning

The set of objects is partitioned so that each partition is an extension of a concept or an extension of a concept of a negated attribute.

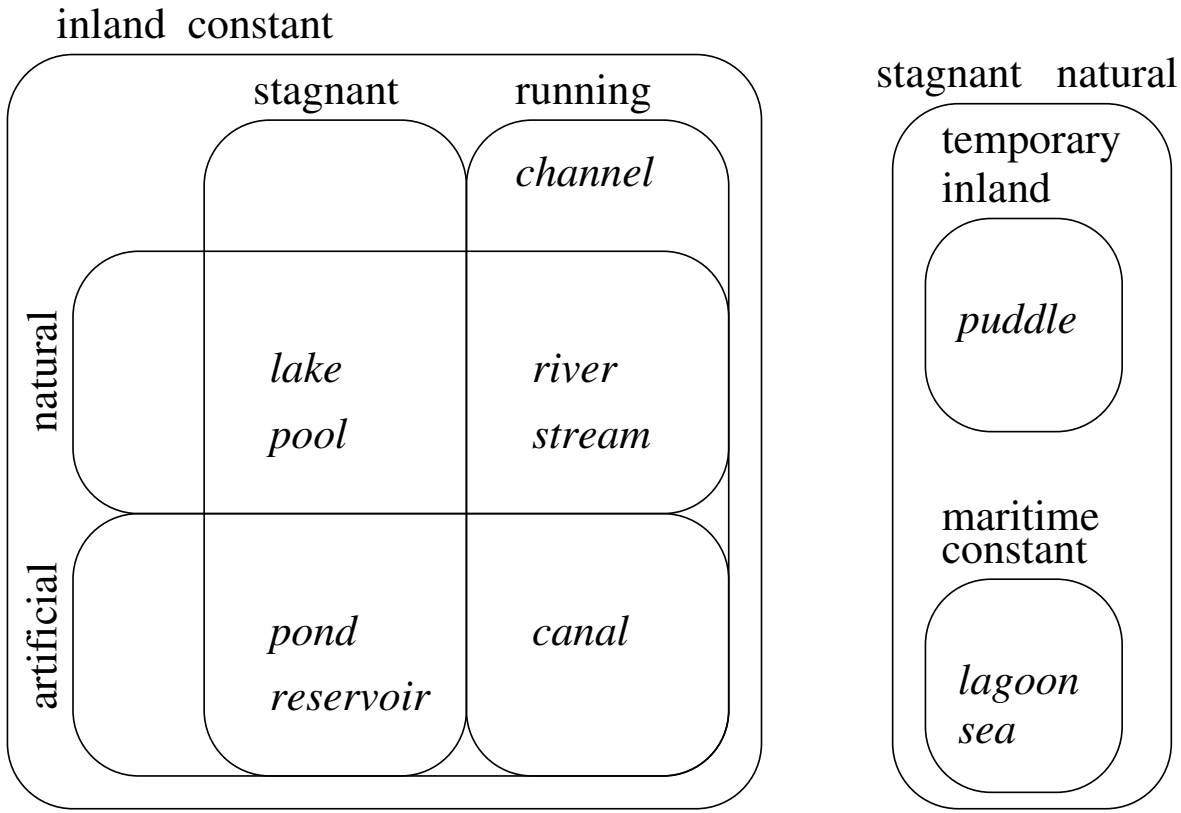
Measures

e.g. how many supplemental concepts have been removed

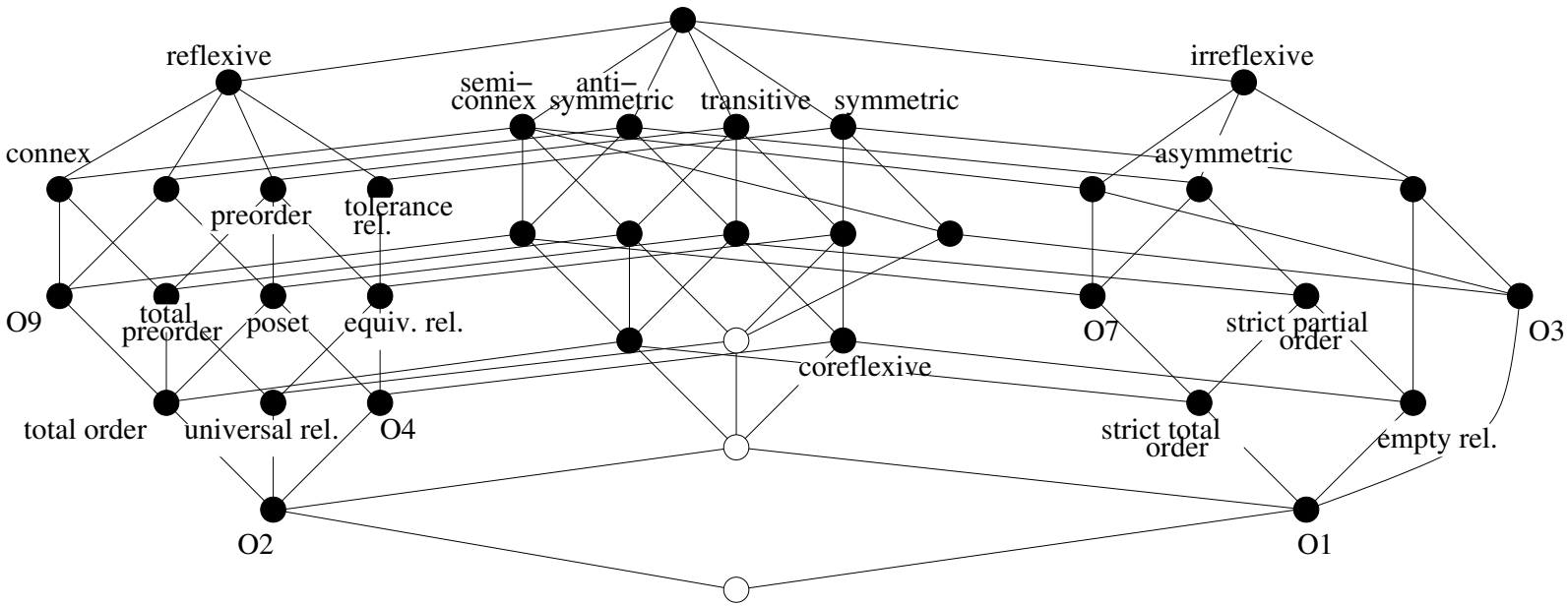
Body of waters lattice



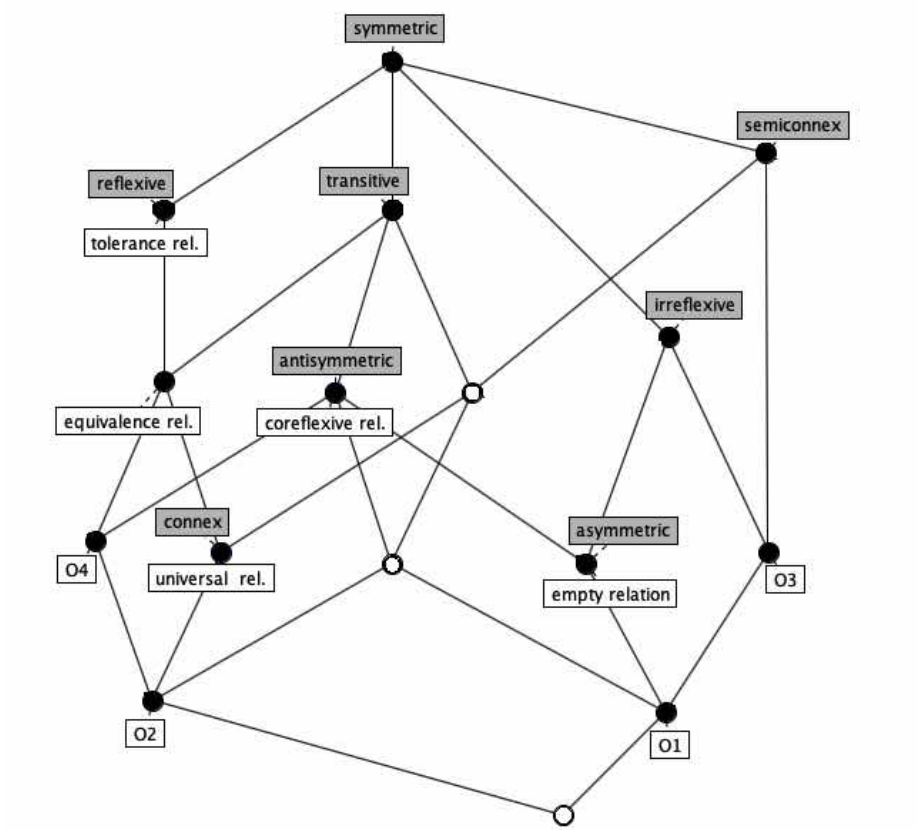
"inland AND constant" – "NOT inland OR NOT constant"

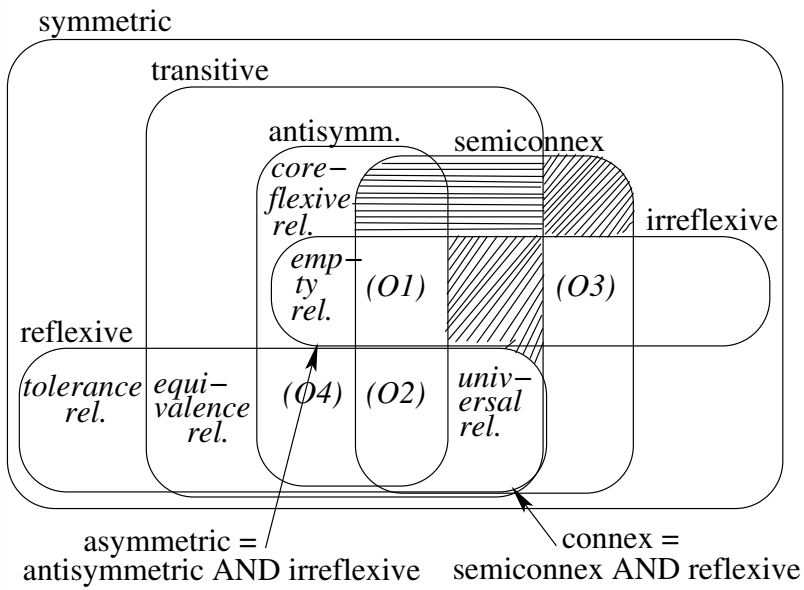


Lattice of "binary relations"

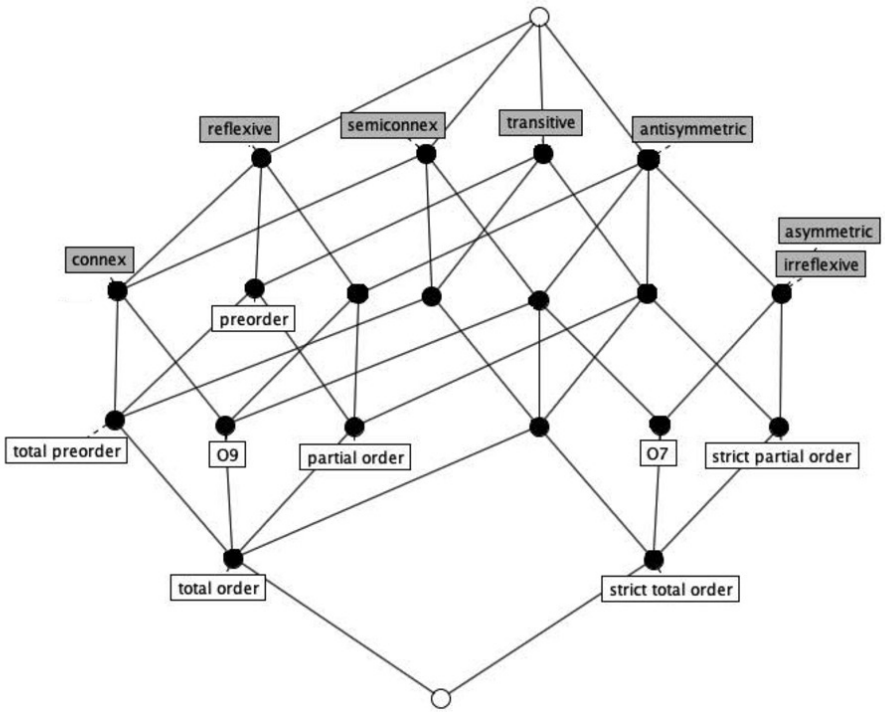


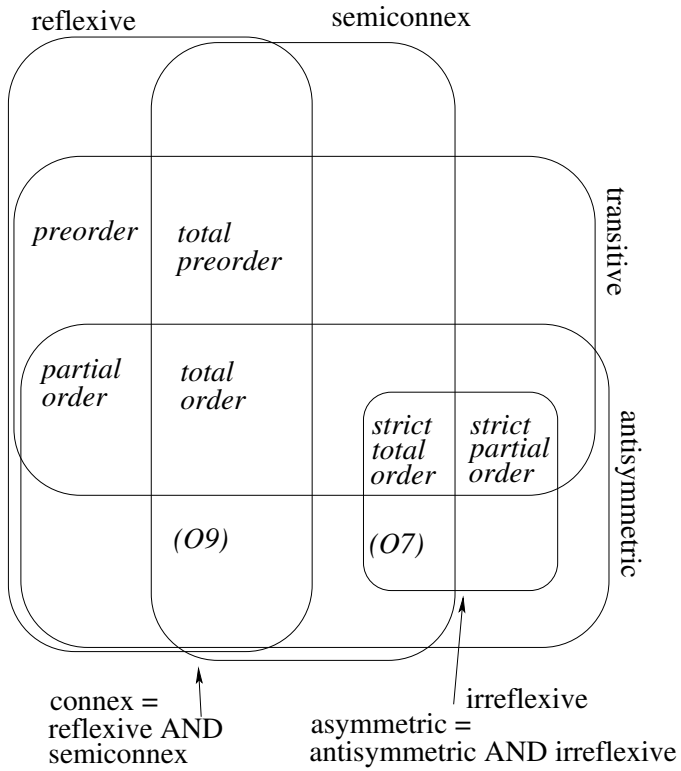
“symmetric”


symmetric
semiconnex
reflexive
transitive
tolerance rel.
irreflexive
antisymmetric
equivalence rel.
coreflexive rel.
connex
asymmetric
O4
universal rel.
empty relation
O3
O2
O1


symmetric
transitive
antisymm.
semiconnex
core–
flexive
rel.
irreflexive
emp–
ty
rel.
(O1)
(O3)
reflexive
tolerance
rel.
equi–
valence
rel.
(O4)
(O2)
univ–
ersal
rel.
asymmetric =
antisymmetric AND irreflexive
connex =
semiconnex AND reflexive

“NOT symmetric”





Conclusion

- Reducing concept lattices
- Euler$^+$diagrams
- Software for Euler$^+$diagrams
- Algorithms for Euler diagram layout?
- Evaluating the usability of Euler$^+$diagrams

Questions?