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| **Figure 1**. Visual representation of mesoscale variables , , and , and processes affecting them, in QLC-1 (as well as the present revision, QLC-2) model. Dashed arrows represent processes affecting how these variables evolve over time. |  |

**QLC-2: A revised quasi-liquid reaction-diffusion model**

Like QLC-1, QLC-2 represents an ice surface as two mesoscale variables and three processes, as shown in Fig. 1. The governing equations are

(1a)

(1b)

Some notes about this model are as follows, with differences between it and QLC-1 noted:

1. represents surface diffusion of the QLL; the underlying ice is considered immobile on time scales considered here.
2. is the rate at which vapor-phase water molecules strike the quasi-liquid; it is assumed that these stick with 100% efficiency, and thermalize on a picosecond time scale (i.e., instantaneously within the time scales of a QLC-2 simulation).
3. prescribes the thickness of quasi-liquid when it is in equilibrium with the underlying ice, according to

(2)

This formulation ensures that the QLL thickness varies continuously from the thin microstate (“surface I”) with thickness , to the thick microstate (“surface II”) with thickness . (Note the connection to work of (Benet et al. 2019, Eq. 8), that also shows a sinusoidal dependence.)

1. Because surfaces I and II have different volatilities, the surface supersaturation at a given point on the surface (designated in Eq. 1a) is a function of both the microstate and the water vapor concentration above it. To compute , we define a variable that quantifies the degree to which a given surface is similar to surface I or II,

(3)

With this definition, surface I will have , while surface II will have . We then express the surface supersaturation as

(4)

where is a measure of the difference in the equilibrium vapor pressure of surfaces I and II, and is the supersaturation relative to surface I, mentioned previously. We will assume here that both (a scalar quantity) and are fixed parameters of a given solution. Eq. 4 is at slight variance with, and simpler than, the corresponding expression in QLC-1.

1. Typically, a parabolic form is used to approximate appearing in Eq. 4,

(5)

where is a “center reduction”, the fractional reduction of supersaturation at facet centers relative to facet corners,

(6)

Negative values of are also used in the simulations presented here, to represent the enhancement of water vapor concentration at facet centers when a crystal is ablating because of exposure to subsaturated conditions.

1. is a first-order relaxation constant describing the time scale at which quasi-liquid/ice equilibrium is achieved. That is, if we imagine a surface with quasi-liquid amount , then equilibration after a time occurs according to

(7)

If one takes the time derivative of Eq. 6, and assumes that is small, the second term on the right-hand side of Eq. 1b results.