Contents

1 Template

1.1 Makefile

```
default:
	g++ -std=c++11 -Wall -Wextra -Wshadow -fsanitize=address -fsanitize=undefined
	↪  -DLOCAL -D_GLIBCXX_DEBUG -g main.cc -o main
```

2 Maths

2.1 Modular Arithmetic

```cpp
// **Really important note**: inputs of the modAdd, modSub, and modMul
// functions must all be normalized (within the range [0..mod - 1]) before use

#pragma once

#include <bits/stdc++.h>

using namespace std;

int modAdd(int a, int b, int mod) {
    a += b;
    if (a >= mod) a -= mod;
    return a;
}

int modSub(int a, int b, int mod) {
    a -= b;
    if (a < 0) a += mod;
    return a;
}

int modMul(int a, int b, int mod) {
    int64_t res = (int64_t) a * b;
    return (int) (res % mod);
}

int64_t binPow(int64_t a, int64_t x) {
    int64_t res = 1;
    while (x) {
        if (x & 1) res *= a;
        a *= a;
        x >>= 1;
    }
    return res;
}

int64_t modPow(int64_t a, int64_t x, int mod) {
    int res = 1;
    while (x) {
        if (x & 1) res = modMul(res, a, mod);
        a = modMul(a, a, mod);
        x >>= 1;
    }
    return res;
}
```

2.2 Modnum

```cpp
#pragma once

#include <bits/stdc++.h>
#include "mod.hpp"

using namespace std;

template <typename T, int md>
struct Modnum {
    using M = Modnum;
    T v;
    Modnum(T _v=0) : v(fix(_v)) {}

    T fix(int64_t x) {
        if (x < -md || x > 2 * md) x %= md;
        if (x >= md) x -= md;
        if (x < 0) x += md;
        return x;
    }
```

```
    M operator+(M o) { return M(v + o.v); }
    M operator-(M o) { return M(v - o.v); }
    M operator*(M o) { return M(fix((int64_t) v * o.v)); }
    M operator/(M o) {
        return *this * modInv(o.v, md);
    }
    M pow(int64_t x) {
        M a(v);
        M res(1);
        while (x) {
            if (x & 1) res = res * a;
            a = a * a;
            x >>= 1;
        }
        return res;
    }
    friend istream& operator>>(istream& is, M& o) {
        is >> o.v; o.v = o.fix(o.v); return is;
    }
    friend ostream& operator<<(ostream& os, const M& o) {
        return os << o.v;
    }
};
```

2.3 Sieve of Eratosthenes

```
#pragma once

#include <bits/stdc++.h>

using namespace std;

/// Sieve of Eratosthenes
/// Benchmark: 3314 ms/188.74 Mib for N = 5 * 1e8
/// Credit: KTH's notebook

namespace eratosthenes {
    constexpr int MAX_N = (int) 5 * 1e8;
    bitset<MAX_N + 1> is_prime;
    vector<int> primes;

    void sieve(int N) {
        is_prime.set();
        is_prime[0] = is_prime[1] = 0;

        for (int i = 4; i <= N; i += 2) is_prime[i] = 0;

        for (int i = 3; i * i <= N; i += 2) {
            if (!is_prime[i]) continue;
            for (int j = i * i; j <= N; j += i * 2) {
                is_prime[j] = 0;
            }
        }

        for (int i = 2; i <= N; i++) {
            if (is_prime[i]) primes.push_back(i);
        }
    }
}
```

2.4 Primality Test

```
// Simple primality test

#pragma once

#include <bits/stdc++.h>

template <typename T>
bool isPrime(T x) {
    for (T d = 2; d * d <= x; d++) {
        if (x % d == 0) return false;
    }
    return true;
}
```

2.5 Euclidean Algorithm

```
#pragma once

#include <bits/stdc++.h>

using namespace std;

template <typename T>
T gcd(T a, T b) {
    if (a < b) swap(a, b);
    while (b != 0) {
        int r = a % b;
```

```
        a = b;
        b = r;
    }
    return a;
}

template <typename T>
int64_t lcm(T a, T b) {
    return (int64_t) a / gcd(a, b) * b;
}
```

2.6 Extended Euclidean Algorithm

```
#pragma once

#include "mod.hpp"

// This solves the equation ax + by = gcd(a, b)
// Input: a, b
// Output: g (returned), x, y (passed by ref)
int64_t extGcd(int64_t a, int64_t b, int64_t& x, int64_t& y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    int64_t x1, y1;
    int64_t g = extGcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    assert(g == 1);
    return g;
}
```

3 Geometry

3.1 Points

```
#pragma once

#include <bits/stdc++.h>
#include "geoutil.hpp"

using namespace std;

template<typename T>
struct Point {
    using P = Point;
    T x, y;

    Point(T x_ = 0, T y_ = 0) : x(x_), y(y_) {}

    P operator+(const P &o) const { return P(x + o.x, y + o.y); }

    P operator-(const P &o) const { return P(x - o.x, y - o.y); }

    P operator*(T d) const { return P(x * d, y * d); }

    P operator/(T d) const { return P(x / d, y / d); }

    T dot(P o) const { return x * o.x + y * o.y; }

    T cross(P o) const { return x * o.y - y * o.x; }

    T abs2() const { return x * x + y * y; }

    long double abs() const { return sqrt((long double) abs2()); }

    double angle() const { return atan2(y, x); } // [-π, π]
    P unit() const { return *this / abs(); } // makes abs()=1
    P perp() const { return P(-y, x); } // rotates +π/2

    P rotate(double a) const { // ccw
        return P(x * cos(a) - y * sin(a), x * sin(a) + y * cos(a));
    }

    friend istream &operator>>(istream &is, P &p) {
        return is >> p.x >> p.y;
    }

    friend ostream &operator<<(ostream &os, P &p) {
        return os << "(" << p.x << ", " << p.y << ")";
    }

    // position of c relative to a->b
    // > 0: c is on the left of a->b
    friend T orient(P a, P b, P c) {
        return (b - a).cross(c - a);
    }

    // Check if u⃗ and v⃗ are parallel
```

```
    // (u = cv) where c ∈ R)
    friend bool parallel(P u, P v) {
        return u.cross(v) == 0;
    }

    // Check if point p lies on the segment ab
    friend bool onSegment(P a, P b, P p) {
        return orient(a, b, p) == 0 &&
               min(a.x, b.x) <= p.x &&
               max(a.x, b.x) >= p.x &&
               min(a.y, b.y) <= p.y &&
               max(a.y, b.y) >= p.y;
    }

    friend bool boundingBox(P p1, P q1, P p2, P q2) {
        if (max(p1.x, q1.x) < min(p2.x, q2.x)) return true;
        if (max(p1.y, q1.y) < min(p2.y, q2.y)) return true;
        if (max(p2.x, q2.x) < min(p1.x, q1.x)) return true;
        if (max(p2.x, q2.x) < min(p1.x, q1.x)) return true;
        return false;
    }

    friend bool intersect(P p1, P p2, P p3, P p4) {
        // Check if two segments are parallel
        if (parallel(p2 - p1, p4 - p3)) {
            // Check if 4 ps are colinear
            if (!parallel(p2 - p1, p3 - p1)) return false;
            if (boundingBox(p1, p2, p3, p4)) return false;
            return true;
        }

        // check if one line is completely on one side of the other
        for (int i = 0; i < 2; i++) {
            if (sgn((p2 - p1).cross(p3 - p1)) == sgn((p2 - p1).cross(p4 - p1))
                && sgn((p2 - p1).cross(p3 - p1)) != 0) {
                return false;
            }
            swap(p1, p3);
            swap(p2, p4);
        }
        return true;
    }

    // Check if p is in ∠bac (including the rays)
    friend bool inAngle(P a, P b, P c, P p) {
        assert(orient(a, b, c) != 0);
        if (orient(a, b, c) < 0) swap(b, c);
        return orient(a, b, p) >= 0 && orient(a, c, p) <= 0;
    }

    // Angle ∠bac (+/-)
    friend double directedAngle(P a, P b, P c) {
        if (orient(a, b, c) >= 0) {
            return (b - a).angle(c - a);
        }
        return 2 * PI - (b - a).angle(c - a);
    }
};
```