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## 1 Template

### 1.1 Makefile

```
1 BASIC := -std=c++11 -Wall -Wextra -Wshadow -g -DLOCAL
2 VERBOSE := -fsanitize=address -fsanitize=undefined
   ↳ -D_GLIBCXX_DEBUG
3
4 main: main.cc
5     g++ $(BASIC) $(VERBOSE) $< -o $@
```

### 1.2 vimrc

```
1 filetype plugin indent on
2 set nu rnu
3 set ai ts=4 shiftwidth=4 sts=4 et
4 set spr sb
5 set clipboard=unnamed,unnamedplus
```

## 2 Graph

### 2.1 Dijkstra

```
1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 struct Edge {
6     int u, v, w;
7     Edge(int u=-1, int v=-1, int w=-1) : u(u), v(v),
   ↳ w(w) {}
8 };
9
10 struct Node {
11     int u;
12     int64_t d;
13     Node(int u_, int64_t d_) : u(u_), d(d_) {}
14     bool operator<(const Node& o) const {
15         return d > o.d; // min-heap
16     }
17 };
18
19 struct Graph {
20     const int64_t inf = 1e18;
21     int n;
22     vector<vector<Edge>> adj;
23     vector<int64_t> dist;
24     vector<Edge> trace; // trace[u]: last edge to get to
   ↳ u from s
25
26     Graph(int n_) : n(n_), adj(n), dist(n, inf),
27         trace(n) {}
28
29     void addEdge(int u, int v, int w) {
30         adj[u].emplace_back(u, v, w);
31     }
32 }
```

```
33 int64_t dijkstra(int s, int t) {
34     priority_queue<Node> pq;
35     pq.emplace(s, 0);
36     dist[s] = 0;
37
38     while (!pq.empty()) {
39         Node cur = pq.top(); pq.pop();
40         int u = cur.u;
41         int64_t d = cur.d;
42
43         if (u == t) return dist[t];
44         if (d > dist[u]) continue;
45
46         for (const Edge& e : adj[u]) {
47             int v = e.v;
48             int w = e.w;
49             if (dist[u] + w < dist[v]) {
50                 dist[v] = dist[u] + w;
51                 trace[v] = e;
52                 pq.emplace(v, dist[v]);
53             }
54         }
55     }
56
57     return inf;
58 }
59
60 vector<Edge> getShortestPath(int s, int t) {
61     assert(dist[t] != inf);
62     vector<Edge> path;
63     int v = t;
64     while (v != s) {
65         Edge e = trace[v];
66         path.push_back(e);
67         v = e.u;
68     }
69     reverse(path.begin(), path.end());
70     return path;
71 }
72 };
73
74 int main() {
75     int n, m, s, t;
76     cin >> n >> m >> s >> t;
77
78     Graph g(n);
79 }
```

```

80
81     for (int i = 0; i < m; i++) {
82         int u, v, w;
83         cin >> u >> v >> w;
84         g.addEdge(u, v, w);
85     }
86
87     int64_t dist = g.dijkstra(s, t);
88
89     if (dist != g.inf) {
90         vector<Edge> path = g.getShortestPath(s, t);
91         cout << dist << ' ' << path.size() << '\n';
92         for (Edge e : path) cout << e.u << ' ' << e.v <<
            ↪ '\n';
93     } else {
94         cout << "-1\n";
95     }
96
97     return 0;
98 }

```

---

```

20     }
21
22     int modMul(int a, int b, int mod) {
23         int64_t res = (int64_t) a * b;
24         return (int) (res % mod);
25     }
26
27     int64_t binPow(int64_t a, int64_t x) {
28         int64_t res = 1;
29         while (x) {
30             if (x & 1) res *= a;
31             a *= a;
32             x >>= 1;
33         }
34         return res;
35     }
36
37     int64_t modPow(int64_t a, int64_t x, int mod) {
38         int res = 1;
39         while (x) {
40             if (x & 1) res = modMul(res, a, mod);
41             a = modMul(a, a, mod);
42             x >>= 1;
43         }
44         return res;
45     }

```

---

### 3.2 Modnum

```

1     #pragma once
2
3     #include <bits/stdc++.h>
4     #include "mod.hpp"
5
6     using namespace std;
7
8     template <typename T, int md>
9     struct Modnum {
10         using M = Modnum;
11         T v;
12         Modnum(T _v=0) : v(fix(_v)) {}
13
14         T fix(int64_t x) {
15             if (x < -md || x > 2 * md) x %= md;
16             if (x >= md) x -= md;
17             if (x < 0) x += md;

```

```

18         return x;
19     }
20
21     M operator+(M o) { return M(v + o.v); }
22     M operator-(M o) { return M(v - o.v); }
23     M operator*(M o) { return M(fix((int64_t) v * o.v));
        ↪ }
24     M operator/(M o) {
25         return *this * modInv(o.v, md);
26     }
27     M pow(int64_t x) {
28         M a(v);
29         M res(1);
30         while (x) {
31             if (x & 1) res = res * a;
32             a = a * a;
33             x >>= 1;
34         }
35         return res;
36     }
37     friend istream& operator>>(istream& is, M& o) {
38         is >> o.v; o.v = o.fix(o.v); return is;
39     }
40     friend ostream& operator<<(ostream& os, const M& o) {
41         return os << o.v;
42     }
43 };

```

---

## 3 Maths

### 3.1 Modular Arithmetic

```

1     // **Really important note**: inputs of the modAdd,
        ↪ modSub, and modMul
2     // functions must all be normalized (within the range
        ↪ [0..mod - 1]) before use
3
4     #pragma once
5
6     #include <bits/stdc++.h>
7
8     using namespace std;
9
10    int modAdd(int a, int b, int mod) {
11        a += b;
12        if (a >= mod) a -= mod;
13        return a;
14    }
15
16    int modSub(int a, int b, int mod) {
17        a -= b;
18        if (a < 0) a += mod;
19        return a;

```

### 3.3 Sieve of Eratosthenes

```

1     #include <bits/stdc++.h>
2
3     using namespace std;
4
5     /// Sieve of Eratosthenes
6     /// Benchmark: 3314 ms/188.74 Mib for N = 5 * 1e8
7     /// Credit: KTH's notebook
8     constexpr int MAX_N = (int) 5 * 1e8;
9     bitset<MAX_N + 1> is_prime;
10    vector<int> primes;
11
12    void sieve(int N) {
13        is_prime.set();
14        is_prime[0] = is_prime[1] = 0;
15

```

```

16     for (int i = 4; i <= N; i += 2) is_prime[i] = 0;
17
18     for (int i = 3; i * i <= N; i += 2) {
19         if (!is_prime[i]) continue;
20         for (int j = i * i; j <= N; j += i * 2) {
21             is_prime[j] = 0;
22         }
23     }
24
25     for (int i = 2; i <= N; i++) {
26         if (is_prime[i]) primes.push_back(i);
27     }
28 }
29
30 // https://judge.yosupo.jp/problem/enumerate_primes
31 int main() {
32     int N, a, b;
33     cin >> N >> a >> b;
34     sieve(N);
35     int num_primes = primes.size();
36     vector<int> res;
37
38     for (int j = 0; a * j + b < num_primes; j++) {
39         res.push_back(primes[a * j + b]);
40     }
41
42     cout << num_primes << ' ' << res.size() << '\n';
43
44     for (int p : res) {
45         cout << p << ' ';
46     }
47     cout << '\n';
48 }

```

### 3.4 Primality Test

```

1 // Simple primality test
2
3 #pragma once
4
5 #include <bits/stdc++.h>
6
7 template <typename T>
8 bool isPrime(T x) {
9     for (T d = 2; d * d <= x; d++) {
10         if (x % d == 0) return false;

```

```

11     }
12     return true;
13 }

```

### 3.5 Euclidean Algorithm

```

1 #pragma once
2
3 #include <bits/stdc++.h>
4
5 using namespace std;
6
7 template <typename T>
8 T gcd(T a, T b) {
9     if (a < b) swap(a, b);
10    while (b != 0) {
11        int r = a % b;
12        a = b;
13        b = r;
14    }
15    return a;
16 }
17
18 template <typename T>
19 int64_t lcm(T a, T b) {
20     return (int64_t) a / gcd(a, b) * b;
21 }

```

### 3.6 Extended Euclidean Algorithm

```

1 #pragma once
2
3 #include "mod.hpp"
4
5 // This solves the equation ax + by = gcd(a, b)
6 // Input: a, b
7 // Output: g (returned), x, y (passed by ref)
8 int64_t extGcd(int64_t a, int64_t b, int64_t& x, int64_t&
    ↪ y) {
9     if (b == 0) {
10         x = 1;
11         y = 0;
12         return a;
13     }
14     int64_t x1, y1;

```

```

15     int64_t g = extGcd(b, a % b, x1, y1);
16     x = y1;
17     y = x1 - y1 * (a / b);
18     assert(g == 1);
19     return g;
20 }

```

### 3.7 Euler's Totient Function

```

1 #pragma once
2
3 #include <bits/stdc++.h>
4
5 using namespace std;
6
7 // Euler's totient function
8 //  $\phi(i)$  = number of coprime numbers of  $n$  in the range
    ↪  $[1..n]$ 
9 // Multiplicative property:  $\phi(a * b) = \phi(a) * \phi(b)$ 
10 // Complexity:  $O(\sqrt{n})$ 
11 int eulerPhi(int n) {
12     int res = n;
13     for (int i = 2; i * i <= n; i++) {
14         if (n % i == 0) {
15             while (n % i == 0) {
16                 n /= i;
17             }
18             res -= res / i;
19         }
20     }
21     if (n > 1) {
22         res -= res / n;
23     }
24     return res;
25 }
26
27 // Complexity:  $O(n \log \log(n))$ 
28 vector<int> eulerPhiN(int n) {
29     vector<int> phi(n + 1);
30     phi[0] = 0;
31     phi[1] = 1;
32
33     for (int i = 2; i <= n; i++) phi[i] = i;
34
35     for (int i = 2; i <= n; i++) {

```

```

36     if (phi[i] == i) {
37         for (int j = i; j <= n; j += i) {
38             phi[j] -= phi[j] / i;
39         }
40     }
41 }
42
43 return phi;
44 }

```

## 4 Strings

### 4.1 Trie

```

1  #pragma once
2
3  #include <bits/stdc++.h>
4
5  using namespace std;
6
7  struct Trie {
8      const int ALPHA = 26;
9      vector<vector<int>> trie;
10     vector<int> eow;
11
12     int ord(char c) { return c - 'a'; }
13
14     Trie() {
15         trie.emplace_back(ALPHA, -1);
16         eow.push_back(0);
17     }
18
19     void add(const string& word) {
20         int node = 0;
21
22         for (char c : word) {
23             int x = ord(c);
24
25             if (trie[node][x] == -1) {
26                 trie[node][x] = trie.size();
27                 trie.emplace_back(ALPHA, -1);
28                 eow.push_back(0);
29             }
30
31             node = trie[node][x];
32             eow[node]++;

```

```

33     }
34 }
35 };

```

### 4.2 Z function

```

1  #pragma once
2
3  #include <bits/stdc++.h>
4
5  using namespace std;
6
7  // z[i]: length of the longest common prefix between s
8  // and
9  // its substrings starting at i
10 vector<int> zFunction(const string& s) {
11     int n = s.length();
12     vector<int> z(n);
13     z[0] = n;
14     int l = 0;
15     int r = 0;
16
17     for (int i = 1; i < n; i++) {
18         if (i <= r) {
19             z[i] = min(z[i - l], r - i + 1);
20         }
21         while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
22             z[i]++;
23         }
24         if (i + z[i] - 1 > r) {
25             l = i;
26             r = i + z[i] - 1;
27         }
28     }
29     return z;
30 }

```

### 4.3 Suffix Array

```

1  #include <bits/stdc++.h>
2
3  using namespace std;
4

```

```

5  // sa[i] = the starting index of the ith suffix (starting
6  // at 0)
7  // sorted in lexicographic order
8  vector<int> suffix_array(const string& s_, int alpha=256)
9  {
10     string s = s_ + '\0';
11     int n = s.size();
12     vector<int> p(n);
13     vector<int> cnt(max(alpha, n), 0);
14
15     for (int i = 0; i < n; i++) cnt[s[i]]++;
16     for (int i = 1; i < alpha; i++) cnt[i] += cnt[i - 1];
17     for (int i = 0; i < n; i++) p[--cnt[s[i]]] = i;
18
19     vector<int> g(n);
20     g[p[0]] = 0;
21
22     for (int i = 1; i < n; i++) {
23         g[p[i]] = g[p[i - 1]] + (s[p[i]] != s[p[i - 1]]);
24     }
25
26     vector<int> pn(n);
27     vector<int> gn(n);
28
29     for (int len = 1; len < n; len <= 1) {
30         for (int i = 0; i < n; i++) {
31             pn[i] = p[i] - len; // transfer the pos from
32             // second to pair
33             if (pn[i] < 0) pn[i] += n; // cyclic
34         }
35
36         int num_groups = g[p[n - 1]] + 1;
37         fill(cnt.begin(), cnt.begin() + num_groups, 0);
38
39         // Radix sort
40         for (int i = 0; i < n; i++) cnt[g[pn[i]]]++;
41         for (int i = 1; i < num_groups; i++) cnt[i] +=
42             cnt[i - 1];
43         for (int i = n - 1; i >= 0; i--)
44             p[--cnt[g[pn[i]]]] = pn[i];
45         gn[p[0]] = 0;
46
47         for (int i = 1; i < n; i++) {
48             pair<int, int> prev, cur;
49             prev.first = g[p[i - 1]];
50             cur.first = g[p[i]];

```

```

46     prev.second = g[p[i - 1] + len - (p[i - 1] +
    ↪ len >= n ? n : 0)];
47     cur.second = g[p[i] + len - (p[i] + len >= n
    ↪ ? n : 0)];
48     gn[p[i]] = gn[p[i - 1]] + (cur != prev);
49 }
50 g.swap(gn);
51 }
52 p.erase(p.begin());
53 return p;
54 }

```

## 5 Flows

### 5.1 Dinic Max Flow

```

1  #pragma once
2
3  #include <bits/stdc++.h>
4
5  using namespace std;
6
7  // Dinic algorithm for max flow
8  // This version should work on flow graph with float
   ↪ capacities
9  // Time complexity:  $O(|V|^2|E|)$ 
10
11 template <typename T>
12 struct FlowEdge {
13     int u, v;
14     T c, f;
15
16     FlowEdge(int _u, int _v, T _c, T _f) :
17         u(_u), v(_v), c(_c), f(_f) {}
18 };
19
20 template <typename T>
21 struct Dinic {
22     static constexpr T inf = numeric_limits<T>::max();
23     static constexpr T eps = (T) 1e-9;
24     int n;
25     int s, t;
26     vector<vector<int>> adj; // stores indices of edges
27     vector<int> level;      // shortest distance from
    ↪ source

```

```

28     vector<int> ptr;        // points to the next edge
    ↪ which can be used
29     vector<FlowEdge<T>> edges;
30
31     Dinic(int _n, int _s, int _t)
32         : n(_n), s(_s), t(_t), adj(_n), level(_n),
    ↪ ptr(_n) {}
33
34     void addEdge(int u, int v, int c, int rc=0) {
35         int eid = (int) edges.size();
36         adj[u].push_back(eid);
37         adj[v].push_back(eid + 1);
38         edges.emplace_back(u, v, c, 0);
39         edges.emplace_back(v, u, rc, 0);
40     }
41
42     bool bfs() {
43         fill(level.begin(), level.end(), -1);
44         level[s] = 0;
45         queue<int> q;
46         q.push(s);
47
48         while (!q.empty()) {
49             int u = q.front();
50             q.pop();
51
52             for (int eid : adj[u]) {
53                 const auto& e = edges[eid];
54                 if (e.c - e.f <= eps || level[e.v] != -1)
    ↪ continue;
55                 level[e.v] = level[u] + 1;
56                 q.push(e.v);
57             }
58         }
59
60         return level[t] != -1;
61     }
62
63     T dfs(int u, T flow) {
64         if (u == t) return flow;
65
66         for (int& j = ptr[u]; j < (int) adj[u].size();
    ↪ j++) {
67             int eid = adj[u][j];
68             const auto& e = edges[eid];
69             if (e.c - e.f > eps && level[e.v] == level[u]
    ↪ + 1) {

```

```

70                 T df = dfs(e.v, min(e.c - e.f, flow));
71                 if (df > eps) {
72                     edges[eid].f += df;
73                     edges[eid ^ 1].f -= df;
74                     return df;
75                 }
76             }
77         }
78         return 0;
79     }
80 }
81
82     T maxFlow() {
83         T f = 0;
84
85         while (bfs()) {
86             fill(ptr.begin(), ptr.end(), 0);
87             T total_df = 0;
88             while (true) {
89                 T df = dfs(s, inf);
90                 if (df <= eps) break;
91                 total_df += df;
92             }
93             if (total_df <= eps) break;
94             f += total_df;
95         }
96         return f;
97     }
98 };
99

```

## 6 Matching

### 6.1 Hopcroft-Karp Bipartite Matching

```

1  #pragma once
2
3  #include <bits/stdc++.h>
4
5  using namespace std;
6
7  #pragma once
8
9  // Bipartite matching. Vertices from both halves start
    ↪ from 0
10 // Time complexity:  $O(\sqrt{|V|}|E|)$ 

```

```

11 struct HopcroftKarp {
12     const int INF = (int) 1e9;
13     int nu;
14     int nv;
15     vector<vector<int>> adj;
16     vector<int> layer;
17     vector<int> u_mate;
18     vector<int> v_mate;
19
20     HopcroftKarp(int nu, int nv) : nu(nu), nv(nv) {
21         adj.resize(nu);
22         layer.resize(nu);
23         u_mate.resize(nu, -1);
24         v_mate.resize(nv, -1);
25     }
26
27     void addEdge(int u, int v) {
28         adj[u].push_back(v);
29     }
30
31     bool bfs() {
32         // Find all possible augmenting paths
33         queue<int> q;
34
35         for (int u = 0; u < nu; u++) {
36             // Consider only unmatched edges
37             if (u_mate[u] == -1) {
38                 layer[u] = 0;
39                 q.push(u);
40             } else {
41                 layer[u] = INF;
42             }
43         }
44
45         bool has_path = false;
46
47         while (!q.empty()) {
48             int u = q.front();
49             q.pop();
50
51             for (int &v : adj[u]) {
52                 if (v_mate[v] == -1) {
53                     has_path = true;
54                 } else if (layer[v_mate[v]] == INF) {
55                     layer[v_mate[v]] = layer[u] + 1;
56                     q.push(v_mate[v]);
57                 }

```

```

58             }
59         }
60
61         return has_path;
62     }
63
64     bool dfs(int u) {
65         if (layer[u] == INF) return false;
66
67         for (int v : adj[u]) {
68             if ((v_mate[v] == -1) ||
69                 (layer[v_mate[v]] == layer[u] + 1 &&
70                  ↪ dfs(v_mate[v]))) {
71                 v_mate[v] = u;
72                 u_mate[u] = v;
73                 return true;
74             }
75
76             return false;
77         }
78
79         vector<pair<int, int>> maxMatching() {
80             int matching = 0;
81
82             while (bfs()) { // there is at least 1 augmenting
83                 ↪ path
84                 for (int u = 0; u < nu; u++) {
85                     if (u_mate[u] == -1 && dfs(u)) {
86                         ++matching;
87                     }
88                 }
89
90                 vector<pair<int, int>> res;
91
92                 for (int u = 0; u < nu; u++) {
93                     if (u_mate[u] == -1) continue;
94                     res.emplace_back(u, u_mate[u]);
95                 }
96                 assert(res.size() == matching);
97                 return res;
98             }
99 };

```

## 7 Geometry

### 7.1 Utility

---

```

1  #pragma once
2
3  #include <bits/stdc++.h>
4
5  using namespace std;
6
7  const double PI = acos(-1);
8
9  template <typename T>
10 int sgn(T x) {
11     if (x > 0) return 1;
12     if (x < 0) return -1;
13     return 0;
14 }
15
16 int inc(int i, int n, int by=1) {
17     i += by;
18     if (i >= n) i -= n;
19     return i;
20 }
21
22 double degToRad(double d) {
23     return d * PI / 180.0;
24 }
25
26 double radToDeg(double r) {
27     return r * 180.0 / PI;
28 }

```

---

### 7.2 Point

---

```

1  #pragma once
2
3  #include <bits/stdc++.h>
4  #include "geoutil.hpp"
5
6  using namespace std;
7
8
9  template<typename T>
10 struct Point {
11     using P = Point;

```

```

12 T x, y;
13
14 Point(T x_ = 0, T y_ = 0) : x(x_), y(y_) {}
15 P operator+(const P &o) const { return P(x + o.x, y +
↪ o.y); }
16 P operator-(const P &o) const { return P(x - o.x, y -
↪ o.y); }
17 P operator*(T d) const { return P(x * d, y * d); }
18 P operator/(T d) const { return P(x / d, y / d); }
19 T dot(P o) const { return x * o.x + y * o.y; }
20 T cross(P o) const { return x * o.y - y * o.x; }
21 T abs2() const { return x * x + y * y; }
22 long double abs() const { return sqrt((long double)
↪ abs2()); }
23 double angle() const { return atan2(y, x); } //
↪  $[-\pi, \pi]$ 
24 P unit() const { return *this / abs(); } // makes
↪  $abs()=1$ 
25 P perp() const { return P(-y, x); } // rotates  $+\pi/2$ 
26
27 P rotate(double a) const { // ccw
28     return P(x * cos(a) - y * sin(a), x * sin(a) + y
↪ * cos(a));
29 }
30
31 friend istream &operator>>(istream &is, P &p) {
32     return is >> p.x >> p.y;
33 }
34
35 friend ostream &operator<<(ostream &os, P &p) {
36     return os << "(" << p.x << ", " << p.y << ")";
37 }
38
39 // position of c relative to a->b
40 // > 0: c is on the left of a->b
41 friend T orient(P a, P b, P c) {
42     return (b - a).cross(c - a);
43 }
44
45 // Check if  $\vec{u}$  and  $\vec{v}$  are parallel
46 // ( $\vec{u} = c\vec{v}$ ) where  $c \in R$ )
47 friend bool parallel(P u, P v) {
48     return u.cross(v) == 0;
49 }
50
51 // Check if point p lies on the segment ab
52 friend bool onSegment(P a, P b, P p) {

```

```

53     return orient(a, b, p) == 0 &&
54         min(a.x, b.x) <= p.x &&
55         max(a.x, b.x) >= p.x &&
56         min(a.y, b.y) <= p.y &&
57         max(a.y, b.y) >= p.y;
58 }
59
60 friend bool boundingBox(P p1, P q1, P p2, P q2) {
61     if (max(p1.x, q1.x) < min(p2.x, q2.x)) return
↪ true;
62     if (max(p1.y, q1.y) < min(p2.y, q2.y)) return
↪ true;
63     if (max(p2.x, q2.x) < min(p1.x, q1.x)) return
↪ true;
64     if (max(p2.y, q2.y) < min(p1.y, q1.y)) return
↪ true;
65     return false;
66 }
67
68 friend bool intersect(P p1, P p2, P p3, P p4) {
69     // Check if two segments are parallel
70     if (parallel(p2 - p1, p4 - p3)) {
71         // Check if 4 ps are colinear
72         if (!parallel(p2 - p1, p3 - p1)) return
↪ false;
73         if (boundingBox(p1, p2, p3, p4)) return
↪ false;
74         return true;
75     }
76
77     // check if one line is completely on one side of
↪ the other
78     for (int i = 0; i < 2; i++) {
79         if (sgn(orient(p1, p2, p3)) == sgn(orient(p1,
↪ p2, p4))
80             && sgn(orient(p1, p2, p3)) != 0) {
81             return false;
82         }
83         swap(p1, p3);
84         swap(p2, p4);
85     }
86     return true;
87 }
88
89 // Check if p is in  $\angle bac$  (including the rays)
90 friend bool inAngle(P a, P b, P c, P p) {
91     assert(orient(a, b, c) != 0);

```

```

92     if (orient(a, b, c) < 0) swap(b, c);
93     return orient(a, b, p) >= 0 && orient(a, c, p) <=
↪ 0;
94 }
95
96 // Angle  $\angle bac$  (+/-)
97 friend double directedAngle(P a, P b, P c) {
98     if (orient(a, b, c) >= 0) {
99         return (b - a).angle(c - a);
100     }
101     return 2 * PI - (b - a).angle(c - a);
102 }
103 };

```

## 7.3 Polygon

```

1 #pragma once
2
3 #include <bits/stdc++.h>
4 #include "point.hpp"
5 #include "geoutil.hpp"
6 #include "../maths/euclidean.hpp"
7
8 using namespace std;
9
10 template <typename T>
11 struct Polygon {
12     using P = Point<T>;
13
14     int n = 0;
15     vector<P> ps;
16     Polygon() : n(0) {}
17     Polygon(vector<P>& ps) : n(ps.size()), ps(ps) {}
18
19     void add(P p) {
20         ps.push_back(p);
21         n++;
22     }
23
24     int64_t twiceArea() {
25         int64_t area = 0;
26         for (int i = 0; i < n; i++) {
27             P p1 = ps[i];
28             P p2 = ps[inc(i, n)];
29             area += p1.cross(p2);

```

```
30     }
31     return abs(area);
32 }
33
34 double area() {
35     return twiceArea() / 2.0;
36 }
37
38 int64_t boundaryLattice() {
39     int64_t res = 0;
40     for (int i = 0; i < n; i++) {
41         int j = i + 1; if (j == n) j = 0;
42         P p1 = ps[i];
43         P p2 = ps[j];
44         P v = p2 - p1;
45         res += gcd(abs(v.x), abs(v.y));
46     }
47     return res;
48 }
49
50 int64_t interiorLattice() {
51     return (twiceArea() - boundaryLattice()) / 2 + 1;
52 }
53
54 bool isConvex() {
55     int pos = 0;
56     int neg = 0;
57
58     for (int i = 0; i < n; i++) {
59         P p1 = ps[i];
60         P p2 = ps[inc(i, n, 1)];
61         P p3 = ps[inc(i, n, 2)];
62         int o = orient(p1, p2, p3);
63         if (o > 0) pos = 1;
64         if (o < 1) neg = 1;
65     }
66
67     return pos ^ neg;
68 }
69
70 // -1: outside; 1: inside; 0: on boundary
71 int vsPoint(P r) {
72     int crossing = 0;
73     for (int i = 0; i < n; i++) {
74         P p1 = ps[i];
75         P p2 = ps[inc(i, n)];
76         if (onSegment(p1, p2, r)) {
```

```
77             return 0;
78         }
79         if (((p2.y >= r.y) - (p1.y >= r.y)) *
80             ↪ orient(r, p1, p2) > 0) {
81             crossing++;
82         }
83     }
84     if (crossing & 1) return 1;
85     return -1;
86 };
87
88 template <typename T>
89 Polygon<T> convexHull(vector<Point<T>> points) {
90     using P = Point<T>;
91
92     sort(points.begin(), points.end(),
93          [](const P& p1, const P& p2) {
94             if (p1.x == p2.x) return p1.y < p2.y;
95             return p1.x < p2.x;
96         });
97
98     vector<P> hull;
99
100     for (int step = 0; step < 2; step++) {
101         int s = hull.size();
102         for (const P& c : points) {
103             while ((int) hull.size() - s >= 2) {
104                 P a = hull.end()[-2];
105                 P b = hull.end()[-1];
106                 // <= if points on the edges are
107                 ↪ accepted, < otherwise
108                 if (orient(a, b, c) <= 0) break;
109                 hull.pop_back();
110             }
111             hull.push_back(c);
112         }
113         hull.pop_back();
114         reverse(points.begin(), points.end());
115     }
116
117     return Polygon<T>(hull);
118 }
```

8 C++ STL

8.1 vector

Underlying implementation: dynamic array

Method	Complexity
size_t size()	$O(1)$
void push_back(T v)	$O(1)$
void emplace_back(Args args...)	$O(1)$
void pop_back()	$O(1)$
T back()	$O(1)$
void erase(iterator position)	$O(n)$

- Resize (values in vector stay unchanged): v.resize(n)
- Resize and fill: v.assign(n, val)
- Fill: fill(v.begin(), v.end(), val)
- Reverse: reverse(v.begin(), v.end())
- Pythonic get element backwards:
  - v.end()[-1]: last element
  - v.end()[-2]: second-last element
- Sort ():

```
1 // by default: non-decreasing, v must be of
  ↪ comparator type
2 sort(v.begin(), v.end());
3 // custom comparator
4 sort(v.begin(), v.end(), [](const Obj& o1, const
  ↪ Obj& o2) {
5     return o1.x < o2.x;
6 });
```

8.2 set

Condition: must be of a comparable type (define the < operator).  
Underlying implementation: self-balancing BST

Method	Complexity
size_t size()	$O(1)$
void insert(T v)	$O(1)$
void emplace(Args args...)	$O(1)$
iterator find(T v)	$O(\log(n))$
void erase(iterator position)	$O(\log(n))$

- Check if an element v is in set s: if (s.find(v) != s.end())
- Get minimum element: \*(m.begin())
- Get maximum element: \*(m.rbegin())



8.3 map

**Condition:** key must be of a comparable type (define the < operator).

**Underlying implementation:** self-balancing BST

Method	Complexity
size_t size()	$O(1)$
void insert(pair<K, V> keyvalpair)	$O(1)$
void emplace(K key, V value)	$O(1)$
iterator find(T v)	$O(\log(n))$
void erase(iterator position)	$O(\log(n))$

- Check if a key k is in map m: if (m.find(k) != m.end())
- Get value of key k in map m: m[k] or m.find(k)→second
- Get minimum key-value pair: \*(m.begin())
- Get key of minimum pair: m.begin()→first
- Get value of minimum pair: m.begin()→second
- Get maximum key-value pair: \*(m.rbegin())
- Get key of maximum pair: m.rbegin()→first
- Get value of maximum pair: m.rbegin()→second

8.4 unordered\_set and unordered\_map

**Underlying implementation:** hash table

**Note:** stay always from these unless you know what you are doing.

There are scenarios where you think these can be faster than set and map, but either:

- The speed-up it will be negligible
- It will actually be unexpectedly slower

**Operations:** pretty much share the same interface with set and map, except for things that require order.

8.5 pair

Lexicographically comparable

8.6 string

- Mutable: s[0] = 'a' is OK.
- Concatenation:
  - s += 'a' takes  $O(1)$ !
  - s += t takes  $O(\text{length}(t))$
- Substring:
  - s.substr(i) returns suffix starting from i
  - s.substr(i, 3) returns suffix starting from i of maximum length 3 (can be shorter if reaches end)

8.7 Other useful utilities

min(x, y), max(x, y), swap(x, y)