

# Veriopt Theories

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## 1 Operator Semantics

```
theory Values
  imports
    HOL-Library.Word
```

*HOL-Library.Signed-Division*  
*HOL-Library.Float*  
*HOL-Library.LaTeXsugar*

**begin**

In order to properly implement the IR semantics we first introduce a type that represents runtime values. These runtime values represent the full range of primitive types currently allowed by our semantics, ranging from basic integer types to object references and arrays.

Note that Java supports 64, 32, 16, 8 signed ints, plus 1 bit (boolean) ints, and char is 16-bit unsigned. E.g. an 8-bit stamp has a default range of -128..+127. And a 1-bit stamp has a default range of -1..0, surprisingly.

During calculations the smaller sizes are sign-extended to 32 bits, but explicit widening nodes will do that, so most binary calculations should see equal input sizes.

An object reference is an option type where the *None* object reference points to the static fields. This is examined more closely in our definition of the heap.

**type-synonym** *int64* = 64 word — long  
**type-synonym** *int32* = 32 word — int  
**type-synonym** *int16* = 16 word — short  
**type-synonym** *int8* = 8 word — char  
**type-synonym** *int1* = 1 word — boolean

**abbreviation** *valid-int-widths* :: nat set **where**  
*valid-int-widths* ≡ {1, 8, 16, 32, 64}

**experiment begin**

Option 2: explicit width stored with each integer value. However, this does not help us to distinguish between short (signed) and char (unsigned).

**typedef** *IntWidth* = { *w* :: nat . *w*=1 ∨ *w*=8 ∨ *w*=16 ∨ *w*=32 ∨ *w*=64 }  
**by** *blast*

**setup-lifting** *type-definition-IntWidth*

**lift-definition** *IntWidthBits* :: *IntWidth* ⇒ nat  
**is** λ*w*. *w* .  
**end**

**experiment begin**

Option 3: explicit type stored with each integer value.

**datatype** *IntType* = *ILong* | *IInt* | *IShort* | *IChar* | *IByte* | *IBoolean*

```

fun int-bits :: IntType  $\Rightarrow$  nat where
  int-bits ILong = 64 |
  int-bits IInt  = 32 |
  int-bits IShort = 16 |
  int-bits IChar  = 16 |
  int-bits IByte  = 8  |
  int-bits IBoolean = 1

```

```

fun int-signed :: IntType  $\Rightarrow$  bool where
  int-signed ILong = True |
  int-signed IInt  = True |
  int-signed IShort = True |
  int-signed IChar  = False |
  int-signed IByte  = True |
  int-signed IBoolean = True
end

```

Option 4: int64 with the number of significant bits.

```

type-synonym iwidth = nat
type-synonym objref = nat option

```

```

datatype (discs-sels) Value =
  UndefVal |

```

```

  IntVal iwidth int64 |

```

```

  ObjRef objref |
  ObjStr string

```

```

fun intval-bits :: Value  $\Rightarrow$  nat where
  intval-bits (IntVal b v) = b

```

```

fun intval-word :: Value  $\Rightarrow$  int64 where
  intval-word (IntVal b v) = v

```

```

fun bit-bounds :: nat  $\Rightarrow$  (int  $\times$  int) where
  bit-bounds bits = (((2  $\wedge$  bits) div 2) * -1, ((2  $\wedge$  bits) div 2) - 1)

```

```

definition logic-negate :: ('a::len) word  $\Rightarrow$  'a word where
  logic-negate x = (if x = 0 then 1 else 0)

```

```

fun int-signed-value :: iwidth  $\Rightarrow$  int64  $\Rightarrow$  int where
  int-signed-value b v = sint (signed-take-bit (b - 1) v)

```

```
fun int-unsigned-value :: iwidth  $\Rightarrow$  int64  $\Rightarrow$  int where
  int-unsigned-value b v = uint v
```

Converts an integer word into a Java value.

```
fun new-int :: iwidth  $\Rightarrow$  int64  $\Rightarrow$  Value where
  new-int b w = IntVal b (take-bit b w)
```

Converts an integer word into a Java value, iff the two types are equal.

```
fun new-int-bin :: iwidth  $\Rightarrow$  iwidth  $\Rightarrow$  int64  $\Rightarrow$  Value where
  new-int-bin b1 b2 w = (if b1=b2 then new-int b1 w else UndefVal)
```

```
fun wf-bool :: Value  $\Rightarrow$  bool where
  wf-bool (IntVal b w) = (b = 1) |
  wf-bool - = False
```

```
fun val-to-bool :: Value  $\Rightarrow$  bool where
  val-to-bool (IntVal b val) = (if val = 0 then False else True) |
  val-to-bool val = False
```

```
fun bool-to-val :: bool  $\Rightarrow$  Value where
  bool-to-val True = (IntVal 32 1) |
  bool-to-val False = (IntVal 32 0)
```

Converts an Isabelle bool into a Java value, iff the two types are equal.

```
fun bool-to-val-bin :: iwidth  $\Rightarrow$  iwidth  $\Rightarrow$  bool  $\Rightarrow$  Value where
  bool-to-val-bin t1 t2 b = (if t1 = t2 then bool-to-val b else UndefVal)
```

```
fun is-int-val :: Value  $\Rightarrow$  bool where
  is-int-val v = is-IntVal v
```

A convenience function for directly constructing -1 values of a given bit size.

```
fun neg-one :: iwidth  $\Rightarrow$  int64 where
  neg-one b = mask b
```

```
lemma neg-one-value[simp]: new-int b (neg-one b) = IntVal b (mask b)
by simp
```

```
lemma neg-one-signed[simp]:
  assumes 0 < b
  shows int-signed-value b (neg-one b) = -1
  by (smt (verit, best) assms diff-le-self diff-less int-signed-value.simps less-one
mask-eq-take-bit-minus-one neg-one.simps nle-le signed-minus-1 signed-take-bit-of-minus-1
signed-take-bit-take-bit verit-comp-simplify1 (1))
```

## 1.1 Arithmetic Operators

We need to introduce arithmetic operations which agree with the JVM.

Within the JVM, bytecode arithmetic operations are performed on 32 or 64 bit integers, unboxing where appropriate.

The following collection of *intval* functions correspond to the JVM arithmetic operations. We merge the 32 and 64 bit operations into a single function, even though the stamp of each *IRNode* tells us exactly what the bit widths will be. These merged functions make it easier to do the instantiation of *Value* as 'plus', etc. It might be worse for reasoning, because it could cause more case analysis, but this does not seem to be a problem in practice.

```
fun intval-add :: Value  $\Rightarrow$  Value  $\Rightarrow$  Value where
  intval-add (IntVal b1 v1) (IntVal b2 v2) =
    (if b1 = b2 then IntVal b1 (take-bit b1 (v1+v2)) else UndefVal) |
  intval-add - - = UndefVal
```

```
fun intval-sub :: Value  $\Rightarrow$  Value  $\Rightarrow$  Value where
  intval-sub (IntVal b1 v1) (IntVal b2 v2) = new-int-bin b1 b2 (v1-v2) |
  intval-sub - - = UndefVal
```

```
fun intval-mul :: Value  $\Rightarrow$  Value  $\Rightarrow$  Value where
  intval-mul (IntVal b1 v1) (IntVal b2 v2) = new-int-bin b1 b2 (v1*v2) |
  intval-mul - - = UndefVal
```

```
fun intval-div :: Value  $\Rightarrow$  Value  $\Rightarrow$  Value where
  intval-div (IntVal b1 v1) (IntVal b2 v2) =
    new-int-bin b1 b2 (word-of-int
      ((int-signed-value b1 v1) sdiv (int-signed-value b2 v2))) |
  intval-div - - = UndefVal
```

```
fun intval-mod :: Value  $\Rightarrow$  Value  $\Rightarrow$  Value where
  intval-mod (IntVal b1 v1) (IntVal b2 v2) =
    new-int-bin b1 b2 (word-of-int
      ((int-signed-value b1 v1) smod (int-signed-value b2 v2))) |
  intval-mod - - = UndefVal
```

```
fun intval-negate :: Value  $\Rightarrow$  Value where
  intval-negate (IntVal t v) = new-int t (- v) |
  intval-negate - = UndefVal
```

```
fun intval-abs :: Value  $\Rightarrow$  Value where
```

```

intval-abs (IntVal t v) = new-int t (if int-signed-value t v < 0 then - v else v) |
intval-abs - = UndefVal

```

TODO: clarify which widths this should work on: just 1-bit or all?

```

fun intval-logic-negation :: Value ⇒ Value where
  intval-logic-negation (IntVal b v) = new-int b (logic-negate v) |
  intval-logic-negation - = UndefVal

```

## 1.2 Bitwise Operators

```

fun intval-and :: Value ⇒ Value ⇒ Value where
  intval-and (IntVal b1 v1) (IntVal b2 v2) = new-int-bin b1 b2 (and v1 v2) |
  intval-and - - = UndefVal

```

```

fun intval-or :: Value ⇒ Value ⇒ Value where
  intval-or (IntVal b1 v1) (IntVal b2 v2) = new-int-bin b1 b2 (or v1 v2) |
  intval-or - - = UndefVal

```

```

fun intval-xor :: Value ⇒ Value ⇒ Value where
  intval-xor (IntVal b1 v1) (IntVal b2 v2) = new-int-bin b1 b2 (xor v1 v2) |
  intval-xor - - = UndefVal

```

```

fun intval-not :: Value ⇒ Value where
  intval-not (IntVal t v) = new-int t (not v) |
  intval-not - = UndefVal

```

## 1.3 Comparison Operators

```

fun intval-short-circuit-or :: Value ⇒ Value ⇒ Value where
  intval-short-circuit-or (IntVal b1 v1) (IntVal b2 v2) = bool-to-val-bin b1 b2 (((v1
  ≠ 0) ∨ (v2 ≠ 0))) |
  intval-short-circuit-or - - = UndefVal

```

```

fun intval-equals :: Value ⇒ Value ⇒ Value where
  intval-equals (IntVal b1 v1) (IntVal b2 v2) = bool-to-val-bin b1 b2 (v1 = v2) |
  intval-equals - - = UndefVal

```

```

fun intval-less-than :: Value ⇒ Value ⇒ Value where
  intval-less-than (IntVal b1 v1) (IntVal b2 v2) =
    bool-to-val-bin b1 b2 (int-signed-value b1 v1 < int-signed-value b2 v2) |
  intval-less-than - - = UndefVal

```

```

fun intval-below :: Value ⇒ Value ⇒ Value where
  intval-below (IntVal b1 v1) (IntVal b2 v2) = bool-to-val-bin b1 b2 (v1 < v2) |
  intval-below - - = UndefVal

```

```

fun intval-conditional :: Value ⇒ Value ⇒ Value ⇒ Value where
  intval-conditional cond tv fv = (if (val-to-bool cond) then tv else fv)

```

## 1.4 Narrowing and Widening Operators

Note: we allow these operators to have  $\text{inBits} = \text{outBits}$ , because the Graal compiler also seems to allow that case, even though it should rarely / never arise in practice.

```
value sint(signed-take-bit 0 (1 :: int32))
```

```
fun intval-narrow :: nat  $\Rightarrow$  nat  $\Rightarrow$  Value  $\Rightarrow$  Value where
  intval-narrow inBits outBits (IntVal b v) =
    (if inBits = b  $\wedge$  0 < outBits  $\wedge$  outBits  $\leq$  inBits  $\wedge$  inBits  $\leq$  64
     then new-int outBits v
     else UndefVal) |
  intval-narrow - - - = UndefVal
```

```
value sint (signed-take-bit 7 ((256 + 128) :: int64))
```

```
fun intval-sign-extend :: nat  $\Rightarrow$  nat  $\Rightarrow$  Value  $\Rightarrow$  Value where
  intval-sign-extend inBits outBits (IntVal b v) =
    (if inBits = b  $\wedge$  0 < inBits  $\wedge$  inBits  $\leq$  outBits  $\wedge$  outBits  $\leq$  64
     then new-int outBits (signed-take-bit (inBits - 1) v)
     else UndefVal) |
  intval-sign-extend - - - = UndefVal
```

```
fun intval-zero-extend :: nat  $\Rightarrow$  nat  $\Rightarrow$  Value  $\Rightarrow$  Value where
  intval-zero-extend inBits outBits (IntVal b v) =
    (if inBits = b  $\wedge$  0 < inBits  $\wedge$  inBits  $\leq$  outBits  $\wedge$  outBits  $\leq$  64
     then new-int outBits (take-bit inBits v)
     else UndefVal) |
  intval-zero-extend - - - = UndefVal
```

Some well-formedness results to help reasoning about narrowing and widening operators

**lemma** *intval-narrow-ok*:

```
assumes intval-narrow inBits outBits val  $\neq$  UndefVal
shows 0 < outBits  $\wedge$  outBits  $\leq$  inBits  $\wedge$  inBits  $\leq$  64  $\wedge$  outBits  $\leq$  64  $\wedge$ 
  is-IntVal val  $\wedge$ 
  intval-bits val = inBits
using assms intval-narrow.simps neq0-conv intval-bits.simps
by (metis Value.disc(2) intval-narrow.elims le-trans)
```

**lemma** *intval-sign-extend-ok*:

```
assumes intval-sign-extend inBits outBits val  $\neq$  UndefVal
shows 0 < inBits  $\wedge$ 
  inBits  $\leq$  outBits  $\wedge$  outBits  $\leq$  64  $\wedge$ 
  is-IntVal val  $\wedge$ 
  intval-bits val = inBits
using assms intval-sign-extend.simps neq0-conv
by (metis intval-bits.simps intval-sign-extend.elims is-IntVal-def)
```

**lemma** *intval-zero-extend-ok*:  
**assumes** *intval-zero-extend inBits outBits val*  $\neq$  *UndefVal*  
**shows**  $0 < \text{inBits} \wedge$   
 $\text{inBits} \leq \text{outBits} \wedge \text{outBits} \leq 64 \wedge$   
 $\text{is-IntVal } \text{val} \wedge$   
 $\text{intval-bits } \text{val} = \text{inBits}$   
**using** *assms intval-sign-extend.simps neq0-conv*  
**by** (*metis intval-bits.simps intval-zero-extend.elims is-IntVal-def*)

## 1.5 Bit-Shifting Operators

**definition** *shiffl* (**infix**  $<<$  75) **where**  
 $\text{shiffl } w \ n = (\text{push-bit } n) \ w$

**lemma** *shiffl-power[simp]*:  $(x::('a::\text{len}) \text{ word}) * (2^j) = x << j$   
**unfolding** *shiffl-def* **apply** (*induction j*)  
**apply** *simp* **unfolding** *funpow-Suc-right*  
**by** (*metis (no-types, opaque-lifting) push-bit-eq-mult*)

**lemma**  $(x::('a::\text{len}) \text{ word}) * ((2^j) + 1) = x << j + x$   
**by** (*simp add: distrib-left*)

**lemma**  $(x::('a::\text{len}) \text{ word}) * ((2^j) - 1) = x << j - x$   
**by** (*simp add: right-diff-distrib*)

**lemma**  $(x::('a::\text{len}) \text{ word}) * ((2^j) + (2^k)) = x << j + x << k$   
**by** (*simp add: distrib-left*)

**lemma**  $(x::('a::\text{len}) \text{ word}) * ((2^j) - (2^k)) = x << j - x << k$   
**by** (*simp add: right-diff-distrib*)

**definition** *shiftr* (**infix**  $>>$  75) **where**  
 $\text{shiftr } w \ n = (\text{drop-bit } n) \ w$

**value**  $(255 :: 8 \text{ word}) >>> (2 :: \text{nat})$

**definition** *sshiftr*  $:: 'a :: \text{len word} \Rightarrow \text{nat} \Rightarrow 'a :: \text{len word}$  (**infix**  $>>$  75) **where**  
 $\text{sshiftr } w \ n = \text{word-of-int } ((\text{sint } w) \text{ div } (2^n))$

**value**  $(128 :: 8 \text{ word}) >> 2$

Note that Java shift operators use unary numeric promotion, unlike other binary operators, which use binary numeric promotion (see the Java language reference manual). This means that the left-hand input determines the output size, while the right-hand input can be any size.

**fun** *shift-amount*  $:: \text{iwidth} \Rightarrow \text{int64} \Rightarrow \text{nat}$  **where**  
 $\text{shift-amount } b \ \text{val} = \text{unat } (\text{and } \text{val } (\text{if } b = 64 \text{ then } 0x3F \text{ else } 0x1f))$



```

fun intval-left-shift :: Value  $\Rightarrow$  Value  $\Rightarrow$  Value where
  intval-left-shift (IntVal b1 v1) (IntVal b2 v2) = new-int b1 (v1 << shift-amount
b1 v2) |
  intval-left-shift - - = UndefVal

```

Signed shift is more complex, because we sometimes have to insert 1 bits at the correct point, which is at b1 bits.

```

fun intval-right-shift :: Value  $\Rightarrow$  Value  $\Rightarrow$  Value where
  intval-right-shift (IntVal b1 v1) (IntVal b2 v2) =
    (let shift = shift-amount b1 v2 in
     let ones = and (mask b1) (not (mask (b1 - shift) :: int64)) in
     (if int-signed-value b1 v1 < 0
      then new-int b1 (or ones (v1 >>> shift))
      else new-int b1 (v1 >>> shift))) |
  intval-right-shift - - = UndefVal

```

```

fun intval-uright-shift :: Value  $\Rightarrow$  Value  $\Rightarrow$  Value where
  intval-uright-shift (IntVal b1 v1) (IntVal b2 v2) = new-int b1 (v1 >>> shift-amount
b1 v2) |
  intval-uright-shift - - = UndefVal

```

### 1.5.1 Examples of Narrowing / Widening Functions

**experiment begin**

**corollary** *intval-narrow 32 8 (IntVal 32 (256 + 128)) = IntVal 8 128 by simp*

**corollary** *intval-narrow 32 8 (IntVal 32 (-2)) = IntVal 8 254 by simp*

**corollary** *intval-narrow 32 1 (IntVal 32 (-2)) = IntVal 1 0 by simp*

**corollary** *intval-narrow 32 1 (IntVal 32 (-3)) = IntVal 1 1 by simp*

**corollary** *intval-narrow 32 8 (IntVal 64 (-2)) = UndefVal by simp*

**corollary** *intval-narrow 64 8 (IntVal 32 (-2)) = UndefVal by simp*

**corollary** *intval-narrow 64 8 (IntVal 64 254) = IntVal 8 254 by simp*

**corollary** *intval-narrow 64 8 (IntVal 64 (256+127)) = IntVal 8 127 by simp*

**corollary** *intval-narrow 64 64 (IntVal 64 (-2)) = IntVal 64 (-2) by simp*

**end**

**experiment begin**

**corollary** *intval-sign-extend 8 32 (IntVal 8 (256 + 128)) = IntVal 32 (2<sup>32</sup> - 128) by simp*

**corollary** *intval-sign-extend 8 32 (IntVal 8 (-2)) = IntVal 32 (2<sup>32</sup> - 2) by simp*

**corollary** *intval-sign-extend 1 32 (IntVal 1 (-2)) = IntVal 32 0 by simp*

**corollary** *intval-sign-extend 1 32 (IntVal 1 (-3)) = IntVal 32 (mask 32) by simp*

**corollary** *intval-sign-extend 8 32 (IntVal 64 254) = UndefVal by simp*

**corollary** *intval-sign-extend 8 64 (IntVal 32 254) = UndefVal by simp*

**corollary** *intval-sign-extend* 8 64 (*IntVal* 8 254) = *IntVal* 64 (-2) **by** *simp*  
**corollary** *intval-sign-extend* 32 64 (*IntVal* 32 ( $2^{32} - 2$ )) = *IntVal* 64 (-2) **by** *simp*  
**corollary** *intval-sign-extend* 64 64 (*IntVal* 64 (-2)) = *IntVal* 64 (-2) **by** *simp*  
**end**

**experiment begin**

**corollary** *intval-zero-extend* 8 32 (*IntVal* 8 (256 + 128)) = *IntVal* 32 128 **by** *simp*  
**corollary** *intval-zero-extend* 8 32 (*IntVal* 8 (-2)) = *IntVal* 32 254 **by** *simp*  
**corollary** *intval-zero-extend* 1 32 (*IntVal* 1 (-1)) = *IntVal* 32 1 **by** *simp*  
**corollary** *intval-zero-extend* 1 32 (*IntVal* 1 (-2)) = *IntVal* 32 0 **by** *simp*

**corollary** *intval-zero-extend* 8 32 (*IntVal* 64 (-2)) = *UndefVal* **by** *simp*  
**corollary** *intval-zero-extend* 8 64 (*IntVal* 64 (-2)) = *UndefVal* **by** *simp*  
**corollary** *intval-zero-extend* 8 64 (*IntVal* 8 254) = *IntVal* 64 254 **by** *simp*  
**corollary** *intval-zero-extend* 32 64 (*IntVal* 32 ( $2^{32} - 2$ )) = *IntVal* 64 ( $2^{32} - 2$ ) **by** *simp*  
**corollary** *intval-zero-extend* 64 64 (*IntVal* 64 (-2)) = *IntVal* 64 (-2) **by** *simp*  
**end**

**experiment begin**

**corollary** *intval-right-shift* (*IntVal* 8 128) (*IntVal* 8 0) = *IntVal* 8 128 **by** *eval*  
**corollary** *intval-right-shift* (*IntVal* 8 128) (*IntVal* 8 1) = *IntVal* 8 192 **by** *eval*  
**corollary** *intval-right-shift* (*IntVal* 8 128) (*IntVal* 8 2) = *IntVal* 8 224 **by** *eval*  
**corollary** *intval-right-shift* (*IntVal* 8 128) (*IntVal* 8 8) = *IntVal* 8 255 **by** *eval*  
**corollary** *intval-right-shift* (*IntVal* 8 128) (*IntVal* 8 31) = *IntVal* 8 255 **by** *eval*  
**end**

**lemma** *intval-add-sym*:

**shows** *intval-add* a b = *intval-add* b a

**by** (*induction* a; *induction* b; *auto simp*: *add commute*)

**code-deps** *intval-add*

**code-thms** *intval-add*

**lemma** *intval-add* (*IntVal* 32 ( $2^{31} - 1$ )) (*IntVal* 32 ( $2^{31} - 1$ )) = *IntVal* 32 ( $2^{32} - 2$ )

**by** *eval*

**lemma** *intval-add* (*IntVal* 64 ( $2^{31} - 1$ )) (*IntVal* 64 ( $2^{31} - 1$ )) = *IntVal* 64 4294967294

**by** *eval*

end

## 1.6 Fixed-width Word Theories

```
theory ValueThms
  imports Values
begin
```

### 1.6.1 Support Lemmas for Upper/Lower Bounds

```
lemma size32: size v = 32 for v :: 32 word
  using size-word.rep-eq
  using One-nat-def add.right-neutral add-Suc-right len-of-numeral-defs(2) len-of-numeral-defs(3)
  mult.right-neutral mult-Suc-right numeral-2-eq-2 numeral-Bit0
  by (smt (verit, del-ists) mult.commute)
```

```
lemma size64: size v = 64 for v :: 64 word
  using size-word.rep-eq
  using One-nat-def add.right-neutral add-Suc-right len-of-numeral-defs(2) len-of-numeral-defs(3)
  mult.right-neutral mult-Suc-right numeral-2-eq-2 numeral-Bit0
  by (smt (verit, del-ists) mult.commute)
```

```
lemma lower-bounds-equiv:
  assumes 0 < N
  shows  $\neg((2::int) \wedge (N-1)) = (2::int) \wedge N \text{ div } 2 * -1$ 
  by (simp add: asms int-power-div-base)
```

```
lemma upper-bounds-equiv:
  assumes 0 < N
  shows  $(2::int) \wedge (N-1) = (2::int) \wedge N \text{ div } 2$ 
  by (simp add: asms int-power-div-base)
```

Some min/max bounds for 64-bit words

```
lemma bit-bounds-min64: ((fst (bit-bounds 64))) ≤ (sint (v::int64))
  unfolding bit-bounds.simps fst-def
  using sint-ge[of v] by simp
```

```
lemma bit-bounds-max64: ((snd (bit-bounds 64))) ≥ (sint (v::int64))
  unfolding bit-bounds.simps fst-def
  using sint-lt[of v] by simp
```

Extend these min/max bounds to extracting smaller signed words using `signed_take_bit`.

Note: we could use `signed` to convert between bit-widths, instead of `signed_take_bit`. But that would have to be done separately for each bit-width type.

**value** *sint*(*signed-take-bit* 7 (128 :: int8))

**ML-val**  $\langle @\{thm\ signed-take-bit-decr-length-iff\} \rangle$

**declare** [[*show-types=true*]]

**ML-val**  $\langle @\{thm\ signed-take-bit-int-less-exp\} \rangle$

**lemma** *signed-take-bit-int-less-exp-word*:

**fixes** *ival* :: 'a :: len word

**assumes**  $n < LENGTH('a)$

**shows**  $sint(signed-take-bit\ n\ ival) < (2::int) ^ n$

**apply** *transfer*

**by** (*smt* (*verit*, *best*) *not-take-bit-negative signed-take-bit-eq-take-bit-shift signed-take-bit-int-less-exp take-bit-int-greater-self-iff*)

**lemma** *signed-take-bit-int-greater-eq-minus-exp-word*:

**fixes** *ival* :: 'a :: len word

**assumes**  $n < LENGTH('a)$

**shows**  $-(2 ^ n) \leq sint(signed-take-bit\ n\ ival)$

**apply** *transfer*

**by** (*smt* (*verit*, *best*) *signed-take-bit-int-greater-eq-minus-exp signed-take-bit-int-greater-eq-self-iff signed-take-bit-int-less-exp*)

**lemma** *signed-take-bit-range*:

**fixes** *ival* :: 'a :: len word

**assumes**  $n < LENGTH('a)$

**assumes**  $val = sint(signed-take-bit\ n\ ival)$

**shows**  $-(2 ^ n) \leq val \wedge val < 2 ^ n$

**using** *signed-take-bit-int-greater-eq-minus-exp-word signed-take-bit-int-less-exp-word*

**using** *assms* **by** *blast*

A *bit\_bounds* version of the above lemma.

**lemma** *signed-take-bit-bounds*:

**fixes** *ival* :: 'a :: len word

**assumes**  $n \leq LENGTH('a)$

**assumes**  $0 < n$

**assumes**  $val = sint(signed-take-bit\ (n - 1)\ ival)$

**shows**  $fst\ (bit-bounds\ n) \leq val \wedge val \leq snd\ (bit-bounds\ n)$

**using** *assms signed-take-bit-range lower-bounds-equiv upper-bounds-equiv*

**by** (*metis bit-bounds.simps fst-conv less-imp-diff-less nat-less-le sint-ge sint-lt snd-conv zle-diff1-eq*)

**lemma** *signed-take-bit-bounds64*:

**fixes** *ival* :: int64

**assumes**  $n \leq 64$

**assumes**  $0 < n$

**assumes**  $val = sint(signed-take-bit\ (n - 1)\ ival)$

**shows**  $\text{fst } (\text{bit-bounds } n) \leq \text{val} \wedge \text{val} \leq \text{snd } (\text{bit-bounds } n)$   
**using** *assms signed-take-bit-bounds*  
**by** (*metis size64 word-size*)

**lemma** *int-signed-value-bounds*:

**assumes**  $b1 \leq 64$   
**assumes**  $0 < b1$   
**shows**  $\text{fst } (\text{bit-bounds } b1) \leq \text{int-signed-value } b1 \text{ } v2 \wedge$   
 $\text{int-signed-value } b1 \text{ } v2 \leq \text{snd } (\text{bit-bounds } b1)$   
**using** *assms int-signed-value.simps signed-take-bit-bounds64* **by** *blast*

**lemma** *int-signed-value-range*:

**fixes** *ival* :: *int64*  
**assumes**  $\text{val} = \text{int-signed-value } n \text{ } \text{ival}$   
**shows**  $-(2^{(n-1)}) \leq \text{val} \wedge \text{val} < 2^{(n-1)}$   
**using** *signed-take-bit-range assms*  
**by** (*smt (verit, ccv-SIG) One-nat-def diff-less int-signed-value.elims len-gt-0*  
*len-num1 power-less-imp-less-exp power-strict-increasing sint-greater-eq sint-less*)

Some lemmas about unsigned words smaller than 64-bit, for zero-extend operators.

**lemma** *take-bit-smaller-range*:

**fixes** *ival* :: '*a* :: *len word*  
**assumes**  $n < \text{LENGTH('a)}$   
**assumes**  $\text{val} = \text{sint}(\text{take-bit } n \text{ } \text{ival})$   
**shows**  $0 \leq \text{val} \wedge \text{val} < (2::\text{int})^n$   
**by** (*simp add: assms signed-take-bit-eq*)

**lemma** *take-bit-same-size-nochange*:

**fixes** *ival* :: '*a* :: *len word*  
**assumes**  $n = \text{LENGTH('a)}$   
**shows**  $\text{ival} = \text{take-bit } n \text{ } \text{ival}$   
**by** (*simp add: assms*)

A simplification lemma for *new\_int*, showing that upper bits can be ignored.

**lemma** *take-bit-redundant[simp]*:

**fixes** *ival* :: '*a* :: *len word*  
**assumes**  $0 < n$   
**assumes**  $n < \text{LENGTH('a)}$   
**shows**  $\text{signed-take-bit } (n-1) (\text{take-bit } n \text{ } \text{ival}) = \text{signed-take-bit } (n-1) \text{ } \text{ival}$

**proof** –

**have**  $\neg (n \leq n-1)$  **using** *assms* **by** *arith*  
**then have**  $\bigwedge i. \text{signed-take-bit } (n-1) (\text{take-bit } n \text{ } i) = \text{signed-take-bit } (n-1) \text{ } i$   
**using** *signed-take-bit-take-bit* **by** (*metis (mono-tags)*)  
**then show** *?thesis*  
**by** *blast*

**qed**

**lemma** *take-bit-same-size-range*:

```

fixes ival :: 'a :: len word
assumes n = LENGTH('a)
assumes ival2 = take-bit n ival
shows - (2 ^ n div 2) ≤ sint ival2 ∧ sint ival2 < 2 ^ n div 2
using assms lower-bounds-equiv sint-ge sint-lt by auto

```

```

lemma take-bit-same-bounds:
  fixes ival :: 'a :: len word
  assumes n = LENGTH('a)
  assumes ival2 = take-bit n ival
  shows fst (bit-bounds n) ≤ sint ival2 ∧ sint ival2 ≤ snd (bit-bounds n)
  unfolding bit-bounds.simps
  using assms take-bit-same-size-range
  by force

```

Next we show that casting a word to a wider word preserves any upper/lower bounds. (These lemmas may not be needed any more, since we are not using scast now?)

```

lemma scast-max-bound:
  assumes sint (v :: 'a :: len word) < M
  assumes LENGTH('a) < LENGTH('b)
  shows sint ((scast v) :: 'b :: len word) < M
  unfolding Word.scast-eq Word.sint-sbintrunc'
  using Bit-Operations.signed-take-bit-int-eq-self-iff
  by (smt (verit, best) One-nat-def assms(1) assms(2) decr-length-less-iff linorder-not-le
    power-strict-increasing-iff signed-take-bit-int-less-self-iff sint-greater-eq)

```

```

lemma scast-min-bound:
  assumes M ≤ sint (v :: 'a :: len word)
  assumes LENGTH('a) < LENGTH('b)
  shows M ≤ sint ((scast v) :: 'b :: len word)
  unfolding Word.scast-eq Word.sint-sbintrunc'
  using Bit-Operations.signed-take-bit-int-eq-self-iff
  by (smt (verit) One-nat-def Suc-pred assms(1) assms(2) len-gt-0 less-Suc-eq or-
    der-less-le order-less-le-trans power-le-imp-le-exp signed-take-bit-int-greater-eq-self-iff
    sint-lt)

```

```

lemma scast-bigger-max-bound:
  assumes (result :: 'b :: len word) = scast (v :: 'a :: len word)
  shows sint result < 2 ^ LENGTH('a) div 2
  using sint-lt upper-bounds-equiv scast-max-bound
  by (smt (verit, best) assms(1) len-gt-0 signed-scast-eq signed-take-bit-int-greater-self-iff
    sint-ge sint-less upper-bounds-equiv)

```

```

lemma scast-bigger-min-bound:
  assumes (result :: 'b :: len word) = scast (v :: 'a :: len word)
  shows - (2 ^ LENGTH('a) div 2) ≤ sint result
  using sint-ge lower-bounds-equiv scast-min-bound

```

**by** (*smt* (*verit*) *assms* *len-gt-0* *nat-less-le* *not-less* *scast-max-bound*)

**lemma** *scast-bigger-bit-bounds*:

**assumes** (*result* :: 'b :: len word) = *scast* (*v* :: 'a :: len word)  
**shows** *fst* (*bit-bounds* (*LENGTH*('a))) ≤ *sint* *result* ∧ *sint* *result* ≤ *snd* (*bit-bounds* (*LENGTH*('a')))  
**using** *assms* *scast-bigger-min-bound* *scast-bigger-max-bound*  
**by** *auto*

Results about *new\_int*.

**lemma** *new-int-take-bits*:

**assumes** *IntVal* *b* *val* = *new-int* *b* *ival*  
**shows** *take-bit* *b* *val* = *val*  
**using** *assms* **by** *force*

### 1.6.2 Support lemmas for take bit and signed take bit.

Lemmas for removing redundant *take\_bit* wrappers.

**lemma** *take-bit-dist-addL*[*simp*]:

**fixes** *x* :: 'a :: len word  
**shows** *take-bit* *b* (*take-bit* *b* *x* + *y*) = *take-bit* *b* (*x* + *y*)  
**proof** (*induction* *b*)  
**case** 0  
**then show** ?*case*  
**by** *simp*  
**next**  
**case** (*Suc* *b*)  
**then show** ?*case*  
**by** (*simp* *add*: *add.commute* *mask-eqs*(2) *take-bit-eq-mask*)  
**qed**

**lemma** *take-bit-dist-addR*[*simp*]:

**fixes** *x* :: 'a :: len word  
**shows** *take-bit* *b* (*x* + *take-bit* *b* *y*) = *take-bit* *b* (*x* + *y*)  
**using** *take-bit-dist-addL* **by** (*metis* *add.commute*)

**lemma** *take-bit-dist-subL*[*simp*]:

**fixes** *x* :: 'a :: len word  
**shows** *take-bit* *b* (*take-bit* *b* *x* - *y*) = *take-bit* *b* (*x* - *y*)  
**by** (*metis* *take-bit-dist-addR* *uminus-add-conv-diff*)

**lemma** *take-bit-dist-subR*[*simp*]:

**fixes** *x* :: 'a :: len word  
**shows** *take-bit* *b* (*x* - *take-bit* *b* *y*) = *take-bit* *b* (*x* - *y*)  
**using** *take-bit-dist-subL*  
**by** (*metis* (*no-types*, *opaque-lifting*) *diff-add-cancel* *diff-right-commute* *diff-self*)

```

lemma take-bit-dist-neg[simp]:
  fixes ix :: 'a :: len word
  shows take-bit b (- take-bit b (ix)) = take-bit b (- ix)
  by (metis diff-0 take-bit-dist-subR)

lemma signed-take-take-bit[simp]:
  fixes x :: 'a :: len word
  assumes 0 < b
  shows signed-take-bit (b - 1) (take-bit b x) = signed-take-bit (b - 1) x
  by (smt (verit, best) Suc-diff-1 assms lessI linorder-not-less signed-take-bit-take-bit)

lemma mod-larger-ignore:
  fixes a :: int
  fixes m n :: nat
  assumes n < m
  shows (a mod 2 ^ m) mod 2 ^ n = a mod 2 ^ n
  by (smt (verit, del-insts) assms exp-mod-exp linorder-not-le mod-0-imp-dvd mod-mod-cancel
mod-self order-less-imp-le)

lemma mod-dist-over-add:
  fixes a b c :: int64
  fixes n :: nat
  assumes 1: 0 < n
  assumes 2: n < 64
  shows (a mod 2 ^ n + b) mod 2 ^ n = (a + b) mod 2 ^ n
proof -
  have 3: (0 :: int64) < 2 ^ n
  using assms by (simp add: size64 word-2p-lem)
  then show ?thesis
  unfolding word-mod-2p-is-mask[OF 3]
  apply transfer
  by (metis (no-types, opaque-lifting) and.right-idem take-bit-add take-bit-eq-mask)
qed

end

```

## 2 Stamp Typing

```

theory Stamp
  imports Values
begin

```

The GraalVM compiler uses the Stamp class to store range and type information for a given node in the IR graph. We model the Stamp class as a datatype, Stamp, and provide a number of functions on the datatype which correspond to the class methods within the compiler.



Stamp information is used in a variety of ways in optimizations, and so, we additionally provide a number of lemmas which help to prove future optimizations.

```
datatype Stamp =
  VoidStamp
| IntegerStamp (stp-bits: nat) (stpi-lower: int) (stpi-upper: int)

| KlassPointerStamp (stp-nonNull: bool) (stp-alwaysNull: bool)
| MethodCountersPointerStamp (stp-nonNull: bool) (stp-alwaysNull: bool)
| MethodPointersStamp (stp-nonNull: bool) (stp-alwaysNull: bool)
| ObjectStamp (stp-type: string) (stp-exactType: bool) (stp-nonNull: bool) (stp-alwaysNull:
bool)
| RawPointerStamp (stp-nonNull: bool) (stp-alwaysNull: bool)
| IllegalStamp
```

```
fun is-stamp-empty :: Stamp  $\Rightarrow$  bool where
  is-stamp-empty (IntegerStamp b lower upper) = (upper < lower) |
```

```
  is-stamp-empty x = False
```

Just like the IntegerStamp class, we need to know that our lo/hi bounds fit into the given number of bits (either signed or unsigned). Our integer stamps have infinite lo/hi bounds, so if the lower bound is non-negative, we can assume that all values are positive, and the integer bits of a related value can be interpreted as unsigned. This is similar (but slightly more general) to what IntegerStamp.java does with its test: if (sameSignBounds()) in the unsignedUpperBound() method.

Note that this is a bit different and more accurate than what StampFactory.forUnsignedInteger does (it widens large unsigned ranges to the max signed range to allow all bit patterns) because its lo/hi values are only 64-bit.

```
fun valid-stamp :: Stamp  $\Rightarrow$  bool where
  valid-stamp (IntegerStamp bits lo hi) =
    (0 < bits  $\wedge$  bits  $\leq$  64  $\wedge$ 
     fst (bit-bounds bits)  $\leq$  lo  $\wedge$  lo  $\leq$  snd (bit-bounds bits)  $\wedge$ 
     fst (bit-bounds bits)  $\leq$  hi  $\wedge$  hi  $\leq$  snd (bit-bounds bits)) |
  valid-stamp s = True
```

**experiment begin**

```
corollary bit-bounds 1 = (-1, 0) by simp
end
```

— A stamp which includes the full range of the type

```
fun unrestricted-stamp :: Stamp  $\Rightarrow$  Stamp where
  unrestricted-stamp VoidStamp = VoidStamp |
  unrestricted-stamp (IntegerStamp bits lower upper) = (IntegerStamp bits (fst
(bit-bounds bits)) (snd (bit-bounds bits))) |

  unrestricted-stamp (KlassPointerStamp nonNull alwaysNull) = (KlassPointerStamp
False False) |
  unrestricted-stamp (MethodCountersPointerStamp nonNull alwaysNull) = (MethodCountersPointerStamp
False False) |
  unrestricted-stamp (MethodPointersStamp nonNull alwaysNull) = (MethodPointersStamp
False False) |
  unrestricted-stamp (ObjectStamp type exactType nonNull alwaysNull) = (ObjectStamp
"" False False False) |
  unrestricted-stamp - = IllegalStamp
```

```
fun is-stamp-unrestricted :: Stamp  $\Rightarrow$  bool where
  is-stamp-unrestricted s = (s = unrestricted-stamp s)
```

— A stamp which provides type information but has an empty range of values

```
fun empty-stamp :: Stamp  $\Rightarrow$  Stamp where
  empty-stamp VoidStamp = VoidStamp |
  empty-stamp (IntegerStamp bits lower upper) = (IntegerStamp bits (snd (bit-bounds
bits)) (fst (bit-bounds bits))) |

  empty-stamp (KlassPointerStamp nonNull alwaysNull) = (KlassPointerStamp
nonNull alwaysNull) |
  empty-stamp (MethodCountersPointerStamp nonNull alwaysNull) = (MethodCountersPointerStamp
nonNull alwaysNull) |
  empty-stamp (MethodPointersStamp nonNull alwaysNull) = (MethodPointersStamp
nonNull alwaysNull) |
  empty-stamp (ObjectStamp type exactType nonNull alwaysNull) = (ObjectStamp
"" True True False) |
  empty-stamp stamp = IllegalStamp
```

— Calculate the meet stamp of two stamps

```
fun meet :: Stamp  $\Rightarrow$  Stamp  $\Rightarrow$  Stamp where
  meet VoidStamp VoidStamp = VoidStamp |
  meet (IntegerStamp b1 l1 u1) (IntegerStamp b2 l2 u2) = (
    if b1  $\neq$  b2 then IllegalStamp else
    (IntegerStamp b1 (min l1 l2) (max u1 u2))
  ) |

  meet (KlassPointerStamp nn1 an1) (KlassPointerStamp nn2 an2) = (
    KlassPointerStamp (nn1  $\wedge$  nn2) (an1  $\wedge$  an2)
```

```

) |
  meet (MethodCountersPointerStamp nn1 an1) (MethodCountersPointerStamp
nn2 an2) = (
  MethodCountersPointerStamp (nn1  $\wedge$  nn2) (an1  $\wedge$  an2)
) |
  meet (MethodPointersStamp nn1 an1) (MethodPointersStamp nn2 an2) = (
  MethodPointersStamp (nn1  $\wedge$  nn2) (an1  $\wedge$  an2)
) |
  meet s1 s2 = IllegalStamp

```

— Calculate the join stamp of two stamps

```

fun join :: Stamp  $\Rightarrow$  Stamp  $\Rightarrow$  Stamp where
  join VoidStamp VoidStamp = VoidStamp |
  join (IntegerStamp b1 l1 u1) (IntegerStamp b2 l2 u2) = (
    if b1  $\neq$  b2 then IllegalStamp else
    (IntegerStamp b1 (max l1 l2) (min u1 u2))
  ) |
  join (KlassPointerStamp nn1 an1) (KlassPointerStamp nn2 an2) = (
    if ((nn1  $\vee$  nn2)  $\wedge$  (an1  $\vee$  an2))
    then (empty-stamp (KlassPointerStamp nn1 an1))
    else (KlassPointerStamp (nn1  $\vee$  nn2) (an1  $\vee$  an2))
  ) |
  join (MethodCountersPointerStamp nn1 an1) (MethodCountersPointerStamp nn2
an2) = (
    if ((nn1  $\vee$  nn2)  $\wedge$  (an1  $\vee$  an2))
    then (empty-stamp (MethodCountersPointerStamp nn1 an1))
    else (MethodCountersPointerStamp (nn1  $\vee$  nn2) (an1  $\vee$  an2))
  ) |
  join (MethodPointersStamp nn1 an1) (MethodPointersStamp nn2 an2) = (
    if ((nn1  $\vee$  nn2)  $\wedge$  (an1  $\vee$  an2))
    then (empty-stamp (MethodPointersStamp nn1 an1))
    else (MethodPointersStamp (nn1  $\vee$  nn2) (an1  $\vee$  an2))
  ) |
  join s1 s2 = IllegalStamp

```

— In certain circumstances a stamp provides enough information to evaluate a value as a stamp, the asConstant function converts the stamp to a value where one can be inferred.

```

fun asConstant :: Stamp  $\Rightarrow$  Value where
  asConstant (IntegerStamp b l h) = (if l = h then IntVal b (word-of-int l) else
  UndefVal) |
  asConstant - = UndefVal

```

— Determine if two stamps never have value overlaps i.e. their join is empty

```

fun alwaysDistinct :: Stamp  $\Rightarrow$  Stamp  $\Rightarrow$  bool where
  alwaysDistinct stamp1 stamp2 = is-stamp-empty (join stamp1 stamp2)

```

— Determine if two stamps must always be the same value i.e. two equal constants

```
fun neverDistinct :: Stamp ⇒ Stamp ⇒ bool where
  neverDistinct stamp1 stamp2 = (asConstant stamp1 = asConstant stamp2 ∧
  asConstant stamp1 ≠ UndefVal)
```

```
fun constantAsStamp :: Value ⇒ Stamp where
  constantAsStamp (IntVal b v) = (IntegerStamp b (int-signed-value b v) (int-signed-value
  b v)) |
```

```
  constantAsStamp - = IllegalStamp
```

— Define when a runtime value is valid for a stamp. The stamp bounds must be valid, and val must be zero-extended.

```
fun valid-value :: Value ⇒ Stamp ⇒ bool where
  valid-value (IntVal b1 val) (IntegerStamp b l h) =
    (if b1 = b then
      valid-stamp (IntegerStamp b l h) ∧
      take-bit b val = val ∧
      l ≤ int-signed-value b val ∧ int-signed-value b val ≤ h
    else False) |

  valid-value (ObjRef ref) (ObjectStamp klass exact nonNull alwaysNull) =
    ((alwaysNull → ref = None) ∧ (ref=Some → ¬ nonNull)) |
  valid-value stamp val = False
```

```
definition wf-value :: Value ⇒ bool where
  wf-value v = valid-value v (constantAsStamp v)
```

```
lemma unfold-wf-value[simp]:
  wf-value v ⇒ valid-value v (constantAsStamp v)
using wf-value-def by auto
```

```
fun compatible :: Stamp ⇒ Stamp ⇒ bool where
  compatible (IntegerStamp b1 lo1 hi1) (IntegerStamp b2 lo2 hi2) =
    (b1 = b2 ∧ valid-stamp (IntegerStamp b1 lo1 hi1) ∧ valid-stamp (IntegerStamp
  b2 lo2 hi2)) |
  compatible (VoidStamp) (VoidStamp) = True |
  compatible - - = False
```

```
fun stamp-under :: Stamp ⇒ Stamp ⇒ bool where
  stamp-under (IntegerStamp b1 lo1 hi1) (IntegerStamp b2 lo2 hi2) = (hi1 < lo2)
  |
  stamp-under - - = False
```

— The most common type of stamp within the compiler (apart from the Void-

Stamp) is a 32 bit integer stamp with an unrestricted range. We use *default-stamp* as it is a frequently used stamp.

**definition** *default-stamp* :: *Stamp* **where**  
*default-stamp* = (*unrestricted-stamp* (*IntegerStamp* 32 0 0))

**value** *valid-value* (*IntVal* 8 (255)) (*IntegerStamp* 8 (−128) 127)  
**end**

### 3 Graph Representation

#### 3.1 IR Graph Nodes

**theory** *IRNodes*  
**imports**  
*Values*  
**begin**

The GraalVM IR is represented using a graph data structure. Here we define the nodes that are contained within the graph. Each node represents a Node subclass in the GraalVM compiler, the node classes have annotated fields to indicate input and successor edges.

We represent these classes with each IRNode constructor explicitly labelling a reference to the node IDs that it stores as inputs and successors.

The *inputs\_of* and *successors\_of* functions partition those labelled references into input edges and successor edges of a node.

To identify each Node, we use a simple natural number index. Zero is always the start node in a graph. For human readability, within nodes we write INPUT (or special case thereof) instead of ID for input edges, and SUCC instead of ID for control-flow successor edges. Optional edges are handled as "INPUT option" etc.

**type-synonym** *ID* = *nat*  
**type-synonym** *INPUT* = *ID*  
**type-synonym** *INPUT-ASSOC* = *ID*  
**type-synonym** *INPUT-STATE* = *ID*  
**type-synonym** *INPUT-GUARD* = *ID*  
**type-synonym** *INPUT-COND* = *ID*  
**type-synonym** *INPUT-EXT* = *ID*  
**type-synonym** *SUCC* = *ID*

**datatype** (*discs-sels*) *IRNode* =  
*AbsNode* (*ir-value*: *INPUT*)  
| *AddNode* (*ir-x*: *INPUT*) (*ir-y*: *INPUT*)  
| *AndNode* (*ir-x*: *INPUT*) (*ir-y*: *INPUT*)  
| *BeginNode* (*ir-next*: *SUCC*)  
| *BytecodeExceptionNode* (*ir-arguments*: *INPUT list*) (*ir-stateAfter-opt*: *INPUT-STATE option*) (*ir-next*: *SUCC*)

- | *ConditionalNode* (*ir-condition*: INPUT-COND) (*ir-trueValue*: INPUT) (*ir-falseValue*: INPUT)
- | *ConstantNode* (*ir-const*: Value)
- | *DynamicNewArrayNode* (*ir-elementType*: INPUT) (*ir-length*: INPUT) (*ir-voidClass-opt*: INPUT option) (*ir-stateBefore-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *EndNode*
- | *ExceptionObjectNode* (*ir-stateAfter-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *FrameState* (*ir-monitorIds*: INPUT-ASSOC list) (*ir-outerFrameState-opt*: INPUT-STATE option) (*ir-values-opt*: INPUT list option) (*ir-virtualObjectMappings-opt*: INPUT-STATE list option)
- | *IfNode* (*ir-condition*: INPUT-COND) (*ir-trueSuccessor*: SUCC) (*ir-falseSuccessor*: SUCC)
- | *IntegerBelowNode* (*ir-x*: INPUT) (*ir-y*: INPUT)
- | *IntegerEqualsNode* (*ir-x*: INPUT) (*ir-y*: INPUT)
- | *IntegerLessThanNode* (*ir-x*: INPUT) (*ir-y*: INPUT)
- | *InvokeNode* (*ir-nid*: ID) (*ir-callTarget*: INPUT-EXT) (*ir-classInit-opt*: INPUT option) (*ir-stateDuring-opt*: INPUT-STATE option) (*ir-stateAfter-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *InvokeWithExceptionNode* (*ir-nid*: ID) (*ir-callTarget*: INPUT-EXT) (*ir-classInit-opt*: INPUT option) (*ir-stateDuring-opt*: INPUT-STATE option) (*ir-stateAfter-opt*: INPUT-STATE option) (*ir-next*: SUCC) (*ir-exceptionEdge*: SUCC)
- | *IsNullNode* (*ir-value*: INPUT)
- | *KillingBeginNode* (*ir-next*: SUCC)
- | *LeftShiftNode* (*ir-x*: INPUT) (*ir-y*: INPUT)
- | *LoadFieldNode* (*ir-nid*: ID) (*ir-field*: string) (*ir-object-opt*: INPUT option) (*ir-next*: SUCC)
- | *LogicNegationNode* (*ir-value*: INPUT-COND)
- | *LoopBeginNode* (*ir-ends*: INPUT-ASSOC list) (*ir-overflowGuard-opt*: INPUT-GUARD option) (*ir-stateAfter-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *LoopEndNode* (*ir-loopBegin*: INPUT-ASSOC)
- | *LoopExitNode* (*ir-loopBegin*: INPUT-ASSOC) (*ir-stateAfter-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *MergeNode* (*ir-ends*: INPUT-ASSOC list) (*ir-stateAfter-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *MethodCallTargetNode* (*ir-targetMethod*: string) (*ir-arguments*: INPUT list)
- | *MulNode* (*ir-x*: INPUT) (*ir-y*: INPUT)
- | *NarrowNode* (*ir-inputBits*: nat) (*ir-resultBits*: nat) (*ir-value*: INPUT)
- | *NegateNode* (*ir-value*: INPUT)
- | *NewArrayNode* (*ir-length*: INPUT) (*ir-stateBefore-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *NewInstanceNode* (*ir-nid*: ID) (*ir-instanceClass*: string) (*ir-stateBefore-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *NotNode* (*ir-value*: INPUT)
- | *OrNode* (*ir-x*: INPUT) (*ir-y*: INPUT)
- | *ParameterNode* (*ir-index*: nat)
- | *PiNode* (*ir-object*: INPUT) (*ir-guard-opt*: INPUT-GUARD option)
- | *ReturnNode* (*ir-result-opt*: INPUT option) (*ir-memoryMap-opt*: INPUT-EXT option)

```

| RightShiftNode (ir-x: INPUT) (ir-y: INPUT)
| ShortCircuitOrNode (ir-x: INPUT-COND) (ir-y: INPUT-COND)
| SignExtendNode (ir-inputBits: nat) (ir-resultBits: nat) (ir-value: INPUT)
| SignedDivNode (ir-nid: ID) (ir-x: INPUT) (ir-y: INPUT) (ir-zeroCheck-opt: IN-
PUT-GUARD option) (ir-stateBefore-opt: INPUT-STATE option) (ir-next: SUCC)

| SignedRemNode (ir-nid: ID) (ir-x: INPUT) (ir-y: INPUT) (ir-zeroCheck-opt:
INPUT-GUARD option) (ir-stateBefore-opt: INPUT-STATE option) (ir-next: SUCC)

| StartNode (ir-stateAfter-opt: INPUT-STATE option) (ir-next: SUCC)
| StoreFieldNode (ir-nid: ID) (ir-field: string) (ir-value: INPUT) (ir-stateAfter-opt:
INPUT-STATE option) (ir-object-opt: INPUT option) (ir-next: SUCC)
| SubNode (ir-x: INPUT) (ir-y: INPUT)
| UnsignedRightShiftNode (ir-x: INPUT) (ir-y: INPUT)
| UnwindNode (ir-exception: INPUT)
| ValuePhiNode (ir-nid: ID) (ir-values: INPUT list) (ir-merge: INPUT-ASSOC)

| ValueProxyNode (ir-value: INPUT) (ir-loopExit: INPUT-ASSOC)
| XorNode (ir-x: INPUT) (ir-y: INPUT)
| ZeroExtendNode (ir-inputBits: nat) (ir-resultBits: nat) (ir-value: INPUT)
| NoNode

| RefNode (ir-ref: ID)

```

```

fun opt-to-list :: 'a option ⇒ 'a list where
  opt-to-list None = [] |
  opt-to-list (Some v) = [v]

```

```

fun opt-list-to-list :: 'a list option ⇒ 'a list where
  opt-list-to-list None = [] |
  opt-list-to-list (Some x) = x

```

The following functions, `inputs_of` and `successors_of`, are automatically generated from the GraalVM compiler. Their purpose is to partition the node edges into input or successor edges.

```

fun inputs-of :: IRNode ⇒ ID list where
  inputs-of-AbsNode:
  inputs-of (AbsNode value) = [value] |
  inputs-of-AddNode:
  inputs-of (AddNode x y) = [x, y] |
  inputs-of-AndNode:
  inputs-of (AndNode x y) = [x, y] |
  inputs-of-BEGINNode:
  inputs-of (BeginNode next) = [] |
  inputs-of-BytecodeExceptionNode:

```

*inputs-of* (*BytecodeExceptionNode* *arguments* *stateAfter* *next*) = *arguments* @  
(*opt-to-list* *stateAfter*) |  
*inputs-of-ConditionalNode*:  
*inputs-of* (*ConditionalNode* *condition* *trueValue* *falseValue*) = [*condition*, *true-*  
*Value*, *falseValue*] |  
*inputs-of-ConstantNode*:  
*inputs-of* (*ConstantNode* *const*) = [] |  
*inputs-of-DynamicNewArrayNode*:  
*inputs-of* (*DynamicNewArrayNode* *elementType* *length0* *voidClass* *stateBefore*  
*next*) = [*elementType*, *length0*] @ (*opt-to-list* *voidClass*) @ (*opt-to-list* *stateBefore*)  
|  
*inputs-of-EndNode*:  
*inputs-of* (*EndNode*) = [] |  
*inputs-of-ExceptionObjectNode*:  
*inputs-of* (*ExceptionObjectNode* *stateAfter* *next*) = (*opt-to-list* *stateAfter*) |  
*inputs-of-FrameState*:  
*inputs-of* (*FrameState* *monitorIds* *outerFrameState* *values* *virtualObjectMappings*)  
= *monitorIds* @ (*opt-to-list* *outerFrameState*) @ (*opt-list-to-list* *values*) @ (*opt-list-to-list*  
*virtualObjectMappings*) |  
*inputs-of-IfNode*:  
*inputs-of* (*IfNode* *condition* *trueSuccessor* *falseSuccessor*) = [*condition*] |  
*inputs-of-IntegerBelowNode*:  
*inputs-of* (*IntegerBelowNode* *x* *y*) = [*x*, *y*] |  
*inputs-of-IntegerEqualsNode*:  
*inputs-of* (*IntegerEqualsNode* *x* *y*) = [*x*, *y*] |  
*inputs-of-IntegerLessThanNode*:  
*inputs-of* (*IntegerLessThanNode* *x* *y*) = [*x*, *y*] |  
*inputs-of-InvokeNode*:  
*inputs-of* (*InvokeNode* *nid0* *callTarget* *classInit* *stateDuring* *stateAfter* *next*) =  
*callTarget* # (*opt-to-list* *classInit*) @ (*opt-to-list* *stateDuring*) @ (*opt-to-list* *stateAfter*)  
|  
*inputs-of-InvokeWithExceptionNode*:  
*inputs-of* (*InvokeWithExceptionNode* *nid0* *callTarget* *classInit* *stateDuring* *stateAfter*  
*next* *exceptionEdge*) = *callTarget* # (*opt-to-list* *classInit*) @ (*opt-to-list* *stateDur-*  
*ing*) @ (*opt-to-list* *stateAfter*) |  
*inputs-of-IsNullNode*:  
*inputs-of* (*IsNullNode* *value*) = [*value*] |  
*inputs-of-KillingBeginNode*:  
*inputs-of* (*KillingBeginNode* *next*) = [] |  
*inputs-of-LeftShiftNode*:  
*inputs-of* (*LeftShiftNode* *x* *y*) = [*x*, *y*] |  
*inputs-of-LoadFieldNode*:  
*inputs-of* (*LoadFieldNode* *nid0* *field* *object* *next*) = (*opt-to-list* *object*) |  
*inputs-of-LogicNegationNode*:  
*inputs-of* (*LogicNegationNode* *value*) = [*value*] |  
*inputs-of-LoopBeginNode*:  
*inputs-of* (*LoopBeginNode* *ends* *overflowGuard* *stateAfter* *next*) = *ends* @ (*opt-to-list*  
*overflowGuard*) @ (*opt-to-list* *stateAfter*) |  
*inputs-of-LoopEndNode*:



*inputs-of* (*LoopEndNode* *loopBegin*) = [*loopBegin*] |  
*inputs-of-LoopExitNode*:  
*inputs-of* (*LoopExitNode* *loopBegin* *stateAfter* *next*) = *loopBegin* # (*opt-to-list* *stateAfter*) |  
*inputs-of-MergeNode*:  
*inputs-of* (*MergeNode* *ends* *stateAfter* *next*) = *ends* @ (*opt-to-list* *stateAfter*) |  
*inputs-of-MethodCallTargetNode*:  
*inputs-of* (*MethodCallTargetNode* *targetMethod* *arguments*) = *arguments* |  
*inputs-of-MulNode*:  
*inputs-of* (*MulNode* *x* *y*) = [*x*, *y*] |  
*inputs-of-NarrowNode*:  
*inputs-of* (*NarrowNode* *inputBits* *resultBits* *value*) = [*value*] |  
*inputs-of-NegateNode*:  
*inputs-of* (*NegateNode* *value*) = [*value*] |  
*inputs-of-NewArrayNode*:  
*inputs-of* (*NewArrayNode* *length0* *stateBefore* *next*) = *length0* # (*opt-to-list* *stateBefore*) |  
*inputs-of-NewInstanceNode*:  
*inputs-of* (*NewInstanceNode* *nid0* *instanceClass* *stateBefore* *next*) = (*opt-to-list* *stateBefore*) |  
*inputs-of-NotNode*:  
*inputs-of* (*NotNode* *value*) = [*value*] |  
*inputs-of-OrNode*:  
*inputs-of* (*OrNode* *x* *y*) = [*x*, *y*] |  
*inputs-of-ParameterNode*:  
*inputs-of* (*ParameterNode* *index*) = [] |  
*inputs-of-PiNode*:  
*inputs-of* (*PiNode* *object* *guard*) = *object* # (*opt-to-list* *guard*) |  
*inputs-of-ReturnNode*:  
*inputs-of* (*ReturnNode* *result* *memoryMap*) = (*opt-to-list* *result*) @ (*opt-to-list* *memoryMap*) |  
*inputs-of-RightShiftNode*:  
*inputs-of* (*RightShiftNode* *x* *y*) = [*x*, *y*] |  
*inputs-of-ShortCircuitOrNode*:  
*inputs-of* (*ShortCircuitOrNode* *x* *y*) = [*x*, *y*] |  
*inputs-of-SignExtendNode*:  
*inputs-of* (*SignExtendNode* *inputBits* *resultBits* *value*) = [*value*] |  
*inputs-of-SignedDivNode*:  
*inputs-of* (*SignedDivNode* *nid0* *x* *y* *zeroCheck* *stateBefore* *next*) = [*x*, *y*] @ (*opt-to-list* *zeroCheck*) @ (*opt-to-list* *stateBefore*) |  
*inputs-of-SignedRemNode*:  
*inputs-of* (*SignedRemNode* *nid0* *x* *y* *zeroCheck* *stateBefore* *next*) = [*x*, *y*] @ (*opt-to-list* *zeroCheck*) @ (*opt-to-list* *stateBefore*) |  
*inputs-of-StartNode*:  
*inputs-of* (*StartNode* *stateAfter* *next*) = (*opt-to-list* *stateAfter*) |  
*inputs-of-StoreFieldNode*:  
*inputs-of* (*StoreFieldNode* *nid0* *field* *value* *stateAfter* *object* *next*) = *value* # (*opt-to-list* *stateAfter*) @ (*opt-to-list* *object*) |  
*inputs-of-SubNode*:

*inputs-of* (*SubNode* *x y*) = [*x*, *y*] |  
*inputs-of* *UnsignedRightShiftNode*:  
*inputs-of* (*UnsignedRightShiftNode* *x y*) = [*x*, *y*] |  
*inputs-of* *UnwindNode*:  
*inputs-of* (*UnwindNode* *exception*) = [*exception*] |  
*inputs-of* *ValuePhiNode*:  
*inputs-of* (*ValuePhiNode* *nid0 values merge*) = *merge* # *values* |  
*inputs-of* *ValueProxyNode*:  
*inputs-of* (*ValueProxyNode* *value loopExit*) = [*value*, *loopExit*] |  
*inputs-of* *XorNode*:  
*inputs-of* (*XorNode* *x y*) = [*x*, *y*] |  
*inputs-of* *ZeroExtendNode*:  
*inputs-of* (*ZeroExtendNode* *inputBits resultBits value*) = [*value*] |  
*inputs-of* *NoNode*: *inputs-of* (*NoNode*) = [] |

*inputs-of* *RefNode*: *inputs-of* (*RefNode* *ref*) = [*ref*]

**fun** *successors-of* :: *IRNode* ⇒ *ID list* **where**

*successors-of* *AbsNode*:  
*successors-of* (*AbsNode* *value*) = [] |  
*successors-of* *AddNode*:  
*successors-of* (*AddNode* *x y*) = [] |  
*successors-of* *AndNode*:  
*successors-of* (*AndNode* *x y*) = [] |  
*successors-of* *BeginNode*:  
*successors-of* (*BeginNode* *next*) = [*next*] |  
*successors-of* *BytecodeExceptionNode*:  
*successors-of* (*BytecodeExceptionNode* *arguments stateAfter next*) = [*next*] |  
*successors-of* *ConditionalNode*:  
*successors-of* (*ConditionalNode* *condition trueValue falseValue*) = [] |  
*successors-of* *ConstantNode*:  
*successors-of* (*ConstantNode* *const*) = [] |  
*successors-of* *DynamicNewArrayNode*:  
*successors-of* (*DynamicNewArrayNode* *elementType length0 voidClass stateBefore next*) = [*next*] |  
*successors-of* *EndNode*:  
*successors-of* (*EndNode*) = [] |  
*successors-of* *ExceptionObjectNode*:  
*successors-of* (*ExceptionObjectNode* *stateAfter next*) = [*next*] |  
*successors-of* *FrameState*:  
*successors-of* (*FrameState* *monitorIds outerFrameState values virtualObjectMappings*) = [] |  
*successors-of* *IfNode*:  
*successors-of* (*IfNode* *condition trueSuccessor falseSuccessor*) = [*trueSuccessor*, *falseSuccessor*] |  
*successors-of* *IntegerBelowNode*:  
*successors-of* (*IntegerBelowNode* *x y*) = [] |

*successors-of-IntegerEqualsNode:*  
*successors-of (IntegerEqualsNode x y) = [] |*  
*successors-of-IntegerLessThanNode:*  
*successors-of (IntegerLessThanNode x y) = [] |*  
*successors-of-InvokeNode:*  
*successors-of (InvokeNode nid0 callTarget classInit stateDuring stateAfter next)*  
*= [next] |*  
*successors-of-InvokeWithExceptionNode:*  
*successors-of (InvokeWithExceptionNode nid0 callTarget classInit stateDuring*  
*stateAfter next exceptionEdge) = [next, exceptionEdge] |*  
*successors-of-IsNullNode:*  
*successors-of (IsNullNode value) = [] |*  
*successors-of-KillingBeginNode:*  
*successors-of (KillingBeginNode next) = [next] |*  
*successors-of-LeftShiftNode:*  
*successors-of (LeftShiftNode x y) = [] |*  
*successors-of-LoadFieldNode:*  
*successors-of (LoadFieldNode nid0 field object next) = [next] |*  
*successors-of-LogicNegationNode:*  
*successors-of (LogicNegationNode value) = [] |*  
*successors-of-LoopBeginNode:*  
*successors-of (LoopBeginNode ends overflowGuard stateAfter next) = [next] |*  
*successors-of-LoopEndNode:*  
*successors-of (LoopEndNode loopBegin) = [] |*  
*successors-of-LoopExitNode:*  
*successors-of (LoopExitNode loopBegin stateAfter next) = [next] |*  
*successors-of-MergeNode:*  
*successors-of (MergeNode ends stateAfter next) = [next] |*  
*successors-of-MethodCallTargetNode:*  
*successors-of (MethodCallTargetNode targetMethod arguments) = [] |*  
*successors-of-MulNode:*  
*successors-of (MulNode x y) = [] |*  
*successors-of-NarrowNode:*  
*successors-of (NarrowNode inputBits resultBits value) = [] |*  
*successors-of-NegateNode:*  
*successors-of (NegateNode value) = [] |*  
*successors-of-NewArrayNode:*  
*successors-of (NewArrayNode length0 stateBefore next) = [next] |*  
*successors-of-NewInstanceNode:*  
*successors-of (NewInstanceNode nid0 instanceClass stateBefore next) = [next] |*  
*successors-of-NotNode:*  
*successors-of (NotNode value) = [] |*  
*successors-of-OrNode:*  
*successors-of (OrNode x y) = [] |*  
*successors-of-ParameterNode:*  
*successors-of (ParameterNode index) = [] |*  
*successors-of-PiNode:*  
*successors-of (PiNode object guard) = [] |*  
*successors-of-ReturnNode:*

*successors-of* (*ReturnNode* *result* *memoryMap*) = [] |  
*successors-of-RightShiftNode*:  
*successors-of* (*RightShiftNode* *x* *y*) = [] |  
*successors-of-ShortCircuitOrNode*:  
*successors-of* (*ShortCircuitOrNode* *x* *y*) = [] |  
*successors-of-SignExtendNode*:  
*successors-of* (*SignExtendNode* *inputBits* *resultBits* *value*) = [] |  
*successors-of-SignedDivNode*:  
*successors-of* (*SignedDivNode* *nid0* *x* *y* *zeroCheck* *stateBefore* *next*) = [*next*] |  
*successors-of-SignedRemNode*:  
*successors-of* (*SignedRemNode* *nid0* *x* *y* *zeroCheck* *stateBefore* *next*) = [*next*] |  
*successors-of-StartNode*:  
*successors-of* (*StartNode* *stateAfter* *next*) = [*next*] |  
*successors-of-StoreFieldNode*:  
*successors-of* (*StoreFieldNode* *nid0* *field* *value* *stateAfter* *object* *next*) = [*next*] |  
*successors-of-SubNode*:  
*successors-of* (*SubNode* *x* *y*) = [] |  
*successors-of-UnsignedRightShiftNode*:  
*successors-of* (*UnsignedRightShiftNode* *x* *y*) = [] |  
*successors-of-UnwindNode*:  
*successors-of* (*UnwindNode* *exception*) = [] |  
*successors-of-ValuePhiNode*:  
*successors-of* (*ValuePhiNode* *nid0* *values* *merge*) = [] |  
*successors-of-ValueProxyNode*:  
*successors-of* (*ValueProxyNode* *value* *loopExit*) = [] |  
*successors-of-XorNode*:  
*successors-of* (*XorNode* *x* *y*) = [] |  
*successors-of-ZeroExtendNode*:  
*successors-of* (*ZeroExtendNode* *inputBits* *resultBits* *value*) = [] |  
*successors-of-NoNode*: *successors-of* (*NoNode*) = [] |

*successors-of-RefNode*: *successors-of* (*RefNode* *ref*) = [*ref*]

**lemma** *inputs-of* (*FrameState* *x* (*Some* *y*) (*Some* *z*) *None*) = *x* @ [*y*] @ *z*

**unfolding** *inputs-of-FrameState* **by** *simp*

**lemma** *successors-of* (*FrameState* *x* (*Some* *y*) (*Some* *z*) *None*) = []

**unfolding** *inputs-of-FrameState* **by** *simp*

**lemma** *inputs-of* (*IfNode* *c* *t* *f*) = [*c*]

**unfolding** *inputs-of-IfNode* **by** *simp*

**lemma** *successors-of* (*IfNode* *c* *t* *f*) = [*t*, *f*]

**unfolding** *successors-of-IfNode* **by** *simp*

**lemma** *inputs-of* (*EndNode*) = [] ∧ *successors-of* (*EndNode*) = []

**unfolding** *inputs-of-EndNode* *successors-of-EndNode* **by** *simp*

**end**

### 3.2 IR Graph Node Hierarchy

```
theory IRNodeHierarchy
imports IRNodes
begin
```

It is helpful to introduce a node hierarchy into our formalization. Often the GraalVM compiler relies on explicit type checks to determine which operations to perform on a given node, we try to mimic the same functionality by using a suite of predicate functions over the *IRNode* class to determine inheritance.

As one would expect, the function *is<ClassName>Type* will be true if the node parameter is a subclass of the *ClassName* within the GraalVM compiler.

These functions have been automatically generated from the compiler.

```
fun is-EndNode :: IRNode  $\Rightarrow$  bool where
  is-EndNode EndNode = True |
  is-EndNode - = False
```

```
fun is-VirtualState :: IRNode  $\Rightarrow$  bool where
  is-VirtualState n = (is-FrameState n)
```

```
fun is-BinaryArithmeticNode :: IRNode  $\Rightarrow$  bool where
  is-BinaryArithmeticNode n = (is-AddNode n)  $\vee$  (is-AndNode n)  $\vee$  (is-MulNode n)
 $\vee$  (is-OrNode n)  $\vee$  (is-SubNode n)  $\vee$  (is-XorNode n)
```

```
fun is-ShiftNode :: IRNode  $\Rightarrow$  bool where
  is-ShiftNode n = (is-LeftShiftNode n)  $\vee$  (is-RightShiftNode n)  $\vee$  (is-UnsignedRightShiftNode n)
```

```
fun is-BinaryNode :: IRNode  $\Rightarrow$  bool where
  is-BinaryNode n = (is-BinaryArithmeticNode n)  $\vee$  (is-ShiftNode n)
```

```
fun is-AbstractLocalNode :: IRNode  $\Rightarrow$  bool where
  is-AbstractLocalNode n = (is-ParameterNode n)
```

```
fun is-IntegerConvertNode :: IRNode  $\Rightarrow$  bool where
  is-IntegerConvertNode n = (is-NarrowNode n)  $\vee$  (is-SignExtendNode n)  $\vee$  (is-ZeroExtendNode n)
```

```
fun is-UnaryArithmeticNode :: IRNode  $\Rightarrow$  bool where
  is-UnaryArithmeticNode n = (is-AbsNode n)  $\vee$  (is-NegateNode n)  $\vee$  (is-NotNode n)
```

```
fun is-UnaryNode :: IRNode  $\Rightarrow$  bool where
```

```

is-UnaryNode n = ((is-IntegerConvertNode n) ∨ (is-UnaryArithmeticNode n))

fun is-PhiNode :: IRNode ⇒ bool where
  is-PhiNode n = ((is-ValuePhiNode n))

fun is-FloatingGuardedNode :: IRNode ⇒ bool where
  is-FloatingGuardedNode n = ((is-PiNode n))

fun is-UnaryOpLogicNode :: IRNode ⇒ bool where
  is-UnaryOpLogicNode n = ((is-IsNullNode n))

fun is-IntegerLowerThanNode :: IRNode ⇒ bool where
  is-IntegerLowerThanNode n = ((is-IntegerBelowNode n) ∨ (is-IntegerLessThanNode n))

fun is-CompareNode :: IRNode ⇒ bool where
  is-CompareNode n = ((is-IntegerEqualsNode n) ∨ (is-IntegerLowerThanNode n))

fun is-BinaryOpLogicNode :: IRNode ⇒ bool where
  is-BinaryOpLogicNode n = ((is-CompareNode n))

fun is-LogicNode :: IRNode ⇒ bool where
  is-LogicNode n = ((is-BinaryOpLogicNode n) ∨ (is-LogicNegationNode n) ∨
    (is-ShortCircuitOrNode n) ∨ (is-UnaryOpLogicNode n))

fun is-ProxyNode :: IRNode ⇒ bool where
  is-ProxyNode n = ((is-ValueProxyNode n))

fun is-FloatingNode :: IRNode ⇒ bool where
  is-FloatingNode n = ((is-AbstractLocalNode n) ∨ (is-BinaryNode n) ∨ (is-ConditionalNode n) ∨
    (is-ConstantNode n) ∨ (is-FloatingGuardedNode n) ∨ (is-LogicNode n) ∨
    (is-PhiNode n) ∨ (is-ProxyNode n) ∨ (is-UnaryNode n))

fun is-AccessFieldNode :: IRNode ⇒ bool where
  is-AccessFieldNode n = ((is-LoadFieldNode n) ∨ (is-StoreFieldNode n))

fun is-AbstractNewArrayNode :: IRNode ⇒ bool where
  is-AbstractNewArrayNode n = ((is-DynamicNewArrayNode n) ∨ (is-NewArrayNode n))

fun is-AbstractNewObjectNode :: IRNode ⇒ bool where
  is-AbstractNewObjectNode n = ((is-AbstractNewArrayNode n) ∨ (is-NewInstanceNode n))

fun is-IntegerDivRemNode :: IRNode ⇒ bool where
  is-IntegerDivRemNode n = ((is-SignedDivNode n) ∨ (is-SignedRemNode n))

fun is-FixedBinaryNode :: IRNode ⇒ bool where
  is-FixedBinaryNode n = ((is-IntegerDivRemNode n))

```

```

fun is-DeoptimizingFixedWithNextNode :: IRNode  $\Rightarrow$  bool where
  is-DeoptimizingFixedWithNextNode n = ((is-AbstractNewObjectNode n)  $\vee$  (is-FixedBinaryNode
n))

fun is-AbstractMemoryCheckpoint :: IRNode  $\Rightarrow$  bool where
  is-AbstractMemoryCheckpoint n = ((is-BytecodeExceptionNode n)  $\vee$  (is-InvokeNode
n))

fun is-AbstractStateSplit :: IRNode  $\Rightarrow$  bool where
  is-AbstractStateSplit n = ((is-AbstractMemoryCheckpoint n))

fun is-AbstractMergeNode :: IRNode  $\Rightarrow$  bool where
  is-AbstractMergeNode n = ((is-LoopBeginNode n)  $\vee$  (is-MergeNode n))

fun is-BeginStateSplitNode :: IRNode  $\Rightarrow$  bool where
  is-BeginStateSplitNode n = ((is-AbstractMergeNode n)  $\vee$  (is-ExceptionObjectNode
n)  $\vee$  (is-LoopExitNode n)  $\vee$  (is-StartNode n))

fun is-AbstractBeginNode :: IRNode  $\Rightarrow$  bool where
  is-AbstractBeginNode n = ((is-BeginNode n)  $\vee$  (is-BeginStateSplitNode n)  $\vee$ 
(is-KillingBeginNode n))

fun is-FixedWithNextNode :: IRNode  $\Rightarrow$  bool where
  is-FixedWithNextNode n = ((is-AbstractBeginNode n)  $\vee$  (is-AbstractStateSplit n)
 $\vee$  (is-AccessFieldNode n)  $\vee$  (is-DeoptimizingFixedWithNextNode n))

fun is-WithExceptionNode :: IRNode  $\Rightarrow$  bool where
  is-WithExceptionNode n = ((is-InvokeWithExceptionNode n))

fun is-ControlSplitNode :: IRNode  $\Rightarrow$  bool where
  is-ControlSplitNode n = ((is-IfNode n)  $\vee$  (is-WithExceptionNode n))

fun is-ControlSinkNode :: IRNode  $\Rightarrow$  bool where
  is-ControlSinkNode n = ((is-ReturnNode n)  $\vee$  (is-UnwindNode n))

fun is-AbstractEndNode :: IRNode  $\Rightarrow$  bool where
  is-AbstractEndNode n = ((is-EndNode n)  $\vee$  (is-LoopEndNode n))

fun is-FixedNode :: IRNode  $\Rightarrow$  bool where
  is-FixedNode n = ((is-AbstractEndNode n)  $\vee$  (is-ControlSinkNode n)  $\vee$  (is-ControlSplitNode
n)  $\vee$  (is-FixedWithNextNode n))

fun is-CallTargetNode :: IRNode  $\Rightarrow$  bool where
  is-CallTargetNode n = ((is-MethodCallTargetNode n))

fun is-ValueNode :: IRNode  $\Rightarrow$  bool where
  is-ValueNode n = ((is-CallTargetNode n)  $\vee$  (is-FixedNode n)  $\vee$  (is-FloatingNode
n))

```

```

fun is-Node :: IRNode  $\Rightarrow$  bool where
  is-Node n = ((is-ValueNode n)  $\vee$  (is-VirtualState n))

fun is-MemoryKill :: IRNode  $\Rightarrow$  bool where
  is-MemoryKill n = ((is-AbstractMemoryCheckpoint n))

fun is-NarrowableArithmeticNode :: IRNode  $\Rightarrow$  bool where
  is-NarrowableArithmeticNode n = ((is-AbsNode n)  $\vee$  (is-AddNode n)  $\vee$  (is-AndNode
n)  $\vee$  (is-MulNode n)  $\vee$  (is-NegateNode n)  $\vee$  (is-NotNode n)  $\vee$  (is-OrNode n)  $\vee$ 
(is-ShiftNode n)  $\vee$  (is-SubNode n)  $\vee$  (is-XorNode n))

fun is-AnchoringNode :: IRNode  $\Rightarrow$  bool where
  is-AnchoringNode n = ((is-AbstractBeginNode n))

fun is-DeoptBefore :: IRNode  $\Rightarrow$  bool where
  is-DeoptBefore n = ((is-DeoptimizingFixedWithNextNode n))

fun is-IndirectCanonicalization :: IRNode  $\Rightarrow$  bool where
  is-IndirectCanonicalization n = ((is-LogicNode n))

fun is-IterableNodeType :: IRNode  $\Rightarrow$  bool where
  is-IterableNodeType n = ((is-AbstractBeginNode n)  $\vee$  (is-AbstractMergeNode n)  $\vee$ 
(is-FrameState n)  $\vee$  (is-IfNode n)  $\vee$  (is-IntegerDivRemNode n)  $\vee$  (is-InvokeWithExceptionNode
n)  $\vee$  (is-LoopBeginNode n)  $\vee$  (is-LoopExitNode n)  $\vee$  (is-MethodCallTargetNode n)
 $\vee$  (is-ParameterNode n)  $\vee$  (is-ReturnNode n)  $\vee$  (is-ShortCircuitOrNode n))

fun is-Invoke :: IRNode  $\Rightarrow$  bool where
  is-Invoke n = ((is-InvokeNode n)  $\vee$  (is-InvokeWithExceptionNode n))

fun is-Proxy :: IRNode  $\Rightarrow$  bool where
  is-Proxy n = ((is-ProxyNode n))

fun is-ValueProxy :: IRNode  $\Rightarrow$  bool where
  is-ValueProxy n = ((is-PiNode n)  $\vee$  (is-ValueProxyNode n))

fun is-ValueNodeInterface :: IRNode  $\Rightarrow$  bool where
  is-ValueNodeInterface n = ((is-ValueNode n))

fun is-ArrayLengthProvider :: IRNode  $\Rightarrow$  bool where
  is-ArrayLengthProvider n = ((is-AbstractNewArrayNode n)  $\vee$  (is-ConstantNode
n))

fun is-StampInverter :: IRNode  $\Rightarrow$  bool where
  is-StampInverter n = ((is-IntegerConvertNode n)  $\vee$  (is-NegateNode n)  $\vee$  (is-NotNode
n))

fun is-GuardingNode :: IRNode  $\Rightarrow$  bool where
  is-GuardingNode n = ((is-AbstractBeginNode n))

```



```

fun is-SingleMemoryKill :: IRNode  $\Rightarrow$  bool where
  is-SingleMemoryKill n = ((is-BytecodeExceptionNode n)  $\vee$  (is-ExceptionObjectNode
n)  $\vee$  (is-InvokeNode n)  $\vee$  (is-InvokeWithExceptionNode n)  $\vee$  (is-KillingBeginNode
n)  $\vee$  (is-StartNode n))

fun is-LIRLowerable :: IRNode  $\Rightarrow$  bool where
  is-LIRLowerable n = ((is-AbstractBeginNode n)  $\vee$  (is-AbstractEndNode n)  $\vee$ 
(is-AbstractMergeNode n)  $\vee$  (is-BinaryOpLogicNode n)  $\vee$  (is-CallTargetNode n)
 $\vee$  (is-ConditionalNode n)  $\vee$  (is-ConstantNode n)  $\vee$  (is-IfNode n)  $\vee$  (is-InvokeNode
n)  $\vee$  (is-InvokeWithExceptionNode n)  $\vee$  (is-IsNullNode n)  $\vee$  (is-LoopBeginNode n)
 $\vee$  (is-PiNode n)  $\vee$  (is-ReturnNode n)  $\vee$  (is-SignedDivNode n)  $\vee$  (is-SignedRemNode
n)  $\vee$  (is-UnaryOpLogicNode n)  $\vee$  (is-UnwindNode n))

fun is-GuardedNode :: IRNode  $\Rightarrow$  bool where
  is-GuardedNode n = ((is-FloatingGuardedNode n))

fun is-ArithmeticLIRLowerable :: IRNode  $\Rightarrow$  bool where
  is-ArithmeticLIRLowerable n = ((is-AbsNode n)  $\vee$  (is-BinaryArithmeticNode n)  $\vee$ 
(is-IntegerConvertNode n)  $\vee$  (is-NotNode n)  $\vee$  (is-ShiftNode n)  $\vee$  (is-UnaryArithmeticNode
n))

fun is-SwitchFoldable :: IRNode  $\Rightarrow$  bool where
  is-SwitchFoldable n = ((is-IfNode n))

fun is-VirtualizableAllocation :: IRNode  $\Rightarrow$  bool where
  is-VirtualizableAllocation n = ((is-NewArrayNode n)  $\vee$  (is-NewInstanceNode n))

fun is-Unary :: IRNode  $\Rightarrow$  bool where
  is-Unary n = ((is-LoadFieldNode n)  $\vee$  (is-LogicNegationNode n)  $\vee$  (is-UnaryNode
n)  $\vee$  (is-UnaryOpLogicNode n))

fun is-FixedNodeInterface :: IRNode  $\Rightarrow$  bool where
  is-FixedNodeInterface n = ((is-FixedNode n))

fun is-BinaryCommutative :: IRNode  $\Rightarrow$  bool where
  is-BinaryCommutative n = ((is-AddNode n)  $\vee$  (is-AndNode n)  $\vee$  (is-IntegerEqualsNode
n)  $\vee$  (is-MulNode n)  $\vee$  (is-OrNode n)  $\vee$  (is-XorNode n))

fun is-Canonicalizable :: IRNode  $\Rightarrow$  bool where
  is-Canonicalizable n = ((is-BytecodeExceptionNode n)  $\vee$  (is-ConditionalNode n)  $\vee$ 
(is-DynamicNewArrayNode n)  $\vee$  (is-PhiNode n)  $\vee$  (is-PiNode n)  $\vee$  (is-ProxyNode
n)  $\vee$  (is-StoreFieldNode n)  $\vee$  (is-ValueProxyNode n))

fun is-UncheckedInterfaceProvider :: IRNode  $\Rightarrow$  bool where
  is-UncheckedInterfaceProvider n = ((is-InvokeNode n)  $\vee$  (is-InvokeWithExceptionNode
n)  $\vee$  (is-LoadFieldNode n)  $\vee$  (is-ParameterNode n))

fun is-Binary :: IRNode  $\Rightarrow$  bool where

```

*is-Binary* *n* = ((*is-BinaryArithmeticNode* *n*) ∨ (*is-BinaryNode* *n*) ∨ (*is-BinaryOpLogicNode* *n*) ∨ (*is-CompareNode* *n*) ∨ (*is-FixedBinaryNode* *n*) ∨ (*is-ShortCircuitOrNode* *n*))

**fun** *is-ArithmeticOperation* :: *IRNode* ⇒ *bool* **where**  
*is-ArithmeticOperation* *n* = ((*is-BinaryArithmeticNode* *n*) ∨ (*is-IntegerConvertNode* *n*) ∨ (*is-ShiftNode* *n*) ∨ (*is-UnaryArithmeticNode* *n*))

**fun** *is-ValueNumberable* :: *IRNode* ⇒ *bool* **where**  
*is-ValueNumberable* *n* = ((*is-FloatingNode* *n*) ∨ (*is-ProxyNode* *n*))

**fun** *is-Lowerable* :: *IRNode* ⇒ *bool* **where**  
*is-Lowerable* *n* = ((*is-AbstractNewObjectNode* *n*) ∨ (*is-AccessFieldNode* *n*) ∨ (*is-BytecodeExceptionNode* *n*) ∨ (*is-ExceptionObjectNode* *n*) ∨ (*is-IntegerDivRemNode* *n*) ∨ (*is-UnwindNode* *n*))

**fun** *is-Virtualizable* :: *IRNode* ⇒ *bool* **where**  
*is-Virtualizable* *n* = ((*is-IsNullNode* *n*) ∨ (*is-LoadFieldNode* *n*) ∨ (*is-PiNode* *n*) ∨ (*is-StoreFieldNode* *n*) ∨ (*is-ValueProxyNode* *n*))

**fun** *is-Simplifiable* :: *IRNode* ⇒ *bool* **where**  
*is-Simplifiable* *n* = ((*is-AbstractMergeNode* *n*) ∨ (*is-BeginNode* *n*) ∨ (*is-IfNode* *n*) ∨ (*is-LoopExitNode* *n*) ∨ (*is-MethodCallTargetNode* *n*) ∨ (*is-NewArrayNode* *n*))

**fun** *is-StateSplit* :: *IRNode* ⇒ *bool* **where**  
*is-StateSplit* *n* = ((*is-AbstractStateSplit* *n*) ∨ (*is-BeginStateSplitNode* *n*) ∨ (*is-StoreFieldNode* *n*))

**fun** *is-ConvertNode* :: *IRNode* ⇒ *bool* **where**  
*is-ConvertNode* *n* = ((*is-IntegerConvertNode* *n*))

**fun** *is-sequential-node* :: *IRNode* ⇒ *bool* **where**  
*is-sequential-node* (*StartNode* -) = *True* |  
*is-sequential-node* (*BeginNode* -) = *True* |  
*is-sequential-node* (*KillingBeginNode* -) = *True* |  
*is-sequential-node* (*LoopBeginNode* - - -) = *True* |  
*is-sequential-node* (*LoopExitNode* - -) = *True* |  
*is-sequential-node* (*MergeNode* - -) = *True* |  
*is-sequential-node* (*RefNode* -) = *True* |  
*is-sequential-node* - = *False*

The following convenience function is useful in determining if two *IRNodes* are of the same type irregardless of their edges. It will return true if both the node parameters are the same node class.

**fun** *is-same-ir-node-type* :: *IRNode* ⇒ *IRNode* ⇒ *bool* **where**  
*is-same-ir-node-type* *n1* *n2* = (  
 ((*is-AbsNode* *n1*) ∧ (*is-AbsNode* *n2*)) ∨  
 ((*is-AddNode* *n1*) ∧ (*is-AddNode* *n2*)) ∨  
 ((*is-AndNode* *n1*) ∧ (*is-AndNode* *n2*)) ∨

$((is-BeginNode\ n1) \wedge (is-BeginNode\ n2)) \vee$   
 $((is-BytecodeExceptionNode\ n1) \wedge (is-BytecodeExceptionNode\ n2)) \vee$   
 $((is-ConditionalNode\ n1) \wedge (is-ConditionalNode\ n2)) \vee$   
 $((is-ConstantNode\ n1) \wedge (is-ConstantNode\ n2)) \vee$   
 $((is-DynamicNewArrayNode\ n1) \wedge (is-DynamicNewArrayNode\ n2)) \vee$   
 $((is-EndNode\ n1) \wedge (is-EndNode\ n2)) \vee$   
 $((is-ExceptionObjectNode\ n1) \wedge (is-ExceptionObjectNode\ n2)) \vee$   
 $((is-FrameState\ n1) \wedge (is-FrameState\ n2)) \vee$   
 $((is-IfNode\ n1) \wedge (is-IfNode\ n2)) \vee$   
 $((is-IntegerBelowNode\ n1) \wedge (is-IntegerBelowNode\ n2)) \vee$   
 $((is-IntegerEqualsNode\ n1) \wedge (is-IntegerEqualsNode\ n2)) \vee$   
 $((is-IntegerLessThanNode\ n1) \wedge (is-IntegerLessThanNode\ n2)) \vee$   
 $((is-InvokeNode\ n1) \wedge (is-InvokeNode\ n2)) \vee$   
 $((is-InvokeWithExceptionNode\ n1) \wedge (is-InvokeWithExceptionNode\ n2)) \vee$   
 $((is-IsNullNode\ n1) \wedge (is-IsNullNode\ n2)) \vee$   
 $((is-KillingBeginNode\ n1) \wedge (is-KillingBeginNode\ n2)) \vee$   
 $((is-LeftShiftNode\ n1) \wedge (is-LeftShiftNode\ n2)) \vee$   
 $((is-LoadFieldNode\ n1) \wedge (is-LoadFieldNode\ n2)) \vee$   
 $((is-LogicNegationNode\ n1) \wedge (is-LogicNegationNode\ n2)) \vee$   
 $((is-LoopBeginNode\ n1) \wedge (is-LoopBeginNode\ n2)) \vee$   
 $((is-LoopEndNode\ n1) \wedge (is-LoopEndNode\ n2)) \vee$   
 $((is-LoopExitNode\ n1) \wedge (is-LoopExitNode\ n2)) \vee$   
 $((is-MergeNode\ n1) \wedge (is-MergeNode\ n2)) \vee$   
 $((is-MethodCallTargetNode\ n1) \wedge (is-MethodCallTargetNode\ n2)) \vee$   
 $((is-MulNode\ n1) \wedge (is-MulNode\ n2)) \vee$   
 $((is-NarrowNode\ n1) \wedge (is-NarrowNode\ n2)) \vee$   
 $((is-NegateNode\ n1) \wedge (is-NegateNode\ n2)) \vee$   
 $((is-NewArrayNode\ n1) \wedge (is-NewArrayNode\ n2)) \vee$   
 $((is-NewInstanceNode\ n1) \wedge (is-NewInstanceNode\ n2)) \vee$   
 $((is-NotNode\ n1) \wedge (is-NotNode\ n2)) \vee$   
 $((is-OrNode\ n1) \wedge (is-OrNode\ n2)) \vee$   
 $((is-ParameterNode\ n1) \wedge (is-ParameterNode\ n2)) \vee$   
 $((is-PiNode\ n1) \wedge (is-PiNode\ n2)) \vee$   
 $((is-ReturnNode\ n1) \wedge (is-ReturnNode\ n2)) \vee$   
 $((is-RightShiftNode\ n1) \wedge (is-RightShiftNode\ n2)) \vee$   
 $((is-ShortCircuitOrNode\ n1) \wedge (is-ShortCircuitOrNode\ n2)) \vee$   
 $((is-SignedDivNode\ n1) \wedge (is-SignedDivNode\ n2)) \vee$   
 $((is-SignedRemNode\ n1) \wedge (is-SignedRemNode\ n2)) \vee$   
 $((is-SignExtendNode\ n1) \wedge (is-SignExtendNode\ n2)) \vee$   
 $((is-StartNode\ n1) \wedge (is-StartNode\ n2)) \vee$   
 $((is-StoreFieldNode\ n1) \wedge (is-StoreFieldNode\ n2)) \vee$   
 $((is-SubNode\ n1) \wedge (is-SubNode\ n2)) \vee$   
 $((is-UnsignedRightShiftNode\ n1) \wedge (is-UnsignedRightShiftNode\ n2)) \vee$   
 $((is-UnwindNode\ n1) \wedge (is-UnwindNode\ n2)) \vee$   
 $((is-ValuePhiNode\ n1) \wedge (is-ValuePhiNode\ n2)) \vee$   
 $((is-ValueProxyNode\ n1) \wedge (is-ValueProxyNode\ n2)) \vee$   
 $((is-XorNode\ n1) \wedge (is-XorNode\ n2)) \vee$   
 $((is-ZeroExtendNode\ n1) \wedge (is-ZeroExtendNode\ n2)))$

**end**

### 3.3 IR Graph Type

```
theory IRGraph
imports
  IRNodeHierarchy
  Stamp
  HOL-Library.FSet
  HOL.Relation
begin
```

This theory defines the main Graal data structure - an entire IR Graph.

IRGraph is defined as a partial map with a finite domain. The finite domain is required to be able to generate code and produce an interpreter.

```
typedef IRGraph = {g :: ID  $\rightarrow$  (IRNode  $\times$  Stamp) . finite (dom g)}
proof –
  have finite(dom(Map.empty))  $\wedge$  ran Map.empty = {} by auto
  then show ?thesis
    by fastforce
qed
```

```
setup-lifting type-definition-IRGraph
```

```
lift-definition ids :: IRGraph  $\Rightarrow$  ID set
is  $\lambda g. \{nid \in \text{dom } g . \nexists s. g \text{ } nid = (\text{Some } (\text{NoNode}, s))\}$  .
```

```
fun with-default :: 'c  $\Rightarrow$  ('b  $\Rightarrow$  'c)  $\Rightarrow$  (('a  $\rightarrow$  'b)  $\Rightarrow$  'a  $\Rightarrow$  'c) where
  with-default def conv = ( $\lambda m k. ($ 
     $\text{case } m \text{ of } \text{None} \Rightarrow \text{def} \mid \text{Some } v \Rightarrow \text{conv } v)$ )
```

```
lift-definition kind :: IRGraph  $\Rightarrow$  (ID  $\Rightarrow$  IRNode)
is with-default NoNode fst .
```

```
lift-definition stamp :: IRGraph  $\Rightarrow$  ID  $\Rightarrow$  Stamp
is with-default IllegalStamp snd .
```

```
lift-definition add-node :: ID  $\Rightarrow$  (IRNode  $\times$  Stamp)  $\Rightarrow$  IRGraph  $\Rightarrow$  IRGraph
is  $\lambda nid \ k \ g. \text{if } \text{fst } k = \text{NoNode} \text{ then } g \text{ else } g(nid \mapsto k)$  by simp
```

```
lift-definition remove-node :: ID  $\Rightarrow$  IRGraph  $\Rightarrow$  IRGraph
is  $\lambda nid \ g. g(nid := \text{None})$  by simp
```

```
lift-definition replace-node :: ID  $\Rightarrow$  (IRNode  $\times$  Stamp)  $\Rightarrow$  IRGraph  $\Rightarrow$  IRGraph
is  $\lambda nid \ k \ g. \text{if } \text{fst } k = \text{NoNode} \text{ then } g \text{ else } g(nid \mapsto k)$  by simp
```

```
lift-definition as-list :: IRGraph  $\Rightarrow$  (ID  $\times$  IRNode  $\times$  Stamp) list
is  $\lambda g. \text{map } (\lambda k. (k, \text{the } (g \ k))) (\text{sorted-list-of-set } (\text{dom } g))$  .
```

**fun** *no-node* :: (*ID* × (*IRNode* × *Stamp*)) *list* ⇒ (*ID* × (*IRNode* × *Stamp*)) *list*  
**where**  
*no-node* *g* = *filter* (λ*n*. *fst* (*snd* *n*) ≠ *NoNode*) *g*

**lift-definition** *irgraph* :: (*ID* × (*IRNode* × *Stamp*)) *list* ⇒ *IRGraph*  
**is** *map-of* ∘ *no-node*  
**by** (*simp add: finite-dom-map-of*)

**definition** *as-set* :: *IRGraph* ⇒ (*ID* × (*IRNode* × *Stamp*)) *set* **where**  
*as-set* *g* = {(*n*, *kind* *g* *n*, *stamp* *g* *n*) | *n* . *n* ∈ *ids* *g*}

**definition** *true-ids* :: *IRGraph* ⇒ *ID* *set* **where**  
*true-ids* *g* = *ids* *g* − {*n* ∈ *ids* *g*. ∃ *n'* . *kind* *g* *n* = *RefNode* *n'*}

**definition** *domain-subtraction* :: '*a* *set* ⇒ ('*a* × '*b*) *set* ⇒ ('*a* × '*b*) *set*  
**(infix** ≤ 30) **where**  
*domain-subtraction* *s* *r* = {(*x*, *y*) . (*x*, *y*) ∈ *r* ∧ *x* ∉ *s*}

**notation** (*latex*)  
*domain-subtraction* (− ◀ −)

**code-datatype** *irgraph*

**fun** *filter-none* **where**  
*filter-none* *g* = {*nid* ∈ *dom* *g* . ∄ *s*. *g* *nid* = (*Some* (*NoNode*, *s*))}

**lemma** *no-node-clears*:  
*res* = *no-node* *xs* ⟶ (∀ *x* ∈ *set* *res*. *fst* (*snd* *x*) ≠ *NoNode*)  
**by** *simp*

**lemma** *dom-eq*:  
**assumes** ∀ *x* ∈ *set* *xs*. *fst* (*snd* *x*) ≠ *NoNode*  
**shows** *filter-none* (*map-of* *xs*) = *dom* (*map-of* *xs*)  
**unfolding** *filter-none.simps* **using** *assms map-of-SomeD*  
**by** *fastforce*

**lemma** *fil-eq*:  
*filter-none* (*map-of* (*no-node* *xs*)) = *set* (*map* *fst* (*no-node* *xs*))  
**using** *no-node-clears*  
**by** (*metis dom-eq dom-map-of-conv-image-fst list.set-map*)

**lemma** *irgraph[code]: ids* (*irgraph* *m*) = *set* (*map* *fst* (*no-node* *m*))  
**unfolding** *irgraph-def ids-def* **using** *fil-eq*  
**by** (*smt Rep-IRGraph comp-apply eq-onp-same-args filter-none.simps ids.abs-eq*  
*ids-def irgraph.abs-eq irgraph.rep-eq irgraph-def mem-Collect-eq*)

**lemma** [*code*]: *Rep-IRGraph* (*irgraph* *m*) = *map-of* (*no-node* *m*)

```

using Abs-IRGraph-inverse
by (simp add: irgraph.rep-eq)

— Get the inputs set of a given node ID
fun inputs :: IRGraph  $\Rightarrow$  ID  $\Rightarrow$  ID set where
  inputs g nid = set (inputs-of (kind g nid))
— Get the successor set of a given node ID
fun succ :: IRGraph  $\Rightarrow$  ID  $\Rightarrow$  ID set where
  succ g nid = set (successors-of (kind g nid))
— Gives a relation between node IDs - between a node and its input nodes
fun input-edges :: IRGraph  $\Rightarrow$  ID rel where
  input-edges g = ( $\bigcup$  i  $\in$  ids g. {(i,j)|j. j  $\in$  (inputs g i)})
— Find all the nodes in the graph that have nid as an input - the usages of nid
fun usages :: IRGraph  $\Rightarrow$  ID  $\Rightarrow$  ID set where
  usages g nid = {i. i  $\in$  ids g  $\wedge$  nid  $\in$  inputs g i}
fun successor-edges :: IRGraph  $\Rightarrow$  ID rel where
  successor-edges g = ( $\bigcup$  i  $\in$  ids g. {(i,j)|j. j  $\in$  (succ g i)})
fun predecessors :: IRGraph  $\Rightarrow$  ID  $\Rightarrow$  ID set where
  predecessors g nid = {i. i  $\in$  ids g  $\wedge$  nid  $\in$  succ g i}
fun nodes-of :: IRGraph  $\Rightarrow$  (IRNode  $\Rightarrow$  bool)  $\Rightarrow$  ID set where
  nodes-of g sel = {nid  $\in$  ids g . sel (kind g nid)}
fun edge :: (IRNode  $\Rightarrow$  'a)  $\Rightarrow$  ID  $\Rightarrow$  IRGraph  $\Rightarrow$  'a where
  edge sel nid g = sel (kind g nid)

fun filtered-inputs :: IRGraph  $\Rightarrow$  ID  $\Rightarrow$  (IRNode  $\Rightarrow$  bool)  $\Rightarrow$  ID list where
  filtered-inputs g nid f = filter (f  $\circ$  (kind g)) (inputs-of (kind g nid))
fun filtered-successors :: IRGraph  $\Rightarrow$  ID  $\Rightarrow$  (IRNode  $\Rightarrow$  bool)  $\Rightarrow$  ID list where
  filtered-successors g nid f = filter (f  $\circ$  (kind g)) (successors-of (kind g nid))
fun filtered-usages :: IRGraph  $\Rightarrow$  ID  $\Rightarrow$  (IRNode  $\Rightarrow$  bool)  $\Rightarrow$  ID set where
  filtered-usages g nid f = {n  $\in$  (usages g nid). f (kind g n)}

fun is-empty :: IRGraph  $\Rightarrow$  bool where
  is-empty g = (ids g = {})

fun any-usage :: IRGraph  $\Rightarrow$  ID  $\Rightarrow$  ID where
  any-usage g nid = hd (sorted-list-of-set (usages g nid))

lemma ids-some[simp]: x  $\in$  ids g  $\longleftrightarrow$  kind g x  $\neq$  NoNode
proof —
  have that: x  $\in$  ids g  $\longrightarrow$  kind g x  $\neq$  NoNode
  using ids.rep-eq kind.rep-eq by force
  have kind g x  $\neq$  NoNode  $\longrightarrow$  x  $\in$  ids g
  unfolding with-default.simps kind-def ids-def
  by (cases Rep-IRGraph g x = None; auto)
  from this that show ?thesis by auto
qed

lemma not-in-g:

```

```

assumes  $nid \notin ids\ g$ 
shows  $kind\ g\ nid = NoNode$ 
using  $assms\ ids\ some$  by  $blast$ 

lemma  $valid\_creation[simp]$ :
   $finite\ (dom\ g) \longleftrightarrow Rep\_IRGraph\ (Abs\_IRGraph\ g) = g$ 
using  $Abs\_IRGraph\_inverse$  by  $(metis\ Rep\_IRGraph\ mem\_Collect\_eq)$ 

lemma  $[simp]$ :  $finite\ (ids\ g)$ 
using  $Rep\_IRGraph\ ids.rep\_eq$  by  $simp$ 

lemma  $[simp]$ :  $finite\ (ids\ (irgraph\ g))$ 
by  $(simp\ add: finite\_dom\_map\_of)$ 

lemma  $[simp]$ :  $finite\ (dom\ g) \longrightarrow ids\ (Abs\_IRGraph\ g) = \{nid \in dom\ g . \nexists s. g\ nid = Some\ (NoNode,\ s)\}$ 
using  $ids.rep\_eq$  by  $simp$ 

lemma  $[simp]$ :  $finite\ (dom\ g) \longrightarrow kind\ (Abs\_IRGraph\ g) = (\lambda x . (case\ g\ x\ of\ None \Rightarrow NoNode\ |\ Some\ n \Rightarrow fst\ n))$ 
by  $(simp\ add: kind.rep\_eq)$ 

lemma  $[simp]$ :  $finite\ (dom\ g) \longrightarrow stamp\ (Abs\_IRGraph\ g) = (\lambda x . (case\ g\ x\ of\ None \Rightarrow IllegalStamp\ |\ Some\ n \Rightarrow snd\ n))$ 
using  $stamp.abs\_eq\ stamp.rep\_eq$  by  $auto$ 

lemma  $[simp]$ :  $ids\ (irgraph\ g) = set\ (map\ fst\ (no\_node\ g))$ 
using  $irgraph$  by  $auto$ 

lemma  $[simp]$ :  $kind\ (irgraph\ g) = (\lambda nid. (case\ (map\_of\ (no\_node\ g))\ nid\ of\ None \Rightarrow NoNode\ |\ Some\ n \Rightarrow fst\ n))$ 
using  $irgraph.rep\_eq\ kind.transfer\ kind.rep\_eq$  by  $auto$ 

lemma  $[simp]$ :  $stamp\ (irgraph\ g) = (\lambda nid. (case\ (map\_of\ (no\_node\ g))\ nid\ of\ None \Rightarrow IllegalStamp\ |\ Some\ n \Rightarrow snd\ n))$ 
using  $irgraph.rep\_eq\ stamp.transfer\ stamp.rep\_eq$  by  $auto$ 

lemma  $map\_of\_upd$ :  $(map\_of\ g)(k \mapsto v) = (map\_of\ ((k,\ v)\ \# \ g))$ 
by  $simp$ 

lemma  $[code]$ :  $replace\_node\ nid\ k\ (irgraph\ g) = (irgraph\ ((nid,\ k)\ \# \ g))$ 
proof  $(cases\ fst\ k = NoNode)$ 
  case  $True$ 
    then show  $?thesis$ 
    by  $(metis\ (mono\_tags,\ lifting)\ Rep\_IRGraph\_inject\ filter.simps(2)\ irgraph.abs\_eq\ no\_node.simps\ replace\_node.rep\_eq\ snd\_conv)$ 
  next
    case  $False$ 

```

**then show** *?thesis unfolding irgraph-def replace-node-def no-node.simps*  
**by** (*smt (verit, best) Rep-IRGraph comp-apply eq-onp-same-args filter.simps(2)*  
*id-def irgraph.rep-eq map-fun-apply map-of-upd mem-Collect-eq no-node.elims re-*  
*place-node.abs-eq replace-node-def snd-eqD)*  
**qed**

**lemma** [code]: *add-node nid k (irgraph g) = (irgraph (((nid, k) # g)))*  
**by** (*smt (z3) Rep-IRGraph-inject add-node.rep-eq filter.simps(2) irgraph.rep-eq*  
*map-of-upd no-node.simps snd-conv)*

**lemma** *add-node-lookup*:  
 $gup = \text{add-node } nid \ (k, s) \ g \longrightarrow$   
 $(\text{if } k \neq \text{NoNode} \text{ then } \text{kind } gup \ nid = k \wedge \text{stamp } gup \ nid = s \text{ else } \text{kind } gup \ nid$   
 $= \text{kind } g \ nid)$   
**proof** (*cases k = NoNode*)  
**case** *True*  
**then show** *?thesis*  
**by** (*simp add: add-node.rep-eq kind.rep-eq*)  
**next**  
**case** *False*  
**then show** *?thesis*  
**by** (*simp add: kind.rep-eq add-node.rep-eq stamp.rep-eq*)  
**qed**

**lemma** *remove-node-lookup*:  
 $gup = \text{remove-node } nid \ g \longrightarrow \text{kind } gup \ nid = \text{NoNode} \wedge \text{stamp } gup \ nid = \text{IllegalStamp}$   
**by** (*simp add: kind.rep-eq remove-node.rep-eq stamp.rep-eq*)

**lemma** *replace-node-lookup[simp]*:  
 $gup = \text{replace-node } nid \ (k, s) \ g \wedge k \neq \text{NoNode} \longrightarrow \text{kind } gup \ nid = k \wedge \text{stamp}$   
 $gup \ nid = s$   
**by** (*simp add: replace-node.rep-eq kind.rep-eq stamp.rep-eq*)

**lemma** *replace-node-unchanged*:  
 $gup = \text{replace-node } nid \ (k, s) \ g \longrightarrow (\forall \ n \in (\text{ids } g - \{nid\}) . n \in \text{ids } g \wedge n \in$   
 $\text{ids } gup \wedge \text{kind } g \ n = \text{kind } gup \ n)$   
**by** (*simp add: kind.rep-eq replace-node.rep-eq*)

### 3.3.1 Example Graphs

Example 1: empty graph (just a start and end node)

**definition** *start-end-graph*:: *IRGraph* **where**  
*start-end-graph = irgraph [(0, StartNode None 1, VoidStamp), (1, ReturnNode*  
*None None, VoidStamp)]*

Example 2: public static int sq(int x) return x \* x;  
 [1 P(0)] / [0 Start] [4 \*] | / V / [5 Return]



```

definition eg2-sq :: IRGraph where
  eg2-sq = irgraph [
    (0, StartNode None 5, VoidStamp),
    (1, ParameterNode 0, default-stamp),
    (4, MulNode 1 1, default-stamp),
    (5, ReturnNode (Some 4) None, default-stamp)
  ]

```

```

value input-edges eg2-sq
value usages eg2-sq 1

```

```

end

```

## 4 java.lang.Long

Utility functions from the Long class that Graal occasionally makes use of.

```

theory Long
  imports ValueThms
begin

```

```

lemma negative-all-set-32:
   $n < 32 \implies \text{bit } (-1::\text{int}32) \ n$ 
  apply transfer by auto

```

```

definition MaxOrNeg :: nat set  $\Rightarrow$  int where
  MaxOrNeg s = (if s = {} then -1 else Max s)

```

```

definition MinOrHighest :: nat set  $\Rightarrow$  nat  $\Rightarrow$  nat where
  MinOrHighest s m = (if s = {} then m else Min s)

```

```

definition highestOneBit :: ('a::len) word  $\Rightarrow$  int where
  highestOneBit v = MaxOrNeg {n . bit v n}

```

```

definition lowestOneBit :: ('a::len) word  $\Rightarrow$  nat where
  lowestOneBit v = MinOrHighest {n . bit v n} (size v)

```

```

lemma max-bit: bit (v::('a::len) word) n  $\implies n < \text{size } v$ 
  by (simp add: bit-imp-le-length size-word.rep-eq)

```

```

lemma max-set-bit: MaxOrNeg {n . bit (v::('a::len) word) n} < Nat.size v
  using max-bit unfolding MaxOrNeg-def
  by force

```

## 4.1 Long.numberOfLeadingZeros

**definition** *numberOfLeadingZeros* :: ('a::len) word  $\Rightarrow$  nat **where**  
*numberOfLeadingZeros* v = nat (Nat.size v - highestOneBit v - 1)

**lemma** *MaxOrNeg-neg*: *MaxOrNeg* {} = -1  
**by** (simp add: *MaxOrNeg-def*)

**lemma** *MaxOrNeg-max*:  $s \neq \{\}$   $\implies$  *MaxOrNeg* s = *Max* s  
**by** (simp add: *MaxOrNeg-def*)

**lemma** *zero-no-bits*:  
 $\{n . \text{bit } 0 \ n\} = \{\}$   
**by** simp

**lemma** *highestOneBit* (0::64 word) = -1  
**by** (simp add: *MaxOrNeg-neg* *highestOneBit-def*)

**lemma** *numberOfLeadingZeros* (0::64 word) = 64  
**unfolding** *numberOfLeadingZeros-def* **using** *MaxOrNeg-neg* *highestOneBit-def*  
*size64*  
**by** (smt (verit) nat-int zero-no-bits)

**lemma** *highestOneBit-top*: *Max* {*highestOneBit* (v::64 word)} < 64  
**unfolding** *highestOneBit-def*  
**by** (metis *Max-singleton* int-eq-iff-numeral max-set-bit size64)

**lemma** *numberOfLeadingZeros-top*: *Max* {*numberOfLeadingZeros* (v::64 word)}  $\leq$  64  
**unfolding** *numberOfLeadingZeros-def*  
**using** *size64*  
**by** (simp add: *MaxOrNeg-def* *highestOneBit-def* nat-le-iff)

**lemma** *numberOfLeadingZeros-range*:  $0 \leq \text{numberOfLeadingZeros } a \wedge \text{numberOfLeadingZeros } a \leq \text{Nat.size } a$   
**unfolding** *numberOfLeadingZeros-def*  
**using** *MaxOrNeg-def* *highestOneBit-def* nat-le-iff  
**by** (smt (verit) bot-nat-0.extremum int-eq-iff)

**lemma** *leadingZerosAddHighestOne*: *numberOfLeadingZeros* v + *highestOneBit* v = *Nat.size* v - 1  
**unfolding** *numberOfLeadingZeros-def* *highestOneBit-def*  
**using** *MaxOrNeg-def* int-nat-eq int-ops(6) max-bit order-less-irrefl **by** fastforce

## 4.2 Long.numberOfTrailingZeros

**definition** *numberOfTrailingZeros* :: ('a::len) word  $\Rightarrow$  nat **where**  
*numberOfTrailingZeros* v = *lowestOneBit* v

**lemma** *lowestOneBit-bot*: *lowestOneBit* (0::64 word) = 64

**unfolding** *lowestOneBit-def MinOrHighest-def*  
**by** (*simp add: size64*)

**lemma** *bit-zero-set-in-top*:  $\text{bit } (-1::'a::\text{len word}) \ 0$   
**by** *auto*

**lemma** *nat-bot-set*:  $(0::\text{nat}) \in xs \longrightarrow (\forall x \in xs . 0 \leq x)$   
**by** *fastforce*

**lemma** *numberOfTrailingZeros*  $(0::64 \text{ word}) = 64$   
**unfolding** *numberOfTrailingZeros-def*  
**using** *lowestOneBit-bot* **by** *simp*

### 4.3 Long.bitCount

**definition** *bitCount* ::  $('a::\text{len}) \text{ word} \Rightarrow \text{nat}$  **where**  
*bitCount*  $v = \text{card } \{n . \text{bit } v \ n\}$

**lemma** *bitCount*  $0 = 0$   
**unfolding** *bitCount-def*  
**by** (*metis card.empty zero-no-bits*)

### 4.4 Long.zeroCount

**definition** *zeroCount* ::  $('a::\text{len}) \text{ word} \Rightarrow \text{nat}$  **where**  
*zeroCount*  $v = \text{card } \{n . n < \text{Nat.size } v \wedge \neg(\text{bit } v \ n)\}$

**lemma** *zeroCount-finite*:  $\text{finite } \{n . n < \text{Nat.size } v \wedge \neg(\text{bit } v \ n)\}$   
**using** *finite-nat-set-iff-bounded* **by** *blast*

**lemma** *negone-set*:  
 $\text{bit } (-1::('a::\text{len}) \text{ word}) \ n \longleftrightarrow n < \text{LENGTH}('a)$   
**by** *simp*

**lemma** *negone-all-bits*:  
 $\{n . \text{bit } (-1::('a::\text{len}) \text{ word}) \ n\} = \{n . 0 \leq n \wedge n < \text{LENGTH}('a)\}$   
**using** *negone-set*  
**by** *auto*

**lemma** *bitCount-finite*:  
 $\text{finite } \{n . \text{bit } (v::('a::\text{len}) \text{ word}) \ n\}$   
**by** *simp*

**lemma** *card-of-range*:  
 $x = \text{card } \{n . 0 \leq n \wedge n < x\}$   
**by** *simp*

**lemma** *range-of-nat*:  
 $\{(n::\text{nat}) . 0 \leq n \wedge n < x\} = \{n . n < x\}$   
**by** *simp*

**lemma** *finite-range*:  
*finite* {*n*::nat . *n* < *x*}  
**by** *simp*

**lemma** *range-eq*:  
**fixes** *x y* :: nat  
**shows** *card* {*y*..*x*} = *card* {*y*<..*x*}  
**using** *card-atLeastLessThan card-greaterThanAtMost* **by** *presburger*

**lemma** *card-of-range-bound*:  
**fixes** *x y* :: nat  
**assumes** *x* > *y*  
**shows** *x* - *y* = *card* {*n* . *y* < *n* ∧ *n* ≤ *x*}  
**proof** -  
**have** *finite*: *finite* {*n* . *y* ≤ *n* ∧ *n* < *x*}  
**by** *auto*  
**have** *nonempty*: {*n* . *y* ≤ *n* ∧ *n* < *x*} ≠ {}  
**using** *assms* **by** *blast*  
**have** *simprep*: {*n* . *y* < *n* ∧ *n* ≤ *x*} = {*y*<..*x*}  
**by** *auto*  
**have** *x* - *y* = *card* {*y*<..*x*}  
**by** *auto*  
**then show** ?*thesis*  
**unfolding** *simprep* **by** *blast*  
**qed**

**lemma** *bitCount* (-1::('a::len) word) = *LENGTH*('a)  
**unfolding** *bitCount-def* **using** *card-of-range*  
**by** (*metis* (*no-types*, *lifting*) *Collect-cong negone-all-bits*)

**lemma** *bitCount-range*:  
**fixes** *n* :: ('a::len) word  
**shows** 0 ≤ *bitCount* *n* ∧ *bitCount* *n* ≤ *Nat.size* *n*  
**unfolding** *bitCount-def*  
**by** (*metis* *atLeastLessThan-iff bot-nat-0.extremum max-bit mem-Collect-eq subsetI subset-eq-atLeast0-lessThan-card*)

**lemma** *zerosAboveHighestOne*:  
*n* > *highestOneBit* *a* ⇒ ¬(*bit* *a* *n*)  
**unfolding** *highestOneBit-def MaxOrNeg-def*  
**by** (*metis* (*mono-tags*, *opaque-lifting*) *Collect-empty-eq Max-ge finite-bit-word less-le-not-le mem-Collect-eq of-nat-le-iff*)

**lemma** *zerosBelowLowestOne*:  
**assumes** *n* < *lowestOneBit* *a*  
**shows** ¬(*bit* *a* *n*)  
**proof** (*cases* {*i*. *bit* *a* *i*} = {})

```

    case True
    then show ?thesis by simp
next
    case False
    have  $n < \text{Min } (\text{Collect } (\text{bit } a)) \implies \neg \text{bit } a \ n$ 
    using False by auto
    then show ?thesis
    by (metis False MinOrHighest-def assms lowestOneBit-def)
qed

lemma union-bit-sets:
  fixes  $a :: ('a::\text{len}) \text{ word}$ 
  shows  $\{n . n < \text{Nat.size } a \wedge \text{bit } a \ n\} \cup \{n . n < \text{Nat.size } a \wedge \neg(\text{bit } a \ n)\} = \{n . n < \text{Nat.size } a\}$ 
  by fastforce

lemma disjoint-bit-sets:
  fixes  $a :: ('a::\text{len}) \text{ word}$ 
  shows  $\{n . n < \text{Nat.size } a \wedge \text{bit } a \ n\} \cap \{n . n < \text{Nat.size } a \wedge \neg(\text{bit } a \ n)\} = \{\}$ 
  by blast

lemma qualified-bitCount:
   $\text{bitCount } v = \text{card } \{n . n < \text{Nat.size } v \wedge \text{bit } v \ n\}$ 
  by (metis (no-types, lifting) Collect-cong bitCount-def max-bit)

lemma card-eq:
  assumes  $\text{finite } x \wedge \text{finite } y \wedge \text{finite } z$ 
  assumes  $x \cup y = z$ 
  assumes  $y \cap x = \{\}$ 
  shows  $\text{card } z - \text{card } y = \text{card } x$ 
  using assms add-diff-cancel-right' card-Un-disjoint
  by (metis inf.commute)

lemma card-add:
  assumes  $\text{finite } x \wedge \text{finite } y \wedge \text{finite } z$ 
  assumes  $x \cup y = z$ 
  assumes  $y \cap x = \{\}$ 
  shows  $\text{card } x + \text{card } y = \text{card } z$ 
  using assms card-Un-disjoint
  by (metis inf.commute)

lemma card-add-inverses:
  assumes  $\text{finite } \{n . Q \ n \wedge \neg(P \ n)\} \wedge \text{finite } \{n . Q \ n \wedge P \ n\} \wedge \text{finite } \{n . Q \ n\}$ 
  shows  $\text{card } \{n . Q \ n \wedge P \ n\} + \text{card } \{n . Q \ n \wedge \neg(P \ n)\} = \text{card } \{n . Q \ n\}$ 
  apply (rule card-add)
  using assms apply simp
  apply auto[1]
  by auto

```

```

lemma ones-zero-sum-to-width:
  bitCount a + zeroCount a = Nat.size a
proof -
  have add-cards: card {n. (λn. n < size a) n ∧ (bit a n)} + card {n. (λn. n <
size a) n ∧ ¬(bit a n)} = card {n. (λn. n < size a) n}
  apply (rule card-add-inverses) by simp
  then have ... = Nat.size a
  by auto
then show ?thesis
  unfolding bitCount-def zeroCount-def using max-bit
  by (metis (mono-tags, lifting) Collect-cong add-cards)
qed

```

```

lemma intersect-bitCount-helper:
  card {n . n < Nat.size a} - bitCount a = card {n . n < Nat.size a ∧ ¬(bit a n)}
proof -
  have size-def: Nat.size a = card {n . n < Nat.size a}
  using card-of-range by simp
  have bitCount-def: bitCount a = card {n . n < Nat.size a ∧ bit a n}
  using qualified-bitCount by auto
  have disjoint: {n . n < Nat.size a ∧ bit a n} ∩ {n . n < Nat.size a ∧ ¬(bit a
n)} = {}
  using disjoint-bit-sets by auto
  have union: {n . n < Nat.size a ∧ bit a n} ∪ {n . n < Nat.size a ∧ ¬(bit a n)}
= {n . n < Nat.size a}
  using union-bit-sets by auto
  show ?thesis
  unfolding bitCount-def
  apply (rule card-eq)
  using finite-range apply simp
  using union apply blast
  using disjoint by simp
qed

```

```

lemma intersect-bitCount:
  Nat.size a - bitCount a = card {n . n < Nat.size a ∧ ¬(bit a n)}
  using card-of-range intersect-bitCount-helper by auto

```

```

hide-fact intersect-bitCount-helper

```

```

end

```

## 4.5 Structural Graph Comparison

```

theory
  Comparison
imports
  IRGraph

```

**begin**

We introduce a form of structural graph comparison that is able to assert structural equivalence of graphs which differ in zero or more reference node chains for any given nodes.

**fun** *find-ref-nodes* :: *IRGraph*  $\Rightarrow$  (*ID*  $\rightarrow$  *ID*) **where**  
*find-ref-nodes* *g* = *map-of*  
 (*map* ( $\lambda n. (n, \text{ir-ref } (\text{kind } g \ n)))$ ) (*filter* ( $\lambda id. \text{is-RefNode } (\text{kind } g \ id)$ ) (*sorted-list-of-set*  
 (*ids* *g*))))

**fun** *replace-ref-nodes* :: *IRGraph*  $\Rightarrow$  (*ID*  $\rightarrow$  *ID*)  $\Rightarrow$  *ID list*  $\Rightarrow$  *ID list* **where**  
*replace-ref-nodes* *g m xs* = *map* ( $\lambda id. (\text{case } (m \ id) \text{ of } \text{Some } other \Rightarrow other \mid \text{None} \Rightarrow id)$ ) *xs*

**fun** *find-next* :: *ID list*  $\Rightarrow$  *ID set*  $\Rightarrow$  *ID option* **where**  
*find-next to-see seen* = (*let* *l* = (*filter* ( $\lambda nid. nid \notin \text{seen}$ ) *to-see*)  
 in (*case* *l* of []  $\Rightarrow$  *None* | *xs*  $\Rightarrow$  *Some* (*hd* *xs*)))

**inductive** *reachables* :: *IRGraph*  $\Rightarrow$  *ID list*  $\Rightarrow$  *ID set*  $\Rightarrow$  *ID set*  $\Rightarrow$  *bool* **where**  
*reachables* *g* [] {} {} |  
 [| *None* = *find-next to-see seen* |]  $\Longrightarrow$  *reachables* *g to-see seen seen* |  
 [| *Some n* = *find-next to-see seen*;  
   *node* = *kind* *g n*;  
   *new* = (*inputs-of node*) @ (*successors-of node*);  
   *reachables* *g (to-see @ new) ({n}  $\cup$  seen) seen'* |]  $\Longrightarrow$  *reachables* *g to-see seen*  
*seen'*

**code-pred** (*modes*: *i*  $\Rightarrow$  *i*  $\Rightarrow$  *i*  $\Rightarrow$  *o*  $\Rightarrow$  *bool*) [*show-steps, show-mode-inference, show-intermediate-results*]  
*reachables* .

**inductive** *nodeEq* :: (*ID*  $\rightarrow$  *ID*)  $\Rightarrow$  *IRGraph*  $\Rightarrow$  *ID*  $\Rightarrow$  *IRGraph*  $\Rightarrow$  *ID*  $\Rightarrow$  *bool*  
**where**  
 [| *kind* *g1 n1* = *RefNode ref*; *nodeEq* *m g1 ref g2 n2* |]  $\Longrightarrow$  *nodeEq* *m g1 n1 g2 n2*  
 |  
 [| *x* = *kind* *g1 n1*;  
   *y* = *kind* *g2 n2*;  
   *is-same-ir-node-type* *x y*;  
   *replace-ref-nodes* *g1 m (successors-of x)* = *successors-of y*;  
   *replace-ref-nodes* *g1 m (inputs-of x)* = *inputs-of y* |]  
 $\Longrightarrow$  *nodeEq* *m g1 n1 g2 n2*

**code-pred** [*show-modes*] *nodeEq* .

**fun** *diffNodesGraph* :: *IRGraph*  $\Rightarrow$  *IRGraph*  $\Rightarrow$  *ID set* **where**  
*diffNodesGraph* *g1 g2* = (*let* *refNodes* = *find-ref-nodes* *g1* in  
 { *n* . *n*  $\in$  *Predicate.the (reachables-i-i-i-o g1 [0] {})*  $\wedge$  (*case* *refNodes n* of *Some*  
 -  $\Rightarrow$  *False* | -  $\Rightarrow$  *True*)  $\wedge$   $\neg(\text{nodeEq } \text{refNodes } g1 \ n \ g2 \ n)$  }

```

fun diffNodesInfo :: IRGraph ⇒ IRGraph ⇒ (ID × IRNode × IRNode) set (infix
  ∩s 20)
  where
    diffNodesInfo g1 g2 = {(nid, kind g1 nid, kind g2 nid) | nid . nid ∈ diffNodesGraph
      g1 g2}

fun eqGraph :: IRGraph ⇒ IRGraph ⇒ bool (infix ≈s 20)
  where
    eqGraph isabelle-graph graal-graph = ((diffNodesGraph isabelle-graph graal-graph)
      = {})

end

```

## 4.6 Control-flow Graph Traversal

```

theory
  Traversal
imports
  IRGraph
begin

```

```

type-synonym Seen = ID set

```

nextEdge helps determine which node to traverse next by returning the first successor edge that isn't in the set of already visited nodes. If there is not an appropriate successor, None is returned instead.

```

fun nextEdge :: Seen ⇒ ID ⇒ IRGraph ⇒ ID option where
  nextEdge seen nid g =
    (let nids = (filter (λnid'. nid' ∉ seen) (successors-of (kind g nid))) in
      (if length nids > 0 then Some (hd nids) else None))

```

pred determines which node, if any, acts as the predecessor of another.

Merge nodes represent a special case where-in the predecessor exists as an input edge of the merge node, to simplify the traversal we treat only the first input end node as the predecessor, ignoring that multiple nodes may act as a successor.

For all other nodes, the predecessor is the first element of the predecessors set. Note that in a well-formed graph there should only be one element in the predecessor set.

```

fun pred :: IRGraph ⇒ ID ⇒ ID option where
  pred g nid = (case kind g nid of
    (MergeNode ends -) ⇒ Some (hd ends) |
    - ⇒
      (if IRGraph.predecessors g nid = {}
        then None else
        Some (hd (sorted-list-of-set (IRGraph.predecessors g nid)))
      )

```



)

Here we try to implement a generic fork of the control-flow traversal algorithm that was initially implemented for the ConditionalElimination phase

**type-synonym**  $'a \text{ TraversalState} = (ID \times Seen \times 'a)$

**inductive Step**

$:: ('a \text{ TraversalState} \Rightarrow 'a) \Rightarrow IRGraph \Rightarrow 'a \text{ TraversalState} \Rightarrow 'a \text{ TraversalState option} \Rightarrow bool$

**for**  $sa \ g$  **where**

— Hit a BeginNode with an IfNode predecessor which represents the start of a basic block for the IfNode. 1.  $nid'$  will be the successor of the begin node. 2. Find the first and only predecessor. 3. Extract condition from the preceding IfNode. 4. Negate condition if the begin node is second branch (we've taken the else branch of the condition) 5. Add the condition or the negated condition to stack 6. Perform any stamp updates based on the condition using the registerNewCondition function and place them on the top of the stack of stamp information

$\llbracket kind \ g \ nid = \text{BeginNode } nid' \rrbracket;$

$nid \notin seen;$   
 $seen' = \{nid\} \cup seen;$

$Some \ ifcond = pred \ g \ nid;$   
 $kind \ g \ ifcond = \text{IfNode } cond \ t \ f;$

$analysis' = sa \ (nid, seen, analysis)$   
 $\implies Step \ sa \ g \ (nid, seen, analysis) \ (Some \ (nid', seen', analysis')) \mid$

— Hit an EndNode 1.  $nid'$  will be the usage of EndNode 2. pop the conditions and stamp stack

$\llbracket kind \ g \ nid = \text{EndNode} \rrbracket;$

$nid \notin seen;$   
 $seen' = \{nid\} \cup seen;$

$nid' = any\text{-usage } g \ nid;$

$analysis' = sa \ (nid, seen, analysis)$   
 $\implies Step \ sa \ g \ (nid, seen, analysis) \ (Some \ (nid', seen', analysis')) \mid$

— We can find a successor edge that is not in seen, go there

$\llbracket \neg(is\text{-EndNode } (kind \ g \ nid));$   
 $\neg(is\text{-BeginNode } (kind \ g \ nid));$

$nid \notin seen;$   
 $seen' = \{nid\} \cup seen;$

$Some \ nid' = nextEdge \ seen' \ nid \ g;$

```

    analysis' = sa (nid, seen, analysis)]
 $\implies$  Step sa g (nid, seen, analysis) (Some (nid', seen', analysis')) |

— We cannot find a successor edge that is not in seen, give back None
 $\llbracket \neg(\text{is-EndNode } (\text{kind } g \text{ nid}))$ ;
 $\neg(\text{is-BEGINNode } (\text{kind } g \text{ nid}))$ ;

    nid  $\notin$  seen;
    seen' = {nid}  $\cup$  seen;

    None = nextEdge seen' nid g]
 $\implies$  Step sa g (nid, seen, analysis) None |

— We've already seen this node, give back None
 $\llbracket \text{nid} \in \text{seen} \rrbracket \implies$  Step sa g (nid, seen, analysis) None

code-pred (modes: i  $\Rightarrow$  i  $\Rightarrow$  i  $\Rightarrow$  o  $\Rightarrow$  bool) Step .

end

```