GraalVM Stamp Theory

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Abstract

The GraalVM compiler uses stamps to track type and range information during program analysis. Type information is recorded by using distinct subclasses of the abstract base class `Stamp`, i.e. `IntegerStamp` is used to represent an integer type. Each subclass introduces facilities for tracking range information. Every subclass of the `Stamp` class forms a lattice, together with an arbitrary top and bottom element each sublattice forms a lattice of all stamps. This Isabelle/HOL theory models stamps as instantiations of a lattice.

Contents

1 Stamps: Type and Range Information

theory *StampLattice*
imports
  Values
  HOL.Lattices
begin

1.1 Void Stamp

The VoidStamp represents a type with no associated values. The VoidStamp lattice is therefore a simple single element lattice.

datatype *void* =
  VoidStamp

instantiation *void* :: *order*
begin

definition *less-eq-void* :: *void* ⇒ *void* ⇒ *bool* **where**
  less-eq-void a b = *True*

definition *less-void* :: *void* ⇒ *void* ⇒ *bool* **where**
  less-void a b = *False*

instance
  ⟨*proof*⟩

end

instantiation *void* :: *semilattice-inf*
begin

definition *inf-void* :: *void* ⇒ *void* ⇒ *void* **where**
  inf-void a b = *VoidStamp*

instance
  ⟨*proof*⟩

end

instantiation *void* :: *semilattice-sup*
begin

definition *sup-void* :: *void* ⇒ *void* ⇒ *void* **where**
  sup-void a b = *VoidStamp*

instance
  ⟨*proof*⟩

end

instantiation *void* :: *bounded-lattice*
begin

definition *bot-void* :: *void* **where**
bot-void = *VoidStamp*

definition *top-void* :: *void* **where**
top-void = *VoidStamp*

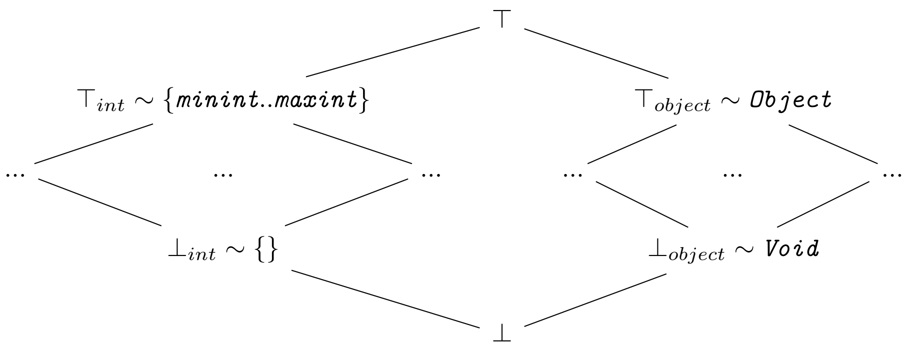
instance
⟨*proof*⟩

end

Definition of the stamp type

datatype *stamp* =
intstamp int64 int64 — Type: Integer; Range: Lower Bound & Upper Bound

1.2 Stamp Lattice



1.2.1 Stamp Order

Defines an ordering on the stamp type.

One stamp is less than another if the valid values for the stamp are a strict subset of the other stamp.

instantiation *stamp* :: *order*
begin

fun *less-eq-stamp* :: *stamp* ⇒ *stamp* ⇒ *bool* **where**
less-eq-stamp (*intstamp l1 u1*) (*intstamp l2 u2*) = ($\{l1..u1\} \subseteq \{l2..u2\}$)

fun *less-stamp* :: *stamp* ⇒ *stamp* ⇒ *bool* **where**
less-stamp (*intstamp l1 u1*) (*intstamp l2 u2*) = ($\{l1..u1\} \subset \{l2..u2\}$)

lemma *less-le-not-le*:
fixes $x\ y$ :: *stamp*
shows $(x < y) = (x \leq y \wedge \neg\, y \leq x)$
⟨*proof*⟩

lemma *order-refl*:
fixes x :: *stamp*
shows $x \leq x$
⟨*proof*⟩

lemma *order-trans*:
fixes $x\ y\ z$ :: *stamp*
shows $x \leq y \Longrightarrow y \leq z \Longrightarrow x \leq z$
⟨*proof*⟩

lemma *antisym*:
fixes $x\ y$ :: *stamp*
shows $x \leq y \Longrightarrow y \leq x \Longrightarrow x = y$
⟨*proof*⟩

instance
⟨*proof*⟩
end

1.2.2 Stamp Join

Defines the *join* operation for stamps.

For any two stamps, the *join* is defined as the intersection of the valid values for the stamp.

instantiation *stamp* :: *semilattice-inf*
begin

notation *inf* (**infix** ⊓ *65*)

fun *inf-stamp* :: *stamp* ⇒ *stamp* ⇒ *stamp* **where**
inf-stamp (*intstamp l1 u1*) (*intstamp l2 u2*) = *intstamp* (*max l1 l2*) (*min u1 u2*)

lemma *inf-le1*:
fixes $x\ y$ :: *stamp*
shows $(x \sqcap y) \leq x$
⟨*proof*⟩

lemma *inf-le2*:
fixes $x\ y$ :: *stamp*
shows $(x \sqcap y) \leq y$
⟨*proof*⟩

lemma *inf-greatest*:
  fixes $x\ y\ z$ :: *stamp*
  shows $x \leq y \Longrightarrow x \leq z \Longrightarrow x \leq (y \sqcap z)$
⟨*proof*⟩

instance
  ⟨*proof*⟩
end

1.2.3 Stamp Meet

Defines the *meet* operation for stamps.

For any two stamps, the *meet* is defined as the union of the valid values for the stamp.

instantiation *stamp* :: *semilattice-sup*
begin

notation *sup* (**infix** $\sqcup$ *65*)

fun *sup-stamp* :: *stamp* ⇒ *stamp* ⇒ *stamp* **where**
  sup-stamp (*intstamp l1 u1*) (*intstamp l2 u2*) = *intstamp* (*min l1 l2*) (*max u1 u2*)

lemma *sup-ge1*:
  fixes $x\ y$ :: *stamp*
  shows $x \leq x \sqcup y$
⟨*proof*⟩

lemma *sup-ge2*:
  fixes $x\ y$ :: *stamp*
  shows $y \leq x \sqcup y$
⟨*proof*⟩

lemma *sup-least*:
  fixes $x\ y\ z$ :: *stamp*
  shows $y \leq x \Longrightarrow z \leq x \Longrightarrow ((y \sqcup z) \leq x)$
⟨*proof*⟩

instance
  ⟨*proof*⟩
end

1.2.4 Stamp Bounds

Defines the top and bottom elements of the stamp lattice.

This poses an interesting question as our stamp type is a union of the various *Stamp* subclasses, e.g. *IntegerStamp*, *ObjectStamp*, etc.

Each subclass should preferably have its own unique top and bottom element, i.e. An *IntegerStamp* would have the top element of the full range of integers allowed by the bit width and a bottom of a range with no integers. While the *ObjectStamp* should have *Object* as the top and *Void* as the bottom element.

instantiation *stamp* :: *bounded-lattice*
begin

notation *bot* ($\perp$ *50*)
notation *top* ($\top$ *50*)

definition *width-min* :: *nat* $\Rightarrow$ *int64* **where**
width-min bits $= -(2\widehat{\ }(bits-1))$

definition *width-max* :: *nat* $\Rightarrow$ *int64* **where**
width-max bits $= (2\widehat{\ }(bits-1)) - 1$

value (*sint* (*width-min 64*), *sint* (*width-max 64*))
value *max-word*::*int64*

lemma
assumes $x =$ *width-min 64*
assumes $y =$ *width-max 64*
shows *sint* $x <$ *sint* y
⟨*proof*⟩

Note that this definition is valid for unsigned integers only.

The bottom and top element for signed integers would be (- 9223372036854775808, 9223372036854775807).

For unsigned we have (0, 18446744073709551615).

For Java we are likely to be more concerned with signed integers. To use the appropriate bottom and top for signed integers we would need to change our definition of less_eq from l1..u1 <= l2..u2 to sint l1..sint u1 <= sint l2..sint u2

We may still find an unsigned integer stamp useful. I plan to investigate the Java code to see if this is useful and then apply the changes to switch to signed integers.

definition *bot-stamp* = *intstamp max-word 0*
definition *top-stamp* = *intstamp 0 max-word*

lemma *bot-least*:
fixes *a* :: *stamp*
shows $(\perp) \leq a$
⟨*proof*⟩

lemma *top-greatest*:
fixes *a* :: *stamp*

shows $a \leq (\top)$
⟨*proof*⟩

instance
⟨*proof*⟩
end

1.3 Java Stamp Methods

The following are methods from the Java Stamp class, they are the methods primarily used for optimizations.

definition *is-unrestricted* :: *stamp* ⇒ *bool* **where**
is-unrestricted s = (⊤ = *s*)

fun *is-empty* :: *stamp* ⇒ *bool* **where**
is-empty s = (⊥ = *s*)

fun *as-constant* :: *stamp* ⇒ *Value option* **where**
as-constant (*intstamp l u*) = (*if* (*card* {*l*..*u*}) = *1*
then Some (*IntVal64* (*SOME x*. *x* ∈ {*l*..*u*}))
else None)

definition *always-distinct* :: *stamp* ⇒ *stamp* ⇒ *bool* **where**
always-distinct stamp1 stamp2 = (⊥ = (*stamp1* ⊓ *stamp2*))

definition *never-distinct* :: *stamp* ⇒ *stamp* ⇒ *bool* **where**
never-distinct stamp1 stamp2 =
(*as-constant stamp1* = *as-constant stamp2* ∧ *as-constant stamp1* ≠ *None*)

1.4 Mapping to Values

fun *valid-value* :: *stamp* => *Value* => *bool* **where**
valid-value (*intstamp l u*) (*IntVal64 v*) = (*v* ∈ {*l*..*u*}) |
valid-value (*intstamp l u*) - = *False*

The *valid-value* function is used to map a stamp instance to the values that are allowed by the stamp.

It would be nice if there was a slightly more integrated way to perform this mapping as it requires some infrastructure to prove some fairly simple properties.

lemma *bottom-range-empty*:
¬(*valid-value* (⊥) *v*)
⟨*proof*⟩

lemma *join-values*:
assumes *joined* = *x-stamp* ⊓ *y-stamp*
shows *valid-value joined x* ⟷ (*valid-value x-stamp x* ∧ *valid-value y-stamp x*)
⟨*proof*⟩

lemma *disjoint-empty*:
  fixes *x-stamp y-stamp* :: *stamp*
  assumes $\bot$ = *x-stamp* $\sqcap$ *y-stamp*
  shows $\neg$(*valid-value x-stamp x* $\wedge$ *valid-value y-stamp x*)
  $\langle$*proof*$\rangle$

experiment begin

A possible equivalent alternative to the definition of less_eq

fun *less-eq-alt* :: *'a::ord* $\times$ *'a* $\Rightarrow$ *'a* $\times$ *'a* $\Rightarrow$ *bool* **where**
  less-eq-alt (*l1*, *u1*) (*l2*, *u2*) = (($\neg$ *l1* $\leq$ *u1*) $\vee$ *l2* $\leq$ *l1* $\wedge$ *u1* $\leq$ *u2*)

Proof equivalence

lemma
  fixes *l1 l2 u1 u2* :: *int*
  assumes *l1* $\leq$ *u1* $\wedge$ *l2* $\leq$ *u2*
  shows {*l1..u1*} $\subseteq$ {*l2..u2*} = ((*l1* $\geq$ *l2*) $\wedge$ (*u1* $\leq$ *u2*))
  $\langle$*proof*$\rangle$

lemma
  fixes *l1 l2 u1 u2* :: *int*
  shows {*l1..u1*} $\subseteq$ {*l2..u2*} = *less-eq-alt* (*l1*, *u1*) (*l2*, *u2*)
  $\langle$*proof*$\rangle$
end

1.5 Generic Integer Stamp

Experimental definition of integer stamps generically, restricting the datatype to only allow valid ranges and the bottom integer element (max_int..min_int).

lemma
  assumes (*x::int*) > *0*
  shows (*2* ^ *x*)/*2* = (*2* ^ (*x* − *1*))
  $\langle$*proof*$\rangle$

definition *max-signed-int* :: *'a::len word* **where**
  max-signed-int = (*2* ^ (*LENGTH('a)* − *1*)) − *1*

definition *min-signed-int* :: *'a::len word* **where**
  min-signed-int = −(*2* ^ (*LENGTH('a)* − *1*))

definition *int-bottom* :: *'a::len word* $\times$ *'a word* **where**
  int-bottom = (*max-signed-int*, *min-signed-int*)

definition *int-top* :: *'a::len word* $\times$ *'a word* **where**
  int-top = (*min-signed-int*, *max-signed-int*)

lemma
  fixes x :: *'a::len word*
  shows *sint* $x \leq$ *sint* $(((2\ \hat{}\ (LENGTH('a) - 1)) - 1)::'a\ word)$
  ⟨*proof*⟩

value *sint* (*0*::*1 word*)
value *sint* (*1*::*1 word*)
value *sint* $(((2\ \hat{}\ 0) - 1)::1\ word)$

value *sint* $(((2\ \hat{}\ 31) - 1)::32\ word)$

lemma *max-signed*:
  fixes *a* :: *'a::len word*
  shows *sint a* $\leq$ *sint* (*max-signed-int*::*'a word*)
⟨*proof*⟩

lemma *min-signed*:
  fixes *a* :: *'a::len word*
  shows *sint a* $\geq$ *sint* (*min-signed-int*::*'a word*)
  ⟨*proof*⟩

value *max-signed-int* :: *32 word*
value *int-bottom*::(*32 word* × *32 word*)
value *sint* (*2147483647*::*32 word*)
value *sint* (*2147483648*::*32 word*)

typedef (**overloaded**) (*'a::len*) *intstamp* =
  {*bounds* :: (*'a word*, *'a word*) *prod* . ((*fst bounds*) $\leq$*s* (*snd bounds*) ∨ *bounds* = *int-bottom*)}
⟨*proof*⟩

setup-lifting *type-definition-intstamp*

lift-definition *lower* :: (*'a::len*) *intstamp* ⇒ *'a word*
  is *prod.fst* ∘ *Rep-intstamp* ⟨*proof*⟩

lift-definition *upper* :: (*'a::len*) *intstamp* ⇒ *'a word*
  is *prod.snd* ∘ *Rep-intstamp* ⟨*proof*⟩

lift-definition *lower-int* :: (*'a::len*) *intstamp* ⇒ *int*
  is *sint* ∘ *prod.fst* ⟨*proof*⟩

lift-definition *upper-int* :: (*'a::len*) *intstamp* ⇒ *int*
  is *sint* ∘ *prod.snd* ⟨*proof*⟩

lift-definition *range* :: ($'a$::*len*) *intstamp* $\Rightarrow$ *int set*
  is λ (l, u). {*sint l*..*sint u*} ⟨*proof*⟩

lift-definition *bounds* :: ($'a$::*len*) *intstamp* $\Rightarrow$ ($'a$ *word* $\times$ $'a$ *word*)
  is *Rep-intstamp* ⟨*proof*⟩

lift-definition *is-bottom* :: ($'a$::*len*) *intstamp* $\Rightarrow$ *bool*
  is λ x. x = *int-bottom* ⟨*proof*⟩

lift-definition *from-bounds* :: ($'a$::*len word* $\times$ $'a$ *word*) $\Rightarrow$ $'a$ *intstamp*
  is *Abs-intstamp* ⟨*proof*⟩

instantiation *intstamp* :: (*len*) *order*
begin

definition *less-eq-intstamp* :: $'a$ *intstamp* $\Rightarrow$ $'a$ *intstamp* $\Rightarrow$ *bool* **where**
  less-eq-intstamp s1 s2 = (*range s1* $\subseteq$ *range s2*)

definition *less-intstamp* :: $'a$ *intstamp* $\Rightarrow$ $'a$ *intstamp* $\Rightarrow$ *bool* **where**
  less-intstamp s1 s2 = (*range s1* $\subset$ *range s2*)

value *int-bottom*::(*1 word* $\times$ *1 word*)
value *sint* (*0*::*1 word*)
value *sint* (*1*::*1 word*)

value *int-bottom*::(*2 word* $\times$ *2 word*)
value *sint* (*1*::*2 word*)
value *sint* (*2*::*2 word*)
value *sint* ((*2* ^(*LENGTH*(*32*) − *1*) − *1*)::*32 word*) > *sint* ((− (*2* ^(*LENGTH*(*32*) − *1*)))::*32 word*)

lemma *bottom-is-bottom*:
  assumes *is-bottom s*
  shows $s \leq a$
⟨*proof*⟩

lemma *bounds-has-value*:
  fixes *x y* :: *int*
  assumes $x < y$
  shows *card* {*x*..*y*} > *0*
  ⟨*proof*⟩

lemma *bounds-has-no-value*:
  fixes *x y* :: *int*
  assumes $x < y$
  shows *card* {*y*..*x*} = *0*

⟨*proof*⟩

lemma *bottom-unique*:
fixes $a\ s$ :: $'a\ intstamp$
assumes *is-bottom* s
shows $a \leq s \longleftrightarrow$ *is-bottom* a
⟨*proof*⟩

lemma *bottom-antisym*:
assumes *is-bottom* x
shows $x \leq y \Longrightarrow y \leq x \Longrightarrow x = y$
⟨*proof*⟩

lemma *int-antisym*:
fixes $x\ y$ :: $'a\ intstamp$
shows $x \leq y \Longrightarrow y \leq x \Longrightarrow x = y$
⟨*proof*⟩

instance
⟨*proof*⟩
end

value *take-bit LENGTH*(*63*) *20*::*int*
value *take-bit LENGTH*(*63*) ((−*20*)::*int*)
value *bit* (*20*::*int64*) (*63*::*nat*)
value *bit* ((−*20*)::*int64*) (*63*::*nat*)

value ((−*20*)::*int64*) < (*20*::*int64*)

value *take-bit LENGTH*(*63*) ((−*20*)::*int*)

lift-definition *smax* :: $'a$::*len word* ⇒ $'a$ *word* ⇒ $'a$ *word*
is λ *a b*. (*if* (*sint a*) ≤ (*sint b*) *then b else a*) ⟨*proof*⟩

lift-definition *smin* :: $'a$::*len word* ⇒ $'a$ *word* ⇒ $'a$ *word*
is λ *a b*. (*if* (*sint a*) ≤ (*sint b*) *then a else b*) ⟨*proof*⟩

instantiation *intstamp* :: (*len*) *semilattice-inf*
begin

notation *inf* (**infix** ⊓ *65*)

definition *join-bounds* :: $'a$ *intstamp* ⇒ $'a$ *intstamp* ⇒ ($'a$ *word* × $'a$ *word*) **where**
join-bounds s1 s2 = (*smax* (*lower s1*) (*lower s2*), *smin* (*upper s1*) (*upper s2*))

definition *join-or-bottom* :: $'a$ *intstamp* ⇒ $'a$ *intstamp* ⇒ ($'a$ *word* × $'a$ *word*)
where

join-or-bottom s1 s2 = (*let bound* = (*join-bounds s1 s2*) *in*
if sint (*fst bound*) $\geq$ *sint* (*snd bound*) *then int-bottom else bound*)

definition *inf-intstamp* :: *'a intstamp* $\Rightarrow$ *'a intstamp* $\Rightarrow$ *'a intstamp* **where**
inf-intstamp s1 s2 = *from-bounds* (*join-or-bottom s1 s2*)

lemma *always-valid*:
fixes *s1 s2* :: *'a intstamp*
shows *Rep-intstamp* (*from-bounds* (*join-or-bottom s1 s2*)) = *join-or-bottom s1 s2*
$\langle proof \rangle$

lemma *invalid-join*:
fixes *s1 s2* :: *'a intstamp*
assumes *bound* = *join-bounds s1 s2*
assumes *sint* (*fst bound*) $\geq$ *sint* (*snd bound*)
shows *from-bounds int-bottom* = *s1* $\sqcap$ *s2*
$\langle proof \rangle$

lemma *unfold-bounds*:
bounds x = (*lower x*, *upper x*)
$\langle proof \rangle$

lemma *int-inf-le1*:
fixes *x y* :: *'a intstamp*
shows $(x \sqcap y) \leq x$
$\langle proof \rangle$

lemma *int-inf-le2*:
fixes *x y* :: *'a intstamp*
shows $(x \sqcap y) \leq y$
$\langle proof \rangle$

lemma
assumes $x \leq y$
assumes *is-bottom y*
shows *is-bottom x*
$\langle proof \rangle$

lemma *int-inf-greatest*:
fixes *x y* :: *'a intstamp*
shows $x \leq y \Longrightarrow x \leq z \Longrightarrow x \leq y \sqcap z$
$\langle proof \rangle$

instance
$\langle proof \rangle$

end

instantiation *intstamp* :: (*len*) *semilattice-sup*
begin

notation *sup* (**infix** ⊔ *65*)

instance ⟨*proof*⟩

end

instantiation *intstamp* :: (*len*) *bounded-lattice*
begin

notation *bot* (⊥ *50*)
notation *top* (⊤ *50*)

definition *bot-intstamp* = *int-bottom*
definition *top-intstamp* = *int-top*

instance ⟨*proof*⟩

end

value *sint* (*0*::*1 word*)
value *sint* (*1*::*1 word*)

datatype *Stamp* =
  BottomStamp |
  TopStamp |
  VoidStamp |

  Int8Stamp 8 intstamp |
  Int16Stamp 16 intstamp |
  Int32Stamp 32 intstamp |
  Int64Stamp 64 intstamp

instantiation *Stamp* :: *order*
begin

fun *less-eq-Stamp* :: *Stamp* ⇒ *Stamp* ⇒ *bool* **where**
  less-eq-Stamp BottomStamp - = *True* |
  less-eq-Stamp - *TopStamp* = *True* |
  less-eq-Stamp VoidStamp VoidStamp = *True* |
  less-eq-Stamp (*Int8Stamp v1*) (*Int8Stamp v2*) = (*v1* ≤ *v2*) |
  less-eq-Stamp (*Int16Stamp v1*) (*Int16Stamp v2*) = (*v1* ≤ *v2*) |
  less-eq-Stamp (*Int32Stamp v1*) (*Int32Stamp v2*) = (*v1* ≤ *v2*) |
  less-eq-Stamp (*Int64Stamp v1*) (*Int64Stamp v2*) = (*v1* ≤ *v2*) |
  less-eq-Stamp - - = *False*

fun *less-Stamp* :: *Stamp* ⇒ *Stamp* ⇒ *bool* **where**

less-Stamp BottomStamp BottomStamp = *False* |
less-Stamp BottomStamp - = *True* |
less-Stamp TopStamp TopStamp = *False* |
less-Stamp - *TopStamp* = *True* |
less-Stamp VoidStamp VoidStamp = *False* |
less-Stamp (*Int8Stamp v1*) (*Int8Stamp v2*) = (*v1* < *v2*) |
less-Stamp (*Int16Stamp v1*) (*Int16Stamp v2*) = (*v1* < *v2*) |
less-Stamp (*Int32Stamp v1*) (*Int32Stamp v2*) = (*v1* < *v2*) |
less-Stamp (*Int64Stamp v1*) (*Int64Stamp v2*) = (*v1* < *v2*) |
less-Stamp - - = *False*

instance
⟨*proof*⟩
end

instantiation *Stamp* :: *semilattice-inf*
begin

notation *inf* (**infix** ⊓ *65*)

fun *inf-Stamp* :: *Stamp* ⇒ *Stamp* ⇒ *Stamp* **where**
inf-Stamp BottomStamp - = *BottomStamp* |
inf-Stamp - *BottomStamp* = *BottomStamp* |
inf-Stamp TopStamp - = *TopStamp* |
inf-Stamp - *TopStamp* = *TopStamp* |
inf-Stamp VoidStamp VoidStamp = *VoidStamp* |
inf-Stamp (*Int8Stamp v1*) (*Int8Stamp v2*) = *Int8Stamp* (*v1* ⊓ *v2*) |
inf-Stamp (*Int16Stamp v1*) (*Int16Stamp v2*) = *Int16Stamp* (*v1* ⊓ *v2*) |
inf-Stamp (*Int32Stamp v1*) (*Int32Stamp v2*) = *Int32Stamp* (*v1* ⊓ *v2*) |
inf-Stamp (*Int64Stamp v1*) (*Int64Stamp v2*) = *Int64Stamp* (*v1* ⊓ *v2*)

instance
⟨*proof*⟩
end

instantiation *Stamp* :: *semilattice-sup*
begin

notation *sup* (**infix** ⊔ *65*)

fun *sup-Stamp* :: *Stamp* ⇒ *Stamp* ⇒ *Stamp* **where**
sup-Stamp BottomStamp - = *BottomStamp* |
sup-Stamp - *BottomStamp* = *BottomStamp* |
sup-Stamp TopStamp - = *TopStamp* |
sup-Stamp - *TopStamp* = *TopStamp* |
sup-Stamp VoidStamp VoidStamp = *VoidStamp* |
sup-Stamp (*Int8Stamp v1*) (*Int8Stamp v2*) = *Int8Stamp* (*v1* ⊔ *v2*) |
sup-Stamp (*Int16Stamp v1*) (*Int16Stamp v2*) = *Int16Stamp* (*v1* ⊔ *v2*) |

$sup\text{-}Stamp\ (Int32Stamp\ v1)\ (Int32Stamp\ v2) = Int32Stamp\ (v1 \sqcup v2)\ |$
$sup\text{-}Stamp\ (Int64Stamp\ v1)\ (Int64Stamp\ v2) = Int64Stamp\ (v1 \sqcup v2)$

instance
⟨*proof*⟩
end

instantiation *Stamp* :: *bounded-lattice*
begin

notation *bot* ($\perp$ *50*)
notation *top* ($\top$ *50*)

definition *top-Stamp* :: *Stamp* **where**
top-Stamp = *TopStamp*
definition *bot-Stamp* :: *Stamp* **where**
bot-Stamp = *BottomStamp*

instance
⟨*proof*⟩
end

lemma [*code*]: *Rep-intstamp* (*from-bounds* (l, u)) = (l, u)
⟨*proof*⟩

code-datatype *Abs-intstamp*

end