

Veriopt Theories

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1 Conditional Elimination Phase

```
theory ConditionalElimination
imports
  Proofs.Rewrites
  Proofs.Bisimulation
begin
```

1.1 Individual Elimination Rules

We introduce a `TriState` as in the Graal compiler to represent when static analysis can tell us information about the value of a boolean expression. `Unknown` = No information can be inferred `KnownTrue/KnownFalse` = We can infer the expression will always be true or false.

```
datatype TriState = Unknown | KnownTrue | KnownFalse
```

The `implies` relation corresponds to the `LogicNode.implies` method from the compiler which attempts to infer when one logic nodes value can be inferred from a known logic node.

```
inductive implies :: IRGraph  $\Rightarrow$  IRNode  $\Rightarrow$  IRNode  $\Rightarrow$  TriState  $\Rightarrow$  bool
  (-  $\vdash$  - & -  $\hookrightarrow$  -) for g where
  eq-imp-less:
    g  $\vdash$  (IntegerEqualsNode x y) & (IntegerLessThanNode x y)  $\hookrightarrow$  KnownFalse |
  eq-imp-less-rev:
    g  $\vdash$  (IntegerEqualsNode x y) & (IntegerLessThanNode y x)  $\hookrightarrow$  KnownFalse |
  less-imp-rev-less:
    g  $\vdash$  (IntegerLessThanNode x y) & (IntegerLessThanNode y x)  $\hookrightarrow$  KnownFalse |
  less-imp-not-eq:
    g  $\vdash$  (IntegerLessThanNode x y) & (IntegerEqualsNode x y)  $\hookrightarrow$  KnownFalse |
```

less-imp-not-eq-rev:

$g \vdash (\text{IntegerLessThanNode } x \ y) \ \& \ (\text{IntegerEqualsNode } y \ x) \hookrightarrow \text{KnownFalse} \mid$

x-imp-x:

$g \vdash x \ \& \ x \hookrightarrow \text{KnownTrue} \mid$

negate-false:

$\llbracket g \vdash x \ \& \ (\text{kind } g \ y) \hookrightarrow \text{KnownTrue} \rrbracket \implies g \vdash x \ \& \ (\text{LogicNegationNode } y) \hookrightarrow \text{KnownFalse} \mid$

negate-true:

$\llbracket g \vdash x \ \& \ (\text{kind } g \ y) \hookrightarrow \text{KnownFalse} \rrbracket \implies g \vdash x \ \& \ (\text{LogicNegationNode } y) \hookrightarrow \text{KnownTrue}$

Total relation over partial implies relation

inductive *condition-implies* :: $\text{IRGraph} \Rightarrow \text{IRNode} \Rightarrow \text{IRNode} \Rightarrow \text{TriState} \Rightarrow \text{bool}$

$(- \vdash - \ \& \ - \hookrightarrow -)$ **for** g **where**

$\llbracket \neg(g \vdash a \ \& \ b \hookrightarrow \text{imp}) \rrbracket \implies (g \vdash a \ \& \ b \hookrightarrow \text{Unknown}) \mid$

$\llbracket (g \vdash a \ \& \ b \hookrightarrow \text{imp}) \rrbracket \implies (g \vdash a \ \& \ b \hookrightarrow \text{imp})$

inductive *implies-tree* :: $\text{IRExpr} \Rightarrow \text{IRExpr} \Rightarrow \text{bool} \Rightarrow \text{bool}$

$(- \ \& \ - \hookrightarrow -)$ **where**

eq-imp-less:

$(\text{BinaryExpr } \text{BinIntegerEquals } x \ y) \ \& \ (\text{BinaryExpr } \text{BinIntegerLessThan } x \ y) \hookrightarrow \text{False} \mid$

eq-imp-less-rev:

$(\text{BinaryExpr } \text{BinIntegerEquals } x \ y) \ \& \ (\text{BinaryExpr } \text{BinIntegerLessThan } y \ x) \hookrightarrow \text{False} \mid$

less-imp-rev-less:

$(\text{BinaryExpr } \text{BinIntegerLessThan } x \ y) \ \& \ (\text{BinaryExpr } \text{BinIntegerLessThan } y \ x) \hookrightarrow \text{False} \mid$

less-imp-not-eq:

$(\text{BinaryExpr } \text{BinIntegerLessThan } x \ y) \ \& \ (\text{BinaryExpr } \text{BinIntegerEquals } x \ y) \hookrightarrow \text{False} \mid$

less-imp-not-eq-rev:

$(\text{BinaryExpr } \text{BinIntegerLessThan } x \ y) \ \& \ (\text{BinaryExpr } \text{BinIntegerEquals } y \ x) \hookrightarrow \text{False} \mid$

x-imp-x:

$x \ \& \ x \hookrightarrow \text{True} \mid$

negate-false:

$\llbracket x \ \& \ y \hookrightarrow \text{True} \rrbracket \implies x \ \& \ (\text{UnaryExpr } \text{UnaryLogicNegation } y) \hookrightarrow \text{False} \mid$

negate-true:

$\llbracket x \ \& \ y \hookrightarrow \text{False} \rrbracket \implies x \ \& \ (\text{UnaryExpr } \text{UnaryLogicNegation } y) \hookrightarrow \text{True}$

Proofs that the implies relation is correct with respect to the existing evaluation semantics.

experiment begin

lemma *logic-negate-type*:

assumes $[m, p] \vdash \text{UnaryExpr UnaryLogicNegation } x \mapsto v$

assumes $v \neq \text{UndefVal}$

shows $\exists v2. [m, p] \vdash x \mapsto \text{IntVal32 } v2$

proof –

obtain ve **where** $ve: [m, p] \vdash x \mapsto ve$

using $\text{assms}(1)$ **by** *blast*

then have $[m, p] \vdash \text{UnaryExpr UnaryLogicNegation } x \mapsto \text{unary-eval UnaryLogicNegation } ve$

by (*metis* $\text{UnaryExprE assms}(1) \text{ evalDet}$)

then show *?thesis* **using** $\text{assms unary-eval.elims evalDet } ve \text{ IRUnaryOp.distinct}$
sorry

qed

lemma *logic-negation-relation-tree*:

assumes $[m, p] \vdash y \mapsto val$

assumes $[m, p] \vdash \text{UnaryExpr UnaryLogicNegation } y \mapsto \text{invval}$

assumes $\text{invval} \neq \text{UndefVal}$

shows $\text{val-to-bool } val \longleftrightarrow \neg(\text{val-to-bool } \text{invval})$

proof –

obtain v **where** $\text{invval} = \text{unary-eval UnaryLogicNegation } v$

using $\text{assms}(2)$ **by** *blast*

then have $[m, p] \vdash y \mapsto v$ **using** $\text{UnaryExprE assms}(1,2)$ **sorry**

then show *?thesis* **sorry**

qed

lemma *logic-negation-relation*:

assumes $[g, m, p] \vdash y \mapsto val$

assumes $\text{kind } g \text{ neg} = \text{LogicNegationNode } y$

assumes $[g, m, p] \vdash \text{neg} \mapsto \text{invval}$

assumes $\text{invval} \neq \text{UndefVal}$

shows $\text{val-to-bool } val \longleftrightarrow \neg(\text{val-to-bool } \text{invval})$

proof –

obtain $y\text{encode}$ **where** $5: g \vdash y \simeq y\text{encode}$

using $\text{assms}(1) \text{ encodeeval-def}$ **by** *auto*

then have $6: g \vdash \text{neg} \simeq \text{UnaryExpr UnaryLogicNegation } y\text{encode}$

using $\text{rep.intros}(7) \text{ assms}(2)$ **by** *simp*

then have $7: [m, p] \vdash \text{UnaryExpr UnaryLogicNegation } y\text{encode} \mapsto \text{invval}$

using $\text{assms}(3) \text{ encodeeval-def}$

by (*metis* repDet)

obtain $v1$ **where** $v1: [g, m, p] \vdash y \mapsto \text{IntVal } 32 \text{ } v1$

using $\text{assms}(1,2,3,4)$ **using** *logic-negate-type* **sorry**

have $\text{invval} = \text{bool-to-val } (\neg(\text{val-to-bool } val))$

using $\text{assms}(1,2,3) \text{ evalDet unary-eval.simps}(4)$

sorry

have $\text{val-to-bool } \text{invval} \longleftrightarrow \neg(\text{val-to-bool } val)$

using $\langle \text{invval} = \text{bool-to-val } (\neg \text{val-to-bool } val) \rangle$ **by** *force*

```

    then show ?thesis
      by simp
qed
end

lemma implies-valid:
  assumes  $x \& y \hookrightarrow imp$ 
  assumes  $[m, p] \vdash x \mapsto v1$ 
  assumes  $[m, p] \vdash y \mapsto v2$ 
  assumes  $v1 \neq UndefinedVal \wedge v2 \neq UndefinedVal$ 
  shows  $(imp \longrightarrow (val\text{-}to\text{-}bool\ v1 \longrightarrow val\text{-}to\text{-}bool\ v2)) \wedge$ 
     $(\neg imp \longrightarrow (val\text{-}to\text{-}bool\ v1 \longrightarrow \neg(val\text{-}to\text{-}bool\ v2)))$ 
     $(is\ (\ ?TP \longrightarrow ?TC) \wedge (\ ?FP \longrightarrow ?FC))$ 
  apply (intro conjI; rule impI)
proof -
  assume KnownTrue: ?TP
  show ?TC
  using assms(1) KnownTrue assms(2-) proof (induct x y imp rule: implies-tree.induct)
    case (eq-imp-less x y)
    then show ?case by simp
  next
    case (eq-imp-less-rev x y)
    then show ?case by simp
  next
    case (less-imp-rev-less x y)
    then show ?case by simp
  next
    case (less-imp-not-eq x y)
    then show ?case by simp
  next
    case (less-imp-not-eq-rev x y)
    then show ?case by simp
  next
    case (x-imp-x)
    then show ?case
      by (metis evalDet)
  next
    case (negate-false x1)
    then show ?case using evalDet
      using assms(2,3) by blast
  next
    case (negate-true y)
    then show ?case
      sorry
  qed
next
  assume KnownFalse: ?FP
  show ?FC using assms KnownFalse proof (induct x y imp rule: implies-tree.induct)
    case (eq-imp-less x y)

```

```

obtain xval where xval:  $[m, p] \vdash x \mapsto xval$ 
  using eq-imp-less(1) eq-imp-less.prems(3)
  by blast
then obtain yval where yval:  $[m, p] \vdash y \mapsto yval$ 
  using eq-imp-less.prems(3)
  using eq-imp-less.prems(2) by blast
have egeval:  $[m, p] \vdash (\text{BinaryExpr BinIntegerEquals } x \ y) \mapsto \text{intval-equals } xval$ 
yval
  using xval yval evaltree.BinaryExpr
  by (metis BinaryExprE bin-eval.simps(11) eq-imp-less.prems(1) evalDet)
have lesseval:  $[m, p] \vdash (\text{BinaryExpr BinIntegerLessThan } x \ y) \mapsto \text{intval-less-than}$ 
xval yval
  using xval yval evaltree.BinaryExpr
  by (metis BinaryExprE bin-eval.simps(12) eq-imp-less.prems(2) evalDet)
have val-to-bool (intval-equals xval yval)  $\longrightarrow \neg(\text{val-to-bool } (\text{intval-less-than } xval$ 
yval))
  using assms(4) apply (cases xval; cases yval; auto) sorry

then show ?case
  using egeval lesseval
  by (metis eq-imp-less.prems(1) eq-imp-less.prems(2) evalDet)
next
case (eq-imp-less-rev x y)
obtain xval where xval:  $[m, p] \vdash x \mapsto xval$ 
  using eq-imp-less-rev.prems(3)
  using eq-imp-less-rev.prems(2) by blast
obtain yval where yval:  $[m, p] \vdash y \mapsto yval$ 
  using eq-imp-less-rev.prems(3)
  using eq-imp-less-rev.prems(2) by blast
have egeval:  $[m, p] \vdash (\text{BinaryExpr BinIntegerEquals } x \ y) \mapsto \text{intval-equals } xval$ 
yval
  using xval yval evaltree.BinaryExpr
  by (metis BinaryExprE bin-eval.simps(11) eq-imp-less-rev.prems(1) evalDet)
have lesseval:  $[m, p] \vdash (\text{BinaryExpr BinIntegerLessThan } y \ x) \mapsto \text{intval-less-than}$ 
yval xval
  using xval yval evaltree.BinaryExpr
  by (metis BinaryExprE bin-eval.simps(12) eq-imp-less-rev.prems(2) evalDet)
have val-to-bool (intval-equals xval yval)  $\longrightarrow \neg(\text{val-to-bool } (\text{intval-less-than } yval$ 
xval))
  using assms(4) apply (cases xval; cases yval; auto) sorry

then show ?case
  using egeval lesseval
  by (metis eq-imp-less-rev.prems(1) eq-imp-less-rev.prems(2) evalDet)
next
case (less-imp-rev-less x y)
obtain xval where xval:  $[m, p] \vdash x \mapsto xval$ 
  using less-imp-rev-less.prems(3)
  using less-imp-rev-less.prems(2) by blast

```

```

obtain yval where yval:  $[m, p] \vdash y \mapsto yval$ 
using less-imp-rev-less.prems(3)
using less-imp-rev-less.prems(2) by blast
have lesseval:  $[m, p] \vdash (\text{BinaryExpr BinIntegerLessThan } x \ y) \mapsto \text{intval-less-than}$ 
xval yval
using xval yval evaltree.BinaryExpr
by (metis BinaryExprE bin-eval.simps(12) evalDet less-imp-rev-less.prems(1))
have revlesseval:  $[m, p] \vdash (\text{BinaryExpr BinIntegerLessThan } y \ x) \mapsto \text{intval-less-than}$ 
yval xval
using xval yval evaltree.BinaryExpr
by (metis BinaryExprE bin-eval.simps(12) evalDet less-imp-rev-less.prems(2))
have val-to-bool (intval-less-than xval yval)  $\longrightarrow \neg(\text{val-to-bool } (\text{intval-less-than}$ 
yval xval))
using assms(4) apply (cases xval; cases yval; auto) sorry

then show ?case
by (metis evalDet less-imp-rev-less.prems(1) less-imp-rev-less.prems(2) lesse-
val revlesseval)
next
case (less-imp-not-eq x y)
obtain xval where xval:  $[m, p] \vdash x \mapsto xval$ 
using less-imp-not-eq.prems(3)
using less-imp-not-eq.prems(1) by blast
obtain yval where yval:  $[m, p] \vdash y \mapsto yval$ 
using less-imp-not-eq.prems(3)
using less-imp-not-eq.prems(1) by blast
have egeval:  $[m, p] \vdash (\text{BinaryExpr BinIntegerEquals } x \ y) \mapsto \text{intval-equals } xval$ 
yval
using xval yval evaltree.BinaryExpr
by (metis BinaryExprE bin-eval.simps(11) evalDet less-imp-not-eq.prems(2))
have lesseval:  $[m, p] \vdash (\text{BinaryExpr BinIntegerLessThan } x \ y) \mapsto \text{intval-less-than}$ 
xval yval
using xval yval evaltree.BinaryExpr
by (metis BinaryExprE bin-eval.simps(12) evalDet less-imp-not-eq.prems(1))
have val-to-bool (intval-less-than xval yval)  $\longrightarrow \neg(\text{val-to-bool } (\text{intval-equals } xval$ 
yval))
using assms(4) apply (cases xval; cases yval; auto) sorry

then show ?case
by (metis egeval evalDet less-imp-not-eq.prems(1) less-imp-not-eq.prems(2)
lesseval)
next
case (less-imp-not-eq-rev x y)
obtain xval where xval:  $[m, p] \vdash x \mapsto xval$ 
using less-imp-not-eq-rev.prems(3)
using less-imp-not-eq-rev.prems(1) by blast
obtain yval where yval:  $[m, p] \vdash y \mapsto yval$ 
using less-imp-not-eq-rev.prems(3)
using less-imp-not-eq-rev.prems(1) by blast

```

```

    have egeval:  $[m, p] \vdash (\text{BinaryExpr BinIntegerEquals } y \ x) \mapsto \text{intval-equals } yval$ 
    xval
    using xval yval evaltree.BinaryExpr
    by (metis BinaryExprE bin-eval.simps(11) evalDet less-imp-not-eq-rev.prem(2))
    have lesseval:  $[m, p] \vdash (\text{BinaryExpr BinIntegerLessThan } x \ y) \mapsto \text{intval-less-than}$ 
    xval yval
    using xval yval evaltree.BinaryExpr
    by (metis BinaryExprE bin-eval.simps(12) evalDet less-imp-not-eq-rev.prem(1))
    have val-to-bool (intval-less-than xval yval)  $\longrightarrow \neg(\text{val-to-bool } (\text{intval-equals } yval$ 
    xval))
    using assms(4) apply (cases xval; cases yval; auto) sorry

    then show ?case
    by (metis egeval evalDet less-imp-not-eq-rev.prem(1) less-imp-not-eq-rev.prem(2)
    lesseval)
  next
    case (x-imp-x x1)
    then show ?case by simp
  next
    case (negate-false x y)
    then show ?case sorry
  next
    case (negate-true x1)
    then show ?case by simp
qed
qed

```

```

lemma implies-true-valid:
  assumes  $x \ \& \ y \hookrightarrow \text{imp}$ 
  assumes imp
  assumes  $[m, p] \vdash x \mapsto v1$ 
  assumes  $[m, p] \vdash y \mapsto v2$ 
  assumes  $v1 \neq \text{UndefVal} \wedge v2 \neq \text{UndefVal}$ 
  shows  $\text{val-to-bool } v1 \longrightarrow \text{val-to-bool } v2$ 
  using assms implies-valid
  by blast

```

```

lemma implies-false-valid:
  assumes  $x \ \& \ y \hookrightarrow \text{imp}$ 
  assumes  $\neg \text{imp}$ 
  assumes  $[m, p] \vdash x \mapsto v1$ 
  assumes  $[m, p] \vdash y \mapsto v2$ 
  assumes  $v1 \neq \text{UndefVal} \wedge v2 \neq \text{UndefVal}$ 
  shows  $\text{val-to-bool } v1 \longrightarrow \neg(\text{val-to-bool } v2)$ 
  using assms implies-valid by blast

```

The following relation corresponds to the `UnaryOpLogicNode.tryFold` and `BinaryOpLogicNode.tryFold` methods and their associated concrete implementations.

The relation determines if a logic operation can be shown true or false through the stamp typing information.

inductive *tryFold* :: *IRNode* \Rightarrow (*ID* \Rightarrow *Stamp*) \Rightarrow *bool* \Rightarrow *bool*

where

```

[[alwaysDistinct (stamps x) (stamps y)]]
   $\Rightarrow$  tryFold (IntegerEqualsNode x y) stamps False |
[[neverDistinct (stamps x) (stamps y)]]
   $\Rightarrow$  tryFold (IntegerEqualsNode x y) stamps True |
[[is-IntegerStamp (stamps x);
  is-IntegerStamp (stamps y);
  stpi-upper (stamps x) < stpi-lower (stamps y)]]
   $\Rightarrow$  tryFold (IntegerLessThanNode x y) stamps True |
[[is-IntegerStamp (stamps x);
  is-IntegerStamp (stamps y);
  stpi-lower (stamps x)  $\geq$  stpi-upper (stamps y)]]
   $\Rightarrow$  tryFold (IntegerLessThanNode x y) stamps False

```

Proofs that show that when the stamp lookup function is well-formed, the tryFold relation correctly predicts the output value with respect to our evaluation semantics.

lemma

```

assumes kind g nid = IntegerEqualsNode x y
assumes [g, m, p]  $\vdash$  nid  $\mapsto$  v
assumes v  $\neq$  UndefVal
assumes ([g, m, p]  $\vdash$  x  $\mapsto$  xval)  $\wedge$  ([g, m, p]  $\vdash$  y  $\mapsto$  yval)
shows val-to-bool (intval-equals xval yval)  $\longleftrightarrow$  v = IntVal32 1

```

proof –

```

have v = intval-equals xval yval
  using asms(1, 2, 3, 4) BinaryExprE IntegerEqualsNode bin-eval.simps(7)
  by (smt (verit) bin-eval.simps(11) encodeeval-def evalDet repDet)
then show ?thesis using intval-equals.simps val-to-bool.simps sorry

```

qed

lemma *tryFoldIntegerEqualsAlwaysDistinct*:

```

assumes wf-stamp g stamps
assumes kind g nid = (IntegerEqualsNode x y)
assumes [g, m, p]  $\vdash$  nid  $\mapsto$  v
assumes alwaysDistinct (stamps x) (stamps y)
shows v = IntVal32 0

```

proof –

```

have  $\forall$  val.  $\neg$ (valid-value val (join (stamps x) (stamps y)))
  using asms(1,4) unfolding alwaysDistinct.simps
  by (smt (verit, best) is-stamp-empty.elims(2) valid-int valid-value.simps(1))
have  $\neg$ ( $\exists$  val . ([g, m, p]  $\vdash$  x  $\mapsto$  val)  $\wedge$  ([g, m, p]  $\vdash$  y  $\mapsto$  val))
  using asms(1,4) unfolding alwaysDistinct.simps wf-stamp.simps encodeeval-def sorry

```

then show ?thesis **sorry**

qed


```

lemma tryFoldIntegerEqualsNeverDistinct:
  assumes wf-stamp g stamps
  assumes kind g nid = (IntegerEqualsNode x y)
  assumes  $[g, m, p] \vdash \text{nid} \mapsto v$ 
  assumes neverDistinct (stamps x) (stamps y)
  shows  $v = \text{IntVal32 } 1$ 
  using assms IntegerEqualsNodeE sorry

lemma tryFoldIntegerLessThanTrue:
  assumes wf-stamp g stamps
  assumes kind g nid = (IntegerLessThanNode x y)
  assumes  $[g, m, p] \vdash \text{nid} \mapsto v$ 
  assumes  $\text{stpi-upper (stamps x)} < \text{stpi-lower (stamps y)}$ 
  shows  $v = \text{IntVal32 } 1$ 
proof –
  have stamp-type: is-IntegerStamp (stamps x)
    using assms
    sorry
  obtain xval where  $xval: [g, m, p] \vdash x \mapsto xval$ 
    using assms(2,3) sorry
  obtain yval where  $yval: [g, m, p] \vdash y \mapsto yval$ 
    using assms(2,3) sorry
  have  $\text{is-IntegerStamp (stamps x)} \wedge \text{is-IntegerStamp (stamps y)}$ 
    using assms(4)
    sorry
  then have  $\text{val-to-bool (intval-less-than xval yval)}$ 
    sorry
  then show ?thesis
    sorry
qed

lemma tryFoldIntegerLessThanFalse:
  assumes wf-stamp g stamps
  assumes kind g nid = (IntegerLessThanNode x y)
  assumes  $[g, m, p] \vdash \text{nid} \mapsto v$ 
  assumes  $\text{stpi-lower (stamps x)} \geq \text{stpi-upper (stamps y)}$ 
  shows  $v = \text{IntVal32 } 0$ 
proof –
  have stamp-type: is-IntegerStamp (stamps x)
    using assms
    sorry
  obtain xval where  $xval: [g, m, p] \vdash x \mapsto xval$ 
    using assms(2,3) sorry
  obtain yval where  $yval: [g, m, p] \vdash y \mapsto yval$ 
    using assms(2,3) sorry
  have  $\text{is-IntegerStamp (stamps x)} \wedge \text{is-IntegerStamp (stamps y)}$ 
    using assms(4)
    sorry
  then have  $\neg(\text{val-to-bool (intval-less-than xval yval)})$ 

```

```

    sorry
  then show ?thesis
    sorry
qed

```

```

theorem tryFoldProofTrue:
  assumes wf-stamp g stamps
  assumes tryFold (kind g nid) stamps True
  assumes  $[g, m, p] \vdash \text{nid} \mapsto v$ 
  shows val-to-bool v
  using assms(2) proof (induction kind g nid stamps True rule: tryFold.induct)
case (1 stamps x y)
  then show ?case using tryFoldIntegerEqualsAlwaysDistinct assms sorry
next
  case (2 stamps x y)
  then show ?case using tryFoldIntegerEqualsAlwaysDistinct assms sorry
next
  case (3 stamps x y)
  then show ?case using tryFoldIntegerLessThanTrue assms sorry
next
  case (4 stamps x y)
  then show ?case using tryFoldIntegerLessThanFalse assms sorry
qed

```

```

theorem tryFoldProofFalse:
  assumes wf-stamp g stamps
  assumes tryFold (kind g nid) stamps False
  assumes  $[g, m, p] \vdash \text{nid} \mapsto v$ 
  shows  $\neg(\text{val-to-bool } v)$ 
  using assms(2) proof (induction kind g nid stamps False rule: tryFold.induct)
case (1 stamps x y)
  then show ?case using tryFoldIntegerEqualsAlwaysDistinct assms sorry
next
  case (2 stamps x y)
  then show ?case using tryFoldIntegerEqualsNeverDistinct assms sorry
next
  case (3 stamps x y)
  then show ?case using tryFoldIntegerLessThanTrue assms sorry
next
  case (4 stamps x y)
  then show ?case using tryFoldIntegerLessThanFalse assms sorry
qed

```

inductive-cases StepE:

$$g, p \vdash (\text{nid}, m, h) \rightarrow (\text{nid}', m', h)$$

Perform conditional elimination rewrites on the graph for a particular node.

In order to determine conditional eliminations appropriately the rule needs two data structures produced by static analysis. The first parameter is the set of IRNodes that we know result in a true value when evaluated. The second parameter is a mapping from node identifiers to the flow-sensitive stamp.

The relation transforms the third parameter to the fifth parameter for a node identifier which represents the fourth parameter.

inductive *ConditionalEliminationStep* ::

IRExpr set \Rightarrow (*ID* \Rightarrow *Stamp*) \Rightarrow *IRGraph* \Rightarrow *ID* \Rightarrow *IRGraph* \Rightarrow *bool* **where**
impliesTrue:

$\llbracket \text{kind } g \text{ ifcond} = (\text{IfNode } cid \text{ } t \text{ } f);$
 $g \vdash cid \simeq cond;$
 $\exists ce \in conds . (ce \ \& \ cond \hookrightarrow True);$
 $g' = \text{constantCondition } True \text{ ifcond } (kind \ g \text{ ifcond}) \ g$
 $\rrbracket \Longrightarrow \text{ConditionalEliminationStep } conds \ stamps \ g \text{ ifcond } g' \mid$

impliesFalse:

$\llbracket \text{kind } g \text{ ifcond} = (\text{IfNode } cid \text{ } t \text{ } f);$
 $g \vdash cid \simeq cond;$
 $\exists ce \in conds . (ce \ \& \ cond \hookrightarrow False);$
 $g' = \text{constantCondition } False \text{ ifcond } (kind \ g \text{ ifcond}) \ g$
 $\rrbracket \Longrightarrow \text{ConditionalEliminationStep } conds \ stamps \ g \text{ ifcond } g' \mid$

tryFoldTrue:

$\llbracket \text{kind } g \text{ ifcond} = (\text{IfNode } cid \text{ } t \text{ } f);$
 $cond = kind \ g \ cid;$
 $\text{tryFold } (kind \ g \ cid) \ stamps \ True;$
 $g' = \text{constantCondition } True \text{ ifcond } (kind \ g \text{ ifcond}) \ g$
 $\rrbracket \Longrightarrow \text{ConditionalEliminationStep } conds \ stamps \ g \text{ ifcond } g' \mid$

tryFoldFalse:

$\llbracket \text{kind } g \text{ ifcond} = (\text{IfNode } cid \text{ } t \text{ } f);$
 $cond = kind \ g \ cid;$
 $\text{tryFold } (kind \ g \ cid) \ stamps \ False;$
 $g' = \text{constantCondition } False \text{ ifcond } (kind \ g \text{ ifcond}) \ g$
 $\rrbracket \Longrightarrow \text{ConditionalEliminationStep } conds \ stamps \ g \text{ ifcond } g' \mid$

code-pred (*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow bool$) *ConditionalEliminationStep* .

thm *ConditionalEliminationStep.equation*

1.2 Control-flow Graph Traversal

type-synonym *Seen* = *ID set*

type-synonym *Condition* = *IRNode*

type-synonym *Conditions* = *Condition list*

type-synonym *StampFlow* = (*ID* \Rightarrow *Stamp*) *list*

nextEdge helps determine which node to traverse next by returning the first successor edge that isn't in the set of already visited nodes. If there is not an appropriate successor, None is returned instead.

```
fun nextEdge :: Seen ⇒ ID ⇒ IRGraph ⇒ ID option where
  nextEdge seen nid g =
    (let nids = (filter (λnid'. nid' ∉ seen) (successors-of (kind g nid))) in
     (if length nids > 0 then Some (hd nids) else None))
```

pred determines which node, if any, acts as the predecessor of another.

Merge nodes represent a special case where-in the predecessor exists as an input edge of the merge node, to simplify the traversal we treat only the first input end node as the predecessor, ignoring that multiple nodes may act as a successor.

For all other nodes, the predecessor is the first element of the predecessors set. Note that in a well-formed graph there should only be one element in the predecessor set.

```
fun pred :: IRGraph ⇒ ID ⇒ ID option where
  pred g nid = (case kind g nid of
    (MergeNode ends -) ⇒ Some (hd ends) |
    - ⇒
      (if IRGraph.predecessors g nid = {}
       then None else
        Some (hd (sorted-list-of-set (IRGraph.predecessors g nid))))
  )
```

When the basic block of an if statement is entered, we know that the condition of the preceding if statement must be true. As in the GraalVM compiler, we introduce the registerNewCondition function which roughly corresponds to the ConditionalEliminationPhase.registerNewCondition. This method updates the flow-sensitive stamp information based on the condition which we know must be true.

```
fun clip-upper :: Stamp ⇒ int ⇒ Stamp where
  clip-upper (IntegerStamp b l h) c = (IntegerStamp b l c) |
  clip-upper s c = s
fun clip-lower :: Stamp ⇒ int ⇒ Stamp where
  clip-lower (IntegerStamp b l h) c = (IntegerStamp b c h) |
  clip-lower s c = s
```

```
fun registerNewCondition :: IRGraph ⇒ Condition ⇒ (ID ⇒ Stamp) ⇒ (ID ⇒ Stamp) where
```

```
  registerNewCondition g (IntegerEqualsNode x y) stamps =
    (stamps(x := join (stamps x) (stamps y)))(y := join (stamps x) (stamps y)) |
  registerNewCondition g (IntegerLessThanNode x y) stamps =
```

```

    (stamps
      (x := clip-upper (stamps x) (stpi-lower (stamps y))))
      (y := clip-lower (stamps y) (stpi-upper (stamps x))) |
    registerNewCondition g - stamps = stamps

```

```

fun hdOr :: 'a list ⇒ 'a ⇒ 'a where
  hdOr (x # xs) de = x |
  hdOr [] de = de

```

The Step relation is a small-step traversal of the graph which handles transitions between individual nodes of the graph.

It relates a pairs of tuple of the current node, the set of seen nodes, the always true stack of IfNode conditions, and the flow-sensitive stamp information.

inductive Step

```

:: IRGraph ⇒ (ID × Seen × Conditions × StampFlow) ⇒ (ID × Seen × Con-
ditions × StampFlow) option ⇒ bool

```

for g **where**

— Hit a BeginNode with an IfNode predecessor which represents the start of a basic block for the IfNode. 1. nid' will be the successor of the begin node. 2. Find the first and only predecessor. 3. Extract condition from the preceding IfNode. 4. Negate condition if the begin node is second branch (we've taken the else branch of the condition) 5. Add the condition or the negated condition to stack 6. Perform any stamp updates based on the condition using the registerNewCondition function and place them on the top of the stack of stamp information

```

[[kind g nid = BeginNode nid';

```

```

  nid ∉ seen;
  seen' = {nid} ∪ seen;

```

```

  Some ifcond = pred g nid;
  kind g ifcond = IfNode cond t f;

```

```

  i = find-index nid (successors-of (kind g ifcond));
  c = (if i = 0 then kind g cond else LogicNegationNode cond);
  conds' = c # conds;

```

```

  flow' = registerNewCondition g c (hdOr flow (stamp g))]
  ⇒⇒ Step g (nid, seen, conds, flow) (Some (nid', seen', conds', flow' # flow)) |

```

— Hit an EndNode 1. nid' will be the usage of EndNode 2. pop the conditions and stamp stack

```

[[kind g nid = EndNode;

```

```

  nid ∉ seen;
  seen' = {nid} ∪ seen;

```

```

  nid' = any-usage g nid;

```

```

  conds' = tl conds;

```

$flow' = tl\ flow$
 $\implies Step\ g\ (nid, seen, conds, flow)\ (Some\ (nid', seen', conds', flow'))\ |$

— We can find a successor edge that is not in seen, go there

$\llbracket \neg(is-EndNode\ (kind\ g\ nid));$
 $\neg(is-BeginNode\ (kind\ g\ nid));$

$nid \notin seen;$
 $seen' = \{nid\} \cup seen;$

$Some\ nid' = nextEdge\ seen'\ nid\ g$
 $\implies Step\ g\ (nid, seen, conds, flow)\ (Some\ (nid', seen', conds', flow'))\ |$

— We cannot find a successor edge that is not in seen, give back None

$\llbracket \neg(is-EndNode\ (kind\ g\ nid));$
 $\neg(is-BeginNode\ (kind\ g\ nid));$

$nid \notin seen;$
 $seen' = \{nid\} \cup seen;$

$None = nextEdge\ seen'\ nid\ g$
 $\implies Step\ g\ (nid, seen, conds, flow)\ None\ |$

— We've already seen this node, give back None

$\llbracket nid \in seen \rrbracket \implies Step\ g\ (nid, seen, conds, flow)\ None$

code-pred (*modes*: $i \Rightarrow i \Rightarrow o \Rightarrow bool$) *Step* .

The ConditionalEliminationPhase relation is responsible for combining the individual traversal steps from the Step relation and the optimizations from the ConditionalEliminationStep relation to perform a transformation of the whole graph.

end