

Veriopt Theories

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theory *TreeSnippets*
imports
 Canonicalizations.ConditionalPhase
 Optimizations.CanonicalizationSyntax
 Semantics.TreeToGraphThms
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begin

no-notation *ConditionalExpr* (- ? - : -)

notation (*latex*)
 kind (-⟨-⟩)

notation (*latex*)
 valid-value (- ∈ -)

notation (*latex*)
 val-to-bool (*bool-of* -)

notation (*latex*)
 constantAsStamp (*stamp-from-value* -)

notation (*latex*)
 size (*trm*(-))

translations
 $y > x \leq x < y$

notation (*latex*)
 greater (- > -)

translations

$n \leq \text{CONST Rep-intExp } n$
 $n \leq \text{CONST Rep-i32e } n$
 $n \leq \text{CONST Rep-i64e } n$

lemma *vminusv*: $\forall vv \ v . vv = \text{IntVal64 } v \longrightarrow v - v = 0$
by *simp*
thm-oracles *vminusv*

lemma *redundant-sub*:
 $\forall vv_1 \ vv_2 \ v_1 \ v_2 . vv_1 = \text{IntVal64 } v_1 \wedge vv_2 = \text{IntVal64 } v_2 \longrightarrow v_1 - (v_1 - v_2) = v_2$
by *simp*
thm-oracles *redundant-sub*

val-eq

$\forall vv \ v . vv = \text{IntVal64 } v \longrightarrow v - v = 0$

$\forall vv_1 \ vv_2 \ v_1 \ v_2 . vv_1 = \text{IntVal64 } v_1 \wedge vv_2 = \text{IntVal64 } v_2 \longrightarrow v_1 - (v_1 - v_2) = v_2$

phase *tmp*
terminating *size*
begin

sub-same-32

optimization *sub-same-32*: $(e::i32e) - e \mapsto \text{const } (\text{IntVal32 } 0)$

apply (*unfold* *rewrite-preservation.simps*, *unfold* *rewrite-termination.simps*,
rule *conjE*, *simp*) **apply** *auto*[1] **using** *Rep-i32e evalDet is-IntVal32-def*
apply (*smt* (*verit*, *del-insts*) *eq-iff-diff-eq-0 evaltree.simps int-constants-valid int-val-sub.simps*(1) *is-int-val.simps*(1) *mem-Collect-eq*)
unfolding *size.simps*
by (*metis* *add-strict-increasing gr-implies-not0 less-one linorder-not-le size-gt-0*)

sub-same-64

optimization *sub-same-64*: $(e::i64e) - e \mapsto \text{const } (\text{IntVal64 } 0)$

apply *auto*
apply (*metis* (*no-types*, *opaque-lifting*) *ConstantExpr bin-eval.simps*(3) *bin-eval-preserves-validity*
cancel-comm-monoid-add-class.diff-cancel evalDet i64e-eval int-and-equal-bits.simps(2)
intval-sub.simps(2))
by (*simp* *add: Suc-le-eq add-strict-increasing size-gt-0*)
end

thm-oracles *sub-same-32*

ast-example

BinaryExpr BinAdd (BinaryExpr BinMul x x) (BinaryExpr BinMul x x)

abstract-syntax-tree

datatype *IRExpr* =
 UnaryExpr IRUnaryOp IRExpr
 | *BinaryExpr IRBinaryOp IRExpr IRExpr*
 | *ConditionalExpr IRExpr IRExpr IRExpr*
 | *ParameterExpr nat Stamp*
 | *LeafExpr nat Stamp*
 | *ConstantExpr Value*
 | *ConstantVar (char list)*
 | *VariableExpr (char list) Stamp*

value

datatype *Value* = *UndefVal*
 | *IntVal32 (32 word)*
 | *IntVal64 (64 word)*
 | *ObjRef (nat option)*
 | *ObjStr (char list)*

eval

unary-eval :: *IRUnaryOp* \Rightarrow *Value* \Rightarrow *Value*
bin-eval :: *IRBinaryOp* \Rightarrow *Value* \Rightarrow *Value* \Rightarrow *Value*

tree-semantics

semantics:unary semantics:binary semantics:conditional seman-
tics:constant semantics:parameter semantics:leaf

tree-evaluation-deterministic

$[m,p] \vdash e \mapsto v_1 \wedge [m,p] \vdash e \mapsto v_2 \implies v_1 = v_2$

thm-oracles *evalDet*

expression-refinement

$$e_1 \sqsubseteq e_2 = (\forall m \ p \ v. [m,p] \vdash e_1 \mapsto v \longrightarrow [m,p] \vdash e_2 \mapsto v)$$

expression-refinement-monotone

$$\begin{aligned} e \sqsubseteq e' &\implies \text{UnaryExpr op } e \sqsubseteq \text{UnaryExpr op } e' \\ x \sqsubseteq x' \wedge y \sqsubseteq y' &\implies \text{BinaryExpr op } x \ y \sqsubseteq \text{BinaryExpr op } x' \ y' \\ ce \sqsubseteq ce' \wedge te \sqsubseteq te' \wedge fe \sqsubseteq fe' &\implies \text{ConditionalExpr } ce \ te \ fe \sqsubseteq \text{ConditionalExpr } ce' \ te' \ fe' \end{aligned}$$

ML <

```
(*fun get-list (phase: phase option) =
```

```
  case phase of
```

```
    NONE => [] |
```

```
    SOME p => (#rewrites p)
```

```
fun get-rewrite name thy =
```

```
  let
```

```
    val (phases, lookup) = (case RWList.get thy of
```

```
      NoPhase store => store |
```

```
      InPhase (name, store, -) => store)
```

```
    val rewrites = (map (fn x => get-list (lookup x)) phases)
```

```
  in
```

```
    rewrites
```

```
  end
```

```
fun rule-print name =
```

```
  Document-Output.antiquotation-pretty name (Args.term)
```

```
  (fn ctxt => fn (rule) => (*Pretty.str hello*))
```

```
  Pretty.block (print-all-phases (Proof-Context.theory-of ctxt)));
```

```
(*
```

```
  Goal-Display.pretty-goal
```

```
  (Config.put Goal-Display.show-main-goal main ctxt)
```

```
  (#goal (Proof.goal (Toplevel.proof-of (Toplevel.presentation-state ctxt)))));
```

```
*)
```

```
val - = Theory.setup
```

```
  (rule-print binding <rule>);*)
```

```
>
```

phase *SnipPhase*

terminating *size*

begin

BinaryFoldConstant

optimization *BinaryFoldConstant*: $\text{BinaryExpr } op \text{ (const } v1 \text{) (const } v2 \text{)} \mapsto \text{ConstantExpr (bin-eval } op \text{ } v1 \text{ } v2 \text{)}$ when *int-and-equal-bits* $v1 \text{ } v2$

unfolding *rewrite-preservation.simps* *rewrite-termination.simps*
apply (rule *conjE*, *simp*, *simp* del: *le-expr-def*)

BinaryFoldConstantObligation

1. *int-and-equal-bits* $v1 \text{ } v2 \longrightarrow$
 $\text{BinaryExpr } op \text{ (ConstantExpr } v1 \text{) (ConstantExpr } v2 \text{)} \sqsubseteq$
 $\text{ConstantExpr (bin-eval } op \text{ } v1 \text{ } v2 \text{)}$
2. *int-and-equal-bits* $v1 \text{ } v2 \longrightarrow$
 $\text{trm}(\text{BinaryExpr } op \text{ (ConstantExpr } v1 \text{)}$
 $\text{(ConstantExpr } v2 \text{)}) > \text{trm}(\text{ConstantExpr (bin-eval } op \text{ } v1 \text{ } v2 \text{)})$

using *BinaryFoldConstant* **by** *auto*

AddCommuteConstantRight

optimization *AddCommuteConstantRight*: $((\text{const } v) + y) \mapsto y + (\text{const } v)$ when $\neg(\text{is-ConstantExpr } y)$

unfolding *rewrite-preservation.simps* *rewrite-termination.simps*
apply (rule *conjE*, *simp*, *simp* del: *le-expr-def*)

AddCommuteConstantRightObligation

1. $\neg \text{is-ConstantExpr } y \longrightarrow$
 $\text{BinaryExpr BinAdd (ConstantExpr } v \text{) } y \sqsubseteq$
 $\text{BinaryExpr BinAdd } y \text{ (ConstantExpr } v \text{)}$
2. $\neg \text{is-ConstantExpr } y \longrightarrow$
 $\text{trm}(\text{BinaryExpr BinAdd (ConstantExpr } v \text{)}$
 $y) > \text{trm}(\text{BinaryExpr BinAdd } y \text{ (ConstantExpr } v \text{)})$

using *AddShiftConstantRight* **by** *auto*

AddNeutral

optimization *AddNeutral*: $((e::i32e) + (\text{const (IntVal32 0)})) \mapsto e$

unfolding *rewrite-preservation.simps* *rewrite-termination.simps*
apply (rule *conjE*, *simp*, *simp* del: *le-expr-def*)

AddNeutralObligation

1. $\text{BinaryExpr BinAdd } e \text{ (ConstantExpr (IntVal32 0))} \sqsupseteq e$
2. $\text{trm}(\text{BinaryExpr BinAdd } e \text{ (ConstantExpr (IntVal32 0))}) > \text{trm}(e)$

using *neutral-zero(1) rewrite-preservation.simps(1)* **apply** *blast*
by *auto*

InverseLeftSub

optimization *InverseLeftSub*: $((e_1::\text{intExp}) - (e_2::\text{intExp})) + e_2 \mapsto e_1$

unfolding *rewrite-preservation.simps rewrite-termination.simps*
apply (*rule conjE, simp, simp del: le-expr-def*)

InverseLeftSubObligation

1. $\text{BinaryExpr BinAdd (BinaryExpr BinSub } e_1 \text{ } e_2) \text{ } e_2 \sqsupseteq e_1$
2. $\text{trm}(\text{BinaryExpr BinAdd (BinaryExpr BinSub } e_1 \text{ } e_2) \text{ } e_2) > \text{trm}(e_1)$

using *neutral-left-add-sub* **by** *auto*

InverseRightSub

optimization *InverseRightSub*: $(e_2::\text{intExp}) + ((e_1::\text{intExp}) - e_2) \mapsto e_1$

unfolding *rewrite-preservation.simps rewrite-termination.simps*
apply (*rule conjE, simp, simp del: le-expr-def*)

InverseRightSubObligation

1. $\text{BinaryExpr BinAdd } e_2 \text{ (BinaryExpr BinSub } e_1 \text{ } e_2) \sqsupseteq e_1$
2. $\text{trm}(\text{BinaryExpr BinAdd } e_2 \text{ (BinaryExpr BinSub } e_1 \text{ } e_2)) > \text{trm}(e_1)$

using *neutral-right-add-sub* **by** *auto*

AddToSub

optimization *AddToSub*: $-e + y \mapsto y - e$

unfolding *rewrite-preservation.simps rewrite-termination.simps*
apply (*rule conjE, simp, simp del: le-expr-def*)

AddToSubObligation

1. $\text{BinaryExpr BinAdd (UnaryExpr UnaryNeg e) y} \sqsubseteq \text{BinaryExpr BinSub y e}$
2. $\text{trm}(\text{BinaryExpr BinAdd (UnaryExpr UnaryNeg e) y}) > \text{trm}(\text{BinaryExpr BinSub y e})$

using *AddLeftNegateToSub* **by** *auto*

end

definition *trm* **where** *trm* = *size*

phase

phase *AddCanonicalizations*
terminating *trm*
begin...end

hide-const (open) *Form.wf-stamp*

phase-example

phase *Conditional*
terminating *trm*
begin

phase-example-1

optimization *negate-condition*: $(\neg e \text{ ? } x : y) \mapsto (e \text{ ? } y : x)$

using *ConditionalPhase.negate-condition*
by (*auto simp: trm-def*)

phase-example-2

optimization *const-true*: $(\text{true} \text{ ? } x : y) \mapsto x$

by (*auto simp: trm-def*)

phase-example-3

optimization *const-false*: $(\text{false} \text{ ? } x : y) \mapsto y$

by (*auto simp: trm-def*)

phase-example-4

optimization *equal-branches*: $(e \text{ ? } x : x) \mapsto x$

by (auto simp: trm-def)

phase-example-5

optimization *condition-bounds-x*: $((x < y) \text{ ? } x : y) \mapsto x$
when (stamp-under (stamp-expr x) (stamp-expr y)
 \wedge wf-stamp x \wedge wf-stamp y)

using ConditionalPhase.condition-bounds-x(1)

by (blast, auto simp: trm-def)

phase-example-6

optimization *condition-bounds-y*: $((x < y) \text{ ? } x : y) \mapsto y$
when (stamp-under (stamp-expr y) (stamp-expr x) \wedge wf-stamp
x \wedge wf-stamp y)

using ConditionalPhase.condition-bounds-y(1)

by (blast, auto simp: trm-def)

phase-example-7

end

termination

$$\begin{aligned} \text{trm}(\text{UnaryExpr } op \ e) &= \text{trm}(e) + 1 \\ \text{trm}(\text{BinaryExpr } \text{BinAdd } x \ y) &= \text{trm}(x) + 2 * \text{trm}(y) \\ \text{trm}(\text{ConditionalExpr } \text{cond } t \ f) &= \text{trm}(\text{cond}) + \text{trm}(t) + \text{trm}(f) + 2 \\ \text{trm}(\text{ConstantExpr } c) &= 1 \\ \text{trm}(\text{ParameterExpr } \text{ind } s) &= 2 \\ \text{trm}(\text{LeafExpr } \text{nid } s) &= 2 \end{aligned}$$

graph-representation

typedef *IRGraph* = $\{g :: ID \rightarrow (IRNode \times Stamp) . \text{finite } (\text{dom } g)\}$

graph2tree

rep:constant rep:parameter rep:conditional rep:unary rep:convert
rep:binary rep:leaf rep:ref

preeval

is-preevaluated (*InvokeNode* *n uu uv uw ux uy*) = *True*
is-preevaluated (*InvokeWithExceptionNode* *n uz va vb vc vd ve*) = *True*
is-preevaluated (*NewInstanceNode* *n vf vg vh*) = *True*
is-preevaluated (*LoadFieldNode* *n vi vj vk*) = *True*
is-preevaluated (*SignedDivNode* *n vl vm vn vo vp*) = *True*
is-preevaluated (*SignedRemNode* *n vq vr vs vt vu*) = *True*
is-preevaluated (*ValuePhiNode* *n vv vw*) = *True*
is-preevaluated (*AbsNode* *v*) = *False*
is-preevaluated (*AddNode* *v va*) = *False*
is-preevaluated (*AndNode* *v va*) = *False*
is-preevaluated (*BeginNode* *v*) = *False*
is-preevaluated (*BytecodeExceptionNode* *v va vb*) = *False*
is-preevaluated (*ConditionalNode* *v va vb*) = *False*
is-preevaluated (*ConstantNode* *v*) = *False*
is-preevaluated (*DynamicNewArrayNode* *v va vb vc vd*) = *False*
is-preevaluated *EndNode* = *False*
is-preevaluated (*ExceptionObjectNode* *v va*) = *False*
is-preevaluated (*FrameState* *v va vb vc*) = *False*
is-preevaluated (*IfNode* *v va vb*) = *False*
is-preevaluated (*IntegerBelowNode* *v va*) = *False*
is-preevaluated (*IntegerEqualsNode* *v va*) = *False*
is-preevaluated (*IntegerLessThanNode* *v va*) = *False*
is-preevaluated (*IsNullNode* *v*) = *False*
is-preevaluated (*KillingBeginNode* *v*) = *False*
is-preevaluated (*LeftShiftNode* *v va*) = *False*
is-preevaluated (*LogicNegationNode* *v*) = *False*
is-preevaluated (*LoopBeginNode* *v va vb vc*) = *False*
is-preevaluated (*LoopEndNode* *v*) = *False*
is-preevaluated (*LoopExitNode* *v va vb*) = *False*
is-preevaluated (*MergeNode* *v va vb*) = *False*
is-preevaluated (*MethodCallTargetNode* *v va*) = *False*
is-preevaluated (*MulNode* *v va*) = *False*
is-preevaluated (*NarrowNode* *v va vb*) = *False*
is-preevaluated (*NegateNode* *v*) = *False*
is-preevaluated (*NewArrayNode* *v va vb*) = *False*
is-preevaluated (*NotNode* *v*) = *False*
is-preevaluated (*OrNode* *v va*) = *False*
is-preevaluated (*ParameterNode* *v*) = *False*
is-preevaluated (*PiNode* *v va*) = *False*
is-preevaluated (*ReturnNode* *v va*) = *False*
is-preevaluated (*RightShiftNode* *v va*) = *False*
is-preevaluated (*ShortCircuitOrNode* *v va*) = *False*
is-preevaluated (*SignExtendNode* *v va vb*) = *False*

deterministic-representation

$$g \vdash n \simeq e_1 \wedge g \vdash n \simeq e_2 \implies e_1 = e_2$$

thm-oracles *repDet*

well-formed-term-graph

$$\exists e. g \vdash n \simeq e \wedge (\exists v. [m,p] \vdash e \mapsto v)$$

graph-semantics

$$([g,m,p] \vdash n \mapsto v) = (\exists e. g \vdash n \simeq e \wedge [m,p] \vdash e \mapsto v)$$

graph-semantics-deterministic

$$[g,m,p] \vdash n \mapsto v_1 \wedge [g,m,p] \vdash n \mapsto v_2 \implies v_1 = v_2$$

thm-oracles *graphDet*

notation (*latex*)

graph-refinement (*term-graph-refinement -*)

graph-refinement

$$\begin{aligned} \text{term-graph-refinement } g_1 \ g_2 = \\ (ids \ g_1 \subseteq ids \ g_2 \wedge \\ (\forall n. n \in ids \ g_1 \longrightarrow (\forall e. g_1 \vdash n \simeq e \longrightarrow g_2 \vdash n \trianglelefteq e))) \end{aligned}$$

translations

$n \leq CONST$ *as-set* n

graph-semantics-preservation

$$\begin{aligned} e_1' \sqsupseteq e_2' \wedge \\ \{n\} \triangleleft g_1 \subseteq g_2 \wedge \\ g_1 \vdash n \simeq e_1' \wedge g_2 \vdash n \simeq e_2' \implies \\ \text{term-graph-refinement } g_1 \ g_2 \end{aligned}$$

thm-oracles *graph-semantics-preservation-subscript*

maximal-sharing

maximal-sharing $g =$
 $(\forall n_1 n_2.$
 $n_1 \in \text{true-ids } g \wedge n_2 \in \text{true-ids } g \longrightarrow$
 $(\forall e. g \vdash n_1 \simeq e \wedge g \vdash n_2 \simeq e \longrightarrow n_1 = n_2))$

tree-to-graph-rewriting

$e_1 \sqsupseteq e_2 \wedge$
 $g_1 \vdash n \simeq e_1 \wedge$
maximal-sharing $g_1 \wedge$
 $\{n\} \triangleleft g_1 \subseteq g_2 \wedge$
 $g_2 \vdash n \simeq e_2 \wedge$
maximal-sharing $g_2 \implies$
term-graph-refinement $g_1 g_2$

thm-oracles *tree-to-graph-rewriting*

term-graph-refines-term

$(g \vdash n \sqsubseteq e) = (\exists e'. g \vdash n \simeq e' \wedge e \sqsupseteq e')$

term-graph-evaluation

$g \vdash n \sqsubseteq e \implies \forall m p v. [m,p] \vdash e \mapsto v \longrightarrow [g,m,p] \vdash n \mapsto v$

graph-construction

$e_1 \sqsupseteq e_2 \wedge g_1 \subseteq g_2 \wedge g_2 \vdash n \simeq e_2 \implies$
 $g_2 \vdash n \sqsubseteq e_1 \wedge \text{term-graph-refinement } g_1 g_2$

thm-oracles *graph-construction*

term-graph-reconstruction

$g \triangleleft e \rightsquigarrow (g', n) \implies g' \vdash n \simeq e$

end

theory *SlideSnippets*

```

imports
  Semantics.TreeToGraphThms
  Snippets.Snipping
begin

```

```

notation (latex)
  kind ( $\langle\!\langle\!-\!\rangle\!\rangle$ )

```

```

notation (latex)
  IRTreeEval.ord-IRExpr-inst.less-eq-IRExpr ( $- \mapsto -$ )

```

abstract-syntax-tree

```

datatype IRExpr =
  UnaryExpr IRUnaryOp IRExpr
| BinaryExpr IRBinaryOp IRExpr IRExpr
| ConditionalExpr IRExpr IRExpr IRExpr
| ParameterExpr nat Stamp
| LeafExpr nat Stamp
| ConstantExpr Value
| ConstantVar (char list)
| VariableExpr (char list) Stamp

```

tree-semantics

```

semantics:constant semantics:parameter semantics:unary seman-
tics:binary semantics:leaf

```

expression-refinement

$$e_1 \sqsupseteq e_2 = (\forall m \ p \ v. [m,p] \vdash e_1 \mapsto v \longrightarrow [m,p] \vdash e_2 \mapsto v)$$

graph2tree

```

semantics:constant semantics:unary semantics:binary

```

graph-semantics

$$([g,m,p] \vdash n \mapsto v) = (\exists e. g \vdash n \simeq e \wedge [m,p] \vdash e \mapsto v)$$

graph-refinement

$graph-refinement\ g_1\ g_2 =$
 $(ids\ g_1 \subseteq ids\ g_2 \wedge$
 $(\forall n. n \in ids\ g_1 \longrightarrow (\forall e. g_1 \vdash n \simeq e \longrightarrow g_2 \vdash n \trianglelefteq e)))$

translations

$n \leq CONST\ as-set\ n$

graph-semantics-preservation

$\llbracket e1' \sqsupseteq e2'; \{n'\} \triangleleft g1 \subseteq g2;$
 $g1 \vdash n' \simeq e1'; g2 \vdash n' \simeq e2' \rrbracket$
 $\implies graph-refinement\ g1\ g2$

maximal-sharing

$maximal-sharing\ g =$
 $(\forall n_1\ n_2.$
 $n_1 \in true-ids\ g \wedge n_2 \in true-ids\ g \longrightarrow$
 $(\forall e. g \vdash n_1 \simeq e \wedge g \vdash n_2 \simeq e \longrightarrow n_1 = n_2))$

tree-to-graph-rewriting

$e_1 \sqsupseteq e_2 \wedge$
 $g_1 \vdash n \simeq e_1 \wedge$
 $maximal-sharing\ g_1 \wedge$
 $\{n\} \triangleleft g_1 \subseteq g_2 \wedge$
 $g_2 \vdash n \simeq e_2 \wedge maximal-sharing\ g_2 \implies$
 $graph-refinement\ g_1\ g_2$

graph-represents-expression

$(g \vdash n \trianglelefteq e) = (\exists e'. g \vdash n \simeq e' \wedge e \sqsupseteq e')$

graph-construction

$e_1 \sqsupseteq e_2 \wedge g_1 \subseteq g_2 \wedge g_2 \vdash n \simeq e_2 \implies$
 $g_2 \vdash n \trianglelefteq e_1 \wedge graph-refinement\ g_1\ g_2$

end