

Veriopt Theories

February 9, 2022

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1 Conditional Elimination Phase

```
theory ConditionalElimination
imports
  Semantics.IRGraphFrames
  Proofs.Stuttering
  Proofs.Rewrites
  Proofs.Bisimulation
begin
```

1.1 Individual Elimination Rules

We introduce a `TriState` as in the Graal compiler to represent when static analysis can tell us information about the value of a boolean expression. `Unknown` = No information can be inferred `KnownTrue`/`KnownFalse` = We can infer the expression will always be true or false.

```
datatype TriState = Unknown | KnownTrue | KnownFalse
```

The `implies` relation corresponds to the `LogicNode.implies` method from the compiler which attempts to infer when one logic nodes value can be inferred from a known logic node.

```
inductive implies :: IRGraph  $\Rightarrow$  IRNode  $\Rightarrow$  IRNode  $\Rightarrow$  TriState  $\Rightarrow$  bool
  (-  $\vdash$  - & -  $\hookrightarrow$  -) for g where
  eq-imp-less:
    g  $\vdash$  (IntegerEqualsNode x y) & (IntegerLessThanNode x y)  $\hookrightarrow$  KnownFalse |
  eq-imp-less-rev:
    g  $\vdash$  (IntegerEqualsNode x y) & (IntegerLessThanNode y x)  $\hookrightarrow$  KnownFalse |
  less-imp-rev-less:
    g  $\vdash$  (IntegerLessThanNode x y) & (IntegerLessThanNode y x)  $\hookrightarrow$  KnownFalse |
```

less-imp-not-eq:
 $g \vdash (\text{IntegerLessThanNode } x \ y) \ \& \ (\text{IntegerEqualsNode } x \ y) \hookrightarrow \text{KnownFalse} \mid$
less-imp-not-eq-rev:
 $g \vdash (\text{IntegerLessThanNode } x \ y) \ \& \ (\text{IntegerEqualsNode } y \ x) \hookrightarrow \text{KnownFalse} \mid$

x-imp-x:
 $g \vdash x \ \& \ x \hookrightarrow \text{KnownTrue} \mid$

negate-false:
 $\llbracket g \vdash x \ \& \ (\text{kind } g \ y) \hookrightarrow \text{KnownTrue} \rrbracket \implies g \vdash x \ \& \ (\text{LogicNegationNode } y) \hookrightarrow \text{KnownFalse} \mid$
negate-true:
 $\llbracket g \vdash x \ \& \ (\text{kind } g \ y) \hookrightarrow \text{KnownFalse} \rrbracket \implies g \vdash x \ \& \ (\text{LogicNegationNode } y) \hookrightarrow \text{KnownTrue}$

Total relation over partial implies relation

inductive *condition-implies* :: *IRGraph* \Rightarrow *IRNode* \Rightarrow *IRNode* \Rightarrow *TriState* \Rightarrow *bool*
 (- \vdash - $\ \& \$ - \hookrightarrow -) **for** *g* **where**
 $\llbracket \neg(g \vdash a \ \& \ b \hookrightarrow \text{imp}) \rrbracket \implies (g \vdash a \ \& \ b \hookrightarrow \text{Unknown}) \mid$
 $\llbracket (g \vdash a \ \& \ b \hookrightarrow \text{imp}) \rrbracket \implies (g \vdash a \ \& \ b \hookrightarrow \text{imp})$

inductive *implies-tree* :: *IRExpr* \Rightarrow *IRExpr* \Rightarrow *bool* \Rightarrow *bool*
 (- $\ \& \$ - \hookrightarrow -) **where**
eq-imp-less:
 $(\text{BinaryExpr } \text{BinIntegerEquals } x \ y) \ \& \ (\text{BinaryExpr } \text{BinIntegerLessThan } x \ y) \hookrightarrow \text{False} \mid$
eq-imp-less-rev:
 $(\text{BinaryExpr } \text{BinIntegerEquals } x \ y) \ \& \ (\text{BinaryExpr } \text{BinIntegerLessThan } y \ x) \hookrightarrow \text{False} \mid$
less-imp-rev-less:
 $(\text{BinaryExpr } \text{BinIntegerLessThan } x \ y) \ \& \ (\text{BinaryExpr } \text{BinIntegerLessThan } y \ x) \hookrightarrow \text{False} \mid$
less-imp-not-eq:
 $(\text{BinaryExpr } \text{BinIntegerLessThan } x \ y) \ \& \ (\text{BinaryExpr } \text{BinIntegerEquals } x \ y) \hookrightarrow \text{False} \mid$
less-imp-not-eq-rev:
 $(\text{BinaryExpr } \text{BinIntegerLessThan } x \ y) \ \& \ (\text{BinaryExpr } \text{BinIntegerEquals } y \ x) \hookrightarrow \text{False} \mid$

x-imp-x:
 $x \ \& \ x \hookrightarrow \text{True} \mid$

negate-false:
 $\llbracket x \ \& \ y \hookrightarrow \text{True} \rrbracket \implies x \ \& \ (\text{UnaryExpr } \text{UnaryLogicNegation } y) \hookrightarrow \text{False} \mid$
negate-true:
 $\llbracket x \ \& \ y \hookrightarrow \text{False} \rrbracket \implies x \ \& \ (\text{UnaryExpr } \text{UnaryLogicNegation } y) \hookrightarrow \text{True}$

Proofs that the implies relation is correct with respect to the existing eval-

uation semantics.

experiment begin

lemma *logic-negate-type*:

assumes $[m, p] \vdash \text{UnaryExpr } \text{UnaryLogicNegation } x \mapsto v$

assumes $v \neq \text{UndefVal}$

shows $\exists v2. [m, p] \vdash x \mapsto \text{IntVal32 } v2$

proof –

obtain ve **where** $ve: [m, p] \vdash x \mapsto ve$

using *assms(1)* **by** *blast*

then have $[m, p] \vdash \text{UnaryExpr } \text{UnaryLogicNegation } x \mapsto \text{unary-eval } \text{UnaryLogicNegation } ve$

by (*metis* *UnaryExprE* *assms(1)* *evalDet*)

then show *?thesis* **using** *assms* *unary-eval.elims* *evalDet* *ve* *IRUnaryOp.distinct*

sorry

qed

lemma *logic-negation-relation-tree*:

assumes $[m, p] \vdash y \mapsto val$

assumes $[m, p] \vdash \text{UnaryExpr } \text{UnaryLogicNegation } y \mapsto invval$

assumes $intval \neq \text{UndefVal}$

shows $\text{val-to-bool } val \longleftrightarrow \neg(\text{val-to-bool } invval)$

proof –

obtain v **where** $invval = \text{unary-eval } \text{UnaryLogicNegation } v$

using *assms(2)* **by** *blast*

then have $[m, p] \vdash y \mapsto v$ **using** *UnaryExprE* *assms(1,2)* **sorry**

then show *?thesis* **sorry**

qed

lemma *logic-negation-relation*:

assumes $[g, m, p] \vdash y \mapsto val$

assumes $\text{kind } g \text{ neg} = \text{LogicNegationNode } y$

assumes $[g, m, p] \vdash \text{neg} \mapsto invval$

assumes $intval \neq \text{UndefVal}$

shows $\text{val-to-bool } val \longleftrightarrow \neg(\text{val-to-bool } invval)$

proof –

obtain $yencode$ **where** $g \vdash y \simeq yencode$

using *assms(1)* *encodeeval-def* **by** *auto*

then have $g \vdash \text{neg} \simeq \text{UnaryExpr } \text{UnaryLogicNegation } yencode$

using *rep.intros(7)* *assms(2)* **by** *simp*

then have $[m, p] \vdash \text{UnaryExpr } \text{UnaryLogicNegation } yencode \mapsto invval$

using *assms(3)* *encodeeval-def*

by (*metis* *repDet*)

obtain $v1$ **where** $[g, m, p] \vdash y \mapsto \text{IntVal32 } v1$

using *assms(1,2,3,4)* **using** *logic-negate-type* **sorry**

have $invval = \text{bool-to-val } (\neg(\text{val-to-bool } val))$

using *assms(1,2,3)* *evalDet* *unary-eval.simps(4)*

by (*smt* (*verit*, *ccfv-SIG*) *UnaryExprE* $\langle [m, p] \vdash \text{UnaryExpr } \text{UnaryLogicNegation } yencode \mapsto invval \rangle \langle g \vdash y \simeq yencode \rangle \text{bool-to-val.simps(1)} \text{bool-to-val.simps(2)} \text{en-}$

```

codeeval-def intval-logic-negation.simps(1) logic-negate-type repDet val-to-bool.simps(1))
  have val-to-bool invval  $\longleftrightarrow \neg(\text{val-to-bool } val)$ 
    using  $\langle \text{invval} = \text{bool-to-val } (\neg \text{ val-to-bool } val) \rangle$  by force
  then show ?thesis
    by simp
qed
end

```

lemma *implies-valid*:

```

  assumes  $x \ \& \ y \hookrightarrow \text{imp}$ 
  assumes  $[m, p] \vdash x \mapsto v1$ 
  assumes  $[m, p] \vdash y \mapsto v2$ 
  assumes  $v1 \neq \text{UndefVal} \wedge v2 \neq \text{UndefVal}$ 
  shows  $(\text{imp} \longrightarrow (\text{val-to-bool } v1 \longrightarrow \text{val-to-bool } v2)) \wedge$ 
     $(\neg \text{imp} \longrightarrow (\text{val-to-bool } v1 \longrightarrow \neg(\text{val-to-bool } v2))) \wedge$ 
     $(\text{is } (?TP \longrightarrow ?TC) \wedge (?FP \longrightarrow ?FC))$ 
  apply (intro conjI; rule impI)
proof -
  assume KnownTrue: ?TP
  show ?TC
  using assms(1) KnownTrue assms(2-) proof (induct x y imp rule: implies-tree.induct)
    case (eq-imp-less x y)
    then show ?case by simp
  next
    case (eq-imp-less-rev x y)
    then show ?case by simp
  next
    case (less-imp-rev-less x y)
    then show ?case by simp
  next
    case (less-imp-not-eq x y)
    then show ?case by simp
  next
    case (less-imp-not-eq-rev x y)
    then show ?case by simp
  next
    case (x-imp-x)
    then show ?case
      by (metis evalDet)
  next
    case (negate-false x1)
    then show ?case using evalDet
      using assms(2,3) by blast
  next
    case (negate-true y)
    then show ?case
      sorry
  qed
next

```

```

assume KnownFalse: ?FP
show ?FC using assms KnownFalse proof (induct x y imp rule: implies-tree.induct)
  case (eq-imp-less x y)
  obtain xval where xval: [m, p] ⊢ x ↦ xval
    using eq-imp-less(1) eq-imp-less.prem(3)
    by blast
  then obtain yval where yval: [m, p] ⊢ y ↦ yval
    using eq-imp-less.prem(3)
    using eq-imp-less.prem(2) by blast
  have egeval: [m, p] ⊢ (BinaryExpr BinIntegerEquals x y) ↦ intval-equals xval
yval
    using xval yval evaltree.BinaryExpr
    by (metis BinaryExprE bin-eval.simp(10) eq-imp-less.prem(1) evalDet)
  have lesseval: [m, p] ⊢ (BinaryExpr BinIntegerLessThan x y) ↦ intval-less-than
xval yval
    using xval yval evaltree.BinaryExpr
    by (metis BinaryExprE bin-eval.simp(11) eq-imp-less.prem(2) evalDet)
  have val-to-bool (intval-equals xval yval) ⟶ ¬(val-to-bool (intval-less-than xval
yval))
    using assms(4) apply (cases xval; cases yval; auto)
    apply (metis (full-types) val-to-bool.simp(1) Values.bool-to-val.simp(2)
signed.less-irrefl)
  by (metis (mono-tags) val-to-bool.simp(1) Values.bool-to-val.elims signed.order.strict-implies-not-eq)
  then show ?case
    using egeval lesseval
    by (metis eq-imp-less.prem(1) eq-imp-less.prem(2) evalDet)
next
  case (eq-imp-less-rev x y)
  obtain xval where xval: [m, p] ⊢ x ↦ xval
    using eq-imp-less-rev.prem(3)
    using eq-imp-less-rev.prem(2) by blast
  obtain yval where yval: [m, p] ⊢ y ↦ yval
    using eq-imp-less-rev.prem(3)
    using eq-imp-less-rev.prem(2) by blast
  have egeval: [m, p] ⊢ (BinaryExpr BinIntegerEquals x y) ↦ intval-equals xval
yval
    using xval yval evaltree.BinaryExpr
    by (metis BinaryExprE bin-eval.simp(10) eq-imp-less-rev.prem(1) evalDet)
  have lesseval: [m, p] ⊢ (BinaryExpr BinIntegerLessThan y x) ↦ intval-less-than
yval xval
    using xval yval evaltree.BinaryExpr
    by (metis BinaryExprE bin-eval.simp(11) eq-imp-less-rev.prem(2) evalDet)
  have val-to-bool (intval-equals xval yval) ⟶ ¬(val-to-bool (intval-less-than yval
xval))
    using assms(4) apply (cases xval; cases yval; auto)
    apply (metis (full-types) val-to-bool.simp(1) Values.bool-to-val.simp(2)
signed.less-irrefl)
  by (metis (full-types) val-to-bool.simp(1) Values.bool-to-val.elims signed.order.strict-implies-not-eq)
  then show ?case

```

```

    using egeval lesseval
    by (metis eq-imp-less-rev.prem(1) eq-imp-less-rev.prem(2) evalDet)
next
case (less-imp-rev-less x y)
obtain xval where xval: [m, p] ⊢ x ↦ xval
    using less-imp-rev-less.prem(3)
    using less-imp-rev-less.prem(2) by blast
obtain yval where yval: [m, p] ⊢ y ↦ yval
    using less-imp-rev-less.prem(3)
    using less-imp-rev-less.prem(2) by blast
have lesseval: [m, p] ⊢ (BinaryExpr BinIntegerLessThan x y) ↦ intval-less-than
xval yval
    using xval yval evaltree.BinaryExpr
    by (metis BinaryExprE bin-eval.simp(11) evalDet less-imp-rev-less.prem(1))
    have revlesseval: [m, p] ⊢ (BinaryExpr BinIntegerLessThan y x) ↦ int-
val-less-than yval xval
        using xval yval evaltree.BinaryExpr
        by (metis BinaryExprE bin-eval.simp(11) evalDet less-imp-rev-less.prem(2))
    have val-to-bool (intval-less-than xval yval) ⟶ ¬(val-to-bool (intval-less-than
yval xval))
        using assms(4) apply (cases xval; cases yval; auto)
    apply (metis val-to-bool.simp(1) Values.bool-to-val.elims signed.not-less-iff-gr-or-eq)
        by (metis val-to-bool.simp(1) Values.bool-to-val.elims signed.less-asym')
    then show ?case
    by (metis evalDet less-imp-rev-less.prem(1) less-imp-rev-less.prem(2) lesseval
revlesseval)
next
case (less-imp-not-eq x y)
obtain xval where xval: [m, p] ⊢ x ↦ xval
    using less-imp-not-eq.prem(3)
    using less-imp-not-eq.prem(1) by blast
obtain yval where yval: [m, p] ⊢ y ↦ yval
    using less-imp-not-eq.prem(3)
    using less-imp-not-eq.prem(1) by blast
have egeval: [m, p] ⊢ (BinaryExpr BinIntegerEquals x y) ↦ intval-equals xval
yval
    using xval yval evaltree.BinaryExpr
    by (metis BinaryExprE bin-eval.simp(10) evalDet less-imp-not-eq.prem(2))
    have lesseval: [m, p] ⊢ (BinaryExpr BinIntegerLessThan x y) ↦ intval-less-than
xval yval
        using xval yval evaltree.BinaryExpr
        by (metis BinaryExprE bin-eval.simp(11) evalDet less-imp-not-eq.prem(1))
    have val-to-bool (intval-less-than xval yval) ⟶ ¬(val-to-bool (intval-equals xval
yval))
        using assms(4) apply (cases xval; cases yval; auto)
    apply (metis (full-types) bool-to-val.simp(2) signed.less-imp-not-eq val-to-bool.simp(1))
    by (metis (full-types) bool-to-val.simp(2) signed.less-imp-not-eq2 val-to-bool.simp(1))
    then show ?case
    by (metis egeval evalDet less-imp-not-eq.prem(1) less-imp-not-eq.prem(2))

```

```

lesseval)
next
  case (less-imp-not-eq-rev x y)
  obtain xval where xval: [m, p] ⊢ x ↦ xval
  using less-imp-not-eq-rev.prem(3)
  using less-imp-not-eq-rev.prem(1) by blast
  obtain yval where yval: [m, p] ⊢ y ↦ yval
  using less-imp-not-eq-rev.prem(3)
  using less-imp-not-eq-rev.prem(1) by blast
  have egeval: [m, p] ⊢ (BinaryExpr BinIntegerEquals y x) ↦ intval-equals yval
xval
  using xval yval evaltree.BinaryExpr
  by (metis BinaryExprE bin-eval.simp(10) evalDet less-imp-not-eq-rev.prem(2))
  have lesseval: [m, p] ⊢ (BinaryExpr BinIntegerLessThan x y) ↦ intval-less-than
xval yval
  using xval yval evaltree.BinaryExpr
  by (metis BinaryExprE bin-eval.simp(11) evalDet less-imp-not-eq-rev.prem(1))
  have val-to-bool (intval-less-than xval yval) ⟶ ¬(val-to-bool (intval-equals yval
xval))
  using assms(4) apply (cases xval; cases yval; auto)
  apply (metis (full-types) bool-to-val.simp(2) signed.less-imp-not-eq2 val-to-bool.simp(1))
  by (metis (full-types, opaque-lifting) val-to-bool.simp(1) Values.bool-to-val.elims
signed.dual-order.strict-implies-not-eq)
  then show ?case
  by (metis egeval evalDet less-imp-not-eq-rev.prem(1) less-imp-not-eq-rev.prem(2))
lesseval)
next
  case (x-imp-x x1)
  then show ?case by simp
next
  case (negate-false x y)
  then show ?case sorry
next
  case (negate-true x1)
  then show ?case by simp
qed
qed

lemma implies-true-valid:
  assumes x & y ⟷ imp
  assumes imp
  assumes [m, p] ⊢ x ↦ v1
  assumes [m, p] ⊢ y ↦ v2
  assumes v1 ≠ UndefVal ∧ v2 ≠ UndefVal
  shows val-to-bool v1 ⟶ val-to-bool v2
  using assms implies-valid
  by blast

lemma implies-false-valid:

```

```

assumes  $x \ \& \ y \hookrightarrow imp$ 
assumes  $\neg imp$ 
assumes  $[m, p] \vdash x \mapsto v1$ 
assumes  $[m, p] \vdash y \mapsto v2$ 
assumes  $v1 \neq UndefinedVal \wedge v2 \neq UndefinedVal$ 
shows  $val\text{-}to\text{-}bool \ v1 \longrightarrow \neg(val\text{-}to\text{-}bool \ v2)$ 
using assms implies-valid by blast

```

The following relation corresponds to the `UnaryOpLogicNode.tryFold` and `BinaryOpLogicNode.tryFold` methods and their associated concrete implementations.

The relation determines if a logic operation can be shown true or false through the stamp typing information.

```

inductive tryFold :: IRNode  $\Rightarrow$  (ID  $\Rightarrow$  Stamp)  $\Rightarrow$  bool  $\Rightarrow$  bool
where
   $\llbracket alwaysDistinct \ (stamps \ x) \ (stamps \ y) \rrbracket$ 
     $\implies tryFold \ (IntegerEqualsNode \ x \ y) \ stamps \ False \mid$ 
   $\llbracket neverDistinct \ (stamps \ x) \ (stamps \ y) \rrbracket$ 
     $\implies tryFold \ (IntegerEqualsNode \ x \ y) \ stamps \ True \mid$ 
   $\llbracket isIntegerStamp \ (stamps \ x);$ 
     $isIntegerStamp \ (stamps \ y);$ 
     $stpi\_upper \ (stamps \ x) < stpi\_lower \ (stamps \ y) \rrbracket$ 
     $\implies tryFold \ (IntegerLessThanNode \ x \ y) \ stamps \ True \mid$ 
   $\llbracket isIntegerStamp \ (stamps \ x);$ 
     $isIntegerStamp \ (stamps \ y);$ 
     $stpi\_lower \ (stamps \ x) \geq stpi\_upper \ (stamps \ y) \rrbracket$ 
     $\implies tryFold \ (IntegerLessThanNode \ x \ y) \ stamps \ False$ 

```

Proofs that show that when the stamp lookup function is well-formed, the `tryFold` relation correctly predicts the output value with respect to our evaluation semantics.

lemma

```

assumes  $kind \ g \ nid = IntegerEqualsNode \ x \ y$ 
assumes  $[g, m, p] \vdash nid \mapsto v$ 
assumes  $v \neq UndefinedVal$ 
assumes  $([g, m, p] \vdash x \mapsto xval) \wedge ([g, m, p] \vdash y \mapsto yval)$ 
shows  $val\text{-}to\text{-}bool \ (intval\text{-}equals \ xval \ yval) \longleftrightarrow v = IntVal32 \ 1$ 

```

proof –

```

have  $v = intval\text{-}equals \ xval \ yval$ 
using assms(1, 2, 3, 4) BinaryExprE IntegerEqualsNode bin-eval.simps(7)
by (smt (verit) bin-eval.simps(10) encodeeval-def evalDet repDet)
then show ?thesis using intval-equals.simps val-to-bool.simps sorry

```

qed

lemma *tryFoldIntegerEqualsAlwaysDistinct*:

```

assumes wf-stamp g stamps
assumes  $kind \ g \ nid = (IntegerEqualsNode \ x \ y)$ 
assumes  $[g, m, p] \vdash nid \mapsto v$ 

```



```

assumes alwaysDistinct (stamps x) (stamps y)
shows v = IntVal32 0
proof –
  have  $\forall$  val.  $\neg$ (valid-value val (join (stamps x) (stamps y)))
    using assms(1,4) unfolding alwaysDistinct.simps
    by (metis is-stamp-empty.elims(2) le-less-trans not-less valid32or64 valid-value.simps(1)
valid-value.simps(2))
  have  $\neg(\exists$  val . ([g, m, p]  $\vdash$  x  $\mapsto$  val)  $\wedge$  ([g, m, p]  $\vdash$  y  $\mapsto$  val))
    using assms(1,4) unfolding alwaysDistinct.simps wf-stamp.simps encodee-
val-def sorry
  then show ?thesis sorry
qed

```

lemma *tryFoldIntegerEqualsNeverDistinct*:

```

assumes wf-stamp g stamps
assumes kind g nid = (IntegerEqualsNode x y)
assumes [g, m, p]  $\vdash$  nid  $\mapsto$  v
assumes neverDistinct (stamps x) (stamps y)
shows v = IntVal32 1
using assms IntegerEqualsNodeE sorry

```

lemma *tryFoldIntegerLessThanTrue*:

```

assumes wf-stamp g stamps
assumes kind g nid = (IntegerLessThanNode x y)
assumes [g, m, p]  $\vdash$  nid  $\mapsto$  v
assumes stpi-upper (stamps x) < stpi-lower (stamps y)
shows v = IntVal32 1
proof –
  have stamp-type: is-IntegerStamp (stamps x)
    using assms
    sorry
  obtain xval where xval: [g, m, p]  $\vdash$  x  $\mapsto$  xval
    using assms(2,3) sorry
  obtain yval where yval: [g, m, p]  $\vdash$  y  $\mapsto$  yval
    using assms(2,3) sorry
  have is-IntegerStamp (stamps x)  $\wedge$  is-IntegerStamp (stamps y)
    using assms(4)
    sorry
  then have val-to-bool (intval-less-than xval yval)
    sorry
  then show ?thesis
    sorry
qed

```

lemma *tryFoldIntegerLessThanFalse*:

```

assumes wf-stamp g stamps
assumes kind g nid = (IntegerLessThanNode x y)
assumes [g, m, p]  $\vdash$  nid  $\mapsto$  v
assumes stpi-lower (stamps x)  $\geq$  stpi-upper (stamps y)

```

```

shows  $v = \text{IntVal32 } 0$ 
proof -
have stamp-type: is-IntegerStamp (stamps  $x$ )
  using assms
  sorry
obtain  $xval$  where  $xval: [g, m, p] \vdash x \mapsto xval$ 
  using assms(2,3) sorry
obtain  $yval$  where  $yval: [g, m, p] \vdash y \mapsto yval$ 
  using assms(2,3) sorry
have is-IntegerStamp (stamps  $x$ )  $\wedge$  is-IntegerStamp (stamps  $y$ )
  using assms(4)
  sorry
then have  $\neg(\text{val-to-bool } (\text{intval-less-than } xval \ yval))$ 
  sorry
then show ?thesis
  sorry
qed

theorem tryFoldProofTrue:
  assumes wf-stamp  $g$  stamps
  assumes tryFold (kind  $g$  nid) stamps True
  assumes  $[g, m, p] \vdash \text{nid} \mapsto v$ 
  shows val-to-bool  $v$ 
  using assms(2) proof (induction kind  $g$  nid stamps True rule: tryFold.induct)
case (1 stamps  $x$   $y$ )
  then show ?case using tryFoldIntegerEqualsAlwaysDistinct assms sorry
next
case (2 stamps  $x$   $y$ )
  then show ?case using tryFoldIntegerEqualsAlwaysDistinct assms sorry
next
case (3 stamps  $x$   $y$ )
  then show ?case using tryFoldIntegerLessThanTrue assms sorry
next
case (4 stamps  $x$   $y$ )
  then show ?case using tryFoldIntegerLessThanFalse assms sorry
qed

theorem tryFoldProofFalse:
  assumes wf-stamp  $g$  stamps
  assumes tryFold (kind  $g$  nid) stamps False
  assumes  $[g, m, p] \vdash \text{nid} \mapsto v$ 
  shows  $\neg(\text{val-to-bool } v)$ 
  using assms(2) proof (induction kind  $g$  nid stamps False rule: tryFold.induct)
case (1 stamps  $x$   $y$ )
  then show ?case using tryFoldIntegerEqualsAlwaysDistinct assms sorry
next
case (2 stamps  $x$   $y$ )
  then show ?case using tryFoldIntegerEqualsNeverDistinct assms sorry
next

```

```

case (3 stamps x y)
then show ?case using tryFoldIntegerLessThanTrue assms sorry
next
case (4 stamps x y)
then show ?case using tryFoldIntegerLessThanFalse assms sorry

qed

```

inductive-cases *StepE*:

$g, p \vdash (nid, m, h) \rightarrow (nid', m', h)$

Perform conditional elimination rewrites on the graph for a particular node. In order to determine conditional eliminations appropriately the rule needs two data structures produced by static analysis. The first parameter is the set of IRNodes that we know result in a true value when evaluated. The second parameter is a mapping from node identifiers to the flow-sensitive stamp.

The relation transforms the third parameter to the fifth parameter for a node identifier which represents the fourth parameter.

inductive *ConditionalEliminationStep* ::

$IRExpr \text{ set} \Rightarrow (ID \Rightarrow Stamp) \Rightarrow IRGraph \Rightarrow ID \Rightarrow IRGraph \Rightarrow bool$ **where**
impliesTrue:

$\llbracket kind\ g\ ifcond = (IfNode\ cid\ t\ f);$
 $g \vdash cid \simeq cond;$
 $\exists\ ce \in conds . (ce \ \&\ cond \hookrightarrow True);$
 $g' = constantCondition\ True\ ifcond\ (kind\ g\ ifcond)\ g$
 $\rrbracket \implies ConditionalEliminationStep\ conds\ stamps\ g\ ifcond\ g' \mid$

impliesFalse:

$\llbracket kind\ g\ ifcond = (IfNode\ cid\ t\ f);$
 $g \vdash cid \simeq cond;$
 $\exists\ ce \in conds . (ce \ \&\ cond \hookrightarrow False);$
 $g' = constantCondition\ False\ ifcond\ (kind\ g\ ifcond)\ g$
 $\rrbracket \implies ConditionalEliminationStep\ conds\ stamps\ g\ ifcond\ g' \mid$

tryFoldTrue:

$\llbracket kind\ g\ ifcond = (IfNode\ cid\ t\ f);$
 $cond = kind\ g\ cid;$
 $tryFold\ (kind\ g\ cid)\ stamps\ True;$
 $g' = constantCondition\ True\ ifcond\ (kind\ g\ ifcond)\ g$
 $\rrbracket \implies ConditionalEliminationStep\ conds\ stamps\ g\ ifcond\ g' \mid$

tryFoldFalse:

$\llbracket kind\ g\ ifcond = (IfNode\ cid\ t\ f);$
 $cond = kind\ g\ cid;$
 $tryFold\ (kind\ g\ cid)\ stamps\ False;$

$g' = \text{constantCondition False ifcond (kind g ifcond) g}$
 $\mathbb{I} \Rightarrow \text{ConditionalEliminationStep conds stamps g ifcond g'}$

code-pred (modes: $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow \text{bool}$) *ConditionalEliminationStep* .

thm *ConditionalEliminationStep.equation*

1.2 Control-flow Graph Traversal

type-synonym *Seen* = *ID set*

type-synonym *Condition* = *IRNode*

type-synonym *Conditions* = *Condition list*

type-synonym *StampFlow* = (*ID* \Rightarrow *Stamp*) *list*

nextEdge helps determine which node to traverse next by returning the first successor edge that isn't in the set of already visited nodes. If there is not an appropriate successor, *None* is returned instead.

fun *nextEdge* :: *Seen* \Rightarrow *ID* \Rightarrow *IRGraph* \Rightarrow *ID option* **where**
nextEdge *seen* *nid* *g* =
 (let *nids* = (filter ($\lambda \text{nid}'$. $\text{nid}' \notin \text{seen}$) (successors-of (kind *g* *nid*))) in
 (if length *nids* > 0 then Some (hd *nids*) else None))

pred determines which node, if any, acts as the predecessor of another.

Merge nodes represent a special case where-in the predecessor exists as an input edge of the merge node, to simplify the traversal we treat only the first input end node as the predecessor, ignoring that multiple nodes may act as a successor.

For all other nodes, the predecessor is the first element of the predecessors set. Note that in a well-formed graph there should only be one element in the predecessor set.

fun *pred* :: *IRGraph* \Rightarrow *ID* \Rightarrow *ID option* **where**
pred *g* *nid* = (case kind *g* *nid* of
 (MergeNode ends -) \Rightarrow Some (hd ends) |
 - \Rightarrow
 (if *IRGraph.predecessors* *g* *nid* = {}
 then None else
 Some (hd (sorted-list-of-set (*IRGraph.predecessors* *g* *nid*)))
)
)

When the basic block of an if statement is entered, we know that the condition of the preceding if statement must be true. As in the GraalVM compiler, we introduce the *registerNewCondition* function which roughly corresponds to the *ConditionalEliminationPhase.registerNewCondition*. This method updates the flow-sensitive stamp information based on the condition which we know must be true.

```

fun clip-upper :: Stamp  $\Rightarrow$  int  $\Rightarrow$  Stamp where
  clip-upper (IntegerStamp b l h) c = (IntegerStamp b l c) |
  clip-upper s c = s
fun clip-lower :: Stamp  $\Rightarrow$  int  $\Rightarrow$  Stamp where
  clip-lower (IntegerStamp b l h) c = (IntegerStamp b c h) |
  clip-lower s c = s

fun registerNewCondition :: IRGraph  $\Rightarrow$  Condition  $\Rightarrow$  (ID  $\Rightarrow$  Stamp)  $\Rightarrow$  (ID  $\Rightarrow$  Stamp) where

  registerNewCondition g (IntegerEqualsNode x y) stamps =
    (stamps(x := join (stamps x) (stamps y)))(y := join (stamps x) (stamps y)) |

  registerNewCondition g (IntegerLessThanNode x y) stamps =
    (stamps
      (x := clip-upper (stamps x) (stpi-lower (stamps y)))
      (y := clip-lower (stamps y) (stpi-upper (stamps x)))
    ) |
  registerNewCondition g - stamps = stamps

fun hdOr :: 'a list  $\Rightarrow$  'a  $\Rightarrow$  'a where
  hdOr (x # xs) de = x |
  hdOr [] de = de

The Step relation is a small-step traversal of the graph which handles transitions between individual nodes of the graph.

It relates a pairs of tuple of the current node, the set of seen nodes, the always true stack of IfNode conditions, and the flow-sensitive stamp information.

inductive Step
  :: IRGraph  $\Rightarrow$  (ID  $\times$  Seen  $\times$  Conditions  $\times$  StampFlow)  $\Rightarrow$  (ID  $\times$  Seen  $\times$  Conditions  $\times$  StampFlow) option  $\Rightarrow$  bool
  for g where
    — Hit a BeginNode with an IfNode predecessor which represents the start of a basic block for the IfNode. 1. nid' will be the successor of the begin node. 2. Find the first and only predecessor. 3. Extract condition from the preceding IfNode. 4. Negate condition if the begin node is second branch (we've taken the else branch of the condition) 5. Add the condition or the negated condition to stack 6. Perform any stamp updates based on the condition using the registerNewCondition function and place them on the top of the stack of stamp information
     $\llbracket$ kind g nid = BeginNode nid';

    nid  $\notin$  seen;
    seen' = {nid}  $\cup$  seen;

    Some ifcond = pred g nid;
    kind g ifcond = IfNode cond t f;

    i = find-index nid (successors-of (kind g ifcond));
    c = (if i = 0 then kind g cond else LogicNegationNode cond);
    conds' = c # conds;

```

$flow' = registerNewCondition\ g\ c\ (hdOr\ flow\ (stamp\ g))$
 $\implies Step\ g\ (nid,\ seen,\ conds,\ flow)\ (Some\ (nid',\ seen',\ conds',\ flow'\ \# \ flow))\ |$

— Hit an EndNode 1. nid' will be the usage of EndNode 2. pop the conditions and stamp stack

$\llbracket kind\ g\ nid = EndNode;$

$nid \notin seen;$
 $seen' = \{nid\} \cup seen;$

$nid' = any-usage\ g\ nid;$

$conds' = tl\ conds;$

$flow' = tl\ flow$

$\implies Step\ g\ (nid,\ seen,\ conds,\ flow)\ (Some\ (nid',\ seen',\ conds',\ flow'))\ |$

— We can find a successor edge that is not in seen, go there

$\llbracket \neg(is-EndNode\ (kind\ g\ nid));$
 $\neg(is-BEGINNode\ (kind\ g\ nid));$

$nid \notin seen;$
 $seen' = \{nid\} \cup seen;$

$Some\ nid' = nextEdge\ seen'\ nid\ g$

$\implies Step\ g\ (nid,\ seen,\ conds,\ flow)\ (Some\ (nid',\ seen',\ conds,\ flow))\ |$

— We can cannot find a successor edge that is not in seen, give back None

$\llbracket \neg(is-EndNode\ (kind\ g\ nid));$
 $\neg(is-BEGINNode\ (kind\ g\ nid));$

$nid \notin seen;$
 $seen' = \{nid\} \cup seen;$

$None = nextEdge\ seen'\ nid\ g$

$\implies Step\ g\ (nid,\ seen,\ conds,\ flow)\ None\ |$

— We've already seen this node, give back None

$\llbracket nid \in seen \rrbracket \implies Step\ g\ (nid,\ seen,\ conds,\ flow)\ None$

code-pred (*modes*: $i \Rightarrow i \Rightarrow o \Rightarrow bool$) *Step* .

The ConditionalEliminationPhase relation is responsible for combining the individual traversal steps from the Step relation and the optimizations from the ConditionalEliminationStep relation to perform a transformation of the whole graph.

end