## Unspecified Veriopt Theory

## April 24, 2021

## Contents

theory ATVA2021

```
imports
    Optimizations. \ Canonicalization Proofs
begin
notation (latex)
  kind (-\langle - \rangle)
syntax (spaced-type-def output)
  -constrain :: logic => type => logic (- :: - [4, 0] 3)
                       is-BinaryArithmeticNode :: IRNode <math>\Rightarrow bool
inputs-of :: IRNode \Rightarrow nat\ list
inputs-of (ConstantNode \ const) = []
inputs-of\ (ParameterNode\ index) = []
inputs-of\ (ValuePhiNode\ nid\ values\ merge) = merge\cdot values
inputs-of (AddNode\ x\ y) = [x,\ y]
inputs-of\ (IfNode\ condition\ trueSuccessor\ falseSuccessor) = [condition]
 typedef \ IRGraph = \{g :: ID \rightarrow IRNode \ . \ finite \ (dom \ g)\}
fun ids-fake :: (ID \rightarrow IRNode) \Rightarrow ID set where
  ids-fake g = \{nid \in dom \ g \ . \ g \ nid \neq (Some \ NoNode)\}
fun kind-fake :: (ID \rightarrow IRNode) \Rightarrow (ID \Rightarrow IRNode) where
  kind\text{-}fake\ g = (\lambda nid.\ (case\ g\ nid\ of\ None \Rightarrow NoNode\ |\ Some\ v \Rightarrow v))
```

```
ids-fake :: (nat \Rightarrow IRNode \ option) \Rightarrow nat \ set
ids-fake g = \{nid \in dom \ g \mid g \ nid \neq Some \ NoNode\}
kind\text{-}fake :: (nat \Rightarrow IRNode \ option) \Rightarrow nat \Rightarrow IRNode
kind-fake g = (\lambda nid. case g \ nid \ of \ None <math>\Rightarrow NoNode \ | \ Some \ v \Rightarrow v)
inputs :: IRGraph \Rightarrow nat \Rightarrow nat set
inputs \ g \ nid = set \ (inputs-of \ g \langle nid \rangle)
succ :: IRGraph \Rightarrow nat \Rightarrow nat set
succ\ g\ nid = set\ (successors-of\ g\langle\!\langle nid\rangle\!\rangle)
input\text{-}edges :: IRGraph \Rightarrow (nat \times nat) set
input\text{-}edges\ g = (\bigcup_{i \in ids\ g} \{(i, j) \mid j \in inputs\ g\ i\})
usages :: IRGraph \Rightarrow nat \Rightarrow nat set
usages\ g\ nid = \{j \in ids\ g \mid (j,\ nid) \in input\text{-}edges\ g\}
successor\text{-}edges :: IRGraph \Rightarrow (nat \times nat) set
successor\text{-}edges\ g = (\bigcup_{i \in ids\ g} \{(i,j) \mid j \in succ\ g\ i\})
predecessors :: IRGraph \Rightarrow nat \Rightarrow nat set
predecessors \ g \ nid = \{j \in \mathit{ids} \ g \mid (j, \ nid) \in \mathit{successor-edges} \ g\}
wf-start g =
(0 \in ids \ g \land is\text{-}StartNode \ g\langle\langle 0\rangle\rangle)
\textit{wf-closed}\ g =
(\forall n \in ids \ g.
     inputs g n \subseteq ids g \land
     succ \ g \ n \subseteq ids \ g \land g\langle\langle n \rangle\rangle \neq NoNode
```

```
wf-phis g =
(\forall n \in ids \ g.
     is-PhiNode g\langle n \rangle \longrightarrow
    |ir\text{-}values\ g\langle\langle n\rangle\rangle| =
     |ir\text{-}ends\ g\langle\langle ir\text{-}merge\ g\langle\langle n\rangle\rangle\rangle|)
wf-ends g =
(\forall n \in ids \ g.
     is-AbstractEndNode g\langle n \rangle \longrightarrow
     0 < |usages\ g\ n|)
wf-graph :: IRGraph \Rightarrow bool
\textit{wf-graph } g = (\textit{wf-start } g \land \textit{wf-closed } g \land \textit{wf-phis } g \land \textit{wf-ends } g)
type-synonym Signature = string
type-synonym Program = Signature 
ightharpoonup IRGraph
print-antiquotations
type-synonym Heap = string \Rightarrow objref \Rightarrow Value
type-synonym Free = nat
type-synonym DynamicHeap = Heap \times Free
h-load-field :: string \Rightarrow objref \Rightarrow DynamicHeap \Rightarrow Value
h-load-field f r (h, n) = h f r
\textit{h-store-field} :: \textit{string} \Rightarrow \textit{objref} \Rightarrow \textit{Value} \Rightarrow \textit{DynamicHeap} \Rightarrow \textit{DynamicHeap}
h-store-field f r v (h, n) = (h(f := (h f)(r := v)), n)
h\text{-}new\text{-}inst::DynamicHeap \Rightarrow (DynamicHeap \times Value)
h-new-inst (h, n) = ((h, n + 1), ObjRef (Some n))
```

eval:const eval:add eval:param eval:phi eval:invoke eval:invoke eval:load

step:seq step:if step:end step:newinst step:load step:store

top:lift top:invoke top:return top:unwind

$$\begin{split} g\langle\!\langle x\rangle\!\rangle &= ConstantNode\ c\text{-}1\\ \underline{g\langle\!\langle y\rangle\!\rangle} &= ConstantNode\ c\text{-}2 \qquad val = intval\text{-}add\ c\text{-}1\ c\text{-}2\\ \overline{CanonicalizeAdd\ g\ (AddNode\ x\ y)\ (ConstantNode\ val)} \\ \underline{g\langle\!\langle x\rangle\!\rangle} &= ConstantNode\ c\text{-}1 \qquad \neg\ is\text{-}ConstantNode\ g\langle\!\langle y\rangle\!\rangle \qquad c\text{-}1 = IntVal\ 32\ 0} \\ \overline{CanonicalizeAdd\ g\ (AddNode\ x\ y)\ (RefNode\ y)} \\ \underline{\neg\ is\text{-}ConstantNode\ g\langle\!\langle x\rangle\!\rangle \qquad g\langle\!\langle y\rangle\!\rangle = ConstantNode\ c\text{-}2 \qquad c\text{-}2 = IntVal\ 32\ 0} \\ \overline{CanonicalizeAdd\ g\ (AddNode\ x\ y)\ (RefNode\ x)} \end{split}$$

 $[CanonicalizeAdd\ g\ before\ after;\ wf-graph\ g \land wf-stamps\ g \land wf-values\ g;\ g\ m \vdash before \mapsto IntVal\ b\ res;\ g\ m \vdash after \mapsto IntVal\ b'\ res"] \Longrightarrow res = res'$ 

$$\frac{g \vdash (nid, \ m, \ h) \rightarrow (nid', \ m, \ h)}{g \ m \ h \vdash nid \leadsto nid'}$$

$$\frac{g \vdash (nid, \ m, \ h) \rightarrow (nid'', \ m, \ h) \qquad g \ m \ h \vdash nid'' \leadsto nid'}{g \ m \ h \vdash nid \leadsto nid'}$$

$$\frac{g \langle\!\langle cond \rangle\!\rangle = ConstantNode\ condv \qquad val\text{-}to\text{-}bool\ condv}{CanonicalizeIf\ g\ (IfNode\ cond\ tb\ fb)\ (RefNode\ tb)}$$
 
$$\frac{g \langle\!\langle cond \rangle\!\rangle = ConstantNode\ condv \qquad \neg\ val\text{-}to\text{-}bool\ condv}{CanonicalizeIf\ g\ (IfNode\ cond\ tb\ fb)\ (RefNode\ fb)}$$
 
$$\frac{\neg\ is\text{-}ConstantNode\ g \langle\!\langle cond \rangle\!\rangle \qquad tb\ =\ fb}{CanonicalizeIf\ g\ (IfNode\ cond\ tb\ fb)\ (RefNode\ tb)}$$

definition replace-node-fake ::  $ID \Rightarrow IRNode \Rightarrow IRGraph \Rightarrow IRGraph$  where replace-node-fake nid node g = replace-node nid (node,default-stamp) g lemma CanonicalizeIfProof-fake: fixes m::MapState and h::FieldRefHeap assumes kind g nid = before assumes CanonicalizeIf g before after assumes g' = replace-node-fake nid after g assumes  $g \vdash (nid, m, h) \rightarrow (nid', m, h)$  shows  $nid \mid g \sim g'$  sorry

```
\llbracket g \langle nid \rangle = before; Canonicalize If g before after;

g' = replace-node-fake \ nid \ after \ g; \ g \vdash (nid, \ m, \ h) \rightarrow (nid', \ m, \ h) \rrbracket

\implies nid \mid g \sim g'
```

```
notation (latex output)
filtered-inputs (inputs - ( - ) - )
notation (latex output)
filtered-successors (succ - ( - ) - )
notation (latex output)
filtered-usages (usages - ( - ) - )

inputs g ( nid ) f

notation (latex output)
Pure.dummy-pattern (-)

notation (latex output)
Int Val (Int Val (2 - ))
end
```