

Veriopt Theories

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1 Optization DSL

1.1 Markup

```
theory Markup
imports Semantics.IRTreeEval Snippets.Snipping
begin
```

```
datatype 'a Rewrite =
  Transform 'a 'a (-  $\mapsto$  - 10) |
  Conditional 'a 'a bool (-  $\mapsto$  - when - 11) |
  Sequential 'a Rewrite 'a Rewrite |
  Transitive 'a Rewrite
```

```
datatype 'a ExtraNotation =
  ConditionalNotation 'a 'a 'a (- ? - : - 50) |
  EqualsNotation 'a 'a (- eq -) |
  ConstantNotation 'a (const - 120) |
  TrueNotation (true) |
  FalseNotation (false) |
  ExclusiveOr 'a 'a (-  $\oplus$  -) |
  LogicNegationNotation 'a (!-) |
```

ShortCircuitOr 'a 'a (- || -)

definition *word* :: ('a::len) *word* \Rightarrow 'a *word* **where**
word *x* = *x*

ML-file *<markup.ML>*

1.1.1 Expression Markup

```
ML <
structure IRExpTranslator : DSL-TRANSLATION =
struct
fun markup DSL-Tokens.Add = @{term BinaryExpr} $ @{term BinAdd}
  | markup DSL-Tokens.Sub = @{term BinaryExpr} $ @{term BinSub}
  | markup DSL-Tokens.Mul = @{term BinaryExpr} $ @{term BinMul}
  | markup DSL-Tokens.And = @{term BinaryExpr} $ @{term BinAnd}
  | markup DSL-Tokens.Or = @{term BinaryExpr} $ @{term BinOr}
  | markup DSL-Tokens.Xor = @{term BinaryExpr} $ @{term BinXor}
  | markup DSL-Tokens.ShortCircuitOr = @{term BinaryExpr} $ @{term Bin-
ShortCircuitOr}
  | markup DSL-Tokens.Abs = @{term UnaryExpr} $ @{term UnaryAbs}
  | markup DSL-Tokens.Less = @{term BinaryExpr} $ @{term BinIntegerLessThan}
  | markup DSL-Tokens.Equals = @{term BinaryExpr} $ @{term BinIntegerEquals}
  | markup DSL-Tokens.Not = @{term UnaryExpr} $ @{term UnaryNot}
  | markup DSL-Tokens.Negate = @{term UnaryExpr} $ @{term UnaryNeg}
  | markup DSL-Tokens.LogicNegate = @{term UnaryExpr} $ @{term UnaryLog-
icNegation}
  | markup DSL-Tokens.LeftShift = @{term BinaryExpr} $ @{term BinLeftShift}
  | markup DSL-Tokens.RightShift = @{term BinaryExpr} $ @{term BinRight-
Shift}
  | markup DSL-Tokens.UnsignedRightShift = @{term BinaryExpr} $ @{term Bin-
URightShift}
  | markup DSL-Tokens.Conditional = @{term ConditionalExpr}
  | markup DSL-Tokens.Constant = @{term ConstantExpr}
  | markup DSL-Tokens.TrueConstant = @{term ConstantExpr (IntVal 32 1)}
  | markup DSL-Tokens.FalseConstant = @{term ConstantExpr (IntVal 32 0)}
end
structure IRExpMarkup = DSL-Markup(IRExpTranslator);
>
```

ir expression translation

```
syntax -expandExpr :: term  $\Rightarrow$  term (exp[-])
parse-translation < [( @{syntax-const -expandExpr} , IREx-
prMarkup.markup-expr []) ] >
```

ir expression example

value $\text{exp}[(e_1 < e_2) ? e_1 : e_2]$

ConditionalExpr (*BinaryExpr* *BinIntegerLessThan* ($e_1::\text{IRExpr}$)
($e_2::\text{IRExpr}$)) e_1 e_2

1.1.2 Value Markup

ML <

structure *IntValTranslator* : *DSL-TRANSLATION* =
struct

fun *markup* *DSL-Tokens.Add* = @{term *intval-add*}
| *markup* *DSL-Tokens.Sub* = @{term *intval-sub*}
| *markup* *DSL-Tokens.Mul* = @{term *intval-mul*}
| *markup* *DSL-Tokens.And* = @{term *intval-and*}
| *markup* *DSL-Tokens.Or* = @{term *intval-or*}
| *markup* *DSL-Tokens.ShortCircuitOr* = @{term *intval-short-circuit-or*}
| *markup* *DSL-Tokens.Xor* = @{term *intval-xor*}
| *markup* *DSL-Tokens.Abs* = @{term *intval-abs*}
| *markup* *DSL-Tokens.Less* = @{term *intval-less-than*}
| *markup* *DSL-Tokens.Equals* = @{term *intval-equals*}
| *markup* *DSL-Tokens.Not* = @{term *intval-not*}
| *markup* *DSL-Tokens.Negate* = @{term *intval-negate*}
| *markup* *DSL-Tokens.LogicNegate* = @{term *intval-logic-negation*}
| *markup* *DSL-Tokens.LeftShift* = @{term *intval-left-shift*}
| *markup* *DSL-Tokens.RightShift* = @{term *intval-right-shift*}
| *markup* *DSL-Tokens.UnsignedRightShift* = @{term *intval-uright-shift*}
| *markup* *DSL-Tokens.Conditional* = @{term *intval-conditional*}
| *markup* *DSL-Tokens.Constant* = @{term *IntVal* 32}
| *markup* *DSL-Tokens.TrueConstant* = @{term *IntVal* 32 1}
| *markup* *DSL-Tokens.FalseConstant* = @{term *IntVal* 32 0}

end

structure *IntValMarkup* = *DSL-Markup*(*IntValTranslator*);

>

value expression translation

syntax *-expandIntVal* :: *term* \Rightarrow *term* (*val*[-])

parse-translation < [(@{syntax-const *-expandIntVal*} , *IntVal-Markup.markup-expr* [])] >

value expression example

value $\text{val}[(e_1 < e_2) ? e_1 : e_2]$

intval-conditional (*intval-less-than* ($e_1::\text{Value}$) ($e_2::\text{Value}$)) e_1 e_2

1.1.3 Word Markup

ML \langle

```
structure WordTranslator : DSL-TRANSLATION =
struct
fun markup DSL-Tokens.Add = @{term plus}
| markup DSL-Tokens.Sub = @{term minus}
| markup DSL-Tokens.Mul = @{term times}
| markup DSL-Tokens.And = @{term Bit-Operations.semiring-bit-operations-class.and}
| markup DSL-Tokens.Or = @{term or}
| markup DSL-Tokens.Xor = @{term xor}
| markup DSL-Tokens.Abs = @{term abs}
| markup DSL-Tokens.Less = @{term less}
| markup DSL-Tokens.Equals = @{term HOL.eq}
| markup DSL-Tokens.Not = @{term not}
| markup DSL-Tokens.Negate = @{term uminus}
| markup DSL-Tokens.LogicNegate = @{term logic-negate}
| markup DSL-Tokens.LeftShift = @{term shiftl}
| markup DSL-Tokens.RightShift = @{term signed-shiftr}
| markup DSL-Tokens.UnsignedRightShift = @{term shiftr}
| markup DSL-Tokens.Constant = @{term word}
| markup DSL-Tokens.TrueConstant = @{term 1}
| markup DSL-Tokens.FalseConstant = @{term 0}
end
structure WordMarkup = DSL-Markup(WordTranslator);
 $\rangle$ 
```

word expression translation

syntax $\text{-expandWord} :: \text{term} \Rightarrow \text{term} \ (\text{bin}[-])$
parse-translation $\langle \ [(\ @\{\text{syntax-const} \ \text{-expandWord}\} \ , \ \text{Word-Markup.markup-expr} \ [])] \rangle$

word expression example

value $\text{bin}[x \ \& \ y \ | \ z]$
intval-conditional $(\text{intval-less-than} \ (e_1 :: \text{Value}) \ (e_2 :: \text{Value})) \ e_1 \ e_2$

value $\text{bin}[\neg x]$
value $\text{val}[\neg x]$
value $\text{exp}[\neg x]$

value $\text{bin}[^!x]$
value $\text{val}[^!x]$
value $\text{exp}[^!x]$

value $\text{bin}[\neg x]$
value $\text{val}[\neg x]$
value $\text{exp}[\neg x]$

```

value bin[ $\sim x$ ]
value val[ $\sim x$ ]
value exp[ $\sim x$ ]

```

```

value  $\sim x$ 

```

```

end

```

1.2 Optimization Phases

```

theory Phase
  imports Main
begin

```

```

ML-file map.ML
ML-file phase.ML

```

```

end

```

1.3 Canonicalization DSL

```

theory Canonicalization
  imports
    Markup
    Phase
    HOL-Eisbach.Eisbach
  keywords
    phase :: thy-decl and
    terminating :: quasi-command and
    print-phases :: diag and
    export-phases :: thy-decl and
    optimization :: thy-goal-defn
begin

```

```

print-methods

```

```

ML <
datatype 'a Rewrite =
  Transform of 'a * 'a |
  Conditional of 'a * 'a * term |
  Sequential of 'a Rewrite * 'a Rewrite |
  Transitive of 'a Rewrite

```

```

type rewrite = {
  name: binding,
  rewrite: term Rewrite,
  proofs: thm list,
  code: thm list,
  source: term

```

```

}

structure RewriteRule : Rule =
struct
type T = rewrite;

(*
fun pretty-rewrite ctxt (Transform (from, to)) =
  Pretty.block [
    Syntax.pretty-term ctxt from,
    Pretty.str  $\mapsto$  ,
    Syntax.pretty-term ctxt to
  ]
| pretty-rewrite ctxt (Conditional (from, to, cond)) =
  Pretty.block [
    Syntax.pretty-term ctxt from,
    Pretty.str  $\mapsto$  ,
    Syntax.pretty-term ctxt to,
    Pretty.str when ,
    Syntax.pretty-term ctxt cond
  ]
| pretty-rewrite - - = Pretty.str not implemented*)

fun pretty-thm ctxt thm =
  (Proof-Context.pretty-fact ctxt (, [thm]))

fun pretty ctxt obligations t =
  let
    val is-skipped = Thm-Deps.has-skip-proof (#proofs t);

    val warning = (if is-skipped
      then [Pretty.str (proof skipped), Pretty.brk 0]
      else []);

    val obligations = (if obligations
      then [Pretty.big-list
        obligations:
        (map (pretty-thm ctxt) (#proofs t)),
        Pretty.brk 0]
      else []);

  fun pretty-bind binding =
    Pretty.markup
      (Position.markup (Binding.pos-of binding) Markup.position)
      [Pretty.str (Binding.name-of binding)];

  in
    Pretty.block ([
      pretty-bind (#name t), Pretty.str : ,

```

```

    Syntax.pretty-term ctxt (#source t), Pretty.fbrk
  ] @ obligations @ warning)
end
end

structure RewritePhase = DSL-Phase(RewriteRule);

val - =
  Outer-Syntax.command command-keyword <phase> enter an optimization phase
  (Parse.binding --| Parse.$$$ terminating -- Parse.const --| Parse.begin
   >> (Toplevel.begin-main-target true o RewritePhase.setup));

fun print-phases print-obligations ctxt =
  let
    val thy = Proof-Context.theory-of ctxt;
    fun print phase = RewritePhase.pretty print-obligations phase ctxt
  in
    map print (RewritePhase.phases thy)
  end

fun print-optimizations print-obligations thy =
  print-phases print-obligations thy |> Pretty.writeln-chunks

val - =
  Outer-Syntax.command command-keyword <print-phases>
  print debug information for optimizations
  (Parse.opt-bang >>
   (fn b => Toplevel.keep ((print-optimizations b) o Toplevel.context-of)));

fun export-phases thy name =
  let
    val state = Toplevel.theory-toplevel thy;
    val ctxt = Toplevel.context-of state;
    val content = Pretty.string-of (Pretty.chunks (print-phases false ctxt));
    val cleaned = YXML.content-of content;

    val filename = Path.explode (name^.rules);
    val directory = Path.explode optimizations;
    val path = Path.binding (
      Path.append directory filename,
      Position.none);
    val thy' = thy |> Generated-Files.add-files (path, (Bytes.string content));

    val - = Export.export thy' path [YXML.parse cleaned];

    val - = writeln (Export.message thy' (Path.basic optimizations));
  in
    thy'
  end

```

```

end

val - =
  Outer-Syntax.command command-keyword <export-phases>
    export information about encoded optimizations
    (Parse.path >>
      (fn name => Toplevel.theory (fn state => export-phases state name)))
  >

```

ML-file *rewrites.ML*

1.3.1 Semantic Preservation Obligation

```

fun rewrite-preservation :: IRExp Rewrite  $\Rightarrow$  bool where
  rewrite-preservation (Transform x y) = (y  $\leq$  x) |
  rewrite-preservation (Conditional x y cond) = (cond  $\longrightarrow$  (y  $\leq$  x)) |
  rewrite-preservation (Sequential x y) = (rewrite-preservation x  $\wedge$  rewrite-preservation
y) |
  rewrite-preservation (Transitive x) = rewrite-preservation x

```

1.3.2 Termination Obligation

```

fun rewrite-termination :: IRExp Rewrite  $\Rightarrow$  (IRExp  $\Rightarrow$  nat)  $\Rightarrow$  bool where
  rewrite-termination (Transform x y) trm = (trm x > trm y) |
  rewrite-termination (Conditional x y cond) trm = (cond  $\longrightarrow$  (trm x > trm y)) |
  rewrite-termination (Sequential x y) trm = (rewrite-termination x trm  $\wedge$  rewrite-termination
y trm) |
  rewrite-termination (Transitive x) trm = rewrite-termination x trm

```

```

fun intval :: Value Rewrite  $\Rightarrow$  bool where
  intval (Transform x y) = (x  $\neq$  UndefVal  $\wedge$  y  $\neq$  UndefVal  $\longrightarrow$  x = y) |
  intval (Conditional x y cond) = (cond  $\longrightarrow$  (x = y)) |
  intval (Sequential x y) = (intval x  $\wedge$  intval y) |
  intval (Transitive x) = intval x

```

1.3.3 Standard Termination Measure

```

fun size :: IRExp  $\Rightarrow$  nat where
  unary-size:
  size (UnaryExpr op x) = (size x) + 2 |

  bin-const-size:
  size (BinaryExpr op x (ConstantExpr cy)) = (size x) + 2 |
  bin-size:
  size (BinaryExpr op x y) = (size x) + (size y) + 2 |
  cond-size:
  size (ConditionalExpr c t f) = (size c) + (size t) + (size f) + 2 |
  const-size:
  size (ConstantExpr c) = 1 |
  param-size:

```



```

size (ParameterExpr ind s) = 2 |
leaf-size:
size (LeafExpr nid s) = 2 |
size (ConstantVar c) = 2 |
size (VariableExpr x s) = 2

```

1.3.4 Automated Tactics

named-theorems *size-simps size simplification rules*

```

method unfold-optimization =
  (unfold rewrite-preservation.simps, unfold rewrite-termination.simps,
   unfold intval.simps,
   rule conjE, simp, simp del: le-expr-def, force?)
| (unfold rewrite-preservation.simps, unfold rewrite-termination.simps,
   rule conjE, simp, simp del: le-expr-def, force?)

```

```

method unfold-size =
  (((unfold size.simps, simp add: size-simps del: le-expr-def)?
   ; (simp add: size-simps del: le-expr-def)?
   ; (auto simp: size-simps)?
   ; (unfold size.simps)?)[1])

```

print-methods

```

ML <
  structure System : RewriteSystem =
  struct
    val preservation = @{const rewrite-preservation};
    val termination = @{const rewrite-termination};
    val intval = @{const intval};
  end

  structure DSL = DSL-Rewrites(System);

  val - =
    Outer-Syntax.local-theory-to-proof command-keyword <optimization>
    define an optimization and open proof obligation
    (Parse-Spec.thm-name : -- Parse.term
     >> DSL.rewrite-cmd);
>

```

end