

Veriopt Theories

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1 Operator Semantics

```
theory Values
  imports
    HOL-Library.Word
```

HOL-Library.Signed-Division
HOL-Library.Float
HOL-Library.LaTeXsugar

begin

In order to properly implement the IR semantics we first introduce a type that represents runtime values. These runtime values represent the full range of primitive types currently allowed by our semantics, ranging from basic integer types to object references and arrays.

Note that Java supports 64, 32, 16, 8 signed ints, plus 1 bit (boolean) ints, and char is 16-bit unsigned. E.g. an 8-bit stamp has a default range of -128..+127. And a 1-bit stamp has a default range of -1..0, surprisingly.

During calculations the smaller sizes are sign-extended to 32 bits, but explicit widening nodes will do that, so most binary calculations should see equal input sizes.

An object reference is an option type where the *None* object reference points to the static fields. This is examined more closely in our definition of the heap.

type-synonym *int64* = 64 word — long
type-synonym *int32* = 32 word — int
type-synonym *int16* = 16 word — short
type-synonym *int8* = 8 word — char
type-synonym *int1* = 1 word — boolean

abbreviation *valid-int-widths* :: nat set **where**
valid-int-widths ≡ {1, 8, 16, 32, 64}

experiment begin

Option 2: explicit width stored with each integer value. However, this does not help us to distinguish between short (signed) and char (unsigned).

typedef *IntWidth* = { *w* :: nat . *w*=1 ∨ *w*=8 ∨ *w*=16 ∨ *w*=32 ∨ *w*=64 }
by *blast*

setup-lifting *type-definition-IntWidth*

lift-definition *IntWidthBits* :: *IntWidth* ⇒ nat
is λ*w*. *w* .
end

experiment begin

Option 3: explicit type stored with each integer value.

datatype *IntType* = *ILong* | *IInt* | *IShort* | *IChar* | *IByte* | *IBoolean*

```

fun int-bits :: IntType  $\Rightarrow$  nat where
  int-bits ILong = 64 |
  int-bits IInt  = 32 |
  int-bits IShort = 16 |
  int-bits IChar  = 16 |
  int-bits IByte  = 8  |
  int-bits IBoolean = 1

```

```

fun int-signed :: IntType  $\Rightarrow$  bool where
  int-signed ILong = True |
  int-signed IInt  = True |
  int-signed IShort = True |
  int-signed IChar  = False |
  int-signed IByte  = True |
  int-signed IBoolean = True
end

```

Option 4: int64 with the number of significant bits.

```

type-synonym iwidth = nat
type-synonym objref = nat option

```

```

datatype (discs-sels) Value =
  UndefVal |

```

```

  IntVal iwidth int64 |

```

```

  ObjRef objref |
  ObjStr string

```

```

fun intval-bits :: Value  $\Rightarrow$  nat where
  intval-bits (IntVal b v) = b

```

```

fun intval-word :: Value  $\Rightarrow$  int64 where
  intval-word (IntVal b v) = v

```

```

fun bit-bounds :: nat  $\Rightarrow$  (int  $\times$  int) where
  bit-bounds bits = (((2  $\wedge$  bits) div 2) * -1, ((2  $\wedge$  bits) div 2) - 1)

```

```

definition logic-negate :: ('a::len) word  $\Rightarrow$  'a word where
  logic-negate x = (if x = 0 then 1 else 0)

```

```

fun int-signed-value :: iwidth  $\Rightarrow$  int64  $\Rightarrow$  int where
  int-signed-value b v = sint (signed-take-bit (b - 1) v)

```

```
fun int-unsigned-value :: iwidth  $\Rightarrow$  int64  $\Rightarrow$  int where
  int-unsigned-value b v = uint v
```

Converts an integer word into a Java value.

```
fun new-int :: iwidth  $\Rightarrow$  int64  $\Rightarrow$  Value where
  new-int b w = IntVal b (take-bit b w)
```

Converts an integer word into a Java value, iff the two types are equal.

```
fun new-int-bin :: iwidth  $\Rightarrow$  iwidth  $\Rightarrow$  int64  $\Rightarrow$  Value where
  new-int-bin b1 b2 w = (if b1=b2 then new-int b1 w else UndefVal)
```

```
fun wf-bool :: Value  $\Rightarrow$  bool where
  wf-bool (IntVal b w) = (b = 1) |
  wf-bool - = False
```

```
fun val-to-bool :: Value  $\Rightarrow$  bool where
  val-to-bool (IntVal b val) = (if val = 0 then False else True) |
  val-to-bool val = False
```

```
fun bool-to-val :: bool  $\Rightarrow$  Value where
  bool-to-val True = (IntVal 32 1) |
  bool-to-val False = (IntVal 32 0)
```

Converts an Isabelle bool into a Java value, iff the two types are equal.

```
fun bool-to-val-bin :: iwidth  $\Rightarrow$  iwidth  $\Rightarrow$  bool  $\Rightarrow$  Value where
  bool-to-val-bin t1 t2 b = (if t1 = t2 then bool-to-val b else UndefVal)
```

```
fun is-int-val :: Value  $\Rightarrow$  bool where
  is-int-val v = is-IntVal v
```

A convenience function for directly constructing -1 values of a given bit size.

```
fun neg-one :: iwidth  $\Rightarrow$  int64 where
  neg-one b = mask b
```

```
lemma neg-one-value[simp]: new-int b (neg-one b) = IntVal b (mask b)
by simp
```

```
lemma neg-one-signed[simp]:
  assumes 0 < b
  shows int-signed-value b (neg-one b) = -1
  by (smt (verit, best) assms diff-le-self diff-less int-signed-value.simps less-one
    mask-eq-take-bit-minus-one neg-one.simps nle-le signed-minus-1 signed-take-bit-of-minus-1
    signed-take-bit-take-bit verit-comp-simplify1 (1))
```

1.1 Arithmetic Operators

We need to introduce arithmetic operations which agree with the JVM.

Within the JVM, bytecode arithmetic operations are performed on 32 or 64 bit integers, unboxing where appropriate.

The following collection of *intval* functions correspond to the JVM arithmetic operations. We merge the 32 and 64 bit operations into a single function, even though the stamp of each *IRNode* tells us exactly what the bit widths will be. These merged functions make it easier to do the instantiation of *Value* as 'plus', etc. It might be worse for reasoning, because it could cause more case analysis, but this does not seem to be a problem in practice.

```
fun intval-add :: Value  $\Rightarrow$  Value  $\Rightarrow$  Value where  
  intval-add (IntVal b1 v1) (IntVal b2 v2) =  
    (if b1 = b2 then IntVal b1 (take-bit b1 (v1+v2)) else UndefVal) |  
  intval-add - - = UndefVal
```

```
fun intval-sub :: Value  $\Rightarrow$  Value  $\Rightarrow$  Value where  
  intval-sub (IntVal b1 v1) (IntVal b2 v2) = new-int-bin b1 b2 (v1-v2) |  
  intval-sub - - = UndefVal
```

```
fun intval-mul :: Value  $\Rightarrow$  Value  $\Rightarrow$  Value where  
  intval-mul (IntVal b1 v1) (IntVal b2 v2) = new-int-bin b1 b2 (v1*v2) |  
  intval-mul - - = UndefVal
```

```
fun intval-div :: Value  $\Rightarrow$  Value  $\Rightarrow$  Value where  
  intval-div (IntVal b1 v1) (IntVal b2 v2) =  
    new-int-bin b1 b2 (word-of-int  
      ((int-signed-value b1 v1) sdiv (int-signed-value b2 v2))) |  
  intval-div - - = UndefVal
```

```
fun intval-mod :: Value  $\Rightarrow$  Value  $\Rightarrow$  Value where  
  intval-mod (IntVal b1 v1) (IntVal b2 v2) =  
    new-int-bin b1 b2 (word-of-int  
      ((int-signed-value b1 v1) smod (int-signed-value b2 v2))) |  
  intval-mod - - = UndefVal
```

```
fun intval-negate :: Value  $\Rightarrow$  Value where  
  intval-negate (IntVal t v) = new-int t (- v) |  
  intval-negate - = UndefVal
```

```
fun intval-abs :: Value  $\Rightarrow$  Value where
```

```

intval-abs (IntVal t v) = new-int t (if int-signed-value t v < 0 then - v else v) |
intval-abs - = UndefVal

```

TODO: clarify which widths this should work on: just 1-bit or all?

```

fun intval-logic-negation :: Value ⇒ Value where
  intval-logic-negation (IntVal b v) = new-int b (logic-negate v) |
  intval-logic-negation - = UndefVal

```

1.2 Bitwise Operators

```

fun intval-and :: Value ⇒ Value ⇒ Value where
  intval-and (IntVal b1 v1) (IntVal b2 v2) = new-int-bin b1 b2 (and v1 v2) |
  intval-and - - = UndefVal

```

```

fun intval-or :: Value ⇒ Value ⇒ Value where
  intval-or (IntVal b1 v1) (IntVal b2 v2) = new-int-bin b1 b2 (or v1 v2) |
  intval-or - - = UndefVal

```

```

fun intval-xor :: Value ⇒ Value ⇒ Value where
  intval-xor (IntVal b1 v1) (IntVal b2 v2) = new-int-bin b1 b2 (xor v1 v2) |
  intval-xor - - = UndefVal

```

```

fun intval-not :: Value ⇒ Value where
  intval-not (IntVal t v) = new-int t (not v) |
  intval-not - = UndefVal

```

1.3 Comparison Operators

```

fun intval-short-circuit-or :: Value ⇒ Value ⇒ Value where
  intval-short-circuit-or (IntVal b1 v1) (IntVal b2 v2) = bool-to-val-bin b1 b2 (((v1
  ≠ 0) ∨ (v2 ≠ 0))) |
  intval-short-circuit-or - - = UndefVal

```

```

fun intval-equals :: Value ⇒ Value ⇒ Value where
  intval-equals (IntVal b1 v1) (IntVal b2 v2) = bool-to-val-bin b1 b2 (v1 = v2) |
  intval-equals - - = UndefVal

```

```

fun intval-less-than :: Value ⇒ Value ⇒ Value where
  intval-less-than (IntVal b1 v1) (IntVal b2 v2) =
    bool-to-val-bin b1 b2 (int-signed-value b1 v1 < int-signed-value b2 v2) |
  intval-less-than - - = UndefVal

```

```

fun intval-below :: Value ⇒ Value ⇒ Value where
  intval-below (IntVal b1 v1) (IntVal b2 v2) = bool-to-val-bin b1 b2 (v1 < v2) |
  intval-below - - = UndefVal

```

```

fun intval-conditional :: Value ⇒ Value ⇒ Value ⇒ Value where
  intval-conditional cond tv fv = (if (val-to-bool cond) then tv else fv)

```

1.4 Narrowing and Widening Operators

Note: we allow these operators to have $\text{inBits} = \text{outBits}$, because the Graal compiler also seems to allow that case, even though it should rarely / never arise in practice.

```
value sint(signed-take-bit 0 (1 :: int32))
```

```
fun intval-narrow :: nat  $\Rightarrow$  nat  $\Rightarrow$  Value  $\Rightarrow$  Value where
  intval-narrow inBits outBits (IntVal b v) =
    (if inBits = b  $\wedge$  0 < outBits  $\wedge$  outBits  $\leq$  inBits  $\wedge$  inBits  $\leq$  64
     then new-int outBits v
     else UndefVal) |
  intval-narrow - - - = UndefVal
```

```
value sint (signed-take-bit 7 ((256 + 128) :: int64))
```

```
fun intval-sign-extend :: nat  $\Rightarrow$  nat  $\Rightarrow$  Value  $\Rightarrow$  Value where
  intval-sign-extend inBits outBits (IntVal b v) =
    (if inBits = b  $\wedge$  0 < inBits  $\wedge$  inBits  $\leq$  outBits  $\wedge$  outBits  $\leq$  64
     then new-int outBits (signed-take-bit (inBits - 1) v)
     else UndefVal) |
  intval-sign-extend - - - = UndefVal
```

```
fun intval-zero-extend :: nat  $\Rightarrow$  nat  $\Rightarrow$  Value  $\Rightarrow$  Value where
  intval-zero-extend inBits outBits (IntVal b v) =
    (if inBits = b  $\wedge$  0 < inBits  $\wedge$  inBits  $\leq$  outBits  $\wedge$  outBits  $\leq$  64
     then new-int outBits (take-bit inBits v)
     else UndefVal) |
  intval-zero-extend - - - = UndefVal
```

Some well-formedness results to help reasoning about narrowing and widening operators

lemma *intval-narrow-ok*:

```
assumes intval-narrow inBits outBits val  $\neq$  UndefVal
shows 0 < outBits  $\wedge$  outBits  $\leq$  inBits  $\wedge$  inBits  $\leq$  64  $\wedge$  outBits  $\leq$  64  $\wedge$ 
  is-IntVal val  $\wedge$ 
  intval-bits val = inBits
using assms intval-narrow.simps neq0-conv intval-bits.simps
by (metis Value.disc(2) intval-narrow.elims le-trans)
```

lemma *intval-sign-extend-ok*:

```
assumes intval-sign-extend inBits outBits val  $\neq$  UndefVal
shows 0 < inBits  $\wedge$ 
  inBits  $\leq$  outBits  $\wedge$  outBits  $\leq$  64  $\wedge$ 
  is-IntVal val  $\wedge$ 
  intval-bits val = inBits
using assms intval-sign-extend.simps neq0-conv
by (metis intval-bits.simps intval-sign-extend.elims is-IntVal-def)
```

```

lemma intval-zero-extend-ok:
  assumes intval-zero-extend inBits outBits val  $\neq$  UndefVal
  shows  $0 < \text{inBits} \wedge$ 
     $\text{inBits} \leq \text{outBits} \wedge \text{outBits} \leq 64 \wedge$ 
     $\text{is-IntVal } \text{val} \wedge$ 
     $\text{intval-bits } \text{val} = \text{inBits}$ 
  using assms intval-sign-extend.simps neq0-conv
  by (metis intval-bits.simps intval-zero-extend.elims is-IntVal-def)

```

1.5 Bit-Shifting Operators

```

definition shiffl (infix << 75) where
  shiffl w n = (push-bit n) w

```

```

lemma shiffl-power[simp]:  $(x::('a::\text{len}) \text{ word}) * (2^j) = x << j$ 
  unfolding shiffl-def apply (induction j)
  apply simp unfolding funpow-Suc-right
  by (metis (no-types, opaque-lifting) push-bit-eq-mult)

```

```

lemma  $(x::('a::\text{len}) \text{ word}) * ((2^j) + 1) = x << j + x$ 
  by (simp add: distrib-left)

```

```

lemma  $(x::('a::\text{len}) \text{ word}) * ((2^j) - 1) = x << j - x$ 
  by (simp add: right-diff-distrib)

```

```

lemma  $(x::('a::\text{len}) \text{ word}) * ((2^j) + (2^k)) = x << j + x << k$ 
  by (simp add: distrib-left)

```

```

lemma  $(x::('a::\text{len}) \text{ word}) * ((2^j) - (2^k)) = x << j - x << k$ 
  by (simp add: right-diff-distrib)

```

```

definition shiftr (infix >>> 75) where
  shiftr w n = (drop-bit n) w

```

```

value (255 :: 8 word) >>> (2 :: nat)

```

```

definition sshiftr :: 'a :: len word  $\Rightarrow$  nat  $\Rightarrow$  'a :: len word (infix >> 75) where
  sshiftr w n = word-of-int ((sint w) div (2^n))

```

```

value (128 :: 8 word) >> 2

```

Note that Java shift operators use unary numeric promotion, unlike other binary operators, which use binary numeric promotion (see the Java language reference manual). This means that the left-hand input determines the output size, while the right-hand input can be any size.

```

fun shift-amount :: iwidth  $\Rightarrow$  int64  $\Rightarrow$  nat where
  shift-amount b val = unat (and val (if b = 64 then 0x3F else 0x1f))

```



```

fun intval-left-shift :: Value ⇒ Value ⇒ Value where
  intval-left-shift (IntVal b1 v1) (IntVal b2 v2) = new-int b1 (v1 << shift-amount
b1 v2) |
  intval-left-shift - - =.UndefVal

```

Signed shift is more complex, because we sometimes have to insert 1 bits at the correct point, which is at b1 bits.

```

fun intval-right-shift :: Value ⇒ Value ⇒ Value where
  intval-right-shift (IntVal b1 v1) (IntVal b2 v2) =
    (let shift = shift-amount b1 v2 in
     let ones = and (mask b1) (not (mask (b1 - shift) :: int64)) in
     (if int-signed-value b1 v1 < 0
      then new-int b1 (or ones (v1 >>> shift))
      else new-int b1 (v1 >>> shift))) |
  intval-right-shift - - =.UndefVal

```

```

fun intval-uright-shift :: Value ⇒ Value ⇒ Value where
  intval-uright-shift (IntVal b1 v1) (IntVal b2 v2) = new-int b1 (v1 >>> shift-amount
b1 v2) |
  intval-uright-shift - - =.UndefVal

```

1.5.1 Examples of Narrowing / Widening Functions

experiment begin

corollary *intval-narrow 32 8 (IntVal 32 (256 + 128)) = IntVal 8 128 by simp*

corollary *intval-narrow 32 8 (IntVal 32 (-2)) = IntVal 8 254 by simp*

corollary *intval-narrow 32 1 (IntVal 32 (-2)) = IntVal 1 0 by simp*

corollary *intval-narrow 32 1 (IntVal 32 (-3)) = IntVal 1 1 by simp*

corollary *intval-narrow 32 8 (IntVal 64 (-2)) =.UndefVal by simp*

corollary *intval-narrow 64 8 (IntVal 32 (-2)) =.UndefVal by simp*

corollary *intval-narrow 64 8 (IntVal 64 254) = IntVal 8 254 by simp*

corollary *intval-narrow 64 8 (IntVal 64 (256+127)) = IntVal 8 127 by simp*

corollary *intval-narrow 64 64 (IntVal 64 (-2)) = IntVal 64 (-2) by simp*

end

experiment begin

corollary *intval-sign-extend 8 32 (IntVal 8 (256 + 128)) = IntVal 32 (2³² - 128) by simp*

corollary *intval-sign-extend 8 32 (IntVal 8 (-2)) = IntVal 32 (2³² - 2) by simp*

corollary *intval-sign-extend 1 32 (IntVal 1 (-2)) = IntVal 32 0 by simp*

corollary *intval-sign-extend 1 32 (IntVal 1 (-3)) = IntVal 32 (mask 32) by simp*

corollary *intval-sign-extend 8 32 (IntVal 64 254) =.UndefVal by simp*

corollary *intval-sign-extend 8 64 (IntVal 32 254) =.UndefVal by simp*

corollary *intval-sign-extend* 8 64 (*IntVal* 8 254) = *IntVal* 64 (-2) **by** *simp*
corollary *intval-sign-extend* 32 64 (*IntVal* 32 ($2^{32} - 2$)) = *IntVal* 64 (-2) **by** *simp*
corollary *intval-sign-extend* 64 64 (*IntVal* 64 (-2)) = *IntVal* 64 (-2) **by** *simp*
end

experiment begin

corollary *intval-zero-extend* 8 32 (*IntVal* 8 (256 + 128)) = *IntVal* 32 128 **by** *simp*
corollary *intval-zero-extend* 8 32 (*IntVal* 8 (-2)) = *IntVal* 32 254 **by** *simp*
corollary *intval-zero-extend* 1 32 (*IntVal* 1 (-1)) = *IntVal* 32 1 **by** *simp*
corollary *intval-zero-extend* 1 32 (*IntVal* 1 (-2)) = *IntVal* 32 0 **by** *simp*

corollary *intval-zero-extend* 8 32 (*IntVal* 64 (-2)) = *UndefVal* **by** *simp*
corollary *intval-zero-extend* 8 64 (*IntVal* 64 (-2)) = *UndefVal* **by** *simp*
corollary *intval-zero-extend* 8 64 (*IntVal* 8 254) = *IntVal* 64 254 **by** *simp*
corollary *intval-zero-extend* 32 64 (*IntVal* 32 ($2^{32} - 2$)) = *IntVal* 64 ($2^{32} - 2$) **by** *simp*
corollary *intval-zero-extend* 64 64 (*IntVal* 64 (-2)) = *IntVal* 64 (-2) **by** *simp*
end

experiment begin

corollary *intval-right-shift* (*IntVal* 8 128) (*IntVal* 8 0) = *IntVal* 8 128 **by** *eval*
corollary *intval-right-shift* (*IntVal* 8 128) (*IntVal* 8 1) = *IntVal* 8 192 **by** *eval*
corollary *intval-right-shift* (*IntVal* 8 128) (*IntVal* 8 2) = *IntVal* 8 224 **by** *eval*
corollary *intval-right-shift* (*IntVal* 8 128) (*IntVal* 8 8) = *IntVal* 8 255 **by** *eval*
corollary *intval-right-shift* (*IntVal* 8 128) (*IntVal* 8 31) = *IntVal* 8 255 **by** *eval*
end

lemma *intval-add-sym*:

shows *intval-add* a b = *intval-add* b a
by (*induction* a; *induction* b; *auto simp: add commute*)

code-deps *intval-add*

code-thms *intval-add*

lemma *intval-add* (*IntVal* 32 ($2^{31} - 1$)) (*IntVal* 32 ($2^{31} - 1$)) = *IntVal* 32 ($2^{32} - 2$)
by *eval*
lemma *intval-add* (*IntVal* 64 ($2^{31} - 1$)) (*IntVal* 64 ($2^{31} - 1$)) = *IntVal* 64 4294967294
by *eval*

end

1.6 Fixed-width Word Theories

```
theory ValueThms
  imports Values
begin
```

1.6.1 Support Lemmas for Upper/Lower Bounds

```
lemma size32: size v = 32 for v :: 32 word
  using size-word.rep-eq
  using One-nat-def add.right-neutral add-Suc-right len-of-numeral-defs(2) len-of-numeral-defs(3)
  mult.right-neutral mult-Suc-right numeral-2-eq-2 numeral-Bit0
  by (smt (verit, del-insts) mult.commute)
```

```
lemma size64: size v = 64 for v :: 64 word
  using size-word.rep-eq
  using One-nat-def add.right-neutral add-Suc-right len-of-numeral-defs(2) len-of-numeral-defs(3)
  mult.right-neutral mult-Suc-right numeral-2-eq-2 numeral-Bit0
  by (smt (verit, del-insts) mult.commute)
```

```
lemma lower-bounds-equiv:
  assumes 0 < N
  shows  $\neg((2::int) \wedge (N-1)) = (2::int) \wedge N \text{ div } 2 * -1$ 
  by (simp add: assms int-power-div-base)
```

```
lemma upper-bounds-equiv:
  assumes 0 < N
  shows  $(2::int) \wedge (N-1) = (2::int) \wedge N \text{ div } 2$ 
  by (simp add: assms int-power-div-base)
```

Some min/max bounds for 64-bit words

```
lemma bit-bounds-min64: ((fst (bit-bounds 64))) ≤ (sint (v::int64))
  unfolding bit-bounds.simps fst-def
  using sint-ge[of v] by simp
```

```
lemma bit-bounds-max64: ((snd (bit-bounds 64))) ≥ (sint (v::int64))
  unfolding bit-bounds.simps fst-def
  using sint-lt[of v] by simp
```

Extend these min/max bounds to extracting smaller signed words using `signed_take_bit`.

Note: we could use `signed` to convert between bit-widths, instead of `signed_take_bit`. But that would have to be done separately for each bit-width type.

value *sint*(*signed-take-bit* 7 (128 :: int8))

ML-val $\langle @\{thm\ signed-take-bit-decr-length-iff\} \rangle$

declare [[*show-types=true*]]

ML-val $\langle @\{thm\ signed-take-bit-int-less-exp\} \rangle$

lemma *signed-take-bit-int-less-exp-word*:

fixes *ival* :: 'a :: len word

assumes $n < LENGTH('a)$

shows $sint(signed-take-bit\ n\ ival) < (2::int) ^ n$

apply *transfer*

by (*smt* (*verit*, *best*) *not-take-bit-negative signed-take-bit-eq-take-bit-shift signed-take-bit-int-less-exp take-bit-int-greater-self-iff*)

lemma *signed-take-bit-int-greater-eq-minus-exp-word*:

fixes *ival* :: 'a :: len word

assumes $n < LENGTH('a)$

shows $-(2 ^ n) \leq sint(signed-take-bit\ n\ ival)$

apply *transfer*

by (*smt* (*verit*, *best*) *signed-take-bit-int-greater-eq-minus-exp signed-take-bit-int-greater-eq-self-iff signed-take-bit-int-less-exp*)

lemma *signed-take-bit-range*:

fixes *ival* :: 'a :: len word

assumes $n < LENGTH('a)$

assumes $val = sint(signed-take-bit\ n\ ival)$

shows $-(2 ^ n) \leq val \wedge val < 2 ^ n$

using *signed-take-bit-int-greater-eq-minus-exp-word signed-take-bit-int-less-exp-word*

using *assms* **by** *blast*

A *bit_bounds* version of the above lemma.

lemma *signed-take-bit-bounds*:

fixes *ival* :: 'a :: len word

assumes $n \leq LENGTH('a)$

assumes $0 < n$

assumes $val = sint(signed-take-bit\ (n - 1)\ ival)$

shows $fst\ (bit-bounds\ n) \leq val \wedge val \leq snd\ (bit-bounds\ n)$

using *assms signed-take-bit-range lower-bounds-equiv upper-bounds-equiv*

by (*metis bit-bounds.simps fst-conv less-imp-diff-less nat-less-le sint-ge sint-lt snd-conv zle-diff1-eq*)

lemma *signed-take-bit-bounds64*:

fixes *ival* :: int64

assumes $n \leq 64$

assumes $0 < n$

assumes $val = sint(signed-take-bit\ (n - 1)\ ival)$

shows $\text{fst } (\text{bit-bounds } n) \leq \text{val} \wedge \text{val} \leq \text{snd } (\text{bit-bounds } n)$
using *assms signed-take-bit-bounds*
by (*metis size64 word-size*)

lemma *int-signed-value-bounds*:

assumes $b1 \leq 64$
assumes $0 < b1$
shows $\text{fst } (\text{bit-bounds } b1) \leq \text{int-signed-value } b1 \ v2 \wedge$
 $\text{int-signed-value } b1 \ v2 \leq \text{snd } (\text{bit-bounds } b1)$
using *assms int-signed-value.simps signed-take-bit-bounds64* **by** *blast*

lemma *int-signed-value-range*:

fixes *ival* :: *int64*
assumes $\text{val} = \text{int-signed-value } n \ \text{ival}$
shows $-(2^{(n-1)}) \leq \text{val} \wedge \text{val} < 2^{(n-1)}$
using *signed-take-bit-range assms*
by (*smt (verit, ccv-SIG) One-nat-def diff-less int-signed-value.elims len-gt-0*
len-num1 power-less-imp-less-exp power-strict-increasing sint-greater-eq sint-less)

Some lemmas about unsigned words smaller than 64-bit, for zero-extend operators.

lemma *take-bit-smaller-range*:

fixes *ival* :: '*a* :: *len word*
assumes $n < \text{LENGTH('a)}$
assumes $\text{val} = \text{sint}(\text{take-bit } n \ \text{ival})$
shows $0 \leq \text{val} \wedge \text{val} < (2::\text{int})^n$
by (*simp add: assms signed-take-bit-eq*)

lemma *take-bit-same-size-nochange*:

fixes *ival* :: '*a* :: *len word*
assumes $n = \text{LENGTH('a)}$
shows $\text{ival} = \text{take-bit } n \ \text{ival}$
by (*simp add: assms*)

A simplification lemma for *new_int*, showing that upper bits can be ignored.

lemma *take-bit-redundant[simp]*:

fixes *ival* :: '*a* :: *len word*
assumes $0 < n$
assumes $n < \text{LENGTH('a)}$
shows $\text{signed-take-bit } (n-1) \ (\text{take-bit } n \ \text{ival}) = \text{signed-take-bit } (n-1) \ \text{ival}$

proof –

have $\neg (n \leq n-1)$ **using** *assms* **by** *arith*
then have $\bigwedge i. \text{signed-take-bit } (n-1) \ (\text{take-bit } n \ i) = \text{signed-take-bit } (n-1) \ i$
using *signed-take-bit-take-bit* **by** (*metis (mono-tags)*)
then show *?thesis*
by *blast*

qed

lemma *take-bit-same-size-range*:

```

fixes ival :: 'a :: len word
assumes n = LENGTH('a)
assumes ival2 = take-bit n ival
shows - (2 ^ n div 2) ≤ sint ival2 ∧ sint ival2 < 2 ^ n div 2
using assms lower-bounds-equiv sint-ge sint-lt by auto

```

```

lemma take-bit-same-bounds:
  fixes ival :: 'a :: len word
  assumes n = LENGTH('a)
  assumes ival2 = take-bit n ival
  shows fst (bit-bounds n) ≤ sint ival2 ∧ sint ival2 ≤ snd (bit-bounds n)
  unfolding bit-bounds.simps
  using assms take-bit-same-size-range
  by force

```

Next we show that casting a word to a wider word preserves any upper/lower bounds. (These lemmas may not be needed any more, since we are not using scast now?)

```

lemma scast-max-bound:
  assumes sint (v :: 'a :: len word) < M
  assumes LENGTH('a) < LENGTH('b)
  shows sint ((scast v) :: 'b :: len word) < M
  unfolding Word.scast-eq Word.sint-sbintrunc'
  using Bit-Operations.signed-take-bit-int-eq-self-iff
  by (smt (verit, best) One-nat-def assms(1) assms(2) decr-length-less-iff linorder-not-le
    power-strict-increasing-iff signed-take-bit-int-less-self-iff sint-greater-eq)

```

```

lemma scast-min-bound:
  assumes M ≤ sint (v :: 'a :: len word)
  assumes LENGTH('a) < LENGTH('b)
  shows M ≤ sint ((scast v) :: 'b :: len word)
  unfolding Word.scast-eq Word.sint-sbintrunc'
  using Bit-Operations.signed-take-bit-int-eq-self-iff
  by (smt (verit) One-nat-def Suc-pred assms(1) assms(2) len-gt-0 less-Suc-eq or-
    der-less-le order-less-le-trans power-le-imp-le-exp signed-take-bit-int-greater-eq-self-iff
    sint-lt)

```

```

lemma scast-bigger-max-bound:
  assumes (result :: 'b :: len word) = scast (v :: 'a :: len word)
  shows sint result < 2 ^ LENGTH('a) div 2
  using sint-lt upper-bounds-equiv scast-max-bound
  by (smt (verit, best) assms(1) len-gt-0 signed-scast-eq signed-take-bit-int-greater-self-iff
    sint-ge sint-less upper-bounds-equiv)

```

```

lemma scast-bigger-min-bound:
  assumes (result :: 'b :: len word) = scast (v :: 'a :: len word)
  shows - (2 ^ LENGTH('a) div 2) ≤ sint result
  using sint-ge lower-bounds-equiv scast-min-bound

```

by (*smt* (*verit*) *assms len-gt-0 nat-less-le not-less scast-max-bound*)

lemma *scast-bigger-bit-bounds*:

assumes (*result* :: 'b :: len word) = *scast* (*v* :: 'a :: len word)
shows *fst* (*bit-bounds* (*LENGTH*('a))) ≤ *sint result* ∧ *sint result* ≤ *snd* (*bit-bounds* (*LENGTH*('a')))
using *assms scast-bigger-min-bound scast-bigger-max-bound*
by *auto*

Results about *new_int*.

lemma *new-int-take-bits*:

assumes *IntVal b val = new-int b ival*
shows *take-bit b val = val*
using *assms* **by** *force*

1.6.2 Support lemmas for take bit and signed take bit.

Lemmas for removing redundant *take_bit* wrappers.

lemma *take-bit-dist-addL[simp]*:

fixes *x* :: 'a :: len word
shows *take-bit b (take-bit b x + y) = take-bit b (x + y)*
proof (*induction b*)
case 0
then show ?*case*
by *simp*
next
case (*Suc b*)
then show ?*case*
by (*simp add: add.commute mask-eqs(2) take-bit-eq-mask*)
qed

lemma *take-bit-dist-addR[simp]*:

fixes *x* :: 'a :: len word
shows *take-bit b (x + take-bit b y) = take-bit b (x + y)*
using *take-bit-dist-addL* **by** (*metis add.commute*)

lemma *take-bit-dist-subL[simp]*:

fixes *x* :: 'a :: len word
shows *take-bit b (take-bit b x - y) = take-bit b (x - y)*
by (*metis take-bit-dist-addR uminus-add-conv-diff*)

lemma *take-bit-dist-subR[simp]*:

fixes *x* :: 'a :: len word
shows *take-bit b (x - take-bit b y) = take-bit b (x - y)*
using *take-bit-dist-subL*
by (*metis (no-types, opaque-lifting) diff-add-cancel diff-right-commute diff-self*)

```

lemma take-bit-dist-neg[simp]:
  fixes ix :: 'a :: len word
  shows take-bit b (- take-bit b (ix)) = take-bit b (- ix)
  by (metis diff-0 take-bit-dist-subR)

lemma signed-take-take-bit[simp]:
  fixes x :: 'a :: len word
  assumes 0 < b
  shows signed-take-bit (b - 1) (take-bit b x) = signed-take-bit (b - 1) x
  by (smt (verit, best) Suc-diff-1 assms lessI linorder-not-less signed-take-bit-take-bit)

lemma mod-larger-ignore:
  fixes a :: int
  fixes m n :: nat
  assumes n < m
  shows (a mod 2 ^ m) mod 2 ^ n = a mod 2 ^ n
  by (smt (verit, del-insts) assms exp-mod-exp linorder-not-le mod-0-imp-dvd mod-mod-cancel
mod-self order-less-imp-le)

lemma mod-dist-over-add:
  fixes a b c :: int64
  fixes n :: nat
  assumes 1: 0 < n
  assumes 2: n < 64
  shows (a mod 2 ^ n + b) mod 2 ^ n = (a + b) mod 2 ^ n
proof -
  have 3: (0 :: int64) < 2 ^ n
  using assms by (simp add: size64 word-2p-lem)
  then show ?thesis
  unfolding word-mod-2p-is-mask[OF 3]
  apply transfer
  by (metis (no-types, opaque-lifting) and.right-idem take-bit-add take-bit-eq-mask)
qed

end

```

2 Stamp Typing

```

theory Stamp
  imports Values
begin

```

The GraalVM compiler uses the Stamp class to store range and type information for a given node in the IR graph. We model the Stamp class as a datatype, Stamp, and provide a number of functions on the datatype which correspond to the class methods within the compiler.

Stamp information is used in a variety of ways in optimizations, and so, we additionally provide a number of lemmas which help to prove future optimizations.

```
datatype Stamp =
  VoidStamp
| IntegerStamp (stp-bits: nat) (stpi-lower: int) (stpi-upper: int)

| KlassPointerStamp (stp-nonNull: bool) (stp-alwaysNull: bool)
| MethodCountersPointerStamp (stp-nonNull: bool) (stp-alwaysNull: bool)
| MethodPointersStamp (stp-nonNull: bool) (stp-alwaysNull: bool)
| ObjectStamp (stp-type: string) (stp-exactType: bool) (stp-nonNull: bool) (stp-alwaysNull:
bool)
| RawPointerStamp (stp-nonNull: bool) (stp-alwaysNull: bool)
| IllegalStamp
```

```
fun is-stamp-empty :: Stamp  $\Rightarrow$  bool where
  is-stamp-empty (IntegerStamp b lower upper) = (upper < lower) |
```

```
  is-stamp-empty x = False
```

Just like the IntegerStamp class, we need to know that our lo/hi bounds fit into the given number of bits (either signed or unsigned). Our integer stamps have infinite lo/hi bounds, so if the lower bound is non-negative, we can assume that all values are positive, and the integer bits of a related value can be interpreted as unsigned. This is similar (but slightly more general) to what IntegerStamp.java does with its test: if (sameSignBounds()) in the unsignedUpperBound() method.

Note that this is a bit different and more accurate than what StampFactory.forUnsignedInteger does (it widens large unsigned ranges to the max signed range to allow all bit patterns) because its lo/hi values are only 64-bit.

```
fun valid-stamp :: Stamp  $\Rightarrow$  bool where
  valid-stamp (IntegerStamp bits lo hi) =
    (0 < bits  $\wedge$  bits  $\leq$  64  $\wedge$ 
     fst (bit-bounds bits)  $\leq$  lo  $\wedge$  lo  $\leq$  snd (bit-bounds bits)  $\wedge$ 
     fst (bit-bounds bits)  $\leq$  hi  $\wedge$  hi  $\leq$  snd (bit-bounds bits)) |
  valid-stamp s = True
```

experiment begin

```
corollary bit-bounds 1 = (-1, 0) by simp
end
```

— A stamp which includes the full range of the type

```
fun unrestricted-stamp :: Stamp  $\Rightarrow$  Stamp where
  unrestricted-stamp VoidStamp = VoidStamp |
  unrestricted-stamp (IntegerStamp bits lower upper) = (IntegerStamp bits (fst
(bit-bounds bits)) (snd (bit-bounds bits))) |

  unrestricted-stamp (KlassPointerStamp nonNull alwaysNull) = (KlassPointerStamp
False False) |
  unrestricted-stamp (MethodCountersPointerStamp nonNull alwaysNull) = (MethodCountersPointerStamp
False False) |
  unrestricted-stamp (MethodPointersStamp nonNull alwaysNull) = (MethodPointersStamp
False False) |
  unrestricted-stamp (ObjectStamp type exactType nonNull alwaysNull) = (ObjectStamp
True True False) |
  unrestricted-stamp - = IllegalStamp
```

```
fun is-stamp-unrestricted :: Stamp  $\Rightarrow$  bool where
  is-stamp-unrestricted s = (s = unrestricted-stamp s)
```

— A stamp which provides type information but has an empty range of values

```
fun empty-stamp :: Stamp  $\Rightarrow$  Stamp where
  empty-stamp VoidStamp = VoidStamp |
  empty-stamp (IntegerStamp bits lower upper) = (IntegerStamp bits (snd (bit-bounds
bits)) (fst (bit-bounds bits))) |

  empty-stamp (KlassPointerStamp nonNull alwaysNull) = (KlassPointerStamp
nonNull alwaysNull) |
  empty-stamp (MethodCountersPointerStamp nonNull alwaysNull) = (MethodCountersPointerStamp
nonNull alwaysNull) |
  empty-stamp (MethodPointersStamp nonNull alwaysNull) = (MethodPointersStamp
nonNull alwaysNull) |
  empty-stamp (ObjectStamp type exactType nonNull alwaysNull) = (ObjectStamp
True True False) |
  empty-stamp stamp = IllegalStamp
```

— Calculate the meet stamp of two stamps

```
fun meet :: Stamp  $\Rightarrow$  Stamp  $\Rightarrow$  Stamp where
  meet VoidStamp VoidStamp = VoidStamp |
  meet (IntegerStamp b1 l1 u1) (IntegerStamp b2 l2 u2) = (
    if b1  $\neq$  b2 then IllegalStamp else
    (IntegerStamp b1 (min l1 l2) (max u1 u2))
  ) |

  meet (KlassPointerStamp nn1 an1) (KlassPointerStamp nn2 an2) = (
    KlassPointerStamp (nn1  $\wedge$  nn2) (an1  $\wedge$  an2)
```

```

) |
  meet (MethodCountersPointerStamp nn1 an1) (MethodCountersPointerStamp
nn2 an2) = (
  MethodCountersPointerStamp (nn1  $\wedge$  nn2) (an1  $\wedge$  an2)
) |
  meet (MethodPointersStamp nn1 an1) (MethodPointersStamp nn2 an2) = (
  MethodPointersStamp (nn1  $\wedge$  nn2) (an1  $\wedge$  an2)
) |
  meet s1 s2 = IllegalStamp

```

— Calculate the join stamp of two stamps

```

fun join :: Stamp  $\Rightarrow$  Stamp  $\Rightarrow$  Stamp where
  join VoidStamp VoidStamp = VoidStamp |
  join (IntegerStamp b1 l1 u1) (IntegerStamp b2 l2 u2) = (
    if b1  $\neq$  b2 then IllegalStamp else
    (IntegerStamp b1 (max l1 l2) (min u1 u2))
  ) |
  join (KlassPointerStamp nn1 an1) (KlassPointerStamp nn2 an2) = (
    if ((nn1  $\vee$  nn2)  $\wedge$  (an1  $\vee$  an2))
    then (empty-stamp (KlassPointerStamp nn1 an1))
    else (KlassPointerStamp (nn1  $\vee$  nn2) (an1  $\vee$  an2))
  ) |
  join (MethodCountersPointerStamp nn1 an1) (MethodCountersPointerStamp nn2
an2) = (
    if ((nn1  $\vee$  nn2)  $\wedge$  (an1  $\vee$  an2))
    then (empty-stamp (MethodCountersPointerStamp nn1 an1))
    else (MethodCountersPointerStamp (nn1  $\vee$  nn2) (an1  $\vee$  an2))
  ) |
  join (MethodPointersStamp nn1 an1) (MethodPointersStamp nn2 an2) = (
    if ((nn1  $\vee$  nn2)  $\wedge$  (an1  $\vee$  an2))
    then (empty-stamp (MethodPointersStamp nn1 an1))
    else (MethodPointersStamp (nn1  $\vee$  nn2) (an1  $\vee$  an2))
  ) |
  join s1 s2 = IllegalStamp

```

— In certain circumstances a stamp provides enough information to evaluate a value as a stamp, the asConstant function converts the stamp to a value where one can be inferred.

```

fun asConstant :: Stamp  $\Rightarrow$  Value where
  asConstant (IntegerStamp b l h) = (if l = h then IntVal b (word-of-int l) else
UndefVal) |
  asConstant - = UndefVal

```

— Determine if two stamps never have value overlaps i.e. their join is empty

```

fun alwaysDistinct :: Stamp  $\Rightarrow$  Stamp  $\Rightarrow$  bool where
  alwaysDistinct stamp1 stamp2 = is-stamp-empty (join stamp1 stamp2)

```

— Determine if two stamps must always be the same value i.e. two equal constants

```
fun neverDistinct :: Stamp ⇒ Stamp ⇒ bool where
  neverDistinct stamp1 stamp2 = (asConstant stamp1 = asConstant stamp2 ∧
  asConstant stamp1 ≠ UndefVal)
```

```
fun constantAsStamp :: Value ⇒ Stamp where
  constantAsStamp (IntVal b v) = (IntegerStamp b (int-signed-value b v) (int-signed-value
  b v)) |
```

```
constantAsStamp - = IllegalStamp
```

— Define when a runtime value is valid for a stamp. The stamp bounds must be valid, and val must be zero-extended.

```
fun valid-value :: Value ⇒ Stamp ⇒ bool where
  valid-value (IntVal b1 val) (IntegerStamp b l h) =
    (if b1 = b then
      valid-stamp (IntegerStamp b l h) ∧
      take-bit b val = val ∧
      l ≤ int-signed-value b val ∧ int-signed-value b val ≤ h
    else False) |
```

```
valid-value (ObjRef ref) (ObjectStamp klass exact nonNull alwaysNull) =
  ((alwaysNull → ref = None) ∧ (ref=Some → ¬ nonNull)) |
  valid-value stamp val = False
```

```
definition wf-value :: Value ⇒ bool where
  wf-value v = valid-value v (constantAsStamp v)
```

```
lemma unfold-wf-value[simp]:
  wf-value v ⇒ valid-value v (constantAsStamp v)
using wf-value-def by auto
```

```
fun compatible :: Stamp ⇒ Stamp ⇒ bool where
  compatible (IntegerStamp b1 lo1 hi1) (IntegerStamp b2 lo2 hi2) =
    (b1 = b2 ∧ valid-stamp (IntegerStamp b1 lo1 hi1) ∧ valid-stamp (IntegerStamp
    b2 lo2 hi2)) |
  compatible (VoidStamp) (VoidStamp) = True |
  compatible - - = False
```

```
fun stamp-under :: Stamp ⇒ Stamp ⇒ bool where
  stamp-under (IntegerStamp b1 lo1 hi1) (IntegerStamp b2 lo2 hi2) = (b1 = b2 ∧
  hi1 < lo2) |
  stamp-under - - = False
```

— The most common type of stamp within the compiler (apart from the Void-

Stamp) is a 32 bit integer stamp with an unrestricted range. We use *default-stamp* as it is a frequently used stamp.

definition *default-stamp* :: *Stamp* **where**
default-stamp = (*unrestricted-stamp* (*IntegerStamp* 32 0 0))

value *valid-value* (*IntVal* 8 (255)) (*IntegerStamp* 8 (−128) 127)
end

3 Graph Representation

3.1 IR Graph Nodes

theory *IRNodes*
imports
Values
begin

The GraalVM IR is represented using a graph data structure. Here we define the nodes that are contained within the graph. Each node represents a Node subclass in the GraalVM compiler, the node classes have annotated fields to indicate input and successor edges.

We represent these classes with each IRNode constructor explicitly labelling a reference to the node IDs that it stores as inputs and successors.

The *inputs_of* and *successors_of* functions partition those labelled references into input edges and successor edges of a node.

To identify each Node, we use a simple natural number index. Zero is always the start node in a graph. For human readability, within nodes we write INPUT (or special case thereof) instead of ID for input edges, and SUCC instead of ID for control-flow successor edges. Optional edges are handled as "INPUT option" etc.

type-synonym *ID* = *nat*
type-synonym *INPUT* = *ID*
type-synonym *INPUT-ASSOC* = *ID*
type-synonym *INPUT-STATE* = *ID*
type-synonym *INPUT-GUARD* = *ID*
type-synonym *INPUT-COND* = *ID*
type-synonym *INPUT-EXT* = *ID*
type-synonym *SUCC* = *ID*

datatype (*discs-sels*) *IRNode* =
AbsNode (*ir-value*: *INPUT*)
| *AddNode* (*ir-x*: *INPUT*) (*ir-y*: *INPUT*)
| *AndNode* (*ir-x*: *INPUT*) (*ir-y*: *INPUT*)
| *BeginNode* (*ir-next*: *SUCC*)
| *BytecodeExceptionNode* (*ir-arguments*: *INPUT list*) (*ir-stateAfter-opt*: *INPUT-STATE option*) (*ir-next*: *SUCC*)

- | *ConditionalNode* (*ir-condition*: INPUT-COND) (*ir-trueValue*: INPUT) (*ir-falseValue*: INPUT)
- | *ConstantNode* (*ir-const*: Value)
- | *DynamicNewArrayNode* (*ir-elementType*: INPUT) (*ir-length*: INPUT) (*ir-voidClass-opt*: INPUT option) (*ir-stateBefore-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *EndNode*
- | *ExceptionObjectNode* (*ir-stateAfter-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *FrameState* (*ir-monitorIds*: INPUT-ASSOC list) (*ir-outerFrameState-opt*: INPUT-STATE option) (*ir-values-opt*: INPUT list option) (*ir-virtualObjectMappings-opt*: INPUT-STATE list option)
- | *IfNode* (*ir-condition*: INPUT-COND) (*ir-trueSuccessor*: SUCC) (*ir-falseSuccessor*: SUCC)
- | *IntegerBelowNode* (*ir-x*: INPUT) (*ir-y*: INPUT)
- | *IntegerEqualsNode* (*ir-x*: INPUT) (*ir-y*: INPUT)
- | *IntegerLessThanNode* (*ir-x*: INPUT) (*ir-y*: INPUT)
- | *InvokeNode* (*ir-nid*: ID) (*ir-callTarget*: INPUT-EXT) (*ir-classInit-opt*: INPUT option) (*ir-stateDuring-opt*: INPUT-STATE option) (*ir-stateAfter-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *InvokeWithExceptionNode* (*ir-nid*: ID) (*ir-callTarget*: INPUT-EXT) (*ir-classInit-opt*: INPUT option) (*ir-stateDuring-opt*: INPUT-STATE option) (*ir-stateAfter-opt*: INPUT-STATE option) (*ir-next*: SUCC) (*ir-exceptionEdge*: SUCC)
- | *IsNullNode* (*ir-value*: INPUT)
- | *KillingBeginNode* (*ir-next*: SUCC)
- | *LeftShiftNode* (*ir-x*: INPUT) (*ir-y*: INPUT)
- | *LoadFieldNode* (*ir-nid*: ID) (*ir-field*: string) (*ir-object-opt*: INPUT option) (*ir-next*: SUCC)
- | *LogicNegationNode* (*ir-value*: INPUT-COND)
- | *LoopBeginNode* (*ir-ends*: INPUT-ASSOC list) (*ir-overflowGuard-opt*: INPUT-GUARD option) (*ir-stateAfter-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *LoopEndNode* (*ir-loopBegin*: INPUT-ASSOC)
- | *LoopExitNode* (*ir-loopBegin*: INPUT-ASSOC) (*ir-stateAfter-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *MergeNode* (*ir-ends*: INPUT-ASSOC list) (*ir-stateAfter-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *MethodCallTargetNode* (*ir-targetMethod*: string) (*ir-arguments*: INPUT list)
- | *MulNode* (*ir-x*: INPUT) (*ir-y*: INPUT)
- | *NarrowNode* (*ir-inputBits*: nat) (*ir-resultBits*: nat) (*ir-value*: INPUT)
- | *NegateNode* (*ir-value*: INPUT)
- | *NewArrayNode* (*ir-length*: INPUT) (*ir-stateBefore-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *NewInstanceNode* (*ir-nid*: ID) (*ir-instanceClass*: string) (*ir-stateBefore-opt*: INPUT-STATE option) (*ir-next*: SUCC)
- | *NotNode* (*ir-value*: INPUT)
- | *OrNode* (*ir-x*: INPUT) (*ir-y*: INPUT)
- | *ParameterNode* (*ir-index*: nat)
- | *PiNode* (*ir-object*: INPUT) (*ir-guard-opt*: INPUT-GUARD option)
- | *ReturnNode* (*ir-result-opt*: INPUT option) (*ir-memoryMap-opt*: INPUT-EXT option)

```

| RightShiftNode (ir-x: INPUT) (ir-y: INPUT)
| ShortCircuitOrNode (ir-x: INPUT-COND) (ir-y: INPUT-COND)
| SignExtendNode (ir-inputBits: nat) (ir-resultBits: nat) (ir-value: INPUT)
| SignedDivNode (ir-nid: ID) (ir-x: INPUT) (ir-y: INPUT) (ir-zeroCheck-opt: IN-
PUT-GUARD option) (ir-stateBefore-opt: INPUT-STATE option) (ir-next: SUCC)

| SignedRemNode (ir-nid: ID) (ir-x: INPUT) (ir-y: INPUT) (ir-zeroCheck-opt:
INPUT-GUARD option) (ir-stateBefore-opt: INPUT-STATE option) (ir-next: SUCC)

| StartNode (ir-stateAfter-opt: INPUT-STATE option) (ir-next: SUCC)
| StoreFieldNode (ir-nid: ID) (ir-field: string) (ir-value: INPUT) (ir-stateAfter-opt:
INPUT-STATE option) (ir-object-opt: INPUT option) (ir-next: SUCC)
| SubNode (ir-x: INPUT) (ir-y: INPUT)
| UnsignedRightShiftNode (ir-x: INPUT) (ir-y: INPUT)
| UnwindNode (ir-exception: INPUT)
| ValuePhiNode (ir-nid: ID) (ir-values: INPUT list) (ir-merge: INPUT-ASSOC)

| ValueProxyNode (ir-value: INPUT) (ir-loopExit: INPUT-ASSOC)
| XorNode (ir-x: INPUT) (ir-y: INPUT)
| ZeroExtendNode (ir-inputBits: nat) (ir-resultBits: nat) (ir-value: INPUT)
| NoNode

| RefNode (ir-ref: ID)

```

```

fun opt-to-list :: 'a option ⇒ 'a list where
  opt-to-list None = [] |
  opt-to-list (Some v) = [v]

```

```

fun opt-list-to-list :: 'a list option ⇒ 'a list where
  opt-list-to-list None = [] |
  opt-list-to-list (Some x) = x

```

The following functions, `inputs_of` and `successors_of`, are automatically generated from the GraalVM compiler. Their purpose is to partition the node edges into input or successor edges.

```

fun inputs-of :: IRNode ⇒ ID list where
  inputs-of-AbsNode:
  inputs-of (AbsNode value) = [value] |
  inputs-of-AddNode:
  inputs-of (AddNode x y) = [x, y] |
  inputs-of-AndNode:
  inputs-of (AndNode x y) = [x, y] |
  inputs-of-BEGINNode:
  inputs-of (BeginNode next) = [] |
  inputs-of-BytecodeExceptionNode:

```

inputs-of (*BytecodeExceptionNode* *arguments* *stateAfter* *next*) = *arguments* @
(*opt-to-list* *stateAfter*) |
inputs-of-ConditionalNode:
inputs-of (*ConditionalNode* *condition* *trueValue* *falseValue*) = [*condition*, *true-*
Value, *falseValue*] |
inputs-of-ConstantNode:
inputs-of (*ConstantNode* *const*) = [] |
inputs-of-DynamicNewArrayNode:
inputs-of (*DynamicNewArrayNode* *elementType* *length0* *voidClass* *stateBefore*
next) = [*elementType*, *length0*] @ (*opt-to-list* *voidClass*) @ (*opt-to-list* *stateBefore*)
|
inputs-of-EndNode:
inputs-of (*EndNode*) = [] |
inputs-of-ExceptionObjectNode:
inputs-of (*ExceptionObjectNode* *stateAfter* *next*) = (*opt-to-list* *stateAfter*) |
inputs-of-FrameState:
inputs-of (*FrameState* *monitorIds* *outerFrameState* *values* *virtualObjectMappings*)
= *monitorIds* @ (*opt-to-list* *outerFrameState*) @ (*opt-list-to-list* *values*) @ (*opt-list-to-list*
virtualObjectMappings) |
inputs-of-IfNode:
inputs-of (*IfNode* *condition* *trueSuccessor* *falseSuccessor*) = [*condition*] |
inputs-of-IntegerBelowNode:
inputs-of (*IntegerBelowNode* *x* *y*) = [*x*, *y*] |
inputs-of-IntegerEqualsNode:
inputs-of (*IntegerEqualsNode* *x* *y*) = [*x*, *y*] |
inputs-of-IntegerLessThanNode:
inputs-of (*IntegerLessThanNode* *x* *y*) = [*x*, *y*] |
inputs-of-InvokeNode:
inputs-of (*InvokeNode* *nid0* *callTarget* *classInit* *stateDuring* *stateAfter* *next*) =
callTarget # (*opt-to-list* *classInit*) @ (*opt-to-list* *stateDuring*) @ (*opt-to-list* *stateAfter*)
|
inputs-of-InvokeWithExceptionNode:
inputs-of (*InvokeWithExceptionNode* *nid0* *callTarget* *classInit* *stateDuring* *stateAfter*
next *exceptionEdge*) = *callTarget* # (*opt-to-list* *classInit*) @ (*opt-to-list* *stateDur-*
ing) @ (*opt-to-list* *stateAfter*) |
inputs-of-IsNullNode:
inputs-of (*IsNullNode* *value*) = [*value*] |
inputs-of-KillingBeginNode:
inputs-of (*KillingBeginNode* *next*) = [] |
inputs-of-LeftShiftNode:
inputs-of (*LeftShiftNode* *x* *y*) = [*x*, *y*] |
inputs-of-LoadFieldNode:
inputs-of (*LoadFieldNode* *nid0* *field* *object* *next*) = (*opt-to-list* *object*) |
inputs-of-LogicNegationNode:
inputs-of (*LogicNegationNode* *value*) = [*value*] |
inputs-of-LoopBeginNode:
inputs-of (*LoopBeginNode* *ends* *overflowGuard* *stateAfter* *next*) = *ends* @ (*opt-to-list*
overflowGuard) @ (*opt-to-list* *stateAfter*) |
inputs-of-LoopEndNode:

inputs-of (*LoopEndNode* *loopBegin*) = [*loopBegin*] |
inputs-of-LoopExitNode:
inputs-of (*LoopExitNode* *loopBegin* *stateAfter* *next*) = *loopBegin* # (*opt-to-list* *stateAfter*) |
inputs-of-MergeNode:
inputs-of (*MergeNode* *ends* *stateAfter* *next*) = *ends* @ (*opt-to-list* *stateAfter*) |
inputs-of-MethodCallTargetNode:
inputs-of (*MethodCallTargetNode* *targetMethod* *arguments*) = *arguments* |
inputs-of-MulNode:
inputs-of (*MulNode* *x* *y*) = [*x*, *y*] |
inputs-of-NarrowNode:
inputs-of (*NarrowNode* *inputBits* *resultBits* *value*) = [*value*] |
inputs-of-NegateNode:
inputs-of (*NegateNode* *value*) = [*value*] |
inputs-of-NewArrayNode:
inputs-of (*NewArrayNode* *length0* *stateBefore* *next*) = *length0* # (*opt-to-list* *stateBefore*) |
inputs-of-NewInstanceNode:
inputs-of (*NewInstanceNode* *nid0* *instanceClass* *stateBefore* *next*) = (*opt-to-list* *stateBefore*) |
inputs-of-NotNode:
inputs-of (*NotNode* *value*) = [*value*] |
inputs-of-OrNode:
inputs-of (*OrNode* *x* *y*) = [*x*, *y*] |
inputs-of-ParameterNode:
inputs-of (*ParameterNode* *index*) = [] |
inputs-of-PiNode:
inputs-of (*PiNode* *object* *guard*) = *object* # (*opt-to-list* *guard*) |
inputs-of-ReturnNode:
inputs-of (*ReturnNode* *result* *memoryMap*) = (*opt-to-list* *result*) @ (*opt-to-list* *memoryMap*) |
inputs-of-RightShiftNode:
inputs-of (*RightShiftNode* *x* *y*) = [*x*, *y*] |
inputs-of-ShortCircuitOrNode:
inputs-of (*ShortCircuitOrNode* *x* *y*) = [*x*, *y*] |
inputs-of-SignExtendNode:
inputs-of (*SignExtendNode* *inputBits* *resultBits* *value*) = [*value*] |
inputs-of-SignedDivNode:
inputs-of (*SignedDivNode* *nid0* *x* *y* *zeroCheck* *stateBefore* *next*) = [*x*, *y*] @ (*opt-to-list* *zeroCheck*) @ (*opt-to-list* *stateBefore*) |
inputs-of-SignedRemNode:
inputs-of (*SignedRemNode* *nid0* *x* *y* *zeroCheck* *stateBefore* *next*) = [*x*, *y*] @ (*opt-to-list* *zeroCheck*) @ (*opt-to-list* *stateBefore*) |
inputs-of-StartNode:
inputs-of (*StartNode* *stateAfter* *next*) = (*opt-to-list* *stateAfter*) |
inputs-of-StoreFieldNode:
inputs-of (*StoreFieldNode* *nid0* *field* *value* *stateAfter* *object* *next*) = *value* # (*opt-to-list* *stateAfter*) @ (*opt-to-list* *object*) |
inputs-of-SubNode:

inputs-of (*SubNode* *x y*) = [*x*, *y*] |
inputs-of *UnsignedRightShiftNode*:
inputs-of (*UnsignedRightShiftNode* *x y*) = [*x*, *y*] |
inputs-of *UnwindNode*:
inputs-of (*UnwindNode* *exception*) = [*exception*] |
inputs-of *ValuePhiNode*:
inputs-of (*ValuePhiNode* *nid0 values merge*) = *merge* # *values* |
inputs-of *ValueProxyNode*:
inputs-of (*ValueProxyNode* *value loopExit*) = [*value*, *loopExit*] |
inputs-of *XorNode*:
inputs-of (*XorNode* *x y*) = [*x*, *y*] |
inputs-of *ZeroExtendNode*:
inputs-of (*ZeroExtendNode* *inputBits resultBits value*) = [*value*] |
inputs-of *NoNode*: *inputs-of* (*NoNode*) = [] |

inputs-of *RefNode*: *inputs-of* (*RefNode* *ref*) = [*ref*]

fun *successors-of* :: *IRNode* ⇒ *ID list* **where**

successors-of *AbsNode*:
successors-of (*AbsNode* *value*) = [] |
successors-of *AddNode*:
successors-of (*AddNode* *x y*) = [] |
successors-of *AndNode*:
successors-of (*AndNode* *x y*) = [] |
successors-of *BeginNode*:
successors-of (*BeginNode* *next*) = [*next*] |
successors-of *BytecodeExceptionNode*:
successors-of (*BytecodeExceptionNode* *arguments stateAfter next*) = [*next*] |
successors-of *ConditionalNode*:
successors-of (*ConditionalNode* *condition trueValue falseValue*) = [] |
successors-of *ConstantNode*:
successors-of (*ConstantNode* *const*) = [] |
successors-of *DynamicNewArrayNode*:
successors-of (*DynamicNewArrayNode* *elementType length0 voidClass stateBefore next*) = [*next*] |
successors-of *EndNode*:
successors-of (*EndNode*) = [] |
successors-of *ExceptionObjectNode*:
successors-of (*ExceptionObjectNode* *stateAfter next*) = [*next*] |
successors-of *FrameState*:
successors-of (*FrameState* *monitorIds outerFrameState values virtualObjectMappings*) = [] |
successors-of *IfNode*:
successors-of (*IfNode* *condition trueSuccessor falseSuccessor*) = [*trueSuccessor*, *falseSuccessor*] |
successors-of *IntegerBelowNode*:
successors-of (*IntegerBelowNode* *x y*) = [] |

successors-of-IntegerEqualsNode:
successors-of (IntegerEqualsNode x y) = [] |
successors-of-IntegerLessThanNode:
successors-of (IntegerLessThanNode x y) = [] |
successors-of-InvokeNode:
successors-of (InvokeNode nid0 callTarget classInit stateDuring stateAfter next)
= [next] |
successors-of-InvokeWithExceptionNode:
successors-of (InvokeWithExceptionNode nid0 callTarget classInit stateDuring
stateAfter next exceptionEdge) = [next, exceptionEdge] |
successors-of-IsNullNode:
successors-of (IsNullNode value) = [] |
successors-of-KillingBeginNode:
successors-of (KillingBeginNode next) = [next] |
successors-of-LeftShiftNode:
successors-of (LeftShiftNode x y) = [] |
successors-of-LoadFieldNode:
successors-of (LoadFieldNode nid0 field object next) = [next] |
successors-of-LogicNegationNode:
successors-of (LogicNegationNode value) = [] |
successors-of-LoopBeginNode:
successors-of (LoopBeginNode ends overflowGuard stateAfter next) = [next] |
successors-of-LoopEndNode:
successors-of (LoopEndNode loopBegin) = [] |
successors-of-LoopExitNode:
successors-of (LoopExitNode loopBegin stateAfter next) = [next] |
successors-of-MergeNode:
successors-of (MergeNode ends stateAfter next) = [next] |
successors-of-MethodCallTargetNode:
successors-of (MethodCallTargetNode targetMethod arguments) = [] |
successors-of-MulNode:
successors-of (MulNode x y) = [] |
successors-of-NarrowNode:
successors-of (NarrowNode inputBits resultBits value) = [] |
successors-of-NegateNode:
successors-of (NegateNode value) = [] |
successors-of-NewArrayNode:
successors-of (NewArrayNode length0 stateBefore next) = [next] |
successors-of-NewInstanceNode:
successors-of (NewInstanceNode nid0 instanceClass stateBefore next) = [next] |
successors-of-NotNode:
successors-of (NotNode value) = [] |
successors-of-OrNode:
successors-of (OrNode x y) = [] |
successors-of-ParameterNode:
successors-of (ParameterNode index) = [] |
successors-of-PiNode:
successors-of (PiNode object guard) = [] |
successors-of-ReturnNode:

successors-of (*ReturnNode* *result* *memoryMap*) = [] |
successors-of-RightShiftNode:
successors-of (*RightShiftNode* *x* *y*) = [] |
successors-of-ShortCircuitOrNode:
successors-of (*ShortCircuitOrNode* *x* *y*) = [] |
successors-of-SignExtendNode:
successors-of (*SignExtendNode* *inputBits* *resultBits* *value*) = [] |
successors-of-SignedDivNode:
successors-of (*SignedDivNode* *nid0* *x* *y* *zeroCheck* *stateBefore* *next*) = [*next*] |
successors-of-SignedRemNode:
successors-of (*SignedRemNode* *nid0* *x* *y* *zeroCheck* *stateBefore* *next*) = [*next*] |
successors-of-StartNode:
successors-of (*StartNode* *stateAfter* *next*) = [*next*] |
successors-of-StoreFieldNode:
successors-of (*StoreFieldNode* *nid0* *field* *value* *stateAfter* *object* *next*) = [*next*] |
successors-of-SubNode:
successors-of (*SubNode* *x* *y*) = [] |
successors-of-UnsignedRightShiftNode:
successors-of (*UnsignedRightShiftNode* *x* *y*) = [] |
successors-of-UnwindNode:
successors-of (*UnwindNode* *exception*) = [] |
successors-of-ValuePhiNode:
successors-of (*ValuePhiNode* *nid0* *values* *merge*) = [] |
successors-of-ValueProxyNode:
successors-of (*ValueProxyNode* *value* *loopExit*) = [] |
successors-of-XorNode:
successors-of (*XorNode* *x* *y*) = [] |
successors-of-ZeroExtendNode:
successors-of (*ZeroExtendNode* *inputBits* *resultBits* *value*) = [] |
successors-of-NoNode: *successors-of* (*NoNode*) = [] |

successors-of-RefNode: *successors-of* (*RefNode* *ref*) = [*ref*]

lemma *inputs-of* (*FrameState* *x* (*Some* *y*) (*Some* *z*) *None*) = *x* @ [*y*] @ *z*

unfolding *inputs-of-FrameState* **by** *simp*

lemma *successors-of* (*FrameState* *x* (*Some* *y*) (*Some* *z*) *None*) = []

unfolding *inputs-of-FrameState* **by** *simp*

lemma *inputs-of* (*IfNode* *c* *t* *f*) = [*c*]

unfolding *inputs-of-IfNode* **by** *simp*

lemma *successors-of* (*IfNode* *c* *t* *f*) = [*t*, *f*]

unfolding *successors-of-IfNode* **by** *simp*

lemma *inputs-of* (*EndNode*) = [] ∧ *successors-of* (*EndNode*) = []

unfolding *inputs-of-EndNode* *successors-of-EndNode* **by** *simp*

end

3.2 IR Graph Node Hierarchy

```
theory IRNodeHierarchy  
imports IRNodes  
begin
```

It is helpful to introduce a node hierarchy into our formalization. Often the GraalVM compiler relies on explicit type checks to determine which operations to perform on a given node, we try to mimic the same functionality by using a suite of predicate functions over the *IRNode* class to determine inheritance.

As one would expect, the function *is*<ClassName>*Type* will be true if the node parameter is a subclass of the *ClassName* within the GraalVM compiler.

These functions have been automatically generated from the compiler.

```
fun is-EndNode :: IRNode  $\Rightarrow$  bool where  
  is-EndNode EndNode = True |  
  is-EndNode - = False
```

```
fun is-VirtualState :: IRNode  $\Rightarrow$  bool where  
  is-VirtualState n = ((is-FrameState n))
```

```
fun is-BinaryArithmeticNode :: IRNode  $\Rightarrow$  bool where  
  is-BinaryArithmeticNode n = ((is-AddNode n)  $\vee$  (is-AndNode n)  $\vee$  (is-MulNode n)  $\vee$  (is-OrNode n)  $\vee$  (is-SubNode n)  $\vee$  (is-XorNode n))
```

```
fun is-ShiftNode :: IRNode  $\Rightarrow$  bool where  
  is-ShiftNode n = ((is-LeftShiftNode n)  $\vee$  (is-RightShiftNode n)  $\vee$  (is-UnsignedRightShiftNode n))
```

```
fun is-BinaryNode :: IRNode  $\Rightarrow$  bool where  
  is-BinaryNode n = ((is-BinaryArithmeticNode n)  $\vee$  (is-ShiftNode n))
```

```
fun is-AbstractLocalNode :: IRNode  $\Rightarrow$  bool where  
  is-AbstractLocalNode n = ((is-ParameterNode n))
```

```
fun is-IntegerConvertNode :: IRNode  $\Rightarrow$  bool where  
  is-IntegerConvertNode n = ((is-NarrowNode n)  $\vee$  (is-SignExtendNode n)  $\vee$  (is-ZeroExtendNode n))
```

```
fun is-UnaryArithmeticNode :: IRNode  $\Rightarrow$  bool where  
  is-UnaryArithmeticNode n = ((is-AbsNode n)  $\vee$  (is-NegateNode n)  $\vee$  (is-NotNode n))
```

```
fun is-UnaryNode :: IRNode  $\Rightarrow$  bool where
```

```

is-UnaryNode n = ((is-IntegerConvertNode n) ∨ (is-UnaryArithmeticNode n))

fun is-PhiNode :: IRNode ⇒ bool where
  is-PhiNode n = ((is-ValuePhiNode n))

fun is-FloatingGuardedNode :: IRNode ⇒ bool where
  is-FloatingGuardedNode n = ((is-PiNode n))

fun is-UnaryOpLogicNode :: IRNode ⇒ bool where
  is-UnaryOpLogicNode n = ((is-IsNullNode n))

fun is-IntegerLowerThanNode :: IRNode ⇒ bool where
  is-IntegerLowerThanNode n = ((is-IntegerBelowNode n) ∨ (is-IntegerLessThanNode n))

fun is-CompareNode :: IRNode ⇒ bool where
  is-CompareNode n = ((is-IntegerEqualsNode n) ∨ (is-IntegerLowerThanNode n))

fun is-BinaryOpLogicNode :: IRNode ⇒ bool where
  is-BinaryOpLogicNode n = ((is-CompareNode n))

fun is-LogicNode :: IRNode ⇒ bool where
  is-LogicNode n = ((is-BinaryOpLogicNode n) ∨ (is-LogicNegationNode n) ∨
    (is-ShortCircuitOrNode n) ∨ (is-UnaryOpLogicNode n))

fun is-ProxyNode :: IRNode ⇒ bool where
  is-ProxyNode n = ((is-ValueProxyNode n))

fun is-FloatingNode :: IRNode ⇒ bool where
  is-FloatingNode n = ((is-AbstractLocalNode n) ∨ (is-BinaryNode n) ∨ (is-ConditionalNode n) ∨
    (is-ConstantNode n) ∨ (is-FloatingGuardedNode n) ∨ (is-LogicNode n) ∨
    (is-PhiNode n) ∨ (is-ProxyNode n) ∨ (is-UnaryNode n))

fun is-AccessFieldNode :: IRNode ⇒ bool where
  is-AccessFieldNode n = ((is-LoadFieldNode n) ∨ (is-StoreFieldNode n))

fun is-AbstractNewArrayNode :: IRNode ⇒ bool where
  is-AbstractNewArrayNode n = ((is-DynamicNewArrayNode n) ∨ (is-NewArrayNode n))

fun is-AbstractNewObjectNode :: IRNode ⇒ bool where
  is-AbstractNewObjectNode n = ((is-AbstractNewArrayNode n) ∨ (is-NewInstanceNode n))

fun is-IntegerDivRemNode :: IRNode ⇒ bool where
  is-IntegerDivRemNode n = ((is-SignedDivNode n) ∨ (is-SignedRemNode n))

fun is-FixedBinaryNode :: IRNode ⇒ bool where
  is-FixedBinaryNode n = ((is-IntegerDivRemNode n))

```

```

fun is-DeoptimizingFixedWithNextNode :: IRNode  $\Rightarrow$  bool where
  is-DeoptimizingFixedWithNextNode n = ((is-AbstractNewObjectNode n)  $\vee$  (is-FixedBinaryNode
n))

fun is-AbstractMemoryCheckpoint :: IRNode  $\Rightarrow$  bool where
  is-AbstractMemoryCheckpoint n = ((is-BytecodeExceptionNode n)  $\vee$  (is-InvokeNode
n))

fun is-AbstractStateSplit :: IRNode  $\Rightarrow$  bool where
  is-AbstractStateSplit n = ((is-AbstractMemoryCheckpoint n))

fun is-AbstractMergeNode :: IRNode  $\Rightarrow$  bool where
  is-AbstractMergeNode n = ((is-LoopBeginNode n)  $\vee$  (is-MergeNode n))

fun is-BeginStateSplitNode :: IRNode  $\Rightarrow$  bool where
  is-BeginStateSplitNode n = ((is-AbstractMergeNode n)  $\vee$  (is-ExceptionObjectNode
n)  $\vee$  (is-LoopExitNode n)  $\vee$  (is-StartNode n))

fun is-AbstractBeginNode :: IRNode  $\Rightarrow$  bool where
  is-AbstractBeginNode n = ((is-BeginNode n)  $\vee$  (is-BeginStateSplitNode n)  $\vee$ 
(is-KillingBeginNode n))

fun is-FixedWithNextNode :: IRNode  $\Rightarrow$  bool where
  is-FixedWithNextNode n = ((is-AbstractBeginNode n)  $\vee$  (is-AbstractStateSplit n)
 $\vee$  (is-AccessFieldNode n)  $\vee$  (is-DeoptimizingFixedWithNextNode n))

fun is-WithExceptionNode :: IRNode  $\Rightarrow$  bool where
  is-WithExceptionNode n = ((is-InvokeWithExceptionNode n))

fun is-ControlSplitNode :: IRNode  $\Rightarrow$  bool where
  is-ControlSplitNode n = ((is-IfNode n)  $\vee$  (is-WithExceptionNode n))

fun is-ControlSinkNode :: IRNode  $\Rightarrow$  bool where
  is-ControlSinkNode n = ((is-ReturnNode n)  $\vee$  (is-UnwindNode n))

fun is-AbstractEndNode :: IRNode  $\Rightarrow$  bool where
  is-AbstractEndNode n = ((is-EndNode n)  $\vee$  (is-LoopEndNode n))

fun is-FixedNode :: IRNode  $\Rightarrow$  bool where
  is-FixedNode n = ((is-AbstractEndNode n)  $\vee$  (is-ControlSinkNode n)  $\vee$  (is-ControlSplitNode
n)  $\vee$  (is-FixedWithNextNode n))

fun is-CallTargetNode :: IRNode  $\Rightarrow$  bool where
  is-CallTargetNode n = ((is-MethodCallTargetNode n))

fun is-ValueNode :: IRNode  $\Rightarrow$  bool where
  is-ValueNode n = ((is-CallTargetNode n)  $\vee$  (is-FixedNode n)  $\vee$  (is-FloatingNode
n))

```

```

fun is-Node :: IRNode  $\Rightarrow$  bool where
  is-Node n = ((is-ValueNode n)  $\vee$  (is-VirtualState n))

fun is-MemoryKill :: IRNode  $\Rightarrow$  bool where
  is-MemoryKill n = ((is-AbstractMemoryCheckpoint n))

fun is-NarrowableArithmeticNode :: IRNode  $\Rightarrow$  bool where
  is-NarrowableArithmeticNode n = ((is-AbsNode n)  $\vee$  (is-AddNode n)  $\vee$  (is-AndNode
n)  $\vee$  (is-MulNode n)  $\vee$  (is-NegateNode n)  $\vee$  (is-NotNode n)  $\vee$  (is-OrNode n)  $\vee$ 
(is-ShiftNode n)  $\vee$  (is-SubNode n)  $\vee$  (is-XorNode n))

fun is-AnchoringNode :: IRNode  $\Rightarrow$  bool where
  is-AnchoringNode n = ((is-AbstractBeginNode n))

fun is-DeoptBefore :: IRNode  $\Rightarrow$  bool where
  is-DeoptBefore n = ((is-DeoptimizingFixedWithNextNode n))

fun is-IndirectCanonicalization :: IRNode  $\Rightarrow$  bool where
  is-IndirectCanonicalization n = ((is-LogicNode n))

fun is-IterableNodeType :: IRNode  $\Rightarrow$  bool where
  is-IterableNodeType n = ((is-AbstractBeginNode n)  $\vee$  (is-AbstractMergeNode n)  $\vee$ 
(is-FrameState n)  $\vee$  (is-IfNode n)  $\vee$  (is-IntegerDivRemNode n)  $\vee$  (is-InvokeWithExceptionNode
n)  $\vee$  (is-LoopBeginNode n)  $\vee$  (is-LoopExitNode n)  $\vee$  (is-MethodCallTargetNode n)
 $\vee$  (is-ParameterNode n)  $\vee$  (is-ReturnNode n)  $\vee$  (is-ShortCircuitOrNode n))

fun is-Invoke :: IRNode  $\Rightarrow$  bool where
  is-Invoke n = ((is-InvokeNode n)  $\vee$  (is-InvokeWithExceptionNode n))

fun is-Proxy :: IRNode  $\Rightarrow$  bool where
  is-Proxy n = ((is-ProxyNode n))

fun is-ValueProxy :: IRNode  $\Rightarrow$  bool where
  is-ValueProxy n = ((is-PiNode n)  $\vee$  (is-ValueProxyNode n))

fun is-ValueNodeInterface :: IRNode  $\Rightarrow$  bool where
  is-ValueNodeInterface n = ((is-ValueNode n))

fun is-ArrayLengthProvider :: IRNode  $\Rightarrow$  bool where
  is-ArrayLengthProvider n = ((is-AbstractNewArrayNode n)  $\vee$  (is-ConstantNode
n))

fun is-StampInverter :: IRNode  $\Rightarrow$  bool where
  is-StampInverter n = ((is-IntegerConvertNode n)  $\vee$  (is-NegateNode n)  $\vee$  (is-NotNode
n))

fun is-GuardingNode :: IRNode  $\Rightarrow$  bool where
  is-GuardingNode n = ((is-AbstractBeginNode n))

```



```

fun is-SingleMemoryKill :: IRNode  $\Rightarrow$  bool where
  is-SingleMemoryKill n = ((is-BytecodeExceptionNode n)  $\vee$  (is-ExceptionObjectNode
n)  $\vee$  (is-InvokeNode n)  $\vee$  (is-InvokeWithExceptionNode n)  $\vee$  (is-KillingBeginNode
n)  $\vee$  (is-StartNode n))

fun is-LIRLowerable :: IRNode  $\Rightarrow$  bool where
  is-LIRLowerable n = ((is-AbstractBeginNode n)  $\vee$  (is-AbstractEndNode n)  $\vee$ 
(is-AbstractMergeNode n)  $\vee$  (is-BinaryOpLogicNode n)  $\vee$  (is-CallTargetNode n)
 $\vee$  (is-ConditionalNode n)  $\vee$  (is-ConstantNode n)  $\vee$  (is-IfNode n)  $\vee$  (is-InvokeNode
n)  $\vee$  (is-InvokeWithExceptionNode n)  $\vee$  (is-IsNullNode n)  $\vee$  (is-LoopBeginNode n)
 $\vee$  (is-PiNode n)  $\vee$  (is-ReturnNode n)  $\vee$  (is-SignedDivNode n)  $\vee$  (is-SignedRemNode
n)  $\vee$  (is-UnaryOpLogicNode n)  $\vee$  (is-UnwindNode n))

fun is-GuardedNode :: IRNode  $\Rightarrow$  bool where
  is-GuardedNode n = ((is-FloatingGuardedNode n))

fun is-ArithmeticLIRLowerable :: IRNode  $\Rightarrow$  bool where
  is-ArithmeticLIRLowerable n = ((is-AbsNode n)  $\vee$  (is-BinaryArithmeticNode n)  $\vee$ 
(is-IntegerConvertNode n)  $\vee$  (is-NotNode n)  $\vee$  (is-ShiftNode n)  $\vee$  (is-UnaryArithmeticNode
n))

fun is-SwitchFoldable :: IRNode  $\Rightarrow$  bool where
  is-SwitchFoldable n = ((is-IfNode n))

fun is-VirtualizableAllocation :: IRNode  $\Rightarrow$  bool where
  is-VirtualizableAllocation n = ((is-NewArrayNode n)  $\vee$  (is-NewInstanceNode n))

fun is-Unary :: IRNode  $\Rightarrow$  bool where
  is-Unary n = ((is-LoadFieldNode n)  $\vee$  (is-LogicNegationNode n)  $\vee$  (is-UnaryNode
n)  $\vee$  (is-UnaryOpLogicNode n))

fun is-FixedNodeInterface :: IRNode  $\Rightarrow$  bool where
  is-FixedNodeInterface n = ((is-FixedNode n))

fun is-BinaryCommutative :: IRNode  $\Rightarrow$  bool where
  is-BinaryCommutative n = ((is-AddNode n)  $\vee$  (is-AndNode n)  $\vee$  (is-IntegerEqualsNode
n)  $\vee$  (is-MulNode n)  $\vee$  (is-OrNode n)  $\vee$  (is-XorNode n))

fun is-Canonicalizable :: IRNode  $\Rightarrow$  bool where
  is-Canonicalizable n = ((is-BytecodeExceptionNode n)  $\vee$  (is-ConditionalNode n)  $\vee$ 
(is-DynamicNewArrayNode n)  $\vee$  (is-PhiNode n)  $\vee$  (is-PiNode n)  $\vee$  (is-ProxyNode
n)  $\vee$  (is-StoreFieldNode n)  $\vee$  (is-ValueProxyNode n))

fun is-UncheckedInterfaceProvider :: IRNode  $\Rightarrow$  bool where
  is-UncheckedInterfaceProvider n = ((is-InvokeNode n)  $\vee$  (is-InvokeWithExceptionNode
n)  $\vee$  (is-LoadFieldNode n)  $\vee$  (is-ParameterNode n))

fun is-Binary :: IRNode  $\Rightarrow$  bool where

```

```

    is-Binary n = ((is-BinaryArithmeticNode n) ∨ (is-BinaryNode n) ∨ (is-BinaryOpLogicNode
n) ∨ (is-CompareNode n) ∨ (is-FixedBinaryNode n) ∨ (is-ShortCircuitOrNode n))

```

```

fun is-ArithmeticOperation :: IRNode ⇒ bool where
    is-ArithmeticOperation n = ((is-BinaryArithmeticNode n) ∨ (is-IntegerConvertNode
n) ∨ (is-ShiftNode n) ∨ (is-UnaryArithmeticNode n))

```

```

fun is-ValueNumberable :: IRNode ⇒ bool where
    is-ValueNumberable n = ((is-FloatingNode n) ∨ (is-ProxyNode n))

```

```

fun is-Lowerable :: IRNode ⇒ bool where
    is-Lowerable n = ((is-AbstractNewObjectNode n) ∨ (is-AccessFieldNode n) ∨
(is-BytecodeExceptionNode n) ∨ (is-ExceptionObjectNode n) ∨ (is-IntegerDivRemNode
n) ∨ (is-UnwindNode n))

```

```

fun is-Virtualizable :: IRNode ⇒ bool where
    is-Virtualizable n = ((is-IsNullNode n) ∨ (is-LoadFieldNode n) ∨ (is-PiNode n)
∨ (is-StoreFieldNode n) ∨ (is-ValueProxyNode n))

```

```

fun is-Simplifiable :: IRNode ⇒ bool where
    is-Simplifiable n = ((is-AbstractMergeNode n) ∨ (is-BeginNode n) ∨ (is-IfNode
n) ∨ (is-LoopExitNode n) ∨ (is-MethodCallTargetNode n) ∨ (is-NewArrayNode n))

```

```

fun is-StateSplit :: IRNode ⇒ bool where
    is-StateSplit n = ((is-AbstractStateSplit n) ∨ (is-BeginStateSplitNode n) ∨ (is-StoreFieldNode
n))

```

```

fun is-ConvertNode :: IRNode ⇒ bool where
    is-ConvertNode n = ((is-IntegerConvertNode n))

```

```

fun is-sequential-node :: IRNode ⇒ bool where
    is-sequential-node (StartNode -) = True |
    is-sequential-node (BeginNode -) = True |
    is-sequential-node (KillingBeginNode -) = True |
    is-sequential-node (LoopBeginNode - - -) = True |
    is-sequential-node (LoopExitNode - -) = True |
    is-sequential-node (MergeNode - -) = True |
    is-sequential-node (RefNode -) = True |
    is-sequential-node - = False

```

The following convenience function is useful in determining if two IRNodes are of the same type irregardless of their edges. It will return true if both the node parameters are the same node class.

```

fun is-same-ir-node-type :: IRNode ⇒ IRNode ⇒ bool where
    is-same-ir-node-type n1 n2 = (
        ((is-AbsNode n1) ∧ (is-AbsNode n2)) ∨
        ((is-AddNode n1) ∧ (is-AddNode n2)) ∨
        ((is-AndNode n1) ∧ (is-AndNode n2)) ∨

```

```

((is-BEGINNode n1) ∧ (is-BEGINNode n2)) ∨
((is-BytecodeExceptionNode n1) ∧ (is-BytecodeExceptionNode n2)) ∨
((is-ConditionalNode n1) ∧ (is-ConditionalNode n2)) ∨
((is-ConstantNode n1) ∧ (is-ConstantNode n2)) ∨
((is-DynamicNewArrayNode n1) ∧ (is-DynamicNewArrayNode n2)) ∨
((is-EndNode n1) ∧ (is-EndNode n2)) ∨
((is-ExceptionObjectNode n1) ∧ (is-ExceptionObjectNode n2)) ∨
((is-FrameState n1) ∧ (is-FrameState n2)) ∨
((is-IfNode n1) ∧ (is-IfNode n2)) ∨
((is-IntegerBelowNode n1) ∧ (is-IntegerBelowNode n2)) ∨
((is-IntegerEqualsNode n1) ∧ (is-IntegerEqualsNode n2)) ∨
((is-IntegerLessThanNode n1) ∧ (is-IntegerLessThanNode n2)) ∨
((is-InvokeNode n1) ∧ (is-InvokeNode n2)) ∨
((is-InvokeWithExceptionNode n1) ∧ (is-InvokeWithExceptionNode n2)) ∨
((is-IsNullNode n1) ∧ (is-IsNullNode n2)) ∨
((is-KillingBeginNode n1) ∧ (is-KillingBeginNode n2)) ∨
((is-LoadFieldNode n1) ∧ (is-LoadFieldNode n2)) ∨
((is-LogicNegationNode n1) ∧ (is-LogicNegationNode n2)) ∨
((is-LoopBeginNode n1) ∧ (is-LoopBeginNode n2)) ∨
((is-LoopEndNode n1) ∧ (is-LoopEndNode n2)) ∨
((is-LoopExitNode n1) ∧ (is-LoopExitNode n2)) ∨
((is-MergeNode n1) ∧ (is-MergeNode n2)) ∨
((is-MethodCallTargetNode n1) ∧ (is-MethodCallTargetNode n2)) ∨
((is-MulNode n1) ∧ (is-MulNode n2)) ∨
((is-NegateNode n1) ∧ (is-NegateNode n2)) ∨
((is-NewArrayNode n1) ∧ (is-NewArrayNode n2)) ∨
((is-NewInstanceNode n1) ∧ (is-NewInstanceNode n2)) ∨
((is-NotNode n1) ∧ (is-NotNode n2)) ∨
((is-OrNode n1) ∧ (is-OrNode n2)) ∨
((is-ParameterNode n1) ∧ (is-ParameterNode n2)) ∨
((is-PiNode n1) ∧ (is-PiNode n2)) ∨
((is-ReturnNode n1) ∧ (is-ReturnNode n2)) ∨
((is-ShortCircuitOrNode n1) ∧ (is-ShortCircuitOrNode n2)) ∨
((is-SignedDivNode n1) ∧ (is-SignedDivNode n2)) ∨
((is-StartNode n1) ∧ (is-StartNode n2)) ∨
((is-StoreFieldNode n1) ∧ (is-StoreFieldNode n2)) ∨
((is-SubNode n1) ∧ (is-SubNode n2)) ∨
((is-UnwindNode n1) ∧ (is-UnwindNode n2)) ∨
((is-ValuePhiNode n1) ∧ (is-ValuePhiNode n2)) ∨
((is-ValueProxyNode n1) ∧ (is-ValueProxyNode n2)) ∨
((is-XorNode n1) ∧ (is-XorNode n2))

```

end

3.3 IR Graph Type

```

theory IRGraph
imports
  IRNodeHierarchy

```

```

    Stamp
    HOL-Library.FSet
    HOL.Relation
begin

```

This theory defines the main Graal data structure - an entire IR Graph.

IRGraph is defined as a partial map with a finite domain. The finite domain is required to be able to generate code and produce an interpreter.

```

typedef IRGraph = {g :: ID  $\rightarrow$  (IRNode  $\times$  Stamp) . finite (dom g)}
proof -
  have finite(dom(Map.empty))  $\wedge$  ran Map.empty = {} by auto
  then show ?thesis
    by fastforce
qed

```

setup-lifting type-definition-IRGraph

```

lift-definition ids :: IRGraph  $\Rightarrow$  ID set
is  $\lambda g. \{nid \in \text{dom } g . \nexists s. g \text{ } nid = (\text{Some } (\text{NoNode}, s))\}$  .

```

```

fun with-default :: 'c  $\Rightarrow$  ('b  $\Rightarrow$  'c)  $\Rightarrow$  (('a  $\rightarrow$  'b)  $\Rightarrow$  'a  $\Rightarrow$  'c) where
  with-default def conv = ( $\lambda m k.$ 
    (case m k of None  $\Rightarrow$  def | Some v  $\Rightarrow$  conv v))

```

```

lift-definition kind :: IRGraph  $\Rightarrow$  (ID  $\Rightarrow$  IRNode)
is with-default NoNode fst .

```

```

lift-definition stamp :: IRGraph  $\Rightarrow$  ID  $\Rightarrow$  Stamp
is with-default IllegalStamp snd .

```

```

lift-definition add-node :: ID  $\Rightarrow$  (IRNode  $\times$  Stamp)  $\Rightarrow$  IRGraph  $\Rightarrow$  IRGraph
is  $\lambda nid k g.$  if fst k = NoNode then g else g(nid  $\mapsto$  k) by simp

```

```

lift-definition remove-node :: ID  $\Rightarrow$  IRGraph  $\Rightarrow$  IRGraph
is  $\lambda nid g.$  g(nid := None) by simp

```

```

lift-definition replace-node :: ID  $\Rightarrow$  (IRNode  $\times$  Stamp)  $\Rightarrow$  IRGraph  $\Rightarrow$  IRGraph
is  $\lambda nid k g.$  if fst k = NoNode then g else g(nid  $\mapsto$  k) by simp

```

```

lift-definition as-list :: IRGraph  $\Rightarrow$  (ID  $\times$  IRNode  $\times$  Stamp) list
is  $\lambda g.$  map ( $\lambda k. (k, \text{the } (g k))$ ) (sorted-list-of-set (dom g)) .

```

```

fun no-node :: (ID  $\times$  (IRNode  $\times$  Stamp)) list  $\Rightarrow$  (ID  $\times$  (IRNode  $\times$  Stamp)) list
where
  no-node g = filter ( $\lambda n.$  fst (snd n)  $\neq$  NoNode) g

```

```

lift-definition irgraph :: (ID  $\times$  (IRNode  $\times$  Stamp)) list  $\Rightarrow$  IRGraph
is map-of  $\circ$  no-node

```

by (*simp add: finite-dom-map-of*)

definition *as-set* :: *IRGraph* \Rightarrow (*ID* \times (*IRNode* \times *Stamp*)) *set* **where**
as-set *g* = $\{(n, \text{kind } g \ n, \text{stamp } g \ n) \mid n . n \in \text{ids } g\}$

definition *true-ids* :: *IRGraph* \Rightarrow *ID set* **where**
true-ids *g* = *ids g* - $\{n \in \text{ids } g. \exists n'. \text{kind } g \ n = \text{RefNode } n'\}$

definition *domain-subtraction* :: '*a set* \Rightarrow ('*a* \times '*b*) *set* \Rightarrow ('*a* \times '*b*) *set*
(infix \trianglelefteq 30) where
domain-subtraction *s r* = $\{(x, y) . (x, y) \in r \wedge x \notin s\}$

notation (*latex*)
domain-subtraction ($- \trianglelefteq -$)

code-datatype *irgraph*

fun *filter-none* **where**
filter-none *g* = $\{nid \in \text{dom } g . \nexists s. g \ nid = (\text{Some } (\text{NoNode}, s))\}$

lemma *no-node-clears*:
 $\text{res} = \text{no-node } xs \longrightarrow (\forall x \in \text{set res}. \text{fst } (\text{snd } x) \neq \text{NoNode})$
by *simp*

lemma *dom-eq*:
assumes $\forall x \in \text{set } xs. \text{fst } (\text{snd } x) \neq \text{NoNode}$
shows *filter-none* (*map-of xs*) = *dom* (*map-of xs*)
unfolding *filter-none.simps* **using** *assms map-of-SomeD*
by *fastforce*

lemma *fil-eq*:
filter-none (*map-of* (*no-node xs*)) = *set* (*map fst* (*no-node xs*))
using *no-node-clears*
by (*metis dom-eq dom-map-of-conv-image-fst list.set-map*)

lemma *irgraph[code]: ids* (*irgraph m*) = *set* (*map fst* (*no-node m*))
unfolding *irgraph-def ids-def* **using** *fil-eq*
by (*smt Rep-IRGraph comp-apply eq-onp-same-args filter-none.simps ids.abs-eq*
ids-def irgraph.abs-eq irgraph.rep-eq irgraph-def mem-Collect-eq)

lemma [*code*]: *Rep-IRGraph* (*irgraph m*) = *map-of* (*no-node m*)
using *Abs-IRGraph-inverse*
by (*simp add: irgraph.rep-eq*)

— Get the inputs set of a given node ID

fun *inputs* :: *IRGraph* \Rightarrow *ID* \Rightarrow *ID set* **where**
inputs *g nid* = *set* (*inputs-of* (*kind g nid*))

— Get the successor set of a given node ID

```
fun succ :: IRGraph ⇒ ID ⇒ ID set where
  succ g nid = set (successors-of (kind g nid))
```

— Gives a relation between node IDs - between a node and its input nodes

```
fun input-edges :: IRGraph ⇒ ID rel where
  input-edges g = (⋃ i ∈ ids g. {(i,j)|j. j ∈ (inputs g i)})
```

— Find all the nodes in the graph that have nid as an input - the usages of nid

```
fun usages :: IRGraph ⇒ ID ⇒ ID set where
  usages g nid = {i. i ∈ ids g ∧ nid ∈ inputs g i}
```

```
fun successor-edges :: IRGraph ⇒ ID rel where
  successor-edges g = (⋃ i ∈ ids g. {(i,j)|j. j ∈ (succ g i)})
```

```
fun predecessors :: IRGraph ⇒ ID ⇒ ID set where
  predecessors g nid = {i. i ∈ ids g ∧ nid ∈ succ g i}
```

```
fun nodes-of :: IRGraph ⇒ (IRNode ⇒ bool) ⇒ ID set where
  nodes-of g sel = {nid ∈ ids g . sel (kind g nid)}
```

```
fun edge :: (IRNode ⇒ 'a) ⇒ ID ⇒ IRGraph ⇒ 'a where
  edge sel nid g = sel (kind g nid)
```



```
fun filtered-inputs :: IRGraph ⇒ ID ⇒ (IRNode ⇒ bool) ⇒ ID list where
  filtered-inputs g nid f = filter (f ∘ (kind g)) (inputs-of (kind g nid))
```

```
fun filtered-successors :: IRGraph ⇒ ID ⇒ (IRNode ⇒ bool) ⇒ ID list where
  filtered-successors g nid f = filter (f ∘ (kind g)) (successors-of (kind g nid))
```

```
fun filtered-usages :: IRGraph ⇒ ID ⇒ (IRNode ⇒ bool) ⇒ ID set where
  filtered-usages g nid f = {n ∈ (usages g nid). f (kind g n)}
```



```
fun is-empty :: IRGraph ⇒ bool where
  is-empty g = (ids g = {})
```



```
fun any-usage :: IRGraph ⇒ ID ⇒ ID where
  any-usage g nid = hd (sorted-list-of-set (usages g nid))
```


lemma ids-some[simp]: $x \in \text{ids } g \longleftrightarrow \text{kind } g \ x \neq \text{NoNode}$

proof —

```
  have that:  $x \in \text{ids } g \longrightarrow \text{kind } g \ x \neq \text{NoNode}$ 
    using ids.rep-eq kind.rep-eq by force
  have  $\text{kind } g \ x \neq \text{NoNode} \longrightarrow x \in \text{ids } g$ 
    unfolding with-default.simps kind-def ids-def
    by (cases Rep-IRGraph g x = None; auto)
  from this that show ?thesis by auto
```

qed

lemma not-in-g:

```
  assumes nid ∉ ids g
  shows kind g nid = NoNode
  using assms ids-some by blast
```


lemma valid-creation[simp]:

```
  finite (dom g) ⟷ Rep-IRGraph (Abs-IRGraph g) = g
  using Abs-IRGraph-inverse by (metis Rep-IRGraph mem-Collect-eq)
```

```

lemma [simp]: finite (ids g)
  using Rep-IRGraph ids.rep-eq by simp

lemma [simp]: finite (ids (irgraph g))
  by (simp add: finite-dom-map-of)

lemma [simp]: finite (dom g)  $\longrightarrow$  ids (Abs-IRGraph g) = {nid  $\in$  dom g .  $\nexists$  s. g
  nid = Some (NoNode, s)}
  using ids.rep-eq by simp

lemma [simp]: finite (dom g)  $\longrightarrow$  kind (Abs-IRGraph g) = ( $\lambda$ x . (case g x of None
 $\Rightarrow$  NoNode | Some n  $\Rightarrow$  fst n))
  by (simp add: kind.rep-eq)

lemma [simp]: finite (dom g)  $\longrightarrow$  stamp (Abs-IRGraph g) = ( $\lambda$ x . (case g x of
  None  $\Rightarrow$  IllegalStamp | Some n  $\Rightarrow$  snd n))
  using stamp.abs-eq stamp.rep-eq by auto

lemma [simp]: ids (irgraph g) = set (map fst (no-node g))
  using irgraph by auto

lemma [simp]: kind (irgraph g) = ( $\lambda$ nid. (case (map-of (no-node g)) nid of None
 $\Rightarrow$  NoNode | Some n  $\Rightarrow$  fst n))
  using irgraph.rep-eq kind.transfer kind.rep-eq by auto

lemma [simp]: stamp (irgraph g) = ( $\lambda$ nid. (case (map-of (no-node g)) nid of None
 $\Rightarrow$  IllegalStamp | Some n  $\Rightarrow$  snd n))
  using irgraph.rep-eq stamp.transfer stamp.rep-eq by auto

lemma map-of-upd: (map-of g)(k  $\mapsto$  v) = (map-of ((k, v) # g))
  by simp

lemma [code]: replace-node nid k (irgraph g) = (irgraph ((nid, k) # g))
proof (cases fst k = NoNode)
  case True
    then show ?thesis
    by (metis (mono-tags, lifting) Rep-IRGraph-inject filter.simps(2) irgraph.abs-eq
  no-node.simps replace-node.rep-eq snd-conv)
  next
    case False
    then show ?thesis unfolding irgraph-def replace-node-def no-node.simps
    by (smt (verit, best) Rep-IRGraph comp-apply eq-onp-same-args filter.simps(2)
  id-def irgraph.rep-eq map-fun-apply map-of-upd mem-Collect-eq no-node.elims re-
  place-node.abs-eq replace-node-def snd-eqD)
qed

lemma [code]: add-node nid k (irgraph g) = (irgraph (((nid, k) # g)))

```

by (*smt* (*z3*) *Rep-IRGraph-inject add-node.rep-eq filter.simps(2) irgraph.rep-eq map-of-upd no-node.simps snd-conv*)

lemma *add-node-lookup*:

gup = *add-node nid (k, s) g* \longrightarrow
 (if *k* \neq *NoNode* then *kind gup nid* = *k* \wedge *stamp gup nid* = *s* else *kind gup nid* = *kind g nid*)

proof (*cases k = NoNode*)

case *True*

then show *?thesis*

by (*simp add: add-node.rep-eq kind.rep-eq*)

next

case *False*

then show *?thesis*

by (*simp add: kind.rep-eq add-node.rep-eq stamp.rep-eq*)

qed

lemma *remove-node-lookup*:

gup = *remove-node nid g* \longrightarrow *kind gup nid* = *NoNode* \wedge *stamp gup nid* = *IllegalStamp*

by (*simp add: kind.rep-eq remove-node.rep-eq stamp.rep-eq*)

lemma *replace-node-lookup[simp]*:

gup = *replace-node nid (k, s) g* \wedge *k* \neq *NoNode* \longrightarrow *kind gup nid* = *k* \wedge *stamp gup nid* = *s*

by (*simp add: replace-node.rep-eq kind.rep-eq stamp.rep-eq*)

lemma *replace-node-unchanged*:

gup = *replace-node nid (k, s) g* \longrightarrow (\forall *n* \in (*ids g* - {*nid*}) . *n* \in *ids g* \wedge *n* \in *ids gup* \wedge *kind g n* = *kind gup n*)

by (*simp add: kind.rep-eq replace-node.rep-eq*)

3.3.1 Example Graphs

Example 1: empty graph (just a start and end node)

definition *start-end-graph*:: *IRGraph* **where**

start-end-graph = *irgraph* [(0, *StartNode* *None* 1, *VoidStamp*), (1, *ReturnNode* *None* *None*, *VoidStamp*)]

Example 2: public static int sq(int x) return x * x;

[1 P(0)] / [0 Start] [4 *] | / V / [5 Return]

definition *eg2-sq* :: *IRGraph* **where**

eg2-sq = *irgraph* [
 (0, *StartNode* *None* 5, *VoidStamp*),
 (1, *ParameterNode* 0, *default-stamp*),
 (4, *MulNode* 1 1, *default-stamp*),
 (5, *ReturnNode* (*Some* 4) *None*, *default-stamp*)
]


```

value input-edges eg2-sq
value usages eg2-sq 1

end

```

4 java.lang.Long

Utility functions from the Long class that Graal occasionally makes use of.

```

theory Long
  imports ValueThms
begin

```

```

lemma negative-all-set-32:
   $n < 32 \implies \text{bit } (-1::\text{int}32) \ n$ 
  apply transfer by auto

```

```

definition MaxOrNeg :: nat set  $\Rightarrow$  int where
  MaxOrNeg s = (if s = {} then -1 else Max s)

```

```

definition MinOrHighest :: nat set  $\Rightarrow$  nat  $\Rightarrow$  nat where
  MinOrHighest s m = (if s = {} then m else Min s)

```

```

definition highestOneBit :: ('a::len) word  $\Rightarrow$  int where
  highestOneBit v = MaxOrNeg {n . bit v n}

```

```

definition lowestOneBit :: ('a::len) word  $\Rightarrow$  nat where
  lowestOneBit v = MinOrHighest {n . bit v n} (size v)

```

```

lemma max-bit: bit (v::('a::len) word) n  $\implies n < \text{size } v$ 
  by (simp add: bit-imp-le-length size-word.rep-eq)

```

```

lemma max-set-bit: MaxOrNeg {n . bit (v::('a::len) word) n} < Nat.size v
  using max-bit unfolding MaxOrNeg-def
  by force

```

4.1 Long.numberOfLeadingZeros

```

definition numberOfLeadingZeros :: ('a::len) word  $\Rightarrow$  nat where
  numberOfLeadingZeros v = nat (Nat.size v - highestOneBit v - 1)

```

```

lemma MaxOrNeg-neg: MaxOrNeg {} = -1
  by (simp add: MaxOrNeg-def)

```

lemma *MaxOrNeg-max*: $s \neq \{\} \implies \text{MaxOrNeg } s = \text{Max } s$
by (*simp add: MaxOrNeg-def*)

lemma *zero-no-bits*:
 $\{n . \text{bit } 0 \ n\} = \{\}$
by *simp*

lemma *highestOneBit* ($0::64 \text{ word}$) = -1
by (*simp add: MaxOrNeg-neg highestOneBit-def*)

lemma *numberOfLeadingZeros* ($0::64 \text{ word}$) = 64
unfolding *numberOfLeadingZeros-def* **using** *MaxOrNeg-neg highestOneBit-def size64*
by (*smt (verit) nat-int zero-no-bits*)

lemma *highestOneBit-top*: $\text{Max } \{\text{highestOneBit } (v::64 \text{ word})\} < 64$
unfolding *highestOneBit-def*
by (*metis Max-singleton int-eq-iff-numeral max-set-bit size64*)

lemma *numberOfLeadingZeros-top*: $\text{Max } \{\text{numberOfLeadingZeros } (v::64 \text{ word})\} \leq 64$
unfolding *numberOfLeadingZeros-def*
using *size64*
by (*simp add: MaxOrNeg-def highestOneBit-def nat-le-iff*)

lemma *numberOfLeadingZeros-range*: $0 \leq \text{numberOfLeadingZeros } a \wedge \text{numberOfLeadingZeros } a \leq \text{Nat.size } a$
unfolding *numberOfLeadingZeros-def*
using *MaxOrNeg-def highestOneBit-def nat-le-iff*
by (*smt (verit) bot-nat-0.extremum int-eq-iff*)

lemma *leadingZerosAddHighestOne*: $\text{numberOfLeadingZeros } v + \text{highestOneBit } v = \text{Nat.size } v - 1$
unfolding *numberOfLeadingZeros-def highestOneBit-def*
using *MaxOrNeg-def int-nat-eq int-ops(6) max-bit order-less-irrefl* **by** *fastforce*

4.2 Long.numberOfTrailingZeros

definition *numberOfTrailingZeros* :: $('a::\text{len}) \text{ word} \Rightarrow \text{nat}$ **where**
 $\text{numberOfTrailingZeros } v = \text{lowestOneBit } v$

lemma *lowestOneBit-bot*: $\text{lowestOneBit } (0::64 \text{ word}) = 64$
unfolding *lowestOneBit-def MinOrHighest-def*
by (*simp add: size64*)

lemma *bit-zero-set-in-top*: $\text{bit } (-1::'a::\text{len} \text{ word}) \ 0$
by *auto*

lemma *nat-bot-set*: $(0::nat) \in xs \longrightarrow (\forall x \in xs . 0 \leq x)$
by *fastforce*

lemma *numberOfTrailingZeros* $(0::64 \text{ word}) = 64$
unfolding *numberOfTrailingZeros-def*
using *lowestOneBit-bot* **by** *simp*

4.3 Long.bitCount

definition *bitCount* :: $('a::len) \text{ word} \Rightarrow nat$ **where**
bitCount $v = \text{card } \{n . \text{bit } v \ n\}$

lemma *bitCount 0 = 0*
unfolding *bitCount-def*
by $(metis \text{card.empty zero-no-bits})$

4.4 Long.zeroCount

definition *zeroCount* :: $('a::len) \text{ word} \Rightarrow nat$ **where**
zeroCount $v = \text{card } \{n . n < \text{Nat.size } v \wedge \neg(\text{bit } v \ n)\}$

lemma *zeroCount-finite*: $\text{finite } \{n . n < \text{Nat.size } v \wedge \neg(\text{bit } v \ n)\}$
using *finite-nat-set-iff-bounded* **by** *blast*

lemma *negone-set*:
 $\text{bit } (-1::('a::len) \text{ word}) \ n \longleftrightarrow n < \text{LENGTH}('a)$
by *simp*

lemma *negone-all-bits*:
 $\{n . \text{bit } (-1::('a::len) \text{ word}) \ n\} = \{n . 0 \leq n \wedge n < \text{LENGTH}('a)\}$
using *negone-set*
by *auto*

lemma *bitCount-finite*:
 $\text{finite } \{n . \text{bit } (v::('a::len) \text{ word}) \ n\}$
by *simp*

lemma *card-of-range*:
 $x = \text{card } \{n . 0 \leq n \wedge n < x\}$
by *simp*

lemma *range-of-nat*:
 $\{(n::nat) . 0 \leq n \wedge n < x\} = \{n . n < x\}$
by *simp*

lemma *finite-range*:
 $\text{finite } \{n::nat . n < x\}$
by *simp*

```

lemma range-eq:
  fixes  $x\ y :: \text{nat}$ 
  shows  $\text{card } \{y..<x\} = \text{card } \{y<..x\}$ 
  using card-atLeastLessThan card-greaterThanAtMost by presburger

lemma card-of-range-bound:
  fixes  $x\ y :: \text{nat}$ 
  assumes  $x > y$ 
  shows  $x - y = \text{card } \{n . y < n \wedge n \leq x\}$ 
proof -
  have finite:  $\text{finite } \{n . y \leq n \wedge n < x\}$ 
    by auto
  have nonempty:  $\{n . y \leq n \wedge n < x\} \neq \{\}$ 
    using assms by blast
  have simp:  $\{n . y < n \wedge n \leq x\} = \{y<..x\}$ 
    by auto
  have  $x - y = \text{card } \{y<..x\}$ 
    by auto
  then show ?thesis
    unfolding simp by blast
qed

lemma bitCount  $(-1::('a::\text{len}) \text{ word}) = \text{LENGTH}('a)$ 
  unfolding bitCount-def using card-of-range
  by (metis (no-types, lifting) Collect-cong negone-all-bits)

lemma bitCount-range:
  fixes  $n :: ('a::\text{len}) \text{ word}$ 
  shows  $0 \leq \text{bitCount } n \wedge \text{bitCount } n \leq \text{Nat.size } n$ 
  unfolding bitCount-def
  by (metis atLeastLessThan-iff bot-nat-0.extremum max-bit mem-Collect-eq subsetI
subset-eq-atLeast0-lessThan-card)

lemma zerosAboveHighestOne:
   $n > \text{highestOneBit } a \implies \neg(\text{bit } a\ n)$ 
  unfolding highestOneBit-def MaxOrNeg-def
  by (metis (mono-tags, opaque-lifting) Collect-empty-eq Max-ge finite-bit-word
less-le-not-le mem-Collect-eq of-nat-le-iff)

lemma zerosBelowLowestOne:
  assumes  $n < \text{lowestOneBit } a$ 
  shows  $\neg(\text{bit } a\ n)$ 
proof (cases  $\{i. \text{bit } a\ i\} = \{\}$ )
  case True
  then show ?thesis by simp
next
  case False
  have  $n < \text{Min } (\text{Collect } (\text{bit } a)) \implies \neg \text{bit } a\ n$ 
    using False by auto

```

then show *?thesis*
by (*metis False MinOrHighest-def assms lowestOneBit-def*)
qed

lemma *union-bit-sets*:
fixes $a :: ('a::len) \text{ word}$
shows $\{n . n < \text{Nat.size } a \wedge \text{bit } a \ n\} \cup \{n . n < \text{Nat.size } a \wedge \neg(\text{bit } a \ n)\} = \{n . n < \text{Nat.size } a\}$
by *fastforce*

lemma *disjoint-bit-sets*:
fixes $a :: ('a::len) \text{ word}$
shows $\{n . n < \text{Nat.size } a \wedge \text{bit } a \ n\} \cap \{n . n < \text{Nat.size } a \wedge \neg(\text{bit } a \ n)\} = \{\}$
by *blast*

lemma *qualified-bitCount*:
 $\text{bitCount } v = \text{card } \{n . n < \text{Nat.size } v \wedge \text{bit } v \ n\}$
by (*metis (no-types, lifting) Collect-cong bitCount-def max-bit*)

lemma *card-eq*:
assumes $\text{finite } x \wedge \text{finite } y \wedge \text{finite } z$
assumes $x \cup y = z$
assumes $y \cap x = \{\}$
shows $\text{card } z - \text{card } y = \text{card } x$
using *assms add-diff-cancel-right' card-Un-disjoint*
by (*metis inf.commute*)

lemma *card-add*:
assumes $\text{finite } x \wedge \text{finite } y \wedge \text{finite } z$
assumes $x \cup y = z$
assumes $y \cap x = \{\}$
shows $\text{card } x + \text{card } y = \text{card } z$
using *assms card-Un-disjoint*
by (*metis inf.commute*)

lemma *card-add-inverses*:
assumes $\text{finite } \{n. Q \ n \wedge \neg(P \ n)\} \wedge \text{finite } \{n. Q \ n \wedge P \ n\} \wedge \text{finite } \{n. Q \ n\}$
shows $\text{card } \{n. Q \ n \wedge P \ n\} + \text{card } \{n. Q \ n \wedge \neg(P \ n)\} = \text{card } \{n. Q \ n\}$
apply (*rule card-add*)
using *assms apply simp*
apply *auto[1]*
by *auto*

lemma *ones-zero-sum-to-width*:
 $\text{bitCount } a + \text{zeroCount } a = \text{Nat.size } a$
proof –
have *add-cards*: $\text{card } \{n. (\lambda n. n < \text{size } a) \ n \wedge (\text{bit } a \ n)\} + \text{card } \{n. (\lambda n. n < \text{size } a) \ n \wedge \neg(\text{bit } a \ n)\} = \text{card } \{n. (\lambda n. n < \text{size } a) \ n\}$

```

    apply (rule card-add-inverses) by simp
  then have ... = Nat.size a
    by auto
  then show ?thesis
    unfolding bitCount-def zeroCount-def using max-bit
    by (metis (mono-tags, lifting) Collect-cong add-cards)
qed

lemma intersect-bitCount-helper:
  card {n . n < Nat.size a} - bitCount a = card {n . n < Nat.size a ∧ ¬(bit a n)}
proof -
  have size-def: Nat.size a = card {n . n < Nat.size a}
    using card-of-range by simp
  have bitCount-def: bitCount a = card {n . n < Nat.size a ∧ bit a n}
    using qualified-bitCount by auto
  have disjoint: {n . n < Nat.size a ∧ bit a n} ∩ {n . n < Nat.size a ∧ ¬(bit a n)} = {}
    using disjoint-bit-sets by auto
  have union: {n . n < Nat.size a ∧ bit a n} ∪ {n . n < Nat.size a ∧ ¬(bit a n)} = {n . n < Nat.size a}
    using union-bit-sets by auto
  show ?thesis
    unfolding bitCount-def
    apply (rule card-eq)
    using finite-range apply simp
    using union apply blast
    using disjoint by simp
qed

lemma intersect-bitCount:
  Nat.size a - bitCount a = card {n . n < Nat.size a ∧ ¬(bit a n)}
  using card-of-range intersect-bitCount-helper by auto

hide-fact intersect-bitCount-helper

end

```

4.5 Structural Graph Comparison

```

theory
  Comparison
imports
  IRGraph
begin

```

We introduce a form of structural graph comparison that is able to assert structural equivalence of graphs which differ in zero or more reference node chains for any given nodes.

```

fun find-ref-nodes :: IRGraph ⇒ (ID ⇒ ID) where

```

```

find-ref-nodes g = map-of
  (map (λn. (n, ir-ref (kind g n))) (filter (λid. is-RefNode (kind g id)) (sorted-list-of-set
    (ids g))))

```

```

fun replace-ref-nodes :: IRGraph ⇒ (ID → ID) ⇒ ID list ⇒ ID list where
replace-ref-nodes g m xs = map (λid. (case (m id) of Some other ⇒ other | None
  ⇒ id)) xs

```

```

fun find-next :: ID list ⇒ ID set ⇒ ID option where
find-next to-see seen = (let l = (filter (λnid. nid ∉ seen) to-see)
  in (case l of [] ⇒ None | xs ⇒ Some (hd xs)))

```

```

inductive reachables :: IRGraph ⇒ ID list ⇒ ID set ⇒ ID set ⇒ bool where
reachables g [] {} {} |
[[None = find-next to-see seen]] ⇒ reachables g to-see seen seen |
[Some n = find-next to-see seen;
 node = kind g n;
 new = (inputs-of node) @ (successors-of node);
 reachables g (to-see @ new) ({n} ∪ seen) seen'] ⇒ reachables g to-see seen
seen'

```

```

code-pred (modes: i ⇒ i ⇒ i ⇒ o ⇒ bool) [show-steps, show-mode-inference, show-intermediate-results]

```

```

reachables .

```

```

inductive nodeEq :: (ID → ID) ⇒ IRGraph ⇒ ID ⇒ IRGraph ⇒ ID ⇒ bool
where
[[ kind g1 n1 = RefNode ref; nodeEq m g1 ref g2 n2 ]] ⇒ nodeEq m g1 n1 g2 n2
|
[[ x = kind g1 n1;
 y = kind g2 n2;
 is-same-ir-node-type x y;
 replace-ref-nodes g1 m (successors-of x) = successors-of y;
 replace-ref-nodes g1 m (inputs-of x) = inputs-of y ]]
⇒ nodeEq m g1 n1 g2 n2

```

```

code-pred [show-modes] nodeEq .

```

```

fun diffNodesGraph :: IRGraph ⇒ IRGraph ⇒ ID set where
diffNodesGraph g1 g2 = (let refNodes = find-ref-nodes g1 in
  { n . n ∈ Predicate.the (reachables-i-i-i-o g1 [0] {}) ∧ (case refNodes n of Some
    - ⇒ False | - ⇒ True) ∧ ¬(nodeEq refNodes g1 n g2 n)})

```

```

fun diffNodesInfo :: IRGraph ⇒ IRGraph ⇒ (ID × IRNode × IRNode) set where
diffNodesInfo g1 g2 = {(nid, kind g1 nid, kind g2 nid) | nid . nid ∈ diffNodesGraph
  g1 g2}

```

```

fun eqGraph :: IRGraph ⇒ IRGraph ⇒ bool (infix ≈s 20)
where

```

```
eqGraph isabelle-graph graal-graph = ((diffNodesGraph isabelle-graph graal-graph)
= {})


```

```
end


```

4.6 Control-flow Graph Traversal

```
theory


```

```
  Traversal


```

```
imports


```

```
  IRGraph


```

```
begin


```

```
type-synonym Seen = ID set


```

nextEdge helps determine which node to traverse next by returning the first successor edge that isn't in the set of already visited nodes. If there is not an appropriate successor, None is returned instead.

```
fun nextEdge :: Seen ⇒ ID ⇒ IRGraph ⇒ ID option where
  nextEdge seen nid g =
    (let nids = (filter (λnid'. nid' ∉ seen) (successors-of (kind g nid))) in
     (if length nids > 0 then Some (hd nids) else None))


```

pred determines which node, if any, acts as the predecessor of another.

Merge nodes represent a special case where-in the predecessor exists as an input edge of the merge node, to simplify the traversal we treat only the first input end node as the predecessor, ignoring that multiple nodes may act as a successor.

For all other nodes, the predecessor is the first element of the predecessors set. Note that in a well-formed graph there should only be one element in the predecessor set.

```
fun pred :: IRGraph ⇒ ID ⇒ ID option where
  pred g nid = (case kind g nid of
    (MergeNode ends -) ⇒ Some (hd ends) |
    - ⇒
      (if IRGraph.predecessors g nid = {}
       then None else
        Some (hd (sorted-list-of-set (IRGraph.predecessors g nid)))
      )
  )


```

Here we try to implement a generic fork of the control-flow traversal algorithm that was initially implemented for the ConditionalElimination phase

```
type-synonym 'a TraversalState = (ID × Seen × 'a)


```

```
inductive Step


```


$:: ('a \text{ TraversalState} \Rightarrow 'a) \Rightarrow \text{IRGraph} \Rightarrow 'a \text{ TraversalState} \Rightarrow 'a \text{ TraversalState}$
 $\text{option} \Rightarrow \text{bool}$

for $sa \ g$ **where**

— Hit a BeginNode with an IfNode predecessor which represents the start of a basic block for the IfNode. 1. nid' will be the successor of the begin node. 2. Find the first and only predecessor. 3. Extract condition from the preceding IfNode. 4. Negate condition if the begin node is second branch (we've taken the else branch of the condition) 5. Add the condition or the negated condition to stack 6. Perform any stamp updates based on the condition using the registerNewCondition function and place them on the top of the stack of stamp information

$\llbracket kind \ g \ nid = \text{BeginNode } nid' \rrbracket;$

$nid \notin \text{seen};$
 $\text{seen}' = \{nid\} \cup \text{seen};$

$\text{Some } ifcond = \text{pred } g \ nid;$
 $kind \ g \ ifcond = \text{IfNode } cond \ t \ f;$

$analysis' = sa \ (nid, \text{seen}, analysis)$
 $\implies \text{Step } sa \ g \ (nid, \text{seen}, analysis) \ (\text{Some } (nid', \text{seen}', analysis')) \mid$

— Hit an EndNode 1. nid' will be the usage of EndNode 2. pop the conditions and stamp stack

$\llbracket kind \ g \ nid = \text{EndNode};$

$nid \notin \text{seen};$
 $\text{seen}' = \{nid\} \cup \text{seen};$

$nid' = \text{any-usage } g \ nid;$

$analysis' = sa \ (nid, \text{seen}, analysis)$
 $\implies \text{Step } sa \ g \ (nid, \text{seen}, analysis) \ (\text{Some } (nid', \text{seen}', analysis')) \mid$

— We can find a successor edge that is not in seen, go there

$\llbracket \neg(is\text{-EndNode } (kind \ g \ nid));$
 $\neg(is\text{-BeginNode } (kind \ g \ nid));$

$nid \notin \text{seen};$
 $\text{seen}' = \{nid\} \cup \text{seen};$

$\text{Some } nid' = \text{nextEdge } \text{seen}' \ nid \ g;$

$analysis' = sa \ (nid, \text{seen}, analysis)$
 $\implies \text{Step } sa \ g \ (nid, \text{seen}, analysis) \ (\text{Some } (nid', \text{seen}', analysis')) \mid$

— We cannot find a successor edge that is not in seen, give back None

$\llbracket \neg(is\text{-EndNode } (kind \ g \ nid));$
 $\neg(is\text{-BeginNode } (kind \ g \ nid));$

```

     $nid \notin seen;$ 
     $seen' = \{nid\} \cup seen;$ 

     $None = nextEdge\ seen'\ nid\ g$ 
     $\implies Step\ sa\ g\ (nid,\ seen,\ analysis)\ None\ |$ 

    — We've already seen this node, give back None
     $\llbracket nid \in seen \rrbracket \implies Step\ sa\ g\ (nid,\ seen,\ analysis)\ None$ 

code-pred ( $modes: i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow bool$ )  $Step$  .

end

```