Veriopt Theories

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Contents

1 Optization DSL

1.1 Markup

theory *Markup*
  imports *Semantics.IRTreeEval Snippets.Snipping*
begin

datatype *'a Rewrite* =
  Transform 'a 'a (- ⟼ - *10*) |
  Conditional 'a 'a bool (- ⟼ - *when* - *11*) |
  Sequential 'a Rewrite 'a Rewrite |
  Transitive 'a Rewrite

datatype *'a ExtraNotation* =
  ConditionalNotation 'a 'a 'a (- *?* - : - *50*) |
  EqualsNotation 'a 'a (- *eq* -) |
  ConstantNotation 'a (*const* - *120*) |
  TrueNotation (*true*) |
  FalseNotation (*false*) |
  ExclusiveOr 'a 'a (- ⊕ -) |
  LogicNegationNotation 'a (!-) |

ShortCircuitOr 'a 'a (- || -) |
Remainder 'a 'a (- % -)

definition *word* :: ('a::*len*) *word* ⇒ 'a *word* **where**
word x = x

ML-val @{*term* ‹*x % x*›}
ML-file ‹*markup.ML*›

1.1.1 Expression Markup

ML ‹
structure IRExprTranslator : DSL-TRANSLATION =
struct
fun markup DSL-Tokens.Add = @{term BinaryExpr} $ @{term BinAdd}
| markup DSL-Tokens.Sub = @{term BinaryExpr} $ @{term BinSub}
| markup DSL-Tokens.Mul = @{term BinaryExpr} $ @{term BinMul}
| markup DSL-Tokens.Div = @{term BinaryExpr} $ @{term BinDiv}
| markup DSL-Tokens.Rem = @{term BinaryExpr} $ @{term BinMod}
| markup DSL-Tokens.And = @{term BinaryExpr} $ @{term BinAnd}
| markup DSL-Tokens.Or = @{term BinaryExpr} $ @{term BinOr}
| markup DSL-Tokens.Xor = @{term BinaryExpr} $ @{term BinXor}
| markup DSL-Tokens.ShortCircuitOr = @{term BinaryExpr} $ @{term BinShortCircuitOr}
| markup DSL-Tokens.Abs = @{term UnaryExpr} $ @{term UnaryAbs}
| markup DSL-Tokens.Less = @{term BinaryExpr} $ @{term BinIntegerLessThan}
| markup DSL-Tokens.Equals = @{term BinaryExpr} $ @{term BinIntegerEquals}
| markup DSL-Tokens.Not = @{term UnaryExpr} $ @{term UnaryNot}
| markup DSL-Tokens.Negate = @{term UnaryExpr} $ @{term UnaryNeg}
| markup DSL-Tokens.LogicNegate = @{term UnaryExpr} $ @{term UnaryLogicNegation}
| markup DSL-Tokens.LeftShift = @{term BinaryExpr} $ @{term BinLeftShift}
| markup DSL-Tokens.RightShift = @{term BinaryExpr} $ @{term BinRightShift}
| markup DSL-Tokens.UnsignedRightShift = @{term BinaryExpr} $ @{term BinURightShift}
| markup DSL-Tokens.Conditional = @{term ConditionalExpr}
| markup DSL-Tokens.Constant = @{term ConstantExpr}
| markup DSL-Tokens.TrueConstant = @{term ConstantExpr (IntVal 32 1)}
| markup DSL-Tokens.FalseConstant = @{term ConstantExpr (IntVal 32 0)}
end
structure IRExprMarkup = DSL-Markup(IRExprTranslator);
›

ir expression translation

syntax *-expandExpr* :: *term* ⇒ *term* (*exp*[-])
parse-translation ‹ [(@{*syntax-const -expandExpr*} , *IRExprMarkup.markup-expr* [])] ›

ir expression example

value *exp*[$(e_1 < e_2)$ *?* e_1 : e_2]

ConditionalExpr (*BinaryExpr* *BinIntegerLessThan* (e_1::*IRExpr*) (e_2::*IRExpr*)) e_1 e_2

1.1.2 Value Markup

ML ‹
structure IntValTranslator : *DSL-TRANSLATION* =
struct
fun markup DSL-Tokens.Add = @{*term intval-add*}
| *markup DSL-Tokens.Sub* = @{*term intval-sub*}
| *markup DSL-Tokens.Mul* = @{*term intval-mul*}
| *markup DSL-Tokens.Div* = @{*term intval-div*}
| *markup DSL-Tokens.Rem* = @{*term intval-mod*}
| *markup DSL-Tokens.And* = @{*term intval-and*}
| *markup DSL-Tokens.Or* = @{*term intval-or*}
| *markup DSL-Tokens.ShortCircuitOr* = @{*term intval-short-circuit-or*}
| *markup DSL-Tokens.Xor* = @{*term intval-xor*}
| *markup DSL-Tokens.Abs* = @{*term intval-abs*}
| *markup DSL-Tokens.Less* = @{*term intval-less-than*}
| *markup DSL-Tokens.Equals* = @{*term intval-equals*}
| *markup DSL-Tokens.Not* = @{*term intval-not*}
| *markup DSL-Tokens.Negate* = @{*term intval-negate*}
| *markup DSL-Tokens.LogicNegate* = @{*term intval-logic-negation*}
| *markup DSL-Tokens.LeftShift* = @{*term intval-left-shift*}
| *markup DSL-Tokens.RightShift* = @{*term intval-right-shift*}
| *markup DSL-Tokens.UnsignedRightShift* = @{*term intval-uright-shift*}
| *markup DSL-Tokens.Conditional* = @{*term intval-conditional*}
| *markup DSL-Tokens.Constant* = @{*term IntVal 32*}
| *markup DSL-Tokens.TrueConstant* = @{*term IntVal 32 1*}
| *markup DSL-Tokens.FalseConstant* = @{*term IntVal 32 0*}
end
structure IntValMarkup = *DSL-Markup*(*IntValTranslator*);
›

value expression translation

syntax *-expandIntVal* :: *term* ⇒ *term* (*val*[-])
parse-translation ‹ [(@{*syntax-const* *-expandIntVal*} , *IntVal-Markup.markup-expr* [])] ›

value expression example

value $val[(e_1 < e_2)\ ?\ e_1 : e_2]$

$intval\text{-}conditional\ (intval\text{-}less\text{-}than\ (e_1 :: Value)\ (e_2 :: Value))\ e_1\ e_2$

1.1.3 Word Markup

ML ‹
structure WordTranslator : DSL-TRANSLATION =
struct
fun markup DSL-Tokens.Add = @{term plus}
| markup DSL-Tokens.Sub = @{term minus}
| markup DSL-Tokens.Mul = @{term times}
| markup DSL-Tokens.Div = @{term signed-divide}
| markup DSL-Tokens.Rem = @{term signed-modulo}
| markup DSL-Tokens.And = @{term Bit-Operations.semiring-bit-operations-class.and}
| markup DSL-Tokens.Or = @{term or}
| markup DSL-Tokens.Xor = @{term xor}
| markup DSL-Tokens.Abs = @{term abs}
| markup DSL-Tokens.Less = @{term less}
| markup DSL-Tokens.Equals = @{term HOL.eq}
| markup DSL-Tokens.Not = @{term not}
| markup DSL-Tokens.Negate = @{term uminus}
| markup DSL-Tokens.LogicNegate = @{term logic-negate}
| markup DSL-Tokens.LeftShift = @{term shiftl}
| markup DSL-Tokens.RightShift = @{term signed-shiftr}
| markup DSL-Tokens.UnsignedRightShift = @{term shiftr}
| markup DSL-Tokens.Constant = @{term word}
| markup DSL-Tokens.TrueConstant = @{term 1}
| markup DSL-Tokens.FalseConstant = @{term 0}
end
structure WordMarkup = DSL-Markup(WordTranslator);
›

word expression translation

syntax *-expandWord :: term ⇒ term* (*bin*[-])
parse-translation ‹ [(@{*syntax-const -expandWord*} , *WordMarkup.markup-expr* [])] ›

word expression example

value $bin[x\ \&\ y \mid z]$

$intval\text{-}conditional\ (intval\text{-}less\text{-}than\ (e_1 :: Value)\ (e_2 :: Value))\ e_1\ e_2$

value $bin[-x]$

value *val*[−*x*]
value *exp*[−*x*]

value *bin*[!*x*]
value *val*[!*x*]
value *exp*[!*x*]

value *bin*[¬*x*]
value *val*[¬*x*]
value *exp*[¬*x*]

value *bin*[~*x*]
value *val*[~*x*]
value *exp*[~*x*]

value ~*x*

end

1.2 Optimization Phases

theory *Phase*
  imports *Main*
begin

ML-file *map.ML*
ML-file *phase.ML*

end

1.3 Canonicalization DSL

theory *Canonicalization*
  imports
    Markup
    Phase
    HOL−Eisbach.Eisbach
  keywords
    phase :: *thy-decl* **and**
    terminating :: *quasi-command* **and**
    print-phases :: *diag* **and**
    export-phases :: *thy-decl* **and**
    optimization :: *thy-goal-defn*
begin

print-methods

ML ‹
datatype 'a *Rewrite* =
  Transform of 'a ∗ 'a |

```
  Conditional of 'a * 'a * term |
  Sequential of 'a Rewrite * 'a Rewrite |
  Transitive of 'a Rewrite

type rewrite = {
  name: binding,
  rewrite: term Rewrite,
  proofs: thm list,
  code: thm list,
  source: term
}

structure RewriteRule : Rule =
struct
type T = rewrite;

(*
fun pretty-rewrite ctxt (Transform (from, to)) =
      Pretty.block [
        Syntax.pretty-term ctxt from,
        Pretty.str ↦ ,
        Syntax.pretty-term ctxt to
      ]
  | pretty-rewrite ctxt (Conditional (from, to, cond)) =
      Pretty.block [
        Syntax.pretty-term ctxt from,
        Pretty.str ↦ ,
        Syntax.pretty-term ctxt to,
        Pretty.str  when ,
        Syntax.pretty-term ctxt cond
      ]
  | pretty-rewrite - - = Pretty.str not implemented*)

fun pretty-thm ctxt thm =
  (Proof-Context.pretty-fact ctxt (, [thm]))

fun pretty ctxt obligations t =
  let
    val is-skipped = Thm-Deps.has-skip-proof (#proofs t);

    val warning = (if is-skipped
      then [Pretty.str (proof skipped), Pretty.brk 0]
      else []);

    val obligations = (if obligations
      then [Pretty.big-list
              obligations:
              (map (pretty-thm ctxt) (#proofs t)),
            Pretty.brk 0]
```

```
      else []);

    fun pretty-bind binding =
      Pretty.markup
        (Position.markup (Binding.pos-of binding) Markup.position)
        [Pretty.str (Binding.name-of binding)];

  in
  Pretty.block ([
    pretty-bind (#name t), Pretty.str : ,
    Syntax.pretty-term ctxt (#source t), Pretty.fbrk
  ] @ obligations @ warning)
  end
end

structure RewritePhase = DSL-Phase(RewriteRule);

val - =
  Outer-Syntax.command command-keyword ‹phase› enter an optimization phase
    (Parse.binding --| Parse.$$$ terminating -- Parse.const --| Parse.begin
      >> (Toplevel.begin-main-target true o RewritePhase.setup));

fun print-phases print-obligations ctxt =
  let
    val thy = Proof-Context.theory-of ctxt;
    fun print phase = RewritePhase.pretty print-obligations phase ctxt
  in
    map print (RewritePhase.phases thy)
  end

fun print-optimizations print-obligations thy =
  print-phases print-obligations thy |> Pretty.writeln-chunks

val - =
  Outer-Syntax.command command-keyword ‹print-phases›
    print debug information for optimizations
    (Parse.opt-bang >>
      (fn b => Toplevel.keep ((print-optimizations b) o Toplevel.context-of)));

fun export-phases thy name =
  let
    val state = Toplevel.theory-toplevel thy;
    val ctxt = Toplevel.context-of state;
    val content = Pretty.string-of (Pretty.chunks (print-phases false ctxt));
    val cleaned = YXML.content-of content;


    val filename = Path.explode (name^.rules);
    val directory = Path.explode optimizations;
```

```
    val path = Path.binding (
              Path.append directory filename,
              Position.none);
    val thy' = thy |> Generated-Files.add-files (path, (Bytes.string content));

    val - = Export.export thy' path [YXML.parse cleaned];

    val - = writeln (Export.message thy' (Path.basic optimizations));
  in
    thy'
  end

val - =
  Outer-Syntax.command command-keyword ‹export-phases›
    export information about encoded optimizations
    (Parse.path >>
      (fn name => Toplevel.theory (fn state => export-phases state name)))
›
```

ML-file *rewrites.ML*

1.3.1 Semantic Preservation Obligation

fun *rewrite-preservation* :: *IRExpr Rewrite* ⇒ *bool* **where**
rewrite-preservation (*Transform x y*) = (*y* ≤ *x*) |
rewrite-preservation (*Conditional x y cond*) = (*cond* ⟶ (*y* ≤ *x*)) |
rewrite-preservation (*Sequential x y*) = (*rewrite-preservation x* ∧ *rewrite-preservation y*) |
rewrite-preservation (*Transitive x*) = *rewrite-preservation x*

1.3.2 Termination Obligation

fun *rewrite-termination* :: *IRExpr Rewrite* ⇒ (*IRExpr* ⇒ *nat*) ⇒ *bool* **where**
rewrite-termination (*Transform x y*) *trm* = (*trm x* > *trm y*) |
rewrite-termination (*Conditional x y cond*) *trm* = (*cond* ⟶ (*trm x* > *trm y*)) |
rewrite-termination (*Sequential x y*) *trm* = (*rewrite-termination x trm* ∧ *rewrite-termination y trm*) |
rewrite-termination (*Transitive x*) *trm* = *rewrite-termination x trm*

fun *intval* :: *Value Rewrite* ⇒ *bool* **where**
intval (*Transform x y*) = (*x* ≠ *UndefVal* ∧ *y* ≠ *UndefVal* ⟶ *x* = *y*) |
intval (*Conditional x y cond*) = (*cond* ⟶ (*x* = *y*)) |
intval (*Sequential x y*) = (*intval x* ∧ *intval y*) |
intval (*Transitive x*) = *intval x*

1.3.3 Standard Termination Measure

fun *size* :: *IRExpr* ⇒ *nat* **where**
unary-size:
size (*UnaryExpr op x*) = (*size x*) + *2* |

bin-const-size:
size (*BinaryExpr op x* (*ConstantExpr cy*)) = (*size x*) + *2* |
bin-size:
size (*BinaryExpr op x y*) = (*size x*) + (*size y*) + *2* |
cond-size:
size (*ConditionalExpr c t f*) = (*size c*) + (*size t*) + (*size f*) + *2* |
const-size:
size (*ConstantExpr c*) = *1* |
param-size:
size (*ParameterExpr ind s*) = *2* |
leaf-size:
size (*LeafExpr nid s*) = *2* |
size (*ConstantVar c*) = *2* |
size (*VariableExpr x s*) = *2*

1.3.4 Automated Tactics

named-theorems *size-simps size simplication rules*

method *unfold-optimization* =
(*unfold rewrite-preservation.simps*, *unfold rewrite-termination.simps*,
unfold intval.simps,
rule conjE, *simp*, *simp del*: *le-expr-def*, *force?*)
| (*unfold rewrite-preservation.simps*, *unfold rewrite-termination.simps*,
rule conjE, *simp*, *simp del*: *le-expr-def*, *force?*)

method *unfold-size* =
(((*unfold size.simps*, *simp add*: *size-simps del*: *le-expr-def*)*?*
; (*simp add*: *size-simps del*: *le-expr-def*)*?*
; (*auto simp*: *size-simps*)*?*
; (*unfold size.simps*)*?*)[*1*])

print-methods

ML ‹
structure System : *RewriteSystem* =
struct
val preservation = @{*const rewrite-preservation*};
val termination = @{*const rewrite-termination*};
val intval = @{*const intval*};
end

structure DSL = *DSL-Rewrites*(*System*);

val - =
Outer-Syntax.local-theory-to-proof ***command-keyword*** ‹*optimization*›
define an optimization and open proof obligation

```
    (Parse-Spec.thm-name : -- Parse.term
        >> DSL.rewrite-cmd);
›

end
```