

Unspecified Veriopt Theory

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begin

notation (*latex*)

kind ($\langle\!\langle\!-\!\rangle\!\rangle$)

notation (*latex*)

IRTreeEval.ord-IRExpr-inst.less-eq-IRExpr ($- \longmapsto -$)

abstract-syntax-tree

```
datatype IRExpr = UnaryExpr IRUnaryOp IRExpr
| BinaryExpr IRBinaryOp IRExpr IRExpr
| ConditionalExpr IRExpr IRExpr IRExpr
| ParameterExpr nat Stamp
| LeafExpr nat Stamp | ConstantExpr Value
| ConstantVar (char list)
| VariableExpr (char list) Stamp
```

tree-semantics

semantics:constant *semantics:parameter* *semantics:conditional* *semantics:unary* *semantics:binary* *semantics:leaf*

tree-evaluation-deterministic

$\llbracket [m,p] \vdash e \mapsto v1; [m,p] \vdash e \mapsto v2 \rrbracket \implies v1 = v2$

expression-refinement

$$(e2 \leq e1) = (\forall m \ p \ v. [m,p] \vdash e1 \mapsto v \longrightarrow [m,p] \vdash e2 \mapsto v)$$

expression-refinement-monotone

$$\begin{aligned} e' \leq e &\implies \text{UnaryExpr } op \ e' \leq \text{UnaryExpr } op \ e \\ \llbracket x' \leq x; y' \leq y \rrbracket &\implies \text{BinaryExpr } op \ x' \ y' \leq \text{BinaryExpr } op \ x \ y \\ \llbracket ce' \leq ce; te' \leq te; fe' \leq fe \rrbracket &\implies \text{ConditionalExpr } ce' \ te' \ fe' \leq \\ &\quad \text{ConditionalExpr } ce \ te \ fe \end{aligned}$$

graph-representation

typedef *IRGraph* = {*g* :: *ID* \rightarrow *IRNode* . *finite* (*dom g*)}

graph2tree

semantics:constant semantics:parameter semantics:conditional semantics:unary semantics:convert semantics:binary semantics:leaf

preeval

is-preevaluated (*InvokeNode* *n uu uv uw ux uy*) = *True*
is-preevaluated (*InvokeWithExceptionNode* *n uz va vb vc vd ve*) = *True*
is-preevaluated (*NewInstanceNode* *n vf vg vh*) = *True*
is-preevaluated (*LoadFieldNode* *n vi vj vk*) = *True*
is-preevaluated (*SignedDivNode* *n vl vm vn vo vp*) = *True*
is-preevaluated (*SignedRemNode* *n vq vr vs vt vu*) = *True*
is-preevaluated (*ValuePhiNode* *n vv vw*) = *True*
is-preevaluated (*AbsNode* *v*) = *False*
is-preevaluated (*AddNode* *v va*) = *False*
is-preevaluated (*AndNode* *v va*) = *False*
is-preevaluated (*BeginNode* *v*) = *False*
is-preevaluated (*BytecodeExceptionNode* *v va vb*) = *False*
is-preevaluated (*ConditionalNode* *v va vb*) = *False*
is-preevaluated (*ConstantNode* *v*) = *False*
is-preevaluated (*DynamicNewArrayNode* *v va vb vc vd*) = *False*
is-preevaluated *EndNode* = *False*
is-preevaluated (*ExceptionObjectNode* *v va*) = *False*
is-preevaluated (*FrameState* *v va vb vc*) = *False*
is-preevaluated (*IfNode* *v va vb*) = *False*
is-preevaluated (*IntegerBelowNode* *v va*) = *False*
is-preevaluated (*IntegerEqualsNode* *v va*) = *False*
is-preevaluated (*IntegerLessThanNode* *v va*) = *False*
is-preevaluated (*IsNullNode* *v*) = *False*
is-preevaluated (*KillingBeginNode* *v*) = *False*
is-preevaluated (*LeftShiftNode* *v va*) = *False*
is-preevaluated (*LogicNegationNode* *v*) = *False*
is-preevaluated (*LoopBeginNode* *v va vb vc*) = *False*
is-preevaluated (*LoopEndNode* *v*) = *False*
is-preevaluated (*LoopExitNode* *v va vb*) = *False*
is-preevaluated (*MergeNode* *v va vb*) = *False*
is-preevaluated (*MethodCallTargetNode* *v va*) = *False*
is-preevaluated (*MulNode* *v va*) = *False*
is-preevaluated (*NarrowNode* *v va vb*) = *False*
is-preevaluated (*NegateNode* *v*) = *False*
is-preevaluated (*NewArrayNode* *v va vb*) = *False*
is-preevaluated (*NotNode* *v*) = *False*
is-preevaluated (*OrNode* *v va*) = *False*
is-preevaluated (*ParameterNode* *v*) = *False*
is-preevaluated (*PiNode* *v va*) = *False*
is-preevaluated (*ReturnNode* *v va*) = *False*
is-preevaluated (*RightShiftNode* *v va*) = *False*
is-preevaluated (*ShortCircuitOrNode* *v va*) = *False*
is-preevaluated (*SignExtendNode* *v va vb*) = *False*

deterministic-representation

$$\llbracket g \vdash n \simeq e1; g \vdash n \simeq e2 \rrbracket \implies e1 = e2$$

graph-semantics

$$([g, m, p] \vdash n \mapsto v) = (\exists e. g \vdash n \simeq e \wedge [m, p] \vdash e \mapsto v)$$

graph-semantics-deterministic

$$[g, m, p] \vdash nid \mapsto v1 \wedge [g, m, p] \vdash nid \mapsto v2 \implies v1 = v2$$

graph-refinement

$$\begin{aligned} \text{graph-refinement } g1 \ g2 = \\ (\forall n. n \in \text{ids } g1 \longrightarrow \\ (\forall e1. g1 \vdash n \simeq e1 \longrightarrow (\exists e2. g2 \vdash n \simeq e2 \wedge e2 \leq e1))) \end{aligned}$$

translations

$$n \leq \text{CONST as-set } n$$

experiment begin

Experimental embedding into a simpler but usable form for expression nodes in a graph

datatype *ExprIRNode* =

ExprUnaryNode IRUnaryOp ID |
ExprBinaryNode IRBinaryOp ID ID |
ExprConditionalNode ID ID ID |
ExprConstantNode Value |
ExprParameterNode nat |
ExprLeafNode ID |
NotExpr

fun *embed-expr* :: *IRNode* \Rightarrow *ExprIRNode* **where**

embed-expr (*ConstantNode v*) = *ExprConstantNode v* |
embed-expr (*ParameterNode i*) = *ExprParameterNode i* |
embed-expr (*ConditionalNode c t f*) = *ExprConditionalNode c t f* |
embed-expr (*AbsNode x*) = *ExprUnaryNode UnaryAbs x* |
embed-expr (*NotNode x*) = *ExprUnaryNode UnaryNot x* |

```

embed-expr (NegateNode x) = ExprUnaryNode UnaryNeg x |
embed-expr (LogicNegationNode x) = ExprUnaryNode UnaryLogicNegation x |
embed-expr (AddNode x y) = ExprBinaryNode BinAdd x y |
embed-expr (MulNode x y) = ExprBinaryNode BinMul x y |
embed-expr (SubNode x y) = ExprBinaryNode BinSub x y |
embed-expr (AndNode x y) = ExprBinaryNode BinAnd x y |
embed-expr (OrNode x y) = ExprBinaryNode BinOr x y |
embed-expr (XorNode x y) = ExprBinaryNode BinXor x y |
embed-expr (IntegerBelowNode x y) = ExprBinaryNode BinIntegerBelow x y |
embed-expr (IntegerEqualsNode x y) = ExprBinaryNode BinIntegerEquals x y |
embed-expr (IntegerLessThanNode x y) = ExprBinaryNode BinIntegerLessThan
x y |
embed-expr (NarrowNode ib rb x) = ExprUnaryNode (UnaryNarrow ib rb) x |
embed-expr (SignExtendNode ib rb x) = ExprUnaryNode (UnarySignExtend ib
rb) x |
embed-expr (ZeroExtendNode ib rb x) = ExprUnaryNode (UnaryZeroExtend ib
rb) x |
embed-expr - = NotExpr

```

lemma unary-embed:

```

assumes g ⊢ n ≃ UnaryExpr op x
shows ∃ x'. embed-expr (kind g n) = ExprUnaryNode op x'
using assms by (induction UnaryExpr op x rule: rep.induct; simp)

```

lemma equal-embedded-x:

```

assumes g ⊢ n ≃ UnaryExpr op xe
assumes embed-expr (kind g n) = ExprUnaryNode op' x
shows g ⊢ x ≃ xe
using assms by (induction UnaryExpr op xe rule: rep.induct; simp)

```

lemma blah:

```

assumes embed-expr (kind g n) = ExprUnaryNode op n'
assumes g ⊢ n' ≃ e
shows (g ⊢ n ≃ UnaryExpr op e)
using assms(2,1) apply (cases kind g n; auto)
using rep.AbsNode apply blast
using rep.LogicNegationNode apply blast
using rep.NarrowNode apply presburger
using rep.NegateNode apply blast
using rep.NotNode apply blast
using rep.SignExtendNode apply blast
using rep.ZeroExtendNode by blast
end

```

graph-semantics-preservation

$$\begin{aligned} & \llbracket e2' \leq e1'; \{n'\} \triangleleft g1 \subseteq g2; \\ & g1 \vdash n' \simeq e1'; g2 \vdash n' \simeq e2' \rrbracket \\ & \implies \text{graph-refinement } g1 \ g2 \end{aligned}$$

maximal-sharing

$$\begin{aligned} & \text{maximal-sharing } g = \\ & (\forall n1 \ n2. \\ & \quad n1 \in \text{ids } g \wedge n2 \in \text{ids } g \longrightarrow \\ & \quad (\forall e. g \vdash n1 \simeq e \wedge g \vdash n2 \simeq e \longrightarrow n1 = n2)) \end{aligned}$$

tree-to-graph-rewriting

$$\begin{aligned} & e2 \leq e1 \wedge \\ & g1 \vdash n \simeq e1 \wedge \\ & \text{maximal-sharing } g1 \wedge \\ & \{n\} \triangleleft g1 \subseteq g2 \wedge \\ & g2 \vdash n \simeq e2 \wedge \text{maximal-sharing } g2 \implies \\ & \text{graph-refinement } g1 \ g2 \end{aligned}$$

graph-represents-expression

$$(g \vdash n \sqsubseteq e) = (\forall m \ p \ v. [m,p] \vdash e \mapsto v \longrightarrow [g,m,p] \vdash n \mapsto v)$$

graph-construction

$$\begin{aligned} & e2 \leq e1 \wedge \\ & g1 \subseteq g2 \wedge \\ & \text{maximal-sharing } g1 \wedge \\ & g2 \vdash n \simeq e2 \wedge \text{maximal-sharing } g2 \implies \\ & g2 \vdash n \sqsubseteq e1 \wedge \text{graph-refinement } g1 \ g2 \end{aligned}$$

```
end
theory SlideSnippets
  imports
    Semantics.TreeToGraphThms
    Veriopt.Snipping
begin
```

notation (*latex*)
kind ($-\langle\!\langle\!-\!\rangle\!\rangle$)

notation (*latex*)
IRTreeEval.ord-IRExpr-inst.less-eq-IRExpr ($- \mapsto -$)

abstract-syntax-tree

datatype *IRExpr* =
UnaryExpr IRUnaryOp IRExpr
| *BinaryExpr IRBinaryOp IRExpr IRExpr*
| *ConditionalExpr IRExpr IRExpr IRExpr*
| *ParameterExpr nat Stamp*
| *LeafExpr nat Stamp*
| *ConstantExpr Value*
| *ConstantVar (char list)*
| *VariableExpr (char list) Stamp*

tree-semantics

semantics:constant semantics:parameter semantics:unary semantics:binary semantics:leaf

expression-refinement

$$(e2 \leq e1) = (\forall m \ p \ v. [m,p] \vdash e1 \mapsto v \longrightarrow [m,p] \vdash e2 \mapsto v)$$

graph2tree

semantics:constant semantics:unary semantics:binary

graph-semantics

$$([g,m,p] \vdash n \mapsto v) = (\exists e. g \vdash n \simeq e \wedge [m,p] \vdash e \mapsto v)$$

graph-refinement

graph-refinement *g1 g2* =
 $(\forall n. n \in \text{ids } g1 \longrightarrow$
 $(\forall e1. g1 \vdash n \simeq e1 \longrightarrow (\exists e2. g2 \vdash n \simeq e2 \wedge e2 \leq e1)))$

translations $n \leq \text{CONST as-set } n$ *graph-antics-preservation*

$$\begin{aligned} & \llbracket e2' \leq e1'; \{n'\} \triangleleft g1 \subseteq g2; \\ & g1 \vdash n' \simeq e1'; g2 \vdash n' \simeq e2' \rrbracket \\ & \implies \text{graph-refinement } g1 \ g2 \end{aligned}$$
maximal-sharing

$$\begin{aligned} \text{maximal-sharing } g = & \\ (\forall n1 \ n2. & \\ n1 \in \text{ids } g \wedge n2 \in \text{ids } g \implies & \\ (\forall e. g \vdash n1 \simeq e \wedge g \vdash n2 \simeq e \implies n1 = n2)) & \end{aligned}$$
tree-to-graph-rewriting

$$\begin{aligned} & e2 \leq e1 \wedge \\ & g1 \vdash n \simeq e1 \wedge \\ & \text{maximal-sharing } g1 \wedge \\ & \{n\} \triangleleft g1 \subseteq g2 \wedge \\ & g2 \vdash n \simeq e2 \wedge \text{maximal-sharing } g2 \implies \\ & \text{graph-refinement } g1 \ g2 \end{aligned}$$
graph-represents-expression

$$(g \vdash n \trianglelefteq e) = (\forall m \ p \ v. [m,p] \vdash e \mapsto v \implies [g,m,p] \vdash n \mapsto v)$$
graph-construction

$$\begin{aligned} & e2 \leq e1 \wedge \\ & g1 \subseteq g2 \wedge \\ & \text{maximal-sharing } g1 \wedge \\ & g2 \vdash n \simeq e2 \wedge \text{maximal-sharing } g2 \implies \\ & g2 \vdash n \trianglelefteq e1 \wedge \text{graph-refinement } g1 \ g2 \end{aligned}$$
end