Veriopt Theories

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oegin				
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		$\begin{array}{cccccccccccccccccccccccccccccccccccc$		
		Rewrite 'a Rewrite		
-	tive 'a I	!		
		ctraNotation =		
		tation 'a 'a (- ? - : - 50)		
		n 'a 'a (- eq -) tion 'a (const - 120)		
	otation	· · · · · · · · · · · · · · · · · · ·		
		$(false) \mid$		
		$a'a (-\oplus -)$		
		Notation 'a (!-)		
-				

```
ShortCircuitOr 'a 'a (- || -)
definition word :: ('a::len) \ word \Rightarrow 'a \ word \ \mathbf{where}
   word x = x
ML-file \langle markup.ML \rangle
1.1.1
                 Expression Markup
ML \ \langle
structure\ IRExprTranslator: DSL-TRANSLATION =
struct
fun\ markup\ DSL\text{-}Tokens.Add = @\{term\ BinaryExpr\} \$ @\{term\ BinAdd\}
      markup\ DSL\text{-}Tokens.Sub = @\{term\ BinaryExpr\} \$ @\{term\ BinSub\}
      markup\ DSL\text{-}Tokens.Mul = @\{term\ BinaryExpr\} \$ @\{term\ BinMul\}
      markup\ DSL\text{-}Tokens.And = @\{term\ BinaryExpr\} \$ @\{term\ BinAnd\}
      markup\ DSL\text{-}Tokens.Or = @\{term\ BinaryExpr\} \$ @\{term\ BinOr\}
      markup\ DSL\text{-}Tokens.Xor = @\{term\ BinaryExpr\} \$ @\{term\ BinXor\}
     \mid markup\ DSL\text{-}Tokens.ShortCircuitOr = @\{term\ BinaryExpr\} \$ @\{term\ Bin-
ShortCircuitOr}
   | markup \ DSL\text{-}Tokens.Abs = @\{term \ UnaryExpr\} \$ @\{term \ UnaryAbs\}
   | markup\ DSL-Tokens.Less = @\{term\ BinaryExpr\} \$ @\{term\ BinIntegerLessThan\}
   | markup\ DSL\text{-}Tokens.Equals = @\{term\ BinaryExpr\} \$ @\{term\ BinIntegerEquals\}
      markup\ DSL\text{-}Tokens.Not = @\{term\ UnaryExpr\} \$ @\{term\ UnaryNot\}
      markup\ DSL\text{-}Tokens.Negate = @\{term\ UnaryExpr\} \$ @\{term\ UnaryNeg\}
      markup\ DSL\text{-}Tokens.LogicNegate = @\{term\ UnaryExpr\} \$ @\{term\ UnaryLog-vareauthered and vareauthered an
icNegation}
   | markup \ DSL\text{-}Tokens.LeftShift = @\{term \ BinaryExpr\} \$ @\{term \ BinLeftShift\}
     | markup \ DSL\text{-}Tokens.RightShift = @\{term \ BinaryExpr\}  $ @\{term \ BinRight\text{-}
Shift
  | markup\ DSL-Tokens. UnsignedRightShift = @\{term\ BinaryExpr\} \$ @\{term\ BinaryExpr\} \}
 URightShift
      markup\ DSL\text{-}Tokens.Conditional = @\{term\ ConditionalExpr\}
      markup\ DSL\text{-}Tokens.Constant = @\{term\ ConstantExpr\}
      markup\ DSL\text{-}Tokens.TrueConstant = @\{term\ ConstantExpr\ (IntVal\ 32\ 1)\}
      markup\ DSL\text{-}Tokens.FalseConstant = @\{term\ ConstantExpr\ (IntVal\ 32\ 0)\}
end
structure\ IRExprMarkup = DSL-Markup(IRExprTranslator);
       ir expression translation
       syntax - expandExpr :: term \Rightarrow term (exp[-])
       parse-translation \leftarrow [(
                                                                      @\{syntax\text{-}const
                                                                                                             -expandExpr
                                                                                                                                                       IREx-
       prMarkup.markup-expr [])] \rightarrow
```

```
ir\ expression\ example value exp[(e_1 < e_2)\ ?\ e_1: e_2] Conditional Expr\ (Binary Expr\ Bin Integer Less Than\ e_1\ e_2)\ e_1\ e_2
```

1.1.2 Value Markup

```
\mathbf{ML}
structure\ IntValTranslator: DSL-TRANSLATION =
struct
fun\ markup\ DSL\text{-}Tokens.Add = @\{term\ intval\text{-}add\}
   markup\ DSL\text{-}Tokens.Sub = @\{term\ intval\text{-}sub\}
   markup\ DSL\text{-}Tokens.Mul = @\{term\ intval\text{-}mul\}
   markup\ DSL\text{-}Tokens.And = @\{term\ intval\text{-}and\}
   markup\ DSL-Tokens.Or = @\{term\ intval\text{-}or\}
   markup\ DSL\text{-}Tokens.ShortCircuitOr = @\{term\ intval\text{-}short\text{-}circuit\text{-}or\}
   markup\ DSL\text{-}Tokens.Xor = @\{term\ intval\text{-}xor\}
   markup\ DSL\text{-}Tokens.Abs = @\{term\ intval\text{-}abs\}
   markup\ DSL\text{-}Tokens.Less = @\{term\ intval\text{-}less\text{-}than\}
   markup\ DSL-Tokens.Equals = @\{term\ intval\text{-}equals\}
   markup\ DSL\text{-}Tokens.Not = @\{term\ intval\text{-}not\}
   markup\ DSL\text{-}Tokens.Negate = @\{term\ intval\text{-}negate\}
   markup\ DSL\text{-}Tokens.LogicNegate = @\{term\ intval\text{-}logic\text{-}negation\}
   markup\ DSL-Tokens.LeftShift = @\{term\ intval-left-shift\}
   markup\ DSL\text{-}Tokens.RightShift = @\{term\ intval\text{-}right\text{-}shift\}
   markup\ DSL\text{-}Tokens.UnsignedRightShift = @\{term\ intval\text{-}uright\text{-}shift\}
   markup\ DSL\text{-}Tokens.Conditional = @\{term\ intval\text{-}conditional\}
   markup\ DSL\text{-}Tokens.Constant = @\{term\ IntVal\ 32\}
   markup\ DSL\text{-}Tokens.TrueConstant = @\{term\ IntVal\ 32\ 1\}
   markup\ DSL-Tokens.FalseConstant = @\{term\ IntVal\ 32\ 0\}
end
structure\ IntValMarkup = DSL-Markup(IntValTranslator);
    value expression translation
    syntax - expandIntVal :: term \Rightarrow term (val[-])
    \mathbf{parse-translation} \quad \leftarrow \quad [(\quad @\{syntax-const \quad -expandIntVal\}]
    Markup.markup-expr [])] \rightarrow
    value expression example
    value val[(e_1 < e_2) ? e_1 : e_2]
    intval-conditional (intval-less-than e_1 e_2) e_1 e_2
```

1.1.3 Word Markup

```
ML \ \ \langle
structure\ WordTranslator: DSL-TRANSLATION =
fun \ markup \ DSL-Tokens.Add = @\{term \ plus\}
   markup\ DSL\text{-}Tokens.Sub = @\{term\ minus\}
   markup\ DSL\text{-}Tokens.Mul = @\{term\ times\}
 | markup\ DSL-Tokens. And = @\{term\ Bit-Operations. semiring-bit-operations-class. and\}
   markup\ DSL\text{-}Tokens.Or = @\{term\ or\}
   markup\ DSL-Tokens.Xor = @\{term\ xor\}
   markup\ DSL-Tokens.Abs = @\{term\ abs\}
   markup\ DSL\text{-}Tokens.Less = @\{term\ less\}
   markup\ DSL\text{-}Tokens.Equals = @\{term\ HOL.eq\}
   markup\ DSL\text{-}Tokens.Not = @\{term\ not\}
   markup\ DSL-Tokens.Negate = @\{term\ uminus\}
   markup\ DSL\text{-}Tokens.LogicNegate = @\{term\ logic-negate\}
   markup\ DSL\text{-}Tokens.LeftShift = @\{term\ shiftl\}
   markup\ DSL\text{-}Tokens.RightShift = @\{term\ signed\text{-}shiftr\}
   markup\ DSL\text{-}Tokens.UnsignedRightShift = @\{term\ shiftr\}
   markup\ DSL-Tokens.Constant = @\{term\ word\}
   markup\ DSL\text{-}Tokens.TrueConstant = @\{term\ 1\}
   markup\ DSL-Tokens.FalseConstant = @\{term\ 0\}
end
structure\ WordMarkup = DSL\text{-}Markup(WordTranslator);
    word expression translation
   syntax - expandWord :: term \Rightarrow term (bin[-])
   parse-translation \leftarrow [( @\{syntax-const\}
                                                       -expand Word}
                                                                              Word-
   Markup.markup-expr [])] \rightarrow
   word expression example
   value bin[x \& y \mid z]
   intval-conditional (intval-less-than e_1 e_2) e_1 e_2
value bin[-x]
value val[-x]
value exp[-x]
value bin[!x]
value val[!x]
value exp[!x]
value bin[\neg x]
value val[\neg x]
value exp[\neg x]
```

```
value bin[^{\sim}x]
value val[^{\sim}x]
value exp[^{\sim}x]
value ^{\sim}x
\quad \text{end} \quad
       Optimization Phases
1.2
{\bf theory}\ {\it Phase}
 \mathbf{imports}\ \mathit{Main}
begin
ML-file map.ML
ML-file phase.ML
end
1.3
        Canonicalization DSL
theory Canonicalization
 imports
   Markup
   Phase
   HOL-Eisbach.Eisbach
  keywords
   phase :: thy-decl and
   terminating:: quasi-command and
   print-phases :: diag and
   export-phases :: thy-decl and
   optimization:: thy\hbox{-} goal\hbox{-} defn
begin
print-methods
\mathbf{ML} \ \ \checkmark
datatype 'a Rewrite =
  Transform of 'a * 'a \mid
  Conditional of 'a*'a*term
  Sequential of 'a Rewrite * 'a Rewrite |
  Transitive of 'a Rewrite
type\ rewrite = \{
  name:\ binding,
  rewrite: term Rewrite,
  proofs: thm list,
```

code: thm list, source: term

```
}
structure\ RewriteRule: Rule =
struct
type T = rewrite;
fun pretty-rewrite ctxt (Transform (from, to)) =
     Pretty.block
       Syntax.pretty\text{-}term\ ctxt\ from,
       Pretty.str \mapsto,
       Syntax.pretty-term ctxt to
  | pretty-rewrite ctxt (Conditional (from, to, cond)) =
     Pretty.block
       Syntax.pretty-term ctxt from,
       Pretty.str \mapsto,
       Syntax.pretty-term ctxt to,
       Pretty.str when,
       Syntax.pretty-term ctxt cond
 | pretty-rewrite - - = Pretty.str not implemented*)
fun\ pretty-thm\ ctxt\ thm =
  (Proof-Context.pretty-fact ctxt (, [thm]))
fun\ pretty\ ctxt\ obligations\ t=
  let
   val is-skipped = Thm-Deps.has-skip-proof (#proofs t);
   val\ warning = (if\ is\text{-}skipped)
     then [Pretty.str (proof skipped), Pretty.brk 0]
     else []);
   val\ obligations = (if\ obligations
     then\ [Pretty.big-list
            obligations:
            (map\ (pretty-thm\ ctxt)\ (\#proofs\ t)),
          Pretty.brk 0]
     else []);
   fun\ pretty-bind\ binding =
     Pretty.markup
       (Position.markup\ (Binding.pos-of\ binding)\ Markup.position)
       [Pretty.str\ (Binding.name-of\ binding)];
  in
  Pretty.block ([
   pretty-bind (#name t), Pretty.str:,
```

```
Syntax.pretty-term ctxt (#source t), Pretty.fbrk
 @ obligations @ warning)
 end
end
structure\ RewritePhase = DSL-Phase(RewriteRule);
val - =
 Outer-Syntax.command command-keyword (phase) enter an optimization phase
  (Parse.binding -- | Parse.\$\$\$ terminating -- Parse.const -- | Parse.begin
    >> (Toplevel.begin-main-target true o RewritePhase.setup));
fun\ print-phases\ print-obligations\ ctxt =
   val thy = Proof\text{-}Context.theory\text{-}of ctxt;
   fun \ print \ phase = RewritePhase.pretty \ print-obligations \ phase \ ctxt
   map print (RewritePhase.phases thy)
fun print-optimizations print-obligations thy =
 print-phases print-obligations thy |> Pretty.writeln-chunks
val - =
 Outer-Syntax.command command-keyword (print-phases)
   print debug information for optimizations
   (Parse.opt-bang >>
     (fn \ b = > Toplevel.keep ((print-optimizations \ b) \ o \ Toplevel.context-of)));
fun \ export\text{-}phases \ thy \ name =
 let
   val \ state = Toplevel.theory-toplevel \ thy;
   val\ ctxt = Toplevel.context-of\ state;
   val\ content = Pretty.string-of\ (Pretty.chunks\ (print-phases\ false\ ctxt));
   val\ cleaned = YXML.content-of\ content;
   val\ filename = Path.explode\ (name \hat{\ }.rules);
   val\ directory = Path.explode\ optimizations;
   val path = Path.binding (
             Path.append directory filename,
             Position.none);
   val thy' = thy \mid > Generated-Files. add-files (path, content);
   val - = Export.export thy' path [YXML.parse cleaned];
   val - = writeln (Export.message thy' (Path.basic optimizations));
 in
   thy'
```

```
end

val - =
  Outer-Syntax.command command-keyword (export-phases)
  export information about encoded optimizations
  (Parse.text >>
        (fn name => Toplevel.theory (fn state => export-phases state name)))
}
```

ML-file rewrites.ML

1.3.1 Semantic Preservation Obligation

```
fun rewrite-preservation :: IRExpr Rewrite \Rightarrow bool where rewrite-preservation (Transform x y) = (y \le x) | rewrite-preservation (Conditional x y cond) = (cond \longrightarrow (y \le x)) | rewrite-preservation (Sequential x y) = (rewrite-preservation x \land x) | rewrite-preservation (Transitive x) = rewrite-preservation x
```

1.3.2 Termination Obligation

```
fun rewrite-termination :: IRExpr Rewrite \Rightarrow (IRExpr \Rightarrow nat) \Rightarrow bool where rewrite-termination (Transform x y) trm = (trm \ x > trm \ y) \mid rewrite-termination (Conditional x y cond) trm = (cond \longrightarrow (trm \ x > trm \ y)) \mid rewrite-termination (Sequential x y) trm = (rewrite-termination \ x trm \land rewrite-termination y trm) \mid rewrite-termination (Transitive x) trm = rewrite-termination \ x trm

fun intval :: Value Rewrite \Rightarrow bool where intval (Transform x y) = (x \neq UndefVal \land y \neq UndefVal \longrightarrow x = y) \mid intval (Conditional x y cond) = (cond \longrightarrow (x = y)) \mid intval (Sequential x y) = (intval \ x \land intval \ y) \mid intval (Transitive x) = intval \ x
```

1.3.3 Standard Termination Measure

```
fun size :: IRExpr \Rightarrow nat where unary-size: size (UnaryExpr op e) = (size e) + 2 \mid bin-const-size: size (BinaryExpr op x (ConstantExpr cy)) = (size x) + 2 \mid bin-size: size (BinaryExpr op x y) = (size x) + (size y) + 2 \mid cond-size: size (ConditionalExpr cond t f) = (size cond) + (size t) + (size f) + 2 \mid const-size: size (ConstantExpr c) = 1 \mid param-size:
```

```
size (ParameterExpr ind s) = 2 \mid leaf-size:
size (LeafExpr nid s) = 2 \mid size (ConstantVar c) = 2 \mid size (VariableExpr x s) = 2
```

1.3.4 Automated Tactics

 ${f method} \ unfold\mbox{-}optimization =$

unfold intval.simps,

end

named-theorems size-simps size simplication rules

```
rule conjE, simp, simp del: le-expr-def, force?)
 (unfold rewrite-preservation.simps, unfold rewrite-termination.simps,
   rule conjE, simp, simp del: le-expr-def, force?)
{f method} \ {\it unfold\text{-}size} =
  (((unfold size.simps, simp add: size-simps del: le-expr-def)?
 ; \ (simp \ add: \ size\text{-}simps \ del: \ le\text{-}expr\text{-}def)?
 ; (auto simp: size-simps)?
 ; (unfold\ size.simps)?)[1])
print-methods
structure\ System: Rewrite System=
val\ preservation = @\{const\ rewrite-preservation\};
val\ termination = @\{const\ rewrite-termination\};
val\ intval = @\{const\ intval\};
end
structure\ DSL = DSL-Rewrites(System);
val - =
  Outer-Syntax.local-theory-to-proof command-keyword < optimization >
   define an optimization and open proof obligation
   (Parse-Spec.thm-name: -- Parse.term
      >> DSL.rewrite-cmd);
```

(unfold rewrite-preservation.simps, unfold rewrite-termination.simps,