Dense Matrix Data Structures

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http://wangshusen.github.io/

Dense Matrix Data Structures

- **Dense matrix:** most of the elements are non-zero.

- Dense matrix can be stored in a fixed-size array.

Array:

Dense Matrix Data Structures

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$

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Dense Matrix Data Structures

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Array:

a_{11}	a_{12}	a_{13}									

Dense Matrix Data Structures

Row-Major Order

Column-Major Order

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$

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Array:

a_{11}	a_{12}	a_{13}	a_{21}	a_{22}	a_{23}						

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Array:

| a_{11} | a_{12} | a_{13} | a_{21} | a_{22} | a_{23} | a_{31} | a_{32} | a_{33} | | | |

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Array: | a_{11} | a_{12} | a_{13} | a_{21} | a_{22} | a_{23} | a_{31} | a_{32} | a_{33} | a_{41} | a_{42} | a_{43} |

Dense Matrix Data Structures

Column-Major Order

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Array:

a_{11}	a_{21}	a_{31}	a_{41}								

Dense Matrix Data Structures

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Array:

a_{11}	a_{21}	a_{31}	a_{41}	a_{12}	a_{22}	a_{32}	a_{42}				

Dense Matrix Data Structures

Column-Major Order

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Array:

a_{11}	a_{21}	a_{31}	a_{41}	a_{12}	a_{22}	a_{32}	a_{42}	a_{13}	a_{23}	a_{33}	a_{43}

Why does layout matter?

Column-Major Order

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$
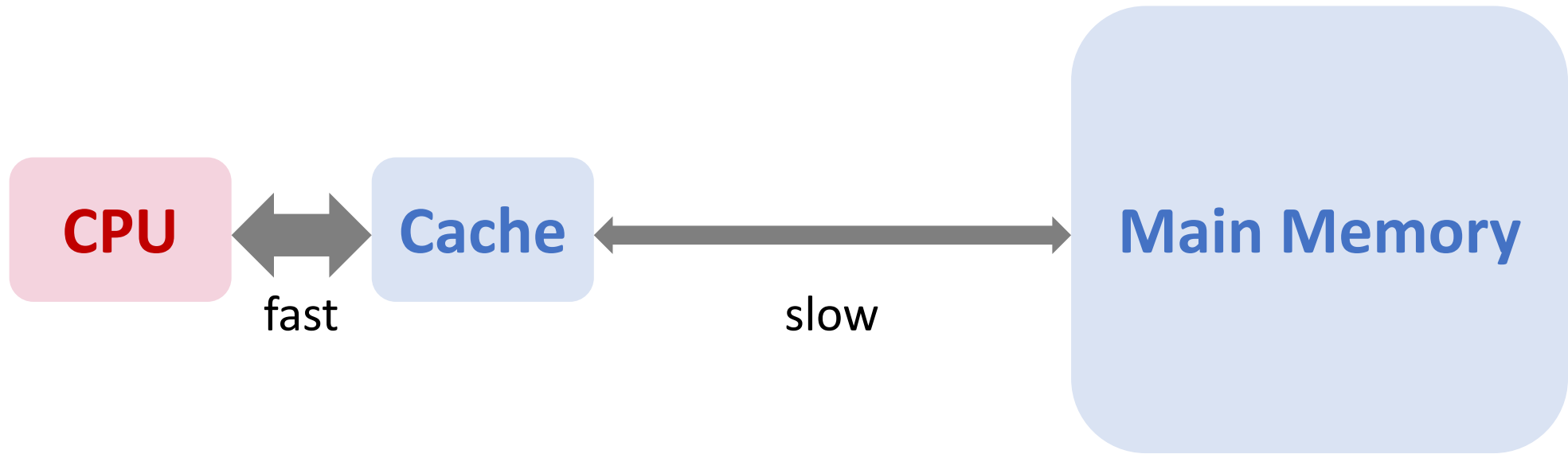
$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$

Array:

a_{11}	a_{12}	a_{13}	a_{21}	a_{22}	a_{23}	a_{31}	a_{32}	a_{33}	a_{41}	a_{42}	a_{43}

Why does layout matter?

Row-Major Order

$$
A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}
$$

$$
A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}
$$

Traversing a row is fast.

Array:

a_{11}	a_{12}	a_{13}	a_{21}	a_{22}	a_{23}	a_{31}	a_{32}	a_{33}	a_{41}	a_{42}	a_{43}

Why does layout matter?

CPU ⟷ Cache ⟷ Main Memory

fast   slow

Traversing a row is fast.

Array: | a_{11} | a_{12} | a_{13} | a_{21} | a_{22} | a_{23} | a_{31} | a_{32} | a_{33} | a_{41} | a_{42} | a_{43} |

Why does layout matter?

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$

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Traversing a column is slow.

Array: | a_{11} | a_{12} | a_{13} | a_{21} | a_{22} | a_{23} | a_{31} | a_{32} | a_{33} | a_{41} | a_{42} | a_{43} |

Thank You!