

Graph essentials

Graph basics

- Nodes(which is called vertices or actors in somewhere)  
Nodes represent entities
- Edges(which is called relationships or social ties)  
Edges indicate inter-node relationships
- Degree
  - The num of edges connected to a node is degree
  - The num of Input directed edges called in-degree
  - The num of Output directed edges called out-degree

Graph Representation

- Goal
  - Does not lose info
  - Can be manipulated easily by computers
  - Can have mathematical methods applied easily
- Adjacency matrix(sociomatrix)  
 $A_{ij} = 1$  (if  $V_i$  is connected to  $V_j$ ) or 0 (otherwise)
- Edge list  
denoted as  $(V_i, V_j)$  which means the edge from  $V_i$  to  $V_j$

Type of graphs

- Null graph  
No nodes
- Empty graph  
There are no nodes but no edge
- Directed/Undirected/Mixed graphs
- Simple or multigraphs  
multiple edges between nodes
- Weighted graphs  
子主题 1

Connectivity in graphs

- Traversing an edge  
Visiting an edge can start at a and end at b.
- Walk
  - Walk is a sequence of incident edges traversed one after another.
  - If a walk traverses edges  $e_1, e_2 \dots e_n$ ,  $V_1$  is starting node and  $V_n$  is ending node.
    - If  $V_1 \neq V_n$ , it is open walk
    - If  $V_1 = V_n$ , it is closed walk
- Trail
  - Trail is a walk where edge is traversed only once.
  - All edges are distinct
  - A closed trail which ends where it started (tour or circuit)
- Path
  - Nodes and edges are distinct
  - Closed path is called cycle

Special graphs

- Trees and forests
  - Tree is a graph structure which has no cycle in it.
  - Forest is a set of disconnected trees.
- Special subgraphs
  - Spanning tree
    - It need guarantee there is a path connects every two nodes.
    - Minimum spanning tree is minimize the total distance of path.
  - Steiner tree
    - It aims to find a tree that it spans all the  $V'$  nodes and the weight of the tree is minimized. ( $V'$  is a subset of  $V$ )
- Planar graphs  
No two edges cross each other
- Bipartite graph
  - A bipartite graph is where the node set can be partitioned into two sets.
    - $V = V_a \cup V_b$
    - $V_a \cap V_b = \text{empty set}$
    - $E = V_a * V_b$
  - Affiliation networks
- Bridge  
The edges between two components

Graph Algorithms

- Graph traversal
  - DFS
  - BFS
- Shortest path
- Minimum Spanning trees
- Network flow
- Maximum Bipartite matching
- Bridge detection