

# Gale-Shapley Algorithm

Shusen Wang

<http://wangshusen.github.io/>

# Example 1

# Inputs

Ada > Becky > Cindy

Alex



Ada



Bob > Alex > Chris

Becky > Ada > Cindy

Bob



Becky



Alex > Bob > Chris

Ada > Becky > Cindy

Chris



Cindy



Alex > Bob > Chris

# Iteration 1

Ada > Becky > Cindy

Alex



Ada



Bob > Alex > Chris

Becky > Ada > Cindy

Bob



Becky



Alex > Bob > Chris

Ada > Becky > Cindy

Chris



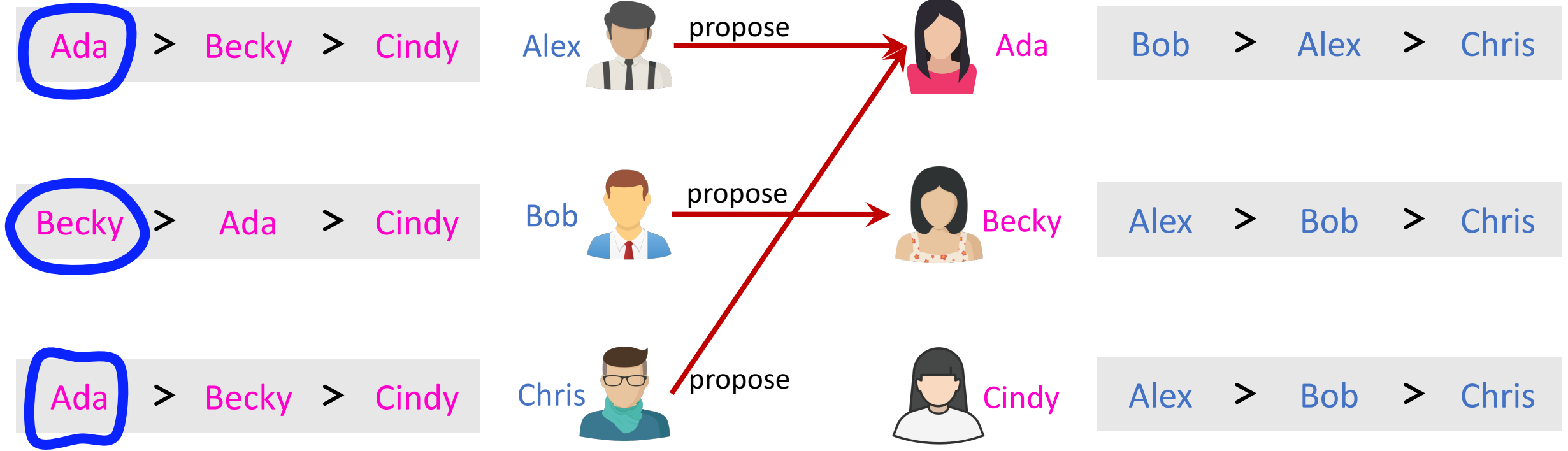
Cindy



Alex > Bob > Chris

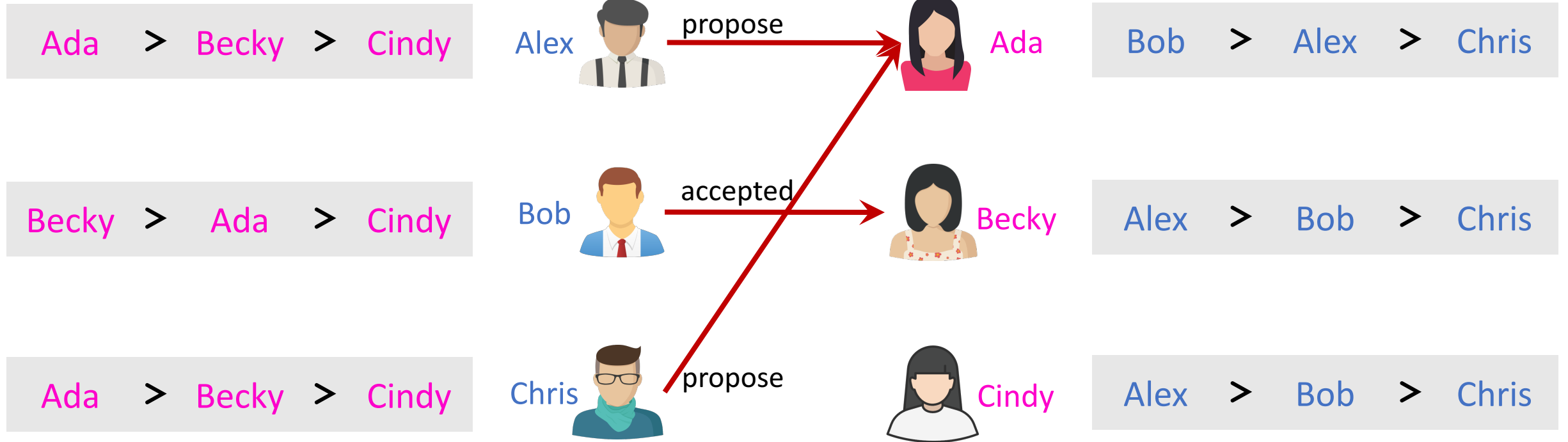
**Step 1:** An **unmarried** man proposes to the woman who is *his most preferred among those he **has not proposed to**.*

# Iteration 1



**Step 1:** An **unmarried** man proposes to the woman who is *his most preferred among those he **has not** proposed to*.

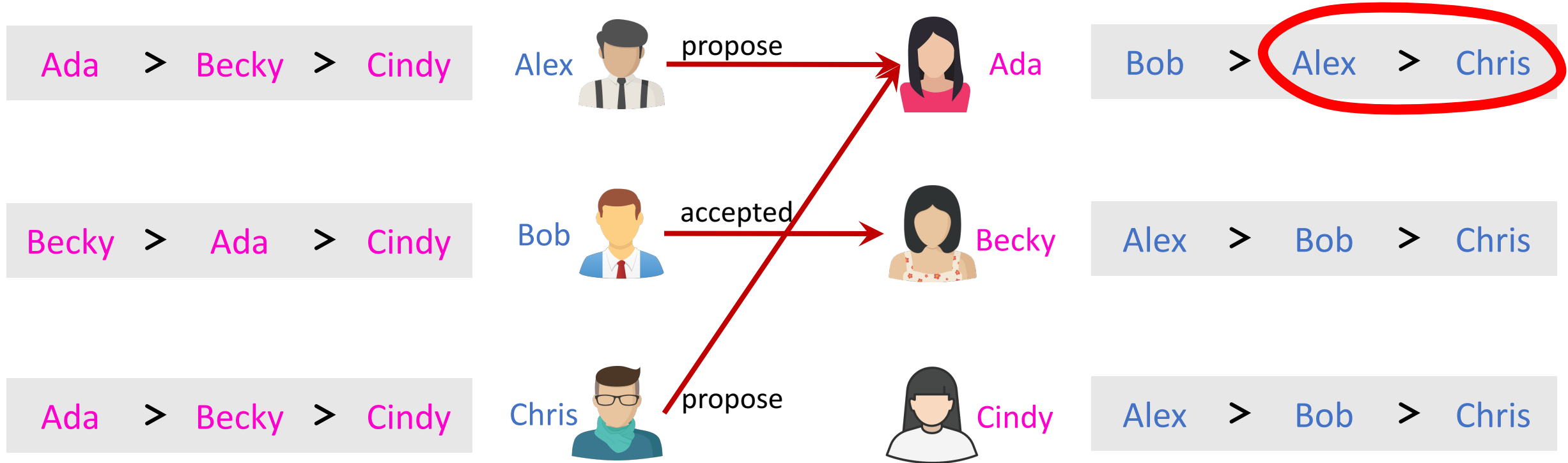
# Iteration 1



**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then divorce.

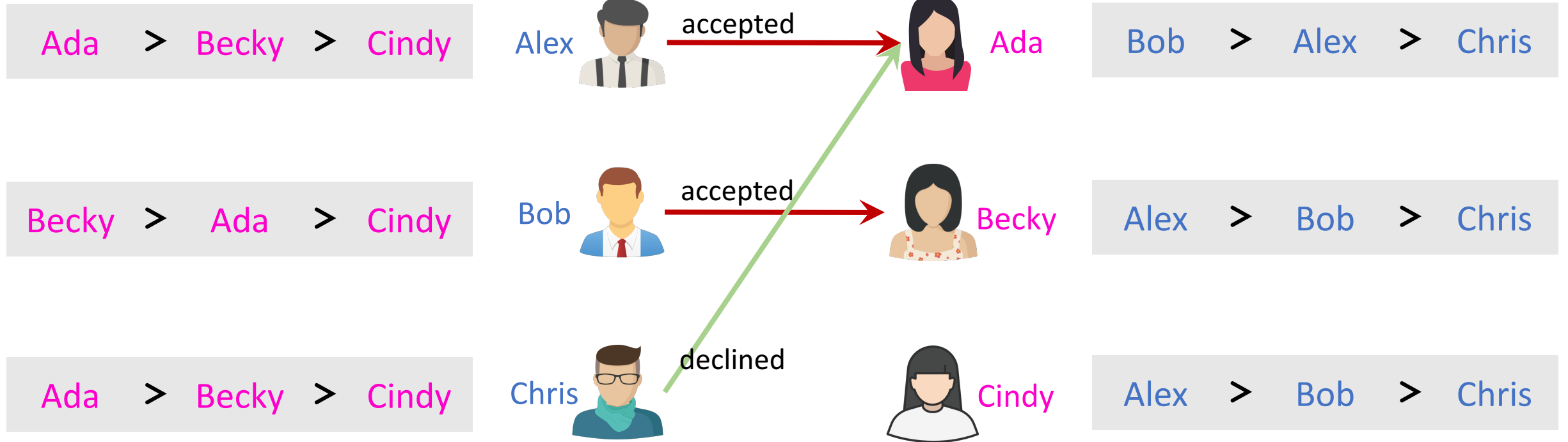
# Iteration 1



**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then divorce.

# Iteration 1

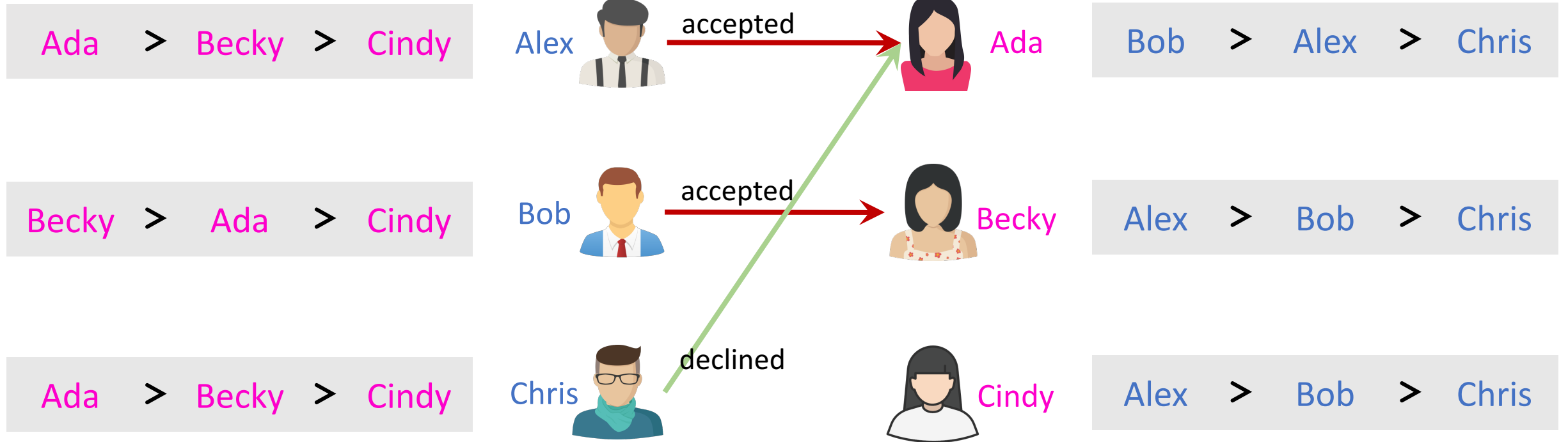


**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then divorce.



# Iteration 1



**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then divorce.

# Iteration 1

~~Ada~~ > Becky > Cindy



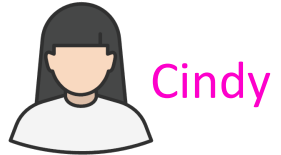
Bob > Alex > Chris

~~Becky~~ > Ada > Cindy



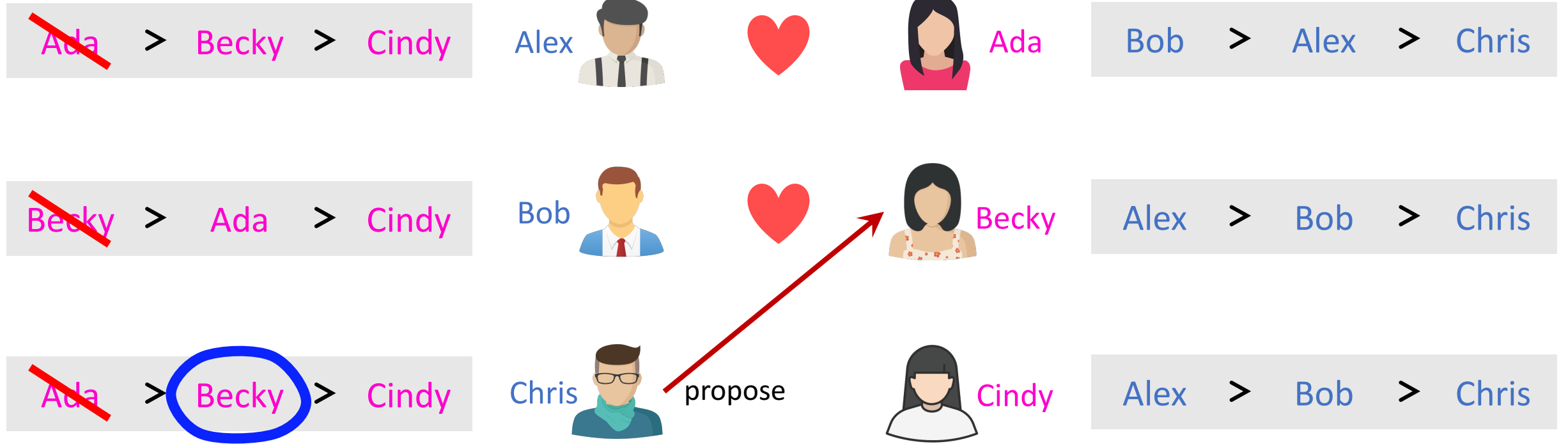
Alex > Bob > Chris

~~Ada~~ > Becky > Cindy



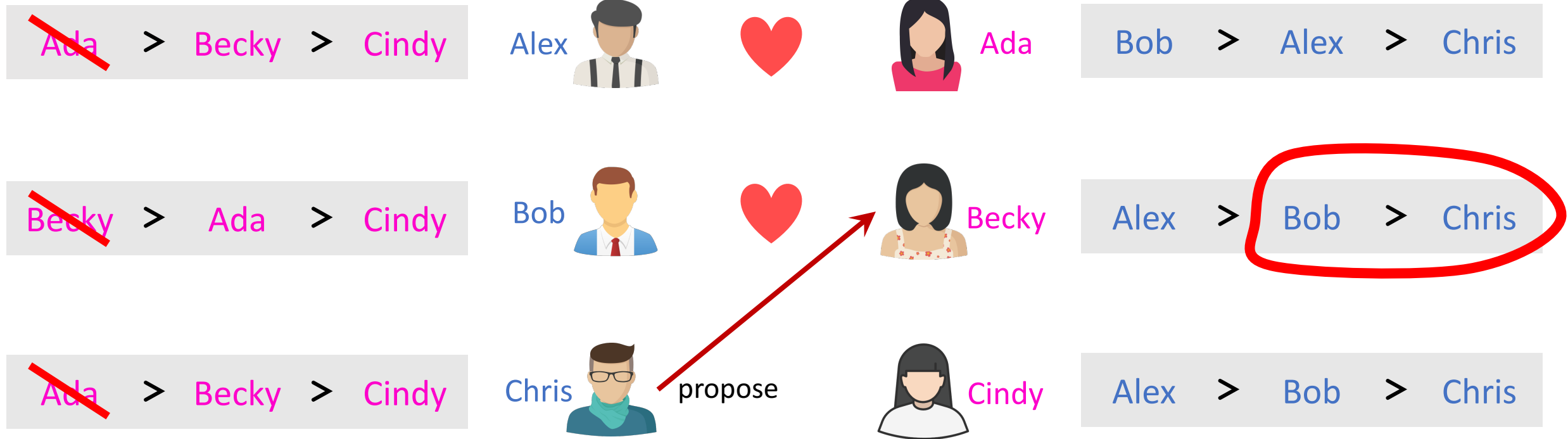
Alex > Bob > Chris

# Iteration 2



**Step 1:** An **unmarried** man proposes to the woman who is *his most preferred among those he **has not proposed to***.

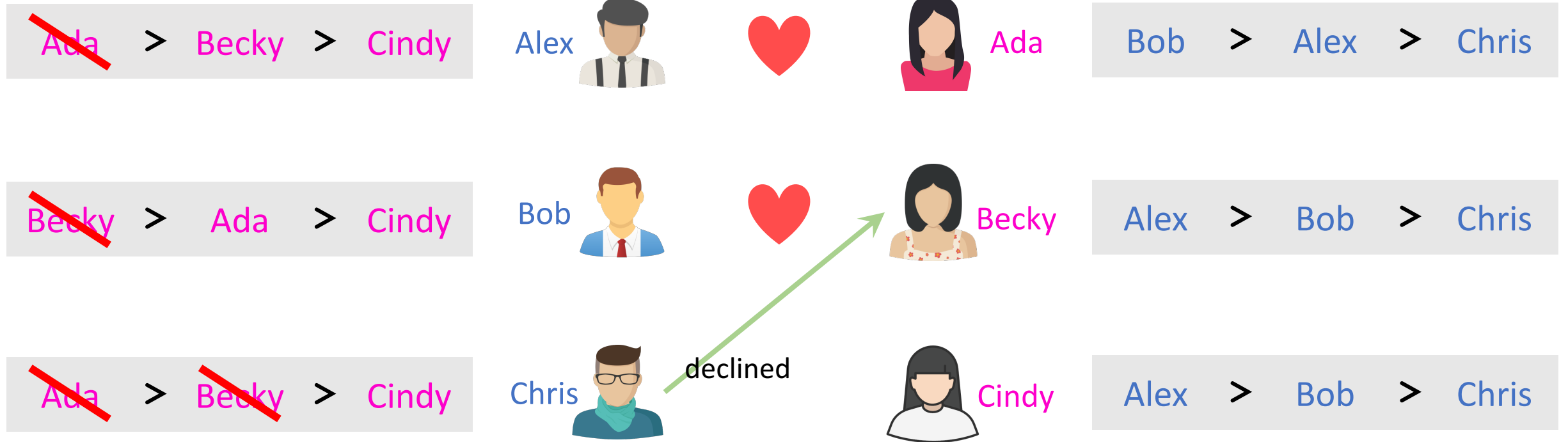
# Iteration 2



**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then divorce.

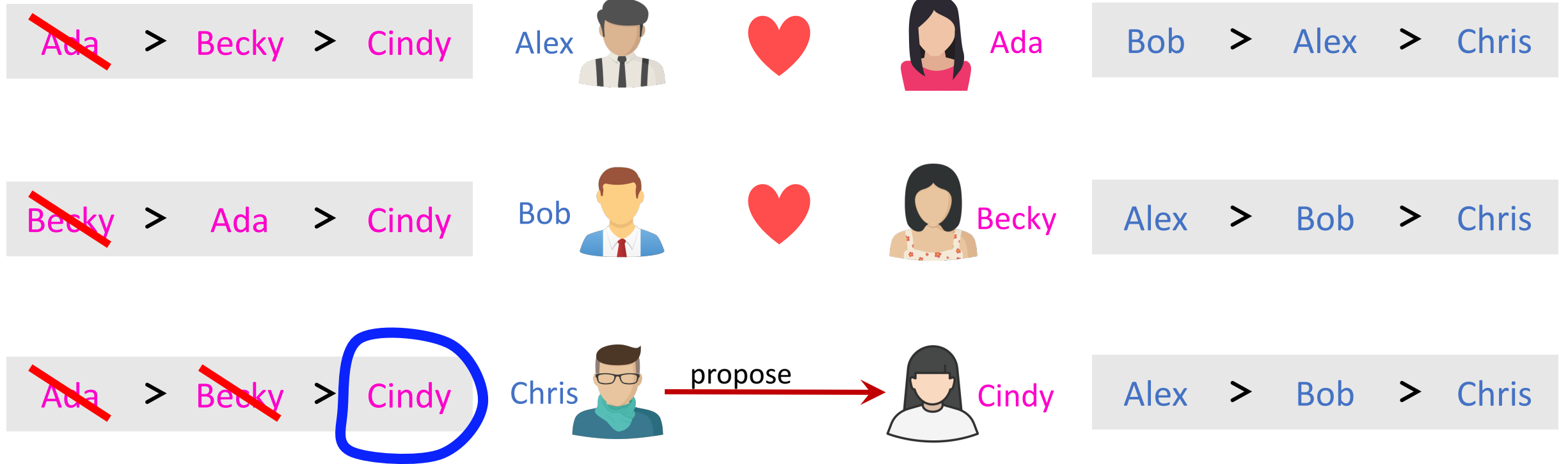
# Iteration 2



**Step 2:** A woman accepts her best offer.

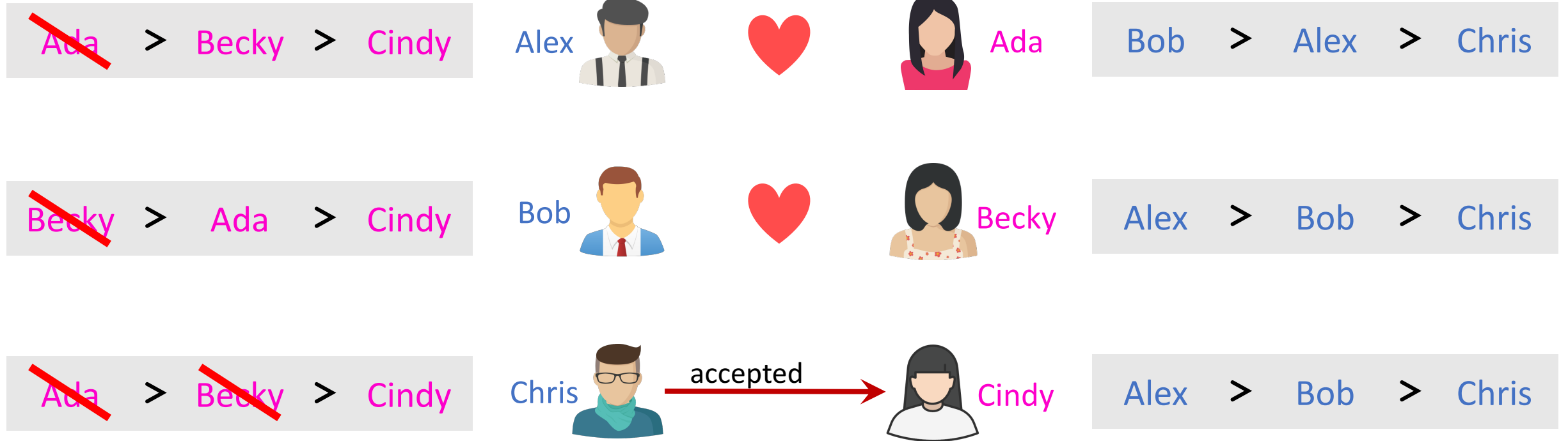
- If the woman finds better matching than her current spouse, then divorce.

# Iteration 3



**Step 1:** An **unmarried** man proposes to the woman who is *his most preferred among those he **has not proposed to***.

# Iteration 3



**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then divorce.

# End of Procedure

~~Ada~~ > Becky > Cindy



Bob > Alex > Chris

~~Becky~~ > Ada > Cindy



Alex > Bob > Chris

~~Ada~~ > ~~Becky~~ > ~~Cindy~~



Alex > Bob > Chris

Stop if everyone is married.



## Example 2

# Inputs

Ada > Diana > Cindy > Becky

Alex



Ada



Chris > Alex > Bob > David

Ada > Becky > Cindy > Diana

Bob



Becky



Alex > David > Chris > Bob

Becky > Diana > Cindy > Ada

Chris



Cindy



Bob > David > Chris > Alex

Cindy > Ada > Becky > Diana

David

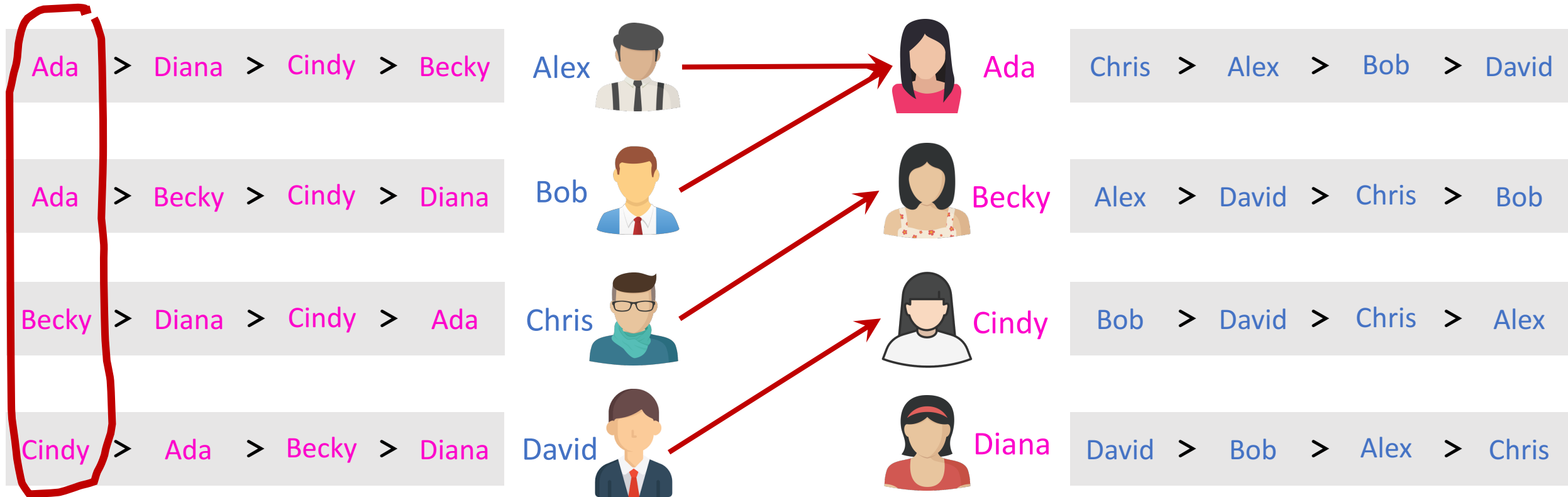


Diana



David > Bob > Alex > Chris

# Iteration 1



**Step 1:** An **unmarried** man proposes to the woman who is *his most preferred among those he **has not** proposed to*.

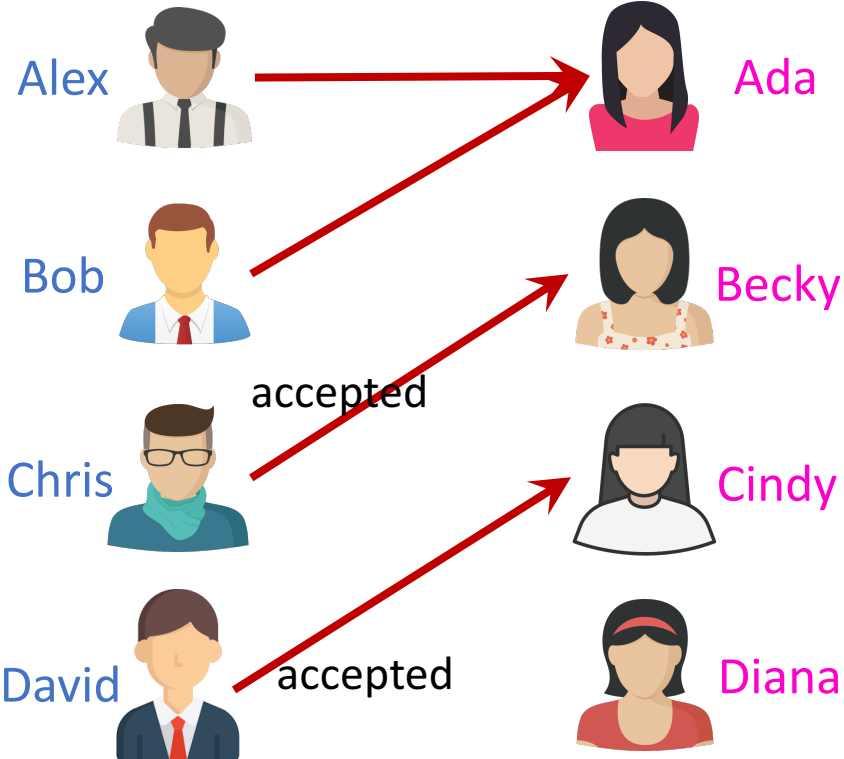
# Iteration 1

Ada > Diana > Cindy > Becky

Ada > Becky > Cindy > Diana

Becky > Diana > Cindy > Ada

Cindy > Ada > Becky > Diana



Chris > Alex > Bob > David

Alex > David > Chris > Bob

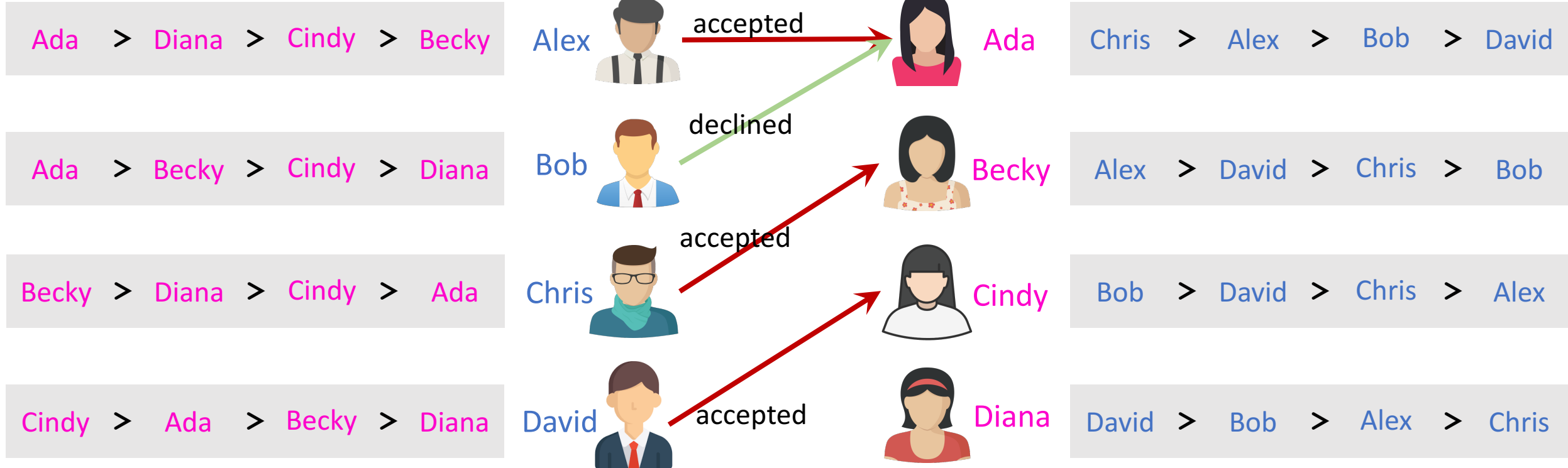
Bob > David > Chris > Alex

David > Bob > Alex > Chris

**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then divorce.

# Iteration 1



**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then divorce.

# Iteration 1

Ada > Diana > Cindy > Becky

Alex



Ada

Chris > Alex > Bob > David

Becky > Diana > Cindy > Ada

Chris



Becky

Alex > David > Chris > Bob

Cindy > Ada > Becky > Diana

David



Cindy

Bob > David > Chris > Alex

Ada > Becky > Cindy > Diana

Bob



Diana

David > Bob > Alex > Chris

# Iteration 1

~~Ada~~ > Diana > Cindy > Becky

Alex



Ada

Chris > Alex > Bob > David

~~Becky~~ > Diana > Cindy > Ada

Chris



Becky

Alex > David > Chris > Bob

~~Cindy~~ > Ada > Becky > Diana

David

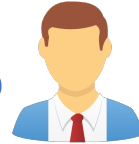


Cindy

Bob > David > Chris > Alex

~~Ada~~ > Becky > Cindy > Diana

Bob



Diana

David > Bob > Alex > Chris

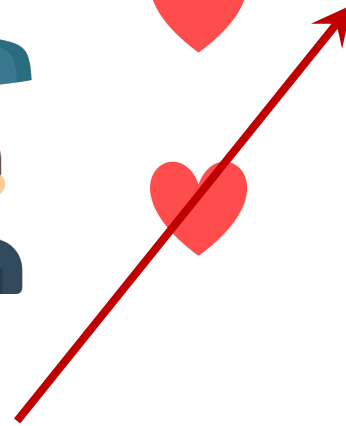
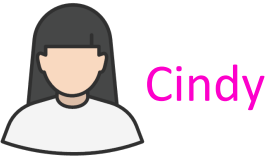
# Iteration 2

~~Ada~~ > Diana > Cindy > Becky

~~Becky~~ > Diana > Cindy > Ada

~~Cindy~~ > Ada > Becky > Diana

~~Ada~~ > **Becky** > Cindy > Diana



Chris > Alex > Bob > David

Alex > David > Chris > Bob

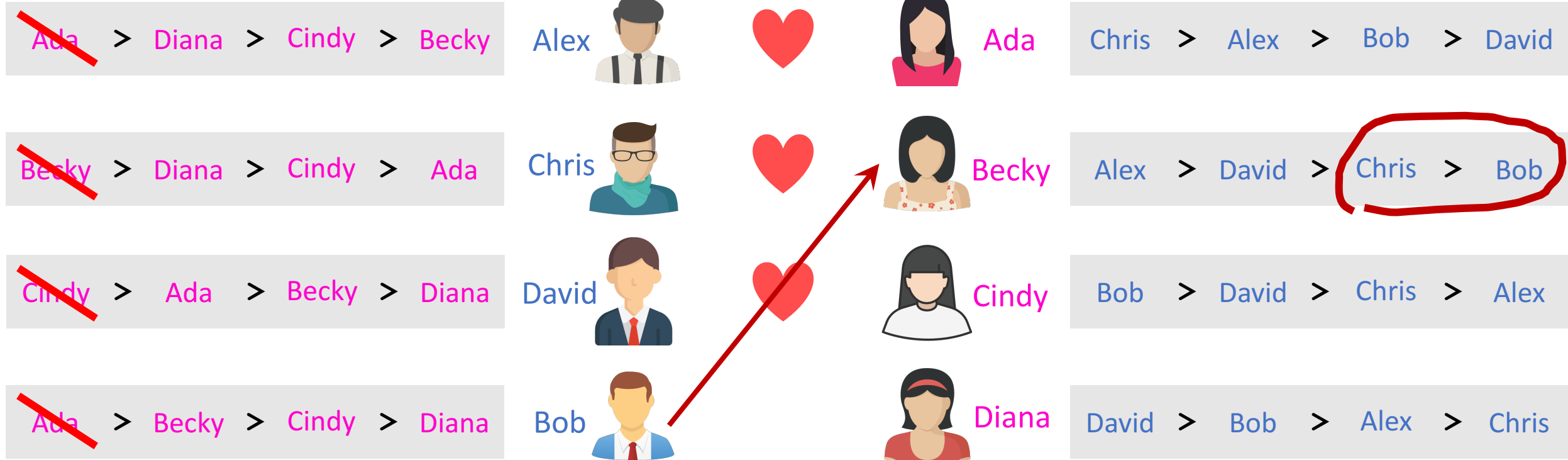
Bob > David > Chris > Alex

David > Bob > Alex > Chris

**Step 1:** An **unmarried** man proposes to the woman who is *his most preferred among those he **has not** proposed to*.



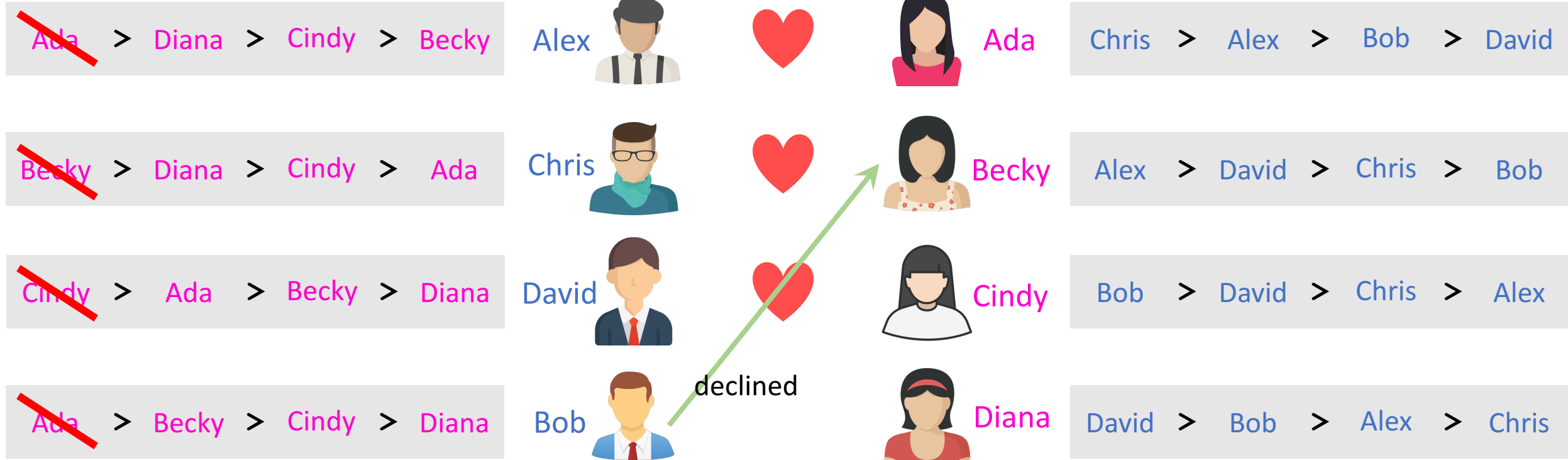
# Iteration 2



**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then divorce.

# Iteration 2



**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then divorce.

# Iteration 2

~~Ada~~ > Diana > Cindy > Becky

Alex



Ada

Chris > Alex > Bob > David

~~Becky~~ > Diana > Cindy > Ada

Chris



Becky

Alex > David > Chris > Bob

~~Cindy~~ > Ada > Becky > Diana

David



Cindy

Bob > David > Chris > Alex

~~Ada~~ > ~~Becky~~ > Cindy > Diana

Bob



Diana

David > Bob > Alex > Chris

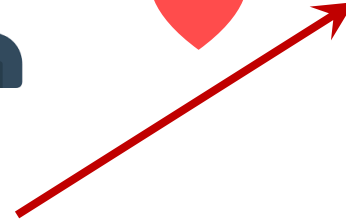
# Iteration 3

~~Ada~~ > Diana > Cindy > Becky

~~Becky~~ > Diana > Cindy > Ada

~~Cindy~~ > Ada > Becky > Diana

~~Ada~~ > ~~Becky~~ > **Cindy** > Diana



Chris > Alex > Bob > David

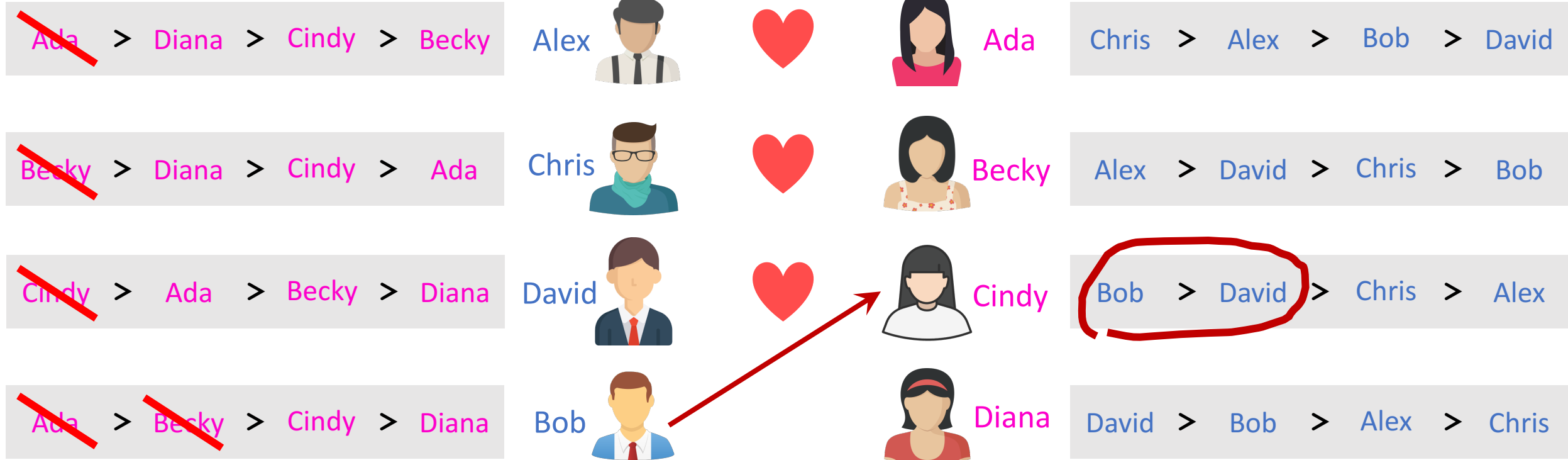
Alex > David > Chris > Bob

Bob > David > Chris > Alex

David > Bob > Alex > Chris

**Step 1:** An **unmarried** man proposes to the woman who is *his most preferred among those he **has not** proposed to*.

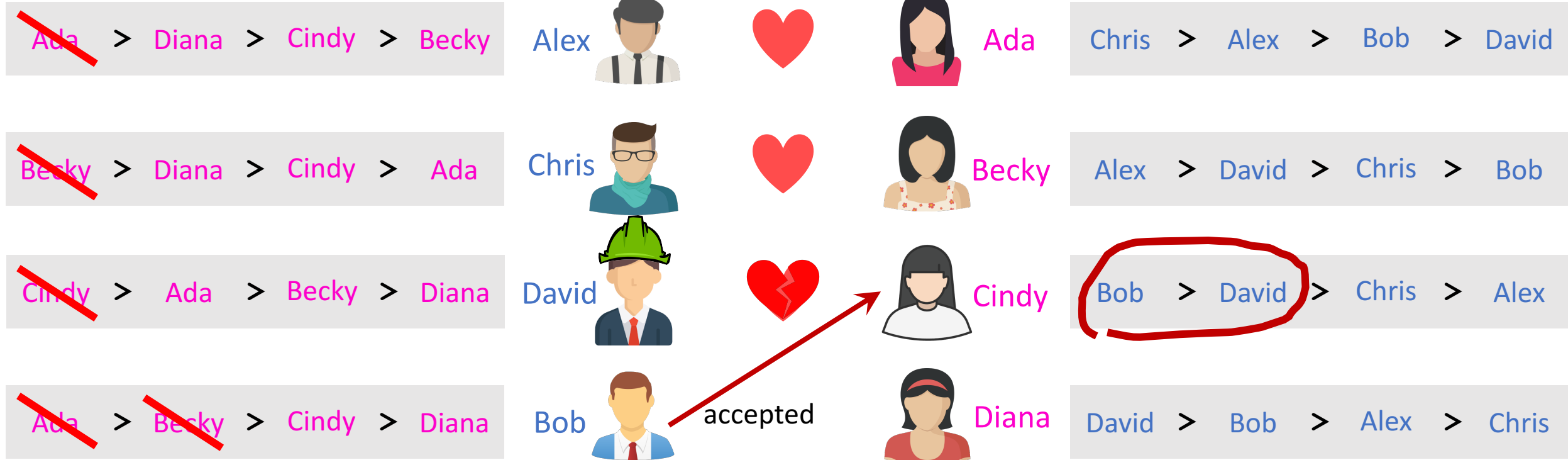
# Iteration 3



**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then **divorce**.

# Iteration 3



**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then **divorce**.

# Iteration 3

~~Ada~~ > Diana > Cindy > Becky

Alex



Ada

Chris > Alex > Bob > David

~~Becky~~ > Diana > Cindy > Ada

Chris



Becky

Alex > David > Chris > Bob

~~Ada~~ > ~~Becky~~ > ~~Cindy~~ > Diana

Bob



Cindy

Bob > David > Chris > Alex

~~Cindy~~ > Ada > Becky > Diana

David



Diana

David > Bob > Alex > Chris

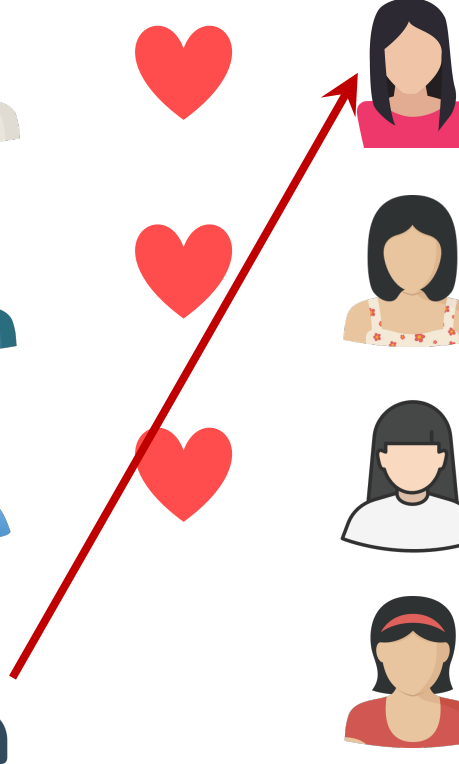
# Iteration 4

~~Ada~~ > Diana > Cindy > Becky

~~Becky~~ > Diana > Cindy > Ada

~~Ada~~ > ~~Becky~~ > ~~Cindy~~ > Diana

~~Cindy~~ > **Ada** > Becky > Diana



Ada

Becky

Cindy

Diana

Chris > Alex > Bob > David

Alex > David > Chris > Bob

Bob > David > Chris > Alex

David > Bob > Alex > Chris

**Step 1:** An **unmarried** man proposes to the woman who is *his most preferred among those he **has not** proposed to.*



# Iteration 4

~~Ada~~ > Diana > Cindy > Becky

~~Becky~~ > Diana > Cindy > Ada

~~Ada~~ > ~~Becky~~ > ~~Cindy~~ > Diana

~~Cindy~~ > Ada > Becky > Diana



declined



Ada

Becky

Cindy

Diana

Chris > **Alex** > Bob > **David**

Alex > David > Chris > Bob

Bob > David > Chris > Alex

David > Bob > Alex > Chris

**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then divorce.

# Iteration 4

~~Ada~~ > Diana > Cindy > Becky

Alex



Ada

Chris > Alex > Bob > David

~~Becky~~ > Diana > Cindy > Ada

Chris



Becky

Alex > David > Chris > Bob

~~Ada~~ > ~~Becky~~ > ~~Cindy~~ > Diana

Bob



Cindy

Bob > David > Chris > Alex

~~Cindy~~ > ~~Ada~~ > Becky > Diana

David



Diana

David > Bob > Alex > Chris

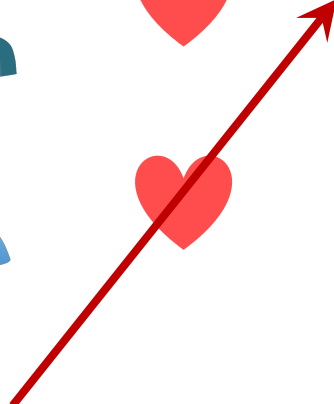
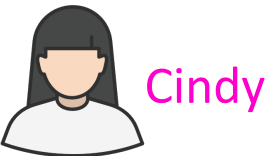
# Iteration 5

~~Ada~~ > Diana > Cindy > Becky

~~Becky~~ > Diana > Cindy > Ada

~~Ada~~ > ~~Becky~~ > ~~Cindy~~ > Diana

~~Cindy~~ > ~~Ada~~ > **Becky** > Diana



Chris > Alex > Bob > David

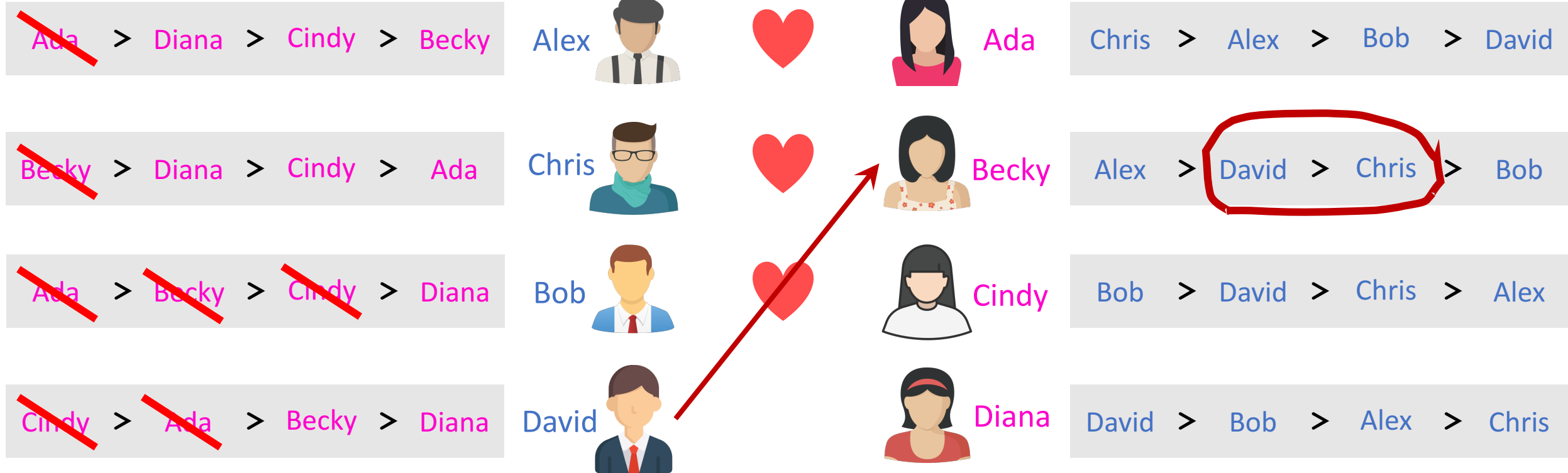
Alex > David > Chris > Bob

Bob > David > Chris > Alex

David > Bob > Alex > Chris

**Step 1:** An **unmarried** man proposes to the woman who is *his most preferred among those he **has not proposed to***.

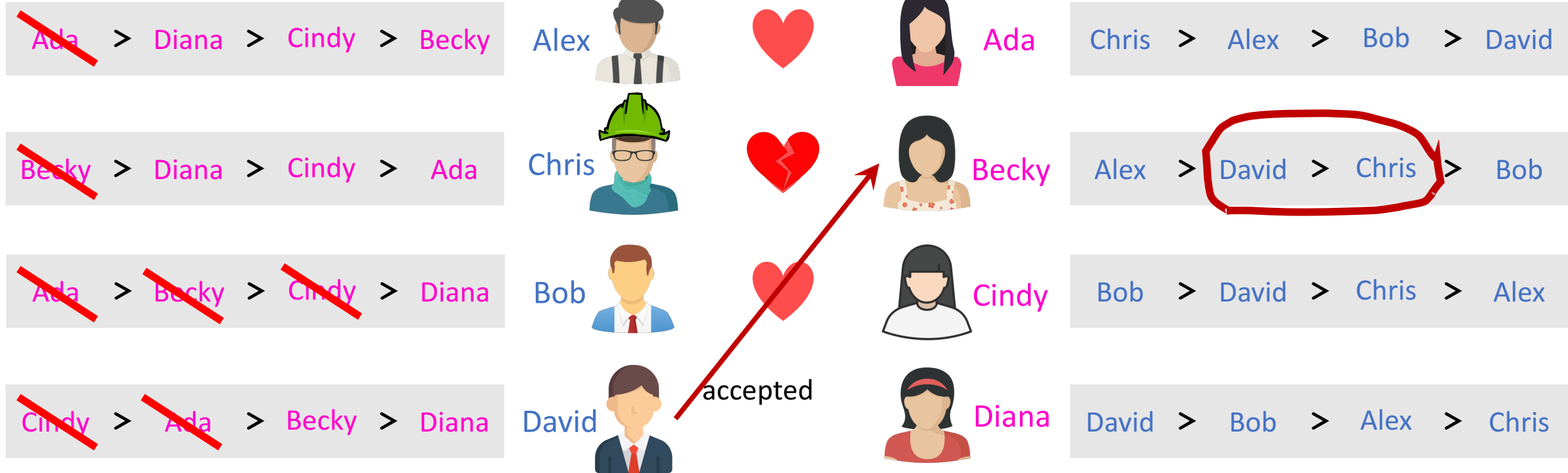
# Iteration 5



**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then **divorce**.

# Iteration 5



**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then **divorce**.

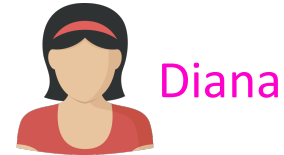
# Iteration 5

~~Ada~~ > Diana > Cindy > Becky

~~Cindy~~ > ~~Ada~~ > ~~Becky~~ > Diana

~~Ada~~ > ~~Becky~~ > ~~Cindy~~ > Diana

~~Becky~~ > Diana > Cindy > Ada



Chris > Alex > Bob > David

Alex > David > Chris > Bob

Bob > David > Chris > Alex

David > Bob > Alex > Chris

# Iteration 6

~~Ada~~ > Diana > Cindy > Becky



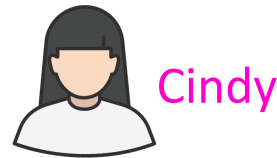
Chris > Alex > Bob > David

~~Cindy~~ > ~~Ada~~ > ~~Becky~~ > Diana



Alex > David > Chris > Bob

~~Ada~~ > ~~Becky~~ > ~~Cindy~~ > Diana



Bob > David > Chris > Alex

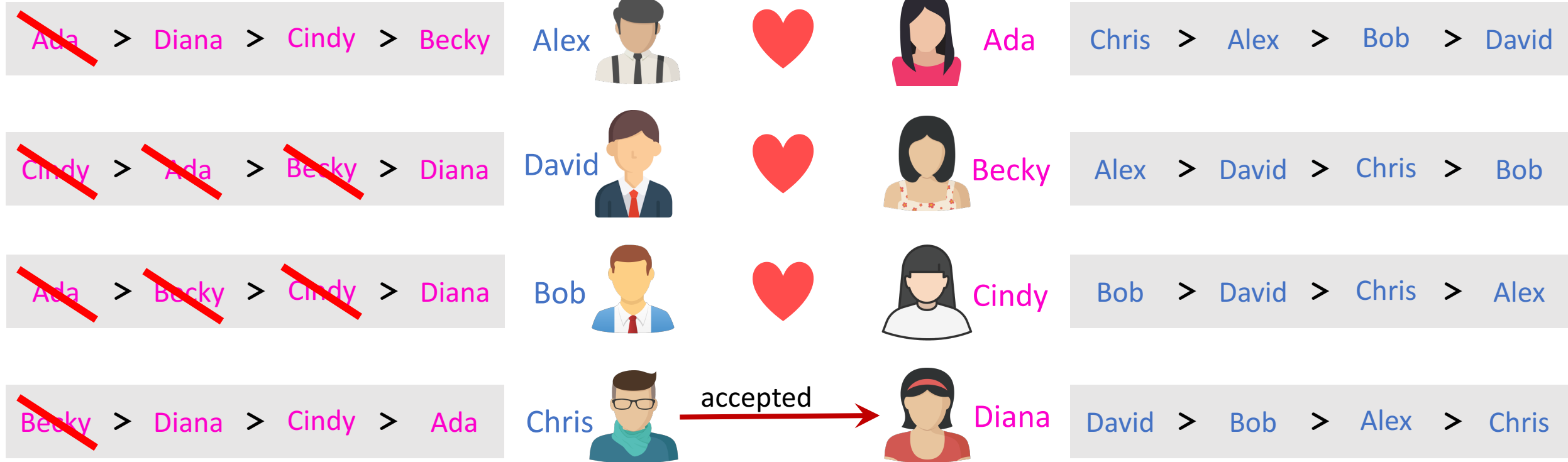
~~Becky~~ > **Diana** > Cindy > Ada



David > Bob > Alex > Chris

**Step 1:** An **unmarried** man proposes to the woman who is *his most preferred among those he **has not proposed to***.

# Iteration 6



**Step 2:** A woman accepts her best offer.

- If the woman finds better matching than her current spouse, then divorce.



# End of Procedure

~~Ada~~ > Diana > Cindy > Becky



Chris > Alex > Bob > David

~~Cindy~~ > ~~Ada~~ > ~~Becky~~ > Diana



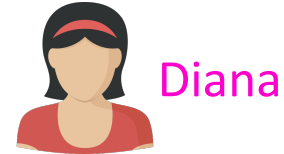
Alex > David > Chris > Bob

~~Ada~~ > ~~Becky~~ > ~~Cindy~~ > Diana



Bob > David > Chris > Alex

~~Becky~~ > ~~Diana~~ > Cindy > Ada



David > Bob > Alex > Chris

Stop if everyone is married.

# Summary

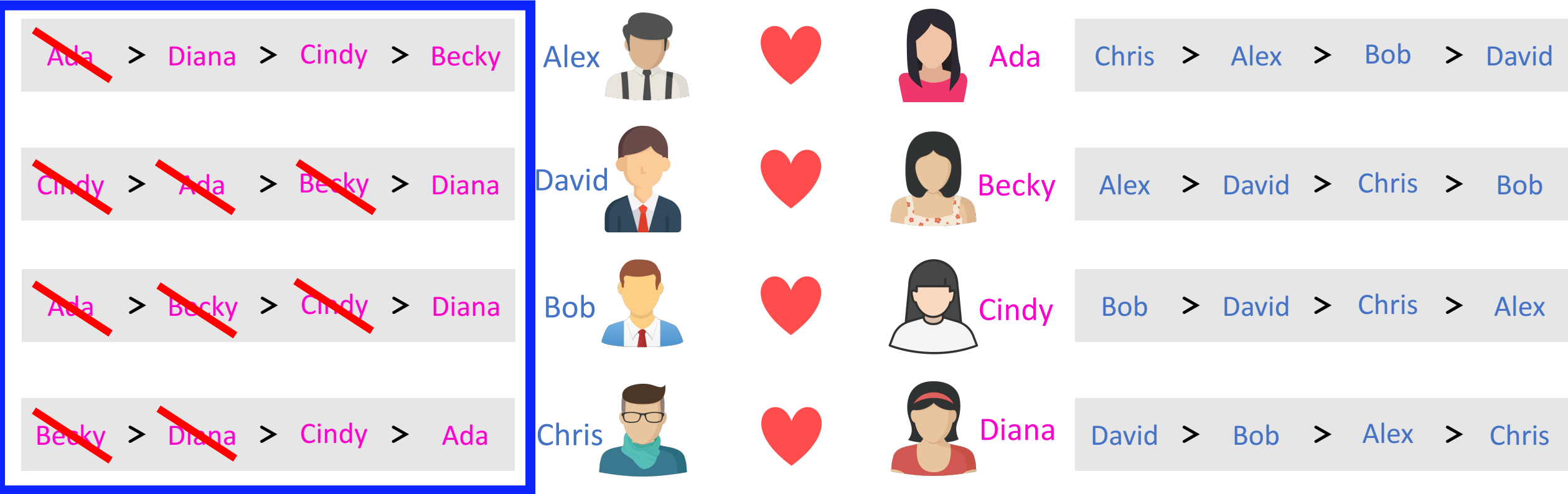
# Gale-Shapley Algorithm

1. Every **unmarried** man proposes to a woman who is his **most preferred** among those he **has not proposed to**.

# Gale-Shapley Algorithm

1. Every **unmarried** man proposes to a woman who is his **most preferred** among those he **has not proposed to**.
2. A woman marries her most preferred suitor.
  - A bad suitor is better than no suitor.
  - If the suitor is better than her current husband, then she will divorce.
3. Stop if everyone is married; otherwise, repeat Steps 1 and 2.

# Time Complexity: $O(n^2)$



- The total number of proposals is at most  $O(n^2)$ .
- A woman takes  $O(1)$  time to decide whether to accept or decline.

**Thank You!**

<http://wangshusen.github.io/>