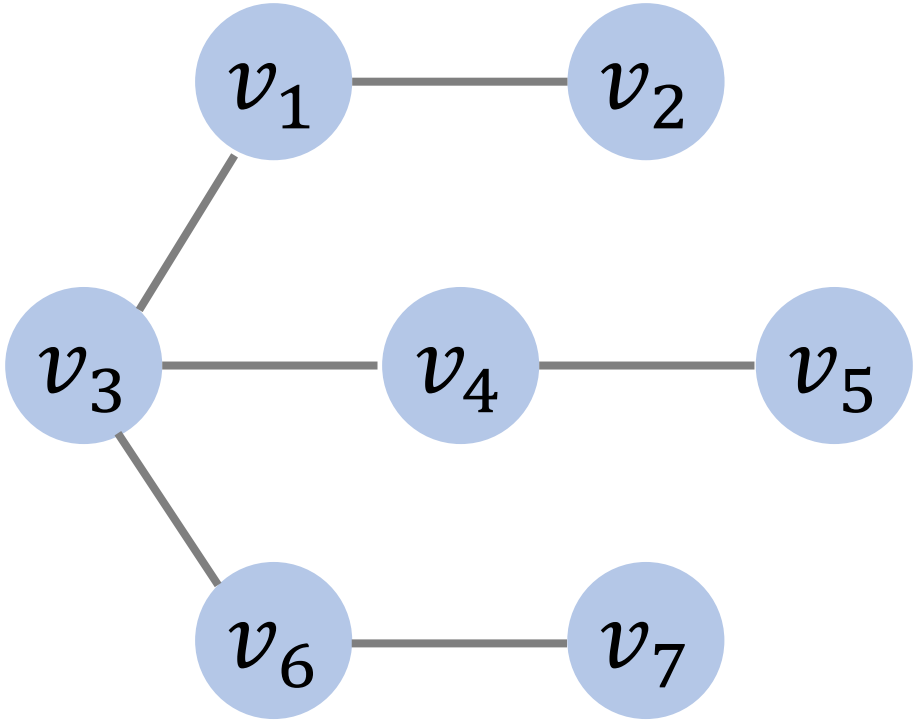
Minimum Spanning Trees

Shusen Wang

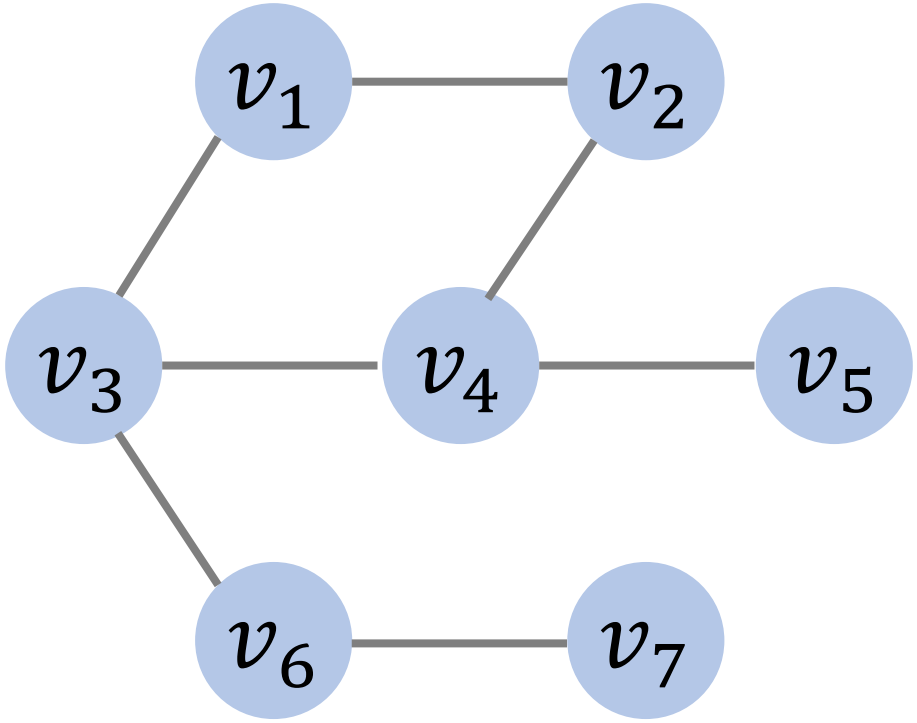
http://wangshusen.github.io/

Trees vs Graphs

Trees are undirected graphs

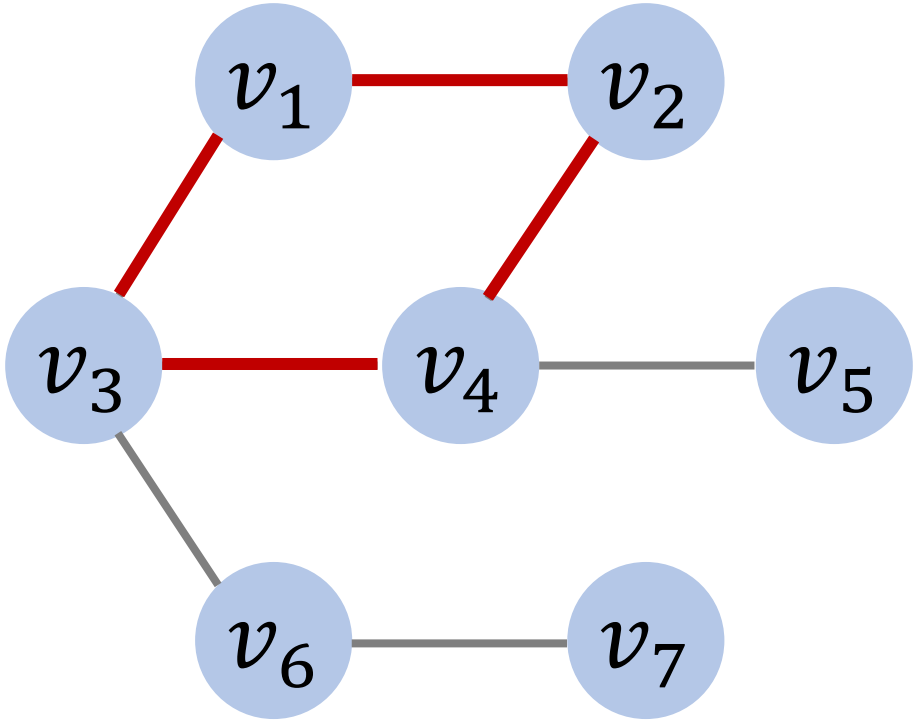


Trees do not have cycles



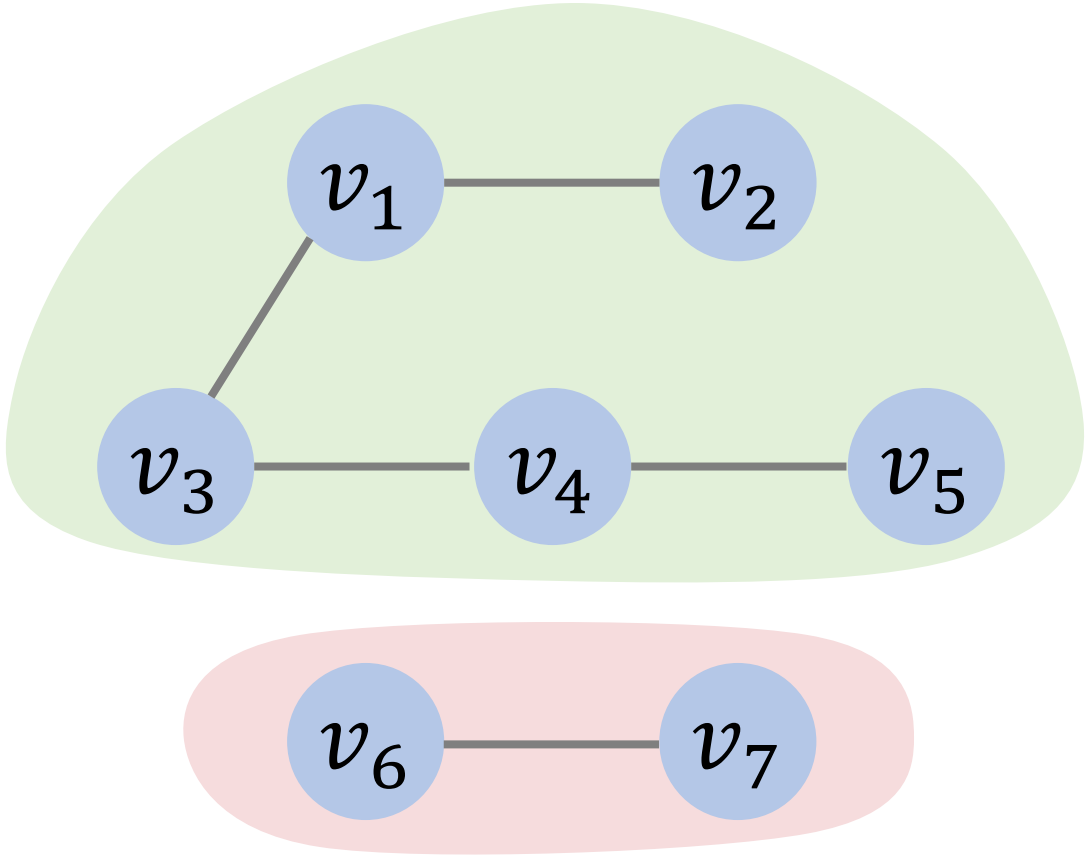
Not a tree

Trees do not have cycles



Not a tree

Trees are connected graphs



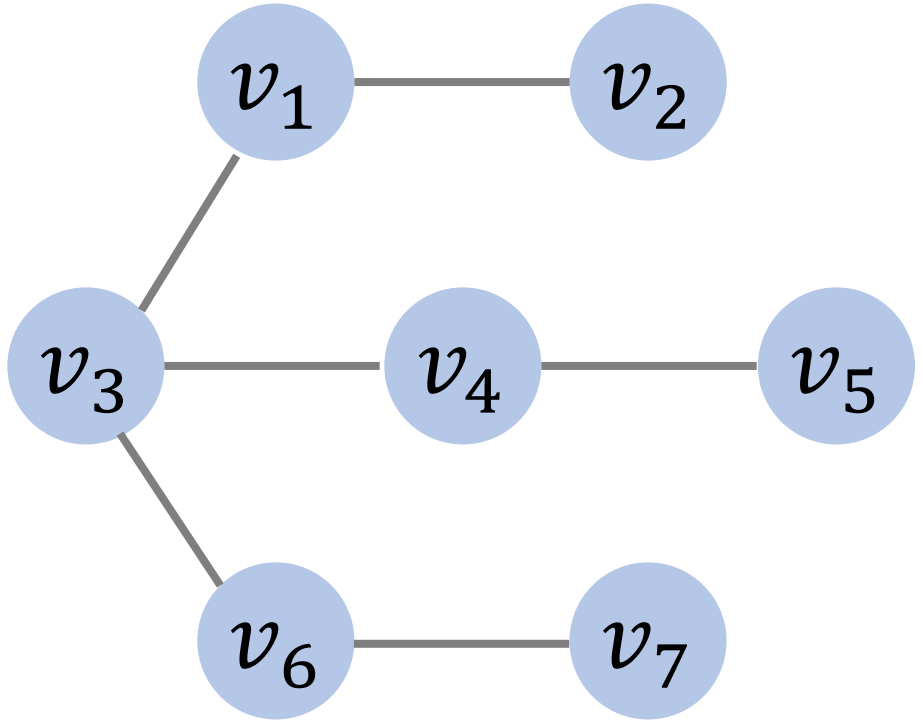
Not a tree

Definition of Trees

Trees are connected acyclic undirected graphs.

Number of Vertices and Edges

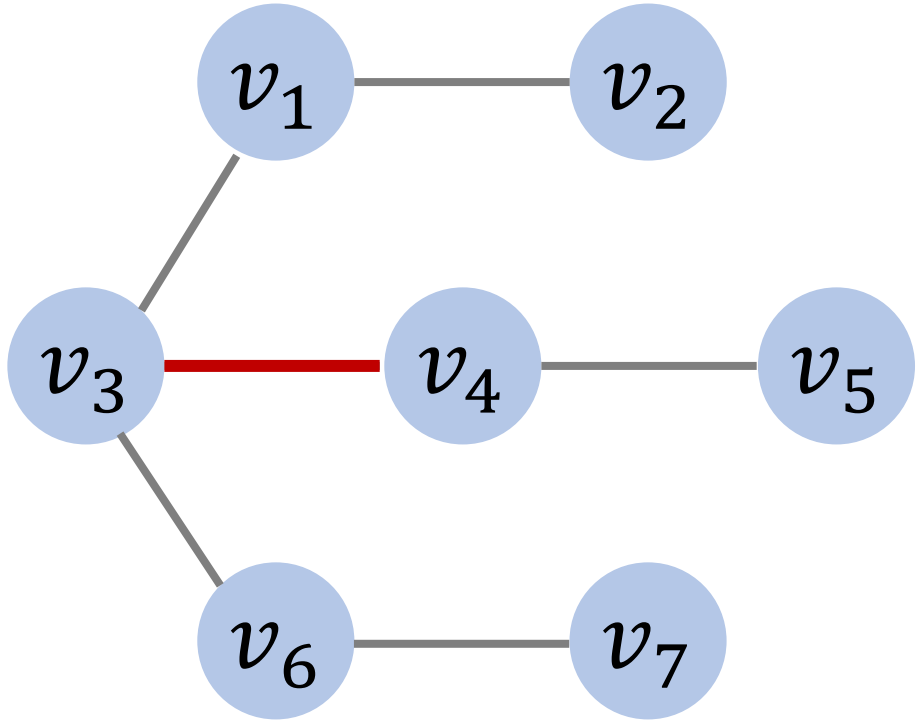
- If a tree has n vertices, then it has $n - 1$ edges.



Tree

Number of Vertices and Edges

- Let n be the number of vertices.
- Less than $n - 1$ edges
  ➔ Disconnected.
- More than $n - 1$ edges
  ➔ There is a cycle.



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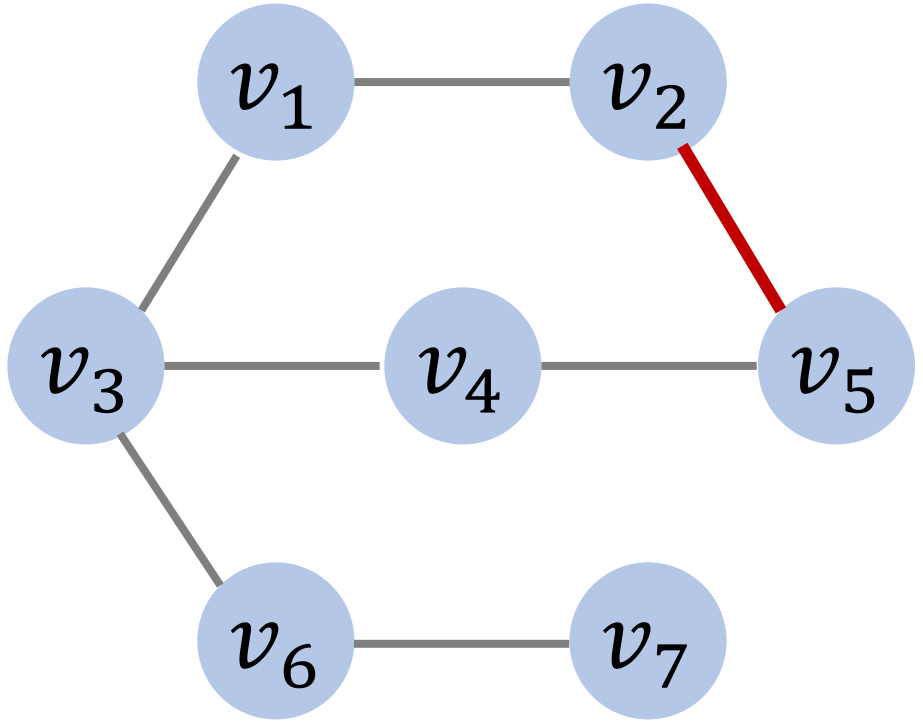


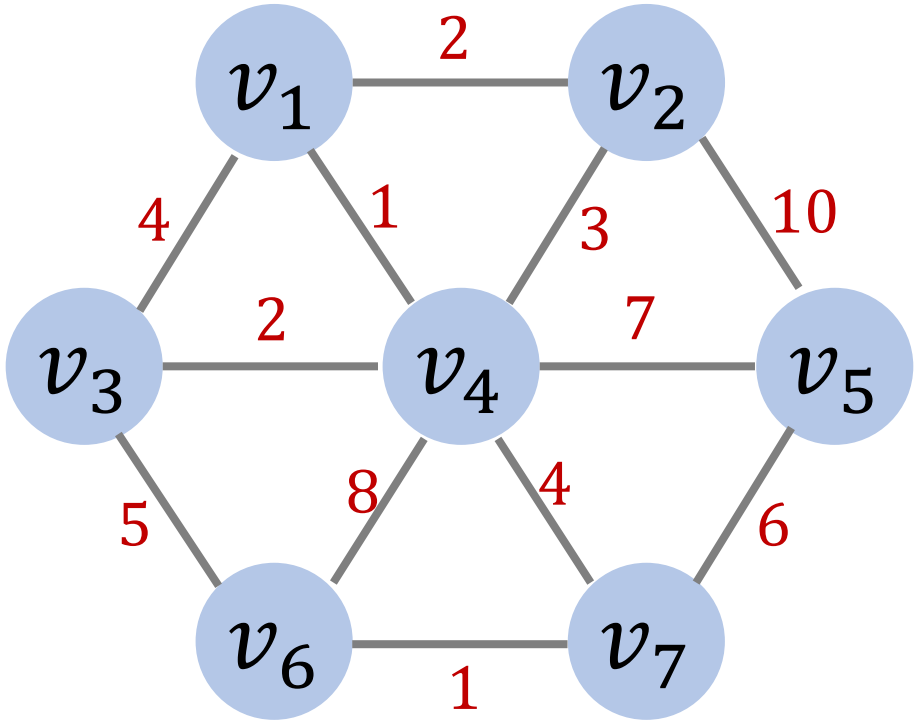
Tree

Spanning Trees

Spanning Trees

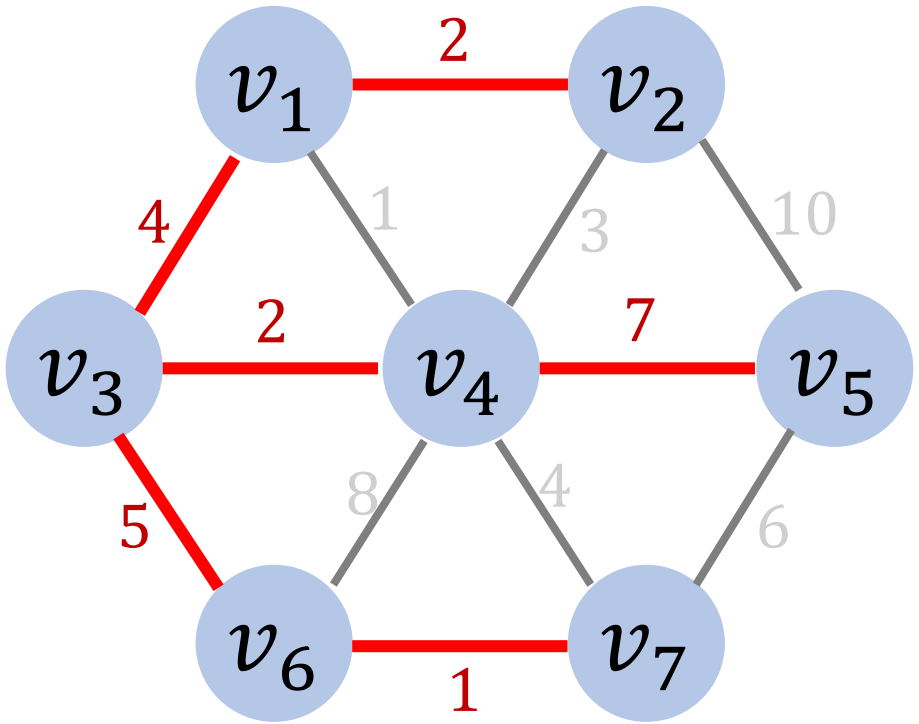
- **Input:** a connected undirected graph with n vertices.
- Find such a subgraph:
  - Keep all the n vertices.
  - Keep $n - 1$ edges.
  - The subgraph is connected.
- The subgraph is a spanning tree.

Spanning Trees



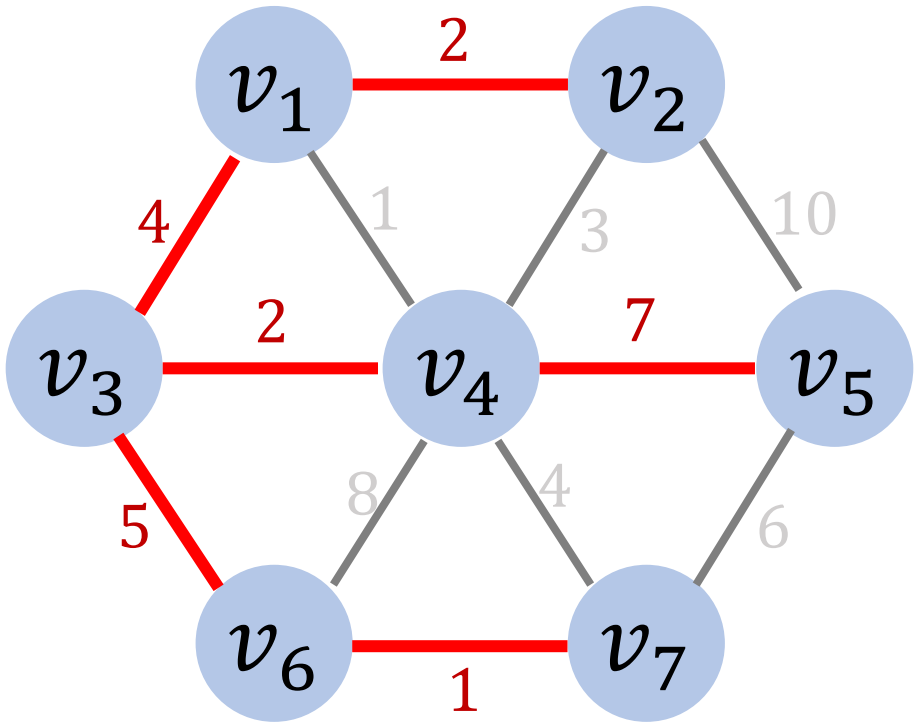
Original Graph

Spanning Trees

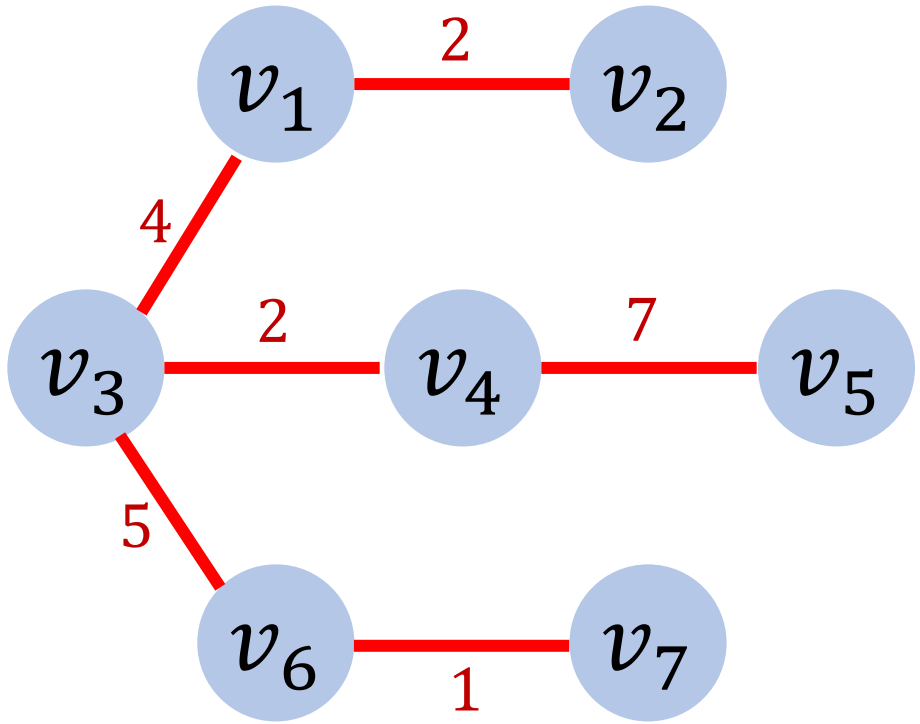


Original Graph

Spanning Trees



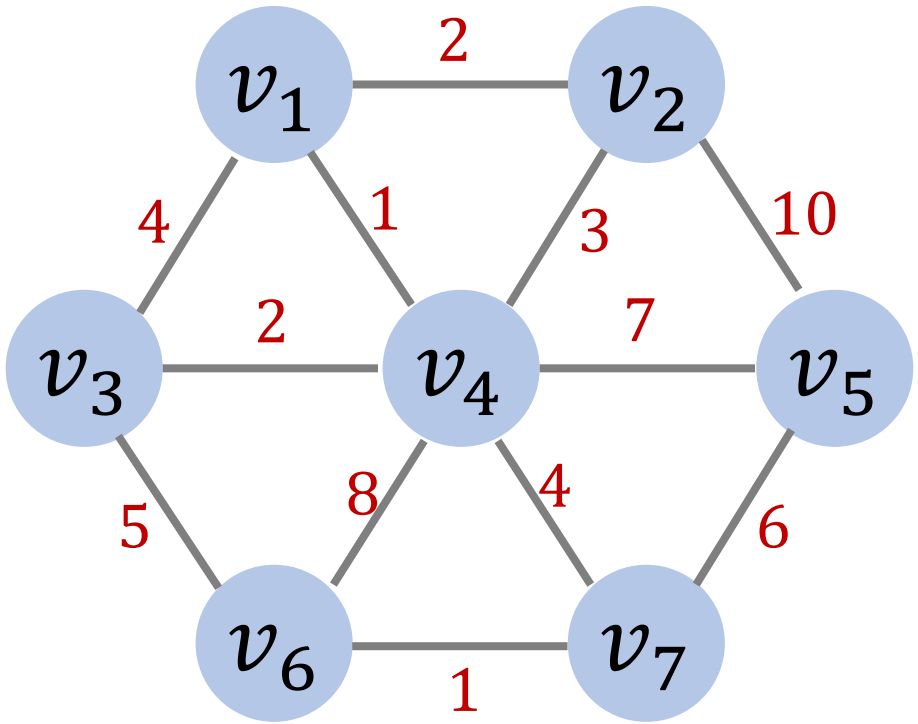
Original Graph



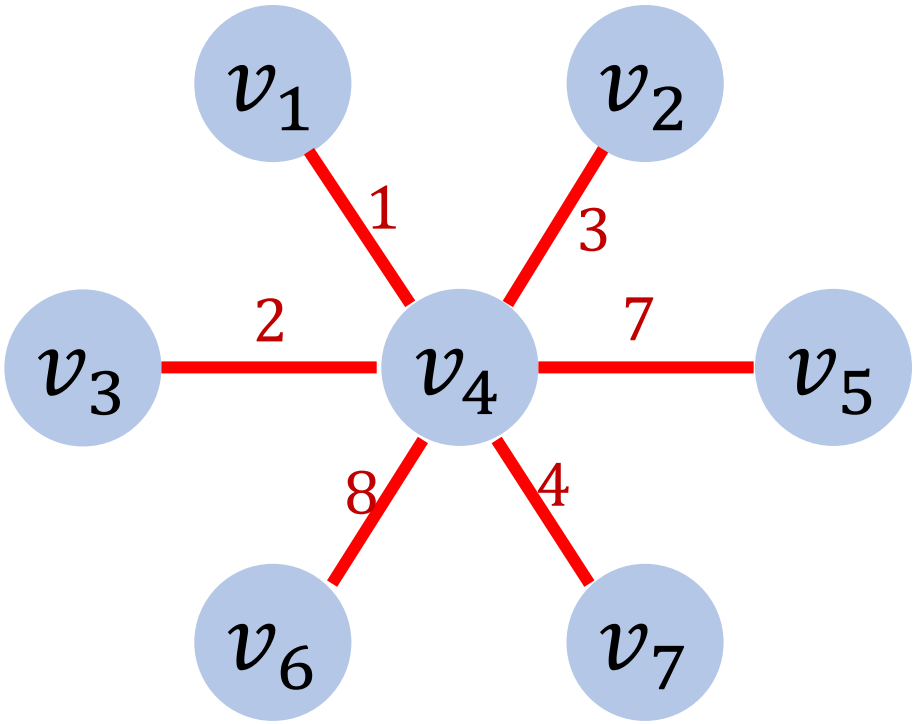
A Spanning Tree

Sum of weights is 21.

Spanning trees are not unique



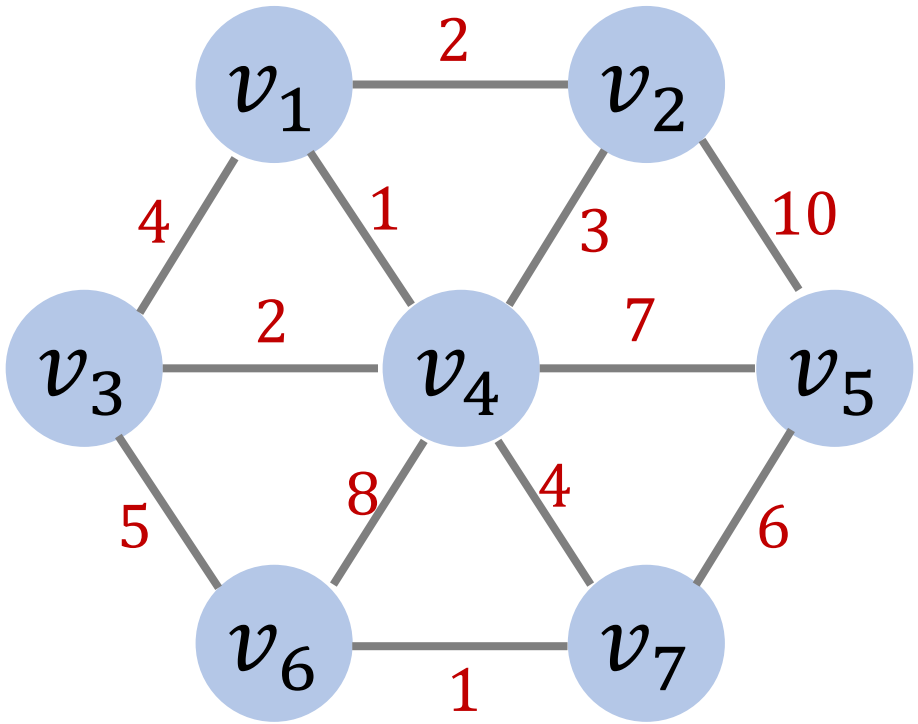
Original Graph



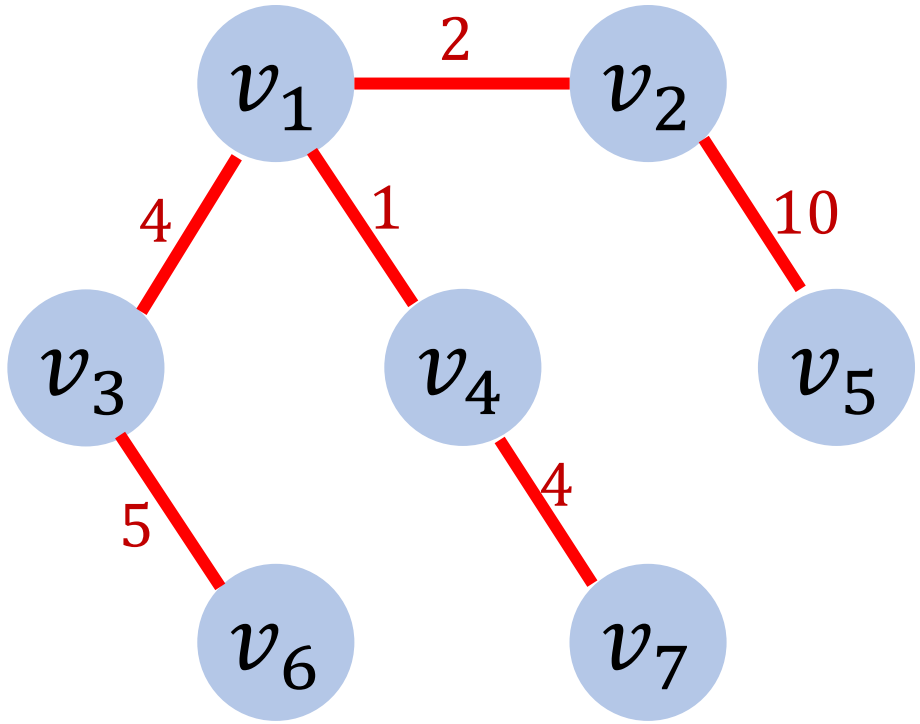
A Spanning Tree

Sum of weights is 25.

Spanning trees are not unique



Original Graph

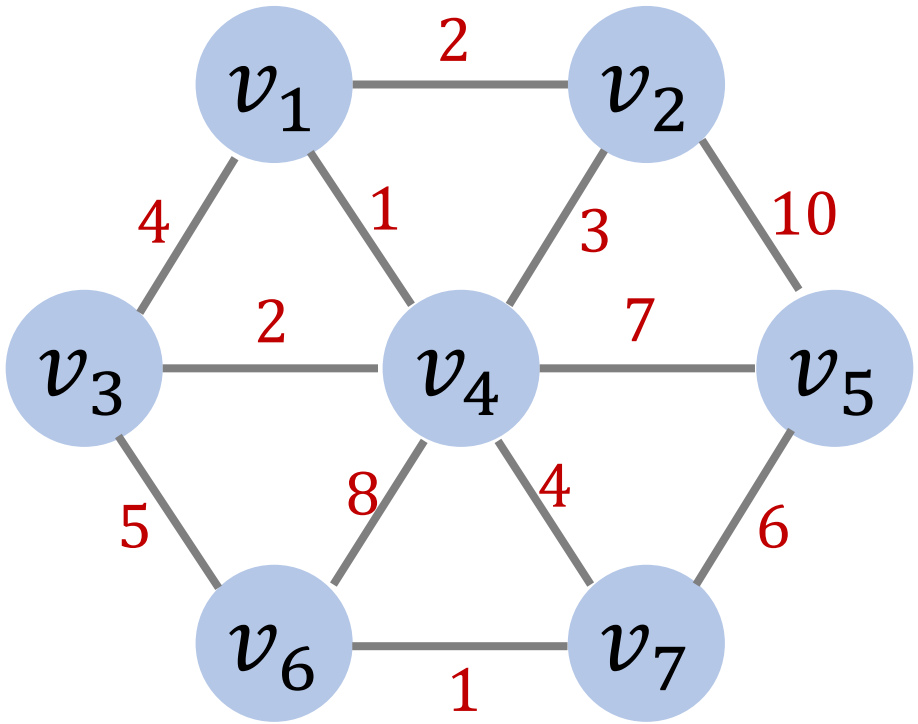


A Spanning Tree

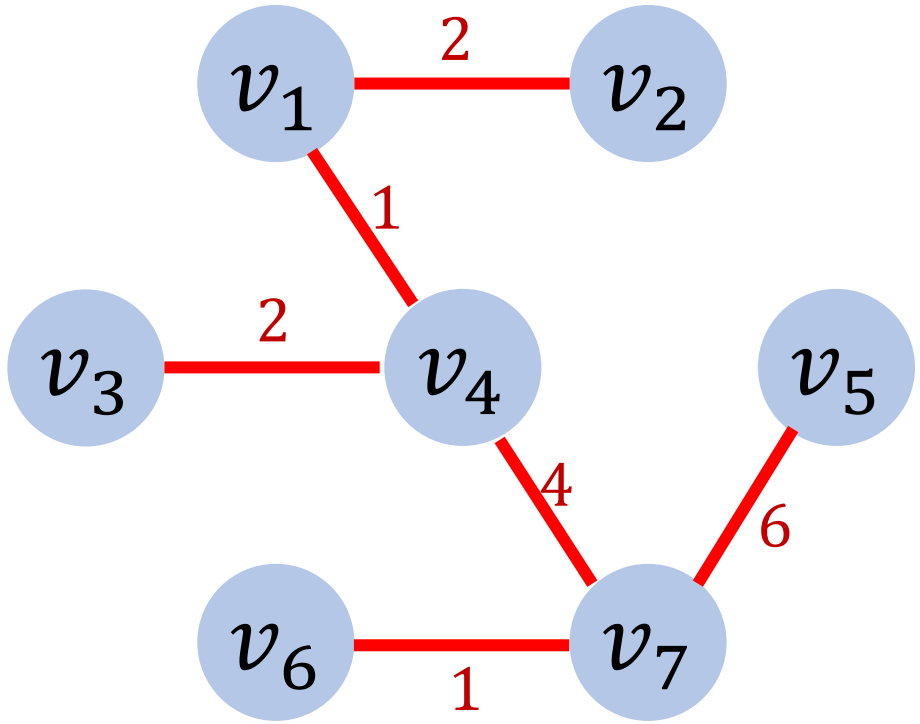
Sum of weights is 26.

Minimum Spanning Trees

Minimum spanning tree is a spanning tree that minimizes the sum of weights.



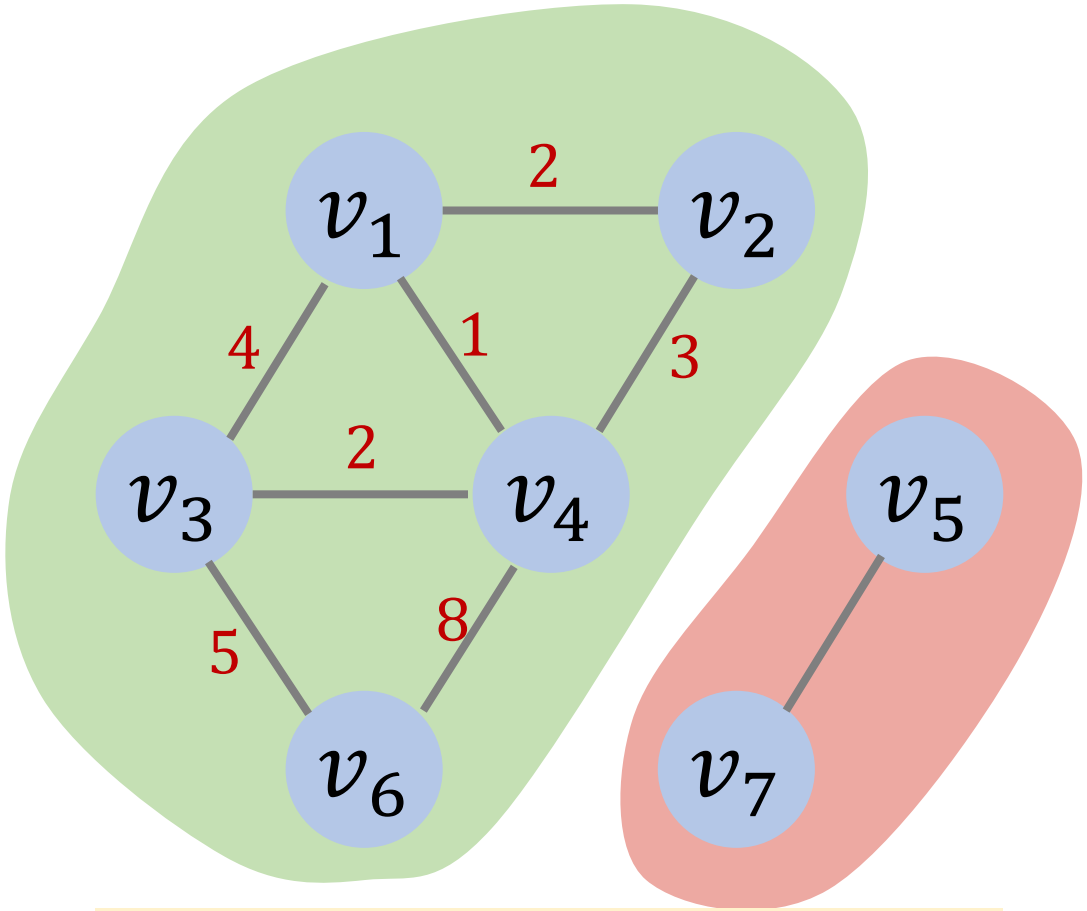
Original Graph



Minimum Spanning Tree

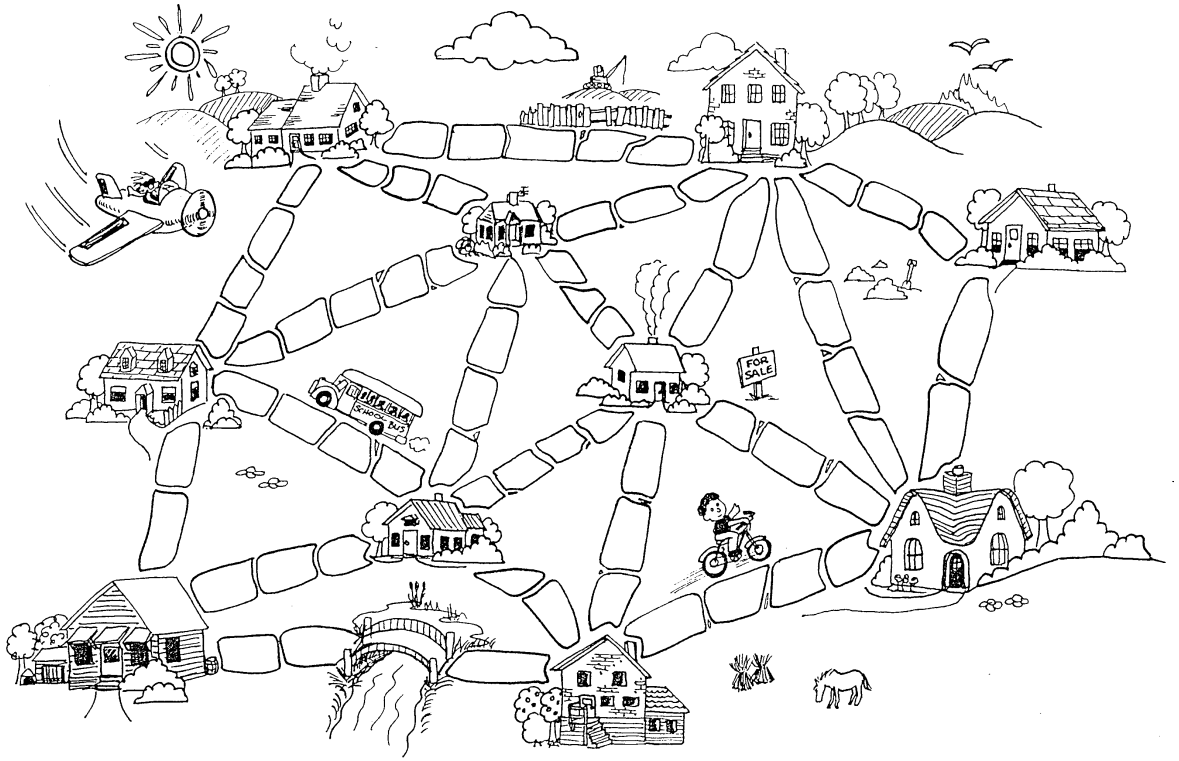
Sum of weights is 16.

A graph may not have spanning tree



Original Graph

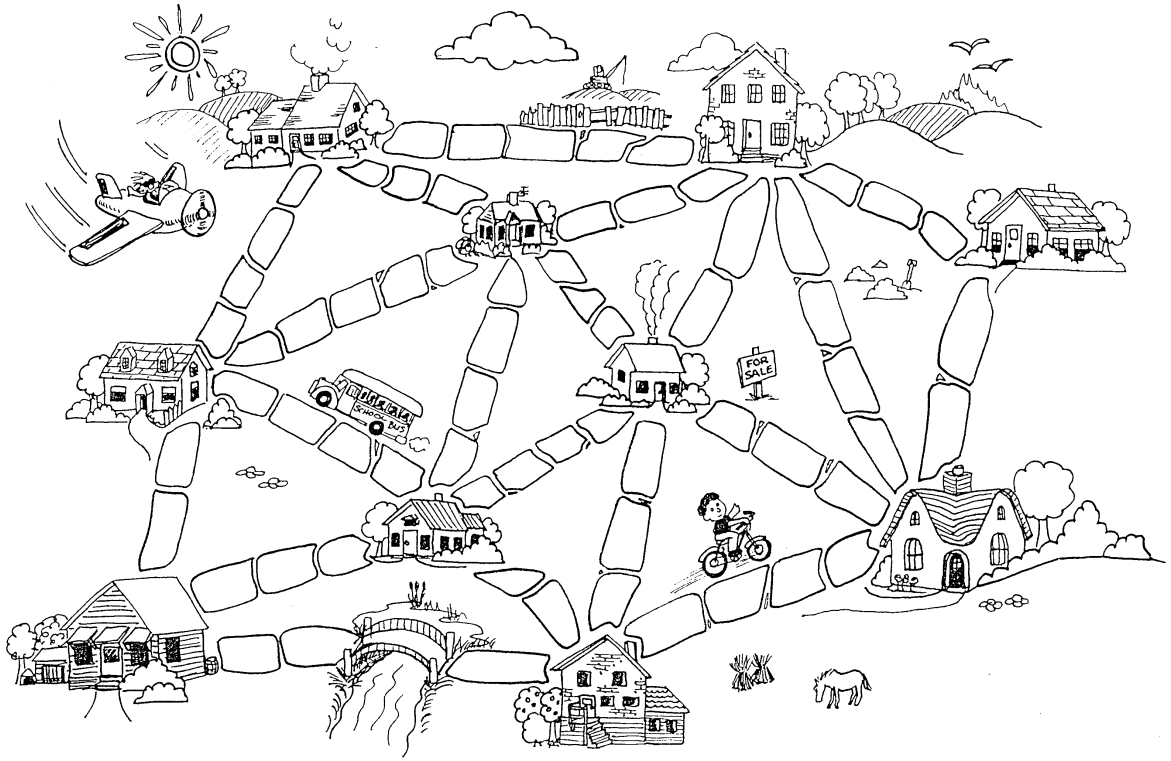
Application: Muddy City Problem



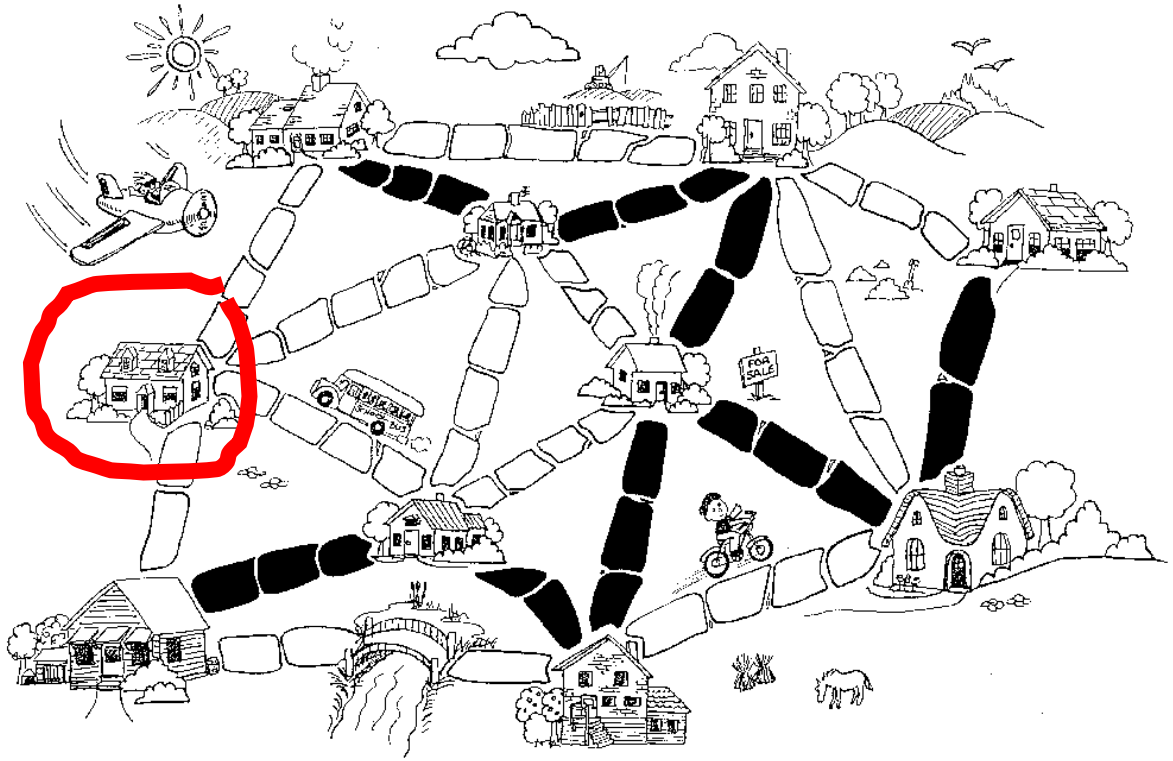
The muddy roads form a graph.

- The city has muddy roads.
- The mayor wants to pave roads.
- Constraints:
  1. Enough roads must be paved so that everyone can travel from his house to anyone else's house.
  2. The paving should cost as little as possible.

Application: Muddy City Problem

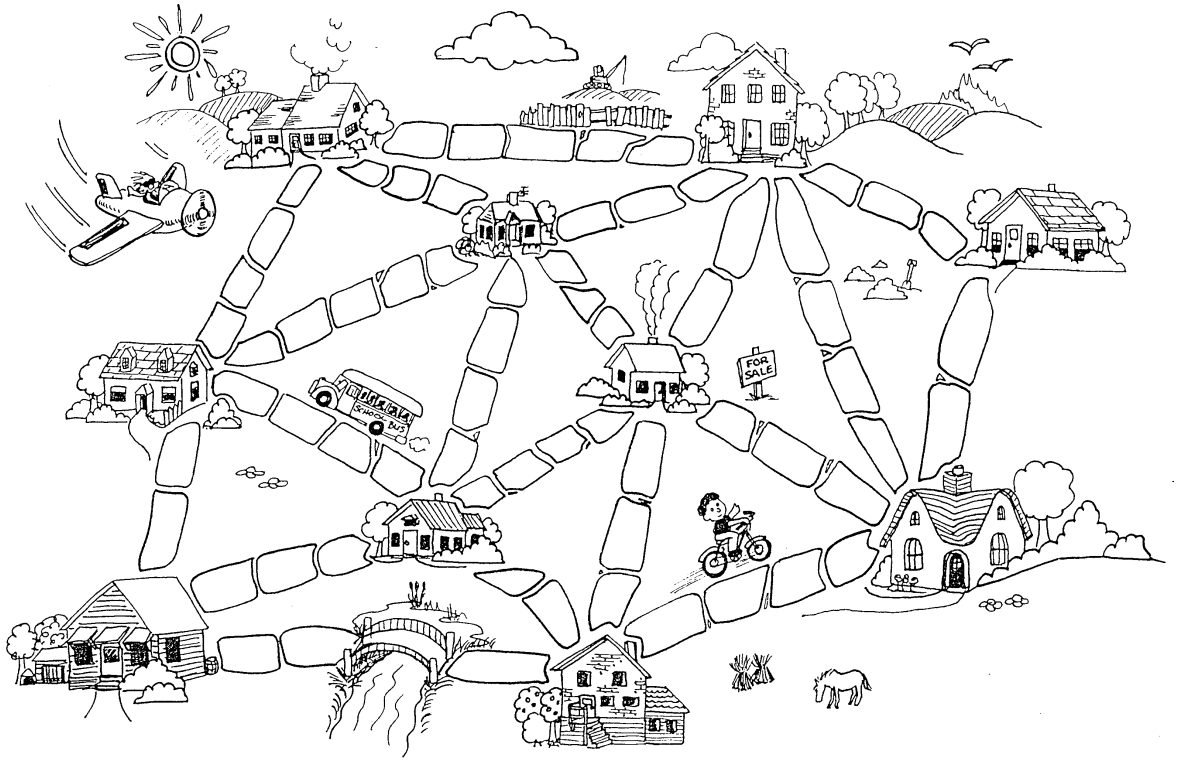


The muddy roads form a graph.



Not a spanning tree.

Application: Muddy City Problem



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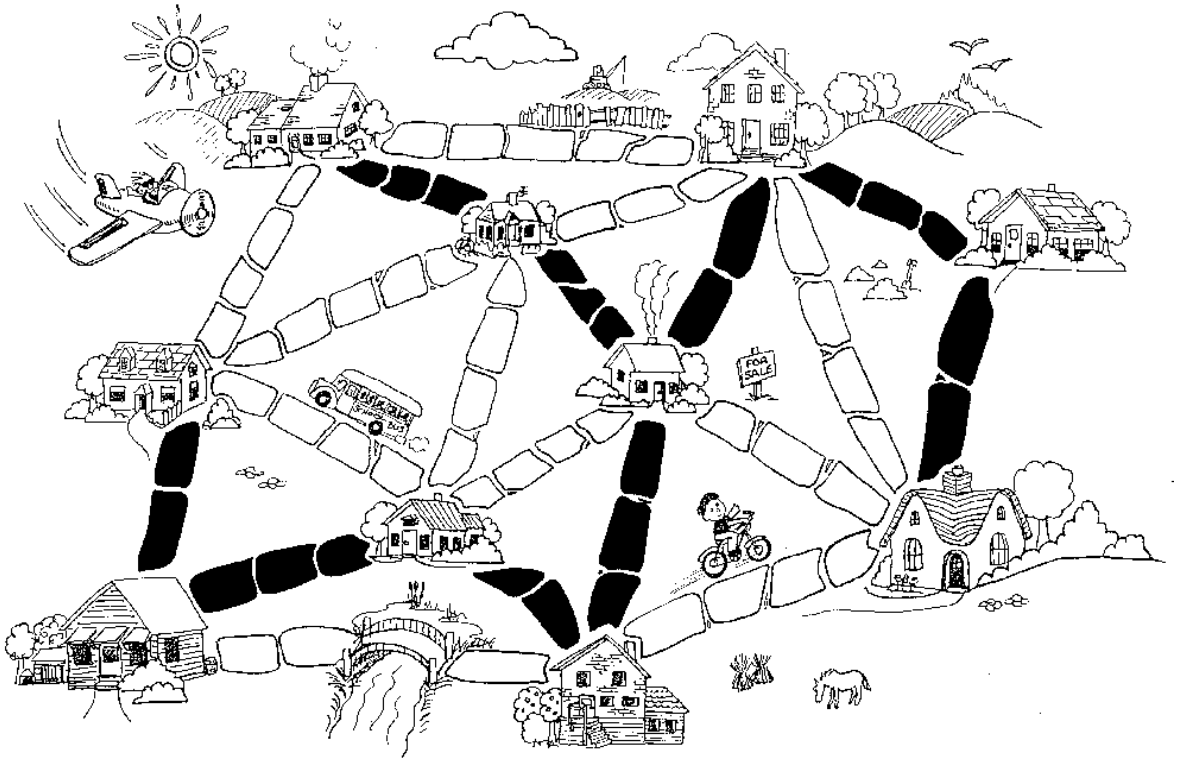

Spanning tree.

Summary

Trees vs Graphs

Trees are undirected graphs. An undirected graph may not be a tree.

Properties of trees:

- There is exactly one path between any two vertices.
- Trees do not have cycles.
- If there are n vertices, then there must be $n - 1$ edges.

Spanning Trees

Input: A connected undirected graph.

- Keep all the n vertices.
- Keep a subset of $n - 1$ edges.
- The subgraph must be connected and acyclic.

Output: The obtained subgraph is called spanning tree.

Minimum spanning tree: The spanning tree with the minimum sum of weights.

Thank You!

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