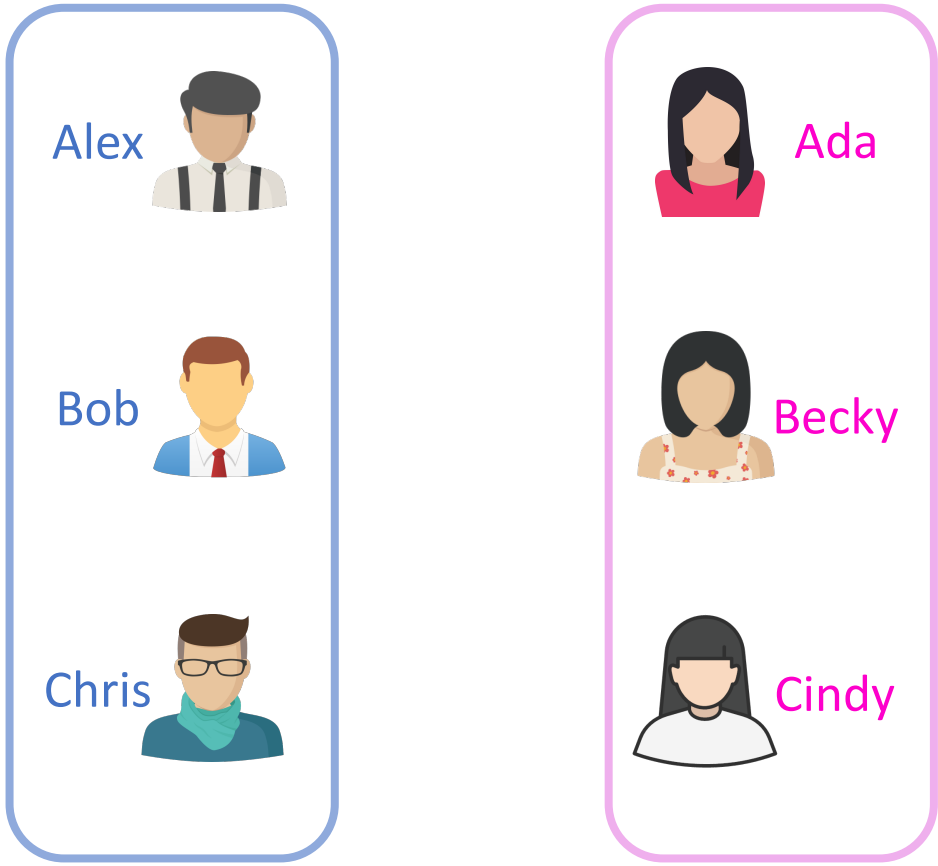
Stable Marriage Problem

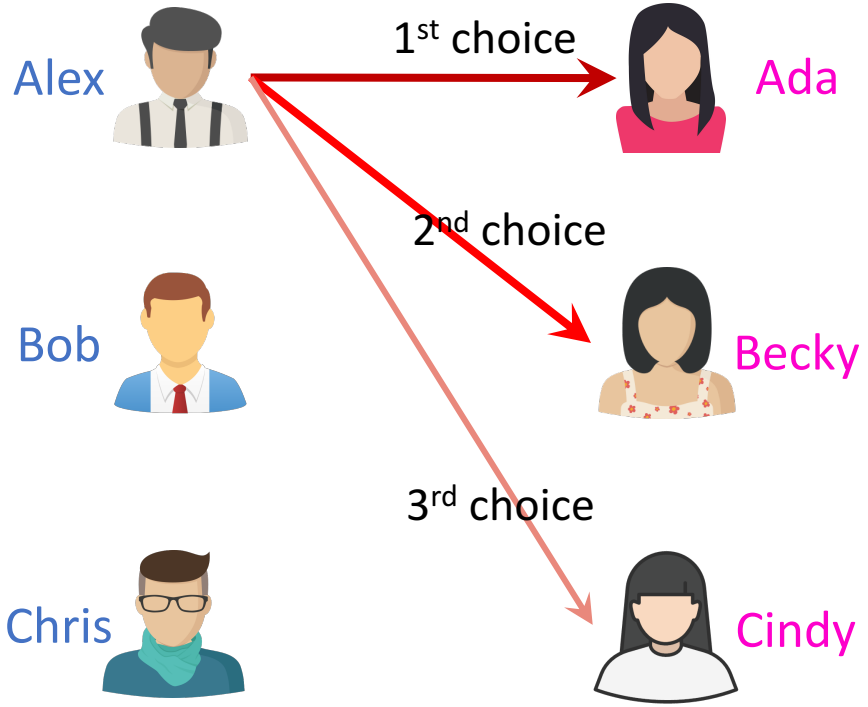
Shusen Wang

http://wangshusen.github.io/

Bipartite Matching



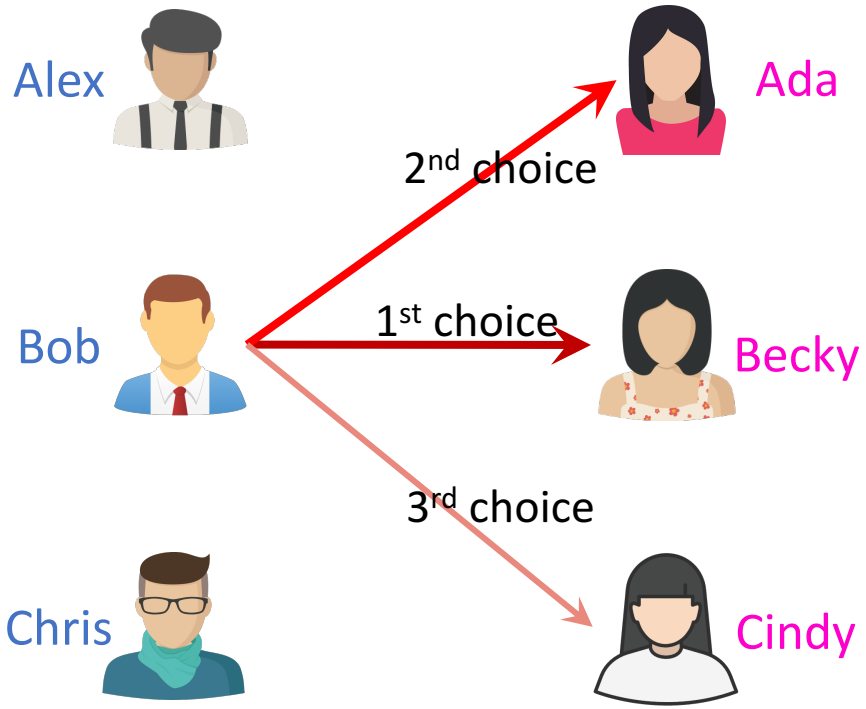
Bipartite Matching



Bipartite Matching

Ada > Becky > Cindy

Becky > Ada > Cindy



Bipartite Matching

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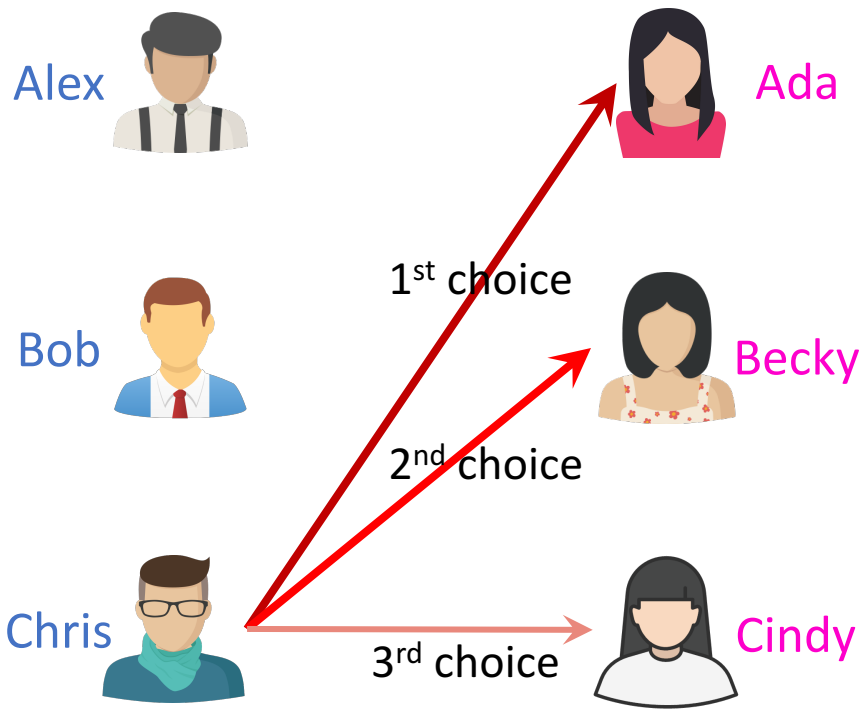
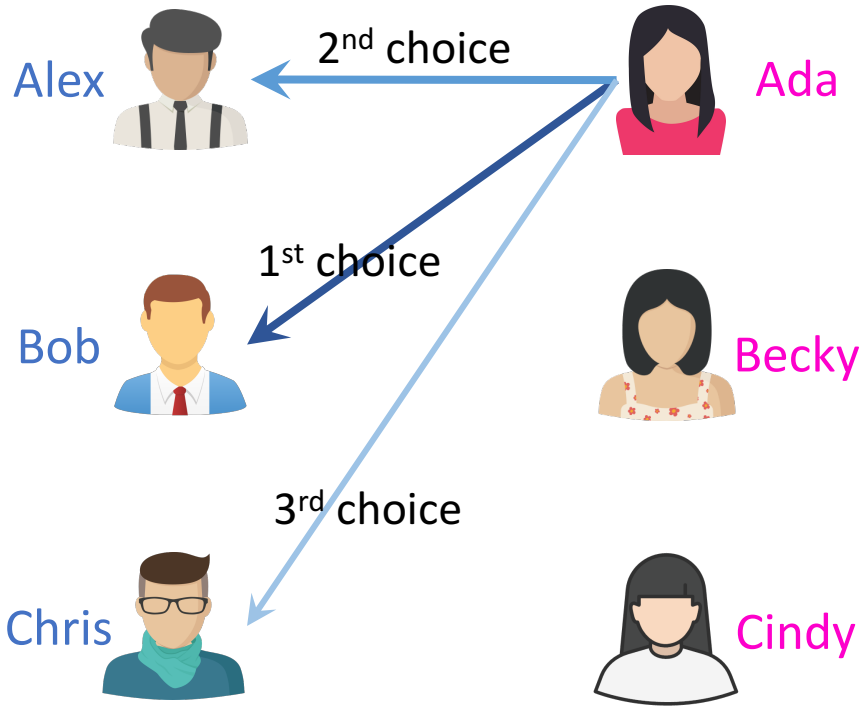


Bipartite Matching

Ada > Becky > Cindy

Becky > Ada > Cindy

Ada > Becky > Cindy



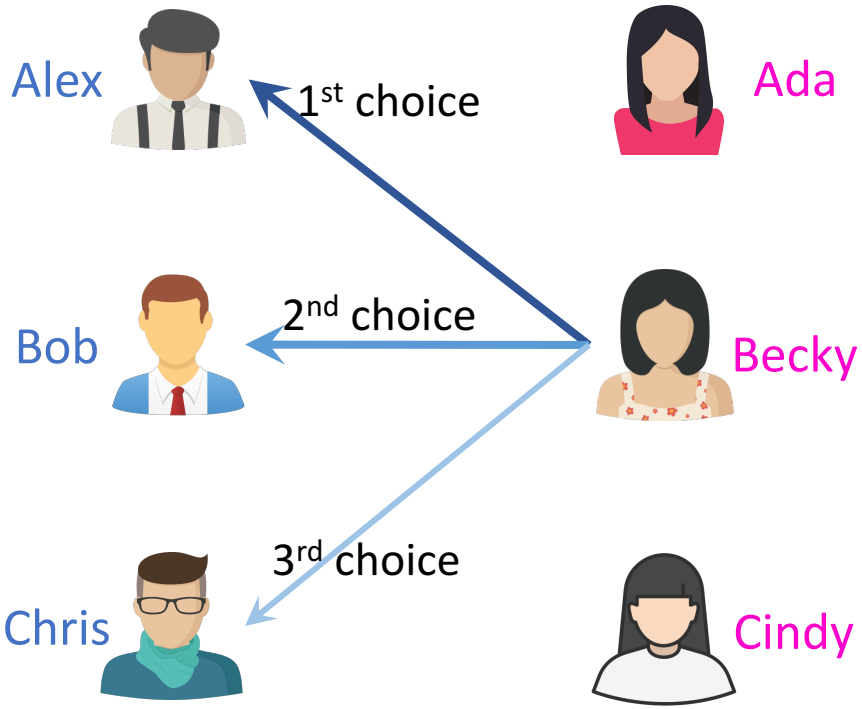
Bob > Alex > Chris

Bipartite Matching

Ada > Becky > Cindy

Becky > Ada > Cindy

Ada > Becky > Cindy



Bob > Alex > Chris

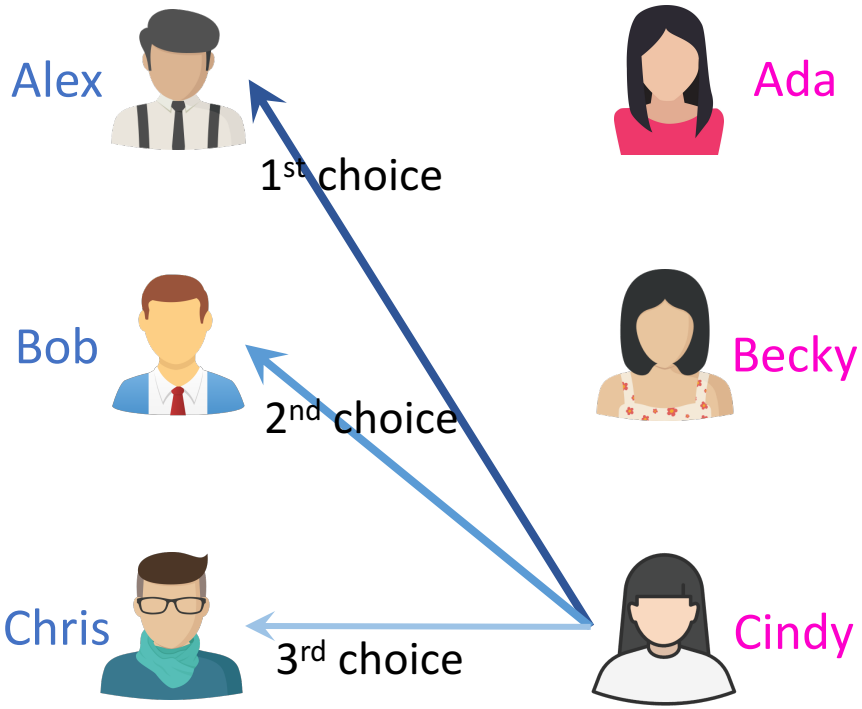
Alex > Bob > Chris

Bipartite Matching

Ada > Becky > Cindy

Becky > Ada > Cindy

Ada > Becky > Cindy



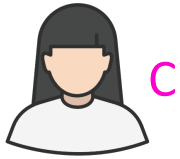
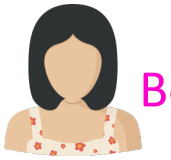
Bob > Alex > Chris

Alex > Bob > Chris

Alex > Bob > Chris

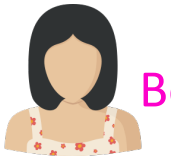
Stable Marriage

Is this a stable marriage?

Ada > Becky > Cindy	Alex 	❤	 Cindy	Alex > Bob > Chris
Becky > Ada > Cindy	Bob 	❤	 Becky	Alex > Bob > Chris
Ada > Becky > Cindy	Chris 	❤	 Ada	Bob > Alex > Chris

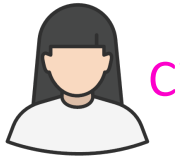
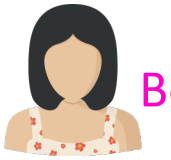
If a man and a woman (who are not spouses) prefer each other over their current spouses, then the marriage is not stable.

Is this a stable marriage?

Ada > Becky > Cindy		Alex	♥		Cindy	Alex > Bob > Chris
Becky > Ada > Cindy		Bob	♥		Becky	Alex > Bob > Chris
Ada > Becky > Cindy		Chris	♥		Ada	Bob > Alex > Chris

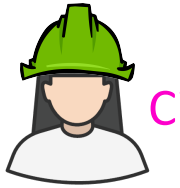
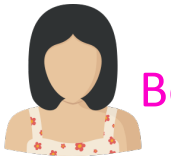
- Alex prefers Ada over his wife, Cindy.

Is this a stable marriage?

Ada > Becky > Cindy	Alex		♥		Cindy	Alex > Bob > Chris
Becky > Ada > Cindy	Bob		♥		Becky	Alex > Bob > Chris
Ada > Becky > Cindy	Chris		♥		Ada	Bob > Alex > Chris

- Alex prefers Ada over his wife, Cindy.
- Ada prefers Alex over her husband, Chris.

This is not a stable marriage!

Ada > Becky > Cindy	Alex 		 Cindy	Alex > Bob > Chris
Becky > Ada > Cindy	Bob 		 Becky	Alex > Bob > Chris
Ada > Becky > Cindy	Chris 		 Ada	Bob > Alex > Chris

- Alex prefers Ada over his wife, Cindy.
- Ada prefers Alex over her husband, Chris.

This is a stable marriage

Ada > Becky > Cindy

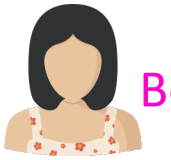
Alex 

Ada

Bob > Alex > Chris

Becky > Ada > Cindy

Bob 

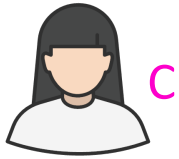


Becky

Alex > Bob > Chris

Ada > Becky > Cindy

Chris 



Cindy

Alex > Bob > Chris

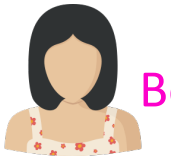
This is a stable marriage

Ada > Becky > Cindy

Alex  ❤  Ada

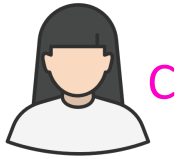
Bob > Alex > Chris

Becky > Ada > Cindy

Bob  ❤  Becky

Alex > Bob > Chris

Ada > Becky > Cindy

Chris  ❤  Cindy

Alex > Bob > Chris

This is a stable marriage

Ada > Becky > Cindy

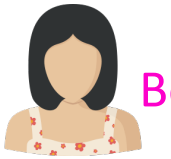
Alex 

 Ada

Bob > Alex > Chris

Becky > Ada > Cindy

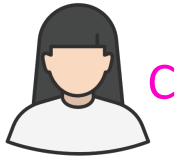
Bob 

 Becky

Alex > Bob > Chris

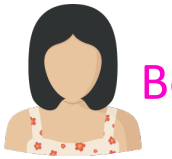
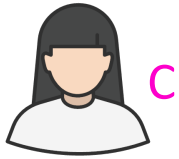
Ada > Becky > Cindy

Chris 

 Cindy

Alex > Bob > Chris

This is another stable marriage

Ada > Becky > Cindy	Alex		❤	 Becky	Alex > Bob > Chris
Becky > Ada > Cindy	Bob		❤	 Ada	Bob > Alex > Chris
Ada > Becky > Cindy	Chris		❤	 Cindy	Alex > Bob > Chris

Stable marriage is not unique.

Stable Marriage Problem

- Directed weighted bipartite graph: $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$.
  - $\mathcal{U}$: a set of men.
  - $\mathcal{V}$: a set of women.
  - The weights are the orders of preference.

Stable Marriage Problem

- Directed weighted bipartite graph: $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$.
  - $\mathcal{U}$: a set of men.
  - $\mathcal{V}$: a set of women.
  - The weights are the orders of preference.
- The numbers of men and women are the same: $|\mathcal{U}| = |\mathcal{V}| = n$.
- If a man and a woman both prefer each other over their current spouses, then the marriage is not stable.

Gale-Shapley Algorithm

- David Gale and Lloyd S. Shapley. College Admissions and the Stability of Marriage. *American Mathematical Monthly*, 69 (1): 9–14, 1962.
- Time complexity: $O(n^2)$.
- To be introduced in the next lecture.

Thank You!

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