Binary Search

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Problem of Search

`arr =`

3_0	5_1	12_2	16_3	17_4	26_5	32_6	51_7	53_8	64_9

Problem of Search

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3 $_0$	5 $_1$	12 $_2$	16 $_3$	17 $_4$	26 $_5$	32 $_6$	51 $_7$	53 $_8$	64 $_9$

- **Inputs:** (i) an array whose elements are in the ascending order and (ii) a key.
- **Goal:** Search for the key in the array. If found, return its index; if not found, return -1.

Problem of Search

`arr =`

3 $_0$	5 $_1$	12 $_2$	16 $_3$	17 $_4$	26 $_5$	32 $_6$	51 $_7$	53 $_8$	64 $_9$

Example 1:

- Search for the element 53.
- Return 8.

Problem of Search

arr =

3 $_0$	5 $_1$	12 $_2$	16 $_3$	17 $_4$	26 $_5$	32 $_6$	51 $_7$	53 $_8$	64 $_9$

Example 2:

- Search for the element 9.
- Return -1.

Example: key=26

arr =

3	5	12	16	17	26	32	51	53	64
0	1	2	3	4	5	6	7	8	9

left=0

right=9

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3	5	12	16	17	26	32	51	53	64
0	1	2	3	4	5	6	7	8	9

left=0

right=9

- mid $\leftarrow \lfloor (\text{left} + \text{right})/2 \rfloor$.
- If arr[mid]==key ==> return mid.
- If arr[mid]>key ==> right ← mid-1.
- If arr[mid]<key ==> left ← mid+1.

Example: key=26

`arr =`

3	5	12	16	17	26	32	51	53	64
0	1	2	3	4	5	6	7	8	9

`left=0` `mid=4` `right=9`

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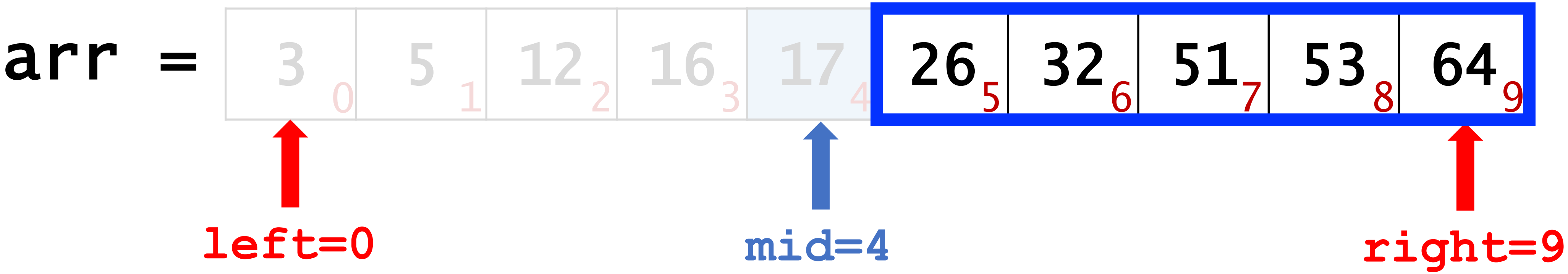

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`left=5` `right=9`

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- `If arr[mid]==key  ==> return mid.`
- `If arr[mid]>key   ==> right` $\leftarrow$ `mid-1.`
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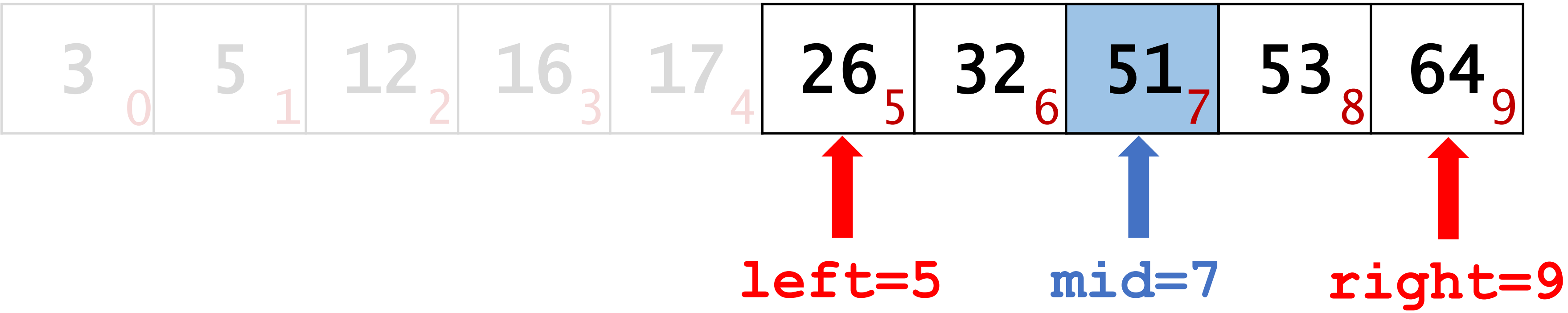

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Example: key=26

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0	1	2	3	4	5	6	7	8	9

`left=5`

`right=6`

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Time Complexity

- Each iteration reduces the size of array by half.
- Let n be the size of the array.
- The total number of iterations is at most $\log_2 n$.
- Per-iteration time complexity: $O(1)$.

Overall time complexity: $O(\log n)$.

```
int search(int arr[], int left, int right, int key) {
    while (left <= right) {
        int mid = (left + right) / 2;
        if (key == arr[mid])
            return mid; // key is found
        if (key > arr[mid])
            left = mid + 1; // search in the right half
        else
            right = mid - 1; // search in the left half
    }
    return -1; // key is not found
}
```

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}
```

How to support both search and insertion?

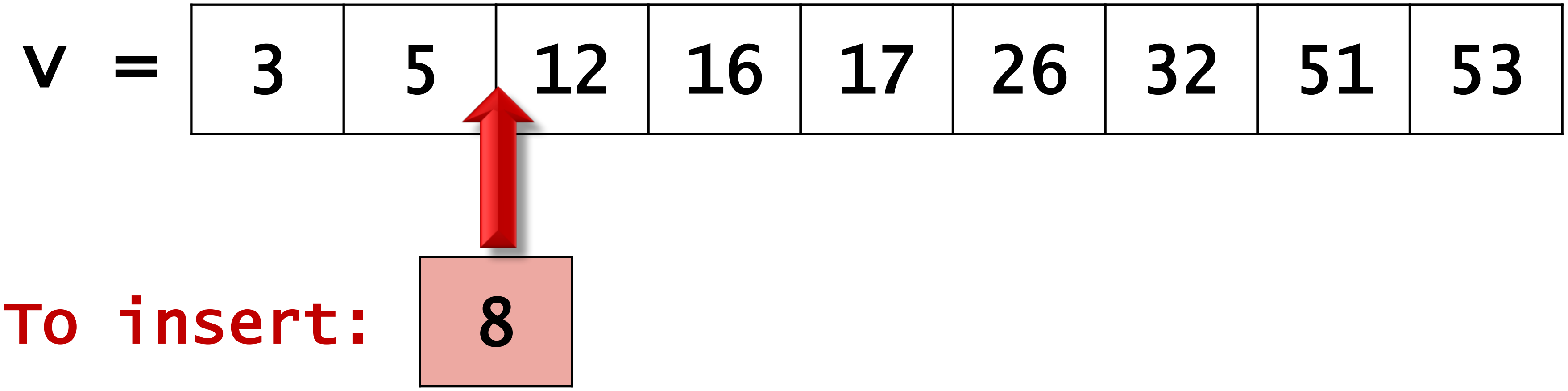
Vector

v =

3	5	12	16	17	26	32	51	53

- The ascending order must be kept; otherwise, search would take $O(n)$ time.
- Inserting an item into the middle has $O(n)$ time complexity (on average).

Vector



- Inserting an item into the middle has $O(n)$ time complexity (on average).

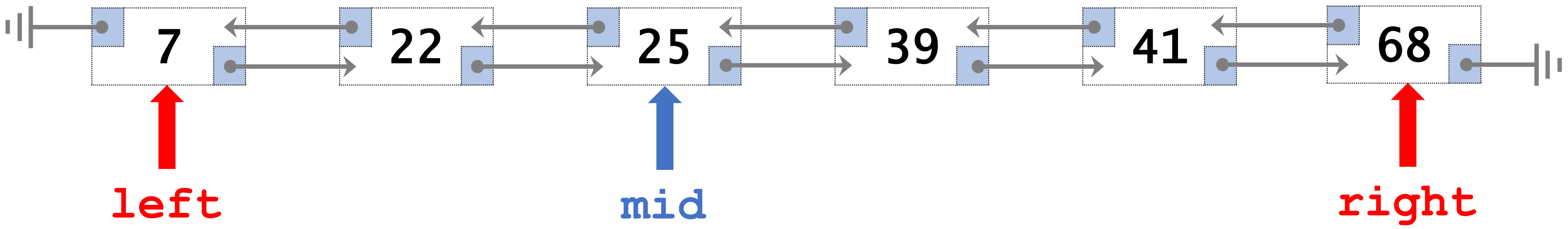
Vector

`v = | 3 | 5 | 12 | 16 | 17 | 26 | 32 | 51 | 53 |`

`To insert: | 8 |`

- Inserting an item into the middle has $O(n)$ time complexity (on average).

List



- Can we perform binary search in the list?
- No. Given left and right, we cannot get mid efficiently.

Comparisons

	Search	Insertion
Vector	$O(\log n)$	$O(n)$
List	$O(n)$	$O(1)$

Comparisons

	Search	Insertion
Vector	$O(\log n)$	$O(n)$
List	$O(n)$	$O(1)$
Skip List	$O(\log n)$	$O(\log n)$

Thank You!