

Dinic's Algorithm

Shusen Wang

<http://wangshusen.github.io/>

Comparisons

- m : the number of edges.
- n : the number of vertices.
- Time complexity of Edmonds–Karp algorithm [1] is $O(m^2 \cdot n)$.

Reference

1. Jack Edmonds and Richard M Karp. [Theoretical improvements in algorithmic efficiency for network flow problems](#). *Journal of the ACM*. 19 (2): 248–264, 1972.

Comparisons

- m : the number of edges.
- n : the number of vertices.
- Time complexity of Edmonds–Karp algorithm [1] is $O(m^2 \cdot n)$.
- Time complexity of Dinic's algorithm [2] is $O(m \cdot n^2)$. (Faster, because n is small than m .)

Reference

1. Jack Edmonds and Richard M Karp. [Theoretical improvements in algorithmic efficiency for network flow problems](#). *Journal of the ACM*. 19 (2): 248–264, 1972.
2. Yefim Dinitz. [Algorithm for solution of a problem of maximum flow in a network with power estimation](#). *Proceedings of the USSR Academy of Sciences*, 11: 1277–1280, 1970.

Dinic's Algorithm

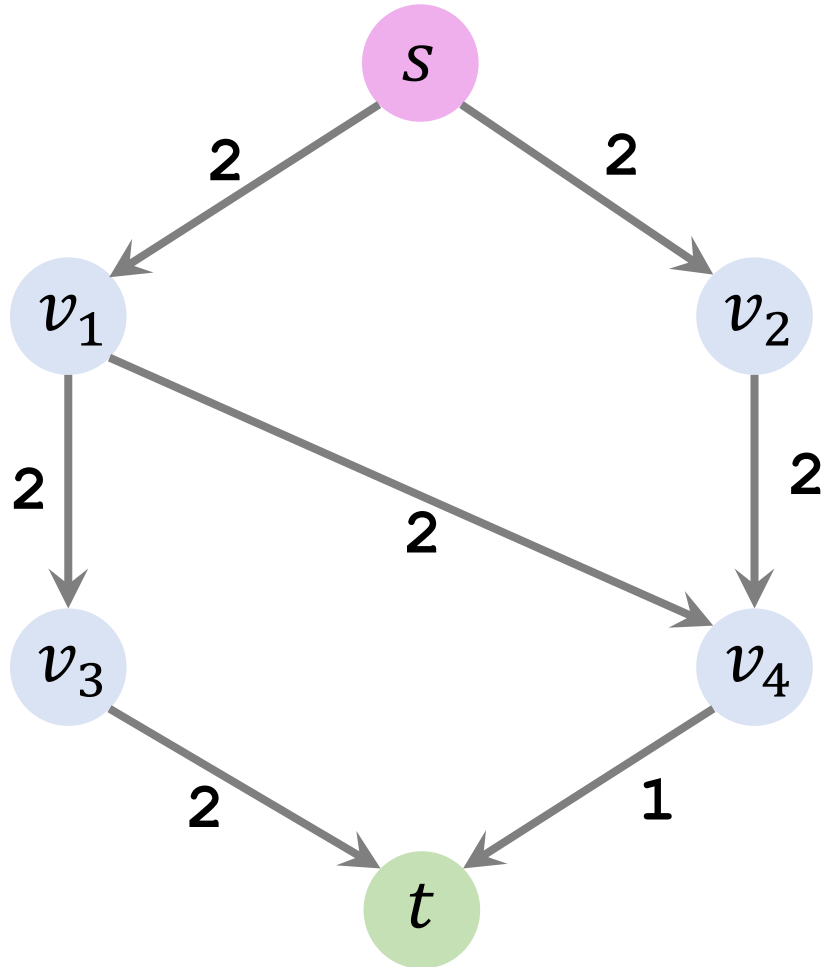
- Yefim Dinitz published “Dinitz’s Algorithm” in USSR, 1970 [1].
- “Dinitz’s Algorithm” was introduced to the westerners by Even & Tarjan’s 1975 paper [2].
- Even & Tarjan misspelled Dinitz’s name as “Dinic” [2].

Reference

1. Yefim Dinitz. [Algorithm for solution of a problem of maximum flow in a network with power estimation](#). *Proceedings of the USSR Academy of Sciences*, 11: 1277–1280, 1970.
2. Shimon Even and R. Endre Tarjan. [Network Flow and Testing Graph Connectivity](#). *SIAM Journal on Computing*, 4 (4): 507–518, 1975.

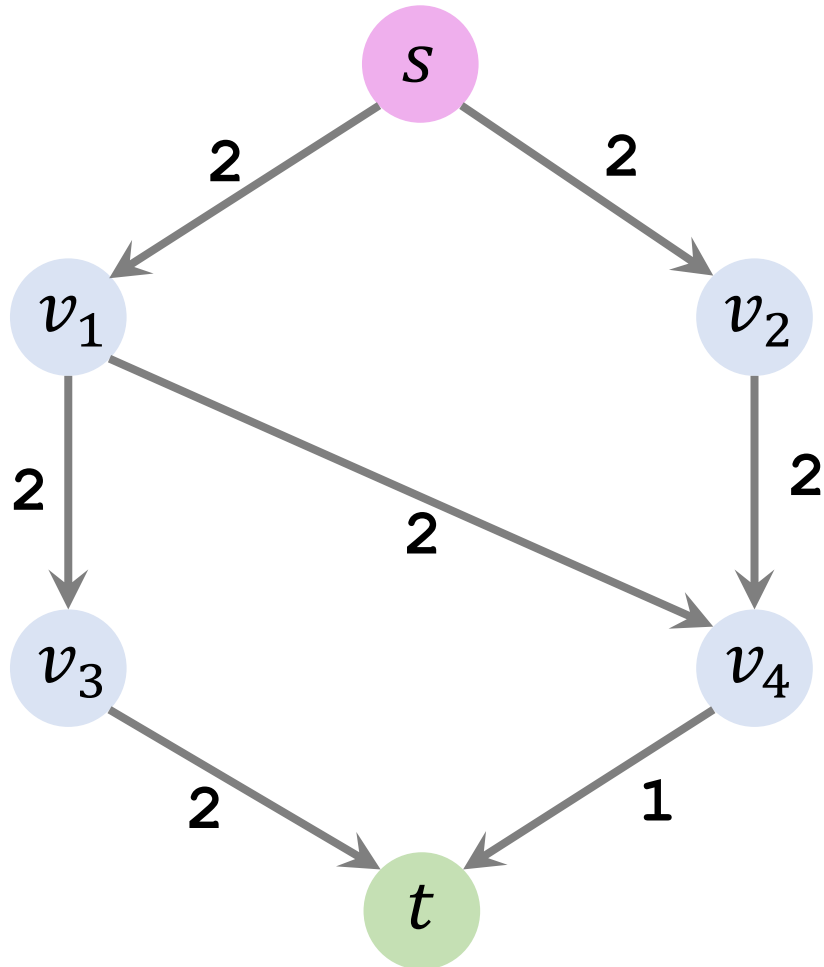
Key Concept: Blocking Flow

Blocking Flow

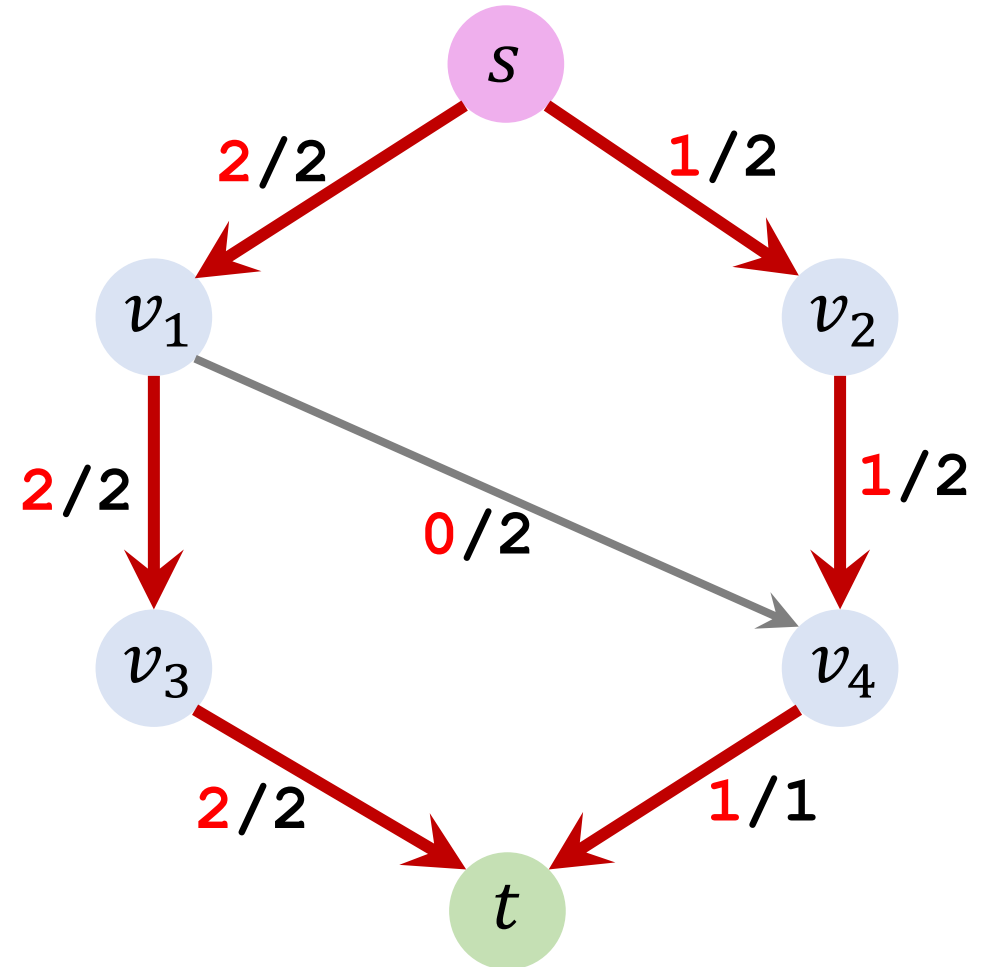


- A flow is **blocking flow** if no more flow from source to sink can be found.
- Max flow is blocking flow; blocking flow may not be max flow.
- Blocking flow can be found by the naïve algorithm.

Blocking Flow

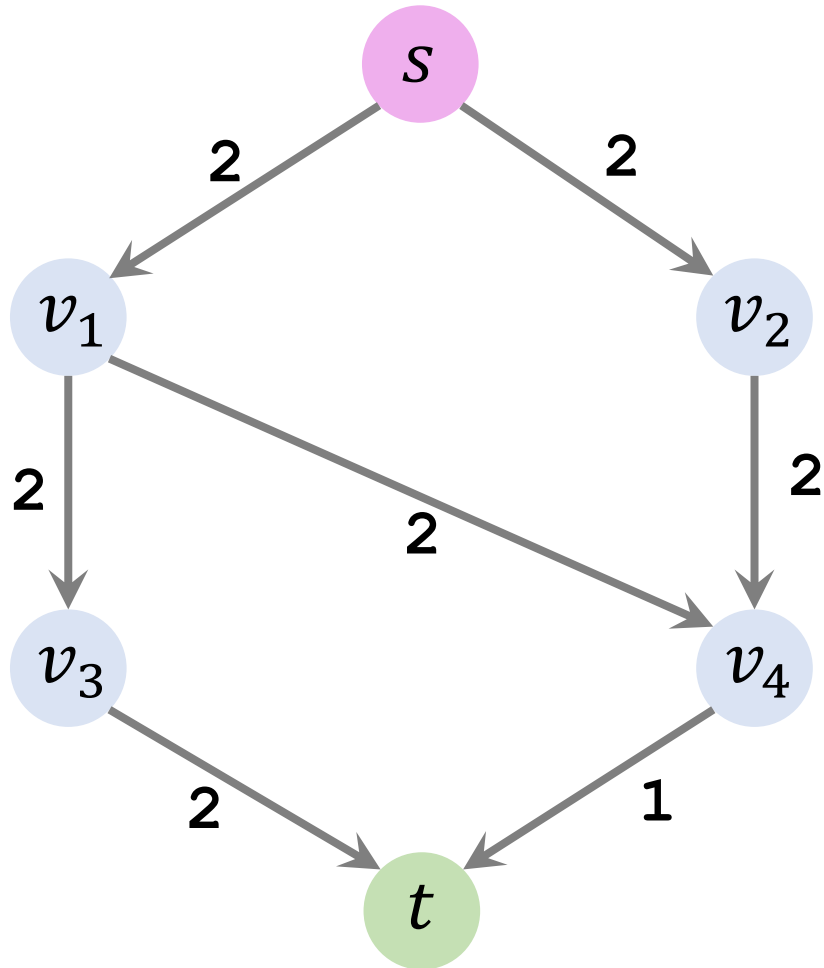


Original Graph

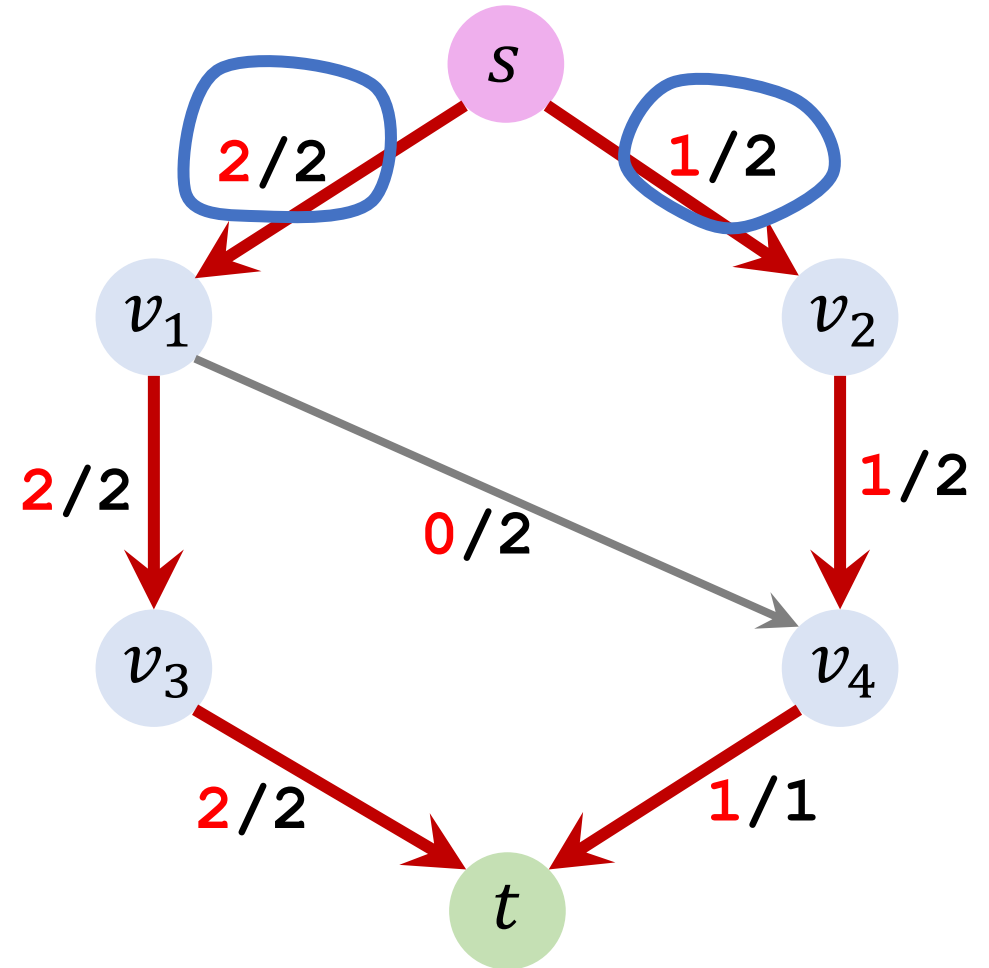


A Blocking Flow

Blocking Flow

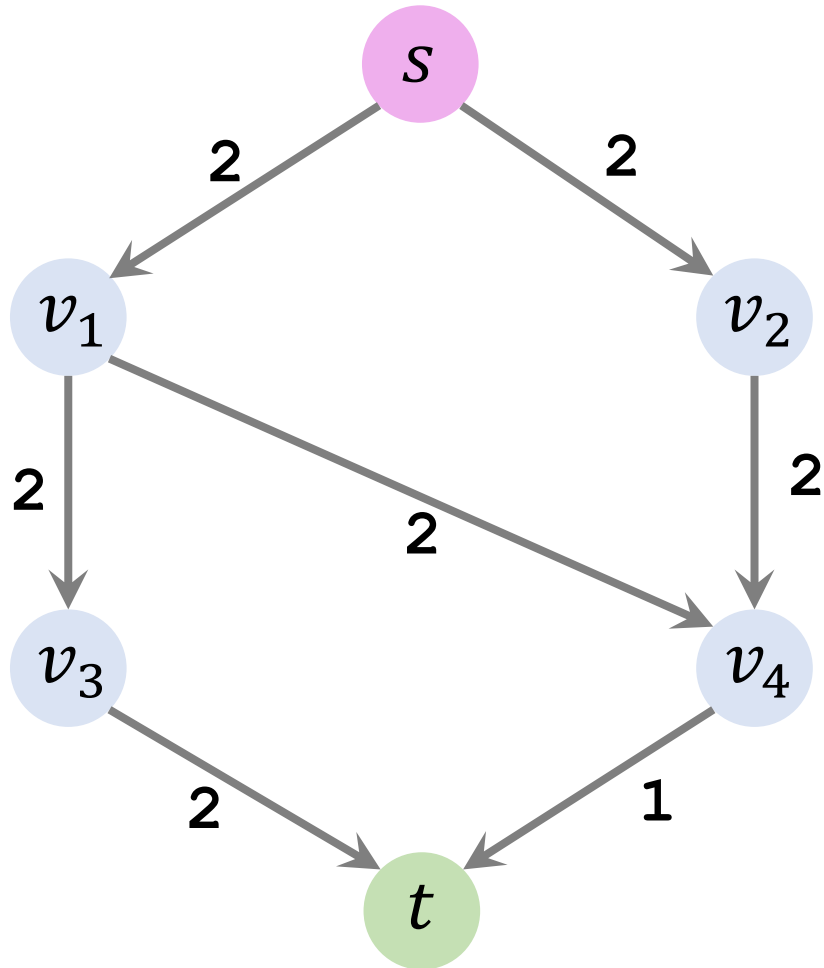


Original Graph

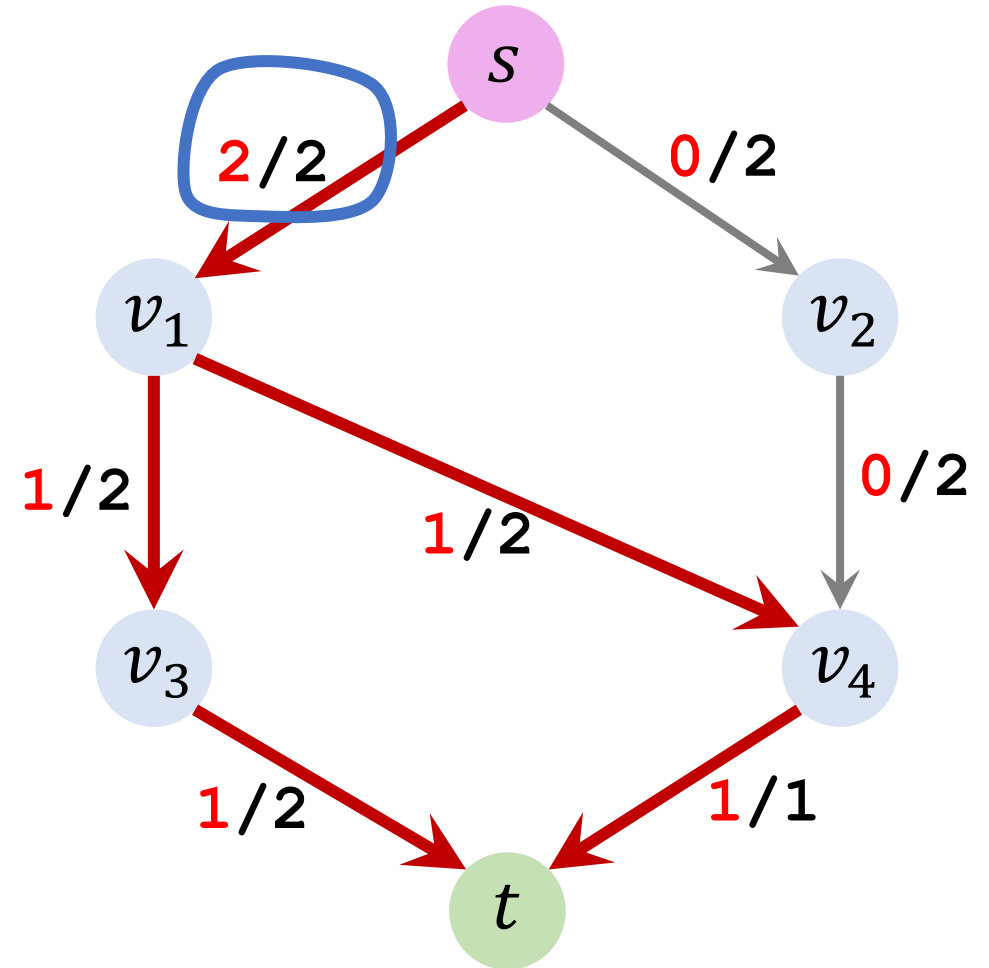


A Blocking Flow

Blocking Flow



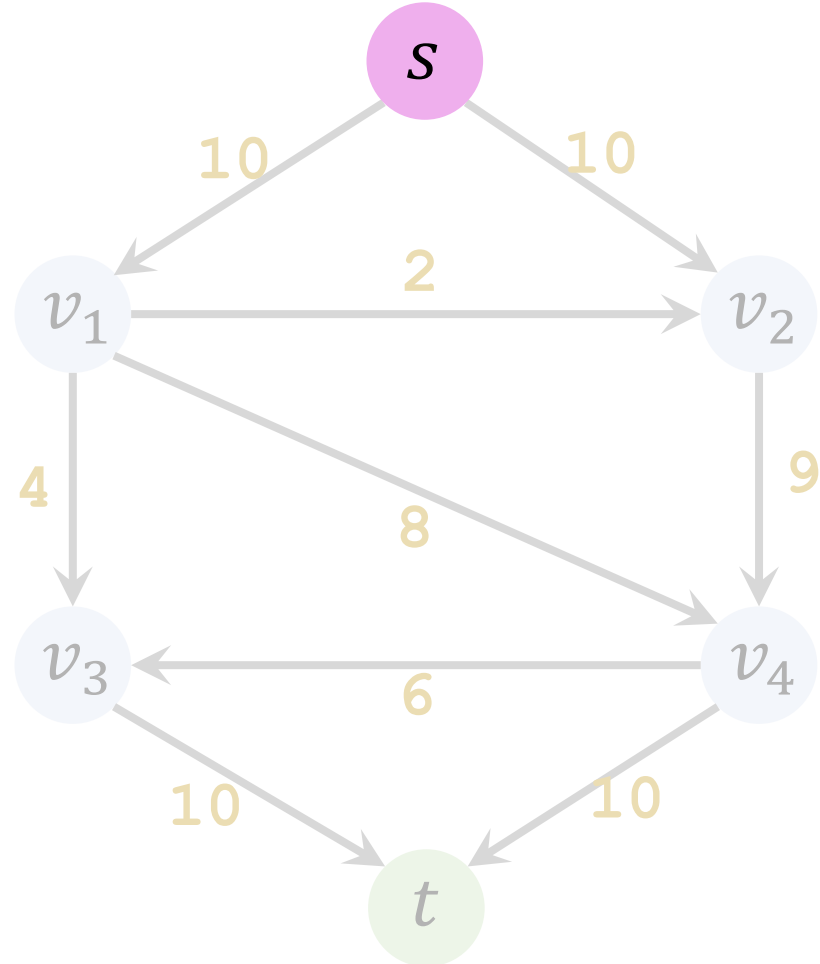
Original Graph



Another Blocking Flow

Key Concept: Level Graph

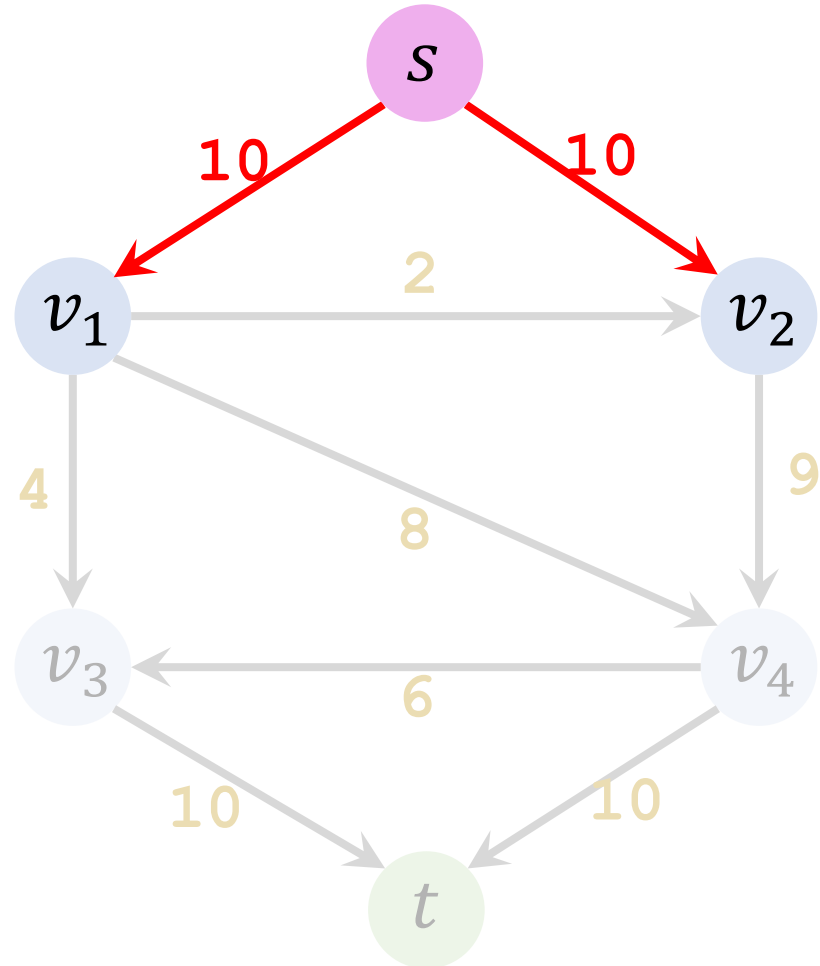
Level Graph: Example 1



Level Graph

Original Graph

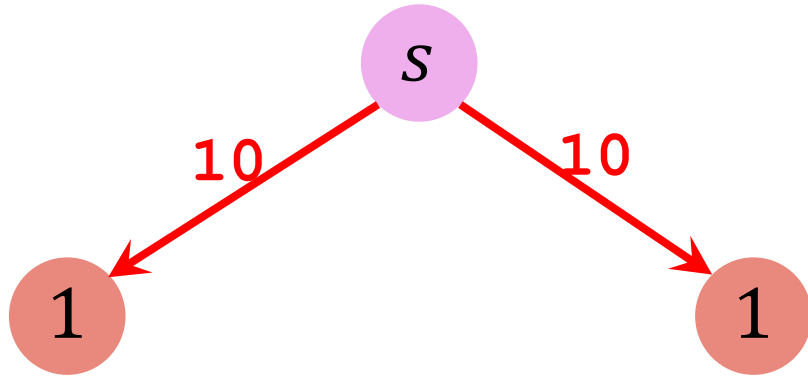
Level Graph: Example 1



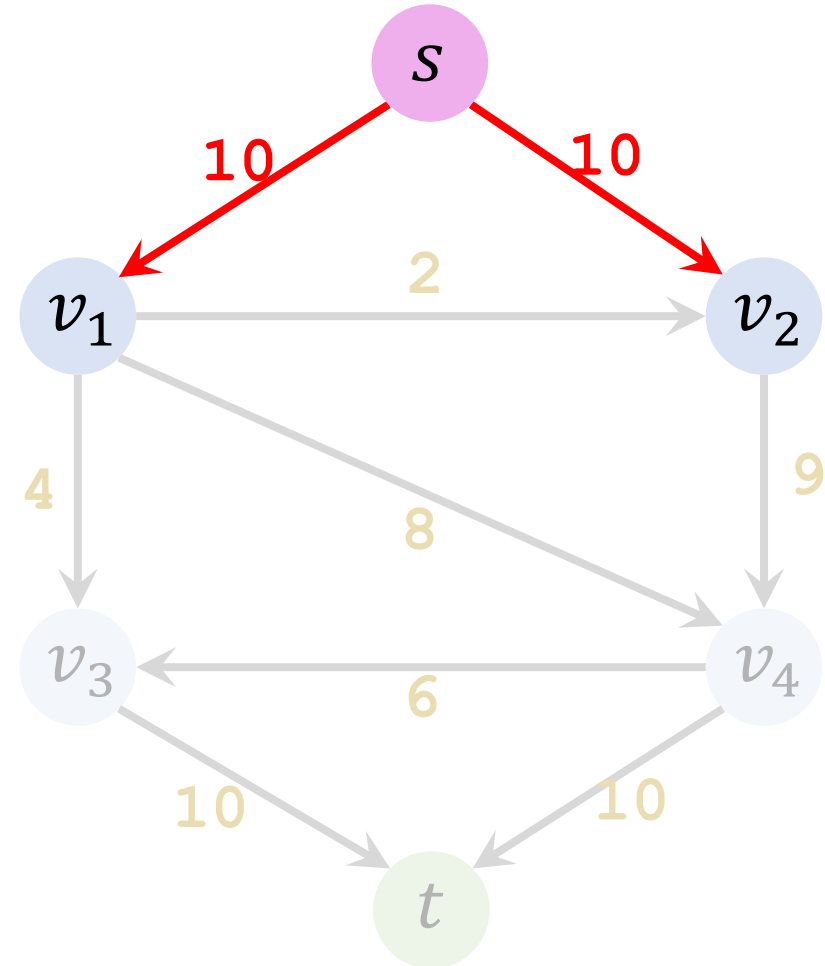
Level Graph

Original Graph

Level Graph: Example 1

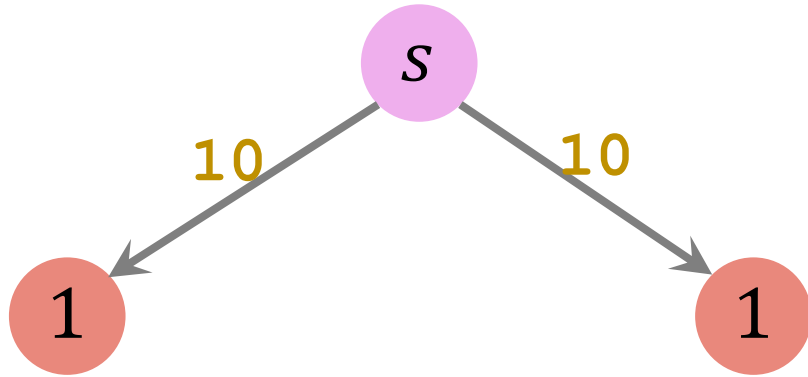


Level Graph

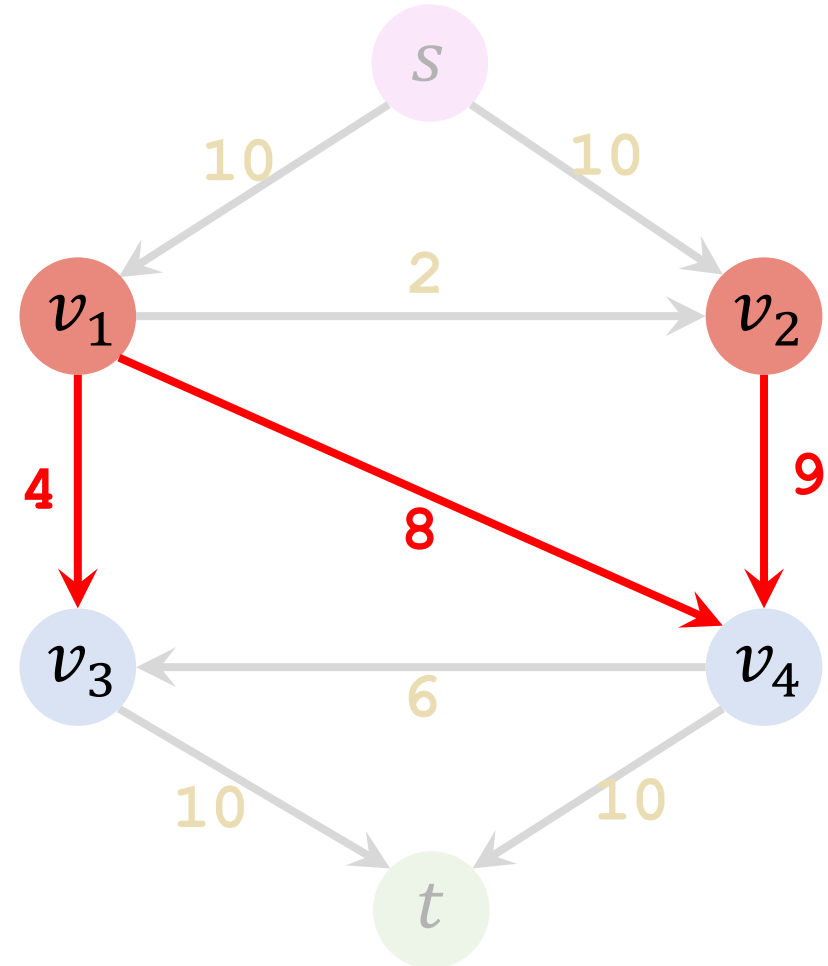


Original Graph

Level Graph: Example 1

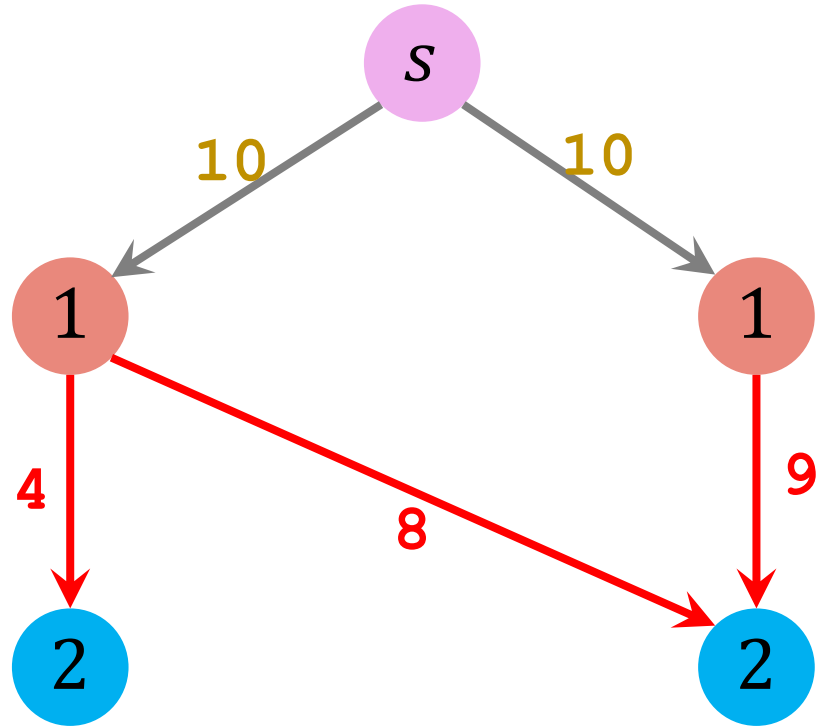


Level Graph

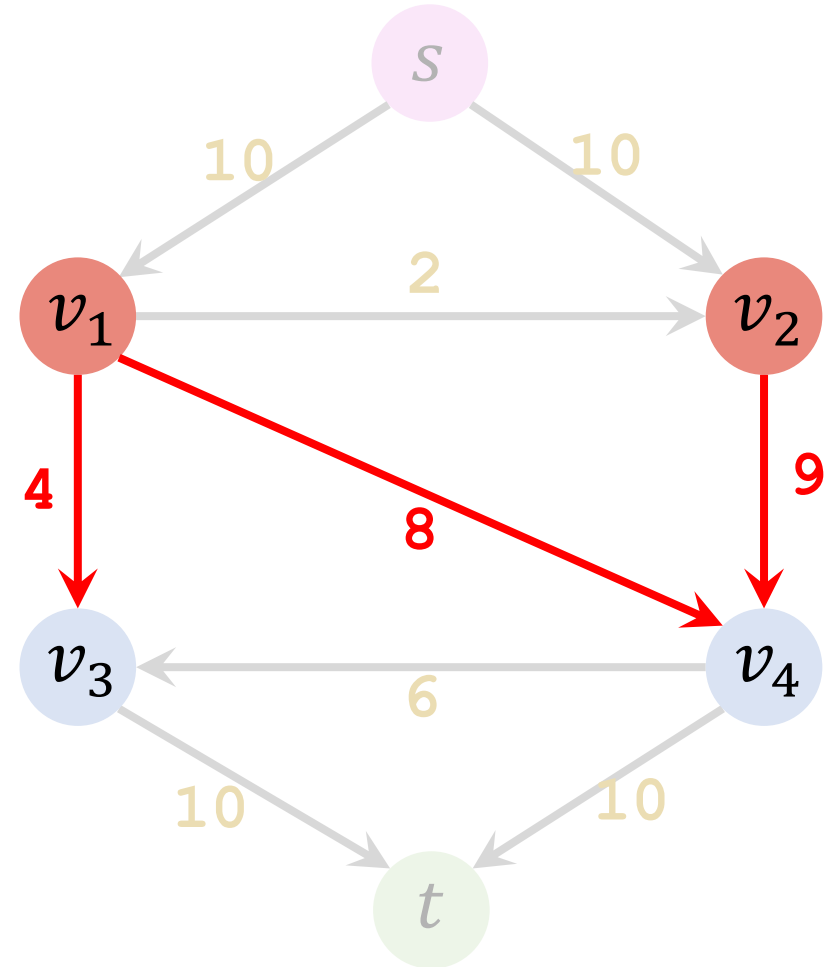


Original Graph

Level Graph: Example 1

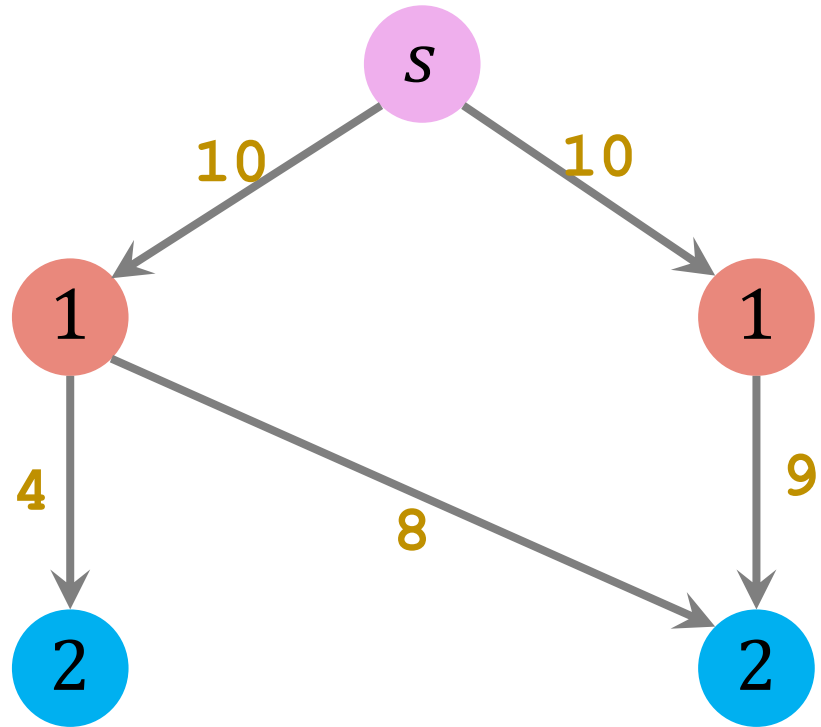


Level Graph

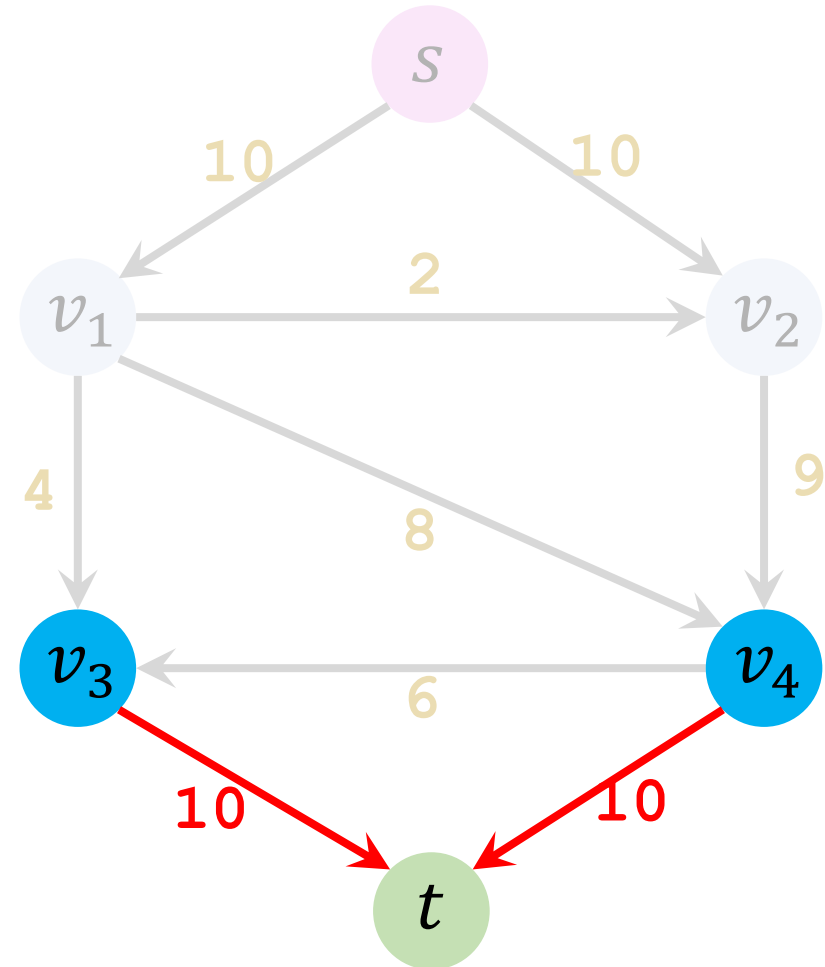


Original Graph

Level Graph: Example 1

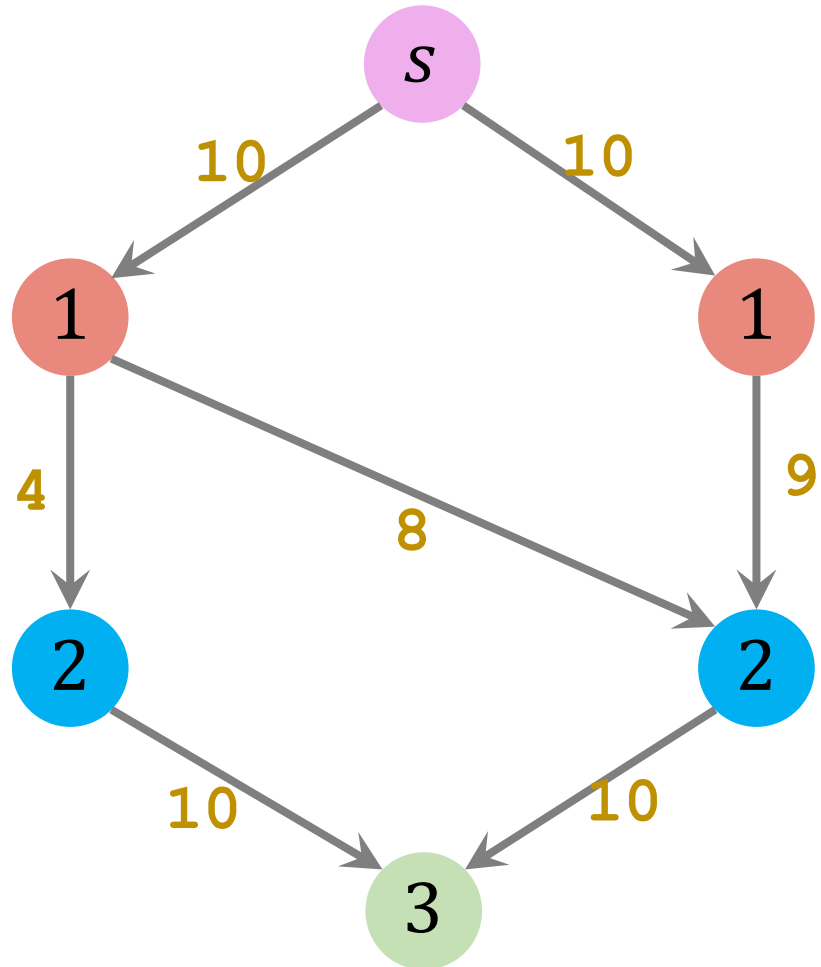


Level Graph

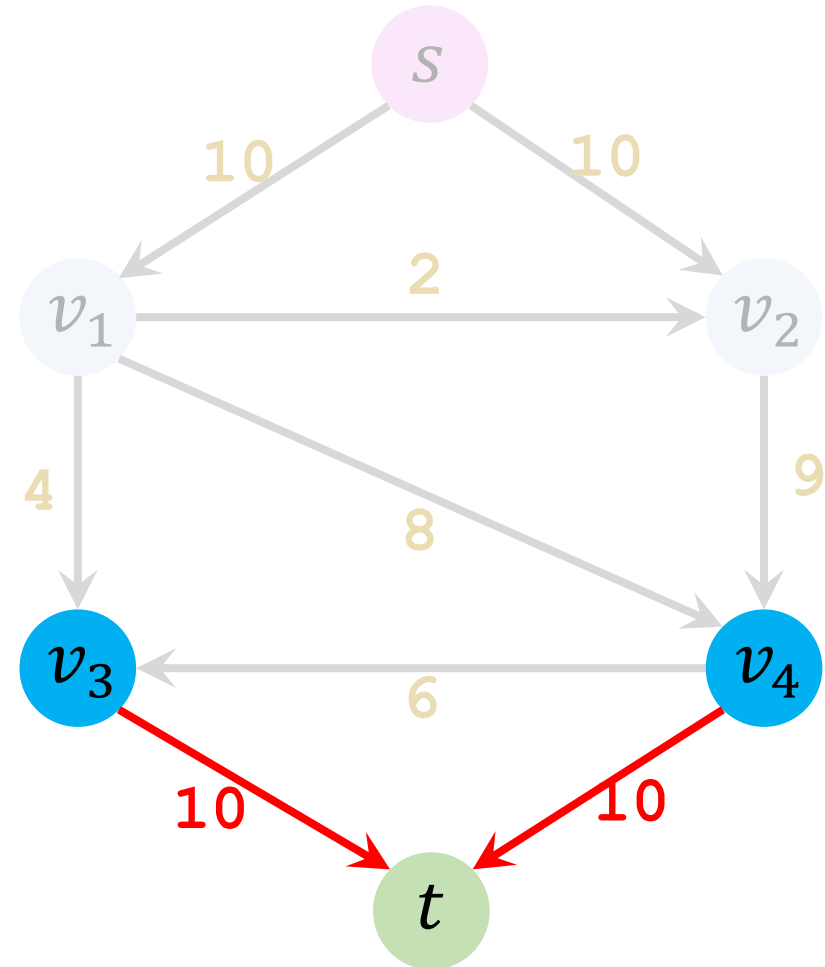


Original Graph

Level Graph: Example 1

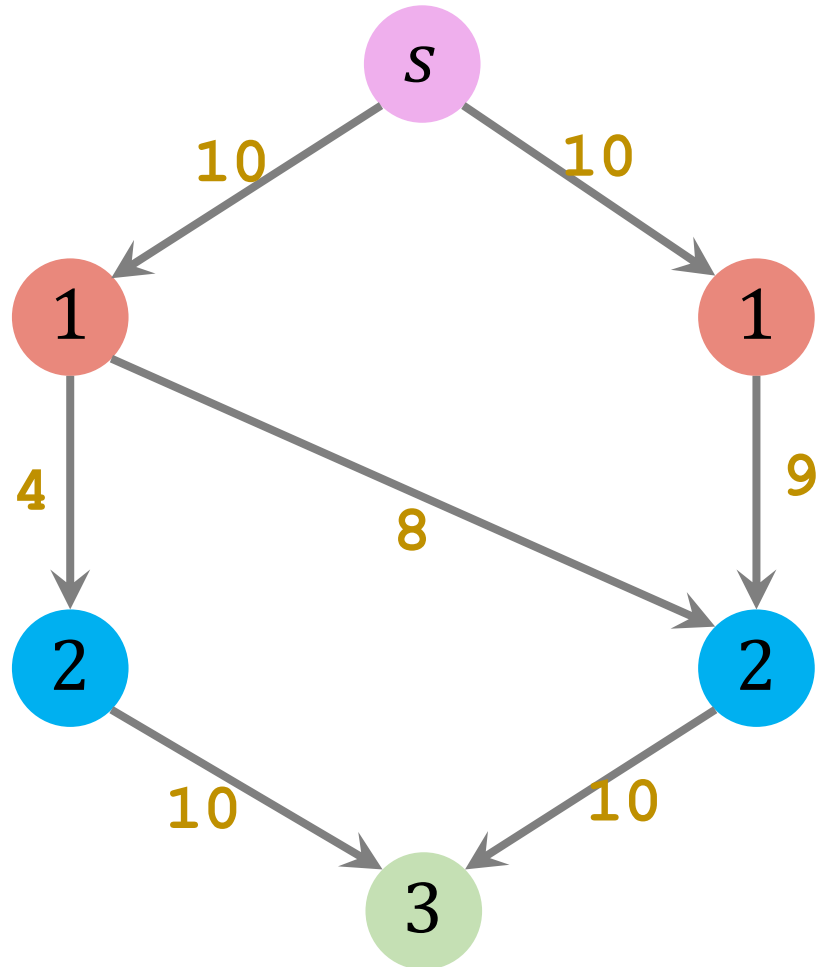


Level Graph

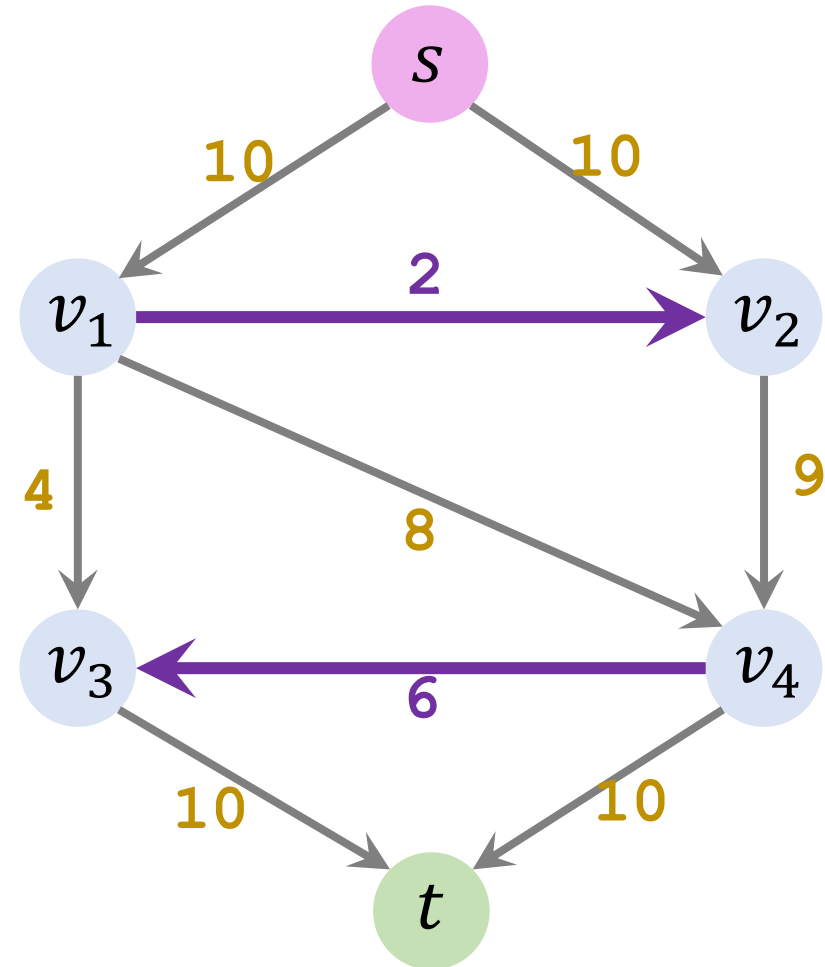


Original Graph

Level Graph: Example 1

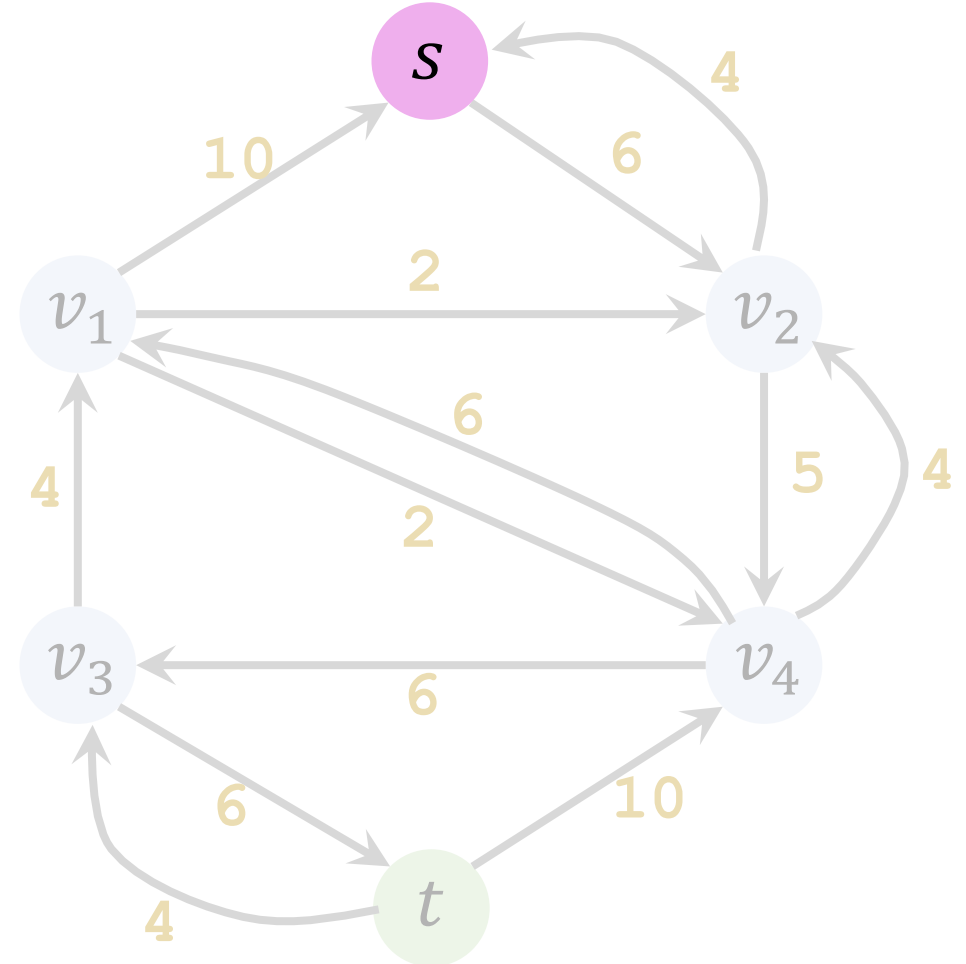


Level Graph



Original Graph

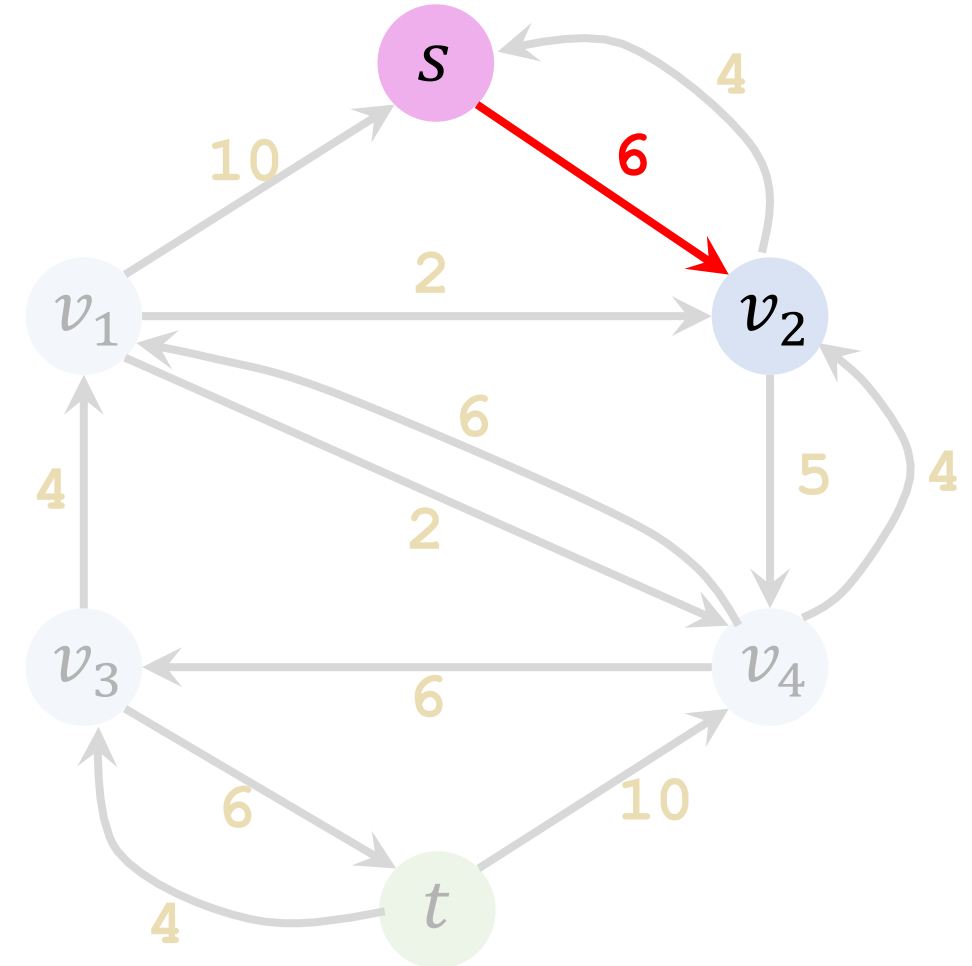
Level Graph: Example 2



Level Graph

Original Graph

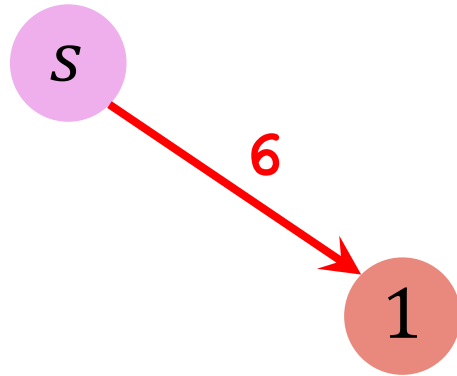
Level Graph: Example 2



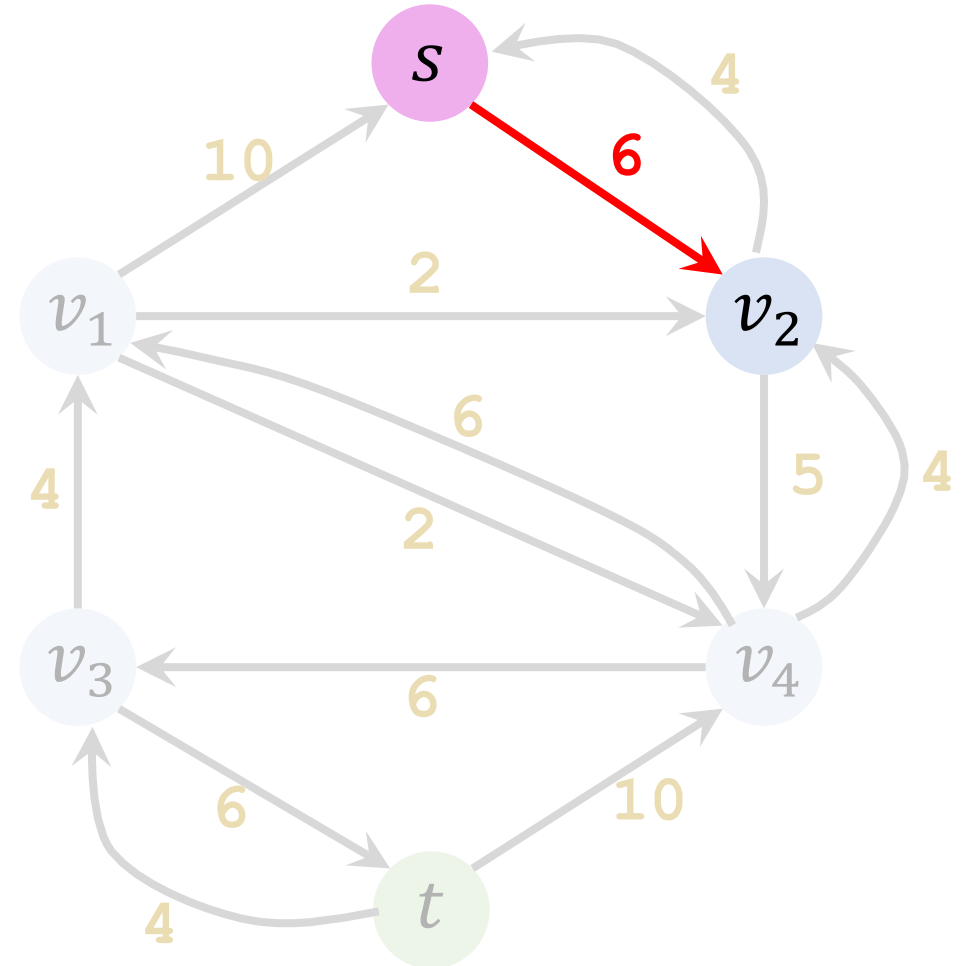
Level Graph

Original Graph

Level Graph: Example 2

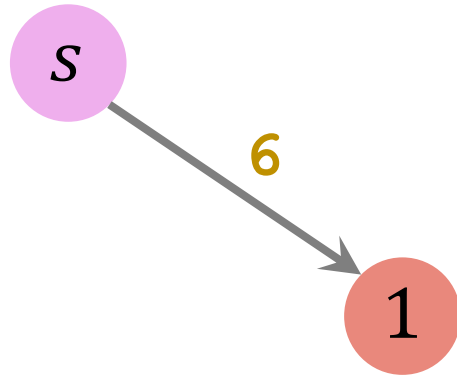


Level Graph

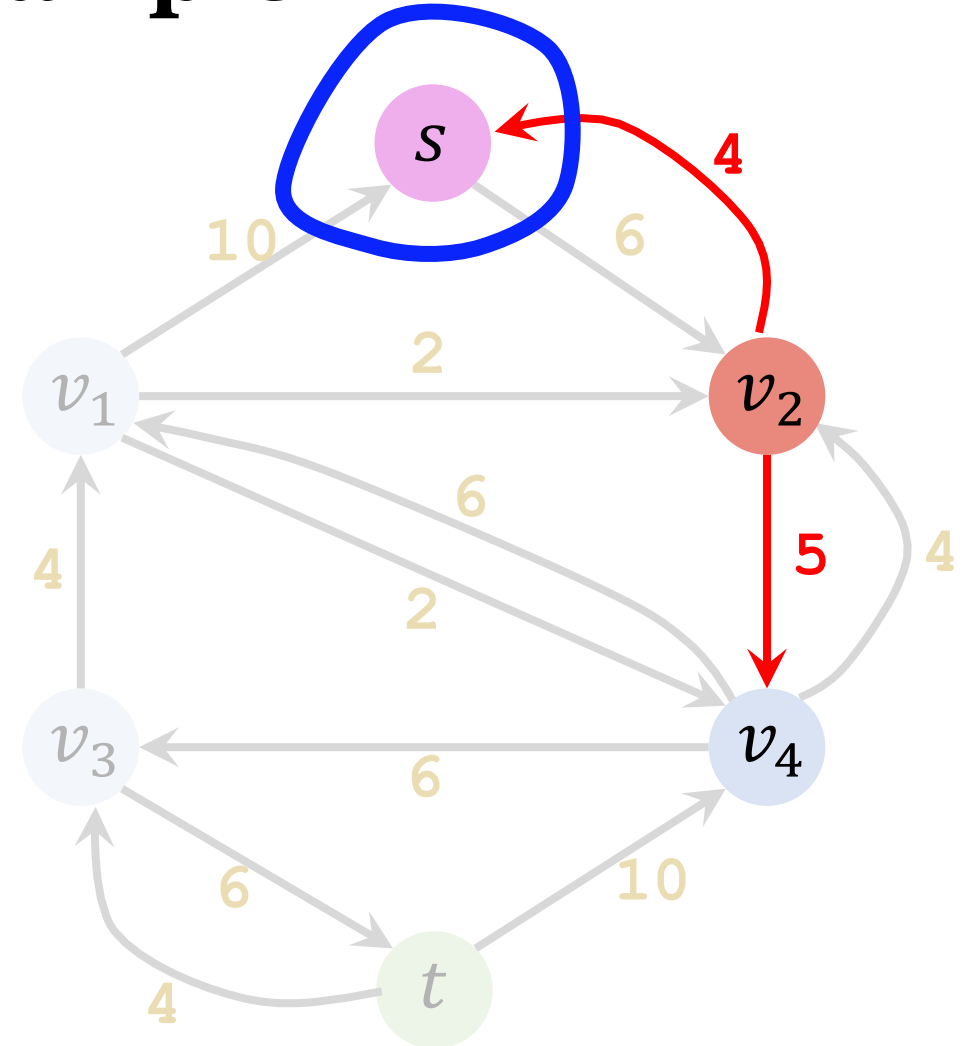


Original Graph

Level Graph: Example 2

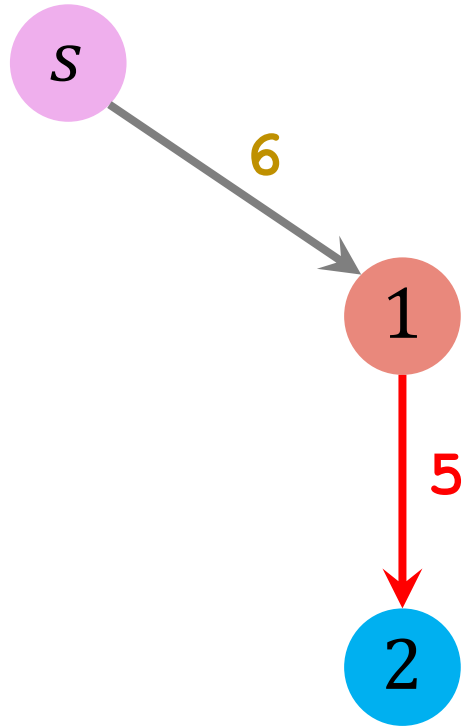


Level Graph

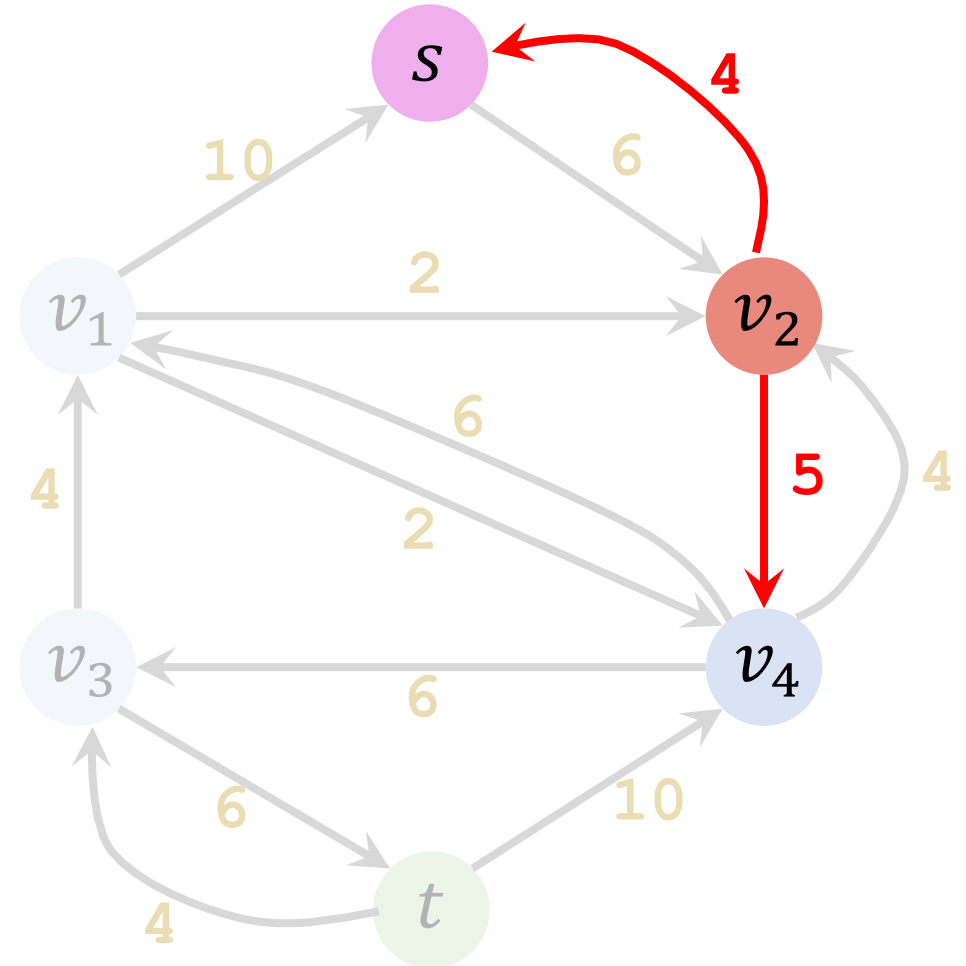


Original Graph

Level Graph: Example 2

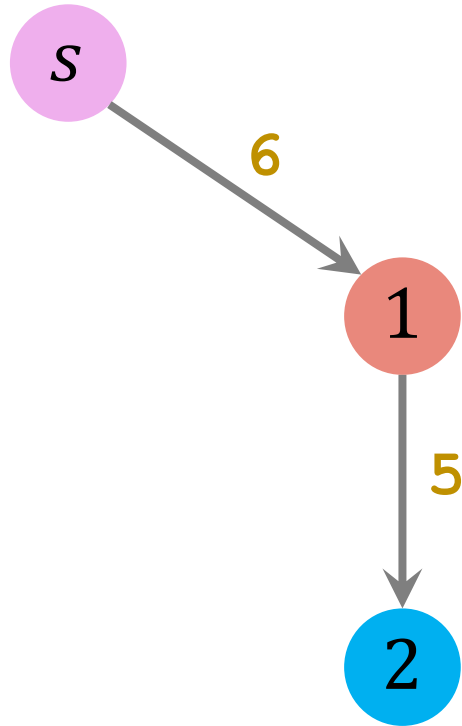


Level Graph

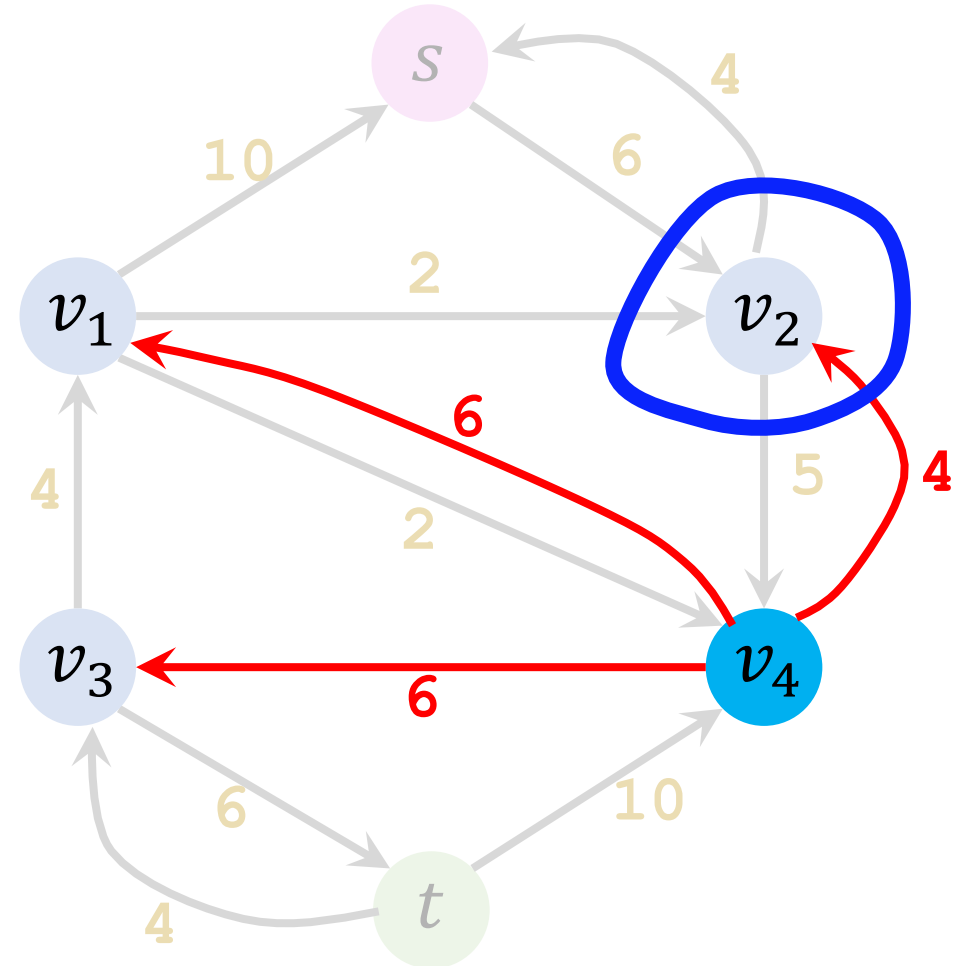


Original Graph

Level Graph: Example 2

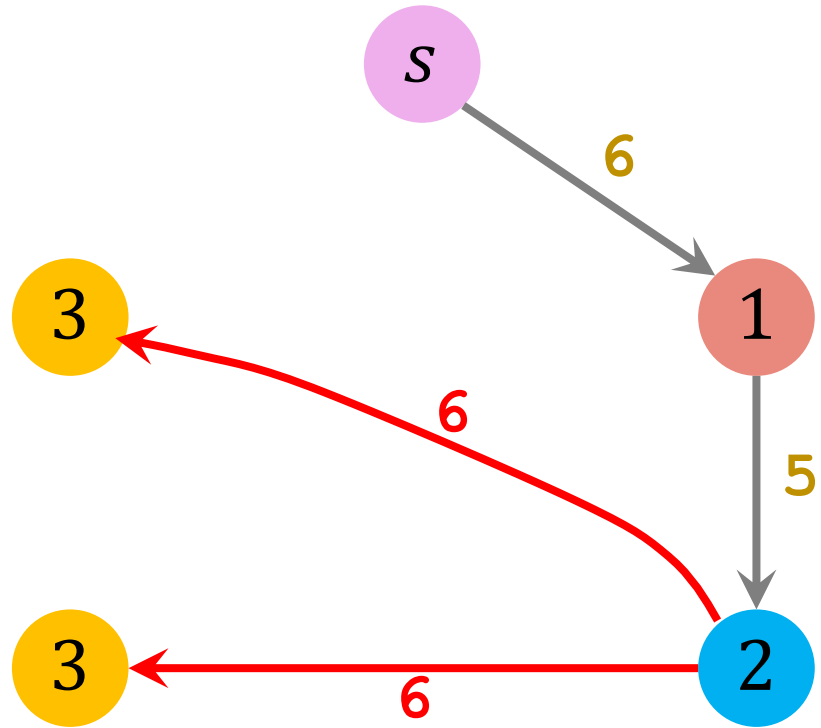


Level Graph

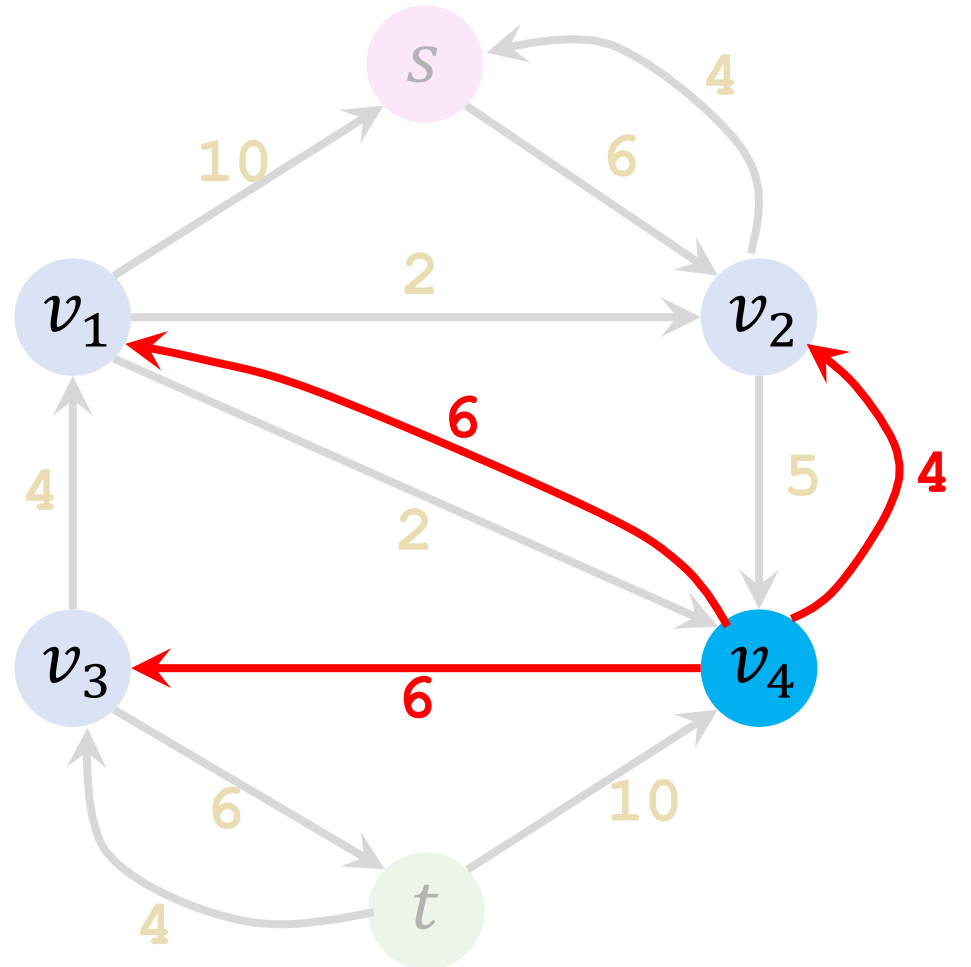


Original Graph

Level Graph: Example 2

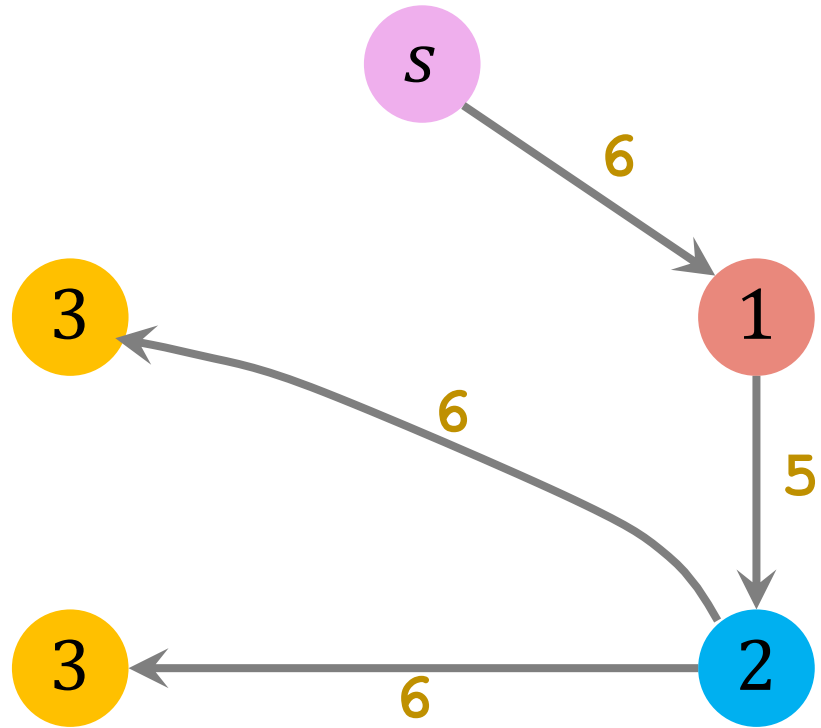


Level Graph

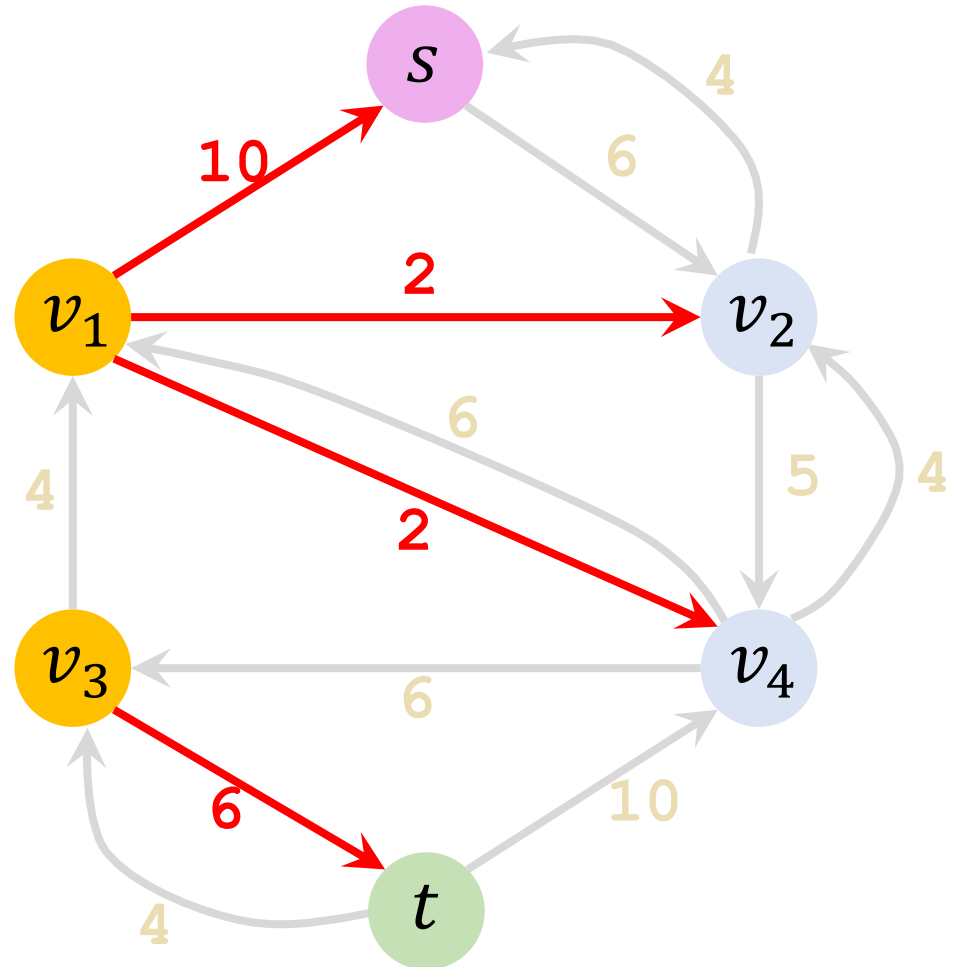


Original Graph

Level Graph: Example 2

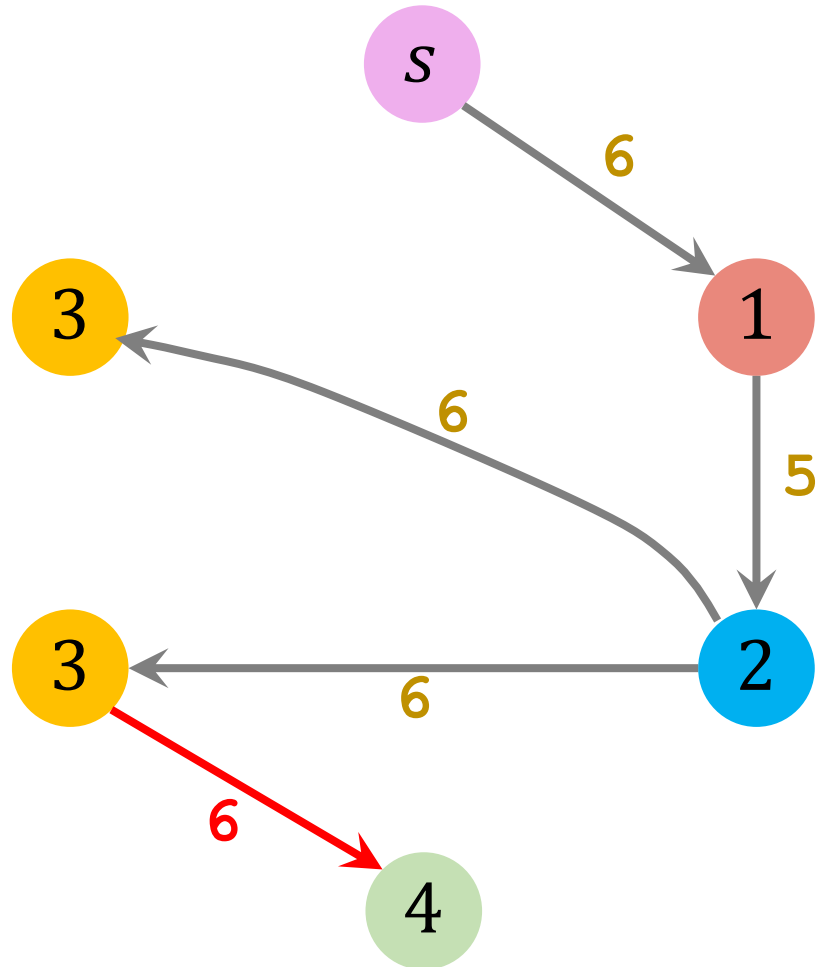


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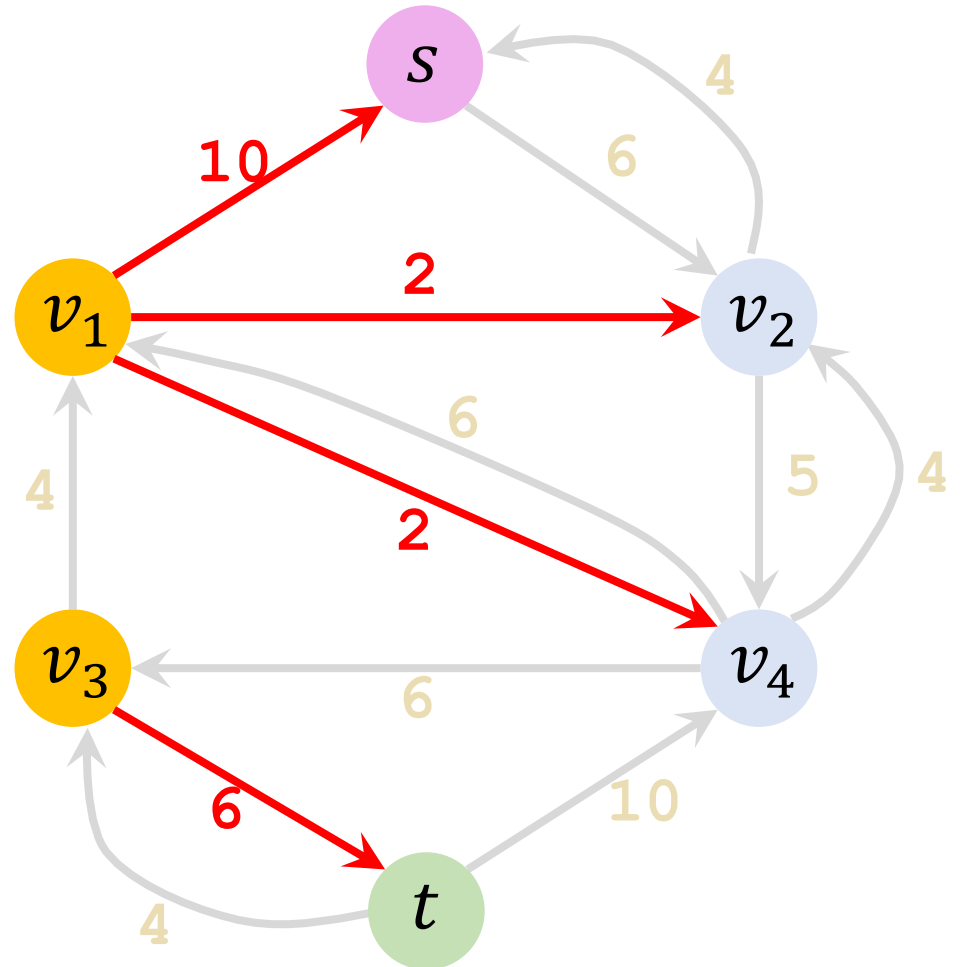


Original Graph

Level Graph: Example 2



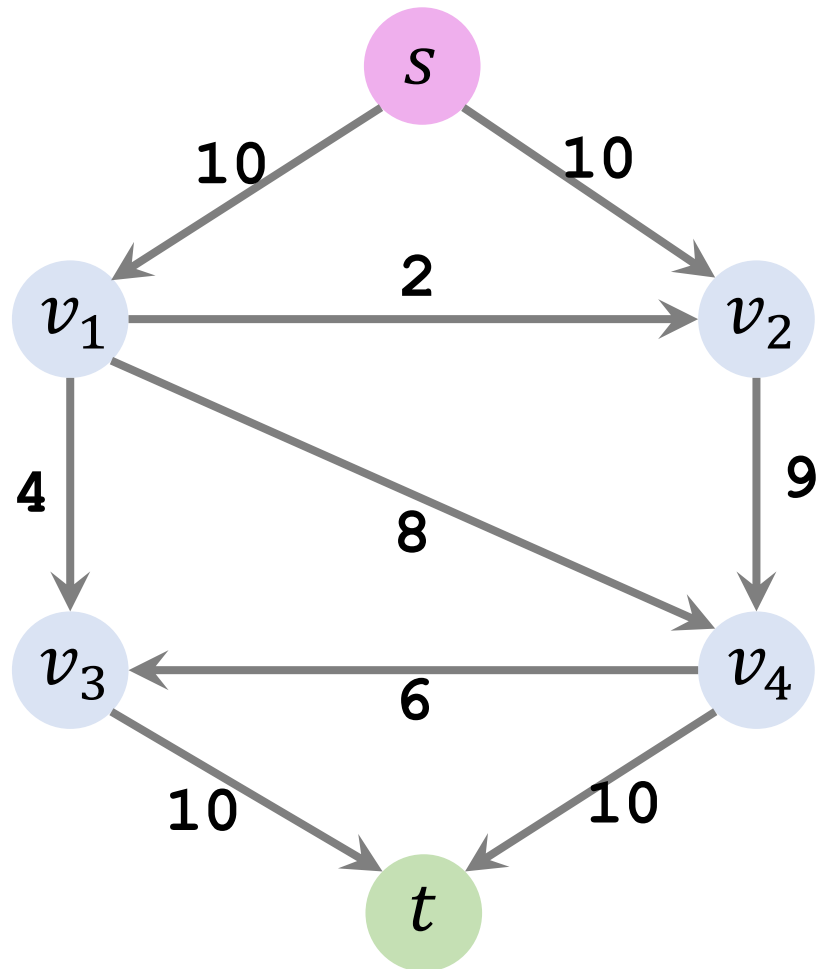
Level Graph



Original Graph

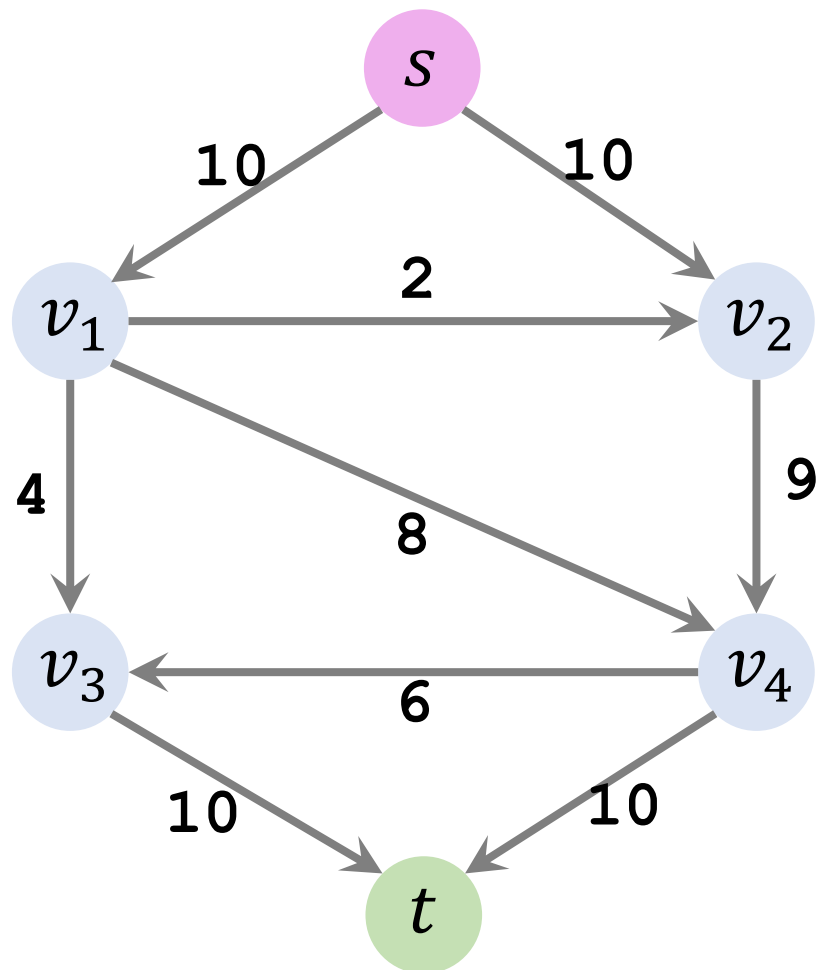
Dinic's Algorithm

Initialization

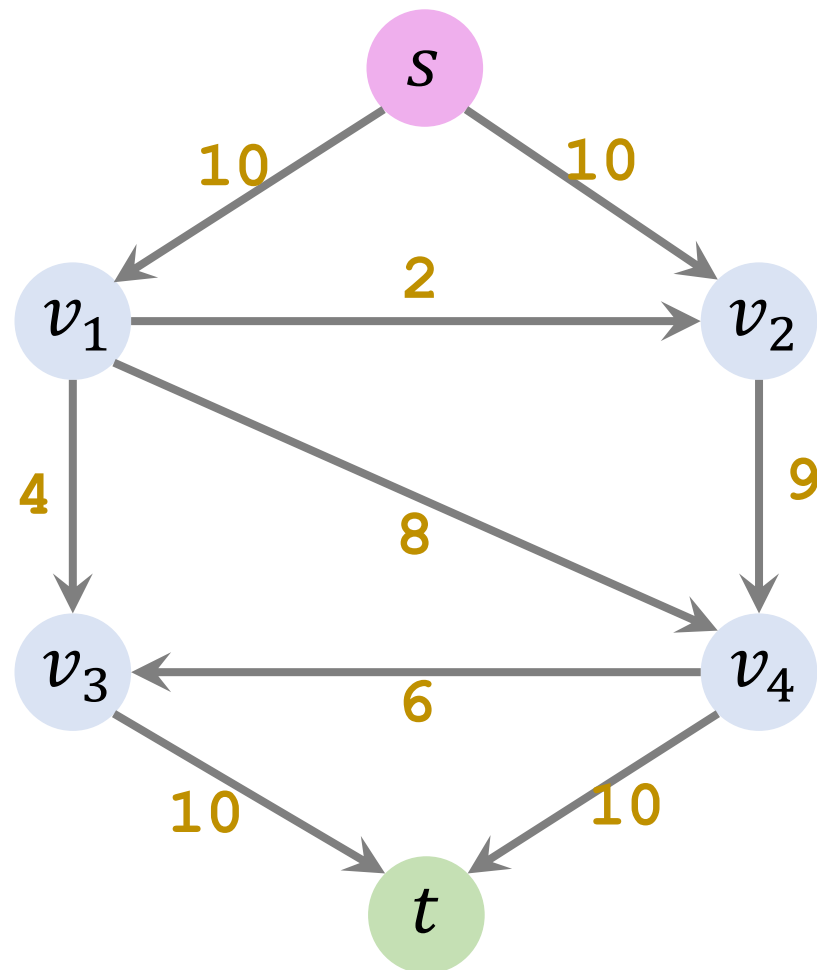


Original Graph

Initialization

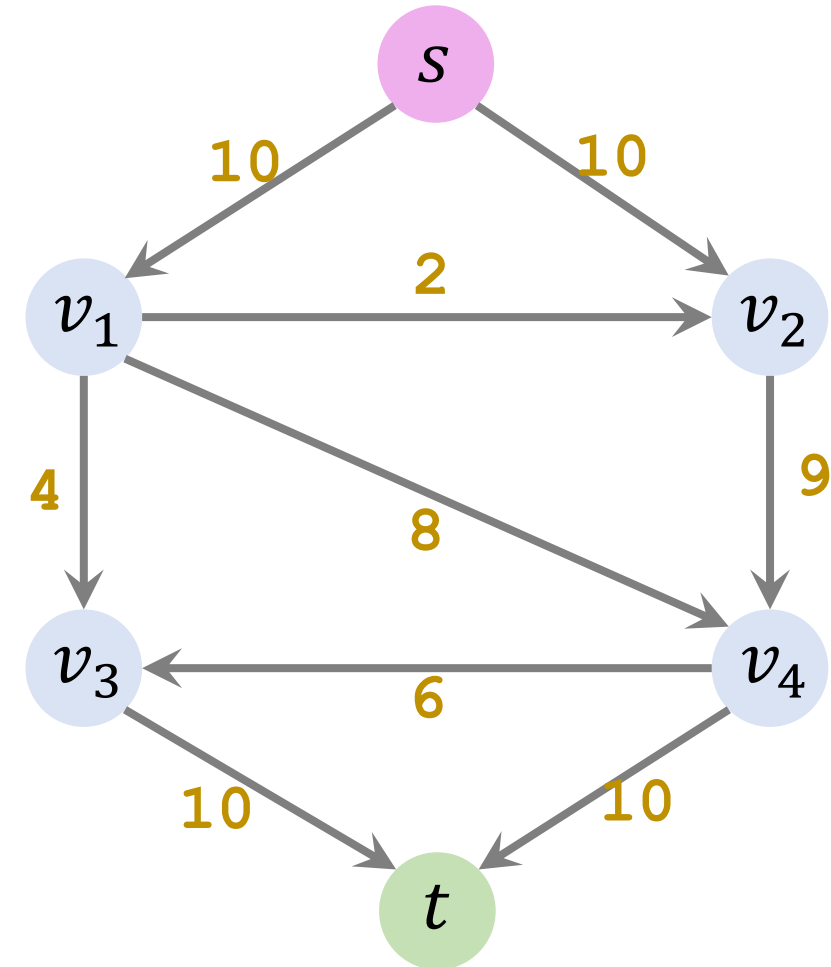


Original Graph



Residual Graph

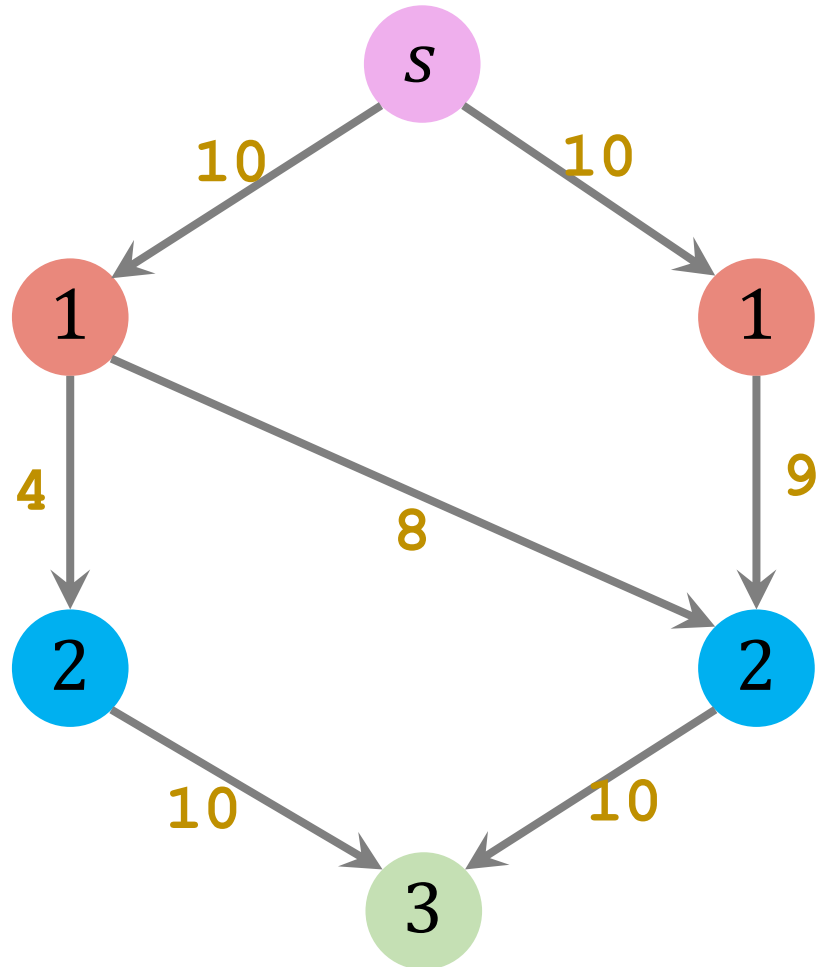
Iteration 1: Construct **level graph**



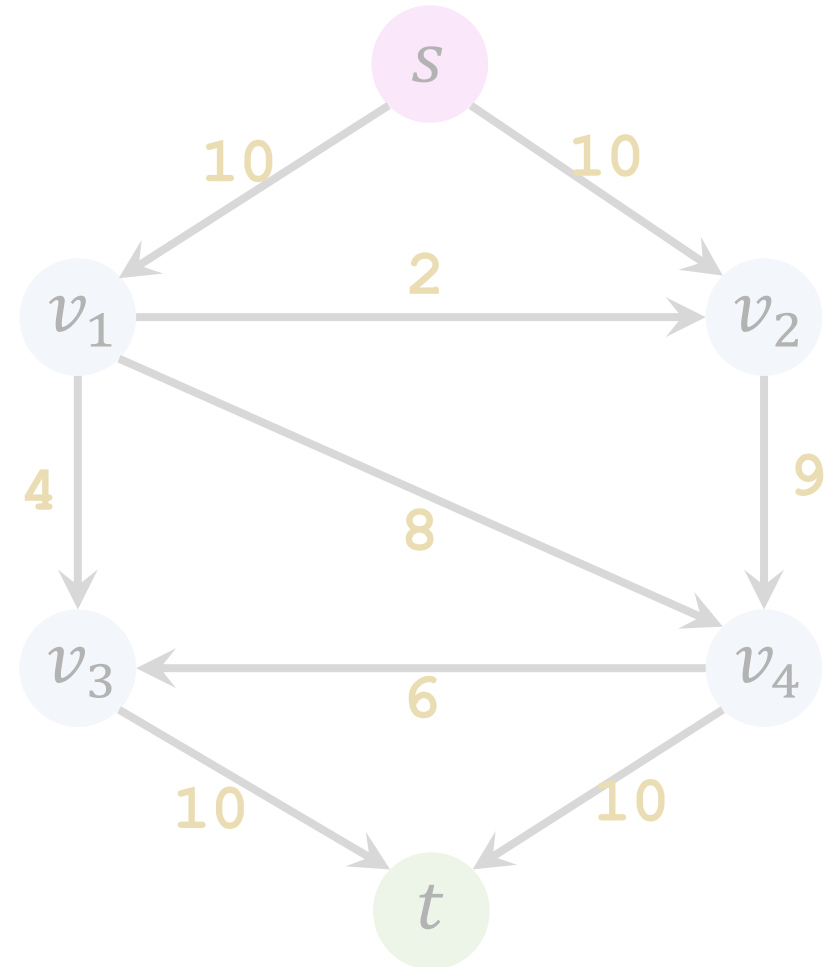
Level Graph

Residual Graph

Iteration 1: Construct **level graph**

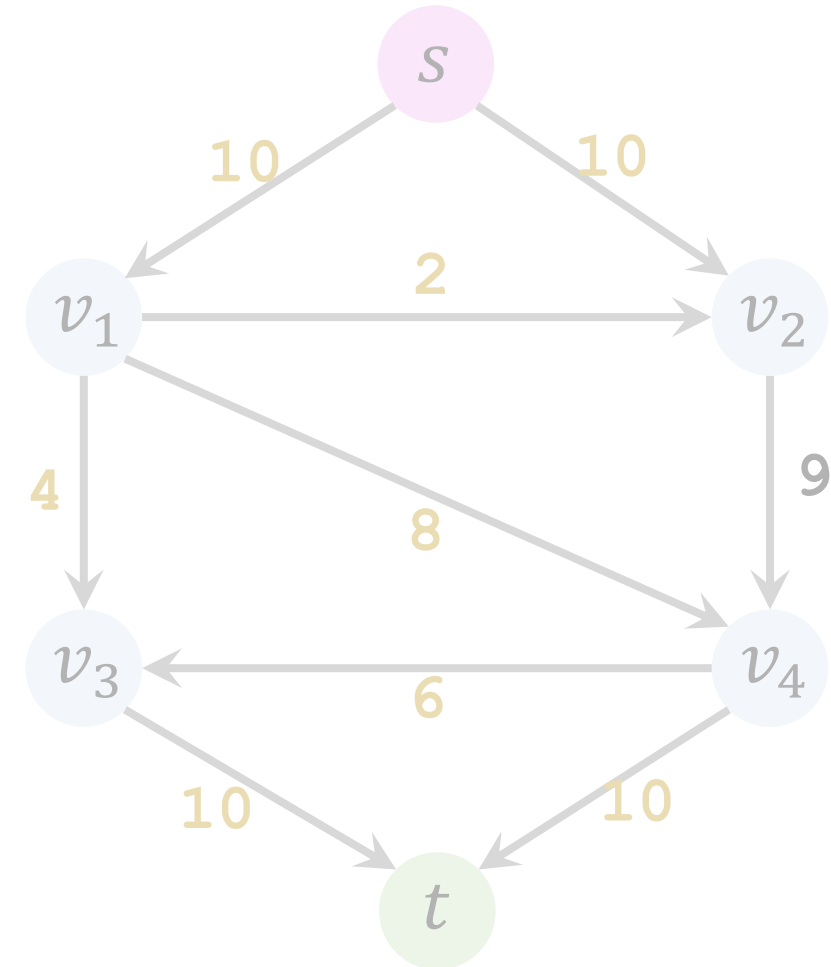
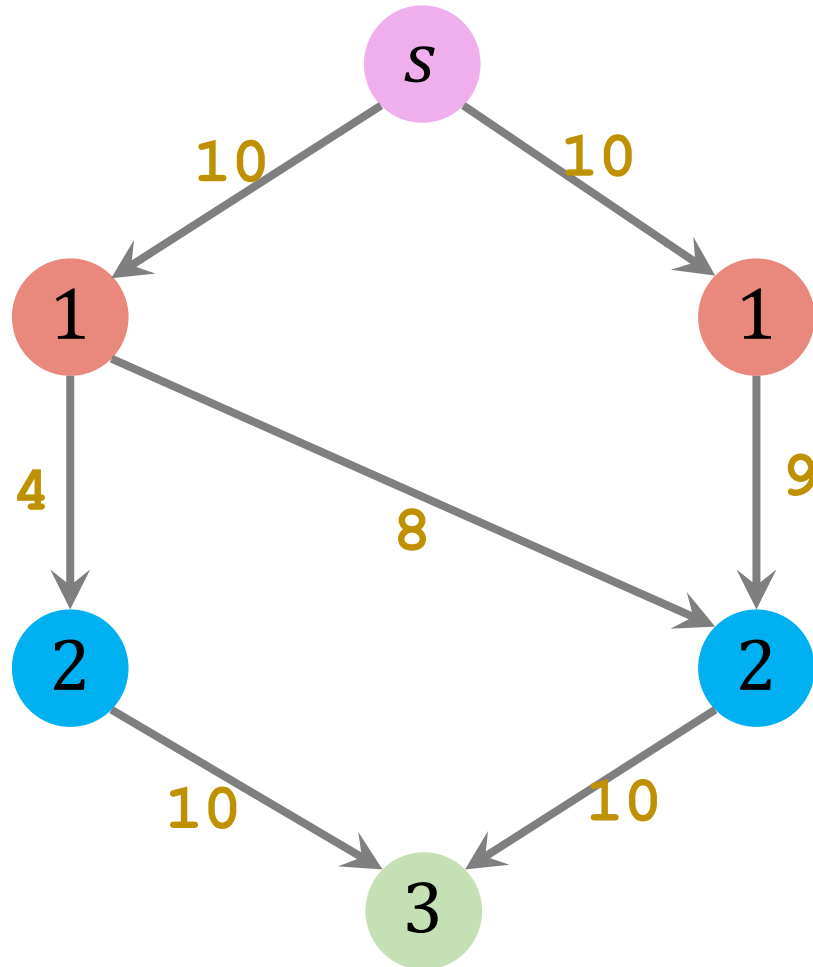


Level Graph



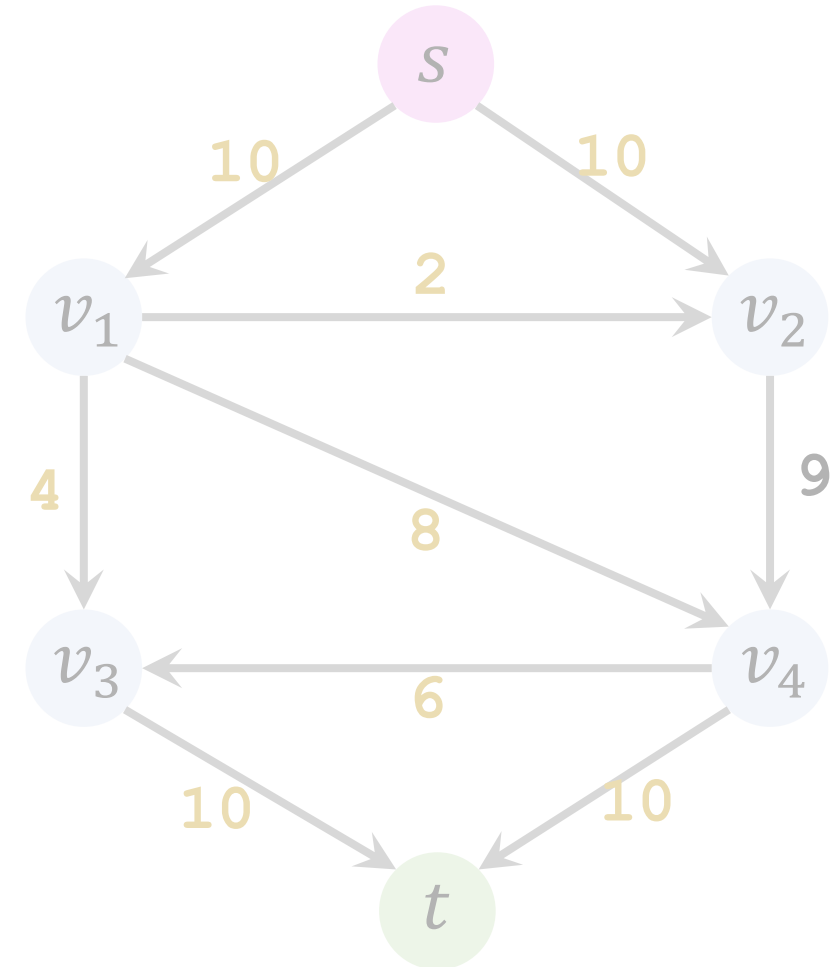
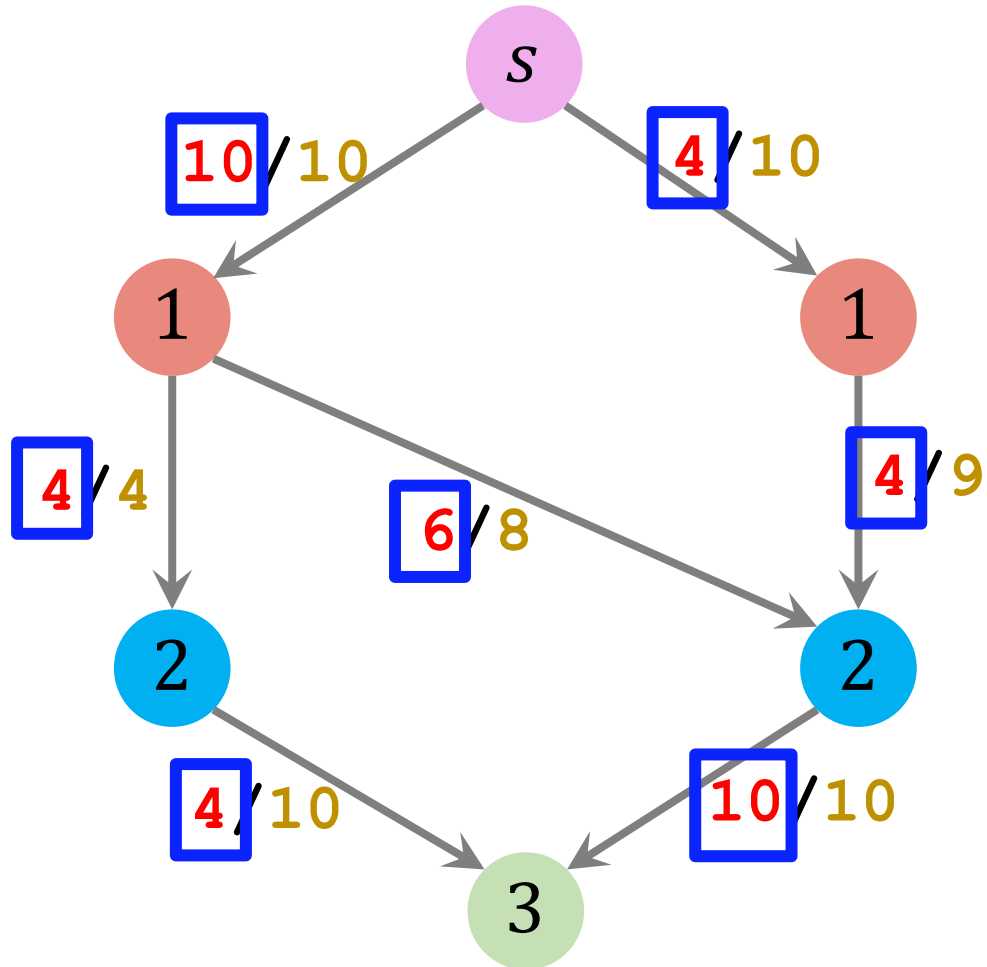
Residual Graph

Iteration 1: Find **blocking flow** in level graph



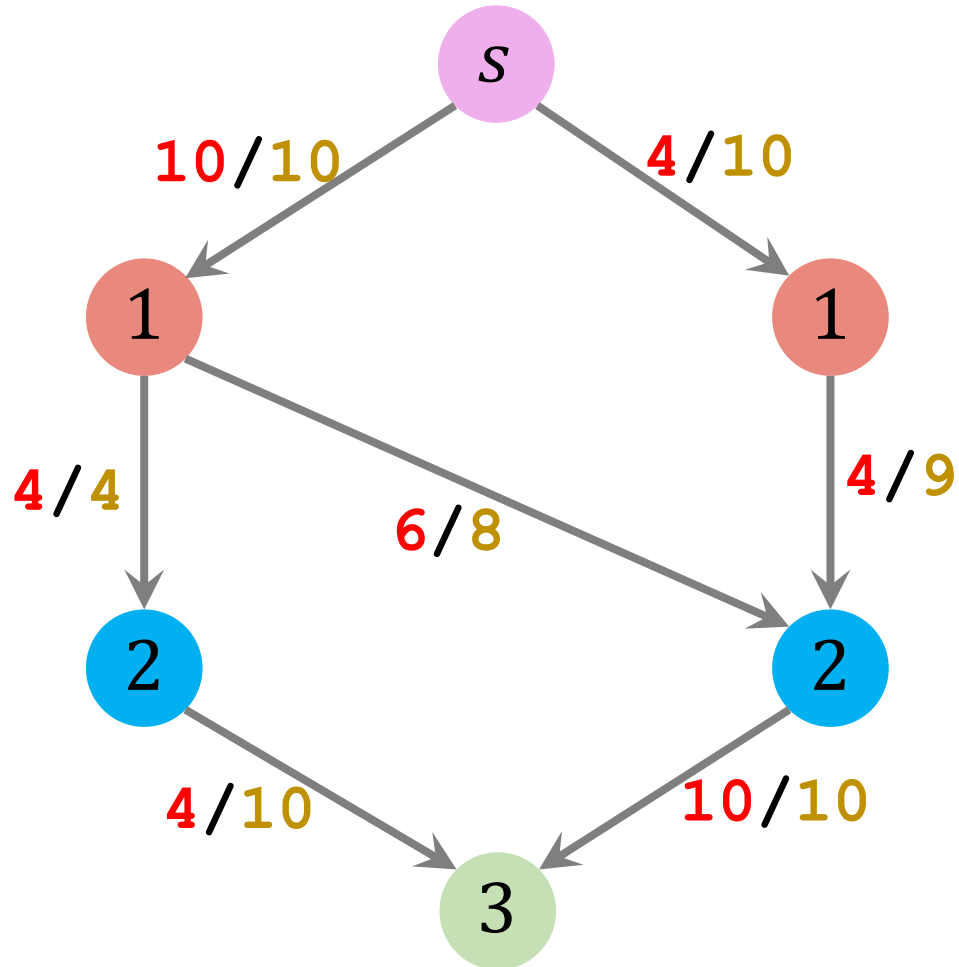
A flow is **blocking flow** if no more flow from source to sink can be found.

Iteration 1: Find **blocking flow** in level graph

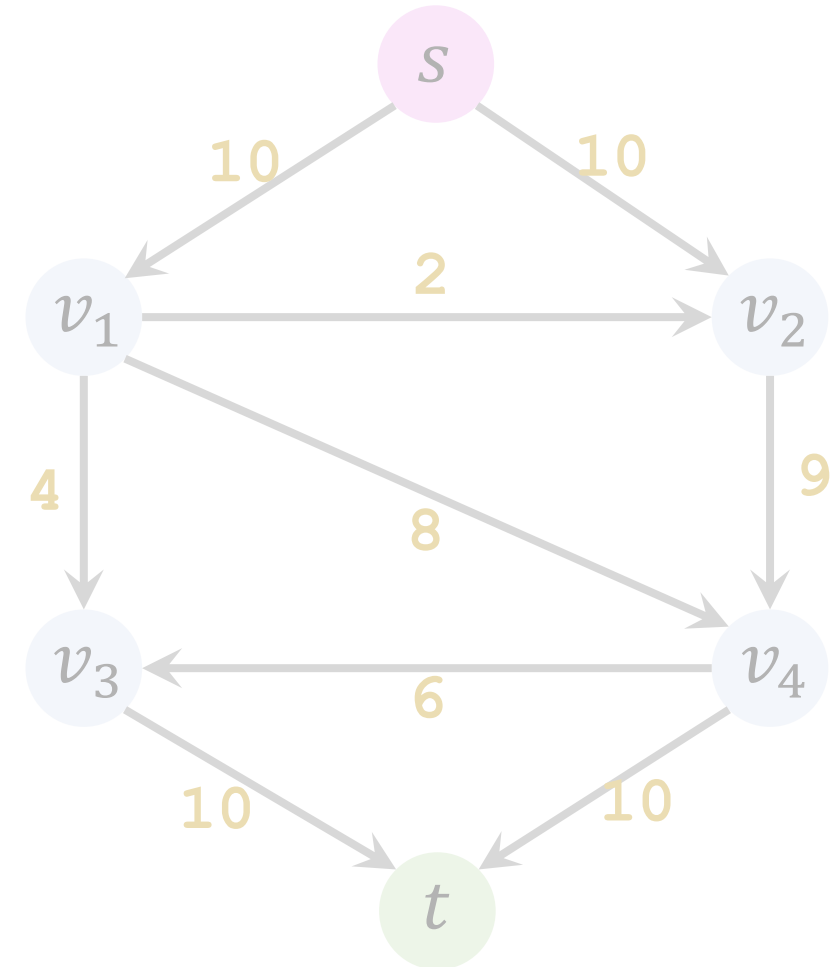


Blocking flow can be found using the naïve algorithm.

Iteration 1: Update the **residual graph**

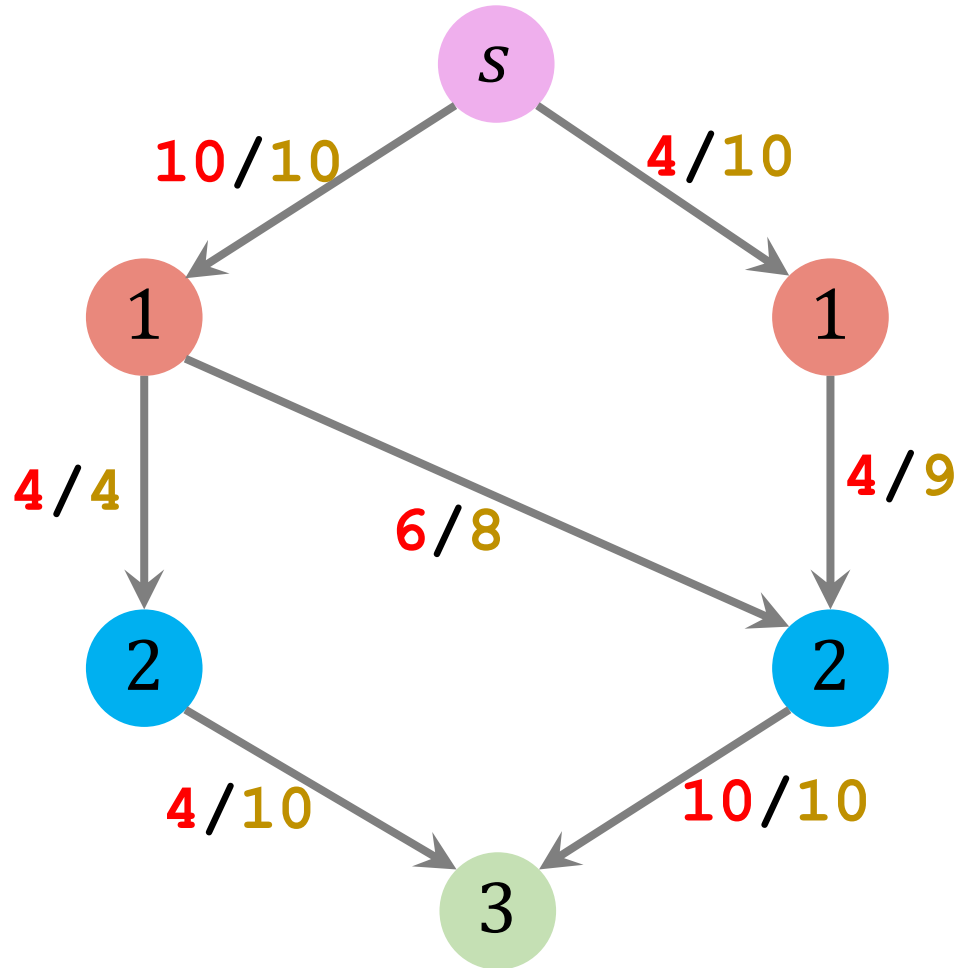


Level Graph

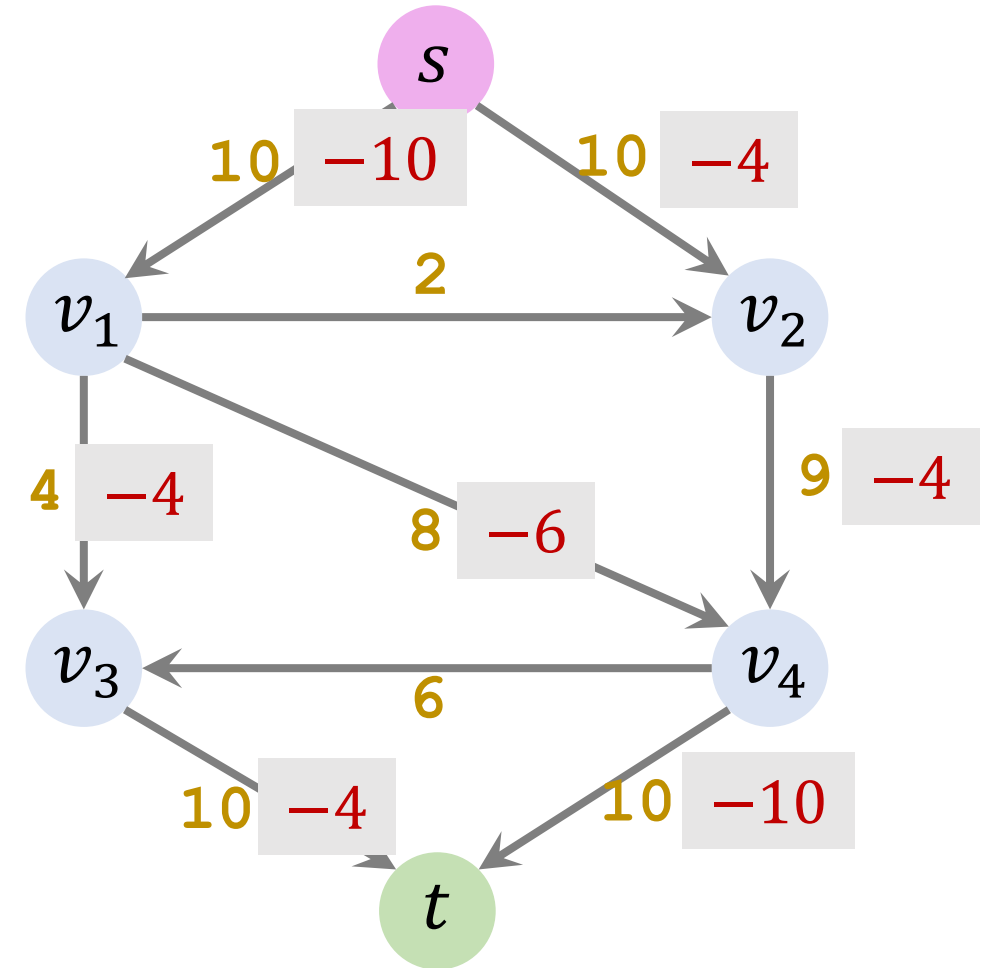


Old Residual Graph

Iteration 1: Update the **residual graph**

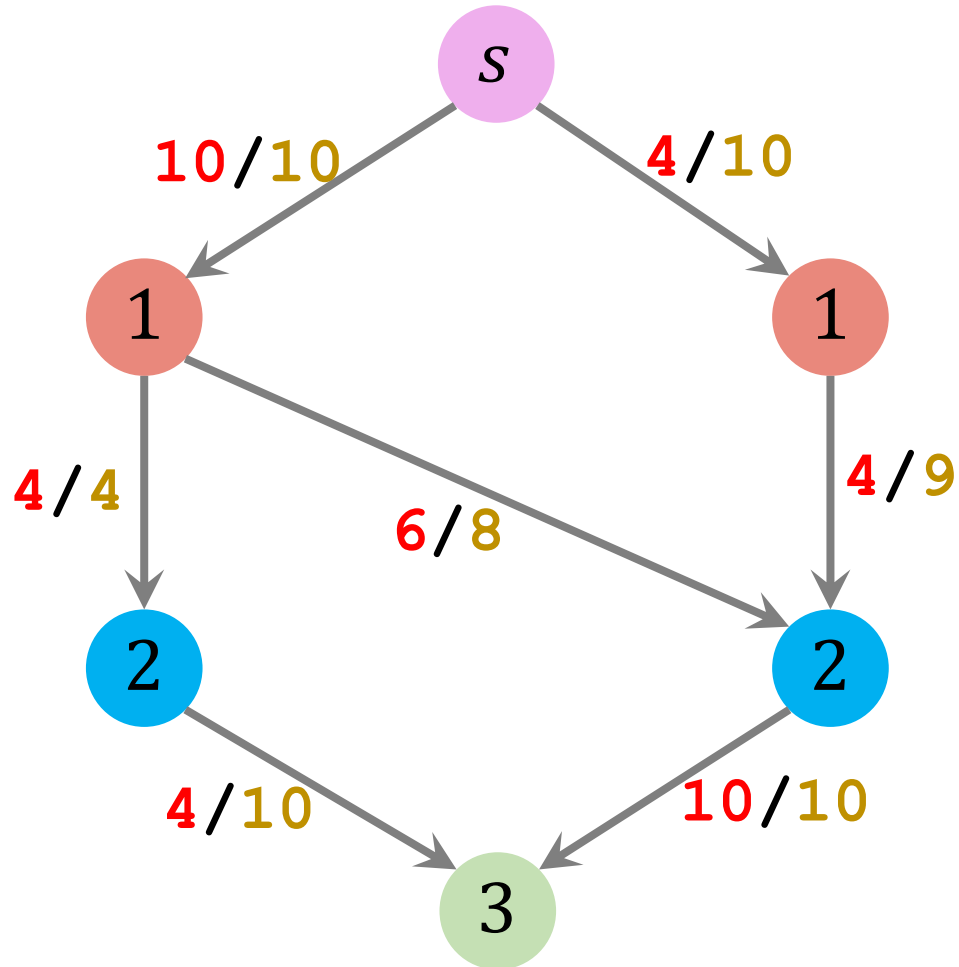


Level Graph

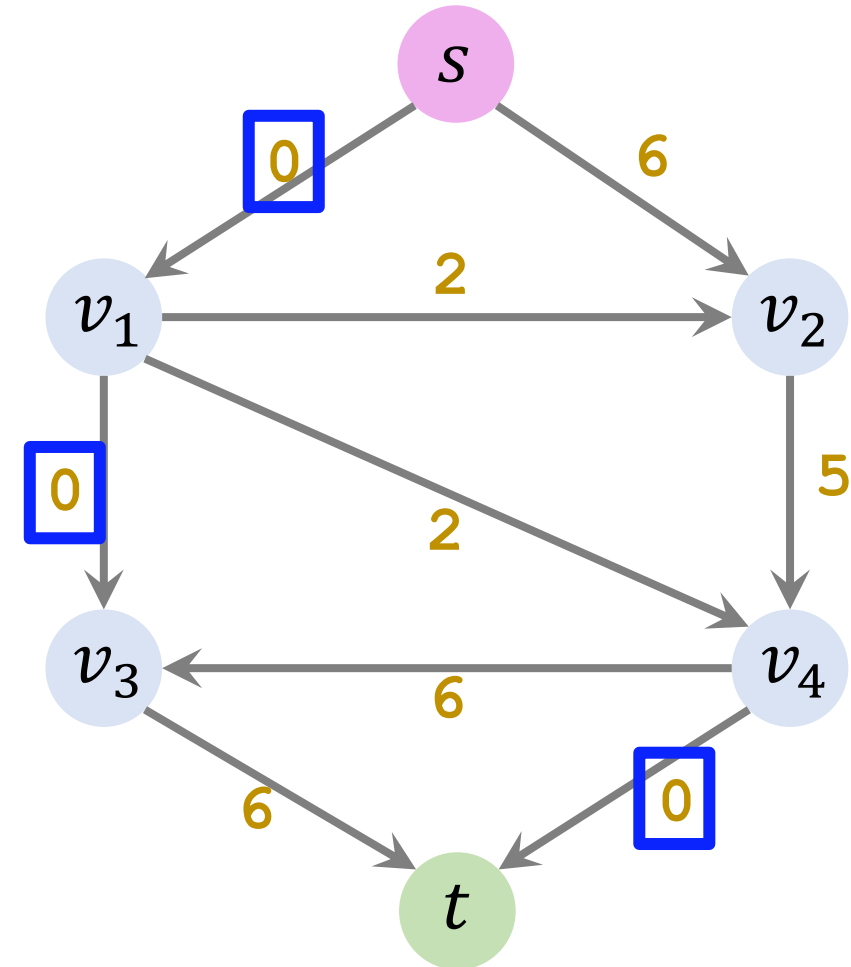


Old Residual Graph

Iteration 1: Update the **residual graph**

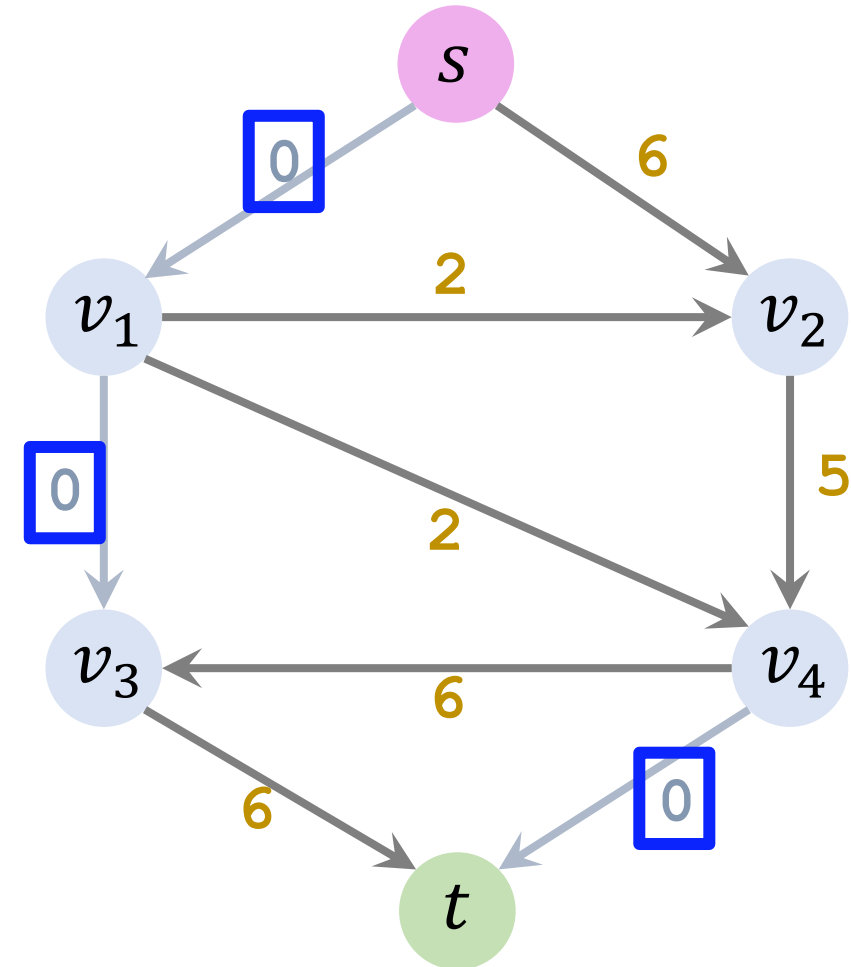
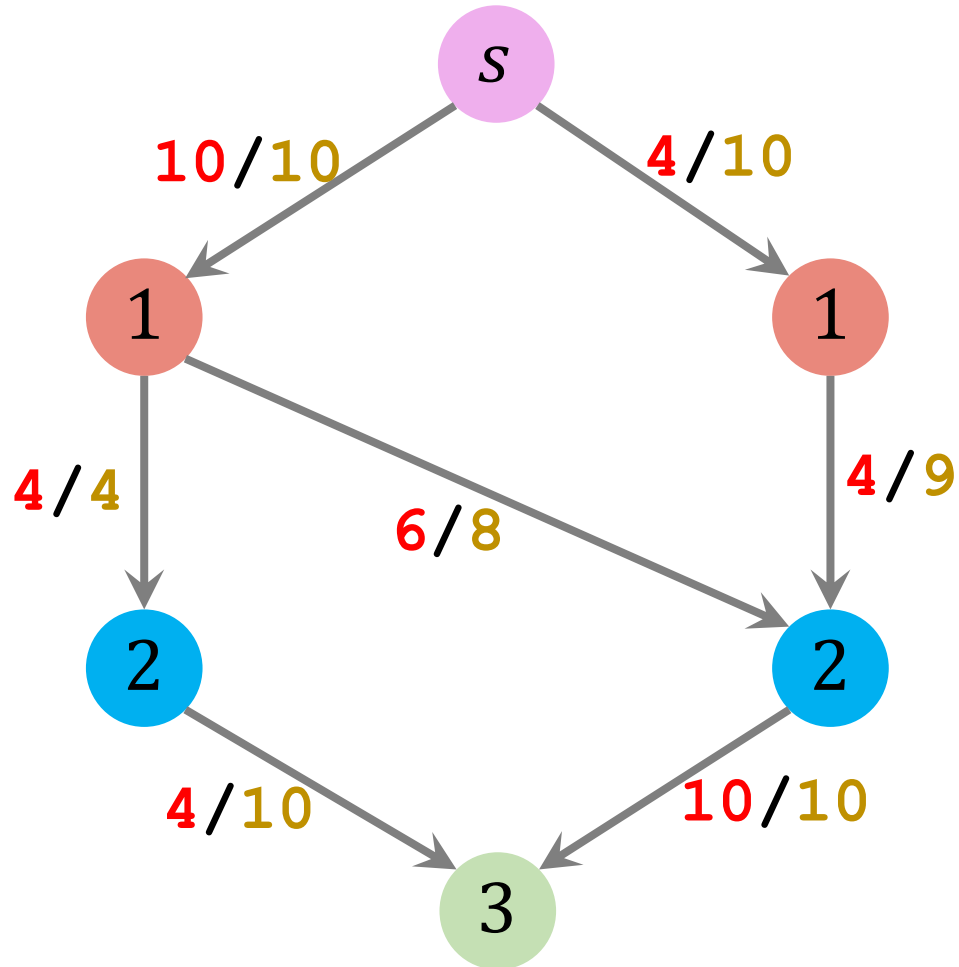


Level Graph



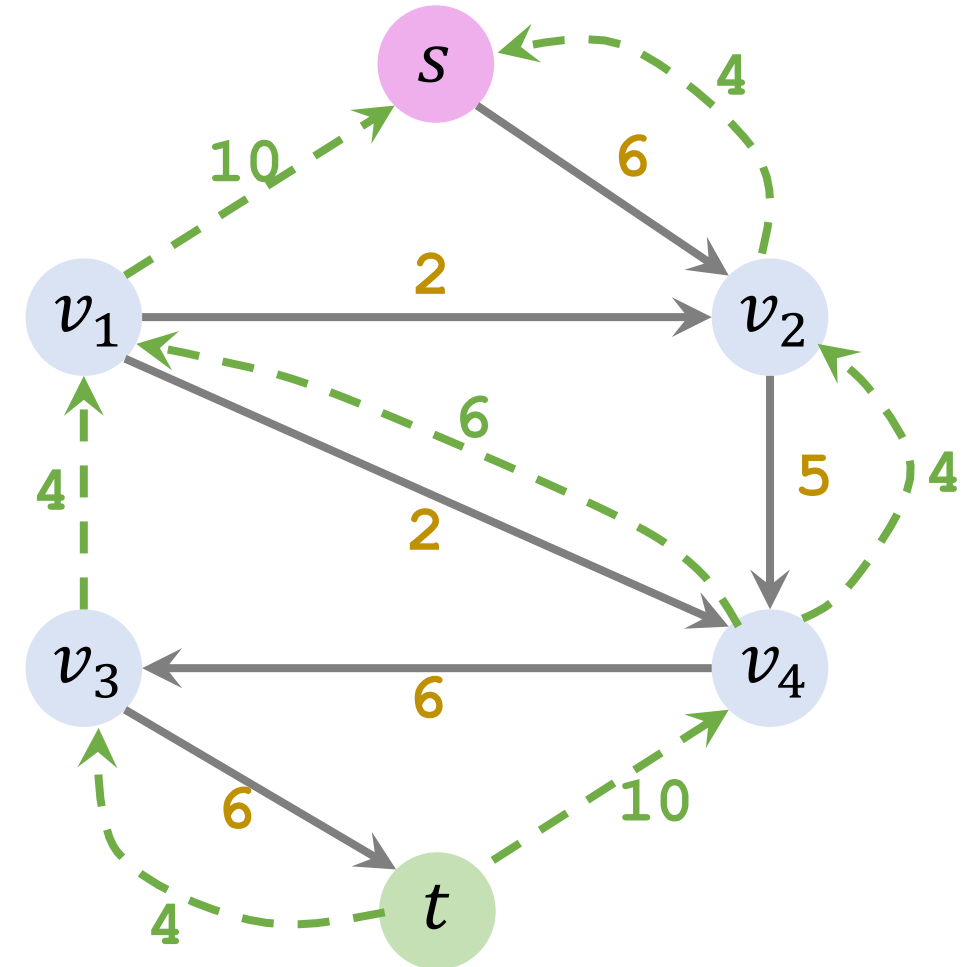
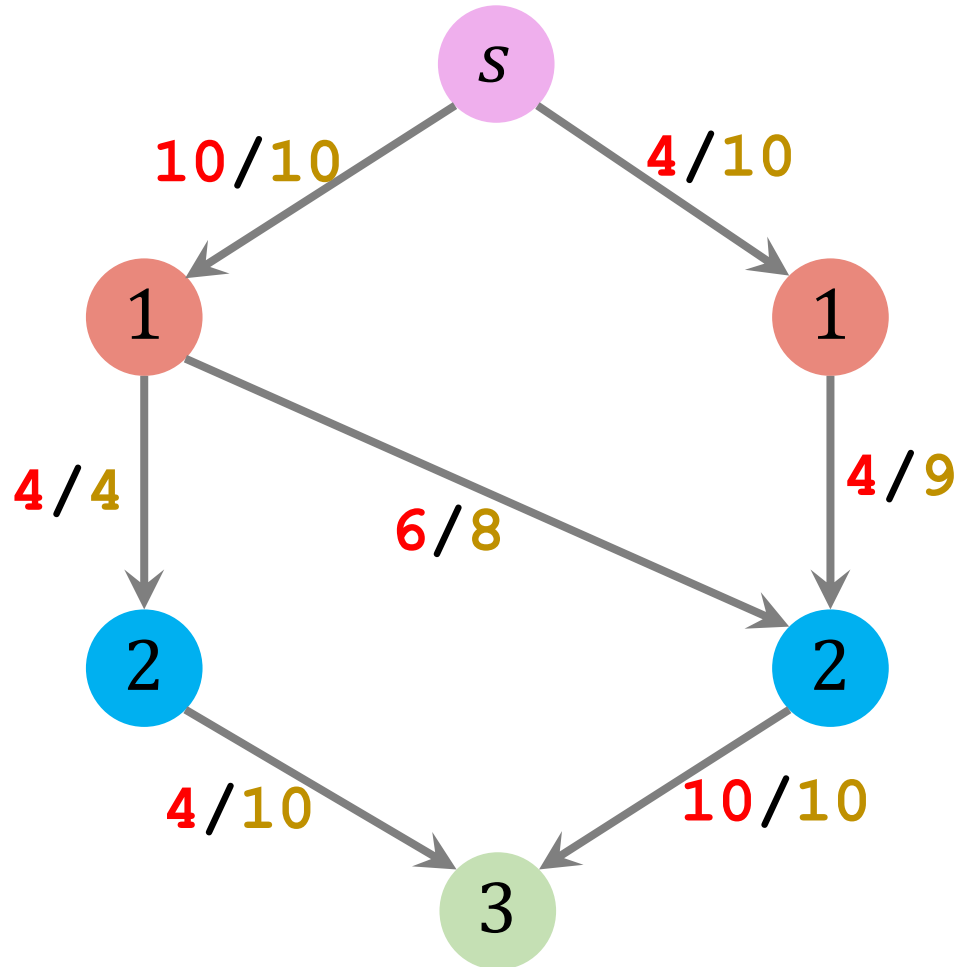
New Residual Graph

Iteration 1: Update the **residual graph**



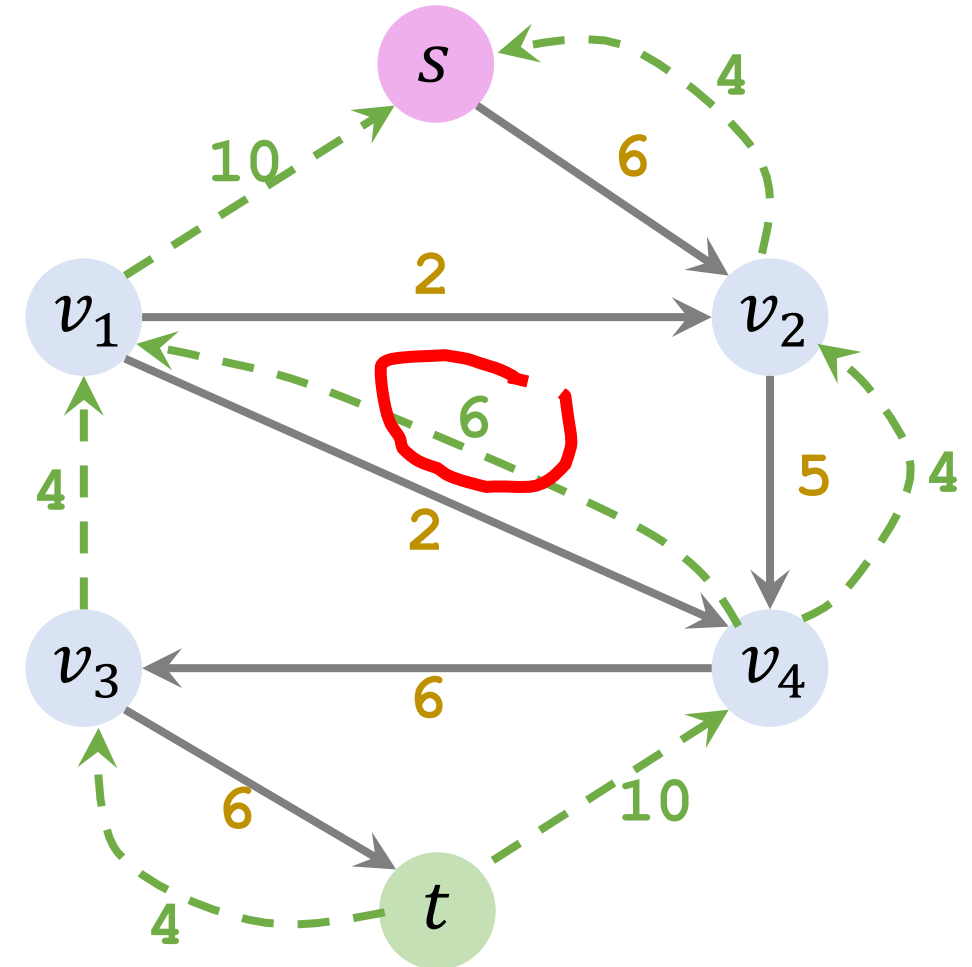
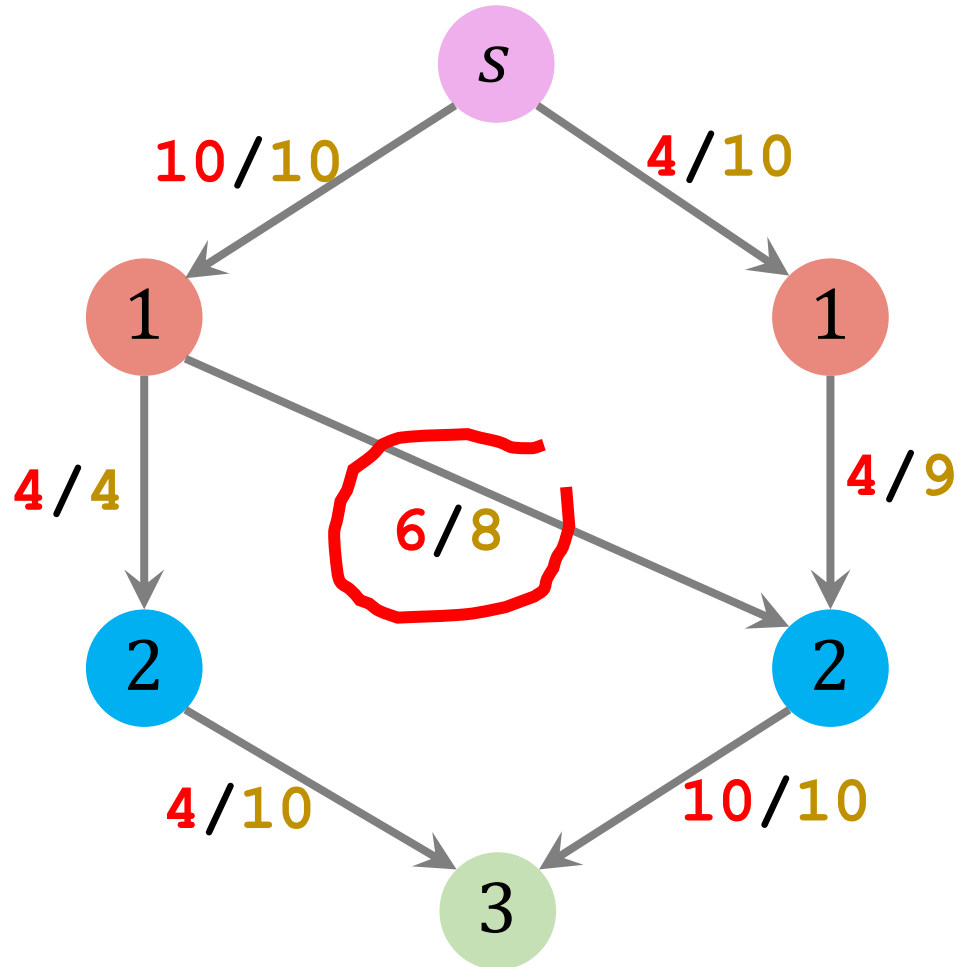
Removed saturated edges from residual graph.

Iteration 1: Update the **residual graph**



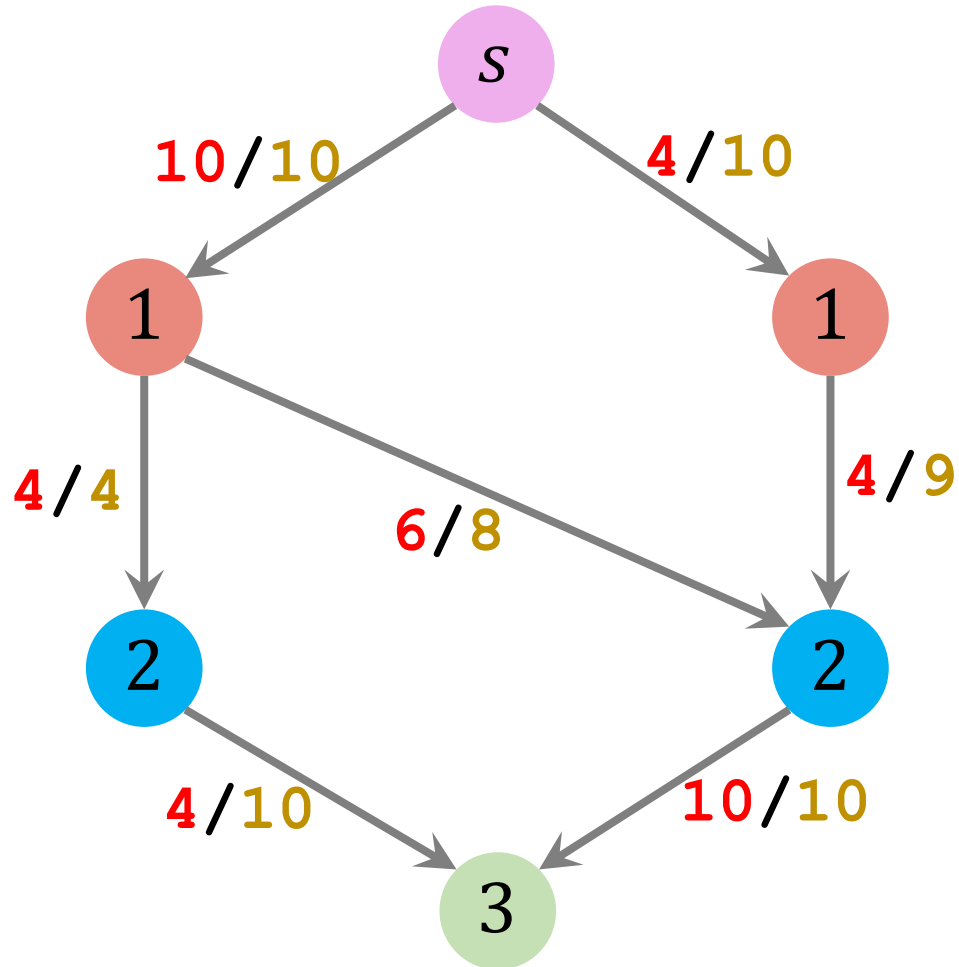
Add **flows** to the residual graph as **backward paths**.

Iteration 1: Update the **residual graph**

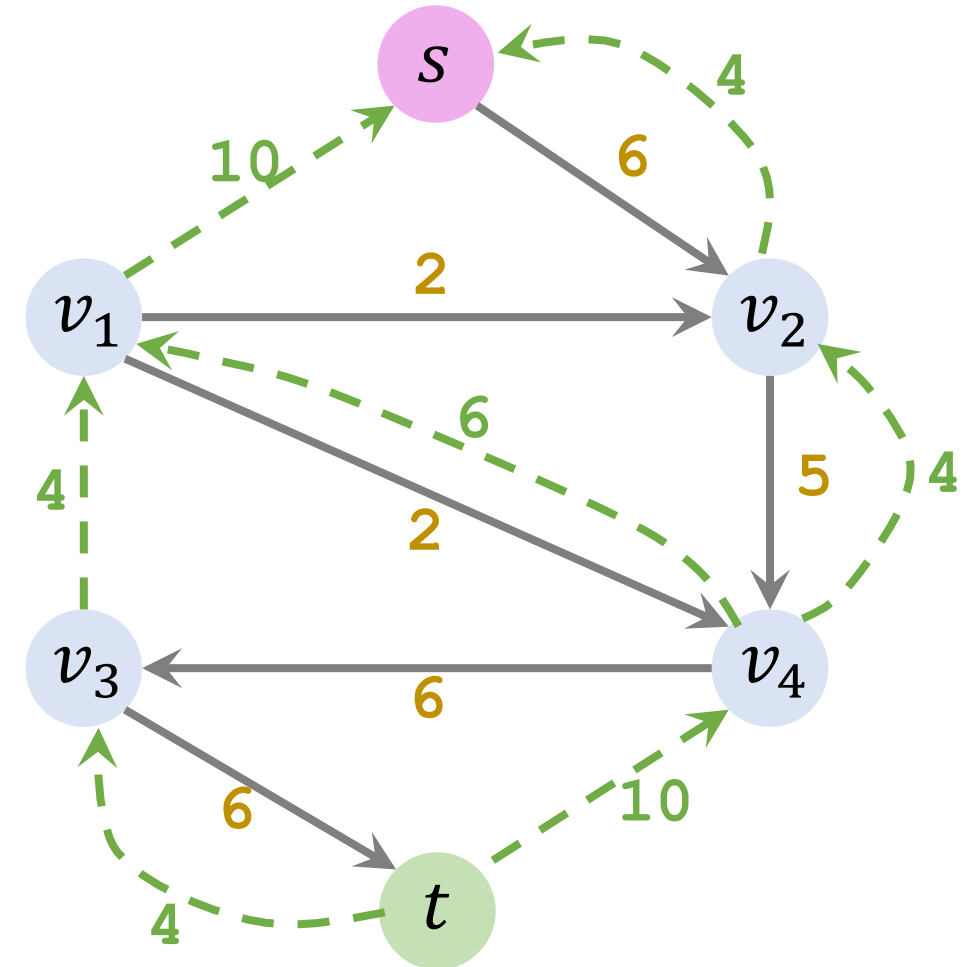


Add **flows** to the residual graph as **backward paths**.

Iteration 1: Finished

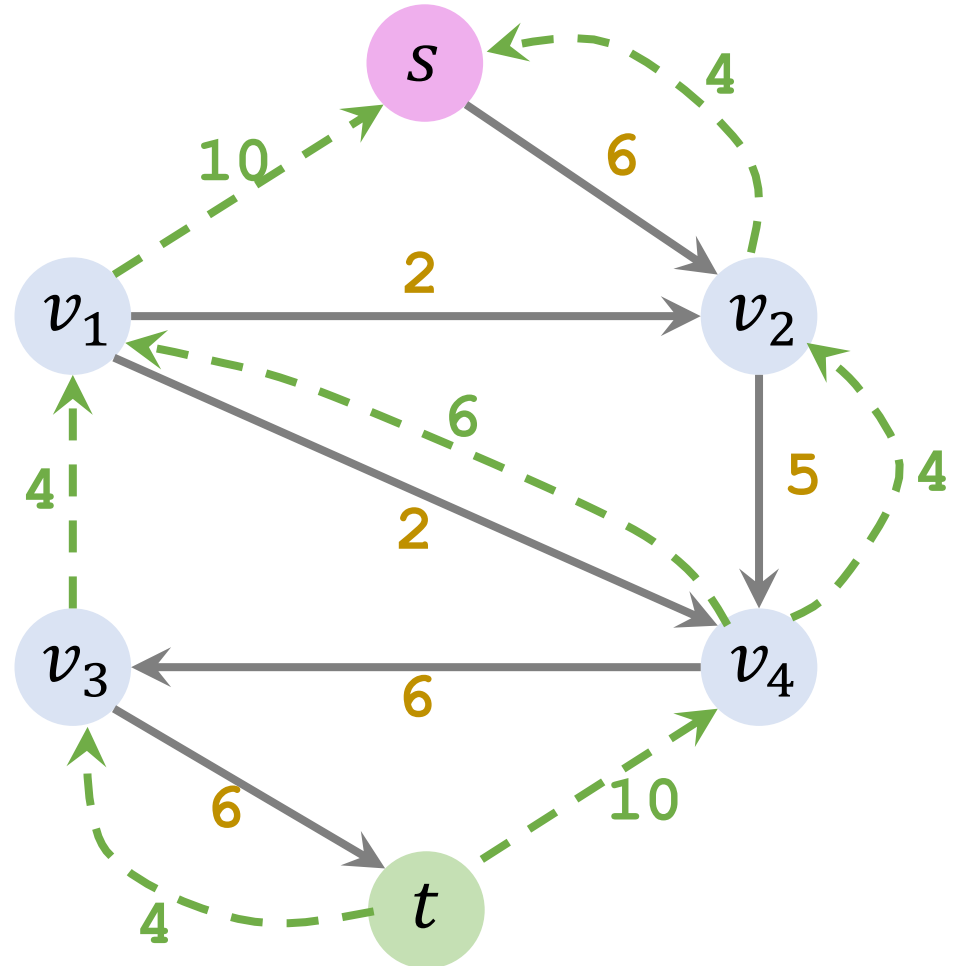


Level Graph



Residual Graph

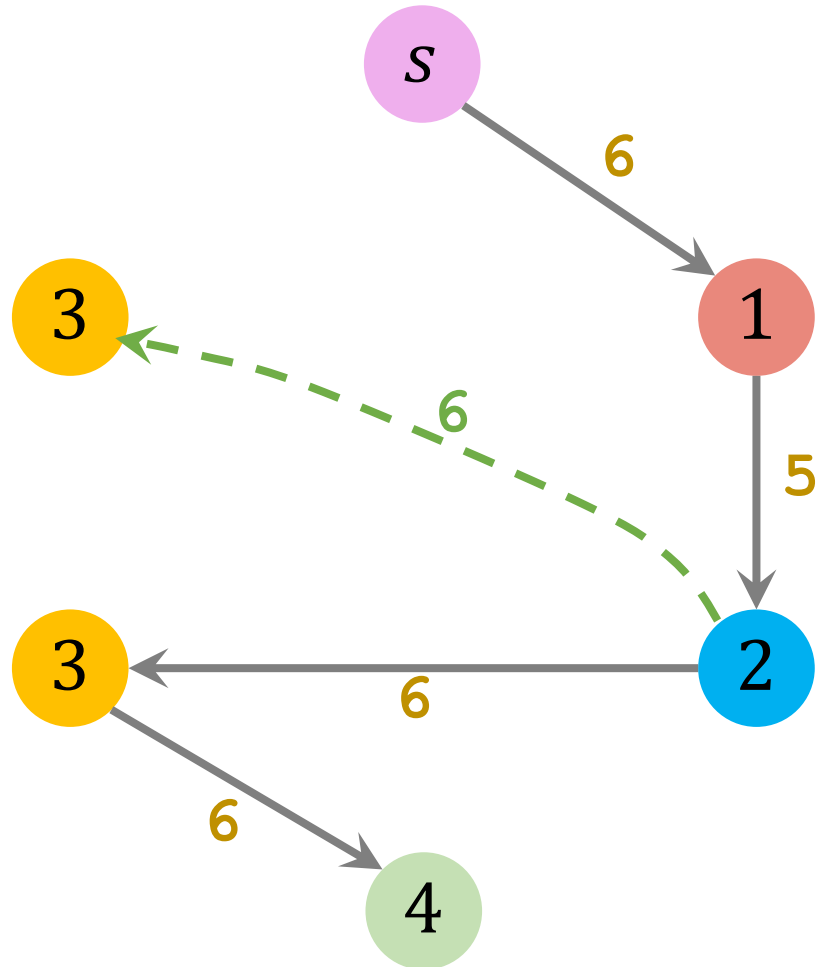
Iteration 2: Construct **level graph**



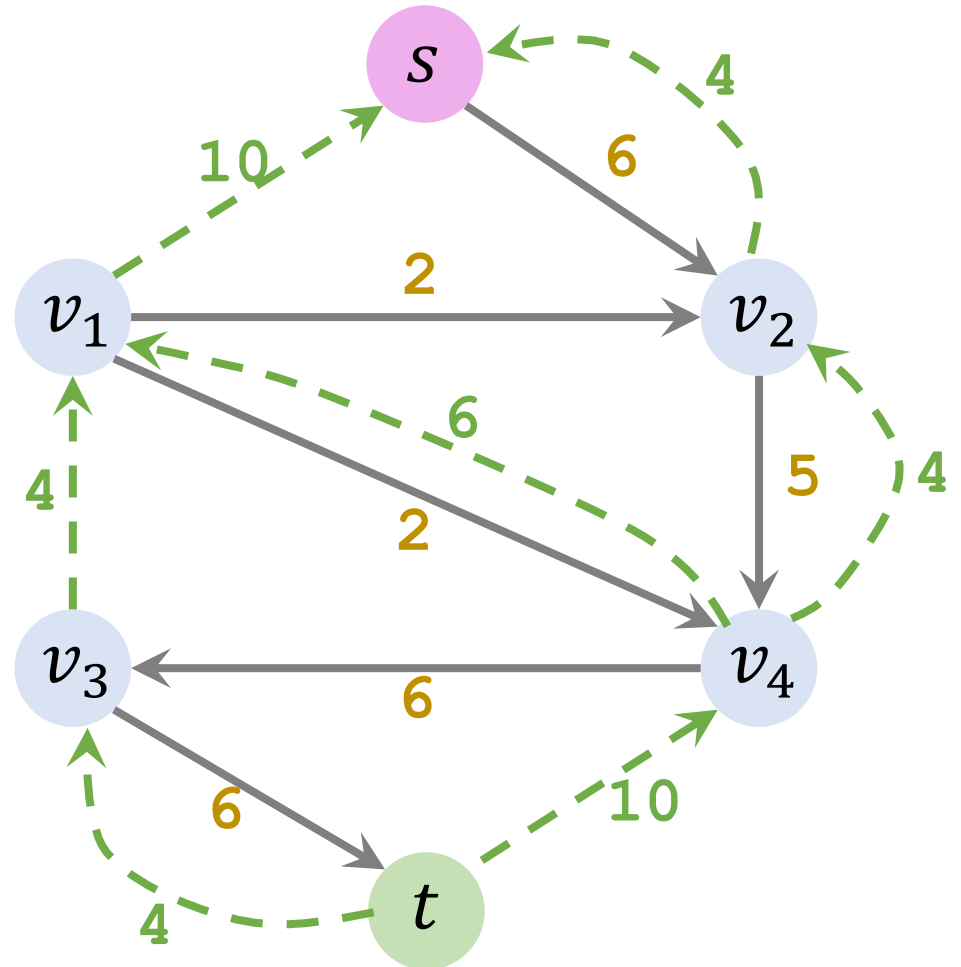
Level Graph

Residual Graph

Iteration 2: Construct **level graph**

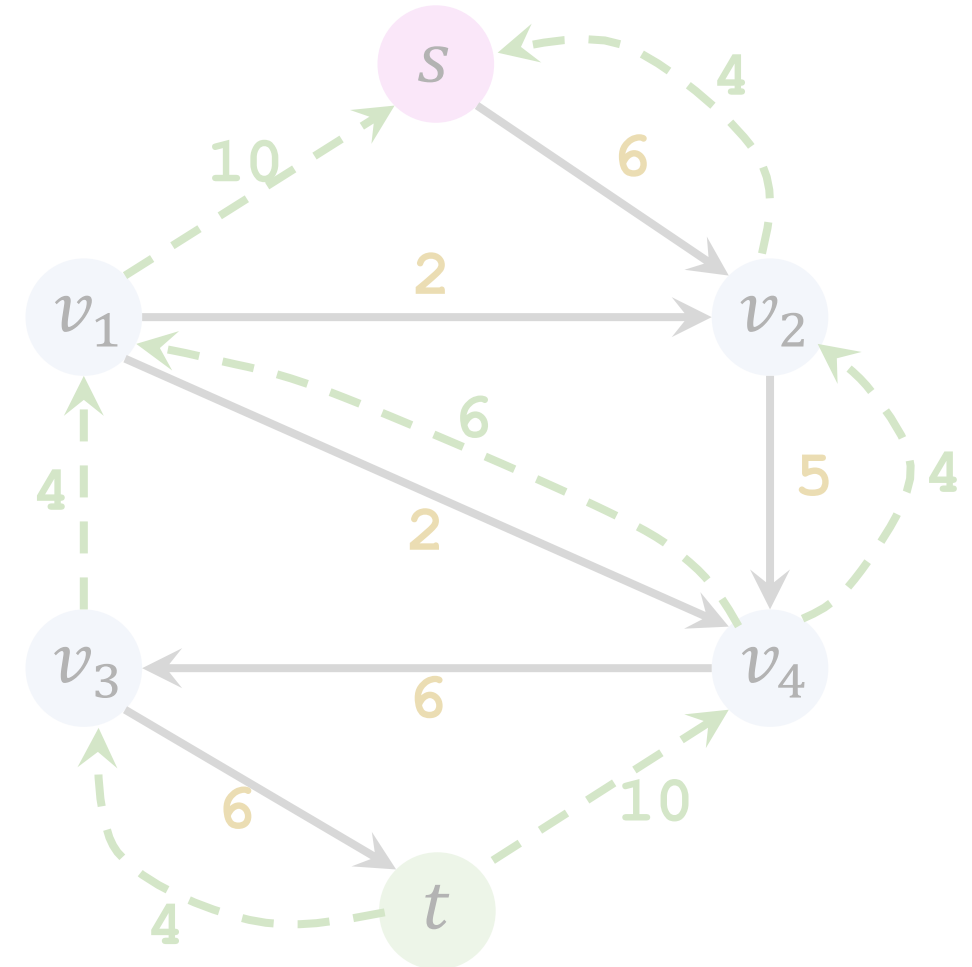
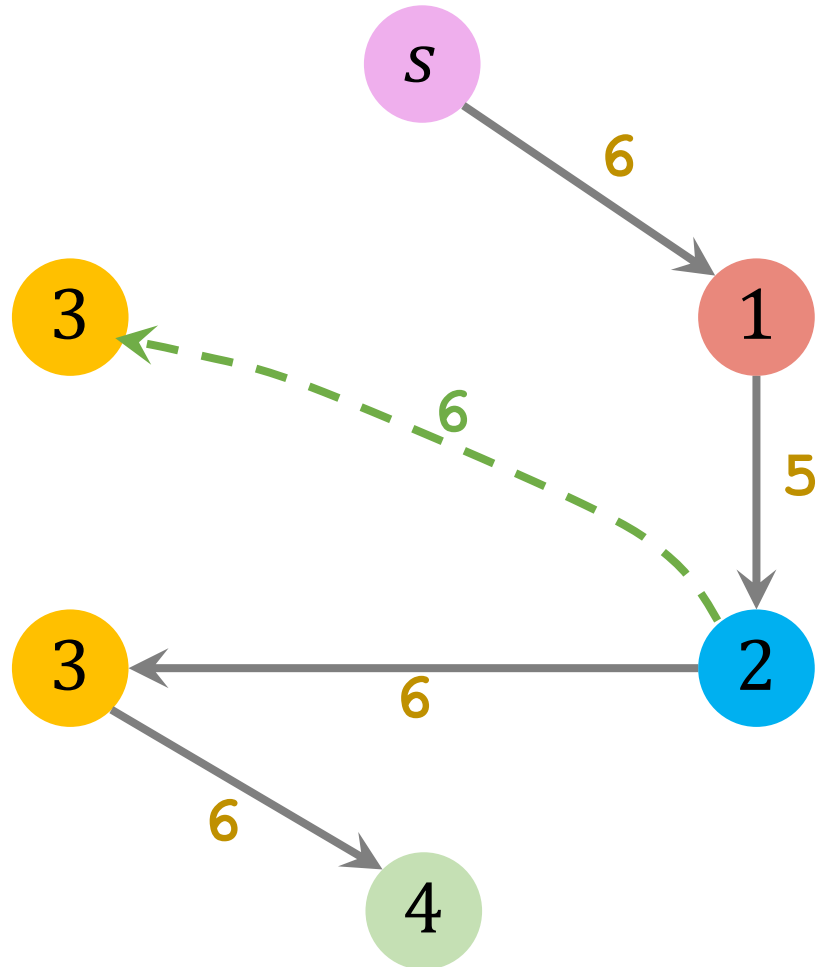


Level Graph



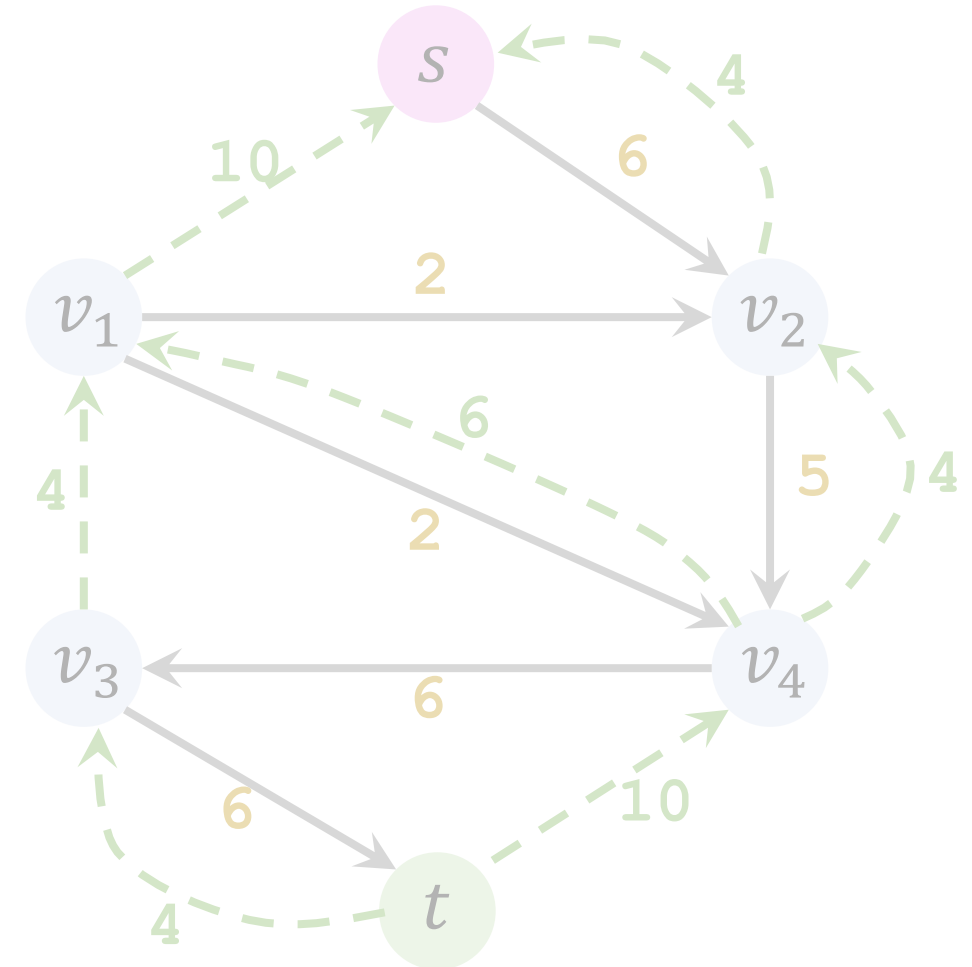
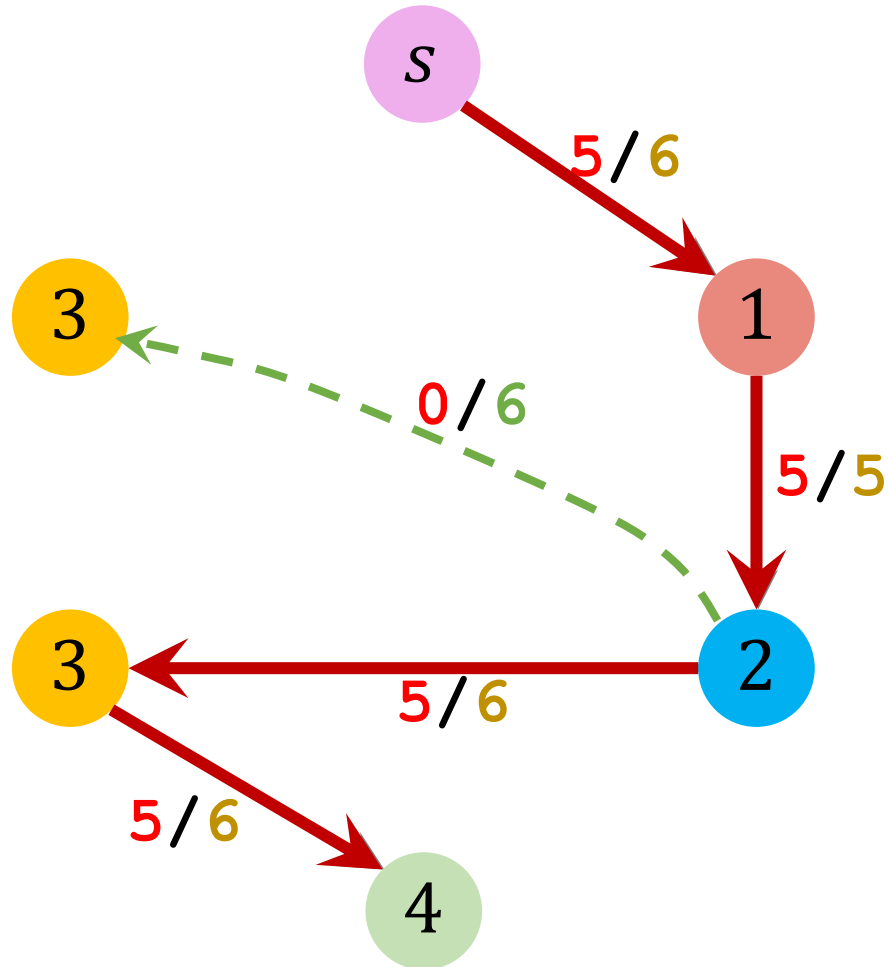
Residual Graph

Iteration 2: Find blocking flow in level graph



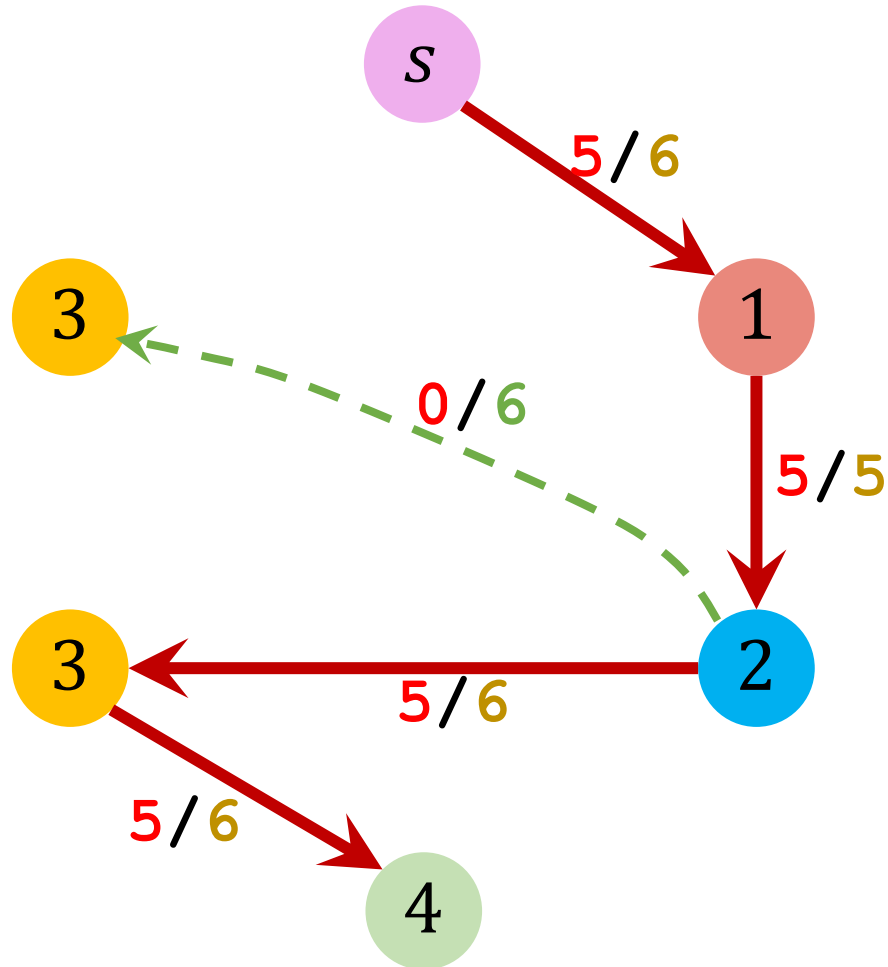
A flow is **blocking flow** if no more flow from source to sink can be found.

Iteration 2: Find blocking flow in level graph

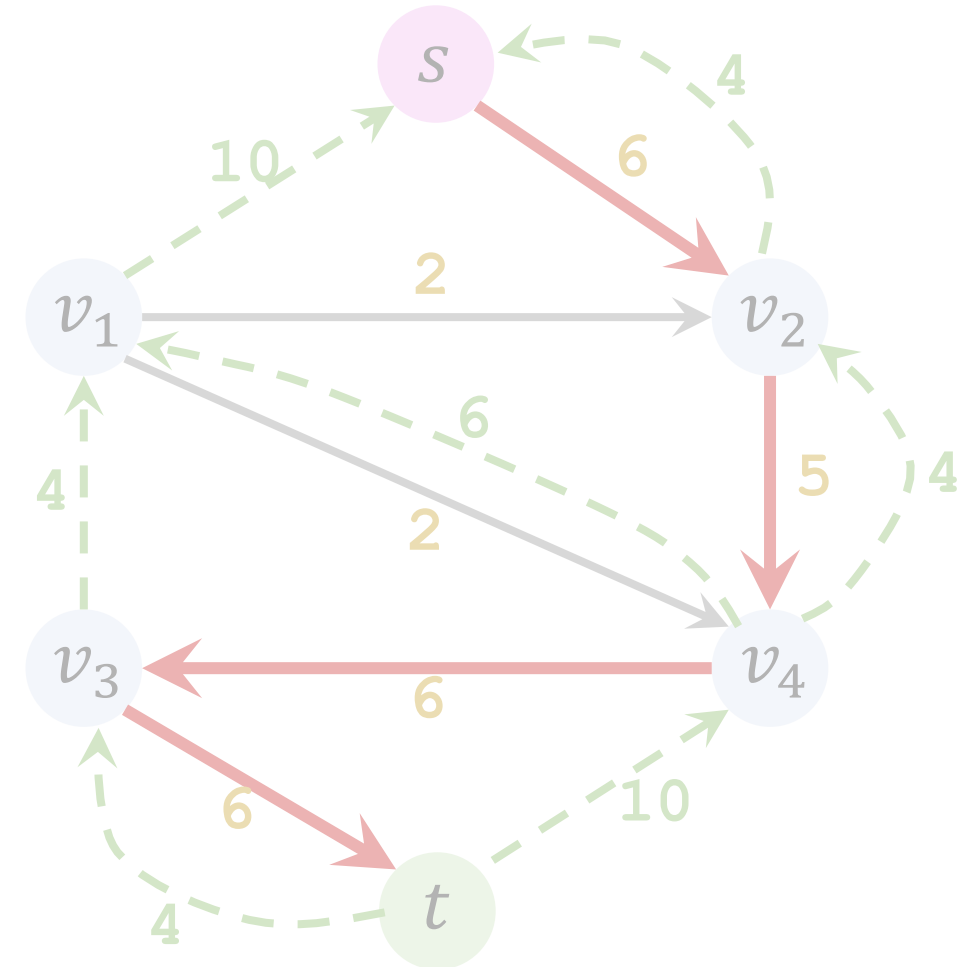


Blocking flow can be found using the naïve algorithm.

Iteration 2: Update the **residual graph**

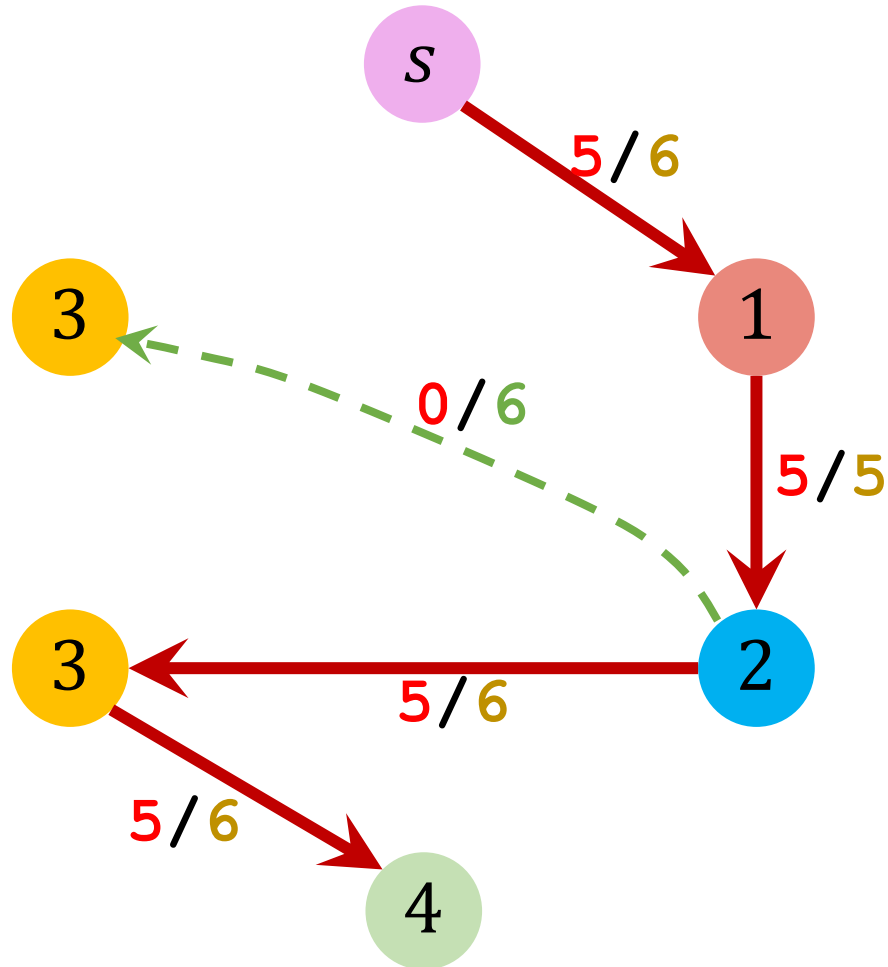


Blocking Flow

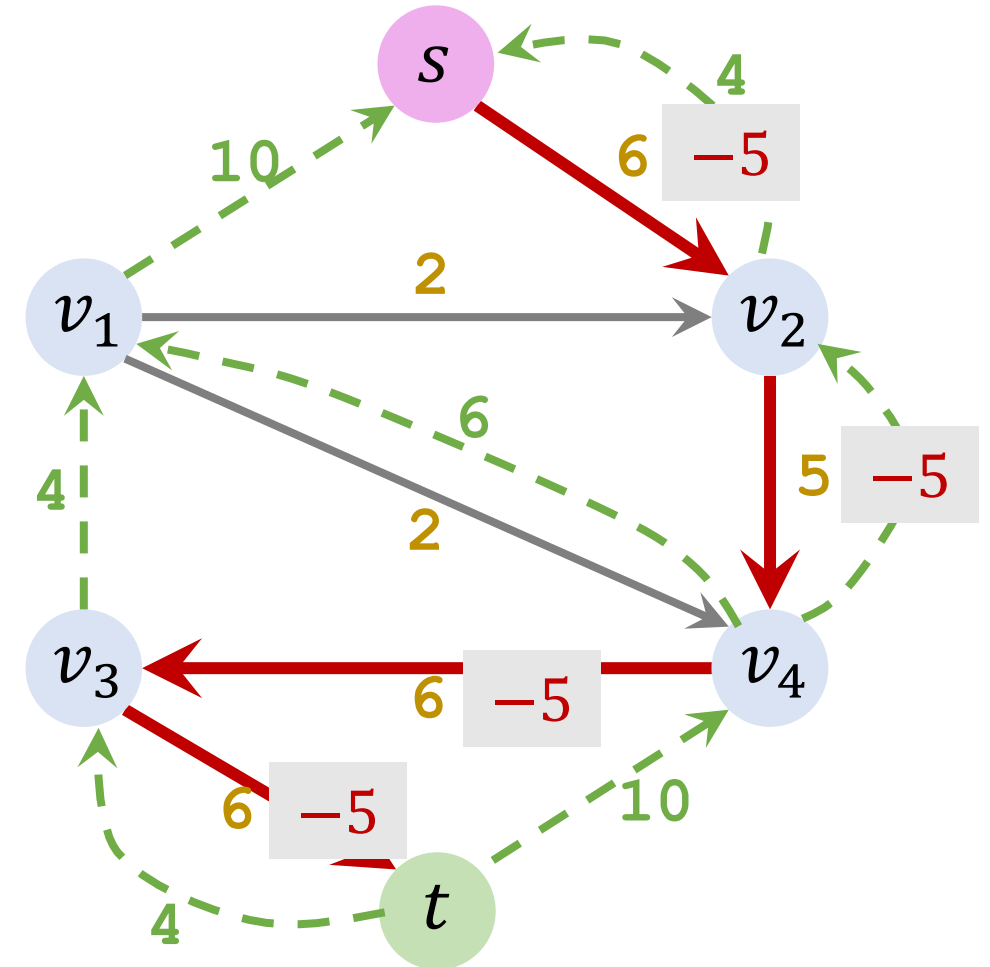


Old Residual Graph

Iteration 2: Update the **residual graph**

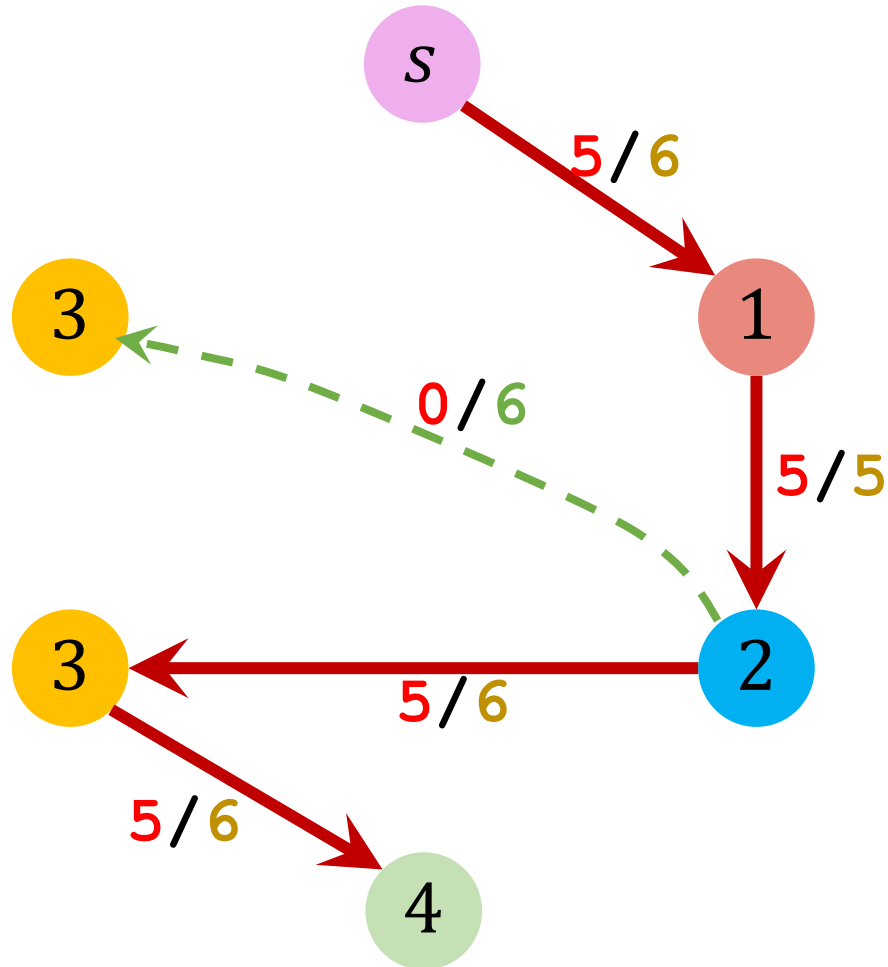


Blocking Flow

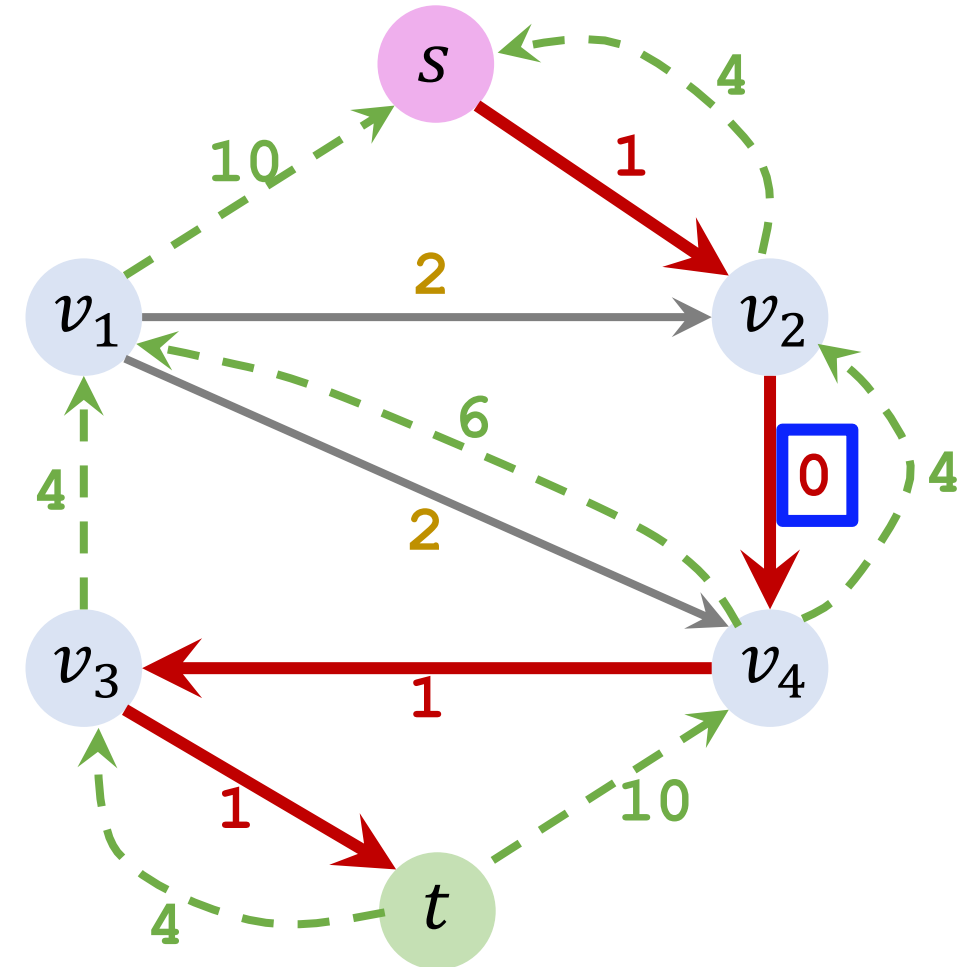


Old Residual Graph

Iteration 2: Update the **residual graph**

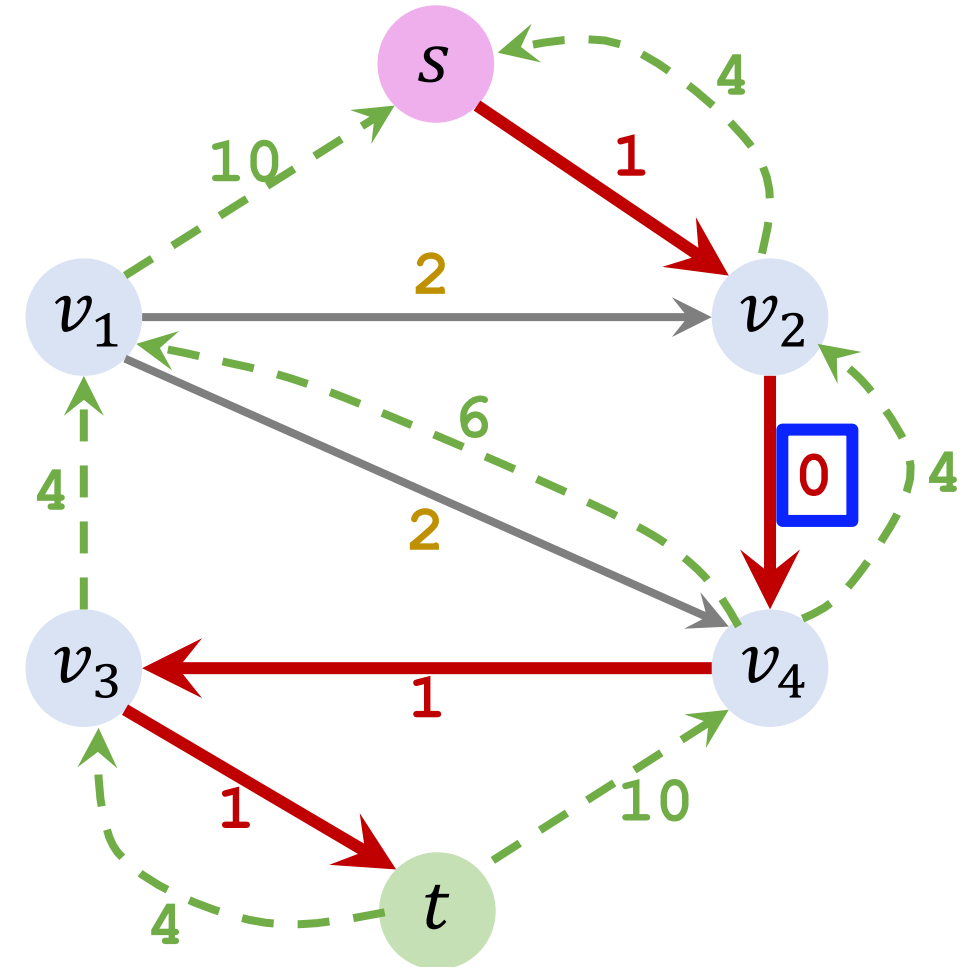
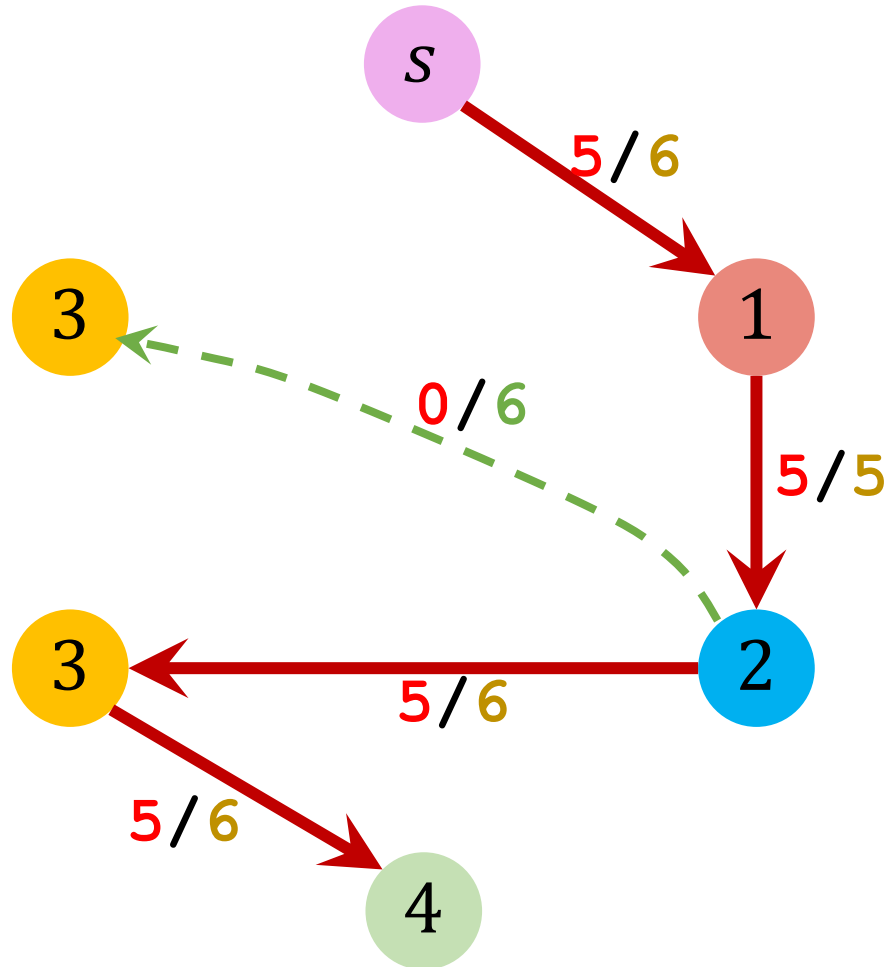


Blocking Flow



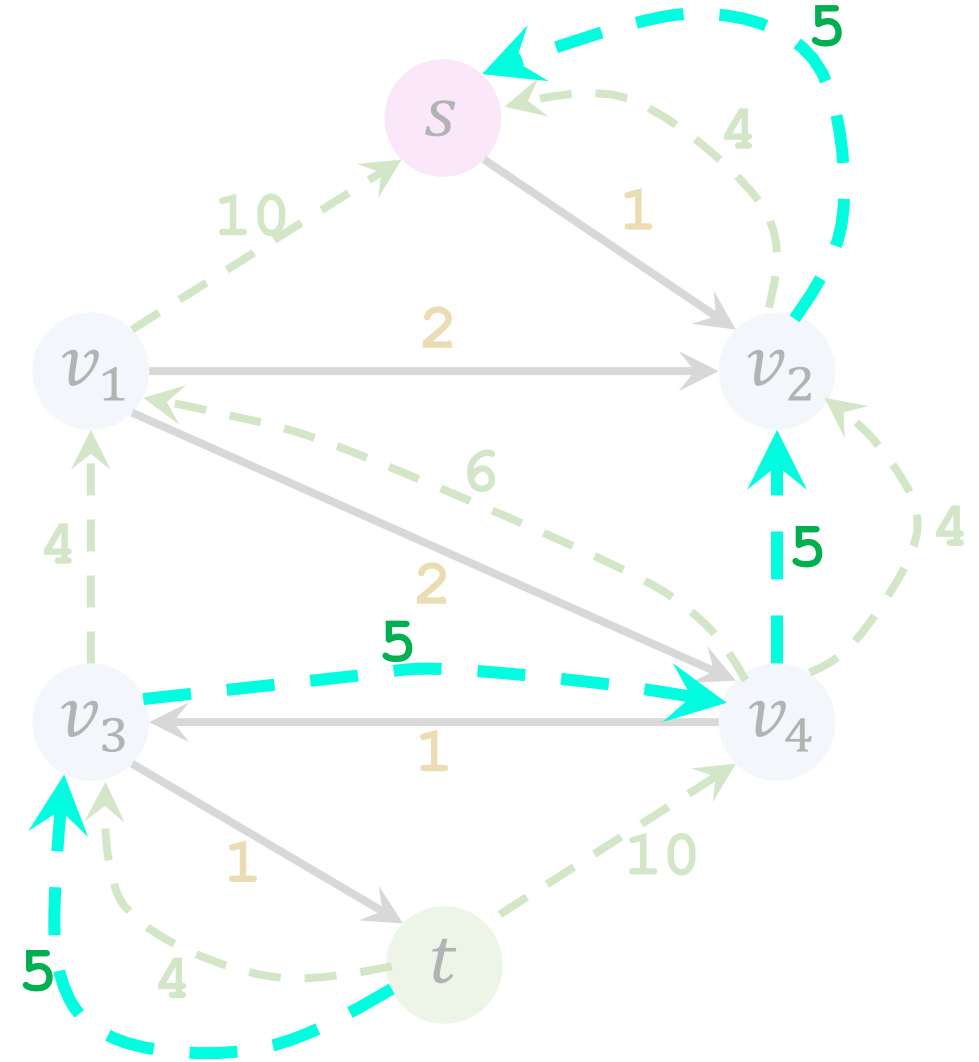
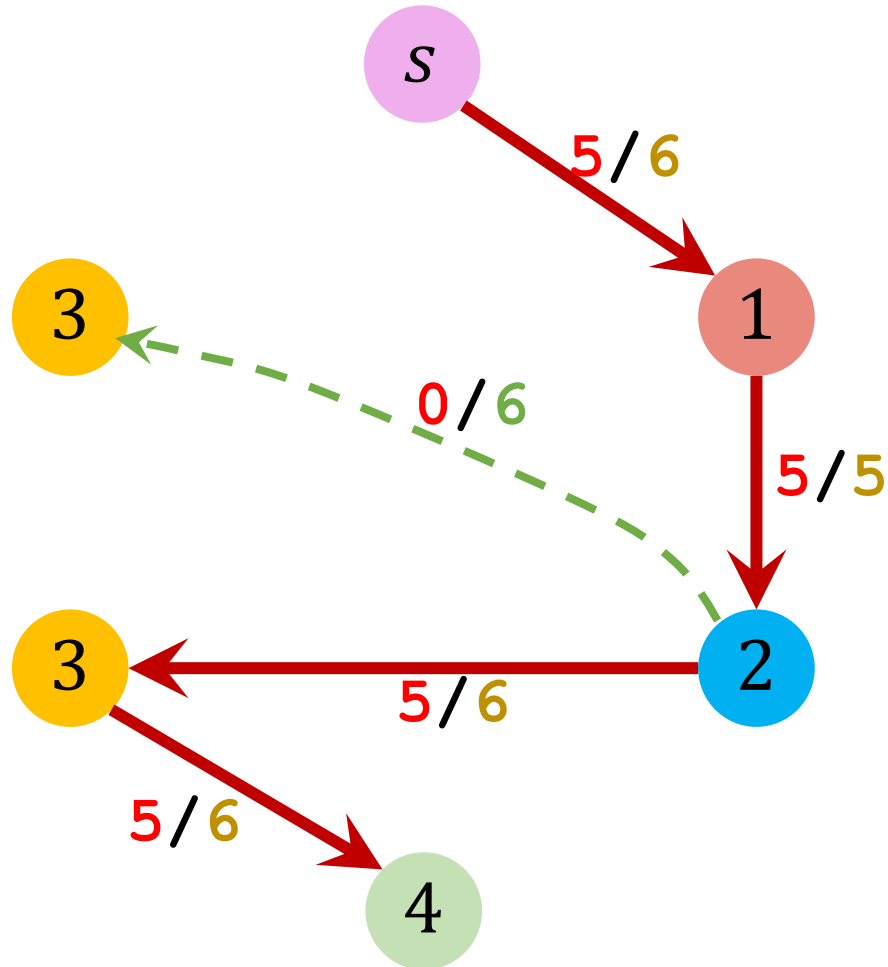
New Residual Graph

Iteration 2: Update the **residual graph**



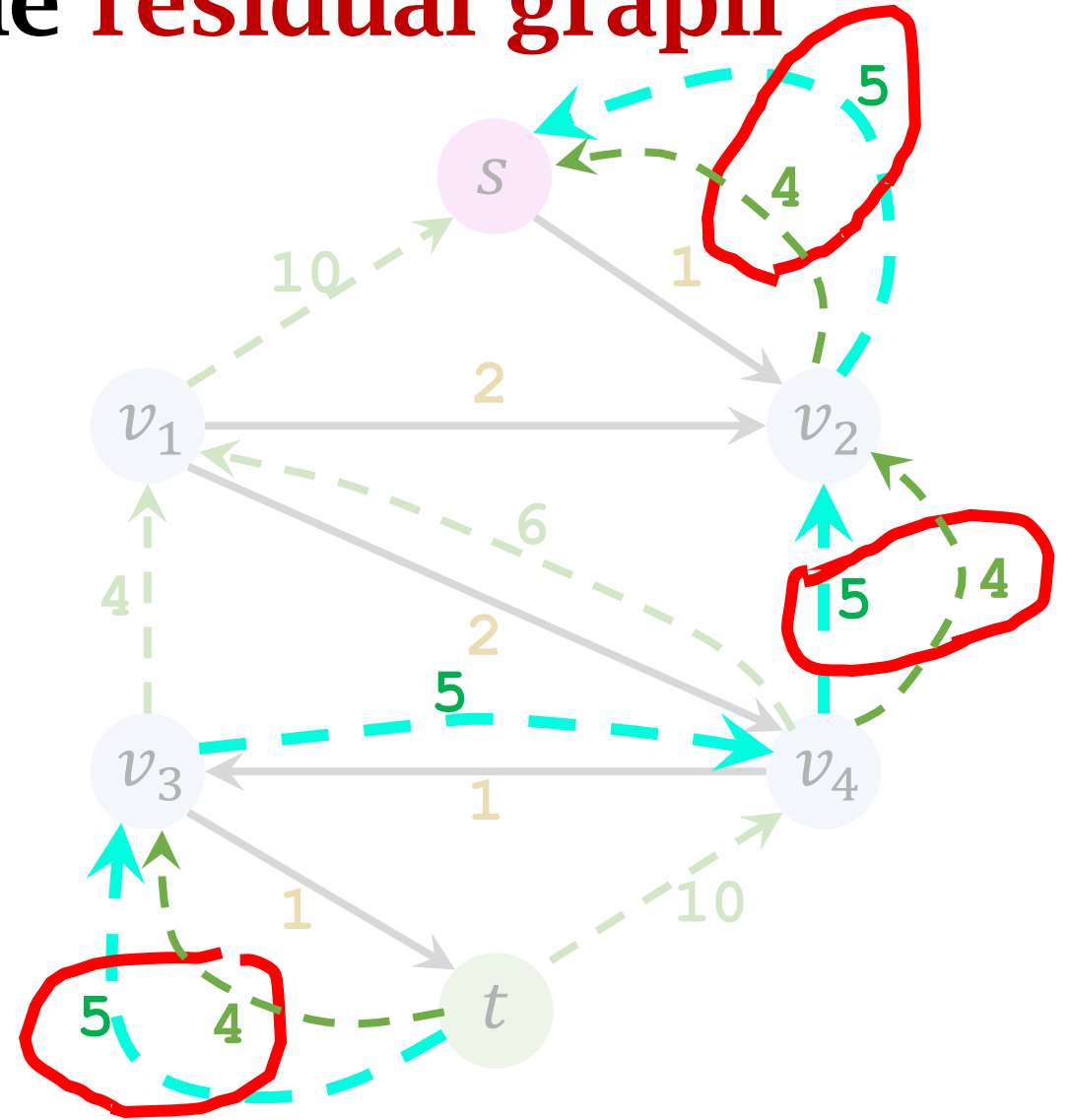
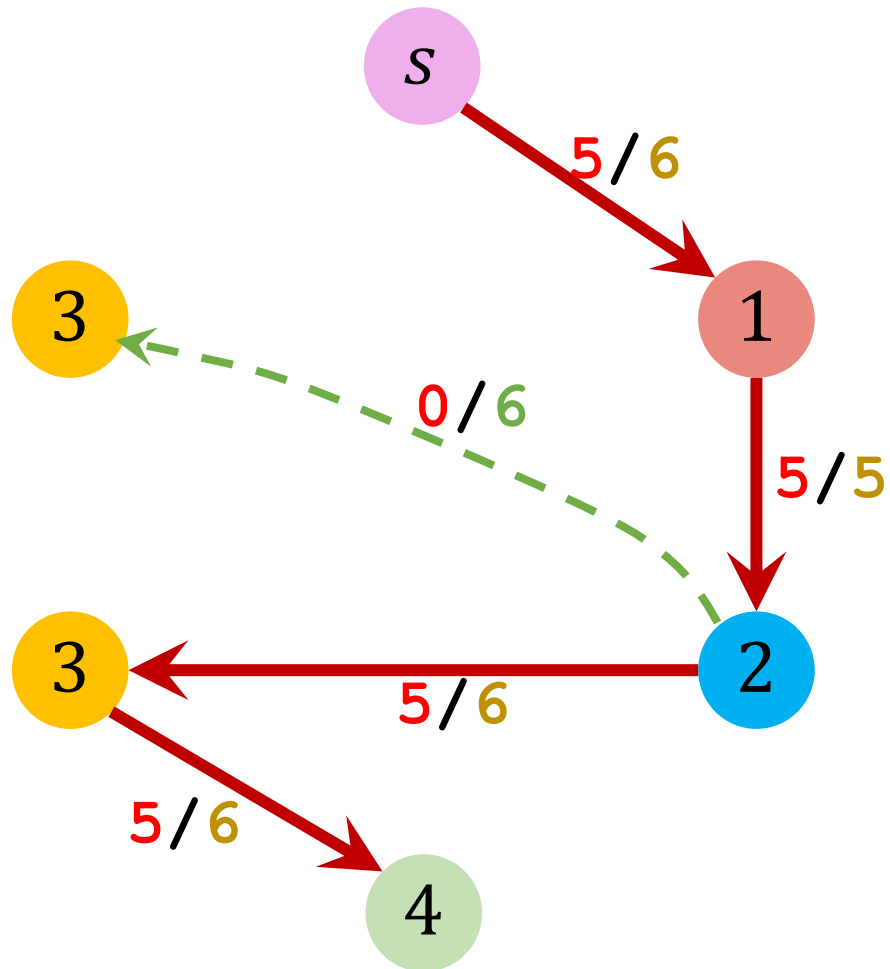
Removed saturated edges from residual graph.

Iteration 2: Update the **residual graph**



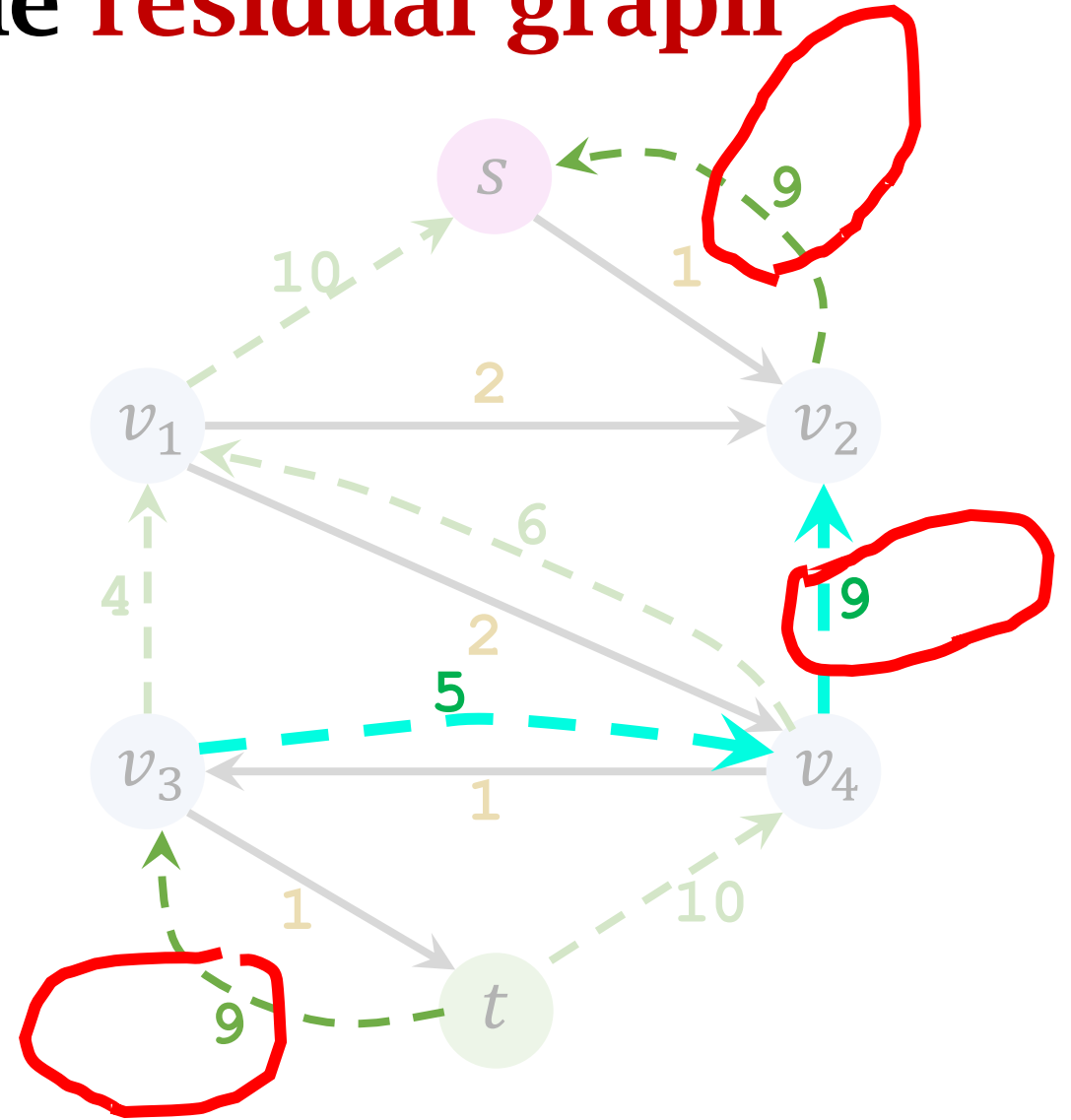
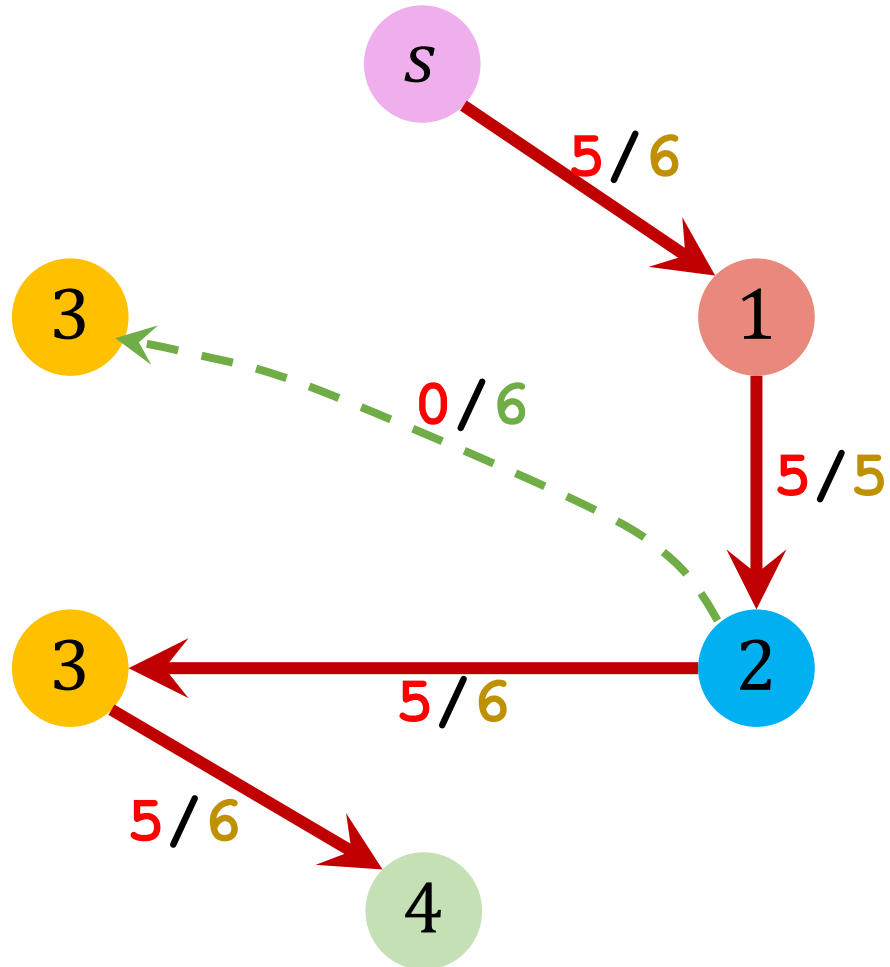
Add **flows** to the residual graph as **backward paths**.

Iteration 2: Update the residual graph



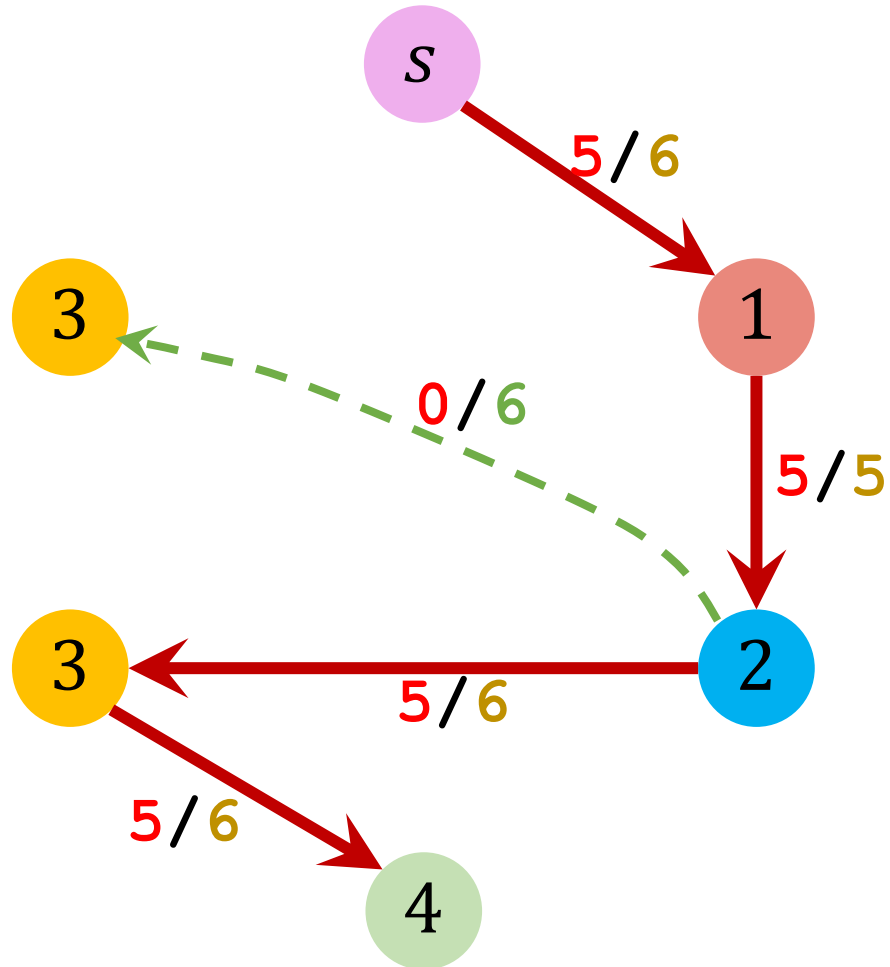
Merge Edges

Iteration 2: Update the residual graph

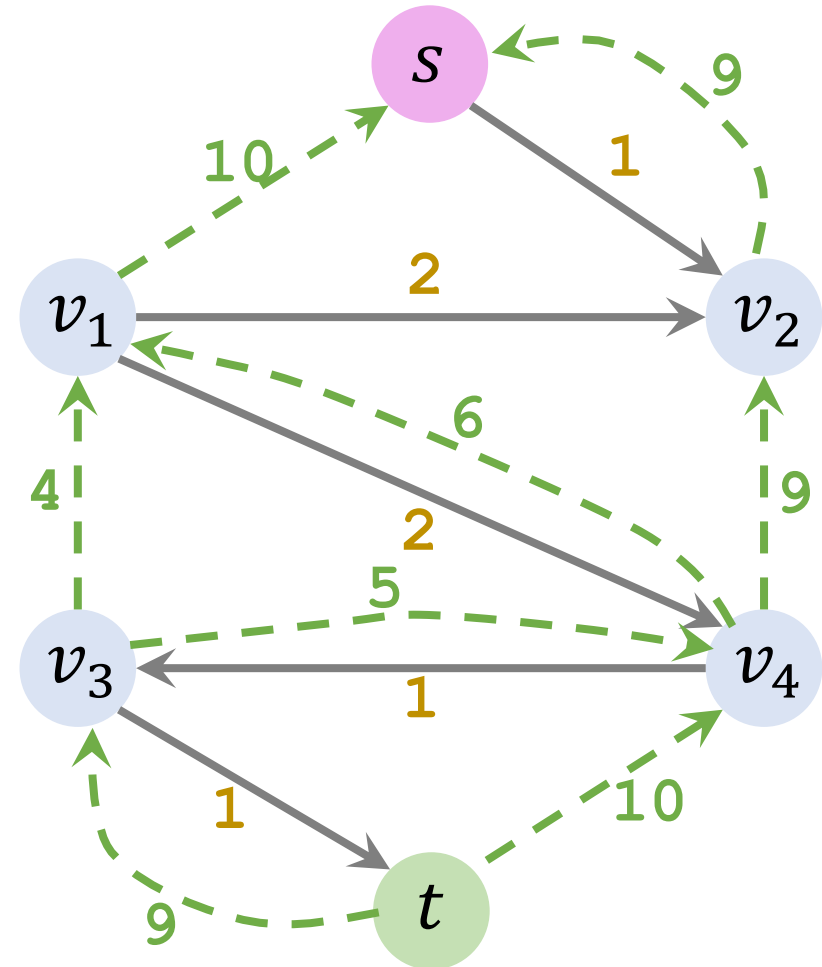


Merge Edges

Iteration 2: Finished

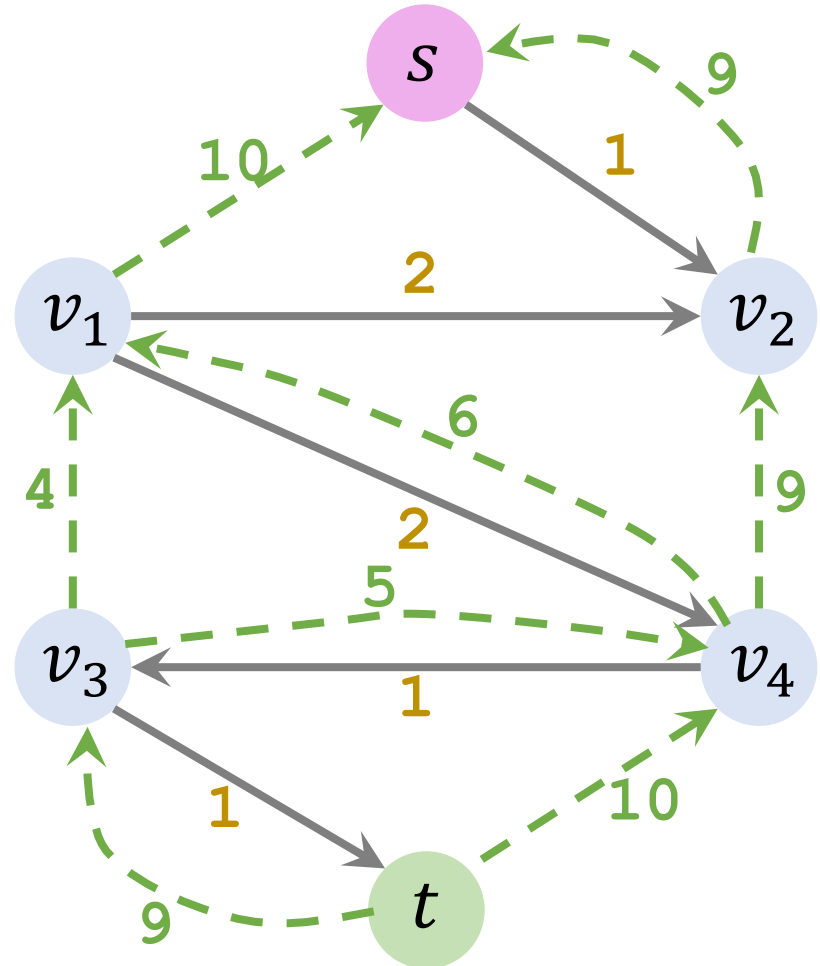


Level Graph



Residual Graph

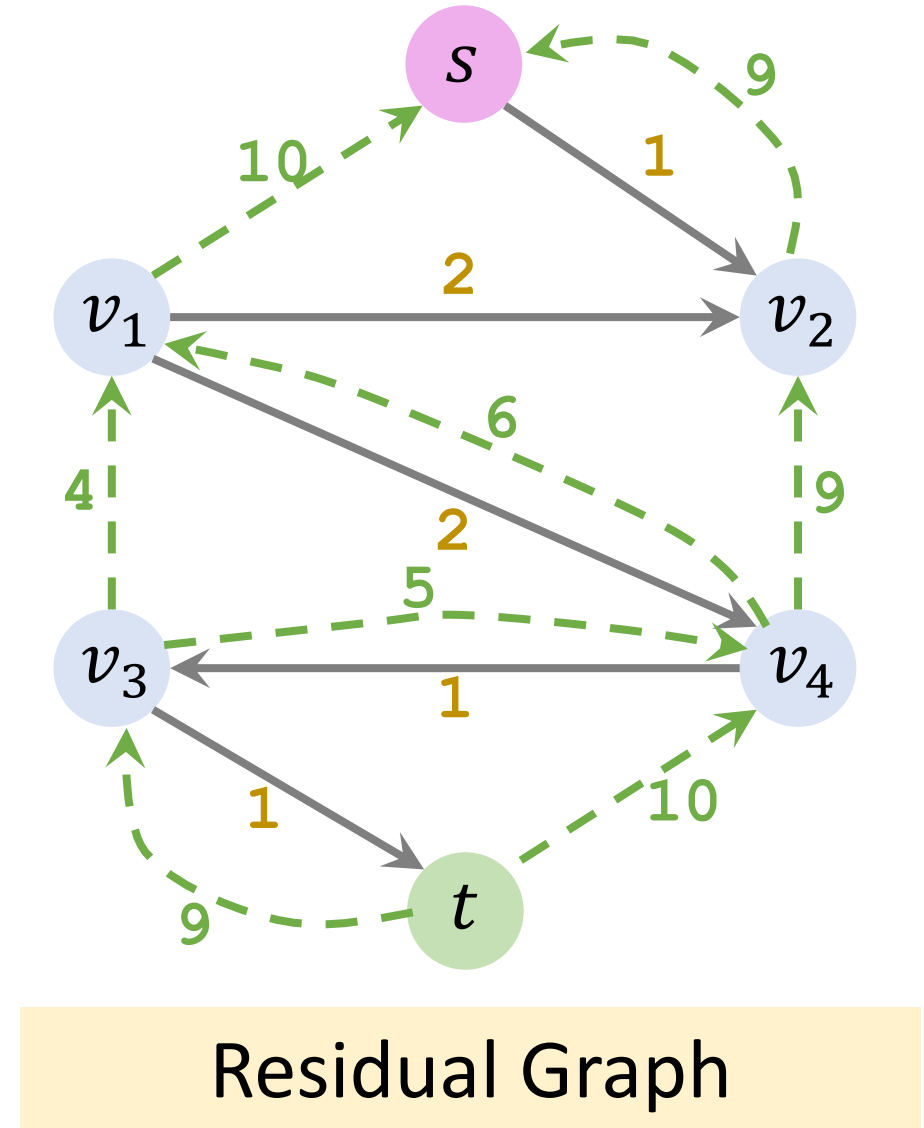
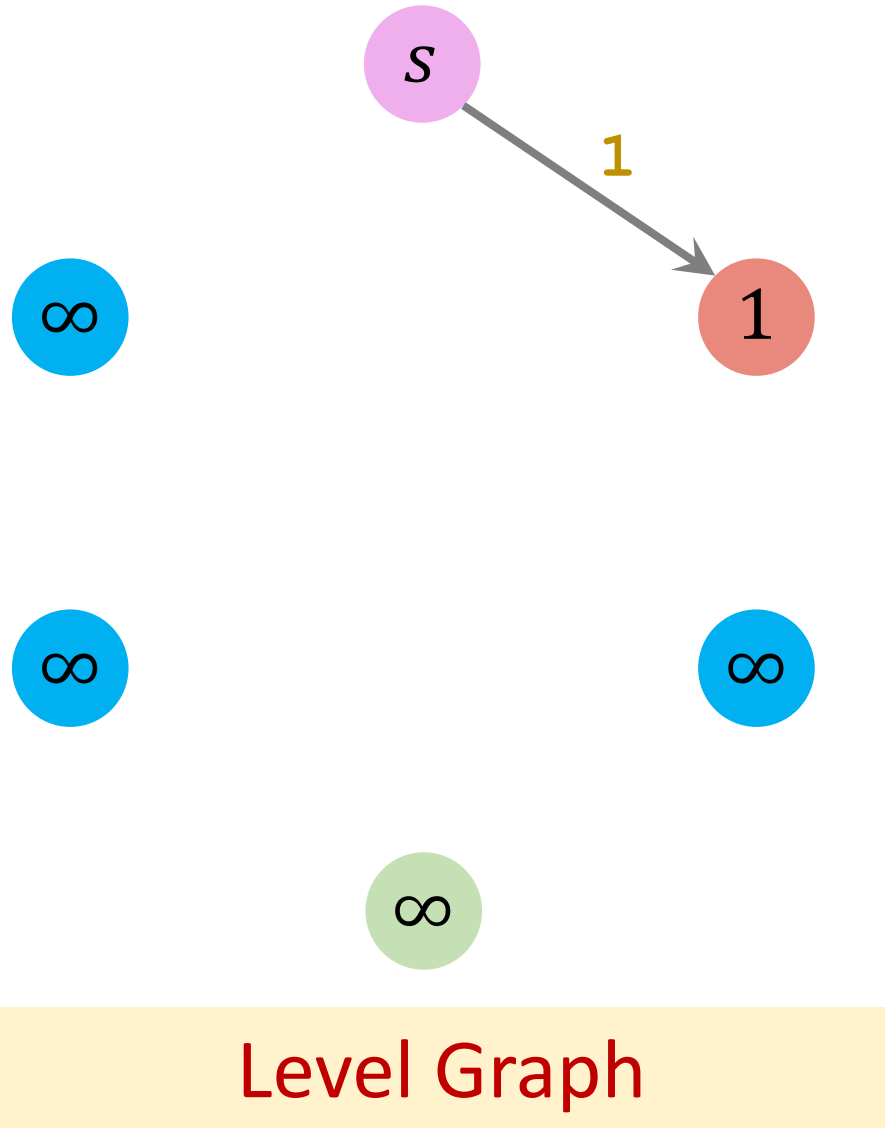
Iteration 3: Construct **level graph**



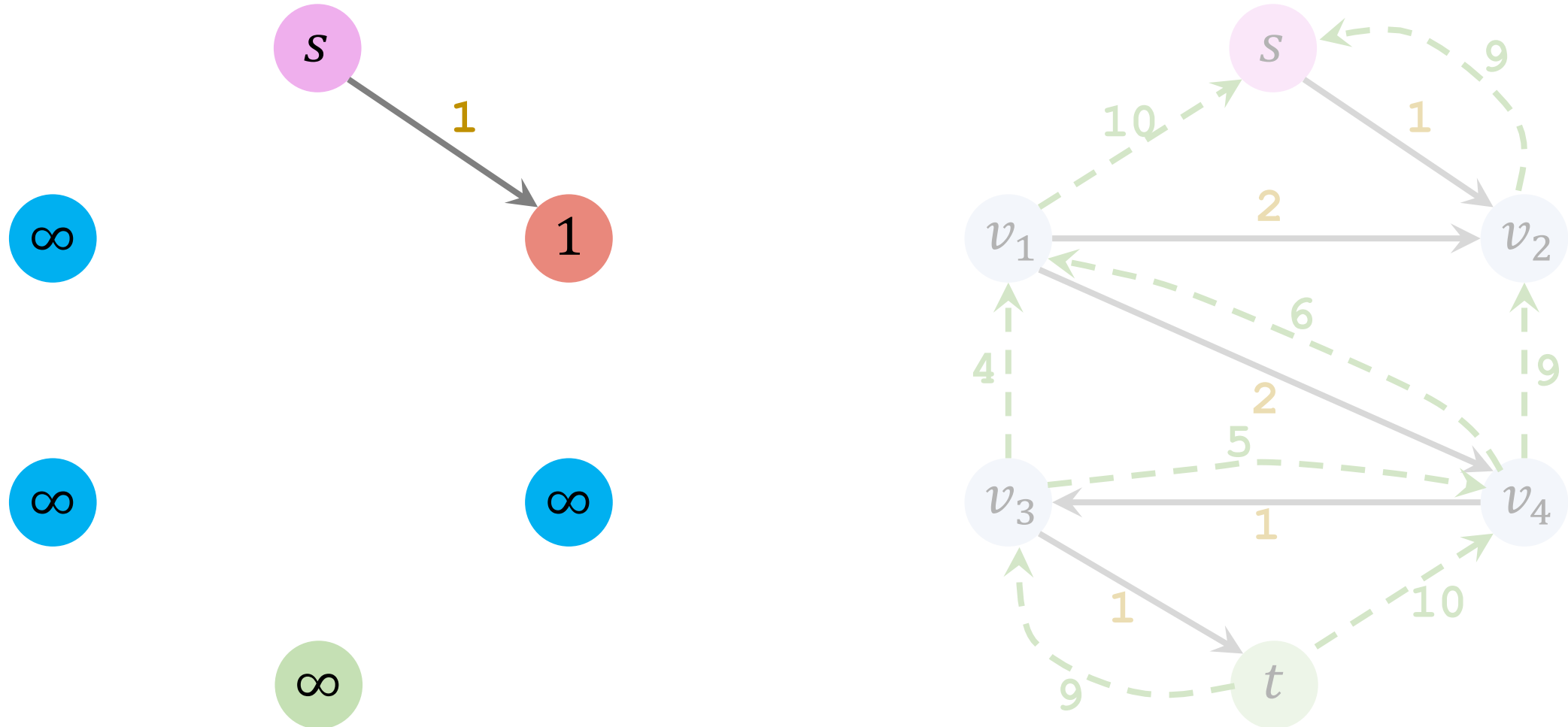
Level Graph

Residual Graph

Iteration 3: Construct **level graph**

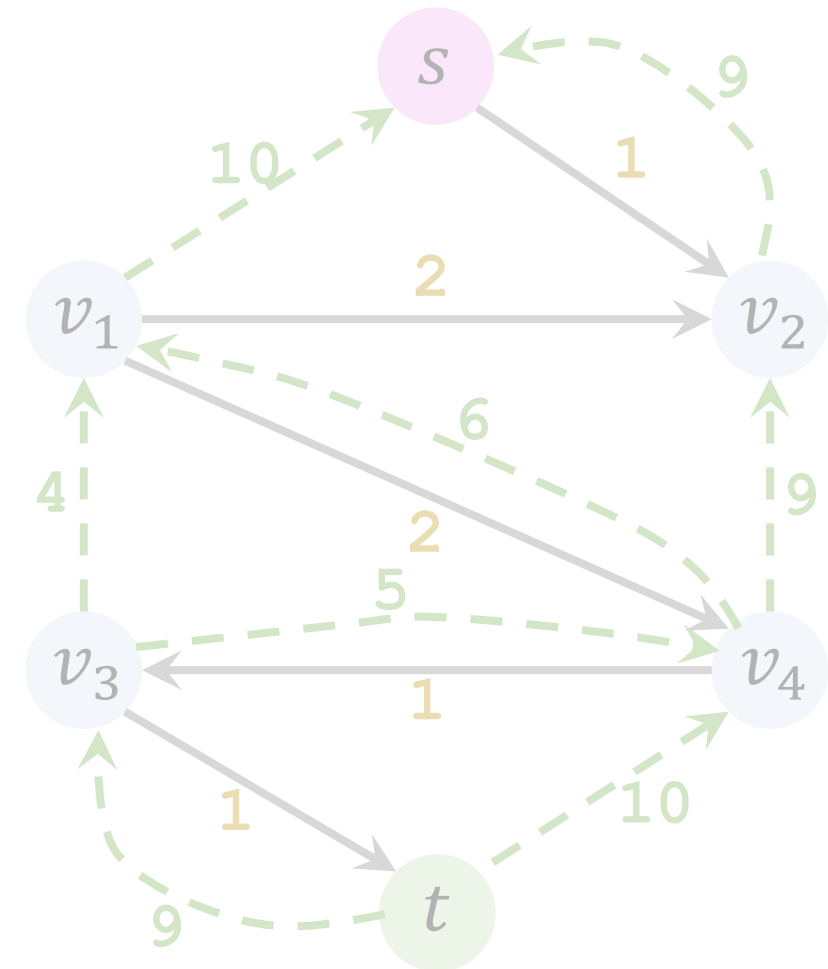


Iteration 3: Find blocking flow in level graph



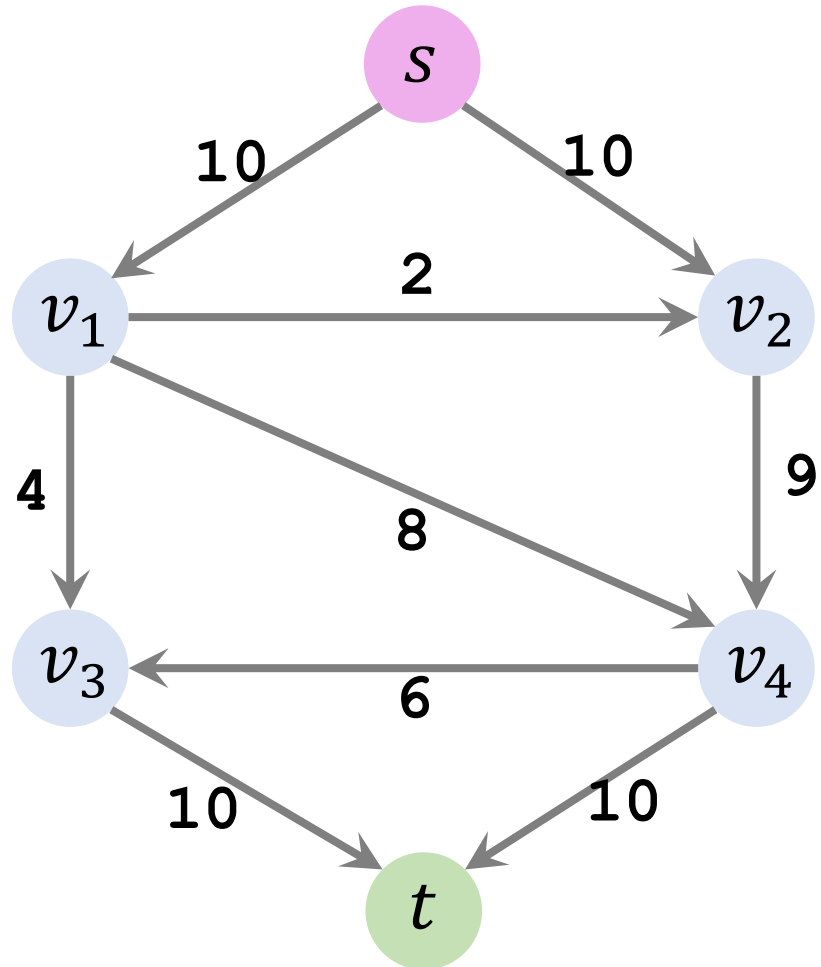
On the level graph, no flow can be found!

End of Procedure

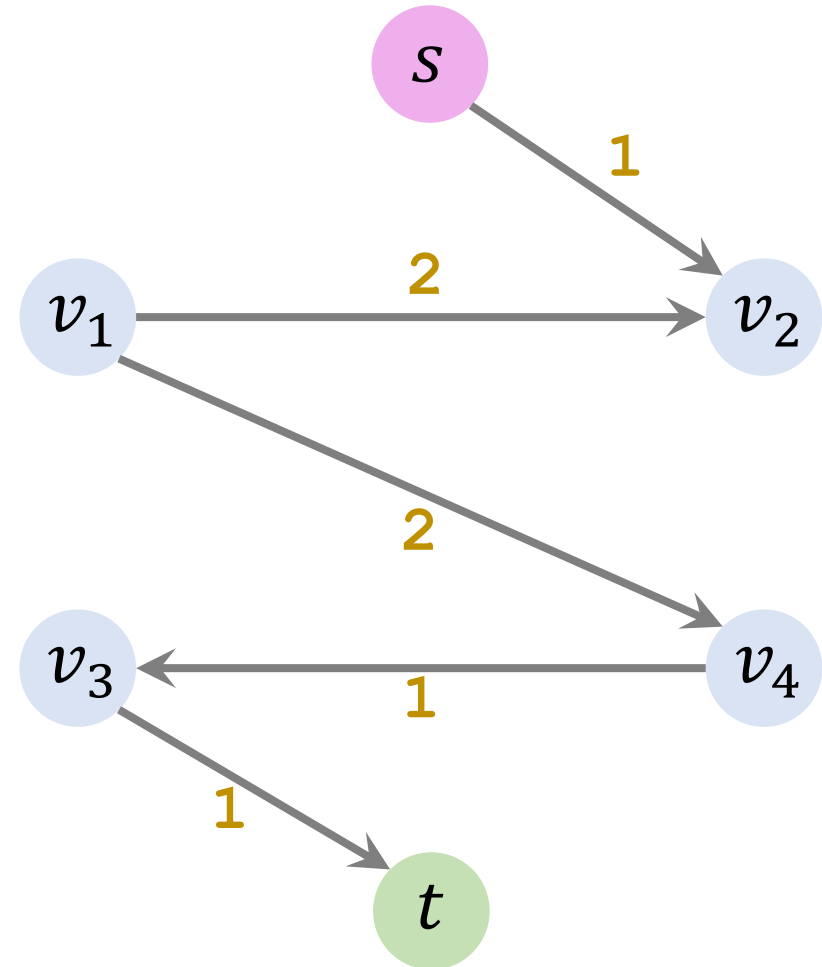


Residual Graph

End of Procedure

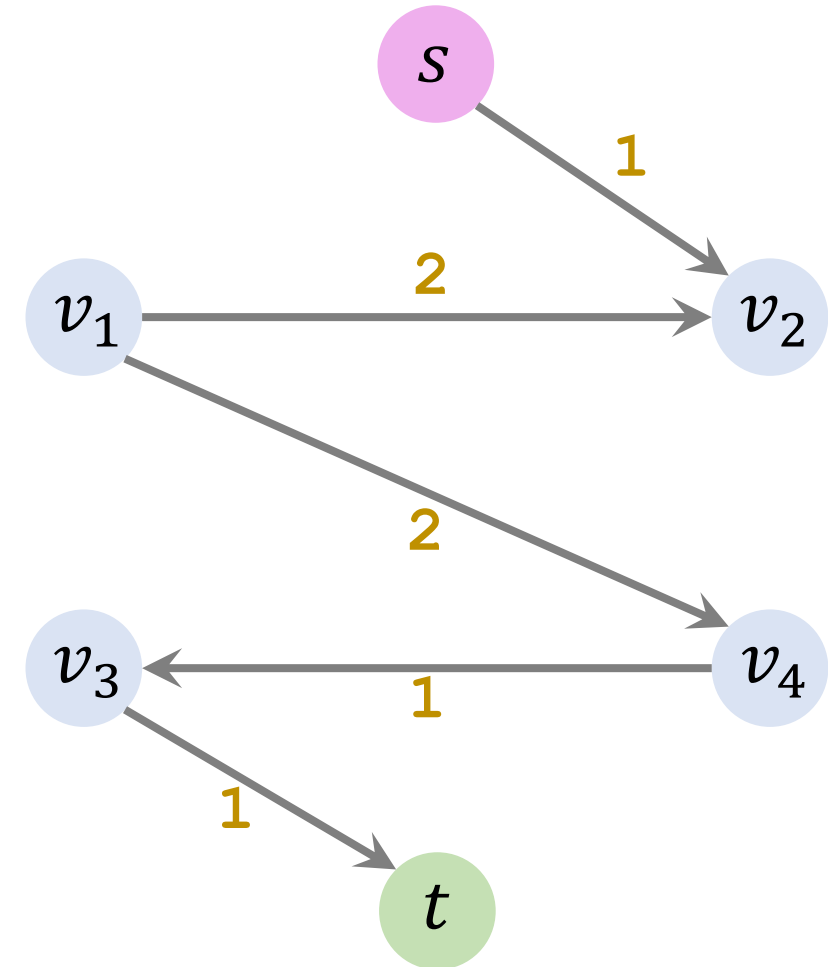
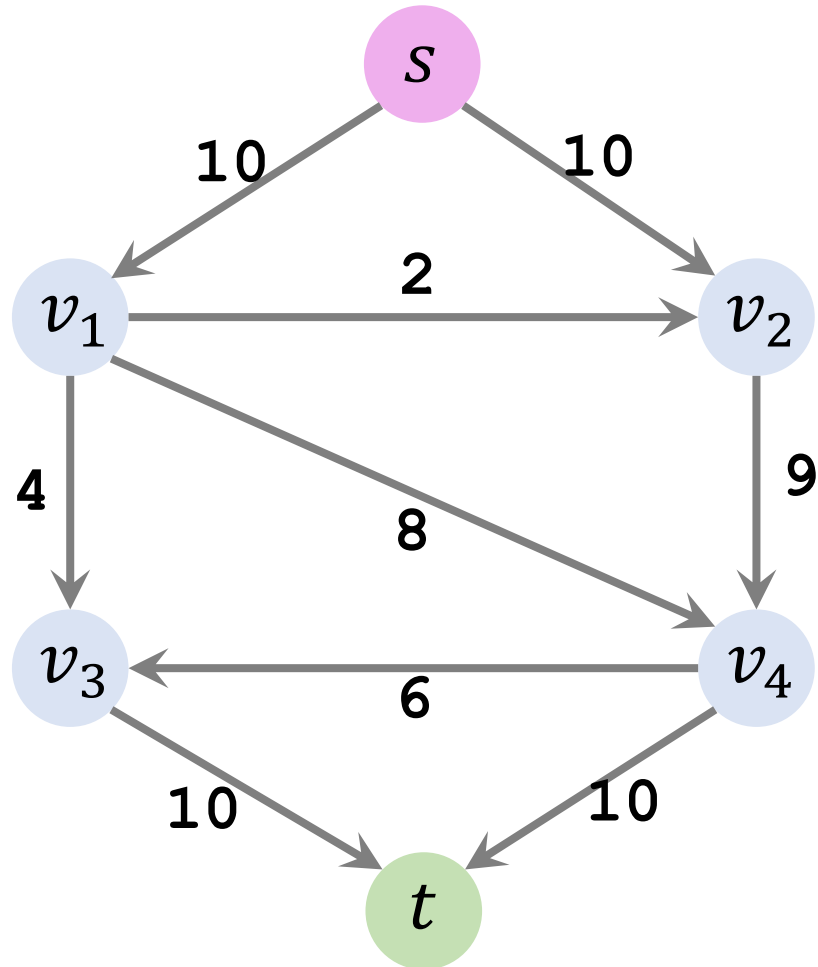


Original Graph



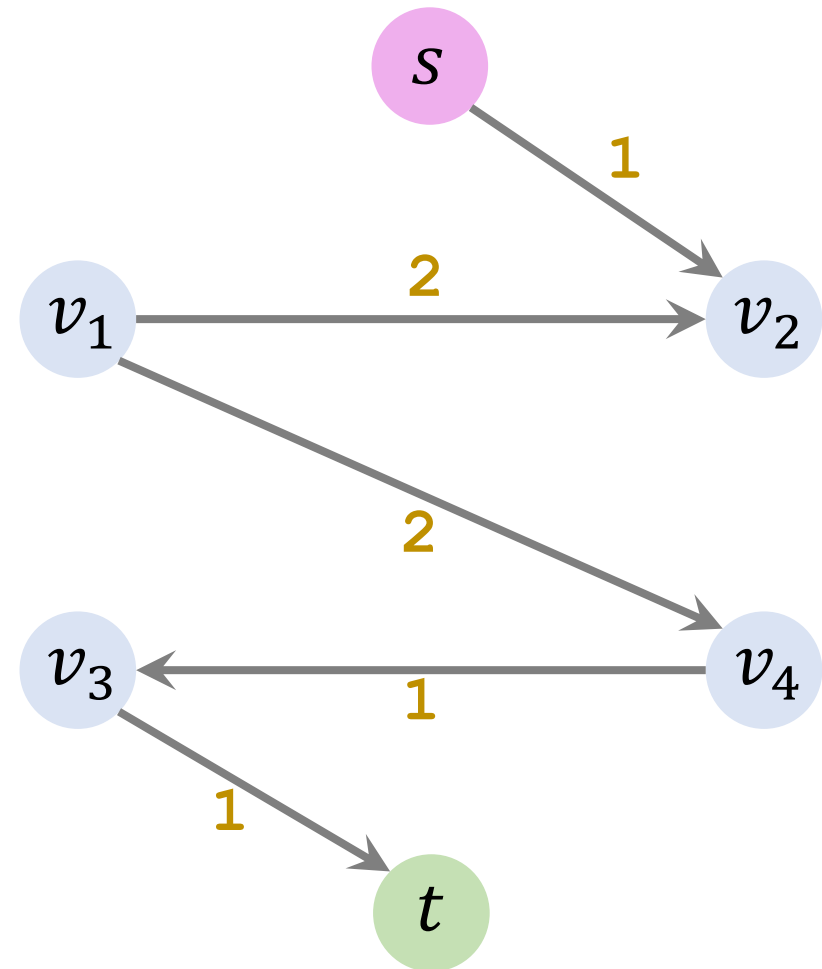
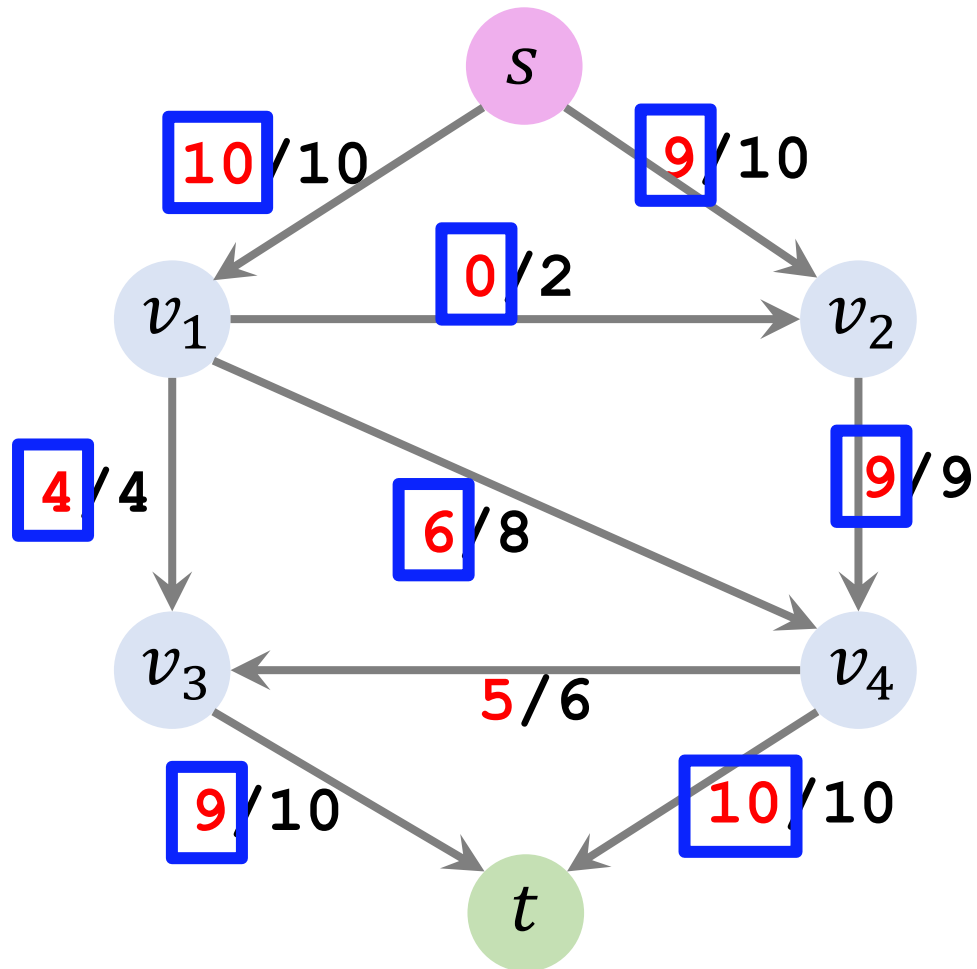
Residual Graph

End of Procedure



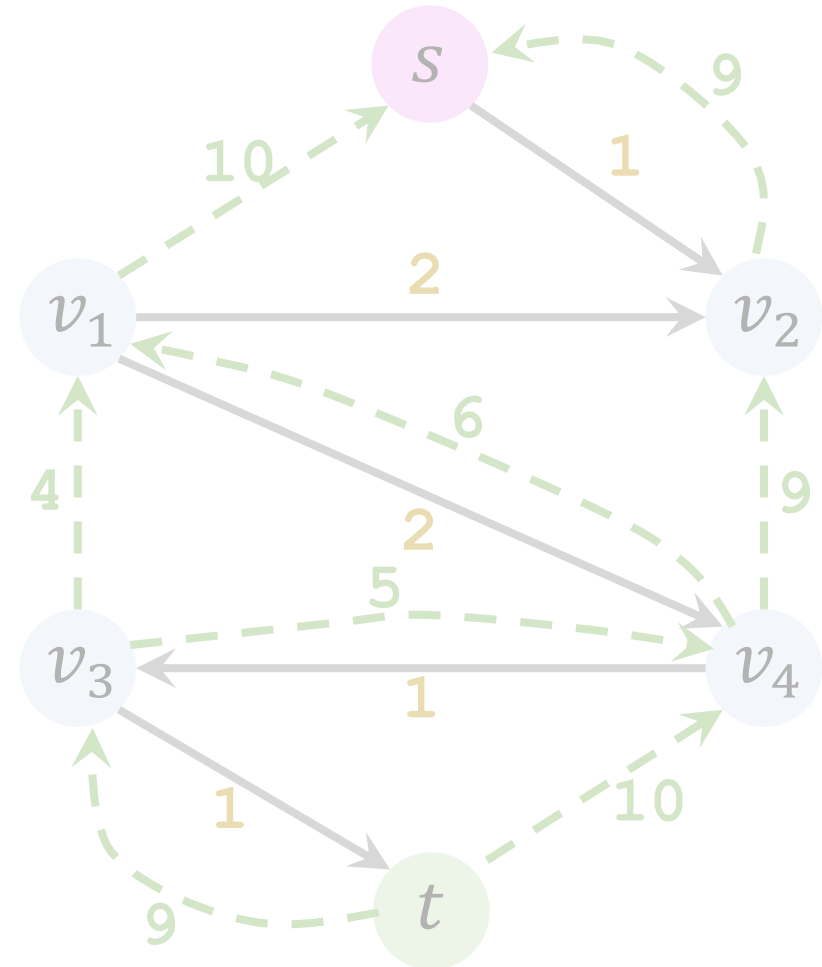
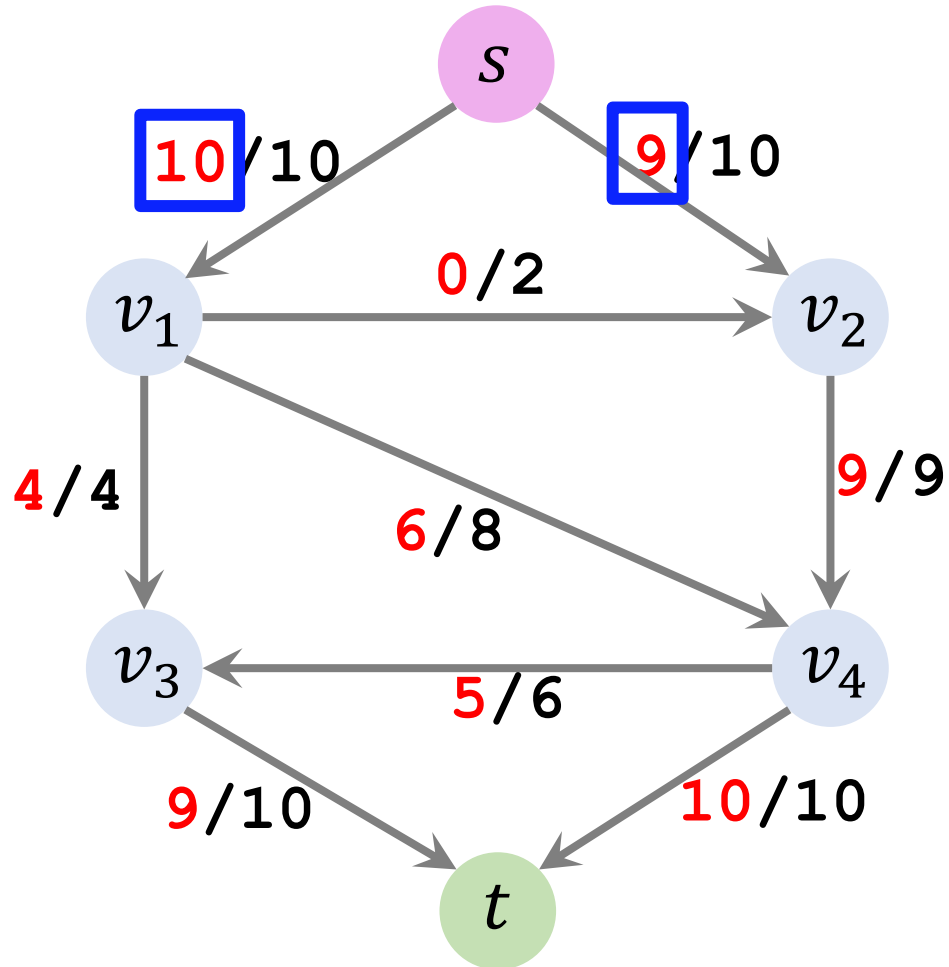
Flow = Capacity - Residual.

End of Procedure



Flow = Capacity - Residual.

End of Procedure




Amount of Max Flow = 19.

Summary

Dinic's Algorithm

1. Initially, the residual graph is a copy of the original graph.

Dinic's Algorithm

1. Initially, the residual graph is a copy of the original graph.
2. Repeat:
 - a. Construct the **level graph** of the residual graph.
 -  b. Find a blocking flow on the level graph.
 - c. Update the residual graph (update the weights, remove saturated edges, and add backward edges.)

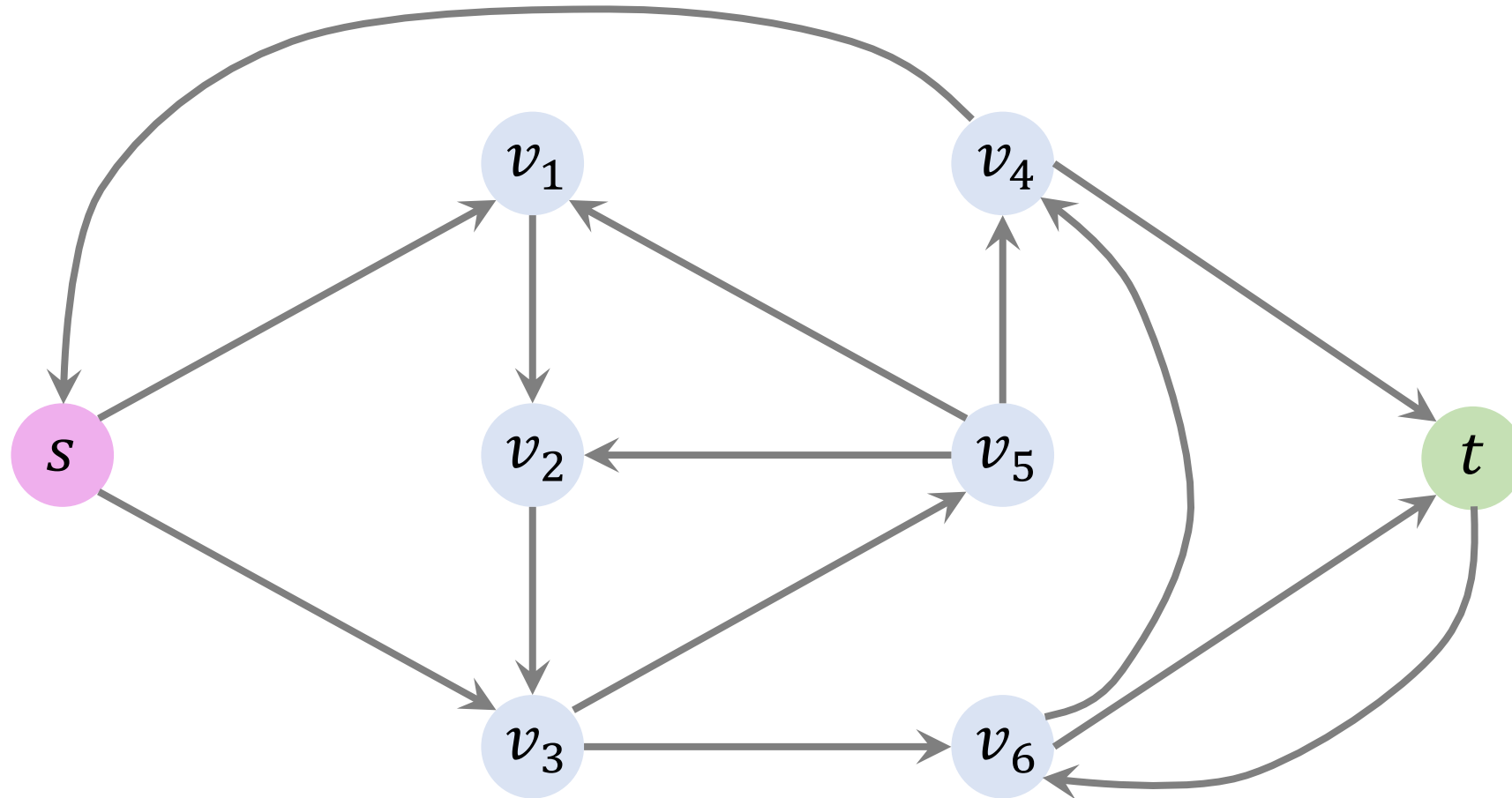
Time Complexity

Time complexity: $O(m \cdot n^2)$. (m is #edges; n is #vertices.)

- Dinic's algorithm has at most $n - 1$ iterations.
- Per-iteration time complexity is $O(mn)$.

Questions

Q1: What is the level graph?



Thank You!

<http://wangshusen.github.io/>