

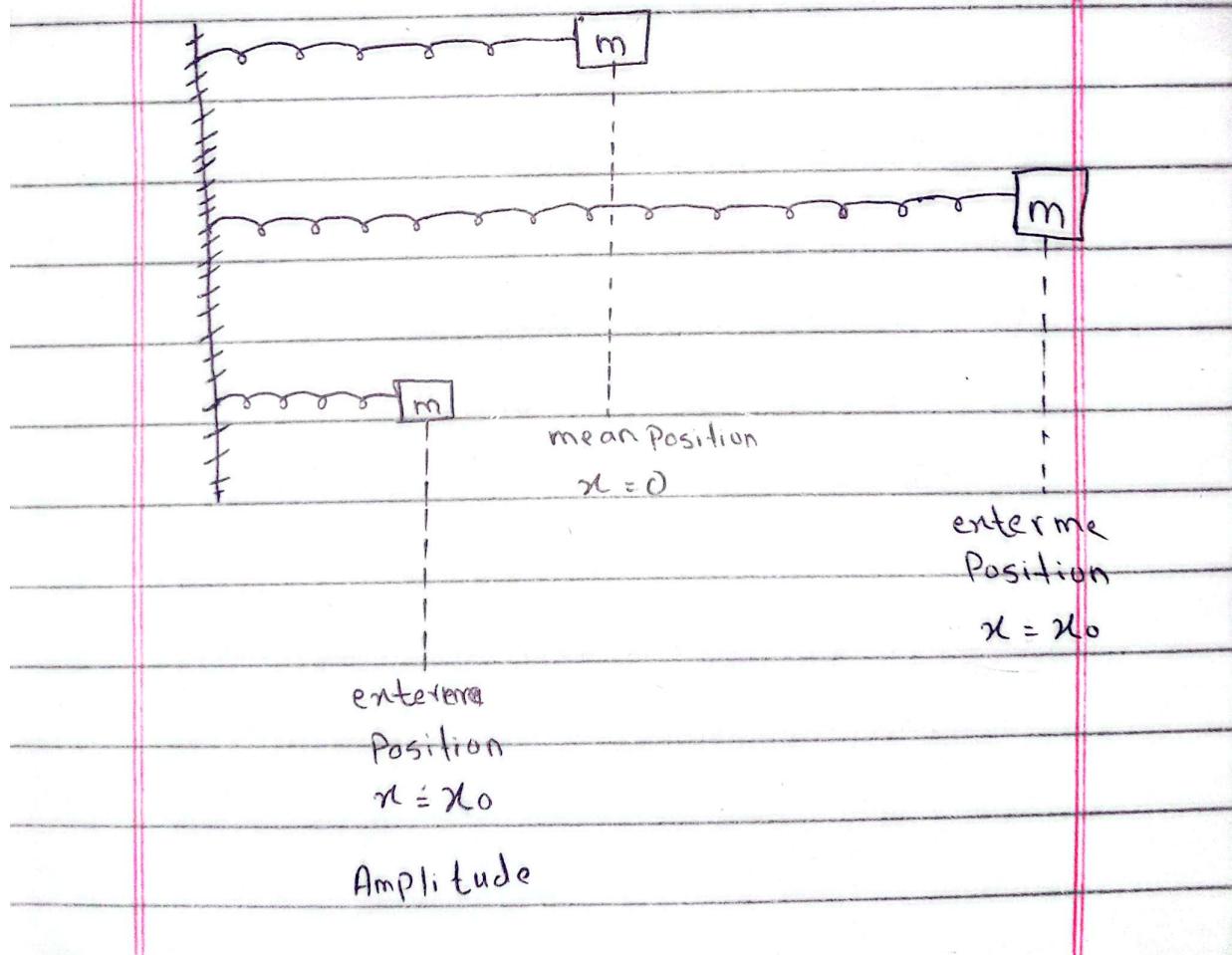
Topic: Chapter # 07
Oscillations

* Simple Harmonic motion:- There are two conditions for a motion to be a simple harmonic motion.

- 1) Acceleration is directly proportional to displacement. $a \propto x$
- 2) It is directed towards the mean position.

$$a \propto -x$$

Mass attached to a spring:



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According to Hooke's Law

$$F = -kx \quad \text{--- (i)}$$

According to Newton's 2nd Law.

$$F = ma \quad \text{--- (ii)}$$

Comparing eqn (i) & (ii)

$$ma = -kx$$

$$a = -\left(\frac{k}{m}\right)x$$

$$a = -\omega^2 x \text{ where}$$

$$\omega^2 = k/m \text{ or } \omega = \sqrt{\frac{k}{m}}$$

$$a = -\omega^2 x$$

So we can say that the motion of mass attached to spring is SHM.

Time Period: Time required to complete one vibration.

$$T = \frac{2\pi}{\omega} \Rightarrow T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Frequency: Numbers of vibrations completed in one second. $F = \frac{1}{T}$

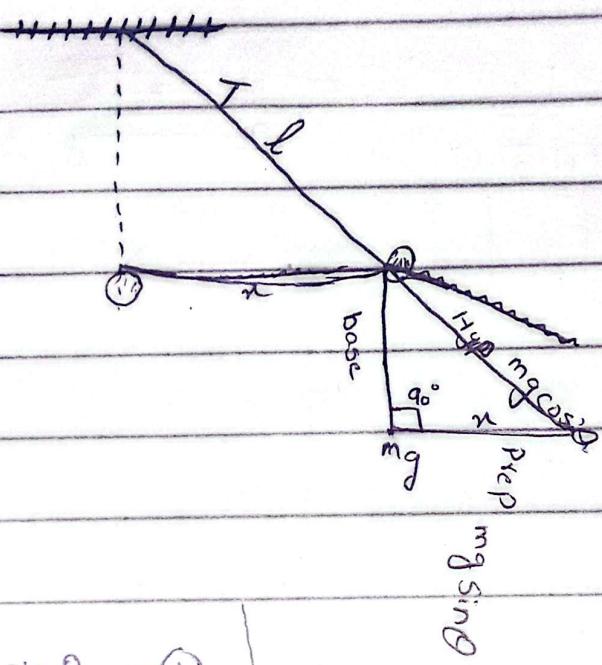
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$$f = \frac{1}{2\pi\sqrt{\frac{m}{k}}}$$

$f = \frac{1}{2\pi\sqrt{\frac{k}{m}}}$

* Simple Pendulum:-



$$F = -mg \sin \theta \quad \text{--- (1)}$$

from the fig

From newton second

$$\sin \theta = \frac{\text{PerP}}{\text{HYP}}$$

law of motion.

$$F = ma \quad \text{--- (II)}$$

$$\sin \theta = \frac{x}{L}$$

comparing equ (I) & (II)

$$rda = -xg \left(\frac{x}{L} \right)$$

Put in equ (I)

$$F = -mg \left(\frac{x}{L} \right) \quad \text{--- (III)}$$

$$a = -\left(\frac{g}{L} \right)x$$

where $\omega^2 = \frac{g}{L}$ or $\omega = \sqrt{\frac{g}{L}}$

$|ax - x|$

Date:

$$\text{Time Period} \dots T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{\frac{g}{L}}}$$

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$\text{Frequency} \Rightarrow F = \frac{1}{T}$$

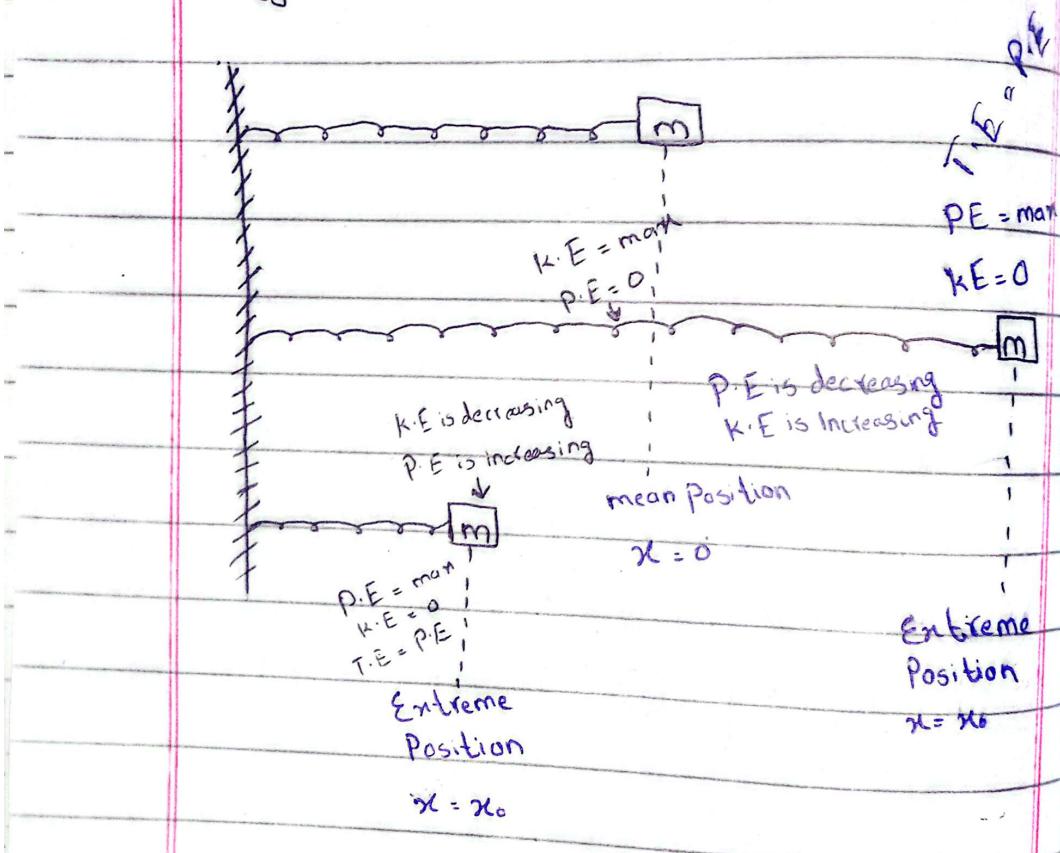
$$F = \frac{1}{2\pi \sqrt{\frac{L}{g}}}$$

$$F = \frac{1}{2} \frac{2\pi}{\sqrt{g}} \sqrt{\frac{1}{L}}$$

$$F = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

$$\frac{1}{2} \frac{2\pi}{\sqrt{g}} \sqrt{\frac{1}{L}}$$

* Energy conservation in SHM.



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According to Hooke's law:-

$$F = kx \quad \text{--- (i)}$$

When $x = x_0$

When $x = x_0$

When $x = 0$

$$F = kx_0$$

$$F = 0$$

$$\text{Average force} = F = \frac{0 + kx_0}{2}$$

$$\boxed{F = \frac{1}{2} kx_0}$$

$$\text{Work} = F \cdot d$$

$$W = \frac{1}{2} kx_0 \cdot x_0 \Rightarrow \boxed{\text{P.E} = \frac{1}{2} kx_0^2} \quad \text{--- (ii)}$$

↓ At extreme

$$* \text{ P.E at any point } = \boxed{\frac{1}{2} kx^2} \quad \text{--- (iii)}$$

$$\text{The K.E is given as : } \text{K.E} = \frac{1}{2} mv^2$$

$$\text{K.E} = \frac{1}{2} m \left[\sqrt{\frac{k}{m}} (x_0^2 - x^2) \right]^2 \quad \therefore v = \omega \sqrt{x_0^2 - x^2}$$

$$\text{K.E} = \frac{1}{2} m \left[\frac{k}{m} (x_0^2 - x^2) \right] \quad v = \sqrt{\frac{k}{m}} \times \sqrt{x_0^2 - x^2}$$
$$= \sqrt{\frac{k}{m}} (x_0^2 - x^2)$$

$$\text{K.E} = \frac{1}{2} k (x_0^2 - x^2)$$

$$\boxed{\text{K.E} = \frac{1}{2} kx_0^2 \left[1 - \frac{x^2}{x_0^2} \right]} \quad \text{--- (iv)}$$

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Now at mean position

$$x = 0$$

$$K.E. = \frac{1}{2} k x_0^2 \left[1 - \frac{x^2}{x_0^2} \right]$$

$$(K.E.)_{\text{max}} = \frac{1}{2} k x_0^2 \quad \text{①}$$

$$T.E. = K.E. + P.E.$$

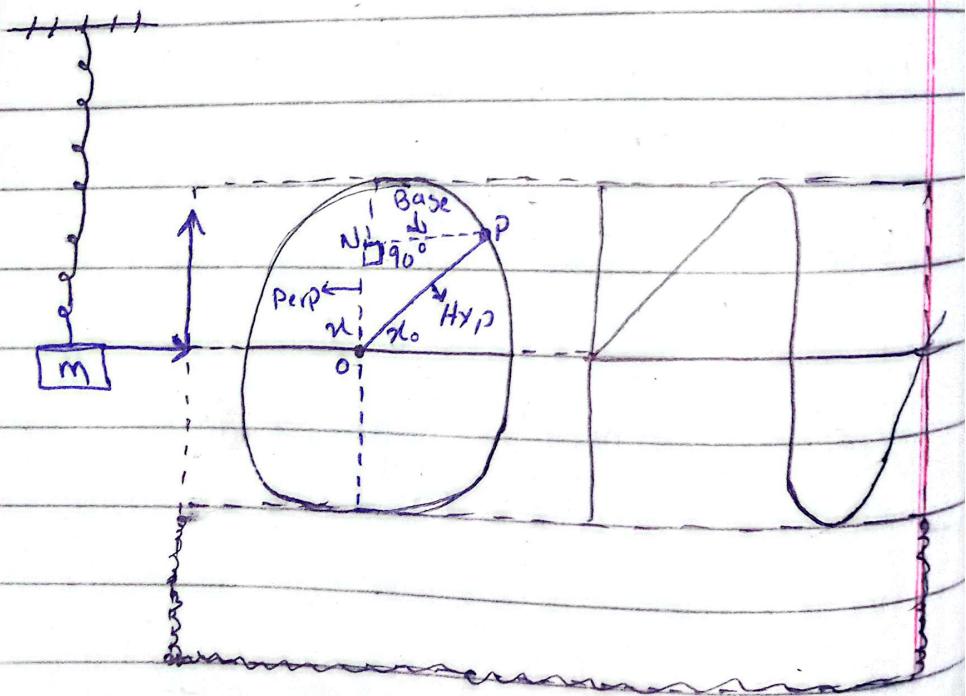
$$= \frac{1}{2} k x_0^2 \left[1 - \frac{x^2}{x_0^2} \right] + \frac{1}{2} k n^2$$

$$= \frac{1}{2} k x_0^2 - \frac{1}{2} k x_0^2 \left(\frac{x^2}{x_0^2} \right) + \frac{1}{2} k n^2$$

$$= \frac{1}{2} k x_0^2 - \frac{1}{2} k x^2 + \frac{1}{2} k n^2$$

$$\Rightarrow F.E. = \frac{1}{2} k x_0^2$$

* Simple Harmonic motion and circular motion.



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* Displacement

$$\sin \theta = \frac{\text{Perp}}{\text{HYP}}$$

HYP

$$\sin \theta = \frac{x}{x_0}$$

 x_0

$$\therefore \theta = \omega t$$

$$n = x_0 \sin \theta$$

$$[n = x_0 \sin \omega t]$$

$$\cos \omega t = \frac{\text{base}}{\text{HYP}}$$

$$(\text{Hyp})^2 = (\text{Perp})^2 + (\text{base})^2$$

$$x_0^2 = x^2 + (NP)^2$$

$$\sqrt{x_0^2 - x^2} = \sqrt{(NP)^2}$$

$$\sqrt{x_0^2 - n^2} = NP$$

$$\cos \omega t = \frac{\sqrt{x_0^2 - n^2}}{x_0}$$

Put in equ ①

$$V = x_0 \omega \cos \omega t$$

$$V = x_0 \omega \times \frac{\sqrt{x_0^2 - n^2}}{x_0}$$

 x_0

$$[V = \omega \sqrt{x_0^2 - n^2}]$$

* Acceleration:-

$$V = x_0 \omega \cos \omega t$$

Diff w.r.t 't'

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$$\frac{dv}{dt} = -x_0 \omega \sin \omega t (\omega)$$

$$dt$$

$$a = -\omega^2 x_0 \sin \omega t$$

$$[a = -\omega^2 x]$$

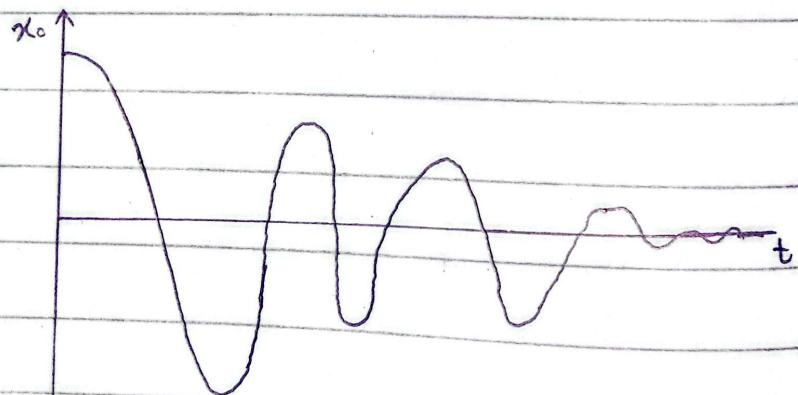
* Free & forced oscillations.

A body is said to be having free oscillations if it oscillate without the interfarance of an external force. e.g:- Motion of Pendulum.

A body is said to be having forced oscillation if it oscillate under the action of an external force. e.g:- Road vibrates due to train movement.

* Damped oscillation.

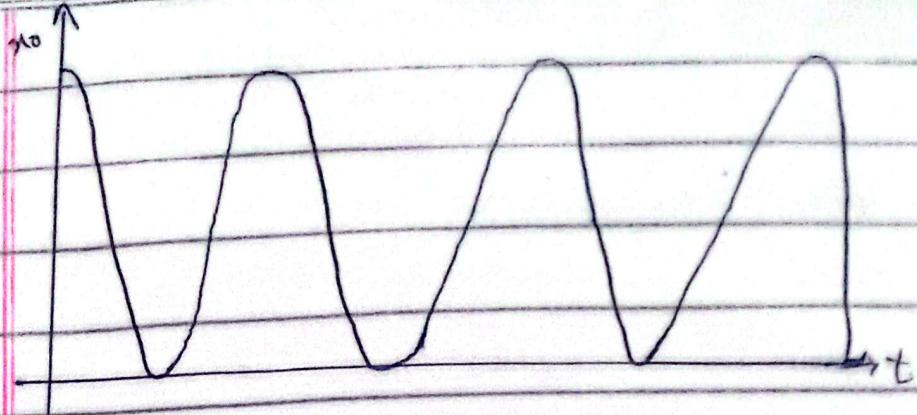
The Oscillation in which Amplitude of vibration decreases with time is called damped oscillation.



- Damped oscillation:-

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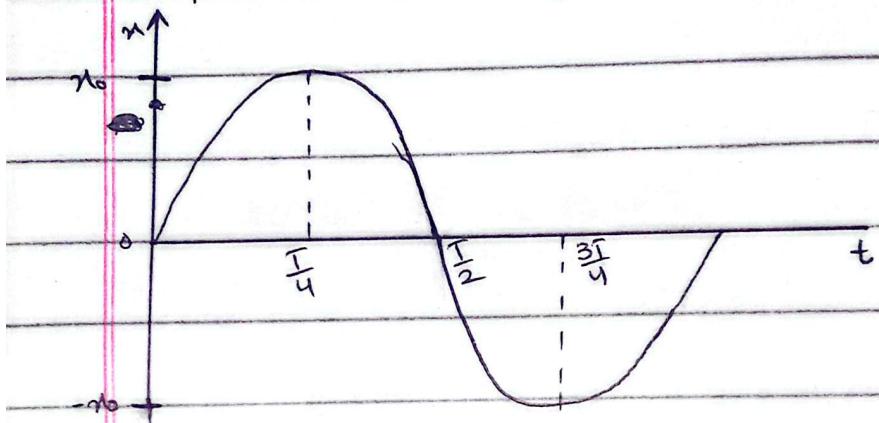
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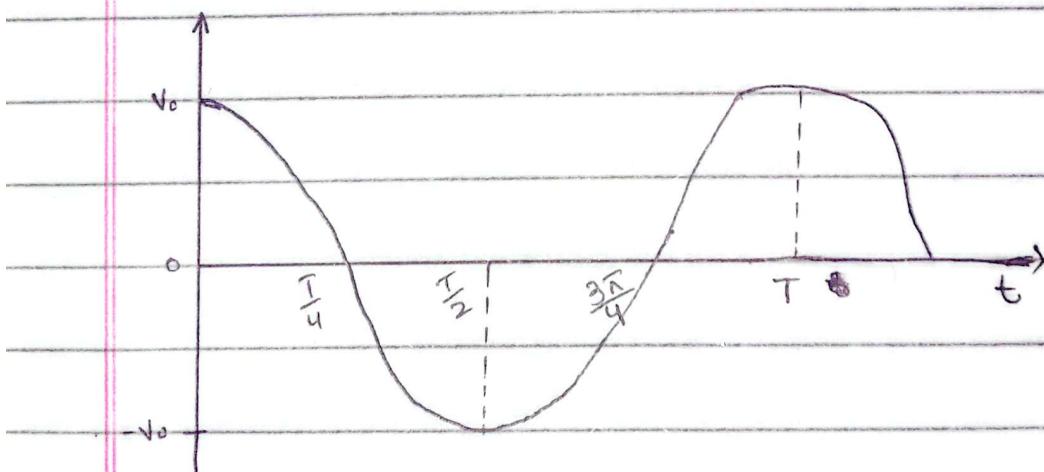
undamped oscillation

:-Graphs :-

i) Displacement - Time graph.



ii) Velocity - Time graph



Date:

- * Resonance It occur when the frequency of applied force is equal to the one of the natural frequencies of a body.

* Application of damped oscillation.

- 1) Simple Pendulum
- 2) swing
- 3) Shock absorbed
- 4) ~~RLC~~ Circuit

* Phase:- The angle $\theta = \omega t$ which specifies the displacement as well as direction of motion having SHM.

* Angular frequency:- It is defined as the angular displacement per unit of time.

$$\omega = \frac{\theta}{t}$$