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1) We can write down the decision version of this problem as follows:

Given a list of cities C, path length k and a list of cities to visit (Sc) before going to a specific city in C ; does there exist a path which visits all the cities in Sc with length < k?

2) We can think of this problem as follows:

If Sc is empty then our problem becomes a one source shortest path problem which can be solved with several algorithms such as Dijkstra's algorithma and Bellman Ford algorithm which takes polynomial time. However when Sc is not empty then it means that we have to visit specific cities before we get to our target destination. We can seperate our general problem to subproblems: For example if Sankara = {Kirikkale} then we can think of this problem as a graph where we need to visit all the vertices of the shortest path from İstanbul to Ankara including Kirikkale.

To prove that this problem is NP Complete firstly we need to prove that it's in NP.

In order to verify the correctness of this problem we check if all the vertices are visited once which can be done in polynomial time and we check if the total cost is less then the length k which can also be done in polynomial time. Therefore we can say that this problem is in NP.

After this we have to prove that this problem is NP Hard. For this we need to make a reduction from another NP Hard problem that we already know. I will make use of Hamiltonian path for our problem since it is the most similar NP Hard problem to our own problem. In Hamiltonian path we also have to find a path that visits all the vertices.

The decision version of the Hamiltonian path problem is that given an undirected graph G=(V,E) determine if there is a path in G which visits all vertices exactly once.

In order to reduce the Hamiltonian path to our problem which I will be calling P from now on, we need to take an instance of Hamiltonian path problem. We take a graph G=(V,E) and we set all of G's edge's weights to 1. Then we take a value k which is equal to the number of vertices (V) in the graph. We will make use of this graph with weighted edges for our own problem P by considering these vertices as cities that we have to visit. We ask the question if there exists a path on G with the cost less than k. If the answer to this question is yes for P then it is also yes for the Hamiltonian path problem. If the answer is no, then it is also no for the Hamiltonian problem.

The reduction of an undirected unweighted graph of Hamiltonian path to our problem P which has weighted edges, is polynomial.

As for the answers of the question of "if there exists a path on G with the cost less than k?", if the answer is yes this means that we have a path which visits all vertices from our starting point to our destination point. For example, in our case it means that we have a path from İstanbul to Ankara which visits all the vertices in Sankara={Bolu,Düzce,Zonguldak}. However if the answer is no to the question, it means that the total weight of the edges are more than k which is the number of vertices. Which means that we are not visiting each vertex exactly once. Since this problem is as hard as Hamiltonian path problem which is NP-Hard we can say that our problem P is also NP-Hard.

Since we've shown that our problem is in NP and it is NP-Hard we can conclude that it is NP-Complete.