Getting up to speed with Double Integrals

Double Integral Review and Practice

Dr. Uma Ravat

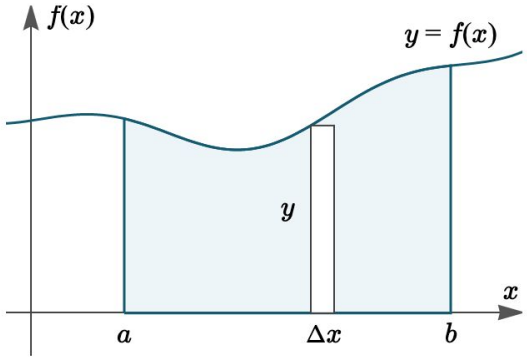
Here

- Review single and double integrals

- Practice strategies for
  a. setting up and
  b. evaluating double integrals

Extra: Review short videos posted in Extra Resources

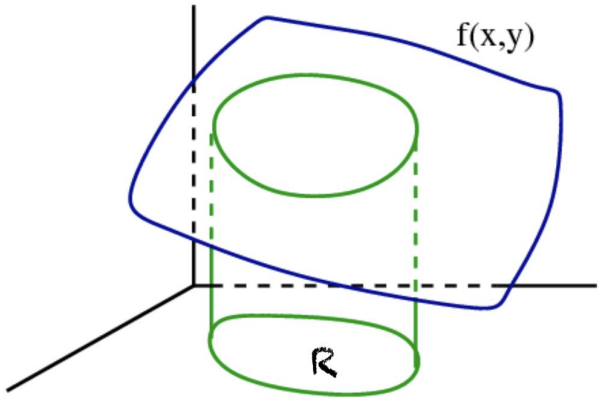
Single integrals application:



area under the curve $y = f(x)$ from $x = a$ to $x = b$.

$$\text{Area} = \int_a^b f(x)\,dx$$

Double integrals:



as the volume between the surface $z = f(x, y)$ and the xy-plane, i.e, the "cylinder" above the region R.

$$\int\int_R f(x,y)dA \qquad \text{OR} \qquad \int\int_R f(x,y)dxdy$$

Double integrals

Basic idea as Reimann sums

Basic idea as a sweep of region of integration

How to evaluate a double integral?

$$\int\int_R f(x,y)dA \quad = ?$$

- **2 steps to success!**

1. setting up the double integral
2. evaluating double integrals

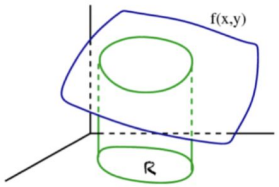
Step 1: Setting up double integrals:

- Horizontal sweep of the region R

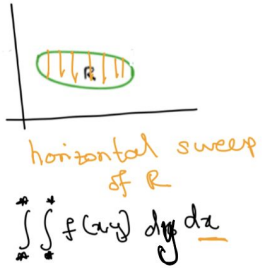
$$\int_*^* \int_*^* f(x,y)dydx$$

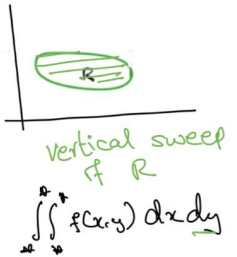
- Vertical sweep of the region R

$$\int_*^* \int_*^* f(x,y)dxdy$$



$\iint f(x,y)\,dA$





Horizontal or vertical sweep of a region of integration in x-y plane

Setting up Double Integrals

- Sketch the region R
- Sweep the region horizontally or vertically.
- Get bounds for x and y that correspond to this sweep.
- Use the bounds to set up the double integral as an iterated integral

Set up the double integral

Set up a double integral of $f(x, y)$ where the region R is given by

1. $0 < x < 1, x < y < x + 1$

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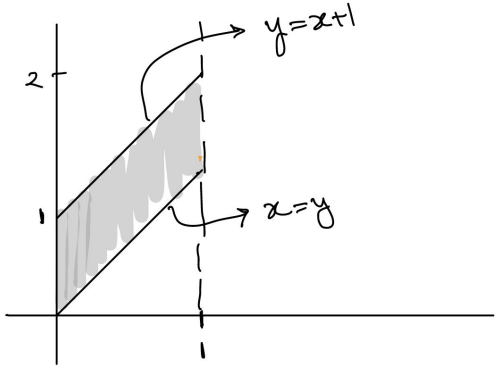
1. Sketch the region



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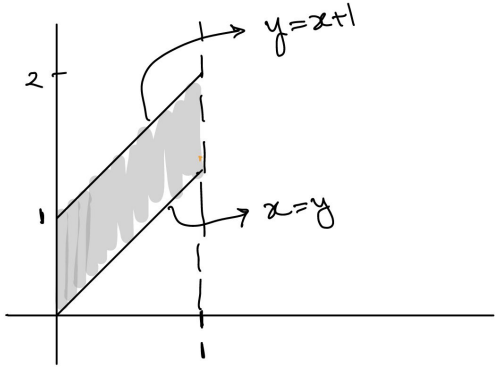

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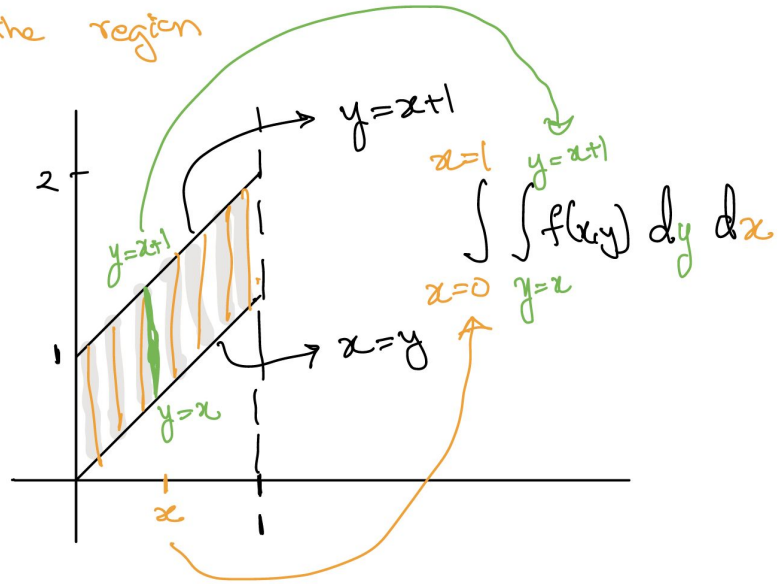
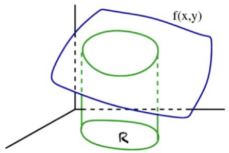
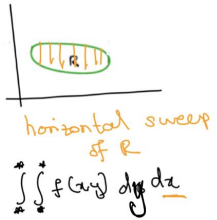


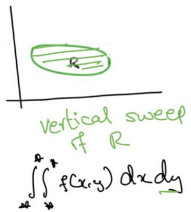
Tips

- you might need to split the region R into two or more pieces, each with it's corresponding double integral.
- The outside integral must have constant limits
  - for a horizontal sweep, x will have fixed(constant) bounds, and bounds for y might depend on x.
  - for a vertical sweep, y will have fixed(constant) bounds, and bounds for x might depend on y.



$\iint f(x,y)\,dA$





Step 2: Evaluate Double Integrals

- Sketch the region R
- Set up the double integral
- **Evaluate the double integral as an integrated integral**
  a. The outer variable of integration will be treated as a constant for the inner integral

Iterated integrals

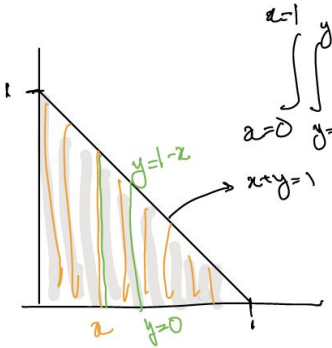
Evaluate the double integral

Evaluate the double integral

$$\int\int_R f(x,y)dxdy$$

where

1. $f(x,y) = e^{-x-y}$, and region R is the first quadrant in which $x + y \leq 1$



$$\int_{x=0}^{1}\int_{y=0}^{1-x} e^{-x-y}\,dy\,dx$$

$$= \int_{x=0}^{1}\int_{y=0}^{1-x} e^{-x}e^{-y}\,dy\,dx$$

$$= \int_{x=0}^{1} e^{-x}\int_{y=0}^{1-x} e^{-y}\,dy\,dx$$

$$= \int_{x=0}^{1} e^{-x}\left(-e^{-y}\Big|_{y=0}^{1-x}\right)dx = \int_{x=0}^{1} e^{-x}\left(1-e^{-(1-x)}\right)dx$$

$$= \int_{0}^{1}\left(e^{-x}-e^{-1}\right)dx$$

$$= 1 - 2e^{-1}$$

Practice Setting up Double Integrals

- sketch the region R
- Sweep the region horizontally or vertically.
- Get bounds for x and y that correspond to this sweep.
- Use the bounds to set up the double integral as an iterated integral - either horizontal or vertical sweep

Set up the double integral

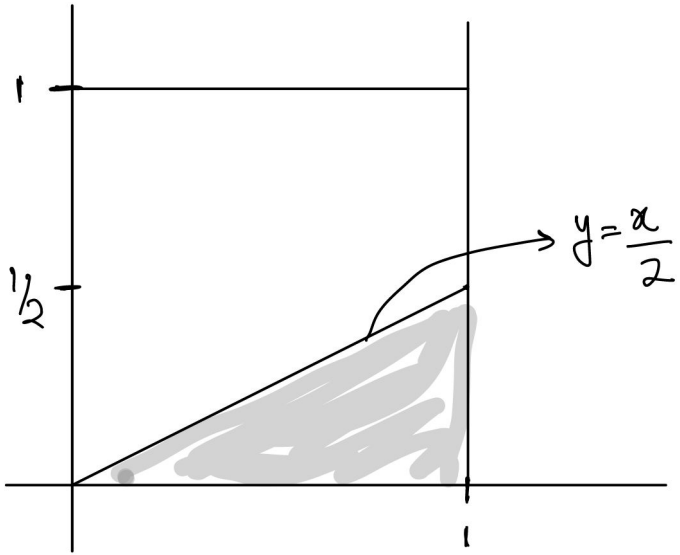
Set up a double integral of $f(x, y)$ where the region R is given by

1. $0 < x < 1, x < y < x + 1$
2. $0 \leq x \leq 1, 0 \leq y \leq 1, y \leq \frac{x}{2}$
3. part of the unit square $0 \leq x \leq 1, 0 \leq y \leq 1$ on which $x + y > 0.5$
4. part of the unit square $0 \leq x \leq 1, 0 \leq y \leq 1$ on which both x and y are greater than 0.5
5. part of the unit square $0 \leq x \leq 1, 0 \leq y \leq 1$ on which at least one of x and y are greater than 0.5
6. $0 < x < 50 - y < 50$ on which both x and y are greater than 20.
7. the points (x, y) in the first quadrant with $|x - y| \leq 1$

Set up the double integral

Set up a double integral of $f(x, y)$ where the region R is given by

$$2.\ 0 \leq x \leq 1, 0 \leq y \leq 1, y \leq \frac{x}{2}$$



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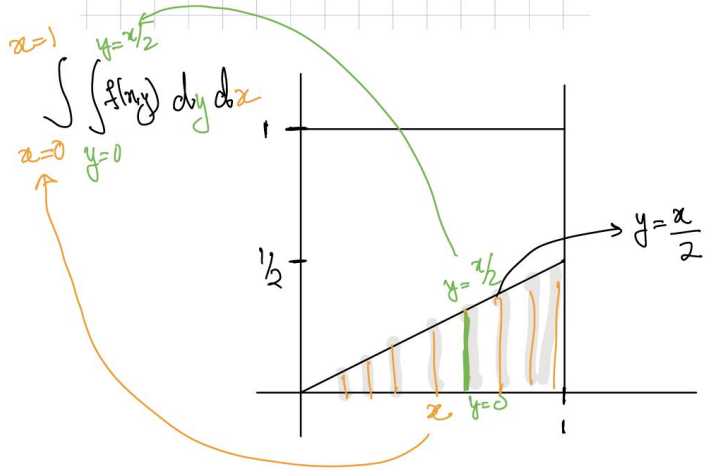
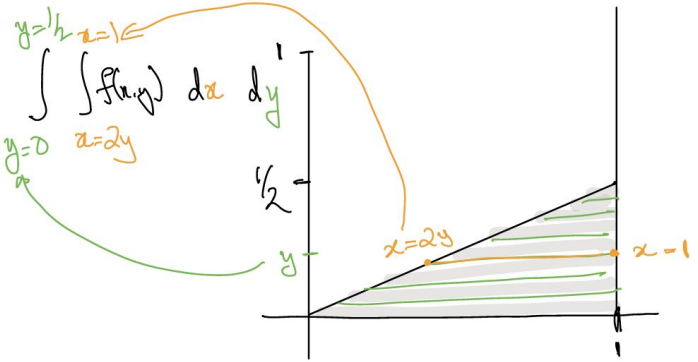
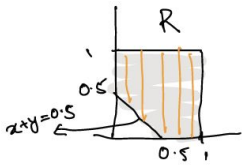




Set up the double integral

Set up a double integral of $f(x, y)$ where the region R is given by

3. part of the unit square $0 \leq x \leq 1, 0 \leq y \leq 1$ on which $x + y > 0.5$



Set up the double integral

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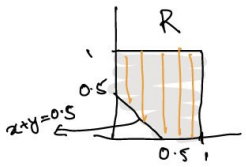
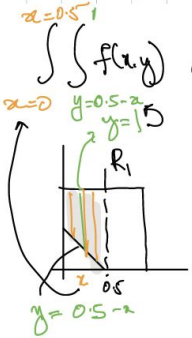
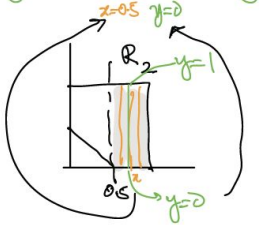
$$\int_{x=0}^{0.5}\int_{y=0.5-x}^{1} f(x,y)\,dy\,dx + \int_{x=0.5}^{1}\int_{y=0}^{1} f(x,y)\,dy\,dx$$







Practice Evaluating Double Integrals

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where

1. $f(x,y) = e^{-x-y}$, and region R is the first quadrant in which $x + y \leq 1$
2. $f(x,y) = e^{-x-2y}$, and region R is the first quadrant in which $x \leq y$
3. $f(x,y) = x - y + 1$, and region R is inside the unit square in which $x + y \geq 0.5$

Evaluate the double integral

Evaluate the double integral

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where

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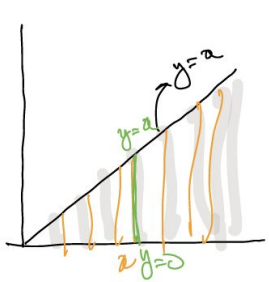

$$\int_{x=0}^{\infty}\int_{y=0}^{y=x} e^{-x-2y}\,dy\,dx$$

$$= \int_{x=0}^{\infty} e^{-x}\left[\frac{e^{-2y}}{-2}\right]_{y=0}^{y=x} dx$$

$$= \int_{x=0}^{\infty} e^{-x}\left(-\frac{1}{2}e^{-2x} + \frac{1}{2}\right) dx$$

$$= \int_{x=0}^{\infty}\left(-\frac{1}{2}e^{-3x} + \frac{e^{-x}}{2}\right) dx$$

$$= \left[\frac{-1}{2}\frac{e^{-3x}}{-3} + \frac{e^{-x}}{-2}\right]_{x=0}^{x=\infty} = \frac{1}{2} - \frac{1}{6} = \frac{3}{6} = \frac{1}{3}$$

Thank you!

UC SANTA BARBARA