1.1 Series Review

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PSTAT 120A: Introduction to Probability

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Review of Series concepts

Geometric Series

If b is any number

$$\begin{aligned}\sum_{k=0}^{\infty} b^k &= b^0 + b^1 + b^2 + \cdots + b^k + \cdots \\ &= 1 + b + b^2 + \cdots + b^k + \cdots \\ &= \frac{1}{1-b}, \textit{if} |b| < 1. \\ &= \frac{\text{first term}}{1 - \text{base}}\end{aligned}$$

* Say: The series $\sum_{k=0}^{\infty} b^k$ converges to $\frac{1}{1-b}$, if $|b| < 1$

Geometric Series

If b is any number

$$\sum_{k=0}^{\infty} b^k = \frac{1}{1-b} = \frac{\text{first term}}{1-\text{base}}, if |b| < 1$$

$$\begin{aligned}
\sum_{k=m}^{\infty} b^k &= b^m + b^{m+1} + b^{m+2} + \cdots + b^{m+k} + \cdots \\
&= b^m \left(1 + b^1 + b^2 + \cdots + b^k + \cdots\right) \\
&= b^m \frac{1}{1-b}, if |b| < 1. \\
&= \frac{b^m}{1-b}, if |b| < 1. \\
&= \frac{\text{first term}}{1-\text{base}}
\end{aligned}$$

Geometric Series

$$\sum_{k=4}^{\infty} 0.6^k \quad =? = \frac{0.6^4}{1-0.6}, since |0.6| < 1.$$

$$\sum_{k=1}^{\infty} 0.6^{2k+1} \quad = \quad ? = 0.6^3 + 0.6^5 + 0.6^7 + 0.6^9 + \cdots$$

$$= \quad \frac{0.6^3}{1 - 0.6^2}$$

since base = $0.6^2$and $|0.6^2| < 1$and first term = 0.6^3

Exponential Series

$$\begin{aligned}\sum_{k=0}^{\infty} \frac{a^k}{k!} &= \frac{a^0}{0!} + \frac{a^1}{1!} + \frac{a^2}{2!} + \cdots + \frac{a^k}{k!} + \cdots \\ &= 1 + a + \frac{a^2}{2!} + \cdots + \frac{a^k}{k!} + \cdots \\ &= e^a\end{aligned}$$

$$\begin{aligned}\sum_{k=1}^{\infty} \frac{a^k}{k!} &= \sum_{k=0}^{\infty} \frac{a^k}{k!} - \frac{a^0}{0!} \\ &= e^a - 1.\end{aligned}$$