Double Integrals Review and Practice

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Problem: Evaluate the integral

$$\int\int_R f(x,y)dA$$

or

$$\int\int_R f(x,y)dxdy$$

This means, depending on the given region, you'll first set up the double integral and then proceed with evaluating the double integal you set up iteratively.

Setting up the double integral means, depending on the given region R, you'll set up the given integral either as either as

a.
$$\int_*^* \int_*^* f(x,y)dxdy$$
ie inner integral wrt x and outer integral wrt y (also referred to as vertical sweep of the region of integration) or

b.
$$\int_*^* \int_*^* f(x,y)dydx$$
ie inner integral wrt y and outer integral wrt x (also referred to as horizontal sweep of the region of integration)

where * above need to be replaced by correct bounds depending on the region R.

Setting up the double integral is the part that causes the greatest difficulties, so you should practice setting up the double integral and pay attention to the steps below to avoid making silly mistakes.

To evaluate a double integral proceed as follows :

1. Sketch the given region R. A simple sketch will help you avoid mistakes made by figuring out limits in your head and will serve as explanation of your work.
2. Sketch the support of the joint density $f(x,y)$ ie the region where $f(x,y) > 0$. (The region where $f(x,y) = 0$ does not matter as it's contribution to the integral is going to be 0.)
3. Sweep the region horizontally or vertically.
4. Get bounds for x and y that correspond to this sweep.
5. Set up the double integral as an iterated integral in the form
$$\int_*^* \int_*^* f(x,y)dxdy$$
or
$$\int_*^* \int_*^* f(x,y)dydx$$
with specific limits in place of $*$
6. In some cases, you might need to split the region R into two or more pieces, each with it's corresponding double integral.

Note that **The outside integral must have constant limits**. This means

– for a horizontal sweep, x will have fixed(constant) bounds, and bounds for y might depend on x.

– for a vertical sweep, y will have fixed(constant) bounds, and bounds for x might depend on y.

Draw pictures to understand why this will be true and reflect on this as you finish each of the practice problems below.

Practice Problems

Set up the double integral

Set up a double integral of $f(x, y)$ where the region R is given by

1. $0 < x < 1, x < y < x + 1$
2. $0 \leq x \leq 1, 0 \leq y \leq 1, y \leq \frac{x}{2}$
3. part of the unit square $0 \leq x \leq 1, 0 \leq y \leq 1$ on which $x + y > 0.5$
4. part of the unit square $0 \leq x \leq 1, 0 \leq y \leq 1$ on which both x and y are greater than 0.5
5. part of the unit square $0 \leq x \leq 1, 0 \leq y \leq 1$ on which at least one of x and y are greater than 0.5
6. $0 < x < 50 - y < 50$ on which both x and y are greater than 20.
7. the points (x, y) in the first quadrant with $|x - y| \leq 1$

Evaluate the double integral

Evaluate the double integral

$$\int \int_R f(x, y) dx dy$$

where

1. $f(x, y) = e^{-x-y}$, and region R is the first quadrant in which $x + y \leq 1$
2. $f(x, y) = e^{-x-2y}$, and region R is the first quadrant in which $x \leq y$
3. $f(x, y) = x - y + 1$, and region R is inside the unit square in which $x + y \geq 0.5$