1.0 Set theory Review

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PSTAT 120A: Introduction to Probability

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Review Basic Set Theory

Set, element, empty set

Definition (Set)
A set is a collection of things. { } notation is used to denote a set

$$S = \{\text{cat, dog, mouse}\},\ T = \{1, 2, 3\},\ U = \{1, 2, a, b\}.$$

Definition (Element)
An element is a member of a set

Notation ($\in$)

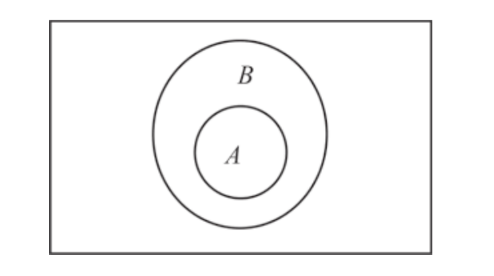
$$cat \in S, 2 \in T, a \in U$$

Definition (Empty set)
The empty set is a set with no elements. Usually denoted by $\emptyset$.

Subset

Definition (Subset)
Let A and B be two sets. A is a subset of B, denoted by $A \subseteq B$ if every element of A is also in B.

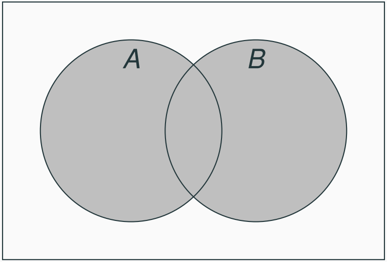
$A = \{3, 4, \pi\}$ is a subset of $B = \{1, 2, 3, 4, 5, \pi\}$ denoted as $A \subseteq B$



More Sets and Venn Diagrams

Definition (Union, OR)
The union of A and B is the set that contains all elements that are in A or in B or both

$$A \cup B = \{x \; ; \; x \in A \text{ or } x \in B\}$$

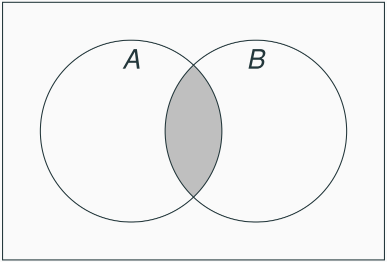


Also denoted as A or B

More Events and Venn Diagrams

Definition (Intersection, AND)
The intersection of A and B is the set that contains all elements that are in A and in B

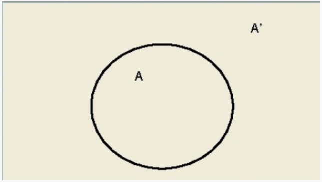
$$A \cap B = \{x \,;\, x \in A \text{ and } x \in B\}$$



*Also denoted as A and B, AB

Definition (Complement)
A^c, the complement of A, contains all elements that are in S and not in A.

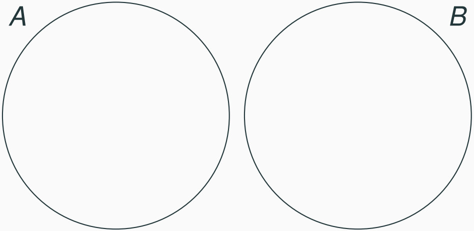
$$A^c = \{x : |x \notin A\}$$



* Also denoted as A', not$A, A', S\backslash A$

More Sets and Venn Diagrams

Definition (Disjoint sets)
If $A \cap B = \emptyset$ then A and B are disjoint.



Pairwise disjoint/mutually exclusive

Definition (Pairwise disjoint/mutually exclusive)
An indexed family of subsets $\{A_i \,;\, i \in I\}$ of the set S, where I is a set of index, is pairwise disjoint if

$$A_i \cap A_j = \emptyset \text{ for every } i \neq j.$$

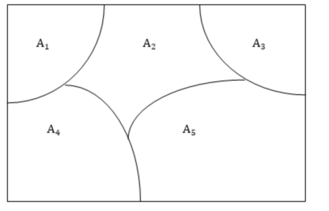
Definition (Exhaustive)
An indexed family of sets $\{A_i \,;\, i \in I\}$ of the set S, where I is a set of index, is exhaustive if

$$\bigcup_{i \in I} A_i = S.$$

Partition

Definition (Partition)
A family of sets $\{A_i\ ;\ i \in I\}$ of set S is a partition of S if

$$\begin{cases} A_i \cap A_j = \emptyset & \text{for every } i \neq j, \text{mutually exclusive} \\ \bigcup_{i \in I} A_i = S. & \text{exhaustive} \end{cases}$$



$\{A, A^c\}$ forms a partition of S because

$$\begin{aligned} A \cap A' &= \emptyset \\ A \cup A' &= S \end{aligned}$$

De Morgan's law

Definition ($(A \cap B)^c$**) and** $(A \cup B)^c$**)**
Let S be a set, and A, B be two subsets of S.

- Recall the complements of A in S by

$$A^c = S \backslash A.$$

- The following identities hold
  - $$(A \cap B)^c = A^c \cup B^c$$
  - $$(A \cup B)^c = A^c \cap B^c.$$

Under complementation, Union and intersection switch.