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Some aspects related to the transformation of Jacobi coordinate sets of the three body wave function built on the Gaussian basis

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The three-body wave function built on the basis of the Gaussian function, calculated using the three-body Hamiltonian with the Pauli blocking operator is studied. Analytical expressions are presented for the matrix elements of the overlap of the basis functions for both basic and alternative set of relative Jacobi coordinates. The correlation densities of the wave function are calculated and illustrated depending on the set of orbital numbers also for the both sets of Jacobi coordinates.

Keywords: the three-body problem, Gaussian basis, relative Jacobi coordinates

Introduction

N years have passed since the skin, halo effects of such exotic nuclei were shown by Tanihata. These discoveries have allowed the existing science to look from a new perspective at the interactions of nucleons in atomic nuclei, and challenged already known theoretical methods. It is difficult to say that there is a unified model describing all the observable characteristics of the exotic nuclei. Nevertheless, it is possible to single out a theoretical method that describe a sufficient number of the observable properties of the being explored nuclei.

It is Gaussian Expansion Method. The essence is in the expansion of the total wave function in terms of the Gaussian basis function. The solution of the Schroedinger equation for the few body problem is reduced to finding the factor, i.e. weight, of the matrix elements of the Hamiltonian calculated through the exponential functions, to set the parameters of the arguments of the exponential function. It is interesting to note that this method is easily applicable for problems of two bodies, three bodies and four bodies. The method has the advantage of expressing matrix elements in the analytical form, which one makes possible to do calculations quite quickly on ordinary desktop computers.

It should also be noted that in this method the parameters of the wave function are varied in order to obtain the minimum eigenvalue of the Hamiltonian matrix. Therefore, the approach is also called as Stochastic Variational Method.

Due to the limited number of existing materials on this topic for practical application, the purpose of this work is to make the formulas for variational calculations accessible and open. In the first section, some vector re-coupling problems in quantum mechanics are given, an expression for the total three-particle wave function and details of the transformation of the basis function from one set to other sets of Jacobi coordinates are given. Then, the second section deduces analytical expressions for the overlap matrix elements, which can further be applied to the matrix elements of other quantum operators. The main conclusions are made in the conclusion section.

The three-body wave function

Angular momentum re-coupling

An addition of two vectors $\mathbf{j}_1$ and $\mathbf{j}_2$ getting $\mathbf{j}$ vector is carried out through the Clebsch-Gordan coefficient as follow

$$|\mathbf{j}\rangle = \sum_{m_1 m_2} \langle\, j_1 m_1\, j_2 m_2 \mid jm \,\rangle\, |\mathbf{j}_1\rangle\, |\mathbf{j}_2\rangle, \tag{1}$$

For non-zero values of the coefficient (1) vectors $\mathbf{j}_1$, $\mathbf{j}_2$ and $\mathbf{j}$ must satisfy the rule of triangle:

$$|j_1 - j_2| \leq j \leq j_1 + j_2 \tag{2}$$

$$|j - j_2| \leq j_1 \leq j + j_2 \tag{3}$$

$$|j_1 - j| \leq j_2 \leq j_1 + j \tag{4}$$

and condition

$$m = m_1 + m_2. \tag{5}$$

If there are three vectors $\mathbf{j}_1, \mathbf{j}_2$ and $\mathbf{j}_3$, one must firstly add $\mathbf{j}_1$ and $\mathbf{j}_2$, which gives $\mathbf{j}_{12}$, then couple $\mathbf{j}_{12}$ with $\mathbf{j}_3$. This is a first scheme of coupling the three vectors. The second scheme takes place by means of adding $\mathbf{j}_2$ and $\mathbf{j}_3$, which gives $\mathbf{j}_{23})$, then coupling it with $\mathbf{j}_1$. A coefficient between two schemes is defined via the $U(\mathbf{j}_1\mathbf{j}_2\mathbf{j}\mathbf{j}_3;\mathbf{j}_{12}\mathbf{j}_{23})$ Racah coefficient as following

The energy dependence of total cross sections of reactions ^{6}He + Si and ^{6,9}Li + Si in the beam energy range 5-30 MeV/nucleon has been measured. An agreement with the published experimental data for the reaction ^{6} He + Si was obtained. For the reaction ^{9}Li + Si new data in the vicinity a local enhancement of the total cross section was obtained. Theoretical analysis of possible reasons of appearance of this peculiarity in the collisions of nuclei ^{6}He and ^{9}Li with Si nuclei has been carried out including the influence of external neutrons of weakly bound projectile nuclei.

Table 1.
Please write your table caption here (*the width of the table should be equal to the width of the text*)

H	q	α_q	$\chi^2/d.o.f.$	Confidence level of fittings
0.3	2	0.56 ± 0 .01	0.38	98.90 %
	3	1.22 ± 0 .03	0.49	95.93 %
	4	1.92 ± 0 .06	0.58	91.30 %
	5	2.63 ± 0 .10	0.70	80.95 %
1	2	0.59 ± 0 .01	0.74	76.62 %

Example of text style and formula design in the text

In our experiments, we employed the Dubna gas-filled recoil separator (DGRFS), that allows the separation of the products of complete fusion reactions from the beam of bombarding ions, elastically-scattered nuclei, and products of incomplete fusion. The detection system includes proportional chambers used to measure the time of flight (TOF) of particles and several semiconductor detectors with position-sensitive strips.

The principle of operation of the separator is selection of products of the complete-fusion reaction by their charge state q in a rare gas and kinematic characteristics (mass of recoil nucleus m and its velocity v) in accordance with the separator magnetic rigidity $B\rho = mv/q$ (note, q depends linearly on v). These values are calculated for the xn-reaction channel when setting the separator's parameters.

The DGFRS strongly separates forward-peaked evaporation residues (ER), products of complete-fusion reactions, within a narrow angle with a huge suppression of the products of the transfer reactions and even incomplete fusion, e.g., αxn reactions. The TOF selection in the existing separators may be complemented and reinforced by the combined measurement of recoil energy and TOF. Note, the production properties "separator", "mass separation", "angular selection", and "TOF selection" were called "assignment properties" in [3].

Subsection title

Formulas should be written follow type:

$$TC(HKL) = \frac{I(hkl)}{I_0(hkl)} \Big/ \frac{1}{n} \sum \frac{I(hkl)}{I_0(hkl)}, \tag{6}$$

Example of figure style

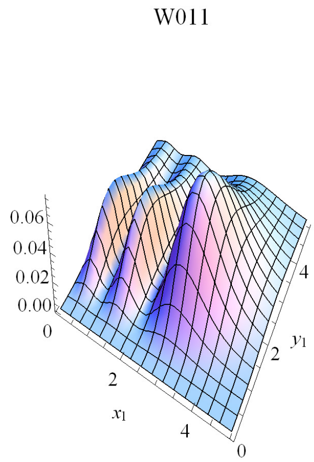

Figure 1. Please write your figure caption here.

The first superheavy nucleus ^{289}Fl was discovered in the ^{244}Pu(^{48}Ca, $3n$) reaction studied at DGFRS (here and after we refer to reviews [1,2] containing references to most of earlier experimental data). The decay properties of ^{289}Fl and descendant nuclei are shown in figure 1.

Authors contributions

All the authors were involved in the preparation of the manuscript. All the authors have read and approved the final manuscript.

Conclusion

Your Conclusion text comes here...

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Also, use this section to provide information about funding by including specific grant numbers and titles. If you need to include funding information, list the name(s) of the funding organization(s) in full, and identify which authors received funding for what.

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For books: Author, *Book title* (Publisher, place year) page numbers.(DOI or ISBN) Example:

[1] Bass R, Nuclear Reactions with Heavy IonsBerlin (Heidelberg, New York: Springer-Verlag, 1980) 410 p.(DOI or ISBN)

For articles from journals: Author, Journal **Volume** (year) page numbers.(DOI) Example:

[2] Tanihata I. et al., Phys.Lett.B. **206** (1988) 592-600.(DOI)

For conference materials, proceedings, etc.: Author, Publication title: Type of publication **Volume** (year) page numbers.(DOI) Example:

[3] Oganessian Y.Ts., Proceeding of the International Conference on Nuclear Physics, Munich **73** (1975) 351-360.(DOI)

Example:

[1] Bass R, Nuclear Reactions with Heavy IonsBerlin (Heidelberg, New York: Springer-Verlag, 1980) 410 p.(DOI or ISBN)
[2] Tanihata I. et al., Phys.Lett.B. **206** (1988) 592-600.(DOI)
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