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ISOSPIN-MIXED STATES IN ^{8}Be†

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Abstract: The isospin mixing of states in ^{8}Be near 17 MeV was investigated by comparison of cross sections for the ^{9}Be(d, t)^{8}Be and ^{9}Be(d, ^{3}He)^{8}Li reactions. For the 2^+ doublet at 17 MeV, our results are inconsistent with previous measurements of the isospin mixing unless the theoretical $T = 0$, $p_{\frac{1}{2}}$ spectroscopic amplitude is changed by $\approx 20\,\%$. For the 1^+ states near 18 MeV we find an isospin amplitude impurity of 0.21 ± 0.03. For the 3^+ states near 19 MeV no new information could be obtained due to the complex peak shape.

E NUCLEAR REACTIONS ^{9}Be(d, t)^{8}Be, ^{9}Be(d, ^{3}He)^{8}Li, $E = 27.97$ MeV; measured $\sigma(E_t, E_{^3\mathrm{He}}, \theta)$. ^{8}Be deduced levels, l, spectroscopic factors, isospin mixing. Natural target.

1. Introduction

Three pairs of states in ^{8}Be near 17 MeV of excitation are known to show considerable isospin mixing [1-3]). The 2^+ states at 16.63 and 16.93 MeV have almost equal $T = 0$ and $T = 1$ amplitudes, the 1^+ states at 17.64 and 18.15 MeV have a 0.24 amplitude impurity and the 3^+ states at 19.06 and 19.22 MeV have a 0.41 mixing amplitude. The charge-dependent matrix elements necessary to account for the experimental energy separations and isospin mixing are -149, -120 and -63 keV for the 2^+, 1^+ and 3^+ states, respectively [3]). Barker [3]) attempted to calculate these matrix elements using a Coulomb force and harmonic oscillator wave functions, but obtained only half of the experimental values. Since this discrepancy could be considered as evidence for a charge-dependent component of the nuclear force, considerable theoretical and experimental effort has gone into studying these states.

Dalton and Robson [4]) pointed out that the 2^+ states are near the proton and neutron thresholds in ^{8}Be. Since the 16.63 and 16.93 MeV states have a large overlap with ^{7}Li+p and ^{7}Be+n, respectively [1]), they suggest the discrepancy could be due to a shift of the Thomas-Ehrman type. Barker [3]) however, says that other nearby open channels also contribute and the net effect on the matrix elements is small.

Since Coulomb matrix elements are sensitive to configuration mixing, Barker [3]) has suggested the discrepancy might be accounted for by admixtures such as $1p^3\,2p$

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to the $1p^4$ basis he used for his calculations. Anderson *et al.* [5]) recalculated the matrix elements using correlated wave functions and realistic nuclear interactions and were able to reproduce the experimental results to a high accuracy. McCarthy and Walker [6]) and Bertsch and Shlomo [7]) however, repeated these calculations taking into account the Pauli operator and obtained matrix elements which did not differ appreciably from the calculations of Barker [3]). Goldhammer [8]) has carried out further calculations using Sussex harmonic oscillator matrix elements and higher order particle-hole excitations than were used in previous calculations. The resulting Coulomb matrix elements agree to within 3 keV with the experimental matrix elements [3]). Hopefully, clarification of the size of the mixing to come from electromagnetic interaction between the nucleons is not beyond the scope of present day theory.

Experimental results for the isospin mixing appear to be as controversial as the theoretical calculations. In order to discuss the results of these experiments, we make use of the notation of Barker [3]). The wave functions for pairs of isospin-mixed states are written as a linear combination of states of good isospin, $\varphi_{T=0}$ and $\varphi_{T=1}$:

$$\begin{aligned} \Psi_a &= \alpha_J \varphi_0 + \beta_J \varphi_1, \\ \Psi_b &= \beta_J \varphi_0 - \alpha_J \varphi_1, \end{aligned} \tag{1}$$

where $E_a < E_b$, $\alpha_J^2 + \beta_J^2 = 1$ and J indicates the spin of the state. Recently Shanley [47]) has suggested that α and β are complex numbers. However, Barker [9]) has pointed out that if φ_0 and φ_1 are shell-model states and R-matrix theory is used to connect the nuclear interior and exterior, α and β are real. We shall use the latter approach with real α and β. Shell-model calculations [3]) indicate in the limit of good isospin, $\alpha = 0$ for all three pairs of states.

The isospin mixing of the 2^+ doublet may be best determined from the widths of the states. Since the γ-widths of these states are very small [≈ 5 eV, ref. [10])], their widths are due almost entirely to isospin-allowed α-decay through the $T = 0$ components of the wave functions:

$$\frac{\Gamma_{16.6}}{\Gamma_{16.9}} = \left[\frac{\alpha_2}{\beta_2}\right]^2.$$

(Here we have ignored the small energy changes in penetrabilities and the level shift parameter.) A recent compilation [11]) lists $\Gamma_{16.6} = 107 \pm 3$ keV and $\Gamma_{16.9} = 77 \pm 3$ keV as an average of results of several experiments. The 2^+ states are overlapping resonances of the same spin and parity, which results in interference effects that change the shapes of the peaks from simple Breit-Wigner shapes. (See sect. 2 for a more detailed discussion.) None of the results averaged by ref. [11]) appear to take these interference effects properly into account, although Barker [3]) estimates the effect on the widths is small. Callender and Browne [12]) studied the (d, α) reaction populating these states. Using the proper R-matrix formulas for interfering resonances, they obtained $\Gamma_{16.6} = 113$ keV and $\Gamma_{16.9} = 80$ keV. They found, however,

a large variation in parameters with beam energy that they suggest may be due to interference with other 2^+ levels. Piluso *et al.* [13]) also used correct *R*-matrix formulas to fit ^{7}Li(^{3}He, d) data, to extract $\Gamma_{16.6} = 103$ keV and $\Gamma_{16.9} = 83$ keV. Ref. [13]) also reanalyzes the ^{7}Li(^{3}He, d) data of Marion *et al.* [14]) with *R*-matrix theory to obtain $\Gamma_{16.6} = 112$ keV and $\Gamma_{16.9} = 78$ keV. The resulting values of α_2 are 0.765 [ref. [12])], 0.744 [ref. [13])] and 0.762 [ref. [11])] versus Barker's [3]) value of 0.74. Using an average value of $\alpha_2 = 0.76$, the charge-dependent matrix element is -149 keV, in agreement with Barker's value [3]) and Goldhammer's calculation [8]).

For the 1^+ states, Barker [3]) obtained a value of $\alpha_1 = 0.24$ from the γ-widths for the states. Two sets of measurements were used. The Maryland group [15–17]) populated the 17.6 and 18.2 MeV states by resonant proton capture and inferred γ-widths for decay to the 2^+ states near 17 MeV by detecting α-particles from those states. The Stanford group [18, 19]) performed a similar experiment, except that the γ-rays were detected in coincidence with both α-particles from the 2^+ decays. Early results [15, 18]) used by Barker [3]) gave a branching ratio for the 17.64 MeV 1^+ to the 16.6 and 16.9 MeV 2^+ states of approximately 7 %. More recently, the decays of the 17.6 MeV and 18.2 MeV 1^+ were remeasured [17, 19]). After the data of ref. [19]) has been renormalized to take into account a new value for the ^{7}Li(p, α) cross section, the two experiments are in reasonable agreement. Ref. [17]) then gives a branching ratio for the 17.64 MeV state of (4.0 ± 0.9) % which would considerably change the value of α_1 given above. Inspecting fig. 9 of ref. [19]) we see in fact that the (renormalized) data is inconsistent with any value of α_1 between 0.0 and 0.44 for the $17.6 \rightarrow 16.6$ and $18.2 \rightarrow 16.9$ transitions. Both experiments are sensitive to background due to direct, nonresonant capture of protons into the 16.6 MeV 2^+ state. Ref. [19]) is sensitive to the Compton edge of the $17.6 \rightarrow 16.9$ transition which lies beneath the weak $17.6 \rightarrow 16.6$ transition peak [17]). Ref. [17]) is sensitive to the shape of the prompt breakup background from the wide 2^+ level at 19.9 MeV [ref. [19])]. In view of the great difficulties involved in these experiments, it may not be surprising that the results are inconsistent with theory. Another method which may be used to determine α_1 is to measure cross sections for the (d, α) reaction to the two states. At sufficiently high energies, the cross sections are primarily direct and proceed through the $T = 0$ components of the wave functions, giving $\sigma_{17.6}(\mathrm{d}, \alpha)/\sigma_{18.2}(\mathrm{d}, \alpha) = (\alpha_1/\beta_1)^2$. From ref. [12]) we find $\sigma_{17.6}/\sigma_{18.2} = 0.090\pm0.016$ for deuteron energies between 10 and 12 MeV, giving $\alpha_1 = 0.29^{+0.02}_{-0.01}$. In view of the difficulties with the γ-ray experiments, we consider this to be the best previous value for α_1. The corresponding charge dependent matrix element is $-(139^{+9}_{-5})$ keV. This result strongly disagrees with the value of -117 keV calculated by Goldhammer [8]).

The isospin mixing of the 3^+ states at 19.06 and 19.22 MeV may be determined from a measurement of γ_n^2/γ_p^2 for the 19.22 MeV state [20]). Using $\gamma_n^2/\gamma_p^2 = 5$, Barker [3]) calculated $\alpha_3 = 0.41$. However, the data is consistent with $3 < \gamma_n^2/\gamma_p^2 < 10$, resulting in $0.31 < \alpha_3 < 0.52$. The corresponding charge-dependent matrix element ranges from -47 to -71 keV. The experimental values for γ_n^2/γ_p^2 depend on penetrabilities

calculated for the neutrons near threshold and γ_n^2/γ_p^2 was extracted assuming the resonance was a simple Breit-Wigner form. Since the penetrabilities are very sensitive to the parameters used (eg. the radius of the well) and the latter assumption is incorrect, these values for α_3 are open to question.

In view of these difficulties for the experimental measurements of isospin mixing, we have attempted to remeasure the experimental quantities. A preliminary report on this work was given in ref. [21]). In sect. 2 we describe our method and quote the correct *R*-matrix results for the shape of overlapping resonances. In sect. 3 we present the details of the experiment and the DWBA calculations. The results are discussed in sect. 4.

2. Experimental method

The mixing coefficient α_J in eq. (1) may be determined by comparison of cross sections for single-nucleon transfer to the states of interest and their "analogs". This technique has been used previously by Garvey *et al.* [22]) and Braithwaite *et al.* [23]). The cross section may be written

$$\sigma = \sum_j |\sum_T \alpha_T C_T S_{jT}^{\frac{1}{2}}|^2 \sigma_j^{\mathrm{DW}}, \tag{2}$$

where j is the angular momentum transfer, C_T is the vector addition coefficient coupling isospin, $S_{jT}^{\frac{1}{2}}$ is the spectroscopic amplitude in a good isospin representation and σ_j^{DW} is calculated in the distorted wave Born approximation. If we populate a pair of isospin-mixed states with the (d, t) reaction, eq. (2) gives a cross section ratio

$$R_{\mathrm{tt}} = \frac{\sigma_{\mathrm{a}}(\mathrm{d,t})}{\sigma_{\mathrm{b}}(\mathrm{d,t})} = Q_{\mathrm{tt}} \frac{\sum_j t_j |\sum_T \alpha_{\mathrm{a}T} C_T S_{jT}^{\frac{1}{2}}|^2}{\sum_j t_j |\sum_T \alpha_{\mathrm{b}T} C_T S_{jT}^{\frac{1}{2}}|^2}, \tag{3}$$

where a (b) labels the lower (upper) state,

$$Q_{\mathrm{tt}} = \sigma_{\frac{1}{2}\mathrm{a}}^{\mathrm{DW}}(\mathrm{d,t})/\sigma_{\frac{1}{2}\mathrm{b}}^{\mathrm{DW}}(\mathrm{d,t}),$$
$$t_j = \sigma_j^{\mathrm{DW}}(\mathrm{d,t})/\sigma_{\frac{1}{2}}^{\mathrm{DW}}(\mathrm{d,t}).$$

Also of interest is the ratio of cross sections for the (d, t) reaction populating an isospin-mixed state and the (d, ^{3}He) reaction populating the analog of its $T = 1$ component

$$R_{\mathrm{th}} = \frac{\sigma(\mathrm{d,t})}{\sigma(\mathrm{d},{}^3\mathrm{He})} = Q_{\mathrm{th}} \frac{\sum_j t_j |\sum_T \alpha_T C_T S_{jT}^{\frac{1}{2}}|^2}{\sum_j t'_j |C'_1 S_{j1}^{\frac{1}{2}}|^2}, \tag{4}$$

where Q_{th} and t_j are ratios similar to those above.

The distorted wave cross sections σ_j^{DW} are given by

$$\sigma_j^{\mathrm{DW}} = N\sigma_j^{\mathrm{DWUCK}}/(2j+1). \tag{5}$$

The quantity $\sigma_j^{\mathrm{DWUCK}}$ was calculated in the zero-range approximation with the DWBA

TABLE 1

Theoretical spectroscopic amplitudes, $S_{JT}^{\frac{1}{2}}$

State JT	l_j	Cohen and Kurath [a]	Barker [b, c]	Boyarkina [c, d]
21	$p_{\frac{1}{2}}$	0.3909	−0.4099	0.4045
	$p_{\frac{3}{2}}$	1.1603	−1.1607	1.1017
20	$p_{\frac{1}{2}}$	0.2882	−0.3184	−0.3019
	$p_{\frac{3}{2}}$	0.5885	−0.5758	−0.5221
11	$p_{\frac{1}{2}}$	0.5445	0.5559	0.5438
	$p_{\frac{3}{2}}$	−0.5559	−0.6024	−0.5586
10	$p_{\frac{1}{2}}$	−0.2057	0.1986	−0.2077
	$p_{\frac{3}{2}}$	0.1248	−0.1780	0.1585
31	$p_{\frac{1}{2}}$			
	$p_{\frac{3}{2}}$	0.7252	−0.7328	0.6051
30	$p_{\frac{1}{2}}$			
	$p_{\frac{3}{2}}$	0.3280	−0.3569	−0.3667

[a]) Ref. [29]).
[b]) Ref. [3]).
[c]) Transformed from *LSJT* coupling and channel-spin representation as described in text.
[d]) Ref. [30]).

code DWUCK [ref. [24])]; details of the calculations are given in sect. 3. The quantity N is a normalization factor related to the light particle spectroscopic factors. We use values given by Bassel [25]): $N(\mathrm{d, t}) = 3.33$ and $N(\mathrm{d, {}^3He}) = 2.95$. More recently Hering *et al.* [26]) have determined the values $N(\mathrm{d, t}) = 3.21$ and $N(\mathrm{d, {}^3He}) = 2.82$. The two determinations give values for the quantity of interest here, $N(\mathrm{d, t})/N(\mathrm{d, {}^3He})$, that agree within 1 %. A recent study [27]) of (d, t) and (d, ^{3}He) reactions on self-conjugate nuclei used the ratio of Hering and obtained agreement with the predicted cross-section ratio to ±10 %. This places a limit on the accuracy we may expect from our calculations.

The spectroscopic amplitudes $S_{jT}^{\frac{1}{2}}$ were taken from three sources. Cohen and Kurath fit energy levels in 1p shell nuclei to obtain *jj* coupling wave functions for these nuclei [28]). From these wave functions, coefficients of fractional parentage (CFP) were calculated [29]). The spectroscopic amplitude of interest here is then

$$S_{jT}^{\frac{1}{2}} = n^{\frac{1}{2}}\mathrm{CFP}_{jT},$$

where n for a pickup reaction is the number of active 1p nucleons in the target. Barker [3]) and Boyarkina [30]) performed an analysis similar to that of Cohen and Kurath but used *LSJT* coupling. The necessary values of $S_{jT}^{\frac{1}{2}}$ were calculated from the wave functions using the *LSJT* CFP of Jahn and Van Wieringen [31]) and the transformation from the channel spin to the angular momentum representation given by Macfarlane and French [32]). The resulting values of $S_{jT}^{\frac{1}{2}}$ are given in table 1. The magnitudes of the $S_{jT}^{\frac{1}{2}}$ are in reasonable agreement, but their signs are frequently different, reflecting different overall phases for the wave functions. To facilitate comparison with Barker's results [3]) for α_j we will change the phases of $S_{jT}^{\frac{1}{2}}$ from refs. [29, 30]) to agree with Barker's.

In order to extract the (d, t) cross sections necessary to calculate experimental values for R_{tt} and R_{th}, proper account of interference effects between the isospin-mixed states must be taken. For a single isolated resonance, the well-known Breit-Wigner formula applies, giving yield as a function of energy in the center of mass,

$$Y(E) = \frac{A}{(E-E_0)^2+(\frac{1}{2}\Gamma)^2}. \tag{6}$$

A natural generalization of eq. (6) for two levels, used by many previous authors [e.g. refs. [12, 14])], is the interfering Breit-Wigner (IBW) form

$$Y(E) = \left| \frac{A_1}{E-E_1+i\frac{1}{2}\Gamma_1} + \frac{A_2}{E-E_2+i\frac{1}{2}\Gamma_2} \right|^2. \tag{7}$$

Barker [33-35, 13]), however, has derived a general formula from R-matrix theory for overlapping levels of the same spin and parity with a single open decay channel

$$Y(E) = \frac{\sum_x c_x \left| \sum_{\lambda=a}^{b} \{g_{\lambda x}\Gamma_\lambda^{\frac{1}{2}}/(E_\lambda-E)\} + J_x \right|^2}{1+\left|\frac{1}{2}\sum_{\lambda=a}^{b}\{\Gamma_\lambda/(E_\lambda-E)\}+K\right|^2}. \tag{8}$$

Here the sum over x is a sum over incoherent contributions (in this case j-values), c_x contains the angular dependence of the yield, $g_{\lambda x}$ is a real positive feeding factor for level λ, E_λ is the energy of the pole λ and Γ_λ is the width of level λ. The parameters J_x and K are contributions due to distant levels, assumed to be independent of energy over the region of interest. It may be shown [35]) that by algebraic manipulations involving the redefinition of the parameters in eq. (8), a new equation may be obtained with the same energy dependence as eq. (8) but with $K \equiv 0$. Further algebraic manipulation will then give the following form for the yield

$$Y(E) = \frac{A^0_{aa}(E_b-E)^2+2A^0_{ab}(E^0_a-E)(E^0_b-E)+A^0_{bb}(E^0_a-E)^2+\mathscr{J}^0(E)}{(E^0_a-E)^2(E^0_b-E)^2+\frac{1}{4}[\Gamma^0_a(E^0_b-E)+\Gamma^0_b(E^0_a-E)]^2}, \tag{9}$$

where

$$A^0_{\lambda\mu} = \Gamma^{\frac{1}{2}}_\lambda \Gamma^{\frac{1}{2}}_\mu \sum_{j=\frac{1}{2}}^{\frac{3}{2}} c_j g^0_{\mu j} g^0_{\lambda j} + A^{\mathrm{cpd}}_{\lambda\mu},$$

$$g^0_{\mu j} = \sum_T \alpha_{\mu T} C_T S^{\frac{1}{2}}_{jT},$$

$\mathscr{J}^0(E)$ includes all terms containing J_x and the superscript zero stands for the $K \equiv 0$ case.

Once the parameters $A^0_{\lambda\lambda}$ and Γ^0_λ are determined by fitting the experimental spectrum, the total yield associated with a specific level may be calculated from

$$Y_\lambda = 2\pi A^0_{\lambda\lambda}/\Gamma^0_\lambda.$$

These total yields then give the (d, t) cross sections necessary to calculate R_{tt} and R_{th}.

The parameters $A^0_{\lambda\mu}$ may also be combined directly to obtain the isospin mixing. Following refs. [13, 14]) we note that there is no interference between levels a and b in the compound nucleus, so $A^{cpd}_{ab} = 0$. Furthermore, since the energies of the two states are approximately the same, $A^{cpd}_{aa} \approx A^{cpd}_{bb}$ and $c(\theta)$ should be the same for all terms. Thus the ratio

$$R_{AA} = \frac{A_{bb} - A_{aa}}{A_{ab}} = \frac{\Gamma^0_b \sum_j |\sum_T \alpha_{bT} C_T S^{\frac{1}{2}}_{jT}|^2 - \Gamma^0_a \sum_j |\sum_T \alpha_{aT} C_T S_{jT}{}^{\frac{1}{2}}|^2}{(\Gamma^0_a \Gamma^0_b)^{\frac{1}{2}} \sum_j [(\sum_T \alpha_{aT} C_T S^{\frac{1}{2}}_{jT})(\sum_T \alpha_{bT} C_T S^{\frac{1}{2}}_{jT})]}, \qquad (10)$$

is purely direct and independent of θ.

In using these techniques to determine α_J, we are assuming (a) only two states contribute to the isospin mixing; (b) the theoretical relative $S^{\frac{1}{2}}_{jT}$ are correct; (c) the reactions are single step, direct; (d) the DWBA correctly accounts for Q-value and other reaction mechanism effects; and (e) the relative (d, t) to (d, ^{3}He) normalization of ref. [25]) is correct. It is clear that R_{AA} is least sensitive to the last three assumptions and should give the best values for α_J. Thus by comparison of results for R_{AA} with those for R_{tt} and R_{th} we may in principle check the last three assumptions. Since the value of α_2 is well determined by the widths of the states, a check on the R_{AA} prescription is also available.

3. Determination of experimental parameters

3.1. EXPERIMENTAL SETUP

A 700 μg/cm^2 self-supporting Be target was bombarded by 27.97 MeV deuterons from the Princeton cyclotron. Tritons and ^{3}He particles were detected simultaneously in a detector telescope located 35 cm from the target with a 0.22 msr solid angle. The telescope consisted of a 52 μ ΔE detector, a 1000 μm E' detector and a 2000 μm veto. Standard slow-fast coincidence electronics were used to process energy signals as well as logic requirements and pile-up rejection. A 3000 μm detector located at -20° was used to monitor target deterioration. The ΔE, E' and monitor signals were fed into ADC units connected to an on-line Σ-2 computer. Data was taken from 6° to 51° in 3° steps. At the conclusion of these runs, elastic scattering data was taken for normalization and to obtain optical parameters.

Since the determination of R^{expt}_{th} depends on the relative triton and ^{3}He collection efficiency, a second run was taken. A somewhat thicker ΔE counter (85 μm) was used so that deuterons, tritons and ^{3}He particles could be observed simultaneously. Extra windows were set in the ID spectrum above and below the nominal d, t and ^{3}He groups to insure no real events were lost. Cross-section ratios amongst the various d, t and ^{3}He peaks for the two runs agreed within statistics, indicating the validity of the original data.

3.2. DEUTERON DATA

Data taken on elastic deuteron scattering showed a small amount of C and O in the target. Fig. 1 shows a plot of ratio to Rutherford for the ^{9}Be(d, d_0) data. The data were fit with an optical model code with search provisions [36]). Searches were made on the optical model parameters and the experimental normalization. The starting parameters were taken from a global parameter set of Perey and Perey [37]). The fit is shown in fig. 1 and optical parameters are given as set D in table 2.

TABLE 2
Optical model parameters

Set	V_R	r_R	a_R	W_D	W_V	r_I	a_I	r_C	λ
				Deuteron parameters					
D	− 77.2	1.15	0.89	57.1		1.35	0.68	1.25	
				Triton and ^{3}He parameters					
H1 [a])	−152	1.14	0.71		−30	1.67	0.78	1.25	
H2 [b])	−177	1.138	0.724		−15	1.602	0.769	1.25	
H3 [c])	−141.2	1.156	0.758		−16.4	1.535	0.99	1.25	
HB	−141.2	1.35	0.85		−16.4	1.40	1.20	1.25	
				Bound state parameters					
	varied	1.25	0.65						25

[a]) Ref. [23]). [b]) Ref. [40]). [c]) Ref. [39]).

3.3. THE ^{3}He DATA

A sample spectrum for the ^{9}Be(d, ^{3}He)^{8}Li data is shown in fig. 2. The resolution was approximately 200 keV due to straggle in the target. The ground state 2^+, 0.981 MeV 1^+ and 2.261 MeV 3^+ states were strongly excited. A weak state is also observed at approximately 6.5 MeV. This is presumably the narrow 6.53 MeV state observed in the ^{7}Li(n, n)^{7}Li and ^{9}Be(t, α) reactions [11]). The 6.5 MeV state was not observed beyond $\theta_{lab} = 21°$ due to detector cutoffs; the other three states were observed at all angles. Peaks from ^{12}C(d, ^{3}He) to the ground and 2.21 MeV levels in ^{11}B were also observed.

Extracting a cross section for the 3^+, 2.26 MeV state was hindered by a discontinuity in the background in the region of the peak. The background change may be due to the three-body phase space from the ^{9}Be(d, ^{3}He n) channel which opens 230 keV below the 3^+ state. Rather than attempting to fit this background, it was assumed that the background changed linearly from the high to low level underneath the peak. In view of the uncertainty in this prescription for the background, a systematic error was assigned to the cross section. This error was taken as half the difference between the high and low background levels. The magnitude of this systematic error was 5–10 % of the cross section, depending on angle.

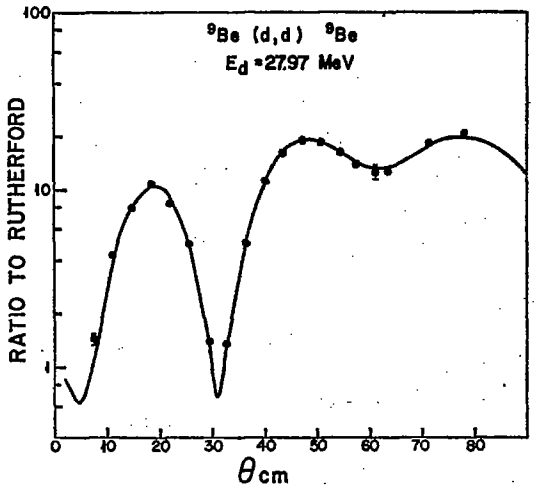

Fig. 1. Ratio to Rutherford for ^{9}Be(d, d)^{9}Be. Error bars are shown where larger than the data points. The theoretical curve is an optical model calculation using parameter set D in table 2.

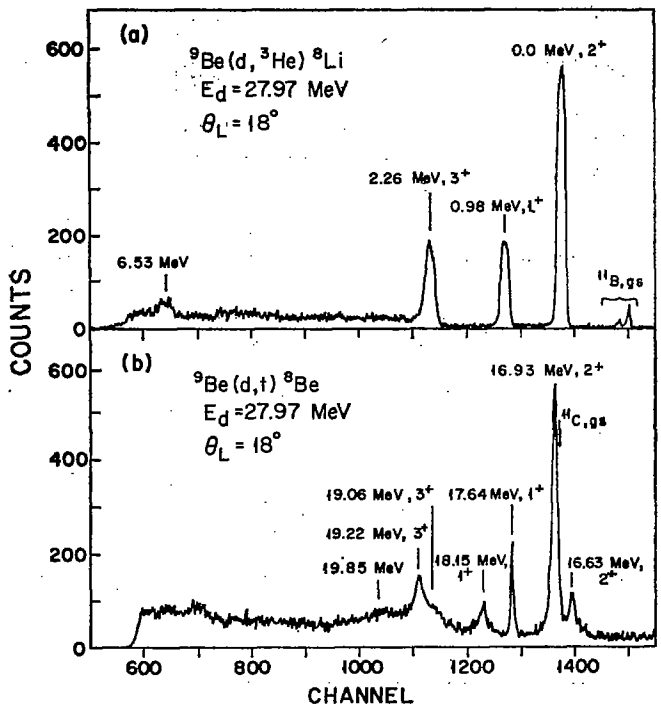

Fig. 2. (a) Spectrum for ^{9}Be(d, ^{3}He)^{8}Li. Observed peaks are indicated by excitation energy and J^{π}. The ^{11}B ground state "doublet" comes from ^{12}C deposited on the front and back of the target. (b) Spectrum for ^{9}Be(d, t)^{8}Be. Observed peaks are indicated by excitation energy and J^{π}. The location of the ^{12}C(d, t_0)^{11}C is also indicated.

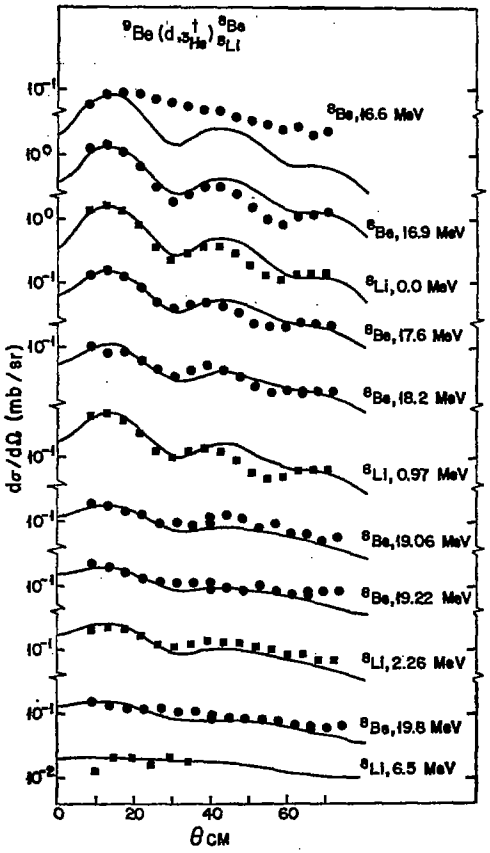

Fig. 3. Angular distributions for single nucleon pickup on ^{9}Be. Solid curves are DWBA calculations using parameter sets D and HB in table 2. See sect. 4 for comments on the validity of the 19.06, 19.22 and 19.8 MeV cross sections.

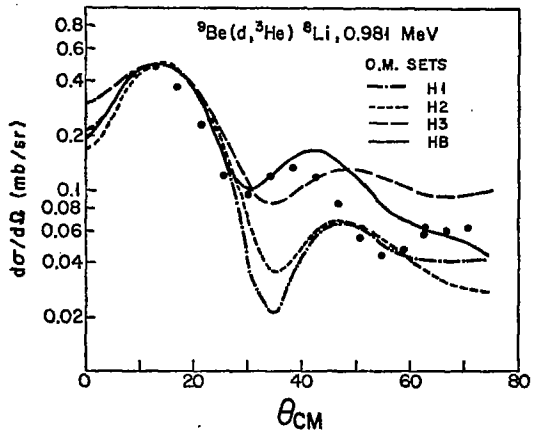

Fig. 4. Comparison of 4 optical model sets. The calculations were done for the ^{9}Be(d, ^{3}He)^{8}Li transition to the 0.980 MeV state. Data is indicated by filled circles. Calculations were done with the deuteron set D of table 2 and the indicated mass-3 sets.

Angular distributions for the four states in ^{8}Li are shown in fig. 3. The 2^+ and 1^+ distributions show a clearly similar structure, while the oscillations in the 3^+ seems to be significantly damped. Over the limited range of data available, the 6.53 MeV angular distribution appears to be rather flat. This could be an artifact of the low ^{3}He energy or could indicate a high l-value for the transfer as suggested by ^{9}Be(t, α) data [38]).

Optical model parameters for mass-three particles were chosen by comparison with the ^{9}Be(d, ^{3}He$_1$)^{8}Li* transition. Calculations were performed using the DWBA code DWUCK [24]) for pure $p_{\frac{3}{2}}$ pickup in the zero-range approximation. Several sets of ^{3}He parameters used in the recent literature to fit elastic ^{3}He scattering and reactions involving ^{3}He particles for the appropriate mass and energy region were tried. Three of the sets are given in table 2. Set HB ("best fit") results from small changes in the geometry parameters of set H3 to improve the phase of the fit with respect to the data. Fig. 4 plots these four parameter sets, normalized to the data at the first maximum. Absolute spectroscopic factors C^2S were extracted from the data using sets D and HB. In table 3 the resulting C^2S are compared to theoretical values obtained by summing contributions from $p_{\frac{1}{2}}$ and $p_{\frac{3}{2}}$ in table 1. To facilitate comparison, the experimental values of C^2S have been renormalized to give best average agreement of experiment with theory of ref. [29]) for the 2^+ and 1^+ states. The renormalized experimental spectroscopic strengths agree very well with those predicted by the calculated wave functions.

3.4. TRITON DATA

A sample ^{9}Be(d, t)^{8}Be spectrum is shown in fig. 2. In order to extract areas for the peaks, it is necessary to fit the spectrum with a formula such as eq. (9). Since the laboratory to c.m. conversion factor is approximately constant over the regions to be fit, it was found to be easier to convert the c.m. form of the yields to a laboratory cross section by integrating it over the energy limits of each channel. This method also makes accounting for experimental energy resolution simpler. The energy calibration was obtained using the narrow 17.642 MeV 1^+ state.

The 1^+ states are sufficiently well separated such that the peaks may be fit independently by simple Breit-Wigner shapes (eq. (6)) rather than by the exact R-matrix form (eq. (9)). The 17.6 MeV 1^+ state was fit with a linear background plus eq. (6) convoluted with a Gaussian to account for experimental energy resolution:

$$Y(E) = B + CE + \int_{-\infty}^{\infty} \frac{A}{(E - E_0 + x)^2 + (\frac{1}{2}\Gamma)^2} \frac{\exp(-x^2/2\Delta^2)}{\Delta(2\pi)^{\frac{1}{2}}} \, dx. \qquad (11)$$

The value of Γ was taken to be 10.7 keV from ref. [11]) and the parameters A, E_0, Δ, B and C were varied using a nonlinear least squares algorithm [41]). Reduced χ^2 (total χ^2 divided by number of free parameters) averaged 1.19 for the spectra fitted. A sample fit is shown in fig. 5. Values for Δ varied from 17 to 37 keV depending on angle. The value of Δ determined from this fit was used for all other fits.

TABLE 3

Experimental and theoretical spectroscopic strengths, C^2S

State [a]	Experiment		Theory		
	absolute	renormalized [b]	[c]	[d]	[e]
^{8}Li, 0.0, 2^+1	1.63	1.03	1.00	1.01	0.92
0.98, 1^+1	0.61	0.39	0.40	0.45	0.41
2.26, 3^+1	0.48	0.30	0.35	0.36	0.24
6.53, ?	0.092	0.058			
^{8}Be, 16.6, 2^+1	0.074	0.047	0.50	0.50	0.46
16.9, 2^+0	1.56	0.99	0.42	0.43	0.36
17.6, 1^+1	0.22	0.14	0.20	0.22	0.20
18.2, 1^+0	0.17	0.11	0.06	0.07	0.07
19.1, 3^+1	0.41	0.26	0.18	0.18	0.12
19.2, 3^+0	0.48	0.30	0.11	0.13	0.13
19.8, ?	0.40	0.25			

[a]) State is denoted by nucleus, excitation energy (MeV) and $J^\pi T$. The ^{8}Be states are here treated as pure states of the isospin indicated.

[b]) Renormalized to give best agreement for the ^{8}Li 2^+1 and 1^+1 states with the calculations of ref. [29]).

[c]) Ref. [29]). [d]) Ref. [3]). [e]) Ref. [30]).

TABLE 4

Values of α_J

State	Ratio used [a]	Spectroscopic amplitude used		Previous value [b]
		[c]	[d]	
2^+	R_{AA}	0.81 ± 0.01	0.82 ± 0.01	
	R_{tt}	0.55–0.87	0.58–0.88	
	R_{th} (16.6)	0.54–0.88	0.56–0.89	0.74–0.76
	R_{th} (16.9)	0.58–0.85	0.72–0.77	
1^+	R_{tt}	0.21 ± 0.03	0.24 ± 0.02	
	R_{th} (17.6)	0.24 ± 0.07	0.26 ± 0.08	$0.29^{+0.02}_{-0.01}$
	R_{th} (18.2)	0.18 ± 0.06	0.22 ± 0.05	
3^+	R_{AA}	0.21 ± 0.04	0.08 ± 0.02	
	R_{tt}	0.14 ± 0.04	0.01 ± 0.03	
	R_{th} (19.1)	0.39–0.84	0.35 ± 0.09	0.31–0.52
	R_{th} (19.2)	[e])	0.32–0.92	

[a]) Numbers in parenthesis for R_{th} denote ^{8}Be state used.

[b]) From sect. 1.

[c]) Average of refs. [3,29]).

[d]) Ref. [30]).

[e]) No solution.

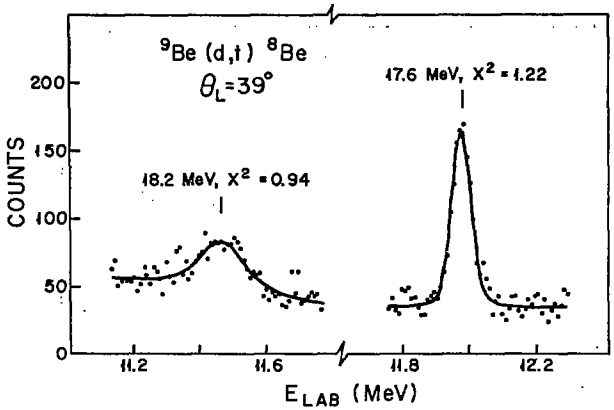

Fig. 5. Typical fits to 1^+ states. The value of the reduced χ^2 for the fit is indicated. The two states were fit independently with eq. (11). For the 17.6 MeV state, Γ was fixed at 10.7 keV. For the 18.2 MeV state, Δ was fixed at the value determined in the 17.6 MeV fit and Γ fixed at 138 keV.

The 18.15 MeV 1^+ was fit with eq. (11) using Γ fixed at 138 keV [ref. [11])]. The parameters A, E_0, B and C were varied, giving an average reduced χ^2 of 1.02. A few fits were made allowing Γ to vary, but did not produce any appreciable improvement in the quality of the fit. A sample fit is shown in fig. 5. If we use an energy E_0 = 17.642 MeV [ref. [11])] for the lower 1^+ state, we find $E_0 = 18.144 \pm 0.005$ MeV for the upper 1^+ state compared with the value of 18.154 ± 0.004 MeV of ref. [11]). Cross sections are plotted in fig. 3. The errors for the cross sections were computed from the errors given by the fitting routine for the parameter A. These errors correspond to the change in the parameter necessary to change *total* χ^2 by one [41]). In practice, the resulting error in the peak areas was very close to the statistical error.

The 2^+ doublet at 17 MeV was fit with a linear background plus eq. (9) convoluted with the Gaussian determined from the fits to the 17.6 MeV 1^+. The ^{12}C(d, t_0)^{11}C reaction overlay the 2^+ states at several angles; since this peak was sharp, the narrow region it obscured was excluded from the fit. Fits were made varying A^0_{aa}, A^0_{ab}, A^0_{bb}, E^0_b and the background parameters B and C. The distant level parameter $\mathscr{J}^0(E)$ was set equal to zero and the remaining parameters were taken from ref. [12]): Γ^0_a = 113 keV, Γ^0_b = 80 keV, $E^0_a = E^0_b - 302$ keV. The resulting fits were very good, with an average reduced χ^2 of 0.99. A sample fit is shown in fig. 6. For comparison a fit using the IBW form (eq. (7)) is also shown. The R-matrix form is clearly superior in fitting the interference region between the peaks. The average energy E^0_b = 16.916 ± 0.005 MeV is in good agreement with the previous value [11]) of 16.911 ± 0.004 MeV. Cross sections are plotted in fig. 3. Several other fits were tried. Allowing Γ^0_b and the separation between the peaks to vary resulted in an average value of Γ^0_b = 78 keV and $E^0_b - E^0_a$ = 304 keV, but did not appreciably alter the areas of the peaks. Callender and Brown [12]) observed that the values for the widths and spacing of these levels showed a systematic variation with beam energy. They attribute this variation to the influence of the distant level parameter $\mathscr{J}^0(E)$ which

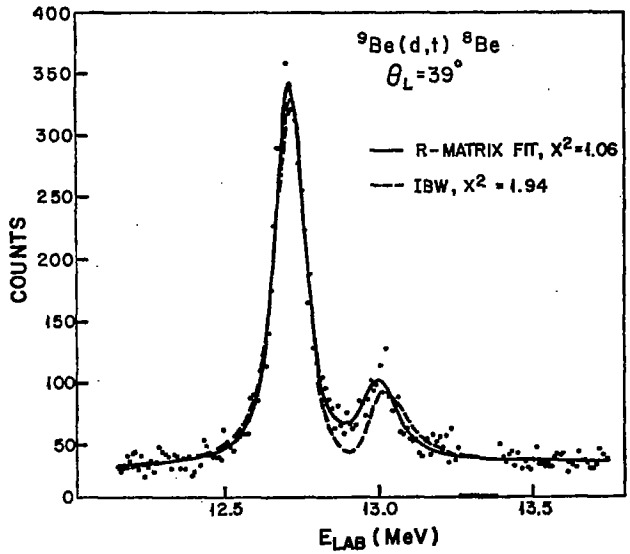

Fig. 6. Typical fit to 2^+ spectrum. The points on the axis correspond to the location of the $^{12}C(d, t_0)$ peak. The fitting functions are a linear background plus the appropriate Gaussian resolution function convoluted with eq. (9) for the solid line and eq. (7) for the dashed line. For the R-matrix fit, the fixed parameters were $\mathcal{S}(E) = 0$, $\Gamma_b^0 = 80$ keV, $\Gamma_a^0 = 113$ keV and $E_b^0 - E_a^0 = 302$ keV. For the IBW fit, the constrained parameters were $\Gamma_a = 113$ keV and $E_b - E_a = 284$ keV. A slightly better IBW fit ($\chi^2 = 1.44$) is obtained with $E_b - E_a = 245$ keV.

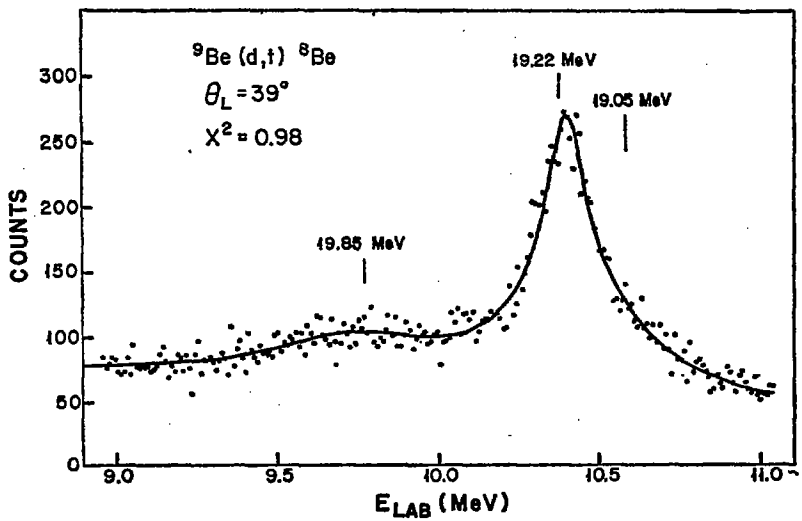

Fig. 7. Typical fit to 3^+ states. The fitting function is a linear background plus the appropriate Gaussian resolution function convoluted with eq. (9) for the 3^+ states and eq. (6) for the 19.85 MeV state. Fixed parameters were $\Gamma_a^0 = 270$ keV, $\Gamma_b^0 = 220$ keV, $\Gamma_3 = 700$ keV and $E_b^0 - E_a^0 = 190$ keV. An additional 1 MeV of spectrum not shown in the figure was fit to improve determination of the background.

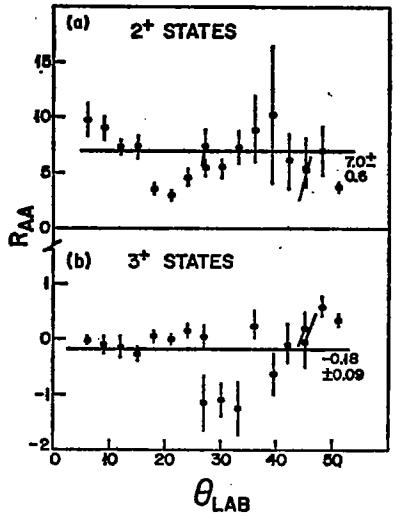

Fig. 8. Experimental values for the ratio R_{AA} defined by eq. (10). The errors shown result from errors assigned to the amplitudes by the fitting routine. Average values are indicated. For the 2^+ states, the points at 18, 21 and 24° are excluded from the average for reasons discussed in the text.

they set equal to zero. We also see a variation in Γ_b^0 (70–90 keV) and $E_b^0 - E_a^0$ (283–323 keV), but the variation is not systematic. Furthermore, allowing $\mathscr{I}^0(E)$ to vary did not appreciably improve the fits here. It is of course possible that the distant levels are populated strongly enough in the (d, α) reaction to affect peak shapes, but not in the (d, t) reaction. Higher resolution work than is presented here would be necessary to observe any effects due to the $\mathscr{I}^0(E)$ term.

As pointed out in sect. 2, the ratio R_{AA} should be independent of angle. In fig. 8a this ratio is plotted for the 2^+ states. The values at lab angles of 18°, 21°, 24° and 51° show an appreciable shift from the average value. However, the first three of these points are at the minimum of the angular distribution where the slight energy difference between the peaks would have its greatest effect on the ratio. Furthermore, the $^{12}C(d, t_0)^{11}C$ peak is in the minimum between the two peaks for these three angles making determination of A_{ab} difficult. Thus we believe the ratio R_{AA} shows the expected behavior.

The spectrum in the region of the 3^+ states (see fig. 7) seems to show two peaks at $\approx$ 19.2 and $\approx$ 19.9 MeV instead of the two expected 3^+ states at 19.06 and 19.22 MeV. An attempt was made to fit several spectra with two independent Breit-Wigners of the form of eq. (11). Reasonable fits ($\chi^2 \approx 1.2$) were obtained with peaks at $\approx$ 19.16 and 19.86 MeV with widths of $\approx$ 230 and $\approx$ 630 keV, respectively. However, these parameters showed a wider variation from angle to angle than was the case for the 2^+ or 1^+ states. On the basis of similar experience with the $^{10}B(d, \alpha)$ reaction, Callender and Brown [12]) have suggested that perhaps only one 3^+ level actually exists at this energy. More recently, however, an analysis [42]) of phase shifts for the

^{7}Li($\vec{p}$, p)^{7}Li reaction indicates that there must be two 3^+ states near 19.06 and 19.22 MeV in ^{8}Be. Furthermore, an anamolous behavior of the phase shifts could indicate interference between the two levels.

On the basis of these phase shift results, we attempted to fit the 3^+ region with eq. (9) convoluted with a Gaussian resolution function plus a linear background. The fits were poor, due to an apparent discontinuity in the background level similar to that seen in the (d, ^{3}He) spectra. Callender and Brown [12]) have suggested that this discontinuity may be due to the three-body phase space associated with the opening of the ^{7}Be+n channel at 18.9 MeV. A phase-space shape with an energy-independent matrix element [43]) was included incoherently with the linear background. Neither ^{7}Be+n nor ^{7}Li+p phase-space shapes gave a suitable fit to the background. The phase-space shapes tend to rise to a maximum near 23 MeV of excitation whereas the data shows a maximum in the background near 20 MeV. Good fits were obtained by replacing the phase-space shape with Breit-Wigner shape incoherent with the 3^+ states. In order to save computer time, a grid on values for E_b^0–E_a^0, Γ_a^0 and Γ_b^0 was performed. The best values obtained were E_b^0–E_a^0 = 190±30 keV, Γ_a^0 = 270±30 keV and Γ_b^0 = 220±30 keV. Previously values [11]) are E_b^0–E_a^0 = 160 keV, Γ_a^0 = 270±20 keV and Γ_b^0 = 208±30 keV. A search on the width of third level gave Γ_3 = 700±100 keV. The entire data set was then fit varying only the parameters A_{aa}^0, A_{bb}^0, A_{ab}^0, E_a^0, A_3, E_3 and the linear background coefficients. The fits were good, with an average χ^2 of 1.05. Fits were then tried for several spectra allowing the previously constrained parameters to vary. No significant changes were seen. A sample fit is shown in fig. 7. We find an average value of E_a^0 = 19.071±0.010 MeV and E_3 = 19.86±0.05 MeV. The value for E_a^0 is in good agreement with the previous value [11]) E_a = 19.06±0.02 MeV. As a check on the fits, a plot of R_{AA} is shown in fig. 8b. The data is independent of angle as expected. Further comments on the validity of the 3^+ fits are made in sect. 4. There are several broad states near our 19.86 MeV state. The most likely candidates [on the basis of energy and widths given in ref. [11])] are the 20.1 MeV 2^+, $T = 0$ ($\Gamma \approx 1.1$ MeV) and the 20.2 MeV 0^+, $T = 0$ ($\Gamma < 1$ MeV). Barker [44]) has suggested that a better energy for the 2^+, $T = 0$ state is 19.9 MeV from ^{7}Li(p, α) data. Thus it would seem that the most likely assignment for the 19.86 MeV state on the basis of this information is 2^+, $T = 0$.

The cross sections resulting from these fits are plotted in fig. 3 along with DWBA calculations. The calculations were done with the same parameters as for the (d, ^{3}He) data. It is clear from the fits that the t and ^{3}He angular distributions are fit equally well, except for the case of the ^{8}Be 16.63 MeV 2^+. Since the 16.63 MeV state is weak and does not have an $l = 1$ shape, it presumably populated primarily through the compound nucleus. Such a result is to be expected on the basis of the picture of Marion [1]) in which the 16.63 MeV level has a parentage chiefly of ^{7}Li+p. Spectroscopic factors were extracted and the results are shown in table 3. Unlike the (d, ^{3}He) case, the renormalized experimental values of C^2S are in gross disagreement with theoretical predictions. It might be assumed that the theoretical values are wrong,

but the good agreement with experiment for the ^{8}Li data gives confidence in at least the relative $T = 1$ spectroscopic factors. As will be shown later, these results are consistent with isospin mixing in the 2^+ and 1^+ ^{8}Be doublets. This may be seen in a simple fashion by noting that the sum of $T = 0$ and $T = 1$ C^2S for experiment and theory are approximately equal. The summed experimental C^2S for the 3^+ states is considerably in excess of that predicted by theory, however. The spectroscopic factor for the 19.9 MeV state is also quite large. The largest unassigned spectroscopic strength calculated by ref. [29]) is 0.067 for a 2^+ $T = 1$ state. The possible assignment previously mentioned (2^+, $T = 0$) would provide only $C^2S = 0.044$.

4. Results and conclusions

Values for α_J may now be obtained by solving eqs. (3), (4) and (10). Values for $t_{\frac{1}{2}}$ and Q in these equations were calculated with the optical-model parameters in sect. 3. Values for $t_{\frac{1}{2}}$ show a strong angular dependence. Fig. 9 shows a plot of $t_{\frac{1}{2}}$ for the ^{8}Be, 16.93 MeV state using the four sets of mass-3 parameters given in table 2. While there is considerable variation among the parameter sets, near the first maximum in the cross section at $\theta_{c.m.} \approx 13^\circ$ the parameter sets agree within 10 %. The solutions for α_J are insensitive to $t_{\frac{1}{2}}$, so it proved sufficient to use the value of $t_{\frac{1}{2}}$ calculated at the first maximum of the cross section using set D-HB. For the 2^+ states (^{8}Li ground state and ^{8}Be, 16.63 and 16.93 MeV) the value of $t_{\frac{1}{2}}$ was 0.89 while for the 1^+ states (^{8}Li, 0.98 MeV and ^{8}Be, 17.6 and 18.2 MeV) the value was 0.93. Since the (d, t) and (d, ^{3}He) data show a significant shift with respect to each other in the c.m., the DWBA calculations for Q_{tt} and Q_{th} were interpolated to the same lab angles. In figs. 10 and 11, the ratios Q_{th} and Q_{tt} are plotted along with the data. The Q-curves are normalized to the data at the first maximum of the angular distribution where compound nuclear contributions should be least important. The ratio R/Q is indicated in the figures. Except for the 1^+, 17.6 MeV/0.98 MeV ratio and the 3^+ ratios, the fits are not very good. The results for the 16.6 MeV 2^+ are further indications that the

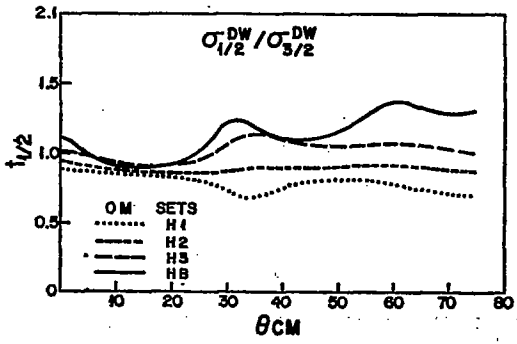

Fig. 9. DWBA calculations for the ratio of cross sections for pickup from $p_{\frac{1}{2}}$ and $p_{\frac{3}{2}}$ orbitals for ^{9}Be(d, t)^{8}Be*, 16.9 MeV state. Optical model parameters used are those of table 2.

reaction is not direct. Results for the 16.9 MeV 2^+ and the 18.2 MeV 1^+ are more puzzling. Calculations for the other sets of optical parameters for the 16.9 MeV state produced somewhat better fits to the R_{th} ratio and a value for R/Q as much as 10 % lower than the fits in fig. 11. Gaillard *et al.* [45]) in fitting σ(d, t)/σ(d, ^{3}He) for $T = 0$ targets found that using the same form factor for both reactions improved fits to the data. Calculations using a common form factor produce better fits to the 16.9 MeV/ 0.0 MeV ratio, but make other fits worse. The resulting R/Q for the 16.9 MeV/ 0 MeV ratio is $\approx$ 10 % lower than normal fits. We choose to use the results of the fits using sets D and HB, but on the basis of variations with optical model sets, we assign a systematic error of $\pm$10 % to Q. Since the errors in cross sections tend to be much smaller than this, the major error in R/Q comes from this uncertainty in the DWBA calculations.

The theoretical and experimental values for R_{AA}, R_{tt} and R_{th} are plotted in figs. 12–14. The values of α_J obtained are listed in table 4 and discussed below. Since the spectroscopic amplitudes from Barker [3]) and Cohen and Kurath [29]) are virtually identical, only results using values of Barker will be given below. Results for α_J

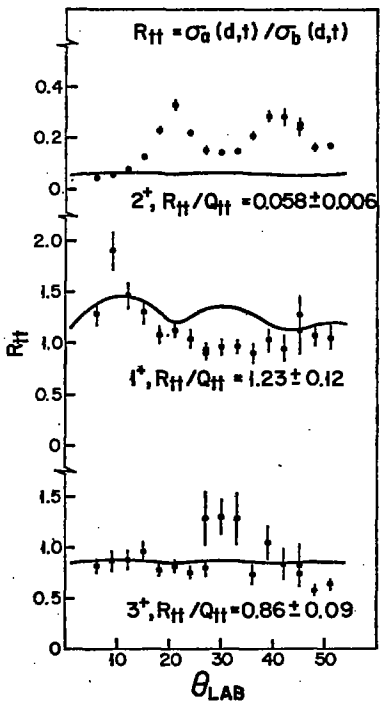

Fig. 10. The ratio $R_{tt} = \sigma_a$(d, t)/σ_b(d, t) where a (b) is the lower (upper) state of the isospin-mixed doublet. The points are data and the lines are DWBA calculations using optical model sets D and HB from table 2 normalized to the data. The curves are labelled by the J^π of the doublet and the ratio R_{tt}/Q_{tt} averaged over the first four data points.

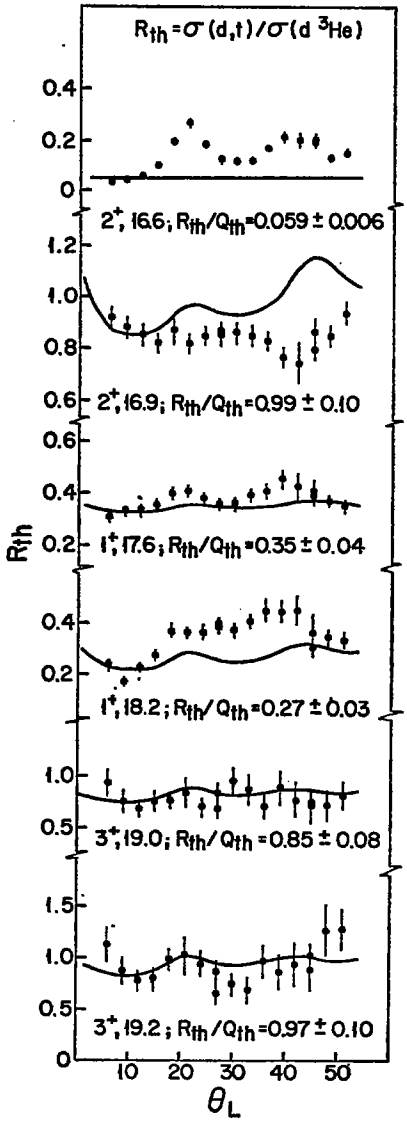

Fig. 11. The ratio $R_{th} = \sigma(d, t)/\sigma(d, {}^3He)$. The points are data and the lines DWBA calculations normalized to the data. The curves are labelled by J^π, excitation energy of the ^8Be state and the ratio R_{th}/Q_{th} averaged over the first four data points.

using Boyarkina's spectroscopic amplitudes [30]) will be given when they differ appreciably from those of Barker.

4.1. THE 2^+ STATES

The values of α_2 obtained using the ratio R_{AA} should be the least dependent on compound nuclear and Q-value effects. From fig. 12 we obtain values for α_2 of

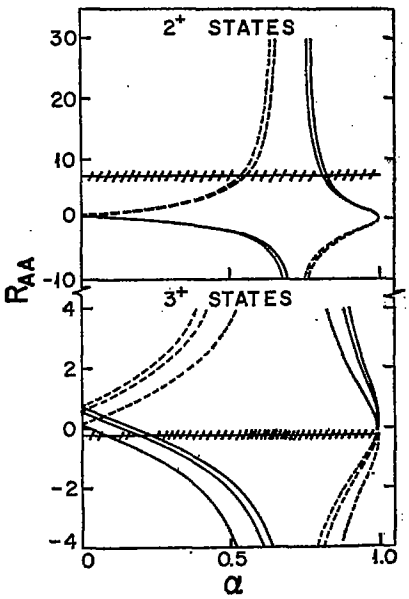

Fig. 12. Theoretical (solid and dashed curves) and experimental (horizontal lines) values for the quantity R_{AA} defined by eq. (10). The cross-hatching on the experimental lines indicates the magnitude of the experimental errors. The three sets of lines correspond to the three sets of spectroscopic amplitudes in table 1. Solid lines correspond to $\beta > 0$, dashed lines to $\beta < 0$.

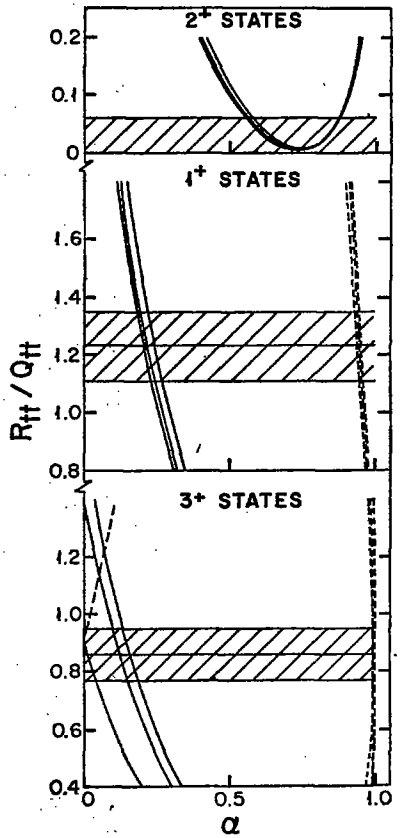

Fig. 13. Theoretical and experimental values for R_{tt}/Q_{tt} defined by eq. (3). For the 2^+ states, the experimental result is an upper limit only. See caption to fig. 12.

0.81 ± 0.01 and 0.54 ± 0.02. Since $\Gamma_a < \Gamma_b$, the former solution is the correct one. This value must be compared with the values 0.74 to 0.76 obtained in sect. 1 from the ratios of the widths of the states. The most probable cause of this large discrepancy is an error in the relative magnitudes of the theoretical spectroscopic amplitudes. The values of α_2 are only weakly dependent on $p_{\frac{1}{2}}$ spectroscopic amplitudes (a change of 20 % in $S^{\frac{1}{2}}_{\frac{1}{2}0}$ produces a change of less than 0.01 in α_2) but strongly dependent on the $p_{\frac{3}{2}}$ amplitudes. If we consider the relative magnitude of $S^{\frac{1}{2}}_{\frac{1}{2}1}$ to be well determined by the (d, ^{3}He) data, then we may vary $S^{\frac{1}{2}}_{\frac{1}{2}0}$ to give the appropriate value of α_2. A 20 % decrease in $S^{\frac{1}{2}}_{\frac{1}{2}0}$ (e.g. from -0.5758 to -0.6910 for Barker's spectroscopic amplitudes [3])) gives $\alpha_2 = 0.765$ in agreement with the results of width measurements. Due to the relatively small size of $S^{\frac{1}{2}}_{\frac{1}{2}0}$, however, the total spectroscopic strength for the 2^+ states would increase only 8 %, which would probably not be detected from

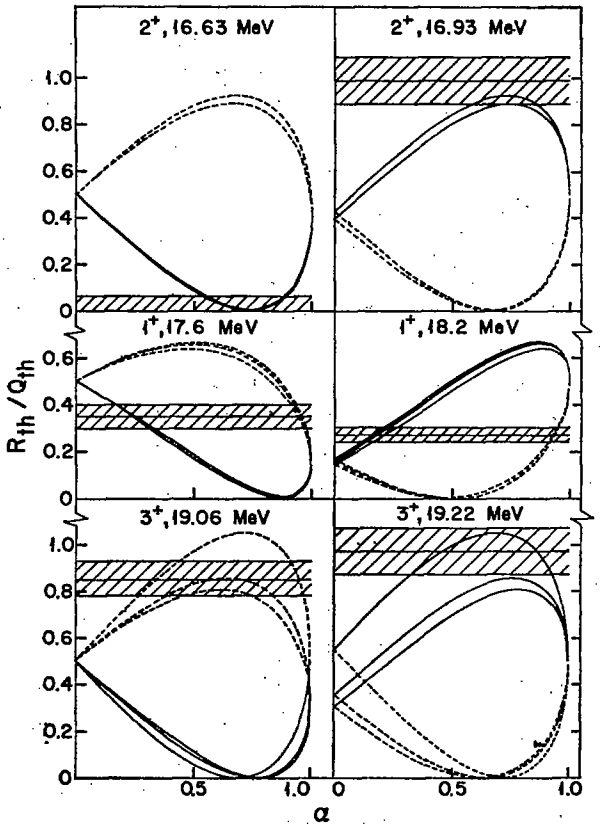

Fig. 14. Theoretical and experimental values for R_{th}/Q_{th} defined by eq. (4). For the 2^+, 16.63 MeV state, the experimental value is an upper limit only. See caption to fig. 12.

cross section measurements. A similar discrepancy in $T = 0$ spectroscopic amplitudes has recently been observed [46]) in ^{12}C.

The ratio R_{tt} gives only limits on the value of α_2 since the 16.6 MeV state is populated primarily through the compound nucleus. If we use $\alpha_2 = 0.75$ we may calculate the value of R_{tt}/Q_{tt} to be ≈ 0.02 whereas the experimental value varies with angle from 0.04 to 0.30. If we take the average value at the first maximum of R_{tt} as an upper limit, we find values of α_2 ranging from 0.55 to 0.88.

The ratio R_{th} for the 16.63 MeV state suffers from the same difficulty as for R_{tt}. Again by using the experimental value as an upper limit, we obtain a range of possible values of $0.54 < \alpha_2 < 0.89$. For the 16.93 MeV state we see from fig. 14 that a large uncertainty in α_2 occurs because of the 10 % uncertainty in Q_{th}. The results, however, are consistent with the results of R_{AA} and R_{tt}.

4.2. THE 1^+ STATES

Since these states were not fit with eq. (9) the ratio R_{AA} cannot be obtained from this experiment. Furthermore, since the three-body breakup background is so large in the region between the two 1^+ states, it is doubtful that an accurate value for A_{ab}^0 could be extracted. The ratio R_{tt} is relatively independent of compound nuclear contributions, since $\sigma_a \approx \sigma_b$ and the compound nuclear contributions to the two states should be approximately equal. From fig. 13 we obtain $\alpha_1 = 0.94 \pm 0.01$ or 0.21 ± 0.03. Results of the ^{10}B(d, α) experiment [12]) indicate the latter solution is the correct one. The ratio R_{th} in fig. 14 gives values of $\alpha_1 = 0.24 \pm 0.07$ or $0.96^{+0.02}_{-0.09}$ for the 17.6 MeV state and $\alpha_1 = 0.18 \pm 0.06$ or 0.93 ± 0.02 for the 18.2 MeV state. The three determinations are thus consistent, with an average value of $\alpha_1 = 0.21 \pm 0.03$, although they are somewhat lower than the previous value of 0.29 given in sect. 1. It should be noted that the value of α_1 obtained here gives a charge dependent matrix element of -103 ± 14 keV. This value is in agreement with the calculation of Goldhammer [8]) which gives -117 keV. The value $\alpha_1 = 0.29$, on the other hand, gives a matrix element of $-(139^{+9}_{-5})$ keV.

4.3. THE 3^+ STATES

From fig. 12 we see that the ratio R_{AA} gives $\alpha_3 = 0.21 \pm 0.04$ (Barker's $S^{\frac{1}{2}}$), 0.08 ± 0.02 (Boyarkina's $S^{\frac{1}{2}}$) or 0.99 ± 0.01 (both). Results of the ^{7}Li(p, γ)^{8}Be (2.90) reaction [3]) indicate the 19.06 MeV state should be primarily $T = 1$, eliminating the third solution. From fig. 13 the ratio R_{tt}/Q_{tt} gives $\alpha_3 = 0.14 \pm 0.04$ (Barker) or 0.01 ± 0.03 (Boyarkina). The 10 % error assigned to Q_{th} produces a large ambiguity in values of α_3 obtained from fig. 14. For the 19.06 MeV state we obtain $0.39 \leqq \alpha_3 \leqq 0.84$ (Barker) and 0.35 ± 0.09 (Boyarkina). For the 19.22 MeV state there is no solution for Barker's $S^{\frac{1}{2}}$ while Boyarkina's $S^{\frac{1}{2}}$ give $0.32 \leqq \alpha_3 \leqq 0.92$. The values of α_3 using R_{th} are thus in disagreement with those from R_{tt} and R_{AA}. On the other hand, R_{th} gives the only result consistent with the previous value of 0.41 obtained [3]) for α_3.

Barker [44]) has pointed out that using eq. (9) to fit the 3^+ doublet is inappropriate. Eq. (9) was derived for the case of a single open decay channel, whereas the 3^+ states may decay in both the p- and n-channels. The correct form for the yield to the 3^+ doublet is given by Barker [44]) as

$$Y(E) = Y_n + Y_p, \tag{12}$$

where

$$Y_n = P_n \sum_x P_x \left| \frac{g_{ax}\gamma_{an}(E_b - E) + g_{bx}\gamma_{bn}(E_a - E) - L_p^0(g_{ax}\gamma_{bp} - g_{bx}\gamma_{ap})(\gamma_{an}\gamma_{bp} - \gamma_{bn}\gamma_{ap})}{D} \right|^2,$$

Y_p similar with $n \leftrightarrow p$

$$D = (E_a - E)(E_b - E) - (E_a - E)\sum_c L_c^0 \gamma_{bc}^2 - (E_b - E)\sum_c L_c^0 \gamma_{ac}^2 + L_p^0 L_n^0 (\gamma_{an}\gamma_{bp} - \gamma_{bn}\gamma_{ap})^2,$$

$$L_c^0 = S_c - B_c + iP_c,$$

where

$$c = \text{n or p.}$$

Since the neutron threshold is nearby, the energy dependence of the penetrability P_n and shift factor S_n might also need to be explicitly considered. Due to the complexity of eq. (12), we did not attempt fits using it. Furthermore, since eq. (9) is a special case of eq. (12) and it already gives good fits ($\chi^2 \approx 1$) to the data, it is not likely that the parameters of eq. (12) could be determined to any great accuracy with our data. It should be noted that the previous determination of α_3 used eq. (6) to describe the shape of the 19.22 MeV state. It is clear that eq. (12) might give a rather different result for this shape and therefore the previous value of $\alpha_3 = 0.41$ is highly suspect. Because eq. (9) rather than eq. (12) was used here to fit the 3^+ doublet, it is not surprising that the experimental C^2S disagree badly with theory for the 3^+ states and the 19.8 MeV state.

In conclusion, we may note that of the methods presented here, only using R_{AA} will give accurate values for α close to maximal mixing ($\alpha \approx 0.7$). The ratios R_{tt} and R_{th} tend to have broad minima or maxima near $\alpha = 0.7$, making determination of α difficult. Conversely, for small mixing ($\alpha \approx 0$ or 1) R_{tt} and R_{th} will give accurate values of α if good wave functions are available even if DWBA calculations can be trusted to no better than 10 %.

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