

PROBLEM 1

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Problem 1 (Wooldridge B.10). Suppose that at a large university, college grade point averages, GPA, and SAT score, SAT, are related by $\mathbb{E}[GPA|SAT] = 0.70 + 0.002SAT$.

- (1) Find the expected GPA when SAT = 800. Find $\mathbb{E}[GPA|SAT = 1400]$. Comment on the difference.
- (2) If the average SAT in the university is 1100, what is the average GPA?
- (3) If a student's SAT score is 1100, does this mean he or she will have the GPA found in (2)? Explain.

Problem 2. A Monet expert is given a painting purported to be a lost Monet. He is asked to assess the chances that it is genuine and has the following information:

- In general, only 1% of the “found” paintings he receives turn out to be genuine, an event we'll call G
- “Found” paintings have a different frequency of use of certain pigments than genuine Monets do:
 - (1) cadmium yellow Y appears in 30% “found” paintings, but only 10% genuine ones
 - (2) raw umber U appears in 90% of “found” paintings, but only 40% of genuine ones
 - (3) burnt sienna S appears in 20% of “found” paintings, but 70% of genuine paintings
- This particular painting uses burnt sienna, but not cadmium yellow or raw umber.

What is the probability that this particular painting is genuine? Do we have to make any additional assumptions to answer the question?

Problem 3. Let $\{(x_i, y_i) : i = 1, 2, \dots, n\}$ be a sample. Let $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ and $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$ be the sample means of X and Y .

- (1) Show that $\sum_{i=1}^n (x_i - \bar{x}) = 0$
- (2) Using the result in part (1), show that

$$\sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n x_i(x_i - \bar{x})$$

- (3) Show that

$$\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^n x_i(y_i - \bar{y}) = \sum_{i=1}^n y_i(x_i - \bar{x})$$