Macroeconomics
Lecture 9 – Small Open Economy RBC; Large Open Economy

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Outline

Balance of payments, International Investment Position: reminders

Trade balance in open economy can be obtained from the GDP decompositon

$$Y_t = C_t + I_t + G_t + TB_t$$
$$TB_t = Y_t - C_t - I_t - G_t$$

We assume $G_t = 0$ for this lecture.

$FA_t = CA_t$

Stock convention for IIP: beginning-of-period. So:

$$\begin{aligned} \underline{IIP_{t+1}} &= \underline{IIP_t} + CA_t \\ &= IIP_t + TB_t + r_{t-1} IIP_t \\ &= TB_t + (1 + r_{t-1}) \underline{IIP_t} \end{aligned}$$

In a **steady state**, $CA_{ss} = 0$ because IIP_{ss} constant

More on International Investment Position

IIP is a difference of assets in the rest of the world and liabilities vis-a-vis rest of the world. So:

1. In an RBC-type model, representative household can be assumed to choose IIP in a same way as it chooses B for saving purposes

2. Negative IIP means net debt of the country with respect to rest of world. Can be an optimal choice

Outline

A SOE RBC model

The RBC framework extended to include exchanges with the rest of the world:

1. Foreign assets held instead of domestic bonds ⇒ *IIP* replaces *B* in the budget constraint

2. Exogenous interest rate ⇒ steady state existence and stability problem ⇒ an additional assumption on foreign asset holding is needed

   ▷ we will assume **interest rate premium**, or external debt-elastic interest rate (EDEIR) assumption

3. **Capital adjustment cost** limits volatility of investment ⇒ limits volatility of **trade balance**

Household problem (in Social Planner form)

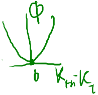
$$\max_{C_t, N_t, IIP_{t+1}, K_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, N_t)$$

subject to

$$C_t + \overbrace{K_{t+1} - (1-\delta)K_t}^{I_t} + IIP_{t+1} + \Phi(K_{t+1} - K_t) = \overbrace{A_t F(K_t, N_t)}^{= w_t N_t + R_t K_t} + (1 + r_{t-1}) IIP_t$$

A_0, K_0, IIP_0 – given; $\quad \ln A_t = \rho \ln A_{t-1} + \epsilon_t$

$\Phi(\cdot)$ is a convex **capital adjustment cost** function with $\Phi(0) = \Phi'(0) = 0$; $\Phi''(\cdot) > 0$



Notice a (yet) another type of utility function: non-separable, with labor and not leisure as argument: $u(\underset{+}{C_t}, \underset{-}{N_t})$

$$IIP_{t+1} = IIP_t + CA_t \underline{= Y_t - C_t - I_t - Cap.adj.cost_t}$$

$$IIP_{t+1} = (1+r_{t-1})\, IIP_t + TB_t$$

$$IIP_{t+1} = (1+r_{t-1})\, IIP_t + Y_t - C_t - I_t - Cap.adj.cost_t$$

$$C_t + I_t + IIP_{t+1} + Cap.adj.cost_t = Y_t + (1+r_{t-1})\, IIP_t$$

$c_t + (1+r)\, d_{t-1} = y_t + d_t$ – seen in TD; d_t - debt at end of t

$$\boxed{IIP_t = -d_{t-1}}$$

$$C_t - (1+r)\, IIP_t = Y_t - IIP_{t+1}$$

$$C_t + IIP_{t+1} = Y_t + (1+r)\, IIP_t$$

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[u(C_t, N_t) + \lambda_t \big(C_t + K_{t+1} - (1-\delta) K_t + ||P_{t+1} + \varphi(K_{t+1} - K_t) - A_t F(K_t, N_t) - (1 + r_{t-1}) ||P_t \big) \Big]$$

$$\mathcal{L}_t = \beta^t \Big[u(C_t, N_t) + \lambda_t \big(C_t + K_{t+1} - (1-\delta) K_t + ||P_{t+1} + \varphi(K_{t+1} - K_t) - A_t F(K_t, N_t) - (1 + r_{t-1}) ||P_t \big) \Big]$$
$$+ \beta^{t+1} \mathbb{E}_t \Big[u(C_{t+1}, N_{t+1}) + \lambda_{t+1} \big(C_{t+1} + K_{t+2} - (1-\delta) K_{t+1} + ||P_{t+2} + \varphi(K_{t+2} - K_{t+1}) - A_{t+1} F(K_{t+1}, N_{t+1}) - (1 + r_t) ||P_{t+1} \big) \Big]$$

$$\mathcal{L}_t = \beta^t \Big[u(C_t, N_t) - \lambda_t \big(C_t + K_{t+1} - (1-\delta)K_t + IIP_{t+1} + \varphi(K_{t+1} - K_t)$$
$$- A_t F(K_t, N_t) - (1 + r_{t-1}) IIP_t\big)\Big]$$
$$+ \beta^{t+1} \mathbb{E}_t \Big[u(C_{t+1}, N_{t+1}) - \lambda_{t+1}\big(C_{t+1} + K_{t+2} - (1-\delta)K_{t+1} + IIP_{t+2}$$
$$+ \varphi(K_{t+2} - K_{t+1})$$
$$- A_{t+1} F(K_{t+1}, N_{t+1}) - (1 + r_t) IIP_{t+1}\big)\Big]$$

FOC:

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t \big[u'_c(C_t, N_t) - \lambda_t\big] = 0$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = \beta^t \big[u'_N(C_t, N_t) + \lambda_t A_t F'_N(K_t, N_t)\big] = 0$$

$$\frac{\partial \mathcal{L}}{\partial IIP_{t+1}} = \beta^t(-\lambda_t) + \beta^{t+1} \mathbb{E}_t \lambda_{t+1}(1 + r_t) = 0 \Rightarrow \text{Euler equation}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = \beta^t(-\lambda_t)\big(1 + \varphi'(K_{t+1} - K_t)\big) - \beta^{t+1} \mathbb{E}_t \Big(\lambda_{t+1}\big(-(1-\delta) + (-1)\varphi'(K_{t+2} - K_{t+1})$$
$$- A_{t+1} F'_K(K_{t+1}, N_{t+1})\big)\Big)$$

Planner's FOC

$\frac{\partial \mathcal{L}}{\partial C_t}$:

$$u'_C(C_t, N_t) = \lambda_t$$

$\frac{\partial \mathcal{L}}{\partial IIP_{t+1}}$:

$$\lambda_t = \beta(1 + r_t)E_t\lambda_{t+1}$$

$\frac{\partial \mathcal{L}}{\partial N_t}$:

$$-u'_N(C_t, N_t) = \lambda_t A_t F'_N(K_t, N_t)$$

$\frac{\partial \mathcal{L}}{\partial K_{t+1}}$:

$$1 + \Phi'(K_{t+1} - K_t) = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[A_{t+1}F'_K(K_{t+1}, N_{t+1}) + 1 - \delta + \Phi'(K_{t+2} - K_{t+1})\right]$$

$\frac{\partial \mathcal{L}}{\partial \lambda_t}$:

$$C_t + K_{t+1} - (1-\delta)K_t + IIP_{t+1} + \Phi(K_{t+1} - K_t) = A_t F(K_t, N_t) + (1 + r_{t-1})IIP_t$$

Euler equation in small open economy – technical problem

$$u'_C(C_t, N_t) = \beta(1 + r_t) E_t u'_C(C_{t+1}, N_{t+1})$$

where r_t should be exogenous in a classical SOE definition
If r_t not constant, s.s. trivially does not exists

Even if $r_t = r$, we get that in s.s.:

$$1 = \beta(1 + r)$$

So we have a strong restriction on preferences for s.s. to exist.

If this restriction is true, then recall that we get
$u'_C(C_t, N_t) = E_t u'_C(C_{t+1}, N_{t+1}) \Rightarrow$ marginal utility must be a **random walk** in equilibrium, does not revert to a steady state

Euler equation problem in SOE: solutions

Several *ad hoc* solutions proposed in literature:

1. Interest rate that depends on economy variables (depart from typcal SOE)
    - ▷ can be interpreted as a **risk premium**
2. Endogenous discount rate
3. Adjustment cost for foreign assets

Interest Rate Premium/ EDEIR

External debt-Elastic Interest Rate (EDEIR) will be used to achieve existence and stability of steady state.

$$r_t = r^* + p(\underset{-}{IIP_t})$$

- r^* = world interest rate, assumed **constant**
- $p(IIP_t)$ = country interest-rate premium, decreasing in IIP (increasing in external debt)
  - intuition: high debt ⇒ repayment risk ⇒ premium
- "External" means that **the household takes the premium as given** i.e. we **do not use the function when obtaining** $\frac{\partial \mathcal{L}}{\partial IIP_{t+1}}$

Balance of Payments indicators revisited

Capital adjustment cost is a "waste" of GDP:

$$Y_t = C_t + I_t + \Phi(K_{t+1} - K_t) + TB_t$$

so we **must use it in TB calculation**:

$$TB_t = Y_t - C_t - I_t - \Phi(K_{t+1} - K_t)$$

Current account reminder

$$\begin{aligned} CA_t &= TB_t + r_{t-1} IIP_t \\ &= IIP_{t+1} - IIP_t \end{aligned}$$

Equilibrium

An equilibrium is a sequence $\{C_t, N_t, K_{t+1}, IIP_{t+1}\}$, such that, given exogenous initial conditions K_0, IIP_0, A_0 and exogenous productivity $\ln A_{t+1} = \rho \ln A_t + \varepsilon_{t+1}$, the following conditions are satisfied:

$$u'_C(C_t, N_t) = \beta(1 + r^* + p(IIP_t))E_t u'_C(C_{t+1}, N_{t+1})$$

$$-\frac{u'_N(C_t, N_t)}{u'_C(C_t, N_t)} = A_t F'_N(K_t, N_t)$$

$$1 = \beta E_t \left\{ \frac{u'_C(C_{t+1}, N_{t+1})}{u'_C(C_t, N_t)} \frac{A_{t+1}F'_K(K_{t+1}, N_{t+1}) + 1 - \delta + \Phi'(K_{t+2} - K_{t+1})}{1 + \Phi'(K_{t+1} - K_t)} \right\}$$

$$C_t + K_{t+1} - (1-\delta)K_t + IIP_{t+1} + \Phi(K_{t+1} - K_t) = A_t F(K_t, N_t) + IIP_t(1 + r^* + p(IIP_t))$$

Steady State

The steady state $(IIP_{ss}, K_{ss}, C_{ss}, N_{ss})$ is solution to a system:

$$-\frac{u'_N(C_{ss}, N_{ss})}{u'_C(C_{ss}, N_{ss})} = F'_N(K_{ss}, N_{ss})$$

$$C_{ss} + \delta K_{ss} - IIP_{ss} \cdot (r^* + p(IIP_{ss})) = F(K_{ss}, N_{ss})$$

$$1 = \beta(1 + r^* + \overbrace{p(IIP_{ss})}^{=0})$$

$$1 = \beta\left[F'_K(K_{ss}, N_{ss}) + 1 - \delta\right]$$

Balance of Payments in Steady State

Since $IIP_t = IIP_{ss}$ constant in s.s., $CA_{ss} = 0$

Recall then $CA_{ss} = TB_{ss} + r_{ss} \cdot IIP_{ss} \Rightarrow TB_{ss} = -r_{ss} \cdot IIP_{ss}$.

Intuition: net **capital flows** (FA balance) are null in s.s., so the flows due to trade balance are exactly compensated by flows due to **income balance**, for identity $CA = FA$ to hold

Functional Forms

Instantaneous utility function – Greenwood–Hercowitz–Huffman (GHH) prefereces :

$$u(C,N) = \frac{(C - N^{\omega}/\omega)^{1-\sigma} - 1}{1-\sigma}; \quad \omega > 1; \sigma > 0$$

⇒ labor supply depends on real wage only (not on consumption) – **prove this using FOC**

Interest rate premium

parameter (equal to s.s. IIP)

$$p(IIP_t) = \psi \cdot \left(e^{\bar{IIP} - IIP_t} - 1\right); \quad \psi > 0$$

$$\Downarrow$$

$$p(IIP_{ss}) = 0$$

Production function – Cobb-Douglas

$$F(K,N) = K^{\alpha} N^{1-\alpha}; \quad \alpha \in (0,1)$$

Capital adjustment cost function – quadratic

$$\Phi(x) = \frac{\phi}{2}x^2; \quad \phi > 0$$

6 structural parameters: σ, ω, ψ, $\bar{IIP}$, α, ϕ

GHH preferences and labor supply:

$$-\frac{u'_N(C_t, N_t)}{u'_C(C_t, N_t)} = A_t \underbrace{F'_N(K_t, N_t)}_{= w_t}$$

$$\begin{cases} u'_N(C_t, N_t) = -N_t^{\omega - 1}\left(C_t - \frac{N_t^{\omega}}{\omega}\right)^{-\sigma} \\ u'_C(C_t, N_t) = \left(C_t - \frac{N_t^{\omega}}{\omega}\right)^{-\sigma} \end{cases}$$

$$-\frac{-N_t^{\omega-1}\left(C_t - \frac{N_t^{\omega}}{\omega}\right)^{-\sigma}}{\left(C_t - \frac{N_t^{\omega}}{\omega}\right)^{-\sigma}} = w_t \Leftrightarrow N_t^{\omega - 1} = w_t$$

$$N_t = w_t^{\frac{1}{\omega - 1}}$$

$\frac{1}{\omega - 1}$ is elasticity of labor supply

Outline

Canadian Economy: Business-Cycle Statistics

Canada – developed economy, but 2.1 % of world GDP $\Rightarrow$ "small" (compare to China – 18.4%; USA – 24% – large open economies).

Historical business-cycle patterns in 1946-1985:

Variable (x_t)	Moments		
	σ_{x_t}	$\text{corr}(x_t, x_{t-1})$	$\text{corr}(x_t, Y_t)$
Y	2.8	0.61	1
C	2.5	0.7	0.59
I	9.8	0.31	0.64
N	2	0.54	0.8
$\frac{TB}{Y}$	1.9	0.66	-0.13

Source: Mendoza AER, 1991. Annual data. Log-quadratically detrended.

Canadian Economy: Business-Cycle Patterns

- Volatility ranking: $\sigma_{TB/Y} < \sigma_C < \sigma_Y < \sigma_I$

- Consumption, investment, and labor hours are **procyclical**

- The trade-balance-to-output ratio is **countercyclical**

- All variables have significant persistence

Calibration~~/estimation~~ strategy

Three categories of parameters have different calibration approaches:

Category a: manual calibration based on literature; **4** parameters: $\sigma = 2$, $\delta = 0.1$, $r^* = 0.04$, $\beta = 1/(1 + r^*)$.

Category b: target **first-order moments** (averages) of the data that the model aims to explain, **2** parameters: α, $\overline{IIP}$

$TB_{ss} = -r_{ss} IIP_{ss}$

$\overline{IIP} = -\frac{TB_{ss}}{r_{ss}}$

$$\text{labor share} = 0.68$$
$$\text{trade-balance-to-output ratio} = 0.02$$

Category c: target **second-order moments** (variances, correlations), **5** parameters: ω, ϕ, ψ, ρ, σ_ε. The targets are: σ_y, σ_N, σ_I, $\sigma_{TB/Y}$, corr($\ln Y_t, \ln Y_{t-1}$)

Calibration/estimation – parameter values

Yearly frequency assumed. **Which parameters would you change for quarterly frequency?**

σ	r^*	δ	α	ω	ϕ	ρ	σ_ϵ	$\bar{IIP}$	ψ
2	0.04	0.1	0.32	1.455	0.028	0.42	0.013	-0.744	0.0007

β follows from r^*: $1/\beta = 1 + r^*$ from Euler equation.

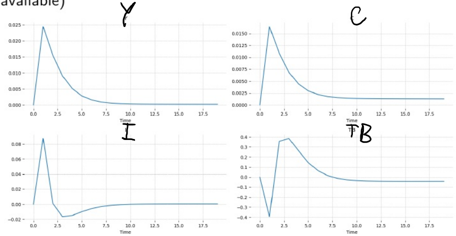
Given parameter values and chosen functional forms, steady state can be calculated by hand.

Outline

TFP shock – impulse responses

Shock of A equal to 0.015 in $t = 1$ and 0 afterwards.

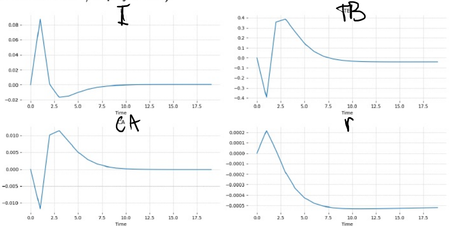
I no longer equivalent to household savings (savings abroad available)



TFP shock – impulse responses cont'd

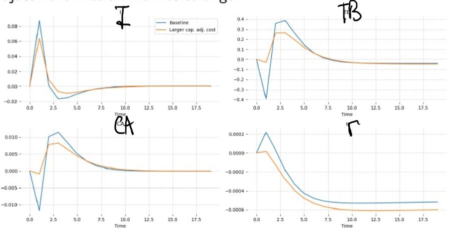
Shock of A equal to 0.015 in $t = 1$ and 0 afterwards

I and CA move in opposite directions. r responds to CA (debt accumulation/repayment)



TFP shock – adjustment cost comparative statics

Compare $\phi = 0.028$ (blue) to $\phi = 0.06$ (orange) i.e. capital adjustment friction 2 times stronger



Second-order Moments: RBC vs. Data

	Canada, 1960-2011			Model		
	σ_{x_t}	$corr(x_t, x_{t-1})$	$corr(x_t, Y_t)$	σ_{x_t}	$corr(x_t, x_{t-1})$	$corr(x_t, Y_t)$
Y	3.7	0.9	1	3.1	0.6	1
C	2.2	0.7	0.6	2.7	0.8	0.8
I	10.3	0.7	0.8	9.0	0.1	0.7
N	3.6	0.7	0.8	2.1	0.6	1
$\frac{TB}{Y}$	1.7	0.8	0.1	1.8	0.5	**-0.04**

- As before, the model correctly ranks $\sigma_C < \sigma_Y < \sigma_I$. The trade balance (normalized by GDP) correctly has $\sigma_{TB/Y} < \sigma_Y$
- model incorrectly makes TB/Y slightly countercyclical

Outline

C, I and interest rate: partial equilibrium

We now consider the **partial equilibrium** effect of r_t on CA_t, through $C_t = C(r_t, \dots)$ and $I_t = I(r_t, \dots)$.

Recall that interest rate influences consumption through substitution effect and income effect.

1. Substitution effect: higher $r_t \Rightarrow C_{t+1}$ relatively cheaper with respect to $C_t \Rightarrow C_t$ decreases
2. Income effect: less savings needed to guarantee a given level of $C_{t+1} \Rightarrow C_t$ might increase

Assume substitution effect dominates (e.g. if $u(C) = \ln(C)$), then $C_t = C_t(\underset{-}{r_t}, \dots)$

Furthermore, $I_t = I(\underset{-}{r_t}, \dots)$, because firms' optimal K_{t+1} depends negatively on R_{t+1} and $r_t = R_{t+1} - \delta$ if no shocks in economy

Trade balance, Current account in parial equilibrium

We get a **positive dependence of both TB and CA on real interest rate**:

$$TB_t = Y_t - C(\underset{-}{r_t}, \dots) - I(\underset{-}{r_t}, \dots) = TB(\underset{+}{r_t}, \dots)$$

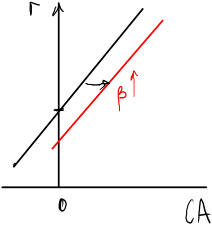
$$CA_t = \underbrace{Y_t - C(\underset{-}{r_t}, \dots) - I(\underset{-}{r_t}, \dots)}_{TB_t} + r_{t-1} IIP_t = CA(\underset{+}{r_t}, \dots)$$

Current account schedule

CA schedule – positively sloped line/curve in (CA, r) space (time indexes dropped).

Shifted by all the factors other than r that affect current account, e.g.:

- Productivity
- Household wealth
- Preferences $(\beta \uparrow)$



Risk premium in partial equilibrium

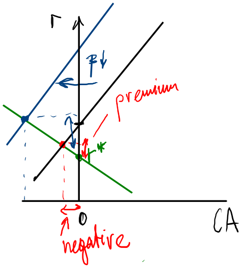
Simple static modelling approach:

$$r = r^* + \underbrace{p(\underline{CA})}_{\text{premium}}$$

$p(0) = 0$

Large negative $CA \Rightarrow$ debts acumulated

Plot function together with CA schedule $\rightarrow$ graphical solution of equilibrium r, CA



Outline

Large Open Economy

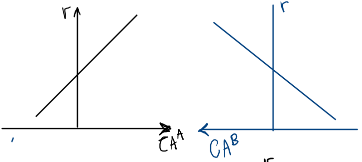
Assume only two economies/economic zones in the world. Typical application: one big open economy + rest of the world as another "economy".

Large open economy can influence world prices. We have focused on one price in this lecture: real interest rate.

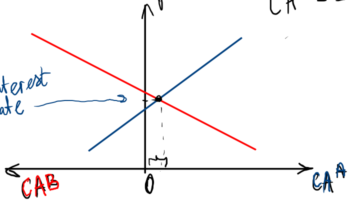
World interest rate will be determined by two large open economies' CA schedules.

Equilibrium condition $CA^A(r) = -CA^B(r)$: all outflows from one country are inflows in the other (nothing else in the world!)

Large open economy partial equilibrium: graph



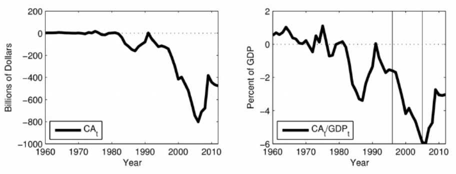
$$CA^A = -CA^B$$



Outline

Persistent CA deficits in US

U.S. current account has been in persistent deficit since 1970s:



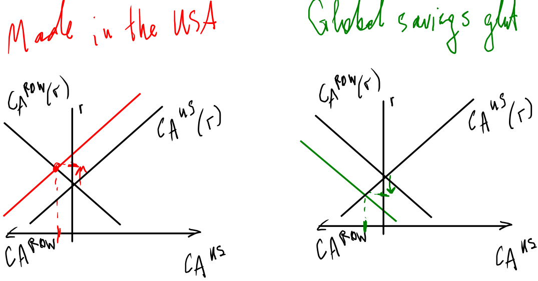
Source: Schmitt-Grohe, Uribe, Woodford, Fig. 6.9

Hypotheses for the CA deficits

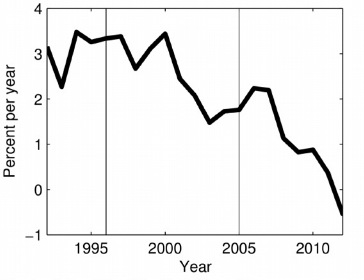
1. "Made in the U.S.A." – common view in early 2000s
   - ▷ Abnormal consumption, e.g. due to developed consumer credit markets
   - ▷ Investment bubbles
2. "Global Savings Glut" – B. Bernanke (2005)
   - ▷ Developing economies accumulate foregn reserves in the aftermath of crises of 1970s and 1990s
   - ▷ Export-led growth through currency depreciations

Which one is right? Look at the large open economy model and at data.

Made in U.S.A. vs. Global Savings Glut – diagrams



World real interest rate



Source: Schmitt-Grohe, Uribe, Woodford, Fig. 6.11

Congratulations, prof. Bernanke!