Macroeconomics
Lecture 5 – Real Business Cycles

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Overview

1. Lucas Critique
2. The RBC model
   - Representative household
   - Representative firm
   - Equilibrium

Outline

Lucas critique

- Big "Keynesian macroeconometric models" group led by Kennedy's Council of Economic Advisers
  - Solow, Tobin, Samuelson
- Application of IS-LM, Mundell-Flemming, AD-AS models to the data. Models linear $\Rightarrow$ represented as system of linear equations and estimated by least-squares methods:

$$
\begin{aligned}
x_{1t} &= \alpha_0 x_{2t} + \alpha_1 x_{3t} + \alpha_2 x_{3t} + \ldots. \\
x_{2t} &= \alpha_3 x_{1t} + \alpha_4 x_{3t} + \alpha_5 x_{4t} + \ldots \\
&\vdots \\
x_{136t} &= \alpha_{5987} x_{1t} + \alpha_{5988} x_{13t} + \alpha_{5989} x_{69t} + \ldots
\end{aligned}
$$

- **R. Lucas' idea (1976)**: The alphas (e.g. marginal propensity to consume) are **endogenous** with respect to government policy

Lucas critique II

Lucas's critique has received a huge response from macro theorists.

Instead of modelling **accounting relationships** such as $Y = C + I + G + PCA\ldots$

...starting to model **rational agents' behaviour**

Two essential elements:

1. All agents **optimize** some objective function. Utility for households, profit for firms.

2. **Rational expectations**: agents know the structure of the economy (the model), errors are possible, but not **systematic**

End of microeconomics vs. macroeconomics divide, everything is **micro-founded**.

Lucas critique: examples

Consider households' reaction to a positive government spending shock:

- If **temporary**, need to know how many years it will last: consumption response is not same if GDP (and incomes) boosted for 1 year and of boosted for 5 years
- If **permanent**, government debt accumulation ⇒ consumers need to know what is the debt reduction strategy of government
  - If taxes will be raised in the future, households need to increase savings now to prepare for decrease of income ⇒ **smaller marginal propensity to consume today**
  - If central bank not independent and finances government deficit with additional money supply, inflation expected to rise ⇒ better buy things now while prices low ⇒ **higher marginal propensity to consume today**

Households' consumption reaction interacts with **labor supply** ⇒ under sticky wages, wage and price setting (and AS) influenced by factors listed above.

The RBC model

Two ways of describing the **Real Business Cycle** model:

1. dynamic Robinson Crusoe economy [*see Micro class*]
2. Ramsey-Cass-Coopmans model [*see Growth course, if enrolled*] with **endogenous labor choice** and **stochastic productivity shocks**

With respect to models seen before:

- Dynamic
- Flexible prices $\Rightarrow$ **money does not matter**
  - hence the word **Real** in **RBC**
  - can think of it as a long-run (trend) model, but is used for short and medium run
- This lecture – closed economy

The RBC model

Structure:

- **Households** with preferences (utility functions) for consumption and labor
- **Firms** produce goods with labor and capital, owned by households
- **General equilibrium** interactions through (at least) 3 markets: goods, labour, capital
- **Uncertainty** about future level of productivity

Assumptions for this lecture, possible to relax in general:

- Closed economy
- Perfect competition
- No government

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Representative agent macroeconomics

A **large number** (population size normalized to 1) of **identical** households populate the economy.

Methodological trick: study a **representative household**: the aggregate outcomes as result of one (big) agent's behaviour.

But the "big" household is actually many small ones $\Rightarrow$ the representative household cannot manipulate aggregate quantities and prices (wage, interest) $\Rightarrow$ **takes prices as given**

Households in RBC model **live forever**:

- Demographics ignored
- To make it seem less crazy, imagine that each household is a sequence of generations of constant size
  - Then, necessary to assume that all offsprings' utility enters ancestors' utility

Representative household

- Households care for amount of good consumed in each period C_t (from $t = 0$ to $t = \infty$) and for hours worked in each period L_t (equivalently, they care for leisure time)
- Two fundamental microeconomic problems:
  1. **Intertemporal choice** of consumption $\leftrightarrow$ **consumption-savings** problem
  2. Choice between consumption and leisure $\leftrightarrow$ labor supply
- Will study the two problems, starting with two periods
- Interaction of the two problems $\Rightarrow$ intertemporal substitution of leisure (Lucas-Rapping effect)

2-period intertemporal consumption choice

Consider a consumer living for two periods, working $\bar{L}$ hours each period. Preferences:

$$U(C_1, C_2) = u(C_1) + \beta u(C_2)$$

where $u(\cdot)$ is **instantaneous utility function** with $u' > 0, u'' < 0$;
β – the **discount factor** $\leftrightarrow$ degree of patience.
Budget constraints of two periods:

$$P_1 \cdot C_1 + P_2 \cdot \Omega_2 \overset{=}{\leq} W_1 \cdot \bar{L}$$
$$P_2 \cdot C_2 \overset{=}{\leq} W_2 \cdot \bar{L} + (1+i) \cdot P_2 \cdot \Omega_2$$

W – **nominal** wage, i **nominal** interest, $\boxed{w \equiv W/P}$ – **real** (in units of good) wage, $1 + r \equiv (1+i)/(1+\pi)$ and **real** interest
Ω_2 – **real** (measured in units of period-2 consumption good) **wealth** at **beginning** of period 2;
<u>Period 1</u>: labor income used for consumption and **savings**
<u>Period 2</u>: consume labor income + savings with a return
If $\Omega_2 < 0$, the consumer borrows (at the same nominal interest i)

2-period intertemporal consumption choice: solution

Both budget constraints hold as equality ⇒
can obtain **intertemporal budget constraint** $\boxed{C_1 + \frac{1}{1+r}C_2 = w_1\bar{L} + \frac{1}{1+r}w_2\bar{L}}$

$$\begin{cases} P_1C_1 + P_2\Omega_2 = W_1\bar{L} & \big|:P_1 \\ P_2C_2 = W_2\bar{L} + (1+i)P_2\Omega_2 & \big|:P_2 \end{cases}$$

$$\begin{cases} C_1 + \frac{P_2}{P_1}\Omega_2 = \overbrace{\frac{W_1}{P_1}}^{w_1}\bar{L} \\ C_2 = \underbrace{\frac{W_2}{P_2}}_{w_2}\bar{L} + (1+i)\Omega_2 \end{cases}$$

$$\frac{P_2}{P_1} = 1 + \frac{P_2 - P_1}{P_1} = 1 + \pi$$

$$\begin{cases} C_1 + (1+\pi)\Omega_2 = w_1\bar{L} \\ C_2 = w_2\bar{L} + (1+i)\Omega_2 \end{cases} \qquad \Omega_2 = \frac{1}{1+\pi}(w_1\bar{L} - C_1)$$

$$\frac{1+i}{1+\pi} \equiv 1+r$$

$$\log: i - \pi^e \approx r$$

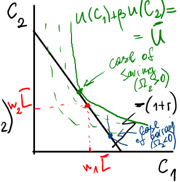
$$C_2 = w_2\bar{L} + \overbrace{\frac{1+i}{1+\pi}}^{1+r}(w_1\bar{L} - C_1) \quad \big|:(1+r)$$

$$\boxed{C_1 + \frac{1}{1+r}C_2 = w_1\bar{L} + \frac{1}{1+r}w_2\bar{L}}$$

2-period intertemporal consumption choice: solution

$$\max_{C_1, C_2} \; u(C_1) + \beta u(C_2)$$

s.t. $C_1 + \frac{1}{1+r} C_2 = w_1 \bar{L} + \frac{1}{1+r} w_2 \bar{L}$



$$\max_{C_1, C_2, \lambda} \mathcal{L}(C_1, C_2, \lambda) = u(C_1) + \beta u(C_2) + \lambda \cdot \left(w_1 \bar{L} + \frac{1}{1+r} w_2 \bar{L} - C_1 - \frac{1}{1+r} C_2\right)$$

FOC: $\frac{\partial \mathcal{L}}{\partial C_1} = 0 \Leftrightarrow u'(C_1) - \lambda = 0 \quad (1)$

$\frac{\partial \mathcal{L}}{\partial C_2} = 0 \Leftrightarrow \beta u'(C_2) - \frac{1}{1+r} \lambda = 0 \quad (2)$

$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Leftrightarrow w_1 \bar{L} + \frac{1}{1+r} w_2 \bar{L} - C_1 - \frac{1}{1+r} C_2 = 0 \quad (3)$

$(1) \& (2) \Rightarrow u'(C_1) = (1+r) \beta u'(C_2) \quad | : \beta u'(C_2)$

$$\frac{u'(C_1)}{\beta u'(C_2)} = 1 + r \Leftrightarrow MRS_{C_1, C_2} = \frac{p_{C_1}}{p_{C_2}}$$

$$1+r = \frac{1}{\frac{1}{1+r}}$$

Consumption-leisure choice

Consider one-period problem of an agent that can work for a total length of time equal to 1 (normalization of work hours $\Rightarrow L$ as **share** of time devoted to work). She cares for consumption C and leisure time $1 - L$: $\longrightarrow$ leisure = 1 − labor

$$U(\underset{+}{C}, \underset{+}{1 - L}) \quad \text{or, equivalently,} \quad U(\underset{+}{C}, \underset{-}{L})$$

Budget constraint:

$$\begin{aligned} PC \leq WL &\Leftrightarrow C \leq wL \\ &\Leftrightarrow C + w(1 - L) \leq w \quad \text{(consumption vs. leisure)} \end{aligned}$$

Wage as **opportunity cost**, or price, of leisure

Consumption-leisure choice: graphs, solution

$$\max_{C,L} U(C,1-L)$$

$$C + w(1-L) = w \quad \rightarrow \quad C = w - w(1-L)$$

$$\mathcal{L} = U(C,1-L) + \lambda\left(w - C - w(1-L)\right)$$

FOC:

$$\frac{\partial \mathcal{L}}{\partial C} = 0 \Leftrightarrow \frac{\partial U}{\partial C} - \lambda = 0 \quad (1)$$

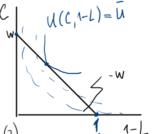
$$\frac{\partial \mathcal{L}}{\partial L} = 0 \Leftrightarrow -\frac{\partial U}{\partial (1-L)} + \lambda \cdot w = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Leftrightarrow w - C - w(1-L) = 0 \quad (3)$$

$$(1)\,\&\,(2) \Rightarrow \frac{\partial U}{\partial C} = \frac{1}{w}\frac{\partial U}{\partial (1-L)} \quad \Big| : \frac{\partial U}{\partial C}$$

$MRS_{(1-L),C}$

$$\boxed{\frac{\partial U / \partial (1-L)}{\partial U / \partial C}} = w$$



Effect of $w\uparrow$:

1. Case 1: income effect > subst. effect $\Rightarrow C\uparrow, L\downarrow$
2. Case 2: substitution effect > income effect $\Rightarrow C\uparrow, L\uparrow$

Consumption and leisure with 2 periods: Lucas-Rapping effect

Now consider a 2-period problem with both consumption and leisure. We will assume a specific form of utility function to simplify analysis:

$U(\underset{+}{C_1}, \underset{-}{L_1}, \underset{+}{C_2}, \underset{-}{L_2})$

$$\max_{C_1,C_2,L_1,L_2} \{\overbrace{\ln C_1 - \gamma\sigma/(1+\sigma)L_1^{(1+\sigma)/\sigma}}^{u(C_1,L_1)}$$

discount factor

$$+ \beta(\overbrace{\ln C_2 - \gamma\sigma/(1+\sigma)L_2^{(1+\sigma)/\sigma}}^{u(C_2,L_2)})\}$$

$$\text{s.t.} \quad C_1 + C_2/(1+r) = w_1 L_1 + w_2 L_2/(1+r)$$

Consumption and leisure with 2 periods: solution

Lagrangian $\rightarrow$ FOC:

$$\begin{cases} C_1 = \frac{1}{\lambda} & (1) \\ C_2 = \frac{1}{\lambda}\beta(1+r) & (2) \\ L_1 = \left(\frac{\lambda w_1}{\gamma}\right)^{\sigma} & (3) \\ L_2 = \left(\frac{\lambda w_2}{\gamma}\right)^{\sigma}\left(\frac{\beta}{1+r}\right)^{\sigma} & (4) \\ C_1 + \frac{C_2}{1+r} = w_1 L_1 + \frac{w_1}{1+r} L_2 \end{cases}$$

$$\frac{L_1}{L_2} = \left(\frac{w_1}{\beta w_2(1+r)}\right)^{\sigma} \qquad \frac{(3)}{(4)}$$

$$\rightarrow \hat{L}_1 = \left(\frac{\cancel{\frac{1}{A}}\lambda \cdot \cancel{A} w_1}{\gamma}\right)^{\sigma} = \left(\frac{\lambda w_1}{\gamma}\right)^{\sigma}$$

$$\rightarrow \hat{L}_2 = \left(\frac{\cancel{\frac{1}{A}}\lambda \cdot \cancel{A} \cdot w_2}{\gamma}\right)\left(\frac{\beta}{1+r}\right)^{\sigma} = L_2$$

$\hookrightarrow$ solve for λ: $\lambda = \text{constant}_1 \cdot \left(w_1^{1+\sigma} + \text{constant}_2 \cdot w_2^{1+\sigma}\right)^{-\frac{1}{1+\sigma}}$

What if $w_1\uparrow, w_2\uparrow$: $A \cdot w_1 \leftarrow$ new wages

$A \cdot w_2 \leftarrow$

$$\text{New}: \hat{\lambda} = \text{constant}_1 \cdot \left(A^{1+\sigma} w_1^{1+\sigma} + \text{const}_2 \cdot A^{1+\sigma} w_2^{1+\sigma}\right)^{-\frac{1}{1+\sigma}} = \frac{1}{A} \cdot \lambda$$

Lucas-Rapping effect

An intertemporal dimension of labor supply through consumption-saving decisions:

- It is the **change in time** and not **average level** of wage that affects labor supply
- In periods with temporary wage increases household works more, makes savings, then works less after wage decrease

Infinitely-lived representative household

$$
\begin{aligned}
U(C_0, L_0, ..., C_t, L_t, C_{t+1}, L_{t+1}, ...) = \\
& u(C_0, 1 - L_0) + \\
& E_0 \, [\, \beta u(C_1, 1 - L_1) + \beta^2 u(C_2, 1 - L_2) + ... \\
& + \beta^t u(C_t, 1 - L_t) + \beta^{t+1} u(C_{t+1}, 1 - L_{t+1}) + ... \,] \\
& = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - L_t)
\end{aligned}
$$

Rational expectations E_0: formed at initial period $t = 0$, take into account the structure of the model (known to household) and distribution of stochastic variables (more on them below)

Properties of $u(.)$

- $u'_c > 0, u'_{1-L} > 0; \quad u''_{cc} < 0, u''_{1-L} < 0$
- $lim_{C_t \to 0} u'_c(C_t, 1 - L_t) = \infty$; $lim_{1-L_t \to 0} u'_{1-L}(C_t, 1 - L_t) = \infty$

Household wealth, budget constraint

We use only **real** quantities for the budget constraint:

$$C_t + \underbrace{\Omega_{t+1} - \Omega_t}_{\text{savings}} = \underbrace{w_t L_t + r_t \Omega_t + \Pi_t}_{\text{income}}$$

$$\Leftrightarrow C_t + \Omega_{t+1} = w_t L_t + (1 + r_t)\Omega_t + \Pi_t$$

- C_t: consumption
- Ω_t: **real wealth** of household **at beginning of period** t (denominated in consumption goods)
- w_t: **real wage** (denominated in consumption goods)
- r_t: **real interest rate** (denominated in consumption goods) – **return on assets available at beginning of period** t
- Π_t: **real profits** of firms (denominated in consumption goods)

Household problem

$$\max_{\{C_t, L_t, \Omega_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - L_t)$$

subject to a **sequence** of budget constraints:

$$\begin{aligned}
C_0 + \Omega_1 &= w_0 L_0 + (1 + r_0)\Omega_0 + \Pi_0 \\
C_1 + \Omega_2 &= w_1 L_1 + (1 + r_1)\Omega_1 + \Pi_1 \text{ and so on:} \\
C_t + \Omega_{t+1} &= w_t L_t + (1 + r_t)\Omega_t + \Pi_t \text{ for } t = 0, 1, 2, \ldots
\end{aligned}$$

Household takes as given:

- Ω_0 – an **initial condition**
- current prices w_t,r_t (cannot influence them by their actions)
- expected prices $\{E_t w_s\}_{s=2}^{\infty}, \{E_t r_s\}_{s=2}^{\infty}$
- $\Pi_t, \{E_t \Pi_s\}_{s=2}^{\infty}$: current and expected firm profits

Consumer optimization

Lagrangian of consumer problem

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \{ u(C_t, 1 - L_t) + \lambda_t \left[w_t L_t + (1 + r_t)\Omega_t + \Pi_t - C_t - \Omega_{t+1} \right] \}$$

Budget constraint holds as equality for all $t \Rightarrow \lambda_t > 0$ for all t.
First-order conditions in period t:

$$(1) \quad \frac{\partial \mathcal{L}}{\partial C_t} \qquad u'_c(C_t, 1 - L_t) - \lambda_t = 0$$

$$(2) \quad \frac{\partial \mathcal{L}}{\partial \cancel{L_t}(1-L_t)} : \qquad u'_{1-L}(C_t, 1 - L_t) \cancel{+} \lambda_t w_t = 0$$

$$(3) \quad \frac{\partial \mathcal{L}}{\partial \Omega_{t+1}} : \qquad - \lambda_t + \beta E_t[\lambda_{t+1}(1 + r_{t+1})] = 0$$

$\lambda_t = u_c(C_t, 1 - L_t)$ follows from (1): λ_t is marginal utility of consumption at t, also known as **shadow price** of wealth.