

Macroeconomics

Lecture 11 – Open Macroeconomy III

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Outline

- 1 Business Cycles in a Small Open Economy: Data
- 2 Real Business Cycles in Small Open Economy (SOE RBC)
- 3 Calibration/estimation
- 4 Model Outputs
- 5 Comparative Statics of Model

Canadian Economy: Business-Cycle Statistics

Canada – developed economy, but 2.1 % of world GDP \Rightarrow “small” (compare to China – 18.4%; USA – 24% – large open economies).

We will seek to explain historical business-cycle patterns: 1946-1985.

Variable (x_t)	Moments		
	σ_{x_t}	$\text{corr}(x_t, x_{t-1})$	$\text{corr}(x_t, Y_t)$
Y	2.8	0.61	1
C	2.5	0.7	0.59
I	9.8	0.31	0.64
L	2	0.54	0.8
$\frac{TB}{Y}$	1.9	0.66	-0.13

Source: Mendoza AER, 1991. Annual data. Log-quadratically detrended.

Canadian Economy: Business-Cycle Patterns

- ▶ Volatility ranking: $\sigma_{TB/Y} < \sigma_C < \sigma_Y < \sigma_I$.
- ▶ Consumption, investment, and labor hours are **procyclical**.
- ▶ The trade-balance-to-output ratio is **countercyclical**.
- ▶ All variables have a positive autoregressive coefficient.
- ▶ Similar patterns hold for other *developed* SOEs.

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A SOE RBC model

We extend the RBC framework to include exchanges with the rest of the world. Specific elements:

1. **Capital adjustment cost** limits impact of investment on **trade balance**
2. **Interest rate premium**, or external debt-elastic interest rate (EDEIR) assumption
 - ▷ Introduced for technical purposes: necessary for model convergence
3. (not obligatory) Utility maximization w.r.t. **resource constraint** of economy instead of budget constraint
 - ⇒ We solve **social planner**'s optimization problem, which is still equivalent to finding decentralized equilibrium (1st Welfare Theorem)

Planner's Maximization Problem

$$\max_{\{C_t, I_t, IIP_{t+1}, K_{t+1}\}_0^\infty} E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t)$$

subject to

$$C_t + I_t + IIP_{t+1} + \Phi(K_{t+1} - K_t) = A_t f(K_t, L_t) + (1 + r_t) IIP_t$$

$$K_{t+1} = (1 - \delta)K_t + I_t$$

$$K_0, IIP_0, A_0 \text{ - given; } \ln A_{t+1} = \rho \ln A_t + \tilde{\eta} \epsilon_{t+1}$$

$\Phi(\cdot)$ is a convex **capital adjustment cost** function with

$$\Phi(0) = \Phi'(0) = 0; \Phi''(\cdot) > 0$$

A transversality condition for IIP applies: $\lim_{j \rightarrow \infty} E_t \frac{IIP_{t+j}}{\prod_{s=0}^j (1+r_s)} \geq 0$

Planner's FOC

$$\frac{\partial \mathcal{L}}{\partial C_t}:$$

$$u'_C(C_t, L_t) = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial IIP_{t+1}}:$$

$$\lambda_t = \beta(1 + r_{t+1})E_t\lambda_{t+1}$$

$$\frac{\partial \mathcal{L}}{\partial L_t}:$$

$$-u'_L(C_t, L_t) = \lambda_t A_t f'_L(K_t, L_t)$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}}:$$

$$1 + \Phi'(K_{t+1} - K_t) = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} [A_{t+1} f'_K(K_{t+1}, L_{t+1}) + 1 - \delta + \Phi'(K_{t+2} - K_{t+1})]$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t}:$$

$$C_t + K_{t+1} - (1 - \delta)K_t + IIP_{t+1} + \Phi(K_{t+1} - K_t) = A_t f(K_t, L_t) + (1 + r_t)IIP_t$$

Interest Rate Premium/ EDEIR

In this model, the assumption of External debt-Elastic Interest Rate (EDEIR) has a technical role: it makes the model converge to a steady state. Formula as before:

$$r_t = r^* + p(\text{IIP}_t)$$

- ▷ r^* = world interest rate, assumed **constant**
- ▷ $p(\text{IIP}_t)$ = country interest-rate premium, decreasing in IIP (increasing in external debt)
- ▷ Note that the function treats negative and positive IIP symmetrically, so the interpretation is wider (and less intuitive) than a risk-based interest rate premium

Balance of Payments Indicators

Trade Balance

$$TB_t = Y_t - C_t - I_t - \Phi(K_{t+1} - K_t)$$

Current Account

$$CA_t = TB_t + r_t IIP_t$$

Equilibrium

An equilibrium is a sequence $\{C_t, L_t, K_{t+1}, IIP_{t+1}\}$, such that, given exogenous initial conditions K_0, IIP_0, A_0 and exogenous productivity $\ln A_{t+1} = \rho \ln A_t + \tilde{\eta}\epsilon_{t+1}$, the following conditions are satisfied:

$$u'_C(C_t, L_t) = \beta(1 + r^* + p(IIP_{t+1}))E_t u'_C(C_{t+1}, L_{t+1})$$

$$- \frac{u'_L(C_t, L_t)}{u'_C(C_t, L_t)} = A_t f'_L(K_t, L_t)$$

$$1 = \beta E_t \left\{ \frac{u'_C(C_{t+1}, L_{t+1})}{u'_C(C_t, L_t)} \frac{A_{t+1} f'_K(K_{t+1}, L_{t+1}) + 1 - \delta + \Phi'(K_{t+2} - K_{t+1})}{1 + \Phi'(K_{t+1} - K_t)} \right\}$$

$$\begin{aligned} C_t + K_{t+1} - (1 - \delta)K_t + IIP_{t+1} + \Phi(K_{t+1} - K_t) \\ = A_t f(K_t, L_t) + IIP_t(1 + r^* + p(IIP_t)) \end{aligned}$$

Steady State

The steady state (IIP, K, C, L) is solution to a system (with $A = 1$):

$$-\frac{u'_L(C, L)}{u'_C(C, L)} = Af'_L(K, L)$$

$$C + \delta K - IIP \cdot (r^* + p(IIP)) = Af(K, L)$$

$$1 = \beta(1 + r^* + p(IIP))$$

$$1 = \beta [Af'_K(K, L) + 1 - \delta]$$

Balance of Payments in Steady State

Recall $IIP_{t+1} - IIP_t = CA_t$ from previous lecture.
Since $IIP_t = IIP$ (constant) in s.s., $CA = 0$ in s.s.

Recall then $CA = TB + r \cdot IIP \Rightarrow TB = -r \cdot IIP$.

Intuition: net **capital flows (FA balance)** are null in s.s., so the flows due to **trade balance** are exactly compensated by flows due to **income balance**, for **Balance of Payments Identity** $CA = FA$ to hold.

Functional Forms

Instantaneous utility function – GHH

$$u(C, L) = \frac{(C - L^\omega/\omega)^{1-\sigma} - 1}{1-\sigma}; \quad \omega > 1; \sigma > 0$$

\Rightarrow labor supply depends on real wage only (not on consumption)

Interest rate premium

$$p(IIP) = \psi \left(e^{\bar{IIP} - IIP} - 1 \right); \quad \psi > 0$$

Production function – Cobb-Douglas

$$f(K, L) = K^\alpha L^{1-\alpha}; \quad \alpha \in (0, 1)$$

Adjustment cost function – quadratic

$$\Phi(x) = \frac{\phi}{2} x^2; \quad \phi > 0$$

6 structural parameters: $\sigma, \omega, \psi, \bar{IIP}, \alpha, \phi$

Steady State

using the assumed functional forms, the steady state (IPP, K, C, L) is solution to a system (with $A = 1$):

$$L^{\omega-1} = A(1 - \alpha)(K/L)^{\alpha}$$

$$C + \delta K - (r^* + \psi(e^{\bar{I}P - IIP}))IIP = A(K/L)^{\alpha}L$$

$$1 = \beta(1 + r^* + \psi(e^{\bar{I}P - IIP} - 1))$$

$$1 = \beta \left[A\alpha(K/L)^{\alpha-1} + 1 - \delta \right]$$

7 parameters in the s.s. system: $\omega, \alpha, \delta, r^*, \psi, \bar{d}, \beta$. + **4** more parameters in model, eliminated in s.s.: $\sigma, \phi, \rho, \tilde{\eta}$

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Calibration/estimation strategy

Three categories of parameters have different calibration approaches:

Category a: manual calibration based on literature consensus/more arbitrary arguments; **4** parameters: $\sigma = 2$, $\delta = 0.1$, $r^* = 0.04$, $\beta = 1/(1 + r^*)$.

Category b: restrictions to match **first moments** (**averages**) of the data that the model aims to explain, **2** parameters: α , \bar{d}

labor share = 0.68

trade-balance-to-output ratio = 0.02

Category c: restrictions to match **second moments** (**variances, correlations**) of the data that the model aims to explain, **5** parameters: ω , ϕ , ψ , ρ , $\tilde{\eta}$. The second moments to be matched are: σ_y , σ_L , σ_I , $\sigma_{TB/Y}$, $\text{corr}(\ln Y_t, \ln Y_{t-1})$

Calibration – details

Simple example of Category b calibration – parameter α :
The labor share, s_L , is defined as

$$s_L = \frac{wL}{Y}$$

In the decentralized economy we have

$$A_t f'_L(K_t, L_t) = w_t$$

Thus, in the steady state:

$$s_L = \frac{A f'_L(K, L) L}{A f(K, L)}$$

Using the Cobb-Douglas form for $f(\cdot)$ yields

$$s_L = (1 - \alpha)$$

Hence we have that $\alpha = 1 - s_L = 1 - 0.68$, that is,

$$\alpha = 0.32$$

Calibration/estimation – details

Let θ denote the vector of structural parameters we still need to assign numerical values to, that is, let

$$\theta \equiv \left[\omega \quad \bar{d} \quad \phi \quad \psi \quad \rho \quad \tilde{\eta} \right]$$

The choice of these parameters is based on **Simulated Method of Moments**. We proceed in 6 steps:

Step 1: set a value (guess) for each element of θ

Step 2: Given the guess for ω find L using s.s. optimality of capital choice:

$$\frac{K}{L} = \left(\frac{r^* + \delta}{A\alpha} \right)^{\frac{1}{\alpha-1}}$$

and replace in s.s. consumption-labor optimality:

$$L = ((1 - \alpha)A(K/L)^\alpha)^{1/(\omega-1)}$$

With L in hand, find K and Y , as $K = (K/L)L$ and $Y = A(K/L)^\alpha L$, respectively.

Calibration/estimation – details

Step 3: Let s_{TB} denote the trade-balance-to-output ratio. In the steady state, $s_{TB} = -\frac{r^* IIP}{Y}$. Solve for IIP to get $IIP = -\frac{s_{TB} Y}{r^*}$. Then use $p(IIP) = 0$ and the restriction that $\beta(1 + r^*) = 1$ to obtain $\bar{IIP} = IIP$

Step 4: Find C from s.s. resource constraint: $C = Y - \delta K - r^* d$

Step 5: With the steady state values of (C, K, L, IIP) and all structural parameters obtained, simulate model (e.g. with **Blanchard-Kahn algorithm**) to get the following **second-order moments**:

$$x(\theta) \equiv \begin{bmatrix} \sigma_y & \sigma_L & \sigma_I & \sigma_{TB/Y} & corr(\ln Y_t, \ln Y_{t-1}) \end{bmatrix}$$

Calibration/estimation – details

$$x(\theta) \equiv \begin{bmatrix} \sigma_Y & \sigma_L & \sigma_I & \sigma_{TB/Y} & \text{corr}(\ln Y_t, \ln Y_{t-1}) \end{bmatrix}$$

Step 6: Find **Euclidean distance** between vector of moments of simulated data $x(\theta)$ and vector of empirical moments x^* :

$$\mathcal{D} = ||x(\theta) - x^*||$$

Step 7: Adjust elements of θ until \mathcal{D} is less than a threshold \mathcal{D}^* .

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Calibration/estimation – parameter values

Assume that one **period** is **one year**; calibrate the model to the **Canadian economy**.

σ	r^*	δ	α	ω	ϕ	ρ	σ_ϵ	IIP	ψ
2	0.04	0.1	0.32	1.455	0.028	0.42	0.013	-0.744	0.0007

β follows from r^* : $1/\beta = 1 + r^*$ from Euler equation.

Given parameter values, the steady state can be computed with any software that solves non-linear systems:

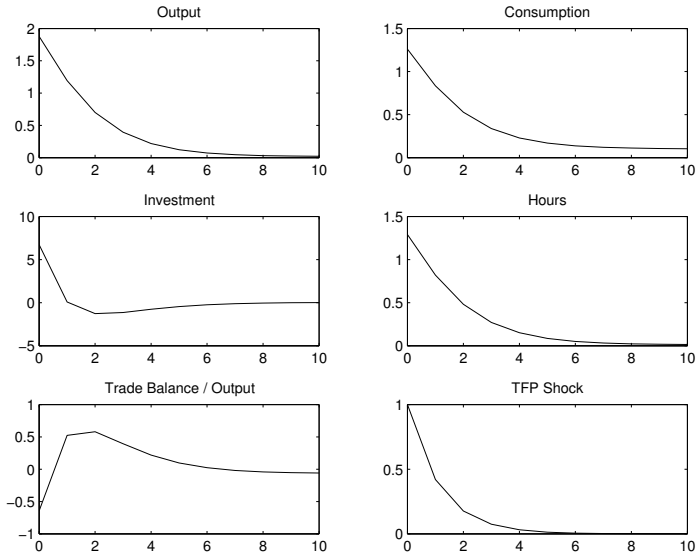
C	IIP	L	k
1.1170	-0.7442	1.0074	3.3977

Empirical vs. Simulated Second Moments

	Canadian Data						Model		
	1946 to 1985			1960 to 2011					
	σ_{x_t}	$\rho_{x_t, x_{t-1}}$	ρ_{x_t, Y_t}	σ_{x_t}	$\rho_{x_t, x_{t-1}}$	ρ_{x_t, Y_t}	σ_{x_t}	$\rho_{x_t, x_{t-1}}$	ρ_{x_t, Y_t}
Y	2.8	0.6	1	3.7	0.9	1	3.1	0.6	1
C	2.5	0.7	0.6	2.2	0.7	0.6	2.7	0.8	0.8
I	9.8	0.3	0.6	10.3	0.7	0.8	9.0	0.1	0.7
L	2.0	0.5	0.8	3.6	0.7	0.8	2.1	0.6	1
$\frac{TB}{Y}$	1.9	0.7	-0.1	1.7	0.8	0.1	1.8	0.5	-0.04
$\frac{CA}{Y}$	-	-	-	-	-	-	1.4	0.3	0.05

- ▶ σ_L , σ_I , σ_Y , $\sigma_{TB/Y}$, and $\rho_{Y_t, Y_{t-1}}$ were targeted by calibration, so no real test here.
- ▶ model correctly places σ_C below σ_Y and σ_I and above σ_L and $\sigma_{TB/Y}$.
- ▶ model correctly makes TB/Y **countercyclical**.
- ▶ model **overestimates** the correlations of hours and consumption with output (due to GHH utility).

Positive Productivity Shock – impulse responses

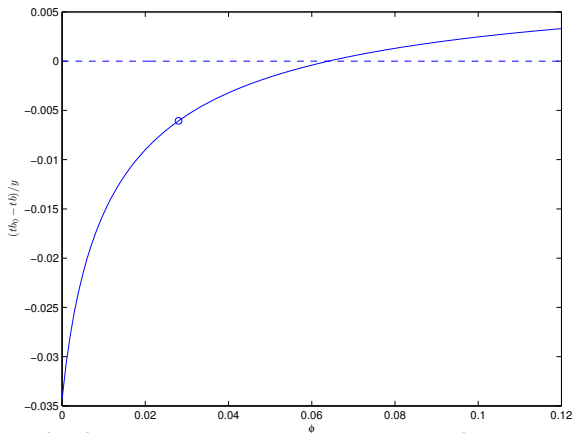


Source: Schmitt-Grohé and Uribe (JIE, 2003)

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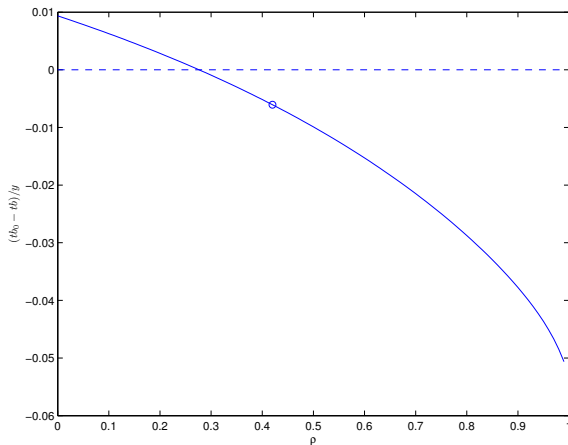
Response of TB and varying capital adjustment cost



Higher capital adj. costs \Rightarrow more positive impulse response of TB.
For $\phi < 0.06$ (scale parameter of adj. cost), the response of the trade balance negative.

Recall $TB_t = Y_t - C_t - I_t$. With large adj. cost, investment response weak \Rightarrow consumption drives TB \Rightarrow intuition from 2-period model with consumption only applies.

Response of TB and varying productivity persistence



More persistent the productivity shock \Rightarrow smaller impulse response of TB (recall temporary vs. permanent shocks in the 2-period model). For $\rho > 0.3$ (productivity persistent enough), the response of TB is negative.