Macroeconomics
Lecture 7 – RBC: additional topics

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Overview

1. Equilibrium optimality
2. Compact specification of model
3. Capital adjustment cost
4. Models vs. data

Social planner's problem

- The Social Planner
  - A fictional entity used to to talk about optimal outcomes
  - Is **benevolent** $\Rightarrow$ maximizes utility of representative household
  - Instead of optimizing w.r.t. prices, can reallocate goods between agents with no transaction cost $\Rightarrow$ prices omitted from optimality analysis
- The Social planner problem $\leftrightarrow$ finding optimal allocations:
  - Maximize a welfare criterion (benevolent $\Rightarrow$ HH utility)
  - ...subject to resource constraints – physical constraints of economy

Social Planner Optimization

$$\max_{\{C_t, L_t, K_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - L_t)$$

s.t. a *sequence* of **resource constraints**:

$$C_t + K_{t+1} - (1-\delta)K_t = Z_t f(K_t, L_t) \quad \text{for each } t = 0, 1, 2, ...$$

FOCs of the Lagrangian with multipliers $\{\lambda_t\}$:

$$(1) \quad \frac{\partial \mathcal{L}}{C_t} = 0: \; u'_c(C_t, 1 - L_t) - \lambda_t = 0$$

$$(2) \quad \frac{\partial \mathcal{L}}{L_t} = 0: \; -u'_{1-L}(C_t, 1 - L_t) + \lambda_t Z_t f'_L(K_t, L_t) = 0$$

$$(3) \quad \frac{\partial \mathcal{L}}{K_{t+1}} = 0: \; -\lambda_t + \beta E_t[\lambda_{t+1}(1 + Z_{t+1} f'_K(K_{t+1}, L_{t+1}) - \delta)] = 0$$

Optimal allocations

Efficient allocation is a sequence $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$, that, given K_0, Z_0 and the exogenous stochastic process $\ln Z_t = \rho \ln Z_{t-1} + \epsilon_t$, solves the social planner's optimization problem, i.e. satisfies the FOCs:

$$\begin{aligned} 1 =& E_t \left[\left(\frac{\beta \lambda_{t+1}}{\lambda_t} \right) (1 + Z_{t+1} f'_K(K_{t+1}, L_{t+1}) - \delta) \right] \\ \frac{u'_{1-L}(C_t, 1 - L_t)}{u'_c(C_t, 1 - L_t)} =& Z_t f'_L(K_t, L_t) \\ C_t + K_{t+1} - (1 - \delta) K_t =& Z_t f(K_t, L_t) \end{aligned}$$

...which also gave the **equilibrium**! $\Rightarrow$ **equilibrium is optimal**

$\rightarrow$ **First Welfare Theorem** (*see Microeconomics*) in a production economy with 1 agent and an ∞ of goods $\{C_t, 1 - L_t\}_{t=0}^{\infty}$.

Note that the social planner does not need prices to achieve the same allocation.

Compact specification of model

Specification of RBC used most often has the household owning the capital stock directly, without the wealth variable Ω.

Investment variable can be substituted from the resource constraint, the HH chooses K_{t+1} directly (like the Social planner).

Firms then **rent** the capital from HH, paying a **rental rate of capital** R_t.

Additionally, existence of **bond market** assumed, where households make savings by holding bonds of real value B_{t+t} in period t. Optimizing w.r.t. B_{t+1} gives the Euler equation. Without the government and with identical households $B_t = 0 \;\; \forall t$ in equilibrium.

Compact specification of model

Representative household:

$$\max_{C_t, L_t, I_t, B_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t (C_t, 1 - L_t)$$

$$\text{s.t.} \quad C_t + I_t + B_{t+1} = w_t L_t + R_t K_t + \Pi_t + (1 + r_t) B_t$$

$$K_{t+1} = (1 - \delta) K_t + I_t$$

or, omitting I_t, household chooses K_{t+1} directly:

$$\max_{C_t, L_t, K_{t+1}, B_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t (C_t, 1 - L_t)$$

$$\text{s.t.} \quad C_t + K_{t+1} + B_{t+1} = w_t L_t + R_t K_t + (1 - \delta) K_t + \Pi_t + (1 + r_t) B_t$$

Representative firm: $\max_{K_t, L_t} Z_t f(K_t, L_t) - w_t L_t - R_t K_t$

Bond market equilibrium: $B_t = 0$ (since no government and identical households)

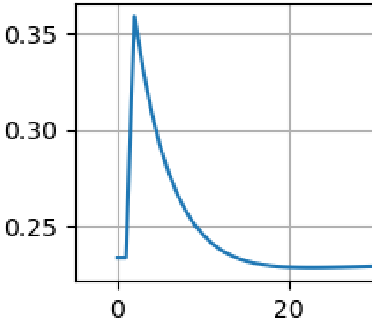
Home task: verify that equilibrium same as before

Outline

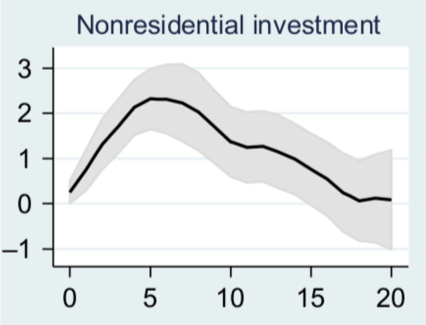
Investment impulse responses: model vs. data

Investment is too "jumpy" in the baseline RBC w.r.t. data

⇒ **capital adjustment costs** introduced to model inertia.



(a) Model IRF



(b) Empirical IRF (Ramey, 2016)

Investment reaction to transitory productivity shock.

Capital law of motion with adjustment costs

A new law of motion for capital:

$$K_{t+1} = I_t - \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t + (1-\delta)K_t$$

If $\phi > 0$, then doing investment above or below steady state (in steady state $\frac{I_{ss}}{K_{ss}} = \delta$) results in a cost. Quadratic function: marginal cost increases as investment further from s.s.

Household problem

New household problem (in a compact form):

$$\max_{C_t, I_t, L_t, K_{t+1}, B_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \theta \frac{L_t^{1+\chi}}{1+\chi} \right)$$

$$\text{s.t.} \quad C_t + I_t + B_{t+1} = w_t L_t + R_t K_t + \Pi_t + (1 + r_t) B_t$$

$$K_{t+1} = I_t - \frac{\phi}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t + (1 - \delta) K_t$$

Household problem – solution

A Lagrangian with two constraints $\Rightarrow$ two multipliers, λ and μ:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \{\ln C_t - \theta \frac{L_t^{1+\chi}}{1+\chi} \\ + \lambda_t (w_t L_t + R_t K_t + \Pi_t + (1+r_t) B_t - C_t - I_t - B_{t+1}) \\ + \mu_t (I_t - \frac{\phi}{2}(\frac{I_t}{K_t} - \delta)^2 K_t + (1-\delta) K_t - K_{t+1})\}$$

Household problem – solution

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \Leftrightarrow \frac{1}{C_t} = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = 0 \Leftrightarrow \theta L_t^{\chi} = \lambda_t w_t$$

$$\frac{\partial \mathcal{L}}{\partial I_t} = 0 \Leftrightarrow \lambda_t = \mu_t (1 - \phi(\frac{I_t}{K_t} - \delta))$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 \Leftrightarrow$$

$$\mu_t = \beta E_t [R_{t+1}\lambda_{t+1} - \mu_{t+1}\frac{\phi}{2}(\frac{I_{t+1}}{K_{t+1}} - \delta)^2 + \mu_{t+1}\phi(\frac{I_{t+1}}{K_{t+1}} - \delta)\frac{I_{t+1}}{K_{t+1}} + \mu_{t+1}(1-\delta)]$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} = 0 \Leftrightarrow \lambda_t = \beta E_t \lambda_{t+1} \left(1 + r_t\right)$$

Tobin's q is back

We define $q_t \equiv \frac{\mu_t}{\lambda_t}$

- μ_t – marginal utility of additional capital installed
- λ_t – marginal utility of consumption
- The ratio q_t – how much consumption you would give up to have some extra future capital
- At the same time, **the price of capital good is 1**, same as consumption good

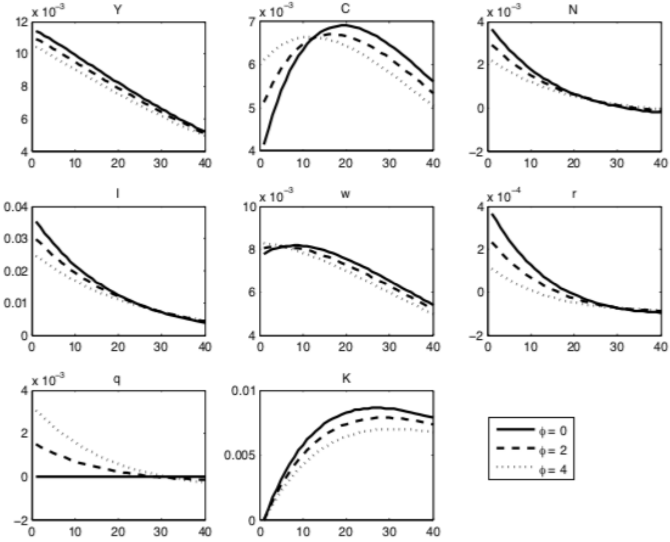
$\Rightarrow$ q_t is theoretical **Tobin's q** – ratio of willingness-to-pay to price of physical capital goods

The FOC $\frac{\partial \mathcal{L}}{\partial I_t} = 0$ can be re-written: $q_t = \left(1 - \phi\left(\frac{I_t}{K_t} - \delta\right)\right)^{-1}$

$\Rightarrow$ positive association between I_t and q_t: $I(\underset{+}{q_t}, \ldots)$

Main implication for equilibrium: additional **inertia** in investment.

Impulse responses with various capital adjustment costs



Response to a transitory productivity shock under different values of ϕ – capital adjustment cost magnitude

Alternative specifications

The form of adjustment cost is the simplest one; investment less reactive, but still not "hump-shaped" as in empirical IRF.
Several other options exist:

1. Specified in HH budget constraint directly:

$$C_t + I_t + B_{t+1} = w_t L_t + R_t K_t - \frac{1}{2}\frac{I_t^2}{K_t}$$

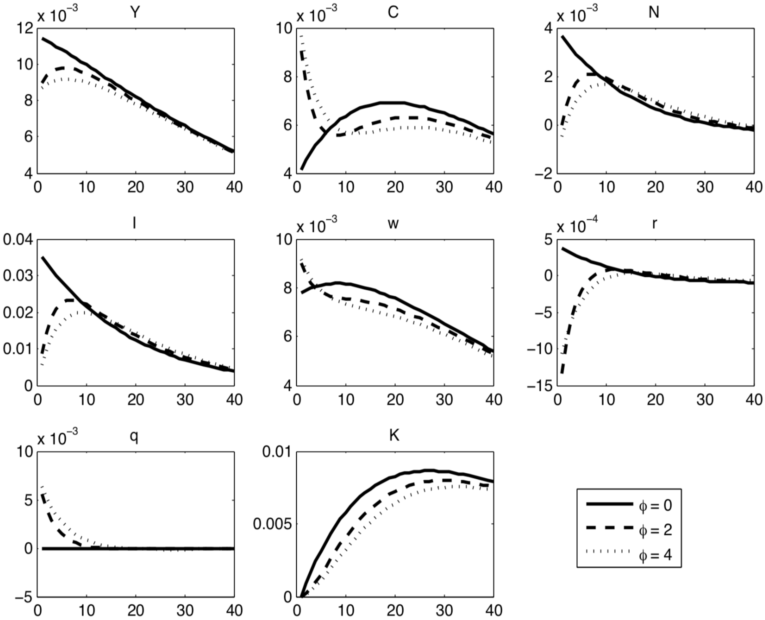
or $$C_t + I_t + B_{t+1} = w_t L_t + R_t K_t - \Phi(K_{t+1} - K_t), \text{ with } \Phi', \Phi'' > 0$$

2. Specified in law of motion of capital, but w.r.t. investment variable only – **investment adjustment cost**:

$$K_{t+1} = \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t + (1-\delta)K_t$$

Here, investment can be called both a **state** and a **control**.
Investment adjustment cost is known to produce hump-shaped reactions of investment to productivity shocks.

Investment adjustment cost – impulse responses



Outline

Model and data

We have seen how modelling assumptions changed to make an RBC model produce more realistic IRF.

How to choose parameter values to make models as realistic as possible? Several approaches for RBC and, more generally, **DSGE** (Dynamic Stochastic General Equilibrium) models:

1. Use values that are established (consensual) in academic literature
   - Example: $\alpha = 1/3$ (share of capital income in national income)
   - must be careful about what is the "literature" we are looking at and what we call "established"!
2. **Calibration** $\rightarrow$ picking parameter values by hand to match some empirical **target**, often about **steady state**
   - Example: labor disutility coefficient chosen such that $L_{ss} = 1/3$, which comes (approximately) from U.S. statistics
   - Related to General Method of Moments (GMM), which is an **estimation** method (see below)

Model and data

3. **Estimation** $\rightarrow$ automated **econometric** procedures; theory not covered in lectures, separate course needed.
   2 main classes of methods:

   - Frequentist: Rely on relationships between theoretical probabilities and empirical frequencies – Law of Large Numbers
     Use **least squares** (less useful for macro than micro), **GMM**, **Maximum Likelihood**, **Kalman filter**

   - Bayesian: Rely on probabilities as **beliefs** of researcher; use data and Bayes rule to update prior beliefs (priors) to posterior beliefs (posteriors).
     Use **Maximum Likelihood**, **Kalman filter**, **Monte Carlo Markov Chains** methods

The TDs after the break will be focused on implementation of ML estimation.

A representation of how you will feel doing model estimation (and it's *normal*)