Macroeconomics
Lecture 9 – New Keynesian Model II

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Outline

Outline

Nominal rigidities, a.k.a. sticky prices

- How sticky are prices?
- Empirical work (Dhyne et al, 2005) $\rightarrow$ average duration of a price spell is **4-5 quarters** in the EU
- How to build this in the New Keynesian model?

$\rightarrow$ Common sticky price mechanisms:

1. **Calvo pricing: exogenous probability that firm can change price at a given period**
2. *Rotemberg pricing*: firm can change price every period, but s.t. a quadratic 'menu cost'
3. *Taylor contracts*: firm can change price every T periods

We will look at Calvo pricing – most common and convenient assumption, although most "unrealistic" one

Aggregate price formula:

$$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon}\, di\right)^{\frac{1}{1-\varepsilon}} = \left(\int_{S(t)} P_{t-1}(i)^{1-\varepsilon}\, di + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

$\boxed{S(t) - \text{set of firms that cannot change price in } t}$

$$\frac{P_t}{P_{t-1}} = \left(\frac{1}{P_{t-1}^{1-\varepsilon}}\right)^{\frac{1}{1-\varepsilon}} \left(\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

$$\frac{P_t}{P_{t-1}} = \left(\theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \Leftrightarrow \Big(\underbrace{\frac{P_t}{P_{t-1}}}_{1+\pi_t}\Big)^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}$$

Linearization (see next slide Taylor series reminder)

around ss: $P_t = P_{t-1} = P \Rightarrow \pi = 0;\ P^* = P$

$$(1+\pi)^{1-\varepsilon} + (1-\varepsilon)(1+\pi)^{1-\varepsilon}(\pi_t - \pi) = \theta + (1-\theta)\cancel{\left(\frac{P}{P}\right)}^{1-\varepsilon} + (1-\theta)(1-\varepsilon)\cancel{\left(\frac{P}{P}\right)}^{1-\varepsilon} \times \cancel{P} \cdot p_t^* \cdot \cancel{\frac{1}{P}} + (1-\theta)(1-\varepsilon)\cancel{\left(\frac{P}{P}\right)}^{1-\varepsilon} \cancel{P}\left(\frac{1}{\cancel{P}}\right) \times p_{t-1} \times \cancel{P}$$

$\ln P_t^* = p_t^*$; $\ln P_{t-1} = p_{t-1}$

Taylor series (first-order approximation)

$$f(x_t) \approx f(x_{ss}) + \boxed{f'(x_{ss}) \cdot (x_t - x_{ss})} + \underbrace{O(x_t - x_{ss})}_{\text{ignore this in first order approx}}$$

$$\boxed{f(x_t) \approx f(x_{ss}) + f'(x_{ss}) x_{ss} \cdot (\ln x_t - \ln x_{ss})}$$

$$\ln x_t - \ln x_{ss} \approx \frac{x_t - x_{ss}}{x_{ss}}$$

$$f(x_t, z_t) \approx f(x_{ss}, z_{ss}) + \frac{\partial f(x_{ss}, z_{ss})}{\partial x} \cdot x_{ss} (\ln x_t - \ln x_{ss})$$
$$+ \frac{\partial f(x_{ss}, z_{ss})}{\partial z} \cdot z_{ss} (\ln z_t - \ln z_{ss})$$

Aggregate price continued:

$$1 + (1-\varepsilon)\pi_t = 1 + (1-\theta)(1-\varepsilon)(p_t^* - p_{t-1})$$

$$\pi_t = (1-\theta)(p_t^* - p_{t-1})$$

Calvo pricing

- Every period, each firm may change price with probability $1 - \theta$

$\Rightarrow$ at a given period, fraction θ of firms keeps price unchanged

$\Rightarrow$ fraction $1 - \theta$ can set new price

- average duration of a price $1/(1 - \theta)$ periods
  - can set θ to match a target duration, e.g. 5 quarters

Aggregate price dynamics

$$\underbrace{(\frac{P_t}{P_{t-1}})}_{1+\pi_t}^{1-\varepsilon} = \theta + (1-\theta)(\frac{P_t^*}{P_{t-1}})^{1-\varepsilon}$$

with P_t^* the optimal price set in period t by firms that can change price. Firms identical $\Rightarrow$ P_t^* same for all.

Log-linearized using Taylor series around steady state with $P_t/P_{t-1} = 1$ and $P_t^*/P_{t-1} = 1$:

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \tag{1}$$

- How are prices P_t^* chosen? $\rightarrow$ coming up next. We will skip some of the algebra; for full derivation, see notes of Drago Bergholt:

  `https://bergholt.weebly.com/uploads/1/1/8/4/11843961/the_basic_new_keynesian_model_-_drago_bergholt.pdf`

Outline

Intertemporal profit maximization: discounting

Firm chooses price for (uncertain) number of periods $\Rightarrow$ profit maximization involves many periods.

How to value future profits? With households' relative utility of future consumption, also known as **stochastic discount factor (SDF)** $Q_{t,t+k}$:

- Consider a small nominal payment ϵ obtained in future period $t+k$. Expected utility in period t:

$$\beta^{t+k} E_t \underbrace{(1-\sigma) C_{t+k}^{-\sigma}}_{\frac{\partial U}{\partial C}} \underbrace{\frac{\epsilon}{P_{t+k}}}_{\text{units of} C_{t+k}}$$

- Suppose the household pays $Q_{t,t+k}\epsilon$ at t to obtain this future payment, so that $Q_{t,t+k}$ is the price of 1 unit of money received k periods in future. Utility:

$$\beta^t (1-\sigma) C_t^{-\sigma} \frac{Q_{t,t+k}\epsilon}{P_t}$$

If $Q_{t,t+k}$ is the right value of future payment, the utilities are equal. The **SDF** is then $Q_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\sigma} (P_t/P_{t+k})$

Firms: Optimal price setting

- Choice of price maximizes stream of expected future profits, discounted with households' SDF:

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} Q_{t,t+k} \Pi_{t+k}^N$$

- Denote $\Pi_{t+k|t}^N$ nominal profit at period $t+k$ of firm that last chose price at t
- For a firm setting price in current period t, maximization of profit of the period is as in flex price model:

$$\Pi_{t|t}^N = P_t^* Y_{t|t} - TC_t^N(Y_{t|t})$$

with $Y_{t|t} = \left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} C_t$

- For a firm that hasn't changed price for a while, it is:

$$\Pi_{t+k|t}^N = P_t^* Y_{t+k|t} - TC_{t+k}^N(Y_{t+k|t})$$

with $Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} C_{t+k}$

Firms: Optimal price setting

Nominal profit Π^N_{t+k} depends on P^*_t only if firm never could change price between t and $t+k$. Probability is θ^k

Expected nominal profit can be rewritten

$$E_t \Pi^N_{t+k} = \theta^k \Pi^N_{t+k|t} + (1-\theta)\theta^{k-1}\Pi^N_{t+k|t+1} + (1-\theta)^2\theta^{k-2}\Pi^N_{t+k|t+2} + \dots$$

- only first term depends on $P^*_t \Rightarrow$ all other terms ignored in period t maximization

Finally, maximization problem can be rewritten:

$$\max_{P^*_t} \sum_{k=0}^{\infty} \theta^k E_t Q_{t,t+k}[P^*_t Y_{t+k|t} - TC^N_{t+k}(Y_{t+k|t})]$$

$$\text{s.t.} \quad Y_{t+k|t} = \left(\frac{P^*_t}{P_{t+k}}\right)^{-\varepsilon} C_{t+k}$$

Firms: Optimal price setting

Solution FOC w.r.t. P_t^*:

$$\sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} Y_{t+k|t} (P_t^* - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k|t}^N) \right] = 0$$

with

- $MC_{t+k|t}^N \equiv \frac{d}{dY} TC_{t+k}^N (Y_{t+k|t})$: nominal marginal cost in period $t + k$
- $\frac{\varepsilon}{\varepsilon - 1}$: desired (flexible-price) mark-up over marginal costs

Flexible price solution obtains under $\theta = 0$:

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} MC_{t|t}^N \quad \text{for each } t$$

Firms: Optimal price setting

$$\sum_{k=0}^{\infty} \theta^k E_t Q_{t,t+k} Y_{t+k|t} P_t^* = \frac{\varepsilon}{\varepsilon - 1} \sum_{k=0}^{\infty} \theta^k E_t Q_{t,t+k} Y_{t+k|t} MC_{t+k|t}^N$$

Using $Q_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\sigma} (P_t/P_{t+k})$ and solving for P_t^*:

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{k=0}^{\infty} \theta^k \beta^k C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon-1} MC_{t+k|t}^N}{E_t \sum_{k=0}^{\infty} \theta^k \beta^k C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon-1}} \qquad (*)$$

marginal (nominal)

Optimal price is markup times weighted sum of expected ~~real~~ costs

$$P_t^* E_t \sum_{k=0}^{\infty} \theta^k \beta^k C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon-1} = \frac{\varepsilon}{\varepsilon-1} \sum_{k=0}^{\infty} \theta^k \beta^k E_t C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon-1} MC_{t+k|t}^N \quad (*)$$

Taylor expansion (first order) of (*):

1) Left hand side of (*):

$$P \sum_{k=0}^{\infty} \theta^k \beta^k C^{1-\sigma} P^{\varepsilon-1} + \overbrace{\sum_{k=0}^{\infty} \theta^k \beta^k C^{1-\sigma} P^{\varepsilon-1}}^{\frac{\partial LHS}{\partial P_t^*}} \cdot P(p_t^* - p)$$

$$+ P \sum_{k=0}^{\infty} \theta^k \beta^k (1-\sigma) C^{-\sigma} P^{\varepsilon-1} \cdot C(c_{t+k} - c)$$

$$+ P \sum_{k=0}^{\infty} \theta^k \beta^k C_{t+k}^{1-\sigma} (\varepsilon-1) P^{\varepsilon-2} \cdot P(p_{t+k} - p)$$

2) Right hand side

$$\frac{\varepsilon}{\varepsilon-1} \sum_{k=0}^{\infty} \theta^k \beta^k C^{1-\sigma} P^{\varepsilon-1} MC^N + \frac{\varepsilon}{\varepsilon-1} \sum_{k=0}^{\infty} \theta^k \beta^k C^{1-\sigma} P^{\varepsilon-1} \cdot MC^N (mc_{t+k|k}^N - mc^N)$$

$$+ \sum_{k=0}^{\infty} \theta^k \beta^k (1-\sigma) C^{\sigma} P^{\varepsilon-1} MC^N \cdot C(c_{t+k} - c)$$

$$+ \sum_{k=0}^{\infty} \theta^k \beta^k C_{t+k}^{1-\sigma} (\varepsilon-1) P^{\varepsilon-2} MC^N \cdot P(p_{t+k} - p)$$

Firms: Optimal price setting

After doing a (long) log-linearization around steady state with $P_t = P_{t-1} = P; \pi = 0$:

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t[\hat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})] \quad (2)$$

where $\hat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc$ denotes the **log deviation** of real marginal cost from its steady state value.
Use $mc = \ln(MC) = \ln(MC^N/P) = \ln \frac{\varepsilon}{\varepsilon - 1} \equiv \mu$ to obtain:

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t[mc_{t+k|t} + p_{t+k}]$$

Outline

Equilibrium – aggregate (log) output

$Y_t(i)$ and $L_t(i)$ vary across firms because of sticky prices $\Rightarrow$ aggregate production function not same as firms' production function. Recall aggregate labor definition: $L_t = \int_0^1 L_t(i)di$; using the firms' production function:

$$L_t = \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di$$

In logs:

$$(1-\alpha)l_t = y_t - a_t + \mathcal{D}_t$$

$$\mathcal{D}_t \equiv (1-\alpha) log \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{\varepsilon}{1-\alpha}}$$

where $\mathcal{D}_t$ is a measure of price dispersion. In a first-order approximation around s.s. with $\pi = 0$, it is null $\Rightarrow$ simple *approximate* aggregate production function: $y_t = a_t + (1-\alpha)l_t$

New Keynesian Phillips Curve

We need some more algebra (again, presented more fully in Bergholt notes) to obtain a micro-founded version of the **AS relationship** linking inflation and the output gap.

Surprisingly, the micro-founded version has been labelled the **New Keynesian Phillips Curve**, although unemployment is out of the picture.

This equation ends up being the only (but crucial) difference between the flexible price and sticky price versions of the model.

Inflation and average marginal cost

Aggregate price dynamics (1) and optimal price setting of firms (2) allow to relate inflation to average marginal costs in the economy.

First, $mc_{t+k|t}$ is expressed with respect to economy's average marginal costs: $mc_{t+k|t} = w_t^N - p_t - \frac{1}{1-\alpha}(a_{t+k} - \alpha y_{t+k|t}) - \ln(1-\alpha)$

mc_{t+k} $\quad$ y_{t+k}

$$mc_{t+k|t} = mc_{t+k} + \frac{\varepsilon\alpha}{1-\alpha}(p_t^* - p_{t+k})$$

After substitution in (2) and re-arranging, we get

$$p_t^* - p_{t-1} = E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left((1-\theta\beta) \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \hat{mc}_{t+k} + \pi_{t+k} \right)$$

which can be written recursively:

$$p_t^* - p_{t-1} = \beta\theta E_t \left[p_{t+1}^* - p_t \right] + (1-\beta\theta)\theta \hat{mc}_t + \pi_t$$

combine with $\pi_t = (1-\theta)(p_t^* - p_{t-1})$ to get:

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda \hat{mc}_t \tag{3}$$

$$\text{with } \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$$

Recursive expressions

$$p_t^* - p_{t-1} \equiv x_t \quad ; \quad \underbrace{(1-\theta\beta)\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\hat{mc}_{t+k} + \pi_{t+k}}_{\xi_{t+k}}$$

$$x_t = E_t \sum_{k=0}^{\infty} \beta^k \theta^k \xi_{t+k}$$

$$x_t = \beta^0 \theta^0 \xi_t + \underbrace{E_t \sum_{k=1}^{\infty} \beta^k \theta^k \xi_{t+k}}_{E_t \underbrace{E_{t+1} \sum_{k=0}^{\infty} \beta^{k+1} \theta^{k+1} \xi_{t+k+1}}_{\beta\theta x_{t+1}}}$$

$$x_t = \xi_t + \beta\theta E x_{t+1}$$

Marginal cost and output gap

As seen in flex. price model, $mc_t^N = w_t^N + \left(\frac{1}{\alpha-1}\right) a_t + \left(\frac{\alpha}{1-\alpha}\right) y_t$.

To get an expression for real log marginal costs $mc_t (= mc_t^N - p_t)$, use households' consumption-labor optimality $w_t^N - p_t = \sigma y_t + \eta l_t$ and approximate aggregate production $y_t = a_t + (1-\alpha) l_t$ to get:

$$mc_t = \left(\sigma + \frac{\eta + \alpha}{1-\alpha}\right) y_t - \frac{1+\eta}{1-\alpha} a_t - log(1-\alpha)$$

The steady state log real marginal cost is $-\mu$ but can also be related to **output under flexible prices** or **natural output** y_t^n:

$$mc = \left(\sigma + \frac{\eta + \alpha}{1-\alpha}\right) y_t^n - \frac{1+\eta}{1-\alpha} a_t - log(1-\alpha)$$

Finally, log deviation of real marginal cost $\hat{mc}_t \equiv mc_t - mc$ is related to **log output gap** $\tilde{y}_t \equiv y_t - y_t^n$:

$$\hat{mc}_t = \left(\sigma + \frac{\eta + \alpha}{1-\alpha}\right) (y_t - y_t^n) \tag{4}$$

And we can obtain the New Keynesian Phillips Curve

The New Keynesian Phillips Curve

Equation (4) for marginal cost & output gap and equation (3) for inflation & marginal cost result in the **New Keynesian Phillips Curve** (NKPC):

$$\pi_t = \beta Et[\pi_{t+1}] + \kappa \tilde{y}_t \tag{5}$$

where $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \left(\sigma + \frac{\eta+\alpha}{1-\alpha}\right)$

Note that κ depends negatively on both θ and β, which get smaller if period length gets larger $\Rightarrow$ relationship gets steeper as horizon gets longer, as with the old **AS** curve

Outline

1. Nominal rigidities
2. Firm problem with Calvo pricing
3. Equilibrium
   - New Keynesian Phillips Curve
   - Dynamic IS curve
   - Monetary policy rule
   - Quantitative analysis

Dynamic IS curve

From the Euler equation and goods market equilibrium

$$y_t = E_t[y_{t+1}] - \frac{1}{\sigma}(\underbrace{i_t - E_t[\pi_{t+1}]}_{r_t} - \rho)$$

discount rate $(-\ln\beta)$

while in the flexible price model:

$$y_t^n = E_t[y_{t+1}] - \frac{1}{\sigma}(r_t^n - \rho)$$

(recall that the **natural real interest rate** r_t^n is entirely driven by fundamentals – parameters and productivity)
Take a difference $\Rightarrow$ **dynamic IS equation** in terms of the output gap $\tilde{y}_t$

$$\tilde{y}_t = -\frac{1}{\sigma}(\underbrace{i_t - E_t[\pi_{t+1}]}_{r_t} - r_t^n) + E_t[\tilde{y}_{t+1}] \qquad (6)$$

$$\tilde{y}_t = -\frac{1}{\sigma}(r_t - r_t^n) + \left(-\frac{1}{\sigma}\right)(r_{t+1} - r_{t+1}^n) + \ldots\ldots + \overbrace{\lim_{T\to\infty}\tilde{y}_T}^{=0}$$

Dynamic IS curve

Note that the dynamic IS is another recursive equation. Iterate forward:

$$\tilde{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}(r_{t+k} - r^n_{t+k}) + \underbrace{\lim_{T\to\infty} E_t[\tilde{y}_{t+T}]}_{=0}$$

Interpretation: the output gap is proportional to the sum of current and anticipated deviations between the real interest rate and its natural counterpart.

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Closing the model: monetary policy rule

- ▷ Output gap $\tilde{y}_t$ and inflation π_t are linked through The NK Phillips Curve (5) and the dynamic IS equation (6)
- ▷ However, the nominal interest rate is not determined by anything $\Rightarrow$ model cannot be solved without an additional monetary rule

$\rightarrow$ Central Bank's **Taylor Rule** used to complete the model

- ▷ Both inflation and output gap are generally added in the Taylor Rule:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \upsilon_t,$$

with coefficients $\phi_\pi, \phi_y > 0$ and an exogenous component υ_t – discretionary part of monetary policy that follows an AR(1) process:

$$\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \varepsilon_t^\upsilon, \text{ where } \rho_\upsilon \in [0, 1)$$

Model dynamics

Complicated micro-foundations, but very simple dynamic properties of model:

- two dynamic endogenous variables only: $\tilde{y}_t$, π_t
- both are jump/control variables (not pre-determined)
- two dynamic equilibrium equations: NKPC and Dynamic IS

Linear state-space form:

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = A \begin{bmatrix} E_t \tilde{y}_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} + B(r_t^n - \rho - \upsilon_t)$$

with $A = \frac{1}{\sigma + \phi_y + \kappa\phi_\pi} \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}$ and $B = \frac{1}{\sigma + \phi_y + \kappa\phi_\pi} \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$

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Calibration

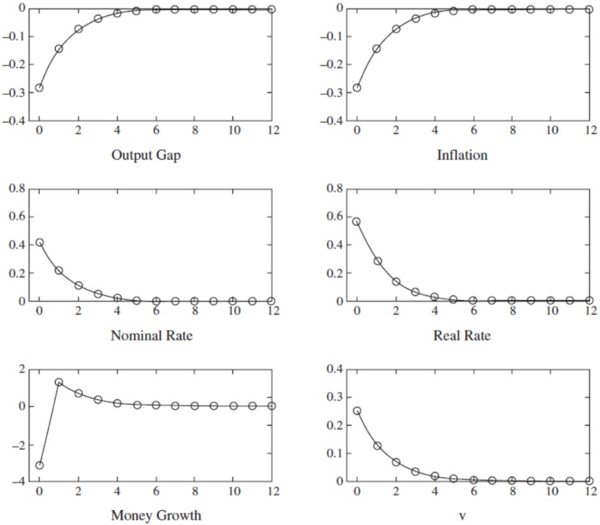
household utility function

$$u(c_t, 1 - n_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\eta}}{1+\eta}$$

- $\beta = 0.99$, time preference
- $\sigma = 1$, (risk aversion) $\rightarrow$ log-utility
- $\eta = 1$, unitary Frisch elasticity of labor supply
- $\alpha = 1/3$, inverse of labour share
- $\varepsilon = 6$, elasticity of substitution in Dixit-Stiglitz aggregator
- $\theta = 2/3$, probability to be able to change price ($\Rightarrow$ average price duration $\approx$ 3 quarters)
- $\phi_\pi = 1.5$, $\phi_y = 0.5/4$, policy rule
- $\rho_v = 0.5$, persistence of monetary policy shock

Effect of monetary policy in basic NK model

Effects of a **contractionary** monetary policy shock, $\varepsilon_t^v > 0$



Source. Galí (2008), Figure 3.1.