Macroeconomics
Lecture 2 — AD-AS, Introduction to Dynamics

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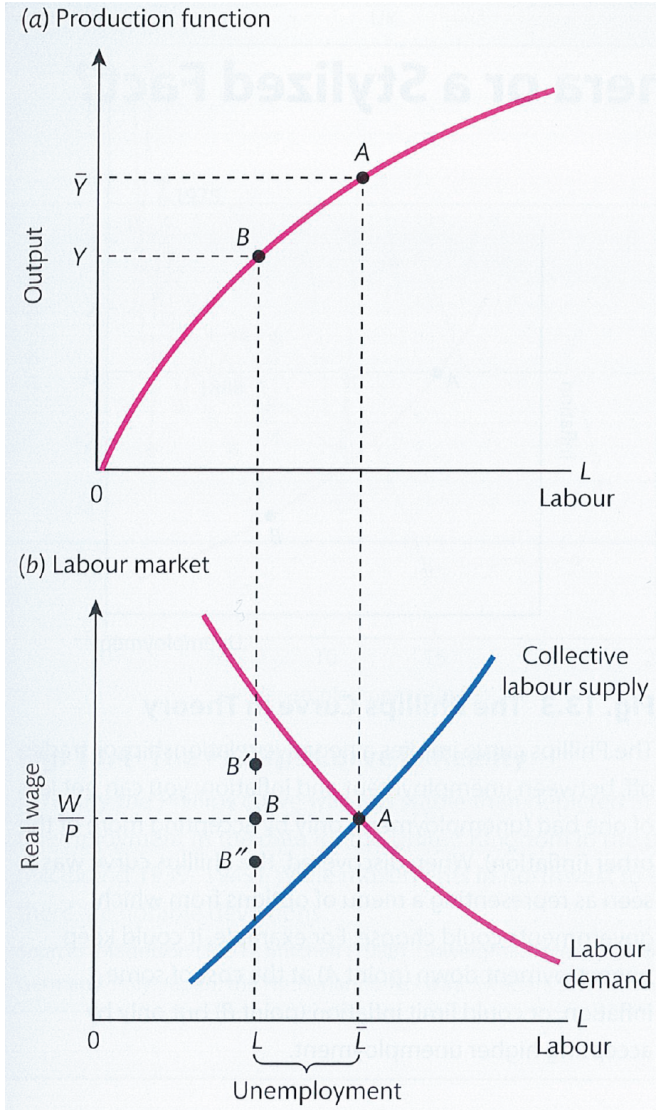
Lecture overview

- So far, very short run $\Rightarrow$ prices fixed $\Rightarrow$ no modelling of *inflation dynamics*
- This lecture:
  - **Long run**: flexible prices $\Rightarrow$ **Monetary Neutrality**
  - **Medium run**: prices adjust in response to excess demand and supply of goods, but not all markets clear fully.

$\Rightarrow$ **Phillips curve** and **AS** curve, the two closely related

Supply-determined output in the long run

- What determines output in the **long run (on the trend)**?
- Simplistic production theory:
  - Only labor (L) used for production
  - L's **marginal product** = **real wage** $\Rightarrow$ equilibrium $\bar{L}$, output $\bar{Y}$
- What about demand?
  - In the medium run, **demand adjusts to supply** via **real wage**



Source. Burda and Wyplosz (2017), Figure 13.2.

Money and inflation in long run

Start with the **Cambridge equation** (Quantity Theory of money):

$$M = kPY$$

$$\Leftrightarrow \quad P = \frac{M}{kY}$$

assuming *money velocity* k constant, logarithms of both sides and a total differential:

$$\ln P = \ln M - \ln k - \ln Y$$

total differential:
$$\frac{dP}{P} = \underbrace{\frac{dM}{M}}_{\equiv \mu} - \underbrace{\frac{dk}{k}}_{=0} - \underbrace{\frac{dY}{Y}}_{\equiv g}$$

$$\Leftrightarrow \pi = \mu - g$$

Monetary policy, interest in the long run

The long-run inflation is the inflation target in the Taylor rule: $\bar{\pi} = \mu - g$. Target depends on:

1. Money growth rate μ, a policy variable
2. Trend growth rate g determined (mostly) by technology and demographics

$\pi = \bar{\pi}$, $Y = \bar{Y}$ in the long run, so from the TR equation:

$$i = \bar{i} + a(\pi - \bar{\pi}) + b\left(\frac{Y - \bar{Y}}{\bar{Y}}\right) = \bar{i}$$

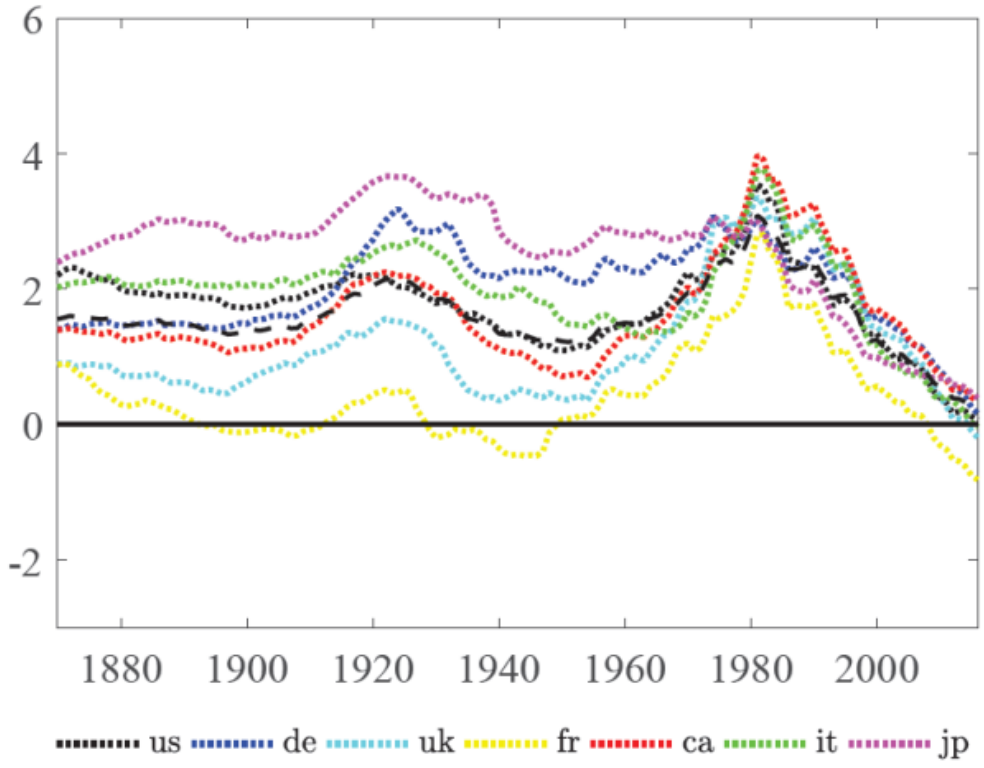
Nominal interest at its long-run target — the **natural interest rate**. Real interest rate:

$$\bar{r} = \bar{i} - \bar{\pi} = \bar{i} - \mu + g$$

Technology and demographics affect the real interest rate: it is high in an economy with faster trend growth.

Real natural interest estimates in developed economies

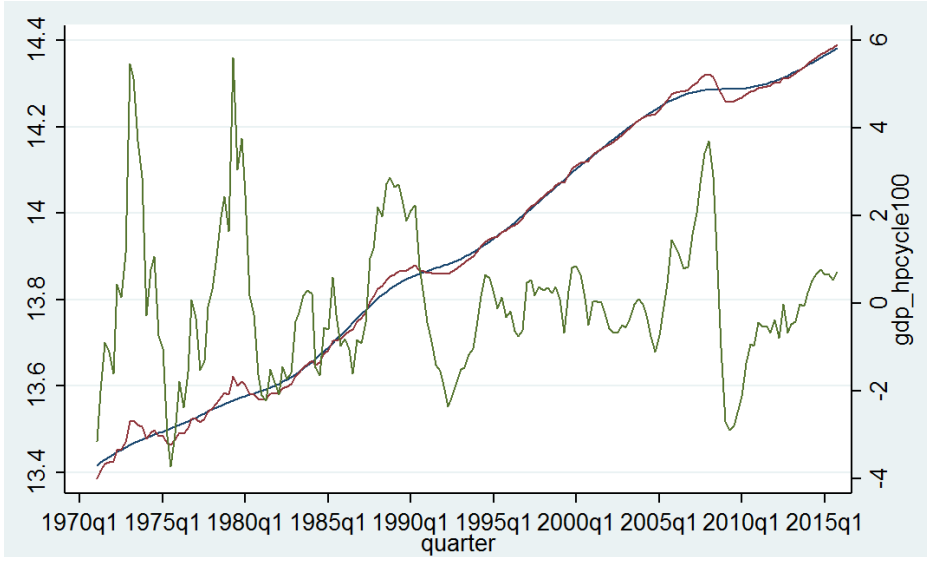
- $\bar{r}$ has a common tendency between countries; very low or negative levels in the 2010s



Note. Estimates from a Bayesian factor model with country-specific and global trend for the real natural rate of interest, r_i^*. **Source.** Del Negro, Giannone, Giannoni and Tambalotti (2019), 'Global trends in interest rates', Figure 3.

Recap: short vs. long run

- Keynesian short run: supply adjusts to demand
- Long run: technology, demographics determine output
  - Keynesian demand mechanisms not relevant



Note. Decomposition of real GDP in the United Kingdom (in logarithm), 1970Q1 - 2018Q4, using the hp-filter for the trend (blue line) and cycle (green line) decomposition. **Source.** OECD, author's calculations.

Next: *How do prices and wages move in response to temporary disequilibria on goods and labour markets?*

Outline

Phillips Curve and Okun's law: role for theory

- Keynesian theory: no relation between aggregate output and inflation — the *missing equation* problem

- two empirical findings used to fill the theory gap:

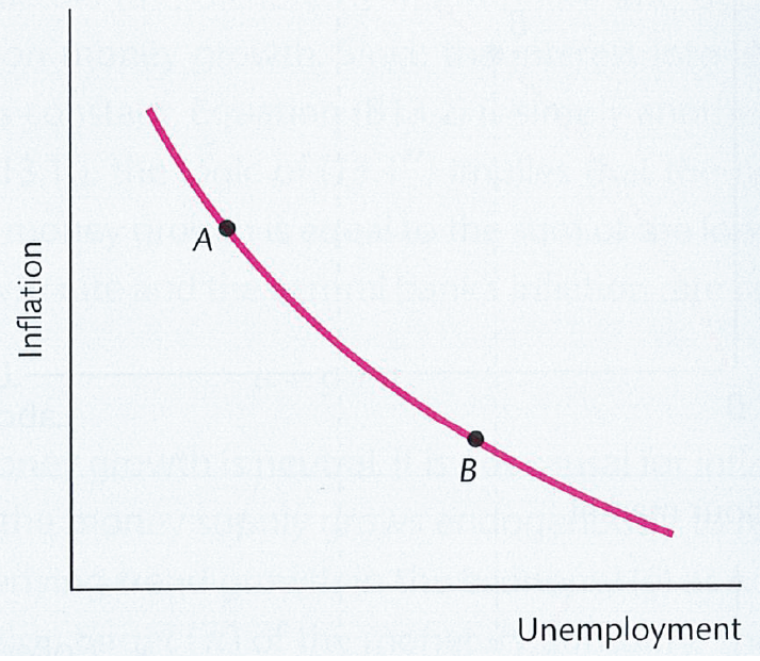
  1. A. W. Phillips (1958): negative correlation between wage inflation and unemployment rate

  2. A. Okun (1962): negative correlation between unemployment rate and GDP changes

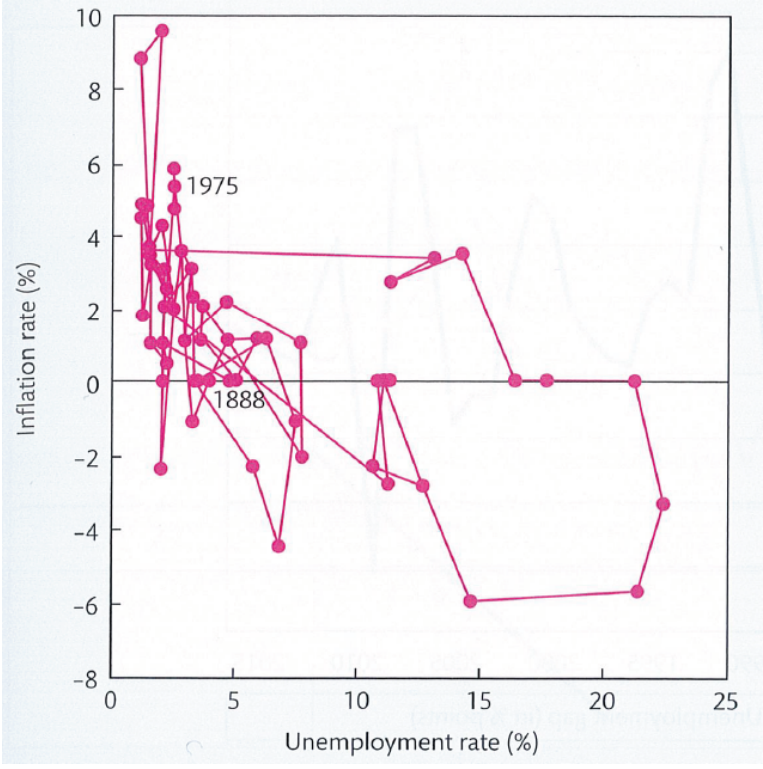
$\Rightarrow$ inflation-output relationship

The Phillips Curve

PC in theory



PC in the data (UK 1921-1973)



Source. Burda and Wyplosz (2017), Figure 13.3, Figure 13.4.

The Phillips Curve: policy trade-off

- Phillips curve as an intuitive and practical idea for policy-makers: cannot achieve all good macro indicators at once:
  - Either lower inflation and higher unemployemt, ...
  - ... or higher inflation, but lower unemployment.
- Slope of Phillips curve referred to as **sacrifice ratio**

"I would rather prefer 5% inflation, then 5% unemployment (...)"

Helmut Schmidt, 1978

Okun's Law

- *Missing element:* Aggregate relationship between output and inflation

$$\text{inflation} \underset{(1)PC}{\longleftrightarrow} \text{unemployment} \underset{(2)OL}{\longleftrightarrow} \text{output}$$

1. (1)**Phillips curve** (PC) not sufficient

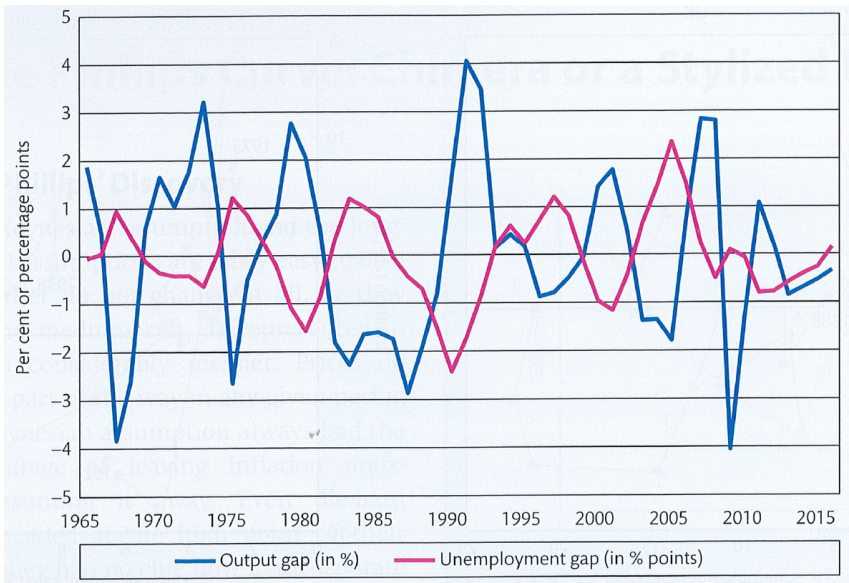
2. (2)**Okun's Law** (OL), Arthur Okun (1928-1980):
   - negative relationship between output and unemployment.

Okun's Law: Data

- **output gap** and **unemployment gap** are negatively correlated, $corr(Y_{gap}, U_{gap}) < 0$, $Y_{gap} \equiv (Y - \bar{Y})/\bar{Y}$ and $U_{gap} \equiv U - \bar{U}$
- equilibrium level of unemployment, $\bar{U}$
- potential level of output, $\bar{Y}$
- Modelling:

$$U_{gap} = -hY_{gap}$$

Figure. The output gap and unemployment in Germany, 1970-2016



Source. Burda and Wyplosz (2017), Figure 13.5.

Outline

Prices and Costs

How are goods' prices (inflation) connected to wages and to output? Consider profit-maximizing firms:

- Firms have some **monopoly power** $\Rightarrow$ set prices
- Cannot set prices too high because demand is **price-elastic**

Mark-up pricing:

- Prices cannot be below **marginal costs**
- Monopoly power $\Rightarrow$ price $=$ marginal costs $+$ **mark-up**
- Size of mark-up depends on **demand elasticity**
- Competitive markets: high elasticity, mark-up $\rightarrow 0$

Marginal cost vs. average cost, share of labor cost

If $TC(Y)$ are total costs, **marginal costs** are $\frac{dTC(Y)}{Y}$.
Marginal costs can be approximated by **average costs** or **unit costs** $\frac{TC(Y)}{Y}$:

$$\text{unit costs} = \frac{TC(Y)}{Y} = \underbrace{\frac{W \cdot L}{Y}}_{\text{unit labour costs}} + \underbrace{\frac{\text{total non-labor costs}}{Y}}_{\text{unit non-labor costs}}$$

Share of labor costs is firms output — S^L — is ratio of labor costs to *value* of output:

$$S^L = \frac{W \cdot L}{P \cdot Y}$$

Share of labour costs ranges from 50% to 70% in firms' output in developed countries and is stable over time — one of Kaldor's stylized facts.

Building theoretical Phillips curve: battle of markups

Prices as mark-up on labour costs

Firms with market power aim to set price as a markup over unit labor cost (or setting labor cost share):

$$P = (1+\theta)\frac{WL}{Y},$$

where $\theta > 0$ the markup.

Wages as a mark-up on prices

Workers (unions) bargain with firms for higher wages. **Sticky wages**: negotiated wage fixed for some periods $\Rightarrow$ firms exposed to change of $\frac{W}{P}$ in the future under a fixed $W \Rightarrow P^e$ used in wage setting. Bargaining determines firms' labor cost share under expected future prices P^e. In the negotiation, firms have reference price $\bar{P}$ and workers have reference (minimum) wage $\bar{W}$.

$$\frac{WL}{Y} = (1+\gamma)\bar{S}_L P^e,$$

where $\bar{S}_L$ is reference labor cost share $\bar{S^L} = \frac{\bar{W}L}{\bar{P}Y}$ and γ is the markup.

Microfoundations: price setting

$\max_P P \cdot Y - W \cdot L$, with

$Y = Y^d(P)$ consumers' goods demand,
$L = L^d(Y)$ firm's labor demand.

Microfoundations: wage setting

$\max_W (W - \bar{W})^{\beta}(\bar{P} \cdot Y - W \cdot L)^{1-\beta}$, where β is workers' negotiation power.

Building theoretical Phillips curve: battle of markups

- Prices depend on wage, wage depends on **expected** prices

$\Rightarrow$ combining wage setting and price setting equations, dependence of current wages on future wage obtained

$$P = (1 + \theta)(1 + \gamma)\bar{S}_L P^e$$

- anchor: expected price level P^e

From markups to output and unemployment

$$P = (1+\theta)(1+\gamma)\bar{S}_L P^e$$

The product $(1+\theta)(1+\gamma)$ most likely *procyclical*.

- Price markup θ: when GDP rises, competition increases ($\theta \downarrow$), but demand elasticity decreases ($\theta \uparrow$)

  $\Rightarrow$ total effect mixed

- Wage markup γ: when GDP rises, higher workers' bargaining power (β), firms hire and incentivise over-time work ($\gamma \uparrow$)

  $\Rightarrow$ γ procyclical

From price to inflation: deriving AS and Phillips curve

Taking logs and total differential of the equation:

$$P = (1+\theta)(1+\gamma)\bar{S}_L P^e$$

$$\ln P = \theta + \gamma + \ln \bar{S}_L + \ln P^e \text{ (as } \ln(1+x) \underset{x\to 0}{\to} x)$$

total differential: $\frac{dP}{P} = d\theta + d\gamma + \underbrace{\frac{d\bar{S}^L}{\bar{S}^L}}_{=0} + \frac{dP^e}{P^e}$

$$\pi = d\theta + d\gamma + \pi^e$$

$\theta + \gamma$ **procyclical** $\rightarrow d\theta + d\gamma = a \cdot Y^{gap} \Rightarrow$

$$\pi = a \cdot Y_{gap} + \pi^e$$

Finally, using **Okun's law** to replace Y^{gap} with U^{gap}:

$$\pi = -b \cdot U_{gap} + \pi^e$$

AS, Phillips Curve — final form

Two elements are added to obtain full **Phillips curve** and **Aggregate Supply** relationships:

1. Underlying rate of inflation $\tilde{\pi}$
   - ▷ generalization of expected inflation rate π^e in wage setting
   - ▷ wages sticky $\Rightarrow$ not only current expectations, but past expectations influence wage setting
   - ▷ explicit rules might exist for adjusting W to π $\Rightarrow$ past inflation rates enter $\tilde{\pi}$
2. Supply shocks s
   - ▷ Shocks to non-wage marginal costs
   - ▷ Taken into account by firms when setting prices

Results:

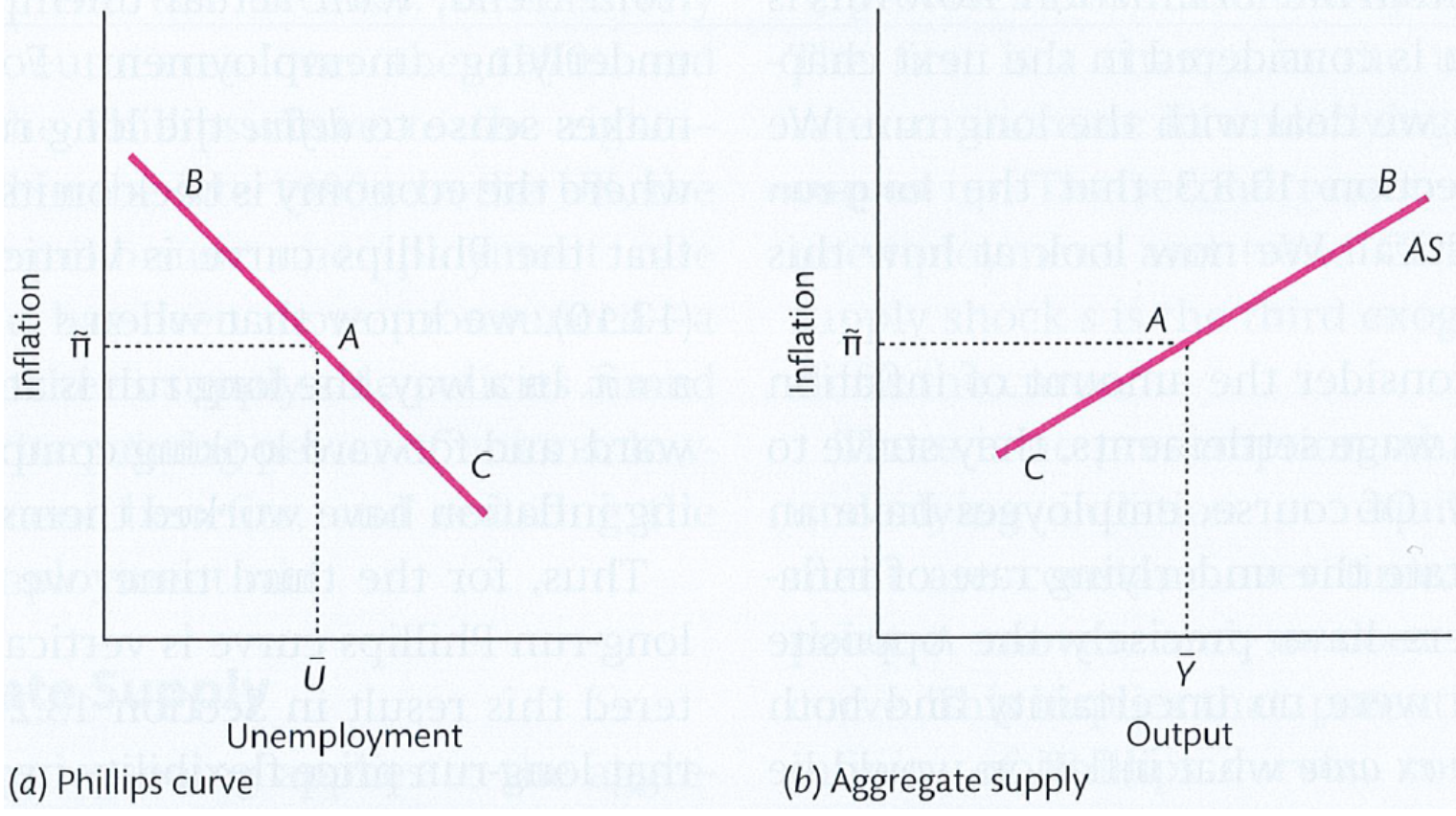
$$\pi = -bU_{gap} + \tilde{\pi} + s \qquad \text{(Phillips Curve)}$$

$$\pi = \quad aY_{gap} + \tilde{\pi} + s \qquad \text{(AS)}$$

AS and Phillips curve: symmetry

$$\pi = -bU_{gap} + \tilde{\pi} + s \qquad \text{(Phillips Curve)}$$

$$\pi = aY_{gap} + \tilde{\pi} + s \qquad \text{(AS)}$$



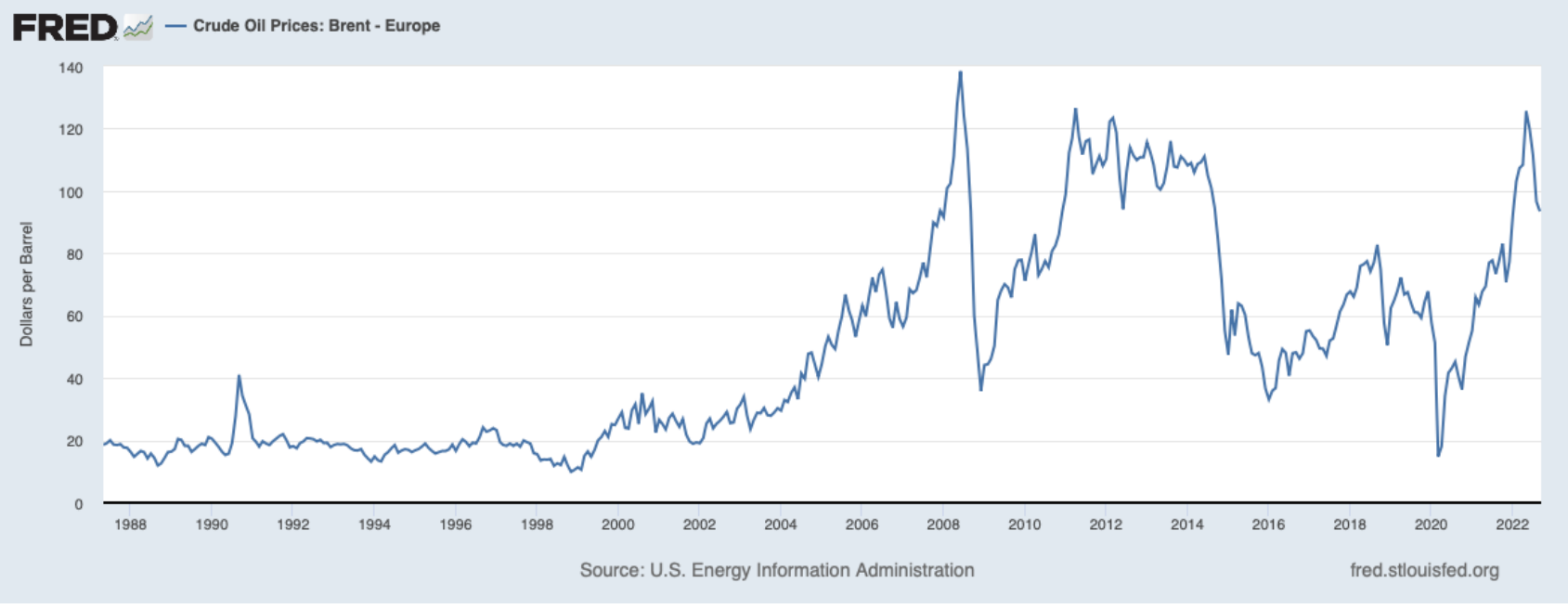
Note. The new Phillips curve. **Source.** Burda and Wyplosz (2017), Figure 13.12.

Non-labour costs and supply shocks

Firms have small supply shocks all the time, but which ones are macroeconomic?

Energy prices, especially fossil fuels, have a big role:

- First oil shocks: 1973/74, 1979/81
- Second sequence of shocks: 1999/2001, 2005/12
- Favourable oil shocks: 1986 and 2015
- Current: war & sanctions $\Rightarrow$ Russian oil shut off: 2022



Brent Europe crude oil price. **Source.** St. Louis Fed.

Underlying inflation, long-run Phillips curve

- **Rational expectations**
  - Forecast errors occur, but must average to zero over longer horizons
  - Differences in π and $\tilde{\pi}$ must be temporary
  - Long-run link equivalence of actual and underlying inflation:
    if $s = 0$ and $U = \bar{U}$, then $\pi = \tilde{\pi}$
- Implies a **vertical Phillips Curve in the long run**

- The level of **long run inflation**?

  $\rightarrow$ $\bar{\pi}$, **inflation target of the central bank**.

Long-run Aggregate Supply (LAS)

- Recall that **trend output** $\bar{Y}$ determined by technology, demographics
  - No relation of $\bar{Y}$ level to $\pi \Rightarrow$ **vertical LAS**
- Another way to obtain — long-run Phillips curve & Okun's law

Implications:

- **Short run**: Actual inflation deviates from underlying inflation $\tilde{\pi}$ in tandem with the business cycle
- **Long-run**: output returns to its growth path, independent of price level.

  Horizontal movement due to trend output $\bar{Y}$ growing at rate g

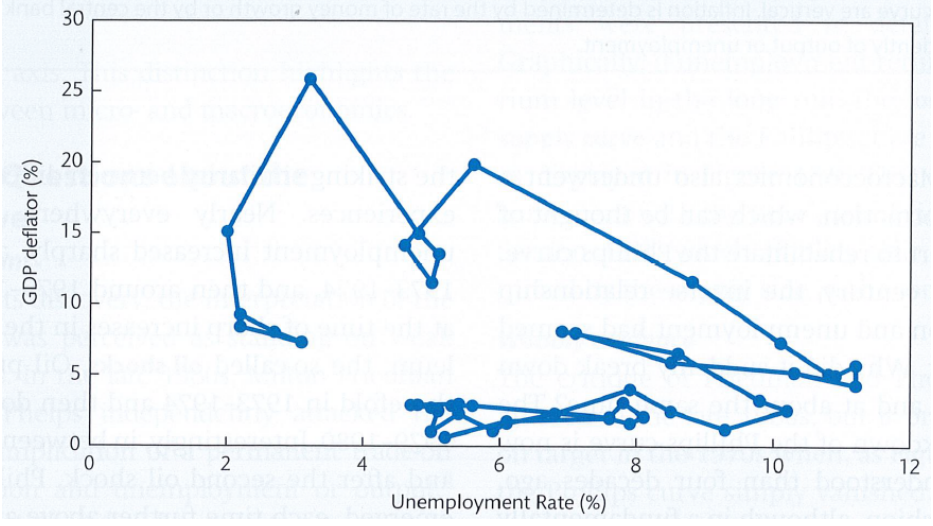
Shifts in Phillips curve and AS — unstable relationships

- $\tilde{\pi}, \bar{U}, \bar{Y}, s$ are taken as exogenous and can shift the curves

$\rightarrow$ a set of Phillips curves, not a stable negative relationship

- A. W. Phillips was **lucky** to find one stable curve
- At least to strong reasons for shifts since 1970s:
  - Supply shocks (oil prices)
  - Shifts in the equilibrium unemployment rate

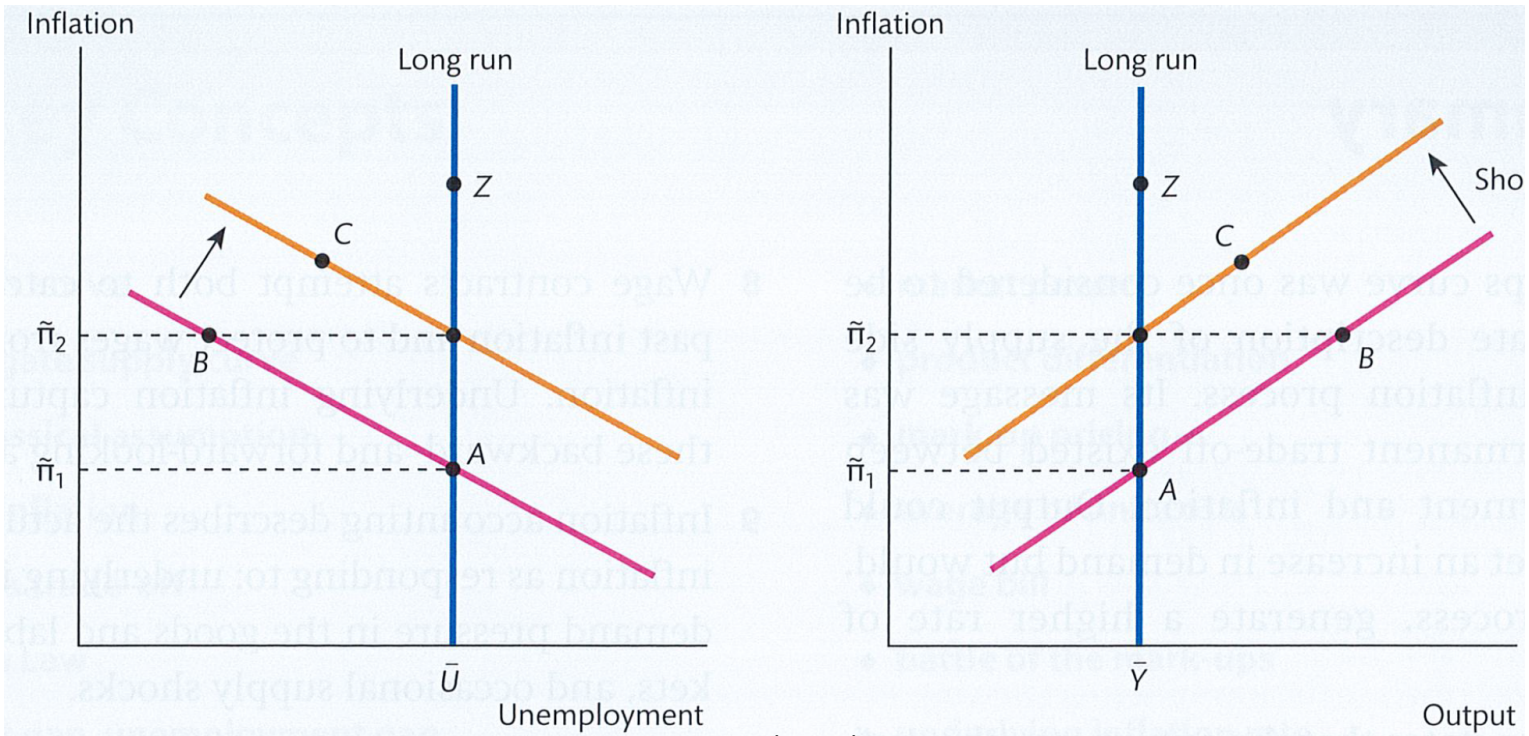
Figure. United Kingdom, 1970-2015



Source. Burda and Wyplosz (2017), Figure 13.10.

Phillips curve and AS: summary

- Instead of a simple U, π relationship found originally, theoretical Phillips curve is more flexible:
  1. Short-run Phillips curve depends on expectations and supply shocks
  2. Long-run Phillips curve is vertical — no inflation-unemployment link
- Via **Okun's law**, short-run and long-run **AS** are obtained



Source. Burda and Wyplosz (2017), Figure 13.13.

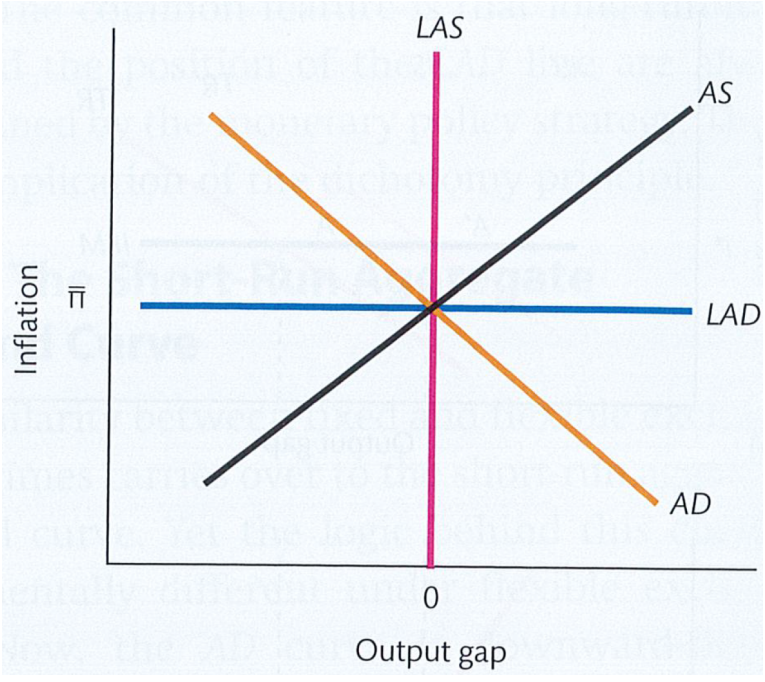
AD-AS model

- Medium-term movements of output and inflation shaped by both supply and demand
- **AS** is obtained above
- **AD** follows immediately from the IS-TR model with full version of TR:

$$i = \bar{i} + a(\pi - \bar{\pi}) + b\left(\frac{Y - \bar{Y}}{\bar{Y}}\right)$$

Long-run AD: Natural rate of interest

- In the long run, central bank assumed to set interest such that $\pi = \bar{\pi}$ for whatever Y (which is $\bar{Y}$ anyways)
  - horizontal **LAD**
  - will become more relevant in open economy analysis



Source. Burda & Wyplosz (2017), Figure 14.12.

Short run AD

AD & IS-TR:

Assume an increase in π:

- TR: Central bank raises interest rate
- Investment decreases, *IS* shifts to the left
- Equilibrium Y lower for higher π: downward sloping short run **AD**

Shifts in AD

- IS: any shift in desired demand
- *TR*: $\bar{Y}, \bar{\pi}$

Policy example: monetary policy

Higher inflation target, $\bar{\pi}' > \bar{\pi}$

Policy example: government spending

Positive fiscal policy shock, $\bar{G}' > \bar{G}$

Multiplier-Accelerator model

- A simple Keynesian model of the goods market
- **NOT** the way we think about cycles today!
- Motivation is methodological: look at a **dynamic** equilibrium

Three equations:

- Consumption depending on **last period** income
- Investment depending on **growth of** income in **last period** — the **accelerator** assumption
- Goods market equilibrium

Multiplier-Accelerator accelerator model

Variables

- $\{C_t\}$ — sequence of levels of aggregate consumption , main endogenous variable in the model.
- $\{I_t\}$ — sequence of rates of investment, another key endogenous variable.
- $\{Y_t\}$ — sequence of levels of national income, yet another endogenous variable.
- $\{G_t\}$ — sequence of levels of government expenditures. Exogenous, assumed constant: $G_t = G$ for all t.

Multiplier-Accelerator accelerator model

Model structure

The model combines the consumption function

$$C_t = aY_{t-1} + \gamma \tag{1}$$

with the investment accelerator

$$I_t = b(Y_{t-1} - Y_{t-2}) \tag{2}$$

and the goods market equilibrium

$$Y_t = C_t + I_t + G_t \tag{3}$$

- The parameter a is people's *marginal propensity to consume* out of income - equation (1) asserts that people consume a fraction of $a \in (0, 1)$ of each additional dollar of income.
- The parameter $b > 0$ is the investment accelerator coefficient - equation (2) asserts that people invest in physical capital when income is increasing and disinvest when it is decreasing.

Solution: a difference equation

Combining the three equations:

$$Y_t = (a + b)Y_{t-1} - bY_{t-2} + (\gamma + G_t)$$

Defining new coefficients gives compact form: assume $\rho_1 = (a + b)$ and $\rho_2 = -b$, then:

$$Y_t = \rho_1 Y_{t-1} + \rho_2 Y_{t-2} + (\gamma + G_t)$$

Assuming initial values to generate Y_t for $t = 0, \ldots, T$:

$$Y_{-1} = \bar{Y}_{-1}, \quad Y_{-2} = \bar{Y}_{-2}$$

When solving numerically, set (a, b) so that starting from $(\bar{Y}_{-1}, \bar{Y}_{-2})$, Y_t converges to a **steady state**

A dynamic equilibrium in explicit form

With eigenvalues λ_1, λ_2, the dynamics of Y_t given by:

$$Y_t = \lambda_1^t c_1 + \lambda_2^t c_2$$

where c_1 and c_2 are constants on parameters and initial conditions. initial conditions and on ρ_1, ρ_2.

When the eigenvalues are complex, can represent them in polar form $\lambda_1 = re^{i\omega}$, $\lambda_2 = re^{-i\omega}$ and rewrite the solution as follows (not necessary to memorize!):

$$Y_t = (c_1 + c_2)r^t \cos(\omega t) + i(c_1 - c_2)r^t \sin(\omega t)$$

Parameters and intital conditions chosen such that $c_1 + c_2$ real, $c_1 - c_2$ complex, so Y_t real and has a form:

$$Y_t = 2vr^t \cos(\omega t + \theta),$$

with v, θ some constants depending on parameters and intial conditions.

Recall graphs of cos function $\Rightarrow$ output (as well as investment and consumption) oscillate around steady state.