Macroeconomics
Lecture ~~4~~ 3 — AD-AS, Time consistency of monetary policy

Ilya Eryzhenskiy

PSME Panthéon-Sorbonne Master in Economics

Fall 2022

Building theoretical Phillips curve: battle of markups

Prices as mark-up on labour costs

Firms with market power aim to set price as a markup over unit labor cost (or setting labor cost share):

$$P = (1+\theta)\frac{WL}{Y}$$

where $\theta > 0$ the markup.

Wages as a mark-up on prices

Workers (unions) bargain with firms for higher wages. **Sticky wages**: negotiated wage fixed for some periods $\Rightarrow$ firms exposed to change of $\frac{W}{P}$ in the future under a fixed $W \Rightarrow P^e$ used in wage setting. Bargaining determines firms' labor cost share under expected future prices P^e. In the negotiation, firms have reference price $\bar{P}$ and workers have reference (minimum) wage $\bar{W}$.

$$\frac{WL}{Y} = (1+\gamma)\bar{S}_L P^e,$$

where $\bar{S}_L$ is reference labor cost share $\bar{S^L} = \frac{\bar{W}L}{\bar{P}Y}$ and γ is the markup.

From price to inflation: deriving AS and Phillips curve

Taking logs and total differential of the equation:

$$P = (1+\theta)(1+\gamma)\bar{S}_L P^e$$

$$\ln P = \theta + \gamma + \ln \bar{S}_L + \ln P^e \text{ (as } \ln(1+x) \underset{x\to 0}{\to} x)$$

total differential: $$\frac{dP}{P} = d\theta + d\gamma + \underbrace{\frac{d\bar{S}^L}{\bar{S}^L}}_{=0} + \frac{dP^e}{P^e}$$

$$\pi = d\theta + d\gamma + \pi^e$$

$\theta + \gamma$ **procyclical** $\rightarrow d\theta + d\gamma = a \cdot Y^{gap} \Rightarrow$

$$\pi = a \cdot Y_{gap} + \pi^e$$

Finally, using **Okun's law** to replace Y^{gap} with U^{gap}:

$$\pi = -b \cdot U_{gap} + \pi^e$$

$Y^{gap} = \frac{Y - \bar{Y}}{\bar{Y}}$

$Y^{gap} = \ln Y - \ln \bar{Y}$

$= y - \bar{y}$

$\approx \frac{Y - \bar{Y}}{\bar{Y}}$

AS, Phillips Curve — final form

Two elements are added to obtain full **Phillips curve** and **Aggregate Supply** relationships:

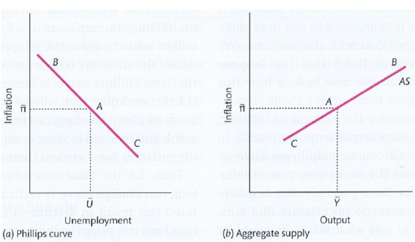
1. Underlying rate of inflation $\tilde{\pi}$
   - generalization of expected inflation rate π^e in wage setting
   - wages sticky $\Rightarrow$ not only current expectations, but past expectations influence wage setting
   - explicit rules might exist for adjusting W to π $\Rightarrow$ past inflation rates enter $\tilde{\pi}$
2. Supply shocks s
   - Shocks to non-wage marginal costs
   - Taken into account by firms when setting prices

Results:

$$\pi = -bU_{gap} + \underline{\tilde{\pi} + s} \qquad \text{(Phillips Curve)}$$
$$\pi = aY_{gap} + \tilde{\pi} + s \qquad \text{(AS)}$$

AS and Phillips curve: symmetry

$$\pi = -\, bU_{gap} + \tilde{\pi} + s \qquad \text{(Phillips Curve)}$$
$$\pi = \; aY_{gap} + \tilde{\pi} + s \qquad \text{(AS)}$$



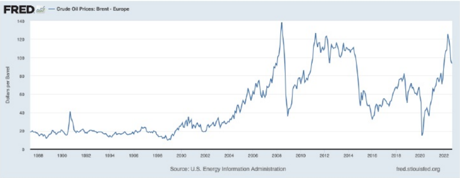
Note. The new Phillips curve. **Source.** Burda and Wyplosz (2017), Figure 13.12.

Non-labor costs and supply shocks

Firms have small supply shocks all the time, but which ones are macroeconomic?

Energy prices, especially fossil fuels, have a big role:

- First oil shocks: 1973/74, 1979/81
- Second sequence of shocks: 1999/2001, 2005/12
- Favourable oil shocks: 1986 and 2015
- Current: war & sanctions ⇒ Russian oil shut off: 2022



Brent Europe crude oil price. **Source.** St. Louis Fed.

Underlying inflation, long-run Phillips curve

- **Rational expectations**
  - Forecast errors occur, but must average to zero over longer horizons
  - Differences in π and $\tilde{\pi}$ must be temporary
  - Long-run link equivalence of actual and underlying inflation:
    if $s = 0$ and $U = \bar{U}$, then $\pi = \tilde{\pi}$
- Implies a **vertical Phillips Curve in the long run**

- The level of **long run inflation**?

  $\rightarrow$ $\bar{\pi}$, **inflation target of the central bank**.

Long-run Aggregate Supply (LAS)

- Recall that **trend output** $\bar{Y}$ determined by technology, demographics
  - No relation of $\bar{Y}$ level to $\pi \Rightarrow$ **vertical LAS**
- Another way to obtain — long-run Phillips curve & Okun's law

Implications:

- **Short run**: Actual inflation deviates from underlying inflation $\tilde{\pi}$ in tandem with the business cycle
- **Long-run**: output returns to its growth path, independent of price level.

  Horizontal movement due to trend output $\bar{Y}$ growing at rate g

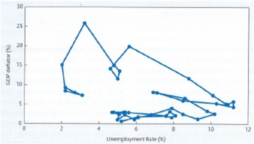
Shifts in Phillips curve and AS — unstable relationships

- $\tilde{\pi}, \bar{U}, \bar{Y}, s$ are taken as exogenous and can shift the curves

→ a set of Phillips curves, not a stable negative relationship

- A. W. Phillips was **lucky** to find one stable curve
- At least to strong reasons for shifts since 1970s:
  - Supply shocks (oil prices)
  - Shifts in the equilibrium unemployment rate

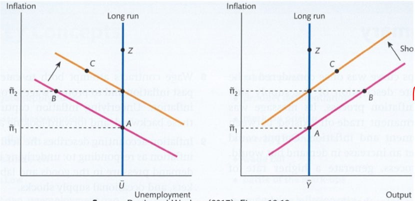
Figure. United Kingdom, 1970-2015



Source. Burda and Wyplosz (2017), Figure 13.10.

Phillips curve and AS: summary

- Instead of a simple U, π relationship found originally, theoretical Phillips curve is more flexible:
  1. Short-run Phillips curve depends on expectations and supply shocks
  2. Long-run Phillips curve is vertical — no inflation-unemployment link
- Via **Okun's law**, short-run and long-run **AS** are obtained



ck of S (supply-side shock)

Source. Burda and Wyplosz (2017), Figure 13.13.

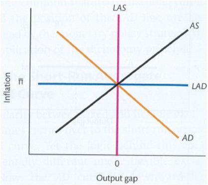
AD-AS model

- Medium-term movements of output and inflation shaped by both supply and demand
- **AS** is obtained above
- **AD** follows immediately from the IS-TR model with full version of TR:

$$i = \bar{i} + \alpha(\pi - \bar{\pi}) + \beta\left(\frac{Y - \bar{Y}}{\bar{Y}}\right)$$

Long-run AD (LAD)

- In the long run, central bank assumed to set interest such that $\pi = \bar{\pi}$ for whatever Y (which is $\bar{Y}$ in equilibrium)
  - horizontal **LAD**
  - will become more relevant in open economy analysis



Source. Burda & Wyplosz (2017), Figure 14.12.

Short run AD

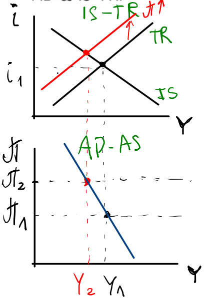
Assume an increase in π:

- TR: Central bank raises interest rate
- Investment decreases, *IS* shifts to the left
- Equilibrium Y lower for higher π: downward sloping short run **AD**

Shifts in AD

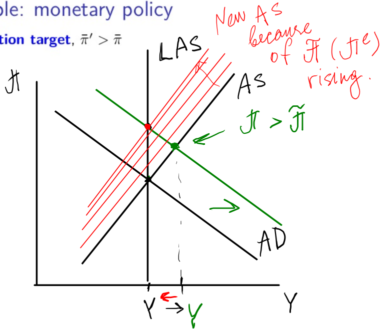
- IS: any shift in desired demand
- *TR*: $\bar{Y}, \bar{\pi}$

AD & IS-TR:



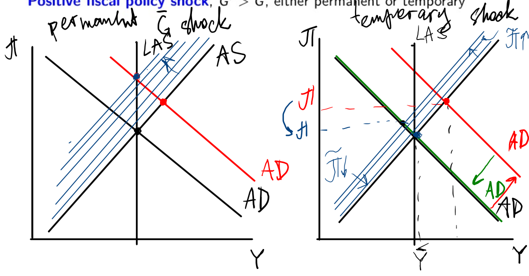
Policy example: monetary policy

Higher inflation target, $\bar{\pi}' > \bar{\pi}$



Policy example: government spending

Positive fiscal policy shock, $\bar{G}' > \bar{G}$, either permanent or temporary



AS once again

A short-medium run AS relationship $\pi = a(y - \bar{y}) + \tilde{\pi} + s$ arises in **many** models, under different assumptions.

- ▷ We have focused on price-wage-expected price link through **markup** price and wage setting, with **pro-cyclical** markups
  - ▷ Story went like output/~~inflation~~ u gap $\rightarrow$ inflation

AS can also go in inflation $\rightarrow$ output direction:

1. Sticky wages $\Rightarrow$ when $\pi \uparrow$, $\frac{W}{P} \downarrow$, so firms more profitable and $(y - \bar{y}) \uparrow$
2. Multiple goods, incomplete information in firms $\Rightarrow$ when $\pi \uparrow$, firms cannot distinguish **inflation** (e.g. due to monetary policy) from **relative price movements** (e.g. due to changes in preferences) $\Rightarrow$ each firm produces more, since demand might be rising for their particular product only (Lucas, 1972)

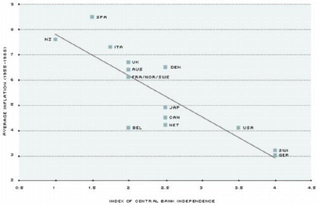
Can rewrite AS to highlight inflation $\rightarrow$ output:
$y = \bar{y} + a'(\pi - \tilde{\pi}) + e, \quad \text{where} \quad a' = 1/a, e = -1/a \cdot s$

Outline

Discretionary policy and time-inconsistency

- Central banks often independent from political influence. **Why?**
- Negative correlation between measures of central bank independence and inflation:

→ An independent central bank with a focus on price stability is a solution to **time consistency** problems in monetary policy



Sources. Federal Reserve Bank of St. Louis Annual Report 2009, Figure 1.

Time consistency of monetary policy: framework

- Central bank (CB) cares for both GDP and inflation
  - influences inflation, but also indirectly GDP though **AS**
- private agents guess CB's actions and form **inflation expectations**
- monetary policy can either be **discretionary** (CB free to change policy) or **rule-based**

Main results

1. CB uses inflation to boost GDP $\Rightarrow$ cannot credibly promise low inflation: **time inconsistency** problem
2. Discretionary policy $\Rightarrow$ higher inflation, which is anticipated $\Rightarrow$ no output gain
3. Rules-based policy $\Rightarrow$ can promise lower inflation, but still respond to supply shocks

The model (Barro, Gordon, 1983)

Central bank: maximize utility/minimize cost:

$$\max_{\pi} U(y - \bar{y}, \pi)$$

AS relationship of inflation and output:

$$\mathbb{E} e = 0$$
$$\mathbb{E} e^2 = \mathbb{E}(e - \mathbb{E} e)^2 = \sigma_e^2$$

$$y = \bar{y} + a(\pi - \pi^e) + e$$

where e supply shock; expected inflation π^e used as **underlying inflation**

Private agents: know CB's preferences, have **rational** inflation expectations π^e

Sequence of events

Timeline very important for understanding the model:

1. Private sector guesses monetary policy based on CB preferences $\Rightarrow$ inflation expectations π^e
2. Supply shock e realizes
3. Monetary policy made: π chosen by CB

Discussion:

- once π^e formed, CB can "surprise" agents with higher inflation
- CB can respond to supply shock — stabilization policy

Expansionist CB

$$U(y - \bar{y}, \pi) = \lambda(y - \bar{y}) - \frac{1}{2}\pi^2$$

- Utility rises in **output gap** (linear relationship) decreases in **inflation** (quadratic)

- λ – relative preference for expansion vs. price stability

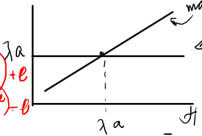
$$\max_{\pi} U(y-\bar{y}, \pi) = \lambda\left(a(\pi-\pi^e)+e\right) - \frac{1}{2}\pi^2$$

$$\text{FOC: } \frac{\partial U}{\partial \pi} = 0 \Leftrightarrow \lambda a - \pi = 0 \Leftrightarrow \boxed{\pi = \lambda a}$$

AS:

$$y = \bar{y} + a(\pi - \pi^e) + e$$

$$y - \bar{y} = a(\pi - \pi^e) + e$$



$$\pi^e = \lambda a \Rightarrow y - \bar{y} = a(\lambda a - \lambda a) + e$$

$$y - \bar{y} = e \Rightarrow \mathbb{E}U = -\frac{1}{2}\lambda^2 a^2$$

Commitment: CB commits $\pi = 0$.

$$\pi^e = 0 \;;\; y - \bar{y} = a(0-0) + e = e$$

$$\mathbb{E}U = 0$$

Expansionist CB: results

- **Discretion**: CB uses inflation to boost medium-run GDP, but effect cancelled by rational expectations
- inflation larger if CB cares more for expansion
- **Commitment**: if null inflation commitment feasible, both CB and economy better off

This verion of model ignores **stability** of GDP as CB's goal, as in the **TR** equation.

Output-stabilizing CB

CB minimizes cost instead of maximizing utility:

$$\min_{\pi} V(y-\bar{y},\pi) = \min_{\pi} \frac{1}{2}\lambda(y - \bar{y} - k)^2 + \frac{1}{2}\pi^2$$

- Cost rises in output gap (quadratic relationship) and in inflation (same)
- New motive – **output gap stability**: both very high and very low y are bad
- CB still **expansion biased**: V is minimal when $y - \bar{y} = k > 0$

$$\min_{\pi} \ \frac{1}{2}\lambda\left(a(\pi-\pi^e)+e-k\right)^2+\frac{1}{2}\pi^2$$

FOC: $\frac{1}{2}\lambda\cdot 2\left(a(\pi-\pi^e)+e-k\right)\cdot a+\frac{1}{2}\cdot 2\cdot\pi=0$

$$\pi=\frac{\lambda a}{1+\lambda a^2}\left(a\pi^e+k-e\right)$$

Public's expectations: $\cancel{\pi=\pi^e} \rightarrow \pi^e=\mathbb{E}\pi$

$\Rightarrow \pi^e=\mathbb{E}\pi=\frac{\lambda a}{1+\lambda a^2}\left(a\pi^e+k\right)$ (because $\mathbb{E}e=0$)

$\Leftrightarrow (1+\lambda a^2)\pi^e=\lambda a^2\pi^e+\lambda a k \Leftrightarrow \pi^e=\lambda a k$

plug this back into CB's optimal π

discretion

$$\pi^d=\frac{\lambda a}{1+\lambda a^2}\left(a\cdot\lambda a k+k-e\right)$$

$$\pi^d=\frac{\lambda a}{1+\lambda a^2}\left(k(\lambda a^2+1)-e\right)$$

$$\pi^d=\lambda a k-\frac{\lambda a}{1+\lambda a^2}e$$

Using π^d in V gives V^d, we will compute its expect. value

$$V^d = \frac{1}{2}\lambda\left(\frac{1}{1+a^2\lambda}e - k\right)^2 + \frac{1}{2}\left(\lambda a k - \frac{\lambda a}{1+a^2\lambda}e\right)^2$$

Taking expectation and opening brackets; all terms involving e (not e^2) are 0 in expectation, terms involving e^2 have σ_e^2:

$$\mathbb{E} V^d = \mathbb{E}\frac{1}{2}\lambda\left(\left(\frac{1}{1+a^2\lambda}\right)^2 \underbrace{e^2}_{\sigma_e^2 \text{ in exp.}} - \underbrace{\frac{1}{1+a^2\lambda}ek}_{0 \text{ in expect.}} + k^2\right) +$$

$$\mathbb{E}\,\frac{1}{2}\left(\lambda^2 a^2 k^2 - \underbrace{\frac{\lambda a}{1+a^2\lambda}e}_{0 \text{ in exp.}} + \left(\frac{\lambda a}{1+a^2\lambda}\right)^2 \underbrace{e^2}_{\sigma_e^2 \text{ in exp.}}\right)$$

$$= \frac{1}{2}\lambda\frac{1}{(1+a^2\lambda)^2}\sigma_e^2 + \frac{1}{2}\lambda k^2 + \frac{1}{2}\lambda^2 a^2 k^2 + \frac{1}{2}\frac{\lambda^2 a^2}{(1+a^2\lambda)^2}\sigma_e^2$$

$\lambda(1+a^2\lambda)$, simplified with denominator

$$= \frac{1}{2}\frac{\lambda + \lambda^2 a^2}{(1+a^2\lambda)^2}\sigma_e^2 + \frac{1}{2}\lambda k^2(1+\lambda a^2)$$

$$= \frac{1}{2}\frac{\lambda}{(1+a^2\lambda)^2}\sigma_e^2 + \frac{1}{2}\lambda k^2(1+\lambda a^2)$$

Commitment equilibrium:

CB commits to rule $\pi^c = b_0 + b_1 e$

Commitment credible $\Rightarrow \pi^e = \mathbb{E}\pi^c = b_0$

b_0, b_1 chosen to minimize $\mathbb{E} V^c$:

$$\min_{b_0, b_1} \mathbb{E} \frac{1}{2} \lambda \big(\underbrace{a(\cancel{b_0} + b_1 e - \cancel{b_0}) - k + e}_{= (ab_1 + 1)e - k} \big)^2 + \frac{1}{2} \left(b_0 + b_1 e \right)^2$$

equation (1)

$$\Leftrightarrow \min_{b_0, b_1} \frac{1}{2} \lambda (ab_1 + 1)^2 \sigma_e^2 + \frac{1}{2} \lambda k^2 + \frac{1}{2} \lambda \sigma_e^2 + \frac{1}{2} b_0^2 + \frac{1}{2} b_1^2 \sigma_e^2$$

FOC: $\begin{cases} \dfrac{\partial \mathbb{E} V^c}{\partial b_0} = 0 \\ \dfrac{\partial \mathbb{E} V^c}{\partial b_1} = 0 \end{cases} \Leftrightarrow \begin{cases} b_0 = 0 \\ a\lambda (ab_1 + 1)\sigma_e^2 + b_1 \sigma_e^2 = 0 \end{cases} \Leftrightarrow \begin{cases} b_0 = 0 \\ b_1 = -\dfrac{a\lambda}{\lambda a^2 + 1} \end{cases}$

$$\Rightarrow \pi^c = -\frac{a\lambda}{\lambda a^2 + 1} \cdot e$$

Expected value of CB cost function under rule-based commitment (replace b_0, b_1 in eq. (1)):

$$\mathbb{E}V^c = \frac{1}{2}\lambda\left(-\frac{a^2\lambda}{\lambda a^2+1}+1\right)^2\sigma_e^2 + \frac{1}{2}\lambda k^2 + \frac{1}{2}\frac{a^2\lambda^2}{(\lambda a^2+1)^2}\sigma_e^2$$

$$= \frac{1}{2}\lambda k^2 + \frac{1}{2}\sigma_e^2\left(\lambda\left(\frac{a^2\lambda}{\lambda a^2+1}\right)^2 - 2\lambda\frac{a^2\lambda}{\lambda a^2+1} + \lambda + \frac{a^2\lambda^2}{(\lambda a^2+1)^2}\right)$$

$$= \frac{1}{2}\lambda k^2 + \frac{1}{2}\sigma_e^2\lambda\frac{a^2\lambda}{\lambda a^2+1}\left(\frac{a^2\lambda}{\lambda a^2+1} - 2 + \frac{1}{\lambda a^2+1}\right) + \frac{1}{2}\lambda\sigma_e^2$$

sum = 1

$$= \frac{1}{2}\lambda k^2 - \frac{1}{2}\sigma_e^2\lambda\frac{a^2\lambda}{\lambda a^2+1} + \frac{1}{2}\lambda\sigma_e^2$$

$$= \frac{1}{2}\lambda k^2 + \frac{1}{2}\sigma_e^2\lambda\Big(\underbrace{-\frac{a^2\lambda}{\lambda a^2+1}+1}_{=\frac{1}{\lambda a^2+1}}\Big) = \boxed{\frac{1}{2}\lambda k^2 + \frac{1}{2}\frac{\lambda\sigma_e^2}{\lambda a^2+1}}$$

(do the common denominator)

Compare discretion and commitment:

$$\underbrace{\frac{1}{2}\lambda k^2(1+\lambda a^2) + \frac{1}{2}\frac{\lambda}{1+a^2\lambda}\sigma_e^2}_{V^{d}} > \underbrace{\frac{1}{2}\lambda k^2 + \frac{1}{2}\frac{\lambda}{1+\lambda a^2}\sigma_e^2}_{V^c}$$

CB better of under rule-based policy. Average inflation lower, too: $\mathbb{E}\pi^d = \lambda a k$, $\mathbb{E}\pi^c = b_0 = 0$.

Output-stabilizing CB: results

- **Discretion**: optimal inflation depends on:
  1. expected inflation $\Rightarrow$ equilibrium interaction of CB and population
  2. output shock realization: CB counteracts it for stabilization, as in Taylor rule (TR)
- **Commitment**: a promise of a fixed inflation level not optimal when output shocks matter
  - introduce commitment to a **rule-based** policy that reacts to output shock, as in TR
  - optimal rule has null average inflation
  - again, commitment policy makes everyone better off with respect to discretion