Macroeconomics
Lecture 5 – Real Business Cycles

Ilya Eryzhenskiy

PSME Panthéon-Sorbonne Master in Economics

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Overview

Outline

Lucas critique

- Big "Keynesian macroeconometric models" group led by Kennedy's Council of Economic Advisers
  - Solow, Tobin, Samuelson
- Application of IS-LM, Mundell-Flemming, AD-AS models to the data. Models linear $\Rightarrow$ represented as system of linear equations and estimated by least-squares methods:

$$
\begin{aligned}
x_{1t} &= \alpha_0 x_{2t} + \alpha_1 x_{3t} + \alpha_2 x_{3t} + \ldots. \\
x_{2t} &= \alpha_3 x_{1t} + \alpha_4 x_{3t} + \alpha_5 x_{4t} + \ldots \\
&\vdots \\
x_{136t} &= \alpha_{5987} x_{1t} + \alpha_{5988} x_{13t} + \alpha_{5989} x_{69t} + \ldots
\end{aligned}
$$

- **R. Lucas' idea (1976)**: The alphas (e.g. marginal propensity to consume) are **endogenous** with respect to government policy

Lucas critique II

Lucas's critique has received a huge response from macro theorists.

Instead of modelling **accounting relationships** such as
$Y = C + I + G + PCA \ldots$

…starting to model **rational agents' behaviour**

Two essential elements:

1. All agents **optimize** some objective function. Utility for households, profit for firms.

2. **Rational expectations**: agents know the structure of the economy (the model), errors are possible, but not **systematic**

End of microeconomics vs. macroeconomics divide, everything is **micro-founded**.

Lucas critique: examples

Consider households' reaction to a positive government spending shock:

- If **temporary**, need to know how many years it will last: consumption response is not same if GDP (and incomes) boosted for 1 year and of boosted for 5 years
- If **permanent**, government debt accumulation ⇒ consumers need to know what is the debt reduction strategy of government
  - If taxes will be raised in the future, households need to increase savings now to prepare for decrease of income ⇒ **smaller marginal propensity to consume today**
  - If central bank not independent and finances government deficit with additional money supply, inflation expected to rise ⇒ better buy things now while prices low ⇒ **higher marginal propensity to consume today**

Households' consumption reaction interacts with **labor supply** ⇒ under sticky wages, wage and price setting (and AS) influenced by factors listed above.

The RBC model

Two ways of describing the **Real Business Cycle** model:

1. dynamic Robinson Crusoe economy [*see Micro class*]
2. Ramsey-Cass-Coopmans model [*see Growth course, if enrolled*] with **endogenous labor choice** and **stochastic productivity shocks**

With respect to models seen before:

- Dynamic
- Flexible prices $\Rightarrow$ **money does not matter**
  - hence the word **Real** in **RBC**
  - can think of it as a long-run (trend) model, but is used for short and medium run
- This lecture – closed economy

The RBC model

Structure:

- **Households** with preferences (utility functions) for consumption and labor
- **Firms** produce goods with labor and capital, owned by households
- **General equilibrium** interactions through (at least) 3 markets: goods, labour, capital
- **Uncertainty** about future level of productivity

Assumptions for this lecture, possible to relax in general:

- Closed economy
- Perfect competition
- No government

Outline

Representative agent macroeconomics

A **large number** (population size normalized to 1) of **identical** households populate the economy.

Methodological trick: study a **representative household**: the aggregate outcomes as result of one (big) agent's behaviour.

But the "big" household is actually many small ones $\Rightarrow$ the representative household cannot manipulate aggregate quantities and prices (wage, interest) $\Rightarrow$ **takes prices as given**

Households in RBC model **live forever**:

- Demographics ignored
- To make it seem less crazy, imagine that each household is a sequence of generations of constant size
  - Then, necessary to assume that all offsprings' utility enters ancestors' utility

Representative household

- Households care for amount of good consumed in each period C_t (from $t = 0$ to $t = \infty$) and for hours worked in each period L_t (equivalently, they care for leisure time)
- Two fundamental microeconomic problems:
    1. **Intertemporal choice** of consumption $\leftrightarrow$ **consumption-savings** problem
    2. Choice between consumption and leisure $\leftrightarrow$ labor supply
- Will study the two problems, starting with two periods
- Interaction of the two problems $\Rightarrow$ intertemporal substitution of leisure (Lucas-Rapping effect)

2-period intertemporal consumption choice

Consider a consumer living for two periods, working $\bar{L}$ hours each period. Preferences:

$$U(C_1, C_2) = u(C_1) + \beta u(C_2)$$

where $u(\cdot)$ is **instantaneous utility function** with $u' > 0, u'' < 0$;
β – the **discount factor** $\leftrightarrow$ degree of patience.
Budget constraints of two periods:

$$P_1 \cdot C_1 + P_2 \cdot \Omega_2 \leq W_1 \cdot \bar{L}$$
$$P_2 \cdot C_2 \leq W_2 \cdot \bar{L} + (1+i) \cdot P_2 \cdot \Omega_2$$

W – **nominal** wage, i **nominal** interest, $w \equiv W/P$ – **real** (in units of good) wage, $1 + r \equiv (1+i)/(1+\pi)$ and **real** interest
Ω_2 – **real** (measured in units of period-2 consumption good) **wealth** at **beginning** of period 2;
Period 1: labor income used for consumption and **savings**
Period 2: consume labor income + savings with a return
If $\Omega_2 < 0$, the consumer borrows (at the same nominal interest i)

2-period intertemporal consumption choice: solution

Both budget constraints hold as equality $\Rightarrow$
can obtain **intertemporal budget constraint** $C_1 + \frac{1}{1+r} C_2 = w_1 + \frac{1}{1+r} w_2$

2-period intertemporal consumption choice: solution

Consumption-leisure choice

Consider one-period problem of an agent that can work for a total length of time equal to 1 (normalization of work hours $\Rightarrow$ L as **share** of time devoted to work). She cares for consumption C and leisure time $1 - L$:

$$U(\underset{+}{C}, \underset{+}{1 - L}) \quad \text{or, equivalently,} \quad U(\underset{+}{C}, \underset{-}{L})$$

Budget constraint:

$$\begin{aligned} PC \leq WL \Leftrightarrow C &\leq wL \\ \Leftrightarrow C + w(1 - L) &\leq w \quad \text{(consumption vs. leisure)} \end{aligned}$$

Wage as **opportunity cost**, or price, of leisure

Consumption-leisure choice: graphs, solution

Consumption and leisure with 2 periods: Lucas-Rapping effect

Now consider a 2-period problem with both consumption and leisure. We will assume a specific form of utility function to simplify analysis:

$$\max_{C_1, C_2, L_1, L_2} \{\ln C_1 - \gamma\sigma/(1+\sigma) L_1^{(1+\sigma)/\sigma} + \beta(\ln C_2 - \gamma\sigma/(1+\sigma) L_2^{(1+\sigma)/\sigma})\}$$

$$\text{s.t.} \quad C_1 + C_2/(1+r) = w_1 L_1 + w_2 L_2/(1+r)$$

Consumption and leisure with 2 periods: solution

Lucas-Rapping effect

An intertemporal dimension of labor supply through consumption-saving decisions:

- It is the **change in time** and not **average level** of wage that affects labor supply
- In periods with temporary wage increases household works more, makes savings, then works less after wage decrease

Infinitely-lived representative household

$$
\begin{aligned}
U(C_0, L_0, ..., C_t, L_t, C_{t+1}, L_{t+1}, ...) = \\
& u(C_0, 1 - L_0) + \\
& E_0 \left[\beta u(C_1, 1 - L_1) + \beta^2 u(C_2, 1 - L_2) + ... \right. \\
& \left. + \beta^t u(C_t, 1 - L_t) + \beta^{t+1} u(C_{t+1}, 1 - L_{t+1}) + ... \right] \\
& = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - L_t)
\end{aligned}
$$

Rational expectations E_0: formed at initial period $t = 0$, take into account the structure of the model (known to household) and distribution of stochastic variables (more on them below)

Properties of $u(.)$

- $u'_c > 0, u'_{1-L} > 0; \quad u''_{cc} < 0, u''_{1-L} < 0$
- $lim_{C_t \to 0} u_c(C_t, 1 - L_t) = \infty$; $lim_{1-L_t \to 0} u_2(C_t, 1 - L_t) = \infty$

Household wealth, budget constraint

We use only **real** quantities for the budget constraint:

$$C_t + \underbrace{\Omega_{t+1} - \Omega_t}_{\text{savings}} = \underbrace{w_t L_t + r_t \Omega_t + \Pi_t}_{\text{income}}$$

$$\Leftrightarrow C_t + \Omega_{t+1} = w_t L_t + (1 + r_t)\Omega_t + \Pi_t$$

- C_t: consumption
- Ω_t: **real wealth** of household **at beginning of period t** (denominated in consumption goods)
- w_t: **real wage** (denominated in consumption goods)
- r_t: **real interest rate** (denominated in consumption goods) – **return on assets available at beginning of period t**
- Π_t: **real profits** of firms (denominated in consumption goods)

Household problem

$$\max_{\{C_t, L_t, \Omega_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - L_t)$$

subject to a **sequence** of budget constraints:

$$C_0 + \Omega_1 = w_0 L_0 + (1 + r_0)\Omega_0 + \Pi_0$$
$$C_1 + \Omega_2 = w_1 L_1 + (1 + r_1)\Omega_1 + \Pi_1 \text{ and so on:}$$
$$C_t + \Omega_{t+1} = w_t L_t + (1 + r_t)\Omega_t + \Pi_t \text{ for } t = 0, 1, 2, \dots$$

Household takes as given:

- Ω_0 – an **initial condition**
- current prices w_t,r_t (cannot influence them by their actions)
- expected prices $\{E_t w_s\}_{s=2}^{\infty}, \{E_t r_s\}_{s=2}^{\infty}$
- $\Pi_t, \{E_t \Pi_s\}_{s=2}^{\infty}$: current and expected firm profits

Consumer optimization

Lagrangian of consumer problem

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \{u(C_t, 1-L_t) \\ + \lambda_t \left[w_t L_t + (1+r_t)\Omega_t + \Pi_t - C_t - \Omega_{t+1}\right]\}$$

Budget constraint holds as equality for all $t \Rightarrow \lambda_t > 0$ for all t.
First-order conditions in period t:

$$\begin{aligned}
&(1) \quad \frac{\partial \mathcal{L}}{\partial C_t} && u_c(C_t, 1-L_t) - \lambda_t = 0 \\
&(2) \quad \frac{\partial \mathcal{L}}{\partial L_t}: && u_{1-L}(C_t, 1-L_t) + \lambda_t w_t = 0 \\
&(3) \quad \frac{\partial \mathcal{L}}{\partial \Omega_{t+1}}: && -\lambda_t + \beta E_t[\lambda_{t+1}(1+r_{t+1})] = 0
\end{aligned}$$

$\lambda_t = u_c(C_t, 1-L_t)$ follows from (1): λ_t is marginal utility of consumption at t, also known as **shadow price** of wealth.

Labour supply

- Eliminate λ_t from FOC (1), (2) to get the **consumption-leisure optimality condition**:

$$\frac{u_n(C_t, 1 - L_t)}{u_c(C_t, 1 - L_t)} = w_t$$

- defines (implicity) the function of **labour supply for a given level of consumption**

$$L_t = L^s(w_t, C_t)$$

- features substitution and income effects: sign of w_t derivative is ambiguous

- number (or share) of hours worked is the *intensive margin* of labor supply. We do not have the *extensive margin* (work vs. unemployment) in this model. If interested, look at Ch. 10 of Romer textbook.

Consumption-savings: the Euler equation

- **(1) and (3)** imply the **consumption-savings optimality condition**

$$1 = E_t\left[\left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)(1+r_{t+1})\right]$$

- **Euler equation**: key equation in modern macro models

- In finance, $E_t\frac{\lambda_{t+1}}{\lambda_t}$ known as **pricing kernel** or **stochastic discount factor**

  - important for finance theory + studied empirically

- Pricing kernel, Euler equation $\rightarrow$ intersection of macro and finance theory

Consumption-savings: the Euler equation

Substituting λ_t from FOC (1) and assuming no uncertainty (for this slide):

$$1 = \left(\frac{\beta u_c(C_{t+1}, 1 - L_{t+1})}{u_c(C_t, 1 - L_t)} \right) (1 + r_{t+1})$$

$$\Leftrightarrow \frac{u_c(C_t, 1 - L_t)}{\beta u_c(C_{t+1}, 1 - L_{t+1})} = 1 + r_{t+1}$$

- LHS: **marginal rate of substitution** (MRS) between period t and $t+1$ consumption
- RHS: ratio of price of period t consumption $(= 1)$ to price of period $t+1$ consumption $\left(= \frac{1}{1+r}\right)$
- Interpretation: Utility of consuming ε units (very small amount) in t and $(1+r)\varepsilon$ units in $t+1$ must be equal

Euler equation with uncertainty

Bringing back uncertainty:

$$1 = E_t \left[\left(\frac{\beta u_c(C_{t+1}, 1 - L_{t+1})}{u_c(C_t, 1 - L_t)} \right) (1 + r_{t+1}) \right]$$

- Utility of consuming ε (very small) in period t...
- ... equal to the **expected** utility of marginal savings with return on them, $(1 + r_{t+1})\varepsilon$, at $t + 1$
- Defines (implicitly) **demand for assets** (or savings supply):

$$\Omega_{t+1} - \Omega_t = G(r_{t+1})$$

Household choices - Summary

Representative household's period t optimal choices of C_t, L_t and Ω_{t+1} characterized by consumption-leisure optimality condition, consumption-savings optimality condition and flow budget constraint:

$$
\begin{aligned}
w_t =& \frac{u_n(C_t, 1-L_t)}{u_c(C_t, 1-L_t)} \\
1 =& E_t \left[\left(\frac{\beta u_c(C_{t+1}, 1-L_{t+1})}{u_c(C_t, 1-L_t)} \right) (1+r_{t+1}) \right] \\
C_t + \Omega_{t+1} =& w_t L_t + r_t \Omega_t + \Pi_t
\end{aligned}
$$

taking as given Ω_t (pre-determined), w_t, r_t, and Π_t

These define:

- demand side of period t goods market (depending on r_t) $\Rightarrow$ close to IS
- supply side of period t labour market
- supply side of period t asset/savings markets (will define **capital formation**)

Outline

Representative firm

A large number (a mass equal to 1) of identical firms in **perfect competition** $\Rightarrow$ study **representative firm**

Firms do not make intertemporal choices. They only:

1. rent factors of production (labour and capital) on markets
2. produce goods according to $Y_t = Z_t f(K_t, L_t)$, with Z_t **stochastic productivity shock** (same for all firms)
3. distribute profits (null in equilibrium) to households

No difference in model if firms live for 1 or ∞ periods

Perfect competition $\Rightarrow$ firm **takes prices** w_t, r_t **as given**

Production function f has $f'_L, f'_K > 0; f''_L, f''_K < 0$

Firm profit maximization

Productivity Z_t **is observed at period** $t-1$. Firm them optimizes period t profit in $t-1$:

$$\Pi_t = Z_t f(K_t, L_t) - w_t L_t - r_t K_t$$

- Static maximization of profit function

$$\max_{\{L_t, K_t\}} Z_t f(K_t, L_t) - w_t L_t - r_t K_t$$

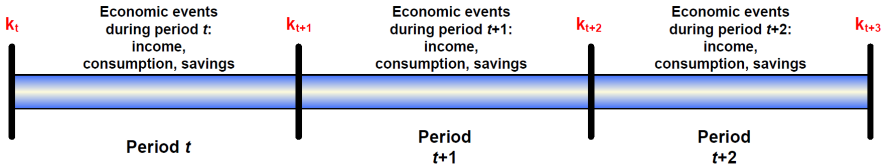
- First-order conditions

$$\begin{aligned} L_t : \quad & Z_t f_L(K_t, L_t) - w_t = 0 \\ K_t : \quad & Z_t f_K(K_t, L_t) - r_t = 0 \end{aligned}$$

FOCs define a downward sloping **labour demand function** $L^d(w_t)$ and **capital demand function** $K^d(r_t)$

Outline

Dynamic equilibrium: diagram



Law of motion of productivity

Productivity is stochastic (random), but has some persistence: it follows a stochastic autoregressive process of order 1 (in logs):

$$\ln Z_t = \rho \ln Z_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

where ϵ is the white noise component of the autoregressiove process – the source of randomness in Z_t.

Productivity of period t is observed at $t-1$ (same timing as the capital variable)– before the firm hires and installs capital.

Law of motion of capital

Capital lasts for more than one period. It partially depreciates and is increased by **investment**:

$$
\begin{aligned}
K_{t+1} &= K_t \underbrace{-\delta K_t}_{\text{depreciation}} + I_t \\
&= (1-\delta)K_t + I_t
\end{aligned}
$$

Closed economy $\Rightarrow$ investment is financed only with households' **savings**

Building the Equilibrium

Capital market clearing

- Capital demand from firm's profit maximization: $K_{t+1} = K^d(r_t)$
- Capital supply is savings supply: $I_t = \underbrace{\Omega_{t+1} - \Omega_t}_{=G(r_t)}$
- Combining with law of motion of capital:

$$K^d(r_t) = (1-\delta)K_t + G(r_t)$$

Goods-market clearing

- Goods aggregate supply is Y_t, given by $Z_t f(K_t, L_t)$
- Goods aggregate demand is $C_t + I_t$ $(= C_t + K_{t+1} - (1-\delta)K_t)$
- Goods market clearing

$$C_t + K_{t+1} - (1-\delta)K_t = Z_t f(K_t, L_t)$$

$\Rightarrow$ **aggregate resource constraint**

Labor market clearing

$$L^s(w_t, C_t) = L^d(w_t)$$