Macroeconomics
Lecture 6 – Labor & Intro to Real Business Cycles

Ilya Eryzhenskiy

PSME Panthéon-Sorbonne Master in Economics

Fall 2023

Overview

1. Labor supply: consumption-leisure choice
   - One-period version
   - Multi-period version

2. Explaining business cycles with a dynamic model
   - Lucas critique
   - RBC framework
   - Macro data processing
   - Basic RBC model vs. data

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Labor: extensive vs. intensive margin

- **Extensive margin**: do I work at all?
  - Not always a choice: voluntary vs. involuntary unemployment
  - **No theory for this in this course**
- **Intensive margin**: how much do I work?
  - An easier question for neoclassical economics
  - **Leisure** modelled as a **good**
    $\Rightarrow$ Labor time $=$ Total time endowment $-$ Leisure time

Consumption-leisure choice

Consider a household with total endowment of time equal to 1 (normalization)
N is **share** of time devoted to work, $L = 1 - N$ is **share** of time devoted to leisure.

Household cares for consumption C and leisure time $1 - N$:

$$U(\underset{+}{C}, \underset{+}{L}) = U(\underset{+}{C}, \underset{+}{1 - N}) \quad \text{or, equivalently,} \quad \tilde{U}(\underset{+}{C}, \underset{-}{N})$$

Denote W nominal wage, $w = W/P$ real wage. $-wN$
Budget constraint:

$+w$ $C + w \leq wN + w \Leftarrow$

$$P \cdot C \leq W \cdot N \Leftrightarrow C \leq wN$$

$C + (1-N)w \leq w$

$$\Leftrightarrow C + w(1 - N) \leq w \quad \text{(consumption vs. leisure)}$$

Real wage acts as **opportunity cost**, or price, of leisure

Consumption-leisure choice: analytical solution

$$\max_{C \geq 0, N \in (0,1)} U(C, 1-N)$$
$$\text{s.t.} \quad C + w(1-N) = w$$

When solving, note that U here is really a function of L, not N, so

$$\frac{\partial U}{\partial C} \Leftrightarrow U'_C$$

$$\frac{\partial U(C, 1-N)}{\partial N} = -U'_L(C, 1-N)$$

$$\frac{d f(g(x))}{dx} = f' \cdot g'(x)$$

$$\mathcal{L} = U(C, 1-N) + \lambda\left(w - C - w(1-N)\right)$$

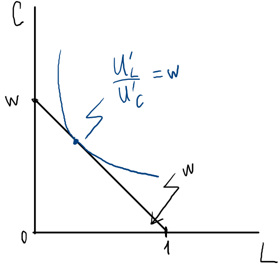
FOC

$$\frac{\partial \mathcal{L}}{\partial C} = U'_C(C, 1-N) - \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial N} = -U'_L(C, 1-N) + \lambda w = 0$$

$$\Leftrightarrow \quad U'_C(C, 1-N) = \lambda, \quad U'_L(C, 1-N) = w\lambda$$

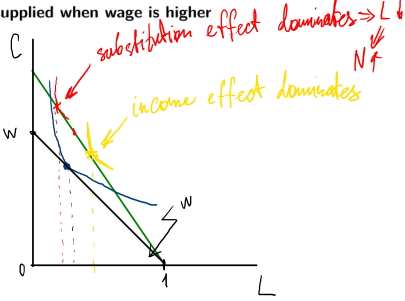
$$\frac{U'_L(C, 1-N)}{U'_C(C, 1-N)} = w \Longleftarrow \frac{U'_C(C, 1-N)}{U'_L(C, 1-N)} = \frac{1}{w}$$

Consumption-leisure choice: graphical solution



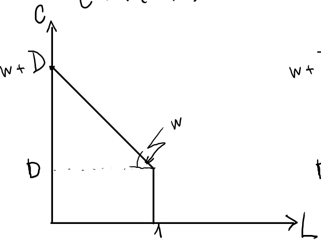
Consumption-leisure choice: comparative statics

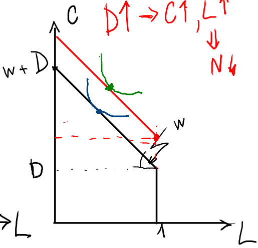
With an increase of w, leisure can either increase or decrease. Typically, substitution effect dominates $\Rightarrow$ leisure decreases $\Rightarrow$ **more labor supplied when wage is higher**



Exogenous income D

$$C + w(1-N) = w + D \Leftrightarrow C + wL = w + D$$





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Consumption-leisure with many periods

$$\max_{\{C_t\}\geq 0, \{N_t\}\in(0,1)} \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - N_t)$$

$$\text{s.t.} \quad C_t + S_t = w_t N_t + (1 + r_{t-1}) S_{t-1}$$

Now the **instanteneous utility function** of every period has two arguments

The consumption-leisure choice is **static**: it only involves variables of one period $\Rightarrow$ same consumption-leisure optimality condition as before:

$$\frac{u'_L(C_t, 1 - N_t)}{u'_C(C_t, 1 - N_t)} = w_t$$

Consumption-leisure with many periods: Lucas-Rapping effect

Consider the following comparative statics with many periods: a one-period increase in wage in t. We will assume that the substitution effect dominates: N_t depends positively on w_t. How is labor supply affected in $t+1, t+2...$?

- With higher wage in t, hours worked are above average, additional savings S_t are made
- From $t+1$ on, wage back to normal. A rational consumer now spends savings from previous period S_t and works **below average** hours
- This continues until the extra savings made in t are consumed

The below-average work effort (**Lucas-Rapping effect**) comes from leisure being **normal good**: with more wealth, you want both **more consumption** and **more leisure**.

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Why micro in macro? Some history

- Empirical macro used to be done as estimation of **big** systems of equations with OLS-like methods

⇒ IS-LM (IS-TR), Mundell-Flemming (IS-TR-IFM), AD-AS models were brought to data like:

$$
\begin{aligned}
x_{1t} &= \alpha_0 x_{2t} + \alpha_1 x_{3t} + \alpha_2 x_{3t} + \ldots. \\
x_{2t} &= \alpha_3 x_{1t} + \alpha_4 x_{3t} + \alpha_5 x_{4t} + \ldots \\
&\vdots \\
x_{136t} &= \alpha_{5987} x_{1t} + \alpha_{5988} x_{13t} + \alpha_{5989} x_{69t} + \ldots
\end{aligned}
$$

- Robert Lucas' idea in 1976 (**Lucas critique**): The parameters (e.g. marginal propensity to consume) are **endogenous** with respect to government policy

Lucas critique: examples

Consider households' reaction to a change in taxes, e.g. a new carbon tax:

- Is it temporary or permanent? Very different consumption responses (mpc's)
- What are implications for the government budget? Households realize that the whole macro equilibrium may change with big changes in tax, e.g. a recession might follow $\Rightarrow$ more savings needed
- Labor supply interacts with consumption demand, as seen above

Lucas critique: response

Two essential elements of modern models address Lucas' critique:

1. All agents optimize some objective function. Utility for households, profit for firms.

2. **Rational expectations**: agents know the structure of the economy (the model), errors are possible, but not **systematic**

This is how macroeconomics became micro-founded and the **Real Business Cycles (RBC)** model was created

Later on, the model was extended to include sticky price components – the **New Keynesian model** was created

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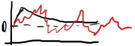
RBC model: structure

- Households maximize expected utility by:
  - supplying labor ⇒ earning wage
  - consuming, saving by investing in (1) bonds (2) firm capital ⇒ earning **interest on bonds** and **return on capital**
- Firms maximize expected profits by:
  - producing goods with labor and capital...
  - ...subject to **productivity shocks**
- Flexible prices ⇒ goods, labor, capital and bond markets in equilibrium

Productivity shocks: a key element

Production with labor N_t and physical capital K_t is modelled as:

$$Y_t = A_t F(K_t, N_t)$$



A_t is **total factor productivity (TFP)** and it is a **random variable** in the model. It is usually assumed to follow an autoregressive process in logs, so that it has some **persistence**:

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_t$$

This is the only source of business cycles in the classical RBC model. One either simulates a one-period shock in $t = 0$ ($\varepsilon_0 > 0, \varepsilon_t = 0$ for $t > 0$) and studies **impulse response** or does a **stochastic simulation** with shocks happening every period and $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$

Possible to introduce other shocks than TFP, but it is TFP shocks that make model variables behave most like the empirical macro time series

Model and data: stationarity

RBC models produce **stationary** paths for macro variables by construction

Stationary $\approx$ mean reverting: variables tend to converge to their mean, which is also **steady state** of the model

This is not usually the case for macro time series: they are non-stationary

Two problems created by non-stationarity:

1. Hard to compare to model output
2. **Spurious correlations**: time series seem related even if they aren't

$\Rightarrow$ Need to transform macro series to have them in stationary form

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Macro data processing

Steps to transform macro data to make it comparable to model output:

1. Remove **seasonality** from data (done by macro data providers in most cases; but you will also learn to de-season in TD)
2. Obtain percentage deviations from **trend** by:
    1. taking a logarithm of your variable (not needed for interest rates – they are already percentages)
    2. subtracting the the log of trend value:
    $$\ln(x_t) - \ln(x_t^{trend}) = \underbrace{\ln\left(\frac{x_t}{x_t^{trend}}\right) = \ln\left(1 + \frac{x_t - x_t^{trend}}{x_t^{trend}}\right)}_{\text{verify this step}}$$
    and $\ln\left(1 + \frac{x_t - x_t^{trend}}{x_t^{trend}}\right) \approx \frac{x_t - x_t^{trend}}{x_t^{trend}}$ using $\ln(1+x) \underset{x \to 0}{\to} x$.

We get $\ln(x_t) - \ln(x_t^{trend}) \approx \frac{x_t - x_t^{trend}}{x_t^{trend}}$: deviation of variable from trend in percent of the trend value of the period $\to$ our preferred measure of business cycle

What is the trend?

Reminder: two different ideas of **trend GDP** in course:

1. Trend GDP is GDP in flexible-price equilibrium – was contrasted with Keynesian equilibria in IS-TR, medium-term AS-AD

2. Trend GDP is literally the trend – GDP time series that has been smoothed using statistical procedures

In empirical business cycle research, the smoothing idea is used

Defined this way, deviations from trend (i.e. business cycles) can exist under flexible prices, too

Trend as smoothing of series: two procedures

Two options for smoothing the series $\{x_t\}_{t=1}^{T}$ – e.g. GDP:

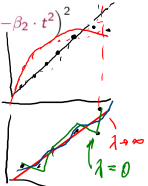
1. Run OLS on linear time trend (possibly quadratic trend, too); use fitted values:

$$(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2) = \text{argmin} \sum_{t=1}^{T} \left(x_t - \beta_0 - \beta_1 \cdot t - \beta_2 \cdot t^2 \right)^2$$

   Then, $x_t^{trend} = \hat{x}_t = \hat{\beta}_0 + \hat{\beta}_1 \cdot t + \beta_2 t^2$

2. Run HP filter:

$$\{x_t^{trend}\}_{t=1}^{T} = \text{argmin} \left[\sum_{t=1}^{T} (x_t - x_t^{trend})^2 + \lambda \sum_{t=2}^{T-1} \left((x_{t+1}^{trend} - x_t^{trend}) - (x_t^{trend} - x_{t-1}^{trend}) \right)^2 \right]$$



   Where λ is a parameter that regulates smoothness of the trend. With $\lambda \to \infty$, trend linear.

Checking stationarity

Once the data is transformed, we check whether it is **stationary** in two ways:

1. Visually – does the series seem to always return to the mean?
2. Statistical tests of Dickey-Fuller (DF) type

Idea of DF test: if process is AR(1): $x_t = \rho x_{t-1} + \varepsilon_t$ with $\rho \leq 1$, then it is non-stationary (random walk) if $\rho = 1$.

Instead of testing $\rho = 1$ directly, subtract x_{t-1} from both sides:

$$\underbrace{x_t - x_{t-1}}_{\Delta x_t} = \underbrace{(\rho - 1)}_{\delta} x_{t-1} + \varepsilon_t$$

and test significance of δ (not t-stat, but a special DF-stat used).

In practice, you will use Augmented Dickey-Fuller (ADF) that also includes several lags of Δx_t on the right hand side, and can allow for a constant term and/or linear time trend (not needed if you have de-trended first).

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RBC models vs. data

RBC models take **long-run** empirical facts about market economies as **assumptions** and then seek to reproduce **short-run** or **medium-run** behaviour of economies

Long-run “stylized facts” (Kaldor, Kuznets):

1. Stable relationship between investment and GDP
2. Stable share of labor income in GDP (less stable recently – see work of Piketty)
3. Stable hours per worker

Short-run **second-order moments**, i.e. variances, correlations:

1. Variance of consumption smaller than variance of GDP, smaller than variance of investment
2. All main macro aggregates **procyclical**: positive correlation to GDP cycle

U.S. economy: Business Cycle moments

Variable x_t	Moments			
	σ_x	$\frac{\sigma_x}{\sigma_{GDP}}$	$\text{corr}(x_t, x_{t-1})$	$\text{corr}(x_t, GDP_t)$
GDP	1.81	1.00	0.84	1.00
Consumption	1.35	0.74	0.80	0.88
Investment	5.30	2.93	0.87	0.80
Work hours	1.79	0.99	0.88	0.88
Real wage	0.68	0.38	0.66	0.12
Real interest	0.30	0.16	0.60	-0.35
Total factor productivity	0.98	0.54	0.74	0.78

All variables except r in logs and with HP filter. Total factor productivity (TFP) computed as a Solow residual:

$$\ln(TFP_t) = \ln Y_t - \hat{\alpha} \ln N_t - (1 - \hat{\alpha}) \ln K_t$$

A basic RBC calibrated to match U.S.

Moments of time series obtained from stochastic simulation (TFP shocks happenning every period)

Variable (x_t)	Moments			
	σ_x	$\frac{\sigma_x}{\sigma_Y}$	corr(x_t, x_{t-1})	corr(x_t, Y_t)
Y	1.39	1.00	0.72	1.00
C	0.61	0.44	0.79	0.94
I	4.09	2.95	0.71	0.99
N	0.67	0.48	0.71	0.97
w	0.75	0.54	0.76	0.98
r	0.05	0.04	0.71	0.95
A	0.94	0.68	0.72	1.00

Key empirical facts matched well: $\sigma_C < \sigma_Y, \sigma_I > \sigma_Y$, high autocorrelation and procyclicality of all variables except r.