Macroeconomics
Lecture 8 – New Keynesian Model

Ilya Eryzhenskiy

PSME Panthéon-Sorbonne Master in Economics

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Outline

The basic New Keynesian model

- Also uses the **microfoundations** as in RBC framework
  - rational expectations
  - representative, infinitely lived agents
  - optimizing behaviour
- But **important differences**
  - a large number (continuum) of consumption goods
    - $\Rightarrow$ not perfectly substitutable for HH $\Rightarrow$ no perfect competition $\rightarrow$ **monopolistic competition**
  - prices for goods not flexible $\rightarrow$ **nominal rigidities**
- We will also make simplifications w.r.t. RBC: no capital accumulation $\rightarrow$ production with labor only
- Versions of this model widespread in central banks, commercial banks, public authorities, international organizations...

The basic New Keynesian model

- **Households**
  - consume **a bundle of diversified goods**
  - supply labour
  - make saving in a **nominal bond** (zero in equilibrium)
- **Firms**
  - a continuum of firms of measure one
  - each **producing a single, imperfectly substitutable good**
  - only using labour as factor input
  - pricing the good
    - under monopolistic competition
    - given **nominal rigidities** (but we start with a **flexible price** version today)

Outline

1. Household

2. Firms
   - Flexible prices

Household

Household utility has the form $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$, and we will work with *isoelastic* utility for both C and L:

$$U(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\eta}}{1+\eta}$$

where C_t is a **consumption indicator** constructed with a large number of goods, each having index i.

C_t calculated with **aggregator function** proposed by Dixit and Stiglitz:

Integrals reminder:

$$\int_0^1 c \cdot x(i)\, di = c \cdot \int_0^1 x(i)\, di$$

$$\int_0^1 x(i)\, x(j)\, di = x(j) \int_0^1 x(i)\, di$$

$$C_t = \left(\sum_{i=1}^{N} C_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$\hookrightarrow$ N discrete goods case

$$C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $C_t(i)$ is the quantity of good i consumed by the household.

Each good has its own price $P_t(i)$ set by a firm producing the good.

Differentiated goods

- imperfectly-substitutable goods combined yield an aggregate good
    - Sometimes assumed that intermediary firms combine the goods for the household $\Rightarrow$ the aggregator is their production function

$$C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

(absolute value)

- ε is the **constant elasticity of substitution (CES)** between any pair of differentiated goods
- **Properties of the aggregator**
    - (1) symmetric, (2) strictly increasing, (3) strictly concave in all arguments, (4) homogeneous of degree one

Household

Households maximize the consumption index C_t for any given level of expenditures $\zeta_t \equiv \int_0^1 P_t(i)C_t(i)di$. The solution yields a set of demand equations

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t \quad \text{for all } i \in [0,1], \tag{1}$$

where $P_t \equiv [\int_0^1 P_t(i)^{1-\varepsilon} di]^{1/(1-\varepsilon)}$ is an aggregate price index. This allows to write total consumption expenditure as

$$\int_0^1 P_t(i)C_t(i)di = P_t C_t$$

Optimization in period t:

- Notation: • no period index
  • i is index, not argument

$$\frac{\varepsilon-1}{\varepsilon}-1=\frac{\varepsilon-1-\varepsilon}{\varepsilon}$$
$$=-\frac{1}{\varepsilon}$$

$$\mathcal{L}=\left(\int_0^1 C_i^{\frac{\varepsilon-1}{\varepsilon}}\,di\right)^{\frac{\varepsilon}{\varepsilon-1}}+\lambda\Big(\underbrace{\zeta}_{\text{total income (zeta)}}-\int_0^1 P_iC_i\Big)$$

$$\frac{\partial\mathcal{L}}{\partial C_i}=\frac{\varepsilon}{\varepsilon-1}\cdot\left(\int_0^1 C_i^{\frac{\varepsilon-1}{\varepsilon}}\,di\right)^{\frac{1}{\varepsilon-1}}\cdot\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot C_i^{-\frac{1}{\varepsilon}}-\lambda P_i$$

$$\frac{d f(g(x))}{dx}=f'(g(x))\cdot g'(x)$$

$$=C^{\frac{1}{\varepsilon}}\cdot C_i^{-\frac{1}{\varepsilon}}-\lambda P_i$$

$$\frac{\partial\mathcal{L}}{\partial C_i}=0\implies C^{\frac{1}{\varepsilon}}C_i^{-\frac{1}{\varepsilon}}=\lambda P_i$$
$$C^{\frac{1}{\varepsilon}}C_j^{-\frac{1}{\varepsilon}}=\lambda P_j$$

$$\frac{C_i}{C_j}=\left(\frac{P_i}{P_j}\right)^{-\varepsilon}\Longleftarrow\boxed{\left(\frac{C_i}{C_j}\right)^{-\frac{1}{\varepsilon}}=\frac{P_i}{P_j}}$$

$$\xi = \int_0^1 P_i C_i = \int_0^1 P_i \underbrace{\left(\frac{P_i}{P_j}\right)^{-\varepsilon} C_j}_{C_i} = \left(\frac{1}{P_j}\right)^{-\varepsilon} C_j \int_0^1 P_i^{1-\varepsilon} d i$$

$$= P_j^{\varepsilon} C_j \int_0^1 P_i^{1-\varepsilon} d i \Rightarrow C_j = \frac{\xi}{P_j^{\varepsilon} \int_0^1 P_i^{1-\varepsilon} d i}$$

$$\underline{C} = \left(\int_0^1 \underline{C_j^{\frac{\varepsilon-1}{\varepsilon}}} d j\right)^{\frac{\varepsilon}{\varepsilon-1}} = \left(\int_0^1 \frac{\xi^{\frac{\varepsilon-1}{\varepsilon}}}{P_j^{\varepsilon-1}\left(\int_0^1 P_i^{1-\varepsilon} d i\right)^{\frac{\varepsilon-1}{\varepsilon}}} d j\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \xi \frac{1}{\int_0^1 P_i^{1-\varepsilon} d i} \left(\int_0^1 \frac{1}{P_j^{\varepsilon-1}} d j\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \xi \left(\int_0^1 P_i^{1-\varepsilon} d i\right)^{-1} \left(\int_0^1 P_j^{1-\varepsilon} d j\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

same ⇒ replace j with i

$$= \underline{\xi} \left(\int_0^1 P_i^{1-\varepsilon} d i\right)^{\frac{1}{\varepsilon-1}}$$

$$C = \xi \left(\int_0^1 P_i^{1-\varepsilon} d i\right)^{\frac{1}{\varepsilon-1}} \Rightarrow \xi = C \underbrace{\left(\int_0^1 P_i^{1-\varepsilon} d i\right)^{\frac{1}{1-\varepsilon}}}_{P}$$

$$P = \left(\int_0^1 P_i^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$$ (see bottom of prev. slide)

$$\mathcal{Z} = \underline{PC}$$

$$\mathcal{Z} = \int_0^1 P_i C_i di = \int_0^1 P_i \left(\frac{P_j}{P_i}\right)^{\varepsilon} C_j di$$

$$= P_j^{\varepsilon} C_j \int_0^1 P_i^{1-\varepsilon} di = P_j^{\varepsilon} C_j P^{1-\varepsilon}$$

$$= \left(\frac{P_j}{P}\right)^{\varepsilon} C_j P$$

$$PC = \left(\frac{P_j}{P}\right)^{\varepsilon} C_j P \Rightarrow \boxed{C_j = C\left(\frac{P_j}{P}\right)^{-\varepsilon}}$$

Household budget constraint

The flow budget constraint is

(*)

$$\int_0^1 P_t(i)C_t(i)di + B_{t+1}^N \leq (1+i_{t-1})B_t^N + W_t^N L_t + \Pi_t^N$$

with $C_t(i)$ period t consumption of good i, $P_t(i)$ price of good i, L_t hours of work, $W_t^{\mathbf{N}}$ **n**ominal (i.e. in units of currency) wage, $B_t^{\mathbf{N}}$ **n**ominal value of bonds held at beginning of t, i_t the nominal interest rate, $\Pi_t^{\mathbf{N}}$ **n**ominal profits.

(*): A new timing assumption (for this model only): i_{t-1} is return on the bonds that are held in beginning of period t (so they have been chosen in t-1).

Using consumption aggregator and price indicator, the constraint can be rewritten:

$$P_t C_t + B_{t+1}^N \leq (1+i_t)B_t^N + W_t^N L_t + \Pi_t^N$$

$$\max_{\{C_t, L_t, B^N_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\eta}}{1+\eta} \right) \quad \text{s.t.}$$

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \Big(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi} +$$

$$+ \lambda_t \big((1+i_{t-1}) B_t^N + W_t^N L_t + \Pi_t^N - P_t C_t - B_{t+1}^N \big) \Big)$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = C_t^{-\sigma} - \lambda_t P_t = 0$$

real wage

$$\frac{\partial \mathcal{L}}{\partial L_t} = - L_t^{\varphi} + \lambda_t W_t^N = 0$$

$$-\frac{C_t^{-\sigma}}{L_t^{\varphi}} = -\frac{P_t}{W_t^N} \Longleftrightarrow C_t^{-\sigma} L_t^{-\varphi} = \frac{P_t}{W_t^N} \Leftrightarrow C_t^{\sigma} L_t^{\varphi} = \frac{W_t^N}{P_t}$$

$\ln C_t = c_t$ · in logs: $\sigma \ln C_t + \varphi \ln L_t = \ln W_t^N - \ln P_t$

$$\boxed{\sigma c_t + \varphi l_t = w_t^N - p_t}$$

Euler equation derivation (attention to timing of i !)

$$\mathcal{L} = \sum_{s=0}^{t-1} \beta^s \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{L_s^{1+\eta}}{1+\eta} + \lambda_s \left((1+i_{s-1}) B_s^N + W_s^N L_s + \Pi_s^N - P_s C_s - B_{s+1}^N \right) \right]$$

$$+ \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\eta}}{1+\eta} + \lambda_t \left((1+i_{t-1}) B_t^N + W_t^N L_t + \Pi_t^N - P_t C_t - B_{t+1}^N \right) \right]$$

$$+ \beta^{t+1} E_t \left[\frac{C_{t+1}^{1-\sigma}}{1-\sigma} - \frac{L_{t+1}^{1+\eta}}{1+\eta} + \lambda_{t+1} \left((1+i_t) B_{t+1}^N + W_{t+1}^N L_{t+1} + \Pi_{t+1}^N - P_{t+1} C_{t+1} - B_{t+2}^N \right) \right]$$

$$+ \sum_{k=t+2}^{\infty} \beta^k E_t \left[\frac{C_k^{1-\sigma}}{1-\sigma} - \frac{L_k^{1+\eta}}{1+\eta} + \lambda_k \left((1+i_{k-1}) B_k^N + W_k^N L_k + \Pi_k^N - P_k C_k - B_{k+1}^N \right) \right]$$

"divide both sides by"

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}^N} = 0 : -\beta^t \lambda_t + \beta^{t+1} E_t \lambda_{t+1} (1+i_t) = 0 \; \Big| : \left(\beta^t \lambda_t (1+i_t) \right)$$

$$-\frac{1}{1+i_t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \right] = 0$$

Use $\lambda_t = C_t^{-\sigma} / P_t$ from $\frac{\partial \mathcal{L}}{\partial C_t} = 0$: $\beta E_t \left[\frac{P_t C_{t+1}^{-\sigma}}{P_{t+1} C_t^{-\sigma}} \right] = \frac{1}{1+i_t}$

Households' optimization

Using same approach as in the RBC, we obtain the FOCs:

$$\beta E_{\cancel{0}\,t}\left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{P_t}{P_{t+1}}\right] = \frac{1}{1+i_{t+1}}$$

$$C_t^{\sigma} L_t^{\eta} \cancel{\frac{L_t^{\eta}}{C_t^{-\sigma}}} = \frac{W_t^N}{P_t}$$

We will use lowercase letters for logs of variables:
$c_t = \ln C_t, l_t = \ln L_t$, etc.:

$$c_t = E_t[c_{t+1}] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - \rho)$$

$\sigma c_t + \eta l_t = w_t^N - p_t$

with $\rho = -\ln \beta$ the discount **rate** (used in continuous time models)

$$\beta^t \longrightarrow e^{-\rho t}$$

Outline

1. Household

2. Firms
   - Flexible prices

Firms

- Continuum of firms indexed by $i \in [0,1]$ (1 firm – 1 good)
- Production with common exogenous productivity for all firms A_t and labor: $Y_t(i) = A_t L_t(i)^{1-\alpha} \Rightarrow$ labor demand trivial: $L_t(i) = \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}}$
- Differentiated goods $\Rightarrow$ monopoly power, setting price $P_t(i)$:

  $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$

  - demand function given by $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$ (from $C_t(i) = Y_t(i)$)
  - continuum of goods $\Rightarrow$ firm i doesn't influence Y_t, C_t, P_t

We will look at firm optimization and model equilibrium under **flexible prices** and **sticky prices (Calvo pricing)** in turn.

Outline

1. Household

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price setting problem (micro)

$$\max_{P} \; P\,Y(P) - TC^N(Y(P))$$

$$\frac{d(f(x)g(x))}{dx} = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

$$\frac{dTC^N}{dY} = MC^N$$

FOC: $Y(P) + Y'(P)\cdot P - MC^N(Y(P))\cdot Y'(P) = 0 \quad | : Y(P)$

$$0 = 1 + \frac{Y'(P)\,P}{Y(P)} - MC^N(Y(P))\frac{Y(P)'}{Y(P)}$$

$$\frac{Y'(P)\cdot P}{Y(P)} = -\varepsilon$$

$$1 \cancel{+}- \varepsilon \cancel{-}+ MC^N(Y(P))\cdot\frac{\varepsilon}{P} = 0$$

$$P \cancel{+}-\varepsilon P = \cancel{-}\varepsilon\cdot MC^N(Y(P))$$

$$P = \frac{\cancel{-}\varepsilon}{1\cancel{+}-\varepsilon}\cdot MC^N(Y(P))$$

$$\frac{\varepsilon}{\varepsilon-1}$$

Firm optimization – flexible prices

Maximize profits:

$$\max_{P_t(i), Y_t(i)} P_t(i) Y_t(i) - TC^N(Y_t(i))$$

Where:

- $TC^{\mathbf{N}}$ is **n**ominal cost function:
  $TC^N(Y_t(i)) = W_t^N L_t^d = W_t^N \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}}$
- $Y_t(i)$ related to $P_t(i)$ via demand: $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$.

Unusual notation, but a familiar problem of monopolistic pricing.
Solution:

$$P_t(i) = \frac{\varepsilon}{\varepsilon - 1} MC^N(Y_t(i))$$

Symmetric solution

All firms symmetric in flexible price equilibrium $\Rightarrow$ drop the i index:

$$P_t = \frac{\varepsilon}{\varepsilon - 1} MC^N(Y_t)$$

and we can get the marginal cost as derivative of total cost:

$$MC^N(Y_t) = \frac{dTC^N(Y_t)}{dY_t} = \frac{d(W_t^N L^d(Y_t))}{dY_t} = \frac{1}{1-\alpha} W_t^N A_t^{\frac{1}{\alpha-1}} Y_t^{\frac{\alpha}{1-\alpha}}$$

so we can use it in the optimal price equation:

$$P_t = \frac{\varepsilon}{\varepsilon - 1} \frac{1}{1-\alpha} W_t^N A_t^{\frac{1}{\alpha-1}} Y_t^{\frac{\alpha}{1-\alpha}}$$

or $\quad p_t = \mu - \ln(1-\alpha) + w_t^N + \left(\frac{1}{\alpha - 1}\right) a_t + \left(\frac{\alpha}{1-\alpha}\right) y_t$ in logs

where μ is log of the price markup: $\mu \equiv \ln(\frac{\varepsilon}{\varepsilon - 1})$

Flexible price equilibrium

i_t

A flexible price equilibrium is a sequence of variables $\{Y(i)_t, C(i)_t, P_t(i), L(i)_t, W_t^N, A_t\}_{t=0}^{\infty}$ and aggregates $C_t = \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$, $Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$, $P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{1/(1-\varepsilon)}$, $L_t = \int_0^1 L_t(i) di$ such that, given an exogenous process for A_t:

1. The **Euler equation** holds: $\beta E_0 \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}}\right] = \frac{1}{1+i_{\cancel{t+1}}}$
2. **Consumption-labor optimality** holds: $C_t^{\sigma} L_t^{\varphi} = \frac{W_t^N}{P_t}$
3. **Optimal price** is set by each firm:
   $P_t(i) = \frac{\varepsilon}{\varepsilon-1} \frac{1}{1-\alpha} W_t^N A_t^{\frac{1}{\alpha-1}} Y_t(i)^{\frac{\alpha}{1-\alpha}}$
4. Goods market clears: $Y_t(i) = C_t(i) \Rightarrow Y_t = C_t$, with $Y_t(i) = A_t L_t(i)^{1-\alpha}$
5. Bonds market clears: $B_t^N = 0$

Technically, we also need to impose a transversality condition in households' optimization: $lim_{T\to\infty} E_t[B_t^N] \geq 0$

Flexible price equilibrium: monetary neutrality

As in RBC, nothing depends on nominal variables P_t, W_t^N, i_t in equilibrium. Consider equilibrium conditions (2)-(4) in logs (written without goods index i):

Real wage : $\frac{W_t^N}{P_t}$

$$\sigma c_t + \eta l_t = \underbrace{w_t^N - p_t}_{w_t}$$

log real wage

$$p_t = \mu - \ln(1-\alpha) + w_t^N + \left(\frac{1}{\alpha - 1}\right) a_t + \left(\frac{\alpha}{1-\alpha}\right) y_t$$

$$y_t = c_t$$

$$y_t = a_t + (1-\alpha) l_t,$$

where the last equation is the production function in logs. $w_t \equiv w_t^N - p_t$ can be introduced in the first two equations. We then have 4 equations, 4 unknowns y_t, c_t, l_t, w_t, that have a static solution each period that depends on a_t. Solution for log GDP is:

$$y_t = \frac{1-\alpha}{(1-\alpha)\sigma + \eta + \alpha}\left(-\mu + \ln(1-\alpha) - \frac{1+\eta}{1-\alpha} \mathbf{a}_t\right)$$

The real interest rate

Real interest rate is a real quantity that can also be obtained in equilibrium using the log Euler equation:

$$\underset{y_t}{c_t} = E_t[\underset{y_{t+1}}{c_{t+1}}] - \frac{1}{\sigma}(\underbrace{i_t - E_t[\pi_{t+1}]}_{r_t} - \rho)$$

Then, recall the definition of the **real interest rate**, a.k.a. the **Fischer equation**:

$$r_t = i_t - E_t\pi_{t+1}$$

combine the two and $y_t = c_t$ to obtain

$$\begin{aligned} r_t &= \rho + \sigma E_t \Delta y_{t+1} \\ &= \rho + \sigma \frac{1+\eta}{\sigma(1-\alpha)+\eta+\alpha} E_t \Delta a_{t+1}, \quad \text{(using the solution for } y\text{)} \end{aligned}$$

So the real interest rate is, too, driven by productivity. In a steady state, $\Delta a_t = 0$, so $r_t = \rho$, the real interest rate is the discount ~~factor.~~ rate

Central Bank in a neutrality economy

Suppose you only know the flexible price model (the sticky price one is much harder!), but your employer **really** wants you to say something about prices, interest rates, central bank, etc.

A neutral central bank with an inflation targeting Taylor Rule can be introduced:

$$i_t = \rho + \phi_\pi \pi_t, \quad \textit{with } \rho = -\ln\beta, \text{ the discount } \cancel{\text{factor}} \text{ rate}$$

and combine ~~the two:~~ with the Fischer equation:

$$\phi_\pi \pi_t = E_t \pi_{t+1} + \hat{r}_t \quad \text{with} \quad \hat{r}_t \equiv r_t - \rho$$

$\hat{r}_t$ is the deviation of the real interest from its steady-state value ρ.

Inflation determinacy – the Taylor Principle

$\phi_\pi \pi_{t+1} = E_{t+1}\pi_{t+2} + \hat{r}_{t+1}$

$$\phi_\pi \pi_t = E_t \pi_{t+1} + \hat{r}_t \quad \text{with} \quad \hat{r}_t \equiv r_t - \rho$$

If $\phi_\pi > 1$, the level of inflation is **determined** as a discounted sum of expected $\hat{r}_t$:

$$\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(s+1)} E_t \hat{r}_{t+s}$$

Otherwise, we can write inflation dynamics as an AR(1)-type process:

$$\pi_{t+1} = \phi_\pi \pi_t - \hat{r}_t + \xi_{t+1}$$

Where ξ is a random variable with $E_t \xi_{t+1} = 0$ and no economic meaning. This is a **sunspot shock** – a random factor affecting economic outcomes such as inflation, but with no economic explanation.

Bottom line: an **active Taylor rule ($\phi_\pi > 1$)** allows to determine level of inflation, otherwise – uncontrollable **sunspot shocks**. Not specific to neutral flexible price economy, – also with nominal rigidity economy, where monetary variables have real effects.