Macroeconomics
Lecture 11 – Open Macroeconomy III

Ilya Eryzhenskiy

PSME Panthéon-Sorbonne Master in Economics

Fall 2022

Outline

Canadian Economy: Business-Cycle Statistics

Canada – developed economy, but 2.1 % of world GDP $\Rightarrow$ "small" (compare to China – 18.4%; USA – 24% – large open economies).

We will seek to explain historical business-cycle patterns: 1946-1985.

Variable (x_t)	Moments		
	σ_{x_t}	corr(x_t, x_{t-1})	corr(x_t, Y_t)
Y	2.8	0.61	1
C	2.5	0.7	0.59
I	9.8	0.31	0.64
L	2	0.54	0.8
$\frac{TB}{Y}$	1.9	0.66	-0.13

Source: Mendoza AER, 1991. Annual data. Log-quadratically detrended.

Canadian Economy: Business-Cycle Patterns

- Volatility ranking: $\sigma_{TB/Y} < \sigma_C < \sigma_Y < \sigma_I$.
- Consumption, investment, and labor hours are **procyclical**.
- The trade-balance-to-output ratio is **countercyclical**.
- All variables have a positive autoregressive coefficient.
- Similar patterns hold for other *developed* SOEs.

Outline

A SOE RBC model

We extend the RBC framework to include exchanges with the rest of the world. Specific elements:

1. **Capital adjustment cost** limits impact of investment on **trade balance**
2. **Interest rate premium**, or external debt-elastic interest rate (EDEIR) assumption
   - Introduced for technical purposes: necessary for model convergence
3. (not obligatory) Utility maximization w.r.t. **resource constraint** of economy instead of budget constraint

$\Rightarrow$ We solve **social planner**'s optimization problem, which is still equivalent to finding decentralized equilibrium (1st Welfare Theorem)

Planner's Maximization Problem

$$\max_{\{C_t, I_t, IIP_{t+1}, K_{t+1}\}_0^\infty} E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t)$$

subject to

$$C_t + I_t + IIP_{t+1} + \Phi(K_{t+1} - K_t) = A_t f(K_t, L_t) + (1 + r_t) IIP_t$$

$$K_{t+1} = (1 - \delta) K_t + I_t$$

$$K_0, IIP_0, A_0 \text{ – given;} \quad \ln A_{t+1} = \rho \ln A_t + \widetilde{\eta} \epsilon_{t+1}$$

$\Phi(\cdot)$ is a convex **capital adjustment cost** function with $\Phi(0) = \Phi'(0) = 0$; $\Phi''(\cdot) > 0$

A transversality condition for IIP applies: $\lim_{j \to \infty} E_t \frac{IIP_{t+j}}{\prod_{s=0}^{j} (1 + r_s)} \geq 0$

$$\mathcal{L} = \sum_{s=0}^{t-1} \beta^s \Big[u(C_s, L_s) + \lambda_s \big(A_s f(K_s, L_s) + (1+r_s) B_s - C_s - K_{s+1} + (1-\delta) K_s - B_{s+1} - \Phi(K_{s+1} - K_s)\big)\Big]$$

$$+ \beta^t \Big[u(C_t, L_t) + \lambda_t \big(A_t f(K_t, L_t) + (1+r_t) B_t - C_t - K_{t+1} + (1-\delta) K_t - B_{t+1} - \Phi(K_{t+1} - K_t)\big)\Big]$$

$$+ E_t \beta^{t+1} \Big[u(C_{t+1}, L_{t+1}) + \lambda_{t+1} \big(A_{t+1} f(K_{t+1}, L_{t+1}) + (1+r_{t+1}) B_{t+1} - C_{t+1} - K_{t+2} + (1-\delta) K_{t+1} - B_{t+2} - \Phi(K_{t+2} - K_{t+1})\big)\Big]$$

$$+ E_t \sum_{k=t+2}^{\infty} \beta^k \Big[u(C_k, L_k) + \lambda_k \big(A_k f(K_k, L_k) + (1+r_k) B_k - C_k - K_{k+1} + (1-\delta) K_k - B_{k+1} - \Phi(K_{k+1} - K_k)\big)\Big]$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 : \beta^t \big[u'_c(C_t, L_t) - \lambda_t\big] = 0$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = 0 : \beta^t \big[u'_L(C_t, L_t) + \lambda_t A_t f'_L(K_t, L_t)\big] = 0$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} = 0 : \beta^t(-\lambda_t) + \beta^{t+1} E_t \lambda_{t+1}(1 + r_{t+1}) = 0 \quad \Big| : \beta^t$$

$$\Leftrightarrow -\lambda_t + \beta E_t \lambda_{t+1}(1 + r_{t+1}) = 0$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 : \beta^t \lambda_t \big(-1 - \Phi'(K_{t+1} - K_t)\big) + \beta^{t+1} E_t \Big[\lambda_{t+1} \big(A_{t+1} f'_K(K_{t+1}, L_{t+1}) + 1 - \delta + \Phi'(K_{t+2} - K_{t+1})\big)\Big] = 0$$

Planner's FOC

$\frac{\partial \mathcal{L}}{\partial C_t}$:

$$u'_C(C_t, L_t) = \lambda_t$$

$\frac{\partial \mathcal{L}}{\partial IIP_{t+1}}$:

$$\lambda_t = \beta(1 + r_{t+1})E_t\lambda_{t+1}$$

$\frac{\partial \mathcal{L}}{\partial L_t}$:

$$-u'_L(C_t, L_t) = \lambda_t A_t f'_L(K_t, L_t)$$

$\frac{\partial \mathcal{L}}{\partial K_{t+1}}$:

$$1 + \Phi'(K_{t+1} - K_t) = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[A_{t+1} f'_K(K_{t+1}, L_{t+1}) + 1 - \delta + \Phi'(K_{t+2} - K_{t+1})\right]$$

$\frac{\partial \mathcal{L}}{\partial \lambda_t}$:

$$C_t + K_{t+1} - (1-\delta)K_t + IIP_{t+1} + \Phi(K_{t+1} - K_t) = A_t f(K_t, L_t) + (1 + r_t)IIP_t$$

Interest Rate Premium/ EDEIR

In this model, the assumption of External debt-Elastic Interest Rate (EDEIR) has a technical role: it makes the model converge to a steady state. Formula as before:

$$r_t = r^* + p(\underset{-}{IIP_t})$$

- r^* = world interest rate, assumed **constant**
- $p(IIP_t)$ = country interest-rate premium, decreasing in IIP (increasing in external debt)
  - Note that the function treats negative and positive IIP symmetrically, so the interpretation is wider (and less intuitive) than a risk-based interest rate premium

Balance of Payments Indicators

Trade Balance

$$TB_t = Y_t - C_t - I_t - \Phi(K_{t+1} - K_t)$$

Current Account

$$CA_t = TB_t + r_t IIP_t$$

$$IIP_{t+1} - IIP_t = FA_t = CA_t$$

Equilibrium

An equilibrium is a sequence $\{C_t, L_t, K_{t+1}, IIP_{t+1}\}$, such that, given exogenous initial conditions K_0, IIP_0, A_0 and exogenous productivity $\ln A_{t+1} = \rho \ln A_t + \widetilde{\eta}\epsilon_{t+1}$, the following conditions are satisfied:

$$u'_C(C_t, L_t) = \beta(1 + r^* + p(IIP_{t+1}))E_t u'_C(C_{t+1}, L_{t+1})$$

$$-\frac{u'_L(C_t, L_t)}{u'_C(C_t, L_t)} = A_t f'_L(K_t, L_t)$$

$$1 = \beta E_t \left\{ \frac{u'_C(C_{t+1}, L_{t+1})}{u'_C(C_t, L_t)} \frac{A_{t+1} f'_K(K_{t+1}, L_{t+1}) + 1 - \delta + \Phi'(K_{t+2} - K_{t+1})}{1 + \Phi'(K_{t+1} - K_t)} \right\}$$

$$C_t + K_{t+1} - (1-\delta)K_t + IIP_{t+1} + \Phi(K_{t+1} - K_t) = A_t f(K_t, L_t) + IIP_t(1 + r^* + p(IIP_t))$$

Steady State

$I_t = K_{t+1} - (1-\delta)K_t \Rightarrow I = \delta K$

The steady state (IIP, K, C, L) is solution to a system (with $A = 1$):

$$-\frac{u'_L(C, L)}{u'_C(C, L)} = Af'_L(K, L)$$

$$C + \delta K - IIP \cdot (r^* + p(IIP)) = Af(K, L)$$

$$1 = \beta(1 + r^* + p(IIP))$$

$$1 = \beta\left[Af'_K(K, L) + 1 - \delta\right]$$

Balance of Payments in Steady State

Recall $IIP_{t+1} - IIP_t = CA_t$ from previous lecture.
Since $IIP_t = IIP$ (constant) in s.s., $CA = 0$ in s.s.

Recall then $CA = TB + r \cdot IIP \Rightarrow TB = -r \cdot IIP$.

Intuition: net **capital flows (FA balance)** are null in s.s., so the flows due to **trade balance** are exactly compensated by flows due to **income balance**, for **Balance of Payments Identity** $CA = FA$ to hold.

Functional Forms

Instantaneous utility function – GHH

$$u(C, L) = \frac{(C - L^{\omega}/\omega)^{1-\sigma} - 1}{1 - \sigma}; \quad \omega > 1; \sigma > 0$$

⇒ labor supply depends on real wage only (not on consumption)

Interest rate premium

$$p(IIP) = \psi\left(e^{\bar{IIP} - IIP} - 1\right); \quad \psi > 0$$

Production function – Cobb-Douglas

$$f(K, L) = K^{\alpha} L^{1-\alpha}; \quad \alpha \in (0, 1)$$

Adjustment cost function – quadratic

$$\Phi(x) = \frac{\phi}{2} x^2; \quad \phi > 0$$

6 structural parameters: σ, ω, ψ, $\bar{IIP}$, α, ϕ

Steady State

using the assumed functional forms, the steady state (IPP, K, C, L) is solution to a system (with $A = 1$):

$$L^{\omega-1} = A(1-\alpha)(K/L)^{\alpha}$$

$$C + \delta K - (r^* + \psi(e^{I\bar{I}P-IIP} - 1))IIP = A(K/L)^{\alpha}L$$

$$1 = \beta(1 + r^* + \psi(e^{I\bar{I}P-IIP} - 1))$$

$$1 = \beta\left[A\alpha(K/L)^{\alpha-1} + 1 - \delta\right]$$

$\overline{IIP}$

7 parameters in the s.s. system: $\omega, \alpha, \delta, r^*, \psi, d, \beta$. **+ 4** more parameters in model, eliminated in s.s.: σ, ϕ, ρ, $\tilde{\eta}$

Outline

Calibration/estimation strategy

Three categories of parameters have different calibration approaches:

Category a: manual calibration based on literature consensus/more arbitrary arguments; **4** parameters: $\sigma = 2$, $\delta = 0.1$, $r^* = 0.04$, $\beta = 1/(1 + r^*)$.

Category b: restrictions to match **first moments** (**averages**) of the data that the model aims to explain, **2** parameters: α, $\cancel{\bar{d}}$ $\overline{IIP}$

$$\text{labor share} = 0.68$$
$$\text{trade-balance-to-output ratio} = 0.02$$

Category c: restrictions to match **second moments** (**variances, correlations**) of the data that the model aims to explain, **5** parameters: ω, ϕ, ψ, ρ, $\tilde{\eta}$. The second moments to be matched are: σ_y, σ_L, σ_I, $\sigma_{TB/Y}$, $\text{corr}(\ln Y_t, \ln Y_{t-1})$

Calibration – details

Simple example of Category b calibration – parameter α:
The labor share, s_L, is defined as

$$s_L = \frac{wL}{Y}$$

In the decentralized economy we have

$$A_t f_L'(K_t, L_t) = w_t$$

Thus, in the steady state:

$$\frac{\cancel{A}\cdot(1-\alpha)\cancel{K^{\alpha}}\cancel{L^{-\alpha}}\cdot\cancel{L}}{\cancel{A}\cancel{K^{\alpha}}\cancel{L^{1-\alpha}}}$$

$$s_L = \frac{Af_L'(K, L)L}{Af(K, L)}$$

Using the Cobb-Douglas form for $f(\cdot)$ yields

$$s_L = (1 - \alpha)$$

Hence we have that $\alpha = 1 - s_L = 1 - 0.68$, that is,

$$\alpha = 0.32$$

Calibration/estimation – details

Let θ denote the vector of structural parameters we still need to assign numerical values to, that is, let

$\overline{TIP}$

$$\theta \equiv \left[\begin{array}{cccccc} \omega & \cancel{\bar{d}} & \phi & \psi & \rho & \tilde{\eta} \end{array} \right]$$

The choice of these parameters is based on **Simulated Method of Moments**. We proceed in 6 steps:

Step 1: set a value (guess) for each element of θ

Step 2: Given the guess for ω find L using s.s. optimality of capital choice:

$$\frac{K}{L} = \left(\frac{r^* + \delta}{A\alpha} \right)^{\frac{1}{\alpha - 1}}$$

and replace in s.s. consumption-labor optimality:

$$L = ((1-\alpha)A(K/L)^{\alpha})^{1/(\omega-1)}$$

With L in hand, find K and Y, as $K = (K/L)L$ and $Y = A(K/L)^{\alpha}L$, respectively.

Calibration/estimation – details

$s_{TB} = \frac{TB}{Y} = -\frac{\bar{r} IIP}{Y}$ $\quad TB = -r\, IIP$

Step 3: Let s_{TB} denote the trade-balance-to-output ratio. In the steady state, $s_{TB} = -\frac{r^* IIP}{Y}$. Solve for IIP to get $IIP = -\frac{s_{TB} Y}{r^*}$. Then use $p(IIP) = 0$ and the restriction that $\beta(1 + r^*) = 1$ to obtain $\bar{IIP} = IIP$

Step 4: Find C from s.s. resource constraint: $C = Y - \delta K - r^* \cancel{d}\, IIP$

Step 5: With the steady state values of (C, K, L, IIP) and all structural parameters obtained, simulate model (e.g. with **Blanchard-Kahn algorithm**) to get the following **second-order moments**:

$$x(\theta) \equiv \left[\begin{array}{ccccc} \sigma_y & \sigma_L & \sigma_I & \sigma_{TB/Y} & corr(\ln Y_t, \ln Y_{t-1}) \end{array} \right]$$

Calibration/estimation – details

$$x(\theta) \equiv \left[\begin{array}{ccccc} \sigma_Y & \sigma_L & \sigma_I & \sigma_{TB/Y} & corr(\ln Y_t, \ln Y_{t-1}) \end{array} \right]$$

Step 6: Find **Euclidean distance** between vector of moments of simulated data $x(\theta)$ and vector of empirical moments x^*:

$$\mathcal{D} = ||x(\theta) - x^*||$$

Step 7: Adjust elements of θ until $\mathcal{D}$ is less than a threshold $\mathcal{D}^*$.

(a_1, b_1)
(a_2, b_2) $\rightarrow \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}$

Outline

Calibration/estimation – parameter values

Assume that one **period** is **one year**; calibrate the model to the **Canadian economy**.

$\tilde{\eta}$

σ	r^*	δ	α	ω	ϕ	ρ	σ_ϵ	$\bar{IIP}$	ψ
2	0.04	0.1	0.32	1.455	0.028	0.42	0.013	-0.744	0.0007

β follows from r^*: $1/\beta = 1 + r^*$ from Euler equation.

Given parameter values, the steady state can be computed with any software that solves non-linear systems:

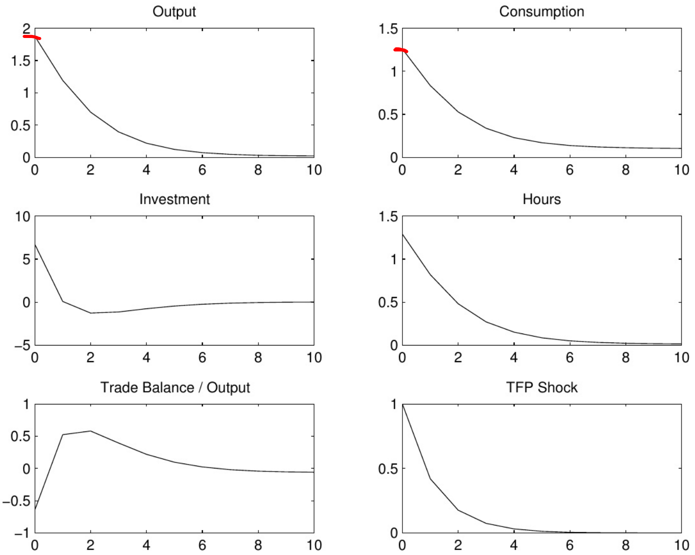
C	IIP	L	K
1.1170	-0.7442	1.0074	3.3977

Empirical vs. Simulated Second Moments

	Canadian Data						Model		
	1946 to 1985			1960 to 2011					
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,Y_t}	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,Y_t}	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,Y_t}
Y	2.8	0.6	1	3.7	0.9	1	3.1	0.6	1
C	2.5	0.7	0.6	2.2	0.7	0.6	2.7	0.8	0.8
I	9.8	0.3	0.6	10.3	0.7	0.8	9.0	0.1	0.7
L	2.0	0.5	**0.8**	3.6	0.7	**0.8**	2.1	0.6	**1**
$\frac{TB}{Y}$	1.9	0.7	-0.1	1.7	0.8	0.1	1.8	0.5	-0.04
$\frac{CA}{Y}$	-	-	-	-	-	-	1.4	0.3	0.05

- σ_L, σ_I, σ_Y, $\sigma_{TB/Y}$, and $\rho_{Y_t,Y_{t-1}}$ were targeted by calibration, so no real test here.
- model correctly places σ_C below σ_Y and σ_I and above σ_L and $\sigma_{TB/Y}$.
- model correctly makes TB/Y **countercyclical**.
- model **overestimates** the correlations of hours and consumption with output (due to GHH utility).

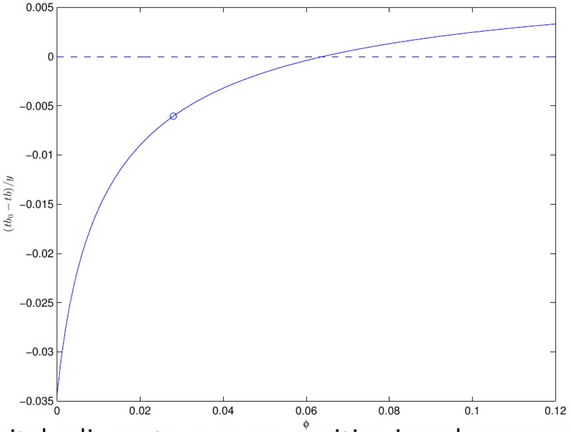
Positive Productivity Shock – impulse responses



Source: Schmitt-Grohé and Uribe (JIE, 2003)

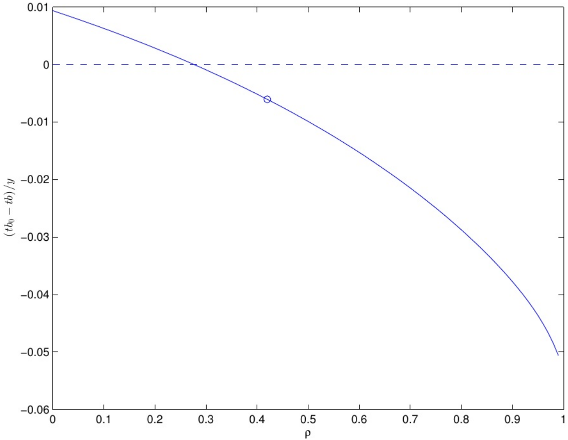
Outline

Response of TB and varying capital adjustment cost



Higher capital adj. costs $\Rightarrow$ more positive impulse response of TB.
For $\phi < 0.06$ (scale parameter of adj. cost), the response of the trade balance negative.
Recall $TB_t = Y_t - C_t - I_t$. With large adj. cost, investment response weak $\Rightarrow$ consumption drives TB $\Rightarrow$ intuition from 2-period model with consumption only applies.

Response of TB and varying productivity persistence



More persistent the productivity shock ⇒ smaller impulse response of TB (recall temporary vs. permanent shocks in the 2-period model). For $\rho > 0.3$ (productivity persistent enough), the response of TB is negative.