Macroeconomics
Lecture 11 – New Keynesian Model II; Time Consistency of Monetary Policy

Ilya Eryzhenskiy

PSME Panthéon-Sorbonne Master in Economics

Fall 2023

Outline

Outline

Sticky prices: data, models

- How sticky/rigid are prices of goods?
  - Empirical work (Dhyne et al, 2005) $\rightarrow$ average duration of a price spell is $\sim$3 quarters in US, $\sim$4 quarters in the EU
- How is this built in the New Keynesian model?

$\rightarrow$ Common sticky price mechanisms:

1. **Calvo pricing**: firm able to change price with a constant probability
2. *Rotemberg pricing*: price change possible, but costly
3. *Taylor contracts*: firm can change price every T periods

We do Calvo pricing – most common and analytically convenient

Calvo pricing

- ▷ Every period, each firm **will be unable to change price with probability** θ
  - ▷ probability same for all firms, independent of their prices

$\Rightarrow$ probability $1-\theta$ to set a new, optimal, price in a given period

- ▷ average duration of a price $\frac{1}{1-\theta}$ periods
  - ▷ θ calibrated to match a target duration, e.g. 3 quarters. In a model with period = quarter:

$$\frac{1}{1-\theta} = 3 \Leftrightarrow \theta = \cancel{\frac{3}{5}} \quad \frac{2}{3}$$

$$1 = 3 - 3\theta$$

Price index under sticky prices

Look at price index again: $P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$

Firms symmetric $\Rightarrow P_t(i) = P_t^*$ for all firms that can set price

Large number of firms $\Rightarrow$ **probability** θ becomes **share** of firms

Denote $S(t)$ the set of indices of unlucky firms (set of size θ) and $\bar{S}(t)$ the set of lucky firms (set size $1-\theta$). Then:

$$P_t = \left(\int_{S(t)} P_t(i)^{1-\varepsilon} di + \int_{\bar{S}(t)} P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$$

$\left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$

$$= \left(\int_{S(t)} P_{t-1}(i)^{1-\varepsilon} di + (1-\theta) P_t^{*\,1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

$$= \left(\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*\,1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

First integral $\rightarrow t-1$ price index (up to a power) because risk of being unlucky (i.e. being in $S(t)$) same for all $\Rightarrow$ price aggregate on $S(t)$ same as price aggregate on $[0,1]$

Aggregate price dynamics

Using the new formula $P_t = \left(\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*\,1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$ we can get:

$$1 + \pi_t = \frac{P_t}{P_{t-1}} = \left[\theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

$p_t = \ln P_t \; ; \quad p_t - p_{ss} = \frac{P_t - P_{ss}}{P_{ss}}$

with P_t^* the optimal price set in period t by firms that **can** change price. Firms symmetric $\Rightarrow$ P_t^* same for all.

Log-linearized using Taylor series around steady state with $P_t/P_{t-1} = 1$ and $P_t^*/P_{t-1} = 1$:

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \tag{1}$$

- ▷ How is optimal P_t^* chosen? Dynamic problem, coming up next

Taylor approximation around a steady-state

$$f(x) \simeq f(x_{ss}) + f'(x_{ss})\cdot(x - x_{ss}) + \cancel{\bar{O}(x - x_{ss})}$$

$$f(x,y) \simeq f(x_{ss}, y_{ss}) + \frac{\partial f(x_{ss}, y_{ss})}{\partial x}\cdot(x - x_{ss})$$
$$+ \frac{\partial f(x_{ss}, y_{ss})}{\partial y}(y - y_{ss}) + \cancel{\bar{O}(x - x_{ss}, y - y_{ss})}$$

$$x - x_{ss} = x_{ss}\cdot\frac{x - x_{ss}}{x_{ss}} = \underline{x_{ss}(\ln x - \ln x_{ss})}$$

$$f(x,y) \simeq f(x_{ss}, y_{ss}) + \frac{\partial f(x_{ss}, y_{ss})}{\partial x}\cdot x_{ss}(\ln x - \ln x_{ss})$$
$$+ \frac{\partial f(x_{ss}, y_{ss})}{\partial y}\cdot y_{ss}(\ln y - \ln y_{ss})$$

$$\left(\frac{P_t}{P_{t-1}}\right)^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}$$

$$\left(\frac{P_t}{P_{t-1}}\right)^{1-\varepsilon} = \left(P_t^{1-\varepsilon} P_{t-1}^{\varepsilon-1}\right) \qquad \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon} = \left(P_t^{*\,1-\varepsilon} P_{t-1}^{\varepsilon-1}\right)$$

$$\cancel{P_{ss}^{1-\varepsilon} P_{ss}^{\varepsilon-1}} +$$

$$(1-\varepsilon) P_{ss}^{1-\varepsilon-1} P_{ss}^{\varepsilon-1} \cdot P_{ss}^{1}(p_t - p_{ss}) + (\varepsilon-1) P_{ss}^{1-\varepsilon} P_{ss}^{\varepsilon-1-1} P_{ss}(p_{t-1} - p_{ss})$$

$$= \cancel{\theta + (1-\theta)\left(\frac{P_{ss}}{P_{ss}}\right)^{1-\varepsilon}} + (1-\theta)\Big((1-\varepsilon)P_{ss}^{1-\varepsilon-1} P_{ss}^{\varepsilon-1} P_{ss}(p_t^* - p_{ss})$$

$$+ (\varepsilon-1) P_{ss}^{1-\varepsilon} P_{ss}^{\varepsilon-1-1} P_{ss}(p_{t-1} - p_{ss})\Big)$$

$$(1-\varepsilon)(p_t - p_{ss}) + (\varepsilon-1)(p_{t-1} - p_{ss}) = (1-\theta)\big((1-\varepsilon)(p_t^* - p_{ss}) + (\varepsilon-1)(p_{t-1} - p_{ss})\big)$$

divide everything by $1-\varepsilon$

$$p_t - \cancel{p_{ss}} - (p_{t-1} - \cancel{p_{ss}}) = (1-\theta)\big(p_t^* - \cancel{p_{ss}} - (p_{t-1} - \cancel{p_{ss}})\big)$$

$$\rightarrow \overbrace{p_t - p_{t-1}}^{=\pi_t} = (1-\theta)(p_t^* - p_{t-1})$$

Outline

Accounting for firm history

Production in $t = k$ of a firm that last optimized prices in $t = s$ with $s \leq k$ is labelled $Y_{k|s}(i)$

Using equilibrium conditions $C_t(i) = Y_t(i)$, $C_t = Y_t$ and the demand formula $C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t$, we can write:

$$Y_{k|s}(i) = \left(\frac{P_s^*}{P_k}\right)^{-\varepsilon} Y_k$$

Profit maximization: stochastic discount factor needs be applied between t (not necessarily $t = 0$) and $t + k$:

$$M_{t,t+k} = E_t \left[\frac{\beta^k C_{t+k}^{-\sigma} P_t}{C_t^{-\sigma} P_{t+k}}\right] = E_t \frac{1}{1+i_t} \frac{1}{1+i_{t+1}} \cdots \frac{1}{1+i_{t+k-1}}$$

$$u(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}$$

Firms: Optimal price setting

P_t^* maximizes present discounted value of firm, i.e. future (nominal) profits:

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} M_{t,t+k} \Pi_{t+k}^n$$

where every expected future period is either:

1. a period where price cannot be set $\Rightarrow$
   $\Pi^n_{t+k|\cancel{t+}s} = P^*_{\cancel{t+}s} Y_{t+k|\cancel{t+}s} - TC^n_{t+k}(Y_{t+k|\cancel{t+}s})$ with some $s \geq 0$

2. or a period where price can be set:

$$\Pi^n_{t+k|t+k} = P^*_{t+k} Y_{t+k|t+k} - TC^N_{t+k}(Y_{t+k|t+k})$$

   which is same profit as in the flexible price model

Firms: Optimal price setting – role of θ

Profit Π^n_{t+k} depends on P^*_t **only if firm never could change price between** t **and** $t+k$ $\Rightarrow$ k times unlucky, probability θ^k

Profit maximization can then be rewritten:

$$\max_{P^*_t} E_t \sum_{k=0}^{\infty} \theta^k M_{t,t+k}[P^*_t Y_{t+k|t} - TC^n_{t+k}(Y_{t+k|t})]$$

$$\text{s.t.} \quad Y_{t+k|t} = \left(\frac{P^*_t}{P_{t+k}}\right)^{-\varepsilon} \overset{Y}{\cancel{C}}_{t+k}$$

Firms: Optimal price setting – FOC

First Order Condition w.r.t. P_t^*:

$$E_t \sum_{k=0}^{\infty} \theta^k \left(M_{t,t+k} Y_{t+k|t} (P_t^* - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k|t}^n) \right) = 0$$

Moving all terms with the marginal cost on right hand side:

$$E_t \sum_{k=0}^{\infty} \theta^k M_{t,t+k} Y_{t+k|t} P_t^* = \frac{\varepsilon}{\varepsilon - 1} E_t \sum_{k=0}^{\infty} \theta^k \overset{M_{t,t+k}}{\cancel{Q}}_{t,t+k} Y_{t+k|t} MC_{t+k|t}^n$$

Using formula for $M_{t,t+k}$ and solving for P_t^*:

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{k=0}^{\infty} \theta^k \beta^k C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon - 1} MC_{t+k|t}^n}{E_t \sum_{k=0}^{\infty} \theta^k \beta^k C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon - 1}}$$

Optimal price is markup × weighted sum of future marginal costs

Firms: Optimal price setting – FOC linearized

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{k=0}^{\infty} \theta^k \beta^k C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon-1} MC_{t+k|t}^n}{E_t \sum_{k=0}^{\infty} \theta^k \beta^k C_{t+k}^{1-\sigma} P_{t+k}^{\varepsilon-1}}$$

$$MC_{t+k|t} = \frac{MC_{t+k|t}^n}{P_{t+k}}$$

Log-linearizing with Taylor expansion around steady state with $P_t = P_{t-1} = P$:

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t[\hat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})] \quad (2)$$

where $\hat{mc}_{t+k|t} = mc_{t+k|t} - mc_{ss}$, $mc_t = \ln\left(\frac{MC_{t+k|t}}{P_t}\right)$ – we use the percentage deviation of real marginal cost from its steady-state level (it's closely related to output gap – to be shown)

Outline

Aggregate production – approximation

$Y_t(i)$ and $N_t(i)$ vary across firms because of sticky prices $\Rightarrow$ aggregate production function not same as firms' function. Aggregate labor definition: $N_t = \int_0^1 N_t(i)di$; using the firms' production function: $Y_t(i) = A_t N_t(i)^{1-\alpha}$

$$N_t = \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di$$

In logs:

$$(1-\alpha)n_t = y_t - a_t \overset{(1-\alpha)}{+} \ln\left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di\right]$$

where last term measures price dispersion. We do Taylor approximation around s.s. with $P_{ss}(i) = P_{ss}$, so null dispersion $\Rightarrow$ **approximate** aggregate production function: $y_t = a_t + (1-\alpha)n_t$

Inflation and marginal cost

Use $p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t[\hat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})]$

and relation between $MC(Y_{t+k|t})$ and $MC(Y_{t+k})$ (using MC formula and demand): $mc_{t+k|t} = mc_{t+k} + \frac{\varepsilon\alpha}{1-\alpha}(p_t^* - p_{t+k})$
...to get a **macro** relationship between marginal cost and prices:

$$\underbrace{p_t^* - p_{t-1}}_{x_t} = E_t \sum_{k=0}^{\infty} \underbrace{\theta^k \beta^k}_{\delta} \left(\overbrace{(1-\theta\beta)\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\hat{mc}_{t+k} + \pi_{t+k}}^{z_{t+k}} \right)$$

$x_t = \delta E_t x_{t+1} + z_t$

which can be written recursively: (try for $x_t = E_t \sum_{k=0}^{\infty} \delta^k z_{t+k}$)

$\rightarrow$ $$p_t^* - p_{t-1} = \beta\theta E_t\left[p_{t+1}^* - p_t\right] + (1-\beta\theta)\theta\hat{mc}_t + \pi_t$$

combine with $\pi_t = (1-\theta)(p_t^* - p_{t-1})$ to get:

$$\pi_t = \beta E_t[\pi_{t+1}] + \lambda\hat{mc}_t \tag{3}$$

with $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$. **Almost the AS relationship!**

$$x_t = \mathbb{E}_t \sum_{k=0}^{\infty} \delta^k z_{t+k} = z_t + \underbrace{\mathbb{E}_t \sum_{k=1}^{\infty} \delta^k z_{t+k}}_{= \delta \mathbb{E}_t x_{t+1}}$$

$= \delta \mathbb{E}_t x_{t+1}$ – $\boxed{\text{to be shown}}$

$$x_{t+1} = \mathbb{E}_{t+1} \sum_{k=0}^{\infty} \delta^k z_{t+1+k}$$

$$\mathbb{E}_t x_{t+1} = \underbrace{\mathbb{E}_t \mathbb{E}_{t+1}}_{\mathbb{E}_t} \sum_{k=0}^{\infty} \delta^k z_{t+1+k} = \mathbb{E}_t \sum_{k=1}^{\infty} \delta^{k-1} z_{t+k}$$

$$\boxed{\delta \mathbb{E}_t x_{t+1} = \mathbb{E}_t \sum_{k=1}^{\infty} \delta^k z_{t+k}}$$

$$x_t = z_t + \delta \mathbb{E}_t x_{t+1}$$

Marginal cost gap $\rightarrow$ output gap

Use $mc_t^n = w_t^n + \left(\frac{1}{\alpha-1}\right) a_t + \left(\frac{\alpha}{1-\alpha}\right) y_t$, consumption-labor optimality $w_t^n - p_t = \sigma y_t + \eta n_t$ and approximate log GDP $y_t = a_t + (1-\alpha) n_t$ to get:

$$mc_t = \left(\sigma + \frac{\eta+\alpha}{1-\alpha}\right) y_t - \frac{1+\eta}{1-\alpha} a_t - \ln(1-\alpha)$$

In s.s., $mc_{ss} = \ln(MC_{ss}) = \ln\left(\frac{MC_{ss}^n}{P_{ss}}\right) = -\ln\left(\frac{\varepsilon}{\varepsilon-1}\right) \equiv -\mu$. Using formula of GDP in flexible-price equilibrium:

$$mc^f = mc_{ss} = -\mu = \left(\sigma + \frac{\eta+\alpha}{1-\alpha}\right) y_t^f - \frac{1+\eta}{1-\alpha} a_t - \ln(1-\alpha)$$

Taking difference of mc_t and mc_{ss}, we obtain marginal cost gap as function of **output gap** $\tilde{y}_t \equiv y_t - y_t^f$:

$$\hat{mc}_t = \left(\sigma + \frac{\eta+\alpha}{1-\alpha}\right) \overbrace{(y_t - y_t^f)}^{\tilde{y}_t} \tag{4}$$

We're back to initial idea of output gap: difference of real (sticky price) GDP and ideal (flexible price) one

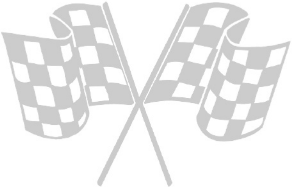
The New Keynesian Phillips Curve (AS)

We finally obtain the **New Keynesian Phillips Curve**:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \hat{mc}_t \quad \& \quad \hat{mc}_t = \left(\sigma + \frac{\eta + \alpha}{1 - \alpha}\right)(y_t - y_t^f)$$

$$\searrow \qquad\qquad \swarrow$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t$$



where $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \left(\sigma + \frac{\eta+\alpha}{1-\alpha}\right)$

κ depends negatively on both θ and β, which get smaller if period length gets larger $\Rightarrow$ the GDP-inflation relationship steeper with longer horizon, like the medium-run AS vs. long-run AS in old models

Outline

Dynamic IS curve

From the Euler equation and goods market equilibrium

$\rho = -\ln\beta$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(\underbrace{i_t - E_t \pi_{t+1}}_{r_t} - \rho)$$

while in the flexible price model:

$$y_t^f = E_t y_{t+1}^f - \frac{1}{\sigma}(r_t^f - \rho)$$

Take a difference $\Rightarrow$ **dynamic IS equation** in terms of the output gap $\tilde{y}_t$

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - r_t^f)$$

Outline

Closing the model: monetary policy rule

- Output gap $\tilde{y}_t$ and inflation π_t are linked through The New Keynesian Phillips Curve and the dynamic IS equation
- However, the **nominal interest rate is not yet determined by anything**

$\rightarrow$ Taylor Rule necessary to solve the model

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \upsilon_t,$$

with reaction coefficients $\phi_\pi, \phi_y > 0$ and an exogenous component υ_t - an "arbitrary" part of monetary policy is random and persistent

$$\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \varepsilon_t^\upsilon, \text{ where } \rho_\upsilon \in [0,1)$$

$\Rightarrow$ ε_t^υ is a new shock – **monetary policy shock**

Complete model

$$\tilde{y}_t = E_t\tilde{y}_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1} - r_t^f) \qquad \text{(Dynamic IS)}$$

$$r_t^f = \rho + \gamma(E_t a_{t+1} - a_t) \qquad \text{(Flex. price real interest)}$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa\tilde{y}_t \qquad \text{(New Keynesian Phillips Curve)}$$

$$i_t = \rho + \phi_\pi\pi_t + \phi_y\tilde{y}_t + \upsilon_t \qquad \text{(Taylor Rule)}$$

$$\upsilon_t = \rho_\upsilon\upsilon_{t-1} + \varepsilon_t^\upsilon \qquad \text{(Exogenous part of interest)}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \qquad \text{(Exogenous productivity)}$$

With $\rho = -\ln\beta$, $\gamma = \sigma\frac{1+\eta}{\sigma(1-\alpha)+\eta+\alpha}$,
$\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\left(\sigma + \frac{\eta+\alpha}{1-\alpha}\right)$

Complete model – dynamics

- two dynamic endogenous variables only: $\tilde{y}_t$, π_t
- both are **jump variables** (not pre-determined)
  $\Rightarrow$ what does Blanchard-Kahn condition require?
- two dynamic equilibrium equations: NKPC and Dynamic IS

Linear state-space form:

$$\begin{bmatrix} E_t \tilde{y}_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = A \begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} + B(r_t^f - \rho - \upsilon_t)$$

Outline

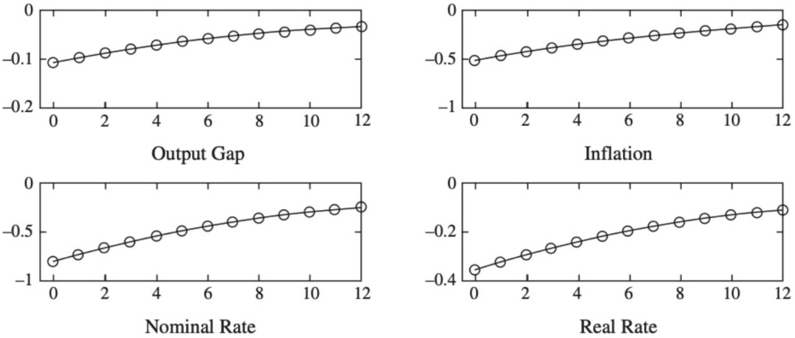
Calibration

Period utility $u(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}$;
production $Y_t(i) = A_t N(i)_t^{1-\alpha}$

- $\beta = 0.99 \rightarrow$ s.s. real interest
- Intertemporal elsticity of substitution $\sigma = 1$
- Frisch elasticity of labor supply $\eta = 1$
- Production function curvature $1 - \alpha = 2/3$
- Elasticity of substitution $\varepsilon = 6$
- Probability of sticky price $\theta = 2/3 \rightarrow$ average price duration
- Taylor rule $\phi_\pi = 1.5$, $\phi_y = 0.5/4 \rightarrow$ active; Blanchard-Kahn holds

Productivity shock: impulse responses

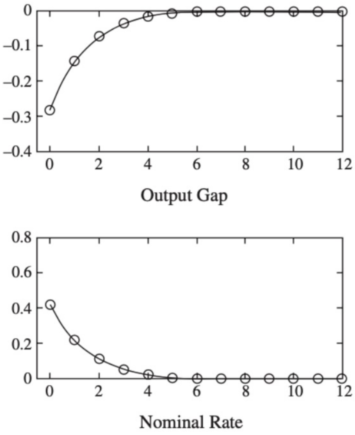
A positive shock of A_t:



Source. Galí (2008), Figure 3.1.

Monetary policy shock: impulse responses

Contractionary monetary policy shock on i_t ($\varepsilon_t^{\upsilon} > 0$)



Source. Galí (2008), Figure 3.1.

Consistent with IS-TR and AD-AS