Macroeconomics
Lecture 4 – Consumption, Savings and Balance of Payments

Ilya Eryzhenskiy

PSME Panthéon-Sorbonne Master in Economics

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Overview

Outline

Stocks and flows

So far, we have only dealt with **flow** variables: what happens **over a given period**. GDP is a flow!

We now introduce **stock** variables: what gets accumulated over time. We sometimes denote $Stock_t$ the stock at **end of period** and sometimes stock **at beginning of period**:

- ▷ End-of-period notation $\Rightarrow Stock_{t+1} = Stock_t + Flow_{t+1}$
- ▷ Beginning-of-period notation $\Rightarrow Stock_{t+1} = Stock_t + Flow_t$

Timing of stock variables: savings

- Saving**s** are a **stock**. Following the GLS[1] textbook, we denote by S_t the savings at **end** of period t, measured in **units of consumption good**
  - savings are part of wealth, and can be composed of different assets
  - negative savings are **debt**

- Saving (no "s") is a **flow**, what is saved over a period. $S_{t+1} - S_t$ is saving of period $t+1$
  - negative saving is **borrowing** or **dissaving**

[1] Garin, Lester, Sims. See reading material.

Consumption problem: budget constraints

Consider a consumer living for two periods t and $t+1$, with no savings initially ($S_{t-1} = 0$)

She has real income (i.e. measured in units of consumption good) Y_t and Y_{t+1} that can be used for consumption and saving

Savings from period t yield a real interest rate r_t in period $t+1$

We get the **budget constraints** of the two periods:

$$\begin{aligned} C_t + S_t &\leq Y_t \\ C_{t+1} + S_{t+1} &\leq Y_{t+1} + (1 + r_t) S_t \end{aligned}$$

Obtaining the inter-temporal budget constraint

Two simplifications of the budget constraints:

1. income is never wasted $\Rightarrow$ constraints verified as equalities
2. end of $t+1$ is "end of life" and there is no motive to leave wealth behind $\Rightarrow S_{t+1} = 0$

Can then obtain one equation instead of two inequalities:

$$\begin{cases} C_t + S_t = Y_t \\ C_{t+1} = Y_{t+1} + (1+r)S_t \end{cases}$$

$$\Leftrightarrow \begin{cases} S_t = Y_t - C_t \\ C_{t+1} = Y_{t+1} + (1+r)(Y_t - C_t) \end{cases}$$

$\Rightarrow \; C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t}$, the **inter-temporal budget constraint**

Inter-temporal budget constraint: interpretation

$$C_t + \frac{C_{t+1}}{1+r_t} = Y_t + \frac{Y_{t+1}}{1+r_t} \quad \Leftrightarrow$$

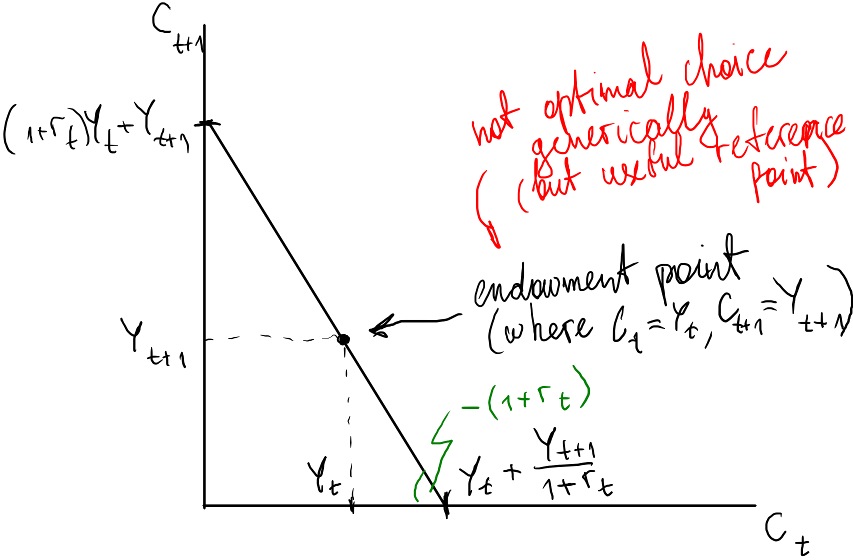
$$C_t + \underbrace{\frac{1}{1+r_t}}_{\text{relative price of } C_{t+1}} C_{t+1} = \underbrace{Y_t + \frac{Y_{t+1}}{1+r_t}}_{\textbf{present value} \text{ of } \textbf{lifetime income}}$$

Both future consumption and future income have lower value in the budget constraint ($\frac{1}{1+r_t} < 1$). Why so?

- Earning 1 unit today means consumption of 1 today, or consumption of $1 + r_t$ in future, if saved $\Rightarrow C_{t+1}$ "cheaper" than C_t
- Earning 1 tomorrow means consuming 1 tomorrow or $\frac{1}{1+r_t}$ today, if borrowed today $\Rightarrow$ future income "less valuable"

Inter-temporal budget constraint: graph

Use form $C_{t+1} = (1 + r_t)Y_t + Y_{t+1} - (1 + r_t)C_t$ for budget line:



Utility maximization

The consumer maximizes lifetime utility, which is a discounted sum of **instantaneous utilities**

$$U(C_t, C_{t+1}) = u(C_t) + \beta u(C_{t+1})$$

where $u(\cdot)$ is instantaneous utility function with $u' > 0$, $u'' < 0$;
β is the **discount factor**, or degree of **patience**

Concave instanteneous utility ($u'' < 0$) $\Rightarrow$ concave U $\Rightarrow$ a unique maximum on a convex budget set

$\Rightarrow$ only need **First Order Conditions** when maximizing utility

Intertemporal consumption choice: analytical solution

Use the **Lagrangian** $U(C_t, C_{t+1}) + \lambda(Y_t + \frac{Y_{t+1}}{1+r_t} - C_t - \frac{C_{t+1}}{1+r_t})$

$$\mathcal{L} = u(C_t) + \beta u(C_{t+1}) + \lambda\left(Y_t + \frac{Y_{t+1}}{1+r_t} - C_t - \frac{C_{t+1}}{1+r_t}\right)$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = u'(C_t) - \lambda = 0 \iff u'(C_t) = \lambda$$

$$\frac{\partial \mathcal{L}}{\partial C_{t+1}} = \beta u'(C_{t+1}) - \frac{1}{1+r_t}\lambda = 0 \iff \beta u'(C_{t+1}) = \frac{\lambda}{1+r_t}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = Y_t + \frac{Y_{t+1}}{1+r_t} - C_t - \frac{C_{t+1}}{1+r_t} = 0$$

First Order Conditions (FOC)

$$\Downarrow$$

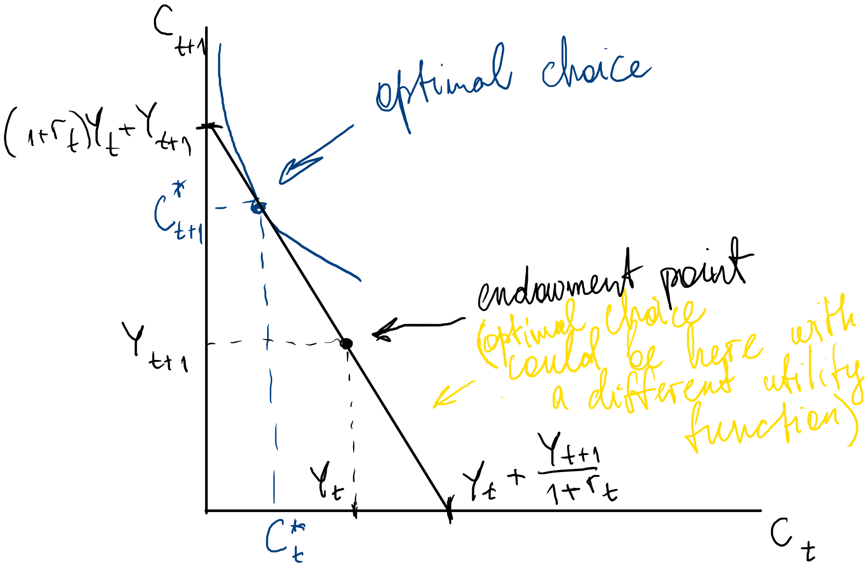
$$\underbrace{\frac{u'(C_t)}{\beta u'(C_{t+1})}}_{MRS(C_t, C_{t+1})} = \underbrace{1 + r_t}_{\text{price ratio}}$$

Euler equation

First order conditions of C_t and C_{t+1} combined $\Rightarrow$
$\frac{u'(C_t)}{\beta u'(C_{t+1})} = 1 + r_t$, the **Euler equation**

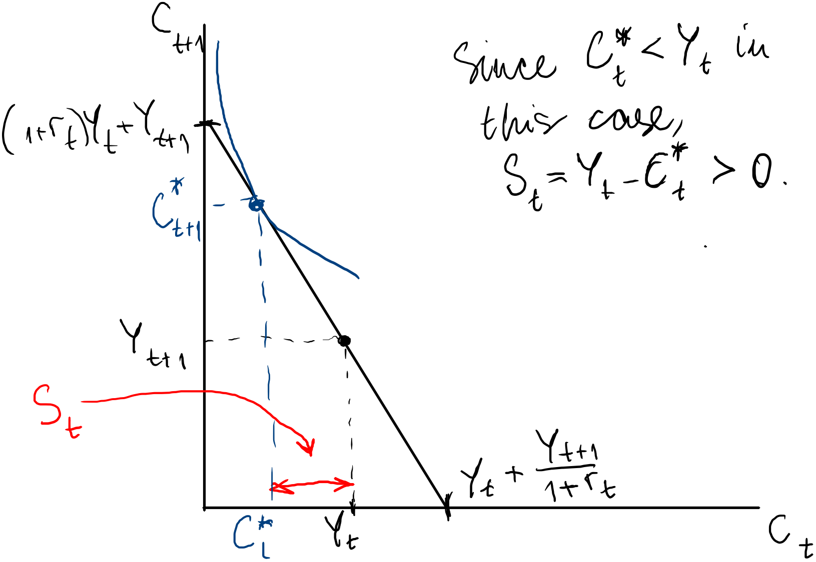
A familiar equality of **marginal rate of substitution** and price ratio (remember, price of C_{t+1} is $\frac{1}{1+r_t}$ if price of C_t taken as 1)

Inter-temporal consumption choice: graphical solution



Graphical analysis: savings

$C_t < Y_t \Rightarrow S_t > 0$ (because $S_{t-1} = 0$ is assumed)



Since $C^*_t < Y_t$ in this case, $S_t = Y_t - C^*_t > 0$.

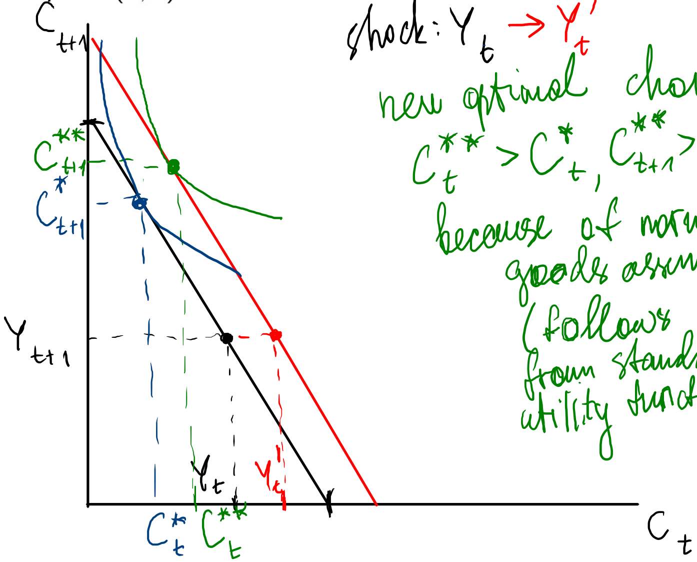
Graphical analysis: current income shock

If C_t, C_{t+1} both **normal goods**, rise of Y_t leads to rise in both C_t and $C_{t+1} \Rightarrow mpc \in (0,1)$

shock: $Y_t \to Y'_t$

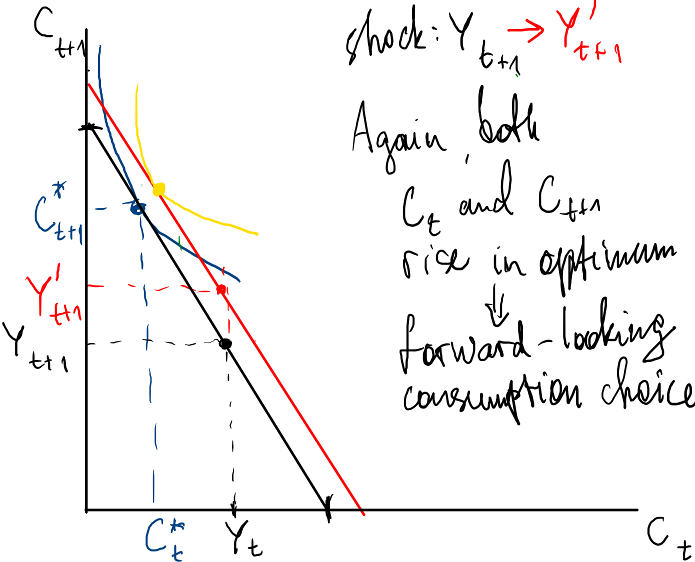
new optimal choice: $C_t^{**} > C_t^{*}$, $C_{t+1}^{**} > C_{t+1}^{*}$

because of normal goods assumption (follows from standard utility functions)



Graphical analysis: future income shock

Shock of Y_{t+1} also leads to a change in both C_t and C_{t+1}



shock: $Y_{t+1} \rightarrow Y'_{t+1}$

Again, both C_t and C_{t+1} rise in optimum

$\Downarrow$

forward-looking consumption choice

Outline

Balance of Payments – full picture

Balance of Payments is an accounting system for **cross-border flows** of an economy. The standard **double entry** accounting principle applies. Structure:

1. **Current account (CA)**:
   - ▷ **Trade Balance (TB)** a.k.a. **Primary Current Account**
   - ▷ **Income Balance**
   - ▷ Net Unilateral Transfers
2. Capital Account
3. **Financial Account**
4. Errors and Omissions

Current Account (CA): full vs. simplified

Current account records international **flows of goods and services** and **incomes from factors of production**.

- Exports minus imports: **Trade Balance**
- Incomes from factors of production: **Income Balance**:
  1. Capital (financial) incomes: Financial incomes of residents abroad **minus** financial incomes of non-residents
  2. Labor incomes: Wages of resident workers abroad **minus** wages of non-residents in the economy
- non-markets flows of goods and services ("gifts"): **Net unilateral transfers**

Simplifying assumptions for course:

- No migration $\Rightarrow$ only **financial incomes** in **income balance**
- No unilateral transfers

Capital account

A financial analogue of unilateral transfers: changes in asset positions that are not due to purchase/sale of assets.

Examples: debt forgiveness, assets of migrants that change residence status.

Not to be confused with Financial Account (see below)!

Assumed **null** for rest of the course.

Financial Account (FA)

Records of changes in asset positions, or **capital flows**. These are financial/monetary transactions **underlying the flows of goods, services, and incomes**

Financial Account balance: change in foreign assets of residents(capital outflows)
minus
change in assets of non-residents in the economy – foreign liabilities of residents (capital inflows).

Sign convention (IMF): + for capital outflows, – for capital inflows.
Attenition the sign convention was opposite before mid-2010s!

Examples: purchase of foreign currency by household (+ FA), foreign direct investment received from abroad (– FA)

Double entry, BOP identity

Most transactions are recorded in CA and in FA. Such transactions **must** have the same sign in the two accounts

Examples of double entry:

1. Goods imported (– CA), payment in check (promise of payment/liability: – FA)
2. Interest on foreign deposit of resident (+ Income Balance in CA), (+ FA)

Balance of Payments Identity must hold:

$$\text{CA} + \underbrace{\text{Capital account} + \text{Errors and Omissions}}_{\text{assumed } 0} = \text{FA}$$

$\Rightarrow$ Net outflows of goods, inflows of incomes $\Leftrightarrow$ net **capital outflows**

Outline

International Investment Position

International Investment Position (IIP) a.k.a. *Net* International Investment Position (NIIP) → **stock** variable corresponding to the flows of the Financial Account (FA)

IIP is assets held by residents abroad **minus** assets held by non-residents in economy
Denote the IIP at **beginning of period** t by IIP_t. [2]

The stock-flow relationship is then:

$$IIP_{t+1} = IIP_t + FA_t + \underbrace{\Delta \text{Asset Valuations}_t}_{\text{assumed 0}}$$

We ignore valuation changes, because we do not model asset markets

[2] The Grohe-Schmitt Uribe Woodford textbook uses a variable B_t

IIP and Trade Balance

Simplified BOP identity is $CA_t = FA_t$ (no capital account, no errors and omissions)

Financial income is return on foreign assets minus return on foreign liabilities, which is $r \cdot IIP_t$ in our model, assuming a constant world interest rate r

Replace in the IPP dynamics formula (without valuation changes):

$$\begin{aligned} IIP_{t+1} &= IIP_t + CA_t \\ &= IIP_t + TB_t + \underbrace{r \cdot IIP_t}_{\text{Income Balance}} \\ \Leftrightarrow IIP_{t+1} &= (1+r)IIP_t + TB_t \end{aligned}$$

A 2-period small open economy

Assume an economy exists for 2 periods, t and $t+1$. It starts with an initial asset position IIP_t. Then, IIP dynamics depends on TB:

$$IIP_{t+1} = (1+r)IIP_t + TB_t$$
$$IIP_{t+2} = (1+r)IIP_{t+1} + TB_{t+1}$$

Country cannot have debts in last period ($IIP_{t+2} \geq 0$) nor hold assets in other countries ($IIP_{t+2} \geq 0$), so $IIP_{t+2} = 0$

One can obtain (**verify this**): $IIP_t = -\frac{TB_t}{1+r} - \frac{TB_{t+1}}{(1+r)^2}$

2-period economy: trade balance and solvency

$$IIP_t = -\frac{TB_t}{1+r} - \frac{TB_{t+1}}{(1+r)^2}$$

According to the initial value of International Investment Position, several scenarios for values of trade balance:

- If $IIP_t = 0$, then $TB_{t+1} = -(1+r)TB_t \Rightarrow TB_t$ and TB_{t+1} have opposite signs. A deficit in one period must be compensated by a surplus in the other
- If $IIP_t > 0$, it is possible to have a trade deficit in each period and remain solvent
- If $IIP_t < 0$, the economy may have to run trade surpluses every period to remain solvent. TB_t has bigger role for debt repayment than TB_{t+1} due to interest accumulation

Model for Balance of Payments and IIP

Recall the GDP decomposition (or desired demand equal to GDP):
$Y_t = C_t + I_t + G_t + TB_t$
Consider a simple model with $I_t = 0$, $G_t = 0$, then:

$$
\begin{aligned}
TB_t &= Y_t - C_t \\
CA_t &= TB_t + r \cdot IIP_t \\
&= \underbrace{Y_t + r \cdot IIP_t}_{\textbf{National Income}} - C_t
\end{aligned}
$$

In this simple model, trade balance is difference of GDP and consumption, while current account is difference of National Income and consumption
Can also use the formula in IIP dynamics:

$$
\begin{aligned}
IIP_{t+1} &= (1+r)IIP_t + TB_t \\
\Leftrightarrow IIP_{t+1} &= (1+r)IIP_t + Y_t - C_t
\end{aligned}
$$

Outline

A representative consumer

We now bring together the consumption problem and the BOP and IIP dynamics

Assume the GDP of the economy is an exogenous value perceived as income of **representative consumer**

The consumer has initial savings that correspond to the international investment position of the economy

Studying consumption choices, we will determine trade balance and international investment position dynamics of the economy

Resource constraint

Since the representative consumer is only constrained by resources of the economy, we call her budget constraint the **resource constraint**:

$$IIP_{t+1} = (1+r)IIP_t + Y_t - C_t$$
$$IIP_{t+2} = (1+r)IIP_{t+1} + Y_{t+1} - C_{t+1}$$

$IIP_{t+2} = 0$ as before, so inter-temporal resource constraint is obtained (by eliminating IIP_{t+1}):

$$C_t + \frac{C_{t+1}}{1+r} = (1+r)IIP_t + Y_t + \frac{Y_{t+1}}{1+r}$$

It can be used as a constraint in utility maximization of representative consumer

2-period consumption in open economy: graph

IIP_{t+1} can be obtained as distance between $Y_t + (1 + r)IIP_t$ and C_t