Macroeconomics
Lecture 2 — IS-TR, Introduction to dynamics

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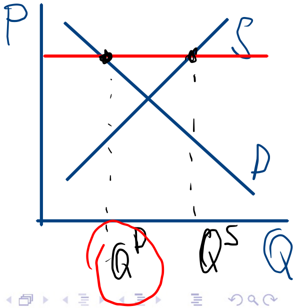
Price rigidity

Central **Keynesian assumption:** Prices do not adjust immediately

- Price rigidity associated with time horizon:
  - very short term — fixed prices (extreme case)
  - short term — *sticky* prices (slow moving)
  - medium or long term — flexible prices
- Assuming demand is insufficient under current prices $\Rightarrow$ supply determines equilibrium
  - Why? Recall micro equilibrium diagram

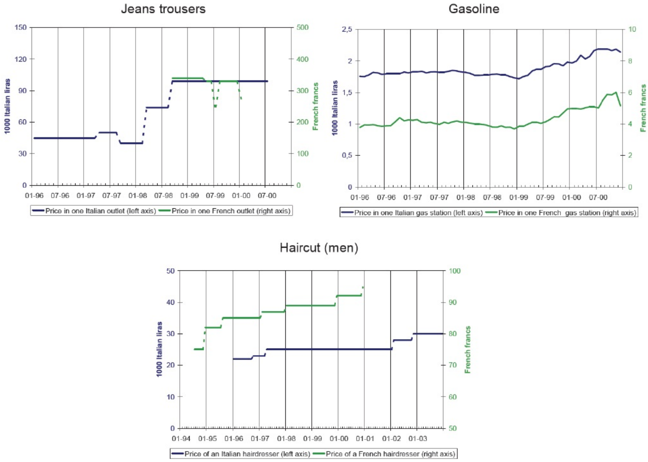


How reasonable is price stickiness?

- Answer: quite reasonable, according to data.

Price rigidities: data

- Evidence from the euro area points to sticky prices



Note. Actual examples of trajectories, extracted from the French and Italian CPI databases. The dotted lines indicate events of price changes. **Source.** Dhyne et al (2005) 'Price setting in the euro area: Some stylized facts from individual consumer price data', Figure 1.

Aggregate demand (closed economy)

$$\underbrace{Y}_{\text{aggregate supply}} = \underbrace{C + I + G}_{\text{aggregate demand}}$$

- An equilibrium condition, rather than a simple decomposition of GDP (that has the same equation!)
- In balance **by assumption**. *Why?*
  - What if aggr. supply $<$ aggr. demand ?
  - price level could adjust ($P \uparrow$)
  - not here, $P = const$ by assumption: aggr. demand *determines* aggr. supply
  - $\Rightarrow$ supply responds to shifts in demand as firms can always accumulate/decumulate **invetories**
  - **Keynesian** demand-driven equilibrium

Aggregate demand

Consumption function

$$C = C(\underset{+}{\Omega}, Y \underset{+}{-} \bar{T}) \tag{11.2}$$

- wealth, (Ω)
- disposable income, $(Y - \bar{T})$, a bar denotes const. variable

Investment function

$$I = I(\underset{+}{q}, \underset{-}{r}) \tag{11.3}$$

$\underset{+}{Y}$

$q(\underset{-}{r}, \underset{+}{\Delta Y})$

- Tobin's q $\left(q = \frac{\text{market value of installed capital}}{\text{replacement cost of capital}}\right)$
- real interest rate, (r)

Goods market equilibrium

Desired demand function

$$DD = C(\underset{+}{\Omega}, \underset{+}{Y - \bar{T}}) + I(\underset{+}{q}, \underset{-}{r}) + G \tag{1}$$

- Assumptions: $r = i - \pi^e$
- (1) r and G exogenous
- (2) goods market is in **equilibrium** (supply=demand)

$$Y = DD(Y, r...)$$

- $Y \equiv$ 'equilibrium GDP'
- This need not be the case!
- What if exogenous variables change?

Goods market equilibrium

Example 1: excess supply

How does a change in Y affect DD?

- What happens if Y' increases by 1 EUR, such that $Y' > Y$?
  - $C \uparrow$
  - ~~The effect on C dominates~~
  - $\rightarrow$ an increase in DD by less then 1 EUR
  - $\Rightarrow Y' > DD'$ **(excess supply)**
- **Dynamic adjustment mechanism**: goods will be stored (*inventories*), future production will be reduced
- ... reductions in income until $Y = DD$ holds again

Discussion

- So far, a number of variables exogenous: P, G, T, Ω, q, $\cancel{Y^*}$
- Intended reduction in complexity, can be *endogenized* later.

Goods market equilibrium

Example 2: Keynesian multiplier

Consider: **Increase in public pending, ΔG**

- $DD\uparrow$, firms will produce more, ..., $Y\uparrow$
- **"Multiplier"**: *By how much does output change, ΔY?*
  - firms will increase production by ΔG
  - this generates new income, thus $C\uparrow$ (I too, if $Y\uparrow\Rightarrow q\uparrow$)
  - $\rightarrow$ $Y\uparrow, C\uparrow, Y\uparrow$
  - additional spending ΔC is smaller due to *leakages*, so $\Delta Y_2 < \Delta Y_1$
  - leakages (closed economy): *savings, taxes (if proportional)*
    - **marginal propensity to consume**, $0 < c < 1$

$C = a + c(Y - T)$

mpc

$$\Delta Y = \Delta G + c\Delta G + c^2\Delta G + ... + c^n\Delta G$$
$$= \underbrace{\frac{1}{1-c}}_{\text{multiplier}>1}\Delta G$$

using $1 + a + a^2 + ... + a^n + ... = 1/(1-a)$.

The *IS* curve

- IS stands for Investment=Saving
- Equivalent relationship to $Y = DD$
  - check it with $S = \underbrace{(Y - T - C)}_{\text{private savings}} + \underbrace{(T - G)}_{\text{gov't savings}}$
- assume $i \downarrow \Rightarrow I \downarrow$
- Also, wealth responds indirectly via q and stock prices
- $\Rightarrow$ **IS-curve**: **downward-sloping**, i.e. negative relationship between output and interest rates (given T, G)

$$Y = C(\underset{+}{\Omega}, Y-T) + I(q, r) + G$$

$$I(q, \underset{-}{i - \pi^e})$$

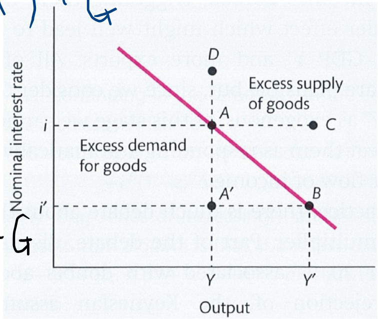
$$Y - C(\Omega, \underset{+}{Y-T}) = I(q, \underset{-}{i - \pi^e}) + G$$

$$I = S$$

$$I = (Y - \cancel{T} - C) + (\cancel{T} - G)$$

$$I = Y - C - G$$

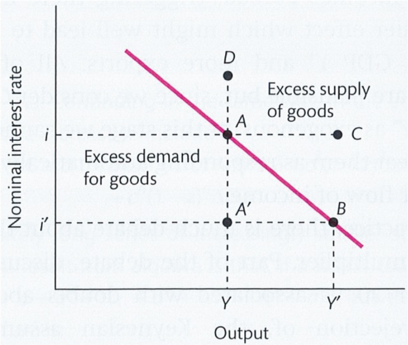
$$Y = C + I + G$$



Source. Burda and Wyplosz (2017), Figure 11.3.

Off the IS curve

- *IS* curve describes the **goods market equilibrium**
- Off the *IS* curve:
  - *excess supply* (point C, D)
  - *excess demand* (point A')
- Temporary deviations from the *IS* curve are possible
- Adjustment will bring output back to **equilibrium level** $Y = DD$
- *more in a couple of slides...*



Taylor rule and the TR curve

Nominal interest rates are set by the central bank as a function of the **inflation gap** and the **output gap** (**Taylor rule**):

$$i = \bar{i} + a(\pi - \bar{\pi}) + b\left(\frac{Y - \bar{Y}}{\bar{Y}}\right)$$

Assuming inflation at its target (for simplicity), this yields

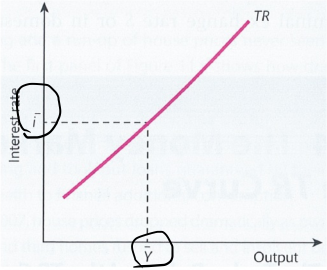
$$i = \bar{i} + b\left(\frac{Y - \bar{Y}}{\bar{Y}}\right)$$

$b > 0$

TR curve

- Monetary policy centered around the **natural rate of interest**, $\bar{i}$, i.e. the nominal interest rate the central bank sets when the economy is on trend output (no demand deficiency).
- (Simplified) Taylor rule: Describing pairs $\{Y, i\}$ consistent with monetary policy

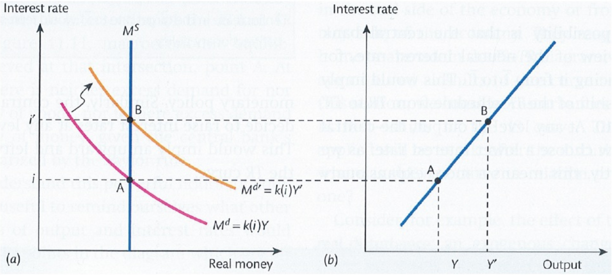
$\Rightarrow$ economy always on the TR curve as long as the central bank does its job



Source. Burda and Wyplosz (2017), Figure 11.6.

Alternative assumption: Targeting Money Supply (LM)

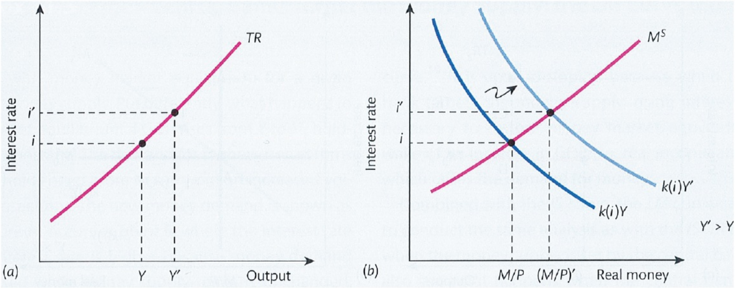
- During the 1980s, monetary targeting as main policy tool (M. Friedman)
- Gives rise to the **LM curve** (Panel b) ($\Rightarrow$ IS-LM model of J.R. Hicks)
- LM curve (Panel b) upward sloping: pairs $\{i, Y\}$ for fixed supply of money M^S



Note. Panel (a) denotes the money market, the vertical M^S line describes the central bank decision. Each money demand curve M^d corresponds to a level of output. Panel (b) **Source.** Burda and Wyplosz (2017), Figure 11.9.

Money market equilibrium with the TR curve

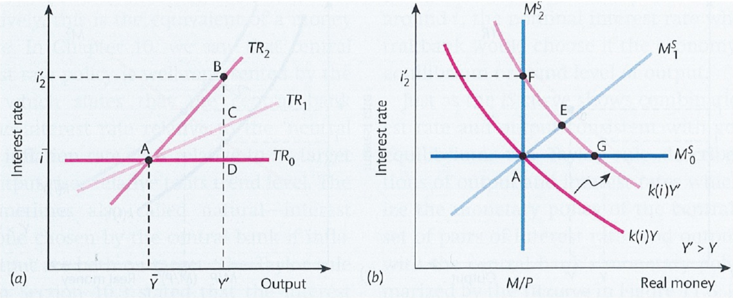
- Expansion in GDP leads to higher interest rates (panel a) and higher demand for real money balances $(M/P)'$ (panel b).
- **Money market equilibrium** implies that the central bank provides this additional money in the form of reserves, which is consistent with i given by the Taylor rule (M^S).



Source. Burda and Wyplosz (2017), Figure 11.7.

Slope of the TR curve

- *How strongly does a central bank react to the output gap?*
- The coefficient b in the Taylor rule captures the response.
  - TR_1: standard case
  - TR_0: perfectly elastic supply of money
  - TR_2: extreme case, implying strong fluctuations in i (equivalent to fixing the money supply M/P)

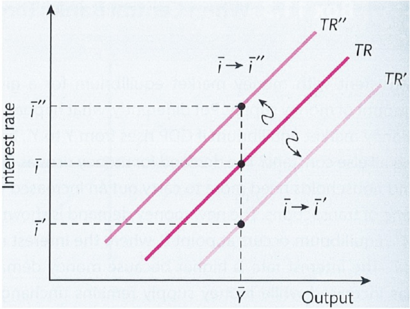


Source. Burda and Wyplosz (2017), Figure 11.8.

Shift of the TR curve

Interest rate given by the simplified Taylor rule
$i = \bar{i} + b\left(Y - \bar{Y}\right)/\bar{Y}$

- Changes in output lead to **movements along the TR curve**
- Changes in the degree of 'leaning against the wind' (coef. b) lead to a **rotation** of the TR curve (cf. Fig. 11.8)
- Changes in the natural rate of interest lead to a **shift in the TR curve** ($\bar{i} = \bar{r} + \bar{\pi}$), $\bar{r}$, natural real rate and $\bar{\pi}$ inflation target



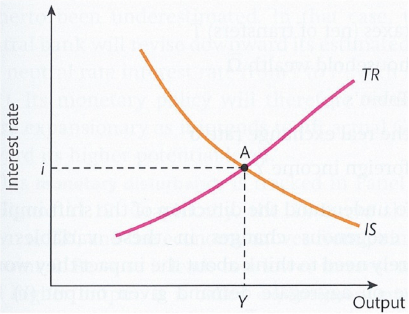
Source. Burda and Wyplosz (2017), Figure 11.10.

Macroeconomic equilibrium in the IS-TR model

- Under which conditions are **goods markets** and **money markets** in equilibrium *at the same time*?
- What are the effects of changes in **exogenous influences** on output and interest rates?

Macroeconomic equilibrium

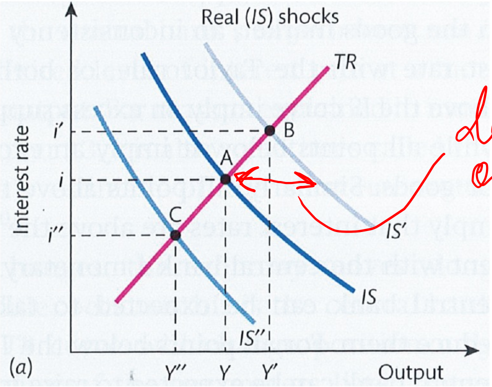
- For goods and money markets to be in equilibrium, economy should be at the **intersection** of the IS and the TR curves, (point A)
- $\rightarrow$ no excess supply or demand for goods or money!



Source. Burda and Wyplosz (2017), Figure 11.11.

Real disturbances: Shifts of the IS curve

- Central question: after a disturbance, where is the *new curve* in relation to the *original one*?
- Consider an increase in government spending, $\bar{G} < \bar{G}'$,
- New equilibrium: IS', point B



depends on Keynesian multiplier

Source. Burda and Wyplosz (2017), Figure 11.12.

Real disturbances: Propagation of the 'shock'

Thinking of cycles in this simple framework:

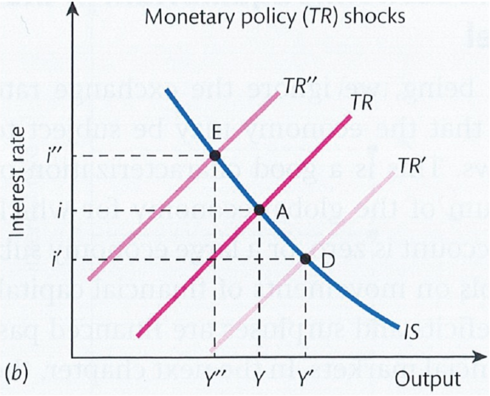
- Assume a positive income shock
- Higher demand leads to higher output
- Keynesian multiplier: higher output = higher income
- Additional income leads to higher demand, thus higher output (leakages):
- *output* $\rightarrow$ *income* $\rightarrow$ *demand* $\rightarrow$ *output* $\rightarrow$ ...
- Where does it end?
  - Leakages: saving, (proportional) taxes...
  - Monetary policy: interest rate i moves along the TR curve upward, dampening demand and output

Sources of "shocks":

- lump-sum taxes $\bar{T}$
- household wealth $\bar{\Omega}$
- Tobin's q

Monetary policy disturbances: Shifts of the TR curve

- Assume a downward revision of $\bar{i}$
  - $\Rightarrow$ downward shift of the TR curve
- New position: TR', equilibrium at point D
  - $\Rightarrow$ **expansionary** monetary policy



Source. Burda and Wyplosz (2017), Figure 11.12.

Monetary policy disturbances: Propagation of the 'shock'

- Lower interest rates lead to higher demand (e.g. Tobin's q)
- higher investment spending leads to more output and higher income
- Keynesian multiplier (net of leakages):
- $output \rightarrow income \rightarrow demand \rightarrow output \rightarrow ...$
- The nominal rate will change by less than the revision of the natural rate, i.e. $i - i' < \bar{i} - \bar{i}' > 0$, due to the **endogenous output response** ($Y \uparrow$) which leads to a muted response in the nominal rate due to the **Taylor rule**.

Summary: How to use the IS-TR framework

- Sometimes, policymakers might ask several questions at the same time:
  - a boost in public spending and a **tax cut**
  - **fall in natural rate** and an **increase in stock prices**
- The **joint effect** has ambiguous sign!
- Follow the steps:
  - **Which curve** is affected by the disturbance?
  - What is the **equilibrium of the new IS and TR curves**

Summary: Macroeconomic equilibrium

- **General equilibrium**: all markets clear simultaneously
- Keynesian assumption: **prices are rigid/sticky**
- **Multiplier** in response to an exogenous increase in demand
- *IS* curve: GDP levels and interest rates compatible with equilibrium in the goods and services market
- *TR* curve: Description of central bank setting of i to stabilize inflation around target and output around potential.
- **Macroeconomic equilibrium:** intersection of *IS* with *TR* curve

Multiplier-Accelerator model

- A simple Keynesian model of the goods market
- **NOT** the way we think about cycles today!
- Motivation is methodological: look at a **dynamic** equilibrium

Three equations:

- Consumption depending on **last period** income
- Investment depending on **growth of** income in **last period** — the **accelerator** assumption
- Goods market equilibrium

Multiplier-Accelerator accelerator model

Variables

- $\{C_t\}$ — sequence of levels of aggregate consumption , main endogenous variable in the model.
- $\{I_t\}$ — sequence of rates of investment, another key endogenous variable.
- $\{Y_t\}$ — sequence of levels of national income, yet another endogenous variable.
- $\{G_t\}$ — sequence of levels of government expenditures. Exogenous, assumed constant: $G_t = G$ for all t.

Multiplier-Accelerator accelerator model

Model structure

The model combines the consumption function

$$C_t = aY_{t-1} + \gamma \tag{1}$$

with the investment accelerator

$I(r, q)$

$$I_t = b(Y_{t-1} - Y_{t-2}) \tag{2}$$

$q(\Delta Y)$

and the goods market equilibrium

$$Y_t = C_t + I_t + G_t \tag{3}$$

- The parameter a is people's *marginal propensity to consume* out of income - equation (1) asserts that people consume a fraction of $a \in (0, 1)$ of each additional dollar of income.
- The parameter $b > 0$ is the investment accelerator coefficient - equation (2) asserts that people invest in physical capital when income is increasing and disinvest when it is decreasing.

Solution: a difference equation

Combining the three equations:

$$Y_t = (a+b)Y_{t-1} - bY_{t-2} + (\gamma + G_t)$$

Defining new coefficients gives compact form: assume $\rho_1 = (a+b)$ and $\rho_2 = -b$, then:

$$Y_t = \rho_1 Y_{t-1} + \rho_2 Y_{t-2} + (\gamma + G_t)$$

$$Y^{ss} = \rho_1 Y^{ss} + \rho_2 Y^{ss} + \gamma + \bar{G}$$

$$\Downarrow$$

$$Y^{ss} = \frac{\gamma + \bar{G}}{1 - \rho_1 - \rho_2}$$

Assuming initial values to generate Y_t for $t = 0, \ldots, T$:

$$Y_{-1} = \bar{Y}_{-1}, \quad Y_{-2} = \bar{Y}_{-2}$$

When solving numerically, set (a, b) so that starting from $(\bar{Y}_{-1}, \bar{Y}_{-2})$, Y_t converges to a **steady state**

Transform equation of order 2 in system of ord. 1:

$$F_t \equiv Y_{t+1} \Rightarrow Y_{t+2} = \rho_1 Y_{t+1} + \rho_2 Y_t + \gamma + \bar{G} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} F_{t+1} = \rho_1 F_t + \rho_2 Y_t + \gamma + \bar{G} \\ Y_{t+1} = F_t \end{cases}$$

Matrix form: $X_t \equiv \begin{bmatrix} F_t \\ Y_t \end{bmatrix} \Rightarrow X_{t+1} = A X_t + E$, where

$$A = \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix}; E = \begin{bmatrix} \gamma + \bar{G} \\ 0 \end{bmatrix}$$

Eigenvalues of A govern dynamics (see slide below).

Finding eigenvalues: $\det(A - \lambda I) = 0 \Leftrightarrow$

$$\Leftrightarrow \det \begin{pmatrix} \rho_1 - \lambda & \rho_2 \\ 1 & -\lambda \end{pmatrix} = 0 \Leftrightarrow (\rho_1 - \lambda)(-\lambda) - \rho_2 = 0$$

$$\Leftrightarrow \lambda^2 - \rho_1 \lambda - \rho_2 = 0$$

$\Rightarrow$ always 2 solutions λ_1, λ_2, as long as complex numbers admitted

A dynamic equilibrium in explicit form

With eigenvalues λ_1, λ_2, the dynamics of Y_t given by:

$$Y_t = \lambda_1^t c_1 + \lambda_2^t c_2$$

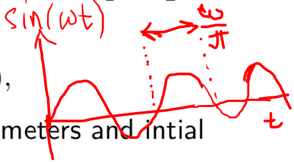
where c_1 and c_2 are constants on parameters and initial conditions. initial conditions and on ρ_1, ρ_2.

When the eigenvalues are complex, can represent them in polar form $\lambda_1 = re^{i\omega}$, $\lambda_2 = re^{-i\omega}$ and rewrite the solution as follows (not necessary to memorize!):

$$Y_t = (c_1 + c_2)r^t \cos(\omega t) + i(c_1 - c_2)r^t \sin(\omega t)$$

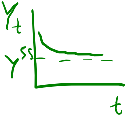
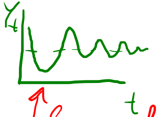
Parameters and intital conditions chosen such that $c_1 + c_2$ real, $c_1 - c_2$ complex, so Y_t real and has a form:

$$Y_t = 2vr^t \cos(\omega t + \theta),$$



with v, θ some constants depending on parameters and intial conditions.

Recall graphs of cos function $\Rightarrow$ output (as well as investment and consumption) oscillate around steady state.

Result can have shapes  

↑ Samuelson's version of business cycles

Intuition:

1) Consider model with $Y_t = C_t$ only

$$Y_t = a Y_{t-1} + \gamma \leftarrow \text{autoregression of degree 1: AR(1) (deterministic)}$$

Change of variable $Y_t^{GAP} \equiv Y_t - Y^{ss}$

$$\Rightarrow Y_t^{GAP} + Y^{ss} = a(Y_{t-1}^{GAP} + Y^{ss}) + \gamma$$

$$Y_t^{GAP} = a Y_{t-1}^{GAP} + (a-1)Y^{ss} + \gamma = 0 \text{ with } Y^{ss} = \frac{\gamma}{1-a}$$

$\Downarrow$

distance to s.s. always decreasing as $a < 1$. $(\bar{G} = 0)$

2) Accelerator of I_t makes Y_t „overshoot" Y^{ss}

$\Downarrow$

oscillations