Macroeconomics
Lecture 2 – IS-TR-IFM, Intro to Dynamics

Ilya Eryzhenskiy

PSME Panthéon-Sorbonne Master in Economics

Fall 2023

Overview

1. Exchange rates & goods market (IS); capital flows (IFM)
2. Fixed exchange rate regime
   - IS-(TR)-IFM equilibrium
3. Flexible exchange rate regime
   - (IS)-TR-IFM equilibrium
4. Introduction to Dynamics: Multiplier-Accelerator

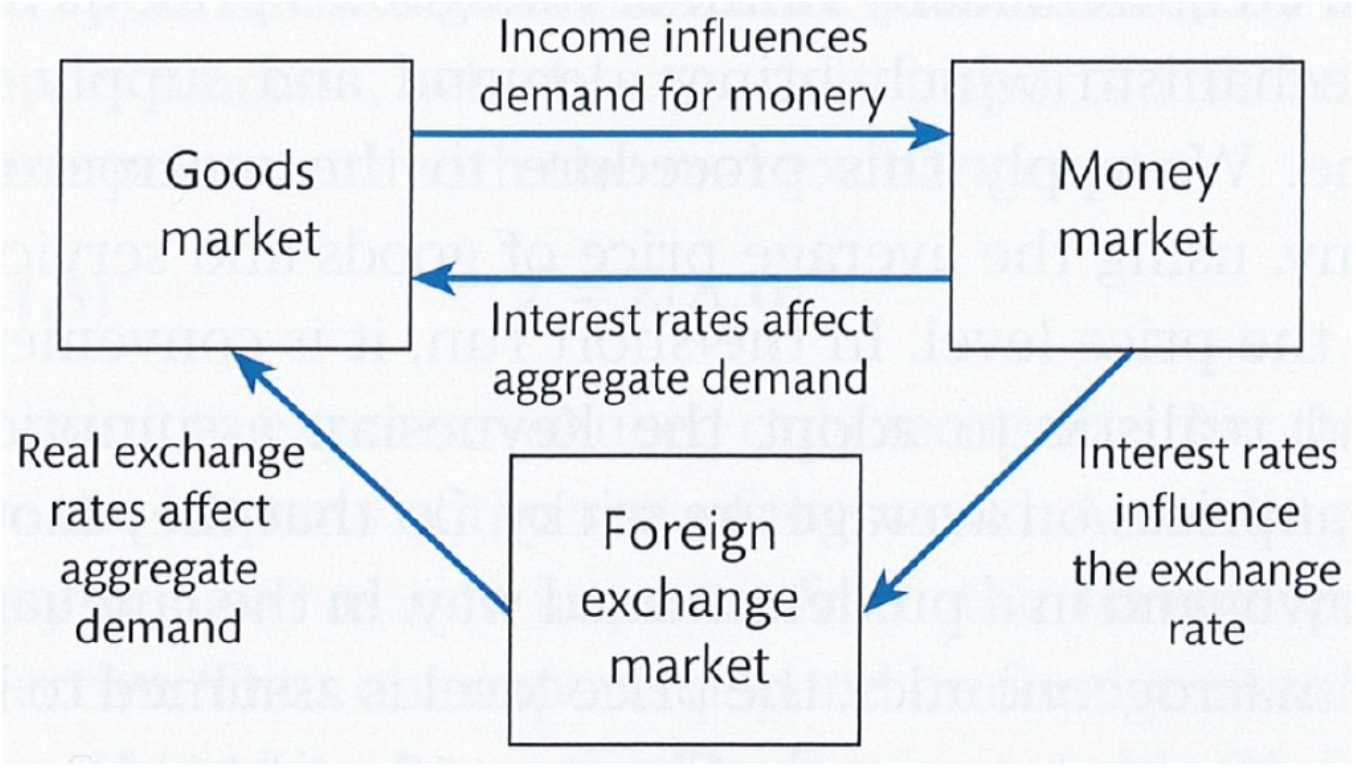
1. Exchange rates & goods market (IS); capital flows (IFM)

2. Fixed exchange rate regime
   - IS-(TR)-IFM equilibrium

3. Flexible exchange rate regime
   - (IS)-TR-IFM equilibrium

4. Introduction to Dynamics: Multiplier-Accelerator

Foreign exchange as a market



Source. Burda and Wyplosz (2017), Figure 10.2.

Lecture overview: opening up IS-TR

We already introduced **trade balance**, but assumed it is null in equilibrium. This lecture:

1. TB is nonzero in equilibrium and it is driven by **real exchange rate**
    - ▷ Real exchange rate **shifts** IS
2. Economy also open to **capital flows**
    - ▷ **International Financial Markets (IFM) line** $\Rightarrow$ IS-TR-IFM model

Equilibrium analysis must be done separately for **fixed** and **flexible** exchange rate regimes.

Exchange rates – quick overview

Nominal vs. **real** exchange rates:

- **Nominal** exchange rate – number of units of one currency per unit of another $\Leftrightarrow$ **relative price of monies**
  - Can be expressed in units of foreign currency per unit of domestic or vice versa
  - Example: EUR-JPY exchange rate is 140 JPY for 1 EUR
- **Real** exchange rate (RER) - ratio of foreign consumption basket value to domestic consumption basket value $\Leftrightarrow$ **relative price of consumption**

Using S as **nominal** exchange rate (number of units of foreign currency per unit of domestic), P the domestic and P^* the foreign price level, define the **real** exchange rate σ as:

$$\sigma = \frac{S \cdot P}{P^*}$$

1. Exchange rates & goods market (IS); capital flows (IFM)

2. Fixed exchange rate regime
   - IS-(TR)-IFM equilibrium

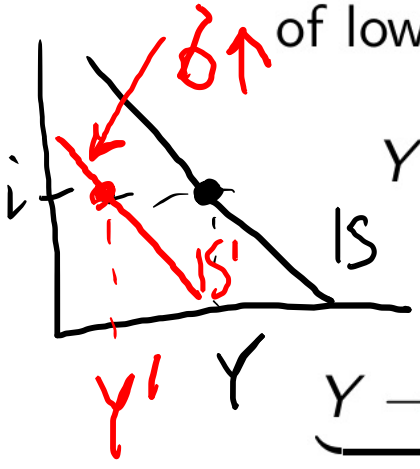
3. Flexible exchange rate regime
   - (IS)-TR-IFM equilibrium

4. Introduction to Dynamics: Multiplier-Accelerator

Open economy IS

How does a change of real exchange rate σ affect IS?

- RER appreciation $\Rightarrow$ more imports, less exports $\Rightarrow$ for a given interest rate, goods market equilibrium has lower Y because of lower aggregate demand



$$Y = C(\underset{+}{\Omega}, \underset{+}{Y - T}) + I(\underset{-}{r}, \underset{+}{q}) + G + TB(\underset{-}{Y}, \underset{+}{Y^*}, \underset{-}{\sigma})$$

$$\underbrace{Y - C(\Omega, Y - T) - TB(Y, Y^*, \sigma)}_{\text{increases in } Y,\ \sigma} = I(\underset{-}{i - \pi^e}, q) + G$$

- Mathematics: left hand side increasing in both Y and σ $\Rightarrow$ for a fixed right hand side, $Y \downarrow$ when $\sigma \uparrow$ for equation to hold

$\Rightarrow$ when $\sigma \uparrow$, $Y \downarrow$ for fixed i $\Rightarrow$ **IS shifts left** in (Y, i) space.

What are capital flows?

Here, a financial notion of capital is used: including any assets + currency.

Capital flows are cross-country operations with capital. They are registered in the **financial account** of Balance of Payments. Examples of capital flows:

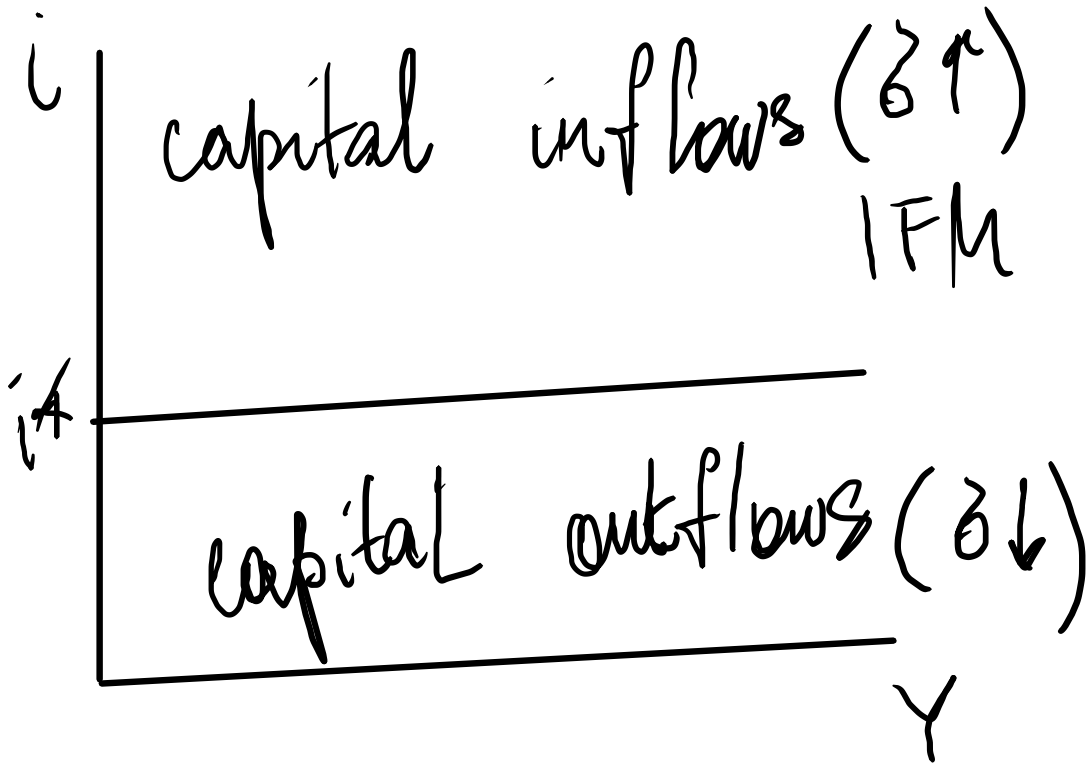
- Foreign exchange market
- Lending/borrowing with non-resident
- Foreign direct investment

International financial market: capital flows

We assume a **small open economy** with free capital flows (no *capital controls*):

- Recall small open economy definition: country does not influence international prices
- ... in particular, international nominal interest, i^*
- free capital flows $\Rightarrow$ no **arbitrage** possibilities between domestic and foreign assets, **returns are equalized**: $i = i^*$

$\Rightarrow$ a new condition in the (Y, i) space: a horizontal IFM line


i
capital inflows (↑)
IFM
i*
capital outflows (↓)
Y

Economy off IFM line – exchange rate regime matters

What happens after capital inflows/outflows? What consequences for IS, TR?

$\rightarrow$ answers depend on **exchange rate regime**

- Under **fixed exchange rate regime**, CB must prevent changes in **nominal exchange rate**
  - Taylor Rule and money supply change such that $i = i^*$ again
- Under **flexible exchange rate regime**, central bank (CB) does not react, nominal (and real) exchange rate changes $\Rightarrow$ IS shifts
  - IS **less influenced by demand shocks** than in closed economy or under fixed exchange rate

1 Exchange rates & goods market (IS); capital flows (IFM)

2 Fixed exchange rate regime
- IS-(TR)-IFM equilibrium

3 Flexible exchange rate regime
- (IS)-TR-IFM equilibrium

4 Introduction to Dynamics: Multiplier-Accelerator

Fixed exchange rate

- Central bank fixes the nominal exchange rate: $S = \bar{S} \Rightarrow \sigma = \frac{\bar{S}P}{P^*}$
- This is done via **foreign exchange interventions** – purchase/sale of foreign currency by the CB
- Change of CB **assets** must be coupled with changes of **liabilities** – the money supply

Simplified structure of a central bank balance sheet:

$$\underbrace{\text{Domestic Credit} + \text{Foreign exchange reserves}}_{\text{Assets}} = \underbrace{\text{Money supply}}_{\text{Liabilities}}$$

Foreign exchange interventions change the reserves $\Rightarrow$ change in money supply

Fixed exchange rate, TR and IFM lines

- Loss of control over i because of capital flows: $i = i^*$; choice of money supply by the CB dictated by exchange rate. Taylor Rule is *de facto* given up

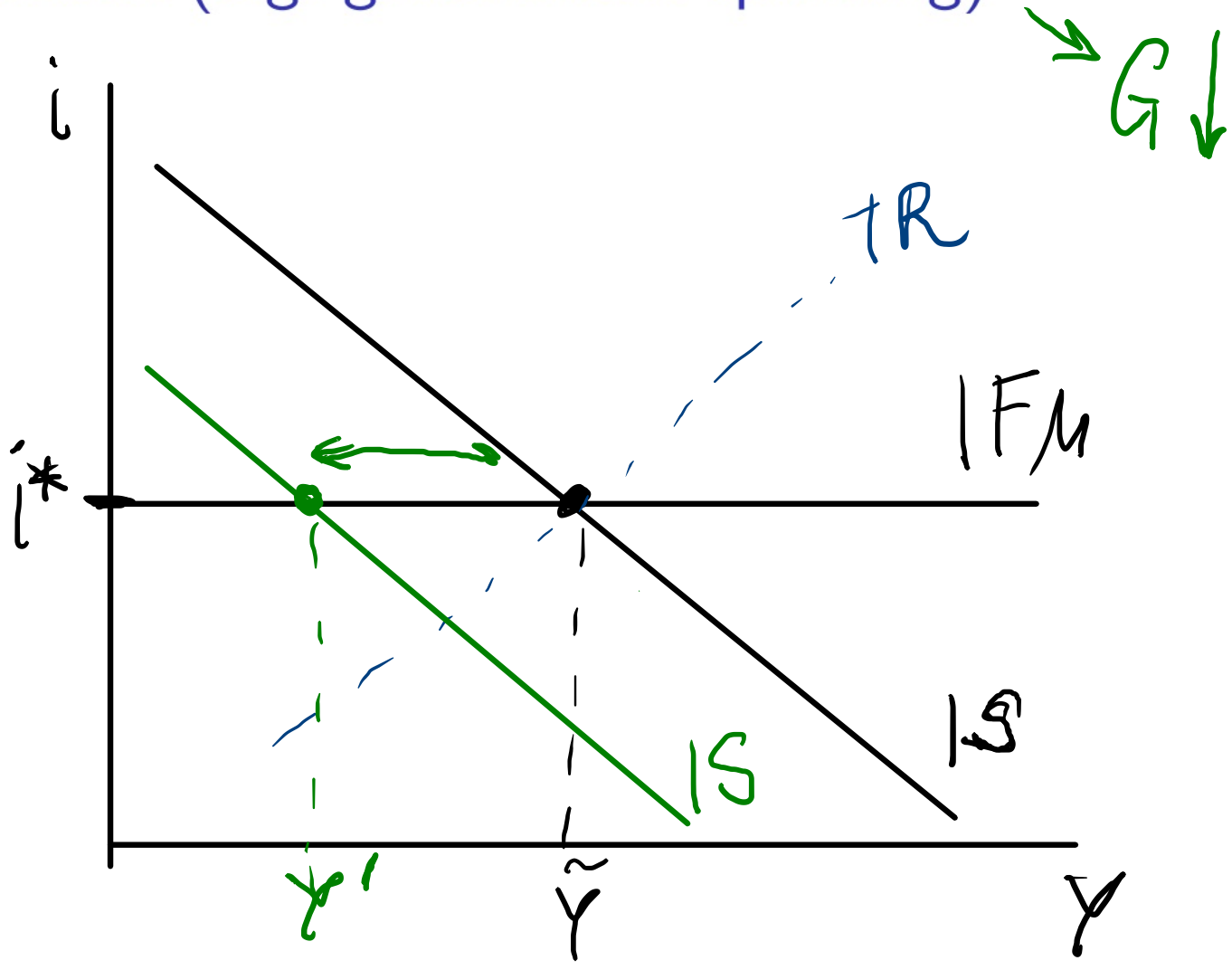
$\Rightarrow$ Graphically, TR is no longer determining equilibrium:

- The horizontal IFM line in (i, Y) space determines equilibrium instead of TR

- Behind the scenes, the foreign exchange interventions and corresponding changes in money supply make TR move wherever the new equilibrium is

We will consider different types of shocks in this IS-(TR)-IFM framework.

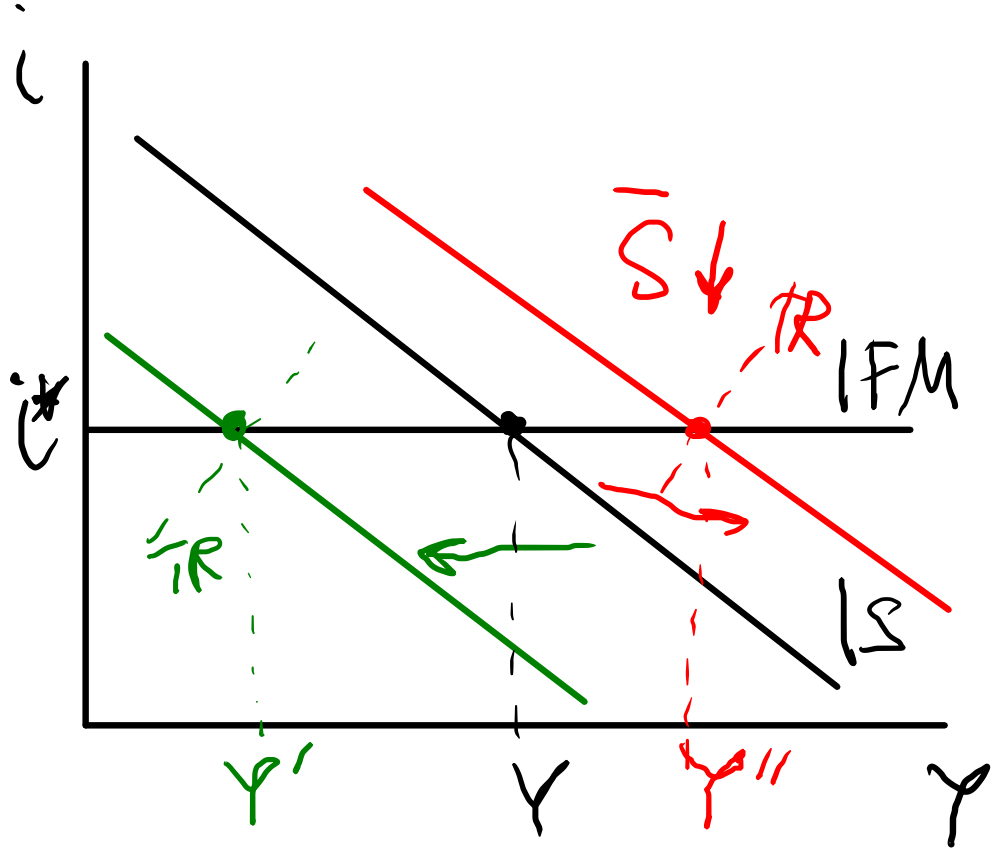
Demand shock (e.g. government spending)



Devaluations/revaluations: explanation

$\bar{S}\uparrow$

$\theta\uparrow = \frac{\bar{S}P}{P^*}$

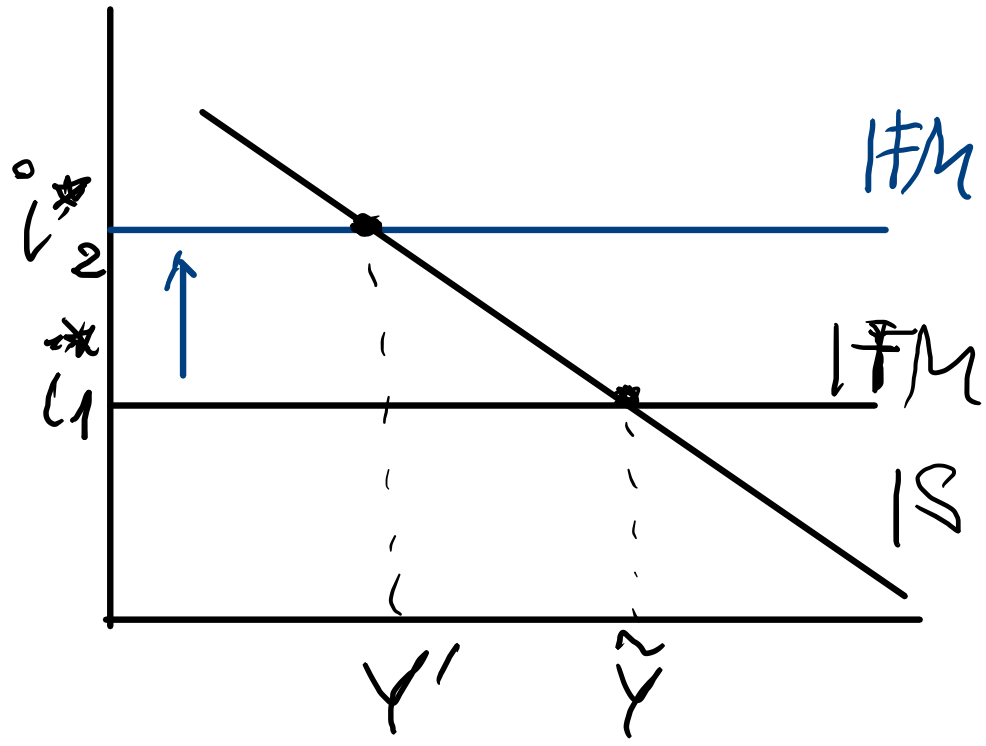


Devaluations/revaluations — changes of $\bar{S}$

- IS moves as explained before
- Central bank must adapt monetary policy – TR shifts
- Behind the scenes – central bank buys or sells foreign currency to keep S fixed – **foreign exchange interventions**

International financial shock – shift of IFM

An increase in world interest rate is contractionary for fixed exchange rate economies



Very relevant shock for 2022-2023: US rate went from 0.33 to over 5 p.p.

1. Exchange rates & goods market (IS); capital flows (IFM)

2. Fixed exchange rate regime
   - IS-(TR)-IFM equilibrium

3. Flexible exchange rate regime
   - (IS)-TR-IFM equilibrium

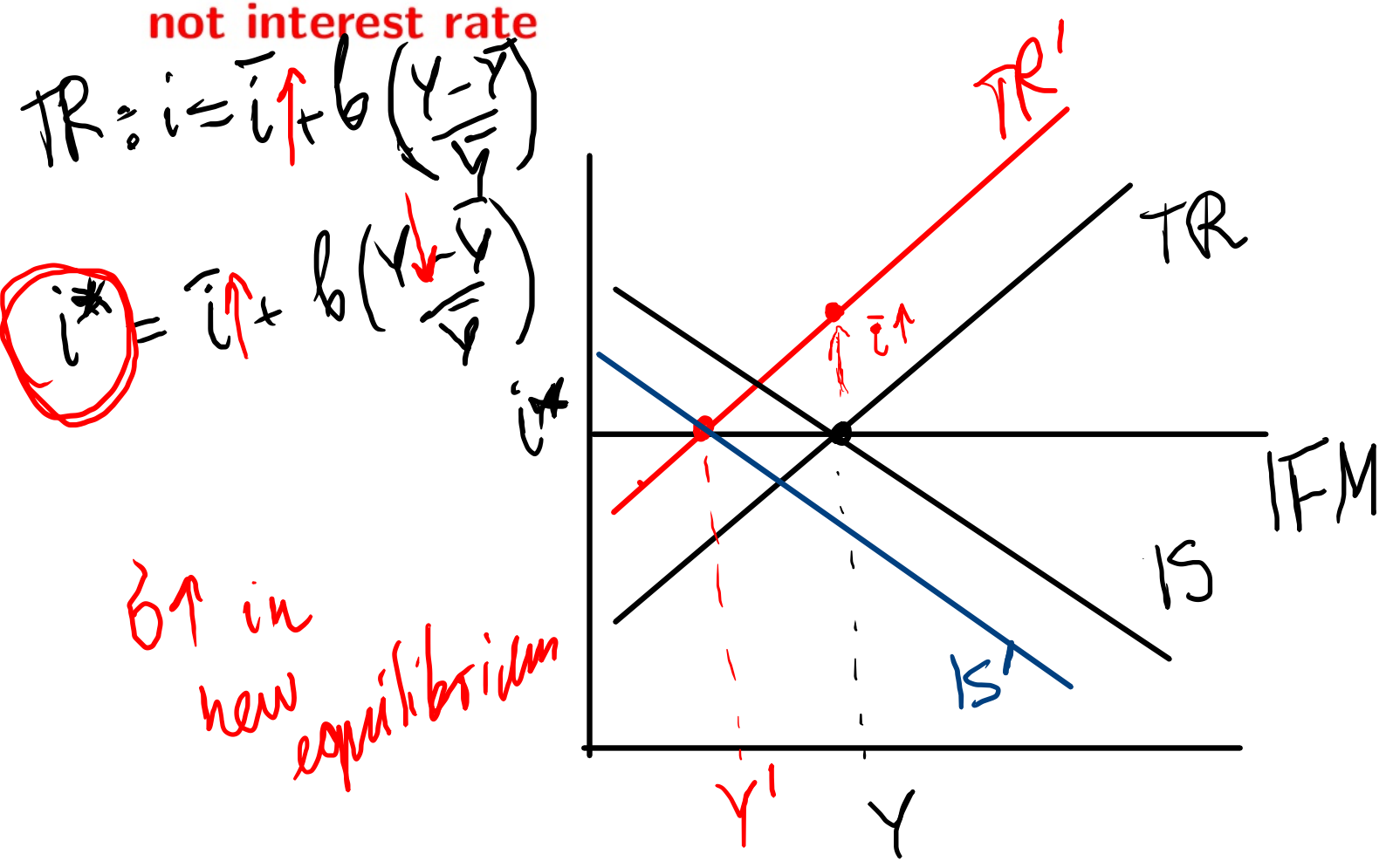
4. Introduction to Dynamics: Multiplier-Accelerator

Flexible exchange rate

- Monetary policy is again independent
- ~~Aggregate~~ Desired demand has additional source of variation: real exchange rate changes through **nominal** exchange rate reaction to **capital flows**
- Position of IS becomes endogenous; TR and IFM determine equilibrium: (IS)-TR-IFM

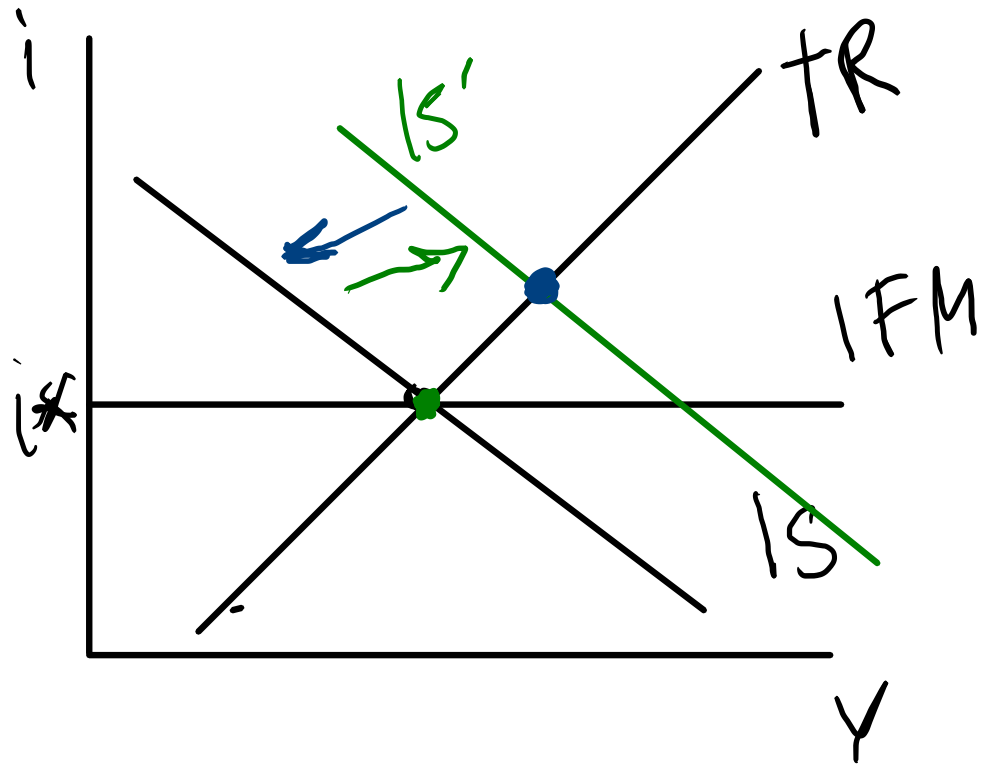
Monetary policy shock with flexible exchange rates

Monetary policy *de facto* operates through **exchange rate** and **not interest rate**



Demand shock (e.g. government spending) with flexible exchange rates

Exchange rate acts as **stabilizer** of aggregate demand

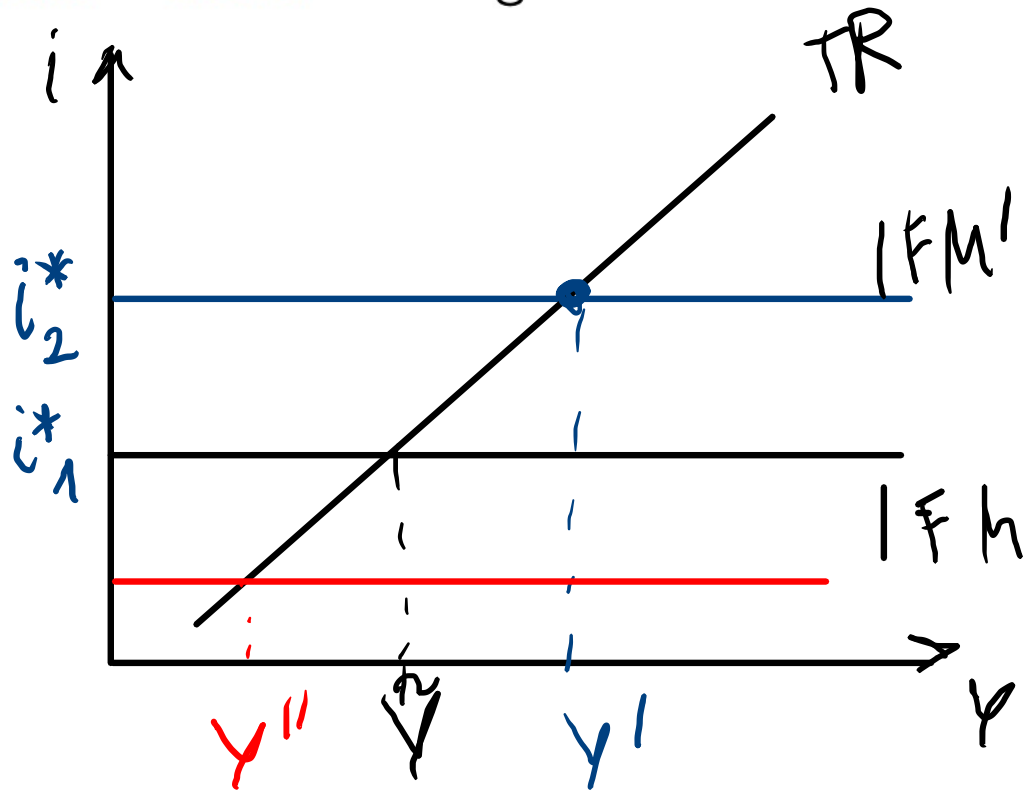


G ↑

G ↑ in new equilibrium

International monetary shock

An increase in foreign interest rate is **expansionary** – **opposite** of **fixed** exchange rate case.



International monetary shock II: beggar-thy-neighbour effect

- Suppose a large foreign economy lowers i^* in expansionary monetary policy
- How does equilibrium change? Draw an (IS)-TR-IFM plot
- Domestic i relatively high $\Rightarrow$ capital inflow $\Rightarrow$ real exchange rate appreciation $\sigma \uparrow$
- IS moves to the left, equilibrium output lower
- Foreign expansionary policy at the expense of neighbours' output. A critique of quantitative easing policies in developed economies post-2008, from smaller economies' perspective

Recap: trade-offs of exchange rate regimes

Each exchange rate regime presents pros and cons. Seen in this lecture:

- **Fixed** exchange rate regime:
  - facilitates trade of goods, services and assets, **but**
  - eliminates independent monetary policy (control over i)
  - not necessarily bad, e.g. if CB has **commitment** problems leading to high inflation
- **Flexible** of floating exchange rate regime:
  - creates problems in trade, **but**
  - allows for monetary policy independency + acts as absorber of demand shocks

Important issues of **foreign exchange reserves** management not covered in lecture – another argument against fixed regimes when CB is not trusted by public.

Beyond fixed vs. flexible: IMF classification

Many regimes exist as countries trade off benefits and costs of fixed and flexible exchange rates.

Type	Categories			
Hard pegs	Exchange arrangement with no separate legal tender	Currency board arrangement		
Soft pegs	Conventional pegged arrangement	Pegged exchange rate within horizontal bands	Stabilized arrangement	Crawling peg
Floating regimes (market-determined rates)	Floating	Free floating		
Residual	Other managed arrangement			

Source: IMF Annual report on exchange arrangements and exchange restrictions (2021).

1. Exchange rates & goods market (IS); capital flows (IFM)

2. Fixed exchange rate regime
   - IS-(TR)-IFM equilibrium

3. Flexible exchange rate regime
   - (IS)-TR-IFM equilibrium

4. Introduction to Dynamics: Multiplier-Accelerator

Multiplier-Accelerator model

- A simple Keynesian model of the goods market that generates endogenous business cycles
- **NOT** the way we think about cycles today...

... but is a useful introduction to **dynamics**

A closed economy model of goods market. 3 equations:

1. Consumption depending on **last period** income
2. Investment depending on **growth of** income in **last period** — the **accelerator** assumption
3. Goods market equilibrium (IS)

Multiplier-Accelerator accelerator model

Variables

Time runs from $t = 1$ to ∞

- $\{C_t\}$ — sequence of consumption
- $\{I_t\}$ — sequence of investment, the key *endogenous* variable
- $\{Y_t\}$ — sequence of GDP
- $\{G_t\}$ — sequence of of **exogenous** government expenditures. We will consider constant expenditure mostly, but it can be a source of shocks

Model structure

The model combines the consumption function

$$C_t = aY_{t-1} + \gamma$$

with the investment accelerator

$$I_t = b(Y_{t-1} - Y_{t-2})$$

and the goods market equilibrium

$$Y_t = C_t + I_t + G_t$$

- The parameter a is **mpc** (past income assumed most relevant for decisions – this assumption is given up by modern models)
- The parameter $b > 0$ is the investment accelerator coefficient: investment in physical capital is done when GDP is increasing and **disinvestment** is done when it is decreasing

Solution: a difference equation

Combining the three equations:

$$Y_t = (a+b)\,Y_{t-1} - bY_{t-2} + (\gamma + G_t)$$

Defining new coefficients gives compact form: assume $\rho_1 = (a+b)$ and $\rho_2 = -b$, then:

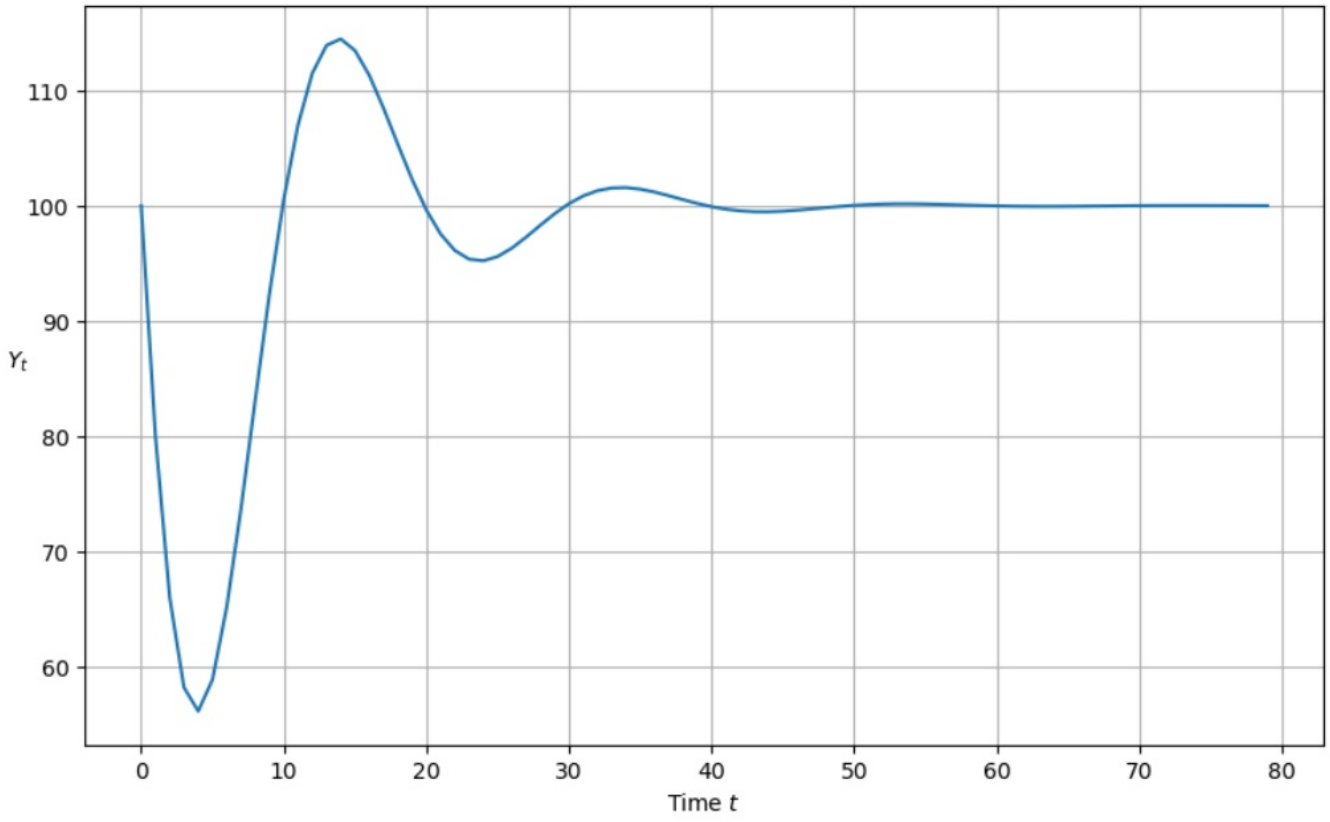
$$Y_t = \rho_1 Y_{t-1} + \rho_2 Y_{t-2} + (\gamma + G_t)$$

Assuming initial values to generate the sequence $\{Y_t\}$ for $t = 0, \ldots, T$:

$$Y_{-1} = \bar{Y}_{-1}, \quad Y_{-2} = \bar{Y}_{-2}$$

When simulating the model, set (a, b) so that starting from $(\bar{Y}_{-1}, \bar{Y}_{-2})$, Y_t converges to a constant value under constant G: this is the **steady state** of the economy

Model dynamics: no shocks



Source: QuantEcon (link)

Why the cycles? Intuition

What if there was no investment in the model? Then:

$$Y_t = aY_{t-1} + \gamma + G$$

where government expenditure is again assumed constant: $G_t = G$)

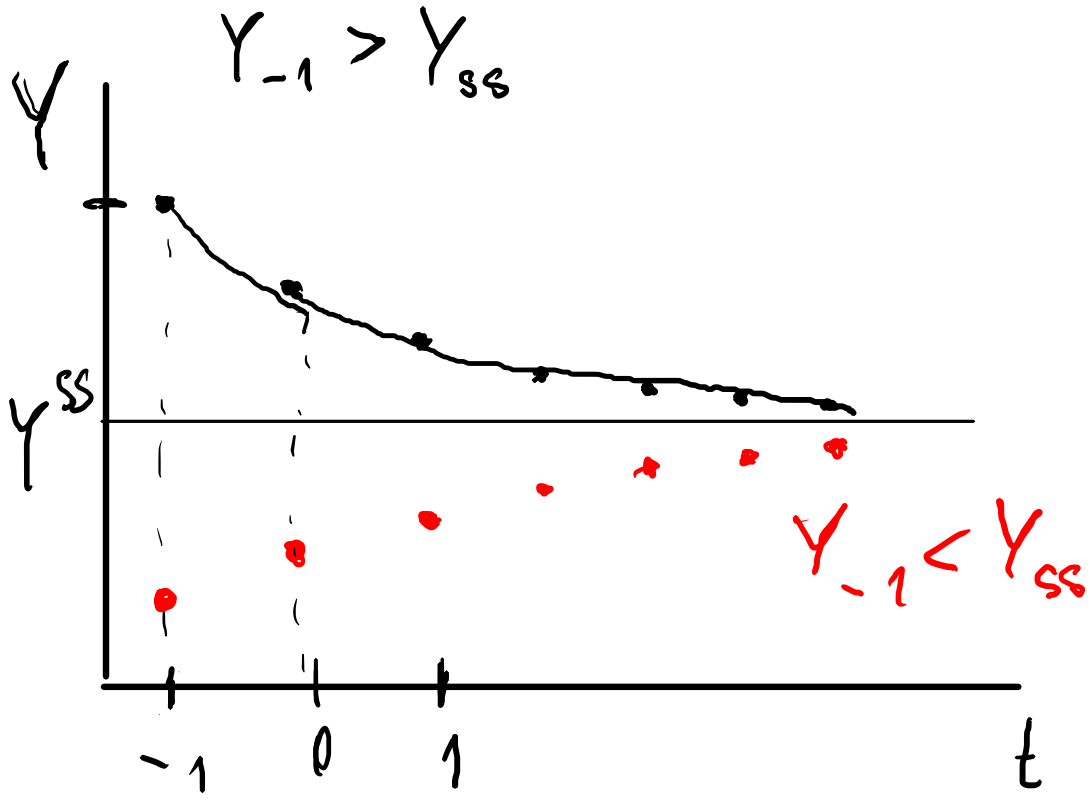
The steady state level of Y is easy to compute:

$$Y^{ss} = aY^{ss} + \gamma + G \Rightarrow Y^{ss} = \frac{\gamma + G}{1 - a}$$

and we get that $Y_t - Y^{ss} = a(Y_{t-1} - Yss)$ (verify this)
$\Rightarrow$ the difference between Y_t and Y^{ss} decreases over time $\Rightarrow$ Y_t converges, no cycles

In the full model, the accelerator $I_t = b(Y_{t-1} - Y_{t-2})$ makes Y_t **overshoot** the steady state, but the consumption dynamics makes it converge back after each overshoot

Why the cycles? Illustration



Why the cycles? Math

Let's solve the full model in a **linear state space form** using matrices

Define a **state vector** $x_t = \begin{pmatrix} Y_{t-1} \\ Y_{t-2} \end{pmatrix}$.

We can write the dynamical equation $Y_t = aY_{t-1} + b(Y_{t-1} - Y_{t-2}) + \gamma + G$ in a matrix form:

$$x_{t+1} = Ax_t + d$$

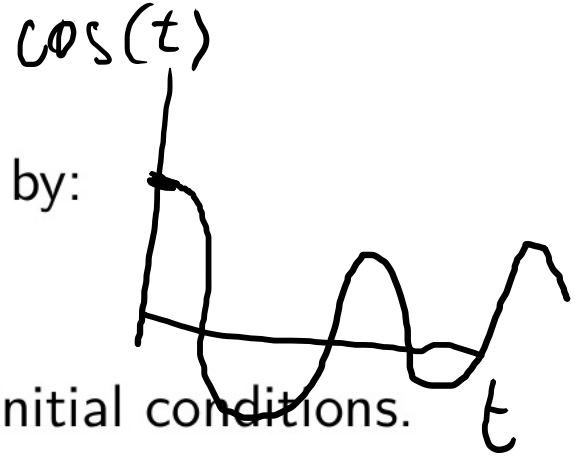
where $A = \begin{pmatrix} a+b & -b \\ 1 & 0 \end{pmatrix}$ $\begin{pmatrix} Y_t \\ Y_{t-1} \end{pmatrix} = \begin{pmatrix} a+b & -b \\ 1 & 0 \end{pmatrix} \begin{pmatrix} Y_{t-1} \\ Y_{t-2} \end{pmatrix} + \begin{pmatrix} \gamma+G \\ 0 \end{pmatrix}$

$d = \begin{pmatrix} \gamma+G \\ 0 \end{pmatrix}$

Model dynamics is then determined by **eigenvalues** of A

$\begin{pmatrix} Y_t \\ Y_{t-1} \end{pmatrix} = \begin{pmatrix} (a+b)Y_{t-1} - bY_{t-2} + \gamma + G \\ Y_{t-1} \end{pmatrix}$

Why the cycles? Math



With eigenvalues λ_1, λ_2, the dynamics of Y_t given by:

$$Y_t = \lambda_1^t c_1 + \lambda_2^t c_2$$

where c_1 and c_2 are constants on parameters and initial conditions.
initial conditions and on ρ_1, ρ_2.
When the eigenvalues are complex, can represent them in polar form $\lambda_1 = re^{i\omega}$, $\lambda_2 = re^{-i\omega}$ and rewrite the solution as follows (not necessary to memorize!):

$$Y_t = (c_1 + c_2)r^t \cos(\omega t) + i(c_1 - c_2)r^t \sin(\omega t)$$

Parameters and intital conditions chosen such that $c_1 + c_2$ real, $c_1 - c_2$ complex, so Y_t real and has a form:

converges to 0?

$$Y_t = 2vr^t \cos(\omega t + \theta),$$

with v, θ some constants depending on parameters and intial conditions.
Graphs of cos function has cycles $\Rightarrow$ Y, C, I oscillate around steady state

Summary

- We studied an open economy extension of IS-TR – the IS-TR-IFM model
- The equilibrium depends crucially on the exchange rate regime: fixed vs. flexible
- The Multiplier-Accelerator model is based on Keynesian assumptions and a trend-follower behaviour of investors
- It exhibits endogenous business cycle dynamics, which we explored intuitively and mathematically with the linear state space form of the model