Problems for Recitation 18

1 Nerditosis

There is a rare and deadly disease called *Nerditosis* which afflicts about 1 person in 1000. One symptom is a compulsion to refer to everything— fields of study, classes, buildings, etc.— using numbers. It's horrible. As victims enter their final, downward spiral, they're awarded a degree from MIT. Two doctors claim that they can diagnose Nerditosis.

1. Doctor X received his degree from Harvard Medical School. He practices at Massachusetts General Hospital and has access to the latest scanners, lab tests, and research. Suppose you ask Doctor X whether you have the disease.

   - If you have Nerditosis, he says "yes" with probability 0.99.
   - If you don't have it, he says "no" with probability 0.97.

   Let D be the event that you have the disease, and let E be the event that the diagnosis is erroneous. Use the Total Probability Law to compute $\Pr\{E\}$, the probability that Doctor X makes a mistake.

   $$P(E) = P(E \mid \text{have disease})\, P(\text{have disease}) + P(E \mid \text{no disease})\, P(\text{no disease})$$
   $$= (0.01)(.001) + (0.03)(0.999) = \underline{0.02998}$$

2. "Doctor" Y received his genuine degree from a fully-accredited university for \$49.95 via a special internet offer. He knows that Nerditosis strikes 1 person in 1000, but is a little shaky on how to interpret this. So if you ask him whether you have the disease, he'll helpfully say "yes" with probability 1 in 1000 regardless of whether you actually do or not.

   Let D be the event that you have the disease, and let F be the event that the diagnosis is faulty. Use the Total Probability Law to compute $\Pr\{F\}$, the probability that Doctor Y made a mistake.

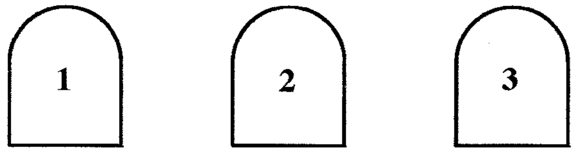
   $$P(F) = P(F \mid D)\, P(D) + P(F \mid \neg D)\, P(\neg D)$$
   $$= (0.999)\, P(0.001) + \cancel{P}(0.001)\, \cancel{P}(0.999)$$
   $$= \underline{0.001998}$$

3. Which doctor is more reliable?

   I prefer Doctor Y.

2 Barglesnort

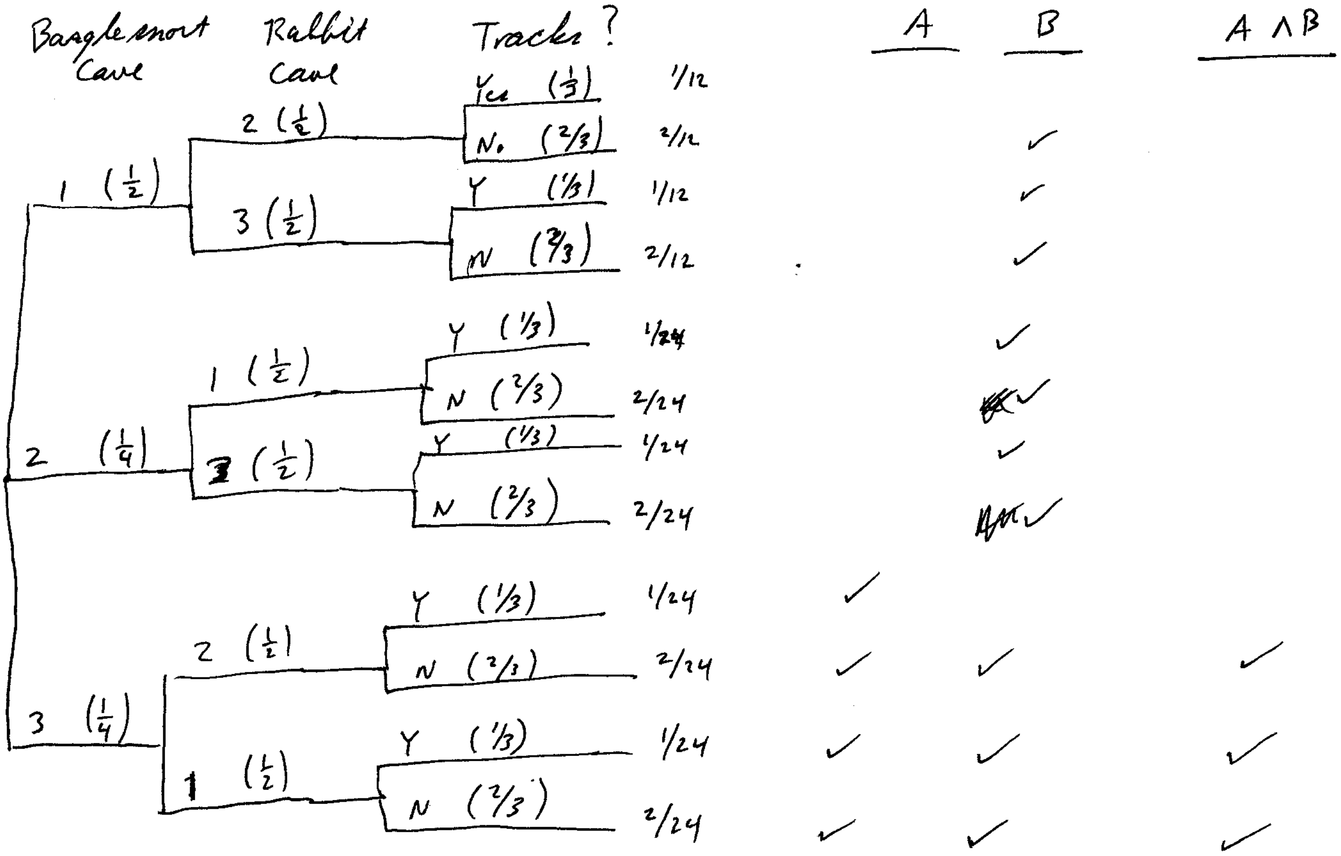
A Barglesnort makes its lair in one of three caves:



The Barglesnort inhabits cave 1 with probability $\frac{1}{2}$, cave 2 with probability $\frac{1}{4}$, and cave 3 with probability $\frac{1}{4}$. A rabbit subsequently moves into one of the two unoccupied caves, selected with equal probability. With probability $\frac{1}{3}$, the rabbit leaves tracks at the entrance to its cave. (Barglesnorts are much too clever to leave tracks.) What is the probability that the Barglesnort lives in cave 3, given that there are no tracks in front of cave 2?

Use a tree diagram and the four-step method.

A: Barglesnort in cave 3
B = No Tracks in front of cave 2



$P(A) = 6/24 = 1/4$

$P(B) = 21/24$

$P(A \wedge B) = 5/24$

$$\boxed{P(A \mid B) = \frac{P(A \wedge B)}{P(B)} = \frac{5/24}{21/24} = 5/21}$$

3 Prisoners

There are three prisoners in a maximum-security prison for fictional villains: the Evil Wizard Voldemort, the Dark Lord Sauron, and Little Bunny Foo-Foo. The parole board has declared that it will release two of the three, chosen uniformly at random, but has not yet released their names. Naturally, Sauron figures that he will be released to his home in Mordor, where the shadows lie, with probability $\frac{2}{3}$.

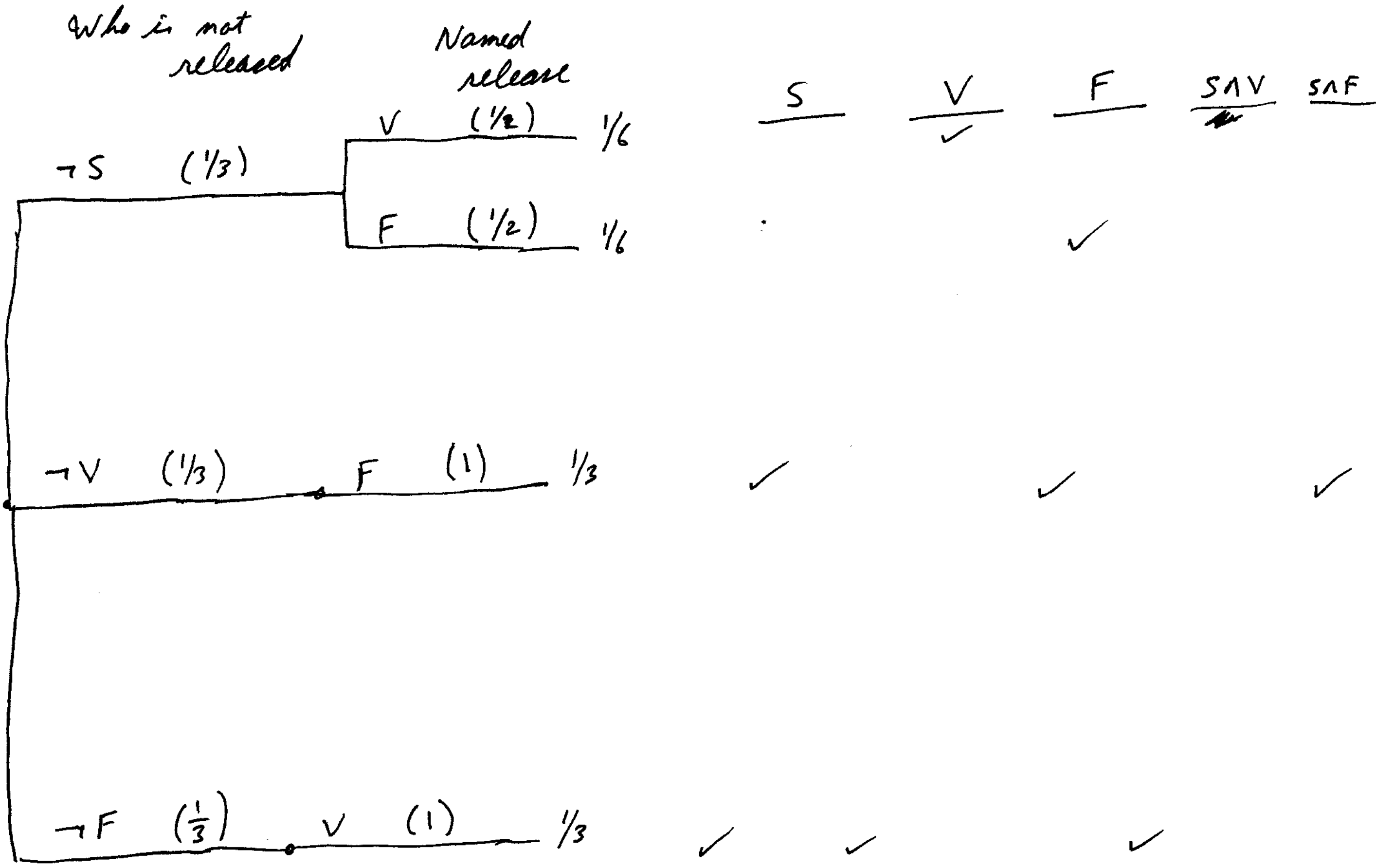
A guard offers to tell Sauron the name of one of the other prisoners who will be released (either Voldemort or Foo-Foo). However, Sauron declines this offer. He reasons that if the guard says, for example, "Little Bunny Foo-Foo will be released", then his own probability of release will drop to $\frac{1}{2}$. This is because he will then know that either he or Voldemort will also be released, and these two events are equally likely.

Using a tree diagram and the four-step method, either prove that the Dark Lord Sauron has reasoned correctly or prove that he is wrong. Assume that if the guard has a choice of naming either Voldemort or Foo-Foo (because both are to be released), then he names one of the two uniformly at random.

S = Sauron is released

F = Bunny Foo Foo is released

V = Voldemort is released



$$P(S) = P(S|V)\cdot P(V) + P(S|F)\cdot P(F) = \frac{P(S\wedge V)}{P(V)}\cdot P(V) + \frac{P(S\wedge F)}{P(F)}\cdot P(F)$$

$$= P(S\wedge V) + P(S\wedge F) = 1/3 + 1/3 = \boxed{2/3}$$

Sauron is wrong

From the solutions

My reasoning about Problem 3 is faulty, even though my numerical answer is correct. Pick one prisoner for the guard to name: "V"

$$P(\text{"V"}) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

$$P(S \mid \text{"V"}) = \frac{P(S \cap \text{"V"})}{P(\text{"V"})} = \frac{1/3}{1/2} = \frac{2}{3}$$

Same is true if "F".