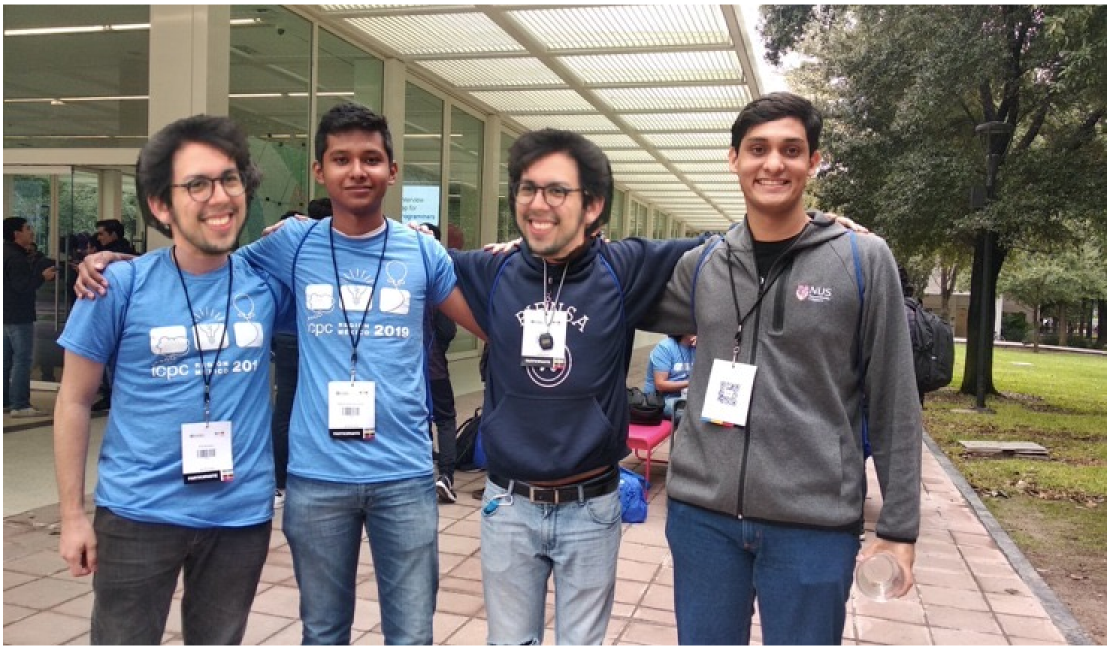

Almost Retired

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Contents

Think twice, code once

Template.cpp

```
#pragma GCC optimize("Ofast,unroll-loops,no-stack-protector
    ")
#include <bits/stdc++.h>
using namespace std;

#define fore(i, l, r) for (auto i = (l) - ((l) > (r)); i !=
      (r) - ((l) > (r)); i += 1 - 2 * ((l) > (r)))
#define sz(x) int(x.size())
#define all(x) begin(x), end(x)
#define f first
#define s second
#define pb push_back

#ifdef LOCAL
#include "debug.h"
#else
#define debug(...)
#endif

using ld = long double;
using lli = long long;
using ii = pair<int, int>;

int main() {
  cin.tie(0)->sync_with_stdio(0), cout.tie(0);
  return 0;
}

/* Please, check the following:
```

Debug.h

```
#include <bits/stdc++.h>
using namespace std;

template <class A, class B>
ostream& operator<<(ostream& os, const pair<A, B>& p) {
  return os << "(" << p.first << ", " << p.second << ")";
}

template <class A, class B, class C>
basic_ostream<A, B>& operator<<(basic_ostream<A, B>& os,
    const C& c) {
  os << "[";
  for (const auto& x : c)
    os << ", " + 2 * (&x == &*begin(c)) << x;
  return os << "]";
}

void print(string s) {
  cout << endl;
}

template <class H, class... T>
void print(string s, const H& h, const T&... t) {
  const static string reset = "\033[0m", blue = "\033[1;34m
    ", purple = "\033[3;95m";
  bool ok = 1;
  do {
    if (s[0] == '\"')
      ok = 0;
    else
      cout << blue << s[0] << reset;
    s = s.substr(1);
  } while (s.size() && s[0] != ',');
  if (ok)
    cout << ": " << purple << h << reset;
  print(s, t...);
}

#define debug(...) print(#__VA_ARGS__, __VA_ARGS__)
```

1 Data structures

1.1 Sparse table

```
template <class T, class F = function<T(const T&, const T&)
    >>
struct Sparse {
  vector<T> sp[21]; // n <= 2^21
  F f;
  int n;

  Sparse(T* begin, T* end, const F& f) : Sparse(vector<T>(
      begin, end), f) {}

  Sparse(const vector<T>& a, const F& f) : f(f), n(sz(a)) {
    sp[0] = a;
    for (int k = 1; (1 << k) <= n; k++) {
      sp[k].resize(n - (1 << k) + 1);
      fore (l, 0, sz(sp[k])) {
        int r = l + (1 << (k - 1));
        sp[k][l] = f(sp[k - 1][l], sp[k - 1][r]);
      }
    }
  }

  T query(int l, int r) {
#warning Can give TLE D:, change it to a log table
    int k = __lg(r - l + 1);
    return f(sp[k][l], sp[k][r - (1 << k) + 1]);
  }

  T queryBits(int l, int r) {
    optional<T> ans;
    for (int len = r - l + 1; len; len -= len & -len) {
      int k = __builtin_ctz(len);
      ans = ans ? f(ans.value(), sp[k][l]) : sp[k][l];
      l += (1 << k);
    }
    return ans.value();
  }
};
```

1.2 Fenwick 2D offline

```
template <class T>
struct Fenwick2D { // add, build then update, query
  vector<vector<T>> fenw;
  vector<vector<int>> ys;
  vector<int> xs;
  vector<ii> pts;

  void add(int x, int y) {
    pts.pb({x, y});
  }

  void build() {
```

```
    sort(all(pts));
    for (auto&& [x, y] : pts) {
      if (xs.empty() || x != xs.back())
        xs.pb(x);
      swap(x, y);
    }
    fenw.resize(sz(xs)), ys.resize(sz(xs));
    sort(all(pts));
    for (auto&& [x, y] : pts) {
      swap(x, y);
      int i = lower_bound(all(xs), x) - xs.begin();
      for (; i < sz(fenw); i |= i + 1)
        if (ys[i].empty() || y != ys[i].back())
          ys[i].pb(y);
    }
    fore (i, 0, sz(fenw))
      fenw[i].resize(sz(ys[i]), T());
  }

  void update(int x, int y, T v) {
    int i = lower_bound(all(xs), x) - xs.begin();
    for (; i < sz(fenw); i |= i + 1) {
      int j = lower_bound(all(ys[i]), y) - ys[i].begin();
      for (; j < sz(fenw[i]); j |= j + 1)
        fenw[i][j] += v;
    }
  }

  T query(int x, int y) {
    T v = T();
    int i = upper_bound(all(xs), x) - xs.begin() - 1;
    for (; i >= 0; i &= i + 1, --i) {
      int j = upper_bound(all(ys[i]), y) - ys[i].begin() -
          1;
      for (; j >= 0; j &= j + 1, --j)
        v += fenw[i][j];
    }
    return v;
  }
};
```

1.3 Persistent segtree

```
template <class T>
struct Per {
  int l, r;
  Per *left, *right;
  T val;

  Per(int l, int r) : l(l), r(r), left(0), right(0) {}

  Per* pull() {
    val = left->val + right->val;
    return this;
  }

  void build() {
    if (l == r)
      return;
    int m = (l + r) >> 1;
    (left = new Per(l, m))->build();
    (right = new Per(m + 1, r))->build();
    pull();
  }

  template <class... Args>
  Per* update(int p, const Args&... args) {
    if (p < l || r < p)
      return this;
    Per* t = new Per(l, r);
    if (l == r) {
      t->val = T(args...);
      return t;
    }
    t->left = left->update(p, args...);
    t->right = right->update(p, args...);
    return t->pull();
  }

  T query(int ll, int rr) {
    if (r < ll || rr < l)
      return T();
    if (ll <= l && r <= rr)
      return val;
    return left->query(ll, rr) + right->query(ll, rr);
  }
};
```

1.4 Li Chao

```
struct LiChao {
  struct Fun {
    lli m = 0, c = -INF;
    lli operator()(lli x) const {
      return m * x + c;
    }
  } f;

  lli l, r;
  LiChao *left, *right;
  LiChao(lli l, lli r, Fun f) : l(l), r(r), f(f), left(0),
      right(0) {}

  void add(Fun& g) {
    lli m = (l + r) >> 1;
    bool bl = g(l) > f(l), bm = g(m) > f(m);
    if (bm)
      swap(f, g);
    if (l == r)
      return;
    if (bl != bm)
      left ? left->add(g) : void(left = new LiChao(l, m, g)
          );
    else
      right ? right->add(g) : void(right = new LiChao(m + 1
          , r, g));
  }

  lli query(lli x) {
    if (l == r)
      return f(x);
    lli m = (l + r) >> 1;
    if (x <= m)
      return max(f(x), left ? left->query(x) : -INF);
    return max(f(x), right ? right->query(x) : -INF);
  }
};
```

1.5 Wavelet

```
struct Wav {
  int lo, hi;
  Wav *left, *right;
  vector<int> amt;

  template <class Iter>
  Wav(int lo, int hi, Iter b, Iter e) : lo(lo), hi(hi) { //
      array 1-indexed
    if (lo == hi || b == e)
      return;
    amt.reserve(e - b + 1);
    amt.pb(0);
    int mid = (lo + hi) >> 1;
    auto leq = [mid](auto x) {
```

```
      return x <= mid;
    };
    for (auto it = b; it != e; it++)
      amt.pb(amt.back() + leq(*it));
    auto p = stable_partition(b, e, leq);
    left = new Wav(lo, mid, b, p);
    right = new Wav(mid + 1, hi, p, e);
  }

  // kth value in [l, r]
  int kth(int l, int r, int k) {
    if (r < l)
      return 0;
    if (lo == hi)
      return lo;
    if (k <= amt[r] - amt[l - 1])
      return left->kth(amt[l - 1] + 1, amt[r], k);
    return right->kth(l - amt[l - 1], r - amt[r], k - amt[r
        ] + amt[l - 1]);
  }

  // Count all values in [l, r] that are in range [x, y]
  int count(int l, int r, int x, int y) {
    if (r < l || y < x || y < lo || hi < x)
      return 0;
    if (x <= lo && hi <= y)
      return r - l + 1;
    return left->count(amt[l - 1] + 1, amt[r], x, y) +
        right->count(l - amt[l - 1], r - amt[r], x, y);
  }
};
```

1.6 Static to dynamic

```
template <class Black, class T>
struct StaticDynamic {
  Black box[25];
  vector<T> st[25];

  void insert(T& x) {
    int p = 0;
    while (p < 25 && !st[p].empty())
      p++;
    st[p].pb(x);
    fore (i, 0, p) {
      st[p].insert(st[p].end(), all(st[i]));
      box[i].clear(), st[i].clear();
    }
    for (auto y : st[p])
      box[p].insert(y);
    box[p].init();
  }
};
```

1.7 Ordered tree

It's a set/map, for a multiset/multimap (? add them as pairs (a[i], i)

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template <class K, class V = null_type>
using OrderedTree = tree<K, V, less<K>, rb_tree_tag,
    tree_order_statistics_node_update>;
#define rank order_of_key
#define kth find_by_order
```

1.8 Treap

```
struct Treap {
  static Treap* null;
  Treap *left, *right;
  unsigned pri = rng(), sz = 0;
  int val = 0;

  void push() {
    // propagate like segtree, key-values aren't modified!!
  }

  Treap* pull() {
    sz = left->sz + right->sz + (this != null);
    // merge(left, this), merge(this, right)
    return this;
  }

  Treap() {
    left = right = null;
  }

  Treap(int val) : val(val) {
    left = right = null;
    pull();
  }

  template <class F>
  pair<Treap*, Treap*> split(const F& leq) { // {<= val, >
       val}
    if (this == null)
      return {null, null};
    push();
    if (leq(this)) {
      auto p = right->split(leq);
      right = p.f;
      return {pull(), p.s};
    } else {
      auto p = left->split(leq);
      left = p.s;
      return {p.f, pull()};
    }
  }

  Treap* merge(Treap* other) {
    if (this == null)
      return other;
    if (other == null)
      return this;
    push(), other->push();
    if (pri > other->pri) {
      return right = right->merge(other), pull();
    } else {
      return other->left = merge(other->left), other->pull
          ();
    }
  }

  pair<Treap*, Treap*> leftmost(int k) { // 1-indexed
    return split([&](Treap* n) {
      int sz = n->left->sz + 1;
      if (k >= sz) {
        k -= sz;
        return true;
      }
      return false;
    });
  }

  auto split(int x) {
    return split([&](Treap* n) {
      return n->val <= x;
    });
  }
```

```
  Treap* insert(int x) {
    auto&& [leq, ge] = split(x);
    // auto &&[le, eq] = split(x); // uncomment for set
    return leq->merge(new Treap(x))->merge(ge); // change
         leq for le for set
  }

  Treap* erase(int x) {
    auto&& [leq, ge] = split(x);
    auto&& [le, eq] = leq->split(x - 1);
    auto&& [kill, keep] = eq->leftmost(1); // comment for
         set
    return le->merge(keep)->merge(ge); // le->merge(ge) for
          set
  }
}* Treap::null = new Treap;
```

1.9 Persistent Treap

```
struct PerTreap {
  static PerTreap* null;
  PerTreap *left, *right;
  unsigned pri = rng(), sz = 0;
  int val;

  void push() {
    // propagate like segtree, key-values aren't modified!!
  }

  PerTreap* pull() {
    sz = left->sz + right->sz + (this != null);
    // merge(left, this), merge(this, right)
    return this;
  }

  PerTreap(int val = 0) : val(val) {
    left = right = null;
    pull();
  }

  PerTreap(PerTreap* t) : left(t->left), right(t->right),
       pri(t->pri), sz(t->sz) {
    val = t->val;
  }

  template <class F>
  pair<PerTreap*, PerTreap*> split(const F& leq) { // {<=
       val, > val}
    if (this == null) return {null, null};
    push();
    PerTreap* t = new PerTreap(this);
    if (leq(this)) {
      auto p = t->right->split(leq);
      t->right = p.f;
      return {t->pull(), p.s};
    } else {
      auto p = t->left->split(leq);
      t->left = p.s;
      return {p.f, t->pull()};
    }
  }

  PerTreap* merge(PerTreap* other) {
    if (this == null) return new PerTreap(other);
    if (other == null) return new PerTreap(this);
    push(), other->push();
    PerTreap* t;
    if (pri > other->pri) {
      t = new PerTreap(this);
      t->right = t->right->merge(other);
    } else {
      t = new PerTreap(other);
      t->left = merge(t->left);
    }
    return t->pull();
  }

  auto leftmost(int k) { // 1-indexed
    return split([&](PerTreap* n) {
      int sz = n->left->sz + 1;
      if (k >= sz) {
        k -= sz;
        return true;
      }
      return false;
    });
  }

  auto split(int x) {
    return split([&](PerTreap* n) {
      return n->val <= x;
    });
  }
}* PerTreap::null = new PerTreap;
```

2 Dynamic programming

2.1 All submasks of a mask

```
for (int B = A; B > 0; B = (B - 1) & A)
```

2.2 Broken profile $\mathcal{O}(n \cdot m \cdot 2^n)$ with $n \leq m$

Cuenta todas las maneras en las que puedes acomodar fichas de 1x2 y 2x1 en un tablero $n \cdot m$

```
// Answer in dp[m][0][0]

lli dp[2][N][1 << N];

dp[0][0][0] = 1;

fore (c, 0, m) {
  fore (r, 0, n + 1)
    fore (mask, 0, 1 << n) {
      if (r == n) {
        dp[~c & 1][0][mask] += dp[c & 1][r][mask];
        continue;
      }

      if (~(mask >> r) & 1) {
        dp[c & 1][r + 1][mask | (1 << r)] += dp[c & 1][r][
             mask];

        if (~(mask >> (r + 1)) & 1)
          dp[c & 1][r + 2][mask] += dp[c & 1][r][mask];
      } else {
        dp[c & 1][r + 1][mask & ~(1 << r)] += dp[c & 1][r][
             mask];
      }
    }

  fore (r, 0, n + 1)
    fore (mask, 0, 1 << n)
      dp[c & 1][r][mask] = 0;
}
```

2.3 Convex hull trick $\mathcal{O}(n^2) \Rightarrow \mathcal{O}(n)$

$dp[i] = \min_{j<i}(dp[j] + b[j] * a[i])$
$dp[i][j] = \min_{k<j}(dp[i-1][k] + b[k] * a[j])$

$b[j] \geq b[j+1]$ optionally $a[i] \leq a[i+1]$

```
// for doubles, use INF = 1/.0, div(a,b) = a / b
struct Line {
  mutable lli m, c, p;
  bool operator<(const Line& l) const {
    return m < l.m;
  }
  bool operator<(lli x) const {
    return p < x;
  }
  lli operator()(lli x) const {
    return m * x + c;
  }
};

template <bool MAX>
struct DynamicHull : multiset<Line, less<>> {
  lli div(lli a, lli b) {
    return a / b - ((a ^ b) < 0 && a % b);
  }

  bool isect(iterator i, iterator j) {
    if (j == end())
      return i->p = INF, 0;
    if (i->m == j->m)
      i->p = i->c > j->c ? INF : -INF;
    else
      i->p = div(i->c - j->c, j->m - i->m);
    return i->p >= j->p;
  }

  void add(lli m, lli c) {
    if (!MAX)
      m = -m, c = -c;
    auto k = insert({m, c, 0}), j = k++, i = j;
    while (isect(j, k))
      k = erase(k);
    if (i != begin() && isect(--i, j))
      isect(i, j = erase(j));
    while ((j = i) != begin() && (--i)->p >= j->p)
      isect(i, erase(j));
  }

  lli query(lli x) {
    if (empty())
      return 0LL;
    auto f = *lower_bound(x);
    return MAX ? f(x) : -f(x);
  }
};
```

2.4 Digit dp

Counts the amount of numbers in $[l, r]$ such are divisible by k. (flag *nonzero* is for different lengths)
It can be reduced to $dp(i, x, small)$, and has to be solved like $f(r) - f(l-1)$

```
#define state [i][x][small][big][nonzero]
int dp(int i, int x, bool small, bool big, bool nonzero) {
  if (i == sz(r))
    return x % k == 0 && nonzero;
  int& ans = mem state;
  if (done state != timer) {
    done state = timer;
    ans = 0;
    int lo = small ? 0 : l[i] - '0';
    int hi = big ? 9 : r[i] - '0';
    fore (y, lo, max(lo, hi) + 1) {
      bool small2 = small | (y > lo);
      bool big2 = big | (y < hi);
      bool nonzero2 = nonzero | (y > 0);
      ans += dp(i + 1, (x * 10 + y) % k, small2, big2,
          nonzero2);
    }
  }
  return ans;
}
```

2.5 Divide and conquer $\mathcal{O}(k \cdot n^2) \Rightarrow \mathcal{O}(k \cdot nlogn)$

Split the array of size n into k continuous groups. $k \leq n$
$cost(a, c) + cost(b, d) \leq cost(a, d) + cost(b, c)$ with $a \leq b \leq c \leq d$

```
lli dp[2][N];

void solve(int cut, int l, int r, int optl, int optr) {
  if (r < l)
    return;
  int mid = (l + r) / 2;
  pair<lli, int> best = {INF, -1};
  fore (p, optl, min(mid, optr) + 1)
    best = min(best, {dp[~cut & 1][p - 1] + cost(p, mid), p
        });
  dp[cut & 1][mid] = best.f;
  solve(cut, l, mid - 1, optl, best.s);
  solve(cut, mid + 1, r, best.s, optr);
}

fore (i, 1, n + 1)
  dp[1][i] = cost(1, i);
fore (cut, 2, k + 1)
  solve(cut, cut, n, cut, n);
```

2.6 Knapsack 01 $\mathcal{O}(n \cdot MaxW)$

```
fore (i, 0, n)
  for (int x = MaxW; x >= w[i]; x--)
    umax(dp[x], dp[x - w[i]] + cost[i]);
```

2.7 Knuth $\mathcal{O}(n^3) \Rightarrow \mathcal{O}(n^2)$

$dp[l][r] = \min_{l \leq k \leq r}\{dp[l][k] + dp[k][r]\} + cost(l, r)$

```
lli dp[N][N];
int opt[N][N];

fore (len, 1, n + 1)
  fore (l, 0, n) {
    int r = l + len - 1;
    if (r > n - 1)
      break;
    if (len <= 2) {
      dp[l][r] = 0;
      opt[l][r] = l;
      continue;
    }
    dp[l][r] = INF;
    fore (k, opt[l][r - 1], opt[l + 1][r] + 1) {
      lli cur = dp[l][k] + dp[k][r] + cost(l, r);
      if (cur < dp[l][r]) {
        dp[l][r] = cur;
        opt[l][r] = k;
      }
    }
  }
```

2.8 Matrix exponentiation $\mathcal{O}(n^3 \cdot logn)$

If TLE change `Mat` to `array<array<T, N>, N>`

```
template <class T>
struct Mat : vector<vector<T>> {
  int n, m;

  Mat(int n, int m) : vector<vector<T>>(n, vector<T>(m)), n
      (n), m(m) {}

  Mat<T> operator*(const Mat<T>& other) {
    assert(m == other.n);
    Mat<T> ans(n, other.m);
    fore (k, 0, m)
      fore (i, 0, n)
        fore (j, 0, other.m)
          ans[i][j] += (*this)[i][k] * other[k][j];
    return ans;
  }

  Mat<T> pow(lli k) {
    assert(n == m);
    Mat<T> ans(n, n);
    fore (i, 0, n)
      ans[i][i] = 1;
    for (; k > 0; k >>= 1) {
      if (k & 1)
        ans = ans * *this;
      *this = *this * *this;
    }
    return ans;
  }
};
```

2.9 SOS dp

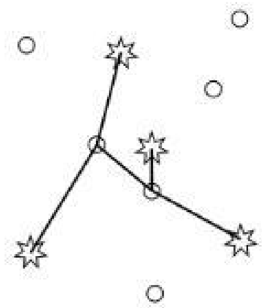
```
// N = amount of bits
// dp[mask] = Sum of all dp[x] such that 'x' is a submask
    of 'mask'
fore (i, 0, N)
  fore (mask, 0, 1 << N)
    if (mask >> i & 1) {
      dp[mask] += dp[mask ^ (1 << i)];
    }
```

2.10 Inverse SOS dp

```
// N = amount of bits
// dp[mask] = Sum of all dp[x] such that 'mask' is a
    submask of 'x'
fore (i, 0, N) {
  for (int mask = (1 << N) - 1; mask >= 0; mask--)
    if (mask >> i & 1) {
      dp[mask ^ (1 << i)] += dp[mask];
    }
```

2.11 Steiner

```
// Connect special nodes by a minimum spanning tree
// special nodes [0, k)
fore (u, k, n)
  fore (a, 0, k)
    umin(dp[u][1 << a], dist[u][a]);
fore (A, 0, (1 << k))
  fore (u, k, n) {
    for (int B = A; B > 0; B = (B - 1) & A)
      umin(dp[u][A], dp[u][B] + dp[u][A ^ B]);
    fore (v, k, n)
      umin(dp[v][A], dp[u][A] + dist[u][v]);
  }
```



3 Geometry

3.1 Geometry

```
const ld EPS = 1e-20;
const ld INF = 1e18;
const ld PI = acos(-1.0);
enum { ON = -1, OUT, IN, OVERLAP };

#define eq(a, b) (abs((a) - (b)) <= +EPS)
#define neq(a, b) (!eq(a, b))
#define geq(a, b) ((a) - (b) >= -EPS)
#define leq(a, b) ((a) - (b) <= +EPS)
#define ge(a, b) ((a) - (b) > +EPS)
#define le(a, b) ((a) - (b) < -EPS)

int sgn(ld a) {
  return (a > EPS) - (a < -EPS);
}
```

3.2 Radial order

```
struct Radial {
  Pt c;
  Radial(Pt c) : c(c) {}

  int cuad(Pt p) const {
    if (p.x > 0 && p.y >= 0)
      return 0;
    if (p.x <= 0 && p.y > 0)
      return 1;
    if (p.x < 0 && p.y <= 0)
      return 2;
    if (p.x >= 0 && p.y < 0)
      return 3;
    return -1;
  }

  bool operator()(Pt a, Pt b) const {
    Pt p = a - c, q = b - c;
    if (cuad(p) == cuad(q))
      return p.y * q.x < p.x * q.y;
    return cuad(p) < cuad(q);
  }
};
```

3.3 Sort along line

```
void sortAlongLine(vector<Pt>& pts, Line l) {
  sort(all(pts), [&](Pt a, Pt b) {
    return a.dot(l.v) < b.dot(l.v);
  });
}
```

4 Point

4.1 Point

```
struct Pt {
  ld x, y;
  explicit Pt(ld x = 0, ld y = 0) : x(x), y(y) {}
```

```cpp
  Pt operator+(Pt p) const {
    return Pt(x + p.x, y + p.y);
  }

  Pt operator-(Pt p) const {
    return Pt(x - p.x, y - p.y);
  }

  Pt operator*(ld k) const {
    return Pt(x * k, y * k);
  }

  Pt operator/(ld k) const {
    return Pt(x / k, y / k);
  }

  ld dot(Pt p) const {
    // 0 if vectors are orthogonal
    // - if vectors are pointing in opposite directions
    // + if vectors are pointing in the same direction
    return x * p.x + y * p.y;
  }

  ld cross(Pt p) const {
    // 0 if collinear
    // - if p is to the right of a
    // + if p is to the left of a
    // gives you 2 * area
    return x * p.y - y * p.x;
  }

  ld norm() const {
    return x * x + y * y;
  }

  ld length() const {
    return sqrtl(norm());
  }

  Pt unit() const {
    return (*this) / length();
  }

  ld angle() const {
    ld ang = atan2(y, x);
    return ang + (ang < 0 ? 2 * acos(-1) : 0);
  }

  Pt perp() const {
    return Pt(-y, x);
  }

  Pt rotate(ld angle) const {
    // counter-clockwise rotation in radians
    // degree = radian * 180 / pi
    return Pt(x * cos(angle) - y * sin(angle), x * sin(
        angle) + y * cos(angle));
  }

  int dir(Pt a, Pt b) const {
    // where am I on the directed line ab
    return sgn((a - *this).cross(b - *this));
  }

  bool operator<(Pt p) const {
    return eq(x, p.x) ? le(y, p.y) : le(x, p.x);
  }

  bool operator==(Pt p) const {
    return eq(x, p.x) && eq(y, p.y);
  }

  bool operator!=(Pt p) const {
    return !(*this == p);
  }

  friend ostream& operator<<(ostream& os, const Pt& p) {
    return os << "(" << p.x << ", " << p.y << ")";
  }

  friend istream& operator>>(istream& is, Pt& p) {
    return is >> p.x >> p.y;
  }
};
```

4.2 Angle between vectors

```cpp
ld angleBetween(Pt a, Pt b) {
  ld x = a.dot(b) / a.length() / b.length();
  return acosl(max(-1.0, min(1.0, x)));
}
```

4.3 Closest pair of points $\mathcal{O}(n \cdot logn)$

```cpp
pair<Pt, Pt> closestPairOfPoints(vector<Pt>& pts) {
  sort(all(pts), [&](Pt a, Pt b) {
    return le(a.y, b.y);
  });
  set<Pt> st;
  ld ans = INF;
  Pt p, q;
  int pos = 0;
  fore (i, 0, sz(pts)) {
    while (pos < i && geq(pts[i].y - pts[pos].y, ans))
      st.erase(pts[pos++]);
    auto lo = st.lower_bound(Pt(pts[i].x - ans - EPS, -INF)
        );
    auto hi = st.upper_bound(Pt(pts[i].x + ans + EPS, -INF)
        );
    for (auto it = lo; it != hi; ++it) {
      ld d = (pts[i] - *it).length();
      if (le(d, ans))
        ans = d, p = pts[i], q = *it;
    }
    st.insert(pts[i]);
  }
  return {p, q};
}
```

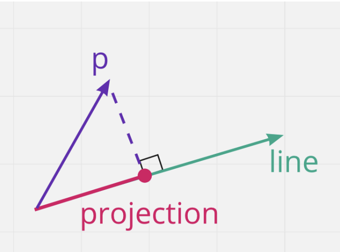
4.4 KD Tree

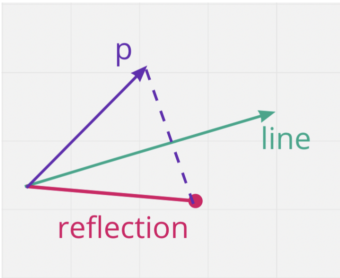
Returns nearest point, to avoid self-nearest add an id to the point

```cpp
struct Pt {
  // Geometry point mostly
  ld operator[](int i) const {
    return i == 0 ? x : y;
  }
};

struct KDTree {
  Pt p;
  int k;
  KDTree *left, *right;

  template <class Iter>
  KDTree(Iter l, Iter r, int k = 0) : k(k), left(0), right(
      0) {
    int n = r - l;
    if (n == 1) {
      p = *l;
      return;
    }
```

```
    nth_element(l, l + n / 2, r, [&](Pt a, Pt b) {
      return a[k] < b[k];
    });
    p = *(l + n / 2);
    left = new KDTree(l, l + n / 2, k ^ 1);
    right = new KDTree(l + n / 2, r, k ^ 1);
  }

  pair<ld, Pt> nearest(Pt x) {
    if (!left && !right)
      return {(p - x).norm(), p};
    vector<KDTree*> go = {left, right};
    auto delta = x[k] - p[k];
    if (delta > 0)
      swap(go[0], go[1]);
    auto best = go[0]->nearest(x);
    if (best.f > delta * delta)
      best = min(best, go[1]->nearest(x));
    return best;
  }
};
```

5 Lines and segments





5.1 Line

```
struct Line {
  Pt a, b, v;

  Line() {}
  Line(Pt a, Pt b) : a(a), b(b), v((b - a).unit()) {}

  bool contains(Pt p) {
    return eq((p - a).cross(b - a), 0);
  }

  int intersects(Line l) {
    if (eq(v.cross(l.v), 0))
      return eq((l.a - a).cross(v), 0) ? 1e9 : 0;
    return 1;
  }

  int intersects(Seg s) {
    if (eq(v.cross(s.v), 0))
      return eq((s.a - a).cross(v), 0) ? 1e9 : 0;
    return a.dir(b, s.a) != a.dir(b, s.b);
  }

  template <class Line>
  Pt intersection(Line l) { // can be a segment too
    return a + v * ((l.a - a).cross(l.v) / v.cross(l.v));
  }

  Pt projection(Pt p) {
    return a + v * proj(p - a, v);
  }

  Pt reflection(Pt p) {
    return a * 2 - p + v * 2 * proj(p - a, v);
  }
};
```

5.2 Segment

```
struct Seg {
  Pt a, b, v;

  Seg() {}
  Seg(Pt a, Pt b) : a(a), b(b), v(b - a) {}

  bool contains(Pt p) {
    return eq(v.cross(p - a), 0) && leq((a - p).dot(b - p),
        0);
  }

  int intersects(Seg s) {
    int d1 = a.dir(b, s.a), d2 = a.dir(b, s.b);
    if (d1 != d2)
      return s.a.dir(s.b, a) != s.a.dir(s.b, b);
    return d1 == 0 && (contains(s.a) || contains(s.b) || s.
        contains(a) || s.contains(b)) ? 1e9 : 0;
  }

  template <class Seg>
  Pt intersection(Seg s) { // can be a line too
    return a + v * ((s.a - a).cross(s.v) / v.cross(s.v));
  }
};
```

5.3 Projection

```
ld proj(Pt a, Pt b) {
  return a.dot(b) / b.length();
}
```

5.4 Distance point line

```
ld distance(Pt p, Line l) {
  Pt q = l.projection(p);
  return (p - q).length();
}
```

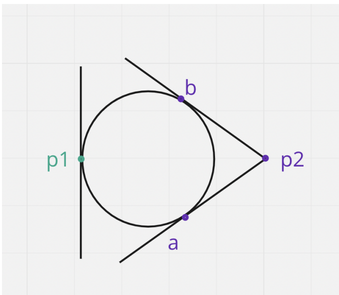
5.5 Distance point segment

```
ld distance(Pt p, Seg s) {
  if (le((p - s.a).dot(s.b - s.a), 0))
    return (p - s.a).length();
  if (le((p - s.b).dot(s.a - s.b), 0))
    return (p - s.b).length();
  return abs((s.a - p).cross(s.b - p) / (s.b - s.a).length
      ());
}
```

5.6 Distance segment segment

```
ld distance(Seg a, Seg b) {
  if (a.intersects(b))
    return 0.L;
  return min({distance(a.a, b), distance(a.b, b), distance(
      b.a, a), distance(b.b, a)});
}
```

6 Circle



6.1 Circle

```
struct Cir : Pt {
  ld r;
  Cir() {}
  Cir(ld x, ld y, ld r) : Pt(x, y), r(r) {}
  Cir(Pt p, ld r) : Pt(p), r(r) {}

  int inside(Cir c) {
    ld l = c.r - r - (*this - c).length();
    return ge(l, 0) ? IN : eq(l, 0) ? ON : OVERLAP;
  }

  int outside(Cir c) {
    ld l = (*this - c).length() - r - c.r;
    return ge(l, 0) ? OUT : eq(l, 0) ? ON : OVERLAP;
  }

  int contains(Pt p) {
    ld l = (p - *this).length() - r;
    return le(l, 0) ? IN : eq(l, 0) ? ON : OUT;
  }

  Pt projection(Pt p) {
    return *this + (p - *this).unit() * r;
  }

  vector<Pt> tangency(Pt p) {
    // point outside the circle
    Pt v = (p - *this).unit() * r;
    ld d2 = (p - *this).norm(), d = sqrt(d2);
    if (leq(d, 0))
      return {}; // on circle, no tangent
    Pt v1 = v * (r / d), v2 = v.perp() * (sqrt(d2 - r * r)
        / d);
    return {*this + v1 - v2, *this + v1 + v2};
  }

  vector<Pt> intersection(Cir c) {
    ld d = (c - *this).length();
    if (eq(d, 0) || ge(d, r + c.r) || le(d, abs(r - c.r)))
      return {}; // circles don't intersect
    Pt v = (c - *this).unit();
    ld a = (r * r + d * d - c.r * c.r) / (2 * d);
    Pt p = *this + v * a;
    if (eq(d, r + c.r) || eq(d, abs(r - c.r)))
      return {p}; // circles touch at one point
    ld h = sqrt(r * r - a * a);
    Pt q = v.perp() * h;
    return {p - q, p + q}; // circles intersects twice
  }

  template <class Line>
  vector<Pt> intersection(Line l) {
    // for a segment you need to check that the point lies
        on the segment
    ld h2 = r * r - l.v.cross(*this - l.a) * l.v.cross(*
        this - l.a) / l.v.norm();
    Pt p = l.a + l.v * l.v.dot(*this - l.a) / l.v.norm();
    if (eq(h2, 0))
      return {p}; // line tangent to circle
    if (le(h2, 0))
      return {}; // no intersection
    Pt q = l.v.unit() * sqrt(h2);
    return {p - q, p + q}; // two points of intersection (
        chord)
  }

  Cir(Pt a, Pt b, Pt c) {
    // find circle that passes through points a, b, c
    Pt mab = (a + b) / 2, mcb = (b + c) / 2;
    Seg ab(mab, mab + (b - a).perp());
    Seg cb(mcb, mcb + (b - c).perp());
    Pt o = ab.intersection(cb);
    *this = Cir(o, (o - a).length());
  }
};
```

6.2 Distance point circle

```
ld distance(Pt p, Cir c) {
  return max(0.L, (p - c).length() - c.r);
}
```

6.3 Common area circle circle

```
ld commonArea(Cir a, Cir b) {
  if (le(a.r, b.r))
    swap(a, b);
  ld d = (a - b).length();
  if (leq(d + b.r, a.r))
    return b.r * b.r * PI;
  if (geq(d, a.r + b.r))
    return 0.0;
  auto angle = [&](ld x, ld y, ld z) {
    return acos((x * x + y * y - z * z) / (2 * x * y));
  };
  auto cut = [&](ld x, ld r) {
    return (x - sin(x)) * r * r / 2;
  };
  ld a1 = angle(d, a.r, b.r), a2 = angle(d, b.r, a.r);
  return cut(a1 * 2, a.r) + cut(a2 * 2, b.r);
}
```

6.4 Minimum enclosing circle $\mathcal{O}(n)$ wow!!

```
Cir minEnclosing(vector<Pt>& pts) { // a bunch of points
  shuffle(all(pts), rng);
  Cir c(0, 0, 0);
  fore (i, 0, sz(pts))
    if (!c.contains(pts[i])) {
      c = Cir(pts[i], 0);
      fore (j, 0, i)
        if (!c.contains(pts[j])) {
          c = Cir((pts[i] + pts[j]) / 2, (pts[i] - pts[j]).
              length() / 2);
          fore (k, 0, j)
            if (!c.contains(pts[k]))
              c = Cir(pts[i], pts[j], pts[k]);
        }
    }
  return c;
}
```

7 Polygon

7.1 Area polygon

```
ld area(const vector<Pt>& pts) {
  ld sum = 0;
  fore (i, 0, sz(pts))
    sum += pts[i].cross(pts[(i + 1) % sz(pts)]);
  return abs(sum / 2);
}
```

7.2 Perimeter

```
ld perimeter(const vector<Pt>& pts) {
  ld sum = 0;
  fore (i, 0, sz(pts))
    sum += (pts[(i + 1) % sz(pts)] - pts[i]).length();
  return sum;
}
```

7.3 Cut polygon line

```
vector<Pt> cut(const vector<Pt>& pts, Line l) {
  vector<Pt> ans;
  int n = sz(pts);
  fore (i, 0, n) {
```

```
    int j = (i + 1) % n;
    if (geq(l.v.cross(pts[i] - l.a), 0)) // left
      ans.pb(pts[i]);
    Seg s(pts[i], pts[j]);
    if (l.intersects(s) == 1) {
      Pt p = l.intersection(s);
      if (p != pts[i] && p != pts[j])
        ans.pb(p);
    }
  }
  return ans;
}
```

7.4 Common area circle polygon $\mathcal{O}(n)$

```
ld commonArea(Cir c, const vector<Pt>& poly) {
  auto arg = [&](Pt p, Pt q) {
    return atan2(p.cross(q), p.dot(q));
  };
  auto tri = [&](Pt p, Pt q) {
    Pt d = q - p;
    ld a = d.dot(p) / d.norm(), b = (p.norm() - c.r * c.r)
        / d.norm();
    ld det = a * a - b;
    if (leq(det, 0))
      return arg(p, q) * c.r * c.r;
    ld s = max(0.L, -a - sqrt(det)), t = min(1.L, -a + sqrt
        (det));
    if (t < 0 || 1 <= s)
      return arg(p, q) * c.r * c.r;
    Pt u = p + d * s, v = p + d * t;
    return u.cross(v) + (arg(p, u) + arg(v, q)) * c.r * c.r
        ;
  };
  ld sum = 0;
  fore (i, 0, sz(poly))
    sum += tri(poly[i] - c, poly[(i + 1) % sz(poly)] - c);
  return abs(sum / 2);
}
```

7.5 Point in polygon

```
int contains(const vector<Pt>& pts, Pt p) {
  int rays = 0, n = sz(pts);
  fore (i, 0, n) {
    Pt a = pts[i], b = pts[(i + 1) % n];
    if (ge(a.y, b.y))
      swap(a, b);
    if (Seg(a, b).contains(p))
      return ON;
    rays ^= (leq(a.y, p.y) && le(p.y, b.y) && p.dir(a, b) >
        0);
  }
  return rays & 1 ? IN : OUT;
}
```

7.6 Convex hull $\mathcal{O}(nlogn)$

```
vector<Pt> convexHull(vector<Pt> pts) {
  vector<Pt> hull;
  sort(all(pts), [&](Pt a, Pt b) {
    return a.x == b.x ? a.y < b.y : a.x < b.x;
  });
  pts.erase(unique(all(pts)), pts.end());
  fore (i, 0, sz(pts)) {
    while (sz(hull) >= 2 && hull.back().dir(pts[i], hull[sz
        (hull) - 2]) < 0)
      hull.pop_back();
    hull.pb(pts[i]);
  }
  hull.pop_back();
  int k = sz(hull);
  fore (i, sz(pts), 0) {
    while (sz(hull) >= k + 2 && hull.back().dir(pts[i],
        hull[sz(hull) - 2]) < 0)
      hull.pop_back();
    hull.pb(pts[i]);
  }
  hull.pop_back();
  return hull;
}
```

7.7 Is convex

```
bool isConvex(const vector<Pt>& pts) {
  int n = sz(pts);
  bool pos = 0, neg = 0;
  fore (i, 0, n) {
    Pt a = pts[(i + 1) % n] - pts[i];
    Pt b = pts[(i + 2) % n] - pts[(i + 1) % n];
    int dir = sgn(a.cross(b));
    if (dir > 0)
      pos = 1;
    if (dir < 0)
      neg = 1;
  }
  return !(pos && neg);
}
```

7.8 Point in convex polygon $\mathcal{O}(logn)$

```
bool contains(const vector<Pt>& a, Pt p) {
  int lo = 1, hi = sz(a) - 1;
  if (a[0].dir(a[lo], a[hi]) > 0)
    swap(lo, hi);
  if (p.dir(a[0], a[lo]) >= 0 || p.dir(a[0], a[hi]) <= 0)
    return false;
  while (abs(lo - hi) > 1) {
    int mid = (lo + hi) >> 1;
    (p.dir(a[0], a[mid]) > 0 ? hi : lo) = mid;
  }
  return p.dir(a[lo], a[hi]) < 0;
}
```

8 Graphs

8.1 Cutpoints and bridges

```
int tin[N], fup[N], timer = 0;

void weakness(int u, int p = -1) {
  tin[u] = fup[u] = ++timer;
  int children = 0;
  for (int v : graph[u])
    if (v != p) {
      if (!tin[v]) {
        ++children;
        weakness(v, u);
        fup[u] = min(fup[u], fup[v]);
        if (fup[v] >= tin[u] && !(p == -1 && children < 2))
              // u is a cutpoint
          if (fup[v] > tin[u]) // bridge u -> v
      }
      fup[u] = min(fup[u], tin[v]);
    }
}
```

8.2 Tarjan

```
int tin[N], fup[N];
bitset<N> still;
stack<int> stk;
int timer = 0;

void tarjan(int u) {
  tin[u] = fup[u] = ++timer;
  still[u] = true;
  stk.push(u);
  for (auto& v : graph[u]) {
    if (!tin[v])
```

```
      tarjan(v);
    if (still[v])
      fup[u] = min(fup[u], fup[v]);
  }
  if (fup[u] == tin[u]) {
    int v;
    do {
      v = stk.top();
      stk.pop();
      still[v] = false;
      // u and v are in the same scc
    } while (v != u);
  }
}
```

8.3 Two sat $\mathcal{O}(2 \cdot n)$

v: true, ~v: false

`implies(a, b)`: if a then b

a	b	a => b
F	F	T
T	T	T
F	T	T
T	F	F

`setVal(a)`: set a = true
`setVal(~a)`: set a = false

```
struct TwoSat {
  int n;
  vector<vector<int>> imp;

  TwoSat(int k) : n(k + 1), imp(2 * n) {} // 1-indexed

  void either(int a, int b) { // a || b
    a = max(2 * a, -1 - 2 * a);
    b = max(2 * b, -1 - 2 * b);
    imp[a ^ 1].pb(b);
    imp[b ^ 1].pb(a);
  }

  void implies(int a, int b) {
    either(~a, b);
  }

  void setVal(int a) {
    either(a, a);
  }

  optional<vector<int>> solve() {
    int k = sz(imp);
    vector<int> s, b, id(sz(imp));
    function<void(int)> dfs = [&](int u) {
      b.pb(id[u] = sz(s)), s.pb(u);
      for (int v : imp[u]) {
        if (!id[v])
          dfs(v);
        else
          while (id[v] < b.back())
            b.pop_back();
      }
      if (id[u] == b.back())
        for (b.pop_back(), ++k; id[u] < sz(s); s.pop_back()
            )
          id[s.back()] = k;
    };
    vector<int> val(n);
    fore (u, 0, sz(imp))
      if (!id[u])
        dfs(u);
    fore (u, 0, n) {
      int x = 2 * u;
      if (id[x] == id[x ^ 1])
        return nullopt;
      val[u] = id[x] < id[x ^ 1];
    }
    return optional(val);
  }
};
```

8.4 LCA

```
const int LogN = 1 + __lg(N);
int par[LogN][N], depth[N];

void dfs(int u, int par[]) {
  for (auto& v : graph[u])
    if (v != par[u]) {
      par[v] = u;
      depth[v] = depth[u] + 1;
      dfs(v, par);
    }
}

int lca(int u, int v) {
  if (depth[u] > depth[v])
    swap(u, v);
  fore (k, LogN, 0)
    if (depth[v] - depth[u] >= (1 << k))
      v = par[k][v];
  if (u == v)
    return u;
  fore (k, LogN, 0)
    if (par[k][v] != par[k][u])
      u = par[k][u], v = par[k][v];
  return par[0][u];
}

int dist(int u, int v) {
  return depth[u] + depth[v] - 2 * depth[lca(u, v)];
}

void init(int r) {
  dfs(r, par[0]);
  fore (k, 1, LogN)
    fore (u, 1, n + 1)
      par[k][u] = par[k - 1][par[k - 1][u]];
}
```

8.5 Virtual tree $\mathcal{O}(n \cdot logn)$ "lca tree"

```
vector<int> virt[N];

int virtualTree(vector<int>& ver) {
  auto byDfs = [&](int u, int v) {
    return tin[u] < tin[v];
  };
  sort(all(ver), byDfs);
  fore (i, sz(ver), 1)
    ver.pb(lca(ver[i - 1], ver[i]));
  sort(all(ver), byDfs);
  ver.erase(unique(all(ver)), ver.end());
  for (int u : ver)
    virt[u].clear();
  fore (i, 1, sz(ver))
    virt[lca(ver[i - 1], ver[i])].pb(ver[i]);
  return ver[0];
}
```

8.6 Dynamic connectivity

```
struct DynamicConnectivity {
  struct Query {
```

```
    int op, u, v, at;
  };

  Dsu dsu; // with rollback
  vector<Query> queries;
  map<ii, int> mp;
  int timer = -1;

  DynamicConnectivity(int n = 0) : dsu(n) {}

  void add(int u, int v) {
    mp[minmax(u, v)] = ++timer;
    queries.pb({'+', u, v, INT_MAX});
  }

  void rem(int u, int v) {
    int in = mp[minmax(u, v)];
    queries.pb({'-', u, v, in});
    queries[in].at = ++timer;
    mp.erase(minmax(u, v));
  }

  void query() {
    queries.push_back({'?', -1, -1, ++timer});
  }

  void solve(int l, int r) {
    if (l == r) {
      if (queries[l].op == '?') // solve the query here
        return;
    }
    int m = (l + r) >> 1;
    int before = sz(dsu.mem);
    for (int i = m + 1; i <= r; i++) {
      Query& q = queries[i];
      if (q.op == '-' && q.at < l)
        dsu.unite(q.u, q.v);
    }
    solve(l, m);
    while (sz(dsu.mem) > before)
      dsu.rollback();
    for (int i = l; i <= m; i++) {
      Query& q = queries[i];
      if (q.op == '+' && q.at > r)
        dsu.unite(q.u, q.v);
    }
    solve(m + 1, r);
    while (sz(dsu.mem) > before)
      dsu.rollback();
  }
};
```

8.7 Euler-tour + HLD + LCA $\mathcal{O}(n \cdot logn)$

Solves subtrees and paths problems

```
int par[N], nxt[N], depth[N], sz[N];
int tin[N], tout[N], who[N], timer = 0;

int dfs(int u) {
  sz[u] = 1;
  for (auto& v : graph[u])
    if (v != par[u]) {
      par[v] = u;
      depth[v] = depth[u] + 1;
      sz[u] += dfs(v);
      if (graph[u][0] == par[u] || sz[v] > sz[graph[u][0]])
        swap(v, graph[u][0]);
    }
  return sz[u];
}

void hld(int u) {
  tin[u] = ++timer, who[timer] = u;
  for (auto& v : graph[u])
    if (v != par[u]) {
      nxt[v] = (v == graph[u][0] ? nxt[u] : v);
      hld(v);
    }
  tout[u] = timer;
}

template <bool OverEdges = 0, class F>
void processPath(int u, int v, F f) {
  for (; nxt[u] != nxt[v]; u = par[nxt[u]]) {
    if (depth[nxt[u]] < depth[nxt[v]])
      swap(u, v);
    f(tin[nxt[u]], tin[u]);
  }
  if (depth[u] < depth[v])
    swap(u, v);
  f(tin[v] + OverEdges, tin[u]);
}

int lca(int u, int v) {
  int last = -1;
  processPath(u, v, [&](int l, int r) {
    last = who[l];
  });
  return last;
}

void updatePath(int u, int v, lli z) {
  processPath(u, v, [&](int l, int r) {
    tree->update(l, r, z);
  });
}

void updateSubtree(int u, lli z) {
  tree->update(tin[u], tout[u], z);
}

lli queryPath(int u, int v) {
  lli sum = 0;
  processPath(u, v, [&](int l, int r) {
    sum += tree->query(l, r);
  });
  return sum;
}

lli queryPathWithOrder(int u, int v, int x) {
  int _lca = lca(u, v); assert(_lca != -1);

  vector<pair<int, int>> firstHalf, secondHalf, ranges;
  processPath(u, _lca, [&] (int l, int r) {
    firstHalf.push_back(make_pair(r, l));
  });

  processPath(_lca, v, [&] (int l, int r) {
    l += tin[_lca] == l;
    if (l <= r) {
      secondHalf.push_back(make_pair(l, r));
    }
  });
  reverse(all(secondHalf));

  ranges = firstHalf;
  ranges.insert(end(ranges), begin(secondHalf), end(
      secondHalf));

  int who = -1;
```

```
  for (auto [begin, end] : ranges) {
    // if begin <= end: left to right, aka. normal
    // if begin > end: right to left,
    // e.g. begin = 3, end = 1
    // order must go 3, 2, 1

    // e.g. first node in the path(u, v) with value less
        than or equal to x
    if ((who = tree->solve(begin, end, x)) != -1) {
      break;
    }
  }

  return who;
}

lli querySubtree(int u) {
  return tree->query(tin[u], tout[u]);
}
```

8.8 Centroid $\mathcal{O}(n \cdot logn)$

Solves "all pairs of nodes" problems

```
int cdp[N], sz[N];
bitset<N> rem;

int dfsz(int u, int p = -1) {
  sz[u] = 1;
  for (int v : graph[u])
    if (v != p && !rem[v])
      sz[u] += dfsz(v, u);
  return sz[u];
}

int centroid(int u, int size, int p = -1) {
  for (int v : graph[u])
    if (v != p && !rem[v] && 2 * sz[v] > size)
      return centroid(v, size, u);
  return u;
}

void solve(int u, int p = -1) {
  cdp[u = centroid(u, dfsz(u))] = p;
  rem[u] = true;
  for (int v : graph[u])
    if (!rem[v])
      solve(v, u);
}
```

8.9 Guni $\mathcal{O}(n \cdot logn)$

Solve subtrees problems

```
int cnt[C], color[N];
int sz[N];

int guni(int u, int p = -1) {
  sz[u] = 1;
  for (auto& v : graph[u])
    if (v != p) {
      sz[u] += guni(v, u);
      if (sz[v] > sz[graph[u][0]] || p == graph[u][0])
        swap(v, graph[u][0]);
    }
  return sz[u];
}

void update(int u, int p, int add, bool skip) {
  cnt[color[u]] += add;
  fore (i, skip, sz(graph[u]))
    if (graph[u][i] != p)
      update(graph[u][i], u, add, 0);
}

void solve(int u, int p = -1, bool keep = 0) {
  fore (i, sz(graph[u]), 0)
    if (graph[u][i] != p)
      solve(graph[u][i], u, !i);
  update(u, p, +1, 1); // add
  // now cnt[i] has how many times the color i appears in
      the subtree of u
  if (!keep)
    update(u, p, -1, 0); // remove
}
```

8.10 Link-Cut tree $\mathcal{O}(n \cdot logn)$

Solves dynamic trees problems, can handle subtrees and paths maybe with a high constant

```
struct LinkCut {
  struct Node {
    Node *left{0}, *right{0}, *par{0};
    bool rev = 0;
    int sz = 1;
    int sub = 0, vsub = 0; // subtree
    lli path = 0; // path
    lli self = 0; // node info

    void push() {
      if (rev) {
        swap(left, right);
        if (left)
          left->rev ^= 1;
        if (right)
          right->rev ^= 1;
        rev = 0;
      }
    }

    void pull() {
      sz = 1;
      sub = vsub + self;
      path = self;
      if (left) {
        sz += left->sz;
        sub += left->sub;
        path += left->path;
      }
      if (right) {
        sz += right->sz;
        sub += right->sub;
        path += right->path;
      }
    }

    void addVsub(Node* v, lli add) {
      if (v)
        vsub += 1LL * add * v->sub;
    }
  };

  vector<Node> a;

  LinkCut(int n = 1) : a(n) {}

  void splay(Node* u) {
    auto assign = [&](Node* u, Node* v, int d) {
      if (v)
```

```
      v->par = u;
    if (d >= 0)
      (d == 0 ? u->left : u->right) = v;
  };
  auto dir = [&](Node* u) {
    if (!u->par)
      return -1;
    return u->par->left == u ? 0 : (u->par->right == u ?
        1 : -1);
  };
  auto rotate = [&](Node* u) {
    Node *p = u->par, *g = p->par;
    int d = dir(u);
    assign(p, d ? u->left : u->right, d);
    assign(g, u, dir(p));
    assign(u, p, !d);
    p->pull(), u->pull();
  };
  while (~dir(u)) {
    Node *p = u->par, *g = p->par;
    if (~dir(p))
      g->push();
    p->push(), u->push();
    if (~dir(p))
      rotate(dir(p) == dir(u) ? p : u);
    rotate(u);
  }
  u->push(), u->pull();
}

void access(int u) {
  Node* last = NULL;
  for (Node* x = &a[u]; x; last = x, x = x->par) {
    splay(x);
    x->addVsub(x->right, +1);
    x->right = last;
    x->addVsub(x->right, -1);
    x->pull();
  }
  splay(&a[u]);
}

void reroot(int u) {
  access(u);
  a[u].rev ^= 1;
}

void link(int u, int v) {
  reroot(v), access(u);
  a[u].addVsub(v, +1);
  a[v].par = &a[u];
  a[u].pull();
}

void cut(int u, int v) {
  reroot(v), access(u);
  a[u].left = a[v].par = NULL;
  a[u].pull();
}

int lca(int u, int v) {
  if (u == v)
    return u;
  access(u), access(v);
  if (!a[u].par)
    return -1;
  return splay(&a[u]), a[u].par ? -1 : u;
}

int depth(int u) {
  access(u);
  return a[u].left ? a[u].left->sz : 0;
}

// get k-th parent on path to root
int ancestor(int u, int k) {
  k = depth(u) - k;
  assert(k >= 0);
  for (;; a[u].push()) {
    int sz = a[u].left->sz;
    if (sz == k)
      return access(u), u;
    if (sz < k)
      k -= sz + 1, u = u->ch[1];
    else
      u = u->ch[0];
  }
  assert(0);
}

lli queryPath(int u, int v) {
  reroot(u), access(v);
  return a[v].path;
}

lli querySubtree(int u, int x) {
  // query subtree of u, x is outside
  reroot(x), access(u);
  return a[u].vsub + a[u].self;
}

void update(int u, lli val) {
  access(u);
  a[u].self = val;
  a[u].pull();
}

Node& operator[](int u) {
  return a[u];
}
};
```

9 Flows

9.1 Blossom $\mathcal{O}(n^3)$

Maximum matching on non-bipartite non-weighted graphs

```
struct Blossom {
  int n, m;
  vector<int> mate, p, d, bl;
  vector<vector<int>> b, g;

  Blossom(int n) : n(n), m(n + n / 2), mate(n, -1), b(m), p
      (m), d(m), bl(m), g(m, vector<int>(m, -1)) {}

  void add(int u, int v) { // 0-indexed!!!!!
    g[u][v] = u;
    g[v][u] = v;
  }

  void match(int u, int v) {
    g[u][v] = g[v][u] = -1;
    mate[u] = v;
    mate[v] = u;
  }

  vector<int> trace(int x) {
    vector<int> vx;
    while (true) {
```

```
    while (bl[x] != x)
      x = bl[x];
    if (!vx.empty() && vx.back() == x)
      break;
    vx.pb(x);
    x = p[x];
  }
  return vx;
}

void contract(int c, int x, int y, vector<int>& vx,
     vector<int>& vy) {
  b[c].clear();
  int r = vx.back();
  while (!vx.empty() && !vy.empty() && vx.back() == vy.
       back()) {
    r = vx.back();
    vx.pop_back();
    vy.pop_back();
  }
  b[c].pb(r);
  b[c].insert(b[c].end(), vx.rbegin(), vx.rend());
  b[c].insert(b[c].end(), vy.begin(), vy.end());
  fore (i, 0, c + 1)
    g[c][i] = g[i][c] = -1;
  for (int z : b[c]) {
    bl[z] = c;
    fore (i, 0, c) {
      if (g[z][i] != -1) {
        g[c][i] = z;
        g[i][c] = g[i][z];
      }
    }
  }
}

vector<int> lift(vector<int>& vx) {
  vector<int> A;
  while (sz(vx) >= 2) {
    int z = vx.back();
    vx.pop_back();
    if (z < n) {
      A.pb(z);
      continue;
    }
    int w = vx.back();
    int i = (sz(A) % 2 == 0 ? find(all(b[z]), g[z][w]) -
         b[z].begin() : 0);
    int j = (sz(A) % 2 == 1 ? find(all(b[z]), g[z][A.back
         ()]) - b[z].begin() : 0);
    int k = sz(b[z]);
    int dif = (sz(A) % 2 == 0 ? i % 2 == 1 : j % 2 == 0)
         ? 1 : k - 1;
    while (i != j) {
      vx.pb(b[z][i]);
      i = (i + dif) % k;
    }
    vx.pb(b[z][i]);
  }
  return A;
}

int solve() {
  for (int ans = 0;; ans++) {
    fill(d.begin(), d.end(), 0);
    queue<int> Q;
    fore (i, 0, m)
      bl[i] = i;
    fore (i, 0, n) {
      if (mate[i] == -1) {
        Q.push(i);
        p[i] = i;
        d[i] = 1;
      }
    }
    int c = n;
    bool aug = false;
    while (!Q.empty() && !aug) {
      int x = Q.front();
      Q.pop();
      if (bl[x] != x)
        continue;
      fore (y, 0, c) {
        if (bl[y] == y && g[x][y] != -1) {
          if (d[y] == 0) {
            p[y] = x;
            d[y] = 2;
            p[mate[y]] = y;
            d[mate[y]] = 1;
            Q.push(mate[y]);
          } else if (d[y] == 1) {
            vector<int> vx = trace(x);
            vector<int> vy = trace(y);
            if (vx.back() == vy.back()) {
              contract(c, x, y, vx, vy);
              Q.push(c);
              p[c] = p[b[c][0]];
              d[c] = 1;
              c++;
            } else {
              aug = true;
              vx.insert(vx.begin(), y);
              vy.insert(vy.begin(), x);
              vector<int> A = lift(vx);
              vector<int> B = lift(vy);
              A.insert(A.end(), B.rbegin(), B.rend());
              for (int i = 0; i < sz(A); i += 2) {
                match(A[i], A[i + 1]);
                if (i + 2 < sz(A))
                  add(A[i + 1], A[i + 2]);
              }
            }
            break;
          }
        }
      }
    }
    if (!aug)
      return ans;
  }
}
};
```

9.2 Hopcroft Karp $\mathcal{O}(e\sqrt{v})$

```
struct HopcroftKarp {
  int n, m;
  vector<vector<int>> graph;
  vector<int> dist, match;

  HopcroftKarp(int k) : n(k + 1), graph(n), dist(n), match(
       n, 0) {} // 1-indexed!!

  void add(int u, int v) {
    graph[u].pb(v), graph[v].pb(u);
  }

  bool bfs() {
    queue<int> qu;
    fill(all(dist), -1);
```

```
    fore (u, 1, n)
      if (!match[u])
        dist[u] = 0, qu.push(u);
    while (!qu.empty()) {
      int u = qu.front();
      qu.pop();
      for (int v : graph[u])
        if (dist[match[v]] == -1) {
          dist[match[v]] = dist[u] + 1;
          if (match[v])
            qu.push(match[v]);
        }
    }
    return dist[0] != -1;
  }

  bool dfs(int u) {
    for (int v : graph[u])
      if (!match[v] || (dist[u] + 1 == dist[match[v]] &&
          dfs(match[v]))) {
        match[u] = v, match[v] = u;
        return 1;
      }
    dist[u] = 1 << 30;
    return 0;
  }

  int maxMatching() {
    int tot = 0;
    while (bfs())
      fore (u, 1, n)
        tot += match[u] ? 0 : dfs(u);
    return tot;
  }
};
```

9.3 Hungarian $\mathcal{O}(n^2 \cdot m)$

n jobs, m people for max assignment

```
template <class C>
pair<C, vector<int>> Hungarian(vector<vector<C>>& a) { //
    max assignment
  int n = sz(a), m = sz(a[0]), p, q, j, k; // n <= m
  vector<C> fx(n, numeric_limits<C>::min()), fy(m, 0);
  vector<int> x(n, -1), y(m, -1);
  fore (i, 0, n)
    fore (j, 0, m)
      fx[i] = max(fx[i], a[i][j]);
  fore (i, 0, n) {
    vector<int> t(m, -1), s(n + 1, i);
    for (p = q = 0; p <= q && x[i] < 0; p++)
      for (k = s[p], j = 0; j < m && x[i] < 0; j++)
        if (abs(fx[k] + fy[j] - a[k][j]) < EPS && t[j] < 0)
            {
          s[++q] = y[j], t[j] = k;
          if (s[q] < 0)
            for (p = j; p >= 0; j = p)
              y[j] = k = t[j], p = x[k], x[k] = j;
        }
    if (x[i] < 0) {
      C d = numeric_limits<C>::max();
      fore (k, 0, q + 1)
        fore (j, 0, m)
          if (t[j] < 0)
            d = min(d, fx[s[k]] + fy[j] - a[s[k]][j]);
      fore (j, 0, m)
        fy[j] += (t[j] < 0 ? 0 : d);
      fore (k, 0, q + 1)
        fx[s[k]] -= d;
      i--;
    }
  }
  C cost = 0;
  fore (i, 0, n)
    cost += a[i][x[i]];
  return make_pair(cost, x);
}
```

9.4 Dinic $\mathcal{O}(\min(e \cdot flow, v^2 \cdot e))$

```
template <class F>
struct Dinic {
  struct Edge {
    int v, inv;
    F cap, flow;
    Edge(int v, F cap, int inv) : v(v), cap(cap), flow(0),
        inv(inv) {}
  };

  F EPS = (F)1e-9;
  int s, t, n;
  vector<vector<Edge>> graph;
  vector<int> dist, ptr;

  Dinic(int n) : n(n), graph(n), dist(n), ptr(n), s(n - 2),
      t(n - 1) {}

  void add(int u, int v, F cap) {
    graph[u].pb(Edge(v, cap, sz(graph[v])));
    graph[v].pb(Edge(u, 0, sz(graph[u]) - 1));
  }

  bool bfs() {
    fill(all(dist), -1);
    queue<int> qu({s});
    dist[s] = 0;
    while (sz(qu) && dist[t] == -1) {
      int u = qu.front();
      qu.pop();
      for (Edge& e : graph[u])
        if (dist[e.v] == -1)
          if (e.cap - e.flow > EPS) {
            dist[e.v] = dist[u] + 1;
            qu.push(e.v);
          }
    }
    return dist[t] != -1;
  }

  F dfs(int u, F flow = numeric_limits<F>::max()) {
    if (flow <= EPS || u == t)
      return max<F>(0, flow);
    for (int& i = ptr[u]; i < sz(graph[u]); i++) {
      Edge& e = graph[u][i];
      if (e.cap - e.flow > EPS && dist[u] + 1 == dist[e.v])
          {
        F pushed = dfs(e.v, min<F>(flow, e.cap - e.flow));
        if (pushed > EPS) {
          e.flow += pushed;
          graph[e.v][e.inv].flow -= pushed;
          return pushed;
        }
      }
    }
    return 0;
  }

  F maxFlow() {
    F flow = 0;
    while (bfs()) {
```

```
      fill(all(ptr), 0);
      while (F pushed = dfs(s))
        flow += pushed;
    }
    return flow;
  }

  bool leftSide(int u) {
    // left side comes from sink
    return dist[u] != -1;
  }
};
```

9.5 Min-Cost flow $\mathcal{O}(\min(e \cdot flow, v^2 \cdot e))$

```
template <class C, class F>
struct Mcmf {
  struct Edge {
    int u, v, inv;
    F cap, flow;
    C cost;
    Edge(int u, int v, C cost, F cap, int inv) : u(u), v(v)
        , cost(cost), cap(cap), flow(0), inv(inv) {}
  };

  F EPS = (F)1e-9;
  int s, t, n;
  vector<vector<Edge>> graph;
  vector<Edge*> prev;
  vector<C> cost;
  vector<int> state;

  Mcmf(int n) : n(n), graph(n), cost(n), state(n), prev(n),
       s(n - 2), t(n - 1) {}

  void add(int u, int v, C cost, F cap) {
    graph[u].pb(Edge(u, v, cost, cap, sz(graph[v])));
    graph[v].pb(Edge(v, u, -cost, 0, sz(graph[u]) - 1));
  }

  bool bfs() {
    fill(all(state), 0);
    fill(all(cost), numeric_limits<C>::max());
    deque<int> qu;
    qu.push_back(s);
    state[s] = 1, cost[s] = 0;
    while (sz(qu)) {
      int u = qu.front();
      qu.pop_front();
      state[u] = 2;
      for (Edge& e : graph[u])
        if (e.cap - e.flow > EPS)
          if (cost[u] + e.cost < cost[e.v]) {
            cost[e.v] = cost[u] + e.cost;
            prev[e.v] = &e;
            if (state[e.v] == 2 || (sz(qu) && cost[qu.front
                ()] > cost[e.v]))
              qu.push_front(e.v);
            else if (state[e.v] == 0)
              qu.push_back(e.v);
            state[e.v] = 1;
          }
    }
    return cost[t] != numeric_limits<C>::max();
  }

  pair<C, F> minCostFlow() {
    C cost = 0;
    F flow = 0;
    while (bfs()) {
      F pushed = numeric_limits<F>::max();
      for (Edge* e = prev[t]; e != nullptr; e = prev[e->u])
        pushed = min(pushed, e->cap - e->flow);
      for (Edge* e = prev[t]; e != nullptr; e = prev[e->u])
          {
        e->flow += pushed;
        graph[e->v][e->inv].flow -= pushed;
        cost += e->cost * pushed;
      }
      flow += pushed;
    }
    return make_pair(cost, flow);
  }
};
```

10 Game theory

10.1 Grundy numbers

If the moves are consecutive $S = \{1, 2, 3, ..., x\}$ the game can be solved like $stackSize \pmod{x+1} \neq 0$

```
int mem[N];

int mex(set<int>& st) {
  int x = 0;
  while (st.count(x))
    x++;
  return x;
}

int grundy(int n) {
  if (n < 0)
    return INF;
  if (n == 0)
    return 0;
  int& g = mem[n];
  if (g == -1) {
    set<int> st;
    for (int x : {a, b})
      st.insert(grundy(n - x));
    g = mex(st);
  }
  return g;
}
```

11 Math

11.1 Bits

Bits++	
Operations on *int*	Function
`x & -x`	Least significant bit in x
`__lg(x)`	Most significant bit in x
`c = x&-x, r = x+c; (((r^x) » 2)/c) | r`	Next number after x with same number of bits set
__builtin_	Function
popcount(x)	Amount of 1's in x
clz(x)	0's to the **left** of biggest bit
ctz(x)	0's to the **right** of smallest bit

11.2 Bitset

Bitset<Size>	
Operation	Function
_Find_first()	Least significant bit
_Find_next(idx)	First set bit after index *idx*
any(), none(), all()	Just what the expression says
set(), reset(), flip()	Just what the expression says x2
to_string('.', 'A')	Print 011010 like .AA.A.

11.3 Fraction

```
struct Frac {
  lli num, den;

  Frac(lli a = 0, lli b = 1) {
    lli g = gcd(a, b);
    num = a / g, den = b / g;
    if (den < 0)
      num *= -1, den *= -1;
  }

  bool operator<(const Frac& f) const {
    return num * f.den < f.num * den;
  }

  bool operator==(const Frac& f) const {
    return num == f.num && den == f.den;
  }

  bool operator!=(const Frac& f) const {
    return !(*this == f);
  }

  friend Frac abs(const Frac& f) {
    return Frac(abs(f.num), f.den);
  }

  friend ostream& operator<<(ostream& os, const Frac& f) {
    return os << f.num << "/" << f.den;
  }

  Frac operator-() const {
    return Frac(-num, den);
  }

  double operator()() const {
    return double(num) / double(den);
  }

  Frac operator*(const Frac& f) {
    return Frac(num * f.num, den * f.den);
  }

  Frac operator/(const Frac& f) {
    return Frac(num * f.den, den * f.num);
  }

  Frac operator+(const Frac& f) {
    lli k = lcm(den, f.den);
    return Frac(num * (k / den) + f.num * (k / f.den), k);
  }

  Frac operator-(const Frac& f) {
    lli k = lcm(den, f.den);
    return Frac(num * (k / den) - f.num * (k / f.den), k);
  }
};
```

11.4 Modular

```
template <const int M>
struct Modular {
  int v;
  Modular(int a = 0) : v(a) {}
  Modular(lli a) : v(a % M) {
    if (v < 0)
      v += M;
  }

  Modular operator+(Modular m) {
    return Modular((v + m.v) % M);
  }

  Modular operator-(Modular m) {
    return Modular((v - m.v + M) % M);
  }

  Modular operator*(Modular m) {
    return Modular((1LL * v * m.v) % M);
  }

  Modular inv() {
    return this->pow(M - 2);
  }

  Modular operator/(Modular m) {
    return *this * m.inv();
  }

  Modular& operator+=(Modular m) {
    return *this = *this + m;
  }

  Modular& operator-=(Modular m) {
    return *this = *this - m;
  }

  Modular& operator*=(Modular m) {
    return *this = *this * m;
  }

  Modular& operator/=(Modular m) {
    return *this = *this / m;
  }

  friend ostream& operator<<(ostream& os, Modular m) {
    return os << m.v;
  }

  Modular pow(lli n) {
    Modular r(1), x = *this;
    for (; n > 0; n >>= 1) {
      if (n & 1)
        r = r * x;
      x = x * x;
    }
    return r;
  }
};
```

11.5 Probability

Conditional

The event A happens and the event B has already happened

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

If **independent** events

$$P(A|B) = P(A), P(B|A) = P(B)$$

Bayes theorem

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Binomial

$$B = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$$

n = number of trials
x = number of **success** from n trials
p = probability of **success** on a single trial

Geometric

Probability of success at the nth-event after failing the others

$$G = (1-p)^{n-1} \cdot p$$

n = number of trials
p = probability of *success* on a single trial

Poisson

$$Po = \frac{\lambda^k \cdot e^{-\lambda}}{k!}$$

λ = number of times an event is expected (occurs / time)
k = number of occurring events in the limited period of time

Example: The event happens 4 times per minute and we want k events to happen in 10 minutes, then $\lambda = 4 \cdot 10 = 40$

Expected value

$$E_x = \sum_{\forall x} x \cdot p(x)$$

11.6 Simplex

Simplex is used for solving system of linear inequalities

Maximize/Minimize $f(x, y) = 3x + 2y$; all variables are ≥ 0

- $2x + y \leq 18$
- $2x + 3y \leq 42$
- $3x + y \leq 24$

$ans = 33, x = 3, y = 12$

$$a = \begin{bmatrix} 2 & 1 \\ 2 & 3 \\ 3 & 1 \end{bmatrix} \quad \text{b} = [18, 42, 24] \quad \text{c} = [3, 2]$$

```
template <class T>
pair<T, vector<T>> simplex(vector<vector<T>> a, vector<T> b
    , vector<T> c) {
  const T EPS = 1e-9;
  T sum = 0;
  int n = b.size(), m = c.size();
  vector<int> p(m), q(n);
  iota(all(p), 0), iota(all(q), m);

  auto pivot = [&](int x, int y) {
    swap(p[y], q[x]);
    b[x] /= a[x][y];
    fore (i, 0, m)
      if (i != y)
        a[x][i] /= a[x][y];
    a[x][y] = 1 / a[x][y];
    fore (i, 0, n)
      if (i != x && abs(a[i][y]) > EPS) {
        b[i] -= a[i][y] * b[x];
        fore (j, 0, m)
          if (j != y)
            a[i][j] -= a[i][y] * a[x][j];
        a[i][y] = -a[i][y] * a[x][y];
      }
    sum += c[y] * b[x];
    fore (i, 0, m)
      if (i != y)
        c[i] -= c[y] * a[x][i];
    c[y] = -c[y] * a[x][y];
  };

  while (1) {
    int x = -1, y = -1;
    ld mn = -EPS;
    fore (i, 0, n)
      if (b[i] < mn)
        mn = b[i], x = i;
    if (x < 0)
      break;
    fore (i, 0, m)
      if (a[x][i] < -EPS) {
        y = i;
        break;
      }
    assert(y >= 0); // no solution to Ax <= b
    pivot(x, y);
  }
  while (1) {
    int x = -1, y = -1;
    ld mx = EPS;
    fore (i, 0, m)
      if (c[i] > mx)
        mx = c[i], y = i;
    if (y < 0)
      break;
    ld mn = 1e200;
    fore (i, 0, n)
      if (a[i][y] > EPS && b[i] / a[i][y] < mn) {
        mn = b[i] / a[i][y], x = i;
      }
    assert(x >= 0); // c^T x is unbounded
    pivot(x, y);
  }

  vector<T> ans(m);
  fore (i, 0, n)
    if (q[i] < m)
      ans[q[i]] = b[i];
  return {sum, ans};
}
```

11.7 Gauss jordan $\mathcal{O}(n^2 \cdot m)$

```
template <class T>
pair<int, vector<T>> gauss(vector<vector<T>> a, vector<T> b
    ) {
  const double EPS = 1e-6;
  int n = a.size(), m = a[0].size();
  for (int i = 0; i < n; i++)
    a[i].push_back(b[i]);
  vector<int> where(m, -1);
  for (int col = 0, row = 0; col < m and row < n; col++) {
    int sel = row;
    for (int i = row; i < n; ++i)
      if (abs(a[i][col]) > abs(a[sel][col]))
        sel = i;
    if (abs(a[sel][col]) < EPS)
```

```
      continue;
    for (int i = col; i <= m; i++)
      swap(a[sel][i], a[row][i]);
    where[col] = row;

    for (int i = 0; i < n; i++)
      if (i != row) {
        T c = a[i][col] / a[row][col];
        for (int j = col; j <= m; j++)
          a[i][j] -= a[row][j] * c;
      }
    row++;
  }
  vector<T> ans(m, 0);
  for (int i = 0; i < m; i++)
    if (where[i] != -1)
      ans[i] = a[where[i]][m] / a[where[i]][i];
  for (int i = 0; i < n; i++) {
    T sum = 0;
    for (int j = 0; j < m; j++)
      sum += ans[j] * a[i][j];
    if (abs(sum - a[i][m]) > EPS)
      return pair(0, vector<T>());
  }
  for (int i = 0; i < m; i++)
    if (where[i] == -1)
      return pair(INF, ans);
  return pair(1, ans);
}
```

11.8 Xor basis

```
template <int D>
struct XorBasis {
  using Num = bitset<D>;
  array<Num, D> basis, keep;
  vector<int> from;
  int n = 0, id = -1;

  XorBasis() : from(D, -1) {
    basis.fill(0);
  }

  bool insert(Num x) {
    ++id;
    Num k;
    fore (i, D, 0)
      if (x[i]) {
        if (!basis[i].any()) {
          k[i] = 1, from[i] = id, keep[i] = k;
          basis[i] = x, n++;
          return 1;
        }
        x ^= basis[i], k ^= keep[i];
      }
    return 0;
  }

  optional<Num> find(Num x) {
    // is x in xor-basis set?
    // v ^ (v ^ x) = x
    Num v;
    fore (i, D, 0)
      if (x[i]) {
        if (!basis[i].any())
          return nullopt;
        x ^= basis[i];
        v[i] = 1;
      }
    return optional(v);
  }

  optional<vector<int>> recover(Num x) {
    auto v = find(x);
    if (!v)
      return nullopt;
    Num tmp;
    fore (i, D, 0)
      if (v.value()[i])
        tmp ^= keep[i];
    vector<int> ans;
    for (int i = tmp._Find_first(); i < D; i = tmp.
        _Find_next(i))
      ans.pb(from[i]);
    return ans;
  }

  optional<Num> operator[](lli k) {
    lli tot = (1LL << n);
    if (k > tot)
      return nullopt;
    Num v = 0;
    fore (i, D, 0)
      if (basis[i]) {
        lli low = tot / 2;
        if ((low < k && v[i] == 0) || (low >= k && v[i]))
          v ^= basis[i];
        if (low < k)
          k -= low;
        tot /= 2;
      }
    return optional(v);
  }
};
```

12 Combinatorics

12.1 Factorial

```
fac[0] = 1LL;
fore (i, 1, N)
  fac[i] = lli(i) * fac[i - 1] % MOD;
ifac[N - 1] = fpow(fac[N - 1], MOD - 2, MOD);
for (int i = N - 2; i >= 0; i--)
  ifac[i] = lli(i + 1) * ifac[i + 1] % MOD;
```

12.2 Factorial mod small prime

```
lli facMod(lli n, int p) {
  lli r = 1LL;
  for (; n > 1; n /= p) {
    r = (r * ((n / p) % 2 ? p - 1 : 1)) % p;
    fore (i, 2, n % p + 1)
      r = r * i % p;
  }
  return r % p;
}
```

12.3 Choose

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

$$\binom{n}{k_1, k_2, ..., k_m} = \frac{n!}{k_1! * k_2! * ... * k_m!}$$

```
lli choose(int n, int k) {
  if (n < 0 || k < 0 || n < k)
    return 0LL;
  return fac[n] * ifac[k] % MOD * ifac[n - k] % MOD;
}

lli choose(int n, int k) {
  lli r = 1;
  int to = min(k, n - k);
```

```
  if (to < 0)
    return 0;
  fore (i, 0, to)
    r = r * (n - i) / (i + 1);
  return r;
}
```

12.4 Pascal

```
fore (i, 0, N) {
  choose[i][0] = choose[i][i] = 1;
  for (int j = 1; j <= i; j++)
    choose[i][j] = choose[i - 1][j - 1] + choose[i - 1][j];
}
```

12.5 Stars and bars

Enclosing n objects in k boxes

$$\binom{n+k-1}{k-1} = \binom{n+k-1}{n}$$

12.6 Lucas

Changes $\binom{n}{k}$ mod p, with $n \geq 2e6, k \geq 2e6$ and $p \leq 1e7$

$$\binom{n}{k} \equiv \prod_{i=0}^{n} \binom{n_i}{k_i} \bmod p$$

```
lli lucas(lli n, lli k) {
  if (k == 0)
    return 1LL;
  return lucas(n / MOD, k / MOD) * choose(n % MOD, k % MOD)
       % MOD;
}
```

12.7 Burnside lemma

Burnside's lemma is a result in group theory that can help when counting objects with symmetry taken into account. It gives a formula to count objects, where two objects that are related by a symmetry (rotation or reflection, for example) are not to be counted as distinct.

let G be a finite group. For each g in G let $f(g)$ denote the set of elements that are fixed by g.

$$|classes| = \frac{1}{|G|} \cdot \sum_{g \in G} f(g)$$

12.8 Catalan

Number of ways to insert n pairs of parentheses in a word of $n+1$ letters.

Consider all the $\binom{2n}{n}$ paths on squared paper that start at (0, 0), end at (n, n) and at each step, either make a (+1,+1) step or a (+1,-1) step. Then the number of such paths that never go below the x-axis.

Number of ordered rooted trees with n nodes, not including the root.

$C_n = \frac{(2n)!}{(n+1)! \cdot n!}$

$C_n = \binom{2n}{n} - \binom{2n}{n+1}$

i	0/1	2	3	4	5	6	7	8	9	10	11
C_i	1	2	5	14	42	132	429	1430	4862	16796	

```
catalan[0] = 1LL;
fore (i, 0, N) {
  catalan[i + 1] = catalan[i] * lli(4 * i + 2) % MOD * fpow
      (i + 2, MOD - 2) % MOD;
}
```

12.9 Bell numbers

The number of ways a set of n elements can be partitioned into **nonempty** subsets

$B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} \cdot B_k$

i	5	6	7	8	9	10	11
B_i	52	203	877	4140	21147	115975	678570

12.10 Stirling numbers

Count the number of permutations of n elements with k disjoint cycles Signed way, $k > 0$

$s(0,0) = 1,\ s(n,0) = s(0,n) = 0$
$s(n,k) = -(n-1) \cdot s(n-1,k) + s(n-1,k-1)$

The unsigned way doesn't have sign $|-(n-1)|$

The sum of products of the $\binom{n}{k}$ subsets of size k of $\{0, 1, ...n-1\}$ is $s(n, n-k)$

12.11 Stirling numbers 2

How many ways are of dividing a set of n **different** objects into k **nonempty** subsets. $\left\{ {n \atop k} \right\}$

$s2(0,0) = 1,\ s2(n,0) = s2(0,n) = 0$
$s2(n,k) = s2(n-1,k-1) + k \cdot s2(n-1,k)$

$s2(n,k) = \frac{1}{k!} \sum_{i=0}^{k} (-1)^i \cdot \binom{k}{i} \cdot (k-i)^n$

```
Mint stirling2(int n, int k) {
  Mint sum = 0;
  fore (i, 0, k + 1)
    sum += fpow<Mint>(-1, i) * choose(k, i) * fpow<Mint>(k
        - i, n);
  return sum * ifac(k);
};
```

13 Number theory

13.1 Amount of divisors $\mathcal{O}(n^{1/3})$

```
ull amountOfDivisors(ull n) {
  ull cnt = 1;
  for (auto p : primes) {
    if (1LL * p * p * p > n)
      break;
    if (n % p == 0) {
      ull k = 0;
      while (n > 1 && n % p == 0)
        n /= p, ++k;
      cnt *= (k + 1);
    }
  }
  ull sq = mysqrt(n); // the last x * x <= n
  if (miller(n))
    cnt *= 2;
  else if (sq * sq == n && miller(sq))
    cnt *= 3;
```

```
  else if (n > 1)
    cnt *= 4;
  return cnt;
}
```

13.2 Chinese remainder theorem

- $x \equiv 3 \pmod 4$
- $x \equiv 5 \pmod 6$
- $x \equiv 2 \pmod 5$

$x \equiv 47 \pmod{60}$

```
pair<lli, lli> crt(pair<lli, lli> a, pair<lli, lli> b) {
  if (a.s < b.s)
    swap(a, b);
  auto p = euclid(a.s, b.s);
  lli g = a.s * p.f + b.s * p.s, l = a.s / g * b.s;
  if ((b.f - a.f) % g != 0)
    return {-1, -1}; // no solution
  p.f = a.f + (b.f - a.f) % b.s * p.f % b.s / g * a.s;
  return {p.f + (p.f < 0) * l, l};
}
```

13.3 Euclid $\mathcal{O}(log(a \cdot b))$

```
pair<lli, lli> euclid(lli a, lli b) {
  if (b == 0)
    return {1, 0};
  auto p = euclid(b, a % b);
  return {p.s, p.f - a / b * p.s};
}
```

13.4 Factorial factors

```
vector<ii> factorialFactors(lli n) {
  vector<ii> fac;
  for (auto p : primes) {
    if (n < p)
      break;
    lli mul = 1LL, k = 0;
    while (mul <= n / p) {
      mul *= p;
      k += n / mul;
    }
    fac.emplace_back(p, k);
  }
  return fac;
}
```

13.5 Inverse

```
lli inv(lli a, lli m) {
  a %= m;
  assert(a);
  return a == 1 ? 1 : m - 1LL * inv(m, a) * m / a;
}
```

13.6 Factorize sieve

```
int factor[N];

void factorizeSieve() {
  iota(factor, factor + N, 0);
  for (int i = 2; i * i < N; i++)
    if (factor[i] == i)
      for (int j = i * i; j < N; j += i)
        factor[j] = i;
}

map<int, int> factorize(int n) {
  map<int, int> cnt;
  while (n > 1) {
    cnt[factor[n]]++;
    n /= factor[n];
  }
  return cnt;
}
```

13.7 Sieve

```
bitset<N> isPrime;
vector<int> primes;

void sieve() {
  isPrime.set();
  isPrime[0] = isPrime[1] = 0;
  for (int i = 2; i * i < N; ++i)
    if (isPrime[i])
      for (int j = i * i; j < N; j += i)
        isPrime[j] = 0;
  fore (i, 2, N)
    if (isPrime[i])
      primes.pb(i);
}
```

13.8 Phi $\mathcal{O}(\sqrt{n})$

```
lli phi(lli n) {
  if (n == 1)
    return 0;
  lli r = n;
  for (lli i = 2; i * i <= n; i++)
    if (n % i == 0) {
      while (n % i == 0)
        n /= i;
      r -= r / i;
    }
  if (n > 1)
    r -= r / n;
  return r;
}
```

13.9 Phi sieve

```
bitset<N> isPrime;
int phi[N];

void phiSieve() {
  isPrime.set();
  iota(phi, phi + N, 0);
  fore (i, 2, N)
    if (isPrime[i])
      for (int j = i; j < N; j += i) {
        isPrime[j] = (i == j);
        phi[j] = phi[j] / i * (i - 1);
      }
}
```

13.10 Miller rabin $\mathcal{O}(Witnesses \cdot (logn)^3)$

```
ull mul(ull x, ull y, ull MOD) {
  lli ans = x * y - MOD * ull(1.L / MOD * x * y);
  return ans + MOD * (ans < 0) - MOD * (ans >= lli(MOD));
}

// use mul(x, y, mod) inside fpow
bool miller(ull n) {
  if (n < 2 || n % 6 % 4 != 1)
    return (n | 1) == 3;
  ull k = __builtin_ctzll(n - 1), d = n >> k;
  for (ull p : {2, 325, 9375, 28178, 450775, 9780504, 17952
      65022}) {
    ull x = fpow(p % n, d, n), i = k;
    while (x != 1 && x != n - 1 && p % n && i--)
      x = mul(x, x, n);
    if (x != n - 1 && i != k)
      return 0;
  }
  return 1;
}
```

13.11 Pollard Rho $\mathcal{O}(n^{1/4})$

```
ull rho(ull n) {
  auto f = [n](ull x) {
    return mul(x, x, n) + 1;
  };
  ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  while (t++ % 40 || __gcd(prd, n) == 1) {
    if (x == y)
      x = ++i, y = f(x);
    if (q = mul(prd, max(x, y) - min(x, y), n))
      prd = q;
    x = f(x), y = f(f(y));
  }
  return __gcd(prd, n);
}

// if used multiple times, try memorization!!
// try factoring small numbers with sieve
void pollard(ull n, map<ull, int>& fac) {
  if (n == 1)
    return;
  if (miller(n)) {
    fac[n]++;
  } else {
    ull x = rho(n);
    pollard(x, fac);
    pollard(n / x, fac);
  }
}
```

14 Polynomials

14.1 Berlekamp Massey

For a linear recurrence of length n you need to feed at least $2n$ terms into Berlekamp-Massey to guarantee getting the same or equivalent recurrence.

```
template <class T>
struct BerlekampMassey {
  int n;
  vector<T> s, t, pw[20];

  vector<T> combine(vector<T> a, vector<T> b) {
    vector<T> ans(sz(t) * 2 + 1);
    for (int i = 0; i <= sz(t); i++)
      for (int j = 0; j <= sz(t); j++)
        ans[i + j] += a[i] * b[j];
    for (int i = 2 * sz(t); i > sz(t); --i)
      for (int j = 0; j < sz(t); j++)
        ans[i - 1 - j] += ans[i] * t[j];
    ans.resize(sz(t) + 1);
    return ans;
  }

  BerlekampMassey(const vector<T>& s) : n(sz(s)), t(n), s(s
      ) {
    vector<T> x(n), tmp;
    t[0] = x[0] = 1;
    T b = 1;
    int len = 0, m = 0;
    fore (i, 0, n) {
      ++m;
      T d = s[i];
      for (int j = 1; j <= len; j++)
        d += t[j] * s[i - j];
      if (d == 0)
        continue;
      tmp = t;
      T coef = d / b;
      for (int j = m; j < n; j++)
        t[j] -= coef * x[j - m];
      if (2 * len > i)
        continue;
      len = i + 1 - len;
      x = tmp;
      b = d;
      m = 0;
    }
    t.resize(len + 1);
    t.erase(t.begin());
    for (auto& x : t)
      x = -x;
    pw[0] = vector<T>(sz(t) + 1), pw[0][1] = 1;
    fore (i, 1, 20)
      pw[i] = combine(pw[i - 1], pw[i - 1]);
  }

  T operator[](lli k) {
    vector<T> ans(sz(t) + 1);
    ans[0] = 1;
    fore (i, 0, 20)
      if (k & (1LL << i))
        ans = combine(ans, pw[i]);
    T val = 0;
    fore (i, 0, sz(t))
      val += ans[i + 1] * s[i];
    return val;
  }
};
```

14.2 Lagrange $\mathcal{O}(n)$

Calculate the extrapolation of $f(k)$, given all the sequence $f(0), f(1), f(2), ..., f(n)$
$\sum_{i=1}^{10} i^5 = 220825$

```
template <class T>
struct Lagrange {
  int n;
  vector<T> y, suf, fac;

  Lagrange(vector<T>& y) : n(sz(y)), y(y), suf(n + 1, 1),
      fac(n, 1) {
    fore (i, 1, n)
      fac[i] = fac[i - 1] * i;
  }

  T operator[](lli k) {
    for (int i = n - 1; i >= 0; i--)
      suf[i] = suf[i + 1] * (k - i);

    T pref = 1, val = 0;
    fore (i, 0, n) {
      T num = pref * suf[i + 1];
      T den = fac[i] * fac[n - 1 - i];
      if ((n - 1 - i) % 2)
        den *= -1;
      val += y[i] * num / den;
      pref *= (k - i);
    }
    return val;
  }
};
```

14.3 FFT

```
template <class Complex>
void FFT(vector<Complex>& a, bool inv = false) {
  const static double PI = acos(-1.0);
  static vector<Complex> root = {0, 1};
```

```
  int n = sz(a);
  for (int i = 1, j = 0; i < n - 1; i++) {
    for (int k = n >> 1; (j ^= k) < k; k >>= 1)
      ;
    if (i < j)
      swap(a[i], a[j]);
  }
  int k = sz(root);
  if (k < n)
    for (root.resize(n); k < n; k <<= 1) {
      Complex z(cos(PI / k), sin(PI / k));
      fore (i, k >> 1, k) {
        root[i << 1] = root[i];
        root[i << 1 | 1] = root[i] * z;
      }
    }
  for (int k = 1; k < n; k <<= 1)
    for (int i = 0; i < n; i += k << 1)
      fore (j, 0, k) {
        Complex t = a[i + j + k] * root[j + k];
        a[i + j + k] = a[i + j] - t;
        a[i + j] = a[i + j] + t;
      }
  if (inv) {
    reverse(1 + all(a));
    for (auto& x : a)
      x /= n;
  }
}

template <class T>
vector<T> convolution(const vector<T>& a, const vector<T>&
     b) {
  if (a.empty() || b.empty())
    return {};

  int n = sz(a) + sz(b) - 1, m = n;
  while (n != (n & -n))
    ++n;

  vector<complex<double>> fa(all(a)), fb(all(b));
  fa.resize(n), fb.resize(n);
  FFT(fa, false), FFT(fb, false);
  fore (i, 0, n)
    fa[i] *= fb[i];
  FFT(fa, true);

  vector<T> ans(m);
  fore (i, 0, m)
    ans[i] = round(real(fa[i]));
  return ans;
}

template <class T>
vector<T> convolutionTrick(const vector<T>& a,
                           const vector<T>& b) { // 2 FFT's
                                 instead of 3!!
  if (a.empty() || b.empty())
    return {};

  int n = sz(a) + sz(b) - 1, m = n;
  while (n != (n & -n))
    ++n;

  vector<complex<double>> in(n), out(n);
  fore (i, 0, sz(a))
    in[i].real(a[i]);
  fore (i, 0, sz(b))
    in[i].imag(b[i]);
  FFT(in, false);
  for (auto& x : in)
    x *= x;
  fore (i, 0, n)
    out[i] = in[-i & (n - 1)] - conj(in[i]);
  FFT(out, false);

  vector<T> ans(m);
  fore (i, 0, m)
    ans[i] = round(imag(out[i]) / (4 * n));
  return ans;
}
```

14.4 Fast Walsh Hadamard Transform

```
template <char op, bool inv = false, class T>
vector<T> FWHT(vector<T> f) {
  int n = f.size();
  for (int k = 0; (n - 1) >> k; k++)
    for (int i = 0; i < n; i++)
      if (i >> k & 1) {
        int j = i ^ (1 << k);
        if (op == '^')
          f[j] += f[i], f[i] = f[j] - 2 * f[i];
        if (op == '|')
          f[i] += (inv ? -1 : 1) * f[j];
        if (op == '&')
          f[j] += (inv ? -1 : 1) * f[i];
      }
  if (op == '^' && inv)
    for (auto& i : f)
      i /= n;
  return f;
}
```

14.5 Primitive root

```
int primitive(int p) {
  auto fpow = [&](lli x, int n) {
    lli r = 1;
    for (; n > 0; n >>= 1) {
      if (n & 1)
        r = r * x % p;
      x = x * x % p;
    }
    return r;
  };

  for (int g = 2; g < p; g++) {
    bool can = true;
    for (int i = 2; i * i < p; i++)
      if ((p - 1) % i == 0) {
        if (fpow(g, i) == 1)
          can = false;
        if (fpow(g, (p - 1) / i) == 1)
          can = false;
      }
    if (can)
      return g;
  }
  return -1;
}
```

14.6 NTT

```
template <const int G, const int M>
void NTT(vector<Modular<M>>& a, bool inv = false) {
  static vector<Modular<M>> root = {0, 1};
  static Modular<M> primitive(G);
  int n = sz(a);
  for (int i = 1, j = 0; i < n - 1; i++) {
    for (int k = n >> 1; (j ^= k) < k; k >>= 1)
      ;
    if (i < j)
```

```cpp
        swap(a[i], a[j]);
    }
    int k = sz(root);
    if (k < n)
      for (root.resize(n); k < n; k <<= 1) {
        auto z = primitive.pow((M - 1) / (k << 1));
        fore (i, k >> 1, k) {
          root[i << 1] = root[i];
          root[i << 1 | 1] = root[i] * z;
        }
      }
    for (int k = 1; k < n; k <<= 1)
      for (int i = 0; i < n; i += k << 1)
        fore (j, 0, k) {
          auto t = a[i + j + k] * root[j + k];
          a[i + j + k] = a[i + j] - t;
          a[i + j] = a[i + j] + t;
        }
    if (inv) {
      reverse(1 + all(a));
      auto invN = Modular<M>(1) / n;
      for (auto& x : a)
        x = x * invN;
    }
}

template <int G = 3, const int M = 998244353>
vector<Modular<M>> convolution(vector<Modular<M>> a, vector
     <Modular<M>> b) {
  // find G using primitive(M)
  // Common NTT couple (3, 998244353)
  if (a.empty() || b.empty())
    return {};

  int n = sz(a) + sz(b) - 1, m = n;
  while (n != (n & -n))
    ++n;
  a.resize(n, 0), b.resize(n, 0);

  NTT<G, M>(a), NTT<G, M>(b);
  fore (i, 0, n)
    a[i] = a[i] * b[i];
  NTT<G, M>(a, true);

  return a;
}
```

15 Strings

15.1 KMP $\mathcal{O}(n)$

- aaabaab - $[0, 1, 2, 0, 1, 2, 0]$
- abacaba - $[0, 0, 1, 0, 1, 2, 3]$

```cpp
template <class T>
vector<int> lps(T s) {
  vector<int> p(sz(s), 0);
  for (int j = 0, i = 1; i < sz(s); i++) {
    while (j && (j == sz(s) || s[i] != s[j]))
      j = p[j - 1];
    if (j < sz(s) && s[i] == s[j])
      j++;
    p[i] = j;
  }
  return p;
}

// positions where t is on s
template <class T>
vector<int> kmp(T& s, T& t) {
  vector<int> p = lps(t), pos;
  debug(lps(t), sz(s));
  for (int j = 0, i = 0; i < sz(s); i++) {
    while (j && (j == sz(t) || s[i] != t[j]))
      j = p[j - 1];
    if (j < sz(t) && s[i] == t[j])
      j++;
    if (j == sz(t))
      pos.pb(i - sz(t) + 1);
  }
  return pos;
}
```

15.2 KMP automaton $\mathcal{O}(Alphabet * n)$

```cpp
template <class T, int ALPHA = 26>
struct KmpAutomaton : vector<vector<int>> {
  KmpAutomaton() {}
  KmpAutomaton(T s) : vector<vector<int>>(sz(s) + 1, vector
      <int>(ALPHA)) {
    s.pb(0);
    vector<int> p = lps(s);
    auto& nxt = *this;
    nxt[0][s[0] - 'a'] = 1;
    fore (i, 1, sz(s))
      fore (c, 0, ALPHA)
        nxt[i][c] = (s[i] - 'a' == c ? i + 1 : nxt[p[i - 1
            ]][c]);
  }
};
```

15.3 Z $\mathcal{O}(n)$

z_i is the length of the longest substring starting from i which is also a prefix of s string will be in range $[i, i + z_i)$

- aaabaab - $[0, 2, 1, 0, 2, 1, 0]$
- abacaba - $[0, 0, 1, 0, 3, 0, 1]$

```cpp
template <class T>
vector<int> zalgorithm(T& s) {
  vector<int> z(sz(s), 0);
  for (int i = 1, l = 0, r = 0; i < sz(s); i++) {
    if (i <= r)
      z[i] = min(r - i + 1, z[i - l]);
    while (i + z[i] < sz(s) && s[i + z[i]] == s[z[i]])
      ++z[i];
    if (i + z[i] - 1 > r)
      l = i, r = i + z[i] - 1;
  }
  return z;
}
```

15.4 Manacher $\mathcal{O}(n)$

- aaabaab - $[[0, 1, 1, 0, 0, 2, 0], [0, 1, 0, 2, 0, 0, 0]]$
- abacaba - $[[0, 0, 0, 0, 0, 0, 0], [0, 1, 0, 3, 0, 1, 0]]$

```cpp
template <class T>
vector<vector<int>> manacher(T& s) {
  vector<vector<int>> pal(2, vector<int>(sz(s), 0));
  fore (k, 0, 2) {
    int l = 0, r = 0;
    fore (i, 0, sz(s)) {
      int t = r - i + !k;
      if (i < r)
        pal[k][i] = min(t, pal[k][l + t]);
      int p = i - pal[k][i], q = i + pal[k][i] - !k;
      while (p >= 1 && q + 1 < sz(s) && s[p - 1] == s[q + 1
          ])
        ++pal[k][i], --p, ++q;
      if (q > r)
        l = p, r = q;
    }
```

```
  }
  return pal;
}
```

15.5 Hash

$bases$ = [1777771, 10006793, 10101283, 10101823, 10136359, 10157387, 10166249]

$mods$ = [999727999, 1000000123, 1000002193, 1000008223, 1000009999, 1000027163, 1070777777]

```
struct Hash : array<int, 2> {
  static constexpr array<int, 2> mod = {1070777777, 1070777
      777};
#define oper(op)                          \
  friend Hash operator op(Hash a, Hash b) {     \
    fore (i, 0, sz(a))                    \
      a[i] = (1LL * a[i] op b[i] + mod[i]) % mod[i]; \
    return a;                             \
  }
  oper(+) oper(-) oper(*)
} pw[N], ipw[N];

struct Hashing {
  vector<Hash> h;

  static void init() {
    #warning "Ensure all base[i] > alphabet"
    pw[0] = ipw[0] = {1, 1};
    Hash base = {12367453, 14567893};
    Hash inv = {::inv(base[0], base.mod[0]), ::inv(base[1],
        base.mod[1])};
    fore (i, 1, N) {
      pw[i] = pw[i - 1] * base;
      ipw[i] = ipw[i - 1] * inv;
    }
  }

  Hashing(string& s) : h(sz(s) + 1) {
    fore (i, 0, sz(s)) {
      int x = s[i] - 'a' + 1;
      h[i + 1] = h[i] + pw[i] * Hash{x, x};
    }
  }

  Hash query(int l, int r) {
    return (h[r + 1] - h[l]) * ipw[l];
  }

  lli queryVal(int l, int r) {
    Hash hash = query(l, r);
    return (1LL * hash[0] << 32) | hash[1];
  }
};

// // Save len in the struct and when you do a cut
// Hash merge(vector<Hash>& cuts) {
//   Hash f = {0, 0};
//   fore (i, sz(cuts), 0) {
//     Hash g = cuts[i];
//     f = g + f * pw[g.len];
//   }
//   return f;
// }
```

15.6 Min rotation $\mathcal{O}(n)$

- baabaaa - 4
- abacaba - 6

```
template <class T>
int minRotation(T& s) {
  int n = sz(s), i = 0, j = 1;
  while (i < n && j < n) {
    int k = 0;
    while (k < n && s[(i + k) % n] == s[(j + k) % n])
      k++;
    (s[(i + k) % n] <= s[(j + k) % n] ? j : i) += k + 1;
    j += i == j;
  }
  return i < n ? i : j;
}
```

15.7 Suffix array $\mathcal{O}(nlogn)$

- Duplicates $\sum_{i=1}^{n} lcp[i]$
- Longest Common Substring of various strings
  Add $notUsed$ characters between strings, i.e. $a + \$ + b + \# + c$
  Use two-pointers to find a range $[l, r]$ such that all $notUsed$ characters are present, then $query(lcp[l+1], .., lcp[r])$ for that window is the common length.

```
template <class T>
struct SuffixArray {
  int n;
  T s;
  vector<int> sa, pos, sp[25];

  SuffixArray(const T& x) : n(sz(x) + 1), s(x), sa(n), pos(
      n) {
    s.pb(0);
    fore (i, 0, n)
      sa[i] = i, pos[i] = s[i];
    vector<int> nsa(sa), npos(n), cnt(max(260, n), 0);
    for (int k = 0; k < n; k ? k *= 2 : k++) {
      fill(all(cnt), 0);
      fore (i, 0, n)
        nsa[i] = (sa[i] - k + n) % n, cnt[pos[i]]++;
      partial_sum(all(cnt), cnt.begin());
      for (int i = n - 1; i >= 0; i--)
        sa[--cnt[pos[nsa[i]]]] = nsa[i];
      for (int i = 1, cur = 0; i < n; i++) {
        cur += (pos[sa[i]] != pos[sa[i - 1]] || pos[(sa[i]
            + k) % n] != pos[(sa[i - 1] + k) % n]);
        npos[sa[i]] = cur;
      }
      pos = npos;
      if (pos[sa[n - 1]] >= n - 1)
        break;
    }
    sp[0].assign(n, 0);
    for (int i = 0, j = pos[0], k = 0; i < n - 1; ++i, ++k)
        {
      while (k >= 0 && s[i] != s[sa[j - 1] + k])
        sp[0][j] = k--, j = pos[sa[j] + 1];
    }
    for (int k = 1, pw = 1; pw < n; k++, pw <<= 1) {
      sp[k].assign(n, 0);
      for (int l = 0; l + pw < n; l++)
        sp[k][l] = min(sp[k - 1][l], sp[k - 1][l + pw]);
    }
  }

  int lcp(int l, int r) {
    if (l == r)
      return n - l;
    tie(l, r) = minmax(pos[l], pos[r]);
```

```
    int k = __lg(r - l);
    return min(sp[k][l + 1], sp[k][r - (1 << k) + 1]);
  }

  auto at(int i, int j) {
    return sa[i] + j < n ? s[sa[i] + j] : 'z' + 1;
  }

  int count(T& t) {
    int l = 0, r = n - 1;
    fore (i, 0, sz(t)) {
      int p = l, q = r;
      for (int k = n; k > 0; k >>= 1) {
        while (p + k < r && at(p + k, i) < t[i])
          p += k;
        while (q - k > l && t[i] < at(q - k, i))
          q -= k;
      }
      l = (at(p, i) == t[i] ? p : p + 1);
      r = (at(q, i) == t[i] ? q : q - 1);
      if (at(l, i) != t[i] && at(r, i) != t[i] || l > r)
        return 0;
    }
    return r - l + 1;
  }

  bool compare(ii a, ii b) {
    // s[a.f ... a.s] < s[b.f ... b.s]
    int common = lcp(a.f, b.f);
    int szA = a.s - a.f + 1, szB = b.s - b.f + 1;
    if (common >= min(szA, szB))
      return tie(szA, a) < tie(szB, b);
    return s[a.f + common] < s[b.f + common];
  }
};
```

15.8 Aho Corasick $\mathcal{O}(\sum s_i)$

```
struct AhoCorasick {
  struct Node : map<char, int> {
    int link = 0, up = 0;
    int cnt = 0, isWord = 0;
  };

  vector<Node> trie;

  AhoCorasick(int n = 1) {
    trie.reserve(n), newNode();
  }

  int newNode() {
    trie.pb({});
    return sz(trie) - 1;
  }

  void insert(string& s, int u = 0) {
    for (char c : s) {
      if (!trie[u][c])
        trie[u][c] = newNode();
      u = trie[u][c];
    }
    trie[u].cnt++, trie[u].isWord = 1;
  }

  int next(int u, char c) {
    while (u && !trie[u].count(c))
      u = trie[u].link;
    return trie[u][c];
  }

  void pushLinks() {
    queue<int> qu;
    qu.push(0);
    while (!qu.empty()) {
      int u = qu.front();
      qu.pop();
      for (auto& [c, v] : trie[u]) {
        int l = (trie[v].link = u ? next(trie[u].link, c) :
            0);
        trie[v].cnt += trie[l].cnt;
        trie[v].up = trie[l].isWord ? l : trie[l].up;
        qu.push(v);
      }
    }
  }

  template <class F>
  void goUp(int u, F f) {
    for (; u != 0; u = trie[u].up)
      f(u);
  }

  int match(string& s, int u = 0) {
    int ans = 0;
    for (char c : s) {
      u = next(u, c);
      ans += trie[u].cnt;
    }
    return ans;
  }

  Node& operator[](int u) {
    return trie[u];
  }
};
```

15.9 Eertree $\mathcal{O}(\sum s_i)$

```
struct Eertree {
  struct Node : map<char, int> {
    int link = 0, len = 0;
  };

  vector<Node> trie;
  string s = "$";
  int last;

  Eertree(int n = 1) {
    trie.reserve(n), last = newNode(), newNode();
    trie[0].link = 1, trie[1].len = -1;
  }

  int newNode() {
    trie.pb({});
    return sz(trie) - 1;
  }

  int next(int u) {
    while (s[sz(s) - trie[u].len - 2] != s.back())
      u = trie[u].link;
    return u;
  }

  void extend(char c) {
    s.push_back(c);
    last = next(last);
    if (!trie[last][c]) {
      int v = newNode();
      trie[v].len = trie[last].len + 2;
      trie[v].link = trie[next(trie[last].link)][c];
      trie[last][c] = v;
```

```
    }
    last = trie[last][c];
  }

  Node& operator[](int u) {
    return trie[u];
  }

  void substringOccurrences() {
    fore (u, sz(s), 0)
      trie[trie[u].link].occ += trie[u].occ;
  }

  lli occurences(string& s, int u = 0) {
    for (char c : s) {
      if (!trie[u].count(c))
        return 0;
      u = trie[u][c];
    }
    return trie[u].occ;
  }
};
```